# GARCH modelling: Daily Spot Prices vs. 24 Hours Separately

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**Abstract**

This paper examine whether the use of the 24 hours separately generates more accurate volatility estimates and forecasts. We have used the GARCH(1,1), GARCH(2,2), GJR-GARCH(1,1), GJR-GARCH(2,2), and EGARCH(1,1). We have tried to solve the problem of explosive volatility process by performing several methods. The results show that the use of the 24 hours separately does not generate more accurate estimates of the conditional volatility. One-day ahead forecasts obtained with the different GARCH models for the daily spot prices are the worst forecasts when comparing with the one-day ahead volatility forecasts of the 24 hours separately. The results also show that the average one-day ahead volatility forecasts of the 24 hours separately generates significant better forecasts for the daily volatility of the returns than using the one-day ahead volatility forecasts of the daily spot prices.

Keywords: Electricity, Returns, Volatility, Outlier, GARCH, GJR-GARCH, EGARCH

# Contents

# 1 Introduction

In the last years electricity markets in numerous countries in the world are experiencing a deregulation process. The aim of this deregulation has been based on encouraging the electricity markets to competition. This process has changed the market pricing mechanism. The electricity prices are now determined by the interaction between supply and demand. The competition in the electricity markets should increase customer value, but it has caused the increase in volatility of electricity prices. The prices in the spot market has been dramatically influenced by this deregulation. Before this deregulation there was little uncertainty in electricity prices and the prices were predictable.

Electricity has become a regular commodity that can be bought and sold in electricity markets. The behaviour of electricity prices are quite complex and volatile when compared with other financial markets or commodities. Escribano et al. (2002) noted that the high volatility of electricity prices can be explained by some characteristics of electricity. One of the characteristics of electricity is the non-storability. Electricity cannot be stored for use in the future. For this reason supply and demand have to be continuously balanced. Sudden increase/ decrease in supply and demand will change the equilibrium price, because changes in supply and demand cannot easily be smoothed out. Another characteristic of electricity is the limited transportability. The limited transportability makes the transmission of electricity among regions impossible. The interaction between supply and demand plays an important role in the volatility of the prices. The demand for electricity is highly inelastic, due to the fact that electricity is a necessary commodity and the consumers do not have substitution possibilities. Electricity demand is also highly weather-dependent. Demand and supply curves are steep for electricity markets. For high levels of demand, an increase in demand or decrease in supply lead to a high price jump. This steepness of demand and supply curves results in price peaks and high volatility.

For risk management, portfolio management and option pricing issues volatility is of crucial importance. Volatility is also of crucial importance for the participants in the highly volatile electricity markets. The volatility process influences the pricing of derivative contracts traded. This highly volatile electricity markets have led to enormous market risks for the participants. It is very important to understand this volatility process in the electricity markets. The predictability of the variation in volatility may be useful for financial application such as Value at Risk (Jorion,2000). Thomas and Mitchell (2005) noted in their paper that high volatility creates uncertainty about the revenues of the generators and the cost of the retailers/distributors. They also emphasize that high price volatility can make capacity planning and investment decisions difficult for generators and operators of the distribution grid. Understanding the volatility is of crucial importance to sytsem operators and regulators to ensure that markets are designed and operated in a way that limits market power and promotes confidence and safety for market participants.

In recent years, researchers have examined the volatility in electricity markets. The volatility of electricity prices in countries, such Spain, California, England, Wales, and the PJM system are analyzed by Benini et al (2002). Mount (2001) noted in his paper that a uniform auction worsens the price volatility as compared to a discriminatory auction in the England & Wales system. Dahlgren et al. (2001) emphasize the need for price risk management for electric utilities, due to the fact that electricity markets are characterized by extreme short-term price volatility. Dahlgren et al. applied the Value at Risk(VaR), a method for quantifying price risk, in the Californian market. Lucia and Schwartz (2002) noted that volatility clustering is a characteristic of electricity markets. Clustered volatility motivate the use of GARCH models. Escribano et al. (2002) have examined the generalized autoregressive conditional heteroscedasticity (GARCH) to focus on the volatility of the daily spot prices. He also included jump-diffusion intensity parameters in their model specification. A multivariate generalized autoregressive conditional heteroscedasticity is also used to identify the source and magnitude of price and price volatility spillovers in Australian electricity spot markets (Worthington et al ,2005). Duffie et al.(1998) and Knittel and Roberts (2001) focus on diferent types of ARCH or GACRH models to energy prices. Thomas and Mitchell (2005) consider the GARCH variants inlcude the basic GARCH, TARCH, EGARCH and PARCH specifications. They examine the volatility process in Australian electricity prices. TARCH models are used to examine the volatility of the electricity prices for five US markets (Hadsel et al , 2004).

Chan and Gray (2006) use the EGACRH models to analyze the returns.

This paper focuses on the volatility in the Norwegian electricity market. The idea is that the volatility is not constant and not directly observable during the considered period. Our purpose is to develop time series models to estimate the volatility ($\sigma_t^2$) and forecast $\sigma_t^2$. We will make use of different GARCH models. The conditional volatility of a time series can be measured with a GARCH model. We will examine GARCH models in two different ways. In the first case we consider GARCH models fit to the returns of the 24 hours separately and in the second case we consider GARCH models fit to the returns of daily spot prices (average of the 24 hours). We will try to examine whether the use of the 24 hours separately generates more accurate volatility estimates and forecasts. The data of 24 hours separately is more valuable data than taking the average of the 24 hours. Our expectation is that the use of the 24 hours separately generates more accurate volatility estimates and forecasts.

This paper is organized as follows. Section 2 describes the dataset and the characteristics of electricity prices. Section 3 discusses the specification of different GARCH models and the evaluation of GARCH models. Section 4 discusses the estimation and the evaluation of volatility with different GARCH models. Section 5 discusses the volatility forecasts with different GARCH models. Section 6 presents the results of GARCH models obtained by two different ways and section 7 states our conclusions.

# 2 Data

In this paper we will work with data from Nord Pool Spot. Nord Pool Spot runs the largest power market in Europe and offers both day-ahead and intraday markets to its customers. There are trading 350 companies from 20 countries on this market. The Norwegian power market is no longer solely Norwegian since 1996. The Norwegian power market has had a common Scandinavian market for buying and selling electricity. Now all of Scandinavia has a single market with daily trading of spot and forward contracts over the Scandinavian power exchange Nord Pool in Oslo.[1]

## 2.1 Data Description

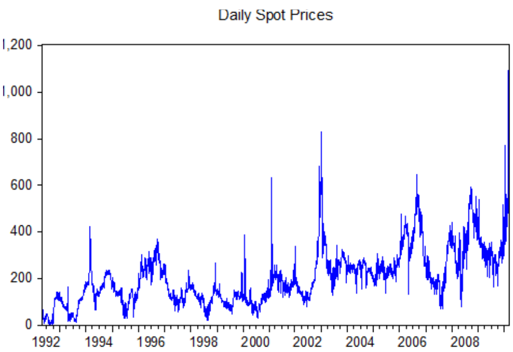
The data obtained from Nord Pool Spot consist twenty-four time series, one for each hour during a day, of daily (seven days a week) system prices. The prices are given in Norwegian kroner( NOK) per MWh with two decimal points. The data comprise electricity prices between May 4,1993 and March 9,2010,that is more than 18 years of data, totaling 6519 daily observations. The obtained data misses some values and the most of the missing values are around the fourth week of March at 3:00 AM. The incomplete data is caused by the transition from standard time to the summer time. The missing values are replaced by the mean of the preceding and following value for the electricity spot prices. We will generate a new time series by calculating the average of the 24 hourly prices and we will refer to this average as the daily spot price.

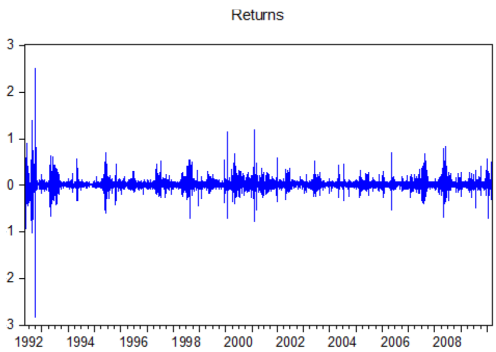
$$P_t = \frac{1}{24}\sum_{m=1}^{24} P_{t,m} \; with \; m = 1, ....., 24 \; and \; t = 1, ....., T$$

The daily spot prices and the first differences of log daily spot prices for the considered period are shown in figure 1. The (log)return is defined as the first differences of spot prices ($r_t = ln(P_t/P_{t-1})$). From the graph of the daily spot prices one can observe several spikes. For example, daily spot prices reached a maximum value of 1090.02 NOK/MWh in February 2010. The dynamics of the returns show clear patterns of volatility clustering. The basic statistical properties of the returns such as mean, skewness and kurtosis of the data are shown in table 1. The returns

[1] see www.nordpoolspot.com for more information

Figure 1: The dynamics of the daily spot prices and returns




*In the left chart the dynamics of the daily spot prices are displayed. The right chart displays the dynamics of the first differences of log daily spot prices (returns). The considered period is from May,4,1992 to March 9,2010,totaling 6519 daily observations.*

are obtained by using the daily spot prices. The mean of the returns takes on a value of 0.000343 (0.03%). The standard deviation of the returns is equal to 0.123043 (12,3%). This results in an annualized volatility of 2,3507 (235,07%), so the returns are very volatile[2]. The kurtosis is equal to 91,02, which indicates that the data frequently contains extreme returns. We reject the null hypothesis of normality because the kurtosis significantly differs from three. The returns have fat tails. Table A.1 in the Appendix shows the basic statistical properties of the returns obtained by using the 24 hours separately. The most volatile hours are 8 AM, 9 AM and 7 AM with a standard deviation of 23,28%, 22,96% and 21,18% respectively. The annualized volatilities for these hours are 444,7%, 438,68% and 404,62% respectively. The kurtosis of all hours significantly differs from three, so we reject the null hypothesis of a normal distribution of the returns. This means that the occurrence of extremely low and high returns has a higher probability than for a normal distribution with the same variance.

