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Elections, Legislative Outcomes, Coalition Formation and Government Stability

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ABSTRACT

In his book ' The theory of Political Coalitions', Riker (1962) introduces the theorem of minimal winning coalition, a game theoretic approach in which rational agents are proved to form coalitions that minimal winning. Mathematical evidence of this theorem have been provided by Austen-Smith and Banks (1988). In this paper we extend Austen-Smith and Banks' model such that, at the legislative coalition formation stage parties take positive externalities, expressed in terms of government stability, of having a broad coalition into account. We then examine the robustness of the minimal winning coalition theorem for this modification. We show that a slight modification of the Austen-Smith and Banks´ three party model may result in a different composition of the coalition in equilibrium and predicts that it is certainly not always that the largest and smallest parties that form a government coalition in equilibrium. We also show that, if individuals preferences are quadratic on policies, large coalitions are more preferable. Moreover, our extension is simple to incorporate in existing models and allows us to predict government stability for any coalition.

INTRODUCTION

In most parliamentary democracies, multi-party coalition governments and post election bargaining to form a government are the norm, as parties seldom have a majority of support in legislature. Across a diverse set of countries, coalitions occur about 65 percent of the time (Armstrong and Duch 2010; Vowles 2010). Moreover, twenty-eight out of the thirty-three OECD member states use some form of proportional representation at national level. Despite the predominance of proportional representation systems, our understanding of the mechanisms behind the behavior of parties and voters in such systems remains incomplete. For many of these countries, if indeed coalition governments are the rule, voters should be aware of this and adjust their voting behavior such that it takes into account coalition preferences. However surprisingly, so far the proportional election system has rarely been subjected to the idea that voters might vote strategically. This view is mainly supported by the notion that the political arena in these systems consists of too many parties in parliament and thus is too difficult for voters to figure out. Due to the coalition formation process, proportional representation systems give additional uncertainty for the voter and can make the voting arena immensely complex. Although voters may be aware of the post election coalition negotiations that occurs between the moment of casting their vote and the government formation (Downs, 1957), making any accurate predictions of expected governments is often rather tricky. Under proportional representation systems voting for a specific coalition can become a highly challenging task because a vote never directly translates into a government's policy. At best, the preferred party becomes a coalition member that compromises its policy with its coalition partners.

It is clear that between the moment that voters cast their vote and the moment policies are implemented, multiple stages must be passed. In order to fully understand the dynamics in proportional representation systems, one therefore must understand all stages that result into a final policy outcome. Before we can understand voting behavior, it is critical to study party behavior in the coalition formation process first, as it determines the composition of the coalition, and therefore policy outcome. A political equilibrium should consist of a legislative equilibrium, government formation equilibrium, and an electoral equilibrium. One of the first formalized models of strategic voting in proportional representation systems was presented by Austen-Smith and Banks (1988). In their study, they present a model of electoral party competition for legislative representation that includes both the electoral and legislative stages. The model consists of three parties in a one-dimensional policy space competing for legislation. In their equilibrium, parties adopt certain policy positions, conditional on voting strategies of the electorate. They show that a significant portion of the electorate votes anticipate to the post election bargaining process, and strategically vote for party other than their favorite one, to ensure a balance at the coalition formation stage. Conditional on the three-step finite bargaining protocol, where the order of proposals matches the order of legislative weights of the parties, they conclude that the equilibrium coalition always consist of the largest and the smallest parties in legislature, also known as minimal winning coalition.

The theorem of minimal winning coalition was first extensively described by Riker (1962), where he deduces the 'size principle', a game theoretic approach in which rational agents are proved to form coalitions that are just large enough to be decisive. Riker conclusions are based on a zero-sum view of the nature of political resources, in which parties within the winning coalition exploit those outside the coalition. In this context, those within the coalition gain more the fewer they are, and the more the number of outsiders are to exploit. The coalition then contains of just over 50 percent, containing 'no surplus members'. Through the years, the validity of the theorem have been doubted by many scholars (Butterworth, 1971; Frohlich, 1975; Shepsle, 1974; Lijphart, 1977; Bandyopadhyay and Oak, 2006). These doubts have been supported by the poor empirical foundation in Western European multi-party systems of the theorem. Druckman and Thies (2002) have analyzed historical data, and find that since World War II, there have been eighty 'oversized coalition' compare to only seventy-four minimal winning coalitions in European parliamentary democracies. This paper belongs to the category that doubts Riker's arguments. We believe that parties do not adopt a minimal winning coalition strategy per se, and that simply assuming the theorem is not only naïve but is also not consistent with empirical evidence. Although we recognize that parties rationally try to form coalitions with as few parties as possible to guarantee a larger share of the total legislative pie, we argue that it certainly cannot be considered a condition in the government formation process.

This paper is built on the study by Austen-Smith and Banks (1988), and therefore does not present much brand new theory, but the aim is to extend the formal literature in order to build a better foundation for understanding strategic party behavior under proportional representation systems. We have modified Austen-Smith and Banks' basic model (1988), such that it takes into account positive externalities for having a large coalition. In their model, Austen-Smith and Banks (1988) define a legislative equilibrium, in which parties make optimal proposals regarding policy outcome and over the distribution of the perks of office, for any distribution of vote shares. Given the legislative equilibrium, the electorate can predict policy implications for any profile of party platforms pre-elections, and therefore face a well-defined electoral decision problem when casting their vote at the ballot. They show that any legislative electoral equilibria must involve a distribution of party platforms with the middle party, adopting the position of the median voter's ideal point and the two extreme parties are located symmetrically around the middle party, neither too close nor too far.

The contributions of this paper are twofold. First, the purpose of our extension is to show that with a slight modification, the theoretical basis of the minimal winning coalition theorem is obscure. We prove our statement by using Austen-Smith and Banks' model (1988), and extend it such that, at the legislative coalition formation stage, parties also take positive externalities (expressed in terms of stability) of having a broad coalition into account and apply this to their equilibrium outcome. When we examine the robustness of the minimal winning coalition theorem for this modification, our results suggests that broad coalitions actually could be more preferable for parties in equilibrium and that, under our conditions, a finite sequential legislative bargaining protocol need not result in a coalition consisting of the largest and smallest parties in legislature. Like Austen-Smith and Banks, we apply a three-step bargaining protocol, and include a fixed externality (government stability) associated with different coalition outcomes, that agents will take into account during the bargaining process, but are not included into the bargaining itself. As will be seen, our modification of the Austen-Smith and Banks´ model may result in a different composition of the coalition in equilibrium. When including our extension, the model predicts a more nuanced view that is more consistent with empirical evidence and shows that in some cases winning parties actually do prefer large coalitions. In our modification, we have included government stability as a positive externality of forming broad coalition but this can be easily substituted by any other externality. Moreover, due to the conceptual design of voter's preferences, our extension could lead to a more stable expected policy outcome in equilibrium, which is more preferable than expected policy outcomes with a greater variance. Secondly, given our specific conditions, we also introduce a method that allows us to predict government stability for any coalition. This could be useful for the considerations during the coalition formation process.

The remainder of this paper is structured as follows. In section two we provide a review of the existing literature on the topic of minimal winning coalitions. Section three presents the basic model of Austen-Smith and Banks (1988) including our modification. Followed by a discussion in section four.

REVIEW

Minimal winning coalitions

The theorem of minimal winning coalition was first put forth by Riker (1962) in his book ' The theory of Political Coalitions', where he deduces the 'size principle', a game theoretic approach in which rational agents are proved to form coalitions that are just large enough to be decisive. He stated that, in a world of complete and perfect information, winning coalitions tend to be minimal in size. His statement followed from the way Riker visualizes politics, in the context of a zero-sum game, in which parties within the winning coalition exploit those outside the coalition. Hence in this context, the gains are only divided among parties that participate in the coalition and as a result, those within the coalition get more the fewer they are, and the more the number of outsiders are to exploit. Although Riker's arguments are beyond doubt, he does seem to overlook important aspects of coalition formation. More specifically, Riker for example, does not take ideological proximity of parties into account. This important aspect was first recognized by De Swaan (1973) and Axelrod (1970). The latter predicts that parties will form 'minimal connected winning coalition', which are not necessarily a minimal winning coalition, but are connected to each other each other along the same policy dimension in the political spectrum. According to De Swaan (1973), parties strive to find coalitions of minimal ideological diversity, rather than minimal in size. The ideological closeness of parties would enable them to form coalitions more easily, as it smoothens the negotiation and bargaining process post elections. He calls this the ‘policy distance theory’. Laver and Schofield (1999) describe it as “a version of the minimal connected winning theory that takes account of the positions of the parties on the policy platform. The coalition outcome is then predicted as the coalition with the smallest ideological range. According to Franklin and Mackie (1984), the ‘combination of the two elements of size and ideology performs considerably better than either of its components alone’.

Brams and Fishburn (1995) have applied Riker’s principle to weighted-majority voting games and also show that players' bargaining power tends to decrease as their weights increases when the minimal winning coalitions that form are "weight-minimal", referred to as least winning coalitions. They argue that in such coalitions, large size may be more harmful than helpful. In latter research Brams and Fishburn (1996) extend and refine their analysis by providing a mathematical foundation for minimal and least winning coalitions. Subsequently, they developed new data and applied more sophisticated measures to these data. Their new analysis indicates that there is a less negative correlation between voting weight and voting power when least winning coalitions form. In this context, players' powers are less insensitive to their voting weights. So being large or small is no longer particularly important for inclusion in a least winning coalition. Austen-Smith and Banks (1988) have provided mathematical evidence of the theorem of minimal winning coalitions, and state that the composition of the governing coalition in equilibrium is always the first and final proposers given the assumption that the order of proposals matches the order of legislative weights of the parties. He describes a model where three homogenous parties compete for legislative representation over a one dimensional policy space. In his model, government formation and legislative policy are decided through a three-step finite sequential bargaining protocol ex ante, which gives the party with the highest weight the first opportunity to form a government by proposing a policy to the others. The legislative bargaining is solved using backward induction and shows that the first proposal is always accepted. Given the legislative equilibrium, the electorate can predict policy implications for any profile of party platforms pre-elections, and therefore face a well-defined electoral decision problem. Furthermore it shows that although the relative locations of parties' ideal points do affect the final policy outcome, they are irrelevant to the composition of the governing coalition as it predicts that it is always the largest and smallest parties that form a government in equilibrium. Although the mathematical evidence of Austen-Smith and Banks (1988) is convincing, we argue that the conclusion of their research is simply due to the chosen bargaining protocol. It is evident that for any fixed three-step sequence bargaining game ex-ante ends up with the first and final proposers forming a government in equilibrium.

The theorem of minimal winning coalition seems controversial and disturbing as it implies that parties have incentives to repel their voters in order to remain small in size. This rather counterintuitive thought, made some scholars doubts about its validity. Butterworth (1971) has modeled a five-person game and shows that parties might form larger than minimal coalition under a symmetric zero-sum game where one of the potential losers bribes himself into the coalition. The model was challenged by Shepsle (1974) who claimed Butterworth did not take into account the possibility of entering a competitive bribery war. He however did recognize that the minimal coalition theorem is unstable. Norman Frohlich (1975) confirms this conclusion. He attempts to generalize and extend the findings of the authors discussed above and shows that larger than minimum winning coalitions are compatible with rational behavior on the part of individuals. More recently, Bandyopadhyay and Oak (2006) analyzed a model under proportional representation, where no party has an absolute majority and examines how the nature of coalitions is affected by the tradeoff parties make if they care both about ideology and the perks from office. They show that in equilibrium parties in coalition may be ideologically disconnected and that coalitions do not need to be minimal winning but also include minority or surplus coalition. Other models that include coalition formation include are Crombez (1996) and Baron and Diermeier (2001). The latter presents a three party equilibrium model under proportional representation for a two-dimensional policy space, producing results on government formation, policy outcome, election outcomes, and representation. They rely on a efficient proto-coalition bargaining model of government formation, where the formateur, first appoints a proto-coalition and asks the involving parties to start negotiating a coalition agreement. In case they accept, the coalition is formed, if not, a caretaker government assumes office. Hence bargaining and therefore policy outcome, depends on the status quo. In turn, voters strictly care about policy outcomes, and anticipate on the status quo accordingly. They have similar conclusions to Austen-Smith and Banks (1988), that representation in legislature do not necessarily reflect the distribution of voter preferences. Crombez (1996) presents a formal model that explain the emergence of minority governments, minimal winning coalitions and surplus majorities, conditional on the vote share of largest party in the legislature and on its position in the policy space. They conclude that as the largest party becomes larger and more central, the government changes from a surplus majority to a minimal winning coalition and from a minimal winning coalition to a minority government. They also find empirical support for these conclusions in eleven parliamentary democracies. Others have come up with newer coalition theories that often include vote-seeking, institutions or multiple dimensions (Strøm, 1990; Strøm, Budge & Laver, 1994; Laver & Shepsle, 1996). The empirical evidence for the theorem is also considered quite poor. For instance, Lijphart (1977) finds grand coalitions to be common in 'consociational societies', such as in Austria, Belgium and the Netherlands. Druckman and Thies (2002) have analyzed historical data, and find that since World War II, there have been eighty 'oversized coalition' compare to only seventy-four minimal winning coalitions in European parliamentary democracies. Besides the disturbing conclusion and the poor empirical foundation in Western European parliamentarian systems, surprisingly the theorem of minimal winning coalition is still one of the most widely recognized theorems.

MODELING

We consider the model of Austen-Smith and Banks (1988), that consists of a world of perfect and complete information, with a one-dimensional policy space, $X = [\underline{x}, \overline{x}]$, where three homogenous parties, *l, c, r,* compete for legislative representation. An electoral strategy for $g \in \{l, m, r\}$ is a platform choice, $a_g \in X$. Hence, a_g is the party position of party g that is located on policy space *X.* Let $V_g(a) \in [0, 1]$ denote g's realized electoral vote share given $g \in \{l, m, r\}$ and $a = (a_l, a_m, a_r) \in X^3$. The electoral system based on proportional representation, where parties need at least a minimal proportion of electoral support s, to achieve a positive level of legislative influence. We assume that s is even and $s \in \left(0, \frac{32}{100}\right)$, where s is supposed to bound away from zero. For even values, the upper bound of s is maximal $\frac{32}{100}$, however in reality is it more likely to be much lower[1]. For each party g and distribution of vote shares $V = (V_l, V_m, V_r)$, let $w_g(V) \in [0, 1]$ be g's legislative weight; then $w_g(V) < 0$ if $V_g < s$ and, if $V_g \geq s$ then,

$$w_g(V) = \frac{V_g}{\sum_{j=l,m,r}\{V_j \mid V_j > s\}}$$

Write $w = (w_l, w_m, w_r)$.

Post elections, legislative decision making is by weighted majority rule. The family of decisive coalitions in legislature, $\tau \subseteq 2^{\{l,m,r\}}$, is described by

$$\tau = \{T \subseteq \{l, m, r\} : \sum_{g \in T} w_g > \frac{1}{2}\}$$

If only two parties receive the minimal threshold s, then, the weights of the parties in the legislature are normalized to reflect this fact. In that case, some party g must have a majority of the seats in legislature, w_g > ½, and as a result party g forms a government on its own and controls all legislative decisions. In case no party g reaches a majority of votes, then by full participation, it must be that all parties have legislative representation $V_g \geq s$ for all g, and any government will involve a coalition of at least two parties. In line with Austen-Smith and Banks (1988), following an election, the process of government formation is decided through a fixed sequential bargaining protocol involving only those parties with a positive legislative weight, where the party with the largest number of seats has the first opportunity to propose a coalition at t = 1. In case of two parties receiving the same amount of vote shares, the party that gets propose first is decided by fair lottery. The proposal consists of a policy outcome $y^1 \in X$ and a distribution of a

[1] The threshold differs across countries and ranges from 0.6% up to 10% in Turkey. For example, for Austria and Sweden, the threshold is 4%. For Belgium, Germany, Polen, and Latvia, the threshold is 5%. While for Greece the threshold is only 3%.

fixed amount of transferable private benefits among the parties, $b^1 = (b_l^1, b_m^1, b_r^1) \in B$, where

$$B = \{b \in [0, B]^3 : \sum_{g=l,m,r} b_g \leq B\} \in [0,1]$$

Hence, private benefits of being in office are assumed to be positive. A proposal by a party g can either be accepted or rejected by the decisive legislative coalition $T \in \tau$. If the decisive legislative coalition accepts the proposal at t=1, then they constitute a winning coalition, and form a government, implementing policy outcome y^1 and distribute the perks b^1. Should the proposal be rejected by all parties then the sequence moves to the second bargaining period t = 2, and the party with the second highest number of seats has a opportunity to form a government, by proposing a pair $(y^2, b^2) \in X \times B$; and again the members of the proposed coalition either accept or reject. If this too fails to receive legislative approval, the sequence goes to the third period t = 3, and the remaining party has the last chance to form a government with a proposal $(y^3, b^3) \in X \times B$. If a government has not formed after the t = 3 proposal, then a fixed "caretaker" government is implemented with decision $(y^0, b^0) \in X \times B$.

