**Introducing retirement flexibility in a population with heterogeneous agents**

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**Abstract**

In this paper we investigate the introduction of flexible retirement in case of heterogeneous agents with different life expectancies. Flexible retirement can help finance pensions when it induces individuals to extend their careers. We define flexible retirement as the opportunity for individuals to choose their own moment of retirement without (large) distortions. In our model we introduce flexible retirement in combination with flexible pensions. The introduction of flexible pensions can be an important step in facilitating flexible retirement. Based on the literature we discuss the importance of flexible pensions for making retirement flexible. Flexible pensions can come with a selection problem. When individuals differ in life expectancy and governments only allow the use of an uniform accrual rate for postponing benefits, flexible pensions can be used by some participants to improve their net benefit from pensions. In our model the effect of flexible retirement on second period labour supply depends on the chosen value of leisure, because the value of leisure of individuals is unclear we can not say what the effect of flexible retirement on second period labour supply is. Only allowing late retirement is possibly a better measure to increase second period labour supply, however this does not enable individuals with a high preference for leisure or a low life expectancy to retire at their preferred retirement age.

**Keywords**

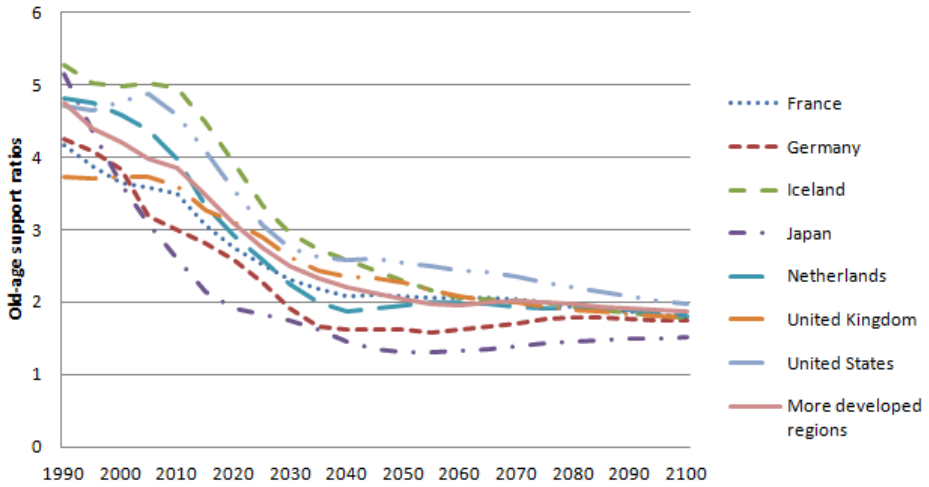
Pension flexibility - Ageing - Retirement – Redistribution

# Index

# 1. Introduction

In the upcoming decades ageing will cause the expected number of retirees to rise quickly in many countries. An ageing population makes it more difficult to finance pensions. In case of a PAYG-pension scheme the working population has to support a larger group of elderly. In case of a funded pension scheme individuals have to save for a longer retirement duration. A good indicator of the effect the population structure of a country has on the difficulty of financing pensions is the old-age support ratio. Figure 1 shows the old-age support ratio for a selection of countries as well as the old-age support ratio of the more developed regions[1]. The old-age support ratio is especially relevant for PAYG-pensions because it shows the number of people aged 20-64 relative to the number of people age 65+. In other words the old-age support ratio gives the number of individuals that are able to economically support others to the number of elderly that may need economic support. The old-age support ratio of the more developed regions will decrease to half of the current ratio by 2050. This implies that for PAYG pension systems the pension contributions will have to double to keep the current level of pension benefits, or pension benefits should be reduced by half to keep the current level of pension contributions, when governments are not able to increase labour force participation.

*Figure 1. Decreasing old-age support ratios 1990-2100 (Source: UN (2011), data 2010)*




In this paper we want to answer the question if the introduction of flexible retirement can contribute to the goal of preparing pension systems for decreasing old-age support ratios. To

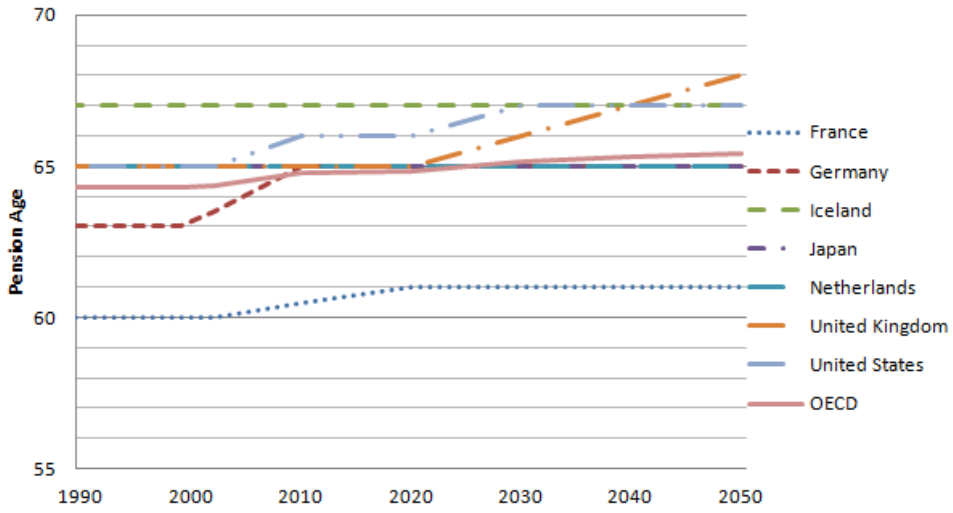
[1] UN world population prospects 2010 defines the more developed regions as: Europe, Northern America, Japan, Australia/New-Zealand

answer this question we will discuss literature about flexible retirement and flexible pensions and we will discuss a simple model in which we introduce endogenous labour supply combined with flexible pensions. Our model shows similarities to the model of Cremer and Pestieau (2003). Cremer and Pestieau (2003) use their model to determine the optimal retirement age and how financial incentives affect the retirement age. Our model differs in the utility function used and in our analysis of the model we focus more on the difference in life expectancies of individuals. Our model also shows similarities to the model of Cremer al. (2010). Cremer et al. (2010) compare pension systems that function as collective annuities with private annuities and discuss when pension systems can improve social welfare. They assume that a private annuity is calculated with information about the survival probability of an individual and pensions without. Further more they assume a positive correlation of income and health. Compared to the model of Cremer et al. (2010) our model differs in the utility function used and in the way we model life expectancy. In our analysis we focus less on social welfare and more on the possibility of a Pareto improvement.

Many countries are planning to reform their pension systems to prevent future problems in financing pensions. In some countries pension reforms have already been made by increasing the pension age. Figure 2 shows the pension age for men in the period 1990-2050 for the previously selected countries as well as the OECD average. Pension age is defined in Figure 2 as the earliest age an individual can collect a full pension when he has been working since age 20. The pension age is sometimes lower than the statutory pension age. This is for example the case in Germany, when the statutory pension age is increased to 67, retirement with unreduced benefits will remain possible after 45 years of contributions. Another potential pension reform is to make retirement more flexible. This is the pension reform we focus on in this paper. Flexible retirement can be part of the solution to prepare pension systems for the ageing population because it might lead to workers extending their careers. Finland and Sweden are examples of countries that have some flexibility in their pensions. In Finland it is possible to postpone pension benefits after age 68 for an increment of 4,8 percent a year. It is also possible in Finland to receive pension benefits while collecting earnings from work. In Sweden pension benefits can be deferred without age limit as well, with the exception of the guarantee pension. The guarantee pension is an additional pension for individuals with a low income. Pension benefits can be collected while collecting earnings from work and it is also possible to collect part of the pension benefits. Early retirement is possible from age 62 in Finland and from age 61 in Sweden. The age restriction on early retirement limits retirement

flexibility in Finland and Sweden. Finland and Sweden are not the only countries where late retirement for additional pension benefits is possible. We have chosen to discuss Finland and Sweden because these countries do not have a statutory retirement age, which makes their pension systems more flexible compared to other countries.

*Figure 2. Increasing pension age, men (Source: OECD, 2011)*




*Definition of flexible retirement*

In this paper we define flexible retirement as the opportunity for individuals to choose their own supply of labour according to their own preferences. An individual can change his labour supply by altering his retirement age or by changing his working hours. Under flexible retirement the retirement decision is unaffected by regulations like mandatory retirement and is unaffected by social norms. In other words individuals base their retirement decision on their own situation and their own preferences for leisure and consumption. Flexible retirement is closely related to flexible pensions. Flexible pensions is defined as the opportunity for individuals to choose their own moment of pension take up without affecting their pension wealth and can be seen as a measure to facilitate flexible retirement. In case of flexible pensions it is possible to collect pension benefits while earning income from work. As a result the moment of pension take up does not have to be the moment of retirement.

Flexible retirement can contribute to the goal of preparing pension systems for decreasing old-age support ratios in a couple of ways. Flexible retirement would in many countries improve financial incentives for postponing retirement. In many countries financial incentives are currently set on shortening careers rather than extending them, for example in Belgium and

Greece. If financial incentives would no longer contribute to shortening careers of individuals this could result in a larger workforce. A larger workforce would help finance pensions and public expenses. Research has shown that retirement incentives have a significant impact on the effective retirement age. This result is found by amongst others Duval (2003). Duval (2003) finds that the implicit tax on continued work has an effect on the retirement decision of all tested age groups 55-59, 60-64 and 65+. Duval (2003) also finds that for the age groups 60-64 and 65+, eligibility ages appear to have a specific impact on the retirement decision. A larger workforce is more important for PAYG-pension schemes compared to funded pension schemes when ageing is not only caused by a higher life-expectancy but also caused by decreasing fertility rates. Flexible retirement can also help preparing pension systems for ageing because flexible retirement in combination with flexible pensions can function as a hedge against pension wealth volatility. Pension flexibility makes it possible for individuals to alter their retirement decision after a wealth shock has appeared. This enables individuals to better cope with volatility of pension wealth. There seems to be an increase in volatility of the final value of pension wealth. The increased volatility in pension wealth is caused by riskier assets of pension funds and underestimated risks involving errors in expected life expectancy. Individuals will be more able to deal with deviations in pension wealth if they are not only able to alter consumption levels but their labour supply as well. The hedge function against pension wealth volatility is more important for funded pension schemes compared to PAYG-pension schemes because of more volatile pension wealth.

A disadvantage of flexible retirement in combination with flexible pensions is the possibility of a selection problem. In case of a selection problem with pensions, participants will have information about themselves that is not known to or can not be used by the pension funds or the government. In case of flexible pensions the accrual rate for postponing benefits has to be calculated using the life expectancy of a participant. There is evidence for differences in the life expectancy of individuals and even for differences in the average life expectancy of population groups. The life expectancy at age 65 differs between men and women by 1,5 to 5 years in the different OECD countries. The life expectancy does not only differ between men and women. Mortality over time and also mortality within countries is examined by Cutler et al. (2006). They conclude that there is a link between income and health. Cutler et al. (2006) also conclude that there is most likely a direct positive effect of education on health, only the exact mechanism is unclear. In Van Vuuren (2011) an example is given for the Netherlands where the median life expectancy at age 65 is more than 7 years higher for medium and higher educated women than for low educated men. Though it is possible for pension funds to

have useful information about the participants, they will not be able to use this information. Current politics will only allow for a uniform accrual rate for postponing benefits to be calculated using the average life expectancy of all participants. Both Finland and Sweden use unisex life expectancy tables to calculate the uniform accrual rate for postponing benefits. As a result there will be many participants for which pension funds or the government use the wrong life expectancy. When a life expectancy is used that is too low, an individual can increase his expected lifetime pension benefits with late retirement. When a life expectancy is used that is too high, an individual can increase his expected lifetime pension benefits with early retirement. In Section 4 we show that it is possible that an individual's lifetime income decreases because of more flexible pensions.

The setup of the rest of this paper is as follows. In the next section we will give a literature overview. In the literature overview we will start by discussing the current situation of pensions and labour force participation. Then we will discuss the effects that can alter the retirement decision. At the end of this section we will discuss the demand for old-age labour. In Section 3 we will explain the model that we will use to investigate effects of retirement flexibility in case of heterogeneous agents. We include individuals with various life expectancies in the model. Because of this we include the welfare effects of the selection problem that comes with flexible pensions. In Section 4 we will discuss the results of the model for different scenarios. In Section 5 we will give a summary of the paper and we will give a conclusion.

## 2. Current situation and literature overview

Most countries have one or two moments individuals more often choose as the moment to retire. It can be hard to explain why so many people retire at these 'standard' retirement ages. Understanding why people retire at the moment that they do is important for figuring out what has to be done for making retirement more flexible. It is also important to know how flexible retirement would alter labour supply. Therefore we will give an overview of the effects that can alter the retirement decision. We will start by focussing on the current situation of pensions and labour force participation. Then we will focus on the supply side of labour. For flexible retirement to contribute to solving future financing problems of pensions, individuals will have to be willing to extend their careers. We will end this section by focussing on the demand side of labour. Elderly workers will have to be able to find or retain jobs at old age to be able to extend their careers.

### 2.1 Current situation

In light of the upcoming ageing, an important goal of flexible retirement is the improvement of labour force participation rates of elderly workers. In Figure 3 we show the labour force participation rates for different age categories for a number of OECD countries. The graph shows large differences in the labour force participation rates across countries. The three countries from continental Europe have labour force participation rates in age category 65-69 of about 10 percent. In the age category 60-64 the labour force participation rates is about 20 percent in France. This is less than one half of the labour force participation rate in the United States in the same age category. Some countries have even higher labour force participation rates. Countries with the highest old age labour force participation rates include Iceland and Japan. In these countries the labour force participation rate in age category 60-64 varies between 60 and 80 percent. Countries that have a higher pension age, according to the pension age definition of Figure 2[2], do have higher labour force participation rates of elderly workers. We will discuss the effect of the pension age later on. We can also look at the financial incentives to retire to explain the current labour force participation rates. Figure 4 shows the change in gross pension wealth for men with average earnings for working an additional year at ages 60 to 64. Parameter values used in Figure 4 are from 2008.

[2] Pension age is defined as the earliest age an individual can collect a full pension when he has been working since age 20.

*Figure 3. Labour force participation rates by age (OECD, 2010), data year 2010.*

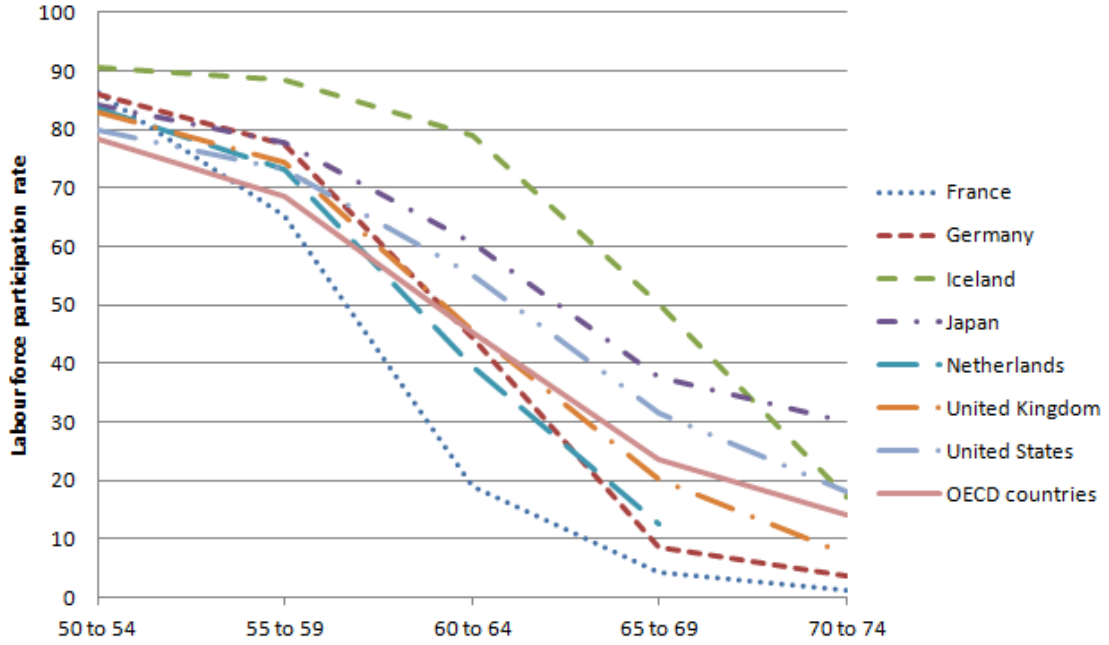
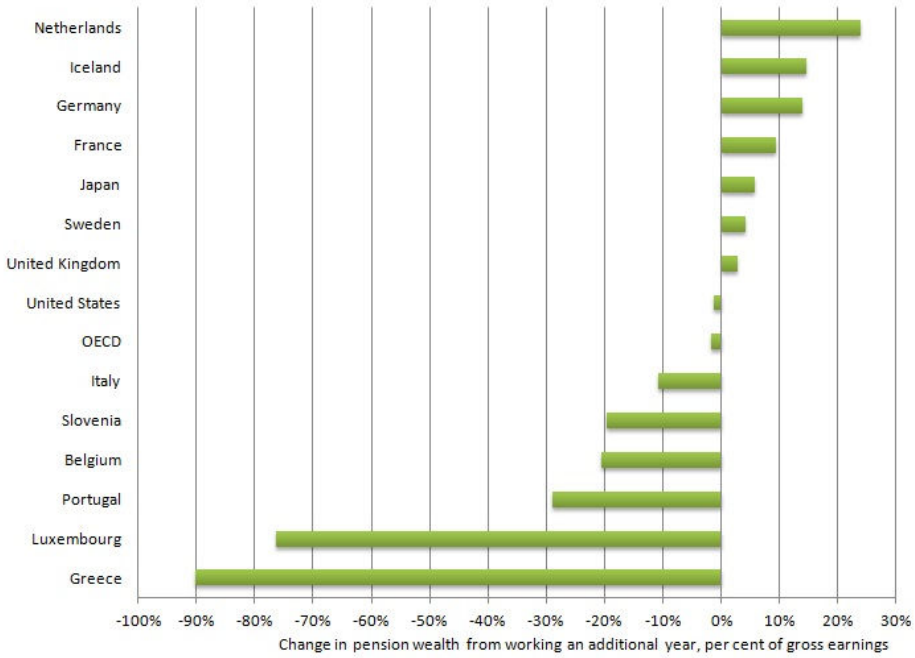



*Figure 4: Gross pension wealth change for working an additional year at age 60-65, men with average earnings (Source: OECD, 2011)*




Good financial incentives to retire do not necessarily translate into higher labour force participation rates. When we compare Figure 3 and Figure 4 we find that the Netherlands, Germany and France offer good financial incentives for working at age 60 to 64 but have relatively low participation rates. In the United States and the United Kingdom there are only

small implicit taxes and implicit subsidies on continued work. The labour force participation rates are about average. Iceland and Japan offer good financial incentives for working at age 60 to 64 and have relatively high participation rates. The differences in the financial incentives for an individual to extend his career are caused by country specific regulations. For instance there are countries where retirement income depends on earnings in the last years before retirement ("final" pay). In Greece for example pension benefits are calculated using the average earnings of the last five years of an individual's career. This can cause a positive (negative) incentive for postponing retirement if wages are increasing (decreasing) in the last years before retirement. An implicit tax on continued work life can also be caused by attractive early pension schemes (Greece, Luxembourg) and unrealistically low correction factors for early retirement (Slovenia, Portugal). Financial incentives to retire can also be affected by flexible pensions in case there is a selection problem. For example, a short living individual can increase his net benefit from pensions when he starts to collect his pension early. This makes pensions more expensive. The selection problem that can come with flexible pensions is discussed in the introduction.