Table 1: Basic statistical properties

| | Returns |
|---|---|
| Mean | 0.000343 |
| Median | -0.005322 |
| Maximum | 2.501104 |
| Minimum | -2.834008 |
| Std. Dev. | 0.123043 |
| Skewness | 0.007391 |
| Kurtosis | 91.025440 |

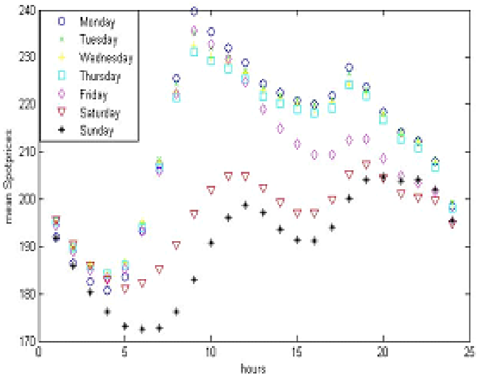
*The basic statistical properties of the (log)returns. The returns are obtained by using the daily spot prices.*

## 2.2 Seasonality

The electricity spot prices show some features of intra-week and intra-year seasonality. Figure 2 shows the average hourly spot prices for every day of the week. The average price for every day of the week are also shown in figure 2.

[2]Annualized volatility is obtained by multiplying the standard deviation by the square root of 365

Figure 2: Intra-week seasonality

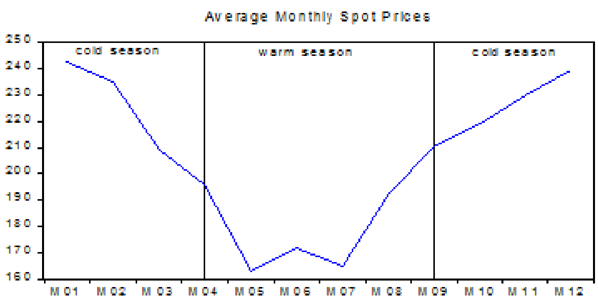



| Days | Mean daily Prices |
|---|---|
| Monday | 212.50 |
| Tuesday | 212.62 |
| Wednesday | 211.96 |
| Thursday | 211.09 |
| Friday | 207.70 |
| Saturday | 196.36 |
| Sunday | 190.77 |

*The left chart displays the average hourly spot prices for every day of the week for the considered period. The left table shows the average price for every day of the week for the considered period.*

From the figure one can observe that the daily spot prices on weekdays differ from the daily spot prices on weekend days. The difference in prices are simply caused by less electricity consumption during the weekends. F-tests confirm that there is a significant difference between the daily spot prices on weekdays and weekend days.[3] The daily electricity spot prices during cold season are higher than during warm season. Cold season is defined as the month October through April and warm season as the month May through September (Johnson et al, 1999). Figure 3 displays the average monthly spot prices and the average spot prices during cold and warm season.

Figure 3: Intra-year seasonality




| Seasons | Mean Season Prices |
|---|---|
| Cold season | 224.72 |
| Warm season | 182.32 |

*The left chart displays the average monthly spot prices. The left table shows the average spot prices during cold and warm season for the considered period.*

A formal F-test is performed to confirm the difference between prices. From the result of the F-test we can conclude that there is a significant difference between the daily spot prices during cold and warm season.[4]

[3] The following regression model $y_t = \alpha + \mu_1 D_{1,t} + \mu_2 D_{2,t} + u_t$ is used to check the presence of seasonality, where $D_{1,t}$ is a dummy variable for Saturday and $D_{2,t}$ for Sunday. The F-tests with null hypothesis that $\alpha = \mu_1$ and $\alpha = \mu_2$ give the values 1951.24 (0,000) and 2048.89 (0,000) respectively with the p-values in the parentheses.

[4] The F-test takes on a value of 234,51 with a p-value of 0,0000

## 2.3 Dealing with data

The existence of price spikes (outliers) during the considered period may affect the values of parameter estimates of the volatility process (Carnero,et al.,2001; Verhoeven and McAleer, 2000). This existence of price spikes may cause explosive volatility process. This is not an appealing result due to the fact that the predictions of the model are meaningless. This problem creates to do any possible risk management analysis based on returns. We will try to solve this problem by performing several methods based on the literature which are explained below:

Method 1:
All returns bigger than 4 standard deviations have been substituted by the sample mean( Carnero, et al 2001).

Method 2:
The seasonal effects and individual spikes are treated pre-whitening the data by removing seasonalities and outlier effects. Thomas and Mitchell (2005) use an OLS framework to remove seasonalities and outlier effects before fitting the various GARCH models to the data. The filtered data are derived by capturing the residuals ($\epsilon_t$) from the following regression model.

$$RP_t = \alpha_0 + \sum_{j=1}^{6} \beta_{2,j} DAY_j + \sum_{k=1,\neq 9}^{12} \beta_{3,k} MTH_k + \sum_{l=1992,\neq 2001}^{2010} \beta_{4,l} YR_l + \sum_{o=0}^{R_s} \beta_{5,o} SPIKE_o \quad (1)$$

where: $RP_t$ represents the returns at time t, $DAY_j$ represents the dummy variable for each day of the week. $MTH_k$ represents the dummy variable for each month. $YR_l$ represents the dummy variable for each year and $SPIKE_s$ represents a set of $R_s$ dummy variables, one for each extreme spike(a filter for outliers at three standard deviations), with $R_s$ representing the number of extreme returns.

Method 3:
This method eliminate the weekly component by applying moving average-based deseasonalization technique beforehand. First, we estimate the trend for the vector of daily spot prices $[x_1, ......, x_{6519}]$ by applying a moving average filter specially chosen to elimate the weekly component and to dampen the noise (Trueck et al, 2007):

$$\hat{m}_t = median(x_{t-3}...., x_t...., x_{t+3}), \quad t = 4......6516 \quad (2)$$

Secondly, the average $w_k$ of the deviations $((x_{k+7j} - \hat{m}_{k+7j}), 3 < k + 7j \leq 6516)$ is calculated for each k=1,...,7. Since these average deviations do not necessarily sum to zero, the seasonal component $s_k$ can be estimated as

$$\hat{s}_k = w_k - \frac{1}{7}\sum_{i=1}^{7} w_i \quad (3)$$

where k=1,....,7 and $\hat{s}_k = \hat{s}_{k-7}$ for $k > 7$. The deseasonalized data is the defined as $y_t = x_t - \hat{s}_t$ for t=1,....,6519 and is used to calculate the returns and fit the GARCH models. In this case the returns are calculated in two different ways.

A) As mentioned before , the returns are defined as $(r_t = ln(P_t/P_{t-1}))$. In this case the negative prices are removed.

B) In the second case the returns are defined as $(P_t - P_{t-1})/|P_{t-1}|)$. The denominator is specified as the absolute value to allow for the presence of negative prices.

Method 4:
This is the same method as method 3, but the deseasonalized data is used to detect and replace the outliers( bigger than 3 standard deviations). The outliers were replaced by the median of all prices having the same weekday and month (Trueck et al, 2007). The (log)return is defined as the first differences of spot prices ($r_t = ln(P_t/P_{t-1})$). In this case there are not negative prices.

# 3 Models

Lucia and Schwartz (2002) noted that volatility clustering is a characteristic of electricity markets. As mentioned before the dynamics of the returns show clear patterns of volatility clustering. Clustered volatility motivate the use of GARCH models.

## 3.1 The mean equation

As mentioned before returns are defined as the first differences of log prices. We assume the following mean equation for the daily price returns (Franses and van Dijk,2000).

$$r_t = \mu + \epsilon_t \tag{4}$$

where $r_t$ is the daily (log)return and $\epsilon_t$ is not white noise with the following properties.

$E[\epsilon_t|I_{t-1}] = 0$ (conditional mean zero)
$E[\epsilon_t^2|I_{t-1}] = \sigma_t^2$ (time-varying conditional variance)

where $I_{t-1}$ is called the information set available at time t-1.

We also note that

$E[r_t|I_{t-1}] = \mu$ and $V[r_t|I_{t-1}] = E[(r_t - \mu)^2|I_{t-1}] = \sigma_t^2$

Equation (1) can also be written as follows;

$$r_t - \mu = \epsilon_t = z_t\sigma_t \tag{5}$$

where $z_t \sim i.i.d.(0, 1)$.
The idea is that the conditional variance ($\sigma_t^2$) of $\epsilon_t$ is not constant. On the other hand we still assume that the unconditional variance is constant.

$$E[\epsilon_t^2] = E[E[\epsilon_t^2|I_{t-1}]]{=}E[\sigma_t^2] = \bar{\sigma}^2$$

## 3.2 The basic GARCH model

For the purpose of this research the GARCH(1,1) will be examined. Hansen and Lunde (2004) concluded that it is difficult to find a volatility model that outperforms the simple GARCH(1,1). The GARCH(1,1) conditional variance equation (3) can be defined as follows, ( Engle, 1982 and Bollerslev,1986).

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{6}$$

where $\omega > 0, \alpha > 0$ *and* $\beta > 0$. These restrictions guarantee that the conditional variance is positive ($\sigma_t^2 > 0$). From equation (3) one can see that the current conditional volatility depends on a mean value, the lagged squared residuals and yesterday's conditional volatility. The intuitive idea behind this model is that the volatility changes only gradually over time, such that $\sigma_t^2$ will be closely related to $\sigma_{t-1}^2$. Given the value of $\beta$, large (small) changes in $\sigma_{t-1}^2$ will be followed by large (small) changes in $\sigma_t^2$. Since $E[\epsilon_t^2|I_{t-1}] = \sigma_t^2$ , the GARCH(1,1) model (3) can be written as an ARMA(1,1) model for $\epsilon_t^2$

$$\epsilon_t^2 = \omega + (\alpha + \beta)\epsilon_{t-1} + v_t + \beta v_{t-1} \tag{7}$$

where $v_t = \epsilon_t^2 - \sigma_t^2$ and $E[v_t|I_{t-1}] = 0$. Equation (4) shows that large (small) changes in $\epsilon_{t-1}$ will be followed by large (small) changes in $\epsilon_t$.

