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Portfolio Optimization using Alternative Investments

-Fact or Fiction? Using Farmland and Timberland to Improve the Risk-Return Relationship of a Traditional Portfolio-

Master Thesis

Ashwin Shantiprekash - 325572

Thesis supervisor: Laurens Swinkels

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Preface and Acknowledgements

What seemed to be an unbridgeable final chapter of my study at the Erasmus University Rotterdam turned into a most interesting and defying research on the analysis of risk and optimization. The end of this thesis never felt close due to almost infinite possibilities of expanding the research. During the course of time however, the pieces of an overwhelming puzzle slowly came together and the thesis started to take shape. First, it must be mentioned that this project would not have become the Master Thesis that I am proud to present without the effort of my thesis supervisor, Laurens Swinkels. He has helped me gain insights in different aspects of economics, finance and financial modeling. I sincerely appreciate all the help and thought provoking feedback that he provided. Furthermore, I want to thank [NAME] (in advance) for reading the thesis and preparing questions for the defense.

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In loving memory of Frans Leeghburch Auwers, father of two dear friends Aram and Nathan.

"*Thanks to God who has made all things possible*" (Mark 10:27)

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Abstract

Over the last two decades, investors have expanded their vision from common stock and bond markets to alternative investments, including farmland and timberland. The aim of this thesis is to provide an overview of the farm and timber asset classes and to explore the effects that farm and timber investments may have on a well diversified portfolio using modern portfolio theory. Using historical return data for various asset classes, a theoretical investment universe was created to assess the impact that farmland and timberland assets have on the efficient frontier. The results clearly reveal that only timberland assets do have an impact on the efficient frontier, which is shown graphically and through mean-variance spanning tests. The results also show that, in times of heavy turmoil, it does not matter to add farmland or timberland to the portfolio in terms of return.

Keywords: Finance, Asset Management, Risk, Correlation, Inflation, Portfolio Optimization, Efficient Frontier Analysis, Mean-Variance Spanning

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Introduction

Due to a range of attractive performance characteristics and diversification opportunities, alternative investments have grown significantly in the last 25 years. Investors are exploring the diversification benefits of including alternative investments like farmland and timberland in the traditional portfolio. This increase of the allocation to alternative investments is the result of more than two decades of increasing liquidity and low interest rates in the major developed economies around the world. Under such economic circumstances it can be difficult for investors to achieve required return targets. Due to this reason investors turn to alternative investment classes (alternative to publicly traded securities), in their quest for higher yield. Another important factor behind the search for alternative investments is globalization. This force has been responsible for a greater interconnection between the traditional financial markets, such as stock and bond markets, around the world. News and important events occurring in today's world have a greater impact on stocks and bonds globally than ever before. Therefore, in order to minimize risk and fully diversify portfolios investors have to look beyond the realm of the traditional stock and bond markets.

People invest their money with the purpose to provide for future consumption and they therefore search for the best investments. Investing is a tradeoff between risk and expected return. In general the rule holds that assets with higher expected returns are riskier. A concept that contributes to a more comprehensive view on the subject is Modern Portfolio Theory (MPT). Markowitz (1952) has had a significant influence in the further development of MPT and on the practice of investment decision making. For a given amount of risk, MPT describes how to select a portfolio with the highest possible expected return. Vice versa, for a given expected return, MPT explains how to select a portfolio with the lowest possible risk. This concept will also form the basis for this thesis.

The main stream of academic literature in general takes a closer look at common stock portfolios, these portfolios purely contain financial assets. As alluded above, instead of only taking financial assets into account, investors started mixing non-conventional assets (or alternative investments) in their portfolios in order to increase returns and reduce risk. There is no formal definition of alternative investments however, it generally encompasses investments in most or all asset classes that are not traded in traditional, liquid markets, such as the stock and bond market. The spectrum of alternative investments is therefore a very broad one.

The total alternative investment universe can be divided into two general groups: financial instruments and real assets (Martin 2010). A financial instrument is a tradable asset of any kind, either cash, evidence of an ownership interest in an entity, or a contractual right to receive or deliver cash or another financial instrument[1]. Alternative financial instruments include for example hedge funds and private equity funds. Real assets include natural recourses such as precious metals, mines, oil fields, as well as real estate and other physical assets, including farmland and timberland, on which the remainder of this thesis will focus.

There are several reasons for specifically taking these two assets into account. One of the reasons is that farmland and timberland have attractive characteristics in the sense that they, for example, have a relatively high return compared to their risk and a low or even negative correlation with other traditional asset classes according to Kaplan (1985) and Scholtens & Spierdijk (2010). Corporate social responsibility is currently a much-discussed subject and is a reason beyond the scope of financial gains. Investors who are inclined toward socially responsible and environmentally friendly investing will find much gain in farmland and timberland. For example, trees replenish the oxygen level in the atmosphere by using greenhouse gases in order to grow. A forest is a sustainable natural resource, of course only if the forest is properly managed. Holding and maintaining (physical) farmland meets the need of the worlds demand for food and with famine in different parts of the world this can also appeal to the social responsibility of investors. It should be noted however that when there is an increase of investments in farmland the price of farmland appreciates. This in turn implies that food can become more expensive which has a counteractive effect, namely, famine increases. In order to make an appeal to the social responsibility of investors regarding famine the food prices should not rise.

The goal of this thesis is to examine the impact of adding farmland and/or timberland to a traditional financial portfolio and, determine which one adds the most value to the risk-return relation of a financial portfolio. In order to realize this goal I am going to examine how the equity sector and real estate asset class in the United States have been used before to enhance the risk-return characteristics of a portfolio. After this, using the observations made, I am going to apply the successful methods to farmland and timberland. A large amount of literature can be found on how real estate and the equity sector can be used as a portfolio investment. However, there is a rather limited supply of literature written on how these methods can be applied on timberland and farmland. This is where the added value lies within the academic literature. The problem statement (main hypothesis) of this thesis is therefore:

Can the risk-return relationship of a portfolio be enhanced by adding farmland and/or timberland to a traditional portfolio?

[1] According to International Accounting Standards (IAS) 32 and 39 a financial instrument is defined as: "any contract that gives rise to a financial asset of one entity and a financial liability or equity instrument of another entity".

The following topics will be covered to provide a step-by-step approach in order to test the main hypothesis. The first chapter will elaborate on key papers that relate to the research done in this thesis. Modern portfolio theory will be discussed to form a basic knowledge for the following chapters. After this follows a discussion of literature regarding real estate investments, benefits of traditional and new real assets, (inflation) hedging benefits using alternative investments, and using farmland/timberland as a portfolio investment.

The second chapter will cover the models used in this thesis and provide a description of the dataset. This information contributes to a greater understanding of figures, tables and interpretation of results to come in the next chapter. In addition to this, limitations of the research will be discussed and an overview will be provided regarding the descriptive statistics of the asset classes used in this thesis.

Results of the empirical research are provided in chapter three. This chapter will contain a thorough analysis of the performance characteristics of farmland and timberland compared to other assets in the predetermined investment universe. Then the correlations between the various asset classes and Sharpe ratio will be discussed. After this, the efficient frontier will be constructed and it will show the effect on the traditional portfolio, the effect will be analyzed though mean-variance spanning tests. Furthermore, a discussion will be provided on how the portfolio behaves in worst-case scenarios when farmland and timberland are added. Finally, an assessment will be made on using farmland and timberland as an inflation hedge and the recommendations for further research will be discussed.

In closing there will be a brief summary of the results.

1. Literature Review

Over the course of time, many papers were written that can be linked somehow to the research done in this thesis. In order to bring structure to this overwhelming quantity of literature, this chapter will be divided in three sections. The three sections will all shed light on the subject, each from a different angle. The first section will discuss the basic concept of modern portfolio theory which essentially lays down the foundation of this thesis. After this, papers regarding the diversification opportunities of real estate will be described. Finally, papers regarding the diversification benefits of farmland and timberland will briefly be discussed. The studies mainly examined the characteristics of these asset classes that can be utilized for the purpose of portfolio optimization.

1.1 Modern Portfolio Theory

Since the pioneering work by Markowitz (1952), Modern Portfolio Theory (MPT) has exerted a profound influence on both the theory and practice of investment decision making. The Markowitz portfolio theory was developed based on the hypothesis that the mean and variance of the probability distributions are sufficient statistics for decision making. Fabbozi et al., (2002) conclude that initially, MPT generated relatively little interest, but with time, the financial community strongly adopted the thesis of Markowitz. Now, more than 50 years later, financial models based on those same principles are constantly being reinvented to incorporate all the new findings that result from seminal work. Until the 1990s, research studies using modern portfolio theory have been confined largely to investments in financial assets, especially common stock. In the last two decades the less-than-robust performance of common stock in terms of both mean-variance efficiency and inflation hedging led to the suggestion that portfolio efficiency may be improved by diversifying across different classes of investments. That is, mixed-asset diversification across both financial and real assets such as real estate, precious metals, farmland, timberland and similar assets.

Markowitz's theory of portfolio selection is a normative theory. A normative theory is one that describes a standard norm of behavior that investors should pursue in constructing a portfolio, in contrast to a theory that is actually followed. Asset pricing theory such as the Capital Asset Pricing Model (CAPM) goes on to formalize the relationship that should exist between asset returns and risk if investors constructed and selected portfolios according to mean-variance analysis. In contrast to a normative theory, asset pricing theory is a positive theory—a theory that hypothesizes how investors behave rather than how investors should behave. Based on that hypothesized behavior of investors, a model that provides the expected return (a key input into constructing portfolios based on mean-variance analysis) is derived and is called an asset pricing model. A framework is provided by the combination of both MPT and asset pricing theory to specify and measure investment risk and to develop relationships between expected asset return and risk.

Diversification has become the Holy Grail in finance. Old wisdom has always dictated not putting all your eggs in one basket. In somewhat more technical terms, this adage is addressing the benefits of diversification. Modern portfolio theory has quantified this concept of diversification by introducing the statistical notion of a covariance or correlation. When projecting the adage in finance, it states in essence that putting all your money in investments that may default at the same time, i.e., whose returns are highly correlated, is not a very wise investment strategy. The concept of diversification is very intuitive and is so strong that is has been continually applied to different aspects in finance. Numerous innovations within the financial world have either been an application of the concept of diversification, or the introduction of new models obtaining improved estimates of the variances and covariances, thereby allowing for a more precise measure of diversification and risk.

Chow et al. (1999) collapses on the Markowitz mean-variance model, they focus especially on experiences with emerging market investments and hedge funds. They address the statement that the risk parameters are unstable in times of crisis and introduced a procedure for identifying multivariate outliers and using them to estimate a new covariance matrix. The suggestion is that a covariance matrix estimated from outliers characterizes a portfolio's riskiness during market turbulence better than a full-sample covariance matrix. The empirical results yielded from the research support the view that volatility and correlations estimated from outliers differ significantly from full-sample estimates. The volatility levels of constructed optimal portfolios estimated from the full-sample covariance matrix nearly doubled when the portfolios where subjected to the outlier-sample covariance matrices. In addition to this, the outlier-sample covariance matrices produced much more conservative optimal mixes than the full-sample matrices but with a lower expected return.

Although the theory behind MPT is relatively straightforward, its implementation can get quite complicated. The theory dictates that given estimates of the returns, volatilities, and correlations of a set of investments and constraints on investment choices (for example, maximum exposures and turnover constraints), it is possible to perform an optimization that results in the mean-variance efficient frontier. This frontier is efficient because underlying every point on this frontier is a portfolio that results in the greatest possible expected return for that level of risk or results in the smallest possible risk for that level of expected return. The portfolios that lie on the frontier make up the set of efficient portfolios. The efficient frontier will also be constructed in section 3.6 of this thesis.

One of the most direct and widely used applications of MPT is asset allocation. According to Chamberlain (1983) a well diversified portfolio, one where asset allocation is properly applied, contains only factor variance, or systematic risk. This type of risk cannot be mitigated through diversification. If the portfolio is not well diversified, it contains unsystematic risk which can be mitigated through diversification. Because the asset allocation decision is so important, almost all asset managers and financial advisors determine an optimal portfolio for their clients by performing an asset allocation analysis using a set of asset classes. Brinson et al. (1986) also provide evidence that leads to the conclusion that asset allocation is a major determinant of portfolio performance. They begin by selecting a set of asset classes (including domestic large cap and small cap stocks, long-term bonds and international stocks). To obtain estimates of the returns, volatilities and correlations, they generally start with the historical performance of the indices representing these asset classes. These estimates are used as inputs in the mean-variance optimization which results in an efficient frontier. Then, using some criterion (for instance, using Monte Carlo simulations to compute the wealth distributions of the candidate portfolios), they pick an optimal portfolio. Finally, this portfolio is implemented using either index or actively managed funds.

1.2 Real Estate as a Diversification Instrument

The gains from mixed-asset diversifications have been analyzed in the context of modern portfolio theory by a series of studies. Webb et al., (1988) investigate the diversification gains from including real estate in mixed-asset portfolios. In order to assess these gains they examined the historical returns on various investment media and estimated potential risk reduction due to mixed-asset diversification. After this, they constructed the optimal mixed-asset portfolio and measured the magnitude of the potential gains in mean-variance efficiency from mixed-asset diversification. The investment media considered include six purely financial investments: New York Stock Exchange (NYSE) common stock, over-the-counter (OTC) common stock, long-term corporate bonds, commercial paper, treasury bills, and long-term municipal bonds. Also included are three real assets, all sub-classifications of the real estate media alluded above: residential housing, farmland, and commercial real estate. The most significant results of their research are threefold:

1. Two-thirds of the investment wealth should be allocated to real estate and one-third to financial assets, this distribution then creates the optimal mixed-asset portfolio.
2. All of the real estate media (residential housing, farmland, and commercial real estate) were included in the optimal mixed-asset portfolio while only three of the financial media (OTC stock, T-bills, and commercial paper) were included.

3. The efficient set of the mixed-asset portfolio dominated the efficient set from the purely financial asset portfolio, thus implying that purely financial diversification produces inefficient portfolios.

The most significant implication that should be noted is that the analyses were conducted on before-tax returns. While the implications would not change for specific institutional investors who are exempt from taxation, such as pension funds, they may vary for other investors depending on their tax status.

An important study has been made by Friedman (1971) regarding real estate investment and portfolio theory. His study intended to show that, using reasonable assumptions, the mathematical models used to select and evaluate common stock portfolios can also be used for the selection and evaluation of real estate portfolios. He also shows that these same models can also be used to choose portfolios containing both real estate and common stock. Two similar models were used for the empirical tests performed. For identifying real estate or common stock efficient frontiers, the Sharpe diagonal model was used. To identify portfolios consisting of both assets, the Cohen & Pogue (1967) multi-index model was used. The latter is a modification of the Sharpe model, in which different indexes are used for different asset classes. The purpose of the research by Cohen & Pogue was to empirically evaluate the performances of a number of single-period portfolio selection models based on the Markowitz formulation but representing successive steps toward simpler models which pose fewer problems of data preparation and computation[2]. When Markowitz introduced his work, the primary hypothesis was that investors want to maximize return for a given level of risk or minimize risk for a given level of return. This means that at each level of return or risk only one set of stocks would satisfy the constraints, these portfolios are denoted to be 'efficient'. It should be noted however, that one efficient portfolio is not clearly better than any other efficient portfolio. The investor's risk-return preference function determines which portfolio is selected. The portfolios identified using a small sample indicate that real estate portfolios can have more return and less risk than common stock portfolios. When the two assets are combined, the portfolios including real estate dominate the resultant portfolios. On an after-tax basis these results are more apparent. Friedman has shown in his paper that the models developed to select common stock portfolios can be adapted to the selection of real estate portfolios and mixed-asset portfolios. The concepts are all identical, and as long as the risk and return can be quantified, the problems are negligible.

[2] These simpler models represent the covariance relationships between individual securities and one or more indexes of industry or market performance.

1.3 Diversification Gains from Including Farmland and Timberland in a Portfolio

Increasingly over the past five years, and particularly in today's environment, investors have a renewed interest in the risk and return properties of "real assets". Martin (2010) investigated the long-horizon benefits of traditional and new real assets in the institutional portfolio. The key focus of his paper was the relationship between asset returns and inflation. The first step he takes is to assess the specific characteristics of the group of assets that comprise real assets. Investors are increasingly seeking real asset investments that have more than one of the properties listed below. They pursue returns that:

- Are positively correlated with the United States or European price inflation.
- Preserve value during periods of financial market contagion or substantial changes in the economic environment.
- Benefit directly from the increasing scarcity of production inputs, particularly in core economic sectors such as energy, manufacturing, and agriculture.
- Offer long-term risk and return properties suitable for investors seeking to fund long-term liabilities.

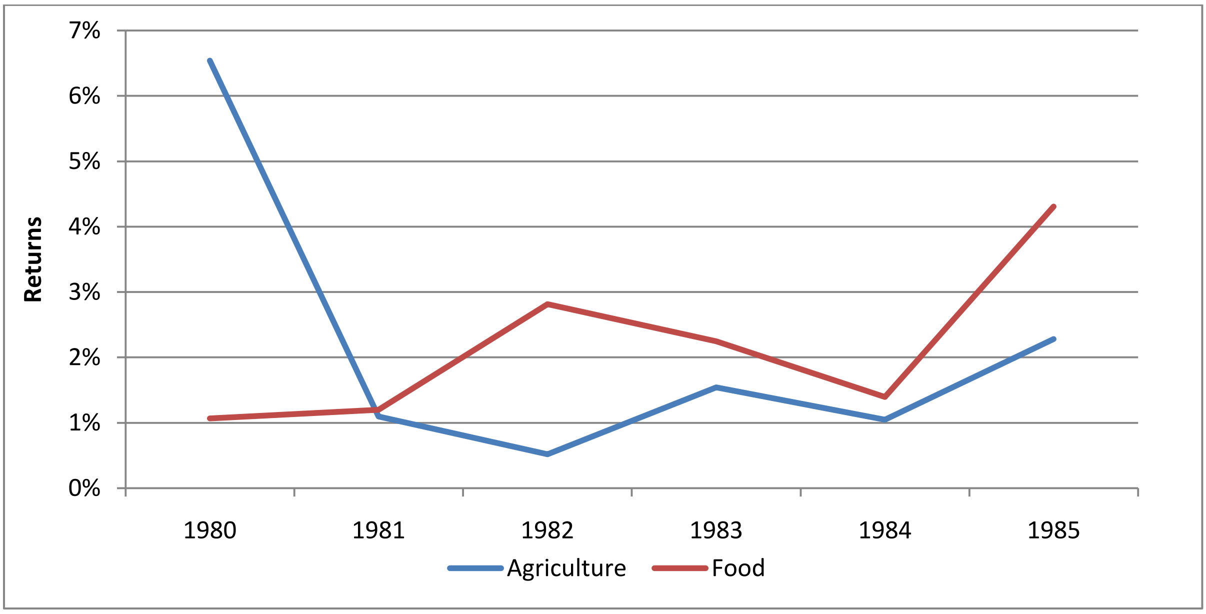
Assets that are typically mentioned in conjunction with these characteristics can be divided into traditional and new real assets. In table 1 an overview is provided of the variables Martin used in his research. The results show that commodities, timberland and farmland can be used for both short- and long-term inflation hedging. Equities, infrastructure and intellectual property do not show significant results for inflation hedging. For real estate the outcome is uncertain.

Table 1. Traditional and new real assets as defined by Martin (2010)

Traditional Real Assets	New Real Assets
1. Equities	1. Infrastructure
2. Inflation-Linked Bonds or Derivatives	2. Farmland
3. Commodities and Commodity-Linked Derivatives	3. Intellectual Property
4. Real Estate	
5. Gold	
6. Timber	

Due to Martin's study we know that farmland and timberland can potentially be used as an inflation hedge. Conrad & Stanton (1993) describe a short history/overview of the Unites States farm sector which will help to form a more comprehensive view on the farmland asset class. The paper starts off by describing the United States farm crisis that occurred in the early to mid-1980s. Figure 1 provides a graphical presentation of the crisis with food and agriculture. The first variable consists of food such as meat, dairy, flour, grain, sugar and roasted coffee. The variable agriculture concerns of crops, livestock and commercial fishing. A number of factors contributed to produce the farm crisis. One of the factors was the increased export demand and reduced grain crops in other parts of the world that led to rapid increases in grain prices and record farm incomes in the Midwest in the 1970s. In addition to this, negative real interest rates stimulated investment in farmland, new farm equipment, and capital improvements. Borrowing increased as farmers and others scrambled to purchase rapidly appreciating land with money borrowed, at times, at rates effectively less than zero.

Figure 1. Food and agriculture equities during the United States "Farm Crisis" early to mid-80s (source: Kenneth French Data Library)



Furthermore, the relatively lax lending standards of the Farm Credit System and other lending institutions contributed to the increase in farm borrowings that resulted in the farm crisis. Finally, one of the most significant factors was a major correction in farmland values. This correction was necessary due to the booming years of the 1970s, a substantial recovery in mid-1980 followed up the crisis and correction in prices. This led to a stabilization of farmland values in 1987. Conrad & Stanton then elaborate on investment considerations. Farmland investments are typically structured as direct equity ownership, although investments can be made through a variety of vehicles. Indirect ownership, individually managed separate accounts, and participations in a limited partnership belong to potential possibilities. Farmland investment managers provide both the portfolio management and asset management services required for a successful farmland investment program. Conrad & Stanton's results show that by altering the asset mix and management style decisions, farmland portfolios can be developed and managed to meet a variety of risk and return objectives.

Kaplan (1985) takes a closer look at utilizing farmland. He states that farmland is not only an excellent hedge against inflation but it can also be seen as a portfolio investment[3]. Modern portfolio theory states that combining two assets whose returns are not fully correlated reduces overall volatility below that of each one taken separately. The selection of assets is thus complicated by the simultaneous desire to increase or maintain return and reduce volatility or risk. The goal of his article is to describe return characteristics of farmland in order to assess its value as a diversification tool in an asset portfolio. Kaplan compares total returns and return correlations for six asset classes: farmland, large capitalization stocks, small capitalization stocks, long-term corporate bonds, long-term government bonds, and treasury bills from 1947 to 1980.

The results of his research show that in terms of total return, farmland was superior to bonds and bills, virtually equivalent to large capitalization stocks and inferior only to small capitalization stocks. The dual favorable attributes of farmland – high total return and low return correlation with other assets – make it an excellent diversification vehicle, based on the data (1947-1980). Another noteworthy result is the high return correlation with the Consumer Price Index (CPI). The CPI can be used to index the real value of wages, salaries and pensions (i.e. adjust for the effect of inflation). The real value is linked to the purchasing power of consumers. This implies that when the purchasing power increases, the number of goods/services that can be purchased per unit of currency also increases. Farmland makes intuitive sense as an inflation hedge, because it is a tangible asset whose replacement cost increases with inflation. Kaplan shows that farmland may be structured as an equity investment that offers a return comparable with the S&P 500, yet appears to be far less volatile. Another advantage of using farmland as a diversification tool is connected to inefficiency opportunities. The objective of security analysis is to uncover undervalued or overvalued securities. The prevalent theory of market behavior is that the stock market, at least in regards to large company stocks, is very nearly efficient. An efficient market is tersely defined as one in which prices fully reflect all relevant information. Adjustments to new information are virtually instantaneous. This means that cost incurred to identify undervalued or overvalued assets rarely produce returns in excess of these costs. Kaplan (1985) states that the market for farmland however fails to meet these criteria for market efficiency. The efficiency of a market is very much a function of the communication network that serves it. When a stock is trading on a major exchange, its transactions are immediately transmitted all over the world. Farmland transactions, on the other hand, are quiet. This relative quiet creates a profitable opportunity for those who are capable of obtaining farmland

[3] This statement is based on the fact that farmland is a tangible asset whose replacement cost increases with inflation. This will be further discussed in the next paragraph.

data faster than others. The main conclusion of Kaplan is that risk-averse investors have much to gain from diversification in farmland.

