

# Search Theory and the Effects of Transparency in a Career Concerns Setting\*

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March 8, 2012

## Abstract

We consider the problem of sequential search where a manager can choose to employ a maximum of two agents that search individually and decide when to stop searching independent of one another. We study the effects of reputational concerns and transparency on the agents' optimal stopping rules and find that more transparency is not always in the interest of the manager. Specifically if the proportion of smart agents is high and agents attach a relatively high weight to their reputation, minimum transparency inevitably leads to a higher outcome than full transparency. Additionally, we find that reputational concerns can mitigate the social loafing effect that may occur if two agents search. This is particularly the case if the manager chooses minimum transparency, as then employing a second agent in fact instigates agents to exert more effort (given that agents care sufficiently about their reputation). We provide welfare implications and a number of discussion topics.

**Keywords:** search theory, reputational concerns, transparency, optimal stopping rules

**JEL codes:** C73; D82; D83

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\*I am grateful to Benoît Crutzen, Florian Ederer, Johannes Spinnewijn, Bauke Visser and in particular Otto Swank for many useful comments.

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# Preface

This thesis concludes my Master's studies in Economics of Markets, Organisations and Policy at the Erasmus University of Rotterdam. I have started working on this research in the spring of 2011, and since it has been an enduring journey of (not seldom pretended) discoveries and momentary triumph, setbacks and frustration, and ultimately achievement. Throughout the process, I have gained valuable insight in doing research in the field of micro-economics, which I will cherish for many years to come. I would like to extend my gratitude to my supervisor prof. dr. Otto Swank for many valuable comments and ideas, and helping me overcome the necessary challenges I have encountered in the progress of my works. I hope this thesis inspires others, and that in the future, someone will be encouraged to pick up where I left off. To anyone who ventures to read this thesis, all that remains for me to say is, in a most sincere way:

Enjoy.

Rens van den Broek, March 2012

Master's thesis Economics and Business  
Economics of Markets, Organizations and Policy

Student ID: 312206

Supervisor: Prof. dr. Otto H. Swank

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# 1 Introduction

Recruiting and cultivating new talent is one of the key ingredients for success in any modern-day sports team. Colleges in the US grant sports scholarships to gifted high school seniors, while many professional sports teams search for promising athletes of an even younger age. The English soccer player Wayne Rooney, for example, was discovered performing on Liverpool's Long Lane fields at an age of eight by Bob Pendleton. Pendleton, then scout of Everton F.C., did not hesitate and immediately asked the youth academy director at Everton to sign the youngster. Rooney would grow up to become a world famous player, yielding Everton an approximate 27 million pounds in transfer revenues.

It is not uncommon for a professional sports team to employ more than one scout. For instance Everton is said to have a network of 200 scouts examining talent domestically and abroad. Only recently it was announced that Steve Calligan would be hired as Everton's new scout, whereby he has been given the assignment to find the best talent the Yorkshire region has to offer. For this task he will have six scouts to his disposal who will get out and about to as many junior football games as possible in the county. This indicates that even in one and the same confined geographical area, one may find several scouts of the same team on the lookout for the next big star.

The phenomenon of collectively yet independently searching is ubiquitous, and can be found also outside the field of sports. We name a few settings in which this may occur:

- (a) special officers of a multinational company that search for a new country in say, south-east Asia to expand their superior's business to;
- (b) two (future) business partners seeking for example (i) a place to set up their business, (ii) an investment opportunity or a line of business to become active in, or something as simple as (iii) a developer to build their new website;
- (c) two bounty hunters searching for the same targets; and
- (d) a couple that has decided to move in together looking for a new house in a certain area. While typically they would make the final decision conjointly, it

is not unthinkable that they first search independently and present each other their (best) results.

In our paper we will explore these settings under the assumption that the individuals that search care (exclusively) about their reputation. The examples set forth in the first two paragraphs and under (a) will be further elaborated on in light of this assumption – and our model in general – in the subsection below. Our paper has two main goals. The first goal is to study under which conditions it is better (from the point of view of the superior) to hire two employees rather than one, in a search-theoretic setting. The second goal is to research the effect of transparency on this decision rule. We find, *inter alia*, that the supervisor may be better off hiring more than one employee if the employees care about their reputation, while this is not necessarily the case, *ceteris paribus*, if they (also) care about the outcome of the assignment itself. Furthermore, arguably unexpectedly, we find that more transparency is not always better from the supervisor’s perspective.

## 1.1 Related Literature

This paper contributes to the literature on the economics of search. The theory of search has been momentous in many areas of economics, including the economics of information and uncertainty. Research on the problem of sequential search conventionally focuses on an individual who periodically draws from a known and exogenous distribution. After each independently and identically distributed draw, the individual can either decide to stop searching, therewithal accepting the payoff he receives from the effectuation of the current draw, or to continue searching for a more lucrative opportunity. The intention of most studies on this subject is to find an optimal stopping rule under various circumstances (see for example the seminal papers Chow and Robbins, 1960/1963a; Kohn and Shavell, 1974). Typically these search models show the optimal way of balancing the value of extending the search against the cost of delay. Alternatively, the cost can be expressed as simply the effort it takes to conduct yet another draw.

This framework is used in many areas of applied economic analysis, with its most notable influence in the field of labor economics. George Stigler (1961/1962) was arguably the first to develop a search model, although he did use a static ap-

proach. Later on, in McCall (1970) the canonical model on sequential job search was developed that gave rise to a wide range of papers that aim to model the labor market. This literature uses search theory to address issues such as frictional unemployment and the determination of wages. McCall and Lippman (1979), Mortensen (1986/1999) and Rogerson et al. (2005) have provided excellent surveys of this string of literature. Noteworthy early additions to this particular field include Mortensen (1970) on the duration of unemployment and the Phillips curve, Gronau (1971) on information and frictional unemployment, Gronau (1973) on wage comparisons, Feldstein (1976) on temporary layoffs and Burdett (1978), in which the on-the-job search model is introduced. More recently a new, macroeconomic application of job search has emerged in the form of matching theory. Influential publications in this respect are Mortensen and Pissarides (1994) and the textbook treatment by Pissarides (2000). A clear overview of this strand of literature can be found in the survey by Petrongolo and Pissarides (2001). Other fields in which search theory is widely applied include consumer theory, industrial organization, monetary economics, finance and economics of the marriage market.<sup>1</sup> We will however not fully cover all of these areas as they are outside the scope of this article.

Originally, in almost all of the search literature the stopping decision was made by an agent acting in isolation. More recent studies on this topic focus on searching in committees, starting with the pioneer article by Compte and Jehiel (2004). This moves the decision of when to stop from a single agent to multiple agents as a group. The common denominator of these studies is that they examine the effect of the committee size and voting rule on the optimal stopping rule and outcome. While Albrecht et al. (2010) mainly focuses on a committee consistent of homogeneous members, Compte and Jehiel (2010a/2010b) and Moldovanu and Shi (2010) also consider the effect of heterogeneity. Guler et al. (2009) takes a different approach and examines the implications of joint-search for jobs in a household setting. Finally, Kamada and Muto (2012), studies a multi-agent search problem with a finite search horizon.

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<sup>1</sup>Many insights from the labor market models are also applicable to consumer theory. Where labor economist McCall introduced the reservation wage, the same analysis underlies the economic concept of the reservation price, used in consumer theory to calculate the consumer surplus. Furthermore, Janssen et al. (2005) offers valuable insights in consumer search and pricing in oligopolistic markets. Examples of papers in other fields are Jovanovic (1982, IO), Kiyotaki and Wright (1993, monetary theory), Weill (2007, finance) and Oppenheimer (1988, marriage timing).

Our model harks back to the classical problem of sequential search, and we try to add to this by including a second agent. The most important difference between this paper and the papers on *joint search theory* or *search by committee* is that in our paper each agent individually and simultaneously searches and decides when to stop searching *in isolation*. As only one of the submitted projects is finally implemented, this setting may create a space of competition. Another effect is that by employing more than one agent, the superior (e.g. a manager or firm) can increase his chances of finding a good project. This is particularly the case if agents differ in ability or skill, and the ability of an agent is not observable for the superior. As mentioned above, one of the main goals of this paper is to examine whether and under which conditions these effects combined are healthy for the final result in terms of the superior's utility.

One can think of various situations in which the assumption of two agents simultaneously searching for projects in competition is realistic. Consider a company where there is a fixed budget for investments, so that only a limited number of investments (e.g., one) can be made. The manager of that company can make a decision whether to put one employee on the task of searching for an investment opportunity, or several employees. There are numerous practical examples of these competitive settings. First bear in mind the story set forth in the first paragraphs of the introduction about a sports team that wants to recruit new talent. We have seen that it is not uncommon that such sports team would give several scouts the assignment to explore the same area. Furthermore, often these scouts will want to be linked to good players. More evocatively, they often care about how their ability is judged by the public.<sup>2</sup> Another example in a different setting is a manufacturing firm that aims to expand to a new country or city. The firm can decide to put one employee on the task of finding a proper location for a new plant, or spread its chances of success by employing two or more employees and letting them search individually. Each employee can visit and investigate as many locations as he desires, and individually decides for which minimum value of a location he settles. The value of a location in this case, for instance, consists of the expected profitability of

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<sup>2</sup>It should be noted that in our model, scouts care only about their *absolute* reputation, and not about their *relative* reputation – whether their ability is assessed to be better or worse than their colleagues' (in a similar fashion, see Sharfstein and Stein, 1990).

the location in terms of future profits. In order to measure this profitability, several components have to be considered and weighed, such as the local market opportunities and competitors, the available distribution channels, the local workforce and (minimum) wages, the distance from the domestic market and the price of land. Needless to explain, executing this task is less costly in terms of effort for more skilled or talented employees. Therefore talented employees can distinguish themselves from less talented employees by submitting a location with a higher expected profitability. Assuming that location values are verifiable for the firm (even though it is not for the public), lying about the value of a submitted location is useless.

In our paper we will generalize this case to a setting with one or more agents, a manager and a public. Importantly, in the crucial part of our paper we assume that agents care only about their reputation. That is, the view the public holds about their type (say, ability) is their only drive to search for any project. Variations of this assumption are common in the literature on reputational concerns, which emerged starting with the groundbreaking Holmström (1982/1999) career concerns model. Reputational concerns are essential in the herding literature, see for instance Scharfstein and Stein (1990). A vital disparity between the herding literature and our paper is that the distinguishing feature of the former is that actors behave sequentially. Herding occurs when a player claims he holds the same private information as the preceding player, as revealed by that player's decision in the previous stage. In our model however, agents act simultaneously so that this kind of mimicking behavior cannot exist. Articles that resemble our model in this respect include Visser and Swank (2007) and Levy (2007), which both focus on decision-making in committees. While reputational concerns are oftentimes regarded as a social burden<sup>3</sup>, other papers such as Suurmond et al. (2004) show that in some settings they may constitute a social blessing. There is also a range of related literature that explores how different transparency levels may improve the agent's incentives and thereby alleviate the harmful effects or intensify the beneficial effects of reputational concerns. For examples see Meade and Stasavage (2008), Prat (2005), Levy (2007) and Swank and Visser (2010). As pointed out in the introduction, we will study the effect of transparency in light of our model as well.

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<sup>3</sup>Notably in the herding literature, where career concerns can lead to agents ignoring valuable information.

To our best knowledge, this paper is the first to combine search theory and reputational concerns on the side of the searching agent. We need to note that in Galenianos et al. (2012) a model with a similar feature is developed. This article contains a search-theoretic model of the retail market for illegal drugs, in which the sellers experience some form of career concerns (with respect to the quality they offer). The important difference with our analysis, however, is that in the Galenianos model no career concerns are incorporated for the players that actually do the searching (the buyers), which is exactly what we will do in our model. In doing so, we hope to take a step in the right direction of closing the gap between the literature on search theory and the literature on career concerns (and transparency).

The remainder of the paper is organized as follows. In the next section we introduce our model. We analyze the model from the perspective of the agents in sections 3 and 4 and use the findings in these sections to obtain welfare implications and optimal decision rules for the manager in section 5. In section 6 we briefly discuss our results and offer some ideas for further research. Section 7 concludes.

## 2 The Model

We consider a manager (she) who supervises  $n$  agents (he),  $n \in \{1, 2\}$ , and a public. The manager can decide at the beginning of the game on how many agents to employ. Agents differ in type: an agent can be either smart or dumb  $t_i \in \{sm, db\}$ , where  $\Pr(t_i = sm) = \pi$  and the agent is dumb with the remaining probability. The pool of agents is sufficiently large to ensure that the agents' types are independently and identically distributed. The manager nor the public observes an agent's type; an agent can only observe his own type. The job of an agent is to search for a proper investment project. Each project yields a certain value  $p$  to the manager and the public. In order to simplify our calculations, we assume that there is a range of projects of which the net present value is uniformly distributed on the interval  $p \in [0, 1]$ . Each agent individually and simultaneously searches for a project. There is however only sufficient budget to execute one project, while only projects with a positive value can be executed. After searching and finding a project with a desired value, each agent submits his results (that is, his most recently found project) to the manager, and she executes the project which yields her the highest payoff. The



manager can costlessly verify the value of the submitted projects (this is ‘hard’ information), whereas the public is unable to do so. We will call the agent whose project is executed the winner of the game and the other (if there is another agent) the loser. If however both agents decide not to search, no project is implemented and they are both considered losers. Before the agents are employed, the manager can decide how much information about the searching process to make public, that is, she can choose a transparency level. Without loss of generality, throughout we assume that she can commit *ex ante* to this decision. We identify two transparency levels: full transparency and minimum transparency. When there is full transparency, the public sees the values that all (employed) agents have submitted (and hence, for each agent whether he is a winner or a loser). When there is minimum transparency, the public only observes for each employed agent whether he is a winner or a loser, and cannot observe any of the submitted values. The message the public receives from the manager is essentially cheap talk: the public cannot verify the information the manager reveals. As the manager does not benefit by lying (*ex post*), however, we can safely assume that – regardless of the transparency level – she reveals all information truthfully.

An agent can search as many times as he desires. However searching is costly: every time an agent decides to search, he has to incur a constant cost of  $c_{t_i} \in [0, \alpha]$ , where  $\alpha < 1$ . The cost of searching depends on the agent’s type: if the agent is smart, this cost is lower than if he is dumb:  $c_{sm} < c_{db}$ . Barring his wage, an agent cares exclusively about the public’s estimation of his ability. The latter is commonly known in literature as reputational or career concerns, and can be defined as the public’s posterior belief of the agent being smart. We denote this by  $\hat{\pi}$ . The weight agents attach to their reputation is measured by the parameter  $\lambda \in [0, \gamma]$ , and is common knowledge. The value of the project submitted by agent  $i$  we denote by  $p_i$ , whereas the value of the project that is implemented is denoted by  $\bar{p}$ . Summarizing, agent  $i$ ’s preferences are represented by:

$$U_{A_i}(p_i) = W + \hat{\pi}\lambda - kc_{t_i} \tag{1}$$

Where  $k$  is the number of searches agent  $i$  conducts and  $W$  is the wage he receives from the manager by participating.

The manager only cares about the value of the project that she eventually implements, which gives us the following utility function for the manager:

$$U_M = \bar{p} - nW \tag{2}$$

Where  $\bar{p}$  is equal to  $p_i$  if  $n = 1$ , and equal to  $\max(p_i, p_j)$  if  $n = 2$ .

We assume that if an agent decides not to participate (e.g., not to take the job), he will be conjectured by the public to be smart with probability  $\pi$  (which is the prior probability that an agent is smart). Meanwhile, to ensure that both types of agents want to participate under all circumstances, we assume that the wage is exogenously determined and  $W \geq \pi\lambda$ . To see that this condition is sufficient, note that an agent can always decide not to search, once participating. We hereby implicitly assume that the manager cannot force the agent to exert effort by contract. The manager cannot offer the agent any kind of variable wage, nor fire or promote his employees. Put differently, the only means by which the manager can influence the agent's effort decision is the (non-)disclosure of certain information.<sup>4</sup> This implies that, as long as  $W$  is equal to or greater than  $\pi\lambda$  (which would be the payoff of an agent if he would not participate), an agent can never be worse off by participating as he always has the choice to exert no effort. Furthermore, we assume that the manager cannot decide to employ zero agents. That is, the manager always participates.

The timing of the model is as follows:

1. The manager chooses the number of agents  $n$  to employ and the transparency level;
2. Nature draws  $t_i$  and, if  $n = 2$ ,  $t_j$ ;
3. Each agent observes his own type, the manager does not observe  $t_i$  nor  $t_j$  (and neither does the public);

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<sup>4</sup>One could argue that the decision to slack off at his current job influences the agent's future prospects on the job market. While this is true given that shirking is bad for the agent's reputation, this is not a reason for the agent not to participate. Assuming that the wage the agent can earn at this job is competitive, his type is static over time and the agent has a negative time preference, there is no reason to decline the job in view of waiting to take another job in the future. If the agent conjectures that he will not search if he participates, he will do so at any other job so that a decline in reputation is inevitable.

