Envy Taxonomy:

Incentive contracts under different tax regimes when the worker envies his boss

Bachelor Thesis IBEB

Erasmus School of Economics – Department of Economics

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Name: Joost van Gemeren

Student number: 332940

Supervisor: Prof. Dr. A.J. Dur

The goal of this paper is to analyze how taxes affect the principal-agent relationship when the worker envies his boss. To do this, an income- and a corporate tax rate are added to a piece rate model where the agent loses utility when the principal earns more than him. Envy is found to increase productivity, as long as risk is not too high. Furthermore, I show that if risk is low or the bonus high, both types of taxes decrease worker productivity. But if risk is high, this conventional wisdom might not hold true. The results imply that it may be desirable for the government to install a higher corporate tax rate.

Key words: Bonus, Envy, Income tax, Corporate tax, Effort, Sensitivity to income, Worker-boss earnings gap, Revenue sharing rule

11,933 words

Contents

1. Introduction

The last decades, economists have come to realize that human behavior is about more than just material payoffs. Back in the seventies, Gary Becker noticed that economists had turned towards an approach where agents maximized the amount of goods and services they could consume. Social interactions among people were consequently ignored (Becker 1974). In response, Becker wrote many about this, for instance on discrimination, marriage and divorce and fertility decisions (Grant and Brue 2007), but also about social relationships such as “charitable behavior” (altruism) and “hatred and envy” (Becker 1974). The interest of Becker in social interactions came, not surprisingly, in a time where behavioral economics was at its dawn (Sent 2004). The psychology-related branch of economics has become more popular ever since, championed by the “Anomalies” column of Thaler and co-authors. These violations of standard economic theory sometimes were explained by previously unconsidered social comparisons between individuals (see for example Dawes and Thaler 1988, Camerer and Thaler 1995). More and more, these types of social relations are integrated into standard economic models (Grant and Brue 2007), comparable to what Becker did.

This paper follows the trend by considering envy towards the boss in a principal-agent model. The key innovation compared to previous research is that tax is included. The boss and the worker can face a different tax rate. By incorporating tax rates into the model, first of all, an effort is made to make the analysis more realistic. Not only because the worker and the boss presumably care about after-tax, rather than gross payoffs, but also because envy and taxes interact. Helmut Schoeck notes in his well-known psychological work “Envy - a theory of social behavior” that envy is to a certain extent responsible for the existence of progressive income taxes, since people do not like to be worse off than others (Steiner 1970). Therefore, it might be beneficial to analyze the effects of taxes and envy in conjunction rather than in isolation. Secondly and perhaps more importantly, the analysis can provide valuable insights for policy making in terms of optimal taxation. Governments across the world are now cutting spending and raising taxes to improve their budgets, which will probably affect the economy. This paper analyzes the effects of corporate and income tax rates on the productivity of a worker, given that this worker is envious towards his boss. Since a nation consists of many individuals like this worker, the consequences of raising taxes may become more transparent. Third, (potential) multinational corporations can benefit from this paper. Given that multinationals face different tax rates in the different countries where they operate, the optimal contract and the resulting worker productivity and thus profitability can be very different in each country.

The results of this paper support the conventional wisdom that taxes decrease economic activity. However, this claim is only valid if risk is low or, analogously, the bonus is sufficiently high. If the bonus is higher than half the output price, a high corporate tax rate indeed decreases productivity. This can be an explanation for the fact that most successful investment banks are located in low corporate tax countries. However, if the per unit bonus is lower than half the output price, high corporate taxes result in higher productivity. In most countries, the bonus on average is lower than half the output price, so increasing the corporate tax might increase national product. This indicates that a "Buffett Tax" on millionaires in the USA could be efficient. Low income taxes were also found to increase productivity, but if risk is large enough, this no longer holds. This can be a reason why poverty in developing countries, where income taxes are generally very low, persists. Envy appeared to be positively related to productivity, but this only holds if risk is not too high or, equivalently, if the worker gets a sufficiently favorable share of additional revenues. This result can be used to explain why the most successful franchisers wait until a few years after foundation before they start "outsourcing" their stores and it also explains why US non-profit hospitals perform better than for-profit clinics.

The remainder of the paper is structured in the following way. Section 2 is dedicated to a review of the relevant literature. Section 3 introduces the model that will be used. In section 4, the model is solved for the special case of contractible effort. In section 5, the optimal effort is determined given the level of the bonus, whereas in section 6 this bonus is endogenous. Section 7 displays cases where this model can be applied. Section 8 concludes.

2. Related literature

The common theory of the effect of income taxes on working hours, as described in for example Rosen and Gayer (2010) and Borjas (2010), identifies two opposing effects. The income effect shows a positive relation between taxes and working hours, because workers need to work more in order to achieve the same level of consumption. The substitution effect, however, yields a negative relationship, since the opportunity cost of leisure in the labor-leisure tradeoff decreases as the real wage declines. It is this substitution effect that leads to a deadweight loss of taxation. For a literature survey on which of the two effects dominates empirically, see Blundell and MaCurdy (1999). The effect of corporate taxes on productivity is, however, not often considered. This is probably due to the fact that it does not have a clear direct relationship with work decisions. This paper, considering the effect of corporate taxes on effort via envy, fills that gap to some extent.

Regarding social preferences, Dupor and Liu (2003) consider jealousy in the labour-leisure model, albeit without taxes. Since more consumption is disliked by others, this drives up the consumption by the others as well, leading to overconsumption in equilibrium. The authors continue by very briefly touching upon the potential of taxes in correcting this inefficiency. The paper they refer to, Ljungqvist and Uhlig (2000), provides a clearer tax framework to correct the inefficiency arising because of “Catching up with the Joneses”. They argue that taxes should be countercyclical in order to lessen overconsumption arising in peaks. This paper does not use the same macro approach as the abovementioned sources, but the results might still be related.

The papers that are closer to this article consider a principal-agent relationship in a setting with either horizontal or vertical social preferences. The papers concerning horizontal social preferences are discussed first. Bartling and Von Siemens (2009) analyze envy in a piece rate setting. They found that envy amongst workers leads an employer to offer a more flat-wage contract. Englmaier and Wambach (2010) derived the same conclusion from their model concerning inequity aversion. Grund and Sliwka (2005) consider envious agents in a tournament setting. They found that envy leads to higher effort but this eliminates the first-best situation, also because envious agents have propensity to sabotage one another. Itoh (2004) considers a model where agents working in a team can have both horizontal and vertical inequity aversion. The paper concludes that inequity aversion gives employers the opportunity to set a contract that gives unequal outcomes if some workers do not perform. Since my research is about horizontal preferences, the link to the above papers is only indirect. A more thorough display of the literature on horizontal social preferences can be found in Dur and Tichem (2012).

The previous research on vertical social preferences has a stronger connection to this paper. Vertical preferences can be on the account of the principal or of the agent, or both. Shchetinin (2010) finds that if the worker is altruistic, incentives will be less effective because the worker also cares about the firm's profit. Altruistic workers do have more intrinsic motivation to provide effort, but high incentives can crowd this out. Non (2012) considers a model where the worker's altruism is conditional on his boss's altruism. He finds that the boss can set a lower bonus and higher base salary in order to signal his altruism towards workers. The workers who accept this contract do not need strong incentives because productivity of altruistic workers is not so sensitive to bonuses. In their research, Dur and Tichem (2012), on the other hand, focus on unconditional social relationship between the principal and the agent. In order for a bonus to be credible, the two players need to have a sufficiently good social relationship. Therefore, better social relationships (altruism) might be associated with a higher, rather than a lower bonus.

With the boss exhibiting no social preferences, Englmaier and Wambach (2010) show that inequality aversion of the worker leads to a tendency to share additional profits equally. Additionally, the desire for equality can be used as incentive: working hard does not only lead to more income, but also to a more equal distribution of profits. Analogously, Itoh (2004) found that agents' inequity aversion leads the boss to share more profits with his employees. This in general leads the principal to be worse off. Lastly, Dur and Glazer (2008) consider a model where an employee is envious towards his boss. They found that envy calls for stronger incentives or a lower effort requirement, since by exerting effort the employee puts a disutility on himself, leading the participation constraint to tighten. My paper has been inspired by the work of Dur and Glazer and thus will contain similarities to it. Overall, the literature suggests that altruism is associated with weaker incentives and envy and inequity aversion lead to stronger incentives.

An important difference with most of the papers presented here is that this paper's focus is on explaining effort, and how this varies for different states of the world. The previous research tries to explain what the optimal contract is, given a set of assumptions about the worker's preferences. In my research, the optimal contract design is seen as merely a mechanism in affecting the level of effort. Despite this, the key innovation with respect to previous research, perhaps except that of Dupor and Liu and Ljunqvist and Uhlig, is that the effects of taxes on effort will be added in conjunction with that of social preferences, in this case envy. The previous principal-agent models with vertical social preferences have consistently disregarded taxes. Still, taxes could definitely affect the optimal contract and therefore effort, because it seems reasonable that envy, inequity aversion and altruism in reality are not merely about gross, but rather about after-tax earnings comparisons.

3. Model and assumptions

The starting point for constructing the model is the standard piece rate model often used in the Personnel Economics literature, described for example in Lazear and Gibbs (2009). In this principal-agent model, a risk-neutral employer can offer an employee a contract by stating a two-part compensation package: a fixed salary a plus a bonus per unit produced b. Having observed the contract, the employee will then choose whether to accept the contract or not, and how much effort he will put in the job. Note that throughout this research, I use the words effort and productivity interchangeably. Effort increases output and thus monetary payoff via the bonus. However, this effort-payoff relation is not deterministic since random external shocks also occur that can influence output in both directions. This exposes the employee to risk, which decreases his utility because of risk-aversion.