Scholtens & Spierdijk (2010) state that the attractiveness of timberland for institutional investors lies within the fact that timberland has a fairly low correlation with more traditional assets like common stock and bonds. This characteristic would make it a suitable diversification instrument. Two papers wherein estimations are made for the Capital Asset Pricing Model (CAPM) are written by Redmond & Cubbage (1988) and Sun & Zhang (2001). They find negative beta values for timberland investments. This negative beta suggests that there can be a potential improvement in the risk-return relationship of a portfolio by adding timberland to the portfolio. Studies that base their results on CAPM focus mainly on excess returns and the risk level relative to the market portfolio. Scholtens & Spierdijk have a different approach because they quantify by how much the risk-adjusted excess return will increase when timberland is included into an institutional portfolio. They analyze the diversification benefits of timberland within the mean-variance framework and include mean-variance spanning and intersection tests based on previous studies by Huberman & Kandel (1987) and Kan & Zhou (2008). With mean-variance spanning and intersection tests it can be measured to what extent the mean-variance frontier improves by including timberland. They have used the S&P500 as a proxy for the market portfolio. The negative betas mentioned before indicate that timberland has a low correlation with this index. The model used in this thesis will also use the S&P 500 as a proxy for the market portfolio. Scholtens & Spierdijk considered the Sharpe ratio in order to quantify the increase in mean-variance efficiency. The Sharpe ratio can be used to assess the economic benefits of adding timberland to the portfolio. It calculates the excess return relative to the volatility and it therefore can be used to compare different portfolios in terms of their risk-adjusted excess return. The results of their research show that, at a first glance, there is a significant improvement of the mean-variance frontier. Property values of timberland change over time, this change in value is reflected in the data and therefore has an effect on the return. This is called the appraisal smoothing bias. After removing the appraisal smoothing bias there is much less evidence for an increased mean-variance efficiency of the portfolio. The appraisal smoothing bias will be further addressed in section 3.7 and the recommendations.

2. Methodology and Dataset

The discussed literature already indicated that farmland and timberland assets posses favorable diversification characteristics. In this thesis, the benefits of including farmland and timberland in a traditional portfolio will be quantified and analyzed through the use of Modern Portfolio Theory (MPT), and efficient frontier analysis based on Pennacchi. Modern Portfolio Theory is principally based on the Markowitz Portfolio Selection Model (1952)[4]. The main focus in this chapter is to describe what model, data, observations and assumptions were used in the research program. After the discussion of the methodology the data will be described. In addition to this, limitations of the research and models will be discussed. Finally, the historical performance and distributions will be elaborated in order to gain a more comprehensive view of the asset classes used in this thesis.

2.1 Methodology

As clearly alluded earlier investing is a trade-off between risk and return. In this research there are several observations and assumptions which are stated below.

- Observation 1: people have money to invest
- Observation 2: there are several assets to invest in and there is a choice between different assets
- Observation 3: people have preferences

These observations indicate that there are multiple possibilities and they are not all equal, some could be better than others.

- Assumption 1: investors prefer more over less (nonsatiation)
- Assumption 2: investors dislike risk (risk-aversion)
- Assumption 3: investors maximize utility and do so for one period
- Assumption 4: Utility is a function of expected return and variance and nothing else
- Assumption 5: No distortion from costs, transaction fees, inflation or taxes
- Assumption 6: all information is available at no costs
- Assumption 7: all investments are infinitely divisible

This gives rise to the two central problems of investing, namely:

- How *should* investors, given their preferences, invest their money?
- What can be said about how the market and its participants actually operate?

The first question is normative and the second question is descriptive. Both revolve around the risk and return relation and both interact with each other. Information about how markets work

[4] Harry Markowitz published his formal model of portfolio selection in 1952, thereby paving the way for his 1990 Nobel Prize in Economics. See the literature section on Modern Portfolio Theory.

influences investment decisions, which in turn influences the market. When optimizing a portfolio given a set of assets and mean-variance preferences, it is likely that the optimum will consist of a lot of relatively small investments.

Mathematically:

$$\bar{r}_p = \omega'\bar{r} \tag{1}$$

The return of a portfolio is equal to the return of the assets in the portfolio times the portfolio weights (ω).

$$\sigma_p^2 = \omega' V \omega \tag{2}$$

The variance of the portfolio is determined by the weights and the covariance matrix of the assets. The variance-covariance matrix is defined as follows for a total of n different risky investments:

$$V = \begin{bmatrix} \sigma_1^2 & \sigma_{1,2} & \cdots & \sigma_{1,n} \\ \sigma_{2,1} & \sigma_{2,2} & & \vdots \\ \vdots & & \ddots & \sigma_{n-1,n} \\ \sigma_{n,1} & \cdots & \sigma_{n,n-1} & \sigma_n^2 \end{bmatrix} \tag{3}$$

where the n diagonal elements are estimates of the variances of the individual investments, σ_i^2, and the $n^2 - n = n(n-1)$ off-diagonal elements are estimates of the covariances between each pair of asset returns, $Cov(r_i, r_j)$. Each covariance measure appears twice in the matrix, once as the covariance between assets i and j and once as the covariance between j and i.

When assumed that there is no risk-free asset then a constraint should be added that the portfolio weights sum to 1:

$$\omega' e = 1 \tag{4}$$

Optimization can then be achieved by constructing the lagrangian:

$$\min_{\omega} \frac{1}{2}\omega' V \omega + \lambda\left[\overline{R_p} - \omega'\bar{r}\right] + \gamma[1 - \omega é] \tag{5}$$

The variance is minimized by adjusting the weights and subject to the restriction that the portfolio return equals R_p and the weights sum to 1. We then construct the first order conditions, set derivatives to zero, with reference to ω and the lagrange multipliers λ and γ:

$$V\omega - \lambda\bar{r} - \gamma e = 0 \tag{6.1}$$

$$\overline{R_p} - \omega'\bar{r} = 0 \tag{6.2}$$

$$1 - \omega é = 0 \tag{6.3}$$

Solving the first one for the portfolio weights gives:

$$\omega^* = \lambda V^{-1}\overline{r} + \gamma V^{-1}e \tag{7}$$

Then the restrictions are used to create the equations that will allow to eliminate λ and γ:

$$\omega^* = \lambda V^{-1}r + \gamma V^{-1}e \tag{8}$$

If we multiply the above by $\overline{r'}$ we get an expression for the required return $\overline{R_p}$ (first restriction). If we multiply by e' we get an equation which equals 1 (see second restriction). Two equations with two unknowns is a solvable system, after rearranging terms we can express the optimal portfolio weights in terms of data that is available. The data contains:

- required return, $\overline{R_p}$
- average return of the assets, $\overline{r}$
- and the (inverse of the) covariance matrix of the assets, V^{-1}

The solution then is the following:

$$\omega^* = \frac{e'V^{-1}e\overline{R_p} - \overline{r'}V^{-1}}{\overline{r'}V^{-1}\overline{r}e'^{V^{-1}}e - \left(\overline{r'}V^{-1}e\right)^2}V^{-1}\overline{r} + \frac{\overline{r'}V^{-1}\overline{r} - \overline{r'}V^{-1}e\overline{R_p}}{\overline{r'}V^{-1}\overline{r}e'^{V^{-1}}e - \left(\overline{r'}V^{-1}e\right)^2}V^{-1}e \tag{9}$$

Also note that with the optimal portfolio for a given required return, we can also find its variance:

$$\sigma_p^2 = \omega^{*\prime}V\omega^* \tag{10}$$

A simplification, where every greek letter is just a number, for formula 9 is stated below:

$$(9) = \frac{\delta\overline{R_p} - \alpha}{\varsigma\delta - \alpha^2}V^{-1}\overline{r} + \frac{\varsigma - \alpha\overline{R_p}}{\varsigma\delta - \alpha^2}V^{-1}e \tag{11}$$

Both formula (9) and (11) provide the same message, the optimal portfolio is based on three factors:

1. What are the average returns
2. How are they related
3. How much return do I want or how much risk am I willing to bear (R_p)

Formula 9 will be used in this thesis for calculations on the portfolio weights and returns. This will lead to the construction of the efficient frontiers with and without farmland and timberland. Also, additional restrictions will be imposed regarding short-sale constraints in the portfolio.

2.2 Data

The data used in this thesis is based on papers in the fields of economics, finance and alternative investments. In addition to these papers, data is used from the library of Kenneth French on the web, DataStream (available in the library of the Erasmus University Rotterdam), and the National Council of Real Estate Investment Fiduciaries (NCREIF). Data that can be found in the Kenneth French library includes the agriculture and food equity sectors. The NCREIF data includes real estate, farmland and timberland and DataStream provides data for the used indices. In table 2 an overview of the used asset classes is provided. For all asset classes I obtained historical data based on quarterly returns between the first quarter of 1990 and the fourth quarter of 2010. In Appendix A the return series used for empirical analysis can be found. As a proxy for global stocks, the large and small cap, inflation and the risk-free rate I used respectively: MSCI World Total Return, S&P 500, Russell 2000, United States CPI and 3-month United States treasury bills. The asset classes are including reinvested dividends and are all concerning the United States except for the global stocks.

Table 2. Asset classes and respective benchmarks (from 1990 Q1 – 2010 Q4)

Asset class	Benchmark
Farmland	NCREIF Farmland Returns*
Timberland	NCREIF Timberland Returns
Global Stocks	MSCI World Total Return
Large Cap	S&P 500
Small Cap	Russell 2000
Real Estate	NCREIF Property Index
Townsend Fund	NCREIF Townsend Fund
Agriculture Equities	Kenneth French Data Library
Food Equities	Kenneth French Data Library
Inflation	United States CPI
Risk-Free Rate	3-Month Treasury Bills

* Data only dates back to the 1st quarter of 1992

Farmland

The NCREIF Farmland Returns index is a quarterly time series composite return measure of investment performance of a large pool of individual agricultural properties acquired in the private market for investment purposes only. All properties in the Farmland Index have been acquired, at least in part, on behalf of tax-exempt institutional investors - the great majority being pension funds. As such, all properties are held in a fiduciary environment. The study by Martin (2010) also uses the NCREIF Farmland Returns Index.

Timberland

The NCREIF Timberland Returns index is a quarterly time series composite return measure of investment performance of a large pool of individual timber properties acquired in the private market for investment purposes only. The NCREIF Timberland Index has been published since 1994 and includes returns dating back to 1987. All properties in the Timberland Index have been acquired, at least in part, on behalf of tax-exempt institutional investors also the great majority being pension funds. As such, all properties are held in a fiduciary environment. A publication by the International Woodland Company (IWC)[5] also used this index for their analysis. They have stated the following limitations to the NCREIF Timberland Index:

1. The number of contributing TIMOs[6] has historically been limited and currently the index has nine contributing members.
2. The index series only dates back to 1987, which is a relatively short period. This will be of less concern over time as more years are added.
3. The index only covers timberland investments in the United States, which is not the only market for timberland investments.
4. Only quarterly appreciation returns are reported by the NCREIF. In quarters when properties are not appraised, the appreciation is reported as zero. As a result, the return series shows a higher volatility than there actually is.

Despite of these limitations, the NCREIF Timberland Index still is the best available measure of the historical performance of timberland returns and the most widely employed. It provides some indication of expected return characteristics for timberland investments. The studies by Martin (2010) and Scholtens & Spierdijk (2010) also used the NCREIF Timberland Returns Index.

Real Estate

The NCREIF Property Index is a quarterly time series composite total rate of return measure of investment performance of a large pool of individual commercial real estate properties acquired in the private market for investment purposes only. The International Woodland Company (2009) also uses the NCREIF Property Index.

[5] Title of the publication: "*Timberland Investments in an Institutional Portfolio*".
[6] The Timberland Investment Management Organization (TIMO) consists of a management group that aids institutional investors in managing their timberland investments.

Townsend Fund

The NCREIF Townsend Fund describes the performance information of private equity real estate funds pursuing core, value added, and opportunistic investment. The performance data is comprised of both active investments, as well as funds that have completed their full lifecycle or discontinued operations.

Large Cap

The S&P 500 index series acts as a proxy for the large cap. The S&P 500 index is a market-capitalization-weighted (large-cap) index. The S&P 500 includes a representative sample of 500 major companies in leading industries of the United States economy actively traded on the largest stock market companies; the NASDAQ OMX in the United States, NYSE and Euro Next. The S&P 500 Index is used in several papers that are included in the literature section such as: the International Woodland Company (2009), Friedman (1971) and Kaplan (1985).

Small Cap

The Russell 2000 index series is a small cap stock market index composed out of the bottom 2000 stocks in the Russell 3000 index. This index represents approximately 8% of the total market capitalization of the Russell 3000. The International Woodland Company (2009) also makes use of the Russell 2000 Index as a proxy for the small cap American stocks.

Global Stocks

The Morgan Stanley Capital International (MSCI) World Total Return index is a stock market index series of over 6000 world stocks, it is therefore often used as a benchmark for ‘world’ or ‘global’ stock funds. This index includes a collection of stocks of all the developed markets in the world, as defined by MSCI. The index includes securities from 24 countries but excludes stocks from emerging and frontier economies making it less worldwide than the name suggests. The International Woodland Company (2009) also uses the MSCI World Total Return Index as it is designed to measure the performance of the global equity markets.

Agriculture & Food Equities

These two sectors are based on the agriculture and food industry and have a corresponding Standard Industrial Classification (SIC) code. The SIC code is a United States government system for classifying industries by a four-digit code, which was established in 1937.

Furthermore, the use of data for the consumer price index and the 3-Month Treasury bills rate are used by several papers including the International Woodland Company (2009) and Martin (2010).

Table 3. Descriptive statistics of return series (quarterly data 1990 Q1 – 2010 Q4)

	Mean	Median	Maximum	Minimum	STDEV	Skewness	Kurtosis
Farmland*	9.81	1.48	22.78	-0.01	6.46	3.98	22.57
Timberland	10.99	1.88	22.34	-6.54	7.82	2.28	11.51
Real Estate	6.78	2.20	5.43	-8.29	5.12	-1.73	6.75
Townsend Fund	5.88	2.11	5.96	-13.44	6.71	-2.23	9.21
Large Cap	6.99	2.63	24.62	-22.20	16.19	-0.40	3.70
Small Cap	9.23	3.10	24.20	-27.62	18.06	-0.58	4.26
Global Stocks	5.27	2.60	24.53	-22.30	17.78	-0.38	3.42
Inflation	2.66	0.73	1.73	-2.36	1.05	-2.29	15.29
Agriculture EQ	12.22	3.13	38.71	-32.45	22.99	-0.23	4.60
Food EQ	10.41	3.84	22.58	-14.15	15.63	-0.16	2.50
3M T-Bills	3.63	4.06	7.78	0.06	2.02	-0.21	2.16

*Data only dates back to the 1st quarter of 1992

In table 3 above the data characteristics are given. The distribution histograms can be found in Appendix B. At a first glance we can see that farmland and timberland have quite interesting characteristics in terms of mean and standard deviation. The returns of the two are similar to that of the small and large cap while the corresponding risk is even less than half. The two real estate parameters, real estate and townsend fund, have similar values in terms of risk but the returns are lower. If we look at the agriculture and food equities we that the opposite holds, the returns are similar but the risk, in terms of standard deviation, are higher.

Table 4. Jarque-Bera test for normality

	Jarque-Bera	Probability
Farmland	1562.67	0.00
Timberland	326.05	0.00
Real Estate	91.15	0.00
Townsend Fund	204.80	0.00
Large Cap	3.99	0.14
Small Cap	10.28	0.01
Global Stocks	2.64	0.27
Inflation	601.63	0.00
Agriculture Equities	9.77	0.01
Food Equities	1.23	0.54
3M T-Bills	3.07	0.22

The Jarque-Bera (1987) test is a goodness-of-fit[7] measure of departure from normality, based on the sample skewness and kurtosis. The test statistic is defined as:

$$JB = \frac{n}{6}\left(S^2 + \frac{1}{4}K^2\right)$$

where n is the number of observations, S is the sample skewness and K is the sample kurtosis. The null hypothesis is a joint hypothesis of the skewness being zero and the kurtosis being three, since samples from a normal distribution have an expected skewness of zero and kurtosis of three. In table 4 we can find the Jarque-Bera test for normality. It shows that all variables are not normally distributed based on this test, except for the large cap, global stocks, food equities and the 3-month treasury bills which will later be addressed.

2.3 Limitations

Naturally, there are multiple limitations in this research of which the most significant ones will be addressed. When looking at the historical data there is a time span of 20 years quarterly data. This can simply be improved by taking a longer time period in consideration. However, the most widely used source of data on farmland and timberland is the NCREIF. Unfortunately, their dataset only dates back to 1992 for farmland and 1987 for timberland.

The second limitation to be addressed is the distribution of the data. It is widely known that financial data is not normally distributed which violates assumptions concerning the disturbance terms and their interpretation. The technical notation of the assumptions is stated below.

$$E(u_t) = 0 \quad (1)$$

$$Var(u_t) = \sigma^2 < \infty \quad (2)$$

$$Cov(u_i, u_j) = 0 \quad (3)$$

$$Cov(u_t, x_t) = 0 \quad (4)$$

$$u_t \sim N(0, \sigma^2) \quad (5)$$

The interpretation of the assumptions is as follows:

1. The errors have zero mean
2. The variance of the errors is constant (homoskedasticity) and finite over all values of x_t
3. The errors are statistically independent of one another
4. There is no relationship between the error and corresponding x variate
5. u_t is normally distributed

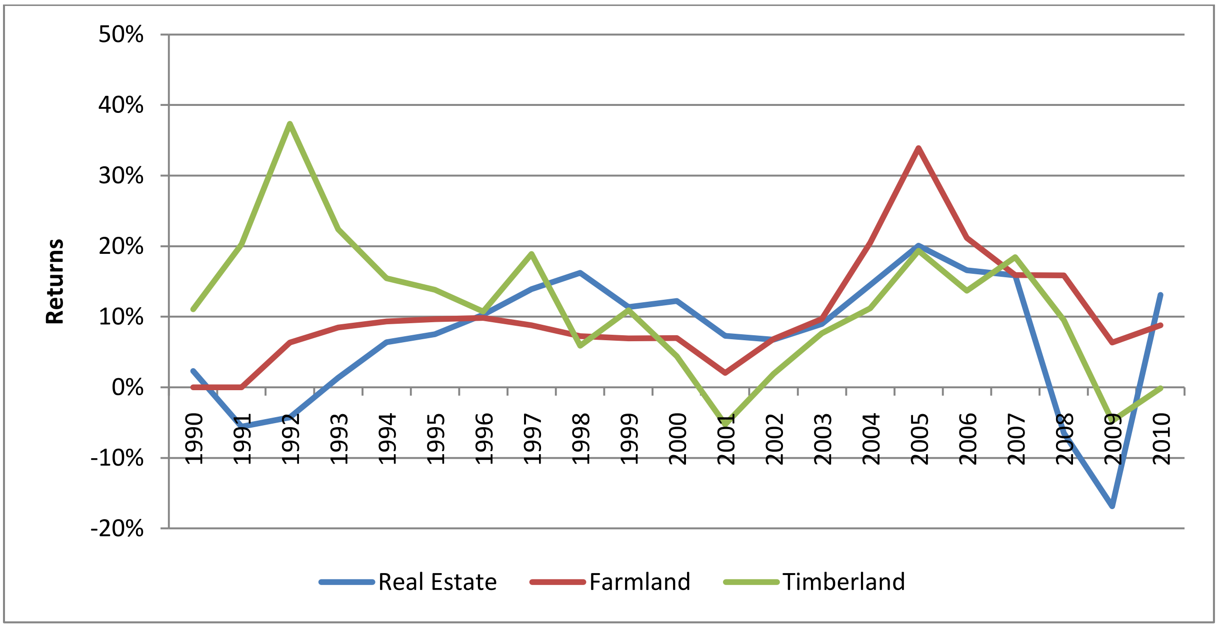
[7] The goodness-of-fit of a statistical model describes how well it fits a set of observations.

The third limitation is the appraisal smoothing bias. Farmland and timberland values rapidly appreciated in recent years. The concern is how to accurately assess the value of these relatively illiquid investments. Unlike publicly traded stocks and fixed-income securities as bonds, the value of farmland and timberland is not determined by a securitized market, nor are there formal transaction-based indices upon which to base pricing decisions. In fact, investors rely on property appraisals to measure the performance. The return series of farmland and timberland are 'smoothed', this is an underestimation to the extent of market value change as alluded earlier by Scholtens & Spierdijk (2010). We would expect in unsmoothed data that the mean is comparable but the standard deviation would be higher. This implies that there is an appraisal smoothing bias which makes the results possibly more positive than reality. The question then arises how this smoothing effect can be adjusted or minimized in order to provide a more accurate representation of true market returns. This will further be addressed in the results chapter section 3.6 by using annual overlapping data for mean-variance spanning tests and more complicated methods of unsmoothing the data will be discussed in the recommendations section.

2.4 Descriptive Statistics and Historical Performance

The annual returns of real estate, farmland and timberland are displayed in figure 2 below. Farmland has a steady annual return as can be seen in the chart. Timberland is more volatile than farmland however it still shows a positive overall return. This volatility can partially be explained by elasticity, timberland is more elastic that farmland. An explanation for this can be that also in times of recession (or crisis) people still eat and therefore the demand for food does not experience the same negative impact as the demand for wood. At the beginning of the financial crisis the housing prices in the United States dropped substantially and construction stagnated. Because of this, also the demand for wood decreased which affected the returns of timberland, as can be seen in figure 2. The financial crisis is clearly visible starting in mid-2007, especially for real estate the chart shows the downfall of returns. Farmland and timberland followed approximately six months later. This delayed affect can be explained in the fact that the consumer confidence in the United States experienced a heavy impact due to the financial crisis. Another noteworthy observation is the negative return for timberland in 2001 which was caused by a recession in manufacturing and a downturn in paper and paperboard products (Ince, 2002). Due to the strong US dollar at that moment imports rose and the growth of exports was limited.

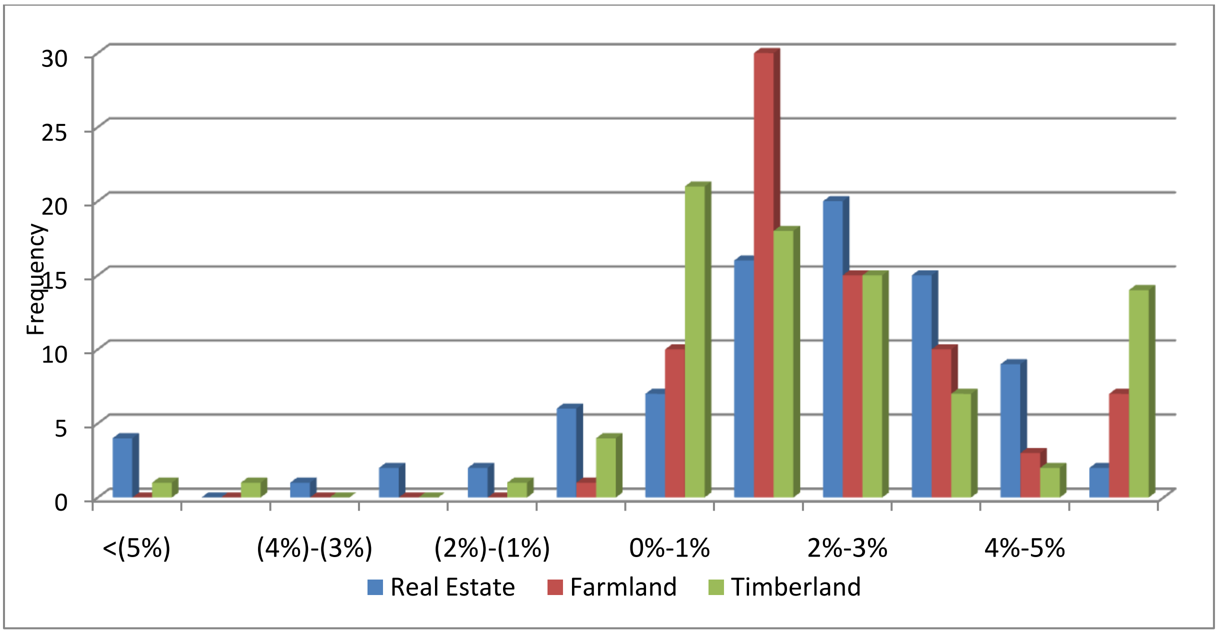
Figure 2. Annual returns of real estate, farmland and timberland (annual data 1990 – 2010)



2.5 Distribution of Returns

Over the scope of 20 years, we can clearly see that the overall returns for real estate, farmland and timberland are positive which could indicate a positive skewness. In order to examine this, the distributions of the return data are displayed below.