## 2.2 Labour Supply

Previously we already made a connection between financial incentives to continue work and the labour force participation rates. Although good financial incentives did not necessarily result in high labour force participation rates, we will now discuss some arguments why financial incentives could affect the retirement decision. First the basic intuition. We start to assume that individuals only care about consumption and leisure and that marginal utilities of consumption and leisure are decreasing when more is consumed. When the price of leisure is constant, an individual can maximise his utility by choosing the optimal ratio between leisure and consumption. In this case a wealth (consumption) increase will lead to a leisure increase. By extending his career, an individual can substitute leisure with consumption. How much consumption an individual has to give up for one unit of leisure we call the price of leisure. A low price of leisure makes early retirement more attractive. In many countries the price of leisure at certain ages is largely affected by the previously mentioned financial incentives of pension systems. In case of the Netherlands the price of leisure at ages 60 to 64 is high. For one additional unit of leisure an individual has to give up additional income and additional pension wealth. For Greece it is the opposite. For one additional unit of leisure an individual has to give up additional income but he will also receive additional pension benefits. Because the price of leisure is largely affected by financial incentives it is likely that individuals alter their labour supply because of financial incentives. If this basic intuition is correct we can also

expect a change in the labour supply of individuals when they receive unexpected additional wealth. Additional wealth would reduce the marginal utility of consumption and would as a result make leisure more attractive. If this holds in the real world is investigated by Imbens et al. (2001). Imbens et al (2001) find that a lottery winning can reduce the number of hours worked by an individual as well as the duration an individual participates in the labour market. A lottery winning is a good example of unexpected additional wealth. Imbens et al. (2001) base their research on questionnaires of participants of the lottery in Massachusetts in the mid-eighties. Questionnaires have been filled in by: "winners" with prices larger than 22'500 USD and "non-winners" with prices between 100 USD and 5000 USD. The positive effect wealth has on leisure consumption is an indication that financial incentives do affect the retirement decision. Imbens et al. (2001) find a stronger effect of lottery winnings on leisure consumption for participants close to retirement but do not find a stronger effect for participants working past the retirement age. This suggests this final group is less affected by financial incentives. Perhaps some participants working past the retirement age prefer work over leisure. The final group that continues to work after the retirement age will consist only of individuals who do not have a strong preference of leisure over working when financial incentives are not focussed on extending careers. Individuals with a strong preference of leisure over working will already have stopped working. Individuals with a strong preference of leisure over working can also have stopped working because there are other reasons to retire that gain in importance after the retirement age. One reason to retire that can gain in importance after the retirement age is social costs. For example the partner/spouse expects the individual to stop working at age 65 and did not expect this from him at age 60. We will discuss the effect of social costs on the retirement decision later on. Instead of looking at how people react to changes in wealth because of lottery winnings we can also look how people react to changes in pension wealth. Krueger and Pischke (1992) measured the impact of an unanticipated reduction in Social Security wealth on old-age labour supply in the United States. However, they found little empirical evidence that linked the fluctuation in social security wealth to the the old-age labour supply.

As financial incentives fail to completely explain why so many individuals retire at certain ages, we need to look at other explanations. In Lumsdaine et al. (1996) the unexplained high age-65 retirement rate in the United States is investigated. By eliminating other possibilities, like the effects of financial incentives, the high age-65 retirement rate is attributed to an "age-65 retirement effect" explained because of the influence of custom or accepted practice. The

effect of Medicare, a social health insurance program for individuals age 65 or older, is investigated by comparing companies that vary in their health benefits for retirees. Lumsdaine et al. (1996) rule out Medicare as possible explanation for the high age-65 retirement rate in the United States. When we compared the labour force participation rates (Figure 3) and the retirement age (Figure 2) we already saw that countries with a higher retirement age had higher participation rates. The research of Lumsdaine et al (1996) confirms the effect the retirement age has on the moment of retirement of individuals. After the research of Lumsdaine et al. (1996) the retirement age in the United States has been increased. We expect that people alter their retirement decision as a reaction to this change in retirement age. Mastrobuoni (2009) investigates the change in the retirement decision due to the increasing retirement age. Mastrobuoni (2009) finds an effect of about 50% of the retirement age on the effective retirement age based on the increasing normal retirement age in the US over the period 2000-2006. The effective retirement age is a measure that shows when people on average retire. The retirement age is the moment that individuals would normally receive a full pension. The retirement age does affect financial incentives to retire. Still the effect of the retirement age on the retirement decision seems large. Mastrobuoni (2009) compares his results to a simulation of the increase in retirement age done by Coile and Gruber (2000). Coile and Gruber (2000) predict that individuals would on average postpone retirement between 0,5 and 2 months in case of an increase in the retirement age of one year. According to Mastrobuoni (2009), the difference in the results can be caused by the effect the retirement age has on the social norms. This effect is not included in the simulations by Coile and Gruber (2000).

We mentioned evidence that the retirement age affects the retirement decision even when it is corrected for financial incentives. Sometimes there are more moments individuals frequently choose as the moment to retire. We call these ages ‘standard’ retirement ages. These ‘standard’ retirement ages can for example be caused by the first age early retirement is possible. We assume that all ‘standard’ retirement ages have a similar effect on the retirement decision. There are a couple explanations for the existence of ‘standard’ retirement ages. Mastrobuoni (2009) and Lumsdaine et al. (1996) both mention social norms as a possible explanation why the retirement age has such a large effect on the effective retirement age. In Lindbeck et al. (1999) social norms are incorporated in a model used to determine an individual’s choice whether or not to work. In this model individuals can experience social costs when they choose not to work because this causes negative externalities to the society.

These social costs also depend on the part of the population that chooses not to work. When more people choose not to work it becomes socially more acceptable and the social costs are reduced. Because of the externalities involved with the retirement decision it is likely that social costs are involved. When deviating from the 'standard' retirement age increases social costs, the 'standard' retirement age becomes a more attractive moment to retire. Social costs can also occur when the social contacts of an individual have little understanding for your retirement decision. The importance of 'standard' retirement ages can also be a sign that individuals do not maximise utility by choosing their optimal retirement age or leisure consumption. A 'standard' retirement age can be a default option. It is profitable for individuals to stick with the default option when the costs of making an informed decision would outweigh the benefits. Lumsdaine et al. (1996) considered this as a possible explanation for the high age-65 retirement rate in the US, but conclude that this explanation is unlikely because of the large welfare loss when an individual does not choose his optimal moment of retirement. Lack of financial knowledge would increase the costs of making an informed decision and makes the default option more attractive. In Lusardi and Mitchell (2011) financial literacy around the world is discussed. To measure financial literacy, a questionnaire was used consisting of three simple financial questions. Financial literacy was measured in eight countries[3]. Lusardi and Mitchell (2011) conclude that financial literacy is very low around the world, that most workers have not even thought much about retirement and that there is a causal relationship between financial literacy and retirement planning. The paper of Lusardi and Mitchell (2011) is not focussed on the moment people retire, but their results suggest that some individuals are not able to make an informed retirement decision.

In case of flexible retirement the 'standard' retirement ages should not be important for individuals and do perhaps not exist. To know how flexible retirement would alter labour supply we need to know what the effect of these 'standard' retirement ages is on the labour supply. We will now discuss research that can give an indication what the net effect of the 'standard' retirement ages on the labour supply is. Brown (2006) examines the effect of usual retirement ages. The usual retirement age can be very similar to the 'standard' retirement age. Brown (2006) uses the results of the health and retirement survey in which respondents are asked what the usual retirement age is for people with the same kind of job. This is often the 'standard' retirement age. Brown (2006) finds evidence that those workers who report that there is no 'usual' retirement age for them, on average retire more early. Workers who report

[3] Results of financial literacy in the Netherlands can be found in the following paper: Alessie et al. (2011)

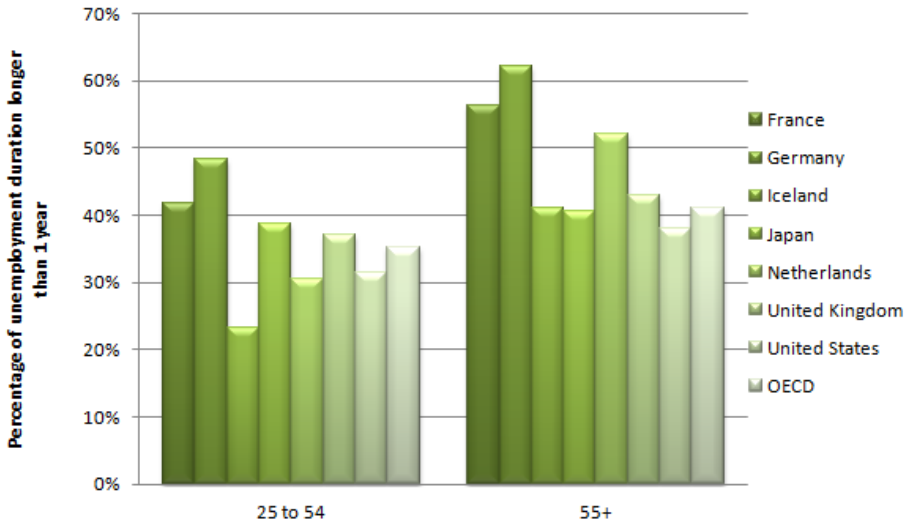
that there is no ‘usual’ retirement age most likely do not represent the entire population, for example due to the nature of their job. However it can not be ruled out that ‘usual’ and ‘standard’ retirement ages have a positive effect on labour supply and that this effect is reduced by more flexible retirement. In this case labour supply can be reduced because of flexible retirement. Wadensjö (2006) looks at the Swedish pension system to get an indication of the effect of more flexible pensions on the labour force participation of elderly. Flexible pensions can give more individuals the opportunity to work part-time. Working part-time can be welfare enhancing for some old-age workers, for example when an individual is not fit enough to work full-time. In Sweden individuals older than 61 can receive a part of their pension benefits. This makes it easier for older workers to reduce their working hours. Wadensjö (2006) concludes that the Swedish pension system has resulted in a higher labour force participation of elderly.

We can conclude that current retirement behaviour is largely affected by previously mentioned effects like social norms, default options and financial incentives. These effects make it difficult to predict retirement behaviour in case of flexible retirement. When some of the effects on the retirement decision lose part of their strength or are changed due to flexible retirement this can lead to individuals making a better retirement decision for themselves. Flexible retirement will not necessarily lead to a larger workforce.

## 2.3 Demand for old-age labour

In the introduction is discussed that ageing will lead to an increase in retirees. Ageing will also lead to an older workforce. To ensure that in the future the workforce remains sufficiently large it is important that the old-age labour market functions properly. The labour force participation of older workers (Figure 3) is in some countries very low. This can indicate that individuals do not want to work at this ages but this can also indicate that people are not able to find and retain jobs at these ages. There are indications that it is more difficult for old-age workers to find and retain jobs. According to D’Addio et al. (2010), there is no doubt some employers discriminate against older workers. Further the unemployment duration is longer for old-age workers. In Figure 5 we show the percentage of unemployment durations longer than 1 year. We use the same selection of countries as in Figures 1-3. We find that the duration of unemployment is more likely longer than 1 year for workers aged 55+ than for workers aged 25-55 in all selected countries.

*Figure 5: Percentage of unemployment duration longer than 1 year by age category (OECD, 2010), data year 2010.*




We will now discuss a couple of reasons why there could be a lower demand for old-age workers. An important factor in the demand for labour is the cost of old age labour (OECD, 2011). Old-age workers are in some countries very expensive. Figure 6 shows the wage index of full time male workers for different age categories. There are some countries where the wage index follows an inverted U shape, for example in Japan and in the United Kingdom. There are also countries were the wages are continuously increasing, for example in France and The Netherlands. There are large differences in the price of old-age labour across countries. This can be an indication that the price of old-age labour in some countries does not match with the productivity of old-age workers. Lazear (2011) mentions that when firms stimulate early retirement it is an indication that elderly workers are overpaid. Lazear (2011) also mentions that he believes that the high wages of old-age workers are not for their current productivity but are a reward for loyalty and a motivator for young employees. Companies can give lower wages to workers when workers can look forward to higher wages in the future. The prospect of higher wage also gives an incentive to stay loyal to the company. As a result companies profit from a reduction in loss of human capital. With high job mobility this explanation becomes less likely. The low demand for old-age workers can be a consequence of their characteristics. Employers can prefer younger workers when employers believe they are more aware of new technology (OECD, 2006). Finally we mention employment protection as a possible explanation for the lower demand for old-age workers. Strictness of employment protection can be found in Figure 7. Employment protection is often stricter for older workers (Van Vuuren, 2011). As a result companies will have more to lose in case of falling productivity rates. Employment protection can also be in favour of young workers

when they have a lower probability of falling productivity rates. For example when young workers are healthier. There is also a positive effect of employment protection on the labour force participation old-age workers because the old-age workers are less likely to get fired.

There is evidence that old-age workers are not preferred by employers. Even when old-age workers are prepared to work longer it remains doubtful if this will result in a higher labour force participation of old-age workers. Especially in countries where wages are increasing over time and employee protection is strict, it is uncertain if individuals can work for as long as they want. To increase the labour force participation it is necessary that measures are taken to make old-age workers more attractive for employers.

*Figure 6. Wage index full time male workers by age (Source: D'Addio et al., 2010, based on OECD earnings database)*

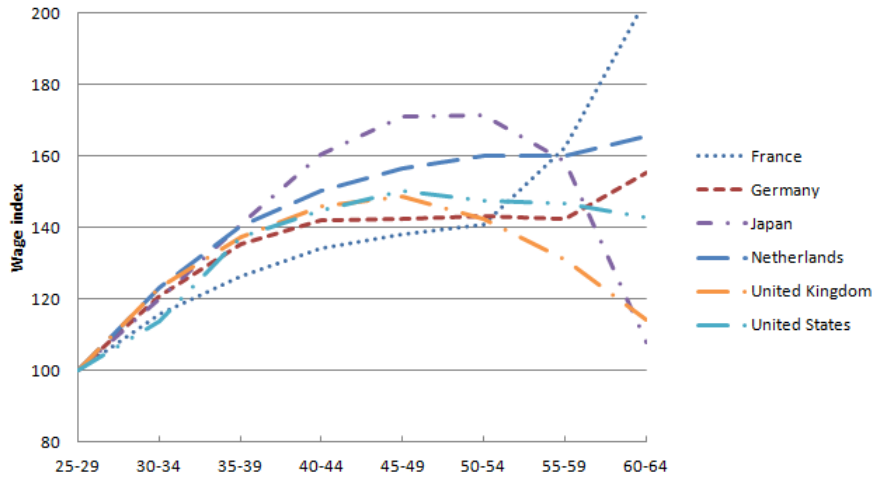
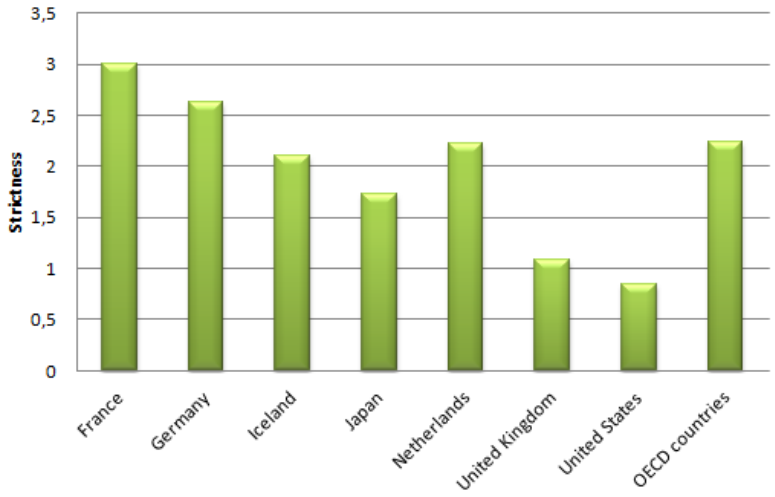



*Figure 7. The Strictness of overall employment protection (OECD, 2010), data 2008, ver. 3.*



# 3. Model

In this section we explain the model we use to simulate the effects of the introduction of retirement flexibility. We want to answer the question if the introduction of flexible retirement can contribute to the goal of preparing pension systems for decreasing old-age support ratios. In this section we discuss which assumptions have been made in the model and we discuss the intuition behind the model. We start with the basic model. In the basic model we include second period labour supply, life-expectancy and a wealth transfer. We extend the basic model with pensions. Pensions determine the size of the wealth transfer between the individuals in the model. We discuss different pension systems. This is because pension flexibility can be an important measure to facilitate flexible retirement and because adding a redistributive element to pensions can compensate individuals who do not gain from flexible pensions. In contrast to the previous section we assume that there is sufficient demand for old-age labour and that 'standard' retirement ages do not affect the retirement decision. We make these assumptions to keep the model simple. In Appendix A.1 we give an overview of the assumptions that have been made in the model. In the next section we give theoretical examples using the model.

## 3.1 Basic model

We consider a two-period overlapping generations model with heterogeneous agents that differ in life expectancy. In the first period of life, an individual spends his time working and decides how much to save for future consumption. In the second period of life, an individual decides how much time he will spend on working and on leisure. In case of fixed retirement the second period labour supply $h_i$ will be given exogenously. In case of flexible retirement individuals are able to choose their second period labour supply individually. We extend the model with pensions. In order to keep the model simple, we assume that pension contributions only have to be paid in the first period of life and pension benefits can only be received in the second period of life. Lifetime pension benefits depend on the life expectancy of individuals. A high life expectancy makes it more likely for an individual to have a higher lifetime income due to pensions, as a long living individual will receive pension income during a longer period.

*Preferences*

The preferences of individuals are represented by the following utility function:

$$U_i = \ln C_{1,i} + z_i \ln \frac{C_{2,i}}{z_i} + \gamma_i \ln(z_i - h_i) \quad (1)$$

The individual is denoted by $i$. First period consumption is $C_{1,i}$ and second-period consumption is $C_{2,i}$. The relative value of leisure compared to consumption is represented by $\gamma_i$. The duration of the second period of life is $z_i$. Finally the amount of leisure consumption is $z_i - h_i$. Leisure consumption is a function of how much is worked in the second period of life $(h_i)$ and duration of the second period of life $(z_i)$. For $z_i$ and $h_i$ must hold: $0 \le z_i \le 1$, $0 \le h_i \le 1$ and $h_i \le z_i$.

In order to let people make interesting retirement decisions, we had to extend the 'standard' utility function in which people only care about consumption. In our utility function individuals care about leisure as well as consumption. We assume that marginal utility of leisure depends on the duration after retirement rather than the duration before retirement. With this assumption the utility from leisure is based on the things individuals want to do during their retirement. However it ignores how the duration of an individual's career can affect his valuation of leisure, for example when an individual has to put in more effort to stay productive at old-age. When marginal utility from leisure depends on the duration after retirement, long living individuals will more likely prefer to work longer than short living individuals because they would still have a sufficient period to enjoy retirement. Cremer and Pestieau (2003) include increasing disutility of labour supply in the utility function, which is based on the duration of an individuals career. In reality the marginal utility from leisure will probably depend on the duration before retirement as well as the duration after retirement. However to include both durations would make the model more difficult. We have chosen to base marginal utility from leisure on the duration after retirement because a lot of young individuals say that they want to retire before the standard retirement age. Young individuals will not know how much effort it will take to work at old-age but can have an idea of what they want to do when they are retired. Young individuals preferring a low retirement age is an indication that the duration of retirement is important for the marginal utility of leisure. In a questionnaire in the Netherlands by Koppes et al. (2010), twenty percent of respondents aged 15-24 responded that they do not want to work until age 65.

