Strategic (re)actions in a Learning Environment:

a theoretical approach

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Abstract

We study a two period model with an incumbent that is already in the market for a long time, an entrant that enters the market in period one and a consumer that decides whether to buy a unit of the product from the incumbent, the entrant or to not buy a product at all. We show that the incumbent has to make a strategic development decision to drive the entrant out of the market at the end of period one. The entrant also has to make a strategic development decision to be profitable in period two. We use a learning model where the consumer learns about the ability of the producers at the end of period one. We show that the optimal strategy for the incumbent is to follow his own private signal in both periods and that the entrant has to distinguish himself from the incumbent in both periods, to be able to make profit in the second period. Furthermore, we show that the incumbent will not change his behaviour in case there is uncertainty about whether the incumbent will be a monopolist after the first period entrant is out of the market, or that a new entrant will enter. Finally, we show that the mere presence of an inferior entrant is enough to discipline the incumbent to always follow his signal and to charge a price that is lower than the monopoly price.

Keywords: Strategic (re)action, duopoly market and social learning

Preface

This thesis concludes my master's studies in Economics of Markets, Organisations and Policy at the Erasmus University Rotterdam. I have been working on this thesis since February 2012 and since then I have learned a lot about doing (theoretical) microeconomic research. I have learned to be more critical, I have improved my analytical and academic writing skills and I gained more insight in microeconomic research in general. I will cherish these useful insights for many years to come, and I am sure this experience is a useful lesson for future PhD research I pursue. I would like to thank my thesis supervisor Prof. dr. Bauke Visser for all the helpful ideas and comments, which enabled me to write this thesis. I hope this thesis enriches the academic literature on strategic (re)action in a duopoly market in a learning environment, and finally I hope someone else will continue my research.

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1. Introduction

Consider the sports drink market and imagine that Aquarius is at this moment the only firm in the market. Before every sport match Aquarius receives a signal about which type of sports drink will give a sportsman additional energy, and offers a bottle of this sports drink to the sportsman at a certain price. The sportsman then decides whether to buy the bottle. The bottle of sports drink has two purposes; it can give the sportsman additional energy (one type of the sports drinks will and the other will not) and the sportsman likes the taste of the drink to a certain extent. Before the sportsman decides to buy a bottle of sports drink, he knows that if the sports drink does not give him additional energy he does not receive any utility from drinking the bottle. Furthermore, he knows that with a certain probability Aquarius knows which type of sports drink is the right sports drink (the drink that gives additional energy) and he knows that if the sports drink gives him additional energy how much he likes the sports drink. If the sportsman has decided to buy a bottle, he drinks the bottle and plays his sports match. After the match, the sportsman observes whether this bottle gave him additional energy or not and updates his beliefs about how good Aquarius is (he updates his beliefs about the probability that Aquarius develops the drink that gives him additional energy).

Now imagine that a second firm, Bullit, enters the sports drink market and also receives a signal about which type of sports drink will give the sportsman additional energy. If both firms produce the type of sports drink that will give the sportsman additional energy he likes the taste of the drink Bullit offers less than the taste of the drink Aquarius offers (the product offered by Aquarius is of higher quality). Bullit observes which type of sports drink Aquarius offers to the sportsman and then decides which type of sports drink to offer to the sportsman. Both producers (Aquarius and Bullit) know that Aquarius receives a better signal (knows more often which drink type he should offer to the sportsman) than Bullit, but the sportsman does not know this. After both producers decided which type of sports drink they will offer, they simultaneously set prices. The sportsman again decides whether or not to buy a bottle of sports drink and if the sportsman decides to buy a bottle, he decides from which producer. After the sportsman played his match he observes whether either of the producers developed the type of sports drink that gave him additional energy (he can for example infer from the ingredients which type both producers offered) and updates his beliefs about the probability that the producers receive the correct signal (that the firm knows which type of sports drink will give the sportsman additional energy).

if the game lasts for two periods (the sportsman will only play two matches) the newcomer in the market (Bullit) produces the type of sports drink that does not give the sportsman additional energy in the first period and Bullit will be out of the market if he produces the ‘wrong’ type of sport drink, will he then, knowing that Aquarius knows more often which type of sports drink should be produced, be willing to ignore his own signal and develop the same type of sports drink as Aquarius does? And if Aquarius knows that Bullit will ignore his signal and will blindly follow Aquarius, is Aquarius willing to develop the 'wrong' type of sports drink on purpose in period one to drive Bullit out of the market and be a monopolist in period two? And is Aquarius (still) willing to develop the 'wrong' type of sports drink if he is uncertain about whether or not a new firm will enter the market in period two after Bullit has left the market? Finally, what does the entry of Bullit mean for the expected utility of the sportsman, knowing that Bullet knows less often which type of sports drink the sportsman needs, and knowing that the quality of the drink (if this drink is of the type the sportsman needs) is less than the quality of the drink Aquarius offers?

In this paper we will propose a theory that captures this situation and answers these questions. In our paper, an incumbent (he) is in the market for a long time and receives every period a private signal about which type of product (A or B) he should produce for the consumer (she). In period one an entrant (he) enters the market and also receives a private signal about which product-type to develop (A or B). Then, first the incumbent decides which product to develop based on his private signal. After the incumbent has decided which product-type to develop, the entrant develops a product. The entrant bases his decision on his private signal (and can also decide to take the development decision of the incumbent into account). After both producers made a development decision, they simultaneously set prices. Finally, the consumer observes the development decision of both producers and the prices they charge and then decides whether or not to buy a unit of the product from one of the producers. This game lasts for two periods.

The phenomenon of an incumbent firm that announces the development of a product-type followed by an entrant that announces the development of a product, that is either very similar of the total opposite, is ubiquitous and can be applied to many other settings, for example:

a) In the market of new electronic devices an established firm as Apple can announce to bring a product on the market that fulfils a kind of needs for his consumers. A new firm can enter the market and decide to develop a product that is similar and fulfils the same needs for the consumers, or it can decide to attract the same group of consumers by developing a product that fulfils different needs. The consumers can only spend their money once, so they have to decide to buy a unit of the product from one of the producers or to buy no product at all.
b) A firm that has to decide whether to use its current consultant to decide whether to invest in a certain project and implement the project or to hire a new consultant. The

firm knows that, because the current consultant already worked for the firm and is more experienced, it takes him less time to make the project a success, if the project is good. But the firm is uncertain which consultant is more able to pick the right project.

We find the following results using backward induction. In period two both producers (incumbent and entrant) no longer have reputational concerns. The incumbent will always follow his private signal in the second period because there is no way in which he can deceive the entrant anymore. The entrant on the other hand, has an incentive to distinguish himself from the incumbent to be able to sell a unit of the product; because in case the producers develop the same product-type the consumer is inclined to buy a unit of the product from the incumbent (due to the fact that the product quality of a product offered by the incumbent is higher than the product quality of a product offered by the entrant if the product matches the state/the preferences of the consumer and is equal to zero if the product-type does not match the state). The only way in which the entrant can distinguish himself from the incumbent and sell a unit of his product to the consumer in period two, is if the producers develop a different product-type in period two, and the consumer believes that it is more likely that the entrant will develop the product-type that matches the state (the consumer preferences) than that the incumbent will. The only way in which the consumer can expect the entrant to know better which product matches the state is if the entrant has developed the product that matched the state in period one, and the incumbent did not. Therefore the entrant also has an incentive to develop the product-type that is the opposite of the product-type developed by the incumbent in period one. On the other hand, the incumbent can make his development decision in period one strategically. But since the entrant has an incentive to develop the product that is the opposite of the product developed by the incumbent and since the incumbent prefers to develop the product that matches the state and having the entrant develop the product that does not match the state over the case where the incumbent developed the 'wrong' product-type and the entrant developed the 'right' product-type, the incumbent will always follows his signal in period one. In both periods the producers set their prices simultaneously after they developed a product. If the incumbent is a monopolist, he can set his price equal to the expected benefit that it will create for the consumer. If the incumbent is in a duopoly market together with the entrant, he has to take the product development decision of the entrant and the price he expects the entrant to charge into account when he determines his price. From this argument, it follows that the expected utility of the consumer already increases only by the presence of the entrant (despite of the fact that the entrant is inferior to the incumbent both in ability to determine which product will match the state, as in the quality of the product if the product matches the state). Furthermore, it turns out that if the incumbent is uncertain about whether or not he will be a monopolist in the second period after the first period entrant has left the market, he will not behave differently compared to the case where he will be a monopolist in the second period with certainty. This contradicts findings in the current literature.

1.1 Related literature

The contribution of this paper to the existing literature is twofold. This paper contributes to the rich literature on entry deterrence and strategic (re)actions of an incumbent to prevent entrants from entry or drive competitors out of the market. Secondly, this paper contributes to the literature on social learning. The existing literature on entry deterrence and strategic (re)actions of incumbent firms dates back to papers of Schelling (1956), Wenders (1971a, 1971b), Spence (1977), Dixit (1979, 1980), and Eastbrook (1981) that analyse entry deterrence through an irrevocable investment decision in the pre-entry period, papers of Salop (1978) and Milgrom and Roberts (1982) about the use of limit pricing, Gilbert and Newbery (1982) about the use of 'sleeping' patents, Kemperer (1987, 1995) about the use of switching costs and Aghion and Bolton (1987) on signing long term contracts. Making a strategic investment in the period before an entrant enters (or wants to enter) is studied extensively. In a seminal paper, Schelling (1956) proposes the following theory. If a monopolist threatens to invest in extra capacity when an entrant considers entering the market, the threat to invest can be a reason for the entrant not to enter. But if the entrant has entered the market, and it turns out that it is better for the monopolist not to invest, and if the entrant knows this on forehand, the thread is no longer credible. The monopolist can only make the threat (to invest in extra capacity) credible by strategically investing in extra capacity in the pre-entry period. Spence (1977) formally shows that in a market with homogeneous goods, the incumbent's capacity is used by the entrant to decide on entry. An incumbent can decide to invest in extra capacity in the pre-entry period so that it can expand output and reduce price in the period the entrant enters (or wants to enter). The effect of capital investment, from an incumbent, on other firms in the same industry is empirically tested by Gilbert and Lieberman (1987) for firms in chemical product industries. Gilbert and Lieberman find that investment, in case there are multiple firms in the market, by one firm reduces the probability of investment (expanding capacity) by the rival firms (only) in the short run. This implies that an incumbent can hold an entrant only temporarily out of the market by strategically investing in extra capacity. Furthermore, Spence shows that in general (also applicable to a market with heterogeneous goods) a firm can deter entry or drive competitors out of the market by increasing the investment in marketing or advertising.

The use of limit pricing to deter entrants from entry is analysed by Salop (1978) among others. Salop shows that entrants try to infer marginal costs of a product from a monopolist, by the price the monopolist charges. Salop shows that in case there are different monopolists in multiple industries, and an entrant wants to enter one of those industries, he infers marginal costs from the price the monopolists charge. Salop discusses a situation where there are monopolists with low marginal costs and there are monopolists with high marginal costs. The entrant only wants to enter a market where a monopolist has high marginal costs. Salop shows that, through strategic actions of the monopolists, there exists a pooling equilibrium where all monopolists charge the same price and the entrant does not enter because if she would want to enter, the low marginal cost firms will lower their price. And there exists a separating equilibrium where the entrant enters a market where the monopolist has high costs and does not enter the markets where the monopolists have low marginal costs.

Aghion and Bolton (1987) show that another way to deter an entrant from entry (or to drive an entrant out of the market) is by having the consumers sign long term contracts. If a consumer stops buying from the incumbent during the contract period he has to pay a fine. In this case the consumer only switches from producer if the price she pays for a unit of the product from the incumbent is larger than the price she pays for a unit of the product from the entrant plus the fine. If the entrant knows that the incumbent uses such kind of contracts to bind consumers, the entrant is less inclined to enter the market. Another possible strategy of the incumbent to bind consumers (as is shown in Aghion and Bolton), is to give discount to loyal consumers. This results in more loyal consumers since they have to give up the discount when they buy from the entrant, and therefore the probability that an entrant enters the market is smaller.

Another way to prevent entrants from entry is by inducing switching costs to consumers that want to switch from producer. Kemperer (1987 and 1995) shows that switching costs can be imposed through different ways, for example there are costs for the consumer from learning the product (an extensive users guide), or there are introduction costs (for example, if one wants to switch from checking accounts between two banks there are costs for opening a new account, while the accounts the banks offer are basically the same). Kemperer creates a two period model with two firms that both impose switching costs to the consumers, and shows that in period one both firms strategically choose a lower price. So that they can bind consumers and can charge a higher price in the second period.

Most of the papers on entry deterrence and strategic behaviour are theoretical papers. This makes sense, since there are not many firms that will admit that they try to deter entrants from entry or try to drive an entrant out of the market. Especially not when the strategy they follow is at the expense of the consumers. On the other hand, Smiley (1987) did such an empirical research on entry deterrence. Smiley sent a questionnaire to product managers of firms in different industries and asked them how and to what extent they tried to deter entrants from entry in their industries. Smiley found that the strategies that are used mostly are the use of patent pre-emption and an increase in advertising costs. Limit pricing is used much less as a strategy to deter entry.

The current literature is mainly based on strategies to deter an entrant from entry or drive an entrant out of the market. But all these papers neglect the fact that an entrant can enter the market with the intention to 'just copy' the incumbent. This 'copying' behaviour of other firms' actions is called herding behaviour, and is extensively explained in for example Bikhchandani, Hirshleifer and Welch (1998). This behaviour and the possible use of herding as a strategic action, is absent in the current models for multiple reasons. In first place, because most of the current models assume that the incumbent and entrant move simultaneously (when firms move simultaneously herding behaviour is impossible to occur). Secondly, because it is very hard to (empirically) proof that a firm behaves in such a way (firms do for example not want to admit that they do an action or produce a product that hurts their consumers on purpose) and finally, that most of the models that describe entry deterrence do not take into account that consumers learn.