Figure 3. Distribution of returns for real estate, farmland and timberland (quarterly data 1990 Q1 - 2010 Q4)



It can be seen that the distribution of the returns is not purely normal as expected by Kaplan (1985). His research implied a positive skewness in the returns. There are, for example, more positive outliers bigger than five percent than negative outliers smaller than five percent for farmland and timberland. Hence, in terms of investing this is a positive result. As can be seen, real estate has slightly less attractive characteristics regarding the frequency of returns. In order to examine this further we can calculate skewness and kurtosis for the three asset classes. Skewness measures the asymmetry of a probability distribution. The value can be positive, negative or undefined. For farmland and timberland we see, in table 5, a positive skewness of respectively 3.98 and 2.28 which indicates that the tails on the right side are longer than the left side. This lies within the expectations because we saw in figure 3 that there are more positive than negative returns.

Table 5. Skewness and kurtosis for real estate, farmland and timberland (quarterly data 1990 Q1 – 2010 Q4)

	Skewness	Kurtosis
Real Estate	-1.73	6.75
Farmland	3.98	22.57
Timberland	2.28	11.51

Real estate on the other hand has a negative skew which indicates that the tail on the left side is longer than the right side. If the value would approach zero, it indicates that the values are relatively evenly distributed on both sides of the mean. Please note that this does not necessarily imply a symmetrical distribution. Another noticeable fact is the kurtosis for all the asset classes in table 5, it measures the 'peakedness' of the distribution. Higher kurtosis implies a sharper peak around the mean and fatter tails, more of the variance is caused by heavy infrequent deviations. Figure 3 indicates a high level of kurtosis because most observations lie between zero and three percent. This is confirmed in table 5 which shows that farmland and timberland have a kurtosis value of respectively 22.57 and 11.51. In order to address this, we will consider the use of annual overlapping data which will be discussed in the results chapter.

3 Results

In order to provide a grounded answer on the main hypothesis we have reviewed key literature and discussed the methodology and dataset. In this chapter we will apply the model on the dataset and interpret the obtained empirical results.

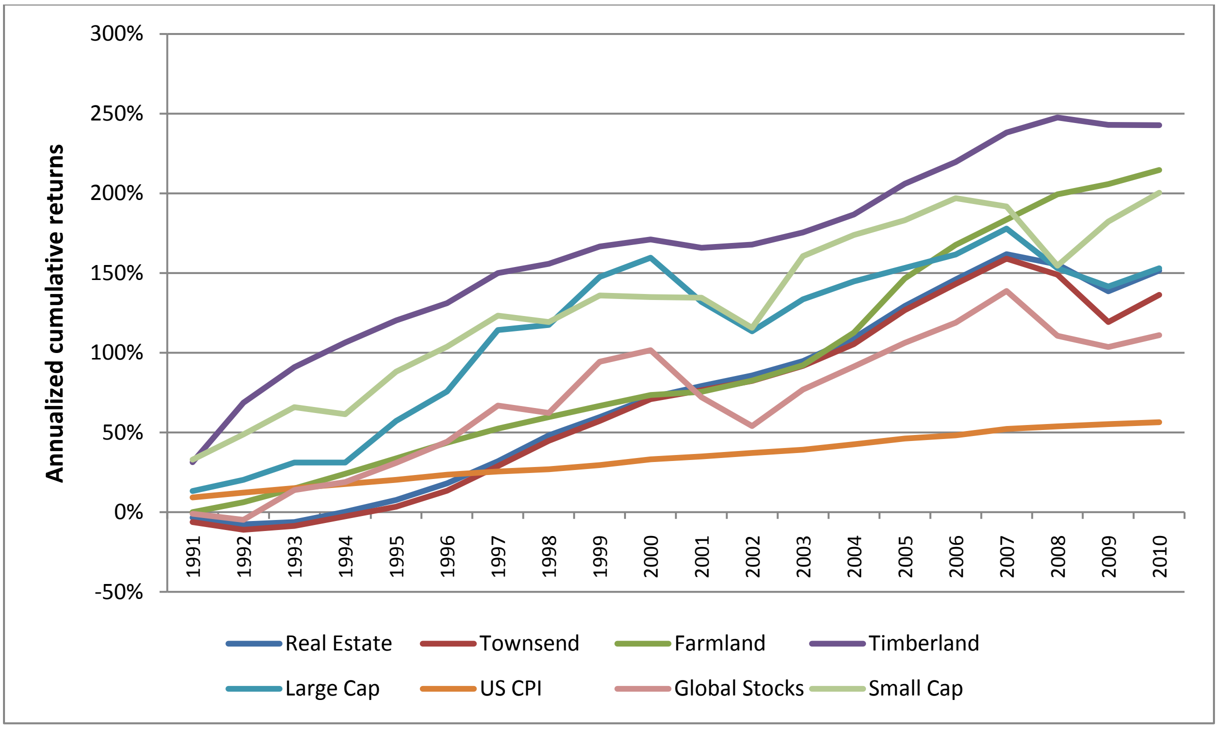
We will start with a short overview of the performance characteristics of farmland, timberland and the other assets classes used in the constructed investment universe. A close-up is taken of the returns of the asset classes and how they relate to their associated risk. After this, we are going to look at the relation of farmland and timberland to the other asset classes. Also, the relationship among the asset classes themselves will be discussed. This will be done by calculating Pearson correlation coefficients, which describes the linear dependence of two variables. The dataset contains data over a twenty-year time span and another interesting part in discussing correlations is to assess the shift in correlations over different time periods. Therefore, the dataset will be divided in two sub-samples of a decade and the correlations will be compared. With this comparison we can see if the correlation between variables changed over time.

If we know the return, associated risk and correlations of the variables the question arises how well the asset classes are performing. We want to know how much return we can gain in excess of the associated risk. In order to check this performance the Sharpe ratio will be calculated and discussed. Also, we want to see and statistically confirm if there is a significant improvement of the risk-return relationship when adding farmland and timberland into the constructed traditional portfolio. The first step in order to realize this is the construction of efficient frontiers and making the shift graphically visible by applying the model discussed in the methodology section. The second step is performing a formal statistical test called mean-variance spanning. This test will show whether there is a significant improvement or not. After this, a worst-case scenario analysis will be presented and the inflation hedging capabilities of farmland and timberland will be discussed. Finally, this chapter will close with a short overview of recommendations for further research.

3.1 Performance Characteristics for Farmland, Timberland and Other Asset Classes

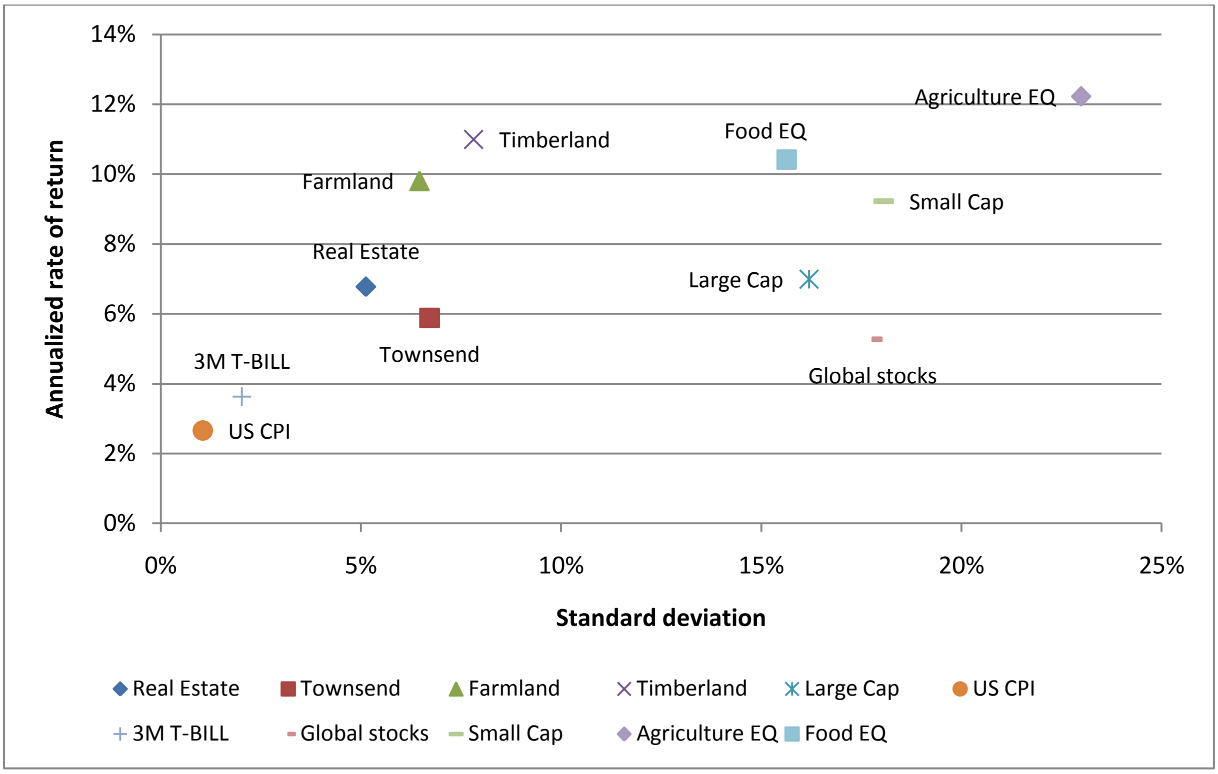
Previous research by Markowitz (1952) and Fabbozi et al., (2002) made clear that the overall performance, in terms of mean-variance efficiency, of common stock has been less-than-robust. It therefore can be interesting to take a closer look at how the returns of the different asset classes developed over time. The cumulative return is the aggregate amount that an investment has gained or lost over time, independent of the period of time involved. A common way to present the effect of performance over time is to show the cumulative returns graphically, figure 4 is a so called "mountain graph" that displays the cumulative returns of farmland, timberland and the other assets classes in the model. It is evident that timberland shows an extraordinary appreciation over time, it has the highest cumulative return since 1990. Farmland shows a steady increase over the years however, since the beginning of 2004 it showed a larger than average appreciation until the financial crisis after which the appreciation went back to its former level. Real estate also appreciated since the beginning of 2003 until the financial crisis in 2007 and 2008, since mid-2009 real estate has shown a substantial recovery. This is in line with the annual returns in figure 2, which also showed a severe turndown of returns during the financial crisis. The large cap show a steadier, but limited, appreciation until 2007 after which the stock markets were severely hit by the financial crisis. The small and large cap together with global stocks show a steep decline in 2008, a slow recovery followed after the crisis for the stock markets. The graph shows a sharper hit for townsend funds with regards to real estate.

Figure 4. Cumulative returns of the assets used in the model (annual data 1990 – 2010)



Although the returns are, of course, quite important it is not realistic or sensible to only look at the returns of an asset class or series and not taking the associated risk into account. Therefore, now that the development of returns for the different asset classes is known, we can take a closer look at how these relate to the corresponding risk in terms of standard deviations. In order to illustrate these characteristics, regarding the risk-return relationship, a chart has been prepared that includes the rate of returns and standard deviations for the assets in the investable universe. The data for this chart is based on the historical return series presented in table 2. The resulting chart is displayed in figure 5 below.

Figure 5. Geometric annual rates of return and standard deviations regarding the used asset classes in the model (quarterly data 1990 Q1 – 2010 Q4)



When looking at the chart, the standard deviation is presented on the x-axis in relation to the corresponding rate of return found on the y-axis. Obviously, the left-upper quadrant is the most desirable region in the chart as it facilitates the highest return at the lowest standard deviation. Once again, farmland and timberland contain very attractive characteristics. They offer a relatively high return with respect to the standard deviation. For a standard deviation around 17 percent, the small and large cap and global stocks provide returns of respectively 9.5, 7.1 and 5.2 percent. The returns of small and large cap and global stocks are smaller than the returns of farmland and timberland, while the standard deviation is higher with a factor two. The characteristics of farmland and timberland therefore dominate those of the small and large cap and global stocks. When only taking the large cap and global stocks into account, we see that the large cap is dominating global stocks due to a 2 percent lower standard deviation and a 2 percent higher rate of return. Real estate lies between farmland, timberland and stocks. The returns are lower but also the standard deviation is below that of farmland and timberland. Townsend fund is dominated by real estate because it has a lower return in addition to the higher standard deviation. If we compare townsend fund with farmland we see that the associated risk of the two is almost equal. However, farmland offers a rate of return that is higher with almost 4 percent. We can also see the expected returns and associated standard deviations for the consumer price index and the 3-month treasury bills. Figure 5 implies that farmland and timberland have interesting characteristics, in terms of return and standard deviation, for the purpose of portfolio optimization.

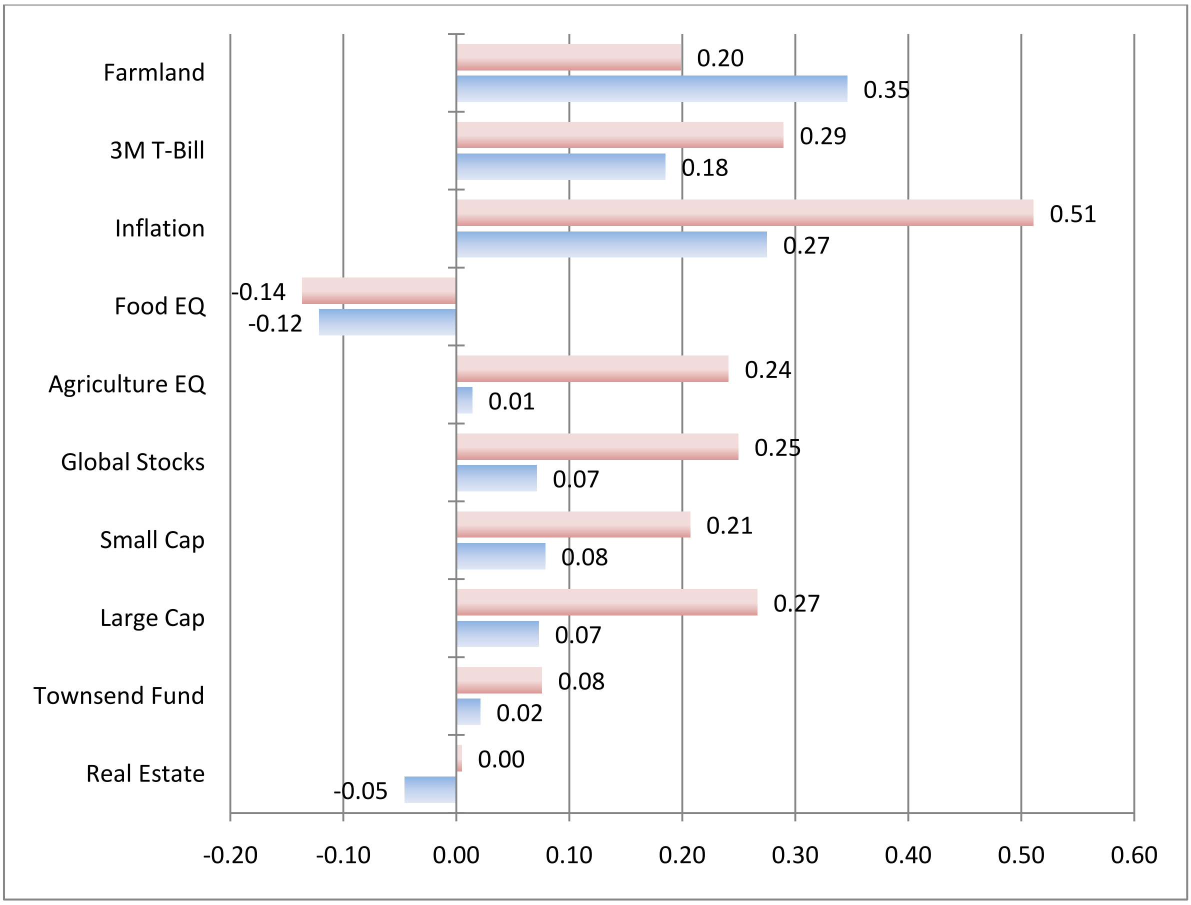
3.2 Correlations of Farmland and Timberland Returns to Other Asset Classes

Besides the attractive risk-return characteristics, it is worthy to take a closer look at how farmland and timberland correlate with the other assets classes. In statistics, dependance refers to any statistical relationship between two variables of two sets of data. The correlation refers to any of a broad class of statistical relationships involving dependance. Correlations are usefull because they can indicate a predictive relationship that can be exploited in practice. The correlation coefficient $\rho_{X,Y}$ between two variables X and Y with expected values μ_X and μ_Y and standard deviations σ_X and σ_Y is defined as:

$$\rho_{X,Y} = corr(X,Y) = \frac{cov(X,Y)}{\sigma_X \sigma_Y} = \frac{E[(X-\mu_X)-(Y-\mu_Y)]}{\sigma_X \sigma_Y}$$

where E is the expected value operator, cov is the covariance and $corr$ is Pearson's correlation. Figure 6 and 7 below, show the correlation between the returns on timberland and farmland investments and the remaining investable universe.

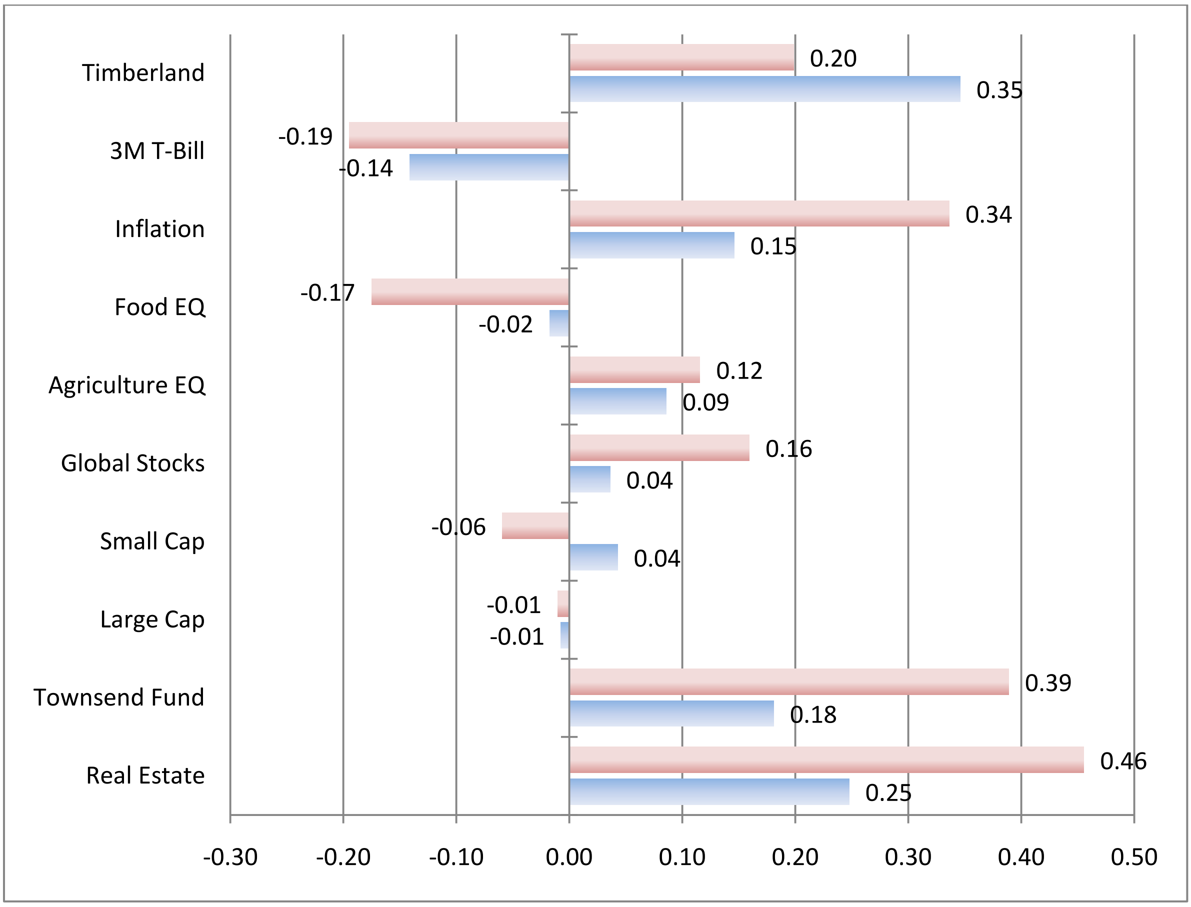
Figure 6. Correlations of the assets from the model with timberland returns



The blue bars in figure 6 and 7 represent the correlations, of respectively timberland and farmland with the other asset classes, based on quarterly data that runs from 1990 Q1 until 2010 Q4. The red bars represent the correlations based on annual overlapping data. The first observation, in the latter, contains overlapping data from 1990 Q1 – 1990 Q4, the second observation contains data from 1990 Q2 – 1991 Q1 and so on.

As can be seen in figure 6, timberland investment returns correlate positively with all of the asset classes, except for food equities and real estate. Timberland therefore, can potentially provide diversification opportunities in a real estate portfolio or portfolios containing food equities. Furthermore, timberland correlates quite well with the consumer price index and farmland. The latter will be addressed in section 3.6. The correlation with the inflation is in accordance to the findings of Martin (2010) where he states that timberland can also be used as a short- and long-term inflation hedge. There is a slightly positive correlation with the other asset classes, this is in accordance with Scholtens & Spierdijk (2010). They state that there is a fairly low correlation of timberland with the more traditional assets like common stock and bonds. This indicates that there are sizable benefits to be achieved by including timberland in a diversified portfolio. Real estate is an exception due to the slightly negative correlation.

Figure 7. Correlations of the assets from the model with farmland returns



We see that farmland has a relatively strong correlation with real estate and townsend fund, this in contrary to timberland. The correlation of farmland with inflation is significant at the 1% level. This result is in accordance with the research of Martin (2010) where he states that farmland can be used as an inflation hedge, both on a short and long horizon. Academics as well as practitioners have considered the positive correlation of farmland with the overall price level to be an important benefit of investing in farm-related assets, see for example Kaplan (1985) and Webb & Rubens (1988). Primary arguments for this to hold include:

1. Farmland is a scarce and tangible asset
2. Land used for crop production can be seeded according to highest-yielding crops, and it therefore can capture price appreciation in particular agricultural sectors (such as food crops or biodiesel crops)
3. The price of farmland, particularly in specific regions in the United States, reflects the option to convert farmland to other uses (such as residential or commercial real estate)
4. Farmland generates a significant amount of cash flow, which is typically not the subject of significant multi-year price contracts, which lowers the inflation risk (in the same way as, ceteris paribus, a floating-rate note has lower duration than a fixed-coupon instrument)
5. The increasing demand for agricultural products from developing economies is broadly linked to demand for other commodities that collectively may contribute to United States price inflation.

