A new model of a market maker

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Abstract

Financial instruments are bought and sold at a financial market. Market makers at a financial market act as intermediaries between the buyer and the seller of the financial instrument. Market makers have the power to quote the bid price and the ask price. This price-setting process is called market-making. With market maker models this price-setting proces of the market maker is studied. To create liquidity in the market the market maker must trade immediately if an order arrives. The market maker bears risk because he does not have optimal portfolios. To protect himself against the losses he quotes the ask price higher than the bid price. The difference between the bid price and the ask price is called the bid-ask spread. The main causes of emergence of bid-ask spread are the fixed costs, the inventory costs and the adverse selection costs. Fixed costs are costs arising from order execution. Inventory costs arise from holding securities in inventory. Adverse selection costs arise from trading with traders who have superior information. In early financial literature the bid-ask spread is modelled with a regression model. Later market makers models are used to study the price setting mechanism of the market maker. There are two types of market makers models: inventory-based models and information-based models. In inventory-based models the behaviour of the market maker as inventory-holder and the inventory cost are studied. In information-based models the adverse selection problem between the market maker and the informed trader is studied. The informed trader has superior information than the market maker, which is why the market maker has adverse selection cost if he trades with the informed trader. Two examples of information based models are the model of Glosten and Milgrom and the Das market maker's model. The market maker of Glosten and Milgrom uses Baysian learning to learn the fundamental value of the underlying security. The market maker of Das expands the Glosten and Milgrom model by keeping a probabilistic density distribution of the fundamental value. We study the market maker's behaviour with a linear pricing strategy and introduce a market maker with bid-ask spread location detection and fundamental value approximation capability. We compare the Das market maker with our market maker with the mean of bid-ask spreads method and the sum of differences between the fundamental value and the bid and ask price method.

Keywords: Market maker's model, market-making, bid-ask spread, order flow, fundamental value, trading probabilities

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Chapter 1

Introduction

Every day people hear news from the financial world. Prices of securities like stocks, bonds or other financial instruments can go up or can go down. Stocks are units of ownership of a company. Bonds are units of ownership of debts of a company. These financial instruments are traded in a financial market. If you want to buy or sell stocks you go to a stockbroker and the stockbroker goes to a dealer to buy or sell stocks for you. Dealers in over-the-counter market are called market makers and on the exchanges they are called specialists. Exchanges are central places where the trading takes place. Exchanges have highly standardised organisational structures and trading rules. Over-the-counter markets have less standardised organisational structures and are strictly traded between two parties. Market makers or specialists are persons or companies that maintain the bid and ask prices for a given stock and process trading orders.

Financial market microstructure is the branch of finance that deals with the organisational structure of financial markets and the rules of trading. Important issues of study are market structure and design, price formation and design, transaction cost and timing cost, information and disclosure, trading behaviour of market participants. There are three classifications of markets: order-driven markets, quote-driven markets and hybrid markets. In order-driven markets the brokers match investor's buy and sell orders, but takes no own position in the traded security. The prices are determined when the orders are placed or thereafter. Market liquidity comes from the continuous flow of orders transmitted by investors. In quote-driven markets designated market makers act as intermediaries and maintain the bid and ask prices by standing ready at any moment to trade at publicly quoted prices. The ask price is higher than or equal to the bid price. This difference between the quoted prices is called the bid-ask spread. The bid price is the price that the market maker is willing to buy the stock. The ask price is the price that the market maker is willing to sell the stock. The market makers provide market liquidity who trade for their own account by always executing the investor's buy and sell orders. The size of the bid-ask spread is a measure of liquidity of the traded stock. More frequently traded stocks have smaller spreads than less frequently traded stocks. Each stock has its own fundamental value which is unknown to the market participants. In hybrid markets components of order-driven and quote-driven market systems are combined. Good works written about financial market microstructure are [24], [25], [26] and [21].

1.1 Major contribution

Why is it important to conduct research to how the market makers do the market making? Because it is important to research the trading behaviour and the pricing mechanism of the market maker. The market maker plays a big role in the financial market. It is also important to know if the trading behaviour of the investors has an effect on the trading behaviour and the price strategies of the market maker. It is also relevant to study the order flow related to the behaviour and price formation of the market maker, because the market maker does not know the real fundamental value. The fundamental value is the value of a security contained in the security itself. This value is usually calculated as all the future income generated by the security. We want to learn which methods the market maker can use to estimate the fundamental value in the market maker's model. We learn how to build a market maker model using an existing market maker's model environment studied by others [1][11][10]. We begin simple first and then gradually making the model more complex. We begin with a simple linear bid and ask pricing rule with the fundamental value in the formula. This indicates that in this model it is assumed that the market maker knows the fundamental value of the security. We model the market maker in such a way that he learns the fundamental value. The market maker learns the place of bid-ask spread with respect to the fundamental value. If the location is determined then the market maker can increase the price or decrease the price to bring the price nearer to the fundamental value. To calculate the location we try first the method where the maximum of the probabilities of the order types of the traders determines the location and then we try the method where we use the density probability distribution of the order flow of the traders to determine the location. We learn how to evaluate the model. We learn how to do simulations with the market maker's models and how to compare the models. We learn how to set up the experiments to compare and analyse the performance between the market maker's models. We use mean of bid-ask spreads and sum of differences between fundamental value and the prices as our performance measure.

1.2 Research questions

These are the research question of our study:

- **How well will the new market maker's trading strategy perform?**
- How can the market maker use the order flow information?
- How can the bid and ask prices be updated if the location of the bid-ask spread with respect to the fundamental value is determined?
- How can the market maker know there is a jump in the fundamental value?
- How to compare and measure the performance between the market makers with different market making strategies?

1.3 Methology

We want to know how a market maker does the market-making. Market-making is the business of quoting bid and ask prices to create a market so that buyers and sellers can trade. We experiment with several market making strategies. We use simulatation as our scientific research method. First we introduce a simple market maker who updates his prices according to the last order executed. If the order is a buy order the ask price is updated, if the order is a sell order the bid price is updated and if there is no order placed then both the ask price and the bid price are updated. Second we introduce another market maker who initially calculates the location of the bid-ask spread with respect to the fundamental value and then updates the prices according to the last arrived order. To calculate the location of bid-ask spread we use the probabilities of executed order types. First the maximum value determines the location, then we use the value that is closest to the probability density distribution of the executed order types. Because the real fundamental value is not known to the market maker, we divide the whole simulation time in short same-sized periods to calculate the probability density distribution. The two models will be compared with the model of Das [11]. The market maker of Das maintains a probability density function of all possible fundamental values to update the bid and ask prices. To control the experiments with the same inputs, we generate exogeneously the fundamental values and a sequence of type of traders and other independent variables we use are in all the three models the same. We measure the performance of the market makers with the mean of bid-ask spreads method and the sum of differences between the fundamental value and the bid and ask prices method. All the market makers are modelled in an environment like that of the Glosten and Milgrom model [1]. The models are discrete in time and the simulations will be modelled in Matlab. Matlab is a high-level language environment that allows performing computer simulations interactively.

1.4 Organisation of the thesis

The following is the outline of this thesis. In chapter 2 we write about financial microstructure and different market maker's models in the financial literature. In chapter 3 we discuss the market maker of the Das model in detail. In chapter 4 we introduce the market maker with a simple market making strategy and the market maker who first calculates the location of the bid-ask spread with respect to the fundamental value and then updates the bid and ask prices. We will find methods to approximate the fundamental value. In chapter 5 we discuss our implementation of the three models and the experimental setups. In chapter 6 we present the results of our experiments. In chapter 7 we draw conclusions and propose directions for further research.

Chapter 2

Financial market microstructure literature

In early financial market microstructure literature the bid-ask spread is determined by a regression model. A regression model is used to explain the relationship of one variable to one or several other variables. An example of such regression model to explain the bid-ask spread is as follows:

$$s_i = \alpha + \beta_1 \ln(M_i) + \beta_2(1/p_i) + \beta_3\sigma_i + \beta_4 \ln(V_i) + \epsilon \qquad (2.1)$$

with

- s_i : average (percentage) bid-ask spread for company i.
- M_i : market capitalization
- p_i : security price level
- σ_i: volatility of the security price
- V_i : trading volume

Studies like [31] and [30] explain that the main causes of bid-ask spread are the fixed costs, adverse selection costs and inventory costs. Fixed costs are costs arising from order execution like administrative cost and compensation for the time of the market maker. Adverse selection costs exist due to assymmetric information between the market maker and informed traders. Uninformed traders have only publicly available information and informed traders have publicly available and some private information. Inventory costs originate from holding unwanted risky securities in inventory by the market maker. The market maker wants to compensate the cost by the bid-ask spread. The studies also show that bid-ask spread is negatively correlated with the security price level and the trading volume and the market capitalization. That means that the bid-ask spread is becoming higher if the security price level or the trading volume of the security or the market capitalization is becoming lower. The market capitalization is the total value of the outstanding shares hold by investors in the company. The reason that the bid-ask spread is negatively correlated with the security price level is that the fixed cost which is fixed for the most part and so per unit price the fixed cost is higher for lower priced securities than the fixed cost for higher priced securities. The reason that the bid-ask spread is negatively correlated with the trading volume is that higher trading volume means that the market maker is less necessary to hold a inventory, so the inventory cost is lower and hence lower bid-ask spread. The reason that the bid-ask spread is negatively correlated with the market capitalization is that lower market capitalization means lower liquidity, so the market maker has to hold securities in inventory. That means higher inventory cost and result is a higher bid-ask spread. And the studies show that the bid-ask spread is positively correlated with the volatility of the security price. That means that the bid-ask spread is becoming lower if the volatility of the security price is becoming higher. The volatility of a security price is the rate that indicates how rapidly the price is going to change over a certain period of time. The reason that the bid-ask spread is positively correlated with the volatility of the security price is that the adverse selection cost is higher for more volatile securities than for less volatile securities. There will be more informed traders willing to trade in volatile securities, because they have superior information. So the bid-ask spread is higher due to higher adverse selection cost, because the market maker has to trade with more informed traders.

2.1 Market maker's models

Two kinds of models have been developed in the literature: inventory-based models which specifically deal with issues surrounding the inventory cost of the market maker for having securities in inventory and information-based models which specifically deal with issues surrounding the adverse selection problem between the informed traders and the market maker. The adverse selection problem arises in a trade between two participants with different levels of information. Chapter 5 of [21] and chapter 2 of [26] discussed about inventory-based models. Chapter 3 and 4 of [21] and [26] discussed about information-based models.

2.1.1 Inventory-based models

In inventory-based models the focus is on a market maker as the provider of market liquidity. The market maker has to trade immediately when an order is placed. Because the buy volume and sell volume differ in each trade the market maker has to hold inventory of securities. The inventory may deviate from the inventory that the market maker wants. Such market maker doesn't want to hold risky portfolios that does not have an optimal expected return given the risk he wants to take. Moreover the market maker has to bare the risk of price changes. The market maker is risk-averse. He prefers low risk securities above high risk securities. The market maker can control the inventory by trading with traders to get back to the portfolio he wants if his portfolio deviates away from the portfolio he wants. That is why the market maker has inventory cost. A modelling approach by Garman [17] to this inventory control problem is done by assuming that the market maker does not want to go bankrupt. The market maker can hold cash or securities in inventory. The market maker goes bankrupt if he has no money or securities in inventory. The goal of the market maker is to maximize the expected profit in each trade. The market maker sets the bid price and the ask price in the beginning of the trading game or simulation. The buy and sell orders are modelled as random Poisson distributions. A Poisson distribution is the probability distribution of a Poisson random variable. A Poisson random variable is the number of times an event takes place during a fixed time period. The event is in this case the placement of buy order or a sell order by traders. The Poisson distribution probabilities can calculated with

$$P(\text{order placed} = x) = \frac{e^{-\mu}\mu^x}{x!} \quad x = 0, 1, 2, ... \tag{2.2}$$

where e is the natural base of logarithms, μ is the expected value of the Poisson distribution and x is the number of orders placed (buy or sell). The inventories of the market maker are modelled as:

$$I_C(t) = I_C(0) + P_a N_a(t) - P_b N_b(t) \tag{2.3}$$

$$I_S(t) = I_S(0) + N_b(t) - N_a(t) \tag{2.4}$$

where $I_C(y)$ is the cash inventory at time t, $I_S(t)$ is the security inventory at time t, $I_C(0)$ and $I_S(0)$ are the begin values in the inventory, P_b is the bid price, P_a is the ask price, $N_a(t)$ is the total executed buy volume at time t and $N_b(t)$ is the total executed sell volume at time t. The goal of Garman's study is to calculate the expected time to bankruptcy. This approach is not realistic because the market maker quotes bid and ask prices only at the beginning of the trading game, so the market maker's price setting decision based not on his inventory.

A more realistic approach is done by Ho and Stoll [18]. The market maker in this model is also a risk-averse liquidity provider. The market maker is assumed that he cannot go banktrupt. The market maker allows temporarily order imbalances, this means that the buy orders and the sell orders do not match. He uses an inventory of securities to do this. The excess of securities is hold in inventory. The market maker maximizes expected utility of total wealth at the end of the trading game. In economics the expected utility is the measure of satisfaction for a person if he has to make choices in an uncertain environment. The buy and sell order flow are modelled as Poisson distributions. The market maker sets the bid and ask prices in reaction to inventory changes and wealth changes. Ho and Stoll solved this trading game using dynamic programming. Dynamic programming is an optimization technique that works as follows: first the problem is divided into a number of subproblems. Each subproblems is then solved with the smallest subproblem solved first. The solutions to the subproblems contribute further to the solution of the whole problem.

Both the model of Garman and the model of Ho and Stoll show that the ask price is higher than the bid price, because the market maker holds an inventory of securities. The market maker of Garman holds an inventory because of the bankruptcy risk and the market maker of Ho and Stoll holds an inventory because of price risk.

2.1.2 Information-based models

Information-based models are used to explain that the bid-ask spread is caused by the adverse selection cost of the market maker. The dynamics of the behaviour and the price setting of the market maker can be modelled. The market maker sets a higher ask price and a lower bid price to protect himself against the adverse selection. The informed traders have superior information than him. If he trades with informed traders he will loose. The difference between the bid and the ask price compensates the lost of the market maker. There are two types of information-based models: strategic trader model and sequential trade model. In strategic trader model the strategic behaviour of the informed trader is stressed. In sequential trade model the information signals in each trade period are stressed.

Strategic trader models

The model of Kyle[22] is a batch and order-driven market. This means that the market maker collects a lot number of different security orders and then executes them simulteneously. In the market there are strategic informed traders and uninformed traders. The traders place only market orders. Market orders are the simplest type of orders a trader can place in a financial market. Market orders are orders to be executed at once at current market prices. The traders can place orders concurrently and are anonymous. The market maker sets the market price p at which the traders buy or sell the security. The uninformed traders place simply random orders of size $\mu \sim N(0, \sigma_\mu^2)$. If $\mu > 0$ then the uninformed trader places a buy order. If $\mu < 0$ then the uninformed trader places a sell order. The informed trader knows the fundamental value v. He trades strategically and chooses the size of the order x such that his expected trading profit $\Pi = E[x(v - p)]$ is the highest. The market price of the security p and the total order flow $D = x + \mu$ are known to all the traders. The market maker collects the total order flow which is equal to $x + \mu$. The market maker has zero-profit expectation. This means that market making is perfectly competitive, in other words the entry cost to become a market maker is very low. The market maker does not know which part of the total order flow is placed by informed traders and which part is placed by uninformed traders. The market maker does not know the true value of the fundamental value. The market maker adjusts his expectation of the price given the total order flow ($p = E[v|D] = E[v|x + \mu]$). An equilibrium in this model can be calculated in which the market maker choses the price setting function such that he earns zero profit given the optimal action of the informed trader. And the informed traders maximize their expected trading profit given the market price that the market maker sets. The equilibrium market price and the equilibrium profit for the informed traders can be calculated (See [22] for the calculation details). Kyle shows that the linear pricing function of the total order flow is the optimal pricing strategy for the market maker. If the market maker chooses a linear price function, then the trading function for the informed trader is also linear. Kyle also shows that the bid-ask spread arises from the adverse selection cost of the market maker. The information of the order flow is also relevant for the market maker's price setting decision.