4. Each agent chooses a strategy based on his type and the other parameters, which strategy can either be to search a specific number of times (including zero), or a threshold value between zero and one for which he stops searching for a higher value;
5. Each agent simultaneously carries out his equilibrium strategy;
6. Each agent submits his most recently found project (or skips this step if the agent's strategy is not to search);
7. The manager implements the project with the highest value (or skips this step if no project is submitted at all);
8. The *ex ante* decided amount of information is disclosed and payoffs are realized.

We solve the game using backward induction. In the next two sections, we will identify several perfect Bayesian-Nash equilibria that exist, given the various possible choices the manager can make. There are in total four possible combinations of decisions by the manager (for instance one employee, full transparency), which combinations we will hereinafter regularly refer to as ‘settings’. An equilibrium consists of an optimal stopping rule for both types of agents, and a set of posterior beliefs. All posteriors are updated according to Bayes’ rule, whenever possible. After having distinguished the equilibria for the different settings, we will use the outcomes to evaluate the manager’s decision criteria with respect to  $n$  and the transparency level. It is important to note that section 4.2 (and consequently part of section 5) is to a certain extent of a speculative nature. That is to say that it is not completely finished, while it does broadly describe the most important aspects of the setting concerned.

### 3 Full transparency

We will first examine the model and evaluate the existing equilibria under the assumption that there is full transparency. With full transparency, the public observes whether an agent is a winner or a loser and all the values of the projects that the respective agents have submitted. Identifying the existing equilibria in this setting

amounts to finding optimal stopping rules for both types of agents under varying sets of parameter values. If an agent behaves according to a ‘threshold strategy’ and finds a project with a value above his threshold value, he will submit this project and stop searching. If an agent’s equilibrium strategy is to conduct a predetermined number of searches (e.g. one or zero), he will simply effectuate this strategy. Because agents only care about their reputation, the optimal stopping rules of both types of agents are interdependent (also if  $n = 1$ ). This will reflect in the (arguably atypical) manner the equilibrium strategies are derived. We will first look at the equilibria that exist if the manager employs only one agent, and from there move on to the situation in which the manager employs two agents. We will find that, as a logical consequence of the assumption of full transparency, the equilibria in both situations are essentially identical.

### 3.1 One agent

As an agent cares only about his reputation and searching is less costly for smart agents, a smart agent is inclined to distinguish himself from his dumb counterparts. For a certain range of parameter values (see below), we encounter a fairly simple signaling game in which the smart type’s optimal stopping rule is a separating value for which a dumb agent never wants to search. This leads to the following separating equilibrium:

**Lemma 1** *Let  $\hat{p}_{sm}^{sep}$  be the threshold value for smart agents in the separating threshold equilibrium. Suppose:*

- (i) *the manager employs one agent ( $n = 1$ );*
- (ii) *the manager chooses a transparency level of full transparency;*
- (iii)  *$\lambda$  is sufficiently high:  $\lambda > \underline{\lambda}^{STE} \equiv c_{db}$ .*

*Then, a separating equilibrium exists in which:*

1. *a dumb agent does not search.*
2. *a smart agent searches until he finds a value which is equal to or greater than  $\hat{p}_{sm}^{sep}$ , where  $\hat{p}_{sm}^{sep}$  solves:*

$$\hat{p}_{sm}^{sep} = 1 - \frac{c_{db}}{\lambda} \tag{3}$$

3. the posteriors are given by:

$$\Pr(t_i = sm|p_i \geq \widehat{p}_{sm}^{sep}) = 1; \Pr(t_i = sm|p_i < \widehat{p}_{sm}^{sep}) = 0 \quad (4)$$

**Proof.** In order to proof that this separating equilibrium exists in this form, we have to show that (i) the postulated strategies are the equilibrium strategies of the agents, so that (ii) both types of agents do not want to deviate and (iii) the posterior beliefs are consistent with the equilibrium. In the above described equilibrium, a smart agent chooses a so called mimicking value, for which a dumb agent just does not want to deviate by searching once or more. This cutoff value can be found by solving the following equation:

$$\begin{aligned} U_{A,db}(\text{not searching}) &= EU_{A,db}(\text{searching once}) \\ 0 &= \left[ \begin{array}{l} \Pr(p' > \widehat{p}_{sm}^{sep}) \Pr(t_i = sm|p_i > \widehat{p}_{sm}^{sep}) + \\ \Pr(p' < \widehat{p}_{sm}^{sep}) \Pr(t_i = sm|p_i < \widehat{p}_{sm}^{sep}) \end{array} \right] \lambda - c_{db} \\ 0 &= (1 - \widehat{p}_{sm}^{sep})\lambda - c_{db} \end{aligned}$$

Where  $p'$  is the value a dumb agent finds after searching (once) and  $p_i$  is the value of the project a particular agent submits. The components  $\Pr(t_i = sm|p_i > \widehat{p}_{sm}^{sep})$  and  $\Pr(t_i = sm|p_i < \widehat{p}_{sm}^{sep})$  constitute the posterior beliefs if an agent submits a project with a value  $p_i$  above  $\widehat{p}_{sm}^{sep}$  or below  $\widehat{p}_{sm}^{sep}$ , respectively.

If smart agents conform to the above posited equilibrium strategy, this ensures that dumb agents do not want to search at all. That is, they do not want to deviate from the separating equilibrium. Next we need to show that smart agents do not want to deviate from this equilibrium either, given the posterior beliefs. It is clear that a smart agent will not want to search another time once he has found a value that is greater than  $\widehat{p}_{sm}^{sep}$ , as he only cares about his reputation and he has already ‘proved’ to the public that he is smart. Searching once more will in such case not contribute any further to his utility and will only lead to higher costs. However, it might be possible that there exists a range of values for which a smart agent wants to deviate from the separating equilibrium by searching less or not searching at all. First note that a smart agent would never deviate from the postulated equilibrium by initially searching, but stopping before he finds a value lower than  $\widehat{p}_{sm}^{sep}$ . Given

the posited equilibrium posteriors, the public would believe the agent is dumb, and the searching would have been in vain (and accordingly the costs the agent would have incurred). Furthermore, if the agent was initially willing to search once, this *mutatis mutandis* implies that he is willing to search once more, each time he has not yet reached  $\widehat{p}_{sm}^{sep}$ .

It cannot indubitably be deemed impossible that a smart agent would want to deviate from the equilibrium by not searching at all. This would be the case if the parameters are such that his *ex ante* utility when not searching is greater than his (expected) *ex ante* utility when conforming to the equilibrium strategy. We will denote the inequality that follows the ‘searching constraint’:<sup>5</sup>

$$\begin{aligned}
U_{A,sm}(\text{not searching}) &> EU_{A,sm}(\text{searching until threshold value } \widehat{p}_{sm}^{sep} \text{ is reached}) \\
0 &> \Pr(t_i = sm | p_i > \widehat{p}_{sm}^{sep})\lambda - E(k | p_i \geq \widehat{p}_{sm}^{sep})c_{sm} \\
0 &> \lambda - \frac{1}{1 - \widehat{p}_{sm}^{sep}}c_{sm} \\
\frac{c_{sm}}{\lambda} &> 1 - \widehat{p}_{sm}^{sep} \\
c_{sm} &> c_{db}
\end{aligned}$$

Obviously, this condition is never satisfied as we have assumed that  $c_{db} > c_{sm}$ . This suggests that there is no set of parameters for which the postulated equilibrium can not exist. Note however, that if  $c_{db} < \lambda$ ,  $\widehat{p}_{sm}^{sep}$  becomes negative. This indicates that  $\lambda$  has to be greater than  $c_{db}$  for equation (3) to make sense. If this is not the case, dumb agents do not search, irrespective of the smart agent’s strategy. In such case it would not make sense for the smart agent to employ a threshold strategy: searching once would be enough to distinguish oneself from dumb agents. More formally, the out-of-equilibrium belief  $\Pr(t_i = sm | 0 < p_i < \widehat{p}_{sm}^{sep}) = 0$  would not be ‘reasonable’ in such case. Thus the equilibrium postulated in this Lemma can only

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<sup>5</sup>Note that this constraint is not the same as the more common participation constraint. In fact, we have assumed in section 2 that the participation constraint is always satisfied for all players. Still, an agent can always choose not to search, even though he did decide to participate (i.e., he has signed an employment agreement). Subsequently, later on we will see that there can even exist equilibria in which no type searches.

We will use a searching constraint for both types to determine for each equilibrium that we encounter the condition(s) under which that equilibrium exists. Note that, as mentioned, in this equilibrium the dumb type’s searching constraint (for which a dumb agent commits to not searching) is automatically met if the smart type adheres to the threshold strategy.

exist if  $\lambda > c_{db}$ . If this is indeed the case,  $\lambda$  is greater than  $c_{sm}$  by assumption and considering the searching constraint smart agents will choose the threshold strategy. This explains condition *iii* in Lemma 1.

As only smart agents report a value that is greater than zero (or: report any value at all) in this equilibrium, the public can instantly deduce the agent's type from the reported project value (with full certainty). This proves that the described equilibrium is indeed a fully separating equilibrium. ■

We will refer to the equilibrium in Lemma 1 as the separating threshold equilibrium with full transparency (and for the purpose of this section as: STE). We have shown that in the current setting, given that agents care sufficiently about their reputation, there exists an equilibrium in which a smart agent will search until he finds a project with a value higher than (or in the putative case, equal to) a certain threshold value, and a dumb agent will not search at all. As this equilibrium behavior is common knowledge, the public can immediately infer from the value of a project that is submitted whether the corresponding agent is smart or dumb. Additionally, we have shown that the separating threshold of smart agents depends on both  $\lambda$  and  $c_{db}$ . As the cost of searching for dumb agents increases, a lower separating threshold is required to make a dumb agent refrain from searching (and vice versa). Conversely, if agents care more about their reputation, i.e.  $\lambda$  increases, dumb agents are more inclined to mimic the smart type and thus a higher separating threshold is needed for the equilibrium to hold together.

We will now briefly discuss other possible equilibria. First it is important to note that in this setting, a semi-separating equilibrium with a different threshold value for each type can not exist. This would mean that dumb agents choose a positive but lower threshold than smart agents. In such a putative equilibrium, if a dumb agent were to find a value between his threshold and  $\hat{p}_{sm}^{sep}$ , he would stop searching. The public would then conjecture that the agent is dumb, while he could get the same result by simply not searching (in which case he would even receive a higher utility). More expressively, his decision after finding such value between the two thresholds is essentially the same as at the beginning of the game. The costs of searching have not increased, and the expected increase in utility when searching once more is also unaltered. This confirms that such a stopping rule for dumb agents

(with a positive threshold lower than  $\widehat{p}_{sm}^{sep}$ ) cannot be part of an equilibrium if there is full transparency. Furthermore, we rule out all equilibria with unreasonable (out-of-equilibrium) beliefs by requiring that beliefs satisfy the intuitive criterion.<sup>6</sup> This ensures that all pooling equilibria are eliminated for any value of  $\lambda$  greater than  $c_{db}$  (cf. Lemma 3), as well as all separating equilibria that are ‘inefficient’.<sup>7</sup> Finally, as we have assumed that  $c_{db} > c_{sm}$ , there cannot exist an equilibrium in which only dumb agents search, nor a pooling equilibrium in which both types search.

There are yet one or more equilibria that remain to be identified. In any case, there must exist at least one equilibrium for values of  $\lambda$  lower than  $c_{db}$ . Even though we have established that dumb agents do not search in this case, regardless of the smart type’s strategy, it is very well possible that smart agents do. Note that the outcome is publicly observable, even if no agent searches. Analogously, if  $\lambda$  is lower than  $c_{sm}$ , there can exist no equilibrium in which either type of agents ever searches. To see this, note that in these cases searching once brings about higher costs than the agent (of the type in question) can ever make up for by improving his reputation. We will now characterize the concomitant equilibria.

**Lemma 2** *Suppose:*

- (i) *the manager employs one agent ( $n = 1$ );*
- (ii) *the manager chooses a transparency level of full transparency;*
- (iii)  *$\lambda$  is at such a level that:  $\underline{\lambda}^{1-0} \leq \lambda \leq \bar{\lambda}^{1-0}$ , where  $\underline{\lambda}^{1-0} \equiv c_{sm}$  and  $\bar{\lambda}^{1-0} \equiv c_{db}$ .*

*Then, a separating equilibrium exists in which:*

1. *a dumb agent does not search.*
2. *a smart agent searches exactly once.*
3. *the posteriors are given by:*

$$\Pr(t_i = sm | p_i \geq 0) = 1; \Pr(t_i = sm | p_i = 0) = 0 \quad (5)$$

**Proof.** In this equilibrium it is never worthwhile for a dumb agent to deviate by searching, given that  $\lambda < c_{db}$ . As long as the condition  $\lambda \geq c_{sm}$  holds as well, a

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<sup>6</sup>Beliefs satisfy the intuitive criterion if, for all out-of-equilibrium actions, zero probability is assigned to player types that can only lose compared to their equilibrium payoff, see Cho and Kreps (1987).

<sup>7</sup>That is, separating equilibria in which smart agents have a higher threshold than is required to make sure that dumb agents refrain from searching.



smart agent will not deviate either, as searching once will sufficiently improve his reputation to outweigh the costs he has to incur, regardless of the outcome of his search. These (non-)searching constraints can be found by solving the following inequalities:

$$\begin{aligned} U_{A,db}(\text{not searching}) &> EU_{A,db}(\text{searching once}) \\ 0 &> \lambda - c_{db} \end{aligned}$$

and

$$\begin{aligned} U_{A,sm}(\text{not searching}) &< EU_{A,sm}(\text{searching once}) \\ 0 &< \lambda - c_{sm} \end{aligned}$$

Submitting a project with any value makes sure the agent is believed to be smart, which makes this a fully separating equilibrium. ■

**Lemma 3** *Suppose:*

- (i) *the manager employs one agent ( $n = 1$ );*
- (ii) *the manager chooses a transparency level of full transparency;*
- (iii)  *$\lambda$  is sufficiently low:  $\lambda \leq \bar{\lambda}^{0-0} \equiv \frac{c_{sm}}{1-\pi}$ .*

*Then, a pooling equilibrium exists in which:*

1. *a dumb agent does not search.*
2. *a smart agent does not search.*
3. *the posteriors are given by:*

$$\Pr(t_i = sm|p_i \geq 0) = 1; \Pr(t_i = sm|p_i = 0) = \pi \tag{6}$$

**Proof.**  $\Pr(t_i = sm|p_i = 0)$  can be calculated by Bayes' Rule, as opposed to the out-of-equilibrium belief  $\Pr(t_i = sm|p_i \geq 0)$ . As  $c_{sm} < c_{db}$ , we consider 1 a 'reasonable' out-of-equilibrium belief for an agent that unexpectedly submits a project with a(ny) positive value. Again we will show that no type of agents wants to deviate from the posited equilibrium, given that the condition  $\lambda \leq \frac{c_{sm}}{1-\pi}$  holds.

This is true if the following inequalities are met:

$$\begin{aligned}
U_{A,db}(\text{not searching}) &> EU_{A,db}(\text{searching once}) \\
\pi\lambda &> \lambda - c_{db} \\
\lambda &< \frac{c_{db}}{1-\pi}
\end{aligned}$$

and

$$\begin{aligned}
U_{A,sm}(\text{not searching}) &> EU_{A,sm}(\text{searching once}) \\
\pi\lambda &> \lambda - c_{sm} \\
\lambda &< \frac{c_{sm}}{1-\pi}
\end{aligned}$$

As  $c_{sm} < c_{db}$  by assumption, the second condition is strictly binding. This proves that if  $\lambda$  is small enough, there exists a pooling equilibrium in which no agent ever searches. ■

We will refer to the equilibria in Lemma 2 and Lemma 3 as the 1–0 separating equilibrium with full transparency and the 0–0 pooling equilibrium with full transparency, respectively (and for the purpose of this section as: 1–0 and 0–0). It is important to note that we have not yet applied any equilibrium refinements (specifically the intuitive criterion) to the equilibria in Lemma 2 and 3. We will do so in the next subsection.

### 3.1.1 Equilibria

In this subsection we summarize the equilibria found in Lemmas 1, 2 and 3. We have seen that for some values of  $\lambda$ , there is a multiplicity of possible equilibria. Specifically, if  $c_{sm} < \lambda < \frac{c_{sm}}{1-\pi}$ , the 0–0 equilibrium and the 1–0 equilibrium or the STE (depending on whether  $c_{db}$  is greater than  $\frac{c_{sm}}{1-\pi}$  or not) exist concurrently. Selecting between the equilibria by refining the equilibrium notion will not help us in this case. We will have to resort to other methods to guarantee that there exists only one equilibrium for every possible  $\lambda$ . While such guarantee is not absolutely necessary, wish to do so in order to be better able to compare the varying settings

in our welfare analysis.

**Remark 1** *Throughout we assume that if – even after refining the equilibrium notion – a multiplicity of equilibria exists, it is common knowledge that the agents will select the payoff dominant equilibrium (Harsanyi and Selten, 1988).*

If  $\lambda < \frac{c_{sm}}{1-\pi}$ , the 0–0 equilibrium is payoff dominant for both types of agents. This leads to Proposition 1:

**Proposition 1** *Suppose:*

- (i) *the manager employs one agent ( $n = 1$ );*
- (ii) *the manager chooses a transparency level of full transparency;*
- (iii)  $\gamma = 1$ .