From now on, it is assumed that the employer always finds it optimal to hire the worker, so the best contract guarantees positive expected profits for the employer[1]. Also, this contract can be constructed in such a way that the worker will accept. These assumptions allow a focus on effort choice and optimal contract construction only, while taking for granted that there will be a contract offer.

To this basic model two proportional tax rates are added, one for the worker (income tax) and one for the employer (corporate tax). The government has full authority in setting these tax rates, and these tax rates are assumed not to vary with income or profits. To avoid lengthy expressions as much as possible without losing generality, I represent each tax as a parameter by which the gross monetary payoffs are multiplied. This factor therefore denotes the fraction of gross monetary payoffs the actor can keep after tax. These parameters are specified as 1 minus the applicable tax

[1] Given that the employer's best alternative would be to not hire the worker for a payoff of zero.

rate. Therefore, from now on, I will refer to this factor as the “after-tax rate”. Both after-tax rates are larger than zero to avoid the model from exploding. The after-tax rates may be a bit larger than one, this would account for the possibility of a labor or corporate subsidy.

Next to that, the model is enriched by a negative envy term following Non’s (2012) example. This envy term consists of a parameter indicating the sensitiveness to envy and an envy function. Unlike Non, I assume an exogenous envy parameter γ like in Dur and Glazer (2008). A second difference compared to Non, following Bartling and Von Siemens (2009), is that the envy function is modeled as the difference in expected monetary payoffs between the boss and the worker, where this paper uses the after tax difference. I will refer to the envy function as the “worker-boss earnings gap”. To make sure envy is modeled and not social comparisons in general, I assume throughout that the expected net payoff of the boss exceeds the expected net monetary payoff to the worker. Furthermore, I assume that the employer does not have social preferences, so he cares purely about his own payoff. Lastly, there is only one worker involved, so the worker is not envious towards peers.

For the sake of clarity, the complete game is represented in schematic form:

Step 1: The government chooses income after-tax rate φ_w and corporate after-tax rate φ_c.

Step 2: The employer observes the after-tax rates and offers a contract to the worker consisting of a fixed compensation a and a per unit bonus b.

Step 3: The worker observes the contract and decides whether or not to accept it. If so, he chooses effort level e. If not, he will take his best outside option for a utility of $\overline{U}$.[2]

Adhering to the model assumptions made above, the model can be represented by the following parametric equations:

$$\pi = \varphi_c\left(\left(P-b\right)e-a\right) \tag{3.1}$$

$$U = \varphi_w\left(a+be\right)-\frac{1}{2}R\sigma^2\left(\varphi_w b\right)^2-\frac{1}{2}\theta e^2-\gamma\left(\varphi_c\left(\left(P-b\right)e-a\right)-\varphi_w\left(a+be\right)\right) \tag{3.2}$$

[2] $\overline{U}$ is considered a fixed parameter, so it will not vary in any tax. One could think of outsides option like being unemployed or taking a job outside the country where other tax rates apply.

Equation (3.1) denotes expected profits, which equals expected utility for the employer. The employer receives a per unit expected income of $P-b$, where P is the unit price determined by perfect competition in the output market. The base salary of the worker is then subtracted to arrive at the gross profit. Multiplying this by the corporate after-tax rate yields expected net profit.

Equation (3.2) describes the worker's expected utility function. The firm term indicates his net monetary payoff. The second term relates to risk aversion, where R denotes the degree of risk aversion and σ^2 the variance of the random shocks to output, which is the measure of risk. Together, the first two terms form the "certainty equivalent" utility of net income[3]. The third term is the cost of effort function, satisfying increasing marginal cost of effort through its convexity. The parameter θ indicates the degree of costliness of effort. The final term is the envy term, as explained above. For a schematic view of all used parameters, please refer to Appendix A.

4. Contractible effort analysis

Before analyzing the principal-agent problem described above, it is useful to look at a case where the employer he can select by himself the amount of effort to be exerted. This amount of effort will be presented to the worker in his contract. If the worker refuses to exert this level of effort, he will be fired or he will not accept the contract in the first place. In this way, there is no more conflict of interest between the players. The literature calls this case "contractible effort". An assumption of this analysis is that effort is perfectly observable. An implication of the above assumptions is that contractible effort results in the first best situation for the employer, whereas without contractible effort, this is not straightforward to achieve.

If effort is contractible, there is no need to provide incentives through a bonus. The only thing the employer needs to do is to compute the optimal level of effort to be exerted by the worker. One feature to this is that the risk for the worker is completely eliminated. The only cost to the worker

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[3] $CE(Income) = Income - \frac{1}{2} R \cdot \mathrm{var}(Income)$

is the cost of effort, which will be reimbursed through the base salary. When these features are included in the utility function, the following model is obtained:

$$\pi_{contr} = \varphi_c \left(Pe - a \right) \tag{4.1}$$

$$U_{contr} = \varphi_w a - \frac{1}{2}\theta e^2 - \gamma\left(\varphi_c \left(Pe - a\right) - \varphi_w a\right) \tag{4.2}$$

From (4.2) the base salary can be determined. Assumed is that the employer has all the bargaining power in setting the contract, so he will extract all surplus from the worker, while still ensuring that the worker participation constraint is satisfied. This yields the following optimal base salary:

$$a^*_{contr} = \frac{\overline{U} + \frac{1}{2}\theta e^2 + \varphi_c \gamma Pe}{\varphi_w \left(1+\gamma\right) + \varphi_c \gamma} \tag{4.3}$$

After substituting (4.3) into (4.1), the first-order condition $d\pi/de = 0$ corresponds to:

$$\frac{d\pi}{de} = \varphi_c P - \frac{\varphi_c \theta e + \varphi_c^2 \gamma P}{\varphi_w \left(1+\gamma\right) + \varphi_c \gamma} = 0 \tag{4.4}$$

Equation (4.4) shows the following: An increase in effort yields the employer net revenue $\varphi_c P$, this is the marginal benefit of exerting effort. The marginal cost is the second part of the expression. For every extra unit of effort, the employee faces cost of effort θe and envy cost $\varphi_c \gamma P$. Multiplying these costs by the corporate after-tax rate yields the "net" marginal costs of effort *to the worker*, denoted by the numerator of the second term. These marginal costs to the worker have to be reimbursed via the base salary in order to keep the worker in the firm. Fortunately for the boss, the worker benefits three times from the base salary, as can be seen in (4.2). Firstly, the worker gets more money, which increases his purchasing power. Getting more money also reduces the earnings-gap that yields the worker disutility. Lastly, the base salary makes the boss's profits lower, which also reduces the earnings-gap. Therefore, the base salary yields utility three times, so the marginal cost of effort *for the worker* can be divided by $\varphi_w(1 +$

$\gamma) + \varphi_c \gamma$[4] to come to the marginal cost of effort for the boss. Equating marginal benefit and cost of effort yields the optimal contractible effort:

$$e^*_{contr} = \frac{\varphi_w P(1+\gamma)}{\theta} \tag{4.5}$$

If effort can be contracted, the optimal effort level depends positively on the income after-tax rate. The reason for this is as follows: if the after-tax rate increases, it becomes cheaper to hire labor. The employee namely demands a certain amount of after-tax salary. If the income tax rate decreases, the pre-tax salary can decrease as well. Thus, if the income after-tax rate increases, the marginal costs of demanding effort decreases, leading to a higher optimal contractible effort. If the unit price increases, similarly, the marginal benefit of demanding effort increases. The marginal costs also go up via envy, but by less. So, the optimal effort level will increase in price. Thirdly, when envy increases, the optimal contractible effort also increases. The reason for this is that base salary is liked by the worker not only because the worker gets richer, but also through envy: the earnings gap between the boss and the worker decreases, which increases utility. If the envy term rises, this second effect becomes larger, meaning that the marginal cost of requiring effort decreases, resulting in a larger optimal contractible effort. Lastly, the optimal contractible effort falls in cost of effort. This makes sense, as cost of effort has to be reimbursed by the employer, making it a (marginal) cost to him. If costs of effort rise, the marginal costs of demanding effort increase, leading to a lower optimal effort. Note that the corporate tax rate is not related to the optimal effort here. The reason is as follows: if φ_c increases, envy cost per unit of effort increases, but the rise in the sensitivity to income exactly compensates this, leading the optimal effort to be unchanged.

I will refer back to this section a couple of times later on, particularly to compare the outcomes obtained with the first-best effort level derived here.

5. Non-contractible effort analysis

From now on, effort will be considered non-contractible, since it is very hard for an employer to demand a precise level of effort and enforce it. This is even more so in this model, where effort is

[4] This term can also be called the “sensitivity to income”.

defined as *expected* output, which is subjective in nature. And if the level of effort is to be enforced, the effort level has to be determined from *realized* output, which is influenced by random shocks.

5.1. Optimal Effort

The game presented in section 3. is solved using backward induction. Therefore, the optimal effort should be derived first, considering all other endogenous variables fixed. Negative effort is not possible, but the corner solution of zero effort is attainable. Zero effort would imply that the boss has an expected profit of $-\varphi_c a$. The boss will therefore only offer a contract with a negative base salary, otherwise he would make a loss[5]. The worker will not face any risk, since he exerts no effort anyway, resulting in a variable pay of zero by definition. Summing up, expected utility of the worker in case of zero effort would be the following:

$$U|_{e=0} = a\left(\varphi_w(1+\gamma)+\varphi_c\gamma\right) < 0 \tag{5.1}$$

In line with what was derived in the previous section, (5.1) also shows that the worker benefits (or loses in this case) three times from the base salary. The worker will only accept this negative utility contract if his outside option yields even lower payoffs. I will not consider such cases, since they are not quite as interesting in terms of incentive contracts. Therefore, I assume from now on that $U|_{e=0} < \bar{U} \leq U|_{e=e^*}$, which is sufficient to let the optimal effort in this model be larger than zero.