Sequential trade models

The model of Glosten and Milgrom [1] captures the adverse selection problem explicitly with a bid-ask spread in his model. The model is a quote-driven market with informed traders and uninformed traders. Informed traders know the true value of the fundamental value and uninformed traders do not know the true fundamental value. In each trade a trader can only place order of one unit of security. In each period only one trade takes place between a trader and the market maker. Trading is anonymous and there are no transaction costs. In each period one type of trader (informed or uninformed) arrives at random. The market maker is uninformed about the fundamental value, is risk-neutral and has zero-profit expectation. Because of the asymmetric information between the market maker and informed traders, the market maker will lose against the informed traders. Result is that the market maker will quote higher prices for ask orders and will quote lower prices for bid orders. Main idea is that the type of order conveys information about the true fundamental value. Informed traders will buy if ask pice is lower than the fundamental value and will sell if the bid price is higher than the fundamental value. The market maker uses the Bayesian learning rule to set the bid and ask prices. Buy order will be interpreted by the market maker as a good signal and updates his expectation $E[v|Buy] > E[v]$ with setting the ask price to $E[v|Buy]$. Likewise, after a sell order the market maker updates his expectation $E[v|Sell] < E[v]$ with setting the bid price to $E[v|Sell]$. The traded security has only two values, high (V_H) and low (V_L), and the expected value of the security is

$$E[v] = P(v = v_H)v_H + P(v = v_L)V_L \tag{2.5}$$

The market maker sets the ask price after a buy order:

$$ask = E[v|Buy] = P(v = v_H|Buy)v_H + P(v = v_L|Buy)v_L \tag{2.6}$$

$P(v = v_H|Buy)$ and $P(v = v_L|Buy)$ can be calculated with the Bayesian learning rule for discrete distribution:

$$P(v = v_H|Buy) = \frac{P(Buy|v = v_H)P(v = v_H)}{P(Buy)} \tag{2.7}$$

$$P(v = v_L|Buy) = \frac{P(Buy|v = v_L)P(v = v_L)}{P(Buy)} \tag{2.8}$$

The conditional probability of buy is

$$P(Buy) = P(Buy|v = v_H)P(v = v_H) + P(Buy|v = v_L)P(v = v_L) \tag{2.9}$$

The market maker sets the bid price after a sell order:

$$bid = E[v|Sell] = P(v = v_H|Sell)v_H + P(v = v_L|Sell)v_L \tag{2.10}$$

$P(v = v_H|Sell)$ and $P(v = v_L|Sell)$ can be calculated using the Bayesian learning rule for discrete distribution again:

$$P(v = v_H|Sell) = \frac{P(Sell|v = v_H)P(v = v_H)}{P(Sell)} \tag{2.11}$$

$$P(v = v_L|Sell) = \frac{P(Sell|v = v_L)P(v = v_L)}{P(Sell)} \tag{2.12}$$

The conditional probability of sell is

$$P(Sell) = P(Sell|v = v_H)P(v = v_H) + P(Sell|v = v_L)P(v = v_L) \tag{2.13}$$

Glosten en Milgrom show that the endogenous bid-ask spread will become visible $S = E[v|Buy] - E[v|Sell]$ in the model. So they use their model to explain that the bid-ask spread is due to the adverse cost of the market maker. And they also show that the bid-ask spread will increase with $v_H - v_L$ (volatility of the security) and the fraction of informed traders.

An extention of this model is the model of Das. This model will be discussed in chapter 3.

Chapter 3

The market maker model of Das

3.1 Model description

The market maker model of Das [10] [11] is an extension of the model of Glosten and Milgrom [1]. The model of Glosten and Milgrom is more theoretical and mathematical while the model of Das has a more realistic setting and is a simple agent-based model. The model is a sequential trading model. The environment wherein the agents operates is a discrete time market. There is only one market maker and traders can be informed, partially or noisy informed or uninformed.

In the market only one security is traded. The security has a real fundamental value V_i at time period i. The market maker sets bid and ask prices (P_b and P_a) in each trading step at which he is willing to buy or sell one unit of the stock. The market maker maintains a probability density function of the fundamental value. After each trade the market maker updates its belief about the fundamental value. The traders have different information about the true value of the stock, so the model is an adverse selection problem. At each time period only one trader is selected for trading. The market maker knows the probability structure of the arrival process. The market maker knows the probability of the traders placing buy or sell orders or decide not to trade. The traders can only place market orders. The informed trader, if selected, will place a buy order if the underlying true value is greater than the buy quote and will place a sell order if the underlying true value is smaller than the sell quote and will place no order if the underlying true value lies between the bid and ask quotes. The uninformed trader, if selected, will place a buy and sell order with the same probability (η) and will place no order with probability $1 - 2\eta$.

The model allows the presence of noisy informed traders, which are absent in the model of Glosten and Milgrom. This is more realistic, because in the real world it is more likely that the investors do not know the underlying value perfectly. A noisy informed trader gets a noisy signal of the true value, but the trader thinks it is the true value. The noisy value for the informed trader is denoted by W_i and is equal to $V_i + \tilde{\eta}(0, \sigma_W)$ where $\tilde{\eta}(0, \sigma_W)$ is drawn from a normal distribution with mean 0 and variance σ_W^2. The noisy informed trader, if selected, will place a buy order when $W_i > P_{i,a}$, will place a sell order when $W_i < P_{i,b}$ and will place no order when $P_{i,b} \leq W_i \leq P_{i,a}$.

During the simulation the true underlying value evolves with probabilistic jumps or changes. The jump process is as the following: at time $i + 1$, a jump in the true value will take place with probability ρ. Then the value at time V_{i+1} will become $V_i + \tilde{\omega}(0, \sigma_W)$ where $\tilde{\omega}(0, \sigma_W)$ is also drawn from the normal distribution with mean 0 and variance σ_W^2.

3.2 The algorithm of the market maker

We describe here the price setting algorithm of the market maker. The algorithm computes approximate solutions to the expected value equations. Glosten and Milgrom derive the bid quote to be the expectation of the true value given that a sell order is received ($P_b = E[V|Sell]$) and the ask quote to be the expectation of the true value given that a buy order is received ($P_a = E[V|Buy]$). Because the true value may change and depends on chance, the value V is a random variable. Das adds a probability distribution function to the market maker to assign probabilities to this random variable V.

3.2.1 Compute the ask quote

The expected value of the discrete random value V given a specific order is defined as the integral of all values of the random variable V, each value multiplied by its conditional probability. For instance for all value $V = x$ given that a market buy order is received:

$$E[V|Buy] = \int_0^\infty xP(V = x|Buy)\, dx \tag{3.1}$$

Because the market is a discrete time market, the x-axis is discretized into intervals to compute the values of equation 3.1 approximately:

$$E[V|Buy] = \sum_{V_i=V_{min}}^{V_i=V_{max}} V_i P(V = V_i|Buy) \tag{3.2}$$

After applying the rule of Bayes and simplifying the equation 3.2:

$$E[V|Buy] = \sum_{V_i=V_{min}}^{V_i=V_{max}} \frac{V_i P(Buy|V = V_i)P(V = V_i)}{P(Buy)} \tag{3.3}$$

And the ask quote is set to the expected value given that a buy order is received:

$$P_a = \frac{1}{P_{Buy}} \sum_{V_i=V_{min}}^{V_i=V_{max}} V_i P(Buy|V = V_i)P(V = V_i) \tag{3.4}$$

Since the ask quote lies always between the minimum value and the maximum value, $V_{min} < P_a < V_{max}$, we divide P_a into two parts:

$$\begin{aligned} P_a = &\frac{1}{P_{Buy}} \sum_{V_i=V_{min}}^{V_i=P_a} V_i P(Buy|V = V_i)P(V = V_i) \\ + &\frac{1}{P_{Buy}} \sum_{V_i=P_{a+1}}^{V_i=V_{max}} V_i P(Buy|V = V_i)P(V = V_i) \end{aligned} \tag{3.5}$$

The probability that a buy order is placed given that $V = V_i$ is constant within each sum. The uninformed trader will always buy independent of what the value is and the informed trader buys only if $V > P_a$. Therefore, $P(Buy|V \leq P_a) = (1 - \alpha)\eta$ and $P(Buy|V > P_a) = (1 - \alpha)\eta + \alpha$. The ask quote equation can be written as this way:

$$P_a = \frac{1}{P_{Buy}} \left(\sum_{V_i=V_{min}}^{V_i=P_a} ((1-\alpha)\eta) V_i P(V = V_i) + \sum_{V_i=P_a+1}^{V_i=V_{max}} ((1-\alpha)\eta + \alpha) V_i P(V = V_i) \right) \tag{3.6}$$

To solve equation 3.6 the a priori probability buy order (P_{Buy}) must be computed with this equation:

$$\begin{aligned} P_{Buy} &= \sum_{V_i=V_{min}}^{V_i=V_{max}} P(V = V_i) \\ &= \sum_{V_i=V_{min}}^{V_i=P_a} ((1 - \alpha)\eta)P(V = V_i) \ + \sum_{V_i=P_{a+1}}^{V_i=V_{max}} ((1 - \alpha)\eta \ + \ \alpha)P(V = V_i) \end{aligned} \tag{3.7}$$

3.2.2 Compute the bid quote

The bid quote equation can be derived in the same way as the ask quote:

$$P_b = \frac{1}{P_{Sell}} \left(\sum_{V_i=V_{min}}^{V_i=P_b-1} ((1-\alpha)\eta + \alpha) V_i P(V = V_i) + \sum_{V_i=P_b}^{V_i=V_{max}} ((1-\alpha)\eta) V_i P(V = V_i) \right) \tag{3.8}$$

The probability that a sell order is placed given that $V = V_i$ is constant within each sum. The uninformed trader will always sell independent of what the value is and the informed trader sells only if $P_b < V$. Therefore, $P(Buy|V \leq P_a) = (1-\alpha)\eta + \alpha$ and $P(Buy|V > P_a) = (1-\alpha)\eta$.

And the a priori probability of a sell order (P_{Sell}) is:

$$P_{Sell} = \sum_{V_i=V_{min}}^{V_i=P_b-1} ((1-\alpha)\eta + \alpha)P(V = V_i) + \sum_{V_i=P_b}^{V_i=V_{max}} ((1-\alpha)\eta)P(V = V_i) \quad (3.9)$$

With the equations 3.6 and 3.8 the market maker can set prices in each time interval.

3.2.3 Compute bid and ask quote with noisy informed traders

The buy quote equation (3.8) and the sell quote equation (3.6) and the probability of buy order equation (3.9) and probability of sell order equation (3.7) for perfectly informed traders can be expanded for noisy informed traders.

The noisy informed investor's selling and buying are determined by a probability, the conditional probabilities for buying and selling are:

$$P(Buy|V = V_i, V_i \leq P_a) = (1-\alpha)\eta + \alpha P(\tilde{\eta}(0, \sigma_W^2)) > (P_a - V_i)) \quad (3.10)$$
$$P(Buy|V = V_i, V_i > P_a) = (1-\alpha)\eta + \alpha P(\tilde{\eta}(0, \sigma_W^2)) < (V_i - P_a)) \quad (3.11)$$
$$P(Sell|V = V_i, V_i \leq P_b) = (1-\alpha)\eta + \alpha P(\tilde{\eta}(0, \sigma_W^2)) < (P_b - V_i)) \quad (3.12)$$
$$P(Sell|V = V_i, V_i > P_b) = (1-\alpha)\eta + \alpha P(\tilde{\eta}(0, \sigma_W^2)) > (V_i - P_b)) \quad (3.13)$$

The second term in the first two equations gives back the probability that a noisy informed trader will buy if the fundamental value is less than or equal to the ask price (equation 3.10) and if the fundamental value is higher than the ask price (equation 3.11). The second term in the last two equations gives back the probability that a noisy informed trader will sell if the fundamental value is less than or equal to the bid price(equation 3.12) and if the fundamental value is higher than the bid price (equation 3.13). The first term in the four equations is the probability that an uninformed trader will buy or sell.

The new buy and sell priors with noisy informed traders are now

$$\begin{aligned} P_{Buy} = &\sum_{V=V_{min}}^{V=Pa} [\alpha P(\tilde{\eta}(0, \sigma_W^2) > (P_a - V_i)) + (1-\alpha)\eta]P(V = V_i) + \\ &\sum_{V_i=P_a+1}^{V_i=V_{max}} P(\tilde{\eta}(0, \sigma_W^2) < (V_i - P_a)) + (1-\alpha)\eta]P(V = V_i) \end{aligned} \quad (3.14)$$

$$P_{Sell} = \sum_{V=V_{min}}^{V=P_b-1} [\alpha P(\tilde{\eta}(0,\sigma_W^2) < (P_b - V_i)) + (1-\alpha)\eta] P(V = V_i) + \sum_{V_i=P_b}^{V_i=V_{max}} P(\tilde{\eta}(0,\sigma_W^2) > (V_i - P_b)) + (1-\alpha)\eta] P(V = V_i) \tag{3.15}$$

Finally, the bid quote and the ask quote can be calculated as

$$P_b = \frac{1}{P_{Sell}} \sum_{V_i=V_{min}}^{V_i=P_b} [((1-\alpha)\eta + \alpha P(\tilde{\eta}(0,\sigma_W^2) < (P_b - V_i)))V_i P(V = V_i)] + \frac{1}{P_{Sell}} \sum_{V_i=P_b+1}^{V_i=Vmax} [((1-\alpha)\eta + \alpha P(\tilde{\eta}(0,\sigma_W^2) > (V_i - P_b)))V_i P(V = V_i)] \tag{3.16}$$

$$P_a = \frac{1}{P_{Buy}} \sum_{V_i=V_{min}}^{V_i=P_a} [((1-\alpha)\eta + \alpha P(\tilde{\eta}(0,\sigma_W^2) > (P_a - V_i)))V_i P(V = V_i)] + \frac{1}{P_{Buy}} \sum_{V_i=P_a+1}^{V_i=Vmax} [((1-\alpha)\eta + \alpha P(\tilde{\eta}(0,\sigma_W^2) < (V_i - P_a)))V_i P(V = V_i)] \tag{3.17}$$

3.2.4 Updating the probabilistic density estimates

Situation of buy or sell order received

Each time the market maker receives an order, he updates the posterior probability on the value of V, in case of the situation with noisy informed traders

When $V_i \leq P_a$ and market buy order:

$$P(V = V_i | Buy, V_i \leq P_a) = \frac{P(Buy | V = V_i, V_i \leq P_a) P(V = V_i)}{P_{Buy}} \tag{3.18}$$

When $V_i > P_a$ and market buy order:

$$P(V = V_i | Buy, V_i > P_a) = \frac{P(Buy | V = V_i, V_i > P_a) P(V = V_i)}{P_{Buy}} \tag{3.19}$$

When $V_i \leq P_b$ and market sell order:

$$P(V = V_i | Sell, V_i \leq P_b) = \frac{P(Sell | V = V_i, V_i \leq P_b) P(V = V_i)}{P_{Sell}} \tag{3.20}$$

When $V_i > P_b$ and market sell order:

$$P(V = V_i|Sell, V_i > P_b) = \frac{P(Sell|V = V_i, V_i > P_b)P(V = V_i)}{P_{Sell}} \tag{3.21}$$

The prior probability $P(V = V_i)$ is known from the probability distribution function, the prior probability of a buy order or a sell order (P_{Buy} and P_{Sell}) can be computed with equations 3.14 and 3.15 respectively, and the conditional probabilities of a buy order or a sell order can computed with equations 3.10, 3.11, 3.12 and 3.13.