*Then, there exist three equilibria, each of which covers an exclusive interval of  $\lambda$ :*

*If  $0 \leq \lambda \leq \frac{c_{sm}}{1-\pi}$ : the 0–0 equilibrium (Lemma 3);*

*If  $\frac{c_{sm}}{1-\pi} \leq \lambda \leq c_{db}$ : the 1–0 equilibrium (Lemma 2);*

*If  $\max(\frac{c_{sm}}{1-\pi}, c_{db}) \leq \lambda \leq 1$ : the STE (Lemma 1).*

Proposition 1 is (abstractly) illustrated in Figure 1 below. It should be noted that, without loss of generality, exclusively for the purpose of this illustration (and analogously for Figures 2 and 6) we assume  $\frac{c_{sm}}{1-\pi} < c_{db}$ .



Figure 1. Illustration of Propositions 1 and 2.

### 3.2 Two agents

We now move on to the case where the manager employs two agents,  $n = 2$ . We still assume full transparency. One can easily see that the mere addition of a second agent does not change the results found in the previous subsection in terms of equilibrium strategies. As all information is released, winning or losing in itself does not influence an agent’s reputation.<sup>8</sup> Moreover, from an agent’s point of view

<sup>8</sup>As opposed to the case of minimum transparency, see section 4.

this situation is identical to the one in which he searches alone.<sup>9</sup> *Ipsa facto*, the equilibrium strategies of both types of agents match the strategies described in Proposition 1. This leads us to Proposition 2:

**Proposition 2** *Suppose:*

- (i) *the manager employs two agents ( $n = 2$ );*
- (ii) *the manager chooses a transparency level of full transparency;*
- (iii)  $\gamma = 1$ .

*Then, there exist three equilibria, each of which covers an exclusive interval of  $\lambda$ :*

*If  $0 \leq \lambda \leq \frac{c_{sm}}{1-\pi}$ : the 0–0 equilibrium;*

*If  $\frac{c_{sm}}{1-\pi} \leq \lambda \leq c_{db}$ : the 1–0 equilibrium;*

*If  $\max\left(\frac{c_{sm}}{1-\pi}, c_{db}\right) \leq \lambda \leq 1$ : the STE.*

## 4 Minimum transparency

We will now explore the setting in which the manager chooses minimum transparency. In this case, the public only observes for each agent whether he is a winner or a loser. None of the values of the submitted projects are made public. Note that, as only projects with positive values can be executed, if there is no agent that searches, there is no winner. Again, we will evaluate this setting by identifying the existing equilibria both in the case where the manager employs one agent and in the case where she employs two agents.

### 4.1 One agent

If there is only one agent who searches, he can win by simply searching once. If both types of agents' equilibrium strategy is to search, this means that the mere observation that an agent wins, contains no information about his ability. In other words, if this is the case, we get a pooling equilibrium in which both types of agents search once. Lemma 4 is dedicated to the posited pooling equilibrium:

**Lemma 4** *Let  $\hat{\pi}_l$  be the posterior belief if an agent loses,  $\Pr(t_i = sm|lose)$  and  $\hat{\pi}_w$  the posterior belief if an agent wins,  $\Pr(t_i = sm|win)$ . Suppose:*

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<sup>9</sup>Evidently, from the manager's point of view, the situation is different. We will return to this topic in the welfare analysis in section 5.

- (i) the manager employs one agent ( $n = 1$ );
- (ii) the manager chooses a transparency level of minimum transparency;
- (iii)  $\lambda$  is sufficiently high:  $\lambda \geq \underline{\lambda}^{1-1} \equiv \frac{c_{db}}{\pi}$ .

Then, a pooling equilibrium exists in which:

1. a dumb agent searches exactly once;
2. a smart agent searches exactly once;
3. the posteriors are given by:

$$\hat{\pi}_w = \pi; \hat{\pi}_l = 0 \tag{7}$$

**Proof.** Given the updating rules described above, neither type of agents wants to deviate from the postulated equilibrium if the following searching constraint holds:

$$\begin{aligned} U_{A,t}(\text{not searching}) &< U_{A,t}(\text{searching once}) \\ \hat{\pi}_l \lambda &< \hat{\pi}_w \lambda - c_t \\ \lambda &> \frac{c_t}{\pi} \end{aligned}$$

The searching constraint for the dumb type is binding, as  $c_{db} > c_{sm}$ . In this equilibrium,  $\hat{\pi}_w$  can be calculated using Bayes' Rule and equals the prior  $\pi$ , as it is a pooling equilibrium. An out-of-equilibrium belief of  $\hat{\pi}_l = 0$  seems 'reasonable', as for  $c_{db} > c_{sm}$ , dumb agents are more inclined to deviate by refraining from searching than smart agents. ■

We will refer to the equilibrium in Lemma 4 as the single agent 1–1 pooling equilibrium with minimum transparency (and for the purpose of this subsection as: 1–1). We have found that if  $\lambda$  is high enough, both types of agents search once. Whether this condition is satisfied, depends negatively on  $c_{db}$  and positively on  $\pi$ . As  $\pi$ , i.e., the proportion of agents that is of the smart type, increases, the posterior belief if an agent searches once increases (while  $\hat{\pi}_l$  remains unaltered). This makes sure that dumb agents are more willing to behave as posited in the pooling equilibrium. On the other hand, smart agents are also more reluctant to deviate from the equilibrium. A higher cost of searching, however, lowers the utility from searching. If  $c_{db}$  is too high, therefore, the above described equilibrium cannot exist.

Note that there are no equilibria in this setting in which either type of agents searches more than once. This will always result in lower utility, as searching once suffices to win when there is minimum transparency. This indicates that there can also be no equilibrium in which smart agents use a threshold strategy, contradictory to what we have seen in the case of full transparency.

All other pooling equilibria (conditional on  $\lambda > c_{sm}$ , which is evidenced in Lemma 7) are eliminated by requiring that the beliefs satisfy the intuitive criterion. *Prima facie*, one might argue that a putative pooling equilibrium in which no type of agents searches and the posteriors are such that  $\hat{\pi}_l = \pi$  is a more ‘efficient’ (or payoff dominant) equilibrium than the one described in Lemma 4. Still, what would be ‘reasonable’ out-of-equilibrium beliefs in the putative pooling equilibrium? If  $\lambda > c_{db}$ , it can only be an equilibrium if  $\hat{\pi}_w$  is either equal to 0 or  $\pi$ . In both cases, no type of agents would have reason to deviate, as searching once would yield lower utility than not searching for both types. However, as we have assumed that  $c_{db} > c_{sm}$ , dumb agents are less likely to deviate from this equilibrium so that any out of equilibrium equal to or lower than  $\pi$  does not satisfy the intuitive criterion. This leads us back to the conclusion that the equilibrium described in Lemma 4 is the only pooling equilibrium in this setting (unless  $\lambda < c_{sm}$ , as pointed out above).

What if the condition  $\lambda > \frac{c_{db}}{\pi}$  does not hold? As we have emphasized in the previous section, there must exist at least one equilibrium for each value  $\lambda$  can take on. Again, we will first look at the equilibria in which either one or both types of agents do not search. These equilibria are described in Lemma 5 and Lemma 6.

**Lemma 5** *Let  $\hat{\pi}_l$  be the posterior belief if an agent loses,  $\Pr(t_i = sm|lose)$  and  $\hat{\pi}_w$  the posterior belief if an agent wins,  $\Pr(t_i = sm|win)$ . Suppose:*

- (i) *the manager employs one agent ( $n = 1$ );*
- (ii) *the manager chooses a transparency level of minimum transparency;*
- (iii)  *$\lambda$  is at such a level that:  $\underline{\lambda}^{1-0} \leq \lambda \leq \bar{\lambda}^{1-0}$ , where  $\underline{\lambda}^{1-0} \equiv c_{sm}$  and  $\bar{\lambda}^{1-0} \equiv c_{db}$ .*

*Then, a separating equilibrium exists in which:*

1. *a dumb agent does not search.*
2. *a smart agent searches exactly once.*
3. *the posteriors are given by:*

$$\hat{\pi}_w = 1; \hat{\pi}_l = 0 \tag{8}$$

**Proof.** This equilibrium is very similar to the one described in Lemma 2. Again, conditional on  $\lambda < c_{db}$  it is never worthwhile for a dumb agent to deviate by searching. In the same sense, as long as  $\lambda \geq c_{sm}$ , there is no reason for smart agents not to behave according to the posited equilibrium strategies. The posteriors show that the postulated equilibrium is fully separating. ■

**Lemma 6** *Let  $\hat{\pi}_l$  be the posterior belief if an agent loses,  $\Pr(t_i = sm|lose)$  and  $\hat{\pi}_w$  the posterior belief if an agent wins,  $\Pr(t_i = sm|win)$ . Suppose:*

- (i) *the manager employs one agent ( $n = 1$ );*
- (ii) *the manager chooses a transparency level of minimum transparency;*
- (iii)  *$\lambda$  is sufficiently low:  $\lambda \leq \bar{\lambda}^{0-0} \equiv \frac{c_{sm}}{1-\pi}$ .*

*Then, a pooling equilibrium exists in which:*

1. *a dumb agent does not search.*
2. *a smart agent does not search.*
3. *the posteriors are given by:*

$$\hat{\pi}_w = 1; \hat{\pi}_l = \pi \tag{9}$$

**Proof.** We refer to the proof appertaining to Lemma 3. Similarly, no type of agents ever wants to deviate from the posited equilibrium strategies by searching, given that  $\lambda \leq \frac{c_{sm}}{1-\pi}$ . As the public's capability to assess the agent's type has not increased at the end of the game compared to the beginning, this is a pooling equilibrium. ■

We refer to the equilibria in Lemma 5 and Lemma 6 as the single agent 1–0 separating equilibrium with minimum transparency, and the single agent 0–0 pooling equilibrium with minimum transparency, respectively (and for the purpose of this subsection as: 1–0 and 0–0). Bearing in mind the analysis in the previous section, it may appear that we have now characterized all possible equilibria. While this may be the case if  $\pi > 0.5$  and  $c_{db} < \frac{c_{sm}}{\pi}$ ,<sup>10</sup> it not universally is. This entails that there is yet another equilibrium we have not typified, which in this case can only be an equilibrium in mixed strategies. This equilibrium is described in Lemma 7:

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<sup>10</sup>Under these circumstances, we have characterized at least one equilibrium for all possible values of  $\lambda$ . Under all other circumstances, however, this is not the case. For example, if  $c_{db} > \frac{c_{sm}}{\pi}$  and  $\pi > \frac{1}{2}$ , we have not yet specified an equilibrium for values of  $\lambda$  between  $c_{db}$  and  $\frac{c_{db}}{\pi}$ . Furthermore, if  $\pi < \frac{1}{2}$  (which implies that  $\frac{c_t}{1-\pi} < \frac{c_t}{\pi}$ ), there is always a range of values of  $\lambda$  for which we have not yet identified any equilibrium.

**Lemma 7** Let  $\hat{\pi}_l$  be the posterior belief if an agent loses,  $\Pr(t_i = sm|lose)$  and  $\hat{\pi}_w$  the posterior belief if an agent wins,  $\Pr(t_i = sm|win)$ . Suppose:

- (i) the manager employs one agent ( $n = 1$ );
- (ii) the manager chooses a transparency level of minimum transparency;
- (iii)  $\lambda$  is such that:  $\underline{\lambda}^{1-\sigma} \leq \lambda \leq \bar{\lambda}^{1-\sigma}$ , where  $\underline{\lambda}^{1-\sigma} \equiv c_{db}$  and  $\bar{\lambda}^{1-\sigma} \equiv \frac{c_{db}}{\pi}$

Then, a semi-separating equilibrium exists in which:

1. a dumb agent randomizes between not searching and searching exactly once; that is he searches once with a probability  $\sigma$ , where  $\sigma$  solves:

$$\sigma = \frac{\pi \left( \frac{\lambda}{c_{db}} - 1 \right)}{1 - \pi} \quad (10)$$

2. a smart agent searches exactly once;
3. the posteriors are given by:

$$\hat{\pi}_w = \frac{c_{db}}{\lambda}; \hat{\pi}_l = 0 \quad (11)$$

**Proof.** If  $\lambda < \frac{c_{db}}{\pi}$ , there cannot be an equilibrium in which both types of agents search once. However, if at the same time  $\lambda$  is greater than  $c_{db}$ , there can also be no equilibrium in which a dumb agent does not search (at least not one in which the beliefs satisfy the intuitive criterion). So the only remaining possibility is an equilibrium in mixed strategies (in which dumb agents randomize). Denote by  $\sigma$  the probability with which a dumb agent searches. This leads to the following posteriors:  $\hat{\pi}_w = \frac{\pi}{\pi + \sigma(1-\pi)}$  and  $\hat{\pi}_l = 0$ , which we then use to calculate  $\sigma$ :

$$\begin{aligned} U_{A,db}(\text{not searching}) &= U_{A,db}(\text{searching once}) \\ \hat{\pi}_l \lambda &= \hat{\pi}_w \lambda - c_{db} \\ 0 &= \frac{\pi}{\pi + \sigma(1-\pi)} \lambda - c_{db} \\ \sigma &= \frac{\pi \left( \frac{\lambda}{c_{db}} - 1 \right)}{1 - \pi} \end{aligned}$$

We can check this solution by entering both  $\lambda = c_{db}$  and  $\lambda = \frac{c_{db}}{\pi}$ . We find that for  $\lambda = c_{db}$ ,  $\sigma = 0$ , and for  $\lambda = \frac{c_{db}}{\pi}$ ,  $\sigma = 1$ . Between these values,  $\sigma$  is strictly increasing



in  $\lambda$ , which is a coherent result. As  $\lambda$  increases, the utility when searching, and consequently  $\sigma$ , increases.

In order to proof that this is indeed an equilibrium, we have to demonstrate that no agent wants to deviate. Obviously, dumb agents are indifferent between searching and not searching in this equilibrium and therefore have no reason to deviate as long as  $\lambda$  is between its set boundaries. The following searching constraint shows us that smart agents never want to deviate from this equilibrium either:

$$\begin{aligned}
 U_{A,sm}(\text{not searching}) &< U_{A,sm}(\text{searching once}) \\
 \hat{\pi}_l \lambda &< \hat{\pi}_w \lambda - c_{db} \\
 0 &< \frac{c_{db}}{\lambda} \lambda - c_{sm} \\
 c_{sm} &< c_{db}
 \end{aligned}$$

Which is strictly satisfied (by assumption). Finally,  $\hat{\pi}_w$  and  $\hat{\pi}_l$  are both calculated using Bayes' Rule (and, as for  $\hat{\pi}_w$ , inserting  $\sigma$ ). ■

We will refer to the equilibrium described in Lemma 7 as the single agent  $1-\sigma$  semi-separating equilibrium with minimum transparency (and for the purpose of this subsection as:  $1-\sigma$ ). We have seen that if  $\lambda$  is between  $c_{db}$  and  $\frac{c_{db}}{\pi}$ , we get a semi-separating equilibrium in which smart agents always search once, and dumb agents randomize between searching once and not searching. Besides we have analyzed the relation between  $\sigma$  and  $\lambda$ , which we have found to be positive. Likewise,  $\sigma$  is strictly increasing in  $\pi$ . The reasoning behind the latter result is straightforward: as  $\pi$  increases, the posterior belief for an agent that searches increases so that dumb agents are more tempted to do so. Finally,  $\sigma$  is negatively related to  $c_{db}$ , as higher costs lower the utility for the dumb type when searching.

#### 4.1.1 Equilibria

We summarize the equilibria found in the previous subsection below in Proposition 3. We apply the same equilibrium refinements and equilibrium selection concepts as in Proposition 1 and 2 (see also Remark 1).

**Proposition 3** *Suppose:*

- (i) *the manager employs one agent ( $n = 1$ );*
- (ii) *the manager chooses a transparency level of minimum transparency;*
- (iii)  $\gamma = 1$ .

*Then, there exist four equilibria, each of which covers an exclusive interval of  $\lambda$ :*

$0 \leq \lambda \leq \frac{c_{sm}}{1-\pi}$ : *the 0–0 equilibrium (Lemma 6);*

$\frac{c_{sm}}{1-\pi} \leq \lambda \leq c_{db}$ : *the 1–0 equilibrium (Lemma 5);*

$\max\left(\frac{c_{sm}}{1-\pi}, c_{db}\right) \leq \lambda \leq \frac{c_{db}}{\pi}$ : *the 1– $\sigma$  equilibrium (Lemma 7);*

$\max\left(\frac{c_{sm}}{1-\pi}, \frac{c_{db}}{\pi}\right) \leq \lambda \leq 1$ : *the 1–1 equilibrium (Lemma 4).*

Proposition 3 is illustrated in Figure 2 below.

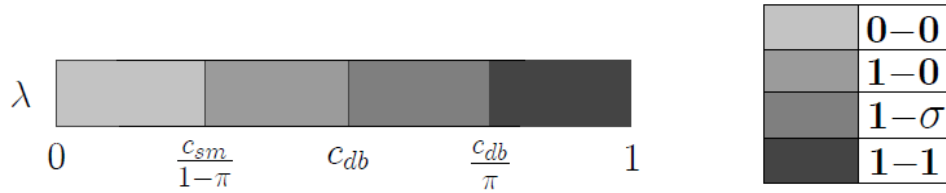


Figure 2. Illustration of Proposition 3.