Although learning is generally not used in the literature on entry deterrence (or driving current entrants out of the market), there is a very rich literature on social learning and herding behaviour in other areas. A major part of this literature focuses on social learning and herding behaviour in financial markets: Scharfstein and Stein (1990), Avery and Zemsky (1998) and Hirshleifer and Hong Teoh (2003) among others. Another part of the literature analyses learning through word-of-mouth communication, for example: Banerjee (1992), Ellison and Fudenberg (1995) and Banerjee and Fudenberg (2004). There are empirical papers on herding behaviour in competitive markets, for instance Kennedy (2002). Kennedy examines decisions by television networks on introducing different genres and finds that television networks copy each other's genre choice. Another empirical paper about herding behaviour by Chang et al. (1997) is about the spatial clustering of bank branches in cities. There are a lot of factors that influence whether it is more profitable for a bank branch to open a store in one area or in the other. Chang et al. (1997) find that after controlling for expected profitability of operating a branch in a certain area, bank branches are still more likely to open a store at a place where another bank is already present. All these papers describe learning and herding behaviour in different contexts using different models, but one part of the analyses that is lacking in all these models is the incentive of one firm/player to deceive the other(s) by developing the wrong product-type or do the wrong action on purpose and thereby leading competitors in the wrong direction (having them choose the wrong action as well).

We analyse a situation where an incumbent makes a development decision based on his private signal prior to the development decision of the entrant. The entrant can decide on his development decision based on his own private signal or he can use the development decision of the incumbent to determine his development decision. Since the producers move here sequentially, and the decisions made are about the same period, herding behaviour is possible. We analyse whether, if the incumbent knows that the entrant herds on the decision of the incumbent, the incumbent has an incentive to use this knowledge to his benefit and drive the entrant out of the market. Secondly, we show what the appearance of an entrant that is clearly inferior to the incumbent, means for social welfare.

The remainder of this paper is organized as follows. In the next chapter we introduce our model. In chapter three we analyse equilibrium behaviour and discuss the results. In chapter four we show what our results imply for consumer welfare. In chapter five we further discuss the results and do suggestions for further research and in chapter six we conclude.

2. The model

In this game there are two producers and one consumer. One producer is the incumbent. The incumbent is in the market for a long time and has a well-known reputation. The other producer is the entrant. The entrant enters the market in period one, and has not yet build up a reputation. In period one both producers receive a private signal about the state of the world. The state of the world is binary and the states are equally likely to occur. After the producers receive a private signal they make a development decision sequentially with the incumbent moving first. The development decision is also binary. After the producers made their development decisions they simultaneously set prices, and the consumer decides on buying a product from none or from one of the producers. After the consumer made her buying decision the period ends. The game lasts for two periods. The profit functions of the two producers are identical:

$$\pi(p) = \begin{cases} p_{i,t} - c \; if\ the\ producer\ sells\ a\ unit\ of\ the\ product\ to\ the\ consumer \\ 0 \qquad\quad if\ the\ producer\ does\ not\ sell\ a\ unit\ of\ the\ product \end{cases}$$

Here $p_{i,t}$ is the price the producer charges. This price can differ per producer (i) and the producer can charge a different price in each period (t). The cost of producing is equal to c, this cost is the same for both producers for both product-types and is constant over time. The cost of producing c is made at the moment the consumer decides to buy a unit of the product from the producer. No costs are made in the development decision phase where the producers only decide which product-type (A or B) they offer. Think for example back at our example about the sports drink, here in the development phase the producer announces what kind of ingredients he will use if the consumer decides to buy a bottle of the sports drink. It is easy to see that the producers offer a unit of their product if and only if the price they can charge is larger than or equal to the cost of production $p_{i,t} \geq c$. The utility function of the consumer is:

$$u(\gamma, p) = \gamma - p_{i,t} \qquad \text{where } \gamma = \begin{cases} R_i, & if: \theta = x \\ 0, & if: \theta \neq x \end{cases}$$

The consumer's utility depends on whether or not the product bought from the producer matches the state of the world (θ). If the product bought (x) matches the state, the utility of the consumer is equal to R. R is the quality of the product, and is a fixed parameter. If the consumer bought a unit of the product from the incumbent (and the product-type matches the state), her utility from the product equals R_I and if the consumer bought a unit of the product from the entrant, then her utility equals R_E. Note that the product qualities $(R_I\ and\ R_E)$ are perfectly known by the consumer before she makes her buying decision. Throughout this game we assume that in case both producers developed the product that matches the state, the product the incumbent developed is of higher quality than the product the entrant developed, $R_I > R_E$. The cost of buying a unit of the product for the consumer equals the price the producer charges.

The timing of the game is as follows. In period one nature determines the state of the world $\theta_1 \in \{0,1\}$ and sends a private signal about the state of the world $s_{I,1} \in \{0,1\}$ to the incumbent and sends a private signal about the state of the world $s_{E,1} \in \{0,1\}$ to the entrant. $\theta_1 = 0$ means that the producers should develop product-type A and $\theta_1 = 1$ means that the producers should develop product-type B. We assume that the signals the producers receive are not correlated, so if one producer receives the wrong signal $s_{i,t} \neq \theta_t$, it does not become more likely that the other producer receives the same (wrong) signal as well. The incumbent receives a signal that matches the state of the world with probability: $\mu_I = pr(s_{I,t} = 0 | \theta_t = 0) = pr(s_{I,t} = 1 | \theta_t = 1)$. Similarly the entrant receives a signal that matches the state of the world with probability: $\mu_E = pr(s_{E,t} = 0 | \theta_t = 0) = pr(s_{E,t} = 1 | \theta_t = 1)$. The probability that a producer receives a correct signal (a signal that matches the state) is equal to the parameter μ, and this probability is what I define as the ability of the producer. Throughout this game I assume that the ability of the incumbent is higher than the ability of the entrant, $\mu_I > \mu_E$. Note that the ability of a producer is a parameter that is only privately known. The consumer and the other producer only have an expectation about the ability of the producer (for the sake of simplicity I assume that the consumer and one of the producers have the same expectation about the ability of the other producer that equals $\mu_{i,t}^c$). The consumer uses the expected ability together with her beliefs about the strategy the producer will follow to determine the reputation of the producer. The reputation of a producer to the public (consumer and the other producer) is a combination of the expected ability $\mu_{i,t}^{c}$ and of the strategy the public expects the producer to follow. The reputation is therefore a variable that is not fixed and depends on the behaviour of the producer and the beliefs of the consumer and represents the probability that the producer develops the product-type that matches the state. Strategy P1 means that the consumer believes that the producer uses a pure strategy to always follow his signal $(x_{i,t} = s_{i,t})$ and M1 means that the consumer believes that the producer uses a mixed strategy where he sometimes follows his signal and decides to develop the other product-type $(x_{i,t} \neq s_{i,t})$ with positive probability. Note that I use as a mixed strategy only the case where the producer develops a product that is opposite to his signal. The reason for this is straightforward; because the only thing that matters to the consumer is that the product matches the state and because the states are equally likely to occur, it is never beneficial to always choose one state over the other, this rules out mixed strategies like: always develop product A. The period one reputation is determined by the consumer on the basis of the behaviour of the producer and the expectation the consumer formed about the ability of the producer in the following way. Before the game starts the consumer has an expectation about the ability of the incumbent $(\mu_{I,1}^{c})$ based on his prior performance and has an expectation about the ability of the entrant $(\mu_{E,1}^{c})$ based on a first impression, recall that subscript c here indicates that this is the expected ability of a producer from the consumer's perspective. When determining the first period reputation of both producers the consumer combines the expected ability of the producers with the expected strategy the producers will follow. If the consumer believes that the incumbent always follows his signal in period one, his reputation is: $\lambda_{I,1}^{P1} = \log\left(\frac{P(\theta_1 = x_{I,1})}{P(\theta_1 \neq x_{I,1})}\right) = \log\left(\frac{\mu_{I,1}^{c}}{1-\mu_{I,1}^{c}}\right)$. To determine the expected ability of the incumbent in period one (from the consumer's perspective) I use that: $\mu_{I,1}^{c,P1} = \frac{e^{\lambda_{I,1}^{P1}}}{1+e^{\lambda_{I,1}^{P1}}}$. This is Bayesian updating using the log likelihood ratio and is extensively described in Chamley (2004) and is applied in for example Fernandez (2007). If the incumbent mixes between following his signal and develop the product-type that is opposite to his signal (with $pr(choose\ x_{I,1} \neq s_{I,1}) = t$) then his period one reputation is: $\lambda_{I,1}^{M1} = \log\left(\frac{\mu_{I,1}^{c}(1-t)+(1-\mu_{I,1}^{c})t}{(1-\mu_{I,1}^{c})(1-t)+\mu_{I,1}^{c}t}\right)$ and the incumbent's expected ability is in this case lower than in the case where the incumbent is always honest (because $\lambda_{I,1}^{M1} < \lambda_{I,1}^{P1}$). To distinguish between ability and reputation, note that the reputation can be used to determine the probability that the product developed matches the state $(x = \theta)$ and that the ability is the probability that the signal of the producer $(s = \theta)$ matches the state. The rules I use for updating the reputation and ability of the entrant are exactly the same as the rules I used for updating the reputation and ability of the incumbent. If the entrant decides to use a mixed strategy he mixes with $pr(x_{E,1} \neq s_{E,1}) = r$. For a more extensive explanation of the role and updating of ability and reputation please refer to appendix A.

After both producers received their private signal they develop a product sequentially, with the incumbent moving first. The incumbent makes development decision $x_{I,1} \in \{0,1\}$, $x_{I,1} = 0$ means that the incumbent develops product-type A and $x_{I,1} = 1$ means that the incumbent develops product-type B. Recall that there are no costs involved with the development decision, costs are only made at the point where the consumer decides to buy a unit of the product. The incumbent makes his development decision based on his private signal ($s_{I,1}$), his ability (μ_I), his reputation ($\lambda_{I,1}^{M1/P1}$) and the reputation of the entrant ($\lambda_{E,1}^{M1/P1}$). After the incumbent made a development decision ($x_{I,1}$) the entrant makes his development decision $x_{E,1} \in \{0,1\}$. The entrant bases his decision on his private signal ($s_{E,1}$), his ability ($\mu_{E,1}$), his reputation ($\lambda_{E,1}^{M1/P1}$), the reputation of the incumbent ($\lambda_{I,1}^{M1/P1}$) and the development-action of the incumbent ($x_{I,1}$).

After both producers made their development decision they set prices simultaneously. After that, the consumer decides to buy a unit of the product from none or from one of the producers based on the expected utility she receives from buying a unit of the product and based on the prices the producers charge. The incumbent chooses price: $p_{I,1}$ and the entrant chooses price: $p_{E,1}$. In their price decision both producers take into account which product they developed, which product their competitor developed ($x_{I,1}\ and\ x_{E,1}$), their own reputation, the reputation of their competitor ($\lambda_{I,1}^{M1/P1}\ and\ \lambda_{E,1}^{M1/P1}$), the expected abilities from the consumer's perspective ($\mu_{I,1}^{c}\ and\ \mu_{E,1}^{c}$), the strategy the consumer expects the producers to follow $(P1_I/M1_I, P1_E/M1_E)$, their own product quality and the product quality of their competitor ($R_I\ and\ R_E$). Recall that the difference between the expected ability $\mu_{i,t}^{c}$ and the reputation $\lambda_{i,t}^{M1/P1}$ is that the expected ability the probability is that the producer receives a 'correct' signal $s_{i,t} = \theta_t$ (from the consumer's perspective) and that the reputation reflects the probability that the producer develops the product-type that matches the state $x_{i,t} = \theta_t$. In setting their prices the producers use this information and calculate perfectly how the consumer determines her expected utility from buying a product from either of the producers. The consumer calculates her expected utility by determining the probability that a certain state takes place given the product-type the producers offer (this calculation involves using the expected ability of both producers and Bayes' rule) times the product quality, $R_I\ or\ R_E$. Calculating the probability that the world is in a certain state given the product development decision from both producers is done in the following way.

If both producers honestly follow their signal in the first period, then the probability that the state of the world is one (product B should be bought by the consumer) given that both producers developed product B is (using Bayes' rule) equal to:

$$pr(\theta_1 = 1 | x_{I,1} = 1, x_{E,1} = 1)$$
$$= \frac{pr(x_{I,1} = 1, x_{E,1} = 1 | \theta_1 = 1) * pr(\theta_1 = 1)}{pr(x_{I,1} = 1, x_{E,1} = 1 | \theta_1 = 1) * pr(\theta_1 = 1) + pr(x_{I,1} = 1, x_{E,1} = 1 | \theta_1 = 0) * pr(\theta_1 = 0)}$$
$$= \frac{\mu_{I,1}^c \mu_{E,1}^c * \frac{1}{2}}{\mu_{I,1}^c \mu_{E,1}^c * \frac{1}{2} + (1 - \mu_{I,1}^c)(1 - \mu_{E,1}^c) * \frac{1}{2}} = \phi_{\lambda_{I,1}^{P1}, \lambda_{E,1}^{P1}}^{1|1-1}$$

In $\phi_{\lambda_{I,1}^{P1}, \lambda_{E,1}^{P1}}^{1|1-1}$, the first 1 means the probability that the state equals 1, given that the incumbent chooses to develop product B (the first one after the | sign), and given that the entrant developed product B (the second one after the | sign), in other words $\phi_{\lambda_{I,1}^{P1}, \lambda_{E,1}^{P1}}^{\theta|x_{I,1}-x_{E,1}}$ where $\lambda_{I,1}^{P1}\ and\ \lambda_{E,1}^{P1}$ are the reputations of the producers from the consumer's perspective. Similarly, if the consumer believes that both producers follow their signal honestly $(P1_I/P_E)$ and observes that the incumbent developed product B and the entrant developed product A, the expected probability that the state equals one (the state where product B should be produced) is:

$$pr(\theta_1 = 1 | x_{I,1} = 1, x_{E,1} = 0)$$
$$= \frac{pr(x_{I,1} = 1, x_{E,1} = 0 | \theta_1 = 1) * pr(\theta_1 = 1)}{pr(x_{I,1} = 1, x_{E,1} = 0 | \theta_1 = 1) * pr(\theta_1 = 1) + pr(x_{I,1} = 1, x_{E,1} = 0 | \theta_1 = 0) * pr(\theta_1 = 0)}$$
$$= \frac{\mu_{I,1}^c 1 - \mu_{E,1}^c) * \frac{1}{2}}{\mu_{I,1}^c (1 - \mu_{E,1}^c) * \frac{1}{2} + (1 - \mu_{I,1}^c)\mu_{E,1}^c * \frac{1}{2}} = \phi_{\lambda_{I,1}^{P1}, \lambda_{E,1}^{P1}}^{1|1-0}$$

If we now imagine that the incumbent does not always follow his signal in the first period, in other words the incumbent develops the product that is opposite to his signal with probability: t. In this case the reputation of the incumbent is: $\lambda_{I,1}^{M1} = \log\left(\frac{\mu_{I,1}^c(1-t)+(1-\mu_{I,1}^c)t}{(1-\mu_{I,1}^c)(1-t)+\mu_{I,1}^c t}\right)$. This reputation is lower than the reputation of the incumbent in case he always follows his signal, $\lambda_{I,1}^{M1} < \lambda_{I,1}^{P1}$. This directly implies that $\mu_{I,1}^{c,M1} < \mu_{I,1}^{c,P1}$ and therefore the probability that the state is one, given that the incumbent developed product-type B (the product that matches with state 1) and the entrant developed product-type A (the product that matches with state 0) is now lower than in case the incumbent is always honest $\phi_{\lambda_{I,1}^{P1}, \lambda_{E,1}^{P1}}^{1|1-0} > \phi_{\lambda_{I,1}^{M1}, \lambda_{E,1}^{P1}}^{1|1-0}$.