Based on the general evidence from the data, we find that farmland serves as an investment asset with positive expected returns for institutional investors that may also function as a hedge against inflation.

When annual overlapping data is used, we see a clear change in most of the correlations. The correlation of farmland with timberland is 0.35 based on quarterly data while it is 0.20 with annual overlapping data. If we look at figure 6 we see that almost all of the correlations became stronger, one noteworthy example is the consumer price index. The correlation is 0.27 based on quarterly data while it is 0.51 based on annual overlapping data. This implies that using timberland as an inflation hedge would make more sense when considering annual overlapping data because if the inflation rises, the value of the timberland investments also rises. The correlations of timberland with agriculture and food equities, the small and large cap, 3m treasury bills and townsend fund all became stronger. Only the small negative correlation of timberland with real estate diminished to a zero correlation. If we look at the correlation of farmland with the other assets we see that all correlations became stronger except for the correlation with timberland. It has been mentioned in the literature section that farmland could be an intuitive hedge against inflation because of the high correlation with inflation. This high correlation can be confirmed based on figure 7 where we see a correlation of 0.34 based on annual overlapping data, this is an increase of 0.19 with regards to the correlation based on quarterly data. Another noteworthy change in correlations is that of farmland with real estate and townsend fund, were an increase of 0.20 can be observed. Furthermore, the negative correlation of farmland with food equities changed from almost zero correlation to a negative correlation of -0.17 based on annual overlapping data.

3.3 Correlations Among the Various Asset Classes

We have investigated the correlations of farmland and timberland with the other asset classes, now this is known we have paved the way to take a closer look at the correlations between the variables themselves. In table 6 we can find the full-sample correlation table which is color-coded, black is not significant, red is significant at the 10% level, blue is significant at the 5% level and green is significant at the 1% level.

Table 6. Full-sample correlation table (quarterly data 1990 Q1 – 2010 Q4)

	Farmland	Timberland	Real Estate	Townsend Fund	Large Cap	Small Cap	Global Stocks	Agriculture EQ	Food EQ	Inflation	3M T-Bill
Farmland	1										
Timberland	0,35	1									
Real Estate	0,25	-0,05	1								
Townsend Fund	0,18	0,02	0,97	1							
Large Cap	-0,01	0,07	0,26	0,27	1						
Small Cap	0,04	0,08	0,09	0,08	0,08	1					
Global Stocks	0,04	0,07	0,26	0,26	0,93	0,08	1				
Agriculture EQ	0,09	0,01	0,20	0,14	0,22	0,45	0,22	1			
Food EQ	-0,02	-0,12	0,05	0,05	0,01	0,49	-0,07	0,28	1		
Inflation	0,15	0,27	0,31	0,39	-0,02	-0,12	-0,03	-0,06	-0,03	1	
3M T-Bill	-0,14	0,18	0,23	0,26	0,14	-0,02	0,04	0,07	0,14	0,42	1

Farmland and timberland have significant correlations with real estate and townsend fund, timberland also has significant correlations with inflation and 3-month treasury bills. These results correspond with various papers from, among others, The International Woodland Company (2009), Kaplan (1985), Martin (2010) and Scholtens & Spierdijk (2010). Real estate and townsend fund have significant correlations with the large cap, global stocks, agriculture equities, inflation and 3-month treasury bills. These results correspond with the research done by Friedman (1971). The small and large cap and global stocks have significant correlations with each other and food and agriculture equities. This can partly be explained by globalization of the stock markets, due to increasing globalization news and current events have a similar impact on these markets throughout the world.

3.4 Sub-Sample Correlations

It can also be interesting to see if there is a shift in correlations over two defined time periods. In this section the full sample is divided into two sets, the first time period runs from 1990 until 2000 and the second runs from 2000 until 2010. This takes us to the sub-sample correlations that can be seen in table 7 and 8 below.

Table 7. Sub-sample correlation table (quarterly data 1990 Q1 – 1999 Q4)

	Farmland	Timberland	Real Estate	Townsend Fund	Large Cap	Small Cap	Global Stocks	Agriculture EQ	Food EQ	Inflation	3M T-Bill
Farmland	1										
Timberland	0,21	1									
Real Estate	0,32	-0,44	1								
Townsend Fund	0,42	-0,32	0,97	1							
Large Cap	-0,02	-0,12	0,21	0,19	1						
Small Cap	0,07	0,15	-0,10	-0,10	-0,21	1					
Global Stocks	-0,02	-0,08	0,12	0,13	0,88	-0,21	1				
Agriculture EQ	0,11	-0,02	0,05	0,03	0,42	0,53	0,30	1			
Food EQ	0,02	-0,13	-0,13	-0,15	-0,29	0,50	-0,37	0,42	1		
Inflation	-0,14	0,25	-0,60	-0,61	-0,17	0,12	-0,15	-0,05	0,14	1	
3M T-Bill	-0,41	-0,28	0,11	0,03	-0,01	-0,05	-0,14	0,01	0,28	0,12	1

As we can see over the course of time the correlation of farmland with real estate, townsend fund and the United States treasury bills diminished to non-significant. The same holds for the correlation of the large cap stocks with agriculture and food equities, the correlation of global stocks with agriculture and food equities, the correlation of agriculture equities with food equities and the correlation of food equities with 3-month treasury bills.

Furthermore, the correlation of timberland with inflation became significant at the 5% level and the correlation of the small cap with global stocks became significant at the 10% level in the second decade.

The correlation of real estate with the large cap, global stocks and agriculture equities became significant, as well as the correlation of townsend fund with the large cap, global stocks and 3-month treasury bills. The same holds for the correlation of inflation with 3-month treasury bills.

Table 8. Sub-sample correlation table (quarterly data 2000 Q1 – 2010 Q4)

	Farmland	Timberland	Real Estate	Townsend Fund	Large Cap	Small Cap	Global Stocks	Agriculture EQ	Food EQ	Inflation	3M T-Bill
Farmland	1										
Timberland	0,73	1									
Real Estate	0,23	0,35	1								
Townsend Fund	0,15	0,28	0,98	1							
Large Cap	0,07	0,16	0,33	0,32	1						
Small Cap	0,08	-0,08	0,21	0,16	0,23	1					
Global Stocks	0,09	0,18	0,35	0,33	0,97	0,26	1				
Agriculture EQ	0,09	0,06	0,26	0,18	0,14	0,41	0,19	1			
Food EQ	-0,01	-0,18	0,19	0,16	0,24	0,49	0,17	0,19	1		
Inflation	0,20	0,36	0,54	0,57	0,01	-0,22	-0,01	-0,06	-0,11	1	
3M T-Bill	0,08	0,29	0,46	0,43	0,04	-0,14	0,03	0,13	0,02	0,56	1

3.5 Performance Measurements

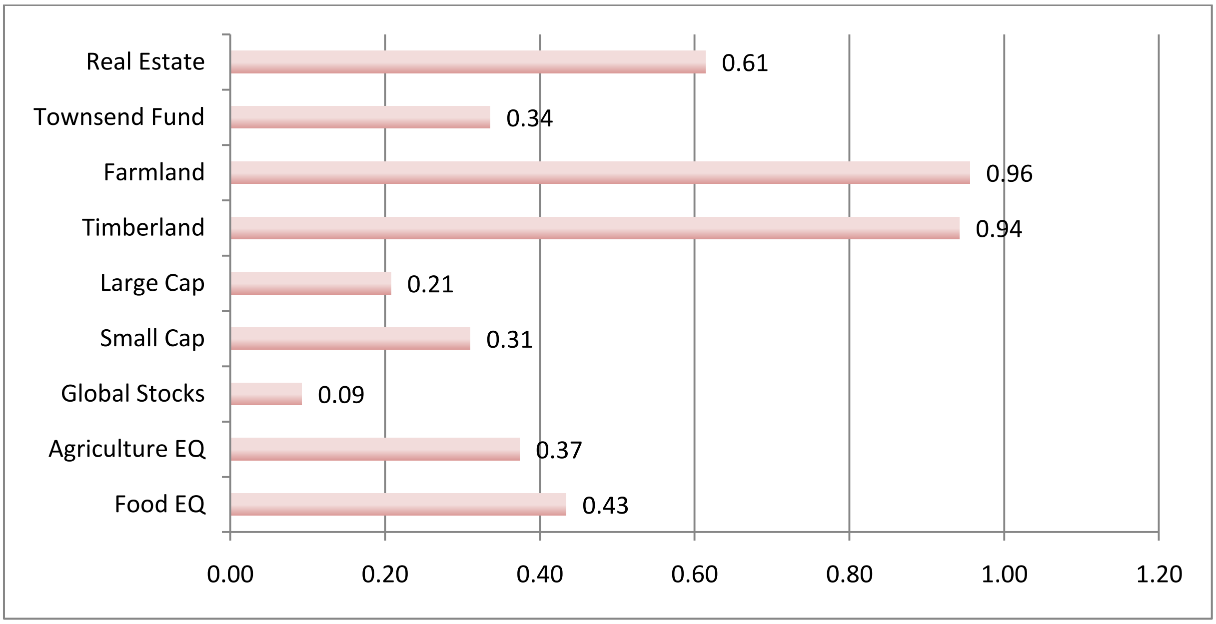
Another aspect of analyzing the performance characteristics is to calculate the excess return to volatility; this can be achieved by calculating the Sharpe ratio for each asset. The Sharpe ratio is often referred to as the reward-to-volatility ratio. It measures the excess return, or risk premium, per unit of risk in an investment asset, trading strategy or portfolio. The Sharpe ratio is defined as:

$$S = \frac{(R_p - R_f)}{\sigma_p}$$

Where R_p is the return on the portfolio, R_f is the risk-free rate and σ_p the standard deviation of the portfolio. The Sharpe ratio is used to characterize how well the return of the portfolio compensates the investor for the risk taken, the higher the ratio number, the better. Furthermore, if we compare the expected return of two portfolios given a fixed standard deviation for both, the portfolio with the highest Sharpe ratio provides the most excess return for the associated risk.

Figure 8 below provides the results of the analysis and we can see that the top three consists of real estate, farmland and timberland. This implies that these three asset classes provide the most return with respect to the associated risk. It is interesting to see is that agriculture and food equities also provide a relatively good Sharpe ratio if we compare them with for example the small and large cap and global stocks. They both offer quite high returns but also high standard deviations, which results in a lower Sharpe ratio but they still outperform the small and large cap and global stocks in terms of return for their risk. Townsend fund is just below the agriculture and food equities with a Sharpe ratio of 0.34 which is still higher than the small and large cap and global stocks.

Figure 8. Sharpe ratios of the assets used in the model (quarterly data 1990 Q1 – 2010 Q4)



Global stocks are performing worst in terms of the Sharpe ratio, this result can also be linked to figure 5 where the position of the asset classes is shown in terms of risk and return. It is quite interesting to see that again farmland and timberland possess good characteristics for investing and portfolio optimization.

3.6 Efficient Frontier Analysis: Construction of the "Bullet"

The data that is necessary for asset allocation and the construction of the efficient frontier which consists of returns, standard deviations and the correlation of every asset combination represented in the model. This data is summarized in table 9 below.

In the top panel of table 9 the returns and standard deviations can be found for the asset classes that were included in the model. At a first glance we see that agriculture and food equities have the highest returns and global stocks and townsend fund have the lowest returns. We would expect that the inflation would lie around the 2 to 2.5 percent and the return of 3-month treasury bills would lie around the 3 to 3.5 percent, which is confirmed in table 9. It can also be concluded from the table that real estate, farmland and timberland are dominating the other assets in terms of return and standard deviation which has already been confirmed by the Sharpe ratios in figure 8. They offer a relatively higher return with a lower corresponding standard deviation. The observed returns and standard deviations can also be linked to figure 5 which shows this graphically. The returns and standard deviations based on annual overlapping data are also provided in the top panel of table 9. It can be seen that the returns, based on quarterly data and on annual overlapping data, lie closely together and that the standard deviations are all higher except for the consumer price index. It has been alluded before in the literature and limitations section that the standard deviation of farmland and timberland could provide a more optimistic vision due to the appraisal smoothing bias. We would expect a higher standard deviation when unsmoothed data is used. In order to unsmooth the data, annual overlapping data is calculated. The returns and standard deviations based on annual overlapping data are shown in the table and will be used in the next section to address the appraisal smoothing bias.

Table 9. Risk, return and correlations for the asset classes included in the model (quarterly data 1990 Q1 – 2010 Q4)

	Farmland	Timberland	Real Estate	Townsend Fund	Large Cao	Small Cap	Global Stocks	Agriculture EQ	Food EQ	Inflation	3M T-Bill
Annual Returns	9.81	10.99	6.78	5.88	6.99	9.23	5.27	12.22	10.41	2.66	3.63
Annual Returns*	10.35	11.87	7.07	6.23	7.61	9.37	5.71	13.11	10.98	3.66	5.02
Annual Standard Deviation	6.46	7.82	5.12	6.71	16.19	18.06	17.78	22.99	15.63	1.05	2.02
Annual Standard Deviation*	7.77	10.52	9.27	11.63	18.49	18.73	18.38	27.59	17.19	0.99	2.92

Correlations on Quarterly Returns	Farmland	Timberland	Real Estate	Townsend Fund	Large Cap	Small Cap	Global Stocks	Agriculture EQ	Food EQ	Inflation	3M T-Bill
Farmland	1										
Timberland	0.35	1									
Real Estate	0.25	-0.05	1								
Townsend Fund	0.18	0.02	0.97	1							
Large Cap	-0.01	0.07	0.26	0.27	1						
Small Cap	0.04	0.08	0.09	0.08	0.08	1					
Global Stocks	0.04	0.07	0.26	0.26	0.93	0.08	1				
Agriculture EQ	0.09	0.01	0.20	0.14	0.22	0.45	0.22	1			
Food EQ	-0.02	-0.12	0.05	0.05	0.01	0.49	-0.07	0.28	1		
Inflation	0.15	0.27	0.31	0.39	-0.02	-0.12	-0.03	-0.06	-0.03	1	
3M T-Bill	-0.14	0.18	0.23	0.26	0.14	-0.02	0.04	0.07	0.14	0.42	1

* Based on annual overlapping data

If we take a look at the correlations stated in table 9 we see a significant correlation at the 1% level between farmland and timberland. A possible explanation for this correlation is provided by Bickley (2007), he concluded that TIMOs[8] associated timberland with farmland. It began from the idea of agricultural lending, as 20 to 30 years ago most American farms had forestland in combination with farmland. Therefore, when companies would lend money to the farms they would consider the trees as part of the collateral asset base of the entire farm. The table also shows a significant correlation at the 5% level between farmland and real estate, this finding is in correspondence to the findings of Webb et al., (1988). It is also noteworthy to mention that there is a highly significant correlation between real estate and townsend fund which makes intuitively sense because both of the variables are regarding real estate.

As earlier mentioned in the data section, the real estate data consists of the total rate of returns of investment performance of a large pool of individual commercial real estate properties acquired in the private market for investment purposes only. Townsend fund is linked with this variable as it includes the performance information of private equity real estate funds pursuing core, value added, and opportunistic investment strategies. Real estate and the large cap have a significant correlation at the 5% level. The same holds for townsend fund and the large cap. Real estate and global stocks also have a significant correlation with each other in the 5% level. The same holds for townsend fund and global stocks. Global stocks and the large cap have a highly significant correlation at the 1% level, partially, due to the fact of globalization. Current news and events have an impact on global stocks and therefore also on the large cap.

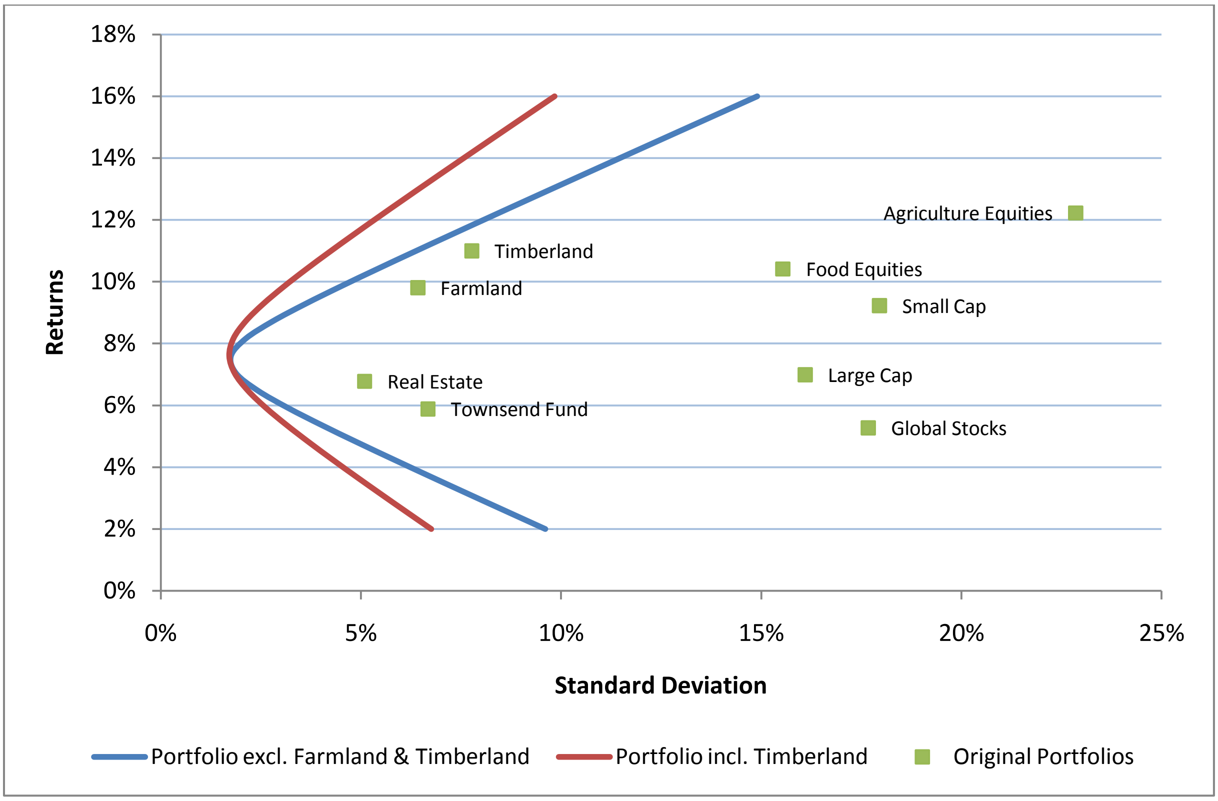
In order to assess the impact of adding farmland or timberland the efficient frontier is constructed. We will first look at the efficient frontier of a traditional portfolio compared with the same portfolio, only including farmland in the mix. After this, the same will be executed only instead of timberland, farmland is included. Additionally, the traditional portfolio will be compared with the same portfolio only including farmland and timberland. Finally, the portfolio weights will be discussed when short-sale constraints are imposed. The constructed efficient frontiers are shown in the figures 9, 10 and 11. The corresponding portfolio weights can be found in Appendix C, in Appendix D and E the portfolio weights can be found when short-sale constraints are imposed.

[8] Timber Investment Management Organization

Figure 9. Efficient frontier of a traditional portfolio versus a mixed-asset portfolio containing financial assets and farmland (quarterly data 1990 Q1 – 2010 Q4)

The efficient frontier contains every possible combination of the assets which is plotted in the risk-return space. The point where the efficient frontier has the lowest standard deviation is called the minimum risk portfolio (Merton, 1972), and is on the furthest point on the left of the efficient frontier. If we go upwards from there, the curve is mean-variance efficient because a higher return can be achieved for the same amount of risk. In figure 9 we see two efficient frontiers and the individual asset classes. If we look at the latter, we see that common stock and equities have standard deviations between 15 and 20 percent with a return between 5 and 13 percent. Real estate, townsend fund, farmland and timberland each have standard deviations between 5 and 10 percent which is considerably lower than common stock and equities with a comparable return. Farmland and timberland offer a relatively high return with respect to the standard deviation and while real estate and townsend fund offer a lower return for the associated risk.

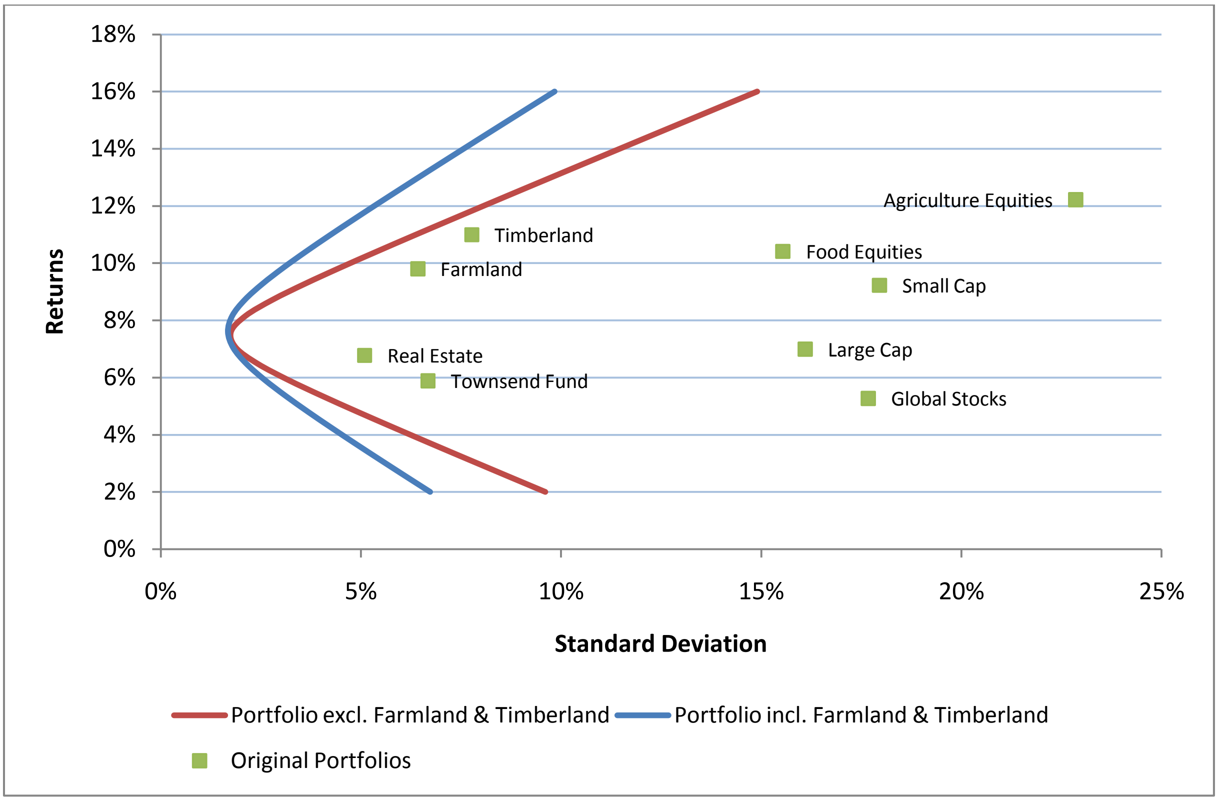
Figure 10. Efficient frontier of a traditional portfolio versus a mixed-asset portfolio containing financial assets and timberland (quarterly data 1990 Q1 – 2010 Q4)



There are two efficient frontiers plotted in the figure, the blue curve represents the possible combinations of all the assets except for farmland and timberland. Intuitively, if farmland is added to the portfolio we would expect a better risk-return relationship. This implies that the efficient frontier should shift diagonally upwards to the left. It can be seen in figure 9 that there is a notable shift in the efficient frontier when farmland is added to the portfolio (shown in red), this implies that diversification benefits arise when farmland is added. However, the efficient frontiers coincide with each other, for standard deviation levels between 2 and 3 percent, which implies that diversification benefits at that range are minimal.