If $\alpha + \beta < 1$, GARCH(1,1) is covariance stationary. This means that the variance reverts back to its unconditional mean $\bar{\sigma}^2 = \frac{\omega}{1-\alpha+\beta}$.
The general GARCH(p,q) conditional variance equation is given by equation 5:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_j \sigma_{t-j}^2 \tag{8}$$

The squared residuals $\epsilon_t^2$ can be written as an ARMA(max(p,q),q) model. If $\sum_{i=1}^{p} \alpha_i + \sum_{i=1}^{q} \beta_i < 1$, GARCH(p,q) is covariance stationary and the unconditional variance of $\epsilon_t$ is

$$\bar{\sigma}^2 = \frac{\omega}{1 - (\sum_{i=1}^{p} \alpha_i + \sum_{i=1}^{q} \beta_i)} \tag{9}$$

## 3.3 The GJR-GARCH model

The GARCH(p,q) model cannot capture the asymmetric effects of positive and negative shock. The sign of the shock have no influence on the conditional volatility. This means that positive and negative shock of the same magnitude have the same effect on the conditional volatility. In practice, we observe that negative shocks (bad news) tend to have a larger impact on volatility than positive shocks( good news). Large negative returns often led to the start of periods of high volatility. For this reason different nonlinear GARCH models are developed over the years (Franses and van Dijk,2000). These nonlinear GARCH models are designed to capture the effects of positive and negative shocks or other types of asymmetries. In our paper we will make use of GJR-GARCH model( Glotsen, Jagannathan and Runkle,1993). The GJR-GARCH model obtained from the GARCH(1,1) model (3) is given by

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \gamma\epsilon_{t-1}^2 I_{t-1} + \beta\sigma_{t-1}^2 \tag{10}$$

where $I_{t-1} = 1 \; if \; \epsilon_{t-1} < 0$ and 0 otherwise. The restrictions $\omega > 0$, $\frac{\alpha+\gamma}{2}$ and $\beta > 0$ guarantee that the conditional variance is positive. A good news has an impact of $\alpha$ and a bad news has a impact of $\alpha + \gamma$ on the conditional variance. If $\gamma > 0$, the impact of bad news is bigger and a leverage effect is present. If $\frac{\alpha+\gamma}{2} + \beta < 1$ is satisfied, the unconditional variance of $\epsilon_t$ is $\sigma^2 = \frac{\omega}{1-\frac{\alpha+\gamma}{2}-\beta}$.
The general GJR-GARCH(p,q) conditional variance equation is given by equation 8:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \sum_{k=1}^{r} \gamma_k \epsilon_{t-k}^2 I_{t-k} + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{11}$$

## 3.4 The EGARCH model

Another extension of the GARCH model which capture the asymmetric effects of positive and negative shock is the Exponential GARCH ( EGARCH) model. The EGARCH model was introduced by Nelson (1991). The specification of the conditional variance is given by equation 9:

$$log(\sigma_t^2) = \omega + \sum_{i=1}^{p} \alpha_i \left| \frac{\epsilon_{t-i}}{\sigma_{t-i}} \right| + \sum_{k=1}^{r} \gamma_k \frac{\epsilon_{t-k}}{\sigma_{t-k}} + \sum_{j=1}^{q} \beta_j log(\sigma_{t-j}^2) \tag{12}$$

The advantage of the EGARCH model over the pure GARCH specification is that the conditional variance are guaranteed to be non-negative, even if the parameters are negative. In the EGARCH the $log(\sigma_t^2)$ is modeled instead of $\sigma_t^2$. If $\gamma_k < 0$, bad news (leverage effects) have a larger impact on the conditional variance. The impact on the conditional variance is asymmetric if $\gamma_k \neq 0$. Covariance stationarity requires $\sum_{j=1}^{q} \beta_j < 1$.

# 4 Estimation of GARCH models

The parameters in GARCH models can be estimated by Maximum Likelihood. Maximum likelihood estimation involves finding the values for the parameters that maximize the following log likelihood function.

$$\hat{\theta}_{ML} = argmax\ lnL(\theta) = argmax \sum_{t=1}^{T} l_t(\theta) \tag{13}$$

where $\theta = (\mu, \omega.\alpha, \beta)$ is the set of all parameters that are to be optimized and $l_t = ln(f(r_t|I_{t-1};\theta)) = lnf(r_t|r_{t-1}, r_{t-2}, ...\theta)$. Maximum Likelihood estimates are consistent and asymptotically normal under certain regularity conditions. To define the specific log likelihood function, GARCH model requires an assumption about the conditional distribution of the error term. The three assumptions are: normal (Gaussian) distribution, Student's t-distribution with fixed degrees of freedom $\nu = 10$, and the generalized error distribution with fixed parameter $\tau = 1.5$. In case $z_t$ (2) is assumed to have a normal distribution, the contribution to the log- likelihood for observation $t$ is:

$$l_t = -\frac{1}{2}log(2\pi) - \frac{1}{2}log\sigma_t^2 - \frac{1}{2}\frac{\epsilon_t^2}{\sigma_t^2} \tag{14}$$

As mentioned in the " Data " section, the returns have a fat-tailed distribution. Huismann and Huurman( 2003) emphasize that a fat-tailed distribution of prices is a characteristic of electricity markets. Due to the fact of fat-tailed distribution, it may be more appropriate to use the student's t-distribution or the generalized error distribution. For the GARCH model with the Student's t-distribution , the log- likelihood contributions are of the form:

$$l_t = -\frac{1}{2}log\left(\frac{\pi(\nu-2)\Gamma(\frac{\upsilon}{2})^2}{\Gamma(\frac{(\upsilon+1)}{2})^2}\right) - \frac{1}{2}log\sigma_t^2 - \frac{(\nu+1)}{2}log\left(1 + \frac{\epsilon_t^2}{\sigma_t^2(\nu-2)}\right) \tag{15}$$

The degree of freedom $\nu > 2$ controls the tail behavior and Student's t-distribution approaches the normal as $\nu \to \infty$. For the GED, the contribution to the log-likelihood for observation $t$ is:

$$l_t = -\frac{1}{2}log\left(\frac{\Gamma(\frac{1}{\tau})^3}{\Gamma(\frac{3}{\tau})(\frac{\tau}{2})^2}\right) - \frac{1}{2}log\sigma_t^2 - \left(\frac{\Gamma(\frac{3}{\tau})(\epsilon_t^2)}{\sigma_t^2\Gamma(\frac{1}{\tau})}\right)^{\frac{\tau}{2}} \tag{16}$$

where the tail parameter $\tau > 0$. The generalized error distribution is fat-tailed if $\tau < 2$ and a normal distribution if $\tau = 2$.

We consider the GARCH model in two different ways. In the first case we consider the GARCH model fit to the returns of the 24 hours separately. We estimate the parameters of the GARCH model and the conditional volatility for each hour separately. In the second case we consider the GARCH model fit to the returns of the daily spot prices. The basic GARCH model and its extensions which examined in this paper are summarized in table 2.

Table 2: GARCH variants

| | |
|---|---|
| GARCH(1,1) | $\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2$ |
| GARCH(2,2) | $\sigma_t^2 = \omega + \alpha_1\epsilon_{t-1}^2 + \alpha_2\epsilon_{t-2}^2 + \beta_1\sigma_{t-1}^2 + \beta_2\sigma_{t-2}^2$ |
| GJR-GARCH(1,1) | $\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \gamma\epsilon_{t-1}^2 I_{t-1} + \beta\sigma_{t-1}^2$ |
| GJR-GARCH(2,2) | $\sigma_t^2 = \omega + \alpha_1\epsilon_{t-1}^2 + \alpha_2\epsilon_{t-2}^2 + \gamma_1\epsilon_{t-1}^2 I_{t-1} + \gamma_2\epsilon_{t-2}^2 I_{t-2} + \beta_1\sigma_{t-1}^2 + \beta_2\sigma_{t-2}^2$ |
| EGARCH(1,1) | $log(\sigma_t^2) = \omega + \alpha\vert\frac{\epsilon_{t-1}}{\sigma_{t-1}}\vert + \gamma\frac{\epsilon_{t-1}}{\sigma_{t-1}} + \beta log(\sigma_{t-1}^2)$ |

*This table gives the summary of the different conditional variance equations which are used for the purpose of this research*

## 4.1 Evaluating GARCH Models

As mentioned in the previous section we will consider the GARCH model for the daily spot prices and the 24 hours separately. The estimated GARCH models can be evaluated using a number of statistical diagnostics. The properties of the standardized residuals can be used to assess whether an estimated GARCH model is any good. The standardized residuals can be described as:

$$\hat{z_t} = \frac{r_t - \hat{\mu}}{\hat{\sigma t}}$$

If the GARCH model is correctly specified, the standardized residuals should have the following properties;

- No autocorrelations in the $\hat{z_t}$: If the mean equation is correctly specified, the standardized residuals should not display serial correlation. We will check the correlogram of the standardized residuals and use Ljung-Box Q-test for remaining autocorrelation in the mean equation.

- No autocorrelations in the squared $\hat{z_t}$: To test the remaining autocorrelation in the variance equation and to check the specification of the variance equation the correlogram of the squared standardized residuals can be plotted. The Engle's LM test can be used to test whether the standardized residuals exhibit additional ARCH (Engle,1992). The ARCH-LM test is a test for presenting of conditional heteroscedasticity in the data. The test is based on a regression of the squared residuals on a constant and q of its lags. ARCH-LM tests the null hypothesis of no remaining ARCH effects. The variance equation is correctly specified if there should be no ARCH left in the standardized residuals. The residuals are obtained by estimating the model for the conditional mean. The LM test for ARCH(1) is performed for the residuals of the different GARCH(1.1). For the residuals of different GARCH(2,2) we perform the LM test for ARCH(2).