The expected product value for the consumer, in case the incumbent develops product B and the entrant develops product A is in case (both producers follow their signal and) she buys from the incumbent: $\phi^{1|1-0}_{\lambda^{P1}_{I,1},\lambda^{P1}_{E,1}} * R_I$ and $\phi^{0|1-0}_{\lambda^{P1}_{I,1},\lambda^{P1}_{E,1}} * R_E$ in case the consumer buys from the entrant. A more extensive explanation and calculations of the expectation the consumer has that a certain state takes place is given in appendix B. Since the producers know $x_{I,1,}, x_{E,1}, \lambda^{M1/P1}_{I,1}, \lambda^{M1/P1}_{E,1}, R_I \ and \ R_E$ they calculate these Bayesian probabilities in the same manner as the consumer does, and base their price setting strategy on this calculation. Recall that the producers will only offer the product as long as the price they can charge is higher than or equal to the cost of producing, $p_{i,t} \geq c$. Since both producers strictly prefer selling a unit of their product to the consumer as long as $p_{i,t} \geq c$ holds and because in case $p_{i,t} < c$ the producer prefers to not sell a unit of his product to the consumer in the first place. Therefore, the producer that offers the product with the lowest expected value from the consumer's perspective $(\phi * R)$ sets his price equal to the minimum price he is willing to charge, c. Since the producers try to maximize their profits, the producer that offers the product with the highest expected value will set his price equal to c plus the difference in expected value between the products developed by both producers. For example, if the incumbent developed product B and the entrant developed product A we see that $\phi^{1|1-0}_{\lambda^{P1}_{I,1},\lambda^{P1}_{E,1}} * R_I > \phi^{0|1-0}_{\lambda^{P1}_{I,1},\lambda^{P1}_{E,1}} * R_E$ (this result is proven in appendix C) and the incumbent sets his price at: $p_{I,1} = p_{E,1} + \phi^{1|1-0}_{\lambda^{P1}_{I,1},\lambda^{P1}_{E,1}} * R_I - \phi^{0|1-0}_{\lambda^{P1}_{I,1},\lambda^{P1}_{E,1}} * R_E$ and the entrant sets his price at: $p_{E,1} = c$. In this case the consumer is indifferent between buying a unit of the product from either of the producers. Throughout this game we assume that in case the consumer is indifferent she will choose to buy a unit of the product from the producer that offers the highest expected quality, in this example to buy a unit of the product from the incumbent. For a more extensive explanation about the price setting strategies of the producers please refer to appendix C.

After the buying decision of the consumer the state of the world is revealed. The consumer updates her beliefs about the reputation ($\lambda^{G/B,P1/M1}_{I,1} \ and \ \lambda^{G/B,P1/M1}_{E,1}$) and ability ($\mu^{c,G/B,P1/M1}_{I,2} \ and \ \mu^{c,G/B,P1/M1}_{E,2}$) of the producers and period one ends. Here subscript G means that the product developed matched the state: $(x_{i,t} = \theta_t)$ and subscript B means that the product developed did not match the state $(x_{i,t} \neq \theta_t)$. Note that I have to distinguish between the case where the producer developed the product-type that matched the state and the consumer believed that this producer always follows his signal and the case where the product-type matched the state and the consumer believed that the producer mixes between following his signal and develop the product-type that is opposite to his signal, because in a mixed strategy it is also possible that the producer developed the correct (or wrong) product accidently. Therefore it is straightforward that $\mu_{i,2}^{c,G,P1} > \mu_{i,2}^{c,G,M1}$ and that $\mu_{i,2}^{c,B,P1} < \mu_{i,2}^{c,B,M1}$. I added all these calculations in appendix A.

In period two, nature determines the state of the world $\theta_2 \in \{0,1\}$ and sends the private signals $s_{I,2} \in \{0,1\}$ to the incumbent and $s_{E,2} \in \{0,1\}$ to the entrant.

After the incumbent observes his period two signal $(s_{I,2})$, he makes his period two development decision $(x_{I,2})$. He makes his development decision using history information $(h_{I,1})$. The history information the incumbent uses consists of: his own ability (μ_I), his updated period one reputation $(\lambda_{I,1}^{G/B,P1/M1})$, the updated period one reputation of the entrant $(\lambda_{E,1}^{G/B,P1/M1})$, the state of the world in period one (θ_1), the production decisions of both producers $(x_{I,1}, x_{E,1})$ the product quality (R_I, R_E), the beliefs of the consumer about the strategies the producers will follow and whether or not the entrant is still in the market and if the entrant is no longer in the market whether or not a new entrant has entered the market. In other words: the incumbent makes a decision on $x_{I,2}$ using $s_{I,2}$ and $h_{I,1}(\mu_I, \lambda_{I,2}^{G/B,P1/M1,M2/P2}, \lambda_{E,2}^{G/B,P1/M1,P2/M2}, \theta_1, x_{I,1}, x_{E,1,}R_I, R_E)$.

If the entrant that entered the market in period one is still in the market in period two, he will receive a signal $(s_{E,2})$ and makes a period two development decision $(x_{E,2})$ based on his signal and on his history information $(h_{E,1})$. The history information of the entrant consists of his own ability, the reputation of himself and the incumbent in period two, the state of the world in period one, the strategy the consumer believes the producers will follow in period two, the development decisions of himself and the incumbent and the product qualities. In other words the entrant makes his development decision $x_{E,2}$ based on $s_{E,2}$ and $h_{E,1}(\mu_E, \lambda_{I,2}^{G/B,P1/M1,M2/P2}, \lambda_{E,2}^{G/B,P1/M1,P2/M2}, \theta_1, x_{I,1}, x_{E,1,}R_I, R_E)$. The entrant is out of the market if his expected product value in period two $(\phi^{\theta_2|x_{I,2}-x_{E,2}} * R_E)$ is smaller than the cost of producing c. In this case the entrant will be out of the market because he will never be able to sell a unit of his product profitable to the consumer. If the entrant that entered the market in the first period is no longer in the market, then there are two distinct cases. Case 1: No new entrant enters and if the period one entrant leaves the market, the incumbent becomes a monopolist. Case 2: A new entrant enters and receives private signal $s_{EN,2}$. After receiving his signal the new entrant makes a development decision $x_{EN,1}$ based on his signal and based on history information: $h_{EN,2}(\mu_{NE}, \lambda_{I,2}^{G/B,P1/M1,P2/M2}, \lambda_{EN,2}^{P2/M2}, \theta_1, x_{I,1}, R_I, R_{NE})$, here we assume that the true ability of the new entrant is equal to the true ability of the old entrant $(\mu_{NE} = \mu_E)$ and that the product quality of the old entrant and the new entrant is the exact same $(R_{NE} = R_E)$.

After both producers made their development decision sequentially (with the again the incumbent moving first), they set prices simultaneously. Similarly to period one, the producers base their prices on their product development decisions $\left(x_{I,2} and\ x_{E,2}\right)$, their own reputation and the reputation of their competitor $(\lambda_{I,2}\ and\ \lambda_{E,2})$, their own product quality and the product quality of their competitor $(R_I\ and\ R_E)$. In case the incumbent became a monopolist in period two he does not have to take the development decision, the reputation and the product quality of the entrant into account in deciding on his price. He therefore only takes his own development decision, reputation and product quality into account when deciding on his price $(p_{I,2})$.

After the producers (or producer in the monopolist case) made their development decision and set their prices, the consumer makes a buying decision. This buying decision is the exact same as the buying decision in period one. After the consumer made her period two buying decision, the game ends.

3. Equilibrium Behaviour

Equilibrium in the second period (without reputational concerns)

This game can be solved using backward induction. The producers enter the second period with a reputation $\lambda_{i,2}$ that is based on their performance and (expected) strategy in period one. Since the game ends after two periods there are no reputational concerns anymore in the second period. Since both states are equally likely to occur and since there are no future gains for the incumbent from deviating from his signal, the incumbent will always follow his signal in the second period (and choose $x_{I,2} = s_{I,2}$).

Now we have to check that if we assume that the consumer believes that both producers always follow their signal honestly in the second period $(x_{I,2} = s_{I,2}\ and\ x_{E,2} = s_{E,2})$, there is no producer that wants to deviate from this strategy. As we just showed, the incumbent will not deviate. To determine whether the entrant has an incentive to deviate we have to check all possible first period situations given that we assume that both producers always follow their signal in the first period, the incumbent followed his signal in the second period and the consumer believes that both producers always follow their signal. We calculated the buying decision of the consumer and the equilibrium prices for all possible information sets in appendix C. From these calculations it follows that in case in period one the incumbent developed the product-type that did not match the state and the entrant developed the product-type that matched the state the entrant is always better off if he develops the product that is opposite to the product developed by the incumbent, this is independent from the entrant's signal since future reputation is unimportant.

Formally this can be showed by comparing the following two cases. The case with history information (from the entrant's point of view): $\lambda_{I,2}^{B,P1,P2}$, $\lambda_{E,2}^{G,P1,P2}, R_I, R_E \; and \; x_{I,2} = 1$ then the payoff for the entrant from choosing $x_{E,2} = 1$ is zero, since: the expected utility from buying a unit of the product from the entrant is: $U_{C,E} = \phi^{1|1-1}_{\lambda_{I,2}^{B,P1,P2}, \lambda_{E,2}^{G,P1,P2}} * R_E - p_{E,2}$ for the consumer and the expected utility of buying a unit of the product from the incumbent is: $U_{C,I} = \phi^{1|1-1}_{,\lambda_{I,2}^{B,P1,P2}, \lambda_{E,2}^{G,P1,P2}} * R_I - p_{I,2}$ for the consumer. Since in case the producers develop the same product-type the probability that this type matches the state is the same and since $R_I > R_E$ by assumption, the consumer will buy a unit of the product from the incumbent at a price of : $p_{I,2} = \phi^{1|1-1}_{\lambda_{I,2}^{B,P1,P2}, \lambda_{E,2}^{G,P1,P2}} * (R_I - R_E) - p_{E,2}$ where in equilibrium $p_{E,2} = c$. (On notation, recall that λ_i is the reputation of the incumbent (I) or the entrant (E), subscript G stands for $(x_{i,t} = \theta_t)$ and subscript B stand for $(x_{i,t} \neq \theta_t)$ and the expected probability that the world is in a certain state is $\phi^{\theta|x_{I,t} - x_{E,t}}$).

In case after observing history information $\lambda_{I,2}^{B,P1,P2}$, $\lambda_{E,2}^{G,P1,P2} \; and \; x_{I,2} = 1$ the entrant decides to develop $x_{E,2} = 0$ the expected utilities for the consumer are: $U_{C,E} = \phi^{0|1-0}_{\lambda_{I,2}^{B,P1,P2}, \lambda_{E,2}^{G,P1,P2}} * R_E - p_{E,2}$ and $U_{C,I} = \phi^{1|1-0}_{\lambda_{I,2}^{B,P1,P2}, \lambda_{E,2}^{G,P1,P2}} * R_I - p_{I,2}$. In this case the consumer will buy a unit of the product from the entrant if the expected value from buying a unit of the product from the entrant exceeds the expected value of buying a unit of the product from the incumbent:

$$\phi^{0|1-0}_{\lambda_{I,2}^{B,P1,P2}, \lambda_{E,2}^{G,P1,P2}} * R_E - p_{E,2} > \phi^{1|1-0}_{\lambda_{I,2}^{B,P1,P2}, \lambda_{E,2}^{G,P1,P2}} * R_I - p_{I,2}$$

Since both producers can set their price minimally equal to the cost of production c, the consumer will buy from the entrant if: $\phi^{0|1-0}_{\lambda_{I,2}^{B,P1,P2}, \lambda_{E,2}^{G,P1,P2}} * R_E > \phi^{1|1-0}_{\lambda_{I,2}^{B,P1,P2}, \lambda_{E,2}^{G,P1,P2}} * R_I$, for a maximum price of: $p_{E,2} = p_{I,2} + \phi^{0|1-0}_{\lambda_{I,2}^{B,P1,P2}, \lambda_{E,2}^{G,P1,P2}} * R_E - \phi^{1|1-0}_{\lambda_{I,2}^{B,P1,P2}, \lambda_{E,2}^{G,P1,P2}} * R_I$ (where $p_{I,2} = c$). Since there are parameter values of the product qualities $R_I \; and \; R_E$ and of the updated expected abilities: $\mu_{E,2}^{c,G,P1}$ and $\mu_{I,2}^{c,B,P1}$ for which the inequality: $\phi^{0|1-0}_{\lambda_{I,2}^{B,P1,P2}, \lambda_{E,2}^{G,P1,P2}} * R_E > \phi^{1|1-0}_{\lambda_{I,2}^{B,P1,P2}, \lambda_{E,2}^{G,P1,P2}} * R_I$ holds, the entrant will choose to develop the product that is opposite to his signal. This result can be summarized in the following proposition:

Proposition 1: *There does not exist an equilibrium where both producers always follow their signal honestly. Instead in period two the entrant has an incentive to make his development decision independent from his period two signal.*

If the incumbent always follows his signal in the second period ($x_{I,2} = s_{I,2}$) and the entrant always develops the product-type that is opposite to the product-type the incumbent developed ($x_{E,2} \neq x_{I,2}$), the consumer knows that the incumbent develops the product that matches the state with probability $\mu^c_{I,2}$ and the entrant develops the correct product with probability $1 - \mu^c_{I,2}$ (namely, the entrant only develops the product that matches the state if the incumbent received a signal that did not match the state). The consumer will never buy a unit of the product from the entrant in this case because $\mu^c_{I,2} \geq \frac{1}{2}$ and $R_I > R_E$ by assumption.