Furthermore, we extend Austen-Smith and Banks' model (1988), such that parties care about the stability of their coalition. The stability $\varphi \in [\widetilde{\varphi}, \bar{\varphi}]$ of a government is determined by, $\varphi = (1 - P_f) \in [0,1]$, where $P_f \in [0,1]$ is the probability of failure of government. Define the size of the legislature as L, and the size of the coalition as c, which is the total number of seats in legislature of two coalition partners. Then let $n \in [0, c]$ be the number of dissidents within that coalition c, and let P_d be the probability of a legislator being a dissident. Here, we let the number of dissidents be determined for coalition failure. However other factors can easily be substituted. A government can fail if it no longer has a strict majority in legislation due to the existence of dissidents within that coalition. Hence the probability of failure P_f is given by the probability that $c - n < L/2$, that is:

$$P_f = \sum_{n=c-\frac{L}{2}}^{c} \binom{c}{n} (P_d)^n (1 - P_d)^{c-n}$$

Given $P_f \in [0,1]$ and $\varphi \in [0,1]$, let $\chi_g \in (0,\infty)$ be the weight party g puts on the importance of stability. And let φ_{gj} denote the stability of coalition g and j. Combined φ_{gj}, multiplied by χ, are additional fixed benefits that come along with the benefits of being in office, with the exception that they are none transferable among parties and is *only* received when a party is participating in a coalition. Also note, that the stability of a large coalition must be greater than for a smaller coalition such that, $\varphi_{12} > \varphi_{13} > \varphi_{23}$. Even though, parties have no influence on the distribution of these benefits, they should however be taken into account in the decision making process by both the proposer as well as the decisive legislative coalition $T \in \tau$ during the coalition formation process.

Given the description above, then, for any party $g \in \{l, m, r\}$ with a positive legislative weight $w_g > 0$, a legislative strategy for party g is described by $\lambda_g = (\pi_g, \delta_g, \varphi_{gj})$. It consists of a proposal $\pi_g = (y^g, b^g) \in X \times B$ conditional on being asked to form a government, and a response strategy δ_g describing the set of other parties' proposals that party g is willing to support,

$$\delta_g : \varphi \times X \times B \times \{1,2,3\} \to [0,1]$$

where $\delta_g(\varphi^{gj}, y^j, b^j, t) \in [0,1]$, is the probability that g supports party j's proposal (y^j, b^j) at legislative bargaining period t, and an additional positive term φ_{gj}, conditional on the proposed decisive legislative parties. In sum, the legislative strategy $\lambda_g = (\pi_g, \delta_g)$, are conditional on the stability of that coalition, the profile of electoral platforms, the distribution of vote shares and the history of the legislative bargaining process. We assume that the legislative strategies are ahistorical at the outset. Including a complete description of legislative strategy would add nothing but notation.

Now let us first formulate party preferences over $\varphi \times X \times B$. Let $a = (a_l, a_m, a_r) \in X^3$ be the profile of electoral platforms, then for any $(z, b) \in X \times B$, $g \in \{l, m, r\}$, and $j \in \{l, m, r\}$, where $g \neq j$, party g's payoff is described by,

$$W_g(z, b; a_g) = \begin{cases} \chi\varphi_{gj} + b_g - (z - a_g)^2 & if \quad w_g > 0 \\ -d \quad if \quad w_g = 0 \end{cases}$$

In case, a party fails to reach the minimal threshold s, then it gains no legislative weight, and bears a nontrivial cost, $d > 0$. We assume that these costs are recovered in case a party does gain legislative weight, $w_g > 0$. If party g has positive legislative representation, $w_g > 0$, then we assume that parties $g \in \{l, m, r\}$ have quasi-linear preferences over private benefits b_g, and final policy outcome z. We characterize a party's legislative policy preferences in terms of the Euclidean distance, so that a party's payoff in office is decreasing in distance in $a_g \in X$, which represents the ideological position of party g, and z^*, which represents the final legislative policy outcome. The perks of being in office b_g is also the upper bounds of $[(z - a_g)^2 - \chi\varphi_{gj}]$ to ensure the lower bound of the payoff is zero and the payoff of being in office is positive.

For a given list of party platforms $a \in X^3$ and a legislative strategy profile, let $W_g^t(\lambda, a_g)$ denote the continuation value of party g under λ from voting against the proposal λ_j offered by party $j \in \{l, m, r\}$ in the t^{th} bargaining period, t = 1, 2, 3. Then a legislative equilibrium at $a \in X^3$ is a list of undominated legislative strategies $\lambda^* = (\lambda_l^*, \lambda_m^*, \lambda_r^*)$ such that for all $g \in \{l, m, r\}$,

(1) For all $(z, b, t) \in X \times B \times \{1,2,3\}$,

$$\delta_g^*(z, b, t) > 0 \Rightarrow W_g\big(z, b; a_g\big) \geq W_g^t(\lambda^*; a_g)\,,$$

(2) For all $\pi_g \in X \times B$

$$E[W_g\big(z, b; a_g\big)|a, (\pi^*, \delta^*)] \geq E[W_g\big(z, b; a_g\big)|a, \big(\pi_g, \pi_{-c}^*, \delta^*\big)]$$

Condition (1) makes sure that parties accept any proposal only if rejection leads to worse expected outcome along the equilibrium path. Condition (2) makes sure parties choose the best response proposal when it is their turn to form a government subject to the responses of the other legislative parties. We assume B is sufficiently large enough such that for the "caretaker" government decision (z^0, b^0), for all $a \in X^3$, $W_g(z^0, b^0, a_c) = 0$ with $\sum_g b_g^0 \leq B$. Hence the utilities for parties is equal to zero in the event of no agreement at t = 1, 2, or 3. The legislative bargaining game can be described as a dynamic game of complete information. Hence legislative equilibria are subgame perfect Nash equilibria in undominated strategies $\lambda^* = (\pi^*, \delta^*)$, and can be solved using backward induction.

All voters are assumed to be rational and vote for exactly one party. Furthermore we assume that voters' preferences are purely policy orientated and that they are quadratic on X. We describe voter's payoff by the following loss function,

$$\forall z \in X, u(z; x) = -(z - x)^2$$

which indicates the distance from final policy outcome z to any voter's ideal point $x \in X$. We assume that the electorate |N| is finite but sufficiently large that the distribution of voter's ideal points on the policy space X is well-approximated by the continuous cumulative distribution function, $F : X \to [0,1]$ with full support on X. Furthermore we assume that the distribution of voters' ideal points is symmetric around the median voter's ideal policy, μ. The voting strategy is a map,

$$v : X^4 \to \Delta\{l, m, r\}$$

such that, for any voter with ideal point $x \in X$ and any list of electoral platforms $a \in X^3$, $v(a; x) = ((v_l(a; x), v_m(a; x), v_r(a; x))$ is the probability distribution over parties; that is, for any party g, $v_g(a; x) \in \{0,1\}$ is the probability that a voter with ideal point x votes for party g and $\sum_g v_g(a; x) = 1$.

For a minimal electoral threshold $s \in \left(0, \frac{32}{100}\right)$ and symmetric distribution of voter' ideal points F, a legislative election equilibrium is a list of undominated party strategies $(a^*, \lambda^*) = ((a_l^*, \lambda_l^*), (a_m^*, \lambda_m^*)(a_r^*, \lambda_r^*))$ and an undominated voting strategy v^* such that,

(1) λ^* is a legislative equilibrium at $a^* \in X^3$;

(2) for all $g \in \{l, m, r\}$ and all $a_g \in X$,

$$E[W_g(z, b; a_g^*)|v^*, a^*, \lambda^*] \geq E[W_g(z, b; a_g)|v^*, (a_c, a_{-c}^*), \lambda^*];$$

(3) for all $x \in X$, all $a \in X^3$ and all $v(\cdot; x) \in \Delta\{l, m, r\}$,

$$E[u(z; x)|v^*, (a, \lambda^*)] \geq E[u(z; x)|(v(a; x), v^*), (a, \lambda^*)].$$

The first condition requires parties choosing legislative equilibrium strategies such that they reach at least the electoral threshold s, and hence have a positive legislative weight. The second condition insists parties take into account, both voters' strategies and other parties' electoral and legislative strategies, when they select their own electoral platform. The final condition requires that every voter's strategy in an undominated best response to other voters' strategies at any given profile of electoral platforms $a \in X^3$, while taking into account of the expected legislative consequences of his or her decision conditional on the legislative strategies of parties. Combined, all three conditions makes sure that the legislative election equilibria is subgame perfect at every stage. Due to the finite property of our bargaining protocol, we identify a no-delay legislative equilibrium, where the proposal of the largest party is always accepted in such equilibrium. Without loss of generality, hereafter we assume $a_l \leq a_m \leq a_r$. Regarding the proposal, the proposed policy must lie between the ideal policy point of the proposer and the potential coalition partner, due to the one-dimensional policy space and quasi-linear party preferences. Furthermore it is rational for parties, that the distribution of benefits is strictly divided among members of the coalition, leaving nothing for the residual party.

With the extension of the model, we have modified lemma 1.1 using the same backward induction arguments as Austen-Smith and Banks (1988). The lemma provides information on the composition of the coalition and details of the proposals, given any distribution of vote shares. It shows that the legislative bargaining protocol induces a function connecting party positions and vote shares to final policy outcomes. The legislative bargaining game can be described as a dynamic game of complete information. Hence legislative equilibria are subgame perfect Nash equilibria and can be solved using backward induction. Recall that in case all parties fail to form a government, the caretaker government's decision yields a payoff equal to zero for all parties. Hence, the continuation values for the last period (t = 3) are known. Given the continuation values, the final proposer has a well defined optimization problem when being asked to make a proposal. His proposal, in turn induces the continuation values for the second bargaining period (t = 2). Hence, the second proposer faces a well-defined optimization problem. Solving the optimization problem for the middle party gives us the continuation values for the first bargaining period. Then finally the party with the vote share makes a proposal, consisting of a policy outcome and a distribution of benefits, that maximizes its utility given the continuation values of t =1. This proposal is the legislative equilibrium and is always accepted, due to the no-delay property of the game. Lemma 1.1 defines party strategy for every distribution of vote shares. In case two parties receive the same amount of votes, the largest party will be decided by a coin flip. Hence we abuses notation somewhat, and write $w_g > w_g$, which could actually mean 'equal to or larger than'. Due to the symmetry argument, we only consider three cases, (1) $w_m > w_l > w_r$, (2) $w_l > w_m > w_r$ and (3) $w_l > w_r > w_m$. For any two parties g, g', we write

$$a_{cc'} = \frac{a_c + a_{c'}}{2}$$

Also for the electoral platform, $a_l \leq a_m \leq a_r$, let ℓ denote the proximity of party positions in the political policy space. Hence, let ℓ_l be the distance from party *l* to party *m*, $\ell_l = (a_m - a_l)$ and let ℓ_r be the distance from party *m* to party *r*, $\ell_r = (a_r - a_m)$. We assume that the party positions of party *l* and *r* are located symmetric around party *m*, hence, $\ell_l = \ell_r$. Furthermore we denote $(z^*, b^*) \in X \times B$ as the equilibrium policy outcome and final distribution of benefits.

Lemma 1.1 *Fix a list of legislative weights w =* (w_l, w_m, w_r) *and stability factor* $\chi\varphi_{12} > \chi\varphi_{13} > \chi\varphi_{23}$. Let the stability factor be restricted such that, $(\ell)^2 \geq \chi\varphi_{lr}$, $\frac{1}{4}(\ell)^2 \geq \chi\varphi_{lm}$, $\frac{1}{4}(\ell)^2 \geq \chi\varphi_{rm}$. *Let the electoral platforms be* $a_l \leq a_m \leq a_r$, *and let* $\ell_l = \ell_r = (a_m - a_l) = (a_r - a_m)$. *Then the following is true of any legislative equilibrium* λ^*.

If $w_c > \frac{1}{2}$ *for some* $c \in \{l, m, r\}$ *then c forms the government on its own and implements the outcome* $\left(z^*, (b_c^*, b_{-c}^*)\right) = (a_c, (B, 0,0))$. *If* $w_c < \frac{1}{2}$ *for all* $c \in \{l, m, r\}$, *then the government is a two party coalition with the following outcomes*

(1) If $w_m > w_l > w_r$ *and*

(1a) if $\chi\varphi_{mr} > \frac{1}{4}(\ell)^2 - B$

and, $\frac{1}{4}(\ell)^2 > \chi\varphi_{lm} > \frac{1}{4}(\ell)^2 - B$,

then $\left(z^*, (b_c^*, b_{-c}^*)\right) = (a_{mr}, (B, 0,0))$

(1b) $if\, \chi\varphi_{mr} < \frac{1}{4}(\ell)^2 - B$

$and, \frac{1}{4}(\ell)^2 > \chi\varphi_{lm} > \frac{1}{4}(\ell)^2 - B,$

$then\ \ z^* = a_{lm}, b_l^* = \chi\varphi_{lm} + B - \frac{1}{4}(\ell)^2, b_m^* = B - b_l^*\ and\ b_r^* = 0$

(2) If $w_l > w_m > w_r$ and

(2a) $if\, \chi\varphi_{lm} < \frac{1}{4}(\ell)^2 - B\ and\ \ \chi\varphi_{lr} < \chi\varphi_{mr} + \frac{3}{4}(\ell)^2,$

$and\, \frac{1}{4}(\ell)^2 > \chi\varphi_{rm} > \frac{1}{4}(\ell)^2 - B$

$then\ \ z^* = a_{lm}, b_l^* = B - b_m^*, b_m^* = \frac{1}{4}(\ell)^2 - \chi\varphi_{lm}\ and\ b_r^* = 0$

(2b) $if\, \chi\varphi_{mr} + \frac{3}{4}(\ell)^2 > \chi\varphi_{lr} > (\ell)^2 - 2B$

$and, \frac{1}{4}(\ell)^2 > \chi\varphi_{rm} > \frac{1}{4}(\ell)^2 - B$

$then\ z^* = a_{lm}, b_l^* = B - b_m^*, b_m^* = B\ and\ b_r^* = 0$

$if\ not,\ then\ z^* = a_{lr}, b_l^* = B - b_r^*, b_m^* = 0\ and\ b_r^* = 0$

2(c) $if\, \chi\varphi_{mr} + \frac{3}{4}(\ell)^2 < \chi\varphi_{lr} < \chi\varphi_{mr} - \chi\varphi_{lm} + (\ell)^2 - B$

$Then,\ z^* = a_{lm}, b_l^* = B - b_m^*, b_m^* = 2\chi\varphi_{mr} - 2\chi\varphi_{lr} - \chi\varphi_{lm} + 1\frac{3}{4}(\ell)^2 and$

$b_r^* = 0.\ Where\ b_m^* > 0$

2(d) $\chi\varphi_{mr} < \frac{1}{4}(\ell)^2 + B\ and\ \ \chi\varphi_{mr} < \chi\varphi_{lm} - \frac{1}{2}B$

$then\ \ z^* = a_{lm}, b_l^* = B - b_m^*, b_m^* = \chi\varphi_{lm} + B - \frac{1}{4}(\ell)^2\ \ and\ b_r^* = 0$

$if\ not,\ then\ z^* = a_{lr}, b_l^* = B - b_r^*, b_m^* = 0\ and\ b_r^* = 0.\ Where\ b_m^* > 0$

(3) If $w_l > w_r > w_m$ and

(3a) $if\ \chi\varphi_{rl} > \chi\varphi_{rm} - \chi\varphi_{ml} - \frac{1}{2}B + \frac{7}{8}(\ell)^2\ and\ \chi\varphi_{lr} > \chi\varphi_{rm} + \frac{5}{8}(\ell)^2$

$and, \frac{3}{4}(\ell)^2 > 2\chi\varphi_{rm} - 2\chi\varphi_{ml} - \chi\varphi_{rl}$

$then\ \ z^* = a_{lr}, b_l^* = B - b_r^*,\ b_m^* = 0, and\ b_r^* = 2\chi\varphi_{rm} - 2\chi\varphi_{ml} - \chi\varphi_{rl} + \frac{3}{4}(\ell)^2.$

$if\ not,\ then\ z^* = a_{lm}, b_l^* = B - b_m^*, b_m^* = \frac{1}{4}(a_l - a_m)^2 - \chi\varphi_{lm}\ and\ b_r^* = 0$

(3b) $\chi\varphi_{rl} > \chi\varphi_{rm} - \chi\varphi_{ml} - \frac{1}{2}B + \frac{7}{8}(\ell)^2\ and\ \ \chi\varphi_{lr} > 2\chi\varphi_{lm} + B + \frac{1}{2}(\ell)^2$

$then\ z^* = a_{lr}, b_l^* = B - b_r^*,\ \ b_m^* = 0, and\ b_r^* = B\ .$

$If\ not,\ then\ z^* = a_{lm}, b_l^* = B - b_m^*, b_m^* = \frac{1}{4}(a_l - a_m)^2 - \chi\varphi_{lm}\ and\ b_r^* = 0$

Proof It is obvious that if a party has a strict majority, $w_c > \frac{1}{2}$, that it forms a government on its own, implements its ideal policy and keeps all the perks to itself. In case there is no such outcome, $0 < w_c < \frac{1}{2}$, for all $c \in \{l, m, r\}$, then the government must consist of is a two party coalition. Recall that $W_c^t(\lambda; a_c)$ is party's continuation value under a legislative strategy profile λ from voting against the proposalπ_j offered by party $j \in \{l, m, r\}$ in the t^{th} bargaining period, *t = 1,2,3*. Also note that some restrictions have been made to ensure that $b_g \geq 0$. According to Austen-Smith and Banks (1988), and as described in lemma 1.2 and theorem 1 hereafter, only equilibria that can occur involves a distribution of vote shares as described in case (3) under lemma 1.1. Hence, below we prove case (3), and relegate the proof for all other (out-of-equilibrium) cases to the appendix.