- Kurtosis 3, if normality is assumed

# 5 Forecasting

In this section we will discuss the out-of-sample forecasting ability of the basic GARCH model and its extensions. The different GARCH models can be used to forecast volatility and can be judged by this ability. We forecast the one-day ahead volatility. The one-day ahead volatility are obtained by static ( a series of rolling single-step-ahead) forecasts using Eviews 7. The data is divided into in-sample and out-of-sample. The in-sample data contains the first 3259 daily observations. The GARCH model will be estimated over the in-sample data. The estimated parameters are used to obtain one-day ahead volatility forecasts.

## 5.1 Forecasts with the basic GARCH

The h-step daily forecast can be formed as follows:

$$\hat{\sigma}_{t+h|t} = \omega \sum_{i=0}^{h-2} (\alpha + \beta)^i + (\alpha + \beta)^{h-1} \sigma_{t+1}^2 \tag{17}$$

We can rewrite equation (6) if $\alpha + \beta < 1$, such that GARCH model is stationary.

$$\hat{\sigma}_{t+h|t} = \bar{\sigma^2} + (\alpha + \beta)^{h-1} (\sigma_{t+1}^2 - \bar{\sigma^2}) \tag{18}$$

where $\frac{\bar{\sigma^2}}{(1-\alpha-\beta)}$ is the unconditional variance of $\epsilon_t$.

The one-step ahead forecast can be described based on GARCH(1,1) model as follows:

$$\hat{\sigma}_{t+1|t}^2 = E[\sigma_{t+1|I_t}^2] = E[\omega + \alpha\epsilon_t^2 + \beta\sigma_t^2 | I_t] = \omega + \alpha\epsilon_t^2 + \beta\sigma_t^2 = \sigma_{t+1}^2 \tag{19}$$

where $\hat{\sigma}_{t+1|t}^2$ is the one day ahead volatility forecast. The one-step ahead conditional variance forecast for the GARCH(2,2) is given by equation 17:

$$\hat{\sigma}_{t+1|t}^2 = \omega + \alpha_1\epsilon_t^2 + \alpha_2\epsilon_{t-1}^2 + \beta_1\sigma_t^2 + \beta_2\sigma_{t-1}^2 \tag{20}$$

## 5.2 Forecasts with the GJR-GARCH

The one-step-ahead conditional variance forecast for the GJR-GARCH(p,q) is given by

$$\hat{\sigma}_{t+1|t}^2 = \omega + \sum_{i=1}^{q} [\alpha_i \epsilon_{t-i+1}] + \gamma\epsilon_t^2 I_t + \sum_{j=1}^{p} [\beta_j \sigma_{t-j+1}] \tag{21}$$

From equation (18), the one-step ahead conditional variance forecast for GJR-GARCH(1,1) and GJR-GARCH(2,2) are given by equation (19) and (20) respectively.

$$\hat{\sigma}_{t+1|t}^2 = \omega + \alpha\epsilon_t + \gamma\epsilon_t^2 I_t + \beta\sigma_t \tag{22}$$

$$\hat{\sigma}_{t+1|t}^2 = \omega + \alpha_1\epsilon_t + \alpha_2\epsilon_{t-1} + \gamma\epsilon_t^2 I_t + \beta_1\sigma_t + \beta_2\sigma_{t-1} \tag{23}$$

## 5.3 Forecasts with the EGARCH

The one-step-ahead conditional variance forecast for the EGARCH(p,q) is given by

$$log\hat{\sigma}_{t+1|t}^2 = \omega + \sum_{i=1}^{q} \left[ \left| \frac{\epsilon_{t-i+1}}{\sigma_{t-i+1}} \right| + \gamma_i \left( \frac{\epsilon_{t-i+1}}{\sigma_{t-i+1}} \right) \right] + \sum_{i=1}^{p} \beta_j log(\sigma_{t-i+1}^2). \tag{24}$$

From equation (18), the one-step ahead conditional variance forecast for EGARCH(1,1) is given by equation (22).

$$log\hat{\sigma}_{t+1|t}^2 = \omega + \left[ \left| \frac{\epsilon_t}{\sigma_t} \right| + \gamma_1 \left( \frac{\epsilon_t}{\sigma} \right) \right] + \beta log(\sigma_t^2). \tag{25}$$

## 5.4 Evaluating volatility forecasts

Now we will describe how the daily volatility forecasts are evaluated. The mean-squared prediction error (MSPE) criterion is used to evaluate out-of-sample volatility forecasts. The MSPE is defined as follows (Zivot,2008);

$$MSPE = \frac{1}{P}\sum_{i=1}^{P}(\sigma^2_{t+i} - \hat{\sigma}^2_{t+i|t+i-1})^2 \tag{26}$$

where P is the out-of-sample data and $\hat{\sigma}^2_{t+i|t+i-1}$ are the one-step forecasts obtained by using the in-sample parameter estimates and the relevant information sets. The MSPE is the average squared difference between the actual conditional variance and the corresponding volatility forecast. Mean absolute error are also used in the literature to evaluate out-of-sample volatility forecasts (Zivot,2008).

$$MAE = \frac{1}{P}\sum_{i=1}^{P}|\sigma^2_{t+i} - \hat{\sigma}^2_{t+i|t+i-1}| \tag{27}$$

Models with smaller MSPE and MAE are " better ". The actual conditional variance $\sigma^2_{t+i}$ is unobservable. Because of this complication the daily squared return $r^2_{t+i}$ is used as a proxy. The MSPE and MAE exhibit an important shortcoming. The shortcoming refers to the use of the proxy. As mentioned before we assume that $r_t = z_t\sigma_t,\ with\ z_t \sim i.i.d.(0,1)$. The daily squared return $r^2_{t+1}$ is an unbiased estimate of $\sigma^2_{t+1}$ due to the fact that $E[r^2_{t+1}|I_t] = \sigma^2_{t+1}$. On the other hand, $r^2_{t+1} = z^2_{t+1}\sigma^2_{t+1}$ is a noisy estimate of $\sigma^2_{t+1}$ due to its asymmetric distribution. This leads to the conclusion that quite accurate volatility forecasts are no good. We will evaluate volatility forecasts by regressing $r^2_{t+1}$ on $\hat{\sigma}^2_{t+1|t}$.

$$r^2_{t+1} = \beta_0 + \beta_1\hat{\sigma}^2_{t+1|t} + u_{t+1}. \tag{28}$$

" Good " volatility forecasts are characterized by $\beta_0 \approx 0$, $\beta_1 \approx 1$ and high regression R-squared.

## 5.5 Evaluating the average volatility forecasts

We will compare the conditional volatility forecasts obtained by the daily spot prices with the average conditional volatility forecasts obtained by the 24 hours separately. In the case of the average volatility forecasts we use the hourly volatility forecasts and average these hours to forecast the daily volatility of the returns. The forecast error can be defined as $e_{t+h|t} = \sigma^2_{t+i} - \hat{\sigma}^2_{t+i|t+i-1}$ where $\sigma^2_{t+i}$ is the squared returns of the daily spot prices. In the first case $\hat{\sigma}^2_{t+i|t+i-1}$ is volatility forecasts obtained with the daily spot prices and in the second case $\hat{\sigma}^2_{t+i|t+i-1}$ is the average volatility forecasts obtained with the 24 hours separately.

We make use of the Diebold Mariano test to determine whether these differences are statistically significant. DM-statistic tests the null hypothesis that one model model has superior predictive performance over another model (Diebold and Mariano,1995). Suppose we have model $i$ and model $j$, respectively with forecast errors $e_{i,t+h|t}\ and\ e_{j,t+h|t}$. The loss differential of the Diebold- Mariano (DM) is defined as

$$d_{t+h} = e^2_{i,t+h|t} - e^2_{j,t+h|t}$$

and the null hypothesis of equal predictive accuracy is $H_0 : E[d_{t+h}] = 0$. The DM test statistic can be defined as:

$$DM = \frac{\bar{d}}{\sqrt{\frac{V(\bar{d}_{t+h})}{P}}} \sim N(0,1) \tag{29}$$

where $\bar{d} = \frac{1}{P}\sum_{t=T}^{T+P} d_{t+h}$, $V(\bar{d}_{t+h}) = \frac{1}{P-1}\sum_{t=T}^{T+P-1}(d_{t+h}-\bar{d})^2$ and P is a set of out-sample observations.

# 6 Empirical Results

The empirical parameter estimates of the basic GARCH(1,1) have $\alpha + \beta > 1$ for the daily spot prices and the 24 hours separately. The fitted GJR-GARCH(1,1) model for the daily spot prices and hours 1-11 and 14-17 are not covariance stationary. We will try to solve this problem by performing several methods based on the literature as described before. The results of several methods to deal with the data are shown in table 3.

Table 3: Non-covariance stationary hours

| | GARCH(1,1) | GJR-GACRH(1,1) |
|---|---|---|
| Original data | Daily spot prices, 1-24 | Daily spot prices, 1-11,14-17 |
| Method 1 | 1-12,14-19,23,24 | 3-9 |
| Method 2 | 1-6 | - |
| Method3a | Daily spot prices, 1-24 | 1-6 |
| Method3b | Daily spot prices, 1-24 | Daily spot prices, 1-7, 10, 14, 18-24 |
| Method 4 | 1-24 | 2-9, 15 |

*This table displays the remaining non-covariance stationary hours. The estimates of the GARCH models are obtained using the returns after dealing the data by different methods. We will assume that the errors follow a Normal distribution.*

From table 3 one can see that method 2 is the best way to solve the problem of non-covariance stationary GARCH processes. The fitted GARCH(1,1) model for the time series of hour 1,2,3,4,5,and 6 are not covariance stationary. The fitted GJR-GARCH(1,1) model for the daily spot prices and 24 hours separately are covariance stationary. From now we will work with returns after applying method 2. The estimation of various GARCH models and the forecasts are based on returns after applying method 2 to the original data. The non-covariance stationary GARCH process can be explained by the deregulation process. The prices in the spot market has been dramatically influenced by this deregulation process and it has caused the increase in volatility of electricity prices. Before the deregulation process the prices were predictable and there was little uncertainty in electricity prices.