Now we should check if there is an equilibrium in which the incumbent always follows his signal and the entrant mixes between following his signal and develop the product-type that is opposite to his signal. For the incumbent there is still no reason to do anything else then to follow his signal. The entrant has an incentive to develop the product-type that is opposite to the product-type developed by the incumbent ($x_{I,2} \neq x_{E,2}$). But if the entrant always follows this strategy, then his product will never be bought by the consumer because the development decision of the entrant does not contain any information about the entrant's signal. Therefore the entrant should mix between following his signal and develop the product-type that differs from the product-type the incumbent developed with positive probability. Recall that the consumer will only buy a unit of the product from the entrant in case the product-types developed differ (because $R_I > R_E$) and the probability that the entrant developed a product that matches the state is higher than the probability that the incumbent developed a product that matched the state, in other words only if $\phi^{1|1-0} * R_I < \phi^{0|1-0} * R_E \; or \; \phi^{0|0-1} * R_I < \phi^{1|0-1} * R_E$. The only case where this is possible is if the entrant distinguished himself from the incumbent in the first period and it turns out that the entrant developed the product that matched the state and the incumbent did not. Therefore in case the incumbent developed the wrong product-type in period one and the entrant developed the product that matched the state, the entrant has an incentive to distinguish himself from the incumbent in period two and will use a mixed strategy. The optimal mixed strategy for the entrant is to follow his signal in case his signal differs from the product-type the incumbent developed ($s_{E,2} \neq x_{I,2}$) and to not follow his signal with positive probability if his signal matches the product-type the incumbent developed ($s_{E,2} = x_{I,2}$). In all the other cases (where for example both producers developed the same product-type in period one) there is no gain for the entrant from deviating from his signal. In fact, the entrant is indifferent between any possible strategy in all other cases, because in equilibrium he will never be able to sell a unit of his product. For convenience we assume that the entrant will follow his signal in all these other cases.

The entrant's second period mixed strategy can be formally stated as follows. If the consumer observes that after the incumbent developed the wrong product-type in the first period and his expected ability is updated to: ($\mu_{I,2}^{c,B,P1}$) and the entrant developed the correct product-type and the entrant's expected ability is updated to: ($\mu_{E,2}^{c,G,P1}$) and the second period development decision differs ($x_{I,2} \neq x_{E,2}$), the expected product value from the entrant's product must be higher compared to the expected product value from the incumbent's product. In other words:

$$pr\left(x_{E,2} = \theta_2 \middle| x_{I,2} \neq x_{E,2}, h_t\left(\lambda_{I,1}^{B,P1}, \lambda_{E,1}^{G,P1}, P2_I, M2_E\right)\right) * R_E \geq pr\left(x_{I,2} = \theta_2 \middle| x_{I,2} \neq x_{E,2}, h_t\left(\lambda_{I,1}^{B,P1}, \lambda_{E,1}^{G,P1}, P2_I, M2_E\right)\right) * R_I$$

$$\phi^{0|1-0}_{\lambda_{I,2}^{B,P1,P2}, \lambda_{E,2}^{G,P1,P2}} * R_E \geq \phi^{1|1-0}_{\lambda_{I,2}^{B,P1,P2}, \lambda_{E,2}^{G,P1,M2}} * R_I$$

There are parameter values of the product qualities (R_E *and* R_I), the updated expected abilities $\mu_{I,2}^{c,B,P1}$ *and* $\mu_{E,2}^{c,G,P1}$ and mix probability w for which this inequality holds. This result can be summarized in the following proposition:

Proposition 2: *There exists a second period equilibrium in which the incumbent always follows his signal, the entrant follows his signal if his signal differs from the product developed by the incumbent and mixes between following his signal and develop the product that is opposite to his signal with positive probability if his signal matches the product developed by the incumbent, in case the entrant developed the product that matched the state in period one and the incumbent developed the product that did not match the state, if the following conditions hold:*

i) *The difference in product quality between the two producers should not be too large:* $R_I - R_E$ *should not be too large*

ii) *The difference in second period updated ability after the entrant developed the product-type that matched the state and the incumbent developed the product-type that did not match the state should be sufficiently large:* $\mu_{E,2}^{c,G,M1} - \mu_{I,2}^{c,B,P1}$ *should be sufficiently large.*

In case these conditions do not hold and in all cases where the incumbent developed the product-type that matched the state, the entrant will be indifferent between all possible strategies, and we will for convenience assume that the entrant will then decide to always follow his signal.

Note that these strategies only hold in equilibrium (where the incumbent never miscalculates). In case the incumbent is uncertain about the product quality (R_I or R_E) of either of the producers or if he miscalculates the updating of the consumer about the probability that the world is in a certain state (ϕ), everything is possible. The entrant might then be inclined to 'herd' the incumbent's development decision, because when using this strategy the entrant's expected product value ($\phi * R_E$) increases in most of the cases.

Equilibrium in the first period (with reputational concerns)

In the first period the consumer will always decide to buy a unit of the product from the incumbent if the two producers develop the same product-type ($x_{I,1} = x_{E,1}$). Because the product quality of the product developed by the incumbent is higher compared to the product quality of the product developed by the entrant ($R_I > R_E$). In case the producers develop a different product-type, the consumer will buy a unit of the product from the incumbent as long as the incumbent always follows his signal honestly ($x_{I,1} = s_{I,1}$), because $\mu_{I,1}^{c} > \mu_{E,1}^{c}$ by assumption.

As one can see from the second period equilibrium, the consumer will also buy a unit of the product from the incumbent in the second period if the producers developed the same product in period one because $\mu_{I,2}^{c,G,P1} > \mu_{E,2}^{c,G,P1}$ and $\mu_{I,2}^{c,B,P1} > \mu_{E,2}^{c,B,P1}$. The incumbent will offer a product in the second period as long as the price the consumer is maximally willing to pay is larger than or equal to the cost of producing c. Important to understand here is that in case the incumbent does not have a competitor in the second period, the incumbent can charge a monopoly price. Therefore the incumbent has an incentive to have the entrant develop the wrong product-type in period one. The entrant will be out of the market in period two if he is never able to sell a unit of his product profitable in the second period. This is most likely in case both producers develop the 'wrong' product-type in period one, because in this case the consumer is least confident (lowest ϕ) that the producers develop the product-type that will match the state in period two (in other words, $\phi_{\lambda_{I,2}^{B},\lambda_{E,2}^{B}}^{\theta|x_{I,2}-x_{E,2}}$ is smaller than in case at least one of the producers' reputation has increased). If both producers develop the same product-type, the consumer is always as least as confident that the products developed match the state than in case the producers develop a different product-type $\left(\phi_{\lambda_{I,2}^{B,P1,P2},\lambda_{E,2}^{B,P1,P2}}^{1|1-1} \geq \phi_{\lambda_{I,2}^{B,P1,P2},\lambda_{E,2}^{B,P1,P2}}^{1|1-0}\right)$. Therefore, the entrant is out of the market if: $\phi_{\lambda_{I,2}^{B,P1,P2},\lambda_{E,2}^{B,P1,P2}}^{1|1-1} * R_E < c$. This means that the incumbent has an incentive to have the entrant develop the same product-type that does not match the state in period one. A second case where the entrant will (for some parameter values of R_I and R_E and some updated expected abilities of$\mu_{I,2}^{c,G,P1}$ and $\mu_{E,2}^{c,B,P1}$) be out of the market in the second period is after the entrant developed the product-type that did not match the state and the incumbent developed the product-type that matched the state ($\phi^{1|1-1}_{\lambda^{G,P1,P2}_{I,2},\lambda^{B,P1,P2}_{E,2}}$ or $\phi^{1|1-0}_{\lambda^{G,P1,P2}_{I,2},\lambda^{B,P1,P2}_{E,2}}$). If $\phi^{1|1-1}_{\lambda^{G,P1,P2}_{I,2},\lambda^{B,P1,P2}_{E,2}} * R_E < c$ holds the incumbent also has an incentive to have the entrant develop the wrong product-type while developing the correct product-type himself. Important here is that the probability that the entrant will no longer be in the market is larger in case both producers developed the product-type that did not match the state in period one, than in case the incumbent developed the product-type that matched the state, and the entrant did not, because $\phi^{1|1-1}_{\lambda^{G,P1,P2}_{I,2},\lambda^{B,P1,P2}_{E,2}} > \phi^{1|1-1}_{\lambda^{B,P1,P2}_{I,2},\lambda^{B,P1,P2}_{E,2}}$.

If we first start by assuming that in period one both producers follow their signal honestly, and the consumer believes that both producers will follow their signal honestly, will one of the producers deviate? As long as the entrant always follows his signal in period one there is no way in which the incumbent can profitably deviate from not following his signal because deviating from his signal will only lower the incumbent's updated expected ability for period two.

If the entrant always follows his own signal in period one, he will possibly be out of the market in period two after both producers developed the product that did not match the state, this occurs with $pr = \left(1 - \mu^{c}_{I,1}\right)(1 - \mu_E)$ from the entrant's point of view, and after the entrant developed the product that did not match the state and the incumbent developed the product that matched the state, this occurs with $pr = \mu^{c}_{I,1}(1 - \mu_E)$ (from the entrant's point of view). Recall that there is a larger scale of parameter values for which the entrant is out of the market after both producers developed the 'wrong' product ($\theta_1 \neq x_{E,1}\ and\ \theta_1 \neq x_{I,1}$) compared to the case where the entrant developed the 'wrong' product and the incumbent developed the 'correct' product ($\theta_1 \neq x_{E,1}\ and\ \theta_1 = x_{I,1}$). The entrant can make a profit in the second period in case in the first period the incumbent developed a product that did not match the state, this happens with pr: $(1 - \mu^{c}_{I,1})$ from the entrant's point of view, and the entrant developed the product-type that matched the state, this happens with pr: μ_E. In other words, if both producers follow their signal honestly the entrant is able to make a profit in the second period with probability: $\left(1 - \mu^{c}_{I,1}\right)\mu_E$.

If the entrant decides to herd the incumbent in period one, he will possibly be out of the market in period two only when both producers developed the 'wrong' product-type in period one, this occurs with pr: $(1 - \mu^{c}_{I,1})$. On the other hand, the entrant is never able to sell a product profitably if he always herds the incumbent because $\mu^{c,G,P1}_{I,2} > \mu^{c,G,P1}_{E,2}$, $\mu^{c,B,P1}_{I,2} > \mu^{c,B,P1}_{E,2}$ and $R_I > R_E$. Whether it is more likely that the entrant has to leave the market if he 'herds' the incumbent or if he follows his own signal crucially depends on the parameter values. But it is obvious that if the incumbent's expected ability at the start of period one $\mu^{c,P1/M1}_{I,1}$ is low (and not much larger than $\mu^{c,P1}_{E,1}$) it is more likely that the entrant has to leave the market if he 'herds' the incumbent. Recall that in equilibrium the entrant is indifferent between having to leave the market after period one and staying in the market but having a lower expected product value than the incumbent $(\phi * R)$ in period two, because in both cases this will result in a profit of zero (and no costs are made). If we now compare the profit perspective from both period one strategies of the entrant it is easy to see that the only way the entrant can make profit (in period two) is to distinguish himself from the incumbent in period one by developing the product-type that differs from the product-type developed by the incumbent $x_{I,1} \neq x_{E,1}$. So although the entrant's ability is expected to be higher when 'herding' the incumbent he will prefer to develop the product-type that is opposite to the product-type developed by the incumbent.

Proposition 3: *Although the expected ability of the entrant will increase in case he 'herds' the incumbent, he will never herd the incumbent in period one. Instead he will have to distinguish himself in period one to be able to make profit in the second period.*

Is it optimal for the entrant to always follow his signal in period one if the incumbent follows his signal in period one and the consumer believes that both producers always follow their signal in period one? To answer this question, recall that the entrant is never able to sell a unit of his product in period one and can only sell a unit of his product in the second period if he distinguished (choose $x_{E,1} \neq x_{I,1}$) himself from the incumbent in period one. Secondly, in equilibrium the incumbent will never miscalculate (will never charge a price that is so high that the consumer buys from the entrant even though the expected product value of the incumbent is higher than the expected product value of the entrant) and therefore in this two period game the entrant is indifferent between having to leave the market after period one and continue to period two with a lower expected ability than the incumbent ($\mu_{I,2}^{c} > \mu_{E,2}^{c}$). Since the entrant is indifferent between leaving the market after period one and stay in the market in the second period knowing that he will never sell a unit of his product, the entrant is inclined to always develop the product-type that is the opposite of the product-type the incumbent developed. If the entrant always develops the product-type that is the opposite of the product-type developed by the incumbent, then his development decision says nothing about his ability. This implies that if it turns out that the entrant developed the 'correct' product $x_{E,1} = \theta_1$, his ability will not increase and the consumer is not inclined to buy a unit of the product from the entrant in period two (because the consumer does not become more confident that the entrant is able to develop the product-type that matches the state). Therefore it is optimal for the entrant to always follow his signal in case his signal differs from the product-type developed by the incumbent ($if\ s_{E,1} \neq x_{I,1}$, the entrant follows his signal $s_{E,1} = x_{E,1}$) and if the entrant's signal matches the product-type developed by the incumbent the entrant mixes (if $s_{E,1} = x_{I,1}$ the entrant mixes and chooses to not follow his signal with $pr(x_{E,1} \neq s_{E,1}) = r$).

Will the incumbent deviate from always following his signal if he knows that the entrant mixes between following his signal and develop the product-type that is opposite to his signal? Note that the incumbent has an incentive to have both producers develop the wrong product-type while the entrant has an incentive to distinguish himself from the incumbent, therefore there is no way in which the incumbent can profit by deviating from the strategy to always follow his signal.

Furthermore, one can see that whether the incumbent becomes a monopolist after the entrant has left the market in period one or that a new entrant enters, does not matter for the behaviour of the incumbent, since the incumbent is always better off following his own signal compared to any other strategy. This result differs from findings of Eastbrook (1981). Eastbrook finds that the strategy of a monopolist, in case an entrant considers entering the market, crucially depends on the expectation of the monopolist about whether a new entrant will try to enter the market if the current entrant is deterred. Eastbrook shows that unless the monopolist is certain that no new entrant will (try to) enter the market the monopolist will be less aggressive in trying to deter the entrant from entry. The reason why our findings differ from Eastbrook's findings is that in our model the incumbent always benefits in the second period from following his signal in period one.