*Life expectancy*

In our model we have heterogeneous agents that differ in life expectancy. The duration of the second period of life can differ between individuals. Individuals live for a period $z_i$ in the second period of their lives. The duration of the first period of life is the same for all individuals. In this chapter we discuss the pension systems in case of two participants. The pension systems have one short living participant $(i = S)$ and one long living participant $(i = L)$. The duration of the second period of life is longer for the long living individual $(z_L \geq z_S)$. In Section 4 we will discuss examples with pension systems with two or three participants. When we discuss pension systems with three participants there will be an additional individual $(i = M)$ with a medium long life expectancy $(z_L \geq z_M \geq z_S)$. We have chosen to model life expectancy as life duration in stead of the probability of reaching the second period of life. When life duration is used in stead of survival probability this results in a different valuation of leisure. Because we use life duration, a higher life expectancy results in a longer period after the 'standard' retirement age. Assuming decreasing marginal utility of leisure the preferred retirement age depends on the life expectancy of an individual. We have chosen to use life duration to include this effect.

*Consumption*

We assume a small open economy, this means that wage $(w_i)$ and the interest rate $(r)$ are exogenously given. First period wage is $w_{1,i}$ and second period wage is $w_{2,i}$. To keep the model simple, we assume that wages are constant over time $(w_{1,i} = w_{2,i} = w_i)$. Individuals can differ in wage because individuals can differ in productivity. By allowing differences in wages we can examine situations where life expectancy and wage are related. Second period labour supply $(h_i)$ in case of fixed retirement is exogenously given and is the same for all individuals. Second period labour supply in case of flexible retirement is determined endogenously and can differ between individuals. Apart from wage income, lifetime income is affected by the wealth transfer gain from pensions $(T_i)$. The individual's budget constraint is represented by equation (2). The individual's lifetime income is represented by equation (3). We maximise the utility function to get first and second period consumption. First period consumption can be found in equation (4) and second period consumption can be found in equation (5).

$$C_{1,i} + \frac{1}{1+r} C_{2,i} = w_{1,i} + \frac{h_i}{1+r} w_{2,i} + T_i \tag{2}$$

$$I_i = w_i + \frac{h_i}{1+r} w_{2,i} + T_i \tag{3}$$

$$C_{1,i} = \frac{1}{1+z_i} I_i \tag{4}$$

$$C_{2,i} = (1+r)\frac{z_i}{1+z_i} I_i \tag{5}$$

*Second period labour supply*

Second period labour supply in case of fixed retirement is exogenously given and is equal for all individuals. Second period labour supply in case of flexible retirement is determined endogenously and can differ between individuals. To solve for second period labour supply we substitute the optimal consumption equations (4) and (5) into the utility function (1). The calculation of the second period labour supply function can be found in Appendix A.2. The second period labour supply function is:

$$h_i = \frac{1+z_i}{1+z_i+\gamma_i} z_i - \frac{\gamma_i}{1+z_i+\gamma_i} \frac{(1+r)(w_{1,i}+T_i)}{w_{2,i}} \tag{6}$$

Second period labour supply is mainly affected by the value individuals attach to leisure $(\gamma_i)$, when individuals attach more value to leisure they will retire more early. If individuals do not care about leisure they will work their entire lives. Second period labour supply is also affected by other variables when individuals care about leisure and income. The welfare gain from having additional income is dependent on the percentage increase of lifetime income, because of the logarithmic utility function. The percentage increase of lifetime income due to working in the second period of life depends on relative value of second period labour income. We find that second period labour supply is positively affected by the value of the time discounted value of second period wage $\left(w_{2,i}(1+r)^{-1}\right)$ compared to the value of income unaffected by second period labour supply $(w_{1,i}+T_i)$. In Appendix A.2 we show the partial derivatives of equation (6) with respect to variables that affect the ratio of income affected by second period labour supply and income unaffected by second period labour supply. When wages are constant over time we find that a higher wage reduces the affect the wealth transfer benefit has on second period labour supply. With higher wages the size of the wealth transfer benefit is lower compared to the time discounted value of second period wage, this has a positive (negative) effect on second period labour supply when the wealth transfer is positive

(negative). Second period labour supply is unaffected by the height of wages when wages are constant over time and there is no wealth transfer. When wages are increasing over time the relative value of second period wage is higher, this has a positive effect on second period labour supply. The interest rate has a negative effect on second period labour supply because it would reduce the time-discounted value of second period wage. The wealth transfer has a negative effect on second period labour supply because it increases income unaffected by second period labour supply. Finally we find that an individual's life expectancy can have a large affect on his second period labour supply. How large this effect is mainly depends on his valuation of leisure. When individuals do not care about leisure than second period labour supply will not be affected by life expectancy. It is possible that the preferred retirement duration shortens due to a higher life expectancy. In Appendix A.2 we give the partial derivative of equation (6) with respect to life expectancy.

## 3.2 Pensions

In this section we discuss different pension systems in case of two participants. Pension contributions have to be paid in the first period of life. Pension benefits are received in the second period of life. Lifetime pension benefits depend on the life expectancy of individuals. In our analysis we assume that the level of pension benefits is constant. For example when life expectancy increases this will increase the pension contributions but the level of pension benefits remains the same. As a result lifetime pension benefits increase due to the higher life expectancy. The moment of pension take-up does not have to be the moment of retirement in case of flexible retirement. Because pension benefits and pension contributions are independent from second period labour supply, pensions systems can only lead to a wealth benefit/loss from pensions. The size of the wealth transfer can differ between pension systems. We assume that pension funds or the government can not use the life expectancy of an individual to adjust pension benefits or contributions. Because lifetime pension benefits depend on life expectancy there is redistribution from short living to long living individuals due to pensions. Further we assume that the return on pension contributions equals the interest rate. In case of PAYG pension systems the population growth equals the interest rate. Finally we assume that all individuals reach the standard retirement age.

*Fixed pensions (lump-sum contribution)*

Individuals pay a fixed amount of pension contribution $(\tau_C)$ in the first period of life. In the second period of life individuals receive pension benefits $(\tau_B)$ for the duration of life after the

pension age $(z_i - \phi)$. The pension age is $\phi$. The pension system is balanced when equation (7) holds. Equation (8) gives the wealth transfer through the pensions system. The calculation of the wealth transfer can be found in Appendix A.3.1.

$$(1+r)2\tau_C = \big((z_S - \phi) + (z_L - \phi)\big)\tau_B \tag{7}$$

$$T_L = -T_S = \frac{1}{2}(z_L - z_S)\tau_B(1+r)^{-1} \tag{8}$$

The wealth transfer through pensions is positive for the long living individual when the individuals differ in life expectancy. In the analysis of equation (6) we have discussed the effect of a wealth transfer on second period labour supply. A positive wealth transfer would reduce second period labour supply and high wages reduce this effect. In case of fixed pensions the size of the wealth transfer depends on the difference in life expectancy, the height of the pension benefits and the interest rate. The difference in life expectancy affects the wealth transfer because it determines the duration that the long living individual receives pension benefits while the short living individual does not. A larger difference in life expectancy results in a larger wealth transfer. The interest rate affects the wealth transfer because we assume that the pension benefits level is constant. A high interest rate would therefore reduce the time discounted value of pension benefits and reduce the wealth transfer. If we had assumed that pension contributions are constant than the interest rate would not have affected the wealth transfer. When the pension benefit level is lowered this also has a negative effect on the wealth transfer. The pension age does not have an effect on the wealth transfer because we have assumed that all individuals reach the pension age. Without this assumption a high pension age would reduce the duration that the long living individual receives pension benefits while the short living individual does not. This reduces the wealth transfer.

*Flexible pensions (lump-sum contribution)*

Individuals pay a fixed amount of pension contribution $(\tau_C)$ in the first period of life. In the second period of life individuals can choose when they want to start to receive pension benefits. Individuals receive higher pension benefits for a shorter duration when they choose to start receiving pension benefits after the standard retirement age. Individuals receive lower pension benefits for a longer duration when they choose to start receiving pension benefits after the standard retirement age. How much altering the moment of pension take-up changes pension benefits is denoted by $\theta$ . The deviation from the standard retirement age is $\alpha_i$ . The

lifetime pension benefit is given in equation (9). Individuals will choose how much they deviate from the standard retirement age by maximising their benefit from the pension system. We assume that pension benefits can not be received before the start of the second period of life $(\alpha_i \geq -\phi)$. The deviation of the moment of pension take-up from the standard retirement age is given in equation (10). Equation (10) is the result of maximising lifetime pension benefits. The calculation of equation (10) can be found in Appendix A.3.2.

$$P_{Benefit\_i} = (z_i - \phi - \alpha_i)\tau_B(1 + \theta\alpha_i) \tag{9}$$

$$\alpha_i = \frac{1}{2}z_i - \frac{1}{2}\theta^{-1} - \frac{1}{2}\phi \tag{10}$$

When maximising lifetime pension benefits this results in three possible outcomes because individuals are unable to receive pension benefits in the first period of life. Both individuals start collecting pension benefits as soon as possible, only the short living individual starts collecting pension benefits as soon as possible or both individuals start collecting pension benefits after the start of the second period of life. Each outcome results in a different wealth transfer equations. The second and third outcomes are the most relevant. Under the first outcome the pension system is not flexible. Under the second outcome pensions are flexible but the short living individuals prefers to receive pension benefits more early than is allowed. Perhaps early pension take-up is only possible after a reaching a certain age. Under the third outcome pensions are completely flexible. The pension system is balanced if equation (11) holds. The wealth transfer when both individuals start collecting pension benefits after the start of the second period of life is given in equation (12). Wealth transfer functions and calculations of the three possible outcomes can be found in Appendix A.3.2.

$$(1+r)2\tau_C = (z_S - \phi - \alpha_S)\tau_B(1 + \theta\alpha_S) + (z_L - \phi - \alpha_L)\tau_B(1 + \theta\alpha_L) \tag{11}$$

$$T_L = -T_S = \left(\frac{1}{4}(1 - \theta\phi)(z_L - z_S) + \frac{1}{8}\left(z_L{}^2 - z_S{}^2\right)\theta\right)\frac{\tau_B}{1+r} \tag{12}$$

The wealth transfer through pensions is positive for the long living individual when the individuals differ in life expectancy. The size of the wealth transfer depends on the height of the pension benefits and the interest rate in the same way as with fixed pensions. There are two differences with fixed pensions. The first difference is that when one or more individuals postpone pension benefits the size of the wealth transfer will depend exponentially on the difference in life expectancy. The second difference is that the size of the wealth transfer depends on the pension age. A higher pension age would reduce the effective pension benefit

level. The wealth transfer can decrease when pensions are made flexible. When there are two individuals, the pensions can be made flexible without changing the wealth transfer.

*Proportional pension contributions*

Because pensions play an important role in providing a minimum income for individuals, they often include redistribution from individuals with high wages to individuals with low wages. We will now discuss pensions with pension contributions that are proportional to the wages that include this redistribution effect. Pension benefits are not contribution based. When both individuals in our model have the same income, pensions with proportional pension contributions will not differ from pensions with lump-sum pension contributions. Note that there are now two forms of redistribution active. Redistribution from short-living individuals to long-living individuals and redistribution from rich individuals to poor individuals. When wealth and life-expectancy are correlated pension systems with proportional contributions can result in a lower wealth transfer than would be the case with pension systems with lump-sum contributions.

*Fixed pensions (proportional contribution)*

Individuals pay the pension contribution rate over their income $(w_i \tau_{CR})$ in the first period of life. In the second period of life individuals receive pension benefits $(\tau_B)$ for the duration of life after the retirement age $(z_i - \phi)$. The pension system is balanced when equation (13) holds. Equation (14) gives the wealth transfer through the pensions system. The calculation of the wealth transfer can be found in Appendix A.3.3.

$$(1+r)\tau_{CR}(w_S + w_L) = \big((z_S - \phi) + (z_L - \phi)\big)\tau_B \tag{13}$$

$$T_L = -T_S = \big(w_S(z_L - \phi) - w_L(z_S - \phi)\big)\frac{1}{w_S + w_L}\frac{\tau_B}{1+r} \tag{14}$$

Compared to the fixed pensions with lump-sum contributions the wealth transfer also depends on the wages of both individuals and on the retirement age. We have previously mentioned that there are two kinds of redistribution active in this pension system. When the relative share of pension benefits of an individual is larger than his relative share of pension contributions his wealth transfer benefit will be positive. In other words, when the relative wage of an individual is lower than the relative retirement duration compared to the other individual his wealth transfer benefit is positive. The wealth transfer can be in favour of the long living individual as well as the short living individual. A higher retirement age has a

positive effect on the wealth transfer for the individual with the highest income. This result only holds when both individuals reach the retirement age. How much the wealth transfer changes due to retirement age depends on the relative difference in wage between the individuals.

*Flexible pensions (proportional contribution)*

Individuals pay the pension contribution rate over their income $(w_i \tau_{CR})$ in the first period of life. In the second period of life individuals can choose when they want to start to receive pension benefits. The pension system is balanced when equation (15) holds. Pension benefits are maximised in the same way as with flexible pensions with lump-sum contributions. The deviation of the moment of pension take-up from the standard retirement age is given in equation (10).

$$(1+r)\tau_{CR}(w_S + w_L) = (z_S - \phi - \alpha_S)\tau_B(1+\theta\alpha_S) + (z_L - \phi - \alpha_L)\tau_B(1+\theta\alpha_L) \qquad (15)$$

As with flexible pensions with lump-sum contributions this results in three possible outcomes. The first outcome is that both individuals will start collecting pension benefits as soon as possible. The second outcome is that only the short living individual will start collecting pension benefits as soon as possible. The third outcome is that both individuals will start collecting pension benefits after the start of the second period of life. The wealth transfer when both individuals start collecting pension benefits after the start of the second period of life is given in equations (16) and (17). Wealth transfer functions and calculations of the three possible outcomes can be found in Appendix A.3.4.

$$P_{Benefit\_i} = \frac{1}{4}\theta^{-1}\tau_B + \frac{1}{4}z_i{}^2\theta\tau_B - \frac{1}{2}\theta\phi z_i\tau_B + \frac{1}{4}\theta\phi^2\tau_B + \frac{1}{2}z_i\tau_B - \frac{1}{2}\phi\tau_B \qquad (16)$$

$$T_L = \frac{w_S}{w_S + w_L}\frac{1}{1+r}P_{Benefit\_L} - \frac{w_L}{w_S + w_L}\frac{1}{1+r}P_{Benefit\_S} \qquad (17)$$

This pensions system is comparable to flexible pensions with lump-sum contribution with the exception that pension contributions are now proportional to wages. As a result the wealth transfer also depends on the wages of both individuals. Individuals with high wages will have to contribute more to the pension system than with flexible pensions with lump-sum contributions. When the relative wage of an individual is lower than the relative lifetime pension benefits compared to the other individual the wealth transfer is positive. The wealth transfer can be in favour of the long living individual as well as the short living individual.

In this section we found that second period labour supply depends mainly on the preference of leisure. When retirement flexibility is introduced it depends on the value individuals attach to leisure whether or not flexible retirement can contribute to the goal of preparing pension systems for the upcoming ageing. In the model we have included pensions. In the Introduction we have mentioned flexible pensions as an important step in making flexible retirement possible. By comparing fixed pension systems with flexible pension systems we see how the introduction of flexible pensions will affect the wealth transfer income of individuals. We found that the introduction of flexible pensions can change the wealth transfer in favour of the short or long living individual. Who will benefit from the introduction of flexible pensions is dependent on the how much pension benefits are adjusted with early or late pension take-up. When incentives are set to stimulate late pension take-up it is likely that the wealth transfer changes in favour of the long living individual. In the next section we will use the model of this section to give theoretical examples of the introduction of flexible retirement in combination with flexible pensions. Because it is likely that incentives will be set to stimulate late pension take-up, we consider making pensions more redistribute from rich to poor individuals to compensate short living individuals.

# 4. Results

In this section we give will give theoretical examples based on our model. With these examples we want to answer the question if the introduction of flexible retirement can contribute to the goal of preparing pension systems for decreasing old-age support ratios. In total we give four theoretical examples. In the first two examples we make retirement completely flexible. People are able to reduce their second period labour supply and are able to increase their second period labour supply. Because individuals are able to reduce their second period labour supply this will not necessarily lead to a larger labour force. A larger workforce would help finance pensions because governments would have a larger tax base. When the goal is to increase labour supply it helps if individuals are not able to retire early. In the last two examples we make retirement partial flexible. People are not able to reduce their second period labour supply but are able to increase their second period labour supply. Making retirement partial flexible is possibly a better measure to increase second period labour supply. In the introduction we have discussed the pension systems of Finland and Sweden because of their flexible pension systems. However both countries have an age restriction for early pension take-up. In the examples we discuss the pension situations for pension systems with two or three participants. The calculation of the wealth transfer through the pension system with three participants can be found in Appendix A.4.

## 4.1 Pension situations

We define a pension situation as the combination of a pension system and the possibility for individuals to choose their own second period labour supply. With the model from Section 3 we can discuss ten pension situations. The results for all ten pension situations can be found in the tables in Appendix A.5. In this section we analyse the change from pension situations with exogenous second period labour supply to pension situations with endogenous second period labour supply. The situations with exogenous labour supply are only discussed in combination with fixed pensions. Flexible pensions would not make much sense when labour supply is fixed. The situations with endogenous labour supply are only discussed in combination with flexible pensions. Based on the literature on the retirement decision of Section 2, we conclude that individuals are affected by social norms, the usual retirement age and lack of financial knowledge. Flexible pensions would reduce these effects on the retirement decision and would make retirement more flexible. The financial consequences of the retirement decision are easier to understand when pensions are flexible and this makes it less attractive to stick with a default option. Flexible pensions would also reduce the effect of the ‘standard’

retirement age when less people retire at the 'standard' retirement age. When less people retire at the 'standard' retirement age it is likely that deviations from the 'standard' retirement age become more acceptable to society.

## 4.2 Setting parameter values examples

In the examples we use parameter values that are able to show differences between the different pension systems and are close to reality. In Section 3 we discussed different pension systems. The results of the discussed pension systems of Section 3 only differ from each other when there are differences between the participating individuals. In all our examples individuals differ in life expectancy and income. We discussed flexible pensions because this is possibly an important measure to facilitate flexible retirement. The effect the introduction of flexible pensions on lifetime income depends on the life expectancy of an individual. When individuals differ in life expectancy this gives more interesting results. We discussed the possibility of proportional pension contributions because some pension systems include wealth redistribution from rich to poor participants. For wealth redistribution to exist individuals will have to differ in income.

The specific parameters values will differ between examples of partial flexible retirement and flexible retirement. The basic assumptions hold for all examples. We start with the simulated ages. We assume that the first period of life starts at age 20. We set the pension age equal to 65 years. The life expectancy of the long living individual is 80 years, the life expectancy of the medium long living individual is 76,5 years and the life expectancy of the short living individuals is 75 years. The preference of leisure of the individuals is chosen to fit the pension age. Because pension age is also the mandatory retirement age under fixed retirement, the pension age should match the preferences of individuals. For example when all individuals prefer to work longer than the mandatory retirement age, the mandatory retirement age may be unrealistically low. When the entire population prefers a higher mandatory retirement age it is likely that it will be changed. An unrealistically low mandatory retirement age will exaggerate the benefits from flexible retirement. In our examples we include a positive correlation between life-expectancy and income. This is based on the article by Cutler et al. (2006) previously mentioned in Section 2.

There are some assumptions specific to flexible pensions. We had to determine the incentive for postponing pension benefits. Preferably we wanted the incentive for postponing pension benefits to be at a level where the moment of pension take-up of each individual is close to the moment of retirement. When we introduce pension flexibility we do not adjust the height of pension benefits. This means that when the moment of pension take-up is not altered under

flexible pensions the received pension benefits are equal to the received benefits under fixed pensions. In our model it is likely that pension contributions increase when pension flexibility is introduced. The first generation of individuals that is able to chose their moment of pension take-up has not paid higher pension contributions when the pension reform is unexpected. We did not discuss the transition generation in this paper to prevent cluttered results.