## 6.1 Evaluating estimates of GARCH models

The GARCH model for the daily spot prices and the 24 hours separately are estimated under the assumption that the errors follow a normal distribution, student's t- distribution, and generalized error distribution. After the parameters of various GARCH models are estimated, the next step will be diagnostic checking on the adequacy for GARCH models. We evaluate the standardized residuals of various GARCH models. We check the correlogram of the standardized residuals for remaining autocorrelation in the mean equation. We check the correlogram of squared standardized residuals which consist of (partial) autocorrelations for remaining autocorrelation in the variance equation. We also test the remaining ARCH in the variance equation. The mean equation of the GARCH(1,1) and GARCH(2,2) fitted to the daily spot prices and the 24 hours separately fails to exclude the autocorrelation in the standardized residuals, regardless the assumptions about the errors. The results of analyzing the specification of the variance equation are shown in table 4. This table also shows the standaridized residuals of hours which are normally distributed. From table 4 one can see that the squared standardized residuals of GARCH(1,1) and GARCH(2,2) fitted to the daily spot prices consist no autocorrelation. We can also not reject the ARCH LM test and conclude that the conditional heteroscedasticity is no longer present in the data. The standardized residuals are not normally distributed. These results are valid regardless the assumption about the errors. The fitted GARCH(1,1) and GARCH(2,2) for most of the hours between hour 6 and hour 19 consist autocorrelation. We conclude the same for the ARCH LM test performed to the hours between 6 and 19. We reject the null hypothesis and can conclude that the conditional heteroscedasticity is present in the data of these hours. The standardized residuals of the fitted GARCH for hours between 20 and 4 consist no autocorrelation and the conditional heteroscedasticity is no longer present, regardless the assumptions about the errors.

Table 4: Diagnostic Checking basic GARCH model

| | GARCH(1,1) | GACRH(2,2) |
|---|---|---|
| error: Normal | | |
| No autocorrelation | Daily spot prices, 1-4, 20-24 | Daily spot prices, 1-5,20-24 |
| ARCH LM | Daily spot prices,1-4, 20-24 | Daily spot prices, 1-7,9,10,12,13,15,18-24 |
| Normally distributed | - | - |
| error: Student's t | | |
| No autocorrelation | Daily spot prices, 1-4,20-24 | Daily spot prices, 1-6,18,20,21,24 |
| ARCH LM | Daily spot prices, 1-5,7,17,19-24 | Daily spot prices, 1-9,11,12,14,15,17-24 |
| Normally distributed | - | - |
| error: GED | | |
| No autocorrelation | Daily spot prices, 1-4,20-24 | Daily spot prices,1-6,19-24 |
| ARCH LM | Daily spot prices,1-5,7,8,20-24 | Daily spot prices, 1-7,10,12,14,15,18-24 |
| Normally distributed | - | - |

*This table shows the hour numbers which squared standardized residuals contain no autocorrelation and do not reject the ARCH-LM test. The standardized residuals of hours which are normally distributed are included in this table.*

Table 5: Diagnostic Checking GJR-GARCH model

| | GJR-GARCH(1,1) | GJR-GACRH(2,2) |
|---|---|---|
| error: Normal | | |
| No autocorrelation | Daily spot prices, 2-4, 21-24 | Daily spot prices, 1-5,19-24 |
| ARCH LM | Daily spot prices,2-4, 21-24 | Daily spot prices, 1-5,7,10,12,18-24 |
| Normally distributed | - | - |
| error: Student's t | | |
| No autocorrelation | Daily spot prices, 2-5,21-24 | Daily spot prices, 1-5,20-24 |
| ARCH LM | Daily spot prices, 1-5,7,8,19-24 | Daily spot prices, 1-6,8-13,17-24 |
| Normally distributed | - | - |
| error: GED | | |
| No autocorrelation | Daily spot prices, 2-5,21-24 | Daily spot prices,1-5,20-24 |
| ARCH LM | Daily spot prices, 2-5,7-9,20-24 | Daily spot prices, 1-5,8-10,12,18-24 |
| Normally distributed | - | - |

*This table shows the hour numbers which squared standardized residuals contain no autocorrelation and do not reject the ARCH-LM test. The standardized residuals of hours which are normally distributed are included in this table.*

The mean equation of the GJR-GARCH(1,1) and GJR-GARCH(2,2) fitted to the daily spot prices and the 24 hours separately fails to exclude the autocorrelation in the standardized residuals, regardless the assumptions about the errors. The results of evaluating the specification of the variance equation and normally distributed hours are shown in table 5. The squared standardized residuals of the fitted GJR-GARCH(1,1) and GJR-GARCH(2,2) for the daily spot prices consist no autocorrelation. From the results in table 5 one can see that the conditional heteroscedasticity is no longer present in the data and the standardized residuals are not normally distributed. These results are valid regardless the assumption about the errors. Analyzing the hours separately we can see that the fitted GJR-GARCH(1,1) and GJR-GARCH(2,2) for the hours between 6 and 19 consist autocorrelation. Most of the hours between 5-20 for the fitted GJR-GARCH(1,1) model present conditional heteroscedasticity in the data. Hours between 18 -5 for the fitted GJR-GARCH(2,2) present no longer conditional heteroscedasticity regardless the assumption about the errors.

Also in the case of the EGARCH(1,1) fitted to the daily spot prices and the 24 hours separately the mean equation fails to exclude the autocorrelation in the standardized residuals, regardless the assumptions about the errors. The results of evaluating the specification of the variance equation are shown in table 6. This table also shows the standardized residuals of hours which are normally distributed.

Table 6: Diagnostic Checking E-GARCH(1,1) model

| | E-GARCH(1,1) |
|---:|---:|
| error: Normal | |
| No autocorrelation | Daily spot prices, 4,11,14, |
| ARCH LM | Daily spot prices,4,11,24 |
| Normally distributed | - |
| error: Student's t | |
| No autocorrelation | Daily spot prices, 3,4,11,24 |
| ARCH LM | Daily spot prices, 3-5,11,24 |
| Normally distributed | - |
| error: GED | |
| No autocorrelation | Daily spot prices, 23,24 |
| ARCH LM | Daily spot prices, 2, 24 |
| Normally distributed | - |

*This table shows the hour numbers which squared standardized residuals contain no autocorrelation and do not reject the ARCH-LM test. The standardized residuals of hours which are normally distributed are included in this table.*

From the results of table 6 one can see that the squared standardized residuals of EGARCH(1,1) fitted to the daily spot prices consist no autocorrelation. We can also not reject the ARCH-LM test and conclude that the conditional heteroscedasticity is no longer present in the data. The standardized residuals are not normally distributed. The results of the fitted EGARCH(1,1) model for the 24 hours separately are also shown in the table 6. Most of the 24 hours consist autocorrelation regardless the assumption of the errors. We can see the same results for ARCH LM test performed to the residuals of the 24 hours separately. We reject the null hypothesis and can conclude that the conditional heteroscedasticity is present in most of the 24 hours data.

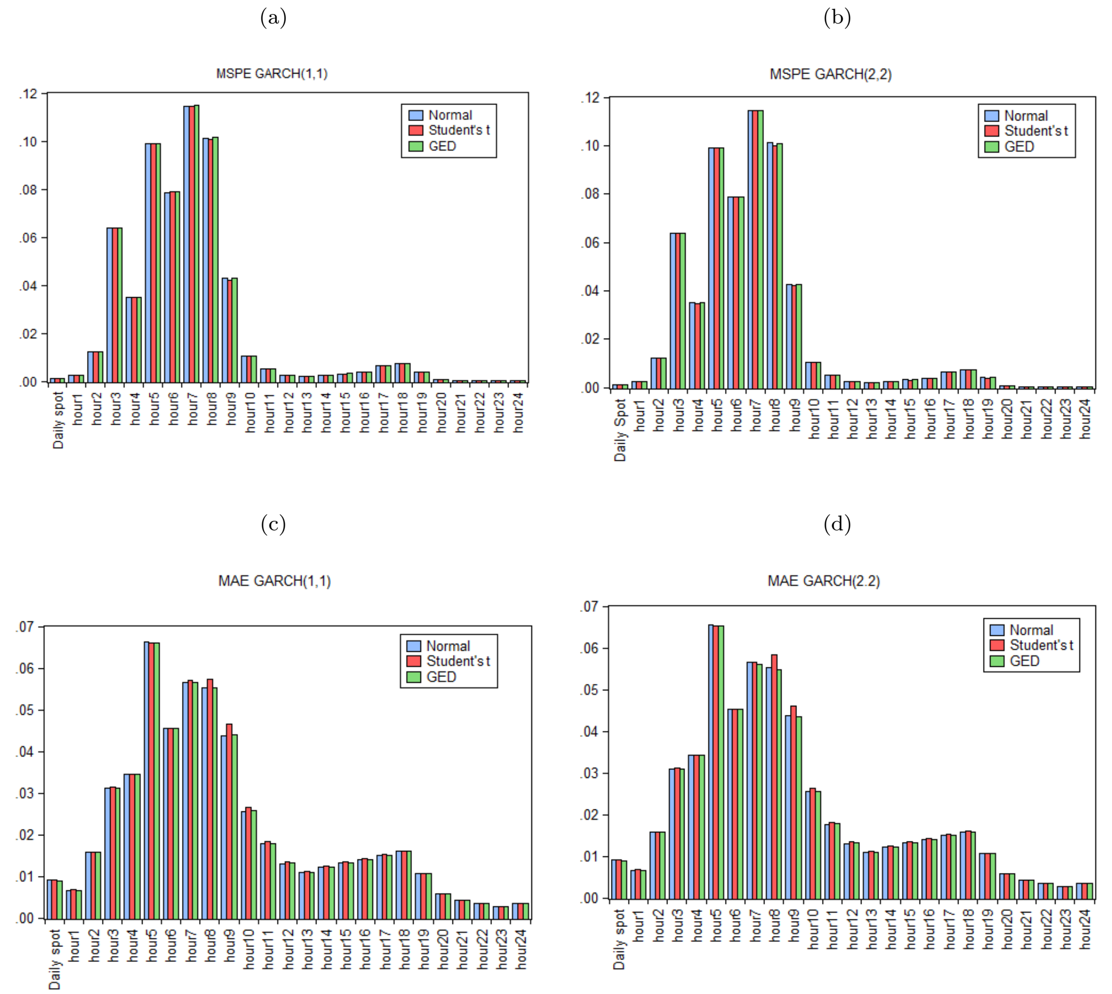
## 6.2 Forecasting performance of various GARCH models

We forecast the one-day ahead conditional volatility of the daily spot prices and the 24 hours separately with various GARCH models. We evaluate the out-of sample by using MSPE and MAE criteria. We also perform a simple linear regression to check for " good " volatility forecast as mentioned before. We will also compare the conditional volatility forecast of the daily spot prices with the conditional volatility forecast of the average of the 24 hours.