Suppose case (3), $w_l > w_r > w_m$, then this also implies $\chi\varphi_{lr} > \chi\varphi_{lm} > \chi\varphi_{rm}$. Now recall that $\ell_l = \ell_r$ and that if all legislative parties fail to form a government, the protocol implements a 'caretaker government'. By construction, for all strategy profiles λ and all parties *c,*

$$W_l^3(\lambda; a_l) = 0$$

$$W_m^3(\lambda; a_m) = 0$$

$$W_r^3(\lambda; a_r) = 0$$

Solving the dynamic game of complete and perfect information, we use backward induction arguments. Hence at t =3, the smallest party, party *m*, choose a proposal $(y^m, b^m) \in X$ x B that maximizes $W_m(\lambda; a_m)$ subject to the proposal being accepted by at least one other party. By assumption, $a_l < a_m < a_r$ and party preferences are quasi-linear. We solve this by maximizing party *m*'s objection function for each potential coalition partner separately and compare them. Because preferences are strictly increasing in perks, this means that no private benefits are allocated to the party that is not in coalition. Also, subgame perfection implies that a proposal is always accepted by a potential coalition partner if it is indifferent between accepting and rejecting. Now consider *m*'s t=3 proposal and suppose, first that party *m* proposes to include party *r* as a coalition partner. Then party *m* solves the following Lagrangian maximization problem:

$$\max \Lambda^m(y, b_r, \zeta^m) = \chi\varphi_{mr} + B - b_r - (y - a_m)^2 + \zeta^m[\chi\varphi_{mr} + b_r - (y - a_r)^2]$$

where . ζ^r is the Lagrange multiplier. Also we have substituted the legislative utilities W_c. By concavity and the assumption that B is sufficiently large to permit all coalitions to form for all $a \in X^3$ the first order conditions suffice for an interior solution:

$$\frac{d\Lambda^m}{dy} = -2(y - a_m) - \zeta^m 2(y - a_r) = 0$$

$$\frac{d\Lambda^m}{db_r} = \zeta^m - 1 = 0$$

$$\frac{d\Lambda^m}{d\zeta^m} = \chi\varphi_{mr} + b_r - (a_{mr} - a_r)^2 = 0$$

Solving yields,

$$y = a_{mr}$$

$$b_r = \frac{1}{4}(a_m - a_r)^2 - \chi\varphi_{mr}$$

By assumption, the stability factor is restricted such that, , $\frac{1}{4}(a_m - a_r)^2 \geq \chi\varphi_{mr}$, hence we ensure that $b_r \geq 0$. This yields a payoff,

$$W_m(a_{mr}, b_r, a_m) = \chi\varphi_{mr} + B - b_r - (a_{mr} - a_m)^2$$

$$W_m(a_{mr}, b_r, a_m) = \chi\varphi_{mr} + B - \frac{1}{4}(a_m - a_r)^2 + \chi\varphi_{mr} - \frac{1}{4}(a_m - a_r)^2$$

$$W_m(a_{mr}, b_r, a_m) = 2\,\chi\varphi_{mr} + B - \frac{1}{2}(a_m - a_r)^2$$

Now, suppose party *m* proposes to include party *l* as a coalition partner, then party *m* solves:

$$\max \Lambda^m(y, b_l, \zeta^m) = \chi\varphi_{ml} + B - b_l - (y - a_m)^2 + \zeta^m[\chi\varphi_{ml} + b_l - (y - a_l)^2]$$

$$\frac{d\Lambda^m}{dy} = -2(y - a_m) - \zeta^m 2(y - a_l) = 0$$

$$\frac{d\Lambda^m}{db_r} = \zeta^m - 1 = 0$$

$$\frac{d\Lambda^m}{d\zeta^m} = \chi\varphi_{ml} + b_l - (a_{ml} - a_l)^2 = 0$$

Solving yields,

$$y = a_{ml}, \qquad b_l = \frac{1}{4}(a_m - a_l)^2 - \chi\varphi_{ml}$$

By assumption, the stability factor is restricted such that, , $\frac{1}{4}(a_m - a_l)^2 \geq \chi\varphi_{ml}$, hence we ensure that $b_l \geq 0$. This yields a payoff,

$$W_m(a_{ml}, b_l, a_m) = \ \chi\varphi_{ml} + B - b_l - (a_{ml} - a_m)^2$$

$$W_m(a_{ml}, b_l, a_m) = \ \chi\varphi_{ml} + B - \frac{1}{4}(a_m - a_l)^2 + \chi\varphi_{ml} - (a_{ml} - a_m)^2$$

$$W_m(a_{ml}, b_l, a_m) = \ 2\chi\varphi_{ml} + B - \frac{1}{2}(a_l - a_m)^2$$

Therefore,

$$W_m(a_{mr}, b_r, a_m) \gtreqless W_m(a_{ml}, b_l, a_m) \Leftrightarrow$$

$$2\,\chi\varphi_{mr} + B - \frac{1}{2}(a_m - a_r)^2 \lesseqgtr 2\chi\varphi_{ml} + B - \frac{1}{4}(a_l - a_m)^2 \Leftrightarrow$$

$$\chi\varphi_{mr} \lesseqgtr \chi\varphi_{ml} \Leftrightarrow$$

By assumption, $\chi\varphi_{lr} > \chi\varphi_{lm} > \chi\varphi_{rm}$. Hence party *m*'s best proposal at t=3 is to suggest a coalition with party *l* by making the proposal $(y^m, (b_m^l, B - b_m^l, 0))$, with $y^m = a_{lm}$and $b_m^l = \frac{1}{4}(a_m - a_l)^2 - \chi\varphi_{ml}$. Hence the t=2 continuation values are,

$$W_l^2(\lambda; a_l) = \quad 0$$

$$W_m^2(\lambda; a_m) = 2\chi\varphi_{ml} + B - \frac{1}{4}(a_l - a_m)^2$$

$$W_r^2(\lambda; a_r) = -(a_{lm} - a_r)^2$$

Now consider party *r*'s proposal at t=2, and suppose first that party *r* proposes to include party *l* as a coalition partner, then party *r* solves the following Lagrangian maximization problem:

$$\max \Lambda^r(y, b_l, \zeta^r) = \chi\varphi_{rl} + B - b_l - (y - a_r)^2 + \zeta^r[\chi\varphi_{rl} + b_l - (y - a_l)^2]$$

then first order conditions are,

$$\frac{d\Lambda^r}{dy} = -2(y - a_r) - \zeta^r 2(y - a_l) = 0$$

$$\frac{d\Lambda^r}{db_l} = \zeta^r - 1 = 0$$

$$\frac{d\Lambda^r}{d\zeta^r} = \chi\varphi_{rl} + b_l - (y - a_l)^2 = 0$$

Solving yields,

$$y = a_{rl}, \qquad b_l = \frac{1}{4}(a_r - a_l)^2 - \chi\varphi_{rl}$$

By assumption, the stability factor is restricted such that, , $\frac{1}{4}(a_r - a_l)^2 \geq \chi\varphi_{rl}$ hence we ensure that $b_l \geq 0$. This yields the following payoff,

$$W_r(a_{rl}, b_l, a_r) = \chi\varphi_{rl} + B - b_l - (y - a_r)^2$$

$$W_r(a_{rl}, b_l, a_r) = \chi\varphi_{rl} + B - \frac{1}{4}(a_r - a_l)^2 + \chi\varphi_{rl} - \frac{1}{4}(a_r - a_l)^2$$

$$W_r(a_{rl}, b_l, a_r) = 2\chi\varphi_{rl} + B - \frac{1}{2}(a_r - a_l)^2$$

On the other hand, if party *r* chooses party *m* as a coalition partner at t=2, then for the proposal to be accepted, party *m* must be offered at least, $W_m^2(\lambda; a_m) = 2\chi\varphi_{ml} + B - \frac{1}{4}(a_l - a_m)^2$. Thus party *r* solves the following Lagrangian maximization problem:

$$\max \Lambda^r(y, b_m, \zeta^r) = \chi\varphi_{rm} + B - b_m - (y - a_r)^2 + \zeta^r[\chi\varphi_{rm} + b_m - (y - a_m)^2 - W_m^2(\lambda; a_m)]$$

then the first order conditions are,

$$\frac{d\Lambda^r}{dy} = -2(y - a_r) - \zeta^r 2(y - a_l) = 0$$

$$\frac{d\Lambda^r}{db_l} = \zeta^r - 1 = 0$$

$$\frac{d\Lambda^r}{d\zeta^r} = \chi\varphi_{rm} + b_m - \frac{1}{4}(a_r - a_m)^2 - 2\chi\varphi_{ml} - B + \frac{1}{4}(a_l - a_m)^2 = 0$$

Solving yields,

$$y = a_{rm},$$

$$b_m = 2\chi\varphi_{ml} - \chi\varphi_{rm} + B$$

By assumption, $\chi\varphi_{lr} > \chi\varphi_{lm} > \chi\varphi_{rm}$, hence $b_m \geq 0$. This yields a payoff,

$$W_r(a_{rm}, b_m, a_r) = \chi\varphi_{rm} + B - b_m - (y - a_r)^2$$

$$W_r(a_{rm}, b_m, a_r) = \chi\varphi_{rm} + B - 2\chi\varphi_{ml} + \chi\varphi_{rm} - B - \frac{1}{4}(a_m - a_r)^2$$

$$W_r(a_{rm}, b_m, a_r) = 2\chi\varphi_{rm} - 2\chi\varphi_{ml} - \frac{1}{4}(a_m - a_r)^2$$

We compare,

$$W_r(a_{rl}, b_l, a_r) \lesseqgtr W_r(a_{rm}, b_m, a_r) \Leftrightarrow$$

$$2\chi\varphi_{rl} + B - \frac{1}{2}(a_r - a_l)^2 \lesseqgtr 2\chi\varphi_{rm} - 2\chi\varphi_{ml} - \frac{1}{4}(a_m - a_r)^2 \Leftrightarrow$$

$$2\chi\varphi_{rl} + B - 2(\ell)^2 \lesseqgtr 2\chi\varphi_{rm} - 2\chi\varphi_{ml} - \frac{1}{4}(\ell)^2 \Leftrightarrow$$

$$\chi\varphi_{rl} \lesseqgtr \chi\varphi_{rm} - \chi\varphi_{ml} - \frac{1}{2}B + \frac{7}{8}(\ell)^2 \Leftrightarrow$$

hence, *r*'s best proposal at t=2 is define by,

$$\begin{cases} if \quad \chi\varphi_{rl} < \chi\varphi_{rm} - \chi\varphi_{ml} - \frac{1}{2}B + \frac{7}{8}(\ell)^2 & then, r\ propose\ to\ m\ (path\ A) \\ if \quad \chi\varphi_{rl} > \chi\varphi_{rm} - \chi\varphi_{ml} - \frac{1}{2}B + \frac{7}{8}(\ell)^2 & then, r\ propose\ to\ l\ (path\ B) \end{cases}$$

Now suppose path (A), $\chi\varphi_{rl} < \chi\varphi_{rm} - \chi\varphi_{ml} - \frac{1}{2}B + \frac{7}{8}(\ell)^2$, hence $W_r(a_{rl}, b_l, a_r) <$ *Wrarm,bm,ar* and as a result, the best coalition for party *r* to propose in t=2 is to include party *m* as a coalition partner. Hence the continuation values for t=1 are,

$$W_l^1(\lambda; a_l) = -(a_{rm} - a_l)^2$$

$$W_m^1(\lambda; a_l) = 0$$

$$W_r^1(\lambda; a_r) = 2\chi\varphi_{rm} - 2\chi\varphi_{ml} - \frac{1}{4}(a_m - a_r)^2$$

Now consider party *l*'s t=1 proposal and suppose, first that *l* proposes to include party *m* as a coalition partner. Then *l* solves the following Lagrangian maximization problem:

$$\max \Lambda^l(y, b_m, \zeta^l) = \chi\varphi_{lm} + B - b_m - (y - a_l)^2 + \zeta^l[\chi\varphi_{lm} + b_m - (y - a_m)^2]$$

$$\frac{d\Lambda^l}{dy} = -2(y - a_l) - \zeta^l 2(y - a_m) = 0$$

$$\frac{d\Lambda^l}{db_m} = \zeta^l - 1 = 0$$

$$\frac{d\Lambda^l}{d\zeta^l} = \chi\varphi_{lm} + b_m - (y - a_m)^2 = 0$$

Solving yields,

$$y = a_{lm}, \qquad b_m = \frac{1}{4}(a_l - a_m)^2 - \chi\varphi_{lm}$$

By assumption, the stability factor is restricted such that, , $\frac{1}{4}(a_l - a_m)^2 \geq \chi\varphi_{lm}$ hence we ensure that $b_l \geq 0$. This yields the following payoff,

$$W_l(a_{lm}, b_m, a_l) = \chi\varphi_{lm} + B - b_m - (a_{lm} - a_l)^2$$

$$W_l(a_{lm}, b_m, a_l) = \chi\varphi_{lm} + B - \frac{1}{4}(a_l - a_m)^2 + \chi\varphi_{lm} - \frac{1}{4}(a_m - a_l)^2$$

$$W_l(a_{lm}, b_m, a_l) = 2\chi\varphi_{lm} + B - \frac{1}{2}(a_l - a_m)^2$$

Alternatively, if party *l* proposes to include party *r* as a coalition partner, then for the proposal to be accepted, party *r* must be offered at least $W_r^2(\lambda; a_r)$. Then party *l* solves:

$$\max \Lambda^l(y, b_r, \zeta^l) = \chi\varphi_{rl} + B - b_r - (y - a_l)^2 + \zeta^l[\chi\varphi_{rl} + b_r - (y - a_r)^2 - W_r^2(\lambda; a_r)]$$

From the first order conditions, ,

$$\frac{d\Lambda^l}{dy} = -2(y - a_l) - \zeta^l 2(y - a_r) = 0$$

$$\frac{d\Lambda^l}{db_r} = \zeta^l - 1 = 0$$

$$\frac{d\Lambda^l}{d\zeta^l} = \chi\varphi_{rl} + b_r - (a_{lr} - a_r)^2 - 2\chi\varphi_{rm} + 2\chi\varphi_{ml} + \frac{1}{4}(a_m - a_r)^2 = 0$$

we obtain party *l*'s t=1 proposal,

$$y = a_{lr}$$

$$b_r = \frac{1}{4}(a_l - a_r)^2 + 2\chi\varphi_{rm} - 2\chi\varphi_{ml} - \chi\varphi_{rl} - \frac{1}{4}(a_m - a_r)^2$$

$$b_r = 2\chi\varphi_{rm} - 2\chi\varphi_{ml} - \chi\varphi_{rl} + \frac{3}{4}(\ell)^2$$

For positive values of b_r, it must be that, $\frac{3}{4}(\ell)^2 + 2\chi\varphi_{rm} > 2\chi\varphi_{ml} + \chi\varphi_{rl}$. If this is not true, then the outcome suggest that b_r is negative. This implies that party *r* is willing to accept party *l*'s proposal, even such that it is willing to give extra benefits to party *l*. However, in our model this cannot happen. For further analysis, we assume b_r is positive. This yields the following payoff,

$$W_l(a_{lr}, b_r, a_l) = \chi\varphi_{lr} + B - b_r - \frac{1}{4}(a_r - a_l)^2$$

$$W_l(a_{lr}, b_r, a_l) = 2\chi\varphi_{lr} + 2\chi\varphi_{ml} - 2\chi\varphi_{rm} + B - 1\frac{3}{4}(\ell)^2$$

Thus,

$$W_l(a_{lm}, b_m, a_l) \gtreqless W_l(a_{lr}, b_r, a_l) \Leftrightarrow$$

$$2\chi\varphi_{lm} + B - \frac{1}{2}(a_l - a_m)^2 \lesseqgtr 2\chi\varphi_{lr} + 2\chi\varphi_{ml} - 2\chi\varphi_{rm} + B - 1\frac{3}{4}(\ell)^2 \Leftrightarrow$$

$$\chi\varphi_{rm} \lesseqgtr \chi\varphi_{lr} - \frac{5}{8}(\ell)^2 \Leftrightarrow$$

Hence, party *l*'s best response at t=1 is defined by,

$$\begin{cases} if \quad \chi\varphi_{rm} + \frac{5}{8}(\ell)^2 < \chi\varphi_{lr} \qquad then, l\ propose\ to\ r\ (AA) \\ if \quad \chi\varphi_{rm} + \frac{5}{8}(\ell)^2 > \chi\varphi_{lr} \qquad then, l\ propose\ to\ m\ (AB) \end{cases}$$

In case party *l* proposes to party *r,* then he makes the following proposal, $(y^r, (b_r^l, B -$ *brl,0)*, with *yr=alr* and *brl=2χφrm−2χφml−χφrl+34ℓ2*. However in case party *l* proposes to party *m* then he proposes $(y^m, (b_m^l, B - b_m^l, 0))$, with $y^m = a_{lm}$ and $b_m^l = \frac{1}{4}(a_l - a_m)^2 - \chi\varphi_{lm}$.