The worker's optimal effort follows from the first-order condition, given by $dU/de = 0$, which corresponds to the following:

$$\frac{dU}{de} = \varphi_w b(1+\gamma) - \theta e - \varphi_c\gamma(P-b) = 0 \tag{5.2}$$

Solving for e yields:

$$e^* = \frac{\varphi_w b(1+\gamma) - \varphi_c\gamma(P-b)}{\theta} \tag{5.3}$$

The second-order condition that checks optimality of this effort level is presented in Appendix B.

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[5] Assuming the boss would refuse to give a contract which yields him expected profits of zero, for example because he faces transaction costs of signing a contract (small enough to be neglected in the model).

5.2. Comparative statics

To grasp the intuition behind the optimal effort level, it is helpful to look at some comparative statics. One by one, all partial derivatives of e^* will be presented in order to do so. Note that all these effects are at the margin and under the ceteris paribus condition. I will start with analyzing the effect of the bonus on effort. After that, it is assumed that the bonus is fixed as well.[6] The analysis starts with the derivatives of the variables that are of key interest in this research, and lastly, I present the derivatives for the other parameters. In Appendix C, the proofs are displayed for those inequalities with a positive or negative sign (without the addition "or equal to").

5.2.1. Bonus

The partial derivative of optimal effort with respect to bonus level is the following:

$$\frac{\delta e^*}{\delta b} = \frac{\varphi_w + \gamma\left(\varphi_w + \varphi_c\right)}{\theta} > 0 \tag{5.4}$$

As can be seen from equation (5.4), effort increases in the bonus for two reasons. The first term of (5.4) shows that a higher bonus leads to higher monetary income per unit of effort to the worker. Effort makes the worker richer, which yields him a higher utility. The other terms indicate that a higher bonus leads an additional gain in utility, namely through the envy term. The net earnings gap between the worker and the boss gives disutility of envy to the worker, which is decreased in two ways in this case: the worker earns more (second term) and the employer earns less (third term) per unit of effort. This is what was discussed also for the base salary in section 3. Overall, the marginal benefit of exerting effort increases as the bonus rises, leading to a higher optimal effort level.

This effect becomes stronger if the worker is more envious or if the after-tax rates are higher. If the worker is more envious, the disutility of the earnings-gap decreases more in each unit of effort. If the after-tax rates are higher, the earnings-gap closes faster in effort and, if it concerns the income after-tax rate, the worker also gets richer than before for each unit of effort. I will refer back to this in section 6.2., where the bonus is allowed to vary.

[6] This assumption will be relaxed in section 6.

5.2.2. Corporate tax

Taking the partial derivative of optimal effort with respect to the corporate after-tax rate yields:

$$\frac{\delta e^{*}|_{b=b_0}}{\delta \varphi_c} = \frac{-\gamma(P-b)}{\theta} \leq 0 \tag{5.5}$$

Increasing the corporate after-tax rate decreases optimal effort, as can be seen in equation (5.5). If the corporate tax rate is decreased, the employer earns more for every unit of effort, while the worker earns an equal amount. This decreases the marginal benefit of effort via envy, because the worker-boss earnings gap is higher for every level of effort. This leads to a lower optimal effort level. The effect can also be zero, namely if $\gamma = 0$ and/or $P = b$. This is so because the worker does not care about the employer's income if $\gamma = 0$, and if $P = b$, exerting effort has no effect on the employer's income anyway.

5.2.3. Income tax

Doing the same as before for the income after-tax rate yields:

$$\frac{\delta e^{*}|_{b=b_0}}{\delta \varphi_w} = \frac{b+\gamma b}{\theta} > 0 \tag{5.6}$$

Equation (5.6) shows that effort increases in the income after-tax rate for again two reasons. Actually, the reasons are very similar to the reasons described in section 5.2.1. The term b indicates that decreasing the income after-tax rate benefits the worker directly through an increase in net monetary payoffs per unit of effort: the worker's purchasing power rises. The term γb reveals that a second positive effect is at work, again through envy. The worker earns more per unit of effort while the income to the employer per unit of effort is unchanged, leading the boss-worker earnings gap to shrink more quickly in effort. Thus, the marginal benefit of exerting effort rises for an increase in the income after-tax rate, leading to a higher optimal effort level.

5.2.4. Envy

So far, all the derivatives with respect to optimal effort had a clear sign. In contrast, it is not immediately clear what happens to the optimal effort if envy increases. To illustrate, the partial derivative of optimal effort with respect to envy is presented:

$$\frac{\delta e^{*}|_{b=b_0}}{\delta \gamma} = \frac{\varphi_w b - \varphi_c (P-b)}{\theta} \tag{5.7}$$

There are two effects here, working in opposite directions. The first term of equation (5.7) shows the positive effect. As seen before in 5.2.1. and 5.2.3., increases in effort lead (via the bonus) to higher utility also because the worker's earnings come closer to that of the boss. That effect is at work here too. If the envy parameter rises, this effect becomes stronger, meaning that the marginal benefits of exerting effort may rise. The second term of (5.7) shows the negative effect on effort of an increase in envy. This works as follows. The worker gets disutility from the earnings gap in the way described in section 5.2.2. If the worker is more envious, he is more sensitive to the earnings gap, so the decrease in utility from envy per unit of effort becomes stronger. This could cause the marginal benefit of exerting effort to decline.

Which effect dominates depends on the distribution of revenues arising from the effort of the worker, or "the additional revenue sharing rule". It is instructive to look at some special cases to explore this. First, if $b = 0$[7], meaning that all the additional revenues are for the boss, the negative effect will dominate. The positive effect is completely non-existent here because exerting effort does not yield any additional income to the worker anyway. Since the only relevant result of effort exertion in this case is that the earnings gap grows, exerting effort yields the worker disutility. If envy is higher, the worker gets even more disutility from working. Therefore, at $b = 0$, higher envy leads to lower effort. However, if $b = P$[8], things are different. Now, the worker can keep all the additional revenues derived from exerting effort. The earnings gap will then clearly decrease. So in this case, only the positive effect described before is at work. If the envy parameter would be larger, the worker would enjoy this decrease in earnings gap more, so the marginal benefits of exerting effort would rise, resulting in a higher optimal effort.

Since equation (5.7) is linear in b, it is clear that the derivative of optimal effort with respect to envy increases from extreme case $b = 0$, where it is negative, until extreme case $b = P$, where it is positive. Somewhere in between, there is a bonus level for which the derivative of optimal effort with respect to envy is equal to zero. Equation (5.7) tells us that this level is exactly at the

[7] An attentive reader will notice that a bonus of zero yields to an optimal effort level of zero in this model, which was flagged as unrealistic before. However unrealistic, this state of the world provides a useful starting point in analyzing the effect of envy on optimal effort.

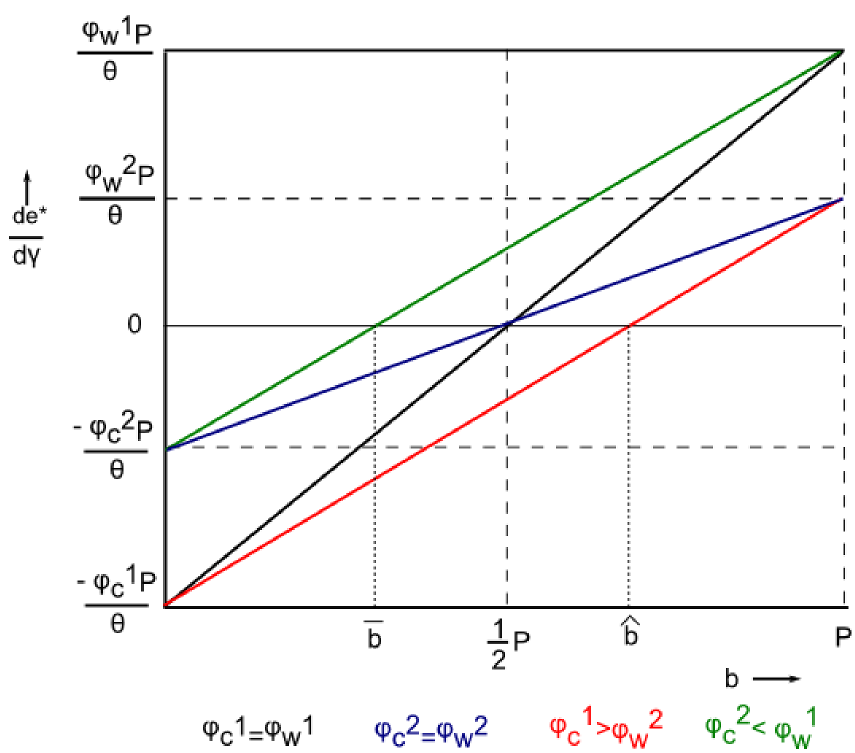
[8] Also this case is used for illustrative purposes, not because this state is highly likely to occur.

point where the distribution of additional revenues is equal after tax, because the first term displays the after-tax share for the worker, and the second term that for the employer. To determine what this level is, the after-tax rates come into play. Consider for example the case where $b = \frac{1}{2}P$, the boss and the worker share additional revenue equally, in pre-tax terms. If that is the case, envy may or may not lead to more effort, depending on which after-tax rate is higher. Equation (5.8) illustrates this:

$$\frac{\delta e^{*}|_{b=P/2}}{\delta\gamma} = \frac{P(\varphi_w - \varphi_c)}{2\theta} \tag{5.8}$$

If $\varphi_w > \varphi_c$, higher envy leads to higher effort at $b = \frac{1}{2}P$ and vice versa. Clearly, it is not the gross distribution that matters in real decisions like the effort decision, but the net distribution. What we learn from the above example is that effort will be more likely to increase in envy if the tax rate for the worker is lower than that for the boss. The intuition is as before: if the worker can keep more money for himself (either through a higher bonus, or in this case, a lower income tax rate), he can reduce the earnings gap more by exerting effort. Similarly, if the corporate tax rate is lowered, the earnings gap is closed less by exerting effort.