Since the incumbent will always follow his signal in the first period and the entrant is inclined to distinguish himself, we find the following results:

Proposition 4: *There exists an equilibrium in period one in which the incumbent always follows his signal, the entrant follows his signal if his signal differed from the product-type developed by the incumbent and mixes between following his signal and develop the product-type that is opposite to the product developed by the incumbent if his signal matches the development decision of the incumbent. The probability with which the entrant decides not to follow his signal depends on:*

i) The difference between the product qualities $(R_I - R_E)$. *If this difference is small, the entrant's expected profit is high, and therefore the mix probability r will be high.*

ii) The differences between the period two updated expected abilities in case the incumbent developed the 'wrong' product and the entrant developed the 'correct' product $\mu_{E,2}^{c,G,M1} - \mu_{I,2}^{c,B,P1}$. *If this difference is large, the expected profit for the entrant is high and therefore the mix probability r will be high.*

The role of the cost of production

The role of ability, strategy (and thereby reputation) and product quality is made clear in propositions 1 to 4. The only part of the analysis that is lacking so far is the role of the cost of production c. The production costs do not play a role for the equilibrium prices in case the incumbent is a monopolist or in case both producers are in the market (duopoly). But the production costs do play a role in when the market exists of two producers and when the incumbent is the only producer in the market. Specifically, the production costs are the participation constraint for both producers. If the production costs are at such a level that the expected product value $(\phi * R)$ of both producers is larger than or equal to the production cost, we have a duopoly with both the incumbent and the entrant in the market. If the production costs are higher, it is possible that one of the producers is no longer in the market, or (when the production costs are higher than the expected product value of both producers) that there is no producer that is willing to develop a unit of the product for the consumer.

Equilibrium behaviour

In equilibrium the game is played as follows. In period one the incumbent always follows his signal because he want his second period reputation to be as high as possible and since there is no better way to have the entrant develop the product-type that does not match the state then to follow his signal. In period one the entrant has an incentive to develop the product-type that is opposite to the product-type developed by the incumbent because distinguishing himself in the first period is the only possible way for the entrant to make a profit in the second period. In equilibrium the entrant cannot always develop the product-type that is opposite to the product-type developed by the incumbent because in this case the consumer knows that the entrant makes his development decision independent of his signal and will therefore not update the entrant’s ability. To make it possible that the consumer’s beliefs about the ability of the entrant increase, the development decision of the entrant must reveal (at least some) information about the signal the entrant received, therefore the entrant has to follow his signal with positive probability. This leads to the following optimal period one strategy for the entrant. The entrant will follow his signal in case it differs from the development-decision of the incumbent, and follows his signal with positive probability if his signal matches the product-type the incumbent developed.

In the second period the incumbent always follows his signal because there is no reason not to. Again, the entrant has an incentive to distinguish himself from the incumbent. The reasons for this are that in period two there is no penalty for developing the ‘wrong’ product-type and that the only possible way to be profitable is to distinguish himself. Therefore the optimal strategy for the entrant in the second period is: to follow his signal in case it differs from the product-type developed by the incumbent and to follow his signal with positive probability in case his signal is the same as the product-type the incumbent developed.

4. Welfare analysis

We can compare the outcome of this model with the case where the incumbent is a monopolist. In a monopoly market the incumbent will always set his price equal to the expected product value $(\phi * R)$. In the monopoly case (where the incumbent has no incentive to do anything else then to follow his signal), the true probability that the product developed matches the state is: $\phi = \mu_I$, and from the consumer's perspective the probability is $\phi = \mu_{I,1}^c$. Therefore the only gain for the consumer in a monopoly market is the difference between the true ability of the incumbent and the expected ability from the consumer's perspective. If the incumbent is in the market for a long time, the expected ability will converge to the true ability (because of the Bayesian learning with the log likelihood ratio) and therefore there will be no profit for the consumer. Formally this can be stated as follows. The utility function of the consumer is $u(\gamma, p) = \gamma - p_{i,t}$ where $\gamma = \begin{cases} R_i, & if: \theta = x \\ 0, & if: \theta \neq x \end{cases}$. Therefore the incumbent can set the price he charges equal to γ. Here $\gamma = R_I \; with \; pr(\theta = x) = \mu_I$ and $\gamma = 0 \; with \; pr(\theta \neq x) = 1 - \mu_I$. Therefore the price the incumbent can charge in equilibrium is $\mu_I * R_I$ and the profit for the consumer is zero.

If an entrant enters we see that the maximum price the incumbent can charge no longer only depends on his own ability (as it is perceived by the consumer) and on the product quality, but also on the ability, strategy and product quality of the entrant. The maximum price the incumbent can charge is lower in case there is an entrant. If the incumbent follows his signal (as he will in equilibrium) we see that the price he can maximally charge is the difference between the expected product value $\phi_{\lambda_{I,t}, \lambda_{E,t}}^{\theta|x_{I,1}-x_{E,1}} * R_I - \phi_{\lambda_{I,t}, \lambda_{E,t}}^{\theta|x_{I,1}-x_{E,1}} * R_E$ plus the cost of production c as long as $\phi_{\lambda_{I,t}, \lambda_{E,t}}^{\theta|x_{I,1}-x_{E,1}} * R_I \geq c$ and $\phi_{\lambda_{I,t}, \lambda_{E,t}}^{\theta|x_{I,1}-x_{E,1}} * R_E \geq c$. This means that the maximum price the incumbent can charge in case there is an entrant is equal to or smaller than in case the incumbent is a monopolist. This implies that only the presence of the entrant is enough to discipline the incumbent to charge a lower price, and thereby increase the utility of the consumer.

Proposition 5: *Even if the entrant is not able to produce and sell any product, only the presence of the entrant already results in a higher expected utility for the consumer compared to a monopoly market.*

5. Discussion

The main results of our model are that the incumbent can only strategically react on the entrant's entry by always following his signal honestly, that the entrant can only be profitable by distinguishing himself from the incumbent in both periods, that the equilibrium behaviour of the incumbent is independent of whether or not there is uncertainty about whether a new entrant enters after the first period entrant is out of the market and that the mere presence of an inferior entrant is enough to discipline the incumbent to always follow his signal and charge a lower price than when he acts in a monopoly market.

In most of the present academic literature we see that the incumbent can strategically react to the entrant's entry by acting in a way that is not optimal for the entrant. For example, impose switching costs or have the consumers sign a long term contract so that for the consumer it is more difficult to buy from another producer when a more efficient producer enters. We show in our model that if the entrant enters the market, the incumbent only has an extra incentive to follow his signal honestly. Furthermore, because of the presence of the entrant the incumbent is disciplined to charge a lower price. This increases the expected utility for the consumer.

We show that the only way in which the entrant can be profitable is by distinguishing himself from the incumbent in both periods. This is remarkable, because the expected utility of the entrant will increase (more) if he 'herds' the incumbent, because the incumbent is more able and therefore the probability that the entrant develops a product-type that matches the state is higher if he always develops the same product-type as the incumbent than if he distinguishes himself.

What happens if we change the assumptions of our model? In our model the producers are heterogeneous in two ways. We did this to show that even an inferior producer can be profitable in a market with an incumbent firm. The producers are heterogeneous in their ability to receive a signal that matches the state, and they are heterogeneous in product quality, where the product quality of the incumbent is higher. If we set the product qualities of the products developed by the producers equal, $R_I = R_E$ then it becomes more attractive for the entrant to distinguish himself from the incumbent. Furthermore, this implies that for the incumbent there is less profit as long as the entrant is also in the market. Since in our two period model the incumbent has an additional incentive to follow his signal (because then the entrant is more likely to develop the 'wrong' product-type), the behaviour of the incumbent will not change.

Another assumption we make is that the product qualities are perfectly known by both the producers and by the consumer. If we instead assume that the product qualities are unknown (differ per producer and per period) and are distributed in a certain way, then the following things will change. If we take for example the normal distribution, the results will not change as long as the consumer is risk neutral. If the consumer is risk averse we will see that also the variance plays a role. The larger the variance, the less inclined the consumer is to buy a unit of the product.

If we stick to the assumption that the product qualities are fixed and perfectly known, and assume that the consumer is risk averse instead of risk neutral, the results of the game will also be somewhat different. If the consumer is risk averse we see that she is even less willing to buy a unit of the product if she expects that the probability that the product-type matches the state is lower. This implies that there is less space for the entrant to not follow his signal, because this reduces the probability that the product-type developed matched the state and this is punished harder by the consumer.

As in Kemperer (1987 and 1995) we can also extend this game to a game with switching costs. In this case, more 'of the battle' will be fought in the first period. This implies two changes in the outcome of our model. In the first place this will lead the entrant to distinguish himself in the first period more often, because he is able to charge a higher price in the second period. Secondly, both producers will lower their prices and are even inclined to sell their product with a loss in the first period, to attract the consumer and be able to make a larger profit in the second period.

A recommendation for further research is the period length of the model. Our model is a two period model. This model perfectly captures the incentive of the entrant to distinguish himself from the incumbent. However, if this game should be extended to for example a ten period game, it can become profitable for the entrant to herd the incumbent in the first period(s) to establish a high reputation, and distinguish himself later on. This situation is interesting due to the fact that it is closer to a real world situation and because this could imply that there is another reason why 'herding' behaviour is bad for the player or firm that 'herds'.

6. Concluding Remarks

In line with the rich literature on entry deterrence and strategic (re)action of the incumbent to deter entry and drive competitors out of the market, we studied the role that social learning and herding behaviour can play in strategic (re)actions in a market.

In a market with an incumbent that is in the market (in a monopoly market) for a long time and has a well-known reputation, the incumbent can use his reputation to drive entrants out of the market. We studied a learning environment where an entrant enters the market that is inferior to the incumbent in two ways. The entrant has a lower ability; this implies that he knows less often which product-type matches the state of the world. And the product quality of the product developed by the entrant (if the product matches the state) is of lower quality compared to the product quality of the product developed by the incumbent. At the end of the period the consumer observes the actions of both producers and updates her beliefs about the ability of the producers.

The main insides our two period model provides, are that the incumbent can only act strategically by always following his signal, this is beneficial for the consumer. The only way in which the entrant can be profitable is by distinguishing himself from the incumbent in both periods. This means, developing the product-type that matches the state in period one while the incumbent develops the product-type that does not match the state, and develop the product-type that differs from the product-type the incumbent developed in period two. The third inside our model provides is that the behaviour of the incumbent is independent of whether or not there is uncertainty about whether the incumbent becomes a monopolist in period two after the first period entrant has left the market. The fourth and last inside our model provides is that even an inferior entrant is able to discipline the incumbent to always follow his signal and to charge a price that is lower than the monopoly price. This results in a higher expected utility for the consumer.

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Appendix A

Determining the reputation and ability of the producers:

<u>If the consumer believes that the incumbent follows his signal in period one</u>

Then the incumbent's reputation is: $\lambda_{I,1}^{P1} = \log\left(\frac{P(\theta_1 = x_{I,1})}{P(\theta_1 \neq x_{I,1})}\right) = \log\left(\frac{\mu_{I,1}^{c}}{1-\mu_{I,1}^{c}}\right)$ when period one starts. Here subscript c means from the consumer's perspective and subscript P1 means that the consumer believes that the producer follows a pure strategy (to always follows his signal $x_{i,t} = s_{i,t}$).

If the consumer believed that the incumbent followed his signal in period one and it turns out that the product the incumbent developed matched the state of the world, then she updates her belief about the producer at the end of the period to: $\lambda_{I,1}^{G,P1} = \lambda_{I,1} + \log\left(\frac{P(x_{i,t}|x_{i,t}=\theta_t)}{P(x_{i,t}|x_{i,t}\neq\theta_t)}\right) = \lambda_{I,1} + \log\left(\frac{\mu_{I,1}^{c}}{1-\mu_{I,1}^{c}}\right)$. Here the subscript G means that the producer developed the product-type that matches the state and subscript P1 means that the consumer believes that the producer follows a pure strategy (to always follows his signal $x_{i,t} = s_{i,t}$).

Note that $P(x_{i,t}|x_{i,t} = \theta_t) = \mu_I^k(1-\mu_I)^{1-k}$, is Bernoulli distributed, where μ_I is the probability of success ($x_{i,t} = \theta_t$) and $1-\mu_i$ is the probability of failure ($x_{i,t} \neq \theta_t$). Using the LLR this results in: $\log\left(\frac{P(x_{i,t}|x_{i,t}=\theta_t)}{P(x_{i,t}|x_{i,t}\neq\theta_t)}\right) = \log\left(\frac{\mu_I^k(1-\mu_I)^{1-k}}{(1-\mu_I)^k\mu_I^{1-k}}\right)$. In case of success ($k = 1$) this results in: $\log\left(\frac{\mu_i}{1-\mu_i}\right)$ and in case of failure ($k = 0$) this results in: $\log\left(\frac{1-\mu_i}{\mu_i}\right) = -\log\left(\frac{\mu_i}{1-\mu_i}\right)$.

The expected ability of the incumbent (from the consumer's perspective) if the incumbent developed the product-type that does not equal the state of the world ($x_{i,t} \neq s_{i,t}$) is: $\lambda_{I,1}^{B,P1} = \lambda_{I,1} + \log\left(\frac{P(x_{i,t}|x_{i,t}=\theta_t)}{P(x_{i,t}|x_{i,t}\neq\theta_t)}\right) = \lambda_{I,1} - \log\left(\frac{\mu_{I,1}^{c}}{1-\mu_{I,1}^{c}}\right)$. Here again, this updating is done at the end of period one.

The ability of the incumbent is updated and the end of the period using $\mu = \frac{e^{\lambda}}{1+e^{\lambda}}$ this is according to the Bayesian log likelihood updating (Chamley 2004). Therefore the reputation $\lambda_{I,1}^{G,P}$ is translated into an expected ability, from the perspective of the consumer, of: $\mu_{I,2}^{c,G,P1} = \frac{e^{\lambda_{I,1}^{G,P1}}}{1+e^{\lambda_{I,1}^{G,P1}}}$ and $\lambda_{I,1}^{B,P1}$ translates into: $\mu_{I,2}^{c,B,P1} = \frac{e^{\lambda_{I,1}^{B,P1}}}{1+e^{\lambda_{I,1}^{B,P1}}}$. Here one can see that the reputation directly after period one is used to determine the expected ability of the producer for period two.