## 4.3 Examples

In this subsection we will give the results of four theoretical examples based on the model of Section 3. In Table 1 we give the variables of the model. We do not discuss all output variables in this section. In the tables in Appendix A.5 the results of all output variables are given.

*Table 1. Variable definitions*

| **Input variables** | | **Output variables** | |
|---|---|---|---|
| $w_i =$ | wage individual $i$ | $T_i =$ | wealth transfer benefit individual $i$ |
| $r =$ | interest rate | $h_i =$ | second period labour supply individual $i$ |
| $\gamma_i =$ | preference for leisure individual $i$ | $I_i =$ | lifetime income individual $i$ |
| $z_i =$ | life expectancy individual $i$ | $U_i =$ | utility individual $i$ |
| $\tau_B =$ | pension benefit | $\alpha_i =$ | deviation from retirement age individual $i$ |
| $\theta =$ | incentive for postponing benefits | $TI =$ | lifetime income all individuals |
| $\phi =$ | 'Standard' retirement age | $P_{ben\_i} =$ | lifetime pension income individual $i$ |

### 4.3.1. First flexible retirement example

In the first example we compare fixed retirement with flexible retirement. By flexible retirement we mean that individuals are able to increase their second period labour supply and individuals are able to reduce their second period labour supply. In the first example we keep the differences between the individuals small. We do this by assuming that all individuals have the same preference of leisure. Because all individuals have the same preference for leisure they do not differ much in their preferred retirement age. In the second example we change the preference of leisure of the long living individual and alter the incentive for postponing benefits. The complete tables of Example 1 can be found in Appendix A.5 (Tables 6-8).

*Table 2. Results first example with flexible retirement*

| Variables | Inputs | **2 Participants** | Fixed retirement Fixed pension | Fixed retirement Fixed pension (Proportional) | Flexible retirement Flexible pension | Flexible retirement Flexible pension (Proportional) | **3 Participants** | Fixed retirement Fixed pension | Fixed retirement Fixed pension (Proportional) | Flexible retirement Flexible pension | Flexible retirement Flexible pension (Proportional) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $w_S$ | 1,00 | $T_S$ | -0,0222 | 0,0148 | -0,0222 | 0,0152 | $T_S$ | -0,0193 | 0,0126 | -0,0190 | 0,0131 |
| $w_M$ | 1,25 | $T_L$ | 0,0222 | -0,0148 | 0,0222 | -0,0152 | $T_M$ | -0,0059 | 0,0068 | -0,0066 | 0,0063 |
| $w_L$ | 2,00 | $h_S$ | 0,5000 | 0,5000 | 0,4214 | 0,4113 | $T_L$ | 0,0252 | -0,0194 | 0,0255 | -0,0194 |
| $R$ | 0,50 | $h_L$ | 0,5000 | 0,5000 | 0,5806 | 0,5852 | $h_S$ | 0,5000 | 0,5000 | 0,4205 | 0,4119 |
| $\gamma_S$ | 0,40 | $U_S$ | -0,7162 | -0,6651 | -0,7063 | -0,6525 | $h_M$ | 0,5000 | 0,5000 | 0,4672 | 0,4645 |
| $\gamma_M$ | 0,40 | $U_L$ | 0,7202 | 0,6924 | 0,7282 | 0,7013 | $h_L$ | 0,5000 | 0,5000 | 0,5802 | 0,5858 |
| $\gamma_L$ | 0,40 | $\alpha_S$ | 0,0000 | 0,0000 | -0,0417 | -0,0417 | $U_S$ | -0,7121 | -0,6682 | -0,7015 | -0,6554 |
| $z_S$ | 0,83 | $\alpha_L$ | 0,0000 | 0,0000 | 0,0417 | 0,0417 | $U_M$ | -0,2623 | -0,2479 | -0,2614 | -0,2467 |
| $z_M$ | 0,88 | | | | | | $U_L$ | 0,7224 | 0,6890 | 0,7306 | 0,6983 |
| $z_L$ | 1,00 | | | | | | $\alpha_S$ | 0,0000 | 0,0000 | -0,0417 | -0,0417 |
| $\tau_B$ | 0,40 | | | | | | $\alpha_M$ | 0,0000 | 0,0000 | -0,0167 | -0,0167 |
| $\theta$ | 2,40 | | | | | | $\alpha_L$ | 0,0000 | 0,0000 | 0,0417 | 0,0417 |
| $\phi$ | 0,50 | | | | | | | | | | |

In the first two examples we compare fixed retirement with flexible retirement. We will first discuss the example in general and we will than discuss possible pension reforms. The main result of this example is that the preferred retirement age depends on the life expectancy of the individuals. The difference in the preferred retirement age without pensions is 0.1677 between the short living individuals and the long living individual (see Table 6 in Appendix A.5), this is approximately 5 years[4]. The difference in the preferred retirement age between individuals strongly depends on the life expectancy because of the effect the life expectancy has on the marginal utility of leisure at a certain age. The life expectancy also has a positive effect on the wealth transfer benefit of an individual. This is because of the longer duration of retirement. Pension systems with proportional pension contributions include redistribution from rich to poor. The redistribution from rich to poor is larger than the redistribution from short living individuals to long living individuals because the differences in wage are large compared to the differences in retirement duration. In this example flexible retirement has a positive effect on total income. However whether or not this result holds depends on the chosen mandatory retirement age compared to the preference of leisure of the individuals.

[4] The difference in preferred retirement age is 0.1677 of a period of 30 years, this equals 0.1677*30=5.031 years.

We start with discussing the transition from fixed retirement with fixed pensions to flexible retirement with flexible pensions. With two participants flexible pensions do not change the wealth transfer. As a result both individuals are able to make a better retirement decision (for themselves) than before without the introduction of flexible pensions having a negative effect on their wealth transfer benefit. Both individuals gain from the pension reform. In case of three participants the wealth transfer is affected by the introduction of flexible pensions. The medium long living individual's wealth transfer benefit will decrease when flexible pensions are introduced because his life expectancy is close to the average life expectancy. In the Introduction we have discussed that the selection problem that comes with flexible pensions can have negative outcomes for medium long living individuals. Both the short living individual and the long living individual benefit from a higher wealth transfer benefit when flexible pensions are introduced. They also benefit from making a better retirement decision so their utility increases due the pension reform. The medium long living individual has a lower wealth transfer benefit which has a negative effect on his utility but he can also make a better retirement age decision. The net effect on utility of the pension reform is also positive for the medium long living individual in this example, however it is only a small improvement. The transition from fixed retirement with fixed pension to flexible retirement with flexible pension when pension contributions are proportional to wage is very similar to the previous transition. Compared to the previous transition introducing flexible pensions is now slightly more attractive for individuals with a low wage and slightly less attractive for individuals with a high wage. All individuals still benefit form the pension reform.

### 4.3.2. Second flexible retirement example

In the second example we again compare fixed retirement with flexible retirement. Compared to the first example we include more differences between the individuals. The individuals no longer have the same preference of leisure. Compared to the previous example the wealthier long living individual has a lower preference for leisure. Wealthier individuals can have a lower preference for leisure when their jobs are more fun or are more suitable for old-age workers. Because the preferred retirement age of the long living individual has changed compared to the previous example we have also altered the incentive for postponing retirement. The results of the second example can be found in Table 3. The complete tables of Example 2 can be found in Appendix A.5 (Tables 9-11).

*Table 3. Results second example with flexible retirement*

| Variables | Inputs | **2 Participants** | Fixed retirement Fixed pension | Fixed retirement Fixed pension (Proportional) | Flexible retirement Flexible pension | Flexible retirement Flexible pension (Proportional) | **3 Participants** | Fixed retirement Fixed pension | Fixed retirement Fixed pension (Proportional) | Flexible retirement Flexible pension | Flexible retirement Flexible pension (Proportional) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $w_S$ | 1,00 | $T_S$ | -0,0222 | 0,0148 | -0,0250 | 0,0130 | $T_S$ | -0,0193 | 0,0125 | -0,0213 | 0,0111 |
| $w_M$ | 1,25 | $T_L$ | 0,0222 | -0,0148 | 0,0250 | -0,0130 | $T_M$ | -0,0059 | 0,0068 | -0,0074 | 0,0055 |
| $w_L$ | 2,00 | $h_S$ | 0,5000 | 0,5000 | 0,4221 | 0,4119 | $T_L$ | 0,0252 | -0,0193 | 0,0287 | -0,0166 |
| $R$ | 0,50 | $h_L$ | 0,5000 | 0,5000 | 0,6715 | 0,6752 | $h_S$ | 0,5000 | 0,5000 | 0,4211 | 0,4124 |
| $\gamma_S$ | 0,40 | $U_S$ | -0,7162 | -0,6651 | -0,7103 | -0,6556 | $h_M$ | 0,5000 | 0,5000 | 0,4674 | 0,4647 |
| $\gamma_M$ | 0,40 | $U_L$ | 0,7895 | 0,7617 | 0,8286 | 0,8025 | $h_L$ | 0,5000 | 0,5000 | 0,6711 | 0,6755 |
| $\gamma_L$ | 0,30 | $\alpha_S$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 | $U_S$ | -0,7121 | -0,6682 | -0,7049 | -0,6582 |
| $z_S$ | 0,83 | $\alpha_L$ | 0,0000 | 0,0000 | 0,0833 | 0,0833 | $U_M$ | -0,2623 | -0,2479 | -0,2624 | -0,2475 |
| $z_M$ | 0,88 | | | | | | $U_L$ | 0,7917 | 0,7583 | 0,8311 | 0,7999 |
| $z_L$ | 1,00 | | | | | | $\alpha_S$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 |
| $\tau_B$ | 0,40 | | | | | | $\alpha_M$ | 0,0000 | 0,0000 | 0,0250 | 0,0250 |
| $\theta$ | 3,00 | | | | | | $\alpha_L$ | 0,0000 | 0,0000 | 0,0833 | 0,0833 |
| $\phi$ | 0,50 | | | | | | | | | | |

The second example is very similar to the first example so we will focus on the differences between the two examples. We will first discuss the general differences of this example compared to the previous example. We will then continue with discussing possible pension reforms. There are two important differences with the previous example. The lower preference of leisure of the long living individual has resulted in a larger difference in the preferred retirement age. The difference between the short living individuals and the long living individual in their preferred retirement age without pensions has increased to 0.2585 (see Table 9 in Appendix A.5), this is approximately 7,8 years[5]. The other important difference is that the change to flexible pensions is much more in favour of long living individuals. The short living individual no longer wishes to alter his moment of pension take-up $(\alpha_S = 0)$ and as a result he can no longer increase his lifetime pension benefit. Compared to the previous example the pension reform has a more positive effect on total income because of the higher preferred retirement age of the long living individual. Other effects discussed in the previous example still hold.

[5] The difference in preferred retirement age is 0.2585 of a period of 30 years, this equals 0.2585*30=7.755 years.

We start with the transition from fixed retirement with fixed pensions to flexible retirement with flexible pensions. Different from the previous example is that the wealth transfer always changes in favour of the long living individual when flexible pensions are introduced. In case of two participants the short living individual's wealth transfer benefit will decrease when flexible pensions are introduced. The short living individual has a lower wealth transfer benefit which has a negative effect on his utility but he can also make a better retirement age decision. The net effect on utility of the pension reform remains positive for the short living individual in this example, but it is small improvement. In case of three participants the wealth transfer benefit will decrease for the short living individual and the medium long living individuals when flexible pensions are introduced. For these two participants there is again a positive effect and a negative effect on utility because of the pension reform. For the short living individual the net effect of the pension reform on utility is positive. However the utility of the medium long living is negatively affected by the pension reform. The medium long living individual has less to gain from choosing his preferred retirement age because it is closer to the mandatory retirement age. If a Pareto improvement is necessary to make the pension reform possible it can be solution to make pension contributions proportional to wage after the pension reform. When pension contribution are made proportional to wage after the pension reform the wealth transfer changes in favour of the short living and medium long living individuals. The long living individual will have a much lower wealth transfer benefit after such a reform which has a negative effect on his utility but he can also make a better retirement age decision. The long living individual gains a lot from choosing his own retirement age because his preferred retirement age differs a lot from the mandatory retirement age. The effect on utility of this pension reform is positive for all individuals. Finally we discuss the transition from fixed retirement with fixed pensions to flexible retirement with flexible pensions when contributions are proportional to wage. The results are similar to the transition from fixed pensions to flexible retirement with flexible pensions when contributions are lump-sum. If pension contributions are proportional to wage making pensions flexible has a less negative effect on the wealth transfer benefit of individuals with a low income. The wealth transfer benefit of the short living individual and the medium long living individual decreases less when flexible pensions are introduced. Due to the lower decrease of the wealth transfer benefit of the medium long living individual his utility increases because of the pension reform in this example and a Pareto improvement is possible.

### 4.3.3. First partial flexible retirement example

In the third example we compare fixed retirement with partial flexible retirement. By partial flexible retirement we mean that individuals are able to increase second period labour supply but individuals are not able to reduce second period labour supply. We have tried to keep the parameters close to the parameters of the first two examples. However to prevent early retirement we have had to alter the duration of the first period of life. As a result the example has changed too much to compare it directly to the previous examples. For example the wages and the interest rate are not corrected for the longer duration of the first period of life. In this example we again keep the differences between the individuals small. All individuals have the same preference of leisure. Because all individuals have the same preference for leisure they do not differ much in their preferred retirement age. In the fourth example we change the preference of leisure of the long living individual and alter the incentive for postponing benefits. The complete tables of Example 3 can be found in Appendix A.5 (Tables 12-14).

*Table 4. Results first example with partial flexible retirement*

| Variables | Inputs | **2 Participants** | Fixed retirement Fixed pension | Fixed retirement Fixed pension (Proportional) | Flexible retirement Flexible pension | Flexible retirement Flexible pension (Proportional) | **3 Participants** | Fixed retirement Fixed pension | Fixed retirement Fixed pension (Proportional) | Flexible retirement Flexible pension | Flexible retirement Flexible pension (Proportional) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $w_S$ | 1,000 | $T_S$ | -0,0148 | 0,0099 | -0,0157 | 0,0093 | $T_S$ | -0,0128 | 0,0084 | -0,0135 | 0,0079 |
| $w_M$ | 1,250 | $T_L$ | 0,0148 | -0,0099 | 0,0157 | -0,0093 | $T_M$ | -0,0040 | 0,0045 | -0,0046 | 0,0040 |
| $w_L$ | 2,000 | $h_S$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 | $T_L$ | 0,0168 | -0,0129 | 0,0180 | -0,0119 |
| $R$ | 0,500 | $h_L$ | 0,0000 | 0,0000 | 0,0420 | 0,0450 | $h_S$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 |
| $\gamma_S$ | 0,250 | $U_S$ | -0,5494 | -0,5192 | -0,5506 | -0,5199 | $h_M$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 |
| $\gamma_M$ | 0,250 | $U_L$ | 0,4110 | 0,3945 | 0,4144 | 0,3983 | $h_L$ | 0,0000 | 0,0000 | 0,0417 | 0,0453 |
| $\gamma_L$ | 0,250 | $\alpha_S$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 | $U_S$ | -0,5470 | -0,5210 | -0,5477 | -0,5215 |
| $z_S$ | 0,222 | $\alpha_L$ | 0,0000 | 0,0000 | 0,0417 | 0,0417 | $U_M$ | -0,2470 | -0,2385 | -0,2476 | -0,2390 |
| $z_M$ | 0,256 | | | | | | $U_L$ | 0,4123 | 0,3925 | 0,4159 | 0,3965 |
| $z_L$ | 0,333 | | | | | | $\alpha_S$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 |
| $\tau_B$ | 0,400 | | | | | | $\alpha_M$ | 0,0000 | 0,0000 | 0,0028 | 0,0028 |
| $\theta$ | 4,000 | | | | | | $\alpha_L$ | 0,0000 | 0,0000 | 0,0417 | 0,0417 |
| $\phi$ | 0,000 | | | | | | | | | | |

Compared to the previous examples this example is easier to interpret. We will only discuss the general results of this example. The main result of this example is that only the long living individual changes his retirement age. Most likely the short living individual and the medium

long living individual prefer to work less but are not allowed to. Without pensions the long living individual prefers to increase his labour supply by 0.0439 (see Table 12 in Appendix A.5), this is approximately 2 years[6]. Another important result is that the short living individual and the medium long living individual make no significant changes to their moment of pension take-up. It is not allowed to collect pension benefits before the start of the second period of life. When only the long living individual is able to increase his lifetime pension benefit the wealth transfer benefit of the short and medium long living individuals will decrease when flexible pensions are introduced. The net effect of introducing flexible retirement in combination with flexible pensions on the utility of the short and medium long living individual is clearly negative. They do not gain utility from making a better retirement age decision and they lose utility because of a lower wealth transfer benefit. When a Pareto improvement is necessary to make the pension reform possible it is necessary to compensate the short and medium long living individual. A possible solution is to make pensions more redistributive from rich to poor. Making pension contributions proportional to wage does not lead to Pareto improvement in this example because this would lower the utility of the long living individual, however making pension contributions partly proportional to wage can be solution.

### 4.3.4. Second partial flexible retirement example

In the fourth example we again compare fixed retirement with partial flexible retirement. Compared to the third example we include more differences between the individuals. The individuals no longer have the same preference of leisure. The long living individual has a lower preference for leisure. As a result the individuals differ more in their preferred retirement age. Because the preferred retirement age of the long living individual has changed compared to the previous example we have also altered the incentive for postponing retirement. The results of the fourth example can be found in Table 5. The complete tables of Example 4 can be found in Appendix A.5 (Tables 15-17).

[6] The preferred increase in retirement age is 0.0439 of a period of 45 years, this equals 0.0439*45=1.976 years.

*Table 5. Results second example with partial flexible retirement*

| Variables | Inputs | **2 Participants** | Fixed retirement Fixed pension | Fixed retirement Fixed pension (Proportional) | Flexible retirement Flexible pension | Flexible retirement Flexible pension (Proportional) | **3 Participants** | Fixed retirement Fixed pension | Fixed retirement Fixed pension (Proportional) | Flexible retirement Flexible pension | Flexible retirement Flexible pension (Proportional) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $w_S$ | 1,00 | $T_S$ | -0,0148 | 0,0099 | -0,0177 | 0,0080 | $T_S$ | -0,0128 | 0,0084 | -0,0150 | 0,0069 |
| $w_M$ | 1,25 | $T_L$ | 0,0148 | -0,0099 | 0,0177 | -0,0080 | $T_M$ | -0,0040 | 0,0045 | -0,0053 | 0,0035 |
| $w_L$ | 2,00 | $h_S$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 | $T_L$ | 0,0168 | -0,0129 | 0,0203 | -0,0103 |
| $R$ | 0,50 | $h_L$ | 0,0000 | 0,0000 | 0,1191 | 0,1213 | $h_S$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 |
| $\gamma_S$ | 0,25 | $U_S$ | -0,5494 | -0,5192 | -0,5530 | -0,5214 | $h_M$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 |
| $\gamma_M$ | 0,25 | $U_L$ | 0,4934 | 0,4769 | 0,5189 | 0,5031 | $h_L$ | 0,0000 | 0,0000 | 0,1189 | 0,1215 |
| $\gamma_L$ | 0,18 | $\alpha_S$ | 0,0000 | 0,0000 | 0,0111 | 0,0111 | $U_S$ | -0,5470 | -0,5210 | -0,5497 | -0,5228 |
| $z_S$ | 0,22 | $\alpha_L$ | 0,0000 | 0,0000 | 0,0667 | 0,0667 | $U_M$ | -0,2470 | -0,2385 | -0,2484 | -0,2396 |
| $z_M$ | 0,26 | | | | | | $U_L$ | 0,4947 | 0,4749 | 0,5205 | 0,5017 |
| $z_L$ | 0,33 | | | | | | $\alpha_S$ | 0,0000 | 0,0000 | 0,0111 | 0,0111 |
| $\tau_B$ | 0,40 | | | | | | $\alpha_M$ | 0,0000 | 0,0000 | 0,0278 | 0,0278 |
| $\theta$ | 5,00 | | | | | | $\alpha_L$ | 0,0000 | 0,0000 | 0,0667 | 0,0667 |
| $\phi$ | 0,00 | | | | | | | | | | |

The fourth example is very similar to the third example so we will focus on the differences between the two examples. Compared to previous example the long living individual prefers to work longer. Without pensions the long living individual prefers to increase his labour supply by 0.1206 (see Table 15 in Appendix A.5), this is approximately 5,4 years[7]. Another difference with the previous example is that it is more attractive to postpone pension benefits. In this example all individuals postpone pension benefits. The long living individual is still the only individual whose wealth transfer benefit increases due to flexible pensions, this increase is larger than in the previous example. The net effect of introducing flexible retirement in combination with flexible pensions on the utility of the short and medium long living individual remains negative. They do not gain utility from making a better retirement age decision and they lose utility because of a lower wealth transfer benefit. When a Pareto improvement is necessary to make the pension reform possible it is necessary to compensate the short and medium long living individual. A possible solution is to make pensions more

[7] The preferred increase in retirement age is 0.1206 of a period of 45 years, this equals 0.1206*45=5.427 years.

redistributive from rich to poor. Making pension contributions proportional to wage after the pension reform does lead to Pareto improvement in this example.