FIGURE 1: Effect of envy on effort



So, both taxes and the bonus are of essence in determining the net effect of higher envy on optimal effort. Putting these two effects together in a single graph (Figure 1) provides a schematic view of how the effect of envy on effort operates. In this figure, both after-tax rates can take two values, namely $\varphi_c^1 = \varphi_w^1 > \varphi_c^2 = \varphi_w^2 > 0$. The graph clearly indicates that the effect of higher envy on effort is increasing in b, whatever values both parameters take. If the tax parameters are equal, as the black and blue lines indicate, the effect is zero at the (gross) equal sharing rule: $b = \frac{1}{2}P$. So, the effect is positive in the range $b \in \left[\frac{1}{2}P, P\right]$. If taxes differ, this is not the case. If the after-tax rate for the worker is higher than that of the boss, as in the green case, the effect of higher envy is positive in the range $b \in [\bar{b}, P]$, where $\bar{b} < \frac{1}{2}P$. Analogously, if the after-tax rate of the worker is lower, as the red line shows, the effect is positive only in the range $b \in [\hat{b}, P]$, where $\hat{b} > \frac{1}{2}P$.

All in all, at low levels of the bonus ($b < \bar{b}$), the effect of higher envy on optimal effort is negative. At medium levels ($\bar{b} < b < \hat{b}$), the sign of the effect depends on how the after-tax rates compare. At high levels ($b > \hat{b}$), the effect is positive. Again, what ultimately makes additional envy increase optimal effort is an after-tax additional revenue sharing rule that yields the worker more income than the boss for each additional unit of effort.

Now that the key variables have been considered, the next two subsections show how effort is affected if the remaining parameters change, given that the bonus is unchanged.

5.2.5. Unit price

Taking the derivative of optimal effort with respect to output price yields:

$$\frac{\delta e^* |_{b=b_0}}{\delta P} = \frac{-\varphi_c \gamma}{\theta} \leq 0 \tag{5.9}$$

An increase in output price yields lower effort. The reason for this is that a higher price gives the employer a higher income per unit of effort, while that for the employee is unchanged. So, a higher unit price decreases marginal benefit of effort through envy. There is no effect if $\gamma = 0$.

5.2.6. Cost of effort parameter

The last of the partial derivatives is the one with respect to the cost of effort parameter:

$$\frac{\delta e^*|_{b=b_0}}{\delta\theta} = \frac{\varphi_c\gamma(P-b)-\varphi_w b(1+\gamma)}{\theta^2} < 0 \tag{5.10}$$

Optimal effort declines as the cost of effort parameter increases. The derivative in this case mainly shows the magnitude of the effect, but not really the intuition. Disregarding (5.10) then, the straightforward intuition behind the lower optimal effort is that a higher θ leads to a higher marginal cost of exerting effort.

5.2.7. Overview

It appears that several parameters have an effect on optimal effort, of which the sign differs per parameter or variable. However interesting this analysis is, it only holds if the bonus level stays fixed. This assumption makes the analysis less general. Still, two cases can be identified where the analysis still holds. The first case is that the worker is risk-neutral[9]. The second is that the bonus has an upper (or lower) limit due to legal, cultural or organizational institutions, and can therefore not increase (decrease) any further. An example of a legal restriction on the bonus is in institutions in the Netherlands of which the state owns all the shares. There, the variable compensation to directors may not exceed their fixed compensation by law (Rijksoverheid 2012). An example of a cultural constraint could be the recent dissatisfaction in the Western World about the "bonus culture" (see, for example, Parker 2012). The relation between the bonus and organizational institutions was researched by Kerr and Slocum (1987), who found that setting a reward system is effective in creating the desired corporate culture. Thus, the bonus is not only used to maximize profit for a given worker, but also to select the appropriate workers into the firm by creating a corporate culture that these employees like. The latter goal might be more important than the former for some firms, leading the bonus to be more or less fixed.

This list of examples is by no means exhaustive, but it illustrates that the analysis of this section is still applicable in many cases. That said, the analysis above will not hold if no restrictions to the bonus are in place. In order to find out what happens then, the fixed bonus assumption is relaxed in the next section.

[9] Risk neutrality would in this model mean that $b^* = P$ and the bonus is therefore "fixed".

6. Contract design analysis

6.1. The optimal contract

Since the optimal effort is now determined and analyzed given the bonus level, it is time to determine the optimal contract. Again, the boss will extract all surplus from the worker, while still ensuring that the worker participation constraint is satisfied. In other words, the employer will set the base salary a so that the worker's utility equals his outside option utility $\overline{U}$. It can be shown that the equilibrium base salary is the following:

$$a^* = \frac{2\overline{U} + R\sigma^2\left(\varphi_w b\right)^2}{2\left(\varphi_w(1+\gamma)+\varphi_c\gamma\right)} - \frac{\left(\varphi_w b(1+\gamma) - \varphi_c\gamma(P-b)\right)^2}{2\theta\left(\varphi_w(1+\gamma)+\varphi_c\gamma\right)} \quad (6.1)$$

However interesting this expression may look, the base salary has no incentive effects on effort: the employee considers this to be fixed. From the employer's point of view, however, the base salary is not fixed, since it varies in the bonus level he sets. Thus, in setting the optimum bonus, the employer should maximize the following objective function:

$$\pi = \varphi_c\left[\left(P-b\right)e^*(b) - a^*(b)\right] \quad (6.2)$$

As can be seen from equation (6.2), the profits are influenced by b in three ways. The first b in line refers to the fact that a higher bonus means a higher marginal cost, keeping output fixed. The second one remembers us about section 5.2.1, namely that optimal effort is positively related to the bonus, so this yields a marginal benefit to the employer. The last one refers to the base salary varying in b. To see whether the base salary part yields a marginal benefit or marginal cost to the employer, its derivative with respect to b is displayed:

$$\frac{da^*}{db} = \frac{R\sigma^2\varphi_w^2 b}{\varphi_w(1+\gamma)+\varphi_c\gamma} - \frac{\left(\varphi_w b(1+\gamma) - \varphi_c\gamma\left(P-b\right)\right)\left(\varphi_w(1+\gamma)+\varphi_c\gamma\right)}{\theta\left(\varphi_w(1+\gamma)+\varphi_c\gamma\right)} = \frac{R\sigma^2\varphi_w^2 b}{\varphi_w(1+\gamma)+\varphi_c\gamma} - e^* \quad (6.3)$$

The expression has two terms working in opposite directions. The sign of this derivative is therefore undetermined a priori. The first term of equation (6.3) shows the negative effect of a bonus increase on utility through increased risk. This increase in risk has to be compensated by an increase in the base salary. From section 3 we know that the worker benefits three times from the base salary so, risk does not have to be compensated one-to-one. The second term shows the positive effect the bonus has on utility. This increase in utility can then be extracted by the employer via a lower base salary. The worker gets a higher utility, since he can keep more of the revenues he produced. This leads him to be better off because his purchasing power rises and because envy cost decreases. The benefit is multiplied by $\varphi_w(1+\gamma)+\varphi_c\gamma$, the sensitiveness of effort to the bonus. This is done because the bonus also increases optimal effort, meaning that the benefit described before does not only count for the effort exerted before, but also for the additional effort induced by the higher bonus. This term, however, cancels out against the denominator[10], giving the negative of the optimal effort as result.

Equation (6.3) is larger than zero if R, σ^2 or θ is sufficiently large. The intuition for this is the following: when the employee sufficiently dislikes risk (through R) or is exposed to a sufficient amount of risk (σ^2), the first term discussed above will weigh more than the second term. If θ is large, effort will be low, as can be seen from section 5.2.6. In that case, the additional utility of increasing the bonus is not so high, because the revenues from which the worker can keep more are not so high in the first place. If the initial effort is low enough, the risk effect of increasing the bonus will dominate.

A remark to this story has to be made. It sounds very unlikely that an increase in the bonus would require the base salary to also increase in order to stay at the same level of utility. If a questionnaire would be held asking whether subjects would prefer the initial base salary plus bonus or a higher base salary and a higher bonus, it would be striking to find anyone that opts for the initial situation, given that the job characteristics do not change. Therefore, the marginal effect of the bonus on a^* will likely be negative.

Now, taking into account the three effects, the optimal bonus can be determined using the following first-order condition: $d\pi/db = 0$, which can be rewritten as:

$$\frac{d\pi}{db} = (P-b)\frac{de^*}{db} - e^* - \frac{da^*}{db} = 0 \tag{6.4}$$

[10] This makes sense, because the worker benefits three times from bonus income (numerator) and also three times from base salary income (denominator).