If the consumer believes that in period 1 the incumbent mixes between following his signal and develop the product-type that is opposite to his signal with: $pr: (x_{I,1} \neq s_{I,1}) = t$

Then the incumbent's reputation is: $\lambda_{I,1}^{M1} = \log\left(\frac{P(\theta_1 = x_{I,1})}{P(\theta_1 \neq x_{I,1})}\right) = \log\left(\frac{\mu_{I,1}^{c}(1-t)+(1-\mu_{I,1}^{c})t}{(1-\mu_{I,1}^{c})(1-t)+\mu_{I,1}^{c}t}\right)$ Here subscript M1 means that the consumer believed that the producer used a mixed strategy (to develop the product-type that is opposite to his signal: $x_{I,1} \neq s_{I,1}$) in period one.

If at the end of the period it turns out that the product developed matched the state of the world, than the consumer updates the reputation of the incumbent to;

$$\lambda_{I,1}^{G,M1} = \lambda_{I,1} + \log\left(\frac{\mu_{I,1}^{c}(1-t)+(1-\mu_{I,1}^{c})t}{(1-\mu_{I,1}^{c})(1-t)+\mu_{I,1}^{c}t}\right) \text{ and } \mu_{I,2}^{c,G,M1} = \frac{e^{\lambda_{I,1}^{G,M1}}}{1+e^{\lambda_{I,1}^{G,M1}}}$$

And if it turned out that the product did not match the state$(x_{I,1} \neq s_{I,1})$:

$$\lambda_{I,1}^{B,M1} = \lambda_{I,1} - \log\left(\frac{\mu_{I,1}^{c}(1-t)+(1-\mu_{I,1}^{c})t}{(1-\mu_{I,1}^{c})(1-t)+\mu_{I,1}^{c}t}\right) \text{ and } \mu_{I,2}^{c,B,M1} = \frac{e^{\lambda_{I,1}^{B,M1}}}{1+e^{\lambda_{I,1}^{B,M1}}}$$

Note here that $\mu_{I,2}^{c,G,P1} > \mu_{I,2}^{c,G,M1} > \mu_{I,2}^{c,B,M1} > \mu_{I,2}^{c,B,P1}$ because if the product developed matched the state (G) the expected ability always increases and if the product developed did not match the state (B) the expected ability always decrease. Furthermore, the increase/decrease in ability is always stronger in case the expected strategy was a pure strategy ($P1$) than in case it was a mixed strategy ($M1$). This makes sense, because in case the incumbent used his mixed strategy and the product matched the state of the world the consumer never knows if this is because the incumbent received the right signal ($s_{I,t} = \theta_t$) or that he received the wrong signal ($s_{I,t} \neq \theta_t$) and decided to not follow his signal ($s_{I,t} \neq x_{I,t}$) and that the development decision therefore matched the state ($x_{I,t} = \theta_t$).

In the second period there are four different cases possible. The case where the consumer believes that: A) The incumbent is honest in period two after being honest in period one, B) The incumbent is honest in period two after mixing in period one, C) the incumbent mixes in period two after being honest in period one and D) the incumbent mixes in period two after he used his mixed strategy in period one. Note that the utility at the end of period two is not interesting because after period two the game ends and there is no benefit in having a high ability at the end of period two. Therefore only the updated reputations are calculated for each of these cases as follows:

A) The consumer believes that: the incumbent is honest in period two after being honest in period one

If the incumbent developed the product that matched the state in period one the incumbent's reputation is updated to: $\lambda_{I,2}^{G,P1,P2} = \log\left(\frac{P(\theta_1 = x_{I,1})}{P(\theta_1 \neq x_{I,1})}\right) = \log\left(\frac{\mu_{I,2}^{c,G,P1}}{1-\mu_{I,2}^{c,G,P1}}\right)$

If the incumbent developed the product that did not match the state in period one the incumbent's reputation is updated to: $\lambda_{I,2}^{B,P1,P2} = \log\left(\frac{P(\theta_1 = x_{I,1})}{P(\theta_1 \neq x_{I,1})}\right) = \log\left(\frac{\mu_{I,2}^{c,B,P1}}{1-\mu_{I,2}^{c,B,P1}}\right)$

B) <u>The consumer believes that: the incumbent is honest in period two after mixing in period one:</u>

$$\lambda_{I,2}^{G,M1,P2} = \log\left(\frac{\mu_{I,2}^{c,G,M1}}{1-\mu_{I,2}^{c,G,M1}}\right)$$

$$\lambda_{I,2}^{B,M1,P2} = \log\left(\frac{\mu_{I,2}^{c,B,M1}}{1-\mu_{I,2}^{c,B,M1}}\right)$$

C) <u>The consumer believes that: the incumbent mixes in period two (with $pr(x_{I,1} \neq s_{I,1}) = v$) after being honest in period one:</u>

$$\lambda_{I,2}^{G,P1,M2} = \log\left(\frac{\mu_{I,2}^{c,G,P1}(1-v) + \left(1-\mu_{I,2}^{c,G,P1}\right)v}{\left(1-\mu_{I,2}^{c,G,P1}\right)(1-v) + \mu_{I,2}^{c,G,P1}v}\right)$$

$$\lambda_{I,2}^{B,P1,M2} = \log\left(\frac{\mu_{I,2}^{c,B,P1}(1-v) + \left(1-\mu_{I,2}^{c,B,P1}\right)v}{\left(1-\mu_{I,2}^{c,B,P1}\right)(1-v) + \mu_{I,2}^{c,B,P1}v}\right)$$

D) <u>The consumer believes that: the incumbent mixes in period two (with $pr(x_{I,1} \neq s_{I,1}) = v$) after he mixed (with $pr(x_{I,1} \neq s_{I,1}) = t$) in period one:</u>

$$\lambda_{I,2}^{G,M1,M2} = \log\left(\frac{\mu_{I,2}^{c,G,M1}(1-v) + \left(1-\mu_{I,2}^{c,G,M1}\right)v}{\left(1-\mu_{I,2}^{c,G,M1}\right)(1-v) + \mu_{I,2}^{c,G,M1}v}\right)$$

$$\lambda_{I,2}^{B,M1,M2} = \log\left(\frac{\mu_{I,2}^{c,B,M1}(1-v) + \left(1-\mu_{I,2}^{c,B,M1}\right)v}{\left(1-\mu_{I,2}^{c,B,M1}\right)(1-v) + \mu_{I,2}^{c,B,M1}v}\right)$$

It is straightforward to see that if in period one $x_{I,1} = \theta_1$, the reputation of the incumbent is the highest in case: $\lambda_{I,2}^{G,P1,P2}$ and the lowest in case $\lambda_{I,2}^{G,M1,M2}$. But whether $\lambda_{I,1}^{G,P1,M2} > \lambda_{I,1}^{G,M1,P2}$ or $\lambda_{I,1}^{G,P1,M2} < \lambda_{I,1}^{G,M1,P2}$ depends on the values of the decision variables t and v. The same argument holds for the case where in period one $x_{I,1} \neq s_{I,1}$. Where the updated reputation of the incumbent is the highest in case $\lambda_{I,2}^{B,M1,P2}$ and the updated reputation is the lowest in case $\lambda_{I,2}^{B,P1,M2}$. This makes sense since developing the wrong product means that the reputation of the producer decreases but the decrease is the smallest if the producer mixed his strategies (because it is possible that the producer did observe the correct signal but made the wrong development decision on purpose). Similarly, the reputation is always higher if the consumer expects the producer to always follow his signal in the current period (P2).

Updating for the entrant works in the exact same way. The entrant only mixes with $pr(x_{E,1} \neq s_{E,1}) = r$ and $pr(x_{E,2} \neq s_{E,2}) = w$

Schematically the updating of the reputation can be presented as follows:

Period 1			2
$\lambda_{i,t}^{P1/M1}$	$x_{i,t}$	θ_t	$\lambda_{i,t+1}^{G/B,P1/M1}$
Consumer's beliefs about producer i.	Development decision by producer i	Revelation state of the world	Updating beliefs about producer i

Appendix B

Determining the probability in which a certain state takes place after the producers made their development decision:

Intuitively it makes sense that if the consumer observes that both producers develop the same product-type it is more likely that they are correct (in other words, it is more likely that their product matches the state of the world). This implies that the consumer is willing to pay more for this product compared to the case where the producers develop a different product-type. To calculate the exact expectation of the consumer about the state I have to use the information the producers provide (their development decision (x)), their expected strategy (pure strategy or mixed strategy) and the reputation and expected ability of the producers and use Bayes' rule.

If I assume that both producers always follow their signal honestly (choose: $x_{i,t} = s_{i,t}$):

Then the probability that the state equals one ($\theta_1 = 1$) after the consumer observed that both producers developed product B (the product that matches state 1) is:

$$\boldsymbol{pr\left(\theta_1 = 1 \middle| x_{I,1} = 1, x_{E,1} = 1, P1_I, P1_E\right)}$$
$$= \frac{pr\left(x_{I,1} = 1, x_{E,1} = 1 \middle| \theta_1 = 1\right) * pr(\theta_1 = 1)}{pr\left(x_{I,1} = 1, x_{E,1} = 1 \middle| \theta_1 = 1\right) * pr(\theta_1 = 1) + pr\left(x_{I,1} = 1, x_{E,1} = 1 \middle| \theta_1 = 0\right) * pr(\theta_1 = 0)}$$
$$= \frac{\mu_{I,1}^{c,P1}\mu_{E,1}^{c,P1} * \frac{1}{2}}{\mu_{I,1}^{c,P1}\mu_{E,1}^{c,P1} * \frac{1}{2} + \left(1 - \mu_{I,1}^{c,P1}\right)\left(1 - \mu_{E,1}^{c,P1}\right) * \frac{1}{2}} = \phi_{\lambda_{I,1}^{P1}\lambda_{E,1}^{P1}}^{1|1-1}$$

On notation; subscripts $P1$ means that the consumer expects both producers to follow their pure strategy (to always follow their own signal in period one) and subscripts 1|1-1 is $\theta|x_I - x_E$. It is easy to see that $\phi_{\lambda_{I,1}^{P1}\lambda_{E,1}^{P1}}^{1|1-1} = \phi_{\lambda_{I,1}^{P1}\lambda_{E,1}^{P1}}^{0|0-0}$ because the states 0 and 1 are equally likely to occur.

The other probabilities that a certain state takes place given the development decision of the producer(s) are:

$$\boldsymbol{pr\left(\theta_1 = 0 \middle| x_{I,1} = 1, x_{E,1} = 1, P1_I, P1_E\right)}$$
$$= \frac{pr\left(x_{I,1} = 1, x_{E,1} = 1 \middle| \theta_1 = 0\right) * pr(\theta_1 = 0)}{pr\left(x_{I,1} = 1, x_{E,1} = 1 \middle| \theta_1 = 0\right) * pr(\theta_1 = 0) + pr\left(x_{I,1} = 1, x_{E,1} = 1 \middle| \theta_1 = 1\right) * pr(\theta_1 = 1)}$$
$$= \frac{\left(1 - \mu_{I,1}^{c,P1}\right)\left(1 - \mu_{E,1}^{c,P1}\right) * \frac{1}{2}}{\left(1 - \mu_{I,1}^{c,P1}\right)\left(1 - \mu_{E,1}^{c,P1}\right) * \frac{1}{2} + \mu_{I,1}^{c,P1}\mu_{E,1}^{c,P1} * \frac{1}{2}} = \phi_{\lambda_{I,1}^{P1}\lambda_{E,1}^{P1}}^{0|1-1} = \phi_{\lambda_{I,1}^{P1}\lambda_{E,1}^{P1}}^{1|0-0}$$

$\boldsymbol{pr(\theta_1 = 1 | x_{I,1} = 1, x_{E,1} = 0, P1_I, P1_E)}$

$$= \frac{pr(x_{I,1} = 1, x_{E,1} = 0 | \theta_1 = 1) * pr(\theta_1 = 1)}{pr(x_{I,1} = 1, x_{E,1} = 0 | \theta_1 = 1) * pr(\theta_1 = 1) + pr(x_{I,1} = 1, x_{E,1} = 0 | \theta_1 = 0) * pr(\theta_1 = 0)}$$

$$= \frac{\mu_{I,1}^{c,P1}(1 - \mu_{E,1}^{c,P1}) * \frac{1}{2}}{\mu_{I,1}^{c,P1}(1 - \mu_{E,1}^{c,P1}) * \frac{1}{2} + (1 - \mu_{I,1}^{c,P1})\mu_{E,1}^{c,P1} * \frac{1}{2}} = \phi^{1|1-0}_{\lambda_{I,1,}^{P1}\lambda_{E,1}^{P1}} = \phi^{0|0-1}_{\lambda_{I,1,}^{P1}\lambda_{E,1}^{P1}}$$

$\boldsymbol{pr(\theta_1 = 0 | x_{I,1} = 1, x_{E,1} = 0, P1_I, P1_E)}$

$$= \frac{pr(x_{I,1} = 1, x_{E,1} = 0 | \theta_1 = 0) * pr(\theta_1 = 0)}{pr(x_{I,1} = 1, x_{E,1} = 0 | \theta_1 = 0) * pr(\theta_1 = 0) + pr(x_{I,1} = 1, x_{E,1} = 0 | \theta_1 = 1) * pr(\theta_1 = 1)}$$

$$= \frac{(1 - \mu_{I,1}^{c,P1})\mu_{E,1}^{c,P1} * \frac{1}{2}}{(1 - \mu_{I,1}^{c,P1})\mu_{E,1}^{c,P1} * \frac{1}{2} + \mu_{I,1}^{c,P1}(1 - \mu_{E,1}^{c,P1}) * \frac{1}{2}} = \phi^{0|1-0}_{\lambda_{I,1,}^{P1}\lambda_{E,1}^{P1}} = \phi^{1|0-1}_{\lambda_{I,1,}^{P1}\lambda_{E,1}^{P1}}$$

From the assumption we made that $\frac{1}{2} < \mu_{E,1}^{c,P1} < \mu_{I,1}^{c,P1} < 1$ it is easy to see that: $\phi^{1|1-1}_{\lambda_{I,1,}^{P1}\lambda_{E,1}^{P1}} = \phi^{0|0-0}_{\lambda_{I,1,}^{P1}\lambda_{E,1}^{P1}} > \phi^{1|1-0}_{\lambda_{I,1,}^{P1}\lambda_{E,1}^{P1}} = \phi^{0|0-1}_{\lambda_{I,1,}^{P1}\lambda_{E,1}^{P1}} > \phi^{0|1-0}_{\lambda_{I,1,}^{P1}\lambda_{E,1}^{P1}} = \phi^{1|0-1}_{\lambda_{I,1,}^{P1}\lambda_{E,1}^{P1}} > \phi^{1|0-0}_{\lambda_{I,1,}^{P1}\lambda_{E,1}^{P1}} = \phi^{0|1-1}_{\lambda_{I,1,}^{P1}\lambda_{E,1}^{P1}}$ always holds.