In case of the situation with perfectly informed traders the signal indicates the value is higher or lower than a certain price. In that case the update equations are

$$P(V = V_i|Buy, V_i > P_a) = ((1-\alpha)\eta + \alpha)P(V = V_i) \tag{3.22}$$
$$P(V = V_i|Buy, V_i \leq P_a) = (1 - \alpha - (1-\alpha)\eta)P(V = V_i) \tag{3.23}$$
$$P(V = V_i|Sell, V_i < P_b) = ((1-\alpha)\eta + \alpha)P(V = V_i) \tag{3.24}$$
$$P(V = V_i|Sell, V_i \geq P_a) = (1 - \alpha - (1-\alpha)\eta)P(V = V_i) \tag{3.25}$$

Situation of no orders received

When there is no order received the posterior probability can be calculated as follows:

$$P(V = V_i|No\,order) = \frac{P(No\,order|V = V_i)P(V = V_i)}{P_{No\,order}} \tag{3.26}$$

and $P(No\,order|V = V_i)$ in case with noisy informed traders can be computed as

$$P(No\,order|V = V_i, V_i < P_b) = (1-\alpha)(1-2\eta) + \alpha P(\tilde{\eta}(0, \sigma_W^2) > (P_b - V_i)) \tag{3.27}$$

$$\begin{aligned} P(No\,order|V = V_i, P_b \leq V_i \leq P_a) = (1-\alpha)(1-2\eta) + \\ \alpha[P(P_b - V_i < \tilde{\eta}(0, \sigma_W^2)) + P(V_i - P_a < \tilde{\eta}(0, \sigma_W^2))] \end{aligned} \tag{3.28}$$

$$P(No\,order|V = V_i, V_i > P_a) = (1-\alpha)(1-2\eta) + \alpha P(V_i - P_a < \tilde{\eta}(0, \sigma_W^2)) \tag{3.29}$$

The second term of the equations 3.27, 3.28 and 3.29 gives back the probability that a noisy informed trader does not place an order. The first term is the probability that an uninformed trader does not place an order. And $P(No\,order|V = V_i)$ in the case with perfectly informed traders can be computed as

$$P(No\,order|V = V_i, V_i < P_b) = 1 - \alpha - (1 - \alpha)(1 - 2\eta) \tag{3.30}$$

$$P(No\,order|V = V_i, P_b \leq V_i \leq P_a) = (1 - \alpha)(1 - 2\eta) + \alpha \tag{3.31}$$

$$P(No\,order|V = V_i, V_i > P_a) = 1 - \alpha - (1 - \alpha)(1 - 2\eta) \tag{3.32}$$

If $V_i < P_b$ and $V_i > P_a$ then the uninformed traders place an order with a probability of 2η and all informed traders place an order. And if $P_b \leq V_i \leq P_a$ then only the uninformed traders place an order with a probability of 2η.

This model extends the Glosten-Milgrom model by introducing an algorithm for keeping a probability density function of the true underlying value of the stock in a dynamic market with probabilistic jumps in the fundamental value. Using the estimation prices are set in a rather realistic framework. The noise in the informed trading is explicitly incorporated into the model which gives a more dynamic market behaviour. In chapter 4 we introduce a market maker in the same environment as the Das model, but without the maintaining of a probabilistic density function of the true fundamental value. The market maker in the model is first assumed to know the true fundamental value and then we modify the market maker to approximate the true fundamental value, because in the real world the true fundamental value is also unknown to the market maker.

Chapter 4

Market maker's strategies and fundamental value approximation

4.1 Introduction

In this chapter we introduce a market maker with a simple market maker price setting strategy and then expand this market maker with a new strategy who detects the fundamental value with respect to the location of the bid-ask spread and then adjusts the bid price and the ask price. Initially, we assume that the market maker knows the fundamental value and uses the fundamental value in the price rule to calculate the future prices. Next the market maker approximates the fundamental value with detection of the location of the fundamental value with respect to the bid-ask spread and then adjusts the bid and ask prices. We generate the fundamental values and a sequence of types of traders exogeneously.

4.2 Financial market microstructure of the model

The financial market microstructure is the same as in the model of Das (See chapter 3). Here we describe it shortly. Only one stock is traded on the market. The stock has a real fundamental value which is known to the informed traders. The fundamental value can change if a jump takes place with a size of σ_v. The change can be positive or negative. There is only trade between the market maker and traders. The traders are informed or uninformed. The portion of informed traders is given by α. The portion of uninformed traders is then of course $1-\alpha$. The measure of informedness of informed traders is σ_w. The model is a sequence trade model. This means that at one point in time only the market maker and one trader can trade with each other. After 'selection' an uninformed trader places a market buy order or a market sell order with the same probability (η) or withhelds an order with probability $(1 - 2\eta)$. An informed trader if selected places a market buy order or a market sell order depending on the fundamental value. The informed trader does not trade if the fundamental value lies between the bid-ask spread. The market maker adjusts his expectation about the fundamental value and sets the bid and ask prices. The initial bid price and ask price can be set to a chosen value. The trading game ends if the stop condition of the number of time steps is satisfied.

4.3 Market maker who uses a simple linear price setting strategy

We describe here the market maker who uses a simple linear price setting strategy. Hereafer we call this market maker simpleMM. This market maker is assumed to know the fundamental value. SimpleMM adjusts the bid price if the order is a sell order and adjusts the ask price if the order is a buy order. If in the timestep there is no trader to trade then the market maker adjusts both the bid price and the ask to make the bid-ask spread closer to the fundamental value. The price schedule for the bid price of SimpleMM is

$$b_t = b_{t-1} + \beta(fv - b_t) \tag{4.1}$$

and for the ask price is

$$a_t = a_{t-1} + \beta(fv - b_t) \tag{4.2}$$

where b_t and b_{t-1} are the bid price at time t and $t-1$ respectively. a_t and a_{t-1} are the ask price at time t and $t-1$ respectively. β is the learning rate or the convergence rate. fv is the fundamental value.

Because in 4.1 b_t and in 4.2 a_t is on both side of the equal sign, we use the following estimated price setting rules

$$b_t = b_{t-1} + \gamma(\hat{fv} - b_{t-1}) \tag{4.3}$$

$$a_t = a_{t-1} + \gamma(\hat{fv} - a_{t-1}) \tag{4.4}$$

where $\hat{fv}$ is the estimated fundamental value.

Equations 4.3 and 4.4 can also be written as

$$b_t = (1 - \gamma)b_{t-1} + \gamma\hat{fv} \tag{4.5}$$

$$a_t = (1 - \gamma)a_{t-1} + \gamma\hat{fv} \tag{4.6}$$

b_t is also equal to $\frac{b_{t-1}+\beta fv}{1+\beta}$ and a_t is equal to $\frac{a_{t-1}+\beta fv}{1+\beta}$, so these also holds

$$\frac{1}{1+\beta} = 1 - \gamma$$

and

$$\gamma = 1 - \frac{1}{1+\beta} = \frac{1+\beta-1}{1+\beta} = \frac{\beta}{1+\beta} \tag{4.7}$$

According to equation 4.7 if γ is the same as $\frac{\beta}{1+\beta}$ then the estimation equations from 4.3 and 4.4 are equal to the equations from 4.1 and 4.2.

4.4 Market maker who uses a bid-ask spread location detection price setting strategy and fundamental value approximation

4.4.1 Using the maximum value of trading probabilities to update the price

Hereafter we call this market maker detectMM. DetectMM computes the executed buy, sell and hold probabilities. These probabilities are calculated by taking the number of each type of order and divides it by the total number of orders. Hold means that during the timestep there is no trader to contact the market maker to place an order.

$$realpBuy = \frac{\text{number of buys}}{\text{total number of orders}} \tag{4.8}$$

$$realpSell = \frac{\text{number of sells}}{\text{total number of orders}} \tag{4.9}$$

$$realpHold = \frac{\text{number of holds}}{\text{total number of orders}} \tag{4.10}$$

In each time the market maker calculates the probabilities $realpBuy$, $realpSell$ and $realpHold$. $RealpBuy$ is the probability of buy order placed from the timestep 0 to timestep t. $RealpSell$ is the probability of sell order placed from timestep 0 to timestep t. $RealpHold$ is the probability of no order received from timestep 0 to timestep t. The location of the bid-ask spread with respect to the fundamental value is determined by the maximum value of these probabilities. If $realpBuy$ is the maximum value then the fundamental value lies above the bid-ask spread. So the market maker needs to increase the bid and ask price,

If a buy order is placed:

$$a_t = a_{t-1} + \frac{\Delta a}{k} \tag{4.11}$$

$$b_t = b_{t-1} \tag{4.12}$$

If a sell order is placed:

$$a_t = a_{t-1} \tag{4.13}$$
$$b_t = b_{t-1} + \frac{\Delta b}{k} \tag{4.14}$$
$$b_t = a_t \qquad (\text{if } b_t > a_t) \tag{4.15}$$

No order received:

$$a_t = a_{t-1} + \frac{\Delta a}{k} \tag{4.16}$$
$$b_t = b_{t-1} + \frac{\Delta b}{k} \tag{4.17}$$

If $realpSell$ is the maximum value then the fundamental value lies under the bid-ask spread. Then the market maker needs to decrease the bid and ask price,

If a buy order is placed:

$$a_t = a_{t-1} - \frac{\Delta a}{k} \tag{4.18}$$
$$b_t = b_{t-1} \tag{4.19}$$
$$a_t = b_t \qquad (\text{if } a_t < b_t) \tag{4.20}$$

If a sell order is placed:

$$a_t = a_{t-1} \tag{4.21}$$
$$b_t = b_{t-1} - \frac{\Delta b}{k} \tag{4.22}$$

No order received:

$$a_t = a_{t-1} - \frac{\Delta a}{k} \tag{4.23}$$
$$b_t = b_{t-1} - \frac{\Delta b}{k} \tag{4.24}$$

If $realpHold$ is the maximum value then the fundamental value lies between the bid-ask spread. In this case the market maker needs to increase the bid price and decrease the ask price,

If a buy order is placed:

$$a_t = a_{t-1} - \frac{\Delta a}{k} \tag{4.25}$$
$$b_t = b_{t-1} \tag{4.26}$$
$$a_t = b_t \qquad (\text{if } a_t < b_t) \tag{4.27}$$

If a sell order is placed:

$$a_t = a_{t-1} \tag{4.28}$$

$$b_t = b_{t-1} + \frac{\Delta b}{k} \tag{4.29}$$

$$b_t = a_t \qquad (\text{if } b_t > a_t) \tag{4.30}$$

No order received:

$$a_t = a_{t-1} - \frac{\Delta a}{k} \tag{4.31}$$

$$b_t = b_{t-1} + \frac{\Delta b}{k} \tag{4.32}$$

$$b_t = a_t \qquad (\text{if } b_t > a_t) \tag{4.33}$$

Δa and Δb is the amount of increase or decrease of the ask price and the bid price respectively in each timestep. For simplicity reason we assume that Δa is equal to Δb. $k = \{1, 2, \dots\}$ and we divide Δa and Δb by k to let the change in the price become smaller and smaller in the hope that it reaches the fundamental value approximately. If a bid order is placed then only the ask price is changed and the bid price is the same as the previous bid price. If a sell order is placed then the bid price is adjust and the ask price is the same as the previous ask price. If no order is received then both the bid price and the ask price are updated. Because the ask price is always greater than or equal to the bid price, the bid price is set to the same value as the ask price or the ask price is set to the same value as the bid price in cases where it is needed.

In the instances where there are more than one maximum value the location of the fundamental value with respect to the bid-ask spread is determined by the most recent order. If the last order is a buy order then $bid \leq ask \leq fv$. If the last order is a sell order then $fv \leq bid \leq ask$. If there is no order received during that particular timestep then $bid \leq fv \leq ask$.

This model has two drawbacks: (1) the market maker does not know if a change in the fundamental value has occurred. (2) the market maker uses the order flow with noise to see if the price is overvalued or undervalued. The order flow is the aggregated orders placed by informed and uninformed traders. The order flow contains noise because the orders placed by uninformed traders are placed randomly. Those orders do not tell something about the price in relationship with the fundamental value. DetectMM uses the maximum of the probability of the order placed. If the market maker with the available information updates the price too slowly and the price is attractive for informed traders to buy or to sell, they will continually only be placing buy orders or only placing sell orders. Then, the market maker will forecast the location of the fundamental value wrong and will not track the true fundamental value right. That's why we introduce another market maker that tries to nullify the two disadvanges of detectMM.

4.4.2 Using the distribution of trading probabilities to update the prices

We use the distribution of trading probabilities to update the prices to nullify the above descibed problems. We call this market maker spreadMM. The distribution of the trading probabilities can be calculated because we know how much informed traders there are and we know the trading probability of the informed traders. If the ask price is lower than the fundamental value then all the informed traders will buy and the uninformed traders will buy with a probability of η, only the uninformed traders will sell with a probability of η and the no order probability is $1 - 2\eta$. If the bid price is higher than the fundamental value then only the uninformed trader will buy with a probability of η, all the informed traders will sell and the uninformed traders will sell with a probability of η and the no order probability is $1-2\eta$. If the fundamental value is between the ask price and the bid price then only the uninformed traders will buy with a probability of η, only the uninformed traders will sell with a probability of η and the no order probability is $1-2\eta$. In mathematical formulas are:

$$\text{trading distribution} = \begin{pmatrix} \alpha + (1-\alpha)\eta & (1-\alpha)\eta & 1 - (\alpha + 2\eta(1-\alpha)) \\ (1-\alpha)\eta & (1-\alpha)\eta & 1 - (2\eta(1-\alpha)) \\ (1-\alpha)\eta & \alpha + (1-\alpha)\eta & 1 - (\alpha + 2\eta(1-\alpha)) \end{pmatrix} \tag{4.34}$$

whereby in each row the first value is the buy probability (p1), the second value is the sell probability (p2) and the third value is the no order probability (p3). The first row is where $fv < ask < bid$. The second row is where $ask < fv < bid$. The third row is where $ask < bid < fv$.

Other difference between spreadMM and detectMM is that spreadMM gets a signal when the fundamental value changes that is when a jump in the fundamental value occurs. This is spreadMM with signalling. Final step is modelling spreadMM in such a way that spreadMM does not get the news signal that the fundamental value has changed (spreadMM without signalling), because in real world market makers do not know when a change in the fundamental value occurs. In the case of detectMM $realpBuy$, $realpSell$ and $realpHold$ in each timestep are computed by taking each order types from the beginning at timestep 0 until timestep t divided by the corresponding number of total orders. Now in the case of spreadMM $realpBuy$, $realpSell$ and $realpHold$ in each timestep are computed by taking each order types from the fundamental value change until timestep t divided by the corresponding number of orders from the fundamental value change until timestep t. The time between the fundamental change until the current timestep is a period. In the case of spreadMM without signalling, we choose a constant period of time steps that spreadMM uses to calculate the distribution of the probabilities of the executed order types. We have to compare the distribution [$realpBuy$ $realpSell$ $realpHold$] with each row of the trading distribution to locate the place of the fundamental value with respect to the bid-ask spread and then update the bid and price accordingly. Prices updating process is the same as detectMM. We compute the minimum distance between [$realpBuy$ $realpSell$ $realpHold$] and each row of the trading distribution with:

$$distance = \sqrt{(realpBuy - p1)^2 + (realpSell - p3)^2 + (realpHold - p2)^2} \tag{4.35}$$

If the minimum value is for row 1 then the fundamental value lies above the bid-ask spread. If the minimum value is for row 2 then the fundamental value lies under the bid-ask spread and if the minimal value is for row 3 then the fundamental value lies between the bid-ask spread.