Now suppose path (B), $\chi\varphi_{rl} > \chi\varphi_{rm} - \chi\varphi_{ml} - \frac{1}{2}B + \frac{7}{8}(\ell)^2$, hence $W_r(a_{rl}, b_l, a_r) > W_r(a_{rm}, b_m, a_r)$. As a result, the best coalition for party *r* to propose in t=2 is party *l* as a coalition partner. Hence the continuation values for t=1 are,

$$W_l^1(\lambda; a_l) = \quad 0$$

$$W_m^1(\lambda; a_l) = -(a_{lr} - a_m)^2$$

$$W_r^1(\lambda; a_r) = \chi\varphi_{rl} + B - \frac{1}{4}(a_r - a_l)^2$$

Now consider party *l*'s t=1 proposal and suppose, first that *l* proposes to include party *m* as a coalition partner. Then *l* solves the following Lagrangian maximization problem:

$$\max \Lambda^l(y, b_m, \zeta^l) = \chi\varphi_{lm} + B - b_m - (y - a_l)^2 + \zeta^l[\chi\varphi_{lm} + b_m - (y - a_m)^2 + (a_{lr} - a_m)^2]$$

$$\frac{d\Lambda^l}{dy} = -2(y - a_l) - \zeta^l 2(y - a_m) = 0$$

$$\frac{d\Lambda^l}{db_m} = \zeta^l - 1 = 0$$

$$\frac{d\Lambda^l}{d\zeta^l} = \chi\varphi_{lm} + b_m - (y - a_m)^2 + (a_{lr} - a_m)^2 = 0$$

Solving yields,

$$y = a_{lm}, \qquad b_m = \frac{1}{4}(a_l - a_m)^2 - \chi\varphi_{lm}$$

Again, by assumption, the stability factor is restricted such that, , $\frac{1}{4}(a_l - a_m)^2 \geq \chi\varphi_{lm}$ hence we ensure that $b_l \geq 0$. Yielding the following payoff,

$$W_l(a_{lm}, b_m, a_l) = \ \chi\varphi_{lm} + B - b_m - (a_{lm} - a_l)^2$$

$$W_l(a_{lm}, b_m, a_l) = \ 2\chi\varphi_{lm} + B - \frac{1}{2}(a_l - a_m)^2$$

Alternatively, if party *l* proposes to include party *r* as a coalition partner, then for the proposal to be accepted, party *r* must be offered at least $W_r^2(\lambda; a_r)$. Then party *l* solves:

$$\max \Lambda^l(y, b_r, \zeta^l) = \chi\varphi_{rl} + B - b_r - (y - a_l)^2 + \zeta^l[\chi\varphi_{rl} + b_r - (y - a_r)^2 - W_r^2(\lambda; a_r)]$$

From the first order conditions, ,

$$\frac{d\Lambda^l}{dy} = -2(y - a_l) - \zeta^l 2(y - a_r) = 0$$

$$\frac{d\Lambda^l}{db_r} = \zeta^l - 1 = 0$$

$$\frac{d\Lambda^l}{d\zeta^l} = \chi\varphi_{rl} + b_r - (a_{lr} - a_r)^2 - \chi\varphi_{rl} - B + \frac{1}{4}(a_r - a_l)^2 = 0$$

we obtain party *l*'s t=1 proposal,

$$y = a_{lr}, \quad b_r = B$$

Hence,

$$W_l(a_{lr}, b_r, a_l) = \ \chi\varphi_{lr} + B - b_r - \frac{1}{4}(a_r - a_l)^2$$

$$W_l(a_{lr}, b_r, a_l) = \ \chi\varphi_{lr} - \frac{1}{4}(a_r - a_l)^2$$

Thus,

$$W_l(a_{lm}, b_m, a_l) \gtreqless W_l(a_{lr}, b_r, a_l) \Leftrightarrow$$

$$2\chi\varphi_{lm} + B - \frac{1}{2}(a_l - a_m)^2 \gtreqless \chi\varphi_{lr} - \frac{1}{4}(a_r - a_l)^2 \Leftrightarrow$$

$$2\chi\varphi_{lm} + B - \frac{1}{2}(\ell)^2 \gtreqless \chi\varphi_{lr} - \frac{1}{4}(2\ell)^2 \Leftrightarrow$$

$$2\chi\varphi_{lm} + B + \frac{1}{2}(\ell)^2 \gtreqless \chi\varphi_{lr} \Leftrightarrow$$

Hence, party *l*'s best response at t=1 is define by,

$$\begin{cases} if \quad \chi\varphi_{lr} > 2\chi\varphi_{lm} + B + \frac{1}{2}(\ell)^2 & then, l\ propose\ to\ r\ (BA) \\ if \quad \chi\varphi_{lr} < 2\chi\varphi_{lm} + B + \frac{1}{2}(\ell)^2 & then, l\ propose\ to\ m\ (BB) \end{cases}$$

In case party *l* proposes to party *r,* then he makes the following proposal, $(y^r, (b_r^l, B -$ *brl,0)*, with *yr=alr* and *brl=B*. However in case party *l* proposes to party *m* then he proposes $(y^m, (b_m^l, B - b_m^l, 0))$, with $y^m = a_{lm}$and $b_m^l = \frac{1}{4}(a_l - a_m)^2 - \chi\varphi_{lm}$. To sum up, according to lemma 1.1, large coalition is possible for path (AA),

If $\begin{cases} \chi\varphi_{rl} < \chi\varphi_{rm} - \chi\varphi_{ml} - \frac{1}{2}B + \frac{7}{8}(\ell)^2 \\ \chi\varphi_{lr} > \chi\varphi_{rm} + \frac{5}{8}(\ell)^2 \end{cases}$

Under the following restrictions, $(\ell)^2 > \ \chi\varphi_{lr}$, $\frac{1}{4}(\ell)^2 > \chi\varphi_{lm}$, $\frac{1}{4}(\ell)^2 > \chi\varphi_{rm}$, and $\frac{3}{4}(\ell)^2 > 2\chi\varphi_{rm} - 2\chi\varphi_{ml} - \chi\varphi_{rl}$

And for path (BA),

If $\begin{cases} \chi\varphi_{rl} > \chi\varphi_{rm} - \chi\varphi_{lm} - \frac{1}{2}B + \frac{7}{8}(\ell)^2 \\ \chi\varphi_{lr} > 2\chi\varphi_{lm} + B + \frac{1}{2}(\ell)^2 \end{cases}$

Under the following restrictions, $(\ell)^2 > \ \chi\varphi_{lr}$, $\frac{1}{4}(\ell)^2 > \chi\varphi_{lm}$, $\frac{1}{4}(\ell)^2 > \chi\varphi_{rm}$.

For visualization of the described proof above, see diagram 1 in the appendix. This proves case (3). The proof for the remaining cases can be found in the appendix.

According to lemma 1.1, voters and parties are able to predict the policy outcomes for any profile of party platforms and distribution of vote shares and therefore face well-defined electoral decision problems at the time of the elections. The legislative equilibrium λ^* induces policy outcome $z^* = z\left(w\big(v(a)\big)\right) \in X$ for a given voting strategy *v**, electoral platforms $a \in X^3$ and realized vector of weights $w(v(a))$ (that is, in case of a tie, a coin flip is decisive). The prediction of Austen-Smith and Banks (1988), that it is always the largest and smallest parties that form a government however, does *no* longer apply. By incorporating a taste of stability, we have shown that in under a fixed three-step bargaining protocol, the legislative equilibrium does not necessarily involve the first and final proposers and could change the composition of a coalition in equilibrium.

We now turn to the analysis of equilibrium voting behavior in Austen-Smith and Banks' model (1988). Because our modification of his model, described above, only affect the legislative equilibrium outcome, leaving the electoral equilibrium intact, we only briefly discuss the remaining parts of the model. For a more in depth analysis or extensive proof, we refer to Austen-Smith and Banks (1988). According to Austen-Smith and Banks, there is an absence of dominated voting strategy at the electoral stage. With three parties and a large number of voters, there exist many undominated Nash voting equilibria for any electoral platforms. He first implements a symmetry assumption for the distribution of ideal points and looks only for the symmetric legislative election equilibria as a whole, and then imposes a non-triviality restriction on individual votes in such equilibria. More specifically, given everyone else's voting behavior, each individual strategy profile in a symmetric legislative equilibria, must be such that every voter must be pivotal with respect to the final policy outcome at the time of the election. Hence every individual can directly affect the symmetric legislative policy equilibrium by switching his or her vote. Recall that, information is perfect and complete, so that voters understand the coalition formation process and the policy outcomes associated with them, for any given election outcome. Given these remarks, Austen-Smith and Banks (1988) fully specifies voting behavior in his model, (v1) through (v6), for every possible list of electoral platforms $a \in X^3$ and concludes that *only* (v6) can consist legislative electoral equilibria. Analyzing (v6), we see that it includes a distribution of vote shares where all three parties have a positive legislative weight, and can only be an equilibrium if $a_m = \mu$, where μ denotes the median voter's ideal point, and where $\ell_l = \ell_r \geq 8\ell^*/3$, meaning that party *l* and *r* are not too closely distributed symmetrically around μ. They show, that in case this is not true (with the exception that all three parties adopt the same platform) parties have incentives to adjust their electoral platform or that the system ends up in a situation where some party g has a strict majority, $w_g > \frac{1}{2}$, which is similar to a three-party plurality election where the winner controls all legislative decisions. According to Austen-Smith and Banks (1988), every single voter is pivotal with respect to the outcome in a legislative electoral equilibrium and the legislative weight is distributed as follows; party *m* attracts just the minimal vote share threshold *s,* and the two extreme parties, *l* and *r,* divide the remaining votes equally. For a full specification of voting behavior, we refer to Austen-Smith and Banks (1988), which describes Nash behavior at every distribution of electoral platforms more extensively, including distributions that are out-of-equilibrium. Conditional on the specification of voting behavior (1) through (6), he comes with the following lemma 1.2,

Lemma 1.2 *Define the voting strategy v^* by properties (v1)-(6). Then for all $a \in X^3$ and $x \in X$,*

$$E[u(z\left(w\left(v^*(a)\right)\right); x)|(a, \lambda^*)] \geq E[u(z\left(w\left(v(a; x), v^*(a)\right)\right); x)|(a, \lambda^*)]$$

Where λ^* is the legislative equilibrium described in lemma 1.1 and $z\big(w(\cdot)\big)$ is the final policy outcome conditional on realized weights $w(\cdot)$.

Lemma 1.2 implies that $v^*(a^*)$ is indeed defined by a list of electoral platforms, where a_m equal the median's ideal point, and where $\ell_l = \ell_r \geq 8\ell^*/3$ under which the two extreme parties attract a equal vote share of (1 - *s*)/2, leaving the remaining votes *s* to the middle party. For extensive proof, we refer to Austen-Smith and Banks (1988).

Theorem 1 *Let λ^* and v^* be defined by lemma 1.1 and 1.2, respectively. Then $((a^*, \lambda^*), v^*)$ is a legislative equilibrium if and only if $\ell_l = \ell_r = \ell$ and $\ell \in [\frac{8}{3} l^*, \hat{\ell}\ )$.*

Again, proof of theorem 1 can be found in Austen-Smith and Banks (1988), where he shows that if $a \in X^3$ is an equilibrium list of party platforms, only the conditions under theorem 1 can constitute equilibrium behavior by both parties and voters. In such equilibrium, parties have no incentive to deviate to another electoral platform, given the best response strategy of voters. We argue that his proof still holds despite of our slight modification of his model. Also, the conclusions of Austen-Smith and Banks' model (1988) towards voting behavior still hold. Although the expected government stability plays an important role in the decision-making process of coalition formation for both the proposer and accepter, and therefore can change the legislative equilibrium, it however does not affect voting behavior. Below we provide description implying that a change in the composition of coalition partners does not imply a change in the expected policy outcome. Recall that, individuals are assumed to be purely policy-motivated with quadratic preferences on *X*. Hence, voters in the model only care about final policy outcomes and not about party platforms per se nor about the distribution of portfolios in any resulting government. According to theorem 1, the distribution of vote shares must be such that, $w_l = w_r > w_m$, then by lemma 1.1, if the outcome is a minimal winning coalition, then the legislative policy outcome that is implemented is described by,

$$\left(z\left(w\left(v^*(a^*)\right)\right), b\left(w\left(v^*(a^*)\right)\right)\right) \in \{(a^*_{lm}, (B - b^*_m, b^*_m, 0), (a^*_{rm}, (0, b^*_m, B - b^*_m)\}$$

Where each outcome occurs with probability one-half and, $b^*_m = 0$

However, in case the outcome is a large coalition, then the legislative outcome is

$$\left(z\left(w\left(v^*(a^*)\right)\right), b\left(w\left(v^*(a^*)\right)\right)\right) \in \{(a^*_{lr}, (B - b^*_r, b^*_r, 0)), (a^*_{rl}, (0, b^*_l, B - b^*_l))\}$$

Where each outcome occurs with probability one-half. Notice that by lemma 1.1, in both cases (a^*_{lr} and a^*_{rl}), the expected legislative policy outcome equals a_m and that despite the change in composition of the coalition, the legislative expected policy outcome remains the same. Hence, we can leave lemma 1.2 and theorem 1 of Austen-Smith and Banks (1988) untouched.

By all the described above, to sum up, for a fixed list of electoral platforms $a_l < a_m < a_r$, legislative election equilibria supported by *v**, must involve a distribution of party platforms with the middle party *m*, adopting the position of the median voter's ideal point and parties, *l* and *r*, are located symmetrically party *m*, neither too close nor too far from party *m*. In equilibrium, the legislative weights are then, $w_l = w_r > w_m$, which is exactly case (3) under lemma 1.1, given that the tie-breaking lottery is won by party *l*. From theorem 1 it follows that if $a \in X^3$ is an equilibrium list of platforms then (v6) must apply, with the following legislative weights, the two relative extreme parties each attracting (1-*s*)/2, leaving the remaining vote share *s* to party *m*. Hence *m* attracts exactly the threshold vote share. Moreover, lemma 1.1 makes clear that it is, the relative stability of different coalitions together with B and ℓ, that is decisive in the decision-making process of coalition formation, and hence for the composition of the coalition outcome in legislative equilibrium. Although the composition may change due to our extension, in a full legislative electoral equilibrium, the expected final policy outcome remains the same, equal to the median voter's ideal point. If parties care about government stability enough and form a large coalition consisting of parties *l* and *r*, then our extension suggests that, the policy outcome is *always* equal to the median voter's ideal point, whereas without the element of stability or different conditions, the final policy outcome in legislative equilibrium is always to the left or to the right of the median voter's ideal point. Under the condition that, voters' preferences are quadratic over policies and are described as a loss function, this would imply that the expected policy outcome of a large coalition is more desirable. This is due to the lack of variance in expected policy outcome for large coalitions, as it yields a median voter's ideal point for sure in equilibrium.

For the realized legislative weights in equilibrium, we now analyze case (3) in lemma 1.1 (that is, given the realization of legislative tie-breaking lottery in favor of party *l*). It follows from lemma 1.1 that the composition of the coalition outcome depends on the difference in stability between the different coalitions $(\chi\varphi_{lr}, \chi\varphi_{lm}, \chi\varphi_{rm})$, the ideological proximity of parties ℓ, and on the total package of benefits *B*, that can be distributed among parties. More specifically, a large coalition consisting of the two extreme parties *l* and *r*, can be the equilibrium outcome under the conditions of path (AA) and path (BA) in lemma 1.1 case 3. In such an outcome, it must be that the weight put on government stability by parties is large enough, that it dominates the positive effect of ideological proximity of party positions such that even ideologically extreme parties are willing to form a coalition together. Now recall that a coalition's stability is determined by, $\varphi = (1 - P_f) \in [0,1]$, where $P_f \in [0,1]$ is the probability of failure of government and P_f is described as,

$$P_f = \sum_{n=c-\frac{L}{2}}^{c} \binom{c}{n} (P_d)^n (1 - P_d)^{c-n}$$

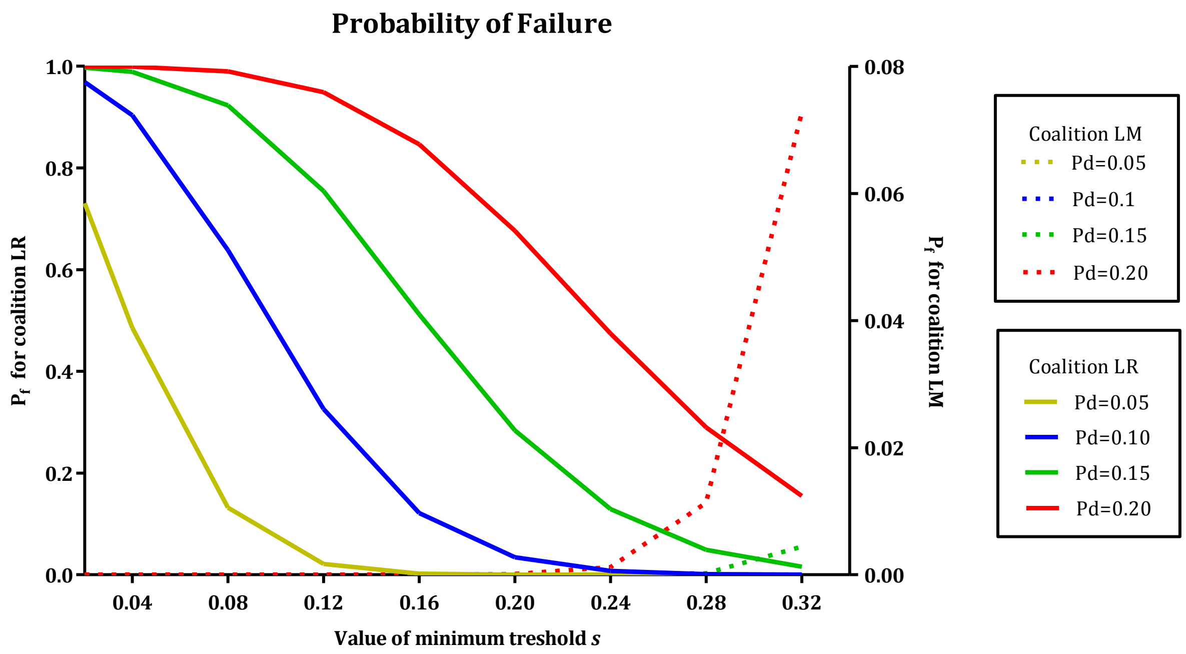
It follows from theorem 1, that the size of the coalitions under case (3) in lemma 1.1, must then be, $c_{lm} = 0.5 + \frac{s}{2}$, $c_{rm} = 0.5 + \frac{s}{2}$, and $c_{lr} = (1 - s)$. Hence, this results in the following, table 1 below, which depicts the probability of failure for the potential coalition under case (3) in lemma 1.1, for different values of the probability of a legislator being a dissident P_d , and for different values of vote share thresholds *s*. The results are also graphically displayed in figure 1.