If the consumer believes that a producer uses a mixed strategy we use $\mu_{I,1}^{c,M1}$ instead of $\mu_{I,1}^{c,P1}$. This rule can both be used for first period probabilities and for second period probabilities (where the expected ability is updated). The order of the probabilities from large to small now crucially depends on the decision variables the producers use to mix (incumbent mixes with $pr(x_{I,1} \neq s_{I,1}) = t\ and$ with $pr(x_{I,2} \neq s_{I,2}) = v$ and the entrant mixes with $pr(x_{E,1} \neq s_{E,1}) = r\ and$ with $pr(x_{E,2} \neq s_{E,2}) = w$).

To determine the second period probabilities that after the consumer observed development decisions $x_{I,2}$ and $x_{E,2}$ the world is in a certain state ($\theta_2 = 0\ or\ \theta_2 = 1$) the consumer uses history information about her period one beliefs about the strategy used by the producers ($P1\ or\ M1$) and the updated beliefs about both producers reputation (λ). Hence, the probability that the world is in a certain state given the development decisions of the producers is calculated in the following way.

If for example the consumer believed that the incumbent mixed his strategy in period one ($M1_I$), the entrant always follows his signal in period one ($P1_E$) and the period one development decisions where ($x_{I,1} = 1, x_{E,1} = 0$) and the state of the world equalled zero ($\theta_1 = 0$). Than the probability that the state of the world is zero in period two $\theta_2 = 0$ given the development decisions ($x_{I,2} = 0, x_{E,2} = 1$) and the beliefs that in period two both follow their signal is:

$$\boldsymbol{pr\left(\theta_2 = 0 \middle| x_{I,2} = 0, x_{E,2} = 1, \lambda_{I,1}^{B,M1}, \lambda_{E,1}^{G,P1}, P2_I, P2_E\right)}$$
$$= \frac{\mu_{I,2}^{c,B,M1}\left(1-\mu_{E,2}^{c,G,P1}\right) * \frac{1}{2}}{\mu_{I,2}^{c,B,M1}\left(1-\mu_{E,2}^{c,G,P1}\right) * \frac{1}{2} + \left(1-\mu_{I,2}^{c,B,M1}\right)\mu_{E,2}^{c,G,P1} * \frac{1}{2}} = \phi^{0|0-1}_{\lambda_{I,2}^{B,M1,P2},\lambda_{E,2}^{G,P1,P2}}$$

Determine the second period probabilities that the period two state equals 1 given that both producers made development decision $x_{I,2} = x_{E,2} = 1$ in the second period in case the consumer beliefs that both producers follow their signal in both periods:

If both producers developed the product that matched the state in period one $x_{I,1} = x_{E,1} = \theta_1$:

$$\boldsymbol{pr\left(\theta_2 = 1 \middle| x_{I,2} = 1, x_{E,2} = 1, \lambda_{I,1}^{G,P1}, \lambda_{E,1}^{G,P1}, P2_I, P2_E\right)}$$
$$= \frac{\mu_{I,2}^{c,G,P1}\mu_{E,2}^{c,G,P1} * \frac{1}{2}}{\mu_{I,2}^{c,G,P1}\mu_{E,2}^{c,G,P1} * \frac{1}{2} + (1-\mu_{I,2}^{c,G,P1})(1-\mu_{E,2}^{c,G,P1}) * \frac{1}{2}} = \phi^{1|1-1}_{\boldsymbol{\lambda_{I,2}^{G,P1,P2}},\boldsymbol{\lambda_{E,2}^{G,P1,P2}}}$$

If the incumbent developed the product that matched the state of the world in period one ($x_{I,1} = \theta_1$) and the entrant did not ($x_{E,1} \neq \theta_1$)**:**

$$\boldsymbol{pr\left(\theta_2 = 1 \middle| x_{I,2} = 1, x_{E,2} = 1, \lambda_{I,1}^{G,P1}, \lambda_{E,1}^{B,P1}, P2_I, P2_E\right)}$$
$$= \frac{\mu_{I,2}^{c,G,P1}\mu_{E,2}^{c,B,P1} * \frac{1}{2}}{\mu_{I,2}^{c,G,P1}\mu_{E,2}^{c,B,P1} * \frac{1}{2} + (1-\mu_{I,2}^{c,G,P1})(1-\mu_{E,2}^{c,B,P1}) * \frac{1}{2}} = \phi^{1|1-1}_{\boldsymbol{\lambda_{I,2}^{G,P1,P2}},\boldsymbol{\lambda_{E,2}^{B,P1,P2}}}$$

If the entrant developed the product that matched the state of the world in period one ($x_{E,1} = \theta_1$) and the incumbent did not ($x_{I,1} \neq \theta_1$)**:**

$$\boldsymbol{pr\left(\theta_2 = 1 \middle| x_{I,2} = 1, x_{E,2} = 1, \lambda_{I,1}^{B,P1}, \lambda_{E,1}^{G,P1}, P2_I, P2_E\right)} \frac{\mu_{I,2}^{c,B,P1}\mu_{E,2}^{c,G,P1} * \frac{1}{2}}{\mu_{I,2}^{c,B,P1}\mu_{E,2}^{c,G,P1} * \frac{1}{2} + (1-\mu_{I,2}^{c,B,P1})(1-\mu_{E,2}^{c,G,P1}) * \frac{1}{2}}$$
$$= \phi^{1|1-1}_{\boldsymbol{\lambda_{I,2}^{B,P1,P2}},\boldsymbol{\lambda_{E,2}^{G,P1,P2}}}$$

If both producers developed a product that did not match the state of the world ($x_{I,1} \neq \theta_1 \ and \ x_{E,1} \neq \theta_1$):

$$\boldsymbol{pr\left(\theta_2 = 1 \middle| x_{I,2} = 1, x_{E,2} = 1, \lambda_{I,1}^{B,P1}, \lambda_{E,1}^{B,P1}, P2_I, P2_E\right)} \frac{\mu_{I,2}^{c,B,P1}\mu_{E,2}^{c,B,P1} * \frac{1}{2}}{\mu_{I,2}^{c,B,P1}\mu_{E,2}^{c,B,P1} * \frac{1}{2} + (1-\mu_{I,2}^{c,B,P1})(1-\mu_{E,2}^{c,B,P1}) * \frac{1}{2}}$$
$$= \phi^{1|1-1}_{\boldsymbol{\lambda_{I,2}^{B,P1,P2}},\boldsymbol{\lambda_{E,2}^{B,P1,P2}}}$$

Here it is easy to see that: $\phi^{1|1-1}_{\lambda^{G,P1,P2}_{I,2},\lambda^{G,P1,P2}_{E,2}} > \phi^{1|1-1}_{\lambda^{G,P1,P2}_{I,2},\lambda^{B,P1,P2}_{E,2}} > \phi^{1|1-1}_{\lambda^{B,P1,P2}_{I,2},\lambda^{G,P1,P2}_{E,2}} > \phi^{1|1-1}_{\lambda^{B,P1,P2}_{I,2},\lambda^{B,P1,P2}_{E,2}}$. This follows from $\mu^{c,G,P1}_{I,2} > \mu^{c,G,P1}_{E,2} > \mu^{c,B,P1}_{I,2} \geq \mu^{c,B,P1}_{E,2}$, the order in expected abilities follows from the updated reputations.

The other probabilities that the world is in a certain state given the development decisions, the updated reputations and expected abilities are calculated in the exact same way. Note that whether one probability is larger than another when mixed strategies are used crucially depends on the value of the decision variables (r, t, v and w).

Here follows one example of the probability that the world is in a certain stated given the development decisions of the producers while a mixed strategy is used in the second period:

If in period one the consumer believed that both producers always followed their signal in period one $(P1_I, P1_E)$ and both developed the product that matched the state $(x_{I,1} = x_{E,1} = \theta_1)$ and in the second period the consumer beliefs that the entrant always follows his signal and the incumbent mixes I $(M2_I, P2_E)$ and both producers developed product 1 $(x_{I,2} = x_{E,2} = 1)$. The probability that the state equals 1 (θ_2) is:

$$\boldsymbol{pr\left(\theta_2 = 1 \middle| x_{I,2} = 1, x_{E,2} = 1, \lambda^{G,P1}_{I,1}, \lambda^{G,P1}_{E,1}, M2_I, P2_E\right)}$$

$$= \frac{(\mu^{c,B,P1}_{I,2}(1-v) + \left(1-\mu^{c,B,P1}_{I,2}\right)v) * \mu^{c,G,P1}_{E,2} * \frac{1}{2}}{(\mu^{c,G,P1}_{I,2}(1-v) + \left(1-\mu^{c,G,P1}_{I,2}\right)v) * \mu^{c,G,P1}_{E,2} * \frac{1}{2} + \left(\mu^{c,G,P1}_{I,2}v + \left(1-\mu^{c,G,P1}_{I,2}\right)(1-v)\right) * (1-\mu^{c,G,P1}_{E,2}) * \frac{1}{2}}$$

$$= \phi^{1|1-1}_{\lambda^{B,P1,M2}_{I,2},\lambda^{B,P1,P2}_{E,2}}$$

Appendix C

Determining price equilibrium in case the consumer believes that the producers follow their signal in both periods ($x_{i,1} = s_{i,1}$):

When the consumer makes a buying decision she always wants to maximize her utility, in other words she tries to maximize: $\phi^{\theta|x_{I,t}-x_{E,t}}_{\lambda^{P1}_{I,1},\lambda^{P1}_{E,1}} * R_i - p_{i,t}$. The producers on the other hand try to maximize their profit: $p_{i,t} - c$. From the profit function of the producers it is clear that both producers want to sell a unit of their product to the consumer as long as $p_{i,t} - c \geq 0$ or as long as $p_{i,t} \geq c$. In equilibrium the producer that creates the lowest expected value for the consumer ($\phi^{\theta|x_{I,t}-x_{E,t}}_{\lambda^{P1}_{I,1},\lambda^{P1}_{E,1}} * R_i$) will set his price equal to c. In equilibrium the following prices will therefore exist:

If the consumer observes that the producers develop the same product-type ($x_{I,1} = x_{E,1} = 1$ or $x_{I,1} = x_{E,1} = 0$) her expected utility is: $\phi^{1|1-1}_{\lambda^{P1}_{I,1},\lambda^{P1}_{E,1}} * R_I - p_{I,1}$ if she buys from the incumbent and $\phi^{1|1-1}_{\lambda^{P1}_{I,1},\lambda^{P1}_{E,1}} * R_E - p_{I,E}$ if she buys from the entrant (remind that $\phi^{1|1-1}_{\lambda^{P1}_{I,1},\lambda^{P1}_{E,1}} = \phi^{0|0-0}_{\lambda^{P1}_{I,1},\lambda^{P1}_{E,1}}$). Since $R_I > R_E$ it is easy to see that the expected value of buying a product from the incumbent is higher than the expected value of buying a product from the entrant. Therefore in equilibrium the entrant will charge a price of: $p_{E,1} = c$ and the incumbent will charge a price of: $p_{I,1} = c + \phi^{1|1-1}_{\lambda^{P1}_{I,1},\lambda^{P1}_{E,1}} * (R_I - R_E)$. Since we assumed that in case the expected utility of buying a unit of the product from the producers is the same she will buy from the incumbent, the consumer will buy from the incumbent for a price of $p_{I,1} = c + \phi^{1|1-1}_{\lambda^{P1}_{I,1},\lambda^{P1}_{E,1}} * (R_I - R_E)$.

If the consumer observes $x_{I,1} = 1 \; and \; x_{E,1} = 0$ or $x_{I,1} = 0 \; and \; x_{E,1} = 1$ then her expected utility from buying a unit of the product from the incumbent is: $\phi^{1|1-0}_{\lambda^{P1}_{I,1},\lambda^{P1}_{E,1}} * R_I - p_{I,1}$ or $\phi^{0|1-0}_{\lambda^{P1}_{I,1},\lambda^{P1}_{E,1}} * R_I - p_{I,1}$ (remind that $\phi^{1|1-0} = \phi^{0|0-1}$ so that these expected utilities are the same). The consumer's expected utility from buying a unit of the product from the entrant is: $\phi^{0|1-0}_{\lambda^{P1}_{I,1},\lambda^{P1}_{E,1}} * R_E - p_{E,1}$. Because $\phi^{1|1-0}_{\lambda^{P1}_{I,1},\lambda^{P1}_{E,1}} > \phi^{0|1-0}_{\lambda^{P1}_{I,1},\lambda^{P1}_{E,1}}$ and $R_I > R_E$ it is easy to see that the expected utility from buying a unit of the product from the incumbent is higher than buying a unit of the product from the entrant. Therefore in equilibrium the producers charge the prices: $p_{E,1} = c$ and $p_{I,1} = c + \phi^{1|1-0}_{\lambda^{P1}_{I,1},\lambda^{P1}_{E,1}} * R_I - \phi^{0|1-0}_{\lambda^{P1}_{I,1},\lambda^{P1}_{E,1}} * R_E$.

In the second period the exact same way of reasoning holds.

After the consumer believed that both producers follow their signal honestly in the first period and developed the product-type that matched the state of the world: $x_{I,1} = x_{E,1} = \theta_1$ the abilities of the producers are updated to $\mu_{I,2}^{c,G,P1}$ and $\mu_{E,2}^{c,G,P1}$. If the producers developed the same product that did not match the state $x_{I,1} = x_{E,1} \neq \theta_1$ the abilities of the producers are updated to $\mu_{I,2}^{c,B,P1}$ and $\mu_{E,2}^{c,B,P1}$.