Chapter 5

Experimental setups

5.1 Fundamental value serie

We have to simulate the value of the fundamental value of the security in each time during the simulation. We allow changes occur in the fundamental value, because in the real world the fundamental value of the security can also change during its lifetime. We generate a sequence of values which represents the fundamental values of the security in each time. We call the sequence the fundamental value serie. The fundamental value serie is generated with the parameters pj, $v0$, $nsteps$ and sv. Pj is the probability that a jump will occur in the fundamental value. $V0$ is the start fundamental value. $Nsteps$ is the number of trades in the simulation. Sv is the standard deviation of the jump of the fundamental value. The idea is that in each time t if a jump takes place with probability pj then the fundamental value at t will be the fundamental value at time $t-1$ plus sv. And if a jump is not taken place then the fundamental value at time t is the same as the fundamental value at time $t-1$. The pseudocode of the function to generate the fundamental value serie is shown in function fv = GenerateFundamentalValues. $jumps$ is a vector of size $nsteps$ with 0s and 1s. 0 means there is no jump in the fundamental value and 1 means there is a jump in the fundamental value. Number of jumps is $nsteps/pj$. Rand is a function to generate random numbers between 0 and 1. If the random number is smaller than pj then the value in the vector is 1 otherwise the value is 0. $Jsize$ is the size of the jump in de fundamental value and it is calculated by sv times a random number drawn from a normal distribution. Each fundamental value in the fundamental value serie is then calculated by $v0$ plus cumulative sum of (jumps * jsize). An example of such fundamental value serie is shown in figure 5.1. This fundamental value serie is randomly chosen, but we use it because the number of jumps in the fundamental value is not too big such that the market maker becomes instable and it is not too small such that we can see what the market maker does when there are more jumps taking place in a short amount of time.

Function fv = GenerateFundamentalValues(nsteps,sv,pj,v0)

---

jumps ← [0,rand(nsteps-1) < pj]; jsize ← sv * randn(nsteps);
fv ← v0 + cumsum(jumps * jsize);

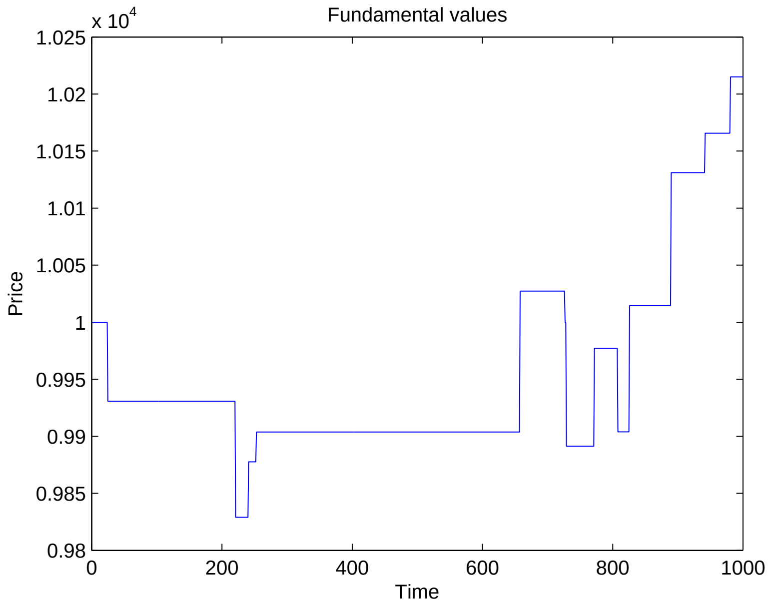


Figure 5.1: A fundamental value serie with 1000 time steps.

5.2 Type of trader serie

We have to simulate the arrival of traders for contacting the market marker and trading with them. Our method is generate at random a sequence of the numbers 0 to 3 to represent the traders. This sequence of numbers we call the type of trader serie. For the simulations we need four groups of traders, which are the informed traders, the uninformed trader who buys, the uninformed trader who sells and the uninformed trader who decides not to trade. The total number of traders generated is determined by the number of time steps during the simulations. The type of trader serie is generated with the parameters α, η and $nsteps$. $Nsteps$ gives the number of time steps. α gives the percentage of informed traders. So the number of informed traders is calculated by $nsteps * \alpha$ and the number of uninformed traders is calculated by $nsteps * (1 - \alpha)$. The uninformed traders will be further classified with the parameter η. η is the buy probability or sell probability of the uninformed trader. The uninformed trader buys or sells with the same probability η. The uninformed trader holds a trade with probability $1 - 2 * \eta$. The informed trader we represent with a 0. The uninformed trader who buys we represent with a 1. The uninformed trader who sells we represent with a 2. The uninformed trader who does not trade we represent with a 3. To illustrate this idea we give here an example: $nsteps = 12$, $\alpha = 0.33$, $\eta = 0.25$ then the type of trader sequence can be

$$0\ 1\ 3\ 2\ 1\ 0\ 3\ 3\ 2\ 0\ 0\ 3$$

There are 4 informed traders and 8 uninformed traders. 4 of the 8 uninformed traders hold a trade, 2 buy and 2 sell.

5.3 Market maker's model parameters estimates

All the three marker maker's models have the following parameters:

α - this is the fraction of informed traders.

η - this is the probability that an uninformed trader buys or sells.

tt - this is the sequence of type of traders.

pj - this is the probability that a jump will occur.

fvp - this is the fundamental values serie.

buy_0 - this is the start buy price at the beginning of the simulation.

$sell\_0$ - this is the start sell price at the beginning of the simulation.

The Das market maker has these additional parameters:

$v0$ - this is the initial fundamental value estimate.

sv - this is the standard deviation in the jump of the fundamental value.

SimpleMM has this additional parameter:

γ - this is the convergence rate.

DetectMM and spreadMM have these additional parameters:

Δa - this is the amount of increase or decrease of the ask price.

Δb - this is the amount of increase or decrease of the bid price.

In the simulations we let most of the parameters constant. We change only a few parameters to analyse the effect of the parameter to the dynamics of the market maker. The parameters that we change in the simulations is explained in detail in section 5.6. There are parameters that the market maker can control like Δa and Δb, because the value of these parameters can be chosen by the market maker. There are parameters that the market maker can't control for example α and η, because these parameters are dependent of the market situation and the traders in the market.

5.4 The pseudocode of detectMM

The pseudocode of detectMM's fundamental value location detection is shown in algorithm 1 and the price quoting mechanism is shown in algorithm 2. In the code it can be seen that detectMM has to choose three cases according to the maximum probabilistic value of $Ptrade$. This is the portion of buys, sells and no orders from the beginning of the simulation until the time i. The simulation stops if the stopcondition (number of time steps) is met. In each case detectMM can choose 3 other decisions according to the type of order received. In total there are 9 deterministic paths in each time step for detectMM to choose for. The ask price is then updated with the function ask and the bid price is updated with the function bid.

Algorithm 1: Pseudocode of DetectMM's fundamental value location detection algorithm

input : tt
output: Ptrade, index

```
ptrade ← ComputePtrade(tt);
index ← Max(ptrade);
```

5.5 The pseudocode of spreadMM

The pseudocode of spreadMM's fundamental value location detection is shown in algorithm 3 and the price quoting mechanism is shown in algorithm 2. In the code it can be seen that spreadMM gets a signal of a change in the fundamental value and sets new ask price to $fv + 200$ and sets the new bid price to $fv - 200$. This is because the market maker receives a change of the fundamental value and does not know what the new bid and ask price are and sets a large bid-ask spread. If there is no change in the fundamental value then spreadMM has to choose three cases according to the minimum value of $DPtrade$. This is the distance we described in section 4.4.2 and is computed with equation 4.35. The simulation stops if the stopcondition (number of time steps) is met. In each case spreadMM can choose 3 other decisions according to the type of order received. In total there are 9 deterministic paths in each time step for spreadMM to choose for. The ask price is then updated with the function ask and the bid price is updated with the function bid.

5.6 Experimental setup

We use Matlab as the programming environment for programming the code, debugging the code, evaluating the model, and doing simulations. Doing simulation is the mainly method we use to see the behaviour of the market maker. The simulations can be categorized according to these criteria:

1. difference in market maker's model: Das market maker, simpleMM, detectMM and spreadMM

2. difference in number of simulations: 1 simulation or 100 simulations

3. difference in the values we set for the parameters: α, γ, Δa, Δb, k, *period*

With simpleMM and detectMM we only do single simulations to see the price dynamics of the market maker mechanism. With single simulation we only run the simulation 1 time. We use the same fundamental value serie for the single simulations. We use α's of 0.25, 0.33, 0.5 and 0.75 to see what kind of effect the number of informed traders has on the prices quoted by the two market makers. We set an η of 0.4. For each α we generate the corresponding trader type serie, because the trader type serie depends on the α chosen. This parameter tells the probability of an uninformed trader to buy or sell. If η is 0.4 then the buy probability is 0.4 and also the sell probability is 0.4 and the no order probability is of course 0.2.

To simulate spreadMM we do 100 simulations. And we calculate in each time step the mean bid price and ask price over the 100 simulations. For example at t = 1 the mean bid price over the 100 simulations is calculated as follows: Adding the bid price in each simulation of t = 1. We do this with all the bid prices in time steps from t = 1 to t = 1000. After this we get 1000 mean bid prices. Likewise, we do the same with the ask price. We plot the mean bid price curve and the mean ask price curve with the fundamental value curve in the same figure. To see what spreadMM will do in situation where the fundamental value is stable for a long time, we generate a fundamental value with only one jump during the simulation. The first 500 time steps the fundamental value is 10000 and last 500 time steps the fundamental steps is 10020. We create this fundamental value with only one change in the fundamental value in the middle of the simulation time and change the parameters to get an insight of what kind of effect have on the parameters to the bid and ask prices and if the market maker using distribution of orders from the order flow can track the fundamental value. We generate 100 type of trader series each with an α of 0.25 and then also the same with an α of 0.33, 0.5, 0.75. The experiments can be categorized into three groups. In the first group of experiments the change in the experiment is the parameter α. There are four experiments in this group. We set η to 0.4, Δa and Δb to 40 and $k = 1, 2, 3, \ldots$ and if the first derivative of the price in time $t = 0$ we set k back to 0 or else we increase k with 1 in the next time step. The goal of this group of experiments is to see what kind of effect the population of informed traders has on the price setting behaviour of the market maker. In the second group of experiments we set α to 0.75 and η to 0.4 and we do with the parameter k the same as in the first group of experiments, but we set Δa and Δb in the simulations to the values of 10, 30, 50, 70, 90 or 200. In the third group of experiments we set α to 0.75 and η to 0.4, Δa and Δb to 90 and we change k in the experiments to 1, 2, 4, or 6, but now in each time step the value of k is fixed and does not change during the simulation as the first group of experiments does. We do the simulations of spreadMM with signalling and without signalling. In case of spreadMM without signalling, we do simulations with constant periods of 5, 10, 20, 25, 30 and 60.

We simulate the Das market maker with α's of 0.25, 0.33, 0.5 and 0.75 and an η of 0.4. We compare the result of the simulation of the Das market maker with the result of the simulation of spreadMM.

5.7 Model evaluation and Performance measure

5.7.1 Model evaluation of detectMM and spreadMM

For evaluation and benchmarking the market maker's model we use two methods. The first method is the mean of bid-ask spreads method. The second method is sum of difference between the fundamental value and the prices as performance measure.

5.7.2 Mean bid-ask spreads

To average the 100 simulations we use for comparison the mean of bid-ask spreads as performance measure. After the 100 simulations we compute the mean of bid-ask spreads of the 100 simulations. The mean of bid-ask spreads is calculated by adding the bid-ask spread in each timestep together and divide by the number of timesteps in the simulation. In mathematical formula this will be:

$$\text{mean of bid-ask spreads} = \frac{\sum a_t - b_t}{\text{Number of timesteps}} \tag{5.1}$$

Sum of diffence between the fundamental value and the prices

For benchmarking the performance of the Das market maker and spreadMM we compute the sum of difference between the fundamental value and the bid price (sumdifbid), the sum of the difference between the fundamental value and the ask price (sumdifask) and the average of the two (avaragesumdif). In mathematical formulas:

$$sumdifbid = \sum (fv_t - bid_t) \tag{5.2}$$

$$sumdifask = \sum (ask_t - fv_t) \tag{5.3}$$

$$averagesumdif = \frac{(sumdifbid + sumdifask)}{2} \tag{5.4}$$

Algorithm 2: Pseudocode of DetectMM and SpreadMM price quoting algorithm

```
input  : index, order, delta_a, delta_b,k
output: ask [i], bid [i]

for i ← 1 to n do
    switch index do
        case index is 1
            if order is 1 then
                ask [i] ← ComputeAsk(ask,i,delta_a,k);
                bid [i] ← bid [i − 1];
            else if order is -1 then
                ask [i] ← ask [i − 1];
                bid [i] ← ComputeBid(bid,i,delta_b,k);
            else order is 0
                ask [i] ← ComputeAsk(ask,i,delta_a,k);
                bid [i] ← ComputeBid(ask,i,delta_b,k);
        case index is 2
            if order is 1 then
                ask [i] ← ComputeAsk(ask,i,-delta_a,k);
                bid [i] ← bid [i − 1];
                if ask [i] < bid [i] then ask [i] = bid [i];
            else if order is -1 then
                ask [i] ← ask [i − 1];
                bid [i] ← ComputeBid(bid,i,-delta_b,k);
                if bid [i] < ask [i] then bid [i] > ask [i];
            else order is 0
                ask [i] ← ComputeAsk(ask,i,-delta_a,k);
                bid [i] ← ComputeBid(ask,i,-delta_b,k);
        case index is 3
            if order is 1 then
                ask [i] ← ComputeAsk(ask,i,-delta_a,k);
                bid [i] ← bid [i − 1];
                if ask [i] < bid [i] then ask [i] = bid [i];
            else if order is -1 then
                ask [i] ← ask [i − 1];
                bid [i] ← ComputeBid(bid,i,delta_b,k);
                if bid [i] < ask [i] then bid [i] > ask [i];
            else order is 0
                ask [i] ← ComputeAsk(ask,i,-delta_a,k);
                bid [i] ← ComputeBid(ask,i,delta_b,k);
```

Function ask(i) = ComputeAsk(ask,i,delta_a,k)

ask$[i] \leftarrow$ ask$[i-1] + \frac{delta_a}{k}$;

Function bid(i) = ComputeBid(bid,i,delta_b,k)

bid$[i] \leftarrow$ bid$[i-1] + \frac{delta_a}{k}$;

Algorithm 3: Pseudocode of spreadMM's fundamental value location detection algorithm

```
input  : priors, txpro, fv
output: ask, bid, dptrade, index

if change in the fundamental value then
    ask ← fv + 200;
    bid ← fv - 200;
    index ← 1;
else
    dptrade ← ComputeDPtrade(priors, txpro);
    index ← Min(dptrade);
```

Function dptrade = ComputeDPtrade(priors, txpro)

```
for i ← 1 to 3 do
    for j ← 1 to 3 do
        dptrade ← sqrt((priors[i][j] − txpro[j])^2);
    end
end
```

Chapter 6

Results

In this chapter we describe the results of the experiments we have simulated with the Das market maker (see chapter 3) and with the market makers simpleMM (see chapter 4.3) , detectMM (see chapter 4.4.1) and spreadMM (see chapter 4.4.2).

6.1 Simulation results of single experiments

6.1.1 Experiment with simpleMM

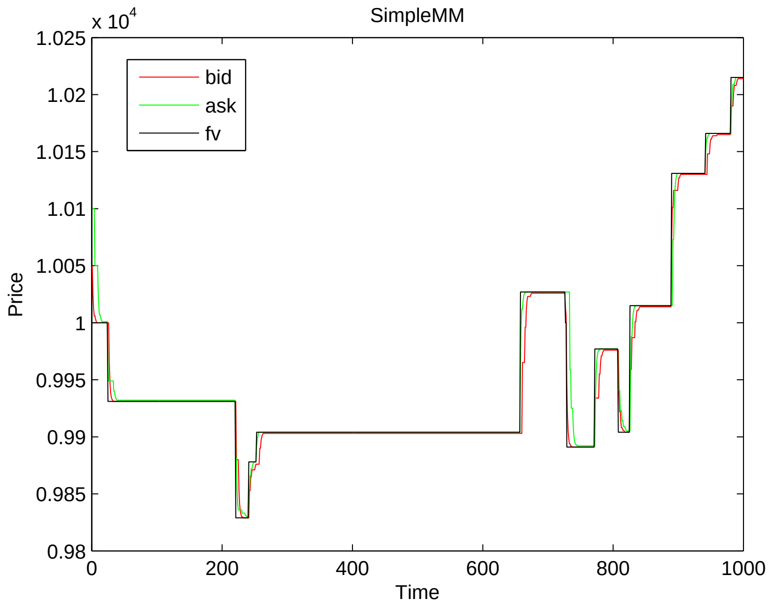
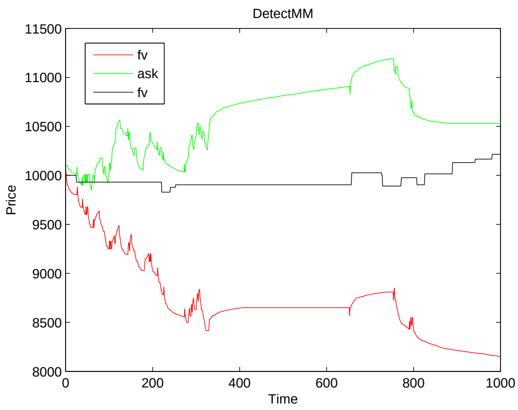


Figure 6.1: This is the result of the experiment with simpleMM($\alpha = 0.25$).