Table 1: Probability of failure

	% Coalition	Pr(dissident)				% Coalition	Pr(dissident)			
s	LR	0.05	0.1	0.15	0.2	LM	0.05	0.1	0.15	0.2
0.02	98	0.00000	0.00000	0.00000	0.00000	51	0.73069	0.96908	0.99749	0.99984
0.04	96	0.00000	0.00000	0.00000	0.00000	52	0.48543	0.90337	0.98900	0.99912
0.08	92	0.00000	0.00000	0.00000	0.00000	54	0.13205	0.63855	0.92310	0.98990
0.12	88	0.00000	0.00000	0.00000	0.00000	56	0.02115	0.32616	0.75440	0.94898
0.16	84	0.00000	0.00000	0.00000	0.00000	58	0.00224	0.12147	0.51195	0.84624
0.2	80	0.00000	0.00000	0.00000	0.00009	60	0.00017	0.03421	0.28371	0.67660
0.24	76	0.00000	0.00000	0.00001	0.00118	62	0.00001	0.00757	0.12919	0.47458
0.28	72	0.00000	0.00000	0.00023	0.01132	64	0.00000	0.00136	0.04909	0.29002
0.32	68	0.00000	0.00003	0.00441	0.07269	66	0.00000	0.00020	0.01584	0.15480

It is clear that the *lower* the minimal vote share threshold *s*, that is required for legislative representation, the *higher* the probability of parties forming a large coalition, as the stability of the large coalition increases for a *lower* values of *s*. . In other words, relative low legislative entry barriers make it more likely that extreme parties find each other in a large coalition. The crucial element here is that a low legislative threshold makes it more attractive for the largest party to propose a large coalition, as it leads to a larger difference in stability between a large coalition on the one hand and a minimal winning coalition on the other hand. For extremely low values of *s*, the probability of failure for a large coalition is marginal, implying the coalition is extremely stable. It is also worth mentioning that in table 1, the size of the coalitions is expressed in terms of percentage of the total legislation *L*. In reality, most legislative representations on the other hand, have more than a hundred seats, which would imply that for those legislative representations the probability of failure would be larger than represented here, because while the values of the probability of being a dissident, P_d remains the same, the absolute size of the coalition is larger and in turn, therefore also the number of potential legislative dissidents is larger.

Figure 1:



DISCUSSION

Models of parliamentary systems under proportional representation are considered extremely complex. The complexity is a result of multiple stages all interconnected to each other; the elections and voting behavior, government formation and party behavior, and policy outcomes, which is usually a compromise between parties only represented in the government coalition. In the past numerous researchers have attempted to reveal the dynamics of both party and voters behavior under proportional representation systems. Our contributions to the existing literature on proportional representation systems are twofold. First, we modify Austen-Smith and Banks' model (1988), such that it takes into account government stability during the coalition formation process. The element of stability is simple to incorporate in existing models, and can help us in a better understanding of the theory of dynamics in proportional representation systems. By linking this extension to Austen-Smith and Banks' model (1988), we attempt to expose the underlying dynamics in government formation, coalition composition and policy outcomes. Unlike previous literature, our model predicts that it is certainly not always that the largest and smallest parties that form a government coalition in equilibrium. The fundamental difference here is that our model includes the advantages (or at least one) of having a large coalition expressed in terms of coalition stability. We argue that the stability of a government is crucial in order to effectively pass legislature and that the undesirable consequences of a coalition failure must be taken into account in the decision-making process of government formation. According to our predictions, the ultimate outcome of the coalition composition depends on the difference in stability between the different coalitions $(\chi\varphi_{lr}, \chi\varphi_{lm}, \chi\varphi_{rm})$, the ideological proximity of parties ℓ, and on the total package of benefits B, that can be distributed among parties. We believe that with our modification of the model, offers a more complete and realistic view and confirms that the coalition outcome is certainly not always minimal winning. Secondly, lemma 1.1 makes clear that although the composition of the coalition may change, in a full legislative electoral equilibrium, the expected final policy outcome remains the same, which is equal to the median voter's ideal point. Our extension illustrate that if parties care about coalition stability enough, the policy outcome is *always* equal to the median voter's ideal point, whereas without the element of stability, the final policy outcome in legislative equilibrium is always to the left or to the right of the median voter's ideal point. Under the condition that voters have quadratic preferences over policies, this would imply that the expected policy outcome of a broad coalition is more desirable, due to the lack of variance in expected policy outcome for broad coalitions, always yielding policy equal to the median voter's ideal point in equilibrium.

When we test our extension for the Dutch minority government anno 2011(consisting of liberals, VVD and the Christen-democrats, CDA, with the support of the rightwing party PVV), which has 76 seats of the total 150 seats in parliament. According to our predictions, we see that having only a slight majority in legislature makes them extremely vulnerable for coalition failure, as only one dissident is needed in order for them to lose their strict majority. Now suppose the probability of being a dissident, $P_d = 0.005$, then our calculations predict that the probability of failure of the Dutch government is approximately 32 percent, and for $P_d = 0.001$, we predict that the probability of failure of the Dutch government is about 7 percent. Based on our predictions, we could argue that the parties involved in the current Dutch government have taken an high risk by forming this vulnerable coalition. This result however should be seen in perspective. We are aware that parties in proportional representation systems usually make agreements during the coalition formation process, and have their party members committing themselves to this agreement before a government is implemented. However, we must keep in mind that legislators are in fact chosen as independent representatives of the voters and therefore have the right to act independently. In the light of this argument, the probability of being a dissident could also be seen as the degree in how well parties can discipline their members to commit to the agreements made.

Any model has its limitations, and the one described in this paper is certainly no exception. Austen-Smith and Banks (1988) have attempted to provide a simplified framework that corresponds as much as possible to the extreme complex environment of proportional representation systems. Important features of the system are still represented here, such as majoritarian legislature, vote share thresholds, and mostly a few core parties that are positioned on an ideological dimension. Such simplified framework, however only exposes components of the dynamics in proportional representation systems. With our extension, we have attempted to add a small, but crucial component to the theory of coalition formation, and therefore contribute to a better understanding of dynamics in proportional representation systems. In reality many more factors play an important role in both party and voters' strategies under proportional representation. The difficulty is that most of these factors are based on irrational subjective elements such as principles or philosophy, and therefore hard to model. Moreover, the model assumes parties to be located on a one dimensional policy space, consisting of leftwing-, centrist-, and rightwing-parties. As mentioned, the political spectrum is extremely complex, and is more likely to consist of multiple axes representing multiple dimensions. Hence, parties differ from each other in several ways, (e.g. progressivity, libertarism, socialism, conservatism) leading to a higher variety among parties and tougher decisions for the electorate.

There is still much to be discovered, in order to fully understand the strategic dynamics of coalition formation in proportional representation systems. We would like to address three possible extensions in future research. First we could increase the number of parties competing for legislative representation. This would increase the number of possible coalitions and hence the complexity of electoral equilibria. Although, we believe that for odd numbers of parties, the dynamics remain the same as for this paper. Interesting would however be, to see how party and voter strategies would change for even number of parties competing in legislature. Secondly, we believe that the model discussed here, could be more refined with an extension that changes the way policy outcomes are determined. Momentarily, in our model, policy outcomes are formed by the sum of policy platform of parties in coalition, divided by two. We would like to suggest a model where policy outcomes are determined by the average weighted vote shares of coalition partners. Hence, parties with a higher legislative weight, that take part in a coalition, have a greater influence in the policy outcome compared to a coalition partner with a lower legislative weight. This would complicate the analysis in the sense that every voter is pivotal and has a direct influence on the final policy outcome. If voters believe that their favorite party is not a viable option to become a coalition partner, they could engage in strategic voting by voting for a 'lesser evil' alternative in order to balance the policy outcome into their preferred direction. Furthermore, it must be mentioned that the model presented in this paper assumes a world of perfect and complete information, where actors face well-defined decisions problems. This is an assumption that will always be questioned by skeptics. It is however, beyond the scope of this paper to discuss the ability of actors to become informed. We simply attempt to expose certain dynamics actors face in the legislative electoral equilibrium. For further research, it would however be interesting to model voter behavior under imperfect and incomplete information, where parties could signal their preferences regarding coalition partners and policy outcome. Using media coverage and actual opinion polls voters receive signals and could forecast these preferences pre-election and act accordingly. The discussing however would then turn to what role opinion polls play, and how that affects the expectations and beliefs of voters and eventually how it influences voters' behavior.

APPENDIX

Proof of lemma 1.1, case (1):

Suppose $w_m > w_l > w_r$, then this implies that, $\chi\varphi_{lm} > \chi\varphi_{mr} > \chi\varphi_{lr}$. By construction the continuation values at t=3,

$$W_l^3(\lambda; a_l) = 0$$

$$W_m^3(\lambda; a_m) = 0$$

$$W_r^3(\lambda; a_r) = 0$$

At t=1, conditional on choosing party *l* as a coalition partner, party *r* solves the following Lagrangian maximization problem:

$$\max \Lambda^r(y, b_l, \zeta^r) = \chi\varphi_{rl} + B - b_l - (y - a_r)^2 + \zeta^r[\chi\varphi_{rl} + b_l - (y - a_l)^2]$$

Where we have substituted for the legislative utilities W_c and ζ^r is the Lagrange multiplier. This yields the following first order conditions,

$$\frac{d\Lambda^r}{dy} = -2(y - a_r) - \zeta^r 2(y - a_l) = 0$$

$$\frac{d\Lambda^r}{db_l} = \zeta^r - 1 = 0$$

$$\frac{d\Lambda^r}{d\zeta^r} = \chi\varphi_{rl} + b_l - (y - a_l)^2 = 0$$

Solving yields,

$$y = a_{rl}, \qquad b_l = \frac{1}{4}(a_r - a_l)^2 - \chi\varphi_{rl}$$

By assumption, the stability factor is restricted such that, , $\frac{1}{4}(a_r - a_l)^2 \geq \chi\varphi_{rl}$ hence we ensure that $b_l \geq 0$. This yields the following payoff,

$$W_r(a_{rl}, b_l, a_r) = \chi\varphi_{rl} + B - b_l - (y - a_r)^2$$

$$W_r(a_{rl}, b_l, a_r) = \chi\varphi_{rl} + B - \frac{1}{4}(a_r - a_l)^2 + \chi\varphi_{rl} - \frac{1}{4}(a_r - a_l)^2$$

$$W_r(a_{rl}, b_l, a_r) = 2\chi\varphi_{rl} + B - \frac{1}{2}(a_r - a_l)^2$$

Similar, conditional on choosing party m as a coalition partner, party r solves the following Lagrangian maximization problem:

$$\max \Lambda^r(y, b_m, \zeta^r) = \chi\varphi_{rm} + B - b_m - (y - a_r)^2 + \zeta^r[\chi\varphi_{rm} + b_m - (y - a_m)^2]$$

party r then makes the following proposal,

$$y = a_{rm}, \qquad b_m = \frac{1}{4}(a_r - a_m)^2 - \chi\varphi_{rm}$$

By assumption, the stability factor is restricted such that, , $\frac{1}{4}(a_r - a_m)^2 \geq \chi\varphi_{rm}$ hence we have ensured that $b_m \geq 0$. This yields the following payoff,

$$W_r(a_{rm}, b_m, a_r) = \chi\varphi_{rm} + B - b_m - \frac{1}{4}(a_r - a_m)^2$$

$$W_r(a_{rm}, b_m, a_r) = \chi\varphi_{rm} + B - \frac{1}{4}(a_r - a_m)^2 + \chi\varphi_{rm} - \frac{1}{4}(a_r - a_m)^2$$

$$W_r(a_{rm}, b_m, a_r) = 2\chi\varphi_{rm} + B - \frac{1}{2}(a_r - a_m)^2$$

Therefore we compare,

$$W_r(a_{rl}, b_l, a_r) \lesseqgtr W_r(a_{rm}, b_m, a_r) \Leftrightarrow$$

$$2\chi\varphi_{rl} + B - \frac{1}{2}(a_r - a_l)^2 \lesseqgtr 2\chi\varphi_{rm} + B - \frac{1}{2}(a_r - a_m)^2 \Leftrightarrow$$

$$2\chi\varphi_{rl} - \frac{1}{2}(2\ell)^2 \lesseqgtr 2\chi\varphi_{rm} - \frac{1}{2}(\ell)^2 \Leftrightarrow$$

$$2\chi\varphi_{rl} - 2(\ell)^2 \lesseqgtr 2\chi\varphi_{rm} - \frac{1}{2}(\ell)^2 \Leftrightarrow$$

$$\chi\varphi_{rl} - (\ell)^2 \lesseqgtr \chi\varphi_{rm} - \frac{1}{4}(\ell)^2 \Leftrightarrow$$

$$\chi\varphi_{rl} - \frac{3}{4}(\ell)^2 \lesseqgtr \chi\varphi_{rm} \Leftrightarrow$$

By assumption, $\chi\varphi_{ml} > \chi\varphi_{rm} > \chi\varphi_{rl}$, so it must be that $W_r(a_{rl}, b_l, a_r) < W_r(a_{rm}, b_m, a_r)$ and party *r*'s best proposal at t=3 is to suggest a coalition with party *m*, by making proposal $(y^m, (b_m^r, B - b_m^r, 0))$, with $y^m = a_{rm}$and $b_m^r = \frac{1}{4}(a_r - a_m)^2 - \chi\varphi_{rm}$. Hence the continuation values for t=2 are,

$$W_l^2(\lambda; a_l) = \quad -(a_{rm} - a_l)^2$$

$$W_m^2(\lambda; a_m) = W_m^3(\lambda; a_m) \quad = 0$$

$$W_r^2(\lambda; a_r) = 2\chi\varphi_{rm} + B - \frac{1}{2}(a_r - a_m)^2$$

Now consider party *l*'s proposal at t=2 and suppose, first that party *l* proposes to include party *m* as a coalition partner. Then party *l* solves the following Lagrangian maximization problem:

$$\max \Lambda^l(y, b_m, \zeta^l) = \chi\varphi_{lm} + B - b_m - (y - a_l)^2 + \zeta^l[\chi\varphi_{lm} + b_m - (y - a_m)^2]$$

Gives the following first order conditions,

$$\frac{d\Lambda^l}{dy} = -2(y - a_l) - \zeta^l 2(y - a_m) = 0$$

$$\frac{d\Lambda^l}{db_m} = \zeta^l - 1 = 0$$

$$\frac{d\Lambda^l}{d\zeta^l} = \chi\varphi_{lm} + b_m - (y - a_m)^2 = 0$$

Solving yields,

$$y = a_{lm}, \qquad b_m = \frac{1}{4}(a_m - a_l)^2 - \chi\varphi_{lm}$$

By assumption, the stability factor is restricted such that, , $\frac{1}{4}(a_m - a_l)^2 \geq \chi\varphi_{lm}$ hence we have ensured that $b_m \geq 0$. This yields the following payoff,

$$W_l(a_{lm}, b_m, a_l) = \chi\varphi_{lm} + B - b_m - (a_{lm} - a_l)^2$$

$$W_l(a_{lm}, b_m, a_l) = 2\chi\varphi_{lm} + B - \frac{1}{2}(a_m - a_l)^2$$

On the other hand, if party *l* proposes to include party *r* as a coalition partner, then for the proposal to be accepted, party *r* must be offered at least $W_r^2(\lambda; a_r)$. Thus party *l* solves:

$$\max \Lambda^l(y, b_r, \zeta^l) = \chi\varphi_{rl} + B - b_r - (y - a_l)^2 + \zeta^l[\chi\varphi_{rl} + b_r - (y - a_r)^2 - W_r^2(\lambda; a_r)]$$