## 4.2 Two agents

If the manager employs two agents, searching in itself is not necessarily sufficient to win – which, as we have seen in the previous subsection, *is* the case if  $n = 1$ . Furthermore, as there is always a chance that a smart agent loses, in this setting  $\hat{\pi}_l$  is strictly higher than zero in any equilibrium.<sup>11</sup> This indicates, *inter alia*, that there can be no separating equilibrium in this setting. For legibility purposes, in this subsection we refer to a (semi-separating) equilibrium in which  $\hat{\pi}_w = 1$  as a ‘quasi-separating equilibrium’. Such equilibrium is separating in the sense that the manager can infer an agent’s type from the value he submits with full certainty (however the public can not). In the remainder of this subsection, we describe the equilibria that exist in this setting (two agents, minimum transparency) and the conditions under which they do.

<sup>11</sup>Note that even in an equilibrium in which dumb agents never search, there is always a chance that a smart has to compete against another smart agent so that at least one smart agent loses.

**Remark 2** *With respect to the setting in which the manager employs two agents and chooses minimum transparency, we limit ourselves to studying equilibria in pure strategies. Specifically, we disregard all mixed equilibria that may exist in this context. Our goal in doing so is to prevent our analysis from becoming convoluted. Meanwhile, be aware that as  $\lambda$  goes to the lower limit of one of the equilibria described in this subsection, the relevant equilibrium for that case may in fact be one in which either type of agents mixes. Having said so, we believe that this does not impair our results (particularly in the welfare analysis) for the cases where the described equilibria do exist.*

We start with the equilibrium that corresponds to low values of  $\lambda$ , and from there move on to the equilibria that exist for increasingly higher values of  $\lambda$ .

**Lemma 8** *Let  $\hat{\pi}_l$  be the posterior belief if an agent loses,  $\Pr(t_i = sm|lose)$  and  $\hat{\pi}_w$  the posterior belief if an agent wins,  $\Pr(t_i = sm|win)$ . Suppose:*

- (i) *the manager employs two agents ( $n = 2$ );*
- (ii) *the manager chooses a transparency level of minimum transparency;*
- (iii)  *$\lambda$  is sufficiently low:  $\lambda \leq \bar{\lambda}^{0-0} \equiv \frac{c_{sm}}{1-\pi}$ .*

*Then, a pooling equilibrium exists in which:*

1. *a dumb agent does not search.*
2. *a smart agent does not search.*
3. *the posteriors are given by:*

$$\hat{\pi}_w = 1; \hat{\pi}_l = \pi \tag{12}$$

**Proof.** This equilibrium is very similar to the ones described in Lemmas 3 and 6, and we therefore also refer to the proofs appertaining to those Lemmas. Apparently, if  $\lambda$  is low enough the transparency level nor the number of agents have any influence on the equilibrium strategies of either type of agents. Even though the underlying line of reasoning may be slightly different for the various settings, the outcome in terms of equilibrium strategies and the corresponding searching constraints are the same. Again, as no agent ever searches in this equilibrium, in equilibrium an agent always loses. Moreover, an agent that does not behave according to the posited equilibrium strategies automatically wins. For such an agent we consider 1 as a

‘reasonable’ out-of-equilibrium belief ( $\widehat{\pi}_w$ ), as  $c_{sm} < c_{db}$  and thus dumb agents are more reluctant to search. ■

We will refer to the equilibrium described in Lemma 8 as the two agent 0–0 pooling equilibrium with minimum transparency (and for the purpose of this subsection as: 0–0). Next we focus on the quasi-separating equilibrium. In this equilibrium, described below in Lemma 9, smart agents choose a threshold level  $\widehat{p}_{sm}^{qs}$  while dumb agents do not search.

**Lemma 9** *Let  $\widehat{\pi}_l$  be the posterior belief if an agent loses,  $\Pr(t_i = sm|lose)$  and  $\widehat{\pi}_w$  the posterior belief if an agent wins,  $\Pr(t_i = sm|win)$ ; and let  $\widehat{p}_{sm}^{qs}$  be the threshold value for smart agents in the quasi-separating equilibrium Suppose:*

- (i) *the manager employs two agents ( $n = 2$ );*
- (ii) *the manager chooses a transparency level of minimum transparency;*
- (iii)  *$\lambda$  is such that:  $\underline{\lambda}^{QSTE} \leq \lambda \leq \overline{\lambda}^{QSTE}$ , where  $\underline{\lambda}^{QSTE} \equiv \frac{c_{sm}}{1-\pi}$ , and  $\overline{\lambda}^{QSTE} \equiv \frac{2-2\pi+\pi^2}{2-4\pi+2\pi^2}(c_{db} - c_{sm})$ .*

*Then, a quasi-separating equilibrium exists in which:*

1. *a dumb agent does not search.*
2. *a smart agent searches until he finds a value which is equals to or greater than  $\widehat{p}_{sm}^{qs}$ , where  $\widehat{p}_{sm}^{qs}$  solves:*

$$\widehat{p}_{sm}^{qs} = 1 - \frac{c_{sm}(\pi^2 - 2\pi + 2)}{\lambda\pi(1 - \pi)} \quad (13)$$

3. *the posteriors are given by:*

$$\widehat{\pi}_w = 1; \widehat{\pi}_l = \frac{\pi^2}{\pi^2 - 2\pi + 2} \quad (14)$$

**Proof.** We will show that this equilibrium exists under certain conditions, and that if these conditions hold, no type of player wants to deviate. To find the equilibrium described in Lemma 9 we first calculate a threshold value for smart agents for which dumb agents do not want to search (and hence do not deviate from the postulated equilibrium). For convenience, we will refer to this threshold value as  $\overline{p}$ . We find this value by solving the following equation for  $\overline{p}$  and entering  $\widehat{\pi}_w$  and  $\widehat{\pi}_l$  (which are both calculated by Bayes’ Rule, see below):

$$\begin{aligned}
U_{A,db}(\text{not searching}) &= EU_{A,db}(\text{searching once}) \\
\hat{\pi}_l \lambda &= \left[ \begin{array}{l} \left( \begin{array}{l} \Pr(t_j = db) \Pr(\text{win}|t_j = db) + \\ \Pr(t_j = sm) \Pr(\text{win}|t_j = sm) \end{array} \right) \hat{\pi}_{w+} \\ \left( \begin{array}{l} \Pr(t_j = db) \Pr(\text{lose}|t_j = db) + \\ \Pr(t_j = sm) \Pr(\text{lose}|t_j = sm) \end{array} \right) \hat{\pi}_l \end{array} \right] \lambda - c_{db} \\
0 &= \left[ \begin{array}{l} ((1 - \pi) + \pi(1 - \bar{p})\frac{1}{2}) \hat{\pi}_{w+} \\ (\pi(\bar{p} + (1 - \bar{p})\frac{1}{2}) - 1) \hat{\pi}_l \end{array} \right] \lambda - c_{db} \\
\bar{p} &= \frac{(\lambda - c_{db})(2 + \pi^2 - 2\pi) - \lambda\pi}{\lambda\pi(1 - \pi)}
\end{aligned}$$

It is easy to see that  $\bar{p}$  is different from the quasi-separating threshold  $\hat{p}_{sm}^{qs}$  as postulated above. We will show that the quasi-separating equilibrium in Lemma 9 can only exist if  $\bar{p} < \hat{p}_{sm}^{qs}$ . In Lemma 1, we have seen that in a full transparency setting there is no reason for a smart agent to search any further after finding a value greater than  $\hat{p}_{sm}^{sep}$  (and any equilibrium in which smart agents do choose a threshold level higher than  $\hat{p}_{sm}^{sep}$ , could be eliminated by requiring that the beliefs satisfy the intuitive criterion). Then why is it that, in this equilibrium, smart agents choose a higher threshold than is needed to make dumb agents refrain from searching? To see this, we emphasize that in the current setting, the value of the project an agent submits is not of crucial importance for his reputation. On the contrary, it is decisive if an agent wins or loses. If we now take  $\bar{p}$  as a starting point, smart agents can increase their chance of winning by searching further and submitting a higher project value. This is especially the case if  $\pi$ , the probability with which a particular smart agent has to compete against another smart agent, is high (for smart agents always win against dumb agents). Note that as dumb agents do not search, the extent to which a smart agent searches (more than once) does not influence the posterior beliefs the public holds about his type. Nonetheless, as a smart agent is more likely to win if he deviates from the putative quasi-separating equilibrium by choosing a higher threshold value than  $\bar{p}$ , smart agents are drawn into a so-called ‘rat race’. In the equilibrium that emerges from this rat race, smart agents choose a threshold level  $\hat{p}_{sm}^{qs}$  for which they are just indifferent between stopping and searching once

more (where searching potentially adds further to the agent's chance of winning). This means that as soon as a smart agent finds a value above  $\widehat{p}_{sm}^{qs}$ , he stops searching; where  $\widehat{p}_{sm}^{qs}$  is proportional to:

$$\begin{aligned}
 U_{A,sm}(\text{stop}) &= EU_{A,db}(\text{search once more}) \\
 [(1 - \pi)\widehat{\pi}_w + \pi\widehat{\pi}_l]\lambda &= \left[ \begin{array}{l} ((1 - \pi) + \pi(1 - \widehat{p}_{sm}^{qs})\frac{1}{2})\widehat{\pi}_w + \\ (\pi(\widehat{p}_{sm}^{qs} + (1 - \widehat{p}_{sm}^{qs})\frac{1}{2}))\widehat{\pi}_l \end{array} \right] \lambda - c_{sm} \\
 \widehat{p}_{sm}^{qs} &= 1 - \frac{c_{sm}(\pi^2 - 2\pi + 2)}{\lambda\pi(1 - \pi)}
 \end{aligned}$$

Note that all smart agents would be better off if the equilibrium threshold value were  $\bar{\bar{p}}$  instead of  $\widehat{p}_{sm}^{qs}$ . Yet the rat race the agents end up in, which stems – *inter alia* – from the premise of minimum transparency, makes sure that smart agents may still continue searching after finding a value above  $\bar{\bar{p}}$ . As mentioned before, dumb agents will not deviate from this rat race equilibrium as long as  $\widehat{p}_{sm}^{qs}$  is greater than  $\bar{\bar{p}}$ . The latter, however, is not always the case, as is illustrated in Figure 3 below. Both values are positively correlated with  $\lambda$ , but as  $\bar{\bar{p}}$  has a steeper slope than  $\widehat{p}_{sm}^{qs}$ , as  $\lambda$  increases  $\bar{\bar{p}}$  converges to  $\widehat{p}_{sm}^{qs}$ . At some point  $\bar{\bar{p}}$  takes on a higher value than  $\widehat{p}_{sm}^{qs}$ , at which point the quasi-separating equilibrium breaks down.

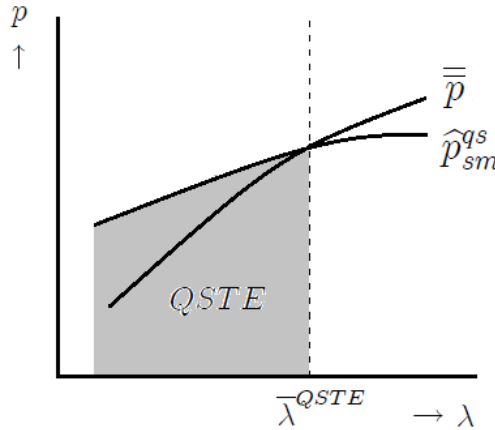


Figure 3. The quasi-separating equilibrium.

Smart agents will never choose a threshold value higher than  $\widehat{p}_{sm}^{qs}$ : this is so to say their ‘maximum’ threshold value.<sup>12</sup> If  $\lambda$  is too high, i.e. agents care relatively

<sup>12</sup>We have seen that for this value smart agents are just indifferent between concluding their quest and searching once more. Consequently, if they find a project with any higher value, searching

much about their reputation, this maximum threshold value is no longer sufficient to make sure that dumb agents do not search. We denote the mentioned cutoff value for  $\lambda$  by  $\bar{\lambda}^{QSTE}$ , and calculate this value by simply setting  $\hat{p}_{sm}^{qs}$  equal to  $\bar{p}$  and solving for  $\lambda$ :

$$\begin{aligned} 1 - \frac{c_{sm}(\pi^2 - 2\pi + 2)}{\lambda\pi(1 - \pi)} &= \frac{(\lambda - c_d)(2 + \pi^2 - 2\pi) - \lambda\pi}{\lambda\pi(1 - \pi)} \\ \bar{\lambda}^{QSTE} &= \frac{2 - 2\pi + \pi^2}{2 - 4\pi + 2\pi^2}(c_{db} - c_{sm}) \end{aligned}$$

We see that if there is a relatively big difference in costs either type of agents has to incur for searching ( $c_{db} - c_{sm}$ ), there is a wider range of values for  $\lambda$  for which the quasi-separating equilibrium exists. In other words, then there is a wider range for  $\lambda$  for which, *ceteris paribus*,  $\hat{p}_{sm}^{qs} > \bar{p}$ . The same goes for relatively high values for  $\pi$ , as then the difference between  $\hat{\pi}_w$  and  $\hat{\pi}_l$  is smaller and there is a greater chance of competing against smart agents (and thus dumb agents are less tempted to deviate by searching).

Finally, both posteriors can be calculated by Bayes' Rule:

$$\begin{aligned} \hat{\pi}_w &= \Pr(t_i = sm|win) \\ &= \frac{\Pr(win|t = sm)\Pr(t = sm)}{\Pr(win|t = sm)\Pr(t = sm) + \Pr(win|t = db)\Pr(t = db)} \\ &= \frac{(\pi\frac{1}{2} + (1 - \pi))\pi}{(\pi\frac{1}{2} + (1 - \pi))\pi} = 1 > \pi \\ \hat{\pi}_l &= \Pr(t_i = sm|lose) \\ &= \frac{\Pr(lose|t = sm)\Pr(t = sm)}{\Pr(lose|t = sm)\Pr(t = sm) + \Pr(lose|t = db)\Pr(t = db)} \\ &= \frac{\frac{1}{2}\pi^2}{\frac{1}{2}\pi^2 + (1 - \pi)} = \frac{\pi^2}{\pi^2 - 2\pi + 2} < \pi \end{aligned}$$

As mentioned above, this cannot be a separating equilibrium, as this would require not only that  $\hat{\pi}_w = 1$ , but also that  $\hat{\pi}_l = 0$ . As agents can either win or lose, the public would then *ex ante* be able to determine an agent's type with full certainty. As  $\pi > \hat{\pi}_l > 0$ , however, the equilibrium in Lemma 9 is semi-separating.

■

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once more does not add sufficiently to their chance of winning against other smart agents to induce them to continue searching.

We will refer to the equilibrium described in Lemma 9 as the two agent quasi-separating threshold equilibrium with minimum transparency (and for the purpose of this subsection as: QSTE). Plausibly, the threshold  $\widehat{p}_{sm}^{qs}$  is higher for lower values of  $c_{sm}$ , as well as for higher values of  $\lambda$ . The effect of  $\pi$  is ambiguous: it is negative in the sense that if  $\pi$  is high, the posterior  $\widehat{\pi}_l$  is closer to the prior  $\pi$  and (smart) agents have less to win by searching more (and vice versa). It is positive, on the other hand, insofar as for a high  $\pi$ , a smart agent has a higher chance of encountering another smart agent, and thus his overall chance of winning is lower. This induces him to increase this probability by choosing a higher threshold (naturally, in equilibrium his chance of winning will always be  $1 - \frac{1}{2}\pi$ ). The combined effect of  $\pi$  on  $\widehat{p}_{sm}^{qs}$  is positive for low values of  $\pi$  and negative for values of  $\pi$  above a certain turning-point value.

We have not yet identified an equilibrium that exists if  $\lambda > \bar{\lambda}^{QSTE}$ . We will characterize this remaining equilibrium below in Lemma 10. What kind of equilibrium can we expect under these circumstances? As  $\lambda$  is relatively high, it is presumably an equilibrium in which at least one type searches. Moreover, there can be no quasi-separating equilibrium, as shown above, and as  $c_{sm} < c_{db}$  there can be no equilibrium in which only dumb agents search nor any pooling equilibrium in which both types search. Thus a semi-separating equilibrium, in which both types apply an optimal stopping rule with a corresponding ('rat race-') threshold value, remains. As a logical result of our assumption that  $c_{sm} < c_{db}$ , we can expect that smart agents will choose a higher threshold value than agents of the dumb type.