### 6.2.1 Forecasting performance of the basic GARCH

Figure 4 shows the forecasting performances of the fitted GARCH(1,1) model for daily spot prices and the 24 hours separately. Smaller MSPE, MAE values are preferred.

Figure 4: MSPE and MAE

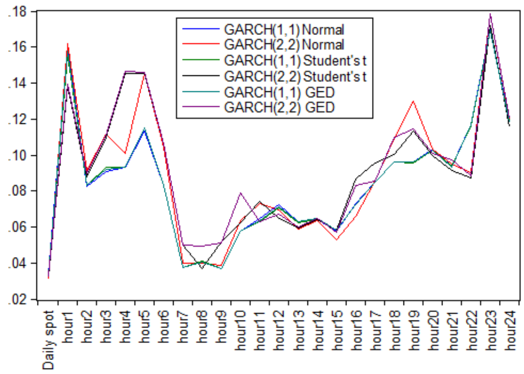



*This figure displays the forecast performances of the fitted GARCH(1,1) and GARCH(2,2) for the daily spot prices and the 24 hours separately. Figure 4(a) and 4(b) shows the MSPE of volatility forecasts obtained with GARCH(1,1) and GARCH(2,2) respectively. Figure 4(c) and 4(d) shows the MAE of the volatility forecasts.*

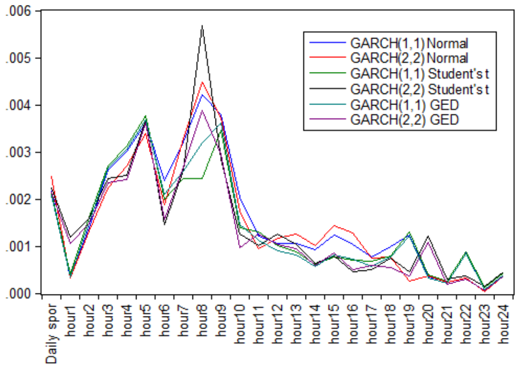
From the figure one can see that the MSPE of hours 20-24 are smaller than the MSPE of the daily spot prices regardless the assumption about the errors. The MAE of the hours 1, 20-24 are smaller than the MAE of the daily spot prices regardless the assumption about the errors. The results of the simple linear regression for the volatility forecasts of the daily spot prices and the 24 hours separately can be found in figure 5. The volatility forecast are good if the $R^2$ is relatively high and $b_0$ and $b_1$ are close to zero and one, respectively. All of the values of $b_0$ is close to zero, but none of the values of $b_1$ is close to 1. Also the $R^2$ values of the daily spot prices and the 24 hours separately are quite low. The highest value is about 0,17 regardless the assumption of the errors. It is notable that the volatility forecast obtained by fitting the GARCH(1,1) for the daily spot prices is the worst forecast with $R^2$ = 0,03, $b_0 = 0,002$, and $b_1$= 0,42 when we assume normally distributed error terms. The $R^2$, $b_0$, and $b_1$ are equal to 0,03, 0,002, and 0,43 respectively, if we assume that the errors follow a Student's t-distribution. When we assume that the errors follow a generalized error distribution, the $R^2$, $b_0$, and $b_1$ are equal to 0,03, 0,002, and 0,46 respectively. The forecast performances of the fitted GARCH(2,2) model for the daily spot prices and the 24 hours separately can be found in figure 4. The results are the same as the results obtained from GARCH(1,1). The results of the simple linear regression for the volatility forecasts of the daily spot prices and the 24 hours separately are shown in figure 5. As noted in the case of GARCH(1,1) all of the values of $b_0$ is close to zero, but none of the values of $b_1$ is close to 1. Also the $R^2$ values of the daily spot prices and the 24 hours separately are quite low. Also in the case of GARCH(2,2) the volatility forecast obtained for the daily spot prices is the worst forecast with the lowest $R^2$. It is notable that the results for GARCH(1,1) are about the same as the results for GARCH(2,2) when comparing the histograms and the charts.

Figure 5: Parameter estimates of $b_0$, $b_1$ and $R^2$

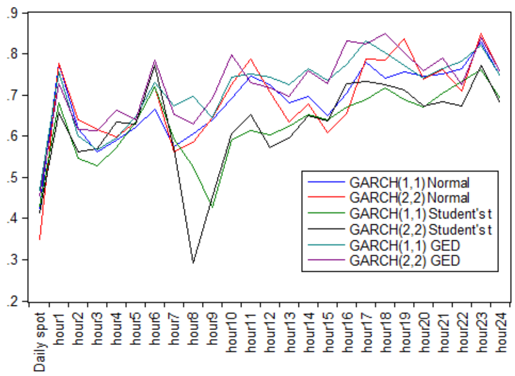
(a)




(b)




(c)




*This figure shows the parameters estimates of the simple linear regression. We regress the squared returns on a constant and the out-of-sample volatility forecasts . Figure 5(a) gives $R^2$ , 5(b) gives $\beta_0$, and 5(c) gives $\beta_1$*

### 6.2.2 Forecasting performance of the GJR-GARCH model

The MSPE and MAE of the one-day ahead volatility forecast obtained with the GJR-GARCH are calculated. Figure A.1 in the Appendix shows the forecast performances of the fitted GJR-GARCH(1,1) model. From the figure one can observe that the MSPE of hours 20-24 and the MAE of the hours 1, 20-24 are smaller than the MSPE and MAE of the daily spot prices. These results are valid regardless the assumption about the errors. Figure A.2 in the Appendix shows the results of the simple linear regression. As noted in the case of GARCH(1,1) and GARCH(2,2) all of the values of $b_0$ is close to zero, but none of the values of $b_1$ is close to 1. The $R^2$ values of the daily spot prices and the 24 hours separately are quite low. The volatility forecast obtained by fitting the GJR-GARCH(1,1) for the daily spot prices is the worst forecast under different assumptions of the error terms. For example, $R^2$ = 0,03, $b_0 = 0,002$, and $b_1$= 0,49 under the assumption that the errors follow a generalized error distribution. There is no clear difference between the results of volatility forecast obtained with GJR-GARCH(1,1) and GJR-GARCH(2,2) model. The forecast performances of the fitted GJR-GARCH(2,2) model for the daily spot prices and the 24 hours separately can be found in figure A.1 in the Appendix. Also, the volatility forecast obtained by fitting the GJR-GARCH(2,2) for the daily spot prices is the worst forecast when comparing the results of the simple regression model. The results are shown in figure A.2 in the Appendix. It is also notable that the results of the basic GARCH are about the same as the results of the GJR-GARCH.

### 6.2.3 Forecasting performance of the EGARCH model

Figure A.3 in the Appendix shows the forecast performances of the fitted EGARCH(1,1) model for daily spot prices and the 24 hours separately. From the figure we can observe that the MSPE of hours 20-24 and the MAE of the hours 1,20-24 are smaller than the MSPE and MAE of the daily spot prices under the different assumption about the errors. The results of the simple linear regression for the volatility forecasts of the daily spot prices and the 24 hours separately can be found in figure A.4 in the Appendix. The volatility forecast obtained by fitting the EGARCH(1,1) for the daily spot prices is the worst forecast when comparing the results of the simple regression model. When comparing the results of the different GARCH models, we can see that there is no clear difference between the basic GARCH,GJR-GARCH and EGARCH models.

### 6.2.4 Forecasting the average volatility forecasts.

We will compare the conditional volatility forecasts obtained by the daily spot prices with the average conditional volatility forecasts obtained by the 24 hours separately. Figure 6 shows the MSPE and MAE for various GARCH models under different assumptions about the errors.