From the first order conditions we obtain *l*'s t=2 proposal,

$$\frac{d\Lambda^l}{dy} = -2(y - a_l) - \zeta^l 2(y - a_r) = 0$$

$$\frac{d\Lambda^l}{db_r} = \zeta^l - 1 = 0$$

$$\frac{d\Lambda^l}{d\zeta^l} = \chi\varphi_{rl} + b_r - (a_{lr} - a_r)^2 - 2\chi\varphi_{rm} - B + \frac{1}{2}(a_r - a_m)^2 = 0$$

Solving yields,

$$y = a_{lr},$$

$$b_r = 2\chi\varphi_{rm} - \chi\varphi_{rl} + B + \frac{1}{4}(a_l - a_r)^2 - \frac{1}{2}(a_r - a_m)^2$$

$$b_r = 2\chi\varphi_{rm} - \chi\varphi_{rl} + B + \frac{1}{2}(\ell)^2$$

By assumption, $\chi\varphi_{lm} > \chi\varphi_{rm} > \chi\varphi_{rl}$, hence b_r must be positive. This yields the following payoff,

$$W_l(a_{lr}, b_r, a_l) = \chi\varphi_{lr} + B - b_r - \frac{1}{4}(a_r - a_l)^2$$

$$W_l(a_{lr}, b_r, a_l) = \chi\varphi_{lr} + B - 2\chi\varphi_{rm} + \chi\varphi_{rl} - B - \frac{1}{2}(\ell)^2 - (\ell)^2$$

$$W_l(a_{lr}, b_r, a_l) = 2\chi\varphi_{lr} - 2\chi\varphi_{rm} - 1\frac{1}{2}(\ell)^2$$

Thus we compare,

$$W_l(a_{lm}, b_m, a_l) \gtreqless W_l(a_{lr}, b_r, a_l) \Leftrightarrow$$

$$2\chi\varphi_{lm} + B - \frac{1}{2}(\ell)^2 \gtreqless 2\chi\varphi_{lr} - 2\chi\varphi_{rm} - 1\frac{1}{2}(\ell)^2 \Leftrightarrow$$

$$2\chi\varphi_{lm} \gtreqless 2\chi\varphi_{lr} - 2\chi\varphi_{rm} - (\ell)^2 - B \Leftrightarrow$$

$$\chi\varphi_{lm} \gtreqless \chi\varphi_{lr} - \chi\varphi_{rm} - \frac{1}{2}(\ell)^2 - \frac{1}{2}B \Leftrightarrow$$

By assumption, $\chi\varphi_{ml} > \chi\varphi_{rm} > \chi\varphi_{rl}$, hence it must be that $W_l(a_{lm}, b_m, a_l) >$ *Wlalr,br,al* and party *l*'s best proposal at t=2 is to suggest a coalition with party *m,* by making proposal $(y^m, (b_m^l, B - b_m^l, 0))$, with $y^m = a_{lm}$ and $b_m^l = \frac{1}{4}(a_m - a_l)^2 - \chi\varphi_{lm}$. Hence the continuation values for t=1 are,

$$W_l^1(\lambda; a_l) = \quad 2\chi\varphi_{lm} + B - \frac{1}{2}(\ell)^2$$

$$W_m^1(\lambda; a_m) = W_m^2(\lambda; a_m) = W_m^3(\lambda; a_m) \quad = 0$$

$$W_r^1(\lambda; a_r) = -(a_{lm} - a_r)^2$$

Now consider party *m*'s proposal at t=1 and suppose, first that party *m* proposes to include party *r* as a coalition partner. Then party *m* solves the following Lagrangian maximization problem:

$$\max \Lambda^m(y, b_r, \zeta^m) = \chi\varphi_{mr} + B - b_r - (y - a_m)^2 + \zeta^m[\chi\varphi_{mr} + b_r - (y - a_r)^2 - W_r^1(\lambda; a_r)]$$

Yields the following first order conditions,

$$\frac{d\Lambda^m}{dy} = -2(y - a_m) - \zeta^m 2(y - a_r) = 0$$

$$\frac{d\Lambda^m}{db_r} = \zeta^m - 1 = 0$$

$$\frac{d\Lambda^m}{d\zeta^m} = \chi\varphi_{mr} + b_r - (a_{mr} - a_r)^2 + (a_{lm} - a_r)^2 = 0$$

Solving yields,

$$y = a_{mr}$$

$$b_r = \frac{1}{4}(a_m - a_r)^2 - (a_{lm} - a_r)^2 - \chi\varphi_{mr}$$

$$b_r = \frac{1}{4}(\ell)^2 - (a_{lm} - a_r)^2 - \chi\varphi_{mr} < 0$$

Recall that, $\ell_l = \ell_r = (a_m - a_l) = (a_r - a_m)$, hence b_r is negative. But because b_r must be positive, hence party *m* offers $b_r = 0$, This yields a payoff,

$$W_m(a_{mr}, b_r, a_m) = \chi\varphi_{mr} + B - b_r - (a_{mr} - a_m)^2$$

$$W_m(a_{mr}, b_r, a_m) = \chi\varphi_{mr} + B - \frac{1}{4}(a_m - a_r)^2$$

Now, if party *m* proposes to include party *l* as a coalition partner, then for the proposal to be accepted, party *l* must be offered at least $W_l^1(\lambda; a_l)$. Thus party *m* solves:

$$\max \Lambda^m(y, b_l, \zeta^m) = \chi\varphi_{ml} + B - b_l - (y - a_m)^2 + \zeta^m\left[\chi\varphi_{ml} + b_l - (y - a_l)^2 - W_l^1(\lambda; a_l)\right]$$

Hence,

$$\frac{d\Lambda^m}{dy} = -2(y - a_m) - \zeta^m 2(y - a_l) = 0$$

$$\frac{d\Lambda^m}{db_l} = \zeta^m - 1 = 0$$

$$\frac{d\Lambda^m}{d\zeta^m} = \chi\varphi_{ml} + b_l - (a_{ml} - a_l)^2 - 2\chi\varphi_{lm} - B + \frac{1}{2}(\ell)^2$$

Solving yields,

$$y = a_{ml}, \qquad b_l = \chi\varphi_{lm} + B - \frac{1}{4}(\ell)^2$$

For positive values of b_m, it must be that, $\chi\varphi_{lm} + B > \frac{1}{4}(\ell)^2$. This yields the following payoff,

$$W_m(a_{ml}, b_l, a_m) = \chi\varphi_{ml} + B - b_l - (a_{ml} - a_m)^2$$

$$W_m(a_{ml}, b_l, a_m) = 0$$

Again we solve,

$$W_m(a_{mr}, b_r, a_m) \gtreqless W_m(a_{ml}, b_l, a_m) \Leftrightarrow$$

$$\chi\varphi_{mr} + B - \frac{1}{4}(a_m - a_r)^2 \gtreqless 0 \Leftrightarrow$$

$$\chi\varphi_{mr} \gtreqless \frac{1}{4}(\ell)^2 - B \Leftrightarrow$$

Hence, party m's best proposal at t=1 is defined by,

$$\begin{cases} if \quad \chi\varphi_{mr} > \frac{1}{4}(\ell)^2 - B & then, m\ propose\ to\ r \\ if \quad \chi\varphi_{mr} < \frac{1}{4}(\ell)^2 - B & then, m\ propose\ to\ l \end{cases}$$

Diagram 1: lemma 1.1, case (1):

Suppose $w_m > w_l > w_r$,

R	$W_l^3(\lambda; a_l) = 0$ $W_m^3(\lambda; a_m) = 0$ $W_r^3(\lambda; a_r) = 0$

$\downarrow$

L	$W_l^2(\lambda; a_l) = -(a_{rm} - a_l)^2$ $W_m^2(\lambda; a_m) = 0$ $W_r^2(\lambda; a_r) = 2\chi\varphi_{rm} + B - \frac{1}{2}B + \frac{7}{8}(\ell)^2$

$\downarrow$

M	$W_l^1(\lambda; a_l) = 2\chi\varphi_{lm} + B - \frac{1}{2}(\ell)^2$ $W_m^1(\lambda; a_m) = 0$ $W_r^1(\lambda; a_r) = -(a_{lm} - a_r)^2$

$\longrightarrow$

$$\begin{array}{l} \mathbf{A} \\ \mathbf{B} \end{array} \begin{cases} if \quad \chi\varphi_{mr} > \frac{1}{4}(\ell)^2 - B & then, m\ propose\ to\ r \\ if \quad \chi\varphi_{mr} < \frac{1}{4}(\ell)^2 - B & then, m\ propose\ to\ l \end{cases}$$

Proof of lemma 1.1 , case (2):

Suppose $w_l > w_m > w_r$, then this implies , $\chi\varphi_{lm} > \chi\varphi_{lr} > \chi\varphi_{mr}$. By construction the continuation values at t=3,

$$W_l^3(\lambda; a_l) = 0$$

$$W_m^3(\lambda; a_m) = 0$$

$$W_r^3(\lambda; a_r) = 0$$

Conditional on choosing party *l* as a coalition partner, party *r* solves the following Lagrangian maximization problem:

$$\max \Lambda^r(y, b_l, \zeta^r) = \chi\varphi_{rl} + B - b_l - (y - a_r)^2 + \zeta^r[\chi\varphi_{rl} + b_l - (y - a_l)^2]$$

Where we have substituted for the legislative utilities W_c and ζ^r is the Lagrange multiplier. This yields the following first order conditions,

$$\frac{d\Lambda^r}{dy} = -2(y - a_r) - \zeta^r 2(y - a_l) = 0$$

$$\frac{d\Lambda^r}{db_l} = \zeta^r - 1 = 0$$

$$\frac{d\Lambda^r}{d\zeta^r} = \chi\varphi_{rl} + b_l - (y - a_l)^2 = 0$$

Solving yields,

$$y = a_{rl}, \qquad b_l = \frac{1}{4}(a_r - a_l)^2 - \chi\varphi_{rl}$$

By assumption, the stability factor is restricted such that, , $\frac{1}{4}(a_r - a_l)^2 \geq \chi\varphi_{rl}$ hence we have ensured that $b_l \geq 0$. This yields the following payoff,

$$W_r(a_{rl}, b_l, a_r) = \chi\varphi_{rl} + B - b_l - (y - a_r)^2$$

$$W_r(a_{rl}, b_l, a_r) = \chi\varphi_{rl} + B - \frac{1}{4}(a_r - a_l)^2 + \chi\varphi_{rl} - \frac{1}{4}(a_r - a_l)^2$$

$$W_r(a_{rl}, b_l, a_r) = 2\chi\varphi_{rl} + B - \frac{1}{2}(a_r - a_l)^2$$

Similar, conditional on choosing party m as a coalition partner, party r solves the following Lagrangian maximization problem:

$$\max \Lambda^r(y, b_m, \zeta^r) = \chi\varphi_{rm} + B - b_m - (y - a_r)^2 + \zeta^r[\chi\varphi_{rm} + b_m - (y - a_m)^2]$$

the first order conditions,

$$\frac{d\Lambda^r}{dy} = -2(y - a_r) - \zeta^r 2(y - a_m) = 0$$

$$\frac{d\Lambda^r}{db_m} = \zeta^r - 1 = 0$$

$$\frac{d\Lambda^r}{d\zeta^r} = \chi\varphi_{rm} + b_m - (y - a_m)^2 = 0$$

Then party r makes proposal,

$$y = a_{rm}, \qquad b_m = \frac{1}{4}(a_r - a_m)^2 - \chi\varphi_{rm}$$

By assumption, the stability factor is restricted such that, , $\frac{1}{4}(a_r - a_m)^2 \geq \chi\varphi_{rm}$ hence we have ensured that $b_m \geq 0$. This yields the following payoff,

$$W_r(a_{rm}, b_m, a_r) = 2\chi\varphi_{rm} + B - \frac{1}{2}(a_r - a_m)^2$$

Therefore we compare,

$$W_r(a_{rl}, b_l, a_r) \lesseqgtr W_r(a_{rm}, b_m, a_r) \Leftrightarrow$$

$$2\chi\varphi_{rl} + B - \frac{1}{2}(a_r - a_l)^2 \lesseqgtr 2\chi\varphi_{rm} + B - \frac{1}{2}(a_r - a_m)^2 \Leftrightarrow$$

$$2\chi\varphi_{rl} - \frac{1}{2}(2\ell)^2 \lesseqgtr 2\chi\varphi_{rm} - \frac{1}{2}(\ell)^2 \Leftrightarrow$$

$$\chi\varphi_{rl} - (\ell)^2 \lesseqgtr \chi\varphi_{rm} - \frac{1}{4}(\ell)^2 \Leftrightarrow$$

$$\chi\varphi_{rl} \lesseqgtr \chi\varphi_{rm} + \frac{3}{4}(\ell)^2 \Leftrightarrow$$

By assumption $\chi\varphi_{lm} > \chi\varphi_{lr} > \chi\varphi_{mr}$, hence, party *r*'s best proposal at t=3 is define by,

$$\begin{cases} if \quad \chi\varphi_{lr} < \chi\varphi_{mr} + \frac{3}{4}(\ell)^2 & then, r\ propose\ to\ m\ (path\ A) \\ if \quad \chi\varphi_{lr} > \chi\varphi_{mr} + \frac{3}{4}(\ell)^2 & then, r\ propose\ to\ l\ (path\ B) \end{cases}$$

Now suppose we follow path (A), that is if $\chi\varphi_{lr} < \chi\varphi_{mr} + \frac{3}{4}(\ell)^2$, and party *r*'s best proposal at t=3 is to propose to party *m.* Then,

$$W_l^2(\lambda; a_l) = \quad -(a_{rm} - a_l)^2$$

$$W_m^2(\lambda; a_m) = W_m^3(\lambda; a_m) \quad = 0$$

$$W_r^2(\lambda; a_r) = 2\chi\varphi_{rm} + B - \frac{1}{2}(a_r - a_m)^2$$

Now, consider party *m*'s t=2 proposal and suppose, first that party *m* proposes to include party *l* as a coalition partner. Then party *m* solves the following Lagrangian maximization problem:

$$\max \Lambda^m(y, b_l, \zeta^m) = \chi\varphi_{lm} + B - b_l - (y - a_m)^2 + \zeta^m[\chi\varphi_{lm} + b_l - (y - a_l)^2 + (a_{rm} - a_l)^2]$$

This gives the following first order conditions,

$$\frac{d\Lambda^m}{dy} = -2(y - a_m) - \zeta^m 2(y - a_l) = 0$$

$$\frac{d\Lambda^l}{db_m} = \zeta^l - 1 = 0$$

$$\frac{d\Lambda^m}{d\zeta^m} = \chi\varphi_{lm} + b_l - (a_{lm} - a_l)^2 + (a_{rm} - a_l)^2 = 0$$

Solving yields,

$$y = a_{lm}, \qquad b_l = \frac{1}{4}(a_m - a_l)^2 - (a_{rm} - a_l)^2 - \chi\varphi_{lm} < 0$$

Given b_l must be positive, we assume party *m* offers $b_l = 0$ to party *l*. Hence solving yields,

$$W_m(a_{lm}, b_l, a_m) = \chi\varphi_{lm} + B - b_l - (a_{lm} - a_m)^2$$

$$W_m(a_{lm}, b_l, a_m) = \chi\varphi_{lm} + B - \frac{1}{4}(a_m - a_l)^2$$

On the other hand, if party *m* proposes to include party *r* as a coalition partner, then for the proposal to be accepted, party *m* must be offered at least $W_r^2(\lambda; a_r)$. Thus party *m* solves:

$$\max \Lambda^m(y, b_r, \zeta^m) = \chi\varphi_{mr} + B - b_r - (y - a_m)^2 + \zeta^m[\chi\varphi_{mr} + b_r - (y - a_r)^2 - W_r^2(\lambda; a_r)]$$

The first order conditions,

$$\frac{d\Lambda^m}{dy} = -2(y - a_m) - \zeta^m 2(y - a_r) = 0$$

$$\frac{d\Lambda^l}{db_m} = \zeta^l - 1 = 0$$

$$\frac{d\Lambda^m}{d\zeta^m} = \chi\varphi_{mr} + b_r - \frac{1}{4}(a_m - a_r)^2 - 2\chi\varphi_{rm} - B + \frac{1}{2}(a_r - a_m)^2 = 0$$

Solving,

$$y = a_{rm}, \qquad b_r = \chi\varphi_{rm} + B - \frac{1}{4}(a_r - a_m)^2$$

Hence we restrict, $\chi\varphi_{rm} \geq \frac{1}{4}(a_r - a_m)^2 - B$ for positive values of b_m. This yields the following payoff,

$$W_m(a_{mr}, b_r, a_m) = \chi\varphi_{mr} + B - b_r - \frac{1}{4}(a_r - a_m)^2$$

$$W_m(a_{mr}, b_r, a_m) = \chi\varphi_{mr} + B - \chi\varphi_{rm} - B + \frac{1}{4}(a_r - a_m)^2 - \frac{1}{4}(a_r - a_m)^2$$

$$W_m(a_{mr}, b_r, a_m) = 0$$

Hence,

$$W_m(a_{lm}, b_l, a_m) \lesseqqgtr W_m(a_{mr}, b_r, a_m) \Leftrightarrow$$

$$\chi\varphi_{lm} \lesseqqgtr \frac{1}{4}(a_m - a_l)^2 - B \Leftrightarrow$$

Hence, party *m*'s best proposal at t=2 is define by,

$$\begin{cases} if \quad \chi\varphi_{lm} < \frac{1}{4}(\ell)^2 - B & then, m\ propose\ to\ l\ (path\ AA) \\ if \quad \chi\varphi_{lm} > \frac{1}{4}(\ell)^2 - B & then, m\ propose\ to\ r\ (path\ AB) \end{cases}$$