**Lemma 10** *Let  $\widehat{\pi}_l$  be the posterior belief if an agent loses,  $\Pr(t_i = sm|lose)$  and  $\widehat{\pi}_w$  the posterior belief if an agent wins,  $\Pr(t_i = sm|win)$ ; let  $\widehat{p}_{sm}^{ss}$  be the threshold value for smart agents in the semi-separating equilibrium and  $\widehat{p}_{db}^{ss}$  the threshold value for dumb agents. Furthermore, let  $\widehat{p}_{sm}^{ss*}$  and  $\widehat{p}_{db}^{ss*}$  be the equilibrium belief about the threshold each corresponding type of agents chooses. Suppose:*

- (i) *the manager employs two agents ( $n = 2$ );*
- (ii) *the manager chooses a transparency level of minimum transparency;*
- (iii)  *$\lambda$  is sufficiently high:  $\lambda \geq \underline{\lambda}^{SSTE} \equiv \frac{2-2\pi+\pi^2}{2-4\pi+2\pi^2}(c_{db} - c_{sm})$ .*

*Then, a quasi-separating equilibrium exists in which:*

1. *dumb agent searches until he finds a value which is equals to or greater than  $\widehat{p}_{db}^{ss}$ , where  $\widehat{p}_{db}^{ss}$  solves:*



$$\widehat{p}_{db}^{ss} = 1 - \frac{(2 - 4\pi + \pi^2)c_{db} + \pi^2 c_{sm} - \pi\sqrt{D}}{\lambda(1 - 3\pi + 2\pi^2)(\widehat{\pi}_w - \widehat{\pi}_l)} \quad (15)$$

Where  $D = \pi^2(c_{db}^2 + c_{sm}^2) + (4 + 2\pi^2 - 8\pi)c_{db}c_{sm}$ .

2. a smart agent searches until he finds a value which is equals to or greater than  $\widehat{p}_{sm}^{ss}$ , where  $\widehat{p}_{sm}^{ss}$  solves:

$$\widehat{p}_{sm}^{ss} = 1 - \frac{\sqrt{D} - \pi(c_{db} + c_{sm})}{\lambda(1 - 2\pi)(\widehat{\pi}_w - \widehat{\pi}_l)} \quad (16)$$

3. the posteriors are given by:

$$\widehat{\pi}_w = \pi + \frac{\pi(1 - \pi)(\widehat{p}_{sm}^{ss*} - \widehat{p}_{db}^{ss*})}{1 - \widehat{p}_{db}^{ss*}}; \widehat{\pi}_l = \frac{\pi^2(\widehat{p}_{sm}^{ss*} - \widehat{p}_{db}^{ss*}) + \pi(1 - \widehat{p}_{sm}^{ss*})}{1 - \widehat{p}_{db}^{ss*}} \quad (17)$$

**Proof.** Both threshold values are the result of a ‘rat-race game’, which is to say that they are calculated in a similar fashion as  $\widehat{p}_{sm}^{qs}$ . We start with deriving  $\widehat{p}_{db}^{ss}$ . For the postulated equilibrium to hold together, an agent has to be indifferent if he finds a value exactly equal to his threshold. This is the case if the following equation holds:

$$\begin{aligned} U_{A,db}(\text{stop}) &= EU_{A,db}(\text{search once more}) \\ \widehat{\pi}_l \lambda &= \left[ \begin{array}{l} ((1 - \pi)(1 - \widehat{p}_{db}^{ss})\frac{1}{2} + \pi(1 - \widehat{p}_{sm}^{ss})\frac{1}{2}) \widehat{\pi}_w + \\ ((1 - \pi)(1 + \widehat{p}_{db}^{ss})\frac{1}{2} + \pi(1 + \widehat{p}_{sm}^{ss})\frac{1}{2}) \widehat{\pi}_l \end{array} \right] \lambda - c_{db} \\ \frac{c_{db}}{\lambda} &= \left( (1 - \pi)(1 - \widehat{p}_{db}^{ss})\frac{1}{2} + \pi(1 - \widehat{p}_{sm}^{ss})\frac{1}{2} \right) (\widehat{\pi}_w - \widehat{\pi}_l) \\ \widehat{p}_{db}^{ss} &= \frac{\lambda(1 - \widehat{p}_{sm}^{ss}\pi)(\widehat{\pi}_w - \widehat{\pi}_l) - 2c_{db}}{\lambda(1 - \pi)(\widehat{\pi}_w - \widehat{\pi}_l)} \end{aligned}$$

We see that  $\widehat{p}_{db}^{ss}$  depends negatively on  $\widehat{p}_{sm}^{ss}$ , which is sensible because  $\widehat{p}_{sm}^{ss}$  has a negative effect on the dumb type’s chance of winning. Furthermore,  $\widehat{p}_{db}^{ss}$  is positively related to  $\lambda$  and negatively related to  $c_{db}$ , both of which results require no further explanation at this point. We continue with deriving  $\widehat{p}_{sm}^{ss}$ . To make the calculations

more readable we will (temporarily) replace  $\widehat{p}_{sm}^{ss}$  and  $\widehat{p}_{db}^{ss}$  with  $s$  and  $d$ , respectively:

$$\begin{aligned}
U_{A,sm}(\text{stop}) &= EU_{A,sm}(\text{search once more}) \\
\left[ \begin{array}{l} ((1-\pi)\frac{d-s}{1-s})\widehat{\pi}_w + \\ ((1-\pi)\frac{1-s}{1-d} + \pi)\widehat{\pi}_l \end{array} \right] \lambda &= \left[ \begin{array}{l} \left[ (1-\pi)\left(s\frac{d-s}{1-s} + (1-s)\left(\frac{d-s}{1-s} + \frac{1-s}{1-d}\frac{1}{2}\right)\right) + \right. \\ \left. \pi(1-s)\frac{1}{2} \right] \widehat{\pi}_w + \\ \left[ (1-\pi)\left(s\frac{1-s}{1-d} + (1-s)\frac{1-s}{1-d}\frac{1}{2}\right) + \right. \\ \left. \pi\left(s + (1-s)\frac{1}{2}\right) \right] \widehat{\pi}_l \end{array} \right] \lambda \\
\frac{c_{sm}}{\lambda} &= \left[ \begin{array}{l} (1-\pi)(1-s)\frac{1-s}{1-d}\frac{1}{2} + \\ \pi(1-s)\frac{1}{2} \end{array} \right] \widehat{\pi}_w + \\
&\quad \left[ \begin{array}{l} (1-\pi)(s-1)\frac{1-s}{1-d}\frac{1}{2} + \\ \pi(s-1)\frac{1}{2} \end{array} \right] \widehat{\pi}_l \\
\frac{c_{sm}}{\lambda} &= \frac{1}{2}(1-\widehat{p}_{sm}^{ss})\left(\left(1-\pi\right)\frac{1-\widehat{p}_{sm}^{ss}}{1-\widehat{p}_{db}^{ss}} + \pi\right)(\widehat{\pi}_w - \widehat{\pi}_l)
\end{aligned}$$

Because the above equation is quadratic in form, we obtain two possible combinations of  $\widehat{p}_{sm}^{ss}$  and  $\widehat{p}_{db}^{ss}$ . There is however only one combination that is consistent with our model. After inserting the formula we found for  $\widehat{p}_{db}^{ss}$ , we solve this equation for  $\widehat{p}_{sm}^{ss}$ , which (after some rewriting) leads to the results in Lemma 10. It is important to ascertain that  $D$  is never negative, as negative numbers cannot have a real square root. It is easy to see that  $D$  is at its minimum when  $\pi = 1$ , which leaves  $c_{db}^2 + c_{sm}^2 - 2c_{db}c_{sm}$ . This reduces to  $(c_{db} - c_{sm})^2$ , which obviously can never be negative. Next we calculate the posteriors by Bayes' Rule. For this purpose  $s^*$  and  $d^*$  represent the equilibrium belief the public holds about  $\widehat{p}_{sm}^{ss}$  and  $\widehat{p}_{db}^{ss}$ , respectively:

$$\begin{aligned}
\widehat{\pi}_w &= \Pr(t_i = sm | win) \\
&= \frac{(\pi\frac{1}{2} + (1-\pi)(\frac{1-s^*}{1-d^*}\frac{1}{2} + \frac{s^*-d^*}{1-d^*}))\pi}{(\pi\frac{1}{2} + (1-\pi)(\frac{1-s^*}{1-d^*}\frac{1}{2} + \frac{s^*-d^*}{1-d^*}))\pi + (\pi\frac{1-s^*}{1-d^*}\frac{1}{2} + (1-\pi)\frac{1}{2})(1-\pi)} \\
&= \pi + \frac{\pi(1-\pi)(s^* - d^*)}{1-d^*} > \pi \\
\widehat{\pi}_l &= \Pr(t_i = sm | lose) \\
&= \frac{(\pi\frac{1}{2} + (1-\pi)\frac{1-s^*}{1-d^*}\frac{1}{2})\pi}{(\pi\frac{1}{2} + (1-\pi)\frac{1-s^*}{1-d^*}\frac{1}{2})\pi + (\pi(\frac{s^*-d^*}{1-d^*} + \frac{1-s^*}{1-d^*}\frac{1}{2}) + (1-\pi)\frac{1}{2})(1-\pi)} \\
&= \frac{\pi}{1-d^*}(\pi(s^* - d^*) + 1 - s^*) < \pi
\end{aligned}$$

The result  $1 > \hat{\pi}_w > \pi > \hat{\pi}_l > 0$ , where  $\pi$  is the prior belief  $\Pr(t = sm)$ , confirms that this is a semi-separating equilibrium. ■

We will refer to the equilibrium described in Lemma 10 as the two agent semi-separating threshold equilibrium with minimum transparency (and for the purpose of this subsection as: SSTE). Comparative statistics show that, given the assumption  $c_{db} > c_{sm}$ ,  $\hat{p}_{db}^{ss}$  is always lower than  $\hat{p}_{sm}^{ss}$ . Logically, both thresholds increase in  $\lambda$  and have a limit of 1 as  $\lambda$  goes to infinity (or  $c_{db}$  and  $c_{sm}$  go to zero). The (isolated) effects of  $c_{db}$  and  $c_{sm}$  are illustrated in Figure 4.

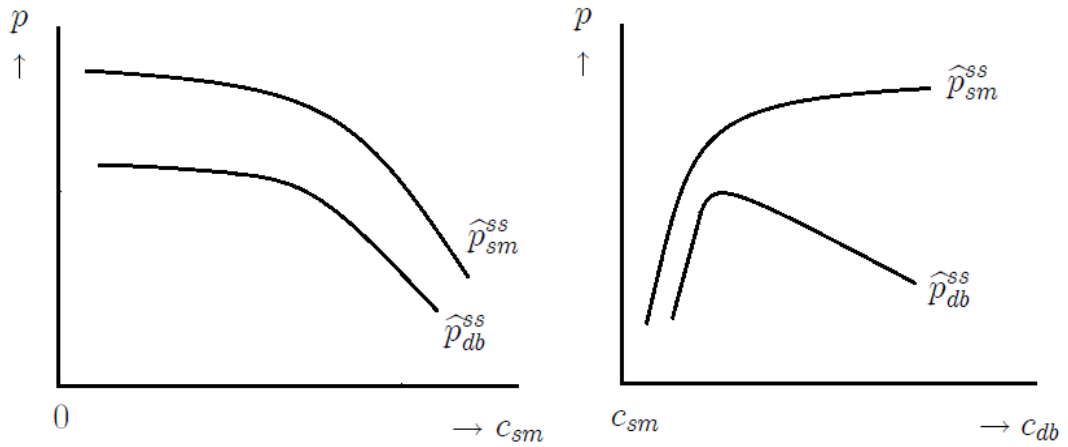


Figure 4. The (isolated) effects of  $c_{sm}$  (left panel) and  $c_{db}$  (right panel) on  $\hat{p}_{sm}^{ss}$  and  $\hat{p}_{db}^{ss}$ .

Firstly we see that the difference between the two thresholds increases as the difference in costs between the two types becomes larger. This is a plausible result: we have seen that if the cost difference is sufficiently large, there exists a quasi-separating equilibrium in which  $\hat{p}_{db}^{ss}$  is effectively equal to zero. Secondly, we see that if the cost difference becomes too small,  $\hat{p}_{db}^{ss}$  and  $\hat{p}_{sm}^{ss}$  get infinitely close and at some point go to zero. One reason for this may be that at some point  $\hat{\pi}_w - \hat{\pi}_l$  becomes so small that there is no longer enough to gain by winning, at which point agents prefer not to search at all. Additionally, as the difference between  $\hat{p}_{db}^{ss}$  and  $\hat{p}_{sm}^{ss}$  becomes smaller, the probability that the winner of the game is decided by chance becomes larger. In sum then, under such circumstances the SSTE cannot subsist and there must exist another equilibrium (which we have not defined). For the scope

of this paper we assume that the difference between  $c_{db}$  and  $c_{sm}$  is sufficiently large to ensure that the SSTE can hold together.

Finally, we briefly examine the effect of  $\pi$  on both threshold values. Again, this effect is twofold: a higher  $\pi$  means a greater chance of competing against a smart agent, which can be either positive or negative for the threshold values. Simultaneously,  $\pi$  directly as well as indirectly influences the difference between the two posteriors,  $\hat{\pi}_w$  and  $\hat{\pi}_l$ , which difference subsequently is reflected in both thresholds. The direct relation between  $\pi$  and  $\hat{\pi}_w - \hat{\pi}_l$  is positive for values of  $\pi$  lower than 0.5 and negative for values greater than 0.5. Furthermore, the difference  $\hat{\pi}_w - \hat{\pi}_l$  is indirectly influenced by  $\pi$  via  $\hat{p}_{sm}^{ss*}$  and  $\hat{p}_{db}^{ss*}$ . The combined effect of  $\pi$  on both threshold values is illustrated in Figure 5:

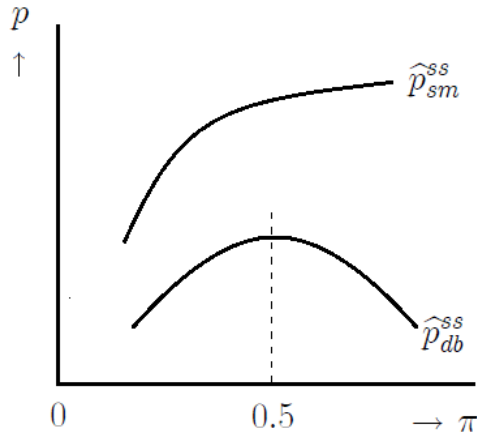


Figure 5. The effect of  $\pi$  on  $\hat{p}_{sm}^{ss}$  and  $\hat{p}_{db}^{ss}$ .

#### 4.2.1 Equilibria

The equilibria found in the previous subsection are summarized in Proposition 4.

**Proposition 4** *Suppose:*

- (i) the manager employs two agents ( $n = 2$ );
- (ii) the manager chooses a transparency level of minimum transparency;
- (iii)  $\gamma = 1$ .

Then, there exist three equilibria, each of which covers an exclusive interval of  $\lambda$ :

If  $0 \leq \lambda \leq \frac{c_{sm}}{1-\pi}$ : 0–0 equilibrium (Lemma 8);

If  $\frac{c_{sm}}{1-\pi} \leq \lambda \leq \frac{2-2\pi+\pi^2}{2-4\pi+2\pi^2}(c_{db} - c_{sm})$ : QSTE (Lemma 9);

If  $\max\left(\frac{c_{sm}}{1-\pi}, \frac{2-2\pi+\pi^2}{2-4\pi+2\pi^2}(c_{db} - c_{sm})\right) \leq \lambda \leq 1$ : SSTE (Lemma 10).

Proposition 4 is (abstractly) illustrated in Figure 6 below.

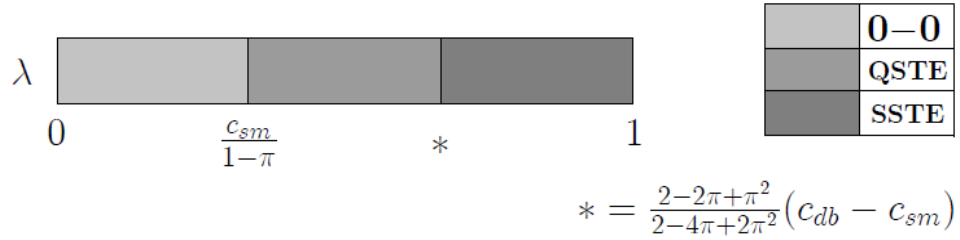


Figure 6. Illustration of Proposition 4.

## 5 Welfare analysis

In this section we address our two main goals formulated in the introduction by analyzing the results found in Propositions 1-4. More specifically, we examine under which conditions it is better for the manager to employ two agents rather than one while scrutinizing the effect of different transparency levels on this decision rule.

### 5.1 One agent versus two agents

We first study which number of agents leads to the highest expected utility for the manager ( $EU_M$ ), holding the transparency decision of the manager fixed. We start with full transparency. Not a very interesting situation is one in which  $\lambda < \frac{c_{sm}}{1-\pi}$ , as under this circumstance no agent ever searches, irrespective of the number of employed agents (or the transparency level). As long as  $W$  is positive (which we have assumed to be the case) it is therefore in the manager's best interest to employ one agent in these cases.

It gets more interesting when we look at the equilibria in which (smart) agents do search: the 1-0 separating equilibrium and the separating threshold equilibrium with full transparency. To begin with we have seen that when there is full transparency, the number of participating agents does not influence the equilibrium strategies the agents employ. Adding a second player, however, does have a positive side as it increases the chance of actually obtaining a project with any positive value; i.e., it increases the probability of employing at least one smart agent. This, and the fact that there is also a chance of employing two smart agents, raises the *ex ante*

expected value of  $\bar{p}$ , the project that will be implemented. The downside is that the manager has to pay out the (exogenously determined) wage  $W$  twice.