In this section we again found that the effect flexible retirement has on second period labour supply depends mainly on the preferences of individuals. It is difficult to determine what the preferences of individuals are and therefore it is hard to say if the introduction of flexible retirement can contribute to the goal of preparing pension systems for ageing. The examples did give an idea of the redistributive effects of making pensions flexible. In the first example making pensions flexible had a negative effect on the wealth transfer for the medium long living individual in favour of the short and long living individual. In the second example making pensions flexible had a negative effect on the wealth transfer for the short and medium long living individual in favour of the long living individual. In this example pension incentives were set to stimulate late pension take up, making the pension system more attractive for the long living individual. In this section we also gave two theoretical examples in which we introduced partial retirement flexibility. Individuals were able to postpone retirement but were not allowed to retire early. In this situation it is likely that some individuals prefer a higher retirement age and that second period labour supply increases. When second period labour supply increases due to partial flexible retirement it would help with financing pensions. Introducing flexible pensions had a negative effect on wealth transfer income of the short and medium long living individual in favour of the long living individual. Because both the short and medium long living individual did not benefit from making a better retirement decision their utility decreased due to the pension reform. When a pension reform includes making pensions more redistributive from high wage individuals to low wage individuals this can compensate individuals with a low life expectancy when there is a positive correlation between income and life expectancy. In Example 4 a Pareto improvement was only possible when the pension reform included introducing proportional pension contributions.

## 5. Conclusion

In this paper we have studied the effectiveness of flexible retirement as a measure to help finance pensions. Flexible retirement can be a measure to help finance pensions when flexible retirement results in a larger workforce. In the literature overview we discussed the effects that alter the retirement decision. This is important for understanding what measures have to be taken to make retirement flexible and for understanding how flexible retirement would affect labour supply. Using our model we have analysed the change from fixed retirement with fixed pensions to flexible retirement with flexible pensions. Our model includes heterogeneous agents that differ in life-expectancy, preference for leisure and earning capabilities.

In the literature overview we have discussed a number of effects that affect labour supply. Examples are social norms, default options and financial incentives. These effects on labour supply lose their strength when flexible retirement in combination with flexible pensions is introduced. It can take time before the introduction of flexible retirement in combination with flexible pensions alters retirement behaviour. Based on the literature it is unclear whether or not flexible retirement would lead to more available workers. Wadensjö (2006) suggests part time retirement could have a positive effect on labour supply. If implicit taxes on continued work are reduced this has a positive effect on labour supply as well. When the effect of an usual retirement age is reduced because of flexible pensions, this can cause workers to prefer a lower retirement age. Brown (2006) concludes that individuals who say that there is no usual retirement age for them retire more early on average. In the literature overview we have also discussed labour demand. In many countries old-age workers are not preferred by employers. Before flexible retirement will result in a larger workforce, the labour market for the elderly has to function properly.

In our model the effect of flexible retirement on labour supply depends on the mandatory retirement age compared to the retirement age preferred by individuals. We assume that the fixed retirement age is somewhere between the preferred retirement age of the individuals. It is likely that at least one of the individuals prefers to work longer under flexible retirement and at least one of the individuals prefers to work less under flexible retirement. The net effect of flexible retirement on labour supply remains unclear. Introducing flexible pensions changes the wealth transfer in favour of long living individuals when incentives are set on late pension take-up, because of this it is possible that short and medium long living individuals lose welfare due to more flexible retirement. The incentive for postponing pension take-up can be set to only cause a small change to the wealth transfer. In the theoretical examples based on

our model we also gave two examples in which we introduced partial flexible retirement. By partial flexible retirement we mean that individuals are able to increase labour supply but are not able to decrease labour supply. When partial flexible retirement is introduced it is likely that second period labour supply increases. The introduction of partial flexible retirement can however have negative welfare effects for some participants. The introduction of partial flexible retirement in combination with flexible pensions changes the wealth transfer in favour of long living individuals and individuals with a low life-expectancy or a high preference of leisure are unable to improve their utility by altering their labour supply. To compensate short living individuals for the loss in wealth transfer income some additional wealth redistribution is necessary for a Pareto improvement.

In this paper we wanted to answer the question if the introduction of flexible retirement can contribute to the goal of preparing pension systems for decreasing old-age support ratios. In our search we mainly looked at the effect the pension reform would have on labour supply. Based on the discussed literature and our model we were unable to determine whether or not the net effect is positive. There are to many effects on current retirement behaviour to determine how much individuals value leisure (retirement). Therefore we do not know if individuals on average want to work longer and if flexible retirement will help finance pensions. Our model did suggest that introducing partial flexible retirement increases second period labour supply. In the literature overview we also discussed that at least in some countries labour demand for old-age workers is low. When the labour market does not function properly it is unlikely that flexible retirement will lead to a larger workforce. When the labour market for elderly functions properly than partial flexible retirement will help finance pensions.

Our paper can benefit from a number of extensions. Using our model we are only able to give results for pension situations with completely fixed or completely flexible retirement. In reality retirement is not completely fixed. Individuals are able to stop working before the retirement age. When flexible retirement is introduced individuals will not directly change their retirement behaviour if they are used to retire at a certain age. Our model could also benefit from more realistic pension systems. We have assumed that participants only pay pension contributions in the first period of life and adjustments in pension benefits in case of flexible pensions are simplified. Our model can also benefit from the addition of pension systems where pension benefits are contribution based. To what extend more complex pension systems change our results is left for future research.

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# A. Appendix

## A.1 Assumptions

In the first part of the Appendix (Appendix A.1-A.4) we explain the steps that have resulted in the formulas in our model. We start with discussing the model without pensions. Then we will discuss the wealth transfer in case of a fixed pension with lump-sum contributions. After that we will discuss a flexible pension with lump-sum contributions. Finally we will discuss pensions with contributions that are proportional to wage. We discuss the pension systems with two or three individuals.

*General Assumptions*

- There is sufficient demand for (old-age) labour
- There is no influence of 'standard' retirement ages
- Labour supply is exogenously given with fixed retirement
- Valuation of leisure is dependent on the duration of retirement
- Small open economy, wages and interest rate are given
- Wages are constant over time
- Individuals can differ in productivity
- Life expectancy is modelled as life duration
- Logarithmic utility function
- Everyone reaches the standard pension age. $z_i > \phi$

*Assumptions Pensions*

- Return on pension contributions equals the interest rate, for PAYG pensions holds that population growth equals the interest rate $(n = r)$.
- Pension contributions only have to be made in the first period of life and pension benefits can only be received in the second period of life.
- The wealth transfer income is the discounted value of life time pension benefits minus the life time pension contributions. $T_i = \frac{P_{benefit\_i}}{1+r} - \tau_C$
- In case of pensions with proportional contributions, lifetime pension contribution is wage multiplied with the pension contribution rate. $\tau_C = w_i \tau_{CR}$
- The wealth transfer benefit does not depend on second period labour supply

- In case of pensions with pension contributions proportional to wage, the wealth transfer can be negative for the long living individual.

*2 individuals*

- We discuss pension systems with two individuals $i = S, L$
- Individuals differ in life duration. $z_L > z_S$
- In case of two individuals, the wealth transfer loss of one individual is the wealth transfer gain of the other individual. $T_L = -T_S$

*3 individuals*

- In case of three individuals $i = S, M, L$
- Individuals differ in life duration. $z_L > z_M > z_S$
- The wealth transfer income is the discounted value of life time pension benefits minus the life time pension contributions. $T_i = \frac{P_{benefit\_i}}{1+r} - \tau_C$

## A.2 Model

The preferences of individuals are represented by the following utility function:

$$U = \ln C_{1,i} + z_i \ln \frac{C_{2,i}}{z_i} + \gamma_i \ln(z_i - h_i) \tag{18}$$

We include the wealth transfer benefit parameter so we can reuse the upcoming calculations later on. The wealth transfer benefit does not depend on second period labour supply. In case there is no pension system the wealth transfer is zero. $T_i = 0$

We will include our assumption that wages are constant over time at the end of this subsection. By not making the assumption that wages are constant over time we can discuss the affect of increasing/decreasing wages on our results.

Total income of individual $i$ is denoted by de following income function

$$I_i = w_{1,i} + \frac{h_i}{1+r} w_{2,i} + T_i \tag{19}$$

Individual's budget constraint is as follows:

$$C_{1,i} + \frac{1}{1+r} C_{2,i} = I_i \tag{20}$$

Consumption as function of total income is derived from the utility function.

$$C_{1,i} = \frac{1}{1+z_i} I_i \tag{21}$$

$$C_{2,i} = (1+r)\frac{z_i}{1+z_i} I_i \tag{22}$$

To determine the second period labour supply we substitute consumption equation (21) and consumption equation (22) into the utility function (18).

$$U = \ln\left(\frac{1}{1+z_i} I_i\right) + z_i \ln\left((1+r)\frac{z_i}{1+z_i} I_i\right) - z_i \ln(z_i) + \gamma_i \ln(z_i - h_i) \tag{23}$$

From the utility function (18) the first derivative is taken to second period labour supply $h_i$.

$$\frac{\partial U}{\partial h_i} = \left(\frac{1}{1+z_i} I_i\right)^{-1} \frac{1}{1+z_i}\frac{1}{1+r} w_{2,i} + z_i \left((1+r)\frac{z_i}{1+z_i} I_i\right)^{-1} \frac{z_i}{1+z_i} w_{2,i} - \gamma_i (z_i - h_i)^{-1} \tag{24}$$

For the utility maximising second period labour supply the first derivative of the utility function is set equal to zero.

$$\left(\frac{1}{1+z_i} I_i\right)^{-1} \frac{1}{1+z_i}\frac{1}{1+r} w_{2,i} + z_i \left((1+r)\frac{z_i}{1+z_i} I_i\right)^{-1} \frac{z_i}{1+z_i} w_{2,i} - \gamma_i (z_i - h_i)^{-1} = 0 \tag{25}$$

$$\left(\frac{1}{1+z_i} I_i\right)^{-1} \frac{1}{1+z_i}\frac{1}{1+r} w_{2,i} + z_i \left((1+r)\frac{z_i}{1+z_i} I_i\right)^{-1} \frac{z_i}{1+z_i} w_{2,i} = \gamma_i (z_i - h_i)^{-1} \tag{26}$$

$$\frac{1}{1+r} I_i^{-1} w_{2,i} + \frac{z_i}{1+r} I_i^{-1} w_{2,i} = \gamma_i (z_i - h_i)^{-1} \tag{27}$$

$$\frac{1+z_i}{1+r} I_i^{-1} w_{2,i} \gamma_i^{-1} = (z_i - h_i)^{-1} \tag{28}$$

$$\frac{1+r}{1+z_i} I_i w_{2,i}^{-1} \gamma_i = z_i - h_i \tag{29}$$

The following equation gives second period labour supply as a function of total income. Note that total income depends on the second period labour supply. This formula can be re-used in case the second period labour supply is independent from the wealth transfer. Because we assume that the moment of pension take-up is independent from the moment of retirement we can re-use this formula when we discuss pensions.

$$z_i - \frac{1+r}{1+z_i}\frac{I_i}{w_{2,i}}\gamma_i = h_i \tag{30}$$

The total income function (19) is substituted into equation (30).

$$z_i - \frac{1+r}{1+z_i}\frac{w_{1,i} + \frac{h_i}{1+r}w_{2,i} + T_i}{w_{2,i}}\gamma_i = h_i \tag{31}$$

$$z_i - \frac{1+r}{1+z_i}\gamma_i\frac{w_{1,i} + T_i}{w_{2,i}} = h_i\left(1 + \frac{1}{1+z_i}\gamma_i\right) \tag{32}$$

$$h_i = \frac{1}{1+\frac{1}{1+z_i}\gamma_i}z_i - \frac{\frac{1+r}{1+z_i}\gamma_i}{1+\frac{1}{1+z_i}\gamma_i}\frac{w_{1,i}+T_i}{w_{2,i}} \tag{33}$$

$$h_i = \frac{1+z_i}{1+z_i+\gamma_i}z_i - \frac{\gamma_i}{1+z_i+\gamma_i}\frac{(1+r)(w_{1,i}+T_i)}{w_{2,i}} \tag{34}$$

Second period labour supply is affected by the wages, the wealth transfer and the interest rate. By taking the first derivative we see the effect of each variable on second period labour supply.

$$\frac{\partial h_i}{\partial w_{1,i}} = -\frac{\gamma_i}{1+z_i+\gamma_i}(1+r)\frac{1}{w_{2,i}} < 0 \tag{35}$$

$$\frac{\partial h_i}{\partial w_{2,i}} = \frac{\gamma_i}{1+z_i+\gamma_i}(1+r)T_i w_{2,i}^{-2} + \frac{\gamma_i}{1+z_i+\gamma_i}(1+r)\frac{w_{1,i}}{w_{2,i}}w_{2,i}^{-1} \tag{36}$$

$$\frac{\partial h_i}{\partial T_i} = -\frac{\gamma_i}{1+z_i+\gamma_i}(1+r)w_{2,i}^{-1} < 0 \tag{37}$$

$$\frac{\partial h_i}{\partial r} = -\frac{\gamma_i}{1+z_i+\gamma_i}(w_{1,i}+T_i)w_{2,i}^{-1} \tag{38}$$

We will now include the assumption that wages are constant over time.

$$h_i = \frac{1+z_i}{1+z_i+\gamma_i}z_i - \frac{\gamma_i}{1+z_i+\gamma_i}\frac{(1+r)(w_i+T_i)}{w_i} \tag{39}$$

$$\frac{\partial h_i}{\partial z_i} = 1 - \left(\gamma_i{}^2 + \gamma_i\right)\left(1 + z_i + \gamma_i\right)^{-2} + \frac{\gamma_i(1+r)(w_i + T_i)}{w_i}\left(1 + z_i + \gamma_i\right)^{-2} \tag{40}$$

$$\frac{\partial h_i}{\partial w_i} = \frac{\gamma_i}{1 + z_i + \gamma_i}(1+r)T_i w_i^{-2} \tag{41}$$

$$\frac{\partial h_i}{\partial T_i} = -\frac{\gamma_i}{1 + z_i + \gamma_i}(1+r)w_i^{-1} < 0 \tag{42}$$

$$\frac{\partial h_i}{\partial r} = -\frac{\gamma_i}{1 + z_i + \gamma_i}\left(1 + T_i w_i^{-1}\right) \tag{43}$$

We substitute $T_i = 0$ in equation (39).

$$h_i = \frac{1 + z_i}{1 + z_i + \gamma_i} z_i - \frac{\gamma_i}{1 + z_i + \gamma_i}(1+r) \tag{44}$$

## A.3 Pensions (2 participants)

### A.3.1 Fixed pension (lump-sum contributions) (2 participants)

- Individual pays $\tau_C$ in period 1, $\tau_C$ is pension contribution

- Individual receives $(z_i - \phi)\tau_B$, $z_i$ is the duration of the second period of life, $\phi$ is the standard pension age, $\tau_B$ is the pension benefit

The pension system is balanced when the following equation holds.

$$(1+r)2\tau_C = \left((z_S - \phi) + (z_L - \phi)\right)\tau_B \tag{46}$$

The pension contribution is as follows:

$$\tau_C = \frac{1}{2}\frac{(z_S - \phi) + (z_L - \phi)}{1+r}\tau_B \tag{47}$$

We use the pension contribution to calculate the wealth transfer benefit.

$$T_L = \frac{(z_L - \phi)}{1+r}\tau_B - \frac{1}{2}\frac{(z_S - \phi) + (z_L - \phi)}{1+r}\tau_B \tag{48}$$

$$T_L = -T_S = \frac{1}{2}\left(z_L - z_S\right)\tau_B (1+r)^{-1} \tag{49}$$

If the pension age does not affect the height of pension benefits the wealth transfer does not change. Total lifetime pension benefits decrease for both individuals in case of an increase of the pension age.