In case $\boldsymbol{x_{I,1} = x_{E,1} = \theta_1 \; and \; x_{I,2} = x_{E,2}}$:

The expected utility of the consumer if she buys from the incumbent is: $U_{C,I} = \phi^{1|1-1}_{\lambda_{I,2}^{G,P1,P2},\lambda_{E,2}^{G,P1,P2}} * R_I - p_{I,2}$, and the expected utility of the consumer if she buys from the entrant is: $U_{C,E} = \phi^{1|1-1}_{\lambda_{I,2}^{G,P1,P2},\lambda_{E,2}^{G,P1,P2}} * R_E - p_{E,2}$. Because $R_I > R_E$ by assumption $p_{E,2} = c$ and $p_{I,2} = \phi^{1|1-1}_{\lambda_{I,2}^{G,P1,P2},\lambda_{E,2}^{G,P1,P2}} * (R_I - R_E) + c$ and the consumer will buy from the incumbent. In this case the restriction that the expected product value $(\phi * R)$ must exceed the product cost c is always fulfilled because the reputation of both producers has increased.

In case $\boldsymbol{x_{I,1} = x_{E,1} = \theta_1 \; and \; x_{I,2} \neq x_{E,2}}$:

$U_{C,I} = \phi^{1|1-0}_{\lambda_{I,2}^{G,P1,P2},\lambda_{E,2}^{G,P1,P2}} * R_I - p_{I,2}$

$U_{C,E} = \phi^{0|1-0}_{\lambda_{I,2}^{G,P1,P2},\lambda_{E,2}^{G,P1,P2}} * R_E - p_{E,2}$

Because $\phi^{1|1-0}_{\lambda_{I,2}^{G,P1,P2},\lambda_{E,2}^{G,P1,P2}} > \phi^{0|1-0}_{\lambda_{I,2}^{G,P1,P2},\lambda_{E,2}^{G,P1,P2}}$ and because $R_I > R_E$ the consumer will buy from the incumbent for: $p_{I,2} = \phi^{1|1-0}_{\lambda_{I,2}^{G,P1,P2},\lambda_{E,2}^{G,P1,P2}} * R_I - \phi^{0|1-0}_{\lambda_{I,2}^{G,P1,P2},\lambda_{E,2}^{G,P1,P2}} * R_E + c$ as long as both producers are still in the market $(\phi * R \geq c)$.

In case $\boldsymbol{x_{I,1} \neq \theta_1 \, , x_{E,1} = \theta_1 \; and \; x_{I,2} = x_{E,2}}$:

$U_{C,I} = \phi^{1|1-1}_{\lambda_{I,2}^{B,P1,P2},\lambda_{E,2}^{G,P1,P2}} * R_I - p_{I,2}$

$U_{C,E} = \phi^{1|1-1}_{\lambda_{I,2}^{B,P1,P2},\lambda_{E,2}^{G,P1,P2}} * R_E - p_{E,2}$

The consumer always buys from the incumbent (if she buys in the first place) because the producers develop the same product-type and the product quality of the product the incumbent developed is higher than the product quality of the product developed by the entrant.

This only holds as long as both producers are still in the market. In case the difference between the product qualities $(R_I \; and/or \; R_E)$ and the cost of production c is small or if $\mu_{I,2}^{c,B,P1} \; and \; \mu_{E,2}^{c,G,P1}$ are low, it is possible that the expected (highest) product value $\phi^{1|1-1}_{\lambda^{B,P1,P2}_{I,2},\lambda^{G,P1,P2}_{E,2}} * R_I < c$ and that no product will be sold to the consumer.

In case $x_{I,1} \neq \theta_1$, $x_{E,1} = \theta_1$ and $x_{I,2} \neq x_{E,2}$:

$U_{C,I} = \phi^{1|1-0}_{\lambda^{B,P1,P2}_{I,2},\lambda^{G,P1,P2}_{E,2}} * R_I - p_{I,2}$

$U_{C,E} = \phi^{0|1-0}_{\lambda^{B,P1,P2}_{I,2},\lambda^{G,P1,P2}_{E,2}} * R_E - p_{E,2}$

Here $\phi^{1|1-0}_{\lambda^{B,P1,P2}_{I,2},\lambda^{G,P1,P2}_{E,2}} = \frac{\mu^{c,B,P1}_{I,2}\left(1-\mu^{c,G,P1}_{E,2}\right)*\frac{1}{2}}{\mu^{c,B,P1}_{I,2}\left(1-\mu^{c,G,P1}_{E,2}\right)*\frac{1}{2}+\left(1-\mu^{c,B,P1}_{I,2}\right)\mu^{c,G,P1}_{E,2}*\frac{1}{2}} < \frac{1}{2}$ because: $\mu^{c,B,P1}_{I,2}\left(1-\mu^{c,G,P1}_{E,2}\right) < \left(1-\mu^{c,B,P1}_{I,2}\right)\mu^{c,G,P1}_{E,2}$

and $\phi^{0|1-0}_{\lambda^{B,P1,P2}_{I,2},\lambda^{G,P1,P2}_{E,2}} = \frac{\left(1-\mu^{c,B,P1}_{I,2}\right)\mu^{c,G,P1}_{E,2}*\frac{1}{2}}{\left(1-\mu^{c,B,P1}_{I,2}\right)\mu^{c,G,P1}_{E,2}*\frac{1}{2}+\mu^{c,B,P1}_{I,2}\left(1-\mu^{c,G,P1}_{E,2}\right)*\frac{1}{2}} > \frac{1}{2}$ for the same reason. Therefore there are values of R_I and R_E where the consumer decides to buy from the entrant.

In case $x_{I,1} = \theta_1$, $x_{E,1} \neq \theta_1$ and $x_{I,2} = x_{E,2}$:

$U_{C,I} = \phi^{1|1-1}_{\lambda^{G,P1,P2}_{I,2},\lambda^{B,P1,P2}_{E,2}} * R_I - p_{I,2}$

$U_{C,E} = \phi^{1|1-1}_{\lambda^{G,P1,P2}_{I,2},\lambda^{B,P1,P2}_{E,2}} * R_E - p_{E,2}$

Consumer buys from the incumbent for price: $p_{I,2} = \phi^{1|1-1}_{\lambda^{G,P1,P2}_{I,2},\lambda^{B,P1,P2}_{E,2}} * (R_I - R_E) + c$

Conditional on the updated expected ability of both producers and on the product quality of the entrant it is also possible that $\phi^{1|1-1}_{\lambda^{G,P1,P2}_{I,2},\lambda^{B,P1,P2}_{E,2}} * R_E < c$, in this case the entrant is not able to sell his product profitable, and the entrant will leave the market. If the incumbent is a monopolist in period two he offers a price of: $p_{I,2} = \phi^{1|1-1}_{\lambda^{G,P1,P2}_{I,2},\lambda^{B,P1,P2}_{E,2}} * R_I$. In case the entrant left the market and a new entrant enters in period two the following case exists:

In case $x_{I,1} = \theta_1$, $x_{E,1} \neq \theta_1$ and $x_{I,2} = x_{EN,2}$: (on notation, note that x_{EN} is the development decision of the new entrant)

$U_{C,I} = \phi^{1|1-1}_{\lambda^{G,P1,P2}_{I,2},\lambda^{P2}_{EN,2}} * R_I - p_{I,2}$

$U_{C,NE} = \phi^{1|1-1}_{\lambda^{G,P1,P2}_{I,2},\lambda^{P2}_{EN,2}} * R_{NE} - p_{EN,2}$

Note that $\phi^{1|1-1}_{\lambda^{G,P1,P2}_{I,2},\lambda^{P2}_{EN,2}} > \phi^{1|1-1}_{\lambda^{G,P1,P2}_{I,2},\lambda^{B,P1,P2}_{E,2}}$ because $\lambda^{P2}_{EN,2} > \lambda^{B,P1,P2}_{E,2}$ and therefore the maximum price the incumbent can charge the consumer ($p_{I,2} = \phi^{1|1-1}_{\lambda^{G,P1,P2}_{I,2},\lambda^{P2}_{EN,2}} * (R_I - R_{NE}) + c$) is larger than in case the same entrant is still in the market.

In case $x_{I,1} = \theta_1$, $x_{E,1} \neq \theta_1$ and $x_{I,2} \neq x_{E,2}$:

$U_{C,I} = \phi^{1|1-0}_{\lambda^{G,P1,P2}_{I,1},\lambda^{B,P1,P2}_{E,1}} * R_I - p_{I,2}$

$U_{C,E} = \phi^{1|0-1}_{\lambda^{G,P1,P2}_{I,1},\lambda^{B,P1,P2}_{E,1}} * R_E - p_{E,2}$

$\phi^{1|1-0}_{\lambda^{G,P1,P2}_{I,1},\lambda^{B,P1,P2}_{E,1}} > \phi^{1|0-1}_{\lambda^{G,P1,P2}_{I,1},\lambda^{B,P1,P2}_{E,1}}$ and $R_I > R_E$ and therefore the expected product value of the product developed by the incumbent is higher than the expected product value of the product developed by the entrant and the consumer will therefore buy from the incumbent for a maximum price of: $p_{I,2} = \phi^{1|1-0}_{\lambda^{G,P1,P2}_{I,1},\lambda^{B,P1,P2}_{E,1}} * R_I - \phi^{1|0-1}_{\lambda^{G,P1,P2}_{I,1},\lambda^{B,P1,P2}_{E,1}} * R_E + c$ (remind that $p_{E,2} = c$).

Note that here it is even more likely that the entrant will leave the market because $\phi^{1|0-1}_{\lambda^{G,P1,P2}_{I,2},\lambda^{B,P1,P2}_{E,2}} * R_E < c$ and the incumbent becomes a monopolist. If the incumbent becomes a monopolist the maximum price he can charge is: $p_{I,2} = \phi^{1|1-0}_{\lambda^{G,P1,P2}_{I,2},\lambda^{B,P1,P2}_{E,2}} * R_I$. If a new entrant enters we have the exact same situation as above, here the consumer buys from the incumbent for a maximum price of:

$p_{I,2} = \phi^{1|1-0}_{\lambda^{G,P1,P2}_{I,2},\lambda^{P2}_{EN,2}} * R_I - \phi^{0|1-0}_{\lambda^{G,P1,P2}_{I,2},\lambda^{P2}_{EN,2}} * R_E + c$

In case $x_{I,1} = x_{E,1} \neq \theta_1$ and $x_{I,2} = x_{E,2}$:

$U_{C,I} = \phi^{1|1-1}_{\lambda^{B,P1,P2}_{I,2},\lambda^{B,P1,P2}_{E,2}} * R_I - p_{I,2}$

$U_{C,E} = \phi^{1|1-1}_{\lambda^{B,P1,P2}_{I,2},\lambda^{B,P1,P2}_{E,2}} * R_E - p_{E,2}$

The consumer will buy from the incumbent as long as $\phi^{1|1-1}_{\lambda^{B,P1,P2}_{I,2},\lambda^{B,P1,P2}_{E,2}} * R_I > c$.

In this case it is also very likely that the entrant will no longer be in the market in period two, this is the case if: $\phi^{1|1-1}_{\lambda^{B,P1,P2}_{I,2},\lambda^{B,P1,P2}_{E,2}} * R_E < c$. If the entrant left the market in period two and no new entrant enters, the incumbent is a monopolist and is able to sell a unit of his product to the consumer for a maximum price of: $p_{I,2} = \phi^{1|1}_{\lambda^{B,P1,P2}_{I,2},} * R_I$. If the entrant left the market in period two and a new entrant enters, we are in the following case:

$x_{I,1} = x_{E,1} \neq \theta_1$ and $x_{I,2} = x_{EN,2}$:

$U_{C,I} = \phi^{1|1-1}_{\lambda^{B,P1,P2}_{I,2},\lambda^{P2}_{EN,2}} * R_I - p_{I,2}$

$U_{C,NE} = \phi^{1|1-1}_{\lambda^{B,P1,P2}_{I,2},\lambda^{P2}_{EN,2}} * R_{NE} - p_{NE,2}$

Note that in this case the expected product value of the incumbent's product is always higher, because the probability that the consumer buys the product that matches the state is the same for buying from either of the producers (since they develop the same product-type) and the product quality of the product from the incumbent is higher ($R_I > R_{NE}$). The consumer will buy from the incumbent for a maximum price of: $p_{I,2} = \phi^{1|1-1}_{\lambda^{B,P1,P2}_{I,2},\lambda^{P2}_{EN,2}} * (R_I - R_{NE}) + c$. But $\phi^{1|1-1}_{\lambda^{B,P1,P2}_{I,2},\lambda^{P2}_{EN,2}} * R_I \geq c$ must hold, if this condition does not hold, then the producers will not offer a product in the first place.

In case $x_{I,1} = x_{E,1} \neq \theta_1$ and $x_{I,2} \neq x_{E,2}$:

$U_{C,I} = \phi^{1|1-0}_{\lambda^{B,P1,P2}_{I,2},\lambda^{B,P1,P2}_{E,2}} * R_I - p_{I,2}$

$U_{C,E} = \phi^{1|0-1}_{\lambda^{B,P1,P2}_{I,2},\lambda^{B,P1,P2}_{E,2}} * R_E - p_{E,2}$

The consumer will buy from the incumbent for a price of $p_{I,2} = \phi^{1|1-0}_{\lambda^{B,P1,P2}_{I,2},\lambda^{B,P1,P2}_{E,2}} * R_I - \phi^{1|0-1}_{\lambda^{B,P1,P2}_{I,2},\lambda^{B,P1,P2}_{E,2}} * R_E + c$ as long as $\phi^{1|1-0}_{\lambda^{B,P1,P2}_{I,2},\lambda^{B,P1,P2}_{E,2}} * R_I > c$. If this condition is not satisfied, then the producers will not offer a product. Therefore if: $\phi^{1|1-0}_{\lambda^{B,P1,P2}_{I,2},\lambda^{B,P1,P2}_{E,2}} * R_E < \phi^{1|1-0}_{\lambda^{B,P1,P2}_{I,2},\lambda^{B,P1,P2}_{E,2}} * R_I < c$ holds, then no product will be offered, if $\phi^{1|1-0}_{\lambda^{B,P1,P2}_{I,2},\lambda^{B,P1,P2}_{E,2}} * R_E < c < \phi^{1|1-0}_{\lambda^{B,P1,P2}_{I,2},\lambda^{B,P1,P2}_{E,2}} * R_I$ holds, the incumbent is a monopolist and if $c < \phi^{1|1-0}_{\lambda^{B,P1,P2}_{I,2},\lambda^{B,P1,P2}_{E,2}} * R_E < \phi^{1|1-0}_{\lambda^{B,P1,P2}_{I,2},\lambda^{B,P1,P2}_{E,2}} * R_I$ holds, than both producers will stay in the market.