**Proposition 5** *Suppose:*

(i) *the manager chooses a transparency level of full transparency;*

(ii-a)  $\frac{c_{sm}}{1-\pi} \leq \lambda \leq c_{db}$ .

*Then, the manager is strictly better off employing two agents rather than one if and only if:*

$$W < \bar{W}^{1-0} \equiv \pi \left( \frac{1}{2} - \frac{1}{3}\pi \right) \quad (18)$$

*Alternatively, suppose:*

(ii-b)  $\lambda \geq c_{db}$ .

*Then, the manager is strictly better off employing two agents rather than one if and only if:*

$$W < \bar{W}^{STE} \equiv \pi(1-\pi) + \frac{(4\pi^2 - 3\pi)c_{db}}{6\lambda} \quad (19)$$

**Proof.** To find the critical values for  $W$  in Proposition 5 we have to compare the (*ex ante*) expected utility the manager obtains when employing one or two agents. We first do this for the 1–0 separating equilibrium with full transparency:

$$\begin{aligned} EU_M^2 &> EU_M^1 \\ \pi^2 \frac{2}{3} + 2\pi(1-\pi) \left( \frac{1}{2} \right) - 2W &> \pi \frac{1}{2} - W \\ \pi \left( \frac{1}{2} - \frac{1}{3}\pi \right) &> W \end{aligned}$$

We do the same for the separating threshold equilibrium with full transparency:

$$\begin{aligned} \pi^2 \frac{1}{3} (2 + \hat{p}_{sm}^{sep}) + 2\pi(1-\pi) \left( \frac{1}{2} (1 + \hat{p}_{sm}^{sep}) \right) - 2W &> \pi \frac{1}{2} (1 + \hat{p}_{sm}^{sep}) - W \\ \frac{1}{6} \pi (3 - 2\pi) + \frac{1}{6} \pi \hat{p}_{sm}^{sep} (3 - 4\pi) &> W \\ \pi(1-\pi) + \frac{(4\pi^2 - 3\pi)c_{db}}{6\lambda} &> W \end{aligned}$$

■

Both critical values of  $W$  are quadratic in  $\pi$ . More specifically,  $\overline{W}^{1-0}$  has an (absolute) maximum of 0.1875 at  $\pi = 0.75$ , while  $\overline{W}^{STE}$  has a limit of 0.25 as  $\lambda$  goes to infinity and  $\pi = 0.5$  (this limit is lower for higher or lower values for  $\pi$ ). This is illustrated in Figures 7 and 8 below.

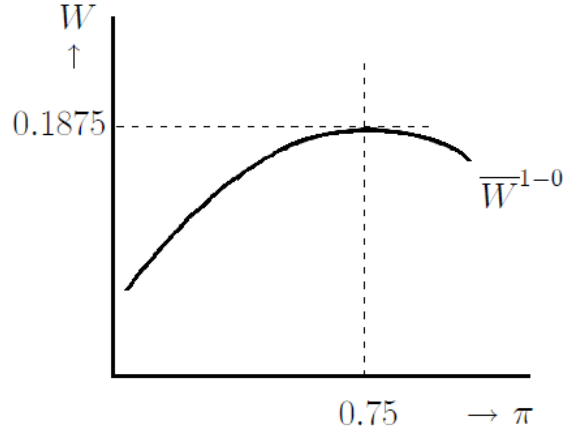


Figure 7. Critical wage levels for full transparency and  $c_{db} < \lambda < c_{sm}$ .

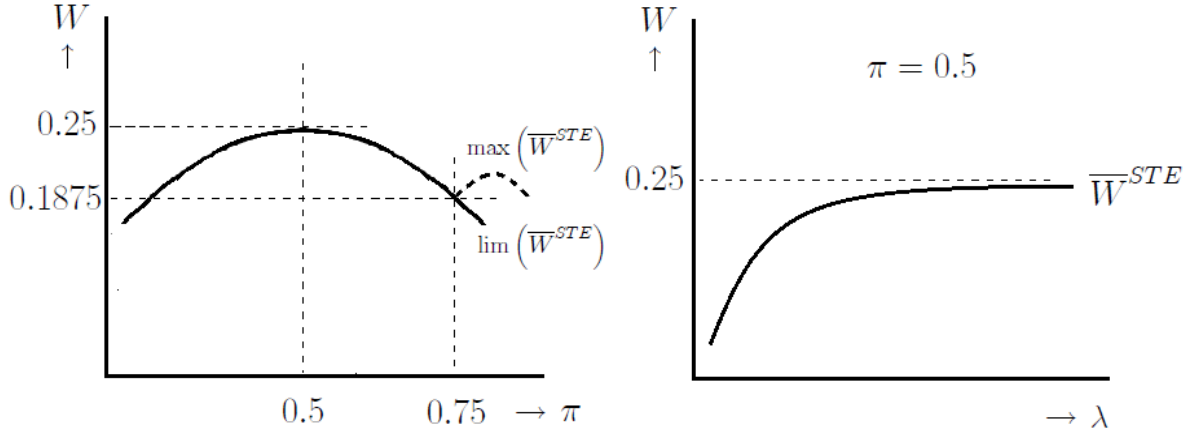


Figure 8. Critical wage levels for full transparency and  $\lambda > c_{db}$ .

One thing the maximum of  $\overline{W}^{1-0}$  and the limit(s) of  $\overline{W}^{STE}$  make clear is that if in a given situation  $W$  is higher than the maximum or the limit that is applicable to that situation, the manager should employ one agent, regardless of the other parameters. Furthermore they can be an instrument to compare the two transparency levels in a certain way, as we will see below. We find that  $\pi$  has an ambiguous effect on the

two critical wage levels. For low values of  $\pi$  (lower than 0.75 and 0.5, respectively), the effect is positive, while for higher values it is negative. A higher  $\pi$  raises the chance of employing a smart agent, and consequently  $E(\bar{p})$ . If  $\pi$  is too high, however, these two effects no longer contribute enough to the manager's utility to outweigh the extra wage cost  $W$  she has to incur for employing a second agent. Because the expected  $E(\bar{p})$  is higher in the separating threshold equilibrium (also if just one agent is employed), the value of  $\pi$  for which the effect on  $\bar{W}^{STE}$  turns negative is lower than in the 1–0 separating equilibrium.  $\lambda$  has no effect on  $\bar{W}^{1-0}$ , as between the given boundaries for  $\lambda$  it does not influence the agents' equilibrium strategies. The effect of  $\lambda$  on  $\bar{W}^{STE}$  depends on  $\pi$ : if  $\pi < 0.75$ , is strictly positive, whereas if  $\pi > 0.75$ , the effect is negative. This can be seen in the left panel of Figure 7 and can also be derived from the definition of  $\bar{W}^{STE}$ .<sup>13</sup> If  $\pi > 0.75$ ,  $4\pi^2 - 3\pi < 0$  and the effect of  $\lambda$  turns negative. The reasoning behind this is as follows:  $\lambda$  influences  $\bar{W}^{STE}$  via  $\hat{p}_{sm}^{sep}$ , where the effect of  $\lambda$  on  $\hat{p}_{sm}^{sep}$  is strictly positive. In turn,  $\hat{p}_{sm}^{sep}$  has two opposite effects on the optimal  $n$ .  $\hat{p}_{sm}^{sep}$  has a (net) positive effect on both  $EU_M^2$  and  $EU_M^1$ , and which effect prevails depends on  $\pi$ . If we take into account only the separate relations between  $\hat{p}_{sm}^{sep}$  and the two expected utility levels, we find that the effect of  $\hat{p}_{sm}^{sep}$  on  $EU_M^2$  equates to  $(\pi - \frac{2}{3}\pi^2)\hat{p}_{sm}^{sep}$  and the effect on  $EU_M^1$  amounts to  $\frac{1}{2}\pi\hat{p}_{sm}^{sep}$ . Equating these two equations and solving for  $\pi$  yields the expected cutoff value of  $\pi = \frac{3}{4}$ .

We move on to minimum transparency. First note that if there are two agents, there exists no equilibrium similar to the single 1–0 separating equilibrium with minimum transparency. The reason for this is that the addition of a second agent opens up the possibility that a (searching) smart agent loses. This ensures that an equilibrium in which a smart agent *always* searches exactly once cannot exist. If the value such a smart agent finds after searching once is sufficiently low, he will be tempted to search (at least) once more to reduce the chance that he indeed loses. In terms of the manager's expected utility, this speaks in favor of employing two agents: a 'threshold equilibrium' always – irrespective of the level of the threshold

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<sup>13</sup>The left panel of Figure 7 displays the (local) maxima of  $\bar{W}^{STE}$ . If  $\pi > 0.75$ , the lower  $\lambda$ , the higher  $\bar{W}^{STE}$ . The reason that there are still maxima stems from the presumption that  $\lambda$  is greater than or equal to  $c_{db}$  (otherwise the STE cannot be an equilibrium). Consequently the dashed line in Figure 7 displays the values of  $\bar{W}^{STE}$  for which  $\lambda$  is equal to  $c_{db}$  (whereas the limits are the values of  $\bar{W}^{STE}$  for which  $\lambda$  goes to infinite).



– leads to a higher (*ex ante*) expected project value *per individual agent* than an equilibrium in which (the same type(s) of) agents always search exactly once. Formally: if  $\frac{c_{sm}}{1-\pi} \leq \lambda \leq \min\left(c_{db}, \frac{2-2\pi+\pi^2}{2-4\pi+2\pi^2}(c_{db} - c_{sm})\right)$ , the expected project value *an individual agent* submits – and thus, evidently, also  $E(\bar{p})$  – is higher if the manager employs two agents rather than one. If  $\lambda$  exceeds the greater of  $c_{db}$  and  $\frac{2-2\pi+\pi^2}{2-4\pi+2\pi^2}(c_{db} - c_{sm})$ , the comparison of one versus two agents is similar. For the sake of clarity we ignore the cases where  $\lambda$  is between these values.

**Proposition 6** *Suppose:*

(i) *the manager chooses a transparency level of minimum transparency;*

(ii-a)  $\frac{c_{sm}}{1-\pi} \leq \lambda \leq \min\left(c_{db}, \frac{2-2\pi+\pi^2}{2-4\pi+2\pi^2}(c_{db} - c_{sm})\right)$ .

*Then, the manager is strictly better off employing two agents rather than one if and only if:*

$$W < \bar{W}^A \equiv \frac{(4\pi^3 - 14\pi^2 + 20\pi - 12)c_{sm} + (6\pi^3 - 15\pi^2\lambda + 9\pi)\lambda}{6\lambda(1 - \pi)} \quad (20)$$

*Alternatively, suppose:*

(ii-b)  $\lambda \geq \max\left(c_{db}, \frac{2-2\pi+\pi^2}{2-4\pi+2\pi^2}(c_{db} - c_{sm})\right)$ .

*Then, the manager is strictly better off employing two agents rather than one if and only if:*

$$W < \bar{W}^B \equiv \left[ \begin{array}{c} \pi^2 \frac{1}{3} (2 + \hat{p}_{sm}^{ss}) + (1 - \pi)^2 \frac{1}{3} (2 + \hat{p}_{db}^{ss}) + \\ 2\pi(1 - \pi) \left( \frac{1}{2} \frac{\hat{p}_{sm}^{ss} - \hat{p}_{db}^{ss}}{1 - \hat{p}_{db}^{ss}} (1 + p_{sm}^{ss}) + \frac{1 - \hat{p}_{sm}^{ss}}{1 - \hat{p}_{db}^{ss}} \frac{1}{3} (2 + \hat{p}_{sm}^{ss}) \right) \end{array} \right] - \frac{1}{2} \quad (21)$$

**Proof.** We first calculate the critical wage level  $\bar{W}^A$ :

$$\begin{aligned} EU_M^2 &> EU_M^1 \\ \pi^2 \frac{1}{3} (2 + \hat{p}_{sm}^{qs}) + 2\pi(1 - \pi) \left( \frac{1}{2} (1 + \hat{p}_{sm}^{qs}) \right) - 2W &> \pi \frac{1}{2} - W \\ \frac{(4\pi^3 - 14\pi^2 + 20\pi - 12)c_{sm} + (6\pi^3 - 15\pi^2\lambda + 9\pi)\lambda}{6\lambda(1 - \pi)} &> W \end{aligned}$$

It gets slightly more difficult if we calculate  $\bar{W}^B$ :

$$\left[ \begin{array}{c} \pi^2 \frac{1}{3} (2 + \widehat{p}_{sm}^{ss}) + (1 - \pi)^2 \frac{1}{3} (2 + \widehat{p}_{db}^{ss}) + \\ 2\pi(1 - \pi) \left( \frac{\widehat{p}_{sm}^{ss} - \widehat{p}_{db}^{ss}}{1 - \widehat{p}_{db}^{ss}} \frac{1}{2} (1 + \widehat{p}_{sm}^{ss}) + \frac{1 - \widehat{p}_{sm}^{ss}}{1 - \widehat{p}_{db}^{ss}} \frac{1}{3} (2 + \widehat{p}_{sm}^{ss}) \right) \end{array} \right] - 2W > \frac{1}{2} - W \quad \blacksquare$$

Both  $\overline{W}^A$  and  $\overline{W}^B$  are positively correlated with  $\pi$  if the latter is low, while the effect of  $\pi$  is negative if  $\pi$  is high. Furthermore, both critical wage-levels – which determine whether the manager should employ one or two agents – logically increase in  $\lambda$ . The cost of searching for dumb agents has no direct effect on  $\overline{W}^A$ , while  $\overline{W}^A$  is negatively related to  $c_{sm}$  as a higher  $c_{sm}$  lowers  $\widehat{p}_{sm}^{qs}$ . Finally,  $\overline{W}^B$  decreases both in  $c_{db}$  and  $c_{sm}$ .

$\overline{W}^A$  has a local maximum of  $\min\left(c_{db}, \frac{2-2\pi+\pi^2}{2-4\pi+2\pi^2}(c_{db} - c_{sm})\right)$ , where  $\overline{W}^B$  has a limit of 0.5 as  $\lambda$  goes to infinity (irrespective of  $\pi$ ). If  $\frac{c_{sm}}{1-\pi} < \lambda < c_{db}$ ,  $\overline{W}^A$  is always greater than  $\overline{W}^{1-0}$ . In other words, if  $\lambda$  is between these boundaries and the manager chooses minimum transparency, there are more cases (i.e., a wider range of wage-levels) for which, *ceteris paribus*, it is favorable to the manager to employ two agents (instead of one) than is the case if she chooses full transparency. This is a rational result: the right hand side of the relevant equations ( $EU_M^1$ ) is equal for both transparency levels, while the left hand side ( $EU_M^2$ ) is always greater for minimum transparency. The latter is the result of the fact that a threshold equilibrium always yields a higher expected project value than an equilibrium in which smart agents always search exactly once. More evocatively, we have seen that in the case of minimum transparency, adding a second agent opens up the possibility that agents search more than once (to avoid losing). One could see this effect as the opposite of the well-known free rider or social loafing effect (which effect(s) we will take a closer look at in subsection 6.2). With full transparency such a (positive) ‘shift’ in individual equilibrium strategies does not occur.

The relatively high limit of  $\overline{W}^B$  (compared to the limit of  $\overline{W}^{STE}$ ) suggests that for higher values for  $\lambda$  the same reasoning as set forth in the previous paragraph is applicable, but this is not necessarily the case. If we again compare  $EU_M^1$  and  $EU_M^2$  for both transparency levels, we see that neither is by definition greater for one of both transparency levels. For instance, the single agent 1–1 pooling equilibrium with minimum transparency may or may not yield a higher expected utility to the manager than the separating threshold equilibrium with full transparency, and vice

versa. The same goes for the semi-separating threshold equilibrium with minimum transparency versus the separating threshold equilibrium with full transparency. Below we will compare both transparency levels in more detail. For now we will conclude by stating that comparative statistics show that only if  $\lambda$  is relatively high and/or the costs of searching are relatively low,  $\bar{W}^B$  is greater than  $\bar{W}^{STE}$ . This is illustrated in Figure 9 (for  $\pi < 0.75$ ).

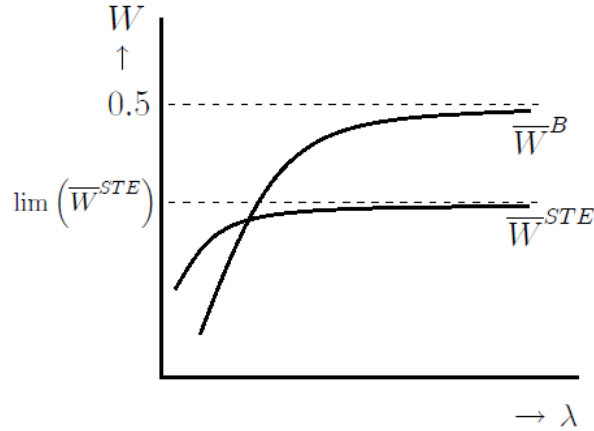


Figure 9. Comparison of  $\bar{W}^B$  and  $\bar{W}^{STE}$ .