Now suppose path (AA), that is if $\varphi_{lm} < \frac{1}{4}(\ell)^2 - B$, then party *m*'s best proposal at t=2 is to suggest a coalition with party *r.* The continuation values for t=1 are,

$$W_l^1(\lambda; a_l) = \quad -(a_{mr} - a_l)^2$$

$$W_m^1(\lambda; a_m) = 0$$

$$W_r^1(\lambda; a_r) = 0$$

Now we can solve party *l*'s best proposal at t=1. Consider party *l* proposes to party *r*, then party *l* solves,

$$\max \Lambda^l(y, b_r, \zeta^l) = \chi\varphi_{rl} + B - b_r - (y - a_l)^2 + \zeta^l[\chi\varphi_{rl} + b_r - (y - a_r)^2 - W_r^1(\lambda; a_r)]$$

the first order conditions,

$$\frac{d\Lambda^l}{dy} = -2(y - a_l) - \zeta^l 2(y - a_r) = 0$$

$$\frac{d\Lambda^l}{db_r} = \zeta^l - 1 = 0$$

$$\frac{d\Lambda^l}{d\zeta^l} = \chi\varphi_{rl} + b_r - (a_{lr} - a_r)^2 = 0$$

Solving yields,

$$y = a_{lr}, \qquad b_r = \frac{1}{4}(a_l - a_r)^2 - \chi\varphi_{rl}$$

By assumption, the stability factor is restricted such that, , $\frac{1}{4}(a_l - a_r)^2 \geq \chi\varphi_{rl}$ hence we ensure that $b_r \geq 0$. This yields the following payoff,

$$W_l(a_{lr}, b_r, a_l) = \chi\varphi_{lr} + B - \frac{1}{4}(a_l - a_r)^2 + \chi\varphi_{rl} - (a_{lr} - a_l)^2$$

$$W_l(a_{lr}, b_r, a_l) = 2\chi\varphi_{lr} + B - \frac{1}{2}(a_l - a_r)^2$$

Now consider party *l*'s proposal to party *m*, then party *l* solves,

$$\max \Lambda^l(y, b_m, \zeta^l) = \chi\varphi_{lm} + B - b_m - (y - a_l)^2 + \zeta^l[\chi\varphi_{lm} + b_m - (y - a_m)^2 - W_m^1(\lambda; a_m)]$$

The first order conditions,

$$\frac{d\Lambda^l}{dy} = -2(y - a_l) - \zeta^l 2(y - a_m) = 0$$

$$\frac{d\Lambda^l}{db_m} = \zeta^l - 1 = 0$$

$$\frac{d\Lambda^l}{d\zeta^l} = \chi\varphi_{lm} + b_m - (a_{lm} - a_m)^2 = 0$$

Solving,

$$y = a_{lm}, \qquad b_m = \frac{1}{4}(a_l - a_m)^2 - \chi\varphi_{lm}$$

By assumption, the stability factor is restricted such that, , $\frac{1}{4}(a_l - a_m)^2 \geq \chi\varphi_{lm}$ hence we ensure that $b_m \geq 0$. This yields the following payoff,

$$W_l(a_{lm}, b_m, a_l) = \chi\varphi_{lm} + B - \frac{1}{4}(a_l - a_m)^2 + \chi\varphi_{lm} - (a_{lm} - a_l)^2$$

$$W_l(a_{lm}, b_m, a_l) = 2\chi\varphi_{lm} + B - \frac{1}{2}(a_l - a_m)^2$$

Hence,

$$W_l(a_{lr}, b_r, a_l) \gtreqless W_l(a_{lm}, b_m, a_l) \Leftrightarrow$$

$$2\chi\varphi_{lr} + B - \frac{1}{2}(a_l - a_r)^2 \gtreqless 2\chi\varphi_{lm} + B - \frac{1}{2}(a_l - a_m)^2 \Leftrightarrow$$

$$2\chi\varphi_{lr} - \frac{1}{2}(2\ell)^2 \gtreqless 2\chi\varphi_{lm} - \frac{1}{2}(\ell)^2 \Leftrightarrow$$

$$2\chi\varphi_{lr} - 1\frac{1}{2}(\ell)^2 \gtreqless 2\chi\varphi_{lm} \Leftrightarrow$$

Given that, $\chi\varphi_{lm} > \chi\varphi_{lr} > \chi\varphi_{mr}$, it must be that $W_l(a_{lr}, b_r, a_l) < W_l(a_{lm}, b_m, a_l)$. Hence party *l*'s best response is to suggest a coalition with party *m,* by making proposal $(y^m, (b_m^l, B - b_m^l, 0))$, with $y^m = a_{lm}$and $b_m^l = \frac{1}{4}(a_l - a_r)^2 - \chi\varphi_{rl}$.

Now suppose path (AB), $\chi\varphi_{lm} > \frac{1}{4}(\ell)^2 - B$, then party *m*'s best proposal at t=2 is to suggest a coalition with party *l.* With, $y^m = a_{lm}$ and $b_l^m = 0$, then party *m*'s best proposal is $(y^m, (0, B, 0))$, keeping all perks to itself. The continuation values for t=1 are,

$$W_l^1(\lambda; a_l) = \quad 0$$

$$W_m^1(\lambda; a_m) = \chi\varphi_{lm} + B - \frac{1}{4}(a_m - a_l)^2$$

$$W_r^1(\lambda; a_r) = -(a_{lm} - a_r)^2$$

Now consider party *l*'s best proposal at t=1 and first consider party *l* proposes to party *r*, then party *l* solves,

$$\max \Lambda^l(y, b_r, \zeta^l) = \chi\varphi_{rl} + B - b_r - (y - a_l)^2 + \zeta^l[\chi\varphi_{rl} + b_r - (y - a_r)^2 - W_r^1(\lambda; a_r)]$$

the first order conditions,

$$\frac{d\Lambda^l}{dy} = -2(y - a_l) - \zeta^l 2(y - a_r) = 0$$

$$\frac{d\Lambda^l}{db_r} = \zeta^l - 1 = 0$$

$$\frac{d\Lambda^l}{d\zeta^l} = \chi\varphi_{rl} + b_r - (a_{lr} - a_r)^2 + (a_{lm} - a_r)^2 = 0$$

Solving yields,

$$y = a_{lr}, \qquad b_r = \frac{1}{4}(a_l - a_r)^2 - (a_{lm} - a_r)^2 - \chi\varphi_{rl} < 0$$

Given b_r must be positive, we assume party *l* offers $b_r = 0$ to party *r*, then the payoff yields,

$$W_l(a_{lr}, b_r, a_l) = \chi\varphi_{lr} + B - (a_{lr} - a_l)^2$$

$$W_l(a_{lr}, b_r, a_l) = \chi\varphi_{lr} + B - \frac{1}{4}(a_l - a_r)^2$$

Now consider party *l*'s proposal to party *m*, then party *l* solves,

$$\max \Lambda^l(y, b_m, \zeta^l) = \chi\varphi_{lm} + B - b_m - (y - a_l)^2 + \zeta^l[\chi\varphi_{lm} + b_m - (y - a_m)^2 - W_m^1(\lambda; a_m)]$$

the first order conditions,

$$\frac{d\Lambda^l}{dy} = -2(y - a_l) - \zeta^l 2(y - a_m) = 0$$

$$\frac{d\Lambda^l}{db_m} = \zeta^l - 1 = 0$$

$$\frac{d\Lambda^l}{d\zeta^l} = \chi\varphi_{lm} + b_m - \frac{1}{4}(a_l - a_m)^2 - \chi\varphi_{lm} - B + \frac{1}{4}(a_m - a_l)^2 = 0$$

Solving,

$$y = a_{lm}, \qquad b_m = B$$

Gives,

$$W_l(a_{lm}, b_m, a_l) = \chi\varphi_{lm} + B - b_m - (a_{lm} - a_l)^2$$

$$W_l(a_{lm}, b_m, a_l) = \chi\varphi_{lm} - \frac{1}{4}(a_l - a_m)^2$$

Hence we compare,

$$W_l(a_{lr}, b_r, a_l) \gtreqless W_l(a_{lm}, b_m, a_l) \Leftrightarrow$$

$$\chi\varphi_{lr} + B - \frac{1}{4}(a_l - a_r)^2 \gtreqless \chi\varphi_{lm} - \frac{1}{4}(a_l - a_m)^2 \Leftrightarrow$$

$$\chi\varphi_{lr} + B - \frac{1}{4}(2\ell)^2 \gtreqless \chi\varphi_{lm} - \frac{1}{4}(\ell)^2 \Leftrightarrow$$

$$\chi\varphi_{lr} \gtreqless \chi\varphi_{lm} - B + \frac{3}{4}(\ell)^2 \Leftrightarrow$$

hence, party *l*'s best proposal at t=1 is define by,

$$\begin{cases} if \quad \chi\varphi_{lr} < \chi\varphi_{lm} - B + \frac{3}{4}(\ell)^2 \qquad then, l\ propose\ to\ m\ (path\ ABA) \\ if \quad \chi\varphi_{lr} > \chi\varphi_{lm} - B + \frac{3}{4}(\ell)^2 \qquad then, l\ propose\ to\ r\ (path\ ABB) \end{cases}$$

Now suppose path (B), that is if $\chi\varphi_{lr} > \chi\varphi_{mr} + \frac{3}{4}(\ell)^2$, and consider party *r*'s best proposal at t=3 to party *l*. Then the continuation values are then,

$$W_l^2(\lambda; a_l) = \quad 0$$

$$W_m^2(\lambda; a_m) = -(a_{rl} - a_m)^2$$

$$W_r^2(\lambda; a_r) = 2\chi\varphi_{rl} + B - \frac{1}{2}(a_r - a_l)^2$$

Now consider party *m*'s t=2 proposal and suppose first that party *m* proposes to include party *l* as a coalition partner. Then party *m* solves the following Lagrangian maximization problem:

$$\max \Lambda^m(y, b_l, \zeta^m) = \chi\varphi_{lm} + B - b_l - (y - a_m)^2 + \zeta^m[\chi\varphi_{lm} + b_l - (y - a_l)^2]$$

the first order conditions,

$$\frac{d\Lambda^m}{dy} = -2(y - a_m) - \zeta^m 2(y - a_l) = 0$$

$$\frac{d\Lambda^l}{db_m} = \zeta^l - 1 = 0$$

$$\frac{d\Lambda^m}{d\zeta^m} = \chi\varphi_{lm} + b_l - \frac{1}{4}(a_m - a_l)^2 = 0$$

Solving yields,

$$y = a_{lm}, \qquad b_l = \frac{1}{4}(a_m - a_l)^2 - \chi\varphi_{lm}$$

By assumption, the stability factor is restricted such that, , $\frac{1}{4}(a_m - a_l)^2 \geq \chi\varphi_{lm}$ hence we ensure that $b_l \geq 0$. This yields the following payoff,

$$W_m(a_{lm}, b_l, a_m) = \chi\varphi_{lm} + B - b_l - (a_{lm} - a_m)^2$$

$$W_m(a_{lm}, b_l, a_m) = \chi\varphi_{lm} + B - \frac{1}{4}(a_m - a_l)^2 + \chi\varphi_{lm} - \frac{1}{4}(a_m - a_l)^2$$

$$W_m(a_{lm}, b_l, a_m) = 2\chi\varphi_{lm} + B - \frac{1}{2}(a_m - a_l)^2$$

Now suppose, if party m proposes to include party r as a coalition partner, then for the proposal to be accepted, party m must be offered at least $W_r^2(\lambda; a_r) = 2\chi\varphi_{rl} + B - \frac{1}{2}(a_r - a_l)^2$. Thus party m solves:

$$\max \Lambda^m(y, b_r, \zeta^m) = \chi\varphi_{mr} + B - b_r - (y - a_m)^2 + \zeta^m[\chi\varphi_{mr} + b_r - (y - a_r)^2 - W_r^2(\lambda; a_r)]$$

yielding the first order conditions,

$$\frac{d\Lambda^m}{dy} = -2(y - a_m) - \zeta^m 2(y - a_r) = 0$$

$$\frac{d\Lambda^l}{db_m} = \zeta^l - 1 = 0$$

$$\frac{d\Lambda^m}{d\zeta^m} = \chi\varphi_{mr} + b_r - \frac{1}{4}(a_m - a_r)^2 - 2\chi\varphi_{rl} - B + \frac{1}{2}(a_r - a_l)^2 = 0$$

solving,

$$y = a_{rm}, \qquad b_r = 2\chi\varphi_{rl} - \chi\varphi_{mr} + B + \frac{1}{4}(a_m - a_r)^2 - \frac{1}{2}(a_r - a_l)^2$$

$$y = a_{rm}, \qquad b_r = 2\chi\varphi_{rl} + B - \chi\varphi_{mr} - 1\frac{3}{4}(\ell)^2$$

Hence we restrict, $2\chi\varphi_{rl} + B > \chi\varphi_{mr} + 1\frac{3}{4}(\ell)^2$ for positive values of b_m. This yields the following payoff,

$$W_m(a_{mr}, b_r, a_m) = \chi\varphi_{mr} + B - b_r - \frac{1}{4}(a_r - a_m)^2$$

$$W_m(a_{mr}, b_r, a_m) = \chi\varphi_{mr} + B - 2\chi\varphi_{rl} - B + \chi\varphi_{mr} + 1\frac{3}{4}(\ell)^2 - \frac{1}{4}(a_r - a_m)^2$$

$$W_m(a_{mr}, b_r, a_m) = 2\chi\varphi_{mr} - 2\chi\varphi_{rl} + 1\frac{1}{2}(\ell)^2$$

Hence,

$$W_m(a_{lm}, b_l, a_m) \lesseqgtr W_m(a_{mr}, b_r, a_m) \iff$$

$$2\chi\varphi_{lm} + B - \frac{1}{2}(\ell)^2 \lesseqgtr 2\chi\varphi_{mr} - 2\chi\varphi_{rl} + 1\frac{1}{2}(\ell)^2 \Leftrightarrow$$

$$2\chi\varphi_{lm} + B - \frac{1}{2}(\ell)^2 \lesseqgtr 2\chi\varphi_{mr} - 2\chi\varphi_{rl} + 1\frac{1}{2}(\ell)^2 \Leftrightarrow$$

$$\chi\varphi_{rl} \lesseqgtr \chi\varphi_{mr} - \chi\varphi_{lm} + (\ell)^2 - \frac{1}{2}B \Leftrightarrow$$

hence, party m's best proposal at t=2 is define by,

$$\begin{cases} if \quad \chi\varphi_{rl} < \chi\varphi_{mr} - \chi\varphi_{lm} + (\ell)^2 - \frac{1}{2}B & then, m\ propose\ to\ r\ (path\ BA) \\ if \quad \chi\varphi_{rl} > \chi\varphi_{mr} - \chi\varphi_{lm} + (\ell)^2 - \frac{1}{2}B & then, m\ propose\ to\ l\ (path\ BB) \end{cases}$$

Let us first assume: $\chi\varphi_{rl} < \chi\varphi_{mr} - \chi\varphi_{lm} + (\ell)^2 - \frac{1}{2}B$, that is path (BA), and at t=2 party m proposes to party r, then the continuation values are,

$$W_l^1(\lambda; a_l) = -(a_{mr} - a_l)^2$$

$$W_m^1(\lambda; a_m) = 2\chi\varphi_{mr} - 2\chi\varphi_{rl} + 1\frac{1}{2}(\ell)^2$$

$$W_r^1(\lambda; a_r) = 0$$

First consider party l proposes to party r, then party l solves,

$$\max \Lambda^l(y, b_r, \zeta^l) = \chi\varphi_{rl} + B - b_r - (y - a_l)^2 + \zeta^l[\chi\varphi_{rl} + b_r - (y - a_r)^2 - W_r^1(\lambda; a_r)]$$

Yielding the first order conditions,

$$\frac{d\Lambda^l}{dy} = -2(y - a_l) - \zeta^l 2(y - a_r) = 0$$

$$\frac{d\Lambda^l}{db_r} = \zeta^l - 1 = 0$$

$$\frac{d\Lambda^l}{d\zeta^l} = \chi\varphi_{rl} + b_r - (y - a_r)^2 = 0$$

Solving yields,

$$y = a_{lr}, \qquad b_r = \frac{1}{4}(a_l - a_r)^2 - \chi\varphi_{rl}$$

Hence we restrict, $\frac{1}{4}(a_l - a_r)^2 > \chi\varphi_{lr}$ for positive values of b_r. This yields the following payoff,

$$W_l(a_{lr}, b_r, a_l) = \chi\varphi_{lr} + B - \frac{1}{4}(a_l - a_r)^2 + \chi\varphi_{rl} - \frac{1}{4}(a_l - a_r)^2$$

$$W_l(a_{lr}, b_r, a_l) = 2\chi\varphi_{lr} + B - \frac{1}{2}(a_l - a_r)^2$$

Now consider party *l*'s proposal to party *m,* then party *l* solves,

$$\max \Lambda^l(y, b_m, \zeta^l) = \chi\varphi_{lm} + B - b_m - (y - a_l)^2 + \zeta^l[\chi\varphi_{lm} + b_m - (y - a_m)^2 - W_m^1(\lambda; a_m)]$$