Figure 6: MSPE and MAE

(a) (b)

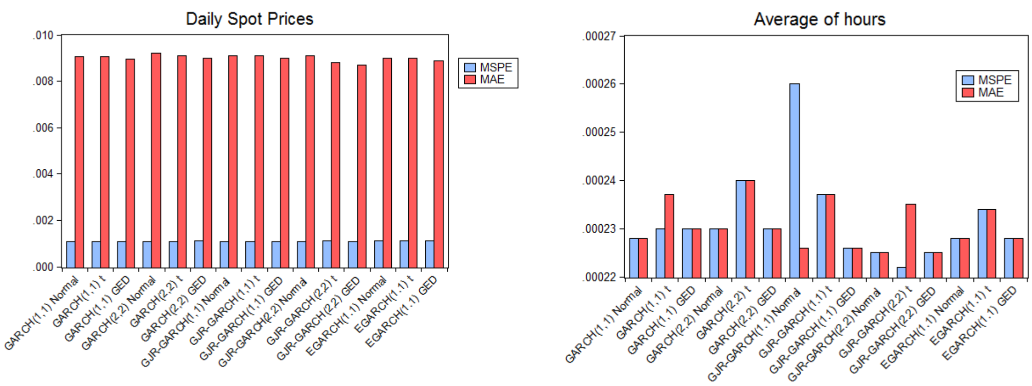
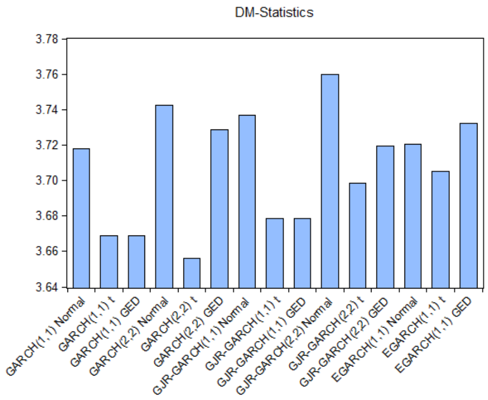



*Figure 6 (a) displays the forecast performances MSPE and MAE of volatility forecasts of daily spot prices. Figure 6(b) shows the MSPE and the MAE of the average volatility forecasts.*

Figure 7: DM-statistics




*This figure exhibits the value of the DM-statistics. DM-statistic tests the null hypothesis that the model with the average volatility forecasts has superior predictive performance over the model with the volatility forecasts of the daily spot prices. The difference between forecast errors are significant if the value of the DM-statistics is bigger than 1,96. This value is based on 5% level of significance*

From the histograms we can observe that the MSPE and the MAE of the average volatility forecasts are smaller than the MSPE and the MAE of the volatility forecasts of the daily spot prices. We test the difference between the forecasts by means of the Diebold-Mariano test statistic. The results can be found in figure 7. From the figure we can see that the difference between the forecasts are significant at 5 % level of significance. The model with the average volatility forecasts generates significant better forecasts for the daily volatility of the returns than the volatility forecast of the daily spot prices.

# 7 Conclusion and future research

This section present the conclusions and summary of the research. We also made suggestions for future work. This research was undertaken to examine whether the use of the time series for the 24 hours separately generates more accurate volatility estimates and forecasts. In our research we have used GARCH models to estimate and forecast the conditional volatility. We assume three different assumption about the conditional distribution error term. The assumptions are the normal (Gaussian) distribution, student's distribution and the generalized error distribution. We have examined the GARCH(1,1), GARCH(2,2), GJR-GARCH(1,1), GJR-GARCH(2,2), and EGARCH(1,1).

First, we have tried to solve the problem of explosive volatility process by performing several methods based on the literature. The method of Thomas and Mitchell (2005) provides the best solution for our data. As mentioned before, the non-covariance stationary GARCH process can be explained by the deregulation process. The estimation of the conditional volatility are based on the returns after applying the method of Thomas and Mitchell (2005). The mean equation of the GARCH(1,1) and the GARCH(2,2) fitted to the daily spot prices and the 24 hours separately are not correctly specified, because the mean equation fails to exlude the autocorrelation in the standardized residuals. The mean equation is not correctly specified under the different assumptions about the errors. The squared standardized residuals of GARCH(1,1) and GARCH(2,2) fitted to the daily spot prices consist no autocorrelation and we also not reject the ARCH-LM test. From this results we can conclude that the the conditional variance equation in both models is correctly specified when we use the daily spot prices. The conditional variance equation in both models is not correctly specified when using most of the hours between 5 and 20. The squared standardized residuals consist autocorrelation and we reject the ARCH-LM test for these hours. These results are valid regardless the assumption about the errors. The estimated GARCH(1,1) and GARCH(2,2) model can not be described as good for the daily spot prices and for the 24 hours separately under the different assumptions of the errors. From the results of the GJR-GARCH model we can conclude that the mean equation of the fitted GJR-GARCH(1,1) and the GJR-GARCH(2,2) for the daily spot prices and the 24 hours separately is not correctly specified. The conditional variance equation of the GJR-GARCH(1,1) and the GJR-GARCH(2,2) is correctly specified when using the daily spot prices. The conditional variance equation of the hours between 6 and 19 is not correctly specified, because the squared standardized residuals consist autocorrelation and we reject the ARCH-LM test for these hours. The mean equation of the EGARCH(1,1) fitted to the daily spot prices and the 24 hours separately are not correctly specified, because the mean equation fails to exlude the autocorrelation in the standardized residuals. The conditional variance equation of the fitted EGARCH(1,1) model for most of the 24 hours is not correctly specified. The final conclusion is that the use of the 24 hours does not generate more accurate estimates of the conditional volatility.

If we analyze the results of the one-day ahead forecast with the GARCH(1,1) and GARCH(2,2), we can conclude that the volatility forecast of daily spot prices is the worst forecast. These results are valid regardless the assumption about the errors. Also, in the case of GJR-GARCH(1,1), GJR-GARCH(2,2), and EGARCH(1,1) the same conclusions can made. The one- day ahead volatility forecast of the daily spot prices is the worst forecast when comparing with the one-day ahead volatility forecast of the 24 hours separately. We have also compared the one-day ahead volatility forecast of the daily spot prices with the average one-day ahead volatility forecast of the 24 hours separately. We conclude that the model with the average one-day ahead volatility forecast generates significant better forecast for the daily volatility of the returns.

The increasing of p and q of the GARCH(p,q) and its extensions is a suggestion for future research. In this paper we focus on p=1,2 and q=1,2. Another suggestion is to correct the mean equation of different GARCH models and create forecast for the returns.

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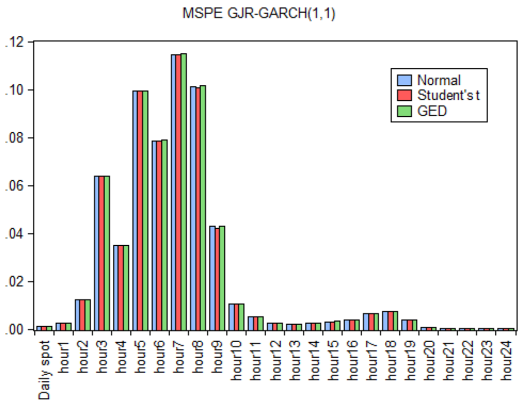
# A Appendix

Table A.1: Basic statistical properties of the 24 hours.

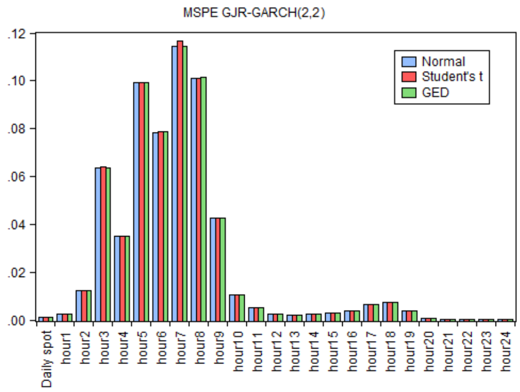
| Hour | Mean | Median | Minimum | Maximum | Std.Dev. | Skewness | Kurtosis |
|---|---|---|---|---|---|---|---|
| 1 | 0,000339 | 0,000285 | 2,462434 | -2,854073 | 0,105647 | -0,047927 | 181,3889 |
| 2 | 0,000338 | 0,000489 | 2,26996 | -2,160926 | 0,126808 | -0,021891 | 66,84951 |
| 3 | 0,000336 | 0,000527 | 2,462434 | -3,40214 | 0,159499 | -1,335476 | 85,20798 |
| 4 | 0,000337 | 0,000649 | 2,462434 | -2,854073 | 0,164226 | -0,327852 | 55,83455 |
| 5 | 0,000339 | 0,0002 | 4,513907 | -7,122105 | 0,206388 | -4,71279 | 286,5153 |
| 6 | 0,000341 | -0,001428 | 2,598682 | -2,854073 | 0,182841 | 0,167012 | 56,08852 |
| 7 | 0,000325 | -0,004738 | 2,97244 | -2,854073 | 0,211788 | 0,93841 | 44,02153 |
| 8 | 0,000337 | -0,007709 | 3,000051 | -2,931107 | 0,232765 | 1,012468 | 35,18209 |
| 9 | 0,000342 | -0,00956 | 2,788606 | -3,126761 | 0,229618 | 0,767591 | 34,42517 |
| 10 | 0,000342 | -0,008849 | 2,46411 | -2,802264 | 0,186919 | 0,52335 | 37,63817 |
| 11 | 0,00034 | -0,007557 | 2,46411 | -2,802264 | 0,164697 | 0,396251 | 44,61251 |
| 12 | 0,000354 | -0,006454 | 2,46411 | -2,802264 | 0,147655 | 0,357046 | 55,14081 |
| 13 | 0,000354 | -0,006927 | 2,456908 | -2,802264 | 0,14053 | 0,498235 | 70,53911 |
| 14 | 0,000348 | -0,007818 | 2,456908 | -2,802264 | 0,143237 | 0,551472 | 66,5262 |
| 15 | 0,000345 | -0,008135 | 2,456908 | -2,802264 | 0,144724 | 0,621075 | 64,29948 |
| 16 | 0,000342 | -0,008062 | 2,456908 | -2,802264 | 0,144952 | 0,638907 | 64,20996 |
| 17 | 0,000344 | -0,006691 | 2,467691 | -2,802264 | 0,140923 | 0,805087 | 70,52185 |
| 18 | 0,000349 | -0,005112 | 2,467691 | -2,802264 | 0,146532 | 0,76933 | 70,2249 |
| 19 | 0,000361 | -0,003497 | 2,497874 | -2,802264 | 0,125883 | 0,252408 | 90,23454 |
| 20 | 0,000363 | -0,001353 | 2,497874 | -2,802264 | 0,10978 | -0,389689 | 137,0247 |
| 21 | 0,000349 | -0,000423 | 2,497874 | -2,802264 | 0,102448 | -0,661249 | 171,0197 |
| 22 | 0,00034 | -0,0000135 | 2,467691 | -2,802264 | 0,103893 | -0,091687 | 187,7173 |
| 23 | 0,000332 | 0,000295 | 2,467691 | -2,802264 | 0,099089 | -0,240596 | 224,6618 |
| 24 | 0,000329 | 0,000831 | 2,467691 | -2,802264 | 0,099331 | -0,238179 | 222,3207 |