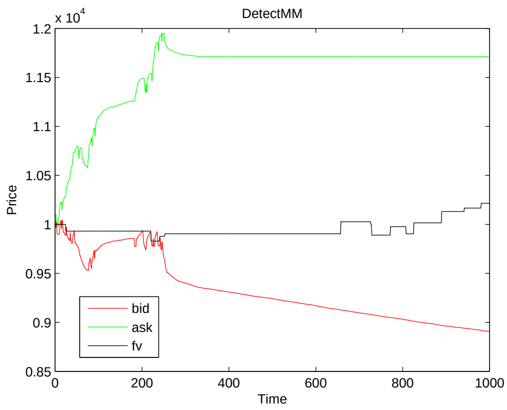
First we did some simulations with simpleMM. Figure 6.1 shows the bid price, the ask price and the fundamental value of the simulations with simpleMM. We only show the figure of the result of the experiment with an α of 0.25 here. The results of the simulations with α's of 0.33, 0.5 and 0.75 is shown in appendix A, because the figures are mostly identical to each other. SimpleMM can track the fundamental value, because the market maker is assumed to know the fundamental value. After a jump in the fundamental value it can be clearly seen that there occurs a little delay in time before the market maker gets to the correct fundamental value. How fast the prices converge to the fundamental value is determined by the parameter γ. If γ is nearer to 0 then the prices will slower converge. If γ is nearer to 1 then the prices will converge faster. The bid price and the ask price in the same time step in the simulations are not the same, due to different trader type series we have used and so the order flow in the four simulation is also different.

6.1.2 Experiments with detectMM

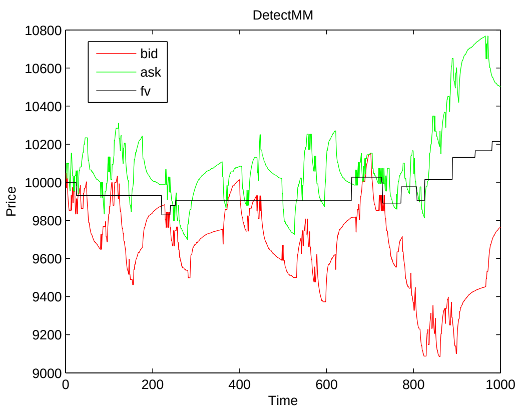
As shown in figure 6.2 the four experiments of DetectMM with α's of 0.25, 0.33, 0.5 and 0.75. The figures show that the detectMM method doesn't work. The detectMM cannot track the fundamental value. In the figures of the experiments with α of 0.25 and 0.33 show similar picture. It seems that information from the uninformed traders who trade randomly cause detectMM to think that the fundamental value is under the bid price, that is why he is continuously decreasing the bid price. DetectMM thinks at the same time that the fundamental value is above the ask price and is continuously increasing the ask price during the whole simulation. The strange thing is that the course of the bid price curve in the experiment with α of 0.25 is almost the same as the course of the ask price curve in the experiment with α of 0.33. And the course of the ask price in the experiment with α of 0.25 is almost the same as the course of the bid price curve in the experiment with α of 0.33. The bid price in the experiment with α of 0.25 decreases fast in the beginning and the ask price in the experiment with α of 0.33 increases fast in the beginning and then the bid price and the ask price go horizontally or go less steep. The ask price in the experiment with α is 0.25 increases fast in the beginning and the bid price in the experiment with α of 0.33 decreases fast in the beginning and then the ask price and the bid price go horizontally or go less steep. The results of the experiments with α of 0.5 and 0.75 also show no good tracking of the fundamental value. Although the bid price and the ask price go up and down around the fundamental value with the difference that the changes in the experiment with α of 0.75 is much faster than the changes in the experiment with α of 0.5. However, detectMM does not know that the fundamental value has changed. We think that more information from the informed traders makes the market maker performs better. The pattern of the order flow has a crucial effect on the price dynamics of the market maker.



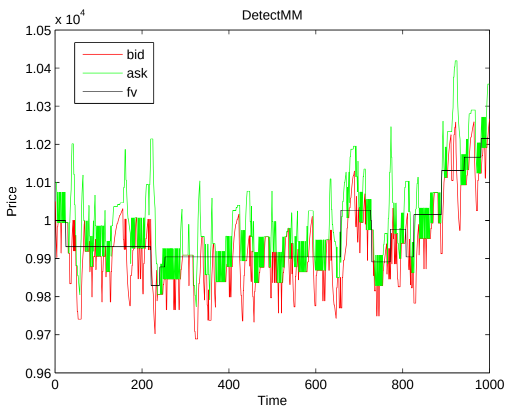
(a) $\alpha = 0.25$



(b) $\alpha = 0.33$



(c) $\alpha = 0.5$



(d) $\alpha = 0.75$

Figure 6.2: These are the results of the experiments of detectMM with α's of 0.25, 0.33, 0.5 and 0.75.

6.2 Simulation results of 100 experiments

We did with each same setting 100 experiments as the single experiment. The parameters are the same in the 100 experiments except that the arrangement of the orders differs. This is randomly generated 100 times. The simulations are runned 100 times and we average the bid and ask price. We compute the mean bid and mean ask price over the 100 experiments and plot the mean prices together with the fundamental value.

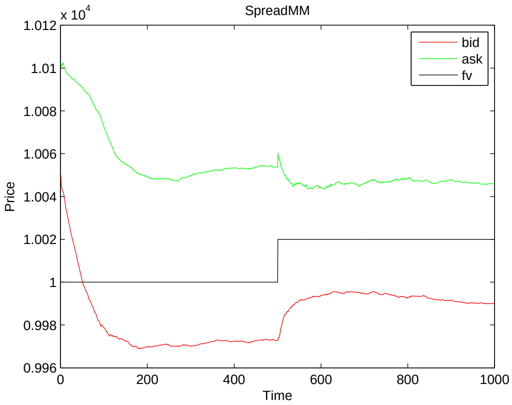
6.2.1 Simulation with SpreadMM

Fundamental value serie with 1 jump

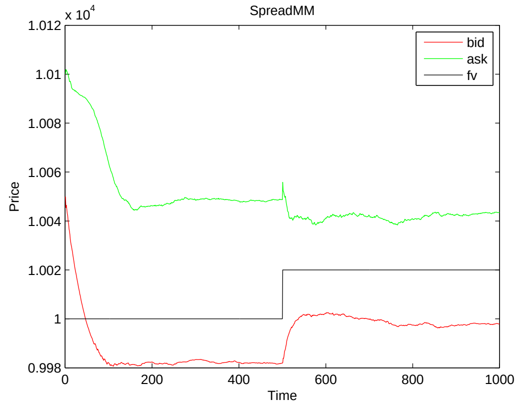
We have done a number of simulations to determine which price updating function fits best and investigate the effect of the different parameters on the price updating process. The simulations can be grouped into three categories. We used a fundamental value serie with only one jump. Figure 6.3 shows the first of experiments where α is changed in each simulation. Figure 6.4 shows the second group of experiments where Δa and Δb are changed in each simulation. These are the small increment of the bid and the ask price. Figure 6.5 shows the third group of experiments where k is changed in each experiment. K is a variable that we choose to divide by the small increment Δa and Δb. In the first group of experiments we increment in each time step the parameter k with 1 and set back to 1 of the derivative of the price function in that time step is equal to 0. We set the small increment fixed to 40. We made 4 experiments: α's are 0.25, 0.33, 0.5 and 0.75. We see that in all four experiments that the ask price function is the same, but the bid price function is closer to the fundamental value, because there are more informed traders trading in the simulation. In the second group of experiments we set α fixed to 0.75. We did the same with the parameter k as in the first group of experiments. Here we did 6 experiments. We increment Δa and Δb in each experiment with 20. The values are respectively 10, 30, 50, 70, and 90. Finally, to see if a large value has an effect on the price behaviour of the market maker, we set Δa and Δb to 200. We see that the parameters Δa and Δb have no effect on the price behaviour of the market maker. In the third group we set α fixed to 0.75 and set Δa and Δb fixed to 90. Here we did not increment the parameter k in each time step with 1. The value does not change during the simulation. We did four experiments with the following values of k: 1, 2, 4 and 6. We see that if k is small then at the beginning of the simulation or after a change in the fundamental value the price converges faster to the fundamental value. A larger k means that the market maker needs more time to set the price closer to the fundamental value. Another characteristic in the experiments is that if k is larger then in a period where there is no change in the fundamental value, then the bid-ask spread is smaller.

Fundamental value serie with multiple jumps

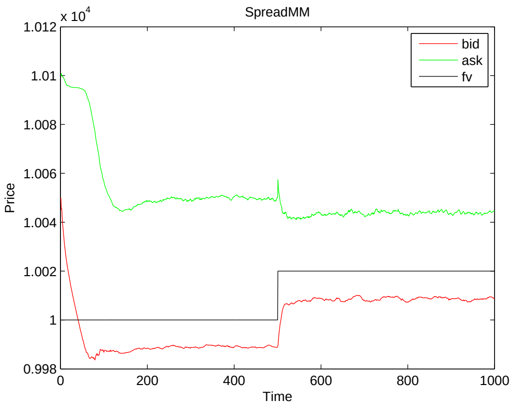
We did with the same experimental settings in simulations but now with more jumps in the fundamental value. We want to see what kind of effect the occurrence of jumps will have in a short period on the price setting of the market maker. First we did 4 experiments with α of 0.25, 0.33, 0.5 and 0.75. The small increment Δa and Δb are set to 90 and the k is set to 1 fixed. The result is shown in figure 6.6. As shown in that figure, if α is larger



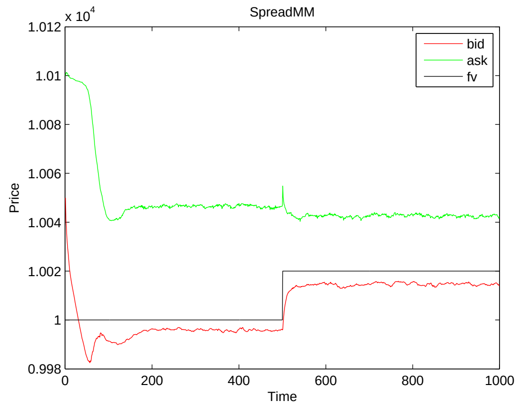
(a) $\alpha = 0.25$



(b) $\alpha = 0.33$

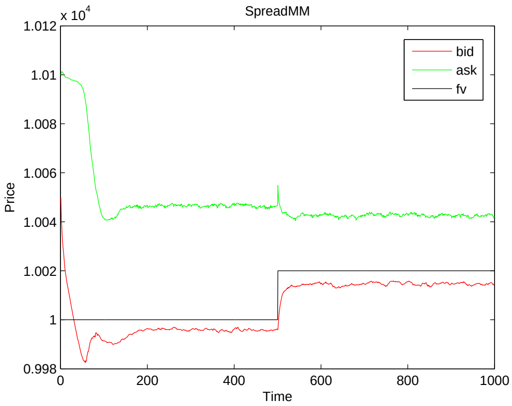


(c) $\alpha = 0.5$

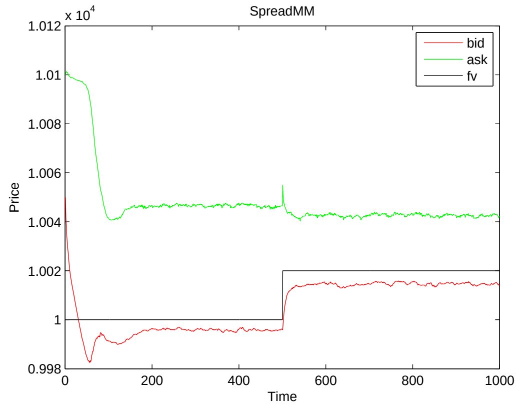


(d) $\alpha = 0.75$

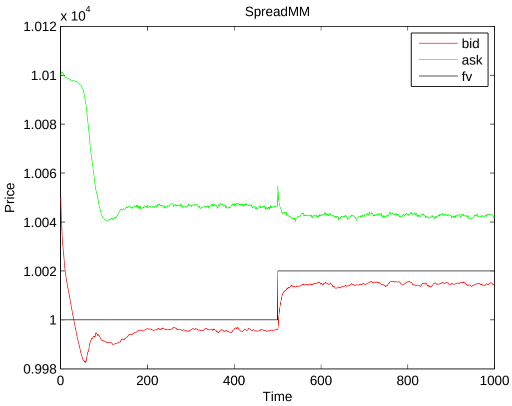
Figure 6.3: These are the results of the experiments of spreadMM with signalling and α's of 0.25, 0.33, 0.5 and 0.75.



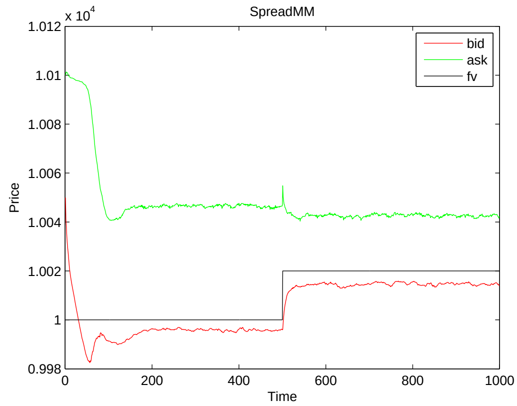
(a) Δa and $\Delta b = 10$



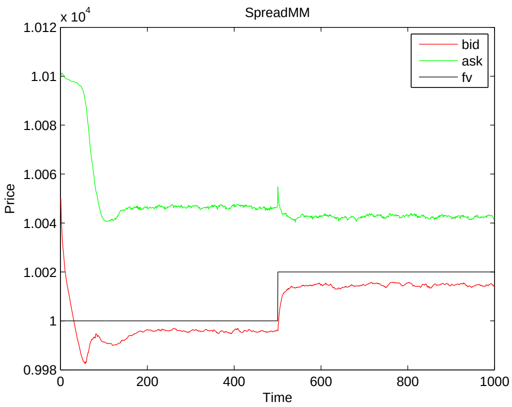
(b) Δa and $\Delta b = 30$



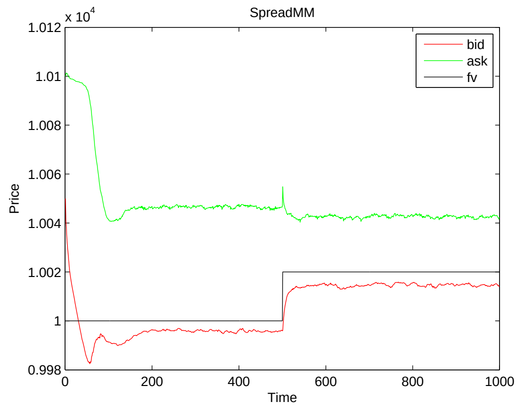
(c) Δa and $\Delta b = 50$



(d) Δa and $\Delta b = 70$

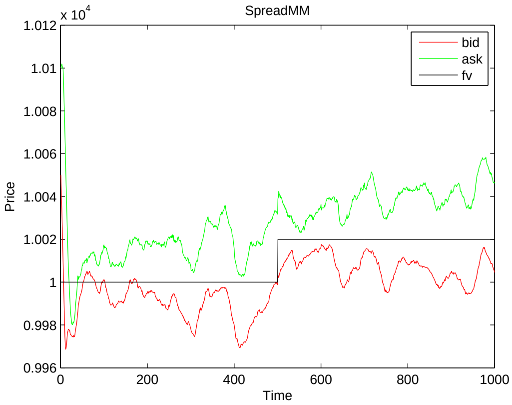


(e) Δa and $\Delta b = 90$

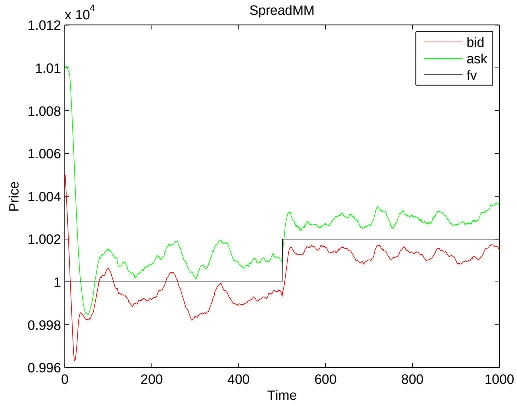


(f) Δa and $\Delta b = 200$

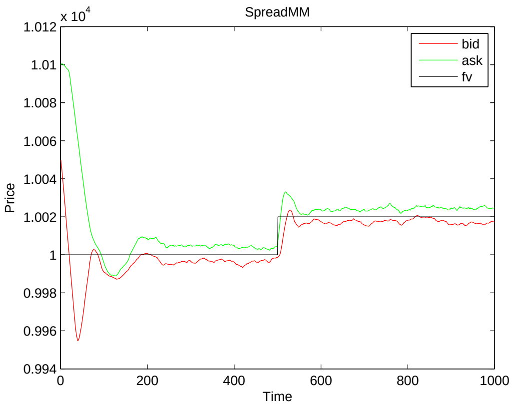
Figure 6.4: These are the results of the experiments of spreadMM with signalling and α is 0.75 and Δa and Δb have values of 10, 30, 50, 70, 90 and 200.