Yielding the first order conditions,

$$\frac{d\Lambda^l}{dy} = -2(y - a_l) - \zeta^l 2(y - a_m) = 0$$

$$\frac{d\Lambda^l}{db_m} = \zeta^l - 1 = 0$$

$$\frac{d\Lambda^l}{d\zeta^l} = \chi\varphi_{lm} + b_m - \frac{1}{4}(\ell)^2 - 2\chi\varphi_{mr} + 2\chi\varphi_{rl} - 1\frac{1}{2}(\ell)^2 = 0$$

Solving,

$$y = a_{lm}, \qquad b_m = 2\chi\varphi_{mr} - 2\chi\varphi_{rl} - \chi\varphi_{lm} + 1\frac{3}{4}(\ell)^2$$

Hence we restrict, $2\chi\varphi_{mr} + 1\frac{3}{4}(\ell)^2 > 2\chi\varphi_{rl} + \chi\varphi_{lm}$ for positive values of b_l. This yields the following payoff,

$$W_l(a_{lm}, b_m, a_l) = \chi\varphi_{lm} + B - b_m - (a_{lm} - a_l)^2$$

$$W_l(a_{lm}, b_m, a_l) = \chi\varphi_{lm} + B - 2\chi\varphi_{mr} + 2\chi\varphi_{rl} + \chi\varphi_{lm} - 1\frac{3}{4}(\ell)^2 - \frac{1}{4}(\ell)^2$$

$$W_l(a_{lm}, b_m, a_l) = 2\chi\varphi_{lm} + 2\chi\varphi_{rl} - 2\chi\varphi_{mr} + B - 2(\ell)^2$$

Hence,

$$W_l(a_{lr}, b_r, a_l) \gtreqless W_l(a_{lm}, b_m, a_l) \Leftrightarrow$$

$$2\chi\varphi_{lr} + B - \frac{1}{2}(2\ell)^2 \gtreqless 2\chi\varphi_{lm} + 2\chi\varphi_{rl} - 2\chi\varphi_{mr} + B - 2(\ell)^2 \Leftrightarrow$$

$$2\chi\varphi_{lr} + B - 2(\ell)^2 \gtreqless 2\chi\varphi_{lm} + 2\chi\varphi_{rl} - 2\chi\varphi_{mr} + B - 2(\ell)^2 \Leftrightarrow$$

$$\chi\varphi_{lr} \gtreqless \chi\varphi_{lm} + \chi\varphi_{rl} - \chi\varphi_{mr} \Leftrightarrow$$

$$\chi\varphi_{mr} \gtreqless \chi\varphi_{lm} \Leftrightarrow$$

By assumption, $\chi\varphi_{lm} > \chi\varphi_{lr} > \chi\varphi_{mr}$, hence it must be that, $W_l(a_{lm}, b_m, a_l) >$ *Wlalr,br,al* and party *l*'s best proposal at t=1 is to suggest a coalition with party *m,* by making proposal $(y^m, (b^l_m, B - b^l_m, 0))$, with $y^m = a_{lm}$and $b^l_m = 2\chi\varphi_{mr} - 2\chi\varphi_{rl} - \chi\varphi_{lm} + 1\frac{3}{4}(\ell)^2$.

Now assume path (BB), that is if $\chi\varphi_{rl} > \chi\varphi_{mr} - \chi\varphi_{lm} + (\ell)^2 - \frac{1}{2}B$, and consider at t=2 party *m* proposes to party *l*, then the continuation values are,

$$W^1_l(\lambda; a_l) = 0$$

$$W^1_m(\lambda; a_m) = 2\chi\varphi_{lm} + B - \frac{1}{2}(\ell)^2$$

$$W^1_r(\lambda; a_r) = -(a_{ml} - a_r)^2$$

Consider if party *l* proposes to party *r*, then party *l* solves,

$$\max \Lambda^l(y, b_r, \zeta^l) = \chi\varphi_{rl} + B - b_r - (y - a_l)^2 + \zeta^l[\chi\varphi_{rl} + b_r - (y - a_r)^2 - W^1_r(\lambda; a_r)]$$

The first order conditions,

$$\frac{d\Lambda^l}{dy} = -2(y - a_l) - \zeta^l 2(y - a_r) = 0$$

$$\frac{d\Lambda^l}{db_r} = \zeta^l - 1 = 0$$

$$\frac{d\Lambda^l}{d\zeta^l} = \chi\varphi_{rl} + b_r - (y - a_r)^2 + (a_{ml} - a_r)^2 = 0$$

Solving yields,

$$y = a_{lr},$$

$$b_r = \frac{1}{4}(2\ell)^2 - \chi\varphi_{rl} - \frac{9}{4}(\ell)^2 < 0$$

$$b_r = \frac{8}{4}(\ell)^2 - \chi\varphi_{rl} - \frac{9}{4}(\ell)^2 < 0$$

Because b_r must be positive, *l* offers $b_r = 0$ to *r*, then the payoff yields,

$$W_l(a_{lr}, b_r, a_l) = \chi\varphi_{lr} + B - \frac{1}{4}(a_l - a_r)^2$$

$$W_l(a_{lr}, b_r, a_l) = \chi\varphi_{lr} + B - (\ell)^2$$

Now consider party *l*'s proposal to party *m,* then party *l* solves,

$$\max \Lambda^l(y, b_m, \zeta^l) = \chi\varphi_{lm} + B - b_m - (y - a_l)^2 + \zeta^l[\chi\varphi_{lm} + b_m - (y - a_m)^2 - W_m^1(\lambda; a_m)]$$

The first order conditions,

$$\frac{d\Lambda^l}{dy} = -2(y - a_l) - \zeta^l 2(y - a_m) = 0$$

$$\frac{d\Lambda^l}{db_m} = \zeta^l - 1 = 0$$

$$\frac{d\Lambda^l}{d\zeta^l} = \chi\varphi_{lm} + b_m - \frac{1}{4}(a_l - a_m)^2 - 2\chi\varphi_{lm} - B + \frac{1}{2}(\ell)^2 = 0$$

Solving,

$$y = a_{lm}, \qquad b_m = \chi\varphi_{lm} + B - \frac{1}{4}(\ell)^2$$

Hence we restrict, $\chi\varphi_{lm} > \frac{1}{4}(\ell)^2 - B$ for positive values of b_m. This yields the following payoff,

$$W_l(a_{lm}, b_m, a_l) = \chi\varphi_{lm} + B - b_m - (a_{lm} - a_l)^2$$

$$W_l(a_{lm}, b_m, a_l) = \chi\varphi_{lm} + B - \chi\varphi_{lm} - B + \frac{1}{4}(\ell)^2 - \frac{1}{4}(a_m - a_l)^2$$

$$W_l(a_{lm}, b_m, a_l) = 0$$

Hence,

$$W_l(a_{lr}, b_r, a_l) \gtreqless W_l(a_{lm}, b_m, a_l) \Leftrightarrow$$

$$\chi\varphi_{lr} + B - (\ell)^2 \lesseqgtr 0 \Leftrightarrow$$

$$\chi\varphi_{lr} \gtreqless (\ell)^2 + B$$

Hence,

$$\begin{cases} if \quad \chi\varphi_{lr} > (\ell)^2 + B \quad then, l\, propose\ to\ r \quad (path\ BBA) \\ if \quad \chi\varphi_{lr} < (\ell)^2 + B \quad then, l\, propose\ to\ m\ (path\ BBB) \end{cases}$$

Diagram 2: Lemma 1.1, case (2)

R

$$W_l^3(\lambda; a_l) = 0$$
$$W_m^3(\lambda; a_m) = 0$$
$$W_r^3(\lambda; a_r) = 0$$

A: $\chi\varphi_{lr} < \chi\varphi_{mr} + \frac{3}{4}(\ell)^2$

M

$$W_l^2(\lambda; a_l) = -(a_{rm} - a_l)^2$$
$$W_m^2(\lambda; a_m) = 0$$
$$W_r^2(\lambda; a_r) = 2\chi\varphi_{rm} + B - \frac{1}{2}(a_r - a_m)^2$$

A: $\chi\varphi_{lm} < \frac{1}{4}(\ell)^2 - B$

L

$$W_l^1(\lambda; a_l) = -(a_{mr} - a_l)^2$$
$$W_m^1(\lambda; a_m) = 0$$
$$W_r^1(\lambda; a_r) = 0$$

2a → *coalition LM*

B: $\chi\varphi_{lm} > \frac{1}{4}(\ell)^2 - B$

L

$$W_l^1(\lambda; a_l) = 0$$
$$W_m^1(\lambda; a_m) = \chi\varphi_{lm} + B - \frac{1}{4}(a_m - a_l)^2$$
$$W_r^1(\lambda; a_r) = -(a_{lm} - a_r)^2$$

2b →

A $\begin{cases} \text{if} \quad \chi\varphi_{lr} < \chi\varphi_{lm} - B + \frac{3}{4}(\ell)^2 & \text{then, coalition } LM \end{cases}$

B $\begin{cases} \text{if} \quad \chi\varsigma B > \chi\varphi_{lm} - B + \frac{3}{4}(\ell)^2 & \text{then, coalition } LR \end{cases}$

B: $\chi\varphi_{lr} > \chi\varphi_{mr} + \frac{3}{4}(\ell)^2$

M

$$W_l^2(\lambda; a_l) = 0$$
$$W_m^2(\lambda; a_m) = -(a_{rl} - a_m)^2$$
$$W_r^2(\lambda; a_r) = 2\chi\varphi_{rl} + B - \frac{1}{2}(a_r - a_l)^2$$

A: $\chi\varphi_{rl} < \chi\varphi_{mr} - \chi\varphi_{lm} + (\ell)^2 - \frac{1}{2}B$

L

$$W_l^1(\lambda; a_l) = -(a_{mr} - a_l)^2$$
$$W_m^1(\lambda; a_m) = 2\chi\varphi_{mr} - 2\chi\varphi_{rl} + 1\tfrac{1}{2}(\ell)^2$$
$$W_r^1(\lambda; a_r) = 0$$

2c → *coalition LM*

B: $\chi\varphi_{rl} > \chi\varphi_{mr} - \chi\varphi_{lm} + (\ell)^2 - \frac{1}{2}B$

L

$$W_l^1(\lambda; a_l) = 0$$
$$W_m^1(\lambda; a_m) = 2\chi\varphi_{lm} + B - \frac{1}{2}(\ell)^2$$
$$W_r^1(\lambda; a_r) = -(a_{ml} - a_r)^2$$

2d →

A $\begin{cases} \text{if} \quad \chi\varphi_{lr} > (\ell)^2 + B & \text{then, } l \text{ propose to } r \end{cases}$

B $\begin{cases} \text{if} \quad \chi\varphi_{lr} < (\ell)^2 + B & \text{then, } l \text{ propose to } m \end{cases}$

Now we computing proposals, $\left(z^*, (b_c^*, b_{-c}^*)\right)$ under case (2), $w_l > w_m > w_r$, for all possible coalitions that do not consist of the largest and smallest parties in legislature. For an overview please see diagram 2.1 in this appendix

Path *AA*:

$$If \begin{cases} \chi\varphi_{lr} < \chi\varphi_{mr} + \frac{3}{4}(\ell)^2 \\ \chi\varphi_{lm} < \frac{1}{4}(\ell)^2 - B \end{cases}$$

$$then\ z^* = a_{lm}, b_l^* = B - b_m^*, b_m^* = \frac{1}{4}(\ell)^2 - \chi\varphi_{lm}\ and\ b_r^* = 0$$

Path *ABA*

$$If \begin{cases} \chi\varphi_{lr} < \chi\varphi_{mr} + \frac{3}{4}(\ell)^2 & (1) \\ \chi\varphi_{lm} > \frac{1}{4}(\ell)^2 - B & (2) \\ \chi\varphi_{lr} \leq \chi\varphi_{lm} - B + \frac{3}{4}(\ell)^2 & (3) \end{cases}$$

$$\chi\varphi_{lr} \leq \chi\varphi_{lm} - B + \frac{3}{4}(\ell)^2 \qquad (3)$$

$$\chi\varphi_{lm} = \chi\varphi_{lr} + B - \frac{3}{4}(\ell)^2 \qquad (3)$$

Substituting (3) into (2),

$$\chi\varphi_{lm} > \frac{1}{4}(\ell)^2 - B \qquad (2)$$

$$\chi\varphi_{lr} + B - \frac{3}{4}(\ell)^2 > \frac{1}{4}(\ell)^2 - B$$

$$\chi\varphi_{lr} > (\ell)^2 - 2B$$

Hence if,

$$\chi\varphi_{mr} + \frac{3}{4}(\ell)^2 > \chi\varphi_{lr} > (\ell)^2 - 2B$$

$then\ z^* = a_{lm}, b_l^* = B - b_m^*, b_m^* = \ B\ and\ b_r^* = 0$

$if\ not,\ then\ z^* = a_{lr}, b_l^* = B - b_r^*, b_m^* = \ 0\ and\ b_r^* = 0$

Path *BA*

$$If \begin{cases} \chi\varphi_{lr} > \ \chi\varphi_{mr} + \frac{3}{4}(\ell)^2 \\ \chi\varphi_{lr} < \ \chi\varphi_{mr} - \chi\varphi_{lm} + (\ell)^2 - \frac{1}{2}B \end{cases}$$

Hence,

$$\chi\varphi_{mr} + \frac{3}{4}(\ell)^2 < \chi\varphi_{lr} < \chi\varphi_{mr} - \chi\varphi_{lm} + (\ell)^2 - B\ then$$

$$z^* = a_{lm}, b_l^* = B - b_m^*, b_m^* = 2\chi\varphi_{mr} - 2\chi\varphi_{lr} - \chi\varphi_{lm} + 1\frac{3}{4}(\ell)^2 and\ b_r^* = 0$$

Path *BBB*

$$If \begin{cases} \chi\varphi_{lr} > \ \chi\varphi_{mr} + \frac{3}{4}(\ell)^2 & (1) \\ \chi\varphi_{lr} > \ \chi\varphi_{mr} - \chi\varphi_{lm} + (\ell)^2 - \frac{1}{2}B & (2) \\ \chi\varphi_{lr} \le \ (\ell)^2 + B & (3) \end{cases}$$

Substitute (3) into (2)

$$\chi\varphi_{lr} = \ (\ell)^2 + B$$

$$(\ell)^2 + B \quad > \ \chi\varphi_{mr} - \chi\varphi_{lm} + (\ell)^2 - \frac{1}{2}B$$

$$\chi\varphi_{lm} + 1\frac{1}{2}B \quad > \ \chi\varphi_{mr}$$

Substituting (3) into (1)

$$\chi\varphi_{lr} = \ (\ell)^2 + B$$

$$\frac{1}{4}(\ell)^2 + B \quad > \ \chi\varphi_{mr}$$

Hence if ,

$$\chi\varphi_{mr} < \frac{1}{4}(\ell)^2 + B \text{ and } \chi\varphi_{mr} < \chi\varphi_{lm} + 1\frac{1}{2}B$$

$$\text{then } z^* = a_{lm}, b_l^* = B - b_m^*, b_m^* = \chi\varphi_{lm} + B - \frac{1}{4}(\ell)^2 \text{ and } b_r^* = 0$$

$$\text{if not, then } z^* = a_{lr}, b_l^* = B - b_r^*, b_m^* = 0 \text{ and } b_r^* = 0$$

Diagram 1: lemma 1.1, case (3)

M	
	$W_l^3(\lambda; a_l) = 0$ $W_m^3(\lambda; a_m) = 0$ $W_r^3(\lambda; a_r) = 0$

$\longrightarrow$

R	
	$W_l^2(\lambda; a_l) = \quad 0$ $W_m^2(\lambda; a_m) = 2\chi\varphi_{ml} + B - \frac{1}{4}(a_l - a_m)^2$ $W_r^2(\lambda; a_r) = -(a_{lm} - a_r)^2$

A $\nearrow$ $\chi\varphi_{rl} < \chi\varphi_{rm} - \chi\varphi_{ml} - \frac{1}{2}B + \frac{7}{8}(\ell)^2$

L	
	$W_l^1(\lambda; a_l) = -(a_{rm} - a_l)^2$ $W_m^1(\lambda; a_m) = 0$ $W_r^1(\lambda; a_r) = +$

$$\begin{cases} if \quad \chi\varphi_{rm} + \frac{5}{8}(\ell)^2 < \chi\varphi_{lr} & then, l\ propose\ to\ r\ (AA) \\ if \quad \chi\varphi_{rm} + \frac{5}{8}(\ell)^2 > \chi\varphi_{lr} & then, l\ propose\ to\ m\ (AB) \end{cases}$$

B $\searrow$ $\chi\varphi_{rl} > \chi\varphi_{rm} - \chi\varphi_{ml} - \frac{1}{2}B + \frac{7}{8}(\ell)^2$

L	
	$W_l^1(\lambda; a_l) = 0$ $W_m^1(\lambda; a_m) = -(a_{lr} - a_m)^2$ $W_r^1(\lambda; a_r) = +$

$$\begin{cases} if \quad \chi\varphi_{lr} > 2\chi\varphi_{lm} + B + \frac{1}{2}(\ell)^2 & then, l\ propose\ to\ r\ (BA) \\ if \quad \chi\varphi_{lr} < 2\chi\varphi_{lm} + B + \frac{1}{2}(\ell)^2 & then, l\ propose\ to\ m\ (BB) \end{cases}$$

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