Notice that $-e^*$ cancels out against the second term of da^*/db (equation (6.3)). After inserting the derivates on the right hand side, the derivative of profits with respect to bonus becomes:

$$\frac{d\pi}{db} = \left(P - b\right)\frac{\varphi_w(1+\gamma)+\varphi_c\gamma}{\theta} - \frac{R\sigma^2\varphi_w^2 b}{\varphi_w(1+\gamma)+\varphi_c\gamma} = 0 \tag{6.5}$$

From this expression, it is straightforward to arrive at the optimal bonus level by solving for b:

$$b^* = \frac{P\left(\varphi_w\left(1+\gamma\right)+\varphi_c\gamma\right)^2}{\left(\varphi_w\left(1+\gamma\right)+\varphi_c\gamma\right)^2 + \theta R\sigma^2{\varphi_w}^2} \tag{6.6}$$

Appendix B shows that this value is indeed the optimal, non-zero bonus.

6.2. Comparative statics

Now that the optimal bonus is determined, it is time to look at what happens to effort if a variable changes or turns out to be different than expected. The same approach as in section 5.2. is adopted to do this analysis, namely via partial derivatives that assume a ceteris paribus environment and show the effect at the margin. The only difference now is that the bonus is no longer exogenous. As displayed in section 5.2.1., effort increases in the bonus. Therefore, if the effect of a change in a variable on effort is to be analyzed, both the direct effect on effort, which was found in section 5.2., and the indirect effect through the bonus have to be considered. It can be shown that the total partial derivative of e^* with respect to variable x_j can be represented by:

$$\frac{\delta e^*}{\delta x_j} = \frac{\delta e^* \,|\, b = b_0}{\delta x_j} + \frac{\delta e^*}{\delta b}\cdot\frac{\delta b^*}{\delta x_j} \forall x_j \in \mathbf{X} \tag{6.7}$$

X represents the vector that includes all variables which influence e^* directly or indirectly through the bonus. The first term of equation (6.7) is the direct effect of variable x_j on e^*, the interpretation of which is already given in section 5.2. The term $\delta b^*/\delta x_j$ denotes how the variable changes the optimal bonus. If this term is then multiplied by $\delta e^*/\delta b$, the sensitiveness of e^* to a change in the bonus (see equation (5.4)), one gets the indirect effect of x_j on e^*. This section will be devoted to displaying and interpreting the effect on the optimal bonus and to determining the sign of the complete derivative for all variables used. I continue the structure of section 5.2. by starting with the derivatives of the key variables.

6.2.1. Corporate tax

The partial derivative of the optimal bonus with respect to the corporate after tax-rate is the following:

$$\frac{\delta b^*}{\delta \varphi_c} = \frac{2\theta R\sigma^2 \varphi_w^2 \gamma P\left(\varphi_w\left(1+\gamma\right)+\varphi_c\gamma\right)}{\left(\left(\varphi_w(1+\gamma)+\varphi_c\gamma\right)^2+\theta R\sigma^2\varphi_w^2\right)^2} \geq 0 \tag{6.8}$$

Equation (6.8) poses that the optimal bonus increases if the corporate tax rate is increased. To grasp the reason behind this, going back to equation (6.6) is helpful. The denominator of the optimal bonus consists of the numerator plus a term without φ_c in it. Consider the illustrative (but fictious) case that $\varphi_c \to \infty$. In that case, the numerator will converge to the same value as the denominator, leading the optimal bonus to become arbitrarily close to P. This conversion to P holds for all values of φ_c.

Now for the intuition. Taking a look back at (5.4), it can be seen that the effect of the bonus on effort increases in φ_c. This makes providing incentives more effective at the margin. Also, if the corporate tax decreases, one of the three sensitivities to income increase, meaning that the worker benefits more from each unit of the base salary. Thus, the salary to ensure the worker still accepts the contract can go down for each level of the bonus. So the optimal bonus increases. If R or σ^2 is equal to zero, the derivative is zero because the optimal bonus would be equal to price already. If $\gamma = 0$, the worker does not care about a change in the after-tax rate of the boss, so incentives will not become more effective as φ_c rises, and the sensitivity to income way also stay equal.

From before, it is known that the direct effect of φ_c on optimal effort is negative, because of an increase in envy. The indirect effect thus works in the other direction. To see which of the two effects dominate, the complete partial derivative of effort with respect to the corporate after-tax rate is displayed:

$$\frac{\delta e^*}{\delta \varphi_c} = \frac{R\sigma^2\varphi_w^2\gamma P\left(\left(\varphi_w(1+\gamma)+\varphi_c\gamma\right)^2-\theta R\sigma^2\varphi_w^2\right)}{\left(\left(\varphi_w(1+\gamma)+\varphi_c\gamma\right)^2+\theta R\sigma^2\varphi_w^2\right)^2} \tag{6.9}$$

This expression is positive if and only if $(\varphi_w(1+\gamma)+\varphi_c\gamma)^2 > \theta R\sigma^2\varphi_w^2$, which is identical to $b^* > P/2$. This makes sense, because the closer the initial bonus is to the price level, the weaker the direct (negative) effect is, as can be seen from equation (5.5). The positive indirect effect through the bonus will thus dominate if the bonus is high. If the bonus is low, the direct negative effect dominates by similar arguments. The threshold appears to be at the (gross) equal sharing rule of revenues.

6.2.2. Income tax

Taking the partial derivative of the bonus with respect to the income after-tax rate yields:

$$\frac{\delta b^*}{\delta \varphi_w} = -\frac{2\theta R\sigma^2\varphi_w\varphi_c\gamma P\left(\varphi_w(1+\gamma)+\varphi_c\gamma\right)}{\left(\left(\varphi_w(1+\gamma)+\varphi_c\gamma\right)^2+\theta R\sigma^2\varphi_w^2\right)^2} \leq 0 \tag{6.10}$$

Decreasing the income tax rate decreases the optimal bonus. At first, this might sound odd, since a decrease in the income tax increases $\delta e^*/\delta b$, as derived in section 5.2.1. From intuition, it would seem logical then to increase the bonus to take advantage of this. However, one must not forget that a decrease in income tax increases volatility of income through the increase in the proportion of the bonus money to be kept (or lost) after-tax. The increased risk has to be compensated via the base salary, which harms the employer. Equation (6.10) shows that the latter effect dominates the positive effect on the bonus mentioned before.

The reason behind this is somewhat less intuitive than what was encountered before. Suppose an initial equilibrium where both players maximize payoffs and a^* is set so that $U(b_1^*, e_1^*, \gamma > 0) = \bar{U}$. Now, suppose the income after-tax rate rises. This increases risk per unit of the bonus quadratically, since the worker is increasingly risk averse. This has to be compensated via the base salary. Recall that the worker benefits three times from the base salary: directly in terms of a purchasing power increase, and indirectly through envy because the worker gets richer and the boss poorer. Following the increase in φ_w, two of the three "sensitivities to income" rise linearly: less tax means that the purchasing power increase and envy decrease are stronger for each unit of income. However, the last "sensitivity", $\varphi_c\gamma$, does not change. This implies that an increase in φ_w yields more utility for each unit of the base salary, but the increase is less than linear. If the after-tax rate rises, the boss therefore does not have to increase the risk compensation (per unit of the bonus) quadratically, but he will have to increase it *more than linearly*. Taking a look at (6.5), this means that the second term, the marginal cost to the boss of providing a bonus, rises more than linearly. A quick look at the first term of that same equation reveals that the marginal benefit of providing the bonus also rises in φ_w. This is just equal to $(P - b)$ times the derivative of effort to the bonus. As can be seen in (5.4), a decrease in income tax increases this derivative, but *less than linearly*. The overall result of these two effects is that the bonus decreases.

This only holds, however, if R, σ^2 are non-zero, otherwise the discussed negative impact through risk is not present. The positive effect also cannot be reaped because bonus then is already equal to price. If $\gamma = 0$, the effect is also absent. In that case, an increase in φ_w will still cause the risk term for each unit of the bonus to grow quadratically. However, the sensitivity to income will now rise linearly in φ_w, since the worker no longer cares about the worker-boss earnings gap, so he only benefits from income because it gives him purchasing power. Therefore, the sensitivity to income is just equal to φ_w. This means that the marginal cost of providing incentives increases linearly in the after-tax rate. The same line of reasoning yields that the marginal benefit of providing incentives also increases linearly in the after-tax rate. Thus, the ratio of marginal benefit to marginal cost will remain unchanged for all levels of φ_w, meaning that the optimal bonus is independent of the income tax after-tax rate if $\gamma = 0$.[11]

From section 5.2.3., we know that the direct effect of a decrease in the income tax on optimal effort is positive because the worker gets more purchasing power and reduces the earnings-gap. It is time to find out whether this positive direct effect is stronger or weaker than the negative indirect effect through the bonus, by representing the complete partial derivative of optimal effort with respect to the income after-tax rate:

$$\frac{\delta e^*}{\delta \varphi_w} = \frac{P\left(\varphi_w(1+\gamma)+\varphi_c\gamma\right)^2 \cdot \left(\left(\left(\varphi_w(1+\gamma)+\varphi_c\gamma\right)^2 + \theta R\sigma^2\varphi_w^2\right)(1+\gamma) - 2\theta R\sigma^2\varphi_w\varphi_c\gamma\right)}{\theta\left(\left(\varphi_w(1+\gamma)+\varphi_c\gamma\right)^2 + \theta R\sigma^2\varphi_w^2\right)^2} \tag{6.11}$$

Thus, the overall effect on effort of a change in the income tax rate is ambiguous. If risk or the sensitivity to risk is low, (6.11) will be positive, since the indirect effect through the bonus will then be small.

[11] Notice that the true condition is $\varphi_c\gamma = 0$, but it was assumed that the corporate after-tax rate is positive.