Figure 10 shows us how the traditional portfolio behaves when timberland is added to the portfolio. Again, we see the individual asset classes in the figure and the efficient frontier which takes into account all of the assets except for farmland and timberland. We can observe a more dramatic shift in the efficient frontier when timberland is added to the portfolio instead of farmland. When we compare the return of a portfolio including farmland, for a given standard deviation of 5 percent, with a portfolio including timberland, we see that the return is just above the 10 percent for the farmland portfolio while the timberland portfolio offers a return of above 11 percent. When the standard deviation rises, this difference becomes larger. This implies that adding timberland to a portfolio provides more diversification benefits than adding farmland.

Figure 11. Efficient frontier of a traditional portfolio versus a mixed-asset portfolio containing financial assets, farmland and timberland (quarterly data 1990 Q1 – 2010 Q4)



We can see in figure 11 that if both farmland and timberland are added to the traditional portfolio there is only a minor shift visible with regards to only adding timberland. If we compare figure 10 and 11 we see the shift around the point of the minimum risk portfolio (MRP). This implies that timberland has the most diversification potential. It should also be noted that the efficient frontier shifts positively when adding farmland, timberland or both to the portfolio. It is a positive shift because the efficient frontier moves towards the left-upper quadrant of the risk-return space. This fact has to be kept in mind when we perform a formal statistical test for the significance of change in the efficient frontier in the next section.

3.7 Mean-Variance Spanning Tests

After seeing a graphical change in the efficient frontier it is now time for a formal statistical test procedure for the significance of the shift. We want to know if the efficient frontier of a portfolio excluding farmland significantly differs from the portfolio including farmland, the same also holds for timberland. With mean-variance spanning we have as null-hypothesis that the efficient frontier of the portfolio without farmland is the same as the efficient frontier of the portfolio with farmland. If this hypothesis is rejected it means that there is a significant difference between the two efficient frontiers. We also know from the previous section that any difference is a positive one because the efficient frontiers always shifted towards the left-upper quadrant of the risk-return space. In table 10 we can see the results of the mean-variance spanning tests (which is also called the F-test or Wald test).

Table 10. Mean-variance spanning test on adding farmland and timberland to a traditional portfolio (quarterly data 1990 Q1 – 2010 Q4)

Dependant variable: Farmland		Prob.	Dependant variable: Timberland		Prob.
α	-0.19	0.73	α	3.07	0.00
b1 Timberland	0.40	0.00	*b1* Farmland	0.59	0.00
b2 Real Estate	2.27	0.00	*b2* Real Estate	-2.73	0.00
b3 Townsend Fund	-1.49	0.00	*b3* Townsend Fund	1.91	0.00
b4 Large Cap	-0.14	0.19	*b4* Large Cap	0.13	0.32
b5 Small Cap	-0.01	0.73	*b5* Small Cap	0.06	0.21
b6 Global Stocks	0.11	0.27	*b6* Global Stocks	-0.10	0.43
b7 Agriculture EQ	-0.01	0.67	*b7* Agriculture EQ	0.02	0.52
b8 Food EQ	0.03	0.47	*b8* Food EQ	-0.11	0.06
Sum of *b*		1.14	Sum of *b*		-0.21
F-statistic		0.17	F-statistic		16.44
Prob(F-statistic)		0.85	Prob(F-statistic)		0.00
Durbin-Watson		2.06	Durbin-Watson		1.94
R-squared		0.34	R-squared		0.33

Huberman & Kandel (1987) and Jobson & Korkie (1989) state that "F" is the test-statistic that tests if α = 0 and *b1+b2+b3+b4+b5+b6+b7+b8* = 1; *prob* is the p-value corresponding to that F-statistic. We can see in table 10 that for farmland the null hypothesis is not rejected and for timberland the null hypothesis is rejected based on the P-values for the F-statistic. This implies that the efficient frontier, or minimum variance frontier, of the constructed portfolio excluding farmland is the same as the efficient frontier including farmland in the portfolio. The opposite holds for timberland and implies that there are sizable benefits to be gained from adding timberland to the portfolio.

However, there is still one problem that should be addressed. The mean-variance spanning test is based on quarterly data and we have seen in the literature and limitations section that there is an appraisal smoothing bias. In order to overcome this bias we use annual overlapping data, as has been suggested in the previous section. We have seen in table 9 that the returns of the asset classes are comparable and that the standard deviations became somewhat higher when annual overlapping data was used. Table 11 provides the results of the mean-variance spanning tests where annual overlapping data was used. It shows that the null-hypothesis is rejected for both farmland and timberland which means that there is a significant positive change in efficient frontiers for adding them to the traditional portfolio. This result is not in accordance with Scholtens & Spierdijk (2010), their results also showed significant results when timberland was added to the portfolio. However, when they used unsmoothed data the results became non-significant which is the opposite of the results regarding farmland in this research. This can possibly be explained due to the de-smoothing procedure Scholtens & Spierdijk used, a further elaboration on their de-smoothing procedure will be provided in the recommendations section.

Table 11. Mean-variance spanning test on adding farmland and timberland to a traditional portfolio based on annual overlapping data

Dependant variable: Farmland		Prob.	Dependant variable: Timberland		Prob.
α	-42.17	0.01	α	148.50	0.00
b1 Timberland	0.43	0.00	*b1* Farmland	0.94	0.00
b2 Real Estate	3.57	0.00	*b2* Real Estate	-5.11	0.00
b3 Townsend Fund	-2.51	0.00	*b3* Townsend Fund	3.68	0.00
b4 Large Cap	-0.41	0.00	*b4* Large Cap	0.50	0.00
b5 Small Cap	0.00	0.98	*b5* Small Cap	0.04	0.53
b6 Global Stocks	0.35	0.00	*b6* Global Stocks	-0.39	0.00
b7 Agriculture EQ	-0.07	0.01	*b7* Agriculture EQ	0.15	0.00
b8 Food EQ	0.03	0.50	*b8* Food EQ	-0.17	0.01
Sum of *b*		1.39	Sum of *b*		-0.35
F-statistic		3.88	F-statistic		45.32
Prob(F-statistic)		0.03	Prob(F-statistic)		0.00
Durbin-Watson		0.68	Durbin-Watson		0.74
R-squared		0.70	R-squared		0.63

3.8 Worst-Case Scenario Analysis

Until now we have only considered the portfolio over the whole dataset but it can be interesting to see how the portfolio behaves in times of heavy turmoil. Therefore the large cap returns are ranked based on the worst quarterly returns in table 12. The first column contains the ranking, the second and third column show the corresponding quarter and return.

Table 12. Worst-case quarterly returns of the large cap (quarterly data 1990 Q1 – 2010 Q4)

Ranking	Quarter	Large Cap Return
1	2009 Q1	-22.20
2	2001 Q4	-16.02
3	2002 Q3	-15.52
4	1998 Q4	-14.12
5	2001 Q2	-13.21
6	2010 Q3	-12.79
7	2002 Q4	-12.46
8	1990 Q4	-12.40
9	2009 Q2	-10.21
10	2008 Q4	-9.64

As we can see in the table, most of the worst-case returns occurred in the recent financial crisis in 2008 and 2009, and the burst of the internet bubble in 2001 and 2002. With table 12 as a foundation, we can have a closer look at how the portfolio returns behave. Table 13 below shows the calculated portfolio returns that correspond with the top-10 worst Large Cap returns for a 10% standard deviation. The first column shows the portfolios with and without farmland and timberland, the second shows the ranking and corresponding quarter, the thirds column shows the portfolio return for that quarter with and without farmland and timberland. Column four and five are a repetition of column two and three. Per quarter, we can see how the portfolio returns change when we add farmland and timberland to the portfolio. We can see that it does not make a clear difference when farmland and/or timberland are added to the portfolio. The portfolio return appreciated only three times out of ten when adding both farmland and timberland to the portfolio. The same analysis, but with a 5% standard deviation, shows that the portfolio return appreciated by a small amount five times out of ten (see Appendix F).

Table 13. Portfolio returns for the top-10 worst quarters based on the large cap for a 10% standard deviation (quarterly data 1990 Q1 – 2010 Q4)

	Worst Quarter Top-10	Portfolio Return	Worst Quarter Top-10	Portfolio Return
With F&T		-9.41		-1.69
With T	1: 2009 Q1	-9.51	6: 2010 Q3	-1.56
With F		-11.58		-1.55
Without F&T		-16.22		-1.56
With F&T		-1.37		-2.03
With T	2: 2001 Q4	-0.42	7: 2002 Q4	-1.67
With F		-0.34		-2.14
Without F&T		-0.65		-1.54
With F&T		-11.09		-1.53
With T	3: 2002 Q3	-11.10	8: 1990 Q4	-2.08
With F		-11.20		-2.15
Without F&T		-11.01		-3.91
With F&T		-0.53		-1.38
With T	4: 1998 Q4	-0.51	9: 2009 Q2	-1.02
With F		-0.27		-1.10
Without F&T		-0.59		-1.12
With F&T		-1.25		-18.34
With T	5: 2001 Q2	-1.15	10: 2008 Q4	-17.73
With F		-1.70		-18.77
Without F&T		-0.87		-17.47

In the first column, “F” and “T” are Farmland and Timberland respectively

3.9 Using Farmland and Timberland as Inflation Hedge

The papers by Kaplan (1985) and Martin (2010) discussed in the literature section, suggested that farmland and timberland could be used as an inflation hedge due to the relatively high correlation with inflation. Also, the results in the paper of the International Woodland Company (2009) show that there is a relatively high correlation between timberland and inflation. We have seen in our earlier results that these statements regarding the correlations are confirmed in this thesis. We would therefore imply that, in times of higher inflation, when farmland and timberland are added to the portfolio, the returns would be higher. In the first column of table 14 and 15, we can see the traditional portfolio which is indicated by 'without F&T'; the traditional portfolio including farmland 'with F'; the traditional portfolio including timberland 'with T'; and the traditional portfolio including farmland and timberland 'with F&T'. This is based on the top-10 highest and lowest inflation quarters which can be seen in the second and fourth column. In addition to this, we also see the calculated portfolio returns of the corresponding quarter in times of high and low inflation for a 7.5 percent standard deviation.

Table 14. Top-10 lowest and highest inflation quarters and portfolio returns for a 7.5% standard deviation

	Top-10 highest inflation quarters	Portfolio returns (high inflation)	Top-10 lowest inflation quarters	Portfolio returns (low inflation)
With F&T	1: 2008 Q3	0.97	1: 2009 Q3	-10.44
With T		-4.23		5.73
With F		-4.34		5.67
Without F&T		-4.38		5.74
With F&T	2: 2008 Q2	3.34	2: 2009 Q2	-7.74
With T		0.09		-0.68
With F		-0.25		-0.91
Without F&T		-0.04		-0.84
With F&T	3: 2008 Q1	4.25	3: 2009Q1	1.43
With T		-1.80		-7.45
With F		-1.48		-7.51
Without F&T		-1.47		-7.13
With F&T	4: 2006 Q2	3.38	4: 2010 Q3	7.97
With T		1.66		-0.78
With F		1.76		-0.90
Without F&T		1.68		-0.96
With F&T	5: 2007 Q4	8.88	5: 2002 Q1	-0.88
With T		4.43		3.97
With F		4.57		3.96
Without F&T		4.87		3.93

Due to the fact that there is a high correlation between farmland and timberland with inflation, the expectation is that the portfolio returns would increase when farmland and timberland are added to the traditional portfolio, especially in times of high inflation. This expectation is only violated two times in times of high inflation which means that adding farmland and timberland to the portfolio has a positive effect on the portfolio returns. For times of low inflation the expectation is violated five times out of ten which indicates that the effect is arbitrary, at least for times of low inflation.

Table 15. Top-10 lowest and highest inflation quarters and portfolio returns for a 7.5% standard deviation (cont'd)

	Top-10 highest inflation quarters	Portfolio returns (high inflation)	Top-10 lowest inflation quarters	Portfolio returns (low inflation)
With F&T	6: 2005 Q3	4.83	6: 2010 Q4	5.00
With T		2.43		10.97
With F		2.22		10.34
Without F&T		2.21		10.57
With F&T	7: 2005 Q4	1.25	7: 2002 Q2	4.51
With T		3.77		-0.32
With F		2.71		-0.47
Without F&T		3.27		-0.44
With F&T	8: 2006 Q1	2.90	8: 2009 Q4	-5.60
With T		4.65		4.39
With F		4.48		3.69
Without F&T		4.52		3.80
With F&T	9: 2000 Q3	3.49	9: 1998 Q1	4.48
With T		1.04		3.42
With F		1.13		3.54
Without F&T		1.07		3.48
With F&T	10: 2000 Q4	9.01	10: 1998 Q4	5.04
With T		1.34		0.42
With F		1.01		0.38
Without F&T		0.93		0.31

Table 14 and 15 confirm that farmland and timberland can be used as an inflation hedge due to the relatively high correlation with inflation. We can also conclude from the table that timberland has a slightly stronger effect than farmland. This is in accordance with results of, among others, Kaplan (1985), Martin (2010), Scholtens & Spierdijk (2010), The International Woodland Company (2009) and Redmond & Cubbage (1988).

3.10 Recommendations for Further Research

The recommendations for further research are closely linked to the assumptions and limitations of this research. The first recommendation is to perform a more thorough analysis of the distribution of the data. As mentioned before, it is widely known that financial data in general is not normally distributed. It has been assumed that the variance of the errors is constant, this is known as the assumption of homoskedasticity. If the errors do not have a constant variance, they are said to be heteroskedastic. It is also possible that the variance of the errors changes over time rather than systematically with one of the explanatory variables; this phenomenon is known as autoregressive conditional heteroskedasticity (ARCH). There are a number of formal statistical tests for heteroskedasticity and one could for example use White's (1980) general test for heteroskedasticity.

Another assumption regarding the data was that errors are uncorrelated with one another. If the errors would be correlated with each other, it would be stated that they are autocorrelated (or that they are serially correlated). A formal test for this assumption would therefore be required. The residual series from the estimated model can be plotted, after which we would look for any patterns. The graphical interpretation can however be quite difficult in practice. A formal statistical test for autocorrelation which has been applied in this thesis is the Durbin & Watson (1951) statistic. The Durbin Watson test however, is for first order autocorrelation which means that it tests only for a relationship between an error and its immediately previous value. The test that is recommended to address the autocorrelation, which can consider greater lags, is the Breusch-Godfrey test. This test could be used in order to see if there is autocorrelation on lags greater than 1, this test was beyond the scope of this thesis.

Additionally, an assumption on the data stated that the disturbances are normally distributed. What should then be done if evidence of non-normality is found? We could leave the models that assume normality, but such methods may be difficult to implement. In economic and financial modeling, it can often be the case that only a few extreme residuals cause a rejection of the normality assumption. Such residuals would then appear in the tails of the distribution, this in turn would imply a high kurtosis. These residuals do not fit within the pattern of the remaining data and are known as outliers. If this is the case, a way to improve the chances of error normality is to use dummy variables to effectively remove those observations.

In this thesis, annual overlapping data was used to address the appraisal smoothing bias which is a fairly simple procedure. A de-smoothing procedure would be recommended that possibly yields returns closer to reality, in order to counter the effect of smoothing in the return series for farmland, timberland and real estate as this may cause a bias in the volatility of the series. De-smoothing processes are further used by Scholtens & Spierdijk (2010) who found that after correcting for this bias, their results on adding timberland to a traditional portfolio were not significant anymore. We have seen just the opposite in this thesis regarding the results of farmland. They use a de-smoothing procedure based on a paper by Fisher et al., (1994) and Cho et al., (2003), which is recommended for further research. The basic idea behind their de-smoothing procedure is stated below.

The smoothed index can be defined as:

$$r_t^* = w_0 r_t + w(B) r_{t-1} \tag{1}$$

where r_t^* is the smoothed return of the index during period t; r_t is the corresponding underlying true or not-smoothed return during period t; w_0 is a weight between 0 and 1; and $w(B)$ is a polynomial function in the lag operator, B:

$$w(B) = w_1 + w_2 B + w_3 B^2 + \cdots \tag{1a}$$

where *B* refers to one lag ($Br_{t-1} = r_{t-2}$), B^2 refers to two lags ($B^2 r_{t-1} = r_{t-3}$), and so on. It can be shown that equation 1 implies that the return of the index can be represented by an autoregressive model of the form:

$$r_t^* = \emptyset(B) r_{t-1}^* + e_t \tag{2}$$

where $\emptyset(B)$ is a lag operator polynomial:

$$\emptyset(B) = \emptyset_1 + \emptyset_2 B + \emptyset_3 B^2 + \cdots \tag{2a}$$

And e_t is given by:

$$e_t = w_0 r_t \tag{2b}$$

The advantage of this representation is that equation 2 can be inverted to obtain an expression for the unobservable underlying return, r_t, as a function of the present and past values of the observable r_t^*:

$$r_t = (r_t^* - \emptyset(B) r_{t-1}^*)/w_0 \tag{3}$$

With formula 3 the unobserved values for the returns of the index can be estimated with the observed values.

Conclusions

The search for yield always was, and still is, the main theme for investors. In order to gain higher yields in a more globalized market than ever, investors started to search beyond the realm of the traditional stock and bond markets. This opened the door to alternative investments such as precious metals and real estate to name just a few, including farmland and timberland. The latter two assets have several attractive characteristics as we have seen in this thesis.

The goal of this thesis was to examine the impact of adding farmland and timberland to a traditional financial portfolio and determine which one adds the most value to the risk-return relation of a financial portfolio. In order to realize this goal we examined how the equity sector and real estate asset class in the United States have been used before to enhance the risk-return characteristics of a portfolio. The main hypothesis of this thesis was:

Can the risk-return relationship of a portfolio be enhanced by adding farmland and/or timberland to a purely financial portfolio?

The first chapter elaborated on key papers that related to the research done in this thesis explaining modern portfolio theory which formed the basic framework of this thesis. Furthermore, topics such as portfolio theory, real estate investments, benefits of traditional and new real assets, inflation hedging benefits using alternative investments, and using farmland and timberland as a portfolio investment were discussed. The literature has shown that both farmland and timberland have interesting characteristics and that they provide potential diversification benefits for the purpose of portfolio optimization.

The second chapter covered the models used in this thesis and provided a description of the dataset. Furthermore, limitations regarding data, distributions and appraisal smoothing were discussed. After this, the descriptive analysis has shown that farmland and timberland have relatively high returns which make them interesting to consider in the traditional portfolio.

Of course, we would not only consider the returns but also the associated risk. Chapter three delved deeper into the associated risk and the relationship between the various asset classes. We have seen that farmland and timberland have the highest cumulative return in comparison with real estate, equities and stocks. When looking at the returns with regards to the risk, farmland and timberland also had the most favorable characteristics when compared to real estate, equities and stocks. In addition to this, the relationship between farmland and timberland and the various asset classes was positive. This relationship was expressed by various correlations which showed that there is a relatively low correlation with stocks and equities and a high correlation with inflation. The positive relationship with inflation is noteworthy because it would imply that farmland and timberland can be used as an inflation hedge. The results showed that when farmland and timberland are included in the traditional portfolio in times of high inflation, the portfolio returns were higher which confirms the inflation hedging capabilities. Furthermore, the excess return per unit of risk was calculated by the Sharpe ratio, which showed that farmland and timberland provide the most return for their risk in comparison with the remaining asset classes.

In order to assess the impact of adding farmland and timberland the efficient frontiers were constructed. We have seen that adding farmland and timberland to the traditional portfolio had a positive influence on the efficient frontier. All of the shifts in the efficient frontier were positive in the sense that they shifted towards more return and less risk. Through mean-variance spanning tests based on quarterly data, we can conclude that the efficient frontier only shifted significantly for timberland which implies that timberland can be used to diversify the traditional portfolio and gain a higher return for a given amount of risk. The results of the mean-variance spanning test for farmland were non-significant which means that farmland does not add significant value to the traditional portfolio. Furthermore, we have seen that in times of heavy turmoil, it does not matter if farmland or timberland is added to the portfolio in terms of portfolio return.

On a marginal note, it should be mentioned that there are more complicated de-smoothing procedures available which yield returns that lie closer to reality then the procedure used in this thesis. This implies that the results of the spanning-tests could be more optimistic than in reality. The research of Scholtens & Spierdijk (2010) showed that the added value of timberland in the portfolio was non-significant when they applied their de-smoothing procedure.

After all these topics that were included in the research, we have arrived at the main question of this thesis. We have investigated the characteristics of farmland and timberland which were quite positive. Like the subtitle stated: *"fact or fiction?"* can farmland and timberland be used for portfolio optimization purposes in terms of risk and return? When considering the data and models used in this thesis, together with the assumption that the de-smoothing procedure yields returns close to reality, it is only a fact for timberland.