We substitute equation (49) in equation (39) to get the second period labour supply:

$$h_S = \frac{1+z_S}{1+z_S+\gamma_S} z_S - \frac{\gamma_S}{1+z_S+\gamma_S} \frac{(1+r)w_S - \frac{1}{2}(z_L - z_S)\tau_B}{w_S} \tag{50}$$

$$h_L = \frac{1+z_L}{1+z_L+\gamma_L} z_L - \frac{\gamma_L}{1+z_L+\gamma_L} \frac{(1+r)w_L + \frac{1}{2}(z_Z - z_S)\tau_B}{w_L} \tag{51}$$

**A.3.2 Flexible pension (lump-sum contributions) (2 participants)**

- Individual pays $\tau_C$ in period 1, $\tau_C$ is pension contribution
- Individual receives $(z_i - \phi - \alpha_i)\tau_B(1+\theta\alpha_A)$, $z_i$ is the duration of the second period of life, $\phi$ is the standard pension age, $\tau_B$ is the pension benefit, $\alpha_i$ is the moment of pension take-up

The pension system is balanced when the following equation holds.

$$(1+r)2\tau_C = (z_S - \phi - \alpha_S)\tau_B(1+\theta\alpha_S) + (z_L - \phi - \alpha_L)\tau_B(1+\theta\alpha_L) \tag{52}$$

To determine the moment of pension take-up we maximise the lifetime pension benefits. Pension contributions only have to be made in first period of life and as a result do not affect the moment of pension take-up.

$$P_{Benefit\_i} = (z_i - \phi - \alpha_i)\tau_B(1+\theta\alpha_i) \tag{53}$$

$$P_{Benefit\_i} = z_i\tau_B - \phi\tau_B - \alpha_i\tau_B + \theta\alpha_i z_i\tau_B - \theta\alpha_i\phi\tau_B - \theta{\alpha_i}^2\tau_B \tag{54}$$

$$\frac{\partial P_{Benefit\_i}}{\partial \alpha_A} = -\tau_B + \theta z_i\tau_B - \theta\phi\tau_B - 2\theta\alpha_i\tau_B \tag{55}$$

$$-\frac{1}{2} + \frac{1}{2}\theta z_i - \frac{1}{2}\theta\phi - \theta\alpha_i = 0 \tag{56}$$

$$\alpha_i = \frac{1}{2}z_i - \frac{1}{2}\theta^{-1} - \frac{1}{2}\phi \tag{57}$$

Because people are only allowed to collect pension benefits in the second period of life we will discuss three situations. In the first situation both individuals will collect pension benefits at the start of the second period of life. In the second situation only the short living individual will collect pension benefits at the start of the second period of life. In the third situation both individuals will collect pension benefits after the start of the second period of life. An individual will collect pension benefits as soon as possible if the following equation holds:

$$\alpha_i = \frac{1}{2} z_i - \frac{1}{2}\theta^{-1} - \frac{1}{2}\phi \leq -\phi \tag{58}$$

$$z_i \leq \theta^{-1} - \phi \tag{59}$$

*Situation 1:* $\left(z_L \leq \theta^{-1} - \phi\right)$ *(Both individuals collect pension benefits as soon as possible)*

$$P_{Benefit\_i} = z_i \tau_B (1 - \theta\phi) \tag{60}$$

$$(1+n)2\tau_C = (z_S - \phi - \alpha_S)\tau_B (1 + \theta\alpha_S) + (z_L - \phi - \alpha_L)\tau_B (1 + \theta\alpha_L) \tag{61}$$

$$\tau_C = \frac{1}{2}\left(z_S \tau_B + z_L \tau_B\right)(1 - \theta\phi)(1+r)^{-1} \tag{62}$$

We use the pension contribution to calculate the wealth transfer benefit.

$$T_L = z_L (1 - \theta\phi)(1+r)^{-1}\tau_B - \frac{1}{2}\left(z_L + z_S\right)(1 - \theta\phi)(1+r)^{-1}\tau_B \tag{63}$$

$$T_L = -T_S = \frac{1}{2}\left(z_L - z_S\right)(1 - \theta\phi)(1+r)^{-1}\tau_B \tag{64}$$

Second period labour supply functions

$$h_S = \frac{1 + z_S}{1 + z_S + \gamma_S} z_S - \frac{\gamma_S}{1 + z_S + \gamma_S} \frac{(1+r)w_S - \frac{1}{2}\left(z_L - z_S\right)(1 - \theta\phi)\tau_B}{w_S} \tag{65}$$

$$h_L = \frac{1 + z_L}{1 + z_L + \gamma_L} z_L - \frac{\gamma_L}{1 + z_L + \gamma_L} \frac{(1+r)w_L + \frac{1}{2}\left(z_L - z_S\right)(1 - \theta\phi)\tau_B}{w_L} \tag{66}$$

*Situation 2:* $(z_L > \theta^{-1} - \phi \geq z_S)$ *(Short living individual collects pension benefits as soon as possible)*

$$P_{Benefit\_i} = (z_i - \phi - \alpha_i)\tau_B (1 + \theta\alpha_i) \tag{67}$$

We substitute equation (57) into equation (67) to get the lifetime pension benefit of the long living individual.

$$P_{Benefit\_L} = z_L\tau_B - \phi\tau_B - \left(\frac{1}{2}z_L - \frac{1}{2}\theta^{-1} - \frac{1}{2}\phi\right)\tau_B + \theta(\alpha_L)z_L\tau_B - \theta(\alpha_L)\phi\tau_B - \theta(\alpha_L)^2\tau_B \quad (68)$$

$$P_{Benefit\_L} = \frac{1}{2}\theta^{-1}\tau_B + \frac{1}{2}{z_L}^2\theta\tau_B - \theta\phi z_L\tau_B + \frac{1}{2}\theta\phi^2\tau_B - \theta\left(\frac{1}{2}z_L - \frac{1}{2}\theta^{-1} - \frac{1}{2}\phi\right)^2\tau_B \quad (69)$$

$$P_{Benefit\_L} = \left(\frac{1}{4}\theta^{-1} + \frac{1}{4}{z_L}^2\theta - \frac{1}{2}\theta\phi z_L + \frac{1}{4}\theta\phi^2 + \frac{1}{2}z_L - \frac{1}{2}\phi\right)\tau_B \quad (70)$$

We use the lifetime pension benefit of the long living individual to determine the pension contribution

$$(1+r)2\tau_C = z_S\tau_B(1-\theta\phi) + P_{Benefit\_L} \quad (71)$$

$$(1+r)2\tau_C = z_S\tau_B(1-\theta\phi) + \frac{1}{4}\theta^{-1}\tau_B + \frac{1}{4}{z_L}^2\theta\tau_B - \frac{1}{2}\theta\phi z_L\tau_B + \frac{1}{4}\theta\phi^2\tau_B + \frac{1}{2}z_L\tau_B - \frac{1}{2}\phi\tau_B \quad (72)$$

$$\tau_C = \left(\frac{1}{8}\theta^{-1} + \frac{1}{8}{z_L}^2\theta - \frac{1}{4}\theta\phi z_L + \frac{1}{8}\theta\phi^2 + \frac{1}{4}z_L - \frac{1}{4}\phi + \frac{1}{2}z_S(1-\theta\phi)\right)\frac{\tau_B}{1+r} \quad (73)$$

The wealth transfer is denoted by the following function

$$T_L = -T_S = \frac{P_{Benefit\_L}}{1+r} - \tau_C \quad (74)$$

$$T_L = -T_S = \left(\frac{1}{8}\theta^{-1} + \frac{1}{8}{z_L}^2\theta - \frac{1}{4}\theta\phi z_L + \frac{1}{8}\theta\phi^2 + \frac{1}{4}z_L - \frac{1}{4}\phi - \frac{1}{2}z_S(1-\theta\phi)\right)\frac{\tau_B}{1+r} \quad (75)$$

Second period labour supply functions

$$h_S = \frac{1+z_S}{1+z_S+\gamma_S}z_S - \frac{\gamma_S}{1+z_S+\gamma_S}\frac{(1+r)(w_S - T_L)}{w_S} \quad (76)$$

$$h_L = \frac{1+z_L}{1+z_L+\gamma_L}z_L - \frac{\gamma_L}{1+z_L+\gamma_L}\frac{(1+r)(w_L + T_L)}{w_L} \quad (77)$$

*Situation 3:* $(z_S > \theta^{-1} - \phi)$

$$(1+n)2\tau_C = \left(\frac{1}{2}\theta^{-1}\tau_B + \frac{1}{4}\left({z_L}^2 + {z_S}^2\right)\theta\tau_B - \frac{1}{2}\theta\phi(z_L + z_S)\tau_B + \frac{1}{2}\theta\phi^2\tau_B + \frac{1}{2}(z_L + z_S)\tau_B - \phi\tau_B\right) \quad (78)$$

$$\tau_C = \frac{\tau_B}{1+r}\left(\frac{1}{4}\theta^{-1} + \frac{1}{8}\left(z_L{}^2 + z_S{}^2\right)\theta - \frac{1}{4}\theta\phi\left(z_L + z_S\right) + \frac{1}{4}\theta\phi^2 + \frac{1}{4}\left(z_L + z_S\right) - \frac{1}{2}\phi\right) \tag{79}$$

$$T_L = \frac{P_{Benefit\_L}}{1+r} - \tau_C = \frac{\tau_B}{1+r}\left(\frac{1}{4}\theta^{-1} + \frac{1}{4}z_L{}^2\theta - \frac{1}{2}\theta\phi z_L + \frac{1}{4}\theta\phi^2 + \frac{1}{2}z_L - \frac{1}{2}\phi\right) - \tau_C \tag{80}$$

$$T_L = -T_S = \left(\frac{1}{4}\left(1-\theta\phi\right)\left(z_L - z_S\right) + \frac{1}{8}\left(z_L{}^2 - z_S{}^2\right)\theta\right)\frac{\tau_B}{1+r} \tag{81}$$

If the following equation holds making pensions flexible will not lead to a change in the wealth transfer.

$$\theta = \left(\frac{1}{2}\left(z_L + z_S\right) - \phi\right)^{-1} \tag{82}$$

Second period labour supply functions

$$h_S = \frac{1+z_S}{1+z_S+\gamma_S}z_S - \frac{\gamma_S}{1+z_S+\gamma_S}\frac{(1+r)\left(w_S - T_L\right)}{w_S} \tag{83}$$

$$h_L = \frac{1+z_L}{1+z_L+\gamma_L}z_L - \frac{\gamma_L}{1+z_L+\gamma_L}\frac{(1+r)\left(w_L + T_L\right)}{w_L} \tag{84}$$

### A.3.3 Fixed pension (proportional contributions) (2 participants)

- Individual pays $w_i\tau_{CR}$ in period 1, $\tau_{CR}$ is the pension contribution rate

- Individual receives $(z_i - \phi)\tau_B$, $z_i$ is the duration of the second period of life, $\phi$ is the standard pension age, $\tau_B$ is the pension benefit

$$(1+r)\tau_{CR}(w_S + w_L) = \left((z_S - \phi) + (z_L - \phi)\right)\tau_B \tag{85}$$

$$\tau_{CR} = \frac{(z_S - \phi)\tau_B + (z_L - \phi)\tau_B}{w_S + w_L}\frac{1}{1+r} \tag{86}$$

Income Transfer

$$T_L = \frac{P_{Benefit\_L}}{1+r} - w_i\tau_{CR} = \left(z_L - \phi\right)\frac{\tau_B}{1+r} - \frac{w_L}{w_S + w_L}\left((z_S - \phi) + (z_L - \phi)\right)\frac{\tau_B}{1+r} \tag{87}$$

$$T_L = -T_S = \left(w_S\left(z_L - \phi\right) - w_L\left(z_S - \phi\right)\right)\frac{1}{w_S + w_L}\frac{\tau_B}{1+r} \tag{88}$$

### A.3.4 Flexible pension (proportional contributions) (2 participants)

- Individual pays $w_i\tau_{CR}$ in period 1, $\tau_{CR}$ is pension contribution rate

- Individual receives $(z_i-\phi-\alpha_i)\tau_B(1+\theta\alpha_A)$, $z_i$ is the duration of the second period of life, $\phi$ is the standard pension age, $\tau_B$ is the pension benefit, $\alpha_i$ is the moment of pension take-up.

$$(1+r)\tau_{CR}(w_S+w_L)=(z_S-\phi-\alpha_S)\tau_B(1+\theta\alpha_S)+(z_L-\phi-\alpha_L)\tau_B(1+\theta\alpha_L) \tag{89}$$

Because individuals only maximise benefits, we can use equation (56) to determine the moment individuals want to start receiving pension benefits.

*Situation 1: ($z_L \le \theta^{-1}-\phi$)*

$$P_{Benefit\_i}=z_i\tau_B(1-\theta\phi) \tag{90}$$

$$(1+r)\tau_{CR}(w_S+w_L)=z_S\tau_B(1-\theta\phi)+z_L\tau_B(1-\theta\phi) \tag{91}$$

$$\tau_{CR}=\frac{z_S+z_L}{w_S+w_L}\frac{\tau_B}{1+r}(1-\theta\phi) \tag{92}$$

$$T_L=\frac{P_{Benefit\_L}}{1+r}-w_L\tau_{CR}=z_L(1-\theta\phi)\frac{\tau_B}{1+r}-\frac{w_L}{w_S+w_L}(z_S+z_L)(1-\theta\phi)\frac{\tau_B}{1+r} \tag{93}$$

*Situation 2: ($z_L > \theta^{-1}-\phi \ge z_S$)*

$$P_{Benefit\_S}=z_S\tau_B(1-\theta\phi) \tag{94}$$

$$P_{Benefit\_L}=\frac{1}{4}\theta^{-1}\tau_B+\frac{1}{4}{z_L}^2\theta\tau_B-\frac{1}{2}\theta\phi z_L\tau_B+\frac{1}{4}\theta\phi^2\tau_B+\frac{1}{2}z_L\tau_B-\frac{1}{2}\phi\tau_B \tag{95}$$

$$\tau_{CR}=\frac{1}{w_S+w_L}\frac{1}{1+r}\left(P_{Benefit\_S}+P_{Benefit\_L}\right) \tag{96}$$

$$T_L=\frac{w_S}{w_S+w_L}\frac{1}{1+r}P_{Benefit\_L}-\frac{w_L}{w_S+w_L}\frac{1}{1+r}P_{Benefit\_S} \tag{97}$$

*Situation 3: ($z_S > \theta^{-1}-\phi$)*

$$P_{Benefit\_i}=\frac{1}{4}\theta^{-1}\tau_B+\frac{1}{4}{z_i}^2\theta\tau_B-\frac{1}{2}\theta\phi z_i\tau_B+\frac{1}{4}\theta\phi^2\tau_B+\frac{1}{2}z_i\tau_B-\frac{1}{2}\phi\tau_B \tag{98}$$

$$\tau_{CR}=\frac{1}{w_S+w_L}\frac{1}{1+r}\left(P_{Benefit\_S}+P_{Benefit\_L}\right) \tag{99}$$

$$T_L=\frac{w_S}{w_S+w_L}\frac{1}{1+r}P_{Benefit\_L}-\frac{w_L}{w_S+w_L}\frac{1}{1+r}P_{Benefit\_S} \tag{100}$$

## A.4 Pensions (3 participants)

### A.4.1 Fixed pension (lump-sum contributions) (3 participants)

- Individual pays $\tau_C$ in period 1, $\tau_C$ is pension contribution

- Individual receives $(z_i - \phi)\tau_B$, $z_i$ is the duration of the second period of life, $\phi$ is the standard pension age, $\tau_B$ is the pension benefit

The pension system is balanced when the following equation holds.

$$(1+r)3\tau_C = \big((z_S - \phi) + (z_M - \phi) + (z_L - \phi)\big)\tau_B \tag{101}$$

The pension contribution is as follows:

$$\tau_C = \frac{1}{3}\frac{(z_S - \phi) + (z_M - \phi) + (z_L - \phi)}{1+r}\tau_B \tag{102}$$

We use the pension contribution to calculate the wealth transfer benefit.

$$T_i = \frac{z_i}{1+r}\tau_B - \frac{1}{3}\frac{z_S + z_M + z_L}{1+r}\tau_B \tag{103}$$

If the pension age does not affect the height of pension benefits the wealth transfer does not change. Total lifetime pension benefits decrease for both individuals in case of an increase of the pension age.

### A.4.2 Flexible pension (lump-sum contributions) (3 participants)

- Individual pays $\tau_C$ in period 1, $\tau_C$ is pension contribution

- Individual receives $(z_i - \phi - \alpha_i)\tau_B(1+\theta\alpha_A)$, $z_i$ is the duration of the second period of life, $\phi$ is the standard pension age, $\tau_B$ is the pension benefit, $\alpha_i$ is the moment of pension take-up.

The moment of pension take-up can be found in equation (57).

Because people are only allowed to collect pension benefits in the second period of life we will discuss four situations. In the first situation all individuals will collect pension benefits at the start of the second period of life. In the second situation only the long living individual will collect pension benefits after the start of the second period of life. In the third situation the long living individual and the medium long living individual will collect pension benefits

after the start of the second period of life. In the fourth situation all individuals will collect pension benefits after the start of the second period of life. An individual will collect pension benefits at the start of the second period of life if the following equation holds:

$$\alpha_i = \frac{1}{2} z_i - \frac{1}{2}\theta^{-1} - \frac{1}{2}\phi \leq -\phi \tag{104}$$

$$z_i \leq \theta^{-1} - \phi \tag{105}$$

*Situation 1:* $\left(z_L \leq \theta^{-1} - \phi\right)$ *(All individuals collect pension benefits as soon as possible)*

$$P_{Benefit\_i} = z_i \tau_B (1 - \theta\phi) \tag{106}$$

$$(1+r)3\tau_C = z_S \tau_B (1-\theta\phi) + z_M \tau_B (1-\theta\phi) + z_L \tau_B (1-\theta\phi) \tag{107}$$

$$\tau_C = \frac{1}{3}(z_S + z_M + z_L)(1-\theta\phi)\frac{\tau_B}{1+r} \tag{108}$$

$$T_i = z_i (1-\theta\phi)\frac{\tau_B}{1+r} - \frac{1}{3}(z_S + z_M + z_L)(1-\theta\phi)\frac{\tau_B}{1+r} \tag{109}$$

*Situation 2:* $(z_L > \theta^{-1} - \phi \geq z_M)$ *(Short living individual and medium long living individual collects pension benefits as soon as possible)*

$$P_{Benefit\_L} = \left(\frac{1}{4}\theta^{-1} + \frac{1}{4}{z_L}^2\theta - \frac{1}{2}\theta\phi z_L + \frac{1}{4}\theta\phi^2 + \frac{1}{2}z_L - \frac{1}{2}\phi\right)\tau_B \tag{110}$$

$$P_{Benefit\_S} = z_S \tau_B (1-\theta\phi) \tag{111}$$

$$P_{Benefit\_M} = z_M \tau_B (1-\theta\phi) \tag{112}$$

$$(1+r)3\tau_C = P_{Benefit\_S} + P_{Benefit\_M} + P_{Benefit\_L} \tag{113}$$

$$\tau_C = \left(\frac{1}{12}\theta^{-1} + \frac{1}{12}{z_L}^2\theta - \frac{1}{6}\theta\phi z_L + \frac{1}{12}\theta\phi^2 + \frac{1}{6}z_L - \frac{1}{6}\phi + \frac{1}{3}(z_S + z_M)(1-\theta\phi)\right)\frac{\tau_B}{1+r} \tag{114}$$

$$T_L = \left(\frac{1}{6}\theta^{-1} + \frac{1}{6}{z_L}^2\theta - \frac{1}{3}\theta\phi z_L + \frac{1}{6}\theta\phi^2 + \frac{1}{3}z_L - \frac{1}{3}\phi - \frac{1}{3}(z_S + z_M)(1-\theta\phi)\right)\frac{\tau_B}{1+r} \tag{115}$$

$$T_M = \left(-\frac{1}{12}\theta^{-1} - \frac{1}{12}{z_L}^2\theta + \frac{1}{6}\theta\phi z_L - \frac{1}{12}\theta\phi^2 - \frac{1}{6}z_L + \frac{1}{6}\phi + \frac{2}{3}\left(z_M - \frac{1}{2}z_S\right)(1-\theta\phi)\right)\frac{\tau_B}{1+r} \tag{116}$$

$$T_S = \left(-\frac{1}{12}\theta^{-1} - \frac{1}{12}{z_L}^2\theta + \frac{1}{6}\theta\phi z_L - \frac{1}{12}\theta\phi^2 - \frac{1}{6}z_L + \frac{1}{6}\phi + \frac{2}{3}\left(z_S - \frac{1}{2}z_M\right)(1-\theta\phi)\right)\frac{\tau_B}{1+r} \tag{117}$$