6.2.3. Envy

To explore the effect of the last of the three main variables, the partial derivative of the optimal bonus with respect to the envy factor is displayed:

$$\frac{\delta b^*}{\delta \gamma} = \frac{2\theta R\sigma^2 \varphi_w^2 P\left(\varphi_w(1+\gamma)+\varphi_c\gamma\right)\cdot\left(\varphi_w+\varphi_c\right)}{\left(\left(\varphi_w(1+\gamma)+\varphi_c\gamma\right)^2+\theta R\sigma^2\varphi_w^2\right)^2} \geq 0 \tag{6.12}$$

The partial derivative turns out to be positive. The reason for this, in terms of mathematics, is the same as with the corporate after-tax rate: $\lim_{\gamma\to\infty} b^* \to P$. The intuition behind the positive sign is that if envy increases, like in section 5.2.1., the effect of an increase in the bonus on effort increases. Also, the sensitivity to income increases, so the base salary can go down for each level of the bonus. Thus, in order to profit from this increase in marginal benefit of the bonus, the boss increases incentives. Again, this result only holds if the bonus was not equal to P already.

Recall from section 5.2.4. that the direct effect of envy on effort depended on the bonus and how the after-tax rates compare. If we combine this direct effect with the positive indirect effect through the bonus, the following complete partial derivative can be derived:

$$\frac{\delta e^*}{\delta \gamma} = \frac{P\left(\varphi_w\left(\varphi_w(1+\gamma)+\varphi_c\gamma\right)^4+\theta R\sigma^2\varphi_w^2\left(\varphi_w(1+\gamma)+\varphi_c\gamma\right)^2\cdot\left(3\varphi_w+\varphi_c\right)-\theta^2R^2\sigma^4\varphi_w^4\varphi_c\right)}{\theta\left(\left(\varphi_w(1+\gamma)+\varphi_c\gamma\right)^2+\theta R\sigma^2\varphi_w^2\right)^2} \tag{6.13}$$

Equation (6.13) is a large expression from which it is tough to interpret each component mathematically. However, the intuition is not so complicated. The expression is positive unless the last term is very large. This is the case if θ, R or σ^2 are very large. This implies that the bonus is low, which will formally be shown in sections 6.4.5. to 6.4.7. If the bonus is low, as seen in section 5.2.4, the direct effect on effort will be strongly negative, enough to dominate the positive effect arising through the bonus.

6.2.4. Unit price

Now that the three main variables have been considered, the focus is shifted to the other parameters. To start off, the unit price is considered. The derivative of the optimal bonus with respect to price is the following:

$$\frac{\delta b^*}{\delta P}=\frac{\left(\varphi_w\left(1+\gamma\right)+\varphi_c\gamma\right)^2}{\left(\varphi_w\left(1+\gamma\right)+\varphi_c\gamma\right)^2+\theta R\sigma^2{\varphi_w}^2}>0 \tag{6.14}$$

The derivative is positive, because an increase in price increases the marginal benefit of effort to the employer. In order to reap these benefits, the worker has to put more effort in, so the boss increases incentives.

Found above was that the direct effect on effort of an increase in price is a decrease in optimal effort. Combining this with the negative effect via the bonus yields the complete derivative:

$$\frac{\delta e^*}{\delta P}=\frac{\varphi_w\left(1+\gamma\right)\left(\varphi_w\left(1+\gamma\right)+\varphi_c\gamma\right)^2-\theta R\sigma^2{\varphi_w}^2\varphi_c\gamma}{\theta\left(\left(\varphi_w\left(1+\gamma\right)+\varphi_c\gamma\right)^2+\theta R\sigma^2{\varphi_w}^2\right)} \tag{6.15}$$

It is unclear whether the direct effect or indirect effect dominates. If θ, R, σ^2 or φ_w is large, the negative effect can dominate, since the bonus will not increase much in price. This leads the positive effect on effort via the bonus to be weak, while the negative direct effect is still operating.

6.2.5. Cost of effort parameter

The derivative of optimal bonus with respect to the cost of effort parameter is the following:

$$\frac{\delta b^*}{\delta \theta}=-\frac{R\sigma^2{\varphi_w}^2P\left(\varphi_w\left(1+\gamma\right)+\varphi_c\gamma\right)^2}{\left(\left(\varphi_w\left(1+\gamma\right)+\varphi_c\gamma\right)^2+\theta R\sigma^2{\varphi_w}^2\right)^2}\le 0 \tag{6.16}$$

Equation (6.16) is weakly negative because an increase in the cost of effort decreases the effect of the bonus on effort, see section 5.2.1. This means that the first term in equation (6.5) decreases, meaning that the relative importance of the risk term increases. This risk term is a marginal cost to the employer via the base salary, so the optimal bonus will decrease. If R or σ^2 is equal to zero, there is no risk that the worker worries about, so the bonus remains the same.

The direct effect of the cost of effort parameter on effort was negative also, so we know that the total effect has to be negative:

$$\frac{\delta e^*}{\delta\theta}=\frac{P}{\theta^2}\left(\varphi_c\gamma-\frac{\left(\varphi_w(1+\gamma)+\varphi_c\gamma\right)^3\left(\left(\varphi_w(1+\gamma)+\varphi_c\gamma\right)^2+\theta R\sigma^2\varphi_w^2\left(1+\theta\right)\right)}{\left(\left(\varphi_w(1+\gamma)+\varphi_c\gamma\right)^2+\theta R\sigma^2\varphi_w^2\right)^2}\right)<0 \qquad (6.17)$$

6.2.6 Risk

Now, the same as before is done for the risk factor, starting with the derivative of the optimal bonus:

$$\frac{\delta b^*}{\delta\sigma^2}=-\frac{\theta R\varphi_w^2 P\left(\varphi_w(1+\gamma)+\varphi_c\gamma\right)^2}{\left(\left(\varphi_w(1+\gamma)+\varphi_c\gamma\right)^2+\theta R\sigma^2\varphi_w^2\right)^2}\leq 0 \qquad (6.18)$$

An increase in risk yields a lower optimal bonus. The reason for the negative sign is that risk is disliked, leading the employer to have to offer a higher base salary per unit of the bonus if that risk increases. Thus, the employer lowers incentives to equate marginal costs to marginal benefits again. The effect is zero if the worker does not care about risk, which is the case if $R = 0$.

As can be seen from equation (5.3), risk is not directly related to effort, only via the bonus. Therefore, the complete derivative of effort with respect to risk is just (6.18) times $\delta e^*/\delta b$:

$$\frac{\delta e^*}{\delta\sigma^2}=-\frac{R\varphi_w^2 P\left(\varphi_w(1+\gamma)+\varphi_c\gamma\right)^3}{\left(\left(\varphi_w(1+\gamma)+\varphi_c\gamma\right)^2+\theta R\sigma^2\varphi_w^2\right)^2}\leq 0 \qquad (6.19)$$

6.2.7. Risk-aversion parameter

Lastly, the derivative of the optimal bonus is displayed for the risk-aversion parameter:

$$\frac{\delta b^*}{\delta R}=-\frac{\theta\sigma^2\varphi_w^2 P\left(\varphi_w(1+\gamma)+\varphi_c\gamma\right)^2}{\left(\left(\varphi_w(1+\gamma)+\varphi_c\gamma\right)^2+\theta R\sigma^2\varphi_w^2\right)^2}\leq 0 \qquad (6.20)$$

An increase in risk aversion leads to a decrease in the optimal bonus. The reason for this is that if risk-aversion rises, risk needs to be compensated more per unit of the bonus. Thus, the employer reduces the bonus promised to the worker. This effect is zero if there is no risk at all ($\sigma^2 = 0$).

As with risk itself, risk aversion is only indirectly related to effort via the bonus, so the derivative of optimal effort with respect to risk-aversion has to be (weakly) negative as well:

$$\frac{\delta e^*}{\delta R} = -\frac{\sigma^2 \varphi_w^2 P\left(\varphi_w(1+\gamma)+\varphi_c\gamma\right)^3}{\left(\left(\varphi_w(1+\gamma)+\varphi_c\gamma\right)^2+\theta R\sigma^2\varphi_w^2\right)^2} \leq 0 \tag{6.21}$$

6.2.8. Overview

Now that the effects of all the parameters on effort have been determined, it is time to compare optimal effort under the first-best, contractible effort (equation (4.5)) and second-best, non-contractible effort. By inserting the optimal bonus into (5.3), the resulting optimal effort can be determined:

$$e^* = \frac{P\left(\varphi_w(1+\gamma)+\varphi_c\gamma\right)^2\left(\varphi_w(1+\gamma)-\theta R\sigma^2\varphi_w^2\varphi_c\gamma\right)}{\theta\left(\left(\varphi_w(1+\gamma)+\varphi_c\gamma\right)^2+\theta R\sigma^2\varphi_w^2\right)} \tag{6.22}$$

Careful examination of equation (6.22) reveals that optimal non-contractible effort is identical to the first-best situation if and only if risk or risk-aversion is not present. This is consistent with the literature: the first-best solution can be obtained by letting the worker fully internalize all the costs and benefits of exerting effort (see, for example, Lazear and Gibbs 2009). If there is risk for the worker, however, some productivity will inevitably be lost.

As can be seen in (6.22), all the used parameters have an effect on effort. Since there are many different parameters at work, it is convenient to have a look at table 1, where all the direct and indirect effects described above on effort are displayed, and their resulting effect. Also, the effects with contractible effort have been repeated there. Note that if risk approaches zero, the resulting effect of the parameters approaches the effect obtained in the contractible effort case. If risk is completely eliminated, all effects of column four are equal to that of column five.