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Appendix

APPENDIX A. Timeseries: Returns of Investment Classes

	Farmland	Timberland	Real Estate	Townsend Fund	Large Cap	Small Cap	Global Stocks	Agriculture EQ	Food EQ	3M T-BILL
1990Q1	0.00	2.66	1.38	1.46	0.72	8.48	3.32	-0.53	-6.32	3.78
1990Q2	0.00	3.06	1.52	1.36	-4.16	4.19	-17.45	-6.99	12.22	3.89
1990Q3	0.00	1.42	0.84	0.49	6.15	-22.92	11.66	-16.20	-7.67	3.87
1990Q4	0.00	3.50	-1.43	-2.02	-12.40	4.89	-17.80	3.02	13.95	3.57
1991Q1	0.00	1.65	0.05	-0.05	4.85	24.20	7.37	20.92	22.58	3.22
1991Q2	0.00	5.70	0.01	-0.80	12.44	3.41	8.68	4.35	-2.47	2.88
1991Q3	0.00	2.47	-0.33	-1.65	1.79	4.93	-1.85	6.33	8.88	2.79
1991Q4	0.00	9.24	-5.33	-5.05	2.98	5.00	5.48	1.98	14.02	2.55
1992Q1	1.36	1.69	-0.03	-0.70	7.17	6.94	3.10	-5.14	-9.11	1.94
1992Q2	1.84	8.34	-1.03	-0.84	-3.08	-4.79	-10.12	-0.32	-0.49	2.01
1992Q3	0.76	1.88	-0.44	-1.10	2.14	-0.21	4.05	-3.30	10.91	1.78
1992Q4	2.25	22.34	-2.81	-2.34	0.83	14.18	-0.33	7.17	-0.40	1.31
1993Q1	1.41	1.92	0.77	0.51	4.66	1.72	-0.38	1.91	-7.97	1.54
1993Q2	2.13	17.38	-0.24	0.52	3.35	4.11	8.97	-1.77	-11.85	1.45
1993Q3	1.07	1.06	1.10	0.92	-0.28	8.02	4.61	6.64	-2.28	1.50
1993Q4	3.64	1.21	-0.25	0.45	2.73	2.27	4.55	3.21	9.42	1.46
1994Q1	1.77	2.47	1.31	1.44	0.90	-0.82	1.07	-0.58	-4.84	1.52
1994Q2	2.84	2.99	1.54	1.51	-4.23	-4.62	-0.17	-6.15	1.60	1.74
1994Q3	1.31	1.48	1.51	1.15	0.10	4.75	2.62	2.73	11.48	2.10
1994Q4	3.13	7.80	1.88	1.75	3.48	-3.51	1.37	-4.38	1.86	2.37
1995Q1	1.02	2.78	2.11	1.75	-0.53	5.67	-0.55	8.63	7.20	2.77
1995Q2	4.25	4.24	2.08	2.00	9.27	8.95	2.90	11.27	10.37	2.85
1995Q3	1.20	0.87	2.06	1.72	9.01	6.83	5.60	15.41	8.43	2.73
1995Q4	2.88	5.34	1.09	0.51	6.33	3.07	3.71	11.62	7.28	2.63
1996Q1	1.21	2.14	2.40	2.18	5.88	6.81	5.34	6.05	0.74	2.48
1996Q2	3.23	0.12	2.29	2.36	6.14	2.04	4.25	3.27	7.79	2.52
1996Q3	1.10	1.99	2.63	2.36	3.39	-0.35	2.32	1.90	6.27	2.54
1996Q4	3.96	6.17	2.61	2.77	1.95	6.40	0.73	7.54	5.51	2.49
1997Q1	1.22	2.89	2.34	2.49	7.50	-3.73	3.99	-0.65	4.43	2.54
1997Q2	3.58	3.12	2.82	3.19	2.55	13.96	-0.81	20.94	13.33	2.59
1997Q3	0.63	2.35	3.38	3.08	17.30	9.98	16.04	13.76	6.94	2.52
1997Q4	3.12	9.50	4.71	5.96	7.23	-1.03	2.58	6.91	8.42	2.49
1998Q1	1.04	2.24	4.14	3.06	1.57	9.90	-3.31	6.45	4.69	2.61
1998Q2	2.62	0.94	4.19	4.48	14.19	-6.43	14.49	16.51	-4.22	2.50
1998Q3	1.21	0.70	3.46	3.71	3.65	-19.05	2.53	-23.06	-10.05	2.48
1998Q4	2.18	1.88	3.55	3.78	-14.12	15.49	-16.01	-2.93	11.17	2.07

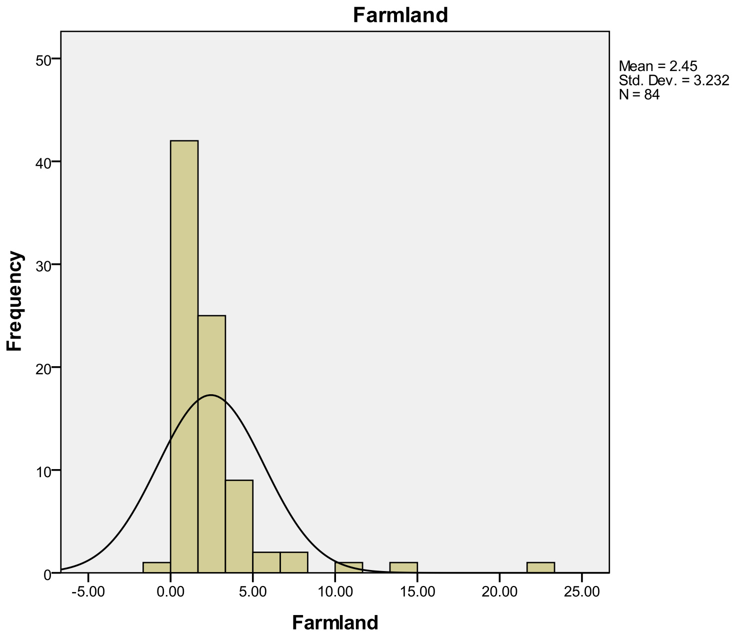
APPENDIX A. Timeseries: Returns of Investment Classes (Cont'd)

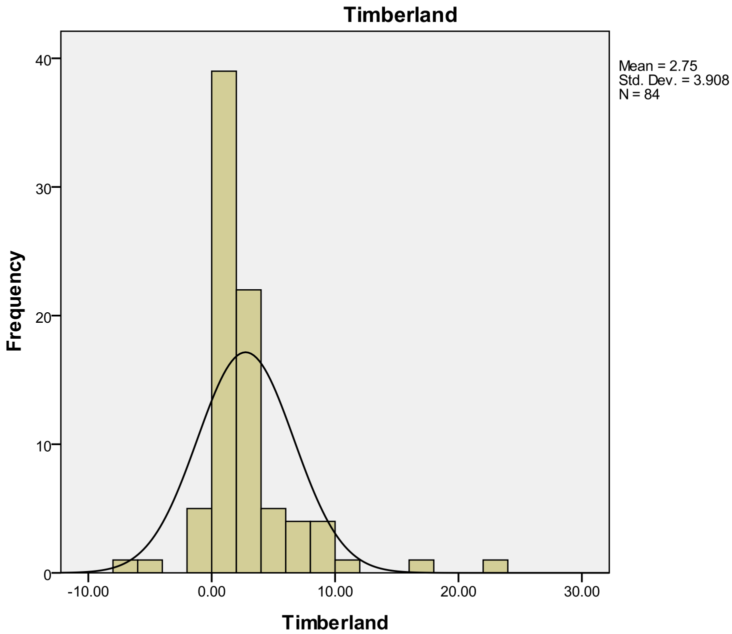
	Farmland	Timberland	Real Estate	Townsend Fund	Large Cap	Small Cap	Global Stocks	Agriculture EQ	Food EQ	3M T-BILL
1999Q1	1.20	1.44	2.59	2.68	24.62	-1.94	24.53	16.45	-11.93	2.19
1999Q2	1.41	0.06	2.62	2.87	5.25	9.69	3.92	4.10	6.37	2.16
1999Q3	0.79	2.46	2.81	2.86	6.74	-4.28	5.08	-4.42	-8.80	2.28
1999Q4	3.39	6.66	2.89	3.41	-7.11	13.34	-2.81	-16.20	-3.63	2.37
2000Q1	1.07	1.64	2.40	2.60	13.44	11.55	16.53	7.99	-8.27	2.64
2000Q2	1.33	0.70	3.05	3.27	3.49	-7.95	0.33	-6.67	13.27	2.85
2000Q3	1.55	2.47	2.94	3.64	-2.42	4.21	-2.67	-0.97	1.08	2.88
2000Q4	2.87	-0.45	3.33	3.43	-2.27	-7.65	-5.88	-0.79	17.41	3.02
2001Q1	0.68	0.49	2.36	2.29	-8.07	-1.92	-6.58	2.52	-7.09	2.87
2001Q2	1.31	0.05	2.47	1.87	-13.21	5.94	-13.98	9.48	0.40	2.06
2001Q3	0.03	0.84	1.60	1.06	7.93	-12.87	4.27	-3.56	7.01	1.80
2001Q4	-0.01	-6.54	0.67	0.30	-16.02	10.05	-15.83	11.01	4.87	1.16
2002Q1	0.65	0.54	1.51	0.77	10.55	3.78	8.84	3.04	4.21	0.86
2002Q2	1.68	0.13	1.61	1.39	-0.13	-9.68	0.15	2.48	-0.47	0.88
2002Q3	0.84	0.50	1.79	1.78	-15.52	-16.07	-10.77	-15.27	-14.15	0.85
2002Q4	3.55	0.70	1.67	1.81	-12.46	3.10	-15.74	3.96	8.95	0.78
2003Q1	1.21	0.61	1.88	1.89	3.76	-3.26	4.84	-2.48	-7.82	0.60
2003Q2	1.74	1.67	2.09	2.41	-2.43	21.50	-4.49	10.34	10.82	0.55
2003Q3	0.95	1.45	1.97	2.39	14.43	10.83	15.57	5.80	1.79	0.44
2003Q4	5.51	3.75	2.76	2.43	3.65	11.29	6.17	23.59	7.58	0.47
2004Q1	1.69	2.04	2.56	2.72	9.20	3.06	11.63	3.46	8.98	0.47
2004Q2	2.61	0.86	3.13	3.10	1.82	-1.42	3.10	5.13	3.41	0.46
2004Q3	0.74	1.97	3.42	3.32	-0.29	-0.38	-1.13	-0.26	-4.69	0.60
2004Q4	14.63	5.96	4.66	3.91	0.23	12.01	0.61	7.47	12.97	0.84
2005Q1	2.08	1.81	3.51	4.36	6.24	-4.23	9.29	0.86	-2.00	1.15
2005Q2	3.66	3.70	5.34	5.18	-2.43	5.89	-1.15	2.84	-0.63	1.37
2005Q3	3.06	0.95	4.44	5.02	1.83	2.31	-0.03	1.92	2.23	1.55
2005Q4	22.78	11.98	5.43	5.13	2.70	5.19	6.36	-8.12	-3.59	1.77
2006Q1	4.02	2.31	3.62	3.88	1.76	8.48	3.14	8.15	5.90	2.00
2006Q2	2.44	3.49	4.01	3.92	3.97	-5.05	6.69	-7.23	5.24	2.28
2006Q3	1.93	0.85	3.51	3.63	-1.36	3.11	-1.15	11.72	4.22	2.48
2006Q4	11.55	6.46	4.51	4.11	3.99	7.28	3.59	10.40	2.50	2.38
2007Q1	2.13	1.86	3.62	3.96	6.53	1.45	7.85	6.22	3.51	2.45
2007Q2	2.33	2.31	4.59	5.08	0.44	2.01	2.33	18.22	2.52	2.46
2007Q3	2.76	3.90	3.56	4.00	6.66	-1.25	6.50	24.55	-1.03	2.41
2007Q4	7.92	9.38	3.21	2.04	1.82	-7.39	1.99	27.43	-0.51	1.92

APPENDIX A. Timeseries: Returns of Investment Classes (Cont'd)

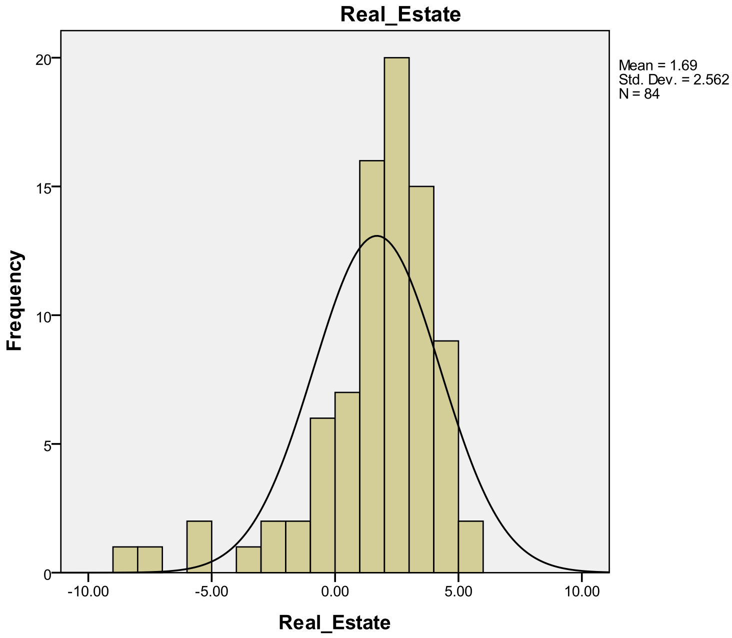
	Farmland	**Timberland**	**Real Estate**	**Townsend Fund**	**Large Cap**	**Small Cap**	**Global Stocks**	**Agriculture EQ**	**Food EQ**	**3M T-BILL**
2008Q1	1.17	4.50	1.60	1.34	-5.09	-6.98	-3.66	-1.67	-2.87	1.65
2008Q2	4.50	1.01	0.56	0.26	-6.69	2.97	-7.62	11.83	-5.59	0.69
2008Q3	2.09	0.99	-0.17	-0.75	-6.22	-9.10	-5.12	-20.23	4.17	0.92
2008Q4	7.33	2.74	-8.29	-10.75	-9.64	-27.62	-14.96	-28.19	-14.04	0.42
2009Q1	1.32	0.73	-7.33	-13.44	-22.20	-8.25	-22.30	16.11	-10.46	0.06
2009Q2	1.15	-1.20	-5.20	-9.20	-10.21	20.57	-11.16	-6.70	14.02	0.11
2009Q3	1.01	0.26	-3.32	-7.30	13.84	11.01	19.00	4.04	9.47	0.09
2009Q4	2.71	-4.55	-2.11	-3.70	11.54	3.91	13.28	5.06	6.37	0.05
2010Q1	1.11	-0.25	0.76	1.00	8.28	11.99	6.03	-10.97	7.97	0.03
2010Q2	0.67	0.99	3.31	4.50	5.65	-5.03	3.74	-32.45	-5.05	0.08
2010Q3	1.03	-0.10	3.86	5.70	-12.79	3.10	-14.52	1.44	6.58	0.09
2010Q4	5.79	-0.79	4.62	4.99	11.57	7.77	14.30	38.71	5.61	0.08

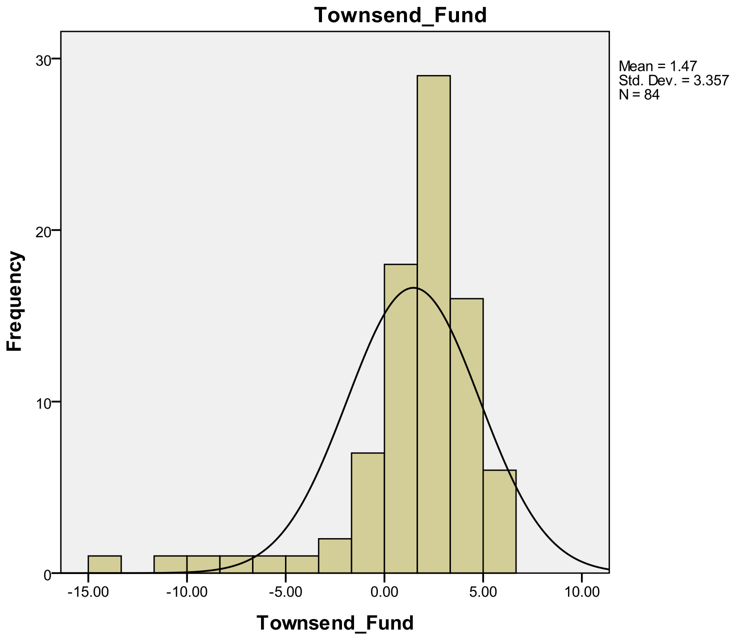
APPENDIX B. Histograms of the variables used in the theoretical portfolio



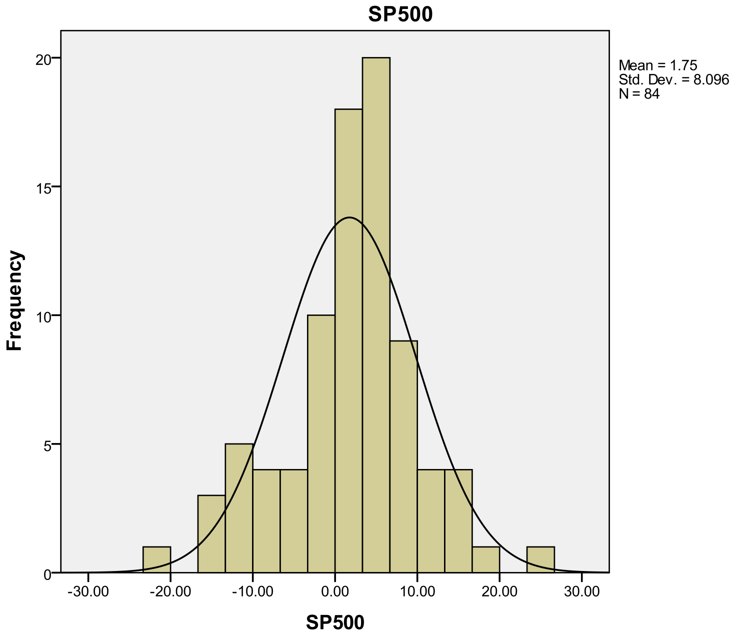


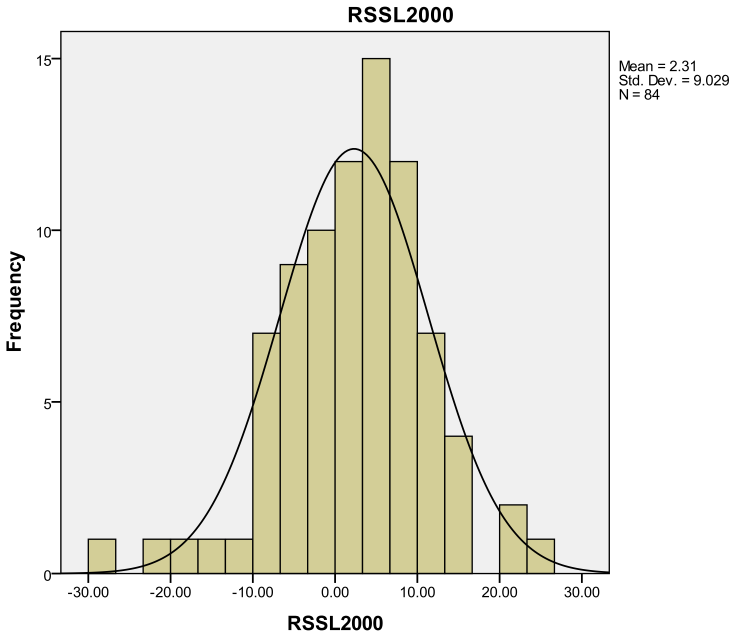
APPENDIX B. Histograms of the variables used in the theoretical portfolio (Cont'd)



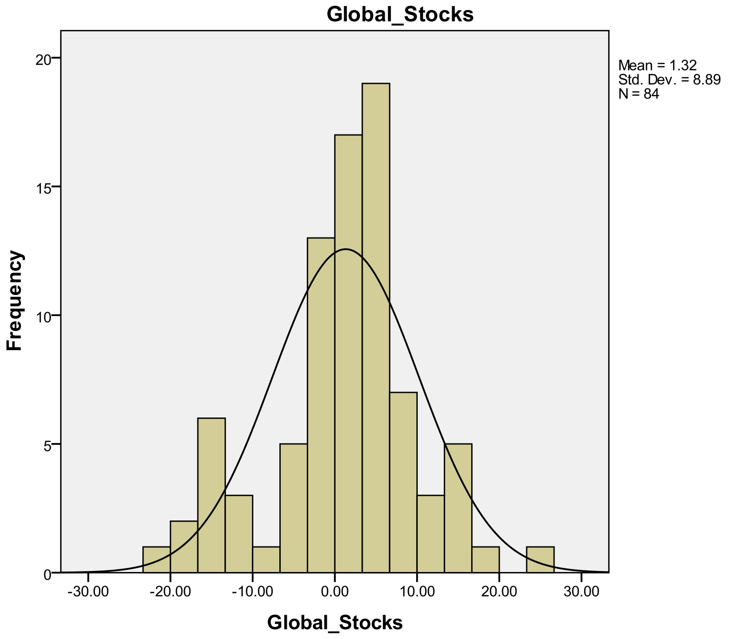


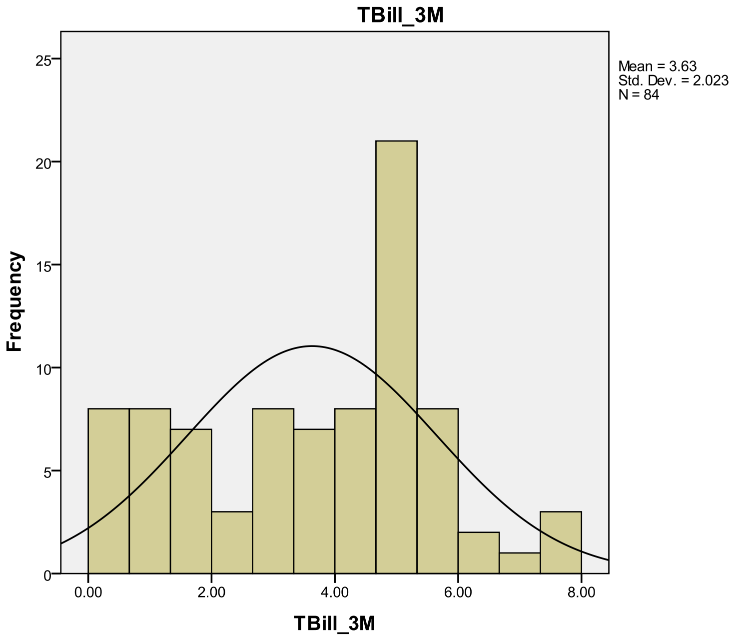
APPENDIX B. Histograms of the variables used in the theoretical portfolio (Cont'd)



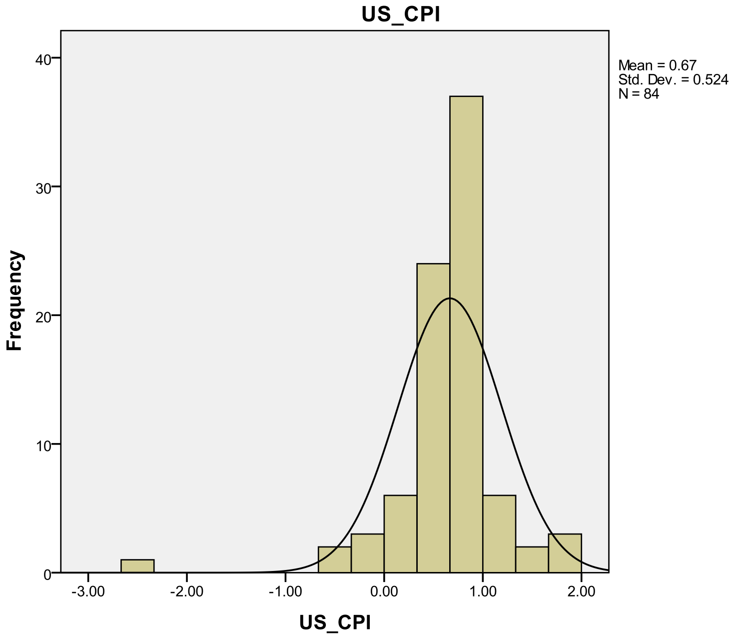


APPENDIX B. Histograms of the variables used in the theoretical portfolio (Cont'd)





APPENDIX B. Histograms of the variables used in the theoretical portfolio (Cont'd)



Appendix C. Portfolio weights with corresponding returns for the constructed portfolios

Portfolio Weights excl. Farmland & Timberland

Return	St.dev.	Farmland	Timberland	Real Estate	Townsend Fund	Large Cap	Small Cap	Global Stocks	Agriculture EQ	Food EQ
0.02	0.10	0.00	0.00	-4.17	3.65	-0.84	0.03	0.80	-0.11	-0.12
0.03	0.09	0.00	0.00	-3.68	3.22	-0.76	0.03	0.73	-0.10	-0.11
0.03	0.08	0.00	0.00	-3.18	2.80	-0.68	0.03	0.65	-0.09	-0.10
0.04	0.07	0.00	0.00	-2.69	2.38	-0.60	0.03	0.57	-0.08	-0.09
0.04	0.06	0.00	0.00	-2.19	1.96	-0.52	0.03	0.50	-0.07	-0.07
0.05	0.05	0.00	0.00	-1.70	1.53	-0.44	0.03	0.42	-0.06	-0.06
0.05	0.04	0.00	0.00	-1.20	1.11	-0.36	0.02	0.34	-0.05	-0.05
0.06	0.03	0.00	0.00	-0.71	0.69	-0.28	0.02	0.27	-0.04	-0.04
0.06	0.02	0.00	0.00	-0.22	0.26	-0.20	0.02	0.19	-0.03	-0.02
0.07	0.02	0.00	0.00	0.28	-0.16	-0.12	0.02	0.11	-0.02	-0.01
0.08	0.02	0.00	0.00	0.77	-0.58	-0.04	0.02	0.04	-0.01	0.00
0.08	0.02	0.00	0.00	1.27	-1.00	0.04	0.02	-0.04	0.00	0.01
0.09	0.03	0.00	0.00	1.76	-1.43	0.12	0.02	-0.12	0.01	0.03
0.09	0.04	0.00	0.00	2.26	-1.85	0.20	0.01	-0.19	0.02	0.04
0.10	0.04	0.00	0.00	2.75	-2.27	0.28	0.01	-0.27	0.03	0.05
0.10	0.05	0.00	0.00	3.24	-2.69	0.37	0.01	-0.35	0.04	0.06
0.11	0.06	0.00	0.00	3.74	-3.12	0.45	0.01	-0.42	0.05	0.07
0.12	0.07	0.00	0.00	4.23	-3.54	0.53	0.01	-0.50	0.06	0.09
0.12	0.08	0.00	0.00	4.73	-3.96	0.61	0.01	-0.58	0.07	0.10
0.13	0.09	0.00	0.00	5.22	-4.39	0.69	0.01	-0.65	0.08	0.11
0.13	0.10	0.00	0.00	5.72	-4.81	0.77	0.00	-0.73	0.09	0.12
0.14	0.11	0.00	0.00	6.21	-5.23	0.85	0.00	-0.81	0.10	0.14
0.14	0.12	0.00	0.00	6.70	-5.65	0.93	0.00	-0.88	0.11	0.15
0.15	0.13	0.00	0.00	7.20	-6.08	1.01	0.00	-0.96	0.12	0.16
0.15	0.14	0.00	0.00	7.69	-6.50	1.09	0.00	-1.04	0.13	0.17
0.16	0.15	0.00	0.00	8.19	-6.92	1.17	0.00	-1.11	0.14	0.19