## 5.2 Full versus minimum transparency

In this subsection we compare both transparency levels in terms of the manager's utility. First we assume the manager employs one agent. In this case, we can ignore the equilibria that exist if  $\lambda < c_{db}$ , as these are the same for both transparency levels. This means that one equilibrium remains for full transparency, namely the separating threshold equilibrium, while the 1- $\sigma$  semi-separating equilibrium and the 1-1 pooling equilibrium have to be taken into account for minimum transparency.

**Proposition 7** *Suppose:*

(i) *the manager employs one agent ( $n = 1$ );*

(ii-a)  *$c_{db} < \lambda < \frac{c_{db}}{\pi}$ .*

*Then, the manager is strictly better off choosing minimum rather than full transparency.*

*Alternatively, suppose:*

(ii-b)  *$\lambda \geq \frac{c_{db}}{\pi}$ .*

Then, the manager is strictly better off choosing minimum rather than full transparency if  $\pi \leq 0.5$ ; and for  $\pi > 0.5$  if and only if:

$$\lambda < \bar{\lambda}^A \equiv \frac{c_{db}}{2 - \frac{1}{\pi}} \quad (22)$$

**Proof.** If  $c_{db} < \lambda < \frac{c_{db}}{\pi}$ , in order to compare both transparency levels we have to weigh the single agent  $1-\sigma$  semi-separating equilibrium with minimum transparency and the separating threshold equilibrium with full transparency against one another:

$$\begin{aligned} EU_M^{min} &> EU_M^{full} \\ \pi \frac{1}{2} + (1-\pi)\sigma \frac{1}{2} - W &> \pi \frac{1}{2} (1 + \widehat{p}_{sm}^{sep}) - W \\ \lambda^2 - 2c_{db}\lambda + c_{db}^2 &> 0 \\ \lambda &> c_{db} \end{aligned}$$

Note that  $\lambda$  is greater than  $c_{db}$  by assumption, so that between the given boundaries for  $\lambda$  minimum transparency always yields the manager a higher utility than full transparency.

We find  $\bar{\lambda}^A$  by solving the following inequality for  $\lambda$ :

$$\begin{aligned} EU_M^{min} &> EU_M^{full} \\ \frac{1}{2} - W &> \pi \frac{1}{2} (1 + \widehat{p}_{sm}^{sep}) - W \\ \lambda &< \frac{c_{db}}{2 - \frac{1}{\pi}} \end{aligned}$$

We see that if  $\pi < 0.5$ ,  $\bar{\lambda}^A$  is negative. If  $\pi = 0.5$ ,  $EU_M^{full}$  has a limit of  $\frac{1}{2} - W$  as  $\widehat{p}_{sm}^{sep}$  goes to (its own limit of) 1, while  $EU_M^{min}$  is always equal to  $\frac{1}{2} - W$  given the posited conditions. This implies that for all values for  $\pi$  lower than 0.5, minimum transparency is always better than full transparency (irrespective of  $\lambda$ ). ■

Provided that  $c_{db} < \lambda < \frac{c_{db}}{\pi}$ , the manager's optimal decision rule regarding the transparency level is straightforward: minimum transparency is the optimal choice under all (additional) circumstances. Next we have shown that if  $\lambda \geq \frac{c_{db}}{\pi}$ ,  $\pi$  is an important factor in determining the optimal decision. It is in fact a decisive factor if  $\pi$  smaller than 0.5, which is shown below in Figure 10. The reason for this is that

in the separating threshold equilibrium with full transparency only smart agents search, whereas if there is minimum transparency – *ceteris paribus* – both types of agents search. Besides  $\pi$  (and supposing that  $\pi > 0.5$ ), the optimal decision depends on  $\lambda$  and  $c_{db}$ . A low  $\lambda$  speaks in favor of minimum transparency, as does a high  $c_{db}$ . This is because  $\lambda$  has a positive and  $c_{db}$  a negative effect on  $\widehat{p}_{sm}^{sep}$  (and through  $\widehat{p}_{sm}^{sep}$  on  $EU_M^{full}$ ), while  $EU_M^{min}$  is uninfluenced by both parameters.

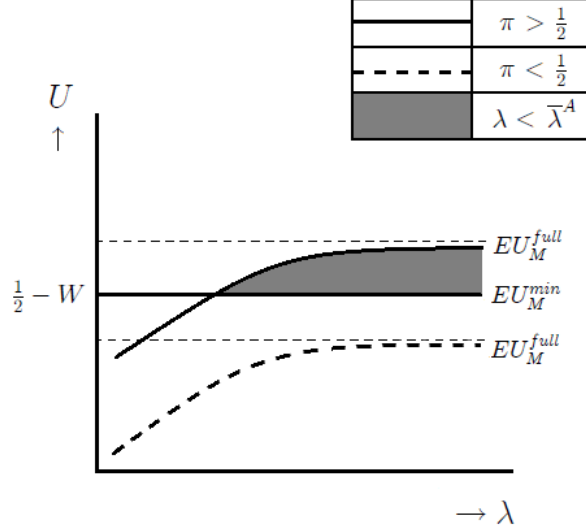


Figure 10. The effects of  $\lambda$  and  $\pi$  on  $EU_M^{min}$ ,  $EU_M^{full}$  and  $\bar{\lambda}^A$ .

We will now compare the two transparency levels under the assumption that the manager employs two agents. Again note that when there is minimum transparency and the manager employs two agents, there is no equilibrium in which smart agents search exactly once. We limit our analysis to the cases in which at least one type of agents searches in equilibrium, both when there is full and minimum transparency. For this it is required that  $\lambda > \frac{c_{sm}}{1-\pi}$ .

**Proposition 8** *Suppose:*

- (i) *the manager employs two agents ( $n = 2$ );*
- (ii-a)  $\frac{c_{sm}}{1-\pi} < \lambda < \frac{2-2\pi+\pi^2}{2-4\pi+2\pi^2}(c_{db} - c_{sm})$ .

*Then, the manager is strictly better off choosing minimum rather than full transparency if and only if:*

$$c_{sm} < \underline{c}_{sm}^A \equiv c_{db} \frac{\pi(1-\pi)}{\pi^2 - 2\pi + 2} \quad (23)$$

Alternatively, suppose:

$$(ii-b) \lambda \geq \frac{2-2\pi+\pi^2}{2-4\pi+2\pi^2}(c_{db} - c_{sm}).$$

Then, the manager is strictly better off choosing minimum rather than full transparency if and only if:

$$\left[ \begin{array}{l} \pi^2 \frac{1}{3}(2 + \widehat{p}_{sm}^{ss}) + (1 - \pi)^2 \frac{1}{3}(2 + \widehat{p}_{db}^{ss}) + \\ 2\pi(1 - \pi) \left( \frac{\widehat{p}_{sm}^{ss} - \widehat{p}_{db}^{ss}}{1 - \widehat{p}_{db}^{ss}} \frac{1}{2}(1 + \widehat{p}_{sm}^{ss}) + \frac{1 - \widehat{p}_{sm}^{ss}}{1 - \widehat{p}_{db}^{ss}} \frac{1}{3}(2 + \widehat{p}_{sm}^{ss}) \right) \end{array} \right] > (2 - \pi)\pi + \frac{(2\pi - 3)\pi c_{db}}{3\lambda} \quad (24)$$

**Proof.** In order to derive  $\underline{c}_{sm}^A$ , we have to compare the manager's expected utility level in case she chooses full transparency with the expected payoff she receives if she chooses minimum transparency. Under the posited conditions, however, in both cases there exists an equilibrium in which dumb agents never search and smart agents apply a threshold strategy. Accordingly, it is possible, and arguably easier, to compare these threshold values instead:

$$\begin{aligned} EU_M^{min} &> EU_M^{full} \\ \widehat{p}_{sm}^{qs} &> \widehat{p}_{sm}^{sep} \\ \frac{c_{sm}(\pi^2 - 2\pi + 2)}{\lambda\pi(1 - \pi)} &< \frac{c_{db}}{\lambda} \\ c_{db} \frac{\pi(1 - \pi)}{\pi^2 - 2\pi + 2} &> c_{sm} \end{aligned}$$

The second situation in Proposition 8 is more difficult to grasp in general terms. The relevant equation will be further explicated below. ■

We will first discuss the first situation in more detail. Note that  $\lambda$  is not one of the determinants of  $\underline{c}_{sm}^A$ . This is because both relevant threshold values are equivalently affected by  $\lambda$ . The manager should, on the other hand, take  $\pi$  into account when choosing the transparency level. Whereas the proportion of smart agents does not have an effect on  $\widehat{p}_{sm}^{sep}$ , the effect of  $\pi$  on  $\widehat{p}_{sm}^{qs}$  is quadratic in nature. This implies (in this case) that there is a  $\pi$  for which  $\widehat{p}_{sm}^{qs}$ , *ceteris paribus*, reaches its maximum. The remaining parameters may be such that this maximum is lower than  $\widehat{p}_{sm}^{sep}$ , in which case full transparency is always better than minimum transparency in terms

of the manager's (expected) utility. This will particularly be the case if the difference between  $c_{db}$  and  $c_{sm}$  is relatively small. In other cases there is a certain range of values for  $\pi$  for which  $\hat{p}_{sm}^{qs}$  is higher than  $\hat{p}_{sm}^{sep}$  (and thus, minimum transparency yields the manager a higher expected utility than full transparency). Both possibilities are illustrated in Figure 11.

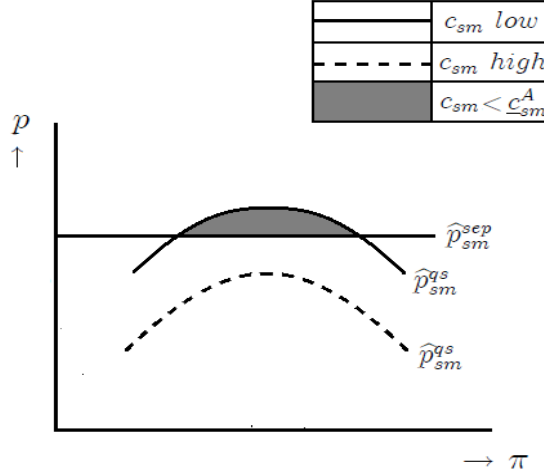


Figure 11. The effect of  $\pi$  on  $\hat{p}_{sm}^{sep}$ ,  $\hat{p}_{sm}^{qs}$  and  $c_{sm}^A$ .

As mentioned the second situation is more complicated, which makes it more challenging to characterize the manager's optimal transparency choice in a straightforward decision rule. Still we can roughly distinguish the effect the different parameters have on this decision rule. Evidently, a higher  $c_{sm}$  speaks in favor of choosing full transparency, for this leads to lower thresholds under minimum transparency while the separating threshold under full transparency remains unaffected (given that  $c_{sm} < c_{db}$ ). The effect of  $c_{db}$  is less distinct though, as it (equivocally) influences the outcome for both transparency levels.  $\lambda$ , on the other hand, clearly has a positive effect on both  $EU_M^{min}$  and  $EU_M^{full}$ . We have seen that in case one agent is employed and  $\lambda$  is relatively high,  $EU_M^{full}$  has a limit of  $\pi - W$  while  $EU_M^{min}$  is equal to  $\frac{1}{2} - W$ , so that  $\pi$  had to be at least greater than  $\frac{1}{2}$  for full transparency ever to be the optimal decision. An analogous reasoning applies to this situation, albeit that the limit of  $EU_M^{full}$  is now (strictly) higher than  $\pi - W$ . The reason for this is simple: if the manager employs two agents, the probability of hiring at least one smart agent is greater than  $\pi$ . To be exact,  $\lim (EU_M^{full})$  is in this case equal to  $\pi(2 - \pi) - 2W$  as  $\lambda$  goes to infinity. On the other hand, while  $EU_M^{min}$  is still at

least  $\frac{1}{2} - 2W$  (as both agents search at least once in equilibrium), the exact level of  $EU_M^{min}$  now also depends on the different parameters (as opposed to the setting where the manager employs one agent). We have already stated that it increases in  $\lambda$  (see the left panel of Figure 12 below), the effect of  $\pi$  is displayed in the right panel of Figure 12.

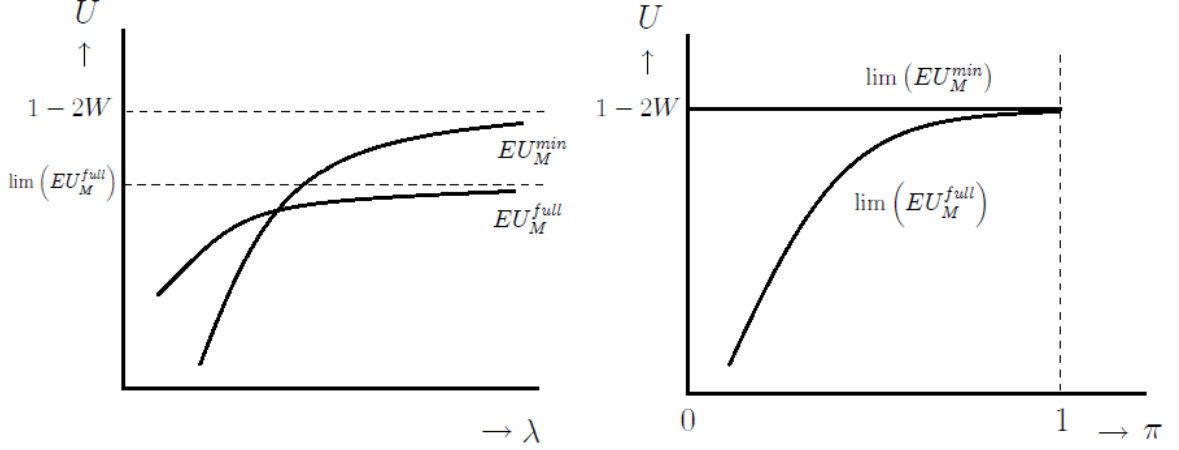


Figure 12. The effects of  $\lambda$  and  $\pi$  on the manager's optimal transparency choice.

Note that  $\lim(EU_M^{full})$  has a maximum of  $1 - 2W$  for  $\pi = 1$ , while  $EU_M^{min}$  has a limit of  $1 - 2W$  as  $\lambda$  goes to infinity (irrespective of  $\pi$ ). This does not mean that  $EU_M^{min}$  is always greater than  $EU_M^{full}$ , which can be seen in the left panel of Figure 12. It does suggest, however, that if  $\pi$  is relatively low, the manager should – in general – feel encouraged to choose for minimum transparency (cf. the right panel of Figure 12).

We conclude by remarking that if  $\lambda$  is relatively high, a risk-averse manager will particularly be tempted to choose for minimum transparency. If she does, she rules out the risk of employing an agent – or only agents – that does/do not search at all (both if she employs one or two agents). Thus there may be cases in which a risk-neutral (or risk-loving) manager would choose full transparency, while a risk-averse manager would rather choose minimum transparency.



## 6 Discussion

In this section, we will briefly explore how the model behaves under alternative assumptions and offer some recommendations for further research.

### 6.1 Alternative transparency levels

Besides the extremes of full and minimum transparency, there may be intermediary transparency levels that are interesting for the manager to consider. The first case we will briefly explore is a transparency level which we will call quasi-full transparency. If the manager chooses this transparency level, we assume that the public sees which agent is the winner, and the manager reveals only the value of the project that will be executed. If there is a second agent that loses, the value he has submitted (if any) will not be revealed.

It is not difficult to show that if only one agent is employed, we get essentially the same result as in the case of full transparency. As opposed to the case of minimum transparency, winning (by searching once) is not sufficient in this case, as the value is always revealed if the agent searches (as an agent that searches always wins). Moreover, if an agent loses, the public will know that he has not searched. This indicates that in this case, if  $\lambda$  is high enough we get the same separating (threshold) equilibrium as described in Lemma 1. For lower  $\lambda$ -levels we find equilibria similar to those described in Lemma 2 and 3. Also, the final result in terms of the manager's utility is the same as with full transparency.