TABLE 1: Overview of parameter effects

Parameter or variable	Direct effect	Indirect effect through bonus	Resulting effect	Contractible effort
b	+	N/A	+	N/A
P	-	+	+ / -	+
R	No	-	-	N/A
γ	+ / -	+	+ / -	+
θ	-	-	-	-
σ^2	No	-	-	N/A
φ_c	-	+	+ / -	No
φ_w	+	-	+ / -	+

7. Applications

Now that both the optimal effort and bonus levels have been determined, it is time to devote some words to where this model can be applied. The focus will be on the main parameters of this paper, namely the corporate- and income after-tax rates and envy. Two cases will be considered: one where the bonus is initially supposed high (or risk is low) and one where the bonus is supposed low (or risk is high). This distinction is made because if the bonus initially is high, the indirect effect on effort via the bonus is more likely to dominate and the other way around. For the income after-tax rate, only a case is considered where the indirect effect dominates, since it is assumed that readers are knowledgeable about cases where a decrease in income tax would lead to more effort. Please note that these examples of the practical applicability of the model are by no means exhaustive.

7.1. Corporate tax

7.1.1 Buffett Tax

In several countries, such as France under the Hollande Administration (Lichfield 2012) and the US under Obama (Calmes, 2011), a special, high tax on millionaire[12] households has been

[12]This concerns households with an income of at least a million euros/dollars a year. In the USA, alternative thresholds have also been proposed.

proposed. This so called “Buffett Tax”, next to its tendency to produce a more equal distribution of income, has been credited for its efficiency, since the rich would have a hard time avoiding this tax which makes the deadweight loss small (see NEC 2012 for an overview of the scientific literature on this).

This research provides another reason why the support for the Buffett Tax is large. It was obtained that productivity increases in the corporate tax rate if and only if the bonus is less than half the output price. It is common knowledge that multiple firms do not pay any bonus at all, especially to lower level employees. And if they do, the per unit bonus is usually less than 50 percent of the price. Therefore, one should expect that an increase in corporate taxes, at least at the national level, increases productivity.

In order to find some crude evidence for this claim, simple regressions were done on data obtained from the OECD[13] concerning total corporate tax as percentage of GDP and real GDP per capita. Table 2 shows the results. The column “OECD” concerns a cross-section study of OECD countries in the year 2009. The last two columns represent time-series regressions concerning the years 2003 to 2009 in The Netherlands and the USA respectively.

Table 2: Regressions

	OECD	Netherlands	USA
Coefficient	3497	-67	1097
P-value	0.003	0.962	0.122
dependent var	Real GDP per capita	*independent var*	Corporate Tax %GDP

Table 2 shows that the effect of corporate taxes on real GDP per capita is significantly positive in the cross-section study, but in both time-series studies there is no effect, perhaps because of limited observations. This gives some evidence for the claim that increasing corporate taxes increases productivity. Given that most millionaires are employer by profession or indirectly through the owning of securities, this Buffett tax can act in the same way as a more general corporate tax, which in the end could mean higher national productivity.

[13] All OECD countries have been used in the dataset except for Mexico and Chile, of which no data was available.

7.1.2. Investment banking

Investment banking is one of those industries characterized by high levels of variable pay. The yearly survey done by www.wallstreetcomps.com (2011) showed that only the most junior investment bankers (Associates) earn more in fixed than in variable compensation. The most senior investment bankers interviewed (Senior Vice Presidents) earn over $550,000 in variable compensation in large US banks, more than twice their average base salary. This sector can therefore be typified as a meritocracy.

An interesting phenomenon can be inferred from the Bloomberg 20 Investment Bank ranking which scores banks according to the level of total fees collected. The ranking of 2009 (Onaran 2010) is presented in Table 3, with the country of headquarters in the last column.

Table 3: Bloomberg 20

Bloomberg 20					
Rank	**Bank name**	**HQ**	**Rank**	**Bank name**	**HQ**
1	JP Morgan Chase	USA	11	Royal Bank of Schotland	UK
2	Goldman Sachs	USA	12	BNP Paribas	France
3	Morgan Stanley	USA	"	HSBC	UK
4	BoA Merrill Lynch	USA	14	RBC Capital markets	Canada
5	Citigroup	USA	15	Lazard	USA
6	Credit Suisse	Switzerland	16	Wells Fargo	USA
7	Deutsche Bank	Germany	17	Société Générale	France
8	UBS	Switzerland	18	TD Securities	Canada
9	Barclays Capital	UK	19	Daiwa Securities	Japan
10	Nomura Holdings	Japan	20	Rothschild	UK

Striking is that 16 out of 20 top investment banks have their headquarters situated in a country where 2009 corporate tax as percentage of GDP is lower than or equal to the OECD average[14]. The banks in Switzerland and Canada are the four odd ones out. A straightforward reason for the relation between corporate taxes and fee collection is that a lower corporate tax results in less tax expenses, making it more likely that a bank will find any given job attractive to accept.

This paper can provide another reason for the observed relation. The investment banking industry could represent a setting where the bonus is high, presumably above 50 percent of the

[14] The same dataset as in section 7.1.1. is used to come to this conclusion.

commission ("selling price") of banking services. This paper showed that if the bonus of the employee was that high, a lower corporate tax rate would yield higher effort. This could have contributed to the flourishing of the banks situated in low corporate tax countries.

7.2. Income tax

Poverty trap

Azariadis (2006) has shown that the income gap between low income countries and advanced economies is ever widening, with a few exceptions. This falsifies the "ergodic growth hypothesis", which assumes that growth in the long run is not affected by institutional factors or periodic shocks. One could therefore say that the least developed countries are in a poverty trap.

Azariadis argues that low income countries are seen as more impatient, and thereby invest less. The required investments to achieve long-term growth cannot adequately be financed from abroad because of limited capital markets. Barrett, Carter and Little (2006) seek the cause of poverty traps in the limited availability and extremely prudent use of assets by the low income population, the latter having to do with building a cushion against the large risks present there.

The model introduced in this paper can also be used to explore why poverty persists, keeping in mind the existence of extreme risks in developing countries. The occurrence of natural disasters, political instability and criminality all contribute to this very high level of risk. Therefore, a low income tax, as derived in section 6.2.2., will likely bring about low effort[15]. Easterly and Rebelo (1993) estimate for all 32 developing countries they considered that the average effective marginal tax rate is below 10 percent. This estimate also includes the rich, which pull the average up disproportionately. Given the low level of the tax and the high level of risk, productivity will according to this paper's model be very low, which can contribute to the persistence of poverty.

7.3. Envy

7.3.1. Non-profit hospitals

In the United States, numerous studies on the comparison between for-profit and non-profit hospitals have been launched in the recent decades. Vaillancourt Rosenau and Linder (2003) did a survey of the 149 studies that compared the performance of the hospitals based on an assessment of access, quality, efficiency and amount of charity care. The authors report that non-

[15] Note that this only holds if the bonus was not already equal to zero.

profit hospitals in the studies come out best 59 percent of the time, as compared to 29 percent for for-profit hospitals. The studies report several reasons for the superiority of non-profit hospitals, among which intrinsic motivation of non-profit hospitals to serve the community, lower administrative costs, more benefit from volunteers and cheaper access to funds. The research done in this paper provides another mechanism why non-profit hospitals might fare better than for-profit hospitals.

Hospitals can be one of the examples mentioned in section 5.2.7. where the bonus is limited because of cultural institutions. Similarly, the costs of medical instruments are so high that it would be too risky to let doctors internalize the costs and benefits of medical care to a large extent. Most doctors therefore do not receive considerable pay for performance. Remember that high envy leads to high productivity if and only if the “additional revenue sharing rule” is favorable enough for the employee. In this case, this sharing rule will probably be unfavorable to the employee, so low envy would lead to high effort. Given that the residual claimant in a non-profit institution is the society at large, envy will likely be low as compared to for-profit hospitals where shareholders or managers are the residual claimant. Thus, reasoning from an envy perspective, in non-profit institutions effort may be higher, leading to better performance.

7.3.2. Franchising

Franchising can be seen as a way to provide high powered incentives to store managers (Lazear and Gibbs 2009, Besanko *et al.* 2010*)*, because they are allowed to independently operate the business of the company only for a fixed fee. (Almost) all other costs and revenues are kept by the franchisee (Michael 1999).

If a look is taken at the Entrepreneur Media Franchise 500 ranking (2012) one finds a typical pattern. This ranking ranks franchise companies in terms of financial performance and growth opportunities. Almost all franchisors in the ranking tend to start their business by operating “normally” for a couple of years or decades, before changing to the franchise structure[16]. The typical economic explanation of this phenomenon is that franchising is a way to counter agency costs (Brickley and Dark 1987, Besanko *et al.* 2010) such as employee monitoring. Given that the companies are growing and store distances become larger, these monitoring costs become larger

[16] A noteworthy exception is McDonalds, who started its franchising immediately after foundation.

(Fladmoe-Lindquist and Jacque 1995), so franchising becomes more likely after an initial growth phase.

This paper's model can add another dimension to this explanation, namely that being a larger company can make franchisees more productive. The argument is as follows. If companies grow, potential franchisees may become more envious. Known from before is that envy leads to larger productivity if the employee has a sufficiently favorable "additional revenue sharing rule". In franchising, this is likely to be the case, since the franchisee by definition will internalize (almost) all benefits and costs. Thus, an explanation for the fact that most companies do not start their franchising activities directly at their foundation is that they first try to increase envy, and after that they take advantage of this by offering high-powered incentives via franchising. There is one caveat to this story, namely that an increase in envy also requires a higher base salary, so the increase in productivity is not necessarily profitable. Perhaps this explains why some companies, such as McDonald's, do start the franchising right at the foundation of the company.