Figure A.1

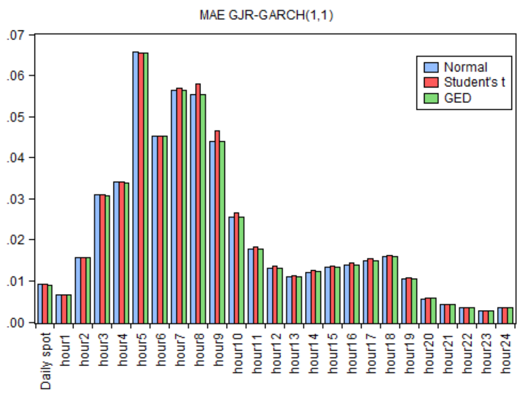
(a)



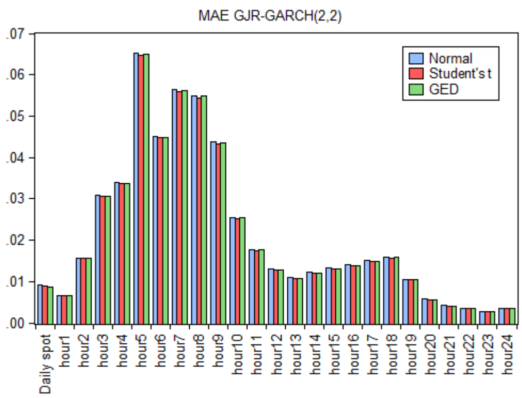

(b)




(c)



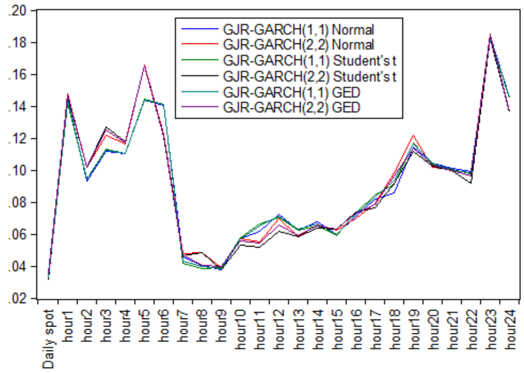

(d)



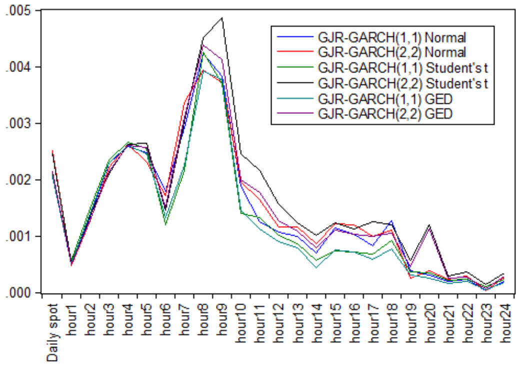

*This figure displays the forecast performances of the fitted GARCH(1,1) and GARCH(2,2) for the daily spot prices and 24 hours sepaarately. Figure 4(a) and 4(b) shows the MSPE of volatility forecasts obtained with GJR-GARCH(1,1) and GJR-GARCH(2,2) respectively. Figure 4(c) and 4(d) shows the MAE of the volatility forecasts.*

Figure A.2

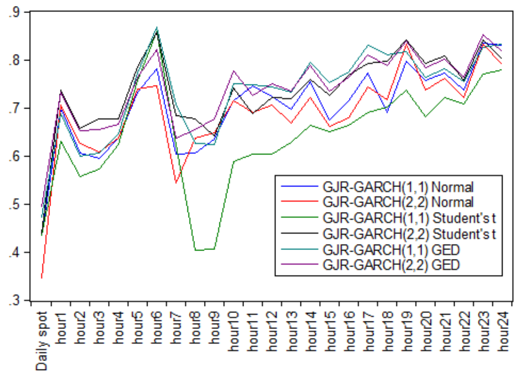
(a)




(b)



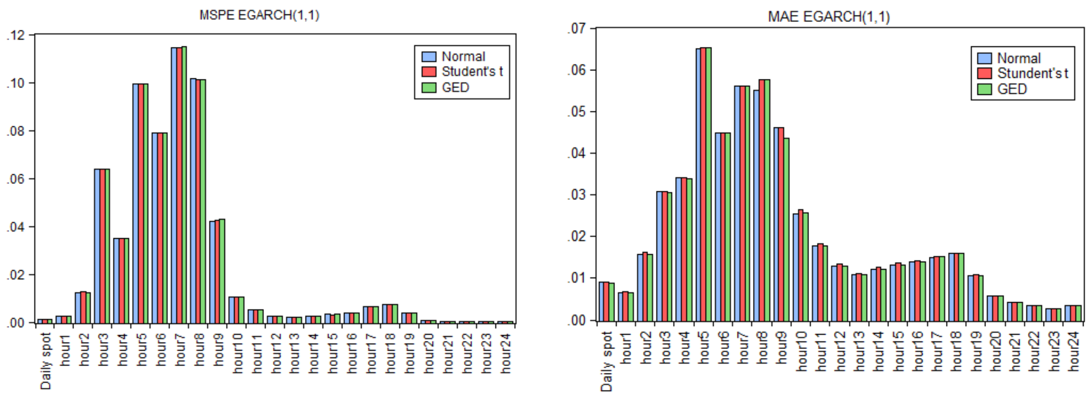

(c)




*This figure shows the parameters estimates of the simple linear regression. We regress the squared returns on a constant and the out-of-sample volatility forecasts . Figure 5(a) gives* $R^2$ *, 5(b) gives* $\beta_0$*, and 5(c) gives* $\beta_1$

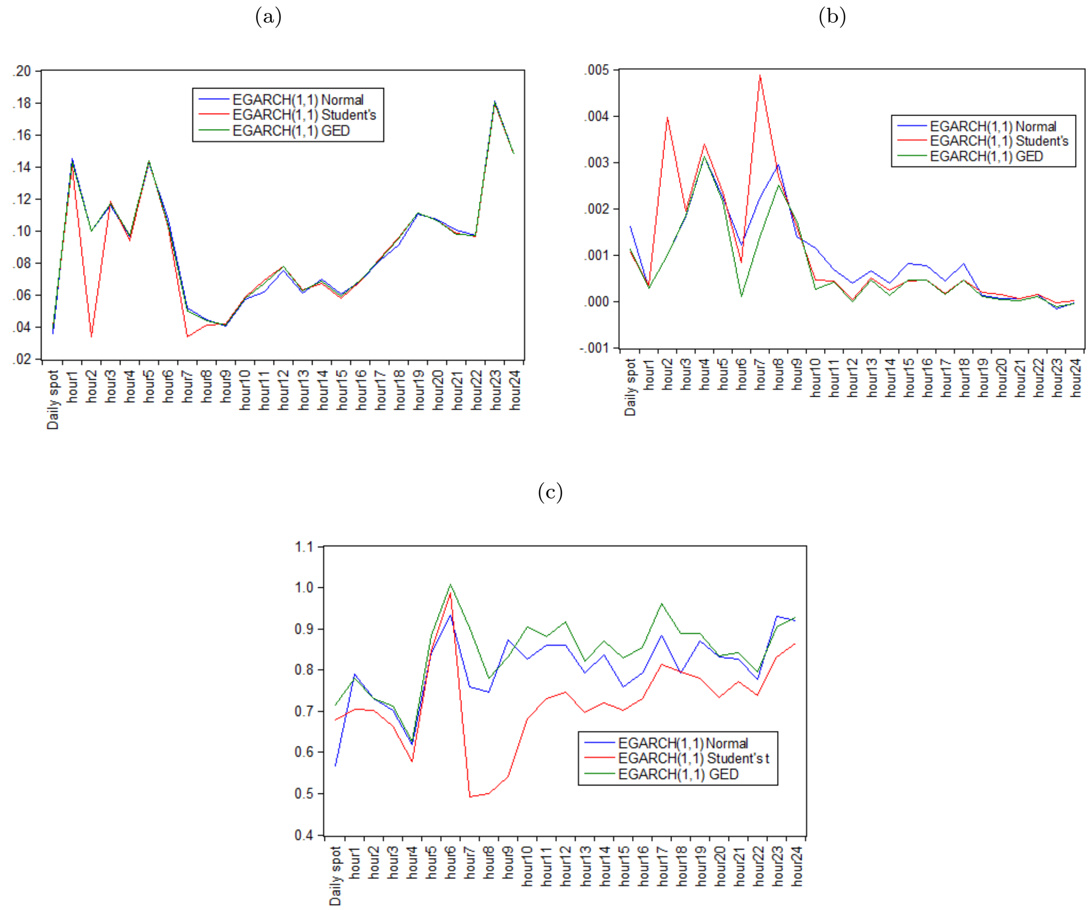
Figure A.3

(a) (b)




*This figure displays the forecast performances of the fitted EGARCH(1,1) for the daily spot prices and 24 hours separately. Figure 4(a) and 4(b) shows the MSPE and the MAE of volatility forecasts obtained with the GARCH(1,1) model respectively.*

Figure A.4




*This figure shows the parameters estimates of the simple linear regression. We regress the squared returns on a constant and the out-of-sample volatility forecasts . Figure 5(a) gives* $R^2$ *, 5(b) gives* $\beta_0$*, and 5(c) gives* $\beta_1$