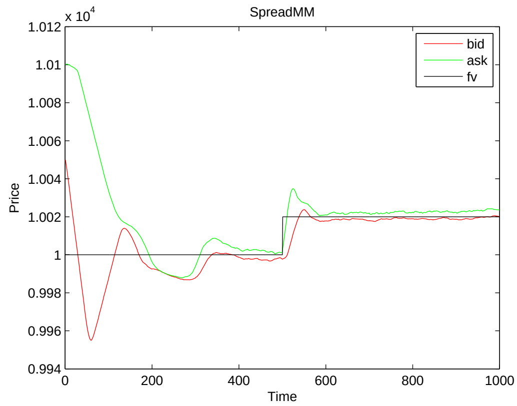
(a) $k = 1$



(b) $k = 2$



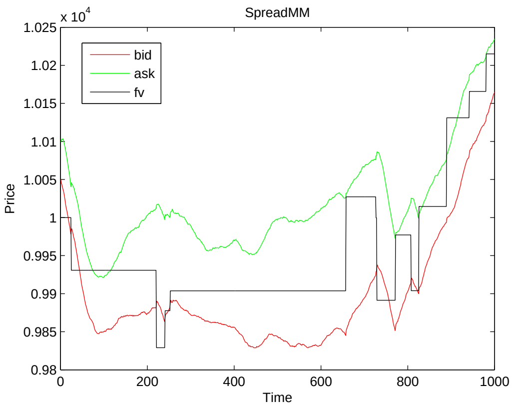
(c) $k = 4$



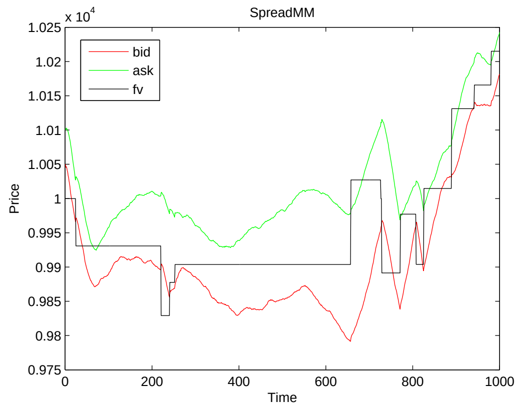
(d) $k = 6$

Figure 6.5: These are the results of the experiments of spreadMM with signalling and α is 0.75 and Δa and Δb are 90 and k is 1, 2, 4 or 6.

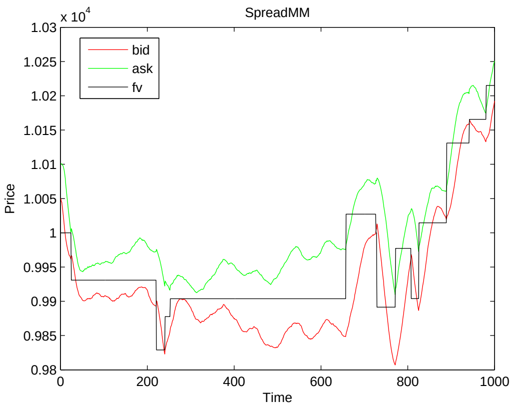
then the bid-ask spread is smaller. In period where many jumps occur, the market maker does not know what to do. The prices are not stable. We did also four experiments and varied the parameter k. The values are 1, 2, 4 and 6. The small increment is set to 90 and α is set to 0.75. The result is shown in figure 6.7. In that figure is shows that if k is larger then the market maker needs more time to converge to the fundamental value and in stable period then the bid-ask spread is smaller. These phenomena are consistent with what we see in figure 6.5, the result of the experiments with only one change in the fundamental value. In instable period the prices are not so stable, but the direction of the prices is the same as the fundamental value.



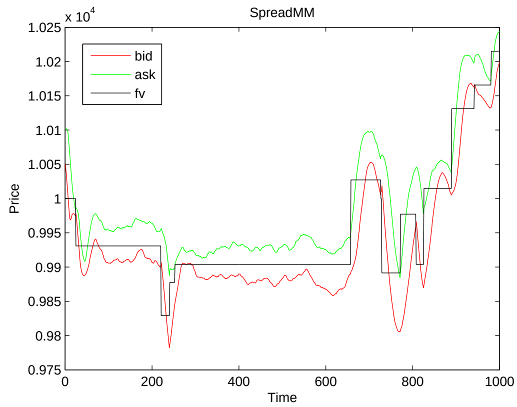
(a) $\alpha = 0.25$



(b) $\alpha = 0.33$

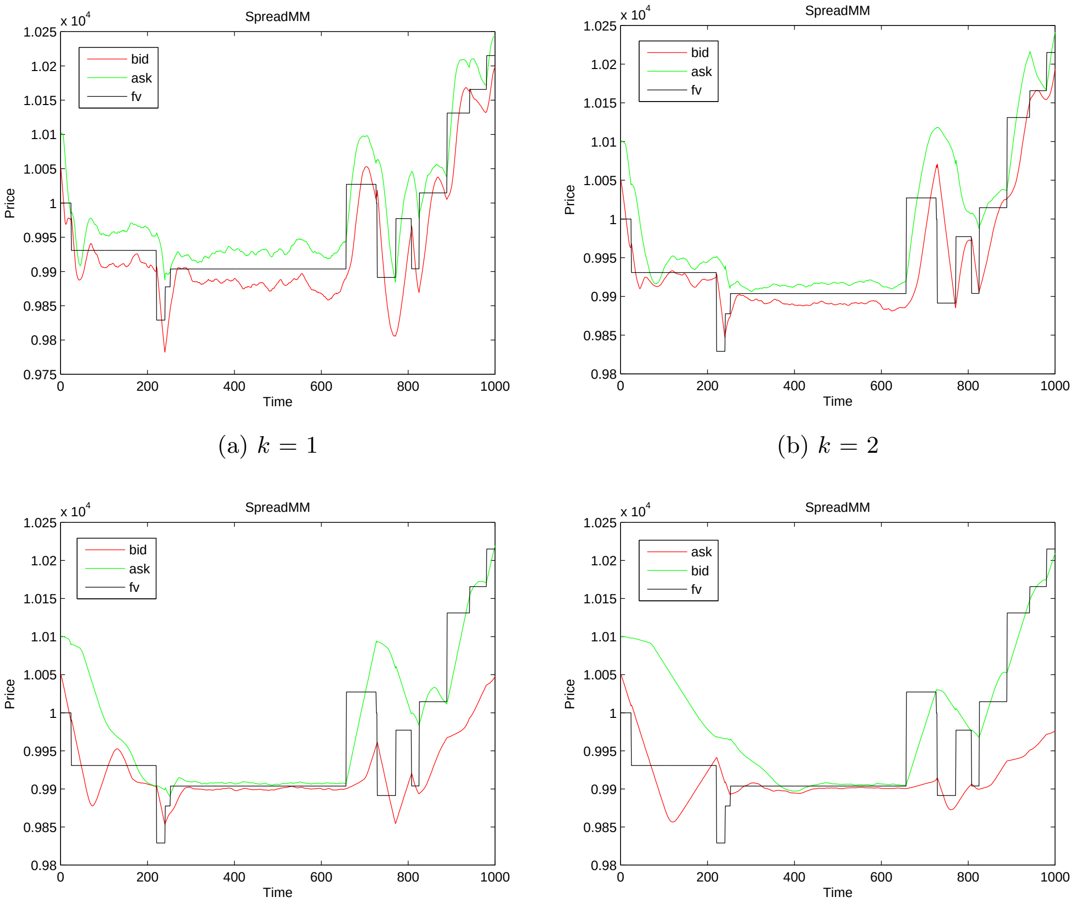


(c) $\alpha = 0.5$



(d) $\alpha = 0.75$

Figure 6.6: These are the results of the experiments of spreadMM with signalling and α's of 0.25, 0.33, 0.5 and 0.75 , k is 1 unchanged.



(a) $k = 1$

(b) $k = 2$

(c) $k = 4$

(d) $k = 6$

Figure 6.7: These are the results of the experiments of spreadMM with signalling and α is 0.75 and Δa and Δb are 90 and k is 1, 2, 4 or 6.

SpreadMM and fundamental value approximation

In the above experiments with spreadMM, the market maker gets the news signal that the fundamental value is changed and from that time step on to the current time step the distribution of the probability of buy, sell and no order is calculated. These calculated probabilities are then compared with the distribution of the prior probability of buy, sell and no order using the distance method (see equation 4.35). However, we want to model in a way such that the market maker does not know whether a fundamental change has taken place or not. The results of the experiments with fundamental value approximation are shown in figure 6.8. We did 6 experiments. Each simulation we varied the size of the period wherein the market maker calculates the probabilities. The total number of orders in equations 4.8, 4.9 and 4.10 in each time step of this period is equal to current timestep minus the number of timesteps since the beginning of the chosen size of the period. As it is seen in the results if the chosen period is large then the market maker will become more uncertain and lead to a large bid-ask spread and unstable prices.

6.2.2 Simulation with the Das market maker

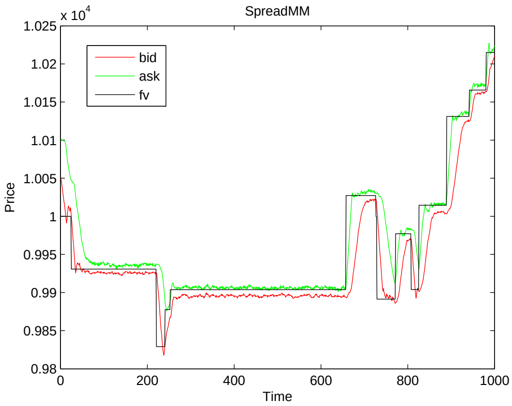
We did experiments with the Das market maker with α is 0.75 and η is 0.4. The result is shown in figure 6.9. The figure shows that after each jump in the fundamental value the market maker sets a very large bid-ask spread. He gets a signal if a change in the fundamental value has taken placed. After a jump the market maker needs time to converge to the new fundamental value. Unlike spreadMM, the Das market maker is in unstable period where there occur more than one jump also very stable and can track the fundamental value.

6.3 Comparison between Das market maker and SpreadMM

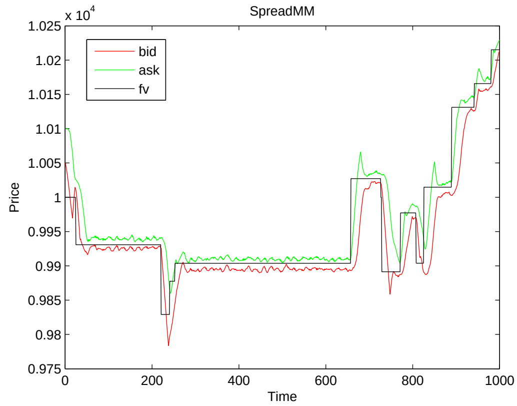
6.3.1 Using the mean of bid-ask spreads method

We use mean of bid-ask spreads method to compare the performance between the Das market maker and spreadMM. We summarise the bid-ask spreads of each time step and divide it by the number of time step. Small bid-ask spreads are preferred by the market maker. So if the mean sum of bid-ask spreads is small, then this means that the market maker is doing a good job. Figure 6.10 shows 4 experiments with the following market makers and settings (from above to below and from left to right):

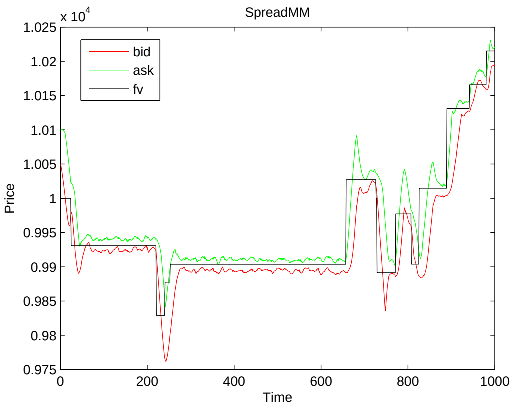
- Das market maker



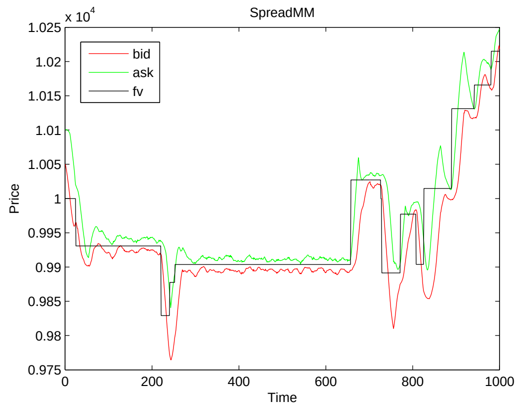
(a) period = 5



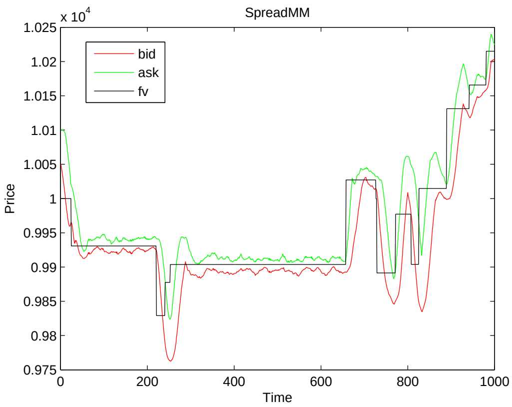
(b) period = 15



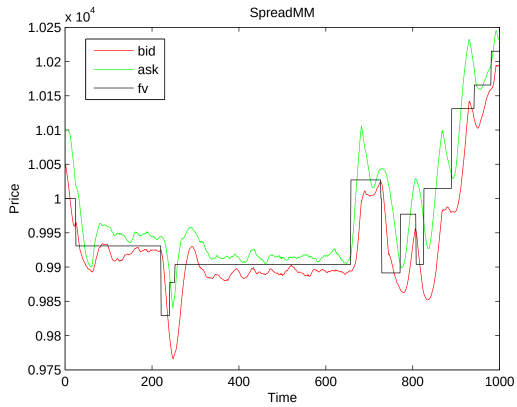
(c) period = 20



(d) period = 25



(e) period = 30



(f) period = 60

Figure 6.8: These are the results of the experiments that spreadMM does not get a signal when a change take place in the fundamental value (without signalling).