8. Conclusion and Discussion

This paper considered a principal-agent model where a boss offers a worker a contract consisting of a base salary and a bonus for each unit produced. The boss faces a proportional corporate tax, whereas the worker faces a proportional income tax. The worker gets utility from income not only because it increases his purchasing power, but also because his earnings compared to that of his boss decrease, since he is envious towards the boss. Envy and the two taxes affect the productivity of the worker in two ways. Firstly, the optimal effort changes, given the level of the bonus. Regarding this "direct effect", it was found that income tax decreases optimal effort, corporate tax increases optimal effort and the effect of envy is ambiguous. The direct effect of envy on effort is positive if and only if the worker receives a higher after-tax share of the additional revenue generated by effort than the boss. This direct effect is the only effect if the bonus is fixed because of cultural, legal or organizational institutions. If this is not the case, productivity will also be affected through the bonus: the "indirect effect". As the bonus is always positively related to productivity, only the effect of a variable on the bonus is required to determine the sign of the indirect effect that variable has on productivity. Income taxes and envy appeared to increase the bonus, so their indirect effects on effort are also positive. Corporate taxes were found to decrease the bonus. Overall, adding the direct effect and the effect through the bonus, all three variables had an ambiguous effect on productivity. This ambiguity has to do with risk coming from exogenous shocks that affect output. If risk is high, the bonus is low, leading envy to decrease productivity, and both taxes to increase productivity. If this risk is low, the effects have the opposite sign. The threshold where the sign changes for the corporate tax was found to be precisely at the case where the bonus is equal to half the output price. Such a well defined threshold was not found for the other parameters. If risk is absent, effort under the optimal contract is exactly the same as when effort is contractible: the first-best situation. In this case, the corporate tax no longer affects productivity, envy increases it and the effect of the income tax is negative. The results provide evidence for the efficiency of a tax on the rich such as the Buffett Tax in the USA. The research also gives an explanation why successful investment banks are situated in low corporate tax countries, and why poverty traps exist in the developing world. Lastly, this paper's model gives a reason for non-profit hospitals in the USA performing better than for-profit hospitals and for the observation that successful franchisors tend not to directly start with franchising upon foundation.

It was found that some effects of the tax rates are absent if envy is absent. Therefore, as expected, envy and tax rates interact. Still, including tax rates into the model does not change the result from previous research that the presence of envy results in more profit sharing through a higher bonus. But because tax rates are also involved, the net profit sharing rule is of more importance than the bonus alone. Also, the taxes affect productivity in line with conventional wisdom if risk is not too high. The conventional wisdom about taxes of any kind is that they decrease economic activity, as for example stated by Arthur Laffer (2004).

In terms of policy recommendations, this research found that it might be a good idea to raise corporate taxes. This can improve the productivity on a national scale, while the government debt and income inequality are reduced. This looks like a panacea to increase welfare. Still, governments should be careful not to increase the tax by too much, because this might induce companies to leave the country altogether. Perhaps a European wide increase in corporate taxes is more effective. This research does not provide strong evidence for or against a low level of income taxes, since it is difficult to determine whether the tax rate will increase or decrease productivity. The resulting effect will depend on the country under scope.

I recognize that my research has multiple limitations. First of all, it assumes that agents do not care about social comparisons amongst workers or amongst occupations. This could affect productivity, at least at the national level. Also, general equilibrium effects will probably arise if the model is interpreted at the national level, so the results should be interpreted with care. Secondly, assumed was that the boss will not leave the country if corporate taxes are too high. Perhaps the model could be augmented to incorporate this, in order to arise at an endogenous optimal corporate tax rate, since it is unlikely optimal for the government to increase the corporate tax rate to 99.99 percent. This can for example be done by installing a reservation utility for the boss as well. Additionally, the best outside option of the worker was assumed to be unemployment or work in another country. In reality, the best outside option of the worker will likely be working inside the same country. If the income tax rate changes, the reservation utility might therefore also change. Further research should take this into account. Another interesting suggestion for future research might be to introduce uncertainty on the account of the worker about the tax bracket where the boss is in. This could lead employers to signal their tax bracket to attract a certain type of worker. Still, these are but a few of the many avenues of future research concerning the principal-agent model with social interaction.

9. References

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10. Appendix

Appendix A: Symbols

Symbol	Restrictions	Meaning
$\boldsymbol{a}$	-	Base salary
$\boldsymbol{b}$	-	Per unit bonus
$\boldsymbol{e}$	≥ 0	Effort, in this case equal to expected output
$\boldsymbol{P}$	> 0	Parameter indicating unit price of output
$\boldsymbol{R}$	≥ 0	Parameter indicating the degree of risk aversion of the worker
$\boldsymbol{U}$	-	Expected utility function of the worker
$\boldsymbol{\bar{U}}$	$U\|_{e=0} < \bar{U} \leq U\|_{e=e^*}$	(Expected) utility of the best outside option of the worker
$\boldsymbol{x_j}$	-	Parameter included in $\boldsymbol{X}$
$\boldsymbol{X}$	-	Set that includes all variables or parameters related to effort, either directly or indirectly
$\boldsymbol{\gamma}$	≥ 0	Parameter indicating the sensitiveness of the worker to envy
$\boldsymbol{\theta}$	> 0	Parameter indicating the costliness of exerting effort
$\boldsymbol{\pi}$	≥ 0	Expected profit function, which equals employer’s expected utility
$\boldsymbol{\sigma^2}$	≥ 0	Parameter indicating the variance of random, exogenous shocks that affect output
$\boldsymbol{\varphi_c}$	> 0	Fraction of gross earnings by the employer retained after-tax, called corporate after-tax rate
$\boldsymbol{\varphi_w}$	> 0	Fraction of gross earnings by the worker retained after-tax, called income after-tax rate

Appendix B: Second-order conditions

B.1. Optimal effort

$$e^* = \frac{\varphi_w b(1+\gamma) - \varphi_c \gamma (P-b)}{\theta} \tag{10.1}$$

For this effort level to be a (global) maximum the second-order condition needs to be satisfied, $d^2U/de^2 < 0$, which can be written as:

$$\frac{d^2U}{de^2} = -\theta < 0 \tag{10.2}$$

This condition is necessary to arrive at an interior solution. This condition does not give up much generality, since exerting effort, at least at the margin, is always costly.

B.2. Optimal bonus

$$b^* = \frac{P\left(\varphi_w(1+\gamma)+\varphi_c\gamma\right)^2}{\left(\varphi_w(1+\gamma)+\varphi_c\gamma\right)^2 + \theta R\sigma^2{\varphi_w}^2} \tag{10.3}$$

This expression is strictly positive, since all parameters are non-negative, and $\varphi_{w,}$ φ_c, γ and θ are positive by assumption. Therefore $b^* > 0$. Section C.2. derives the same conclusion in a different way.

The second-order condition, $d^2\pi/db^2 < 0$, needs to hold for this value to be a (global) maximum. The condition boils down to:

$$\frac{d^2\pi}{db^2} = -\frac{\varphi_w(1+\gamma)+\varphi_c\gamma}{\theta} - \frac{R\sigma^2\varphi_w^2}{\varphi_w(1+\gamma)+\varphi_c\gamma} < 0 \tag{10.4}$$

Given that all parameters are nonnegative and $\theta, \gamma, \varphi_c$ and φ_w are positive, condition (10.4) must hold. Note that this condition also holds if γ is negative but close to 0. Thus, this condition will lose some generality, namely in the case that the worker gets utility from receiving less net income than his boss. But in any other case, the condition holds. *Q.E.D.*

Apppendix C: Proofs of inequalities

Known by assumption is that all parameters are non-negative.

C.1. Derivative of optimal effort to the Bonus

$$\frac{\delta e^*}{\delta b} = \frac{\varphi_w + \varphi_w \gamma + \varphi_c \gamma}{\theta} > 0 \tag{10.5}$$

Known is that $e^* = \frac{\varphi_w b(1+\gamma) - \varphi_c \gamma (P-b)}{\theta} > 0$. Given that $P \geq b \geq 0$, which is derived in (6.6), the second term of e^*, $\frac{-\varphi_c \gamma (P-b)}{\theta}$, is non-positive. Then, for the optimal effort to be positive, $\frac{\varphi_w b(1+\gamma)}{\theta}$ must be positive. This implies that φ_w is also positive. If this holds, $\delta e^*/\delta b$ must be positive also. *Q.E.D.*

C.2. Derivative of optimal effort to the Income after-tax rate

$$\frac{\delta e^* \mid b = b_0}{\delta \varphi_w} = \frac{b + \gamma b}{\theta} > 0 \tag{10.6}$$

From section C.1., known is that $\frac{\varphi_w b(1+\gamma)}{\theta} > 0$. For this to hold, $b > 0$. *Q.E.D.*

C.3. Derivative of optimal effort to the Cost of effort parameter

$$\frac{\delta e^* \mid b = b_0}{\delta \theta} = \frac{\varphi_c \gamma (P-b) - \varphi_w b(1+\gamma)}{\theta^2} < 0 \tag{10.7}$$

The derivative is just the optimal effort level divided by $-\theta$. Known from section C.1. is that $e^* = \frac{\varphi_w b(1+\gamma) - \varphi_c \gamma (P-b)}{\theta} > 0$. Any negative factor or quotient will invert the sign. *Q.E.D.*

C.4. Derivative of optimal bonus to unit price

$$\frac{\delta b^*}{\delta P} = \frac{\left(\varphi_w (1+\gamma) + \varphi_c \gamma\right)^2}{\left(\varphi_w (1+\gamma) + \varphi_c \gamma\right)^2 + \theta R \sigma^2 {\varphi_w}^2} > 0 \tag{10.8}$$

Known from sections B.2. and C.2. is that $b^* > 0$. The derivative is just equal to $\frac{b^*}{P}$**.** Given that b^* and P are positive, equation (10.8) also has to be positive. *Q.E.D.*