Appendix C. Portfolio weights with corresponding returns for the constructed portfolios (Cont'd)

Portfolio Weights incl. Farmland & Timberland

Return	St.dev.	Farmland	Timberland	Real Estate	Townsend Fund	Large Cap	Small Cap	Global Stocks	Agriculture EQ	Food EQ
0.02	0.07	0.12	-0.69	-2.72	2.21	-0.43	0.09	0.43	-0.05	-0.14
0.03	0.06	0.12	-0.62	-2.40	1.95	-0.40	0.08	0.39	-0.05	-0.13
0.03	0.05	0.11	-0.55	-2.08	1.69	-0.36	0.08	0.35	-0.05	-0.12
0.04	0.05	0.11	-0.48	-1.75	1.43	-0.32	0.07	0.31	-0.04	-0.10
0.04	0.04	0.10	-0.41	-1.43	1.17	-0.28	0.06	0.28	-0.04	-0.09
0.05	0.04	0.10	-0.34	-1.11	0.92	-0.24	0.05	0.24	-0.03	-0.07
0.05	0.03	0.09	-0.27	-0.79	0.66	-0.20	0.05	0.20	-0.03	-0.06
0.06	0.03	0.09	-0.20	-0.47	0.40	-0.17	0.04	0.16	-0.03	-0.04
0.06	0.02	0.08	-0.13	-0.15	0.14	-0.13	0.03	0.13	-0.02	-0.03
0.07	0.02	0.07	-0.06	0.18	-0.12	-0.09	0.02	0.09	-0.02	-0.02
0.08	0.02	0.07	0.00	0.50	-0.38	-0.05	0.02	0.05	-0.01	0.00
0.08	0.02	0.06	0.07	0.82	-0.63	-0.01	0.01	0.01	-0.01	0.01
0.09	0.02	0.06	0.14	1.14	-0.89	0.03	0.00	-0.03	-0.01	0.03
0.09	0.03	0.05	0.21	1.46	-1.15	0.06	-0.01	-0.06	0.00	0.04
0.10	0.03	0.05	0.28	1.78	-1.41	0.10	-0.01	-0.10	0.00	0.06
0.10	0.04	0.04	0.35	2.11	-1.67	0.14	-0.02	-0.14	0.01	0.07
0.11	0.04	0.04	0.42	2.43	-1.93	0.18	-0.03	-0.18	0.01	0.08
0.12	0.05	0.03	0.49	2.75	-2.18	0.22	-0.03	-0.21	0.01	0.10
0.12	0.05	0.03	0.56	3.07	-2.44	0.26	-0.04	-0.25	0.02	0.11
0.13	0.06	0.02	0.63	3.39	-2.70	0.29	-0.05	-0.29	0.02	0.13
0.13	0.07	0.01	0.70	3.71	-2.96	0.33	-0.06	-0.33	0.03	0.14
0.14	0.07	0.01	0.77	4.04	-3.22	0.37	-0.06	-0.36	0.03	0.16
0.14	0.08	0.00	0.84	4.36	-3.48	0.41	-0.07	-0.40	0.03	0.17
0.15	0.09	0.00	0.91	4.68	-3.73	0.45	-0.08	-0.44	0.04	0.18
0.15	0.09	-0.01	0.97	5.00	-3.99	0.49	-0.09	-0.48	0.04	0.20
0.16	0.10	-0.01	0.98	5.00	-3.99	0.49	-0.09	-0.48	0.04	0.20

Appendix C. Portfolio weights with corresponding returns for the constructed portfolios (Cont'd)

Portfolio Weights incl. Farmland

Return	St.dev.	Farmland	Timberland	Real Estate	Townsend Fund	Large Cap	Small Cap	Global Stocks	Agriculture EQ	Food EQ
0.02	0.09	-0.60	0.00	-1.87	1.96	-0.75	0.04	0.71	-0.10	-0.10
0.03	0.08	-0.54	0.00	-1.63	1.72	-0.68	0.04	0.64	-0.09	-0.09
0.03	0.07	-0.47	0.00	-1.39	1.49	-0.61	0.04	0.57	-0.08	-0.08
0.04	0.06	-0.40	0.00	-1.16	1.26	-0.54	0.04	0.51	-0.08	-0.07
0.04	0.05	-0.33	0.00	-0.92	1.02	-0.47	0.03	0.44	-0.07	-0.06
0.05	0.05	-0.26	0.00	-0.69	0.79	-0.40	0.03	0.38	-0.06	-0.05
0.05	0.04	-0.20	0.00	-0.45	0.56	-0.33	0.03	0.31	-0.05	-0.04
0.06	0.03	-0.13	0.00	-0.22	0.33	-0.26	0.03	0.25	-0.04	-0.03
0.06	0.02	-0.06	0.00	0.02	0.09	-0.19	0.02	0.18	-0.03	-0.02
0.07	0.02	0.01	0.00	0.26	-0.14	-0.12	0.02	0.11	-0.02	-0.01
0.08	0.02	0.07	0.00	0.49	-0.37	-0.05	0.02	0.05	-0.01	0.00
0.08	0.02	0.14	0.00	0.73	-0.61	0.02	0.01	-0.02	0.00	0.01
0.09	0.02	0.21	0.00	0.96	-0.84	0.09	0.01	-0.08	0.00	0.02
0.09	0.03	0.28	0.00	1.20	-1.07	0.16	0.01	-0.15	0.01	0.03
0.10	0.04	0.34	0.00	1.43	-1.31	0.23	0.01	-0.21	0.02	0.04
0.10	0.05	0.41	0.00	1.67	-1.54	0.30	0.00	-0.28	0.03	0.05
0.11	0.05	0.48	0.00	1.91	-1.77	0.37	0.00	-0.35	0.04	0.06
0.12	0.06	0.55	0.00	2.14	-2.01	0.44	0.00	-0.41	0.05	0.07
0.12	0.07	0.62	0.00	2.38	-2.24	0.51	0.00	-0.48	0.06	0.08
0.13	0.08	0.68	0.00	2.61	-2.47	0.58	-0.01	-0.54	0.07	0.08
0.13	0.09	0.75	0.00	2.85	-2.71	0.65	-0.01	-0.61	0.07	0.09
0.14	0.09	0.82	0.00	3.08	-2.94	0.72	-0.01	-0.68	0.08	0.10
0.14	0.10	0.89	0.00	3.32	-3.17	0.79	-0.02	-0.74	0.09	0.11
0.15	0.11	0.95	0.00	3.56	-3.41	0.86	-0.02	-0.81	0.10	0.12
0.15	0.12	1.02	0.00	3.79	-3.64	0.93	-0.02	-0.87	0.11	0.13
0.16	0.13	1.09	0.00	4.03	-3.87	1.00	-0.02	-0.94	0.12	0.14

Appendix C. Portfolio weights with corresponding returns for the constructed portfolios (Cont'd)

Portfolio Weights incl. Timberland

Return	St.dev.	Farmland	Timberland	Real Estate	Townsend Fund	Large Cap	Small Cao	Global Stocks	Agriculture EQ	Food EQ
0.02	0.07	0.00	-0.63	-2.40	2.00	-0.45	0.09	0.44	-0.06	-0.14
0.03	0.06	0.00	-0.57	-2.09	1.75	-0.41	0.08	0.40	-0.05	-0.12
0.03	0.06	0.00	-0.50	-1.79	1.51	-0.37	0.07	0.36	-0.05	-0.11
0.04	0.05	0.00	-0.43	-1.48	1.26	-0.33	0.07	0.32	-0.04	-0.10
0.04	0.04	0.00	-0.37	-1.17	1.01	-0.29	0.06	0.29	-0.04	-0.08
0.05	0.04	0.00	-0.30	-0.86	0.76	-0.25	0.05	0.25	-0.04	-0.07
0.05	0.03	0.00	-0.23	-0.56	0.51	-0.21	0.04	0.21	-0.03	-0.05
0.06	0.03	0.00	-0.17	-0.25	0.26	-0.18	0.04	0.17	-0.03	-0.04
0.06	0.02	0.00	-0.10	0.06	0.01	-0.14	0.03	0.13	-0.02	-0.03
0.07	0.02	0.00	-0.03	0.37	-0.24	-0.10	0.02	0.09	-0.02	-0.01
0.08	0.02	0.00	0.04	0.67	-0.49	-0.06	0.02	0.06	-0.02	0.00
0.08	0.02	0.00	0.10	0.98	-0.74	-0.02	0.01	0.02	-0.01	0.02
0.09	0.02	0.00	0.17	1.29	-0.99	0.02	0.00	-0.02	-0.01	0.03
0.09	0.03	0.00	0.24	1.60	-1.24	0.06	-0.01	-0.06	0.00	0.04
0.10	0.03	0.00	0.30	1.91	-1.49	0.10	-0.01	-0.10	0.00	0.06
0.10	0.04	0.00	0.37	2.21	-1.74	0.14	-0.02	-0.13	0.01	0.07
0.11	0.04	0.00	0.44	2.52	-1.99	0.18	-0.03	-0.17	0.01	0.09
0.12	0.05	0.00	0.50	2.83	-2.24	0.21	-0.04	-0.21	0.01	0.10
0.12	0.05	0.00	0.57	3.14	-2.48	0.25	-0.04	-0.25	0.02	0.11
0.13	0.06	0.00	0.64	3.44	-2.73	0.29	-0.05	-0.29	0.02	0.13
0.13	0.07	0.00	0.70	3.75	-2.98	0.33	-0.06	-0.33	0.03	0.14
0.14	0.07	0.00	0.77	4.06	-3.23	0.37	-0.06	-0.36	0.03	0.16
0.14	0.08	0.00	0.84	4.37	-3.48	0.41	-0.07	-0.40	0.03	0.17
0.15	0.09	0.00	0.90	4.67	-3.73	0.45	-0.08	-0.44	0.04	0.18
0.15	0.09	0.00	0.97	4.98	-3.98	0.49	-0.09	-0.48	0.04	0.20
0.16	0.10	0.00	1.04	5.29	-4.23	0.53	-0.09	-0.52	0.05	0.21

Appendix D. Portfolio weights with corresponding returns for the constructed portfolios when short-sale constraints are imposed

Portfolio weights incl. Farmland and Timberland with a short-sale constraint on Farmland, Timberland and Global Stocks

Return	St.dev.	Farmland	Timberland	Real Estate	Townsend Fund	Large Cap	Small Cap	Global Stocks	Agriculture EQ	Food EQ
0.02	0.10	0.00	0.00	-4.17	3.65	-0.84	0.03	0.80	-0.11	-0.12
0.03	0.07	0.00	0.00	-2.93	2.59	-0.64	0.03	0.61	-0.09	-0.09
0.05	0.05	0.00	0.00	-1.70	1.53	-0.44	0.03	0.42	-0.06	-0.06
0.06	0.03	0.00	0.00	-0.46	0.48	-0.24	0.02	0.23	-0.04	-0.03
0.08	0.02	0.07	0.00	0.50	-0.38	-0.05	0.02	0.05	-0.01	0.00
0.09	0.02	0.05	0.19	1.34	-1.06	0.00	-0.01	0.00	0.00	0.04
0.10	0.04	0.02	0.39	2.22	-1.77	0.00	-0.03	0.00	0.01	0.10
0.12	0.05	0.00	0.58	3.08	-2.48	0.00	-0.06	0.00	0.02	0.15
0.13	0.07	0.00	0.77	3.90	-3.16	0.00	-0.08	0.00	0.03	0.20
0.15	0.09	0.00	0.96	4.72	-3.83	0.00	-0.11	0.00	0.04	0.25
0.16	0.10	0.00	1.14	5.53	-4.51	0.00	-0.14	0.00	0.05	0.30
0.17	0.12	0.00	1.33	6.35	-5.18	0.00	-0.16	0.00	0.06	0.36
0.19	0.14	0.00	1.52	7.16	-5.86	0.00	-0.19	0.00	0.07	0.41
0.20	0.16	0.00	1.71	7.97	-6.53	0.00	-0.21	0.00	0.08	0.46
0.22	0.17	0.00	1.89	8.79	-7.21	0.00	-0.24	0.00	0.09	0.51
0.23	0.19	0.00	2.08	9.60	-7.88	0.00	-0.26	0.00	0.10	0.57
0.24	0.21	0.00	2.27	10.42	-8.55	0.00	-0.29	0.00	0.12	0.62
0.26	0.22	0.00	2.45	11.23	-9.23	0.00	-0.32	0.00	0.13	0.67
0.27	0.24	0.00	2.64	12.04	-9.90	0.00	-0.34	0.00	0.14	0.72
0.29	0.26	0.00	2.83	12.86	-10.58	0.00	-0.37	0.00	0.15	0.77
0.30	0.28	0.00	3.01	13.67	-11.25	0.00	-0.39	0.00	0.16	0.83
0.31	0.29	0.00	3.20	14.49	-11.93	0.00	-0.42	0.00	0.17	0.88
0.33	0.31	0.00	3.39	15.30	-12.60	0.00	-0.45	0.00	0.18	0.93
0.34	0.33	0.00	3.57	16.11	-13.28	0.00	-0.47	0.00	0.19	0.98
0.36	0.35	0.00	3.76	16.93	-13.95	0.00	-0.50	0.00	0.20	1.03
0.37	0.36	0.00	3.95	17.74	-14.63	0.00	-0.52	0.00	0.21	1.09

Appendix D. Portfolio weights with corresponding returns for the constructed portfolios when short-sale constraints are imposed (Cont'd)

Portfolio weights incl. Farmland and Timberland with a short-sale constraint on Farmland and Timberland

Return	St.dev.	Farmland	Timberland	Real Estate	Townsend Fund	Large Cap	Small Cap	Global Stocks	Agriculture EQ	Food EQ
0.02	0.10	0.00	0.00	-4.17	3.65	-0.84	0.03	0.80	-0.11	-0.12
0.03	0.07	0.00	0.00	-2.93	2.59	-0.64	0.03	0.61	-0.09	-0.09
0.05	0.05	0.00	0.00	-1.70	1.53	-0.44	0.03	0.42	-0.06	-0.06
0.06	0.03	0.00	0.00	-0.46	0.48	-0.24	0.02	0.23	-0.04	-0.03
0.08	0.02	0.07	0.00	0.50	-0.38	-0.05	0.02	0.05	-0.01	0.00
0.09	0.02	0.06	0.18	1.30	-1.02	0.05	0.00	-0.04	0.00	0.03
0.10	0.04	0.04	0.35	2.11	-1.67	0.14	-0.02	-0.14	0.01	0.07
0.12	0.05	0.03	0.52	2.91	-2.31	0.24	-0.04	-0.23	0.02	0.11
0.13	0.07	0.01	0.70	3.71	-2.96	0.33	-0.06	-0.33	0.03	0.14
0.15	0.08	0.00	0.87	4.52	-3.61	0.43	-0.07	-0.42	0.04	0.18
0.16	0.10	0.00	1.04	5.29	-4.23	0.53	-0.09	-0.52	0.05	0.21
0.17	0.11	0.00	1.21	5.97	-4.79	0.62	-0.11	-0.61	0.06	0.25
0.19	0.13	0.00	1.37	6.83	-5.48	0.72	-0.13	-0.71	0.07	0.28
0.20	0.15	0.00	1.54	7.60	-6.10	0.82	-0.15	-0.80	0.08	0.32
0.22	0.16	0.00	1.71	8.37	-6.72	0.92	-0.16	-0.90	0.09	0.35
0.23	0.18	0.00	1.87	9.14	-7.35	1.01	-0.18	-0.99	0.10	0.39
0.24	0.19	0.00	2.04	9.91	-7.97	1.11	-0.20	-1.09	0.11	0.42
0.26	0.21	0.00	2.21	10.67	-8.59	1.21	-0.22	-1.19	0.12	0.46
0.27	0.23	0.00	2.38	11.44	-9.22	1.31	-0.24	-1.28	0.13	0.49
0.29	0.24	0.00	2.54	12.21	-9.84	1.40	-0.25	-1.38	0.14	0.53
0.30	0.26	0.00	2.71	12.98	-10.46	1.50	-0.27	-1.47	0.15	0.56
0.31	0.28	0.00	2.88	13.75	-11.09	1.60	-0.29	-1.57	0.16	0.60
0.33	0.29	0.00	3.05	14.52	-11.71	1.70	-0.31	-1.66	0.17	0.63
0.34	0.31	0.00	3.21	15.29	-12.33	1.79	-0.33	-1.76	0.18	0.67
0.36	0.32	0.00	3.38	16.06	-12.96	1.89	-0.34	-1.85	0.19	0.70
0.37	0.34	0.00	3.55	16.83	-13.58	1.99	-0.36	-1.95	0.20	0.74

Appendix D. Portfolio weights with corresponding returns for the constructed portfolios when short-sale constraints are imposed (Cont'd)

Portfolio weights incl. Farmland and Timberland with a short-sale constraint on Farmland

Return	**St.dev.**	**Farmland**	**Timberland**	**Real Estate**	**Townsend Fund**	**Large Cap**	**Small Cap**	**Global Stocks**	**Agriculture EQ**	**Food EQ**
0.02	0.07	0.12	-0.69	-2.72	2.21	-0.43	0.09	0.43	-0.05	-0.14
0.03	0.05	0.11	-0.52	-1.92	1.56	-0.34	0.07	0.33	-0.04	-0.11
0.05	0.04	0.10	-0.34	-1.11	0.92	-0.24	0.05	0.24	-0.03	-0.07
0.06	0.02	0.08	-0.17	-0.31	0.27	-0.15	0.03	0.14	-0.02	-0.04
0.08	0.02	0.07	0.00	0.50	-0.38	-0.05	0.02	0.05	-0.01	0.00
0.09	0.02	0.06	0.18	1.30	-1.02	0.05	0.00	-0.04	0.00	0.03
0.10	0.04	0.04	0.35	2.11	-1.67	0.14	-0.02	-0.14	0.01	0.07
0.12	0.05	0.03	0.52	2.91	-2.31	0.24	-0.04	-0.23	0.02	0.11
0.13	0.07	0.01	0.70	3.71	-2.96	0.33	-0.06	-0.33	0.03	0.14
0.15	0.08	0.00	0.87	4.52	-3.61	0.43	-0.07	-0.42	0.04	0.18
0.16	0.10	0.00	1.04	5.29	-4.23	0.53	-0.09	-0.52	0.05	0.21
0.17	0.11	0.00	1.21	6.06	-4.85	0.62	-0.11	-0.61	0.06	0.25
0.19	0.13	0.00	1.37	6.83	-5.48	0.72	-0.13	-0.71	0.07	0.28
0.20	0.15	0.00	1.54	7.60	-6.10	0.82	-0.15	-0.80	0.08	0.32
0.22	0.16	0.00	1.71	8.37	-6.72	0.92	-0.16	-0.90	0.09	0.35
0.23	0.18	0.00	1.87	9.14	-7.35	1.01	-0.18	-0.99	0.10	0.39
0.24	0.19	0.00	2.04	9.91	-7.97	1.11	-0.20	-1.09	0.11	0.42
0.26	0.21	0.00	2.21	10.67	-8.59	1.21	-0.22	-1.19	0.12	0.46
0.27	0.23	0.00	2.38	11.44	-9.22	1.31	-0.24	-1.28	0.13	0.49
0.29	0.24	0.00	2.54	12.21	-9.84	1.40	-0.25	-1.38	0.14	0.53
0.30	0.26	0.00	2.71	12.98	-10.46	1.50	-0.27	-1.47	0.15	0.56
0.31	0.28	0.00	2.88	13.75	-11.09	1.60	-0.29	-1.57	0.16	0.60
0.33	0.29	0.00	3.05	14.52	-11.71	1.70	-0.31	-1.66	0.17	0.63
0.34	0.31	0.00	3.21	15.29	-12.33	1.79	-0.33	-1.76	0.18	0.67
0.36	0.32	0.00	3.38	16.06	-12.96	1.89	-0.34	-1.85	0.19	0.70
0.37	0.34	0.00	3.55	16.83	-13.58	1.99	-0.36	-1.95	0.20	0.74

Appendix D. Portfolio weights with corresponding returns for the constructed portfolios when short-sale constraints are imposed (Cont'd)

Portfolio weights incl. Farmland and Timberland with a short-sale constraint on Timberland

Return	St.dev.	Farmland	Timberland	Real Estate	Townsend Fund	Large Cap	Small Cap	Global Stocks	Agriculture EQ	Food EQ
0.02	0.09	-0.60	0.00	-1.87	1.96	-0.75	0.04	0.71	-0.10	-0.10
0.03	0.07	-0.43	0.00	-1.28	1.37	-0.57	0.04	0.54	-0.08	-0.07
0.05	0.05	-0.26	0.00	-0.69	0.79	-0.40	0.03	0.38	-0.06	-0.05
0.06	0.03	-0.10	0.00	-0.10	0.21	-0.22	0.02	0.21	-0.04	-0.03
0.08	0.02	0.09	0.00	0.38	-0.29	-0.05	0.02	0.05	-0.01	0.00
0.09	0.02	0.06	0.18	1.30	-1.02	0.05	0.00	-0.04	0.00	0.03
0.10	0.04	0.04	0.35	2.11	-1.67	0.14	-0.02	-0.14	0.01	0.07
0.12	0.05	0.03	0.52	2.91	-2.31	0.24	-0.04	-0.23	0.02	0.11
0.13	0.07	0.01	0.70	3.71	-2.96	0.33	-0.06	-0.33	0.03	0.14
0.15	0.08	0.00	0.87	4.52	-3.60	0.43	-0.07	-0.42	0.04	0.18
0.16	0.10	-0.01	1.04	5.32	-4.25	0.53	-0.09	-0.52	0.05	0.21
0.17	0.11	-0.03	1.22	6.13	-4.90	0.62	-0.11	-0.61	0.06	0.25
0.19	0.13	-0.04	1.39	6.93	-5.54	0.72	-0.13	-0.70	0.07	0.28
0.20	0.15	-0.05	1.56	7.73	-6.19	0.81	-0.15	-0.80	0.08	0.32
0.22	0.16	-0.07	1.74	8.54	-6.83	0.91	-0.17	-0.89	0.09	0.35
0.23	0.18	-0.08	1.91	9.34	-7.48	1.00	-0.18	-0.99	0.10	0.39
0.24	0.19	-0.09	2.08	10.15	-8.12	1.10	-0.20	-1.08	0.11	0.43
0.26	0.21	-0.11	2.26	10.95	-8.77	1.20	-0.22	-1.18	0.12	0.46
0.27	0.23	-0.12	2.43	11.76	-9.42	1.29	-0.24	-1.27	0.13	0.50
0.29	0.24	-0.13	2.60	12.56	-10.06	1.39	-0.26	-1.36	0.14	0.53
0.30	0.26	-0.15	2.78	13.36	-10.71	1.48	-0.28	-1.46	0.15	0.57
0.31	0.28	-0.16	2.95	14.17	-11.35	1.58	-0.29	-1.55	0.16	0.60
0.33	0.29	-0.18	3.12	14.97	-12.00	1.68	-0.31	-1.65	0.17	0.64
0.34	0.31	-0.19	3.30	15.78	-12.64	1.77	-0.33	-1.74	0.18	0.68
0.36	0.32	-0.20	3.47	16.58	-13.29	1.87	-0.35	-1.84	0.19	0.71
0.37	0.34	-0.22	3.64	17.38	-13.94	1.96	-0.37	-1.93	0.20	0.75