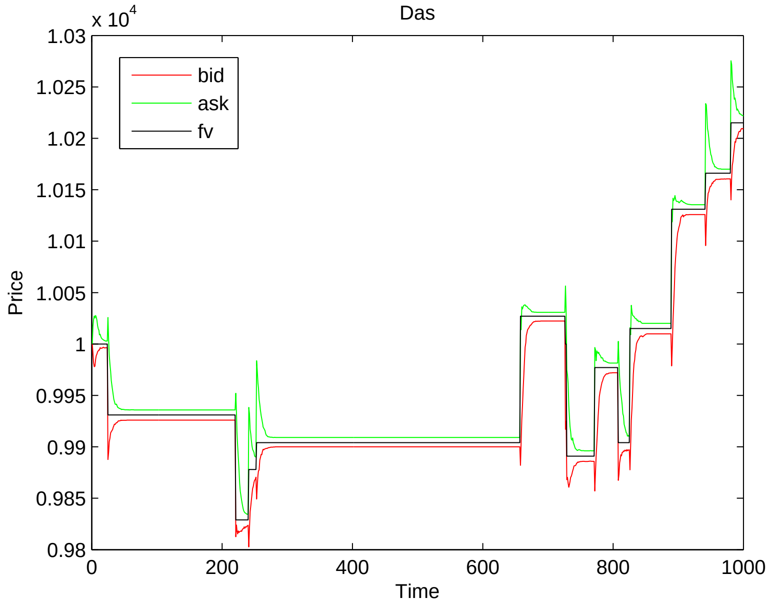
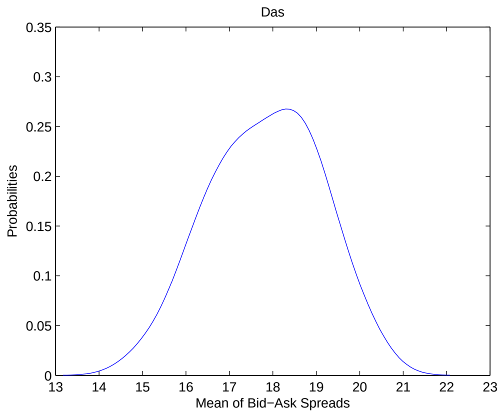


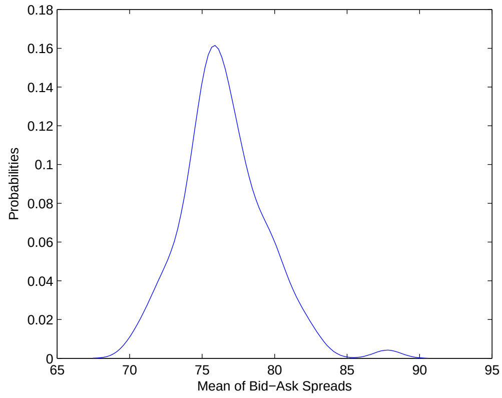
Figure 6.9: This is the result of the experiment with the Das market maker ($\alpha = 0.75$).

- SpreadMM with fundamental value changing signalling (k is 6 and is fixed in each time step)
- SpreadMM with fundamental value changing signalling (k is incremented in time step)
- SpreadMM without fundamental value changing signalling (period is 10)

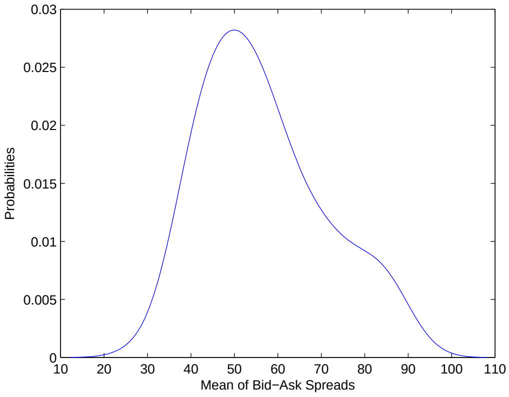
Each figure plots the probabilistic density function of the mean of bid-ask spreads over the 100 simulations generated by the market maker. The Das market maker has generated mean of bid-ask spreads between 14 and 22. SpreadMM with fundamental value changing signalling and fixed k has generated mean of bid-ask spreads between 68 and 90. SpreadMM with fundamental value changing signalling and k is incremented has generated mean of bid-ask spreads between 20 and 110. The spreadMM without fundamental value changing signalling has generated mean of bid-ask spreads between 16 and 36. We also put the result of the experiments of spreadMM without signalling, but varied the size of the period wherein the market maker calculates the probabilities to adjust the prices in figure 6.11. As shown in figure 6.11 if the period is large, for example 30 and 60, then the mean of bid-ask spreads is also increasing.



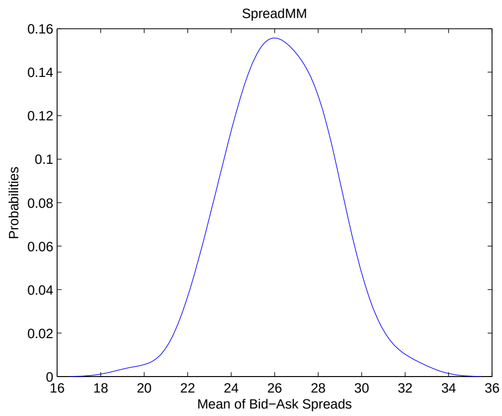
(a) Das market maker



(b) SpreadMM with signalling (k is fixed)

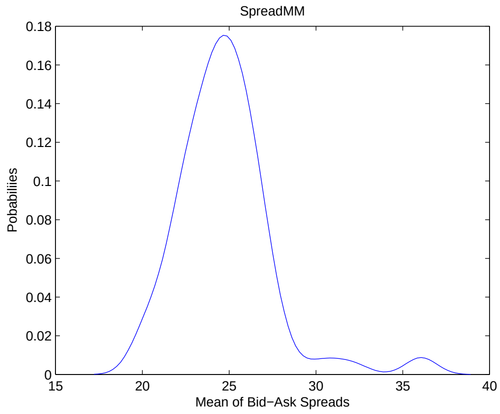


(c) SpreadMM with signalling (k is incremented

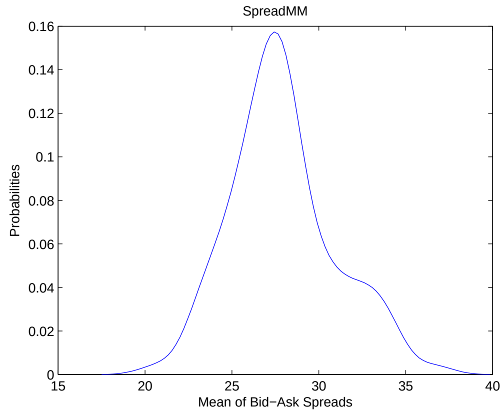


(d) SpreadMM without signalling

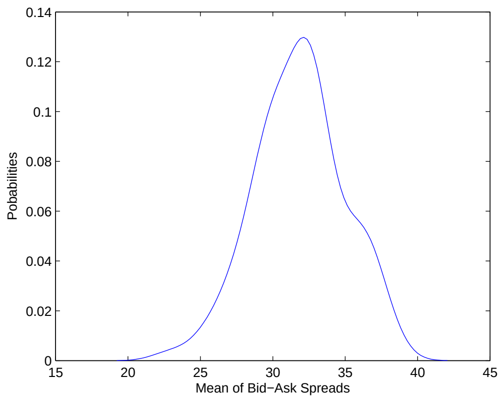
Figure 6.10: These are the results of the probabilistic density function of the mean of bid-ask spreads (with α is 0.75).



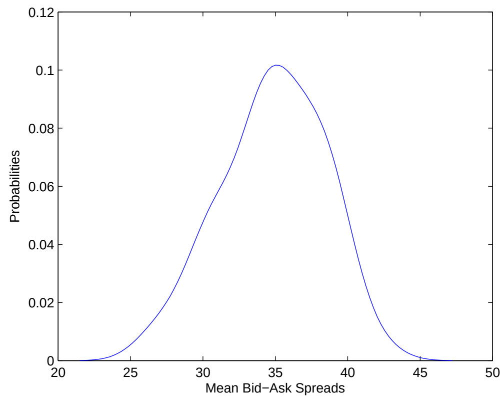
(a) period = 5



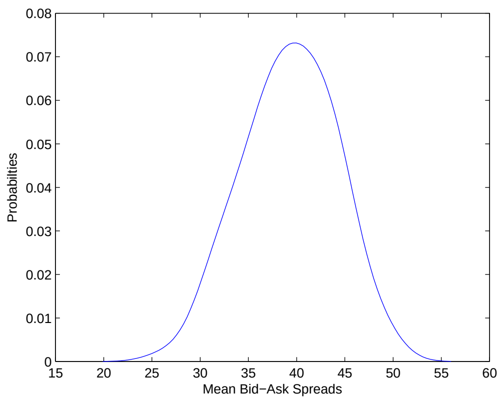
(b) period = 15



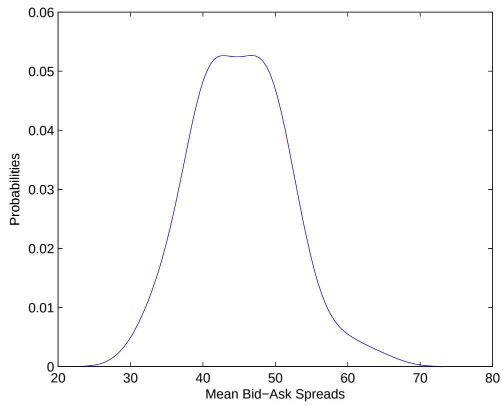
(c) period = 20



(d) period = 25



(e) period = 30



(f) period = 60

Figure 6.11: These are the results of the probabilistic density function of the mean of bid-ask spreads done with the simulation with spreadMM when he does not get a signal when a change takes place in the fundamental value.

6.3.2 Using the sum of difference between the fundamental value and the price method

We use the sum of difference between the fundamental value and the prices method to compare the performance of Das market maker and the diverse settings of spreadMM, because the mean of bid-ask spreads method says nothing about how close the market maker is tracking the real fundamental value. We use the same models described in 6.3.1 about the mean of bid-ask spreads method for camparisons. Table 6.1 and table 6.2 list the sum of difference between fundamental value and bid price, the sum of difference between fundamental value and ask price and the mean of the models. The tables show that the Das market maker tracks the fundamental value better than spreadMM. SpreadMM with signalling where k is incremented has a larger difference than spreadMM with signalling where k is fixed. As we look at the results of spreadMM without signalling then we can see that the larger the period that spreadMM uses to calculate the probabilities the larger the difference is.

Table 6.1: Table sum of difference between fv and prices 1

Model	diffbid	diffask	diffmean
Das market maker	9173.01	8772.91	8972.96
SpreadMM with signalling (k is fixed)	32081.67	36807.07	34444.60
SpreadMM with signalling(k is incremented)	53611.97	47586.65	50599.31
SpreadMM without signalling (period is 10)	20551.46	17329.24	18940.35

Table 6.2: Table sum of difference between fv and prices 2

Model	diffbid	diffask	diffmean
SpreadMM without signalling (period is 5)	19345.90	16909.15	18127.53
SpreadMM without signalling (period is 15)	21250.09	19432.14	20341.12
SpreadMM without signalling (period is 20)	23063.50	20625.36	21844.43
SpreadMM without signalling (period is 25)	26505.41	23727.31	25116.36
SpreadMM without signalling (period is 30)	28929.41	26512.80	27721.10
SpreadMM without signalling (period is 60)	32939.29	32425.62	32682.46

6.4 Conclusion

SimpleMM can quote prices which reflect the true fundamental value, because it is assumed that he knows the fundamental value. DetectMM does not work, because of the shortcomings of the model. If we compare the Das market maker with spreadMM both with the mean of bid-ask spreads method and the sum difference between the fundamental value and the bid and ask price then we see that the Das market maker performs better, because the Das market maker keeps a probability distribution of all possible fundamental values. Das market maker has more information to approximate the fundamental value than spreadMM. SpreadMM can track the fundamental value, but it is not perfect. Advantage of spreadMM over the Das market maker is that spreadMM uses a simpler fundamental value approximation method so that spreadMM uses less computation time.

Chapter 7

Conclusion and future research

7.1 Conclusion

Market makers play a very important role in today's financial markets. One of the main task of the market maker is to execute investors' orders at the best possible price. How does the market maker know what is the best price to create liquidity and at the same time profits from the trade? How does a market maker quote the bid and ask prices? Using a model we can study the pricing mechanism of the market maker. Market makers are using inside knowledge, experience to set prices on a daily basis. The order flow is one of the information a market maker uses. The question is whether the order flow contains information for the market maker to track the underlying fundamental value. In this thesis we have conducted a research on market marker's models. We introduced a simple linear market maker's price setting function with the fundamental value in it. This is not consistent with the real world. Then we introduced the method of using the order flow information to locate the position of the fundamental value with respect to the bid-ask spread. We first use the maximum value of probability order type in the order flow to determine the position of the fundamental value. However, because of the drawbacks of this method we introduced another method to use. Then we used the distribution of the probability of the order type in the order flow to determine the fundamental value. And the market maker gets a signal if the fundamental value changes. In the real world the market maker does not know in advance that the fundamental value is going to change. That's why the market maker calculates the probabilities not since a change occurs in the fundamental value, but from each small periods of time the market maker begins to compute the probabilities. We compared this market maker with the Das market maker using the mean of bid-ask spreads method and the difference between the fundamental value and the prices method. We conclude that the information in order flow does indeed contain information that the market maker can use to update the prices, but this only works in certain situations. For example if the period is relatively small then the information is more useful. Using the mean of bid-ask spreads method to compare the performance of the Das market maker and the in this paper introduced market maker spreadMM shows that the Das market maker is better, because he generates smaller bid-ask spreads. If we compare the mean of bid-ask spreads that is generated by spreadMM but with varied size of periods that the market maker is using to calculates the probabilities to adjust the prices then the results show that the larger the period the larger the bid-ask spread. If we compare the models with the sum difference between the fundamental value and the prices we also see that the Das market maker performs better than spreadMM.

7.2 Future research

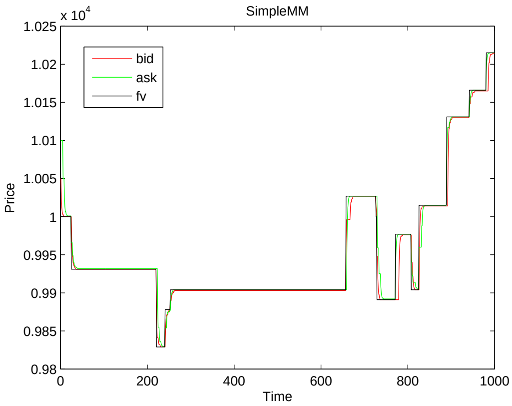
These are the possible recommendations we suggest to conduct further research:

- We suggest to use probabilistic fuzzy system to model the probability density of the order types in the order flow. It is good to model the order flow process as a probabilistic fuzzy system, because the probabilistic uncertainty of the orders placed by investors can be modelled and better understanding of the order flow process can be gained.