*Situation 3: ($z_M > \theta^{-1} - \phi \geq z_S$) (Short living individual collects pension benefits as soon as possible)*

$$P_{Benefit\_L} = \left(\frac{1}{4}\theta^{-1} + \frac{1}{4}z_L{}^2\theta - \frac{1}{2}\theta\phi z_L + \frac{1}{4}\theta\phi^2 + \frac{1}{2}z_L - \frac{1}{2}\phi\right)\tau_B \tag{118}$$

$$P_{Benefit\_M} = \left(\frac{1}{4}\theta^{-1} + \frac{1}{4}z_L{}^2\theta - \frac{1}{2}\theta\phi z_L + \frac{1}{4}\theta\phi^2 + \frac{1}{2}z_L - \frac{1}{2}\phi\right)\tau_B \tag{119}$$

$$P_{Benefit\_S} = z_S\tau_B(1-\theta\phi) \tag{120}$$

$$(1+r)3\tau_C = P_{Benefit\_S} + P_{Benefit\_M} + P_{Benefit\_L} \tag{121}$$

$$\tau_C = \left(\frac{1}{6}\theta^{-1} + \frac{1}{12}\left(z_L{}^2 + z_M{}^2\right)\theta - \frac{1}{6}\theta\phi\left(z_L + z_M\right) + \frac{1}{6}\theta\phi^2 + \frac{1}{6}\left(z_L + z_M\right) - \frac{1}{3}\phi + \frac{1}{3}z_S(1-\theta\phi)\right)\frac{\tau_B}{1+r} \tag{122}$$

$$T_L = \frac{P_{Benefit\_L}}{1+r} - \tau_C = \frac{\tau_B}{1+r}\left(\frac{1}{4}\theta^{-1} + \frac{1}{4}z_L{}^2\theta - \frac{1}{2}\theta\phi z_L + \frac{1}{4}\theta\phi^2 + \frac{1}{2}z_L - \frac{1}{2}\phi\right) - \tau_C \tag{123}$$

$$T_L = \frac{1}{3}\left(\frac{1}{4}\theta^{-1} + \frac{1}{2}\left(z_L{}^2 - \frac{1}{2}z_M{}^2\right)\theta - \theta\phi\left(z_L - \frac{1}{2}z_M\right) + \frac{1}{4}\theta\phi^2 + z_L - \frac{1}{2}z_M - \frac{1}{2}\phi - z_S(1-\theta\phi)\right)\frac{\tau_B}{1+r} \tag{124}$$

$$T_M = \frac{1}{3}\left(\frac{1}{4}\theta^{-1} + \frac{1}{2}\left(z_M{}^2 - \frac{1}{2}z_L{}^2\right)\theta - \theta\phi\left(z_M - \frac{1}{2}z_L\right) + \frac{1}{4}\theta\phi^2 + z_M - \frac{1}{2}z_L - \frac{1}{2}\phi - z_S(1-\theta\phi)\right)\frac{\tau_B}{1+r} \tag{125}$$

$$T_S = \left(-\frac{1}{6}\theta^{-1} - \frac{1}{12}\left(z_L{}^2 + z_M{}^2\right)\theta + \frac{1}{6}\theta\phi\left(z_L + z_M\right) - \frac{1}{6}\theta\phi^2 - \frac{1}{6}\left(z_L + z_M\right) + \frac{1}{3}\phi + \frac{2}{3}z_S(1-\theta\phi)\right)\frac{\tau_B}{1+r} \tag{126}$$

*Situation 4: ($z_S > \theta^{-1} - \phi$) (All individuals collect pension benefits after the start of the second period of life)*

$$P_{Benefit\_i} = \left(\frac{1}{4}\theta^{-1} + \frac{1}{4}z_i{}^2\theta - \frac{1}{2}\theta\phi z_i + \frac{1}{4}\theta\phi^2 + \frac{1}{2}z_i - \frac{1}{2}\phi\right)\tau_B \tag{127}$$

$$(1+r)3\tau_C = P_{Benefit\_S} + P_{Benefit\_M} + P_{Benefit\_L} \tag{128}$$

$$\tau_C = \frac{\tau_B}{1+r}\left(\frac{1}{4}\theta^{-1} + \frac{1}{12}\left(z_L{}^2 + z_M{}^2 + z_S{}^2\right)\theta - \frac{1}{6}\theta\phi\left(z_L + z_M + z_S\right) + \frac{1}{4}\theta\phi^2 + \frac{1}{6}\left(z_L + z_M + z_S\right) - \frac{1}{2}\phi\right) \tag{129}$$

$$T_L = \frac{P_{Benefit\_L}}{1+r} - \tau = \frac{\tau_B}{1+r}\left(\frac{1}{4}\theta^{-1} + \frac{1}{4}z_L{}^2\theta - \frac{1}{2}\theta\phi z_L + \frac{1}{4}\theta\phi^2 + \frac{1}{2}z_L - \frac{1}{2}\phi\right) - \tau_C \tag{130}$$

$$T_L = \left(\frac{1}{6}\left(z_L{}^2 - \frac{1}{2}z_M{}^2 - \frac{1}{2}z_S{}^2\right)\theta - \frac{1}{3}\theta\phi\left(z_L - \frac{1}{2}z_M - \frac{1}{2}z_S\right) + \frac{1}{3}\left(z_L - \frac{1}{2}z_M - \frac{1}{2}z_S\right)\right)\frac{\tau_B}{1+r} \quad (131)$$

$$T_M = \left(\frac{1}{6}\left(z_M{}^2 - \frac{1}{2}z_L{}^2 - \frac{1}{2}z_S{}^2\right)\theta - \frac{1}{3}\theta\phi\left(z_M - \frac{1}{2}z_L - \frac{1}{2}z_S\right) + \frac{1}{3}\left(z_M - \frac{1}{2}z_L - \frac{1}{2}z_S\right)\right)\frac{\tau_B}{1+r} \quad (132)$$

$$T_S = \left(\frac{1}{6}\left(z_S{}^2 - \frac{1}{2}z_L{}^2 - \frac{1}{2}z_M{}^2\right)\theta - \frac{1}{3}\theta\phi\left(z_S - \frac{1}{2}z_L - \frac{1}{2}z_M\right) + \frac{1}{3}\left(z_S - \frac{1}{2}z_L - \frac{1}{2}z_M\right)\right)\frac{\tau_B}{1+r} \quad (133)$$

### A.4.3 Fixed pension (proportional contributions) (3 participants)

- Individual pays $w_i\tau_{CR}$ in period 1, $\tau_{CR}$ is the pension contribution rate

- Individual receives $(z_i - \phi)\tau_B$, $z_i$ is the duration of the second period of life, $\phi$ is the standard pension age, $\tau_B$ is the pension benefit

$$(1+r)\tau_{CR}(w_S + w_M + w_L) = \big((z_S - \phi) + (z_L - \phi)\big)\tau_B \quad (134)$$

$$\tau_{CR} = \frac{(z_S - \phi) + (z_M - \phi) + (z_L - \phi)}{w_S + w_M + w_L}\frac{\tau_B}{1+r} \quad (135)$$

Income Transfer

$$T_i = (z_i - \phi)\frac{\tau_B}{1+r} + \frac{w_i}{w_S + w_M + w_L}\big((z_S - \phi) + (z_M - \phi) + (z_L - \phi)\big)\frac{\tau_B}{1+r} \quad (136)$$

### A.4.4 Flexible pension (proportional contributions) (3 participants)

- Individual pays $w_i\tau_{CR}$ in period 1, $\tau_{CR}$ is pension contribution rate

- Individual receives $(z_i - \phi - \alpha_i)\tau_B(1 + \theta\alpha_A)$, $z_i$ is the duration of the second period of life, $\phi$ is the standard pension age, $\tau_B$ is the pension benefit, $\alpha_i$ is the moment of pension take-up.

$$(1+r)\tau_{CR}(w_S + w_M + w_L) = P_{Benefit\_S} + P_{Benefit\_M} + P_{Benefit\_L} \quad (137)$$

The moment of pension take-up can be found in equation (56).

*Situation 1: ($z_L \le \theta^{-1} - \phi$)*

$$P_{Benefit\_i} = z_i\tau_B(1 - \theta\phi) \quad (138)$$

$$(1+r)\tau_{CR}(w_S + w_M w_L) = (z_S + z_M + z_L)\tau_B(1 - \theta\phi) \quad (139)$$

$$\tau_{CR} = \frac{z_S + z_M + z_L}{w_S + w_M + w_L} \frac{\tau_B}{1+r} (1 - \theta\phi) \tag{140}$$

$$T_i = z_i (1 - \theta\phi) \frac{\tau_B}{1+r} - \frac{w_i}{w_S + w_M + w_L} (z_S + z_M + z_L)(1 - \theta\phi) \frac{\tau_B}{1+r} \tag{141}$$

*Situation 2:* $(z_L > \theta^{-1} - \phi \geq z_M)$

$$P_{Benefit\_S} = (z_S - \phi)\tau_B \tag{142}$$

$$P_{Benefit\_M} = (z_S - \phi)\tau_B \tag{143}$$

$$P_{Benefit\_L} = \frac{1}{4}\theta^{-1}\tau_B + \frac{1}{4}{z_L}^2\theta\tau_B - \frac{1}{2}\theta\phi z_L\tau_B + \frac{1}{4}\theta\phi^2\tau_B + \frac{1}{2}z_L\tau_B - \frac{1}{2}\phi\tau_B \tag{144}$$

$$\tau_{CR} = \frac{1}{w_S + w_M + w_L} \frac{1}{1+r} \left( P_{Benefit\_S} + P_{Benefit\_M} + P_{Benefit\_L} \right) \tag{145}$$

$$T_i = P_{Benefit\_i} - \frac{w_i}{w_S + w_M + w_L} \frac{1}{1+r} \left( P_{Benefit\_S} + P_{Benefit\_M} + P_{Benefit\_L} \right) \tag{146}$$

*Situation 3:* $(z_M > \theta^{-1} - \phi \geq z_S)$

$$P_{Benefit\_S} = (z_S - \phi)\tau_B \tag{147}$$

$$P_{Benefit\_M} = \frac{1}{4}\theta^{-1}\tau_B + \frac{1}{4}{z_M}^2\theta\tau_B - \frac{1}{2}\theta\phi z_M\tau_B + \frac{1}{4}\theta\phi^2\tau_B + \frac{1}{2}z_M\tau_B - \frac{1}{2}\phi\tau_B \tag{148}$$

$$P_{Benefit\_L} = \frac{1}{4}\theta^{-1}\tau_B + \frac{1}{4}{z_L}^2\theta\tau_B - \frac{1}{2}\theta\phi z_L\tau_B + \frac{1}{4}\theta\phi^2\tau_B + \frac{1}{2}z_L\tau_B - \frac{1}{2}\phi\tau_B \tag{149}$$

$$\tau_{CR} = \frac{1}{w_S + w_M + w_L} \frac{1}{1+r} \left( P_{Benefit\_S} + P_{Benefit\_M} + P_{Benefit\_L} \right) \tag{150}$$

$$T_i = P_{Benefit\_i} - \frac{w_i}{w_S + w_M + w_L} \frac{1}{1+r} \left( P_{Benefit\_S} + P_{Benefit\_M} + P_{Benefit\_L} \right) \tag{151}$$

*Situation 4:* $(z_S > \theta^{-1} - \phi)$

$$P_{Benefit\_i} = \frac{1}{4}\theta^{-1}\tau_B + \frac{1}{4}{z_i}^2\theta\tau_B - \frac{1}{2}\theta\phi z_i\tau_B + \frac{1}{4}\theta\phi^2\tau_B + \frac{1}{2}z_i\tau_B - \frac{1}{2}\phi\tau_B \tag{152}$$

$$\tau_{CR} = \frac{1}{w_S + w_M + w_L} \frac{1}{1+r} \left( P_{Benefit\_S} + P_{Benefit\_M} + P_{Benefit\_L} \right) \tag{153}$$

$$T_i = P_{Benefit\_i} - \frac{w_i}{w_S + w_M + w_L} \frac{1}{1+r} \left( P_{Benefit\_S} + P_{Benefit\_M} + P_{Benefit\_L} \right) \tag{154}$$

## A.5 Examples

### Example 1

Table 6. Parameter values example 1

| Parameter Input | | | |
|---|---|---|---|
| $w_S$ | 1,00 | $z_S$ | 0,83 |
| $w_M$ | 1,25 | $z_M$ | 0,88 |
| $w_L$ | 2,00 | $z_L$ | 1,00 |
| $R$ | 0,50 | $\tau_B$ | 0,40 |
| $\gamma_S$ | 0,40 | $\theta$ | 2,40 |
| $\gamma_M$ | 0,40 | $\phi$ | 0,50 |
| $\gamma_L$ | 0,40 | | |

*Table 7. Results first example with flexible retirement (2 Participants)*

| 2 Participants | Fixed retirement No pension | Flexible retirement No pension | Fixed retirement Fixed pension | Flexible retirement Fixed pension | Fixed retirement Flexible pension | Flexible retirement Flexible pension | Fixed retirement Fixed pension (Proportional) | Flexible retirement Fixed pension (Proportional) | Fixed retirement Flexible pension (Proportional) | Flexible retirement Flexible pension (Proportional) |
|---|---|---|---|---|---|---|---|---|---|---|
| $P_{Ben\_S}$ | 0,0000 | 0,0000 | 0,1333 | 0,1333 | 0,1350 | 0,1350 | 0,1333 | 0,1333 | 0,1350 | 0,1350 |
| $P_{Ben\_L}$ | 0,0000 | 0,0000 | 0,2000 | 0,2000 | 0,2017 | 0,2017 | 0,2000 | 0,2000 | 0,2017 | 0,2017 |
| $T_S$ | 0,0000 | 0,0000 | -0,0222 | -0,0222 | -0,0222 | -0,0222 | 0,0148 | 0,0148 | 0,0152 | 0,0152 |
| $T_L$ | 0,0000 | 0,0000 | 0,0222 | 0,0222 | 0,0222 | 0,0222 | -0,0148 | -0,0148 | -0,0152 | -0,0152 |
| $h_S$ | 0,5000 | 0,4154 | 0,5000 | 0,4214 | 0,5000 | 0,4214 | 0,5000 | 0,4114 | 0,5000 | 0,4113 |
| $h_L$ | 0,5000 | 0,5833 | 0,5000 | 0,5806 | 0,5000 | 0,5806 | 0,5000 | 0,5852 | 0,5000 | 0,5852 |
| $I_S$ | 1,3333 | 1,2769 | 1,3111 | 1,2587 | 1,3111 | 1,2587 | 1,3481 | 1,2891 | 1,3485 | 1,2894 |
| $I_L$ | 2,6667 | 2,7778 | 2,6889 | 2,7963 | 2,6889 | 2,7963 | 2,6519 | 2,7654 | 2,6515 | 2,7651 |
| $U_S$ | -0,6854 | -0,6742 | -0,7162 | -0,7063 | -0,7162 | -0,7063 | -0,6651 | -0,6530 | -0,6646 | -0,6525 |
| $U_L$ | 0,7036 | 0,7123 | 0,7202 | 0,7282 | 0,7202 | 0,7282 | 0,6924 | 0,7016 | 0,6921 | 0,7013 |
| $\alpha_S$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 | -0,0417 | -0,0417 | 0,0000 | 0,0000 | -0,0417 | -0,0417 |
| $\alpha_L$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0417 | 0,0417 | 0,0000 | 0,0000 | 0,0417 | 0,0417 |
| $TI$ | 4,0000 | 4,0547 | 4,0000 | 4,0550 | 4,0000 | 4,0550 | 4,0000 | 4,0545 | 4,0000 | 4,0545 |

*Table 8. Results first example with flexible retirement (3 Participants)*

| 3 Participants | Fixed retirement No pension | Flexible retirement No pension | Fixed retirement Fixed pension | Flexible retirement Fixed pension | Fixed retirement Flexible pension | Flexible retirement Flexible pension | Fixed retirement Fixed pension (Proportional) | Flexible retirement Fixed pension (Proportional) | Fixed retirement Flexible pension (Proportional) | Flexible retirement Flexible pension (Proportional) |
|---|---|---|---|---|---|---|---|---|---|---|
| $P_{Ben\_S}$ | 0,0000 | 0,0000 | 0,1333 | 0,1333 | 0,1350 | 0,1350 | 0,1333 | 0,1333 | 0,1350 | 0,1350 |
| $P_{Ben\_M}$ | 0,0000 | 0,0000 | 0,1533 | 0,1533 | 0,1536 | 0,1536 | 0,1533 | 0,1533 | 0,1536 | 0,1536 |
| $P_{Ben\_L}$ | 0,0000 | 0,0000 | 0,2000 | 0,2000 | 0,2017 | 0,2017 | 0,2000 | 0,2000 | 0,2017 | 0,2017 |
| $T_S$ | 0,0000 | 0,0000 | -0,0193 | -0,0193 | -0,0189 | -0,0189 | 0,0125 | 0,0125 | 0,0131 | 0,0131 |
| $T_M$ | 0,0000 | 0,0000 | -0,0059 | -0,0059 | -0,0065 | -0,0065 | 0,0068 | 0,0068 | 0,0063 | 0,0063 |
| $T_L$ | 0,0000 | 0,0000 | 0,0252 | 0,0252 | 0,0255 | 0,0255 | -0,0193 | -0,0193 | -0,0194 | -0,0194 |
| $h_S$ | 0,5000 | 0,4154 | 0,5000 | 0,4206 | 0,5000 | 0,4205 | 0,5000 | 0,4121 | 0,5000 | 0,4119 |
| $h_M$ | 0,5000 | 0,4658 | 0,5000 | 0,4671 | 0,5000 | 0,4672 | 0,5000 | 0,4644 | 0,5000 | 0,4645 |
| $h_L$ | 0,5000 | 0,5833 | 0,5000 | 0,5802 | 0,5000 | 0,5801 | 0,5000 | 0,5858 | 0,5000 | 0,5858 |
| $I_S$ | 1,3333 | 1,2769 | 1,3141 | 1,2611 | 1,3144 | 1,2614 | 1,3459 | 1,2873 | 1,3464 | 1,2877 |
| $I_M$ | 1,6667 | 1,6382 | 1,6607 | 1,6333 | 1,6601 | 1,6328 | 1,6735 | 1,6438 | 1,6729 | 1,6434 |
| $I_L$ | 2,6667 | 2,7778 | 2,6919 | 2,7988 | 2,6922 | 2,7990 | 2,6473 | 2,7617 | 2,6473 | 2,7616 |
| $U_S$ | -0,6854 | -0,6742 | -0,7121 | -0,7020 | -0,7116 | -0,7015 | -0,6682 | -0,6562 | -0,6675 | -0,6554 |
| $U_M$ | -0,2556 | -0,2539 | -0,2623 | -0,2607 | -0,2630 | -0,2614 | -0,2479 | -0,2461 | -0,2485 | -0,2467 |
| $U_L$ | 0,7036 | 0,7123 | 0,7224 | 0,7304 | 0,7226 | 0,7306 | 0,6890 | 0,6983 | 0,6890 | 0,6983 |
| $\alpha_S$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 | -0,0417 | -0,0417 | 0,0000 | 0,0000 | -0,0417 | -0,0417 |
| $\alpha_M$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 | -0,0167 | -0,0167 | 0,0000 | 0,0000 | -0,0167 | -0,0167 |
| $\alpha_L$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0417 | 0,0417 | 0,0000 | 0,0000 | 0,0417 | 0,0417 |
| $TI$ | 5,6667 | 5,6929 | 5,6667 | 5,6932 | 5,6667 | 5,6932 | 5,6667 | 5,6927 | 5,6667 | 5,6927 |

## Example 2

*Table 9. Parameter values example 2*

| **Parameter Input** | | | |
|---|---|---|---|
| $w_S$ | 1,00 | $z_S$ | 0,83 |
| $w_M$ | 1,25 | $z_M$ | 0,88 |
| $w_L$ | 2,00 | $z_L$ | 1,00 |
| $R$ | 0,50 | $\tau_B$ | 0,40 |
| $\gamma_S$ | 0,40 | $\theta$ | 3,00 |
| $\gamma_M$ | 0,40 | $\phi$ | 0,50 |
| $\gamma_L$ | 0,30 | | |