Appendix D. Portfolio weights with corresponding returns for the constructed portfolios when short-sale constraints are imposed (Cont'd)

Portfolio weights incl. Farmland and Timberland with a short-sale constraint on Farmland and Large Cap

Return	St.dev.	Farmland	Timberland	Real Estate	Townsend Fund	Large Cap	Small Cap	Global Stocks	Agriculture EQ	Food EQ
0.02	0.07	0.19	-0.80	-3.02	2.45	0.00	0.12	0.07	-0.07	-0.21
0.03	0.06	0.16	-0.60	-2.15	1.75	0.00	0.10	0.05	-0.05	-0.16
0.05	0.04	0.13	-0.40	-1.28	1.05	0.00	0.07	0.04	-0.04	-0.11
0.06	0.02	0.11	-0.21	-0.41	0.35	0.00	0.05	0.02	-0.03	-0.06
0.08	0.02	0.08	-0.01	0.46	-0.35	0.00	0.02	0.01	-0.02	-0.01
0.09	0.02	0.06	0.18	1.30	-1.02	0.05	0.00	-0.04	0.00	0.03
0.10	0.04	0.04	0.35	2.11	-1.67	0.14	-0.02	-0.14	0.01	0.07
0.12	0.05	0.03	0.52	2.91	-2.31	0.24	-0.04	-0.23	0.02	0.11
0.13	0.07	0.01	0.70	3.71	-2.96	0.33	-0.06	-0.33	0.03	0.14
0.15	0.08	0.00	0.87	4.52	-3.61	0.43	-0.07	-0.42	0.04	0.18
0.16	0.10	0.00	1.04	5.29	-4.23	0.53	-0.09	-0.52	0.05	0.21
0.17	0.11	0.00	1.21	6.06	-4.85	0.62	-0.11	-0.61	0.06	0.25
0.19	0.13	0.00	1.37	6.83	-5.48	0.72	-0.13	-0.71	0.07	0.28
0.20	0.15	0.00	1.54	7.60	-6.10	0.82	-0.15	-0.80	0.08	0.32
0.22	0.16	0.00	1.71	8.37	-6.72	0.92	-0.16	-0.90	0.09	0.35
0.23	0.18	0.00	1.87	9.14	-7.35	1.01	-0.18	-0.99	0.10	0.39
0.24	0.19	0.00	2.04	9.91	-7.97	1.11	-0.20	-1.09	0.11	0.42
0.26	0.21	0.00	2.21	10.67	-8.59	1.21	-0.22	-1.19	0.12	0.46
0.27	0.23	0.00	2.38	11.44	-9.22	1.31	-0.24	-1.28	0.13	0.49
0.29	0.24	0.00	2.54	12.21	-9.84	1.40	-0.25	-1.38	0.14	0.53
0.30	0.26	0.00	2.71	12.98	-10.46	1.50	-0.27	-1.47	0.15	0.56
0.31	0.28	0.00	2.88	13.75	-11.09	1.60	-0.29	-1.57	0.16	0.60
0.33	0.29	0.00	3.05	14.52	-11.71	1.70	-0.31	-1.66	0.17	0.63
0.34	0.31	0.00	3.21	15.29	-12.33	1.79	-0.33	-1.76	0.18	0.67
0.36	0.32	0.00	3.38	16.06	-12.96	1.89	-0.34	-1.85	0.19	0.70
0.37	0.34	0.00	3.55	16.83	-13.58	1.99	-0.36	-1.95	0.20	0.74

Appendix D. Portfolio weights with corresponding returns for the constructed portfolios when short-sale constraints are imposed (Cont'd)

Portfolio weights incl. Farmland and Timberland with a short-sale constraint on Farmland and Small Cap

Return	St.dev.	Farmland	Timberland	Real Estate	Townsend Fund	Large Cap	Small Cap	Global Stocks	Agriculture EQ	Food EQ
0.02	0.07	0.12	-0.69	-2.72	2.21	-0.43	0.09	0.43	-0.05	-0.14
0.03	0.05	0.11	-0.52	-1.92	1.56	-0.34	0.07	0.33	-0.04	-0.11
0.05	0.04	0.10	-0.34	-1.11	0.92	-0.24	0.05	0.24	-0.03	-0.07
0.06	0.02	0.08	-0.17	-0.31	0.27	-0.15	0.03	0.14	-0.02	-0.04
0.08	0.02	0.07	0.00	0.50	-0.38	-0.05	0.02	0.05	-0.01	0.00
0.09	0.02	0.06	0.18	1.30	-1.02	0.05	0.00	-0.04	0.00	0.03
0.10	0.04	0.05	0.35	2.11	-1.68	0.15	0.00	-0.15	0.00	0.06
0.12	0.05	0.03	0.52	2.92	-2.33	0.25	0.00	-0.25	0.01	0.09
0.13	0.07	0.02	0.69	3.73	-2.98	0.36	0.00	-0.35	0.01	0.11
0.15	0.08	0.01	0.86	4.56	-3.65	0.46	0.00	-0.45	0.02	0.14
0.16	0.10	0.00	1.02	5.35	-4.30	0.57	0.00	-0.55	0.02	0.17
0.17	0.12	0.00	1.19	6.15	-4.94	0.67	0.00	-0.66	0.03	0.19
0.19	0.13	0.00	1.36	6.83	-5.51	0.78	0.00	-0.77	0.04	0.22
0.20	0.15	0.00	1.52	7.70	-6.21	0.88	0.00	-0.86	0.04	0.24
0.22	0.16	0.00	1.68	8.49	-6.84	0.99	0.00	-0.97	0.05	0.27
0.23	0.18	0.00	1.85	9.27	-7.48	1.09	0.00	-1.07	0.05	0.30
0.24	0.20	0.00	2.01	10.05	-8.12	1.20	0.00	-1.17	0.06	0.32
0.26	0.21	0.00	2.18	10.83	-8.75	1.30	0.00	-1.28	0.07	0.35
0.27	0.23	0.00	2.34	11.61	-9.39	1.41	0.00	-1.38	0.07	0.37
0.29	0.25	0.00	2.50	12.40	-10.02	1.51	0.00	-1.48	0.08	0.40
0.30	0.26	0.00	2.67	13.18	-10.66	1.62	0.00	-1.58	0.08	0.43
0.31	0.28	0.00	2.83	13.96	-11.30	1.72	0.00	-1.69	0.09	0.45
0.33	0.30	0.00	3.00	14.74	-11.93	1.83	0.00	-1.79	0.10	0.48
0.34	0.31	0.00	3.16	15.53	-12.57	1.93	0.00	-1.89	0.10	0.50
0.36	0.33	0.00	3.33	16.31	-13.21	2.04	0.00	-2.00	0.11	0.53
0.37	0.34	0.00	3.49	17.09	-13.84	2.14	0.00	-2.10	0.11	0.56

Appendix D. Portfolio weights with corresponding returns for the constructed portfolios when short-sale constraints are imposed (Cont'd)

Portfolio weights incl. Farmland and Timberland with a short-sale constraint on Farmland and Global Stocks

Return	St.dev.	Farmland	Timberland	Real Estate	Townsend Fund	Large Cap	Small Cap	Global Stocks	Agriculture EQ	Food EQ
0.02	0.07	0.04	-0.66	-2.41	1.99	-0.45	0.09	0.43	-0.06	-0.15
0.03	0.05	0.11	-0.52	-1.92	1.56	-0.34	0.07	0.33	-0.04	-0.11
0.05	0.04	0.10	-0.34	-1.11	0.92	-0.24	0.05	0.24	-0.03	-0.07
0.06	0.02	0.08	-0.17	-0.31	0.27	-0.15	0.03	0.14	-0.02	-0.04
0.08	0.02	0.07	0.00	0.50	-0.38	-0.05	0.02	0.05	-0.01	0.00
0.09	0.02	0.05	0.19	1.34	-1.06	0.00	-0.01	0.00	0.00	0.04
0.10	0.04	0.02	0.39	2.22	-1.77	0.00	-0.03	0.00	0.01	0.10
0.12	0.05	0.00	0.58	3.08	-2.48	0.00	-0.06	0.00	0.02	0.15
0.13	0.07	0.00	0.77	3.90	-3.16	0.00	-0.08	0.00	0.03	0.20
0.15	0.09	0.00	0.96	4.72	-3.83	0.00	-0.11	0.00	0.04	0.25
0.16	0.10	0.00	1.14	5.53	-4.51	0.00	-0.14	0.00	0.05	0.30
0.17	0.12	0.00	1.33	6.35	-5.18	0.00	-0.16	0.00	0.06	0.36
0.19	0.14	0.00	1.52	7.16	-5.86	0.00	-0.19	0.00	0.07	0.41
0.20	0.16	0.00	1.71	7.97	-6.53	0.00	-0.21	0.00	0.08	0.46
0.22	0.17	0.00	1.89	8.79	-7.21	0.00	-0.24	0.00	0.09	0.51
0.23	0.19	0.00	2.08	9.60	-7.88	0.00	-0.26	0.00	0.10	0.57
0.24	0.21	0.00	2.27	10.42	-8.55	0.00	-0.29	0.00	0.12	0.62
0.26	0.22	0.00	2.45	11.23	-9.23	0.00	-0.32	0.00	0.13	0.67
0.27	0.24	0.00	2.64	12.04	-9.90	0.00	-0.34	0.00	0.14	0.72
0.29	0.26	0.00	2.83	12.86	-10.58	0.00	-0.37	0.00	0.15	0.77
0.30	0.28	0.00	3.01	13.67	-11.25	0.00	-0.39	0.00	0.16	0.83
0.31	0.29	0.00	3.20	14.49	-11.93	0.00	-0.42	0.00	0.17	0.88
0.33	0.31	0.00	3.39	15.30	-12.60	0.00	-0.45	0.00	0.18	0.93
0.34	0.33	0.00	3.57	16.11	-13.28	0.00	-0.47	0.00	0.19	0.98
0.36	0.35	0.00	3.76	16.93	-13.95	0.00	-0.50	0.00	0.20	1.03
0.37	0.36	0.00	3.95	17.74	-14.63	0.00	-0.52	0.00	0.21	1.09

Appendix D. Portfolio weights with corresponding returns for the constructed portfolios when short-sale constraints are imposed (Cont'd)

Portfolio weights incl. Farmland and Timberland with a short-sale constraint on Timberland and Large Cap

Return	St.dev.	Farmland	Timberland	Real Estate	Townsend Fund	Large Cap	Small Cap	Global Stocks	Agriculture EQ	Food EQ
0.02	0.10	-0.69	0.00	-2.19	2.35	0.00	0.09	0.09	-0.14	-0.21
0.03	0.07	-0.50	0.00	-1.53	1.67	0.00	0.07	0.07	-0.11	-0.16
0.05	0.05	-0.31	0.00	-0.86	1.00	0.00	0.06	0.05	-0.08	-0.11
0.06	0.03	-0.12	0.00	-0.20	0.32	0.00	0.04	0.03	-0.05	-0.06
0.08	0.02	0.07	0.00	0.47	-0.35	0.00	0.02	0.01	-0.02	-0.01
0.09	0.02	0.06	0.18	1.30	-1.02	0.05	0.00	-0.04	0.00	0.03
0.10	0.04	0.04	0.35	2.11	-1.67	0.14	-0.02	-0.14	0.01	0.07
0.12	0.05	0.03	0.52	2.91	-2.31	0.24	-0.04	-0.23	0.02	0.11
0.13	0.07	0.01	0.70	3.71	-2.96	0.33	-0.06	-0.33	0.03	0.14
0.15	0.08	0.00	0.87	4.52	-3.60	0.43	-0.07	-0.42	0.04	0.18
0.16	0.10	-0.01	1.04	5.32	-4.25	0.53	-0.09	-0.52	0.05	0.21
0.17	0.11	-0.03	1.22	6.13	-4.90	0.62	-0.11	-0.61	0.06	0.25
0.19	0.13	-0.04	1.39	6.93	-5.54	0.72	-0.13	-0.70	0.07	0.28
0.20	0.15	-0.05	1.56	7.73	-6.19	0.81	-0.15	-0.80	0.08	0.32
0.22	0.16	-0.07	1.74	8.54	-6.83	0.91	-0.17	-0.89	0.09	0.35
0.23	0.18	-0.08	1.91	9.34	-7.48	1.00	-0.18	-0.99	0.10	0.39
0.24	0.19	-0.09	2.08	10.15	-8.12	1.10	-0.20	-1.08	0.11	0.43
0.26	0.21	-0.11	2.26	10.95	-8.77	1.20	-0.22	-1.18	0.12	0.46
0.27	0.23	-0.12	2.43	11.76	-9.42	1.29	-0.24	-1.27	0.13	0.50
0.29	0.24	-0.13	2.60	12.56	-10.06	1.39	-0.26	-1.36	0.14	0.53
0.30	0.26	-0.15	2.78	13.36	-10.71	1.48	-0.28	-1.46	0.15	0.57
0.31	0.28	-0.16	2.95	14.17	-11.35	1.58	-0.29	-1.55	0.16	0.60
0.33	0.29	-0.18	3.12	14.97	-12.00	1.68	-0.31	-1.65	0.17	0.64
0.34	0.31	-0.19	3.30	15.78	-12.64	1.77	-0.33	-1.74	0.18	0.68
0.36	0.32	-0.20	3.47	16.58	-13.29	1.87	-0.35	-1.84	0.19	0.71
0.37	0.34	-0.22	3.64	17.38	-13.94	1.96	-0.37	-1.93	0.20	0.75

Appendix D. Portfolio weights with corresponding returns for the constructed portfolios when short-sale constraints are imposed (Cont'd)

Portfolio weights incl. Farmland and Timberland with a short-sale constraint on Timberland and Small Cap

Return	St.dev.	Farmland	Timberland	Real Estate	Townsend Fund	Large Cap	Small Cap	Global Stocks	Agriculture EQ	Food EQ
0.02	0.09	-0.60	0.00	-1.87	1.96	-0.75	0.04	0.71	-0.10	-0.10
0.03	0.07	-0.43	0.00	-1.28	1.37	-0.57	0.04	0.54	-0.08	-0.07
0.05	0.05	-0.26	0.00	-0.69	0.79	-0.40	0.03	0.38	-0.06	-0.05
0.06	0.03	-0.10	0.00	-0.10	0.21	-0.22	0.02	0.21	-0.04	-0.03
0.08	0.02	0.07	0.00	0.50	-0.38	-0.05	0.02	0.05	-0.01	0.00
0.09	0.02	0.06	0.18	1.30	-1.02	0.05	0.00	-0.04	0.00	0.03
0.10	0.04	0.05	0.35	2.11	-1.68	0.15	0.00	-0.15	0.00	0.06
0.12	0.05	0.03	0.52	2.92	-2.33	0.25	0.00	-0.25	0.01	0.09
0.13	0.07	0.02	0.69	3.73	-2.98	0.36	0.00	-0.35	0.01	0.11
0.15	0.08	0.01	0.85	4.54	-3.64	0.46	0.00	-0.45	0.02	0.14
0.16	0.10	0.00	1.02	5.35	-4.29	0.57	0.00	-0.55	0.02	0.17
0.17	0.12	-0.01	1.19	6.16	-4.95	0.67	0.00	-0.66	0.03	0.19
0.19	0.13	-0.02	1.36	6.97	-5.60	0.77	0.00	-0.76	0.04	0.22
0.20	0.15	-0.03	1.53	7.78	-6.26	0.88	0.00	-0.86	0.04	0.25
0.22	0.16	-0.04	1.70	8.59	-6.91	0.98	0.00	-0.96	0.05	0.27
0.23	0.18	-0.05	1.87	9.40	-7.56	1.09	0.00	-1.06	0.05	0.30
0.24	0.20	-0.06	2.04	10.21	-8.22	1.19	0.00	-1.17	0.06	0.32
0.26	0.21	-0.07	2.21	11.02	-8.87	1.29	0.00	-1.27	0.06	0.35
0.27	0.23	-0.08	2.38	11.83	-9.53	1.40	0.00	-1.37	0.07	0.38
0.29	0.25	-0.09	2.55	12.64	-10.18	1.50	0.00	-1.47	0.08	0.40
0.30	0.26	-0.11	2.72	13.45	-10.84	1.60	0.00	-1.57	0.08	0.43
0.31	0.28	-0.12	2.89	14.26	-11.49	1.71	0.00	-1.68	0.09	0.46
0.33	0.30	-0.13	3.05	15.07	-12.14	1.81	0.00	-1.78	0.09	0.48
0.34	0.31	-0.14	3.22	15.88	-12.80	1.92	0.00	-1.88	0.10	0.51
0.36	0.33	-0.15	3.39	16.69	-13.45	2.02	0.00	-1.98	0.10	0.54
0.37	0.34	-0.16	3.56	17.50	-14.11	2.12	0.00	-2.08	0.11	0.56

Appendix D. Portfolio weights with corresponding returns for the constructed portfolios when short-sale constraints are imposed (Cont'd)

Portfolio weights incl. Farmland and Timberland with a short-sale constraint on Timberland and Global Stocks

Return	St.dev.	Farmland	Timberland	Real Estate	Townsend Fund	Large Cap	Small Cap	Global Stocks	Agriculture EQ	Food EQ
0.02	0.09	-0.60	0.00	-1.87	1.96	-0.75	0.04	0.71	-0.10	-0.10
0.03	0.07	-0.43	0.00	-1.28	1.37	-0.57	0.04	0.54	-0.08	-0.07
0.05	0.05	-0.26	0.00	-0.69	0.79	-0.40	0.03	0.38	-0.06	-0.05
0.06	0.03	-0.10	0.00	-0.10	0.21	-0.22	0.02	0.21	-0.04	-0.03
0.08	0.02	0.09	0.00	0.38	-0.29	-0.05	0.02	0.05	-0.01	0.00
0.09	0.02	0.05	0.19	1.34	-1.06	0.00	-0.01	0.00	0.00	0.04
0.10	0.04	0.02	0.39	2.22	-1.77	0.00	-0.03	0.00	0.01	0.10
0.12	0.05	-0.01	0.59	3.11	-2.49	0.00	-0.06	0.00	0.02	0.15
0.13	0.07	-0.03	0.79	3.99	-3.21	0.00	-0.08	0.00	0.03	0.20
0.15	0.09	-0.06	0.98	4.87	-3.93	0.00	-0.11	0.00	0.04	0.25
0.16	0.10	-0.09	1.18	5.76	-4.65	0.00	-0.14	0.00	0.05	0.31
0.17	0.12	-0.12	1.38	6.64	-5.37	0.00	-0.16	0.00	0.06	0.36
0.19	0.14	-0.14	1.58	7.53	-6.09	0.00	-0.19	0.00	0.07	0.41
0.20	0.16	-0.17	1.78	8.41	-6.81	0.00	-0.21	0.00	0.08	0.47
0.22	0.17	-0.20	1.98	9.30	-7.52	-0.01	-0.24	0.00	0.09	0.52
0.23	0.19	-0.23	2.18	10.18	-8.24	-0.01	-0.27	0.00	0.10	0.57
0.24	0.21	-0.26	2.37	11.06	-8.96	-0.01	-0.29	0.00	0.11	0.63
0.26	0.22	-0.28	2.57	11.95	-9.68	-0.01	-0.32	0.00	0.12	0.68
0.27	0.24	-0.31	2.77	12.83	-10.40	-0.01	-0.35	0.00	0.13	0.73
0.29	0.26	-0.34	2.97	13.72	-11.12	-0.01	-0.37	0.00	0.14	0.78
0.30	0.28	-0.37	3.17	14.60	-11.84	-0.01	-0.40	0.00	0.15	0.84
0.31	0.29	-0.39	3.37	15.48	-12.56	-0.01	-0.42	0.00	0.16	0.89
0.33	0.31	-0.42	3.57	16.37	-13.27	-0.01	-0.45	0.00	0.17	0.94
0.34	0.33	-0.45	3.77	17.25	-13.99	-0.01	-0.48	0.00	0.18	1.00
0.36	0.34	-0.48	3.96	18.14	-14.71	-0.01	-0.50	0.00	0.19	1.05
0.37	0.36	-0.50	4.16	19.02	-15.43	-0.01	-0.53	0.00	0.20	1.10

Appendix E. Portfolio weights with corresponding returns for the constructed portfolios when short-sale constraints are imposed on all the asset classes

Portfolio weights with a short-sale constraint on all asset classes

Return	St.dev.	Farmland	Timberland	Real Estate	Townsend Fund	Large Cap	Small Cap	Global Stocks	Agriculture EQ	Food EQ
0.010		0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
0.015		0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
0.020		0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
0.025		0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
0.030		0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
0.035		0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
0.040		0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
0.045		0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
0.050	0.177	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
0.055	0.120	0.00	0.00	0.00	0.37	0.00	0.00	0.63	0.00	0.00
0.060	0.063	0.00	0.00	0.18	0.75	0.00	0.00	0.07	0.00	0.00
0.065	0.055	0.00	0.00	0.73	0.22	0.00	0.00	0.06	0.00	0.00
0.070	0.048	0.00	0.05	0.90	0.00	0.00	0.00	0.03	0.00	0.01
0.075	0.043	0.05	0.12	0.78	0.00	0.00	0.00	0.02	0.00	0.03
0.080	0.040	0.11	0.18	0.65	0.00	0.00	0.00	0.01	0.00	0.05
0.085	0.039	0.16	0.23	0.53	0.00	0.01	0.00	0.00	0.00	0.07
0.090	0.040	0.22	0.29	0.39	0.00	0.00	0.00	0.00	0.00	0.09
0.095	0.043	0.28	0.34	0.27	0.00	0.00	0.00	0.00	0.01	0.10
0.100	0.048	0.33	0.39	0.13	0.00	0.00	0.00	0.00	0.03	0.12
0.105	0.053	0.38	0.44	0.00	0.00	0.00	0.00	0.00	0.04	0.13
0.110	0.066	0.05	0.72	0.00	0.00	0.00	0.00	0.00	0.11	0.12
0.115	0.105	0.00	0.59	0.00	0.00	0.00	0.00	0.00	0.41	0.00
0.120	0.188	0.00	0.18	0.00	0.00	0.00	0.00	0.00	0.82	0.00
0.125	0.229	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
0.130		0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
0.135		0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00

Appendix F. Portfolio returns for the top-10 worst quarters based on the Large Cap for a 5% standard deviation

Top 10 worst quarters of the large cap analysis for a 5% STDEV				
	Worst Quarter Top-10	**Portfolio Return**	**Worst Quarter Top-10**	**Portfolio Return**
With F&T	1: 2009 Q1	-6.65	6: 2010 Q3	1.32
With T		-8.17		1.48
With F		-7.95		1.68
Without F&T		-9.23		3.79
With F&T	2: 2001 Q4	2.52	7: 2002 Q4	1.23
With T		-1.93		0.62
With F		-0.37		1.31
Without F&T		0.55		1.87
With F&T	3: 2002 Q3	-3.87	8: 1990 Q4	0.55
With T		-3.33		0.32
With F		-3.08		-0.67
Without F&T		-0.27		-0.40
With F&T	4: 1998 Q4	2.69	9: 2009 Q2	-0.50
With T		2.43		-2.14
With F		2.53		-1.42
Without F&T		4.25		-3.17
With F&T	5: 2001 Q2	0.31	10: 2008 Q4	-7.12
With T		0.01		-8.51
With F		2.50		-6.86
Without F&T		1.70		-9.21