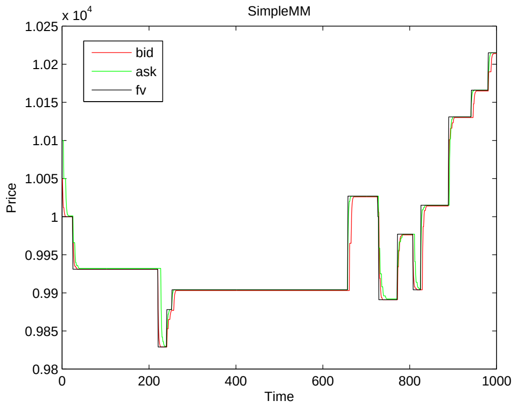
- The smallest increments Δa and Δb in the price updating function are fixed in each time step. We suggest to change this amount of incrementation in each time step, because the distance between the prices and the fundamental value differs in each time step, so the incrementation should also be different in each time step to update the prices.

Appendix A

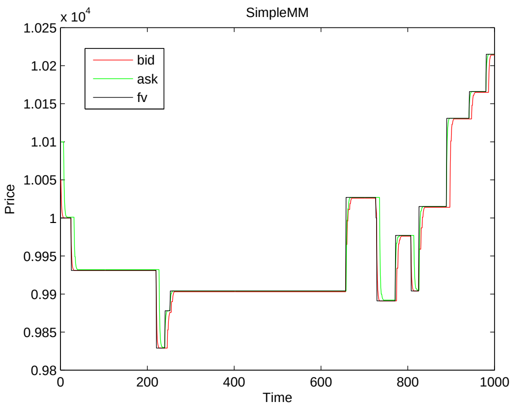
Figures



(a) $\alpha = 0.33$



(b) $\alpha = 0.5$



(c) $\alpha = 0.75$

Figure A.1: These are the results of the experiments with simpleMM.

Appendix B

Matlab codes

SpreadMM code

```
function [fv,ba,of,in] = xuemm2(fvp,tt,v0,bid0,ask0,a,eta,n,r,delta_a,delta_b);

%% Initialisation
% Error checking
if (nargin < 1), error( 'Not enough input arguments.' ); end;
if (nargin < 2), v0 = 10000; elseif isempty(v0), v0 = 10000; end;
if (nargin < 3), bid0 = 9950; elseif isempty(bid0), bid0 = 9950; end;
if (nargin < 4), ask0 = 10050; elseif isempty(ask0), ask0 = 10050; end;
if (nargin < 5), a = 0.3; elseif isempty(a), a = 0.3; end;
if (nargin < 6), eta = 0.5; elseif isempty(eta), eta = 0.5; end;
if (nargin < 7), pj = 0.001; elseif isempty(pj), pj = 0.001; end;
if (nargin < 8), sw = 0; elseif isempty(sw), sw = 0; end;
if (nargin < 9), r = 0; elseif isempty(r), r = 0; end;
if (nargin < 10), delta_b = 10; elseif isempty(delta_b), delta_b = 10; end;
if (nargin < 11), delta_a = 10; elseif isempty(delta_a), delta_a = 10; end;
if ( a < 0 ) | ( a > 1 ), error( 'A must be between 0 and 1. '); end;
if ( eta < 0 ) | ( eta > 0.5 ), error( 'ETA must be between 0 and 0.5.' ); end;

% Generate fundamental value series
nt = size(fvp,1);
if (nt == 1), nt = fvp; fvp = generate_fvp(nt,v0,pj,sv); end;
if (r == 1), fvp = round(fvp); end;

% Initialise output storage
ba = zeros(nt,2);
fv = fvp;
of = zeros(nt,1) - 2;
in = zeros(nt,1);
```

```
index = -1;
bid = bid0; ask = ask0;
ba(1,:) = [bid,ask];
asksign(1) = NaN;
bidsign(1) = NaN;
k = 1;

% priors when bid <= ask <= fv
fv_above = [(1-a)*eta + a, (1-a)*eta, 1-(a+2*eta(1-a))];
% priors when bid <= fv <= ask
fv_between = [(1-a)*eta, (1-a)*eta, 1-(2*eta(1-a))];
% priors when fv <= bid <= ask
fv_beneath = [(1-a)*eta, (1-a)*eta + a, 1-(a+2*eta(1-a))];

priors = [fv_above; fv_between; fv_beneath];

for i = 2:nt,

    trader = tt(i);

    if (tt(i) == 0)
      order = informed_trader(fvp(i),ba(i-1,:));
    elseif (tt(i) == 1)
      order = 1;
    elseif (tt(i) == 2)
      order = -1;
    else
      order = 0;
    end

    of(i) = order;
    trades = compute_trades(of);

    maximum = find(trades == max(trades));

    if (size(maximum,2) > 1)
      if order == 1,
        index = 1;
      elseif order == -1,
        index = 2;
      else
        index = 3;
      end
    else
```

```
  index = maximum;
end

in(i) = index;

switch index,

case 1
  if order == 1,
    if (asksign(i-1) == 1)
      k = 1;
    end
    ask = compute_ask(ba,i,delta_a,k);
    bid = ba(i-1,1);
    asksign(i) = 0;
    bidsign(i) = bidsign(i-1);

  else if order == -1,
    if (bidsign(i-1) == 1)
      k = 1;
    end
    bid = compute_bid(ba,i,delta_b,k);
    ask = ba(i-1,2);
    if (bid > ask)
      bid = ask;
    end
    bidsign(i) = 0;
    asksign(i) = asksign(i-1);
  else
    if (bidsign(i-1) == 1)
      k = 1;
    end
    bid = compute_bid(ba,i,delta_b,k);
    bidsign(i) = 0;
    if (asksign(i-1) == 1)
      k = 1;
    end
    ask = compute_ask(ba,i,delta_a,k);
    asksign(i) = 0;
  end
end

case 2
  if order == 1,
```

```
    if (asksign(i-1) == 0)
      k = 1;
    end
    ask = compute_ask(ba,i,-delta_a,k);
    bid = ba(i-1,1);
    asksign(i) = 1;
    bidsign(i) = bidsign(i-1);
    if (ask < bid)
      ask = bid;
    end
  else if order == -1,
    if (bidsign(i-1) == 0)
      k = 1;
    end
    bid = compute_bid(ba,i,-delta_b,k);
    ask = ba(i-1,2);
    bidsign(i) = 1;
    asksign(i) = asksign(i-1);
  else
    if (bidsign(i-1) == 0)
      k = 1;
    end
    bid = compute_bid(ba,i,-delta_b,k);
    bidsign(i) = 1;
    if (asksign(i-1) == 0)
      k = 1;
    end
    ask = compute_ask(ba,i,-delta_a,k);
    asksign(i) = 1;
  end
end

case 3
  if order == 1,
    if (asksign(i-1) == 0)
      k = 1;
    end
    ask = compute_ask(ba,i,-delta_a,k);
    bid = ba(i-1,1);
    asksign(i) = 1;
    bidsign(i) = bidsign(i-1);
    if (ask < bid)
      ask = bid;
    end
```

```
      else if order == -1,
        if (bidsign(i-1) == 1)
          k = 1;
        end
        bid = compute_bid(ba,i,delta_b,k);
        ask = ba(i-1,2);
        bidsign(i) = 0;
        asksign(i) = asksign(i-1);
        if (bid > ask)
          bid = ask;
        end
      else
        if (bidsign(i-1) == 1)
          k = 1;
        end
        bid = compute_bid(ba,i,delta_b,k);
        bidsign(i) = 0;
        if (asksign(i-1) == 0)
          k = 1;
        end
        ask = compute_ask(ba,i,-delta_a,k);
        asksign(i) = 1;
        if (bid > ask)
          bid = ask;
        end
      end
    end

    otherwise
      error('There are only three cases');
    end

  if ( r == 1 ), bid = floor(bid); ask = ceil(ask); end;
  ba(i,:) = [bid, ask]; k = k + 1;
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% compute the number of buys, sells or holds
%% tradesRatio = [buys ratio, sells ratio, holds ratio]
function trades = compute_trades(tt);
trades = [size(find(tt == 1),1)/size(tt,1),...
          size(find(tt == -1),1)/size(tt,1),...
          size(find(tt == 0),1)/size(tt,1)];
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Function to generate fundamental values to be used in the
% simulation
function fvp = generate_fvp(nsteps,v0,pj,sv);
jumps = [0;rand(nsteps-1,1) < pj]; jsize = sv*randn(nsteps,1);
fvp = v0 + cumsum(jumps .* jsize);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Function to determine the order type for the informed trader
function order = informed_trader(fv,ba);
if ( fv > ba(2)),
    order = 1;     % buy
elseif ( fv < ba(1) ),
    order = -1;    % sell
else
    order = 0;     % hold
end

%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function ask = compute_ask(ba,i,delta_a,k);
ask = ba(i-1,2) + (delta_a/k);

%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function bid = compute_bid(ba,i,delta_b,k);
bid = ba(i-1,1) + (delta_b/k);
```

SimpleMM Code

```
for i = 2:nt,

    trader = tt(i);

    if (tt(i) == 0)
      order = informed_trader(fv(i),ba(i-1,:));
    elseif (tt(i) == 1)
      order = 1;
    elseif (tt(i) == 2)
      order = -1;
    else
      order = 0;
    end

    of(i) = order;
```

```
    if (order == 1)
      ask = compute_ask(gamma,fv(i),ba(i-1,:));
    elseif (order == -1)
      bid = compute_bid(gamma,fv(i),ba(i-1,:));
    else
      bid = compute_bid(gamma,fv(i),ba(i-1,:));
      ask = compute_ask(gamma,fv(i),ba(i-1,:));
    end

  if ( r == 1 ), bid = floor(bid); ask = ceil(ask); end;
  ba(i,:) = [bid,ask];
end
```

Generate type of traders serie code

```
function tt = generate_traders(nsteps,a,eta);
%% Generate traders
% 0 informed trader
% 1 uninformed trader who buys
% 2 uninformed trader who sells
% 3 uninformed trader who holds

tt = zeros(nsteps,1);
tt(1) = -2;

for i = 2:nsteps,
  % determine type of trader

  if ( rand(1) > a ) % uninformed trader
    r = rand(1);
    if ( r <= eta ),
      tt(i) = 1;    % buy
    elseif ( (r > eta) & (r <= 2*eta) ),
      tt(i) = 2;    % sell
    else
      tt(i) = 3;   % hold
  % else: trader is informed
  end

end

end
```

Different fundamental value with bid or ask price code

```
clear;
clc;

map = uigetdir;

dirListing = dir(map);

sumbid = 0;
sumask = 0;
meanba = ones(1000,2);
nsteps = 1000;

for i = 1:nsteps,
  for d = 3:length(dirListing),
    filename = dirListing(d).name;
    load(filename);
    sumbid = sumbid + ba(i,1);
    sumask = sumask + ba(i,2);
  end
  meanba(i,1) = sumbid / 100;
  meanba(i,2) = sumask / 100;
  sumbid = 0;
  sumask = 0;
end

diffbid = sum(abs(fv - meanba(:,1)));
diffask = sum(abs(meanba(:,2) - fv));
diffmean = (diffbid + diffask) / 2;
```

Calculate mean 100 simulations to plot

```
clear;
clc;

map = uigetdir;

dirListing = dir(map);

sumbid = 0;
sumask = 0;
```

```
meanba = ones(1000,2);
nsteps = 1000;

for i = 1:nsteps,
  for d = 3:length(dirListing),
    filename = dirListing(d).name;
    load(filename);
    sumbid = sumbid + ba(i,1);
    sumask = sumask + ba(i,2);
  end
  meanba(i,1) = sumbid / 100;
  meanba(i,2) = sumask / 100;
  sumbid = 0;
  sumask = 0;
end
```

Calculate the mean sum of bid-ask spreads

```
clear;
clc;

map = uigetdir;

dirListing = dir(map);

for d = 3:length(dirListing),

  filename = dirListing(d).name;
  load(filename);
  meanba(d) = sum((ba(:,2) - ba(:,1))) / size(ba,1);
end
```

Bibliography

[1] L. Glosten abd P. Milgrom. Bid, ask and transaction orices in a specialist market with heterogeneously informed traders. *Journal of Financial Economics*, 14(1):70–100, 1985.

[2] A.R. Admati and P. Pfleiderer. A theory of intraday patterns: volume and price variability. *The Review of Financial Studies*, 1(1):2–40, 1988.

[3] Y. Amihud and H. Mendelson. Dealership market: market making with inventory. *Journal of Financial Economics*, 8(1):31–53, 1980.

[4] L. Glosten B. Biais and C. Spatt. Market microstructure: A survey of microfoundations, empirical results, and policy implications. *Journal of Financial Markets*, 8:217–264, 2005.

[5] W. Bagehot. The only game in town. *Financial Analysts Journal*, 27(2):12–14,22, 1971.

[6] M. Brunnermeier. Asset pricing under asymmetric information: bubbles, crashes, technical analysis, and herding. *Oxford University Press*, 2001.

[7] Nicholas T. Chan and Christian R. Shelton. An electronic market-maker. In *Seventh International Conference of the Society for Computational Economics*, 2001.

[8] N.T. Chan. *Artificial markets and intelligent agents.* PhD thesis, Massachusetts Institute of Technology, 2001.

[9] M. Chhabra and S. Das. Learning the demand curve in posted-price digital to market auctions. In *Proceedings of the Tenth International Joint Conference on Autonomous Agents and Multi-Agent System*, pages 63–70, 2011.

[10] S. Das. Intelligent market-making in artificial financial markets. Master's thesis, Department of Electical Engineering and Computer Science, Massachusetts Institute of Technology, 2003.

[11] S. Das. A learning market maker in the glosten-milgrom model. *Quantitative Finance*, 5(2):169–180, 2005.

[12] S. Das. Learning to trade with insider information. In *Proceedings of the Ninth International Conference on E-Commerc*, 2007.

[13] S. Das. The effects of market-making on price dynamics. In *Proceedings of the Seventh International Joint Conference on Autonomous Agents and Multi-Agent System*, pages 997–894, 2008.

[14] S. Das and M. Magdon-Ismail. Adapting to a market shock: Optimal sequential market-making. In *Advances in Neural Information Processing Systems 21*, pages 361–368, 2009.

[15] S. Das and M. Magdon-Ismail. Comparing prediction market structures, with an application to market making. *Quantitative Finance Papers arXiv.org*, 2010.

[16] F.D. Foster and S. iswanathan. Strategic trading when agents forecast the forecasts of others. *Journal of Finance*, 51(4):1437–1478, 1996.

[17] M. Garman. Market microstructure. *Journal of Financial Economics*, 3(3):257–275, 1976.

[18] T. Ho and H. Stoll. Optimal dealer pricing under transactions and return uncertainty. *Journal of Financial Economics*, 9(1):47–73, 1981.

[19] T. Ho and H. Stoll. The dynamics of dealer markets under competition. *Journal of Finance*, 38(4):1053–1074, 1983.

[20] C.W. Holden and A. Subrahmanyam. Long-lived private information and imperfect competition. *Journal of Finance*, 47(1):247–270, 1992.

[21] F. De Jong and B. Rindi. *The Microstructure of Financial Markets.* Cambridge University Press, 2009.

[22] A. Kyle. Continuous auctions and insider trading. *Econometrica*, 53(6):1315–1336, 1985.

[23] A. Kyle. Informed speculation with imperfect competition. *Review of Economic Studies*, 56:317–355, 1989.

[24] A. Madhaven. Market microstructur: a survey. *Journal of financial markets*, 3(3):205–258, 2000.

[25] A. Madhaven. Market microstructur: a practioner's guide. *Financial Analysts Journal*, 58(2):1–28, 2002.

[26] M. O'Hara. *Market Microstructure Theory.* Blackwell Publishers, 2000.

[27] C. Parlour and D. Seppi. Limit order markets: A survey. *Handbook of Financial Intermediation and Banking*, 2008.

[28] R.A. Schwartz. *Reshaping the equity markets: a guide for the 1990s.* HarperBusiness, 1991.

[29] S. Smidt. The road to an efficient stock market. *Financial Analysts Journal*, 27:18–20, 64–69, 1971.

[30] H.R. Stoll. The supply of dealer services in securities markets. *Journal of Finance*, 33(4):1133–1151, 1978.

[31] S. Tinic and R. West. Competition and the pricing of dealer services in the over-the-counter market. *Journal of Financial and Quantitative Analysis*, 8:1707–1727, 1972.