*Table 10. Results second example with flexible retirement (2 Participants)*

| 2 Participants | Fixed retirement No pension | Flexible retirement No pension | Fixed retirement Fixed pension | Flexible retirement Fixed pension | Fixed retirement Flexible pension | Flexible retirement Flexible pension | Fixed retirement Fixed pension (Proportional) | Flexible retirement Fixed pension (Proportional) | Fixed retirement Flexible pension (Proportional) | Flexible retirement Flexible pension (Proportional) |
|---|---|---|---|---|---|---|---|---|---|---|
| $P_{Ben\_S}$ | 0,0000 | 0,0000 | 0,1333 | 0,1333 | 0,1333 | 0,1333 | 0,1333 | 0,1333 | 0,1333 | 0,1333 |
| $P_{Ben\_L}$ | 0,0000 | 0,0000 | 0,2000 | 0,2000 | 0,2083 | 0,2083 | 0,2000 | 0,2000 | 0,2083 | 0,2083 |
| $T_S$ | 0,0000 | 0,0000 | -0,0222 | -0,0222 | -0,0250 | -0,0250 | 0,0148 | 0,0148 | 0,0130 | 0,0130 |
| $T_L$ | 0,0000 | 0,0000 | 0,0222 | 0,0222 | 0,0250 | 0,0250 | -0,0148 | -0,0148 | -0,0130 | -0,0130 |
| $h_S$ | 0,5000 | 0,4154 | 0,5000 | 0,4214 | 0,5000 | 0,4221 | 0,5000 | 0,4114 | 0,5000 | 0,4119 |
| $h_L$ | 0,5000 | 0,6739 | 0,5000 | 0,6717 | 0,5000 | 0,6715 | 0,5000 | 0,6754 | 0,5000 | 0,6752 |
| $I_S$ | 1,3333 | 1,2769 | 1,3111 | 1,2587 | 1,3083 | 1,2564 | 1,3481 | 1,2891 | 1,3463 | 1,2876 |
| $I_L$ | 2,6667 | 2,8986 | 2,6889 | 2,9179 | 2,6917 | 2,9203 | 2,6519 | 2,8857 | 2,6537 | 2,8873 |
| $U_S$ | -0,6854 | -0,6742 | -0,7162 | -0,7063 | -0,7201 | -0,7103 | -0,6651 | -0,6530 | -0,6677 | -0,6556 |
| $U_L$ | 0,7729 | 0,8114 | 0,7895 | 0,8267 | 0,7915 | 0,8286 | 0,7617 | 0,8012 | 0,7631 | 0,8025 |
| $\alpha_S$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 |
| $\alpha_L$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0833 | 0,0833 | 0,0000 | 0,0000 | 0,0833 | 0,0833 |
| $TI$ | 4,0000 | 4,1755 | 4,0000 | 4,1766 | 4,0000 | 4,1767 | 4,0000 | 4,1748 | 4,0000 | 4,1749 |

*Table 11. Results second example with flexible retirement (3 Participants)*

| 3 Participants | Fixed retirement No pension | Flexible retirement No pension | Fixed retirement Fixed pension | Flexible retirement Fixed pension | Fixed retirement Flexible pension | Flexible retirement Flexible pension | Fixed retirement Fixed pension (Proportional) | Flexible retirement Fixed pension (Proportional) | Fixed retirement Flexible pension (Proportional) | Flexible retirement Flexible pension (Proportional) |
|---|---|---|---|---|---|---|---|---|---|---|
| $P_{Ben\_S}$ | 0,0000 | 0,0000 | 0,1333 | 0,1333 | 0,1333 | 0,1333 | 0,1333 | 0,1333 | 0,1333 | 0,1333 |
| $P_{Ben\_M}$ | 0,0000 | 0,0000 | 0,1533 | 0,1533 | 0,1541 | 0,1541 | 0,1533 | 0,1533 | 0,1541 | 0,1541 |
| $P_{Ben\_L}$ | 0,0000 | 0,0000 | 0,2000 | 0,2000 | 0,2083 | 0,2083 | 0,2000 | 0,2000 | 0,2083 | 0,2083 |
| $T_S$ | 0,0000 | 0,0000 | -0,0193 | -0,0193 | -0,0213 | -0,0213 | 0,0125 | 0,0125 | 0,0111 | 0,0111 |
| $T_M$ | 0,0000 | 0,0000 | -0,0059 | -0,0059 | -0,0074 | -0,0074 | 0,0068 | 0,0068 | 0,0055 | 0,0055 |
| $T_L$ | 0,0000 | 0,0000 | 0,0252 | 0,0252 | 0,0287 | 0,0287 | -0,0193 | -0,0193 | -0,0166 | -0,0166 |
| $h_S$ | 0,5000 | 0,4154 | 0,5000 | 0,4206 | 0,5000 | 0,4211 | 0,5000 | 0,4121 | 0,5000 | 0,4124 |
| $h_M$ | 0,5000 | 0,4658 | 0,5000 | 0,4671 | 0,5000 | 0,4674 | 0,5000 | 0,4644 | 0,5000 | 0,4647 |
| $h_L$ | 0,5000 | 0,6739 | 0,5000 | 0,6714 | 0,5000 | 0,6711 | 0,5000 | 0,6758 | 0,5000 | 0,6755 |
| $I_S$ | 1,3333 | 1,2769 | 1,3141 | 1,2611 | 1,3121 | 1,2595 | 1,3459 | 1,2873 | 1,3445 | 1,2861 |
| $I_M$ | 1,6667 | 1,6382 | 1,6607 | 1,6333 | 1,6592 | 1,6320 | 1,6735 | 1,6438 | 1,6722 | 1,6427 |
| $I_L$ | 2,6667 | 2,8986 | 2,6919 | 2,9205 | 2,6954 | 2,9235 | 2,6473 | 2,8817 | 2,6500 | 2,8841 |
| $U_S$ | -0,6854 | -0,6742 | -0,7121 | -0,7020 | -0,7149 | -0,7049 | -0,6682 | -0,6562 | -0,6702 | -0,6582 |
| $U_M$ | -0,2556 | -0,2539 | -0,2623 | -0,2607 | -0,2640 | -0,2624 | -0,2479 | -0,2461 | -0,2493 | -0,2475 |
| $U_L$ | 0,7729 | 0,8114 | 0,7917 | 0,8287 | 0,7943 | 0,8311 | 0,7583 | 0,7980 | 0,7604 | 0,7999 |
| $\alpha_S$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 |
| $\alpha_M$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0250 | 0,0250 | 0,0000 | 0,0000 | 0,0250 | 0,0250 |
| $\alpha_L$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0833 | 0,0833 | 0,0000 | 0,0000 | 0,0833 | 0,0833 |
| $TI$ | 5,6667 | 5,8137 | 5,6667 | 5,8149 | 5,6667 | 5,8150 | 5,6667 | 5,8128 | 5,6667 | 5,8129 |

## Example 3

*Table 12. Parameter values example 3*

| Parameter Input | | | |
|---|---|---|---|
| $w_S$ | 1,00 | $z_S$ | 0,22 |
| $w_M$ | 1,25 | $z_M$ | 0,26 |
| $w_L$ | 2,00 | $z_L$ | 0,33 |
| $R$ | 0,50 | $\tau_B$ | 0,40 |
| $\gamma_S$ | 0,25 | $\theta$ | 4,00 |
| $\gamma_M$ | 0,25 | $\phi$ | 0,00 |
| $\gamma_L$ | 0,25 | | |

*Table 13. Results first example with partial flexible retirement (2 Participants)*

| 2 Participants | Fixed retirement No pension | Flexible retirement No pension | Fixed retirement Fixed pension | Flexible retirement Fixed pension | Fixed retirement Flexible pension | Flexible retirement Flexible pension | Fixed retirement Fixed pension (Proportional) | Flexible retirement Fixed pension (Proportional) | Fixed retirement Flexible pension (Proportional) | Flexible retirement Flexible pension (Proportional) |
|---|---|---|---|---|---|---|---|---|---|---|
| $P_{Ben\_S}$ | 0,0000 | 0,0000 | 0,0889 | 0,0889 | 0,0889 | 0,0889 | 0,0889 | 0,0889 | 0,0889 | 0,0889 |
| $P_{Ben\_L}$ | 0,0000 | 0,0000 | 0,1333 | 0,1333 | 0,1361 | 0,1361 | 0,1333 | 0,1333 | 0,1361 | 0,1361 |
| $T_S$ | 0,0000 | 0,0000 | -0,0148 | -0,0148 | -0,0157 | -0,0157 | 0,0099 | 0,0099 | 0,0093 | 0,0093 |
| $T_L$ | 0,0000 | 0,0000 | 0,0148 | 0,0148 | 0,0157 | 0,0157 | -0,0099 | -0,0099 | -0,0093 | -0,0093 |
| $h_S$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 |
| $h_L$ | 0,0000 | 0,0439 | 0,0000 | 0,0421 | 0,0000 | 0,0420 | 0,0000 | 0,0450 | 0,0000 | 0,0450 |
| $I_S$ | 1,0000 | 1,0000 | 0,9852 | 0,9852 | 0,9843 | 0,9843 | 1,0099 | 1,0099 | 1,0093 | 1,0093 |
| $I_L$ | 2,0000 | 2,0585 | 2,0148 | 2,0710 | 2,0157 | 2,0717 | 1,9901 | 2,0502 | 1,9907 | 2,0507 |
| $U_S$ | -0,5312 | -0,5312 | -0,5494 | -0,5494 | -0,5506 | -0,5506 | -0,5192 | -0,5192 | -0,5199 | -0,5199 |
| $U_L$ | 0,4011 | 0,4043 | 0,4110 | 0,4138 | 0,4116 | 0,4144 | 0,3945 | 0,3979 | 0,3949 | 0,3983 |
| $\alpha_S$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 |
| $\alpha_L$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0417 | 0,0417 | 0,0000 | 0,0000 | 0,0417 | 0,0417 |
| $TI$ | 3,0000 | 3,0585 | 3,0000 | 3,0561 | 3,0000 | 3,0560 | 3,0000 | 3,0600 | 3,0000 | 3,0599 |

*Table 14. Results first example with partial flexible retirement (3 Participants)*

| 3 Participants | Fixed retirement No pension | Flexible retirement No pension | Fixed retirement Fixed pension | Flexible retirement Fixed pension | Fixed retirement Flexible pension | Flexible retirement Flexible pension | Fixed retirement Fixed pension (Proportional) | Flexible retirement Fixed pension (Proportional) | Fixed retirement Flexible pension (Proportional) | Flexible retirement Flexible pension (Proportional) |
|---|---|---|---|---|---|---|---|---|---|---|
| $P_{Ben\_S}$ | 0,0000 | 0,0000 | 0,0889 | 0,0889 | 0,0889 | 0,0889 | 0,0889 | 0,0889 | 0,0889 | 0,0889 |
| $P_{Ben\_M}$ | 0,0000 | 0,0000 | 0,1022 | 0,1022 | 0,1022 | 0,1022 | 0,1022 | 0,1022 | 0,1022 | 0,1022 |
| $P_{Ben\_L}$ | 0,0000 | 0,0000 | 0,1333 | 0,1333 | 0,1361 | 0,1361 | 0,1333 | 0,1333 | 0,1361 | 0,1361 |
| $T_S$ | 0,0000 | 0,0000 | -0,0128 | -0,0128 | -0,0135 | -0,0135 | 0,0084 | 0,0084 | 0,0079 | 0,0079 |
| $T_M$ | 0,0000 | 0,0000 | -0,0040 | -0,0040 | -0,0046 | -0,0046 | 0,0045 | 0,0045 | 0,0040 | 0,0040 |
| $T_L$ | 0,0000 | 0,0000 | 0,0168 | 0,0168 | 0,0180 | 0,0180 | -0,0129 | -0,0129 | -0,0119 | -0,0119 |
| $h_S$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 |
| $h_M$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 |
| $h_L$ | 0,0000 | 0,0439 | 0,0000 | 0,0419 | 0,0000 | 0,0417 | 0,0000 | 0,0454 | 0,0000 | 0,0453 |
| $I_S$ | 1,0000 | 1,0000 | 0,9872 | 0,9872 | 0,9865 | 0,9865 | 1,0084 | 1,0084 | 1,0079 | 1,0079 |
| $I_M$ | 1,2500 | 1,2500 | 1,2460 | 1,2460 | 1,2454 | 1,2454 | 1,2545 | 1,2545 | 1,2540 | 1,2540 |
| $I_L$ | 2,0000 | 2,0585 | 2,0168 | 2,0726 | 2,0180 | 2,0737 | 1,9871 | 2,0476 | 1,9881 | 2,0484 |
| $U_S$ | -0,5312 | -0,5312 | -0,5470 | -0,5470 | -0,5477 | -0,5477 | -0,5210 | -0,5210 | -0,5215 | -0,5215 |
| $U_M$ | -0,2430 | -0,2430 | -0,2470 | -0,2470 | -0,2476 | -0,2476 | -0,2385 | -0,2385 | -0,2390 | -0,2390 |
| $U_L$ | 0,4011 | 0,4043 | 0,4123 | 0,4151 | 0,4131 | 0,4159 | 0,3925 | 0,3959 | 0,3932 | 0,3965 |
| $\alpha_S$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 |
| $\alpha_M$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0028 | 0,0028 | 0,0000 | 0,0000 | 0,0028 | 0,0028 |
| $\alpha_L$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0417 | 0,0417 | 0,0000 | 0,0000 | 0,0417 | 0,0417 |
| $TI$ | 4,2500 | 4,3085 | 4,2500 | 4,3058 | 4,2500 | 4,3056 | 4,2500 | 4,3105 | 4,2500 | 4,3104 |

## Example 4

*Table 15. Parameter values example 4*

| Parameter Input | | | |
|---|---|---|---|
| $w_S$ | 1,00 | $z_S$ | 0,22 |
| $w_M$ | 1,25 | $z_M$ | 0,26 |
| $w_L$ | 2,00 | $z_L$ | 0,33 |
| $R$ | 0,50 | $\tau_B$ | 0,40 |
| $\gamma_S$ | 0,25 | $\theta$ | 5,00 |
| $\gamma_M$ | 0,25 | $\phi$ | 0,00 |
| $\gamma_L$ | 0,18 | | |

*Table 16. Results second example with partial flexible retirement (2 Participants)*

| 2 Participants | Fixed retirement No pension | Flexible retirement No pension | Fixed retirement Fixed pension | Flexible retirement Fixed pension | Fixed retirement Flexible pension | Flexible retirement Flexible pension | Fixed retirement Fixed pension (Proportional) | Flexible retirement Fixed pension (Proportional) | Fixed retirement Flexible pension (Proportional) | Flexible retirement Flexible pension (Proportional) |
|---|---|---|---|---|---|---|---|---|---|---|
| $P_{Ben\_S}$ | 0,0000 | 0,0000 | 0,0889 | 0,0889 | 0,0891 | 0,0891 | 0,0889 | 0,0889 | 0,0891 | 0,0891 |
| $P_{Ben\_L}$ | 0,0000 | 0,0000 | 0,1333 | 0,1333 | 0,1422 | 0,1422 | 0,1333 | 0,1333 | 0,1422 | 0,1422 |
| $T_S$ | 0,0000 | 0,0000 | -0,0148 | -0,0148 | -0,0177 | -0,0177 | 0,0099 | 0,0099 | 0,0080 | 0,0080 |
| $T_L$ | 0,0000 | 0,0000 | 0,0148 | 0,0148 | 0,0177 | 0,0177 | -0,0099 | -0,0099 | -0,0080 | -0,0080 |
| $h_S$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 |
| $h_L$ | 0,0000 | 0,1206 | 0,0000 | 0,1193 | 0,0000 | 0,1191 | 0,0000 | 0,1215 | 0,0000 | 0,1213 |
| $I_S$ | 1,0000 | 1,0000 | 0,9852 | 0,9852 | 0,9823 | 0,9823 | 1,0099 | 1,0099 | 1,0080 | 1,0080 |
| $I_L$ | 2,0000 | 2,1608 | 2,0148 | 2,1739 | 2,0177 | 2,1765 | 1,9901 | 2,1521 | 1,9920 | 2,1538 |
| $U_S$ | -0,5312 | -0,5312 | -0,5494 | -0,5494 | -0,5530 | -0,5530 | -0,5192 | -0,5192 | -0,5214 | -0,5214 |
| $U_L$ | 0,4835 | 0,5080 | 0,4934 | 0,5171 | 0,4953 | 0,5189 | 0,4769 | 0,5019 | 0,4782 | 0,5031 |
| $\alpha_S$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0111 | 0,0111 | 0,0000 | 0,0000 | 0,0111 | 0,0111 |
| $\alpha_L$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0667 | 0,0667 | 0,0000 | 0,0000 | 0,0667 | 0,0667 |
| $TI$ | 3,0000 | 3,1608 | 3,0000 | 3,1591 | 3,0000 | 3,1588 | 3,0000 | 3,1620 | 3,0000 | 3,1618 |

*Table 17. Results second example with partial flexible retirement (3 Participants)*

| 3 Participants | Fixed retirement No pension | Flexible retirement No pension | Fixed retirement Fixed pension | Flexible retirement Fixed pension | Fixed retirement Flexible pension | Flexible retirement Flexible pension | Fixed retirement Fixed pension (Proportional) | Flexible retirement Fixed pension (Proportional) | Fixed retirement Flexible pension (Proportional) | Flexible retirement Flexible pension (Proportional) |
|---|---|---|---|---|---|---|---|---|---|---|
| $P_{Ben\_S}$ | 0,0000 | 0,0000 | 0,0889 | 0,0889 | 0,0891 | 0,0891 | 0,0889 | 0,0889 | 0,0891 | 0,0891 |
| $P_{Ben\_M}$ | 0,0000 | 0,0000 | 0,1022 | 0,1022 | 0,1038 | 0,1038 | 0,1022 | 0,1022 | 0,1038 | 0,1038 |
| $P_{Ben\_L}$ | 0,0000 | 0,0000 | 0,1333 | 0,1333 | 0,1422 | 0,1422 | 0,1333 | 0,1333 | 0,1422 | 0,1422 |
| $T_S$ | 0,0000 | 0,0000 | -0,0128 | -0,0128 | -0,0150 | -0,0150 | 0,0084 | 0,0084 | 0,0069 | 0,0069 |
| $T_M$ | 0,0000 | 0,0000 | -0,0040 | -0,0040 | -0,0053 | -0,0053 | 0,0045 | 0,0045 | 0,0035 | 0,0035 |
| $T_L$ | 0,0000 | 0,0000 | 0,0168 | 0,0168 | 0,0203 | 0,0203 | -0,0129 | -0,0129 | -0,0103 | -0,0103 |
| $h_S$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 |
| $h_M$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 |
| $h_L$ | 0,0000 | 0,1206 | 0,0000 | 0,1192 | 0,0000 | 0,1189 | 0,0000 | 0,1217 | 0,0000 | 0,1215 |
| $I_S$ | 1,0000 | 1,0000 | 0,9872 | 0,9872 | 0,9850 | 0,9850 | 1,0084 | 1,0084 | 1,0069 | 1,0069 |
| $I_M$ | 1,2500 | 1,2500 | 1,2460 | 1,2460 | 1,2447 | 1,2447 | 1,2545 | 1,2545 | 1,2535 | 1,2535 |
| $I_L$ | 2,0000 | 2,1608 | 2,0168 | 2,1757 | 2,0203 | 2,1788 | 1,9871 | 2,1494 | 1,9897 | 2,1517 |
| $U_S$ | -0,5312 | -0,5312 | -0,5470 | -0,5470 | -0,5497 | -0,5497 | -0,5210 | -0,5210 | -0,5228 | -0,5228 |
| $U_M$ | -0,2430 | -0,2430 | -0,2470 | -0,2470 | -0,2484 | -0,2484 | -0,2385 | -0,2385 | -0,2396 | -0,2396 |
| $U_L$ | 0,4835 | 0,5080 | 0,4947 | 0,5184 | 0,4970 | 0,5205 | 0,4749 | 0,5001 | 0,4766 | 0,5017 |
| $\alpha_S$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0111 | 0,0111 | 0,0000 | 0,0000 | 0,0111 | 0,0111 |
| $\alpha_M$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0278 | 0,0278 | 0,0000 | 0,0000 | 0,0278 | 0,0278 |
| $\alpha_L$ | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0667 | 0,0667 | 0,0000 | 0,0000 | 0,0667 | 0,0667 |
| $TI$ | 4,2500 | 4,4108 | 4,2500 | 4,4089 | 4,2500 | 4,4085 | 4,2500 | 4,4123 | 4,2500 | 4,4120 |