It gets more interesting, and as we will see more difficult, when we 'add' a second agent. While winning may be crucial for an agent that wants to maximize the view the public holds about his ability, the value of the project with which he does is also of importance. This indicates that in an equilibrium in which smart agents exercise a threshold strategy, a (dumb) agent that wins by submitting a project with a value lower than this threshold value will still be seen as dumb with absolute certainty. This in turn implies that with this transparency level, an equilibrium in which dumb agents search (either pooling or semi-separating) cannot exist. Furthermore, for relatively low values for  $\lambda$ , we will not encounter an equilibrium similar to the 1–0 separating equilibrium with full transparency. Instead, if  $\lambda$  is such that smart agents are willing to search, they will automatically be drawn into a rat race equi-

librium in which they use a threshold for which increasing their chance of winning no longer outweighs the costs of searching once more. This equilibrium is similar to the quasi-separating threshold equilibrium with minimum transparency, albeit that this equilibrium exists for all values of  $\lambda$  greater than  $\frac{c_{sm}}{1-\pi}$ . For the sake of clarity, let  $\hat{p}_{sm}^{qt}$  denote the smart agents' threshold in this equilibrium, which equilibrium we will refer to as the quasi-full transparency threshold equilibrium (or: QTTE). In the QTTE, the posterior of an agent that wins (say,  $\hat{\pi}_w$ ), given that he has found a value greater than  $\hat{p}_{sm}^{qt}$  equals 1 (which will generally be the case in equilibrium). If both agents lose, the posterior belief about each agent's ability (say,  $\hat{\pi}_l$ ) is equal to 0. If there is an agent that wins, however,  $\hat{\pi}_l$  will depend on the value with which that agent has won (say,  $p_w$ ). If it is just above  $\hat{p}_{sm}^{qt}$ , it is not very likely that the other agent is (also) smart and has found a value between  $\hat{p}_{sm}^{qt}$  and  $p_w$ . Put differently, if  $p_w$  is relatively close to  $\hat{p}_{sm}^{qt}$ , the chance that the losing agent is smart, i.e.  $\hat{\pi}_l$ , is relatively low and vice versa. Yet, the *ex ante* expected posterior belief  $E(\hat{\pi}_l)$  must be equal to  $\frac{\pi^2}{\pi^2-2\pi+2}$ , the posterior belief associated to a losing agent in the two agent quasi-separating threshold equilibrium with minimum transparency. Therefore the equilibrium strategies and the final outcome in terms of the manager's utility will be the same as in that equilibrium.<sup>14</sup> As already mentioned, still, a major difference between minimum and quasi-full transparency is that the QTTE also exists for high values of  $\lambda$ . A vital advantage for the manager of choosing minimum over full transparency (especially if she is risk-averse), is that if  $\lambda$  is high enough, both types of agents search in equilibrium (irrespective of  $n$ ). Such advantage is absent for quasi-full transparency. This indicates, for instance, that a risk-averse manager will be tempted to choose minimum over quasi-full transparency. Additionally,  $\hat{p}_{sm}^{qt}$  largely depends on  $c_{sm}$ , whereas  $\hat{p}_{sm}^{sep}$ , for instance, depends on  $c_{db}$ . This is one important difference between full and quasi-full transparency, which we also came across when comparing full with minimum transparency. Whether quasi-full transparency is better for the manager than full transparency therefore *ipso facto* depends principally on the difference  $c_{db}$  and  $c_{sm}$ . Winding up: if the manager chooses quasi-full transparency and she employs two agents, there exist two equilibria: if  $\lambda$  is low an

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<sup>14</sup>In order to finalize the formal description of this equilibrium we have to specify the out-of-equilibrium beliefs: what would the public reasonably think if it would receive an unexpected message, specifically a reported value between zero and  $\hat{p}_{sm}^{qt}$ ? Analogous to the rest of our paper we assume the posterior in such case would be equal to zero.

equilibrium in which no type of agents search (cf. the two agent 0–0 equilibrium with minimum transparency), and if  $\lambda$  is high the QTTE. Whether it is favorable for the manager to choose quasi-full transparency depends, besides  $\lambda$ , on her attitude towards risk,  $\pi$  and the difference between the costs of searching of either type of agents. The first two factors are particularly important for comparing quasi-full with minimum transparency, whereas the last is crucial for comparing quasi-full with full transparency.

One last transparency level that may be interesting is one in which only the value of an agent that loses, as well as the identity of the agent that submitted that value, is made known to the public. In that case a separating equilibrium can exist, in which smart agents that search and lose will still be seen as smart with full certainty. As dumb agents can still win from other dumb agents, however, this equilibrium will probably not exist for all combinations of  $c_{db}$  and  $c_{sm}$ . If these are relatively close, we expect to find again a semi-separating equilibrium in which both types of agents apply a threshold strategy. Simultaneously we expect that the difference between both threshold levels is (*ceteris paribus*) greater than the difference between  $\hat{p}_{sm}^{ss}$  and  $\hat{p}_{db}^{ss}$ , as the value smart agents find will under this transparency level be revealed if they lose. It might be an interesting recommendation for further research to examine both transparency levels we have briefly touched upon in this subsection more thoroughly.

## 6.2 Agents care about the project value

In many real-life situations agents will to some extent care about the value of the project that is implemented (take for example situation (d) in the introduction). As a benchmark, in this subsection we will derive the agents' equilibrium strategies under the assumption that there are no reputational concerns – that is, agents care exclusively about the project value. We will show that in such a setting, there exists a social loafing effect if two agents search. In the social psychology of groups, the term social loafing refers to the phenomenon of people exerting less effort to achieve a goal when they work in a group than when they work alone (Karau and Williams, 1993; Gilovich et al., 2006).<sup>15</sup>

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<sup>15</sup>Kidwell and Bennett (1993) argue that social loafing and the well-known free rider effect are similar in the sense that both phenomena describe agents that do not provide the maximum effort

If an agent cares (only) about the project value, he searches until he finds a value that is such, that the chance of finding a higher value is so low that the expected gain in utility from searching once more is equal to or lower than the cost of searching. We can find this threshold for each type by solving the following equation for  $p$ :

$$p = p^2 + (1 - p) \frac{1}{2} (1 + p) - c_t \quad (25)$$

Which reduces to  $\widehat{p}_t^1 = 1 - \sqrt{2c_t}$ , where  $\widehat{p}_t^1$  is the threshold value of an agent of type  $t$  for the case where the manager employs one agent. This leads to an expected outcome  $\bar{p}$  of:

$$\bar{p}^1 = \pi \frac{1}{2} (2 - \sqrt{2c_{sm}}) + (1 - \pi) \frac{1}{2} (2 - \sqrt{2c_{db}}) \quad (26)$$

If the manager employs a second agent, whether an agent contributes to the final outcome  $\bar{p}$  by searching once more also depends on the value the other agent submits. Put differently, the probability that searching again leads to a higher project value is smaller than if he searches alone. Moreover, an agent has nothing to gain by inducing the other agent to search less (or not at all). Conversely, we have seen that this is an important feature of the situation where agents care about their reputation, as an agent's reputation is influenced by the other agent's (or other type's) performance.  $\widehat{p}_{sm}^2$  and  $\widehat{p}_{db}^2$  can be found by solving the following system of equations, where  $\widehat{p}_{sm}^2$  and  $\widehat{p}_{db}^2$  are represented by  $s$  and  $d$ , respectively:

$$\begin{aligned} \left[ (1 - \pi) \left( \begin{array}{c} \frac{s-d}{1-d} s + \\ \frac{1-s}{1-d} \frac{1+s}{2} \end{array} \right) + \right. &= \left[ (1 - \pi) \left( \begin{array}{c} \frac{s-d}{1-d} (s^2 + (1-s) \frac{1+s}{2}) \\ \frac{1-s}{1-d} \left( s \frac{1+s}{2} + \right. \right. \\ \left. \left. (1-s) \frac{1}{3} (2+s) \right) \right) \right. &+ \left. \right] - c_{sm} \\ \left. \pi \frac{1+s}{2} \right] & \\ (1 - \pi) \frac{1+d}{2} + \pi \frac{1+s}{2} &= \left[ (1 - \pi) \left( d \frac{1+d}{2} + (1-d) \frac{1}{3} (2+d) \right) + \right. \\ &\left. \pi \left( s \frac{1+s}{2} + (1-s) \frac{1}{3} (2+s) \right) \right] - c_{db} \end{aligned}$$

Because, *inter alia*, the first equation is a polynomial of degree three (i.e. a cubic

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if they participate in a group. On the other hand, they claim that the crucial difference between the two phenomena is the reason for the drop in effort or lack of group participation.

equation), this system is virtually impossible to solve analytically. This implies we cannot compare  $\bar{p}^1$  and  $\bar{p}^2$  directly, so that we need to use an alternative approach to show that there exists a social loafing effect in this setting. We compare the extent to which both types of agents contribute to  $E(\bar{p})$  by searching once more after (exactly) reaching their respective thresholds (and the probability with which they do) for  $n = 2$  and  $n = 1$ . If  $n = 1$ , an agent finds a higher value than  $\hat{p}_t^1$  with a probability of  $1 - \hat{p}_t^1$ . If he does, the expected value he adds to  $\bar{p}$  is equal to  $\frac{1+\hat{p}_t^1}{2} - \hat{p}_t^1 = \frac{1}{2} - \frac{1}{2}\hat{p}_t^1$ . If  $n = 2$ , the expected extra amount an agent can contribute to  $\bar{p}$  by searching another time depends on his type. For a smart agent the probability that he finds a value higher than  $\hat{p}_{sm}^2$  is again  $1 - \hat{p}_{sm}^2$ . He adds an expected amount of  $\frac{1}{2} - \frac{1}{2}\hat{p}_{sm}^2$  if the other agent is dumb and submits a value lower than  $\hat{p}_{sm}^2$ , and  $\frac{1}{6} - \frac{1}{6}\hat{p}_{sm}^2$  if the other agent is smart, or if the other agent is dumb and finds a value higher than the smart agents' threshold value. On average, this is less than if a smart agent searches alone, while the costs of searching are identical in both settings. This proves that  $\hat{p}_{sm}^2 < \hat{p}_{sm}^1$ , and thus for smart agents a social loafing effect occurs. The same applies to dumb agents: a dumb agent adds nothing to  $\bar{p}$  by continuing his search if he finds a value lower than  $\hat{p}_{db}^2$  and the other agent is dumb, or if the other is smart and he finds a value lower than  $\hat{p}_{sm}^2$ . Furthermore he adds an expected amount of  $\frac{1}{6} - \frac{1}{6}\hat{p}_{db}^2$  if he finds a value greater than  $\hat{p}_{db}^2$ . While  $\frac{1}{6} - \frac{1}{6}\hat{p}_{sm}^2$  may in theory be higher than  $\frac{1}{2} - \frac{1}{2}\hat{p}_{db}^1$ , it is easy to see that on average the expected benefit of searching once more after reaching the relevant threshold is lower for  $n = 2$  than for  $n = 1$ . Therefore  $\hat{p}_{db}^2$  will be lower than  $\hat{p}_{db}^1$ , which shows that dumb agents social loaf as well. This means that a group of two agents is on average less productive than the combined performance of both agents searching individually (with  $n = 1$ ). This negative effect for the manager can be mitigated, as we have seen, by employing agents that care about their reputation (preferably with a high  $\lambda$ ). Specifically for minimum transparency we have seen that adding a second agent may in fact lead to a *higher* productivity per individual agent. It might be an interesting topic for further research to study the effect of the social loafing phenomenon on the optimal number of agents and transparency level, given that agents care about their reputation as well as place some weight on the expected project value in their objective functions.

### 6.3 $n > 2$ agents

In this paper we have analyzed the differences between employing one and two agents. In practice, there are numerous similar settings in which managers employ *more* than two agents. In this subsection, we will briefly discuss the expected consequences of this both in terms of the agents' equilibrium strategies and the resulting utility for the manager. If the manager chooses full transparency, the equilibrium strategies will be the same as in Lemma 1, 2 and 3, irrespective of  $n$ . The optimal number of agents therefore depends directly on  $W$ . The higher  $n$ , the less employing another agent adds to  $E(\bar{p})$  and  $E(U_M)$ . The following equations show this (under the assumption that  $\lambda > c_{db}$ ):

$$\begin{aligned}
 EU_M^0 &= 0 \\
 EU_M^1 &= \pi \frac{1}{2}(1 + \hat{p}_{sm}^{sep}) + (1 - \pi) \frac{1}{2}(0) - W \\
 EU_M^2 &= \pi^2 \frac{1}{3}(2 + \hat{p}_{sm}^{sep}) + 2\pi(1 - \pi) \left( \frac{1}{2}(1 + \hat{p}_{sm}^{sep}) \right) + (1 - \pi)^2(0) - 2W \\
 EU_M^3 &= \pi^3 \frac{1}{4}(3 + \hat{p}_{sm}^{sep}) + 3\pi^2(1 - \pi) \left( \frac{1}{3}(2 + \hat{p}_{sm}^{sep}) \right) + 3\pi(1 - \pi)^2 \left( \frac{1}{2}(1 + \hat{p}_{sm}^{sep}) \right) - 3W \\
 EU_M^n &= \pi^n \frac{1}{n+1}(n + \hat{p}_{sm}^{sep}) + n\pi^{n-1}(1 - \pi)^{n-(n-1)} \left( \frac{1}{n}(n - 1 + \hat{p}_{sm}^{sep}) \right) + (\dots) + n\pi(1 - \pi)^{n-1} \left( \frac{1}{2}(1 + \hat{p}_{sm}^{sep}) \right) + (1 - \pi)^n(0) - nW
 \end{aligned}$$

If the manager chooses no transparency, adding more agents decreases the probability with which an agent (of any type) wins. As a result  $\hat{\pi}_l$  will be closer to  $\pi$  as  $n$  increases (in any equilibrium). Consequently any threshold will be lower in equilibrium if more agents are employed (except for when  $n = 1$  and the manager employs a second agent, for in the former case agents cannot lose at all). If we repeatedly apply this line of reasoning for any number of subsequent agents, at some point agents will not want to search at all. This cannot be an equilibrium however, as agents would always deviate from it if  $\lambda$  is sufficiently high. We must think of alternative equilibrium concepts, such as equilibria in which agents mix and search with a probability  $\rho$  (cf. Remark 2). If such equilibria exist we would probably find that  $\rho$  increases in  $\lambda$ . Even though it is easy to show that under most circumstances it will be more favorable for the manager to choose full over minimum transparency if she is to employ a high number of agents, it might be interesting to look into the aforementioned equilibria in more detail, also to get a better understanding of the set of equilibria described in Proposition 4.

## 6.4 Other suggestions for further research

There are yet some questions and alternative settings we are unable to address in full within the scope of this paper, but which may form some interesting ideas for further research. By way of concluding our paper, we will name a few:

- (a) The manager can have different attitudes towards risk, e.g. she can be risk-averse to a certain extent (cf. subsection 5.2).
- (b) Agents know the other agent's ability (if  $n = 2$ ).
- (c) Agents are effort averse:  $c'_i > 0$ ,  $c''_i > 0$ .
- (d) The agents can differ in the weight they attach to their reputation: i.e., each agent has his own  $\lambda_i$ .
- (e) Endogenously determined wage: in our paper we have assumed  $W$  to be fixed, positive and exogenously determined. Under some circumstances these assumptions may be a realistic. These circumstances may, *inter alia*, include certain legislation ( $W$  fixed and positive), labor market power and organizational bureaucracy ( $W$  exogenously determined). There are also situations in which it is sensible that the manager can determine  $W$  herself. Videlicet,  $W$  is endogenously determined. This means on the one hand that  $W$  may also be negative (but still fixed), and on the other hand that  $W$  may be (in part) variable instead of fixed. The former can be used to capture (a share of) the (expected) utility surplus the agents receive from participating: the manager can require agents to pay him for the opportunity to be in the spotlight and boost their reputation. He may even use this as a screening mechanism to attract only smart agents; be aware however that if the public recognizes this and this becomes known to the agents, they no longer have a reason to search (assuming that they care only about their reputation). A piece-rate wage – or for instance a promotion opportunity – can be an alternative instrument to restrain the social loafing effect discussed above, for it assures that agents (also) care about the value they submit (and, for example, not just about  $\bar{p}$  and/or their reputation). Though such a variable wage may be costly for the manager, these costs can be offset by a negative fixed wage (cf. the bounty hunter example (c) in the introduction of this paper).

(f) Involve the agents' and the public's welfare in the welfare analysis: which transparency level and number of employed agents are the most favorable to the public and (each type of) agents? Note that for the same set of parameters, the choices that optimize the agents' welfare are rarely the same for both types – let alone identical to the choices that optimize the manager's welfare. Conversely, if the public cares only about  $\bar{p}$ , its preferences are perfectly aligned with the manager's. It is not unconceivable however that the public also cares, to some extent, about being able to accurately assess the agents' types. This may lead to an interesting trade-off and the manager not always choosing in the interest of the public. Moreover, it might even be in the manager's interest not to disclose too much information about his agents, for instance if she wants to retain them for her own business.

## 7 Concluding remarks

We have shown that all four combinations of choices by the manager (settings) can be an optimal decision. That being said, there are cases where the manager can immediately see that she should employ one agent and/or choose minimum transparency. The former is the case if  $W$  is above a maximum level applicable to that situation, the latter if  $\pi$  is low and  $\lambda$  is high. There are two main advantages to choosing minimum over full transparency: (i) if  $\lambda$  is high both types of agents search hoping to increase their chances of winning; and (ii) employing a second agent under some circumstances induces agents to exert more effort (which may counteract the social loafing effect). With full transparency dumb agents never search as just winning in that case is not enough to be seen as smart with some positive probability. This explains why if  $\pi$  is low and  $\lambda$  is high, minimum transparency is generally better than full. On the other hand if there is minimum transparency there is always a chance that an agent loses, which potentially leads to lower effort levels (thresholds) than for full transparency. Furthermore, if just one agent is employed he will never search more than once, given that there is minimum transparency, for continuing his search will not increase his chance of winning (this chance is already equal to 1). The results found in the discussion suggest, although not fully conclusive, that there exists a social loafing effect if agents attach some weight to the project



outcome, which effect can be alleviated (*inter alia*) by reputational concerns. This is a practical implication which may be interesting to look into on another occasion. Finally we would like to remark that these positive effects of reputational concerns would be inexistent if all agents were smart (that is,  $\pi = 1$ ). In such case, searching to proof one's type would be futile. This shows us, once again, that dumb agents in their own way may in fact be a social blessing.

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