MORAL HAZARD AND TEMPORAL RESOLUTION OF UNCERTAINTY IN CONTRACT THEORY

Behavioral Economics

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Abstract

Moral hazard and temporal resolution of uncertainty are both well explored topics in behavioral economics. However a combination of both theories in principal-agent contracts is not yet available. Combining these theories in principal-agent contracts leads to a better understanding of the optimal wages in multi-period contracts with resolution of uncertainty. In this paper Kreps' model of moral hazard (1990) and Kreps' and Porteus' model of temporal resolution of uncertainty (1978) are combined. Introducing temporal resolution of uncertainty in the moral hazard model leads to the same wages when probability and function changes are accounted for, but only when the uncertainty is resolved from the start. If this is not done, less optimal risk sharing will occur between principal and agent due to a more concave utility function of the agent. Therefore, the model creates a smaller share for the principal in principal-agent contracts. Due to the restrictive nature of Kreps' moral hazard model, a generalization is made using Holmstrom's (1979) moral hazard model, which gives the same results.

KEYWORDS: Moral hazard, temporal resolution of uncertainty, contract theory, principal-agent problem, optimal sharing, coin game, risk aversion.

I Introduction

In contract theory many research is done on the moral hazard problem and temporal resolution of uncertainty.

Moral hazard is well explored in the area of performance based contracts, also known as principal-agent contracts, where the output depends on (semi-)observable effort of the agent and the incentives are positioned in such a way that the agent behaves in the best interest of the principal (Shapiro, Stiglitz 1984). Here the agent is the entity working on behalf of the principal, respectively the employee and employer.

Temporal resolution of uncertainty is used for the preference/valuation of early or late resolution of uncertainty in a multi-period environment. *"Here typically the agent needs to make a decision/valuation in period one subject to uncertainty in period two"* (Epstein 2000).

Both theories appear numerously in microeconomic and behavioral economic studies. However a combination of the theories in principal-agent contracts is not yet available. Ergin and Sarver (2011) relate hidden actions to preference of timing of resolution of uncertainty, but do not incorporate risk aversion of the agent, therefore not giving a solution for the moral hazard problem, since risk aversion of the agent is needed for a solution in the moral hazard problem. The reason risk aversion is needed for a solution of the moral hazard problem to occur is that *"a best contract is not attainable under the traditional principal-agent framework when effort is unobservable and the agent is risk averse. Under these conditions and the traditional assumption of zero moral sensitivity (i.e., self-interested opportunism), financial incentives based on performance measures are required to induce any effort from the agent, creating the moral hazard problem"* (Ross 1973; Demski & Feltham 1978; Feltham & Xie 1994). On the other hand, Epstein and Zin (1989) relate risk aversion and moral hazard to temporal consumption, but do not take early or late resolution of uncertainty into account.

Combining both theories in one model will therefore give a new insight, which have not been given before, in the optimal wages for multi-period principal-agent contracts with temporal uncertainty. In other words, it helps principals, not only determine the optimal wage in a multi-period contract with uncertainty, but also shows if uncertainty rather needs to be resolved in an early or late stage in relation to the wage optimization of the principal.

The solutions for moral hazard and temporal resolution of uncertainty have been criticized of being complex and inconsistent with contracts commonly found in practice (Baiman 1990). Arrow (1985) gives critique on the moral hazard principle, because of the low realism of the model due to the assumption of narrow self-interest and ignoring non-monetary preferences, such as job esteem, fairness and ethics. However the model gives a basic insight in the expectations about the occurrence of self-interested behavior and the usefulness of financial incentives in solving the moral hazard problem (Bohren 1998). Therefore the model, evaluated in this paper, will still hold explanatory power if the assumptions are taken into account.

Furthermore testing for moral hazard or temporal resolution of uncertainty for the labour market in practice has been proven difficult, since data requirements are very restrictive to micro-level observations of agents (Rice and Sen, 2008). Tests for the combination of moral hazard and temporal resolution of uncertainty are not yet available. Therefore this paper focusses only on the implications of a theoretical moral hazard model in combination with a theoretical model of temporal resolution of uncertainty.

Dembe et al. (2000) describe moral hazard theory first being used in relation with the definition: *"a tendency of insurance plans to encourage behavior that increases the risk of insured loss".* The effect of moral hazard theory on financial incentives on partially controlled outcomes, subject to this paper, is found in several research papers investigating economic decision making, probability and uncertainty analyses. As the first, Dreze (1963), Arrow (1963) and Pauly (1986) introduced and did research on financial incentives based on outcomes only partially in control of the individual. This was done, using expected utility functions also found 200 years earlier in Bernoulli's (1738) St. Petersburg Paradox. Stiglitz (1987) describes the principal-agent problem, which gives the basis for the moral hazard problem in this paper in the following manner: *"The principal–agent problem or agency dilemma treats the difficulties that arise under conditions of incomplete and asymmetric information when a principal hires an agent,* *such as the problem of potential moral hazard and conflict of interest, because the principal is hiring the agent to pursue the principal's interests"*. Wilson (1969), Leland (1977) and Ross (1973, 1974) enhanced this theory by investigating optimal risk sharing in situations where the risk distribution could be influenced by the actions of the agent, creating the general model of an agent, who takes some action, thereby influencing the probability distribution of a random output. Since the agent influences also the utility of the principal and the effort of the agent cannot be (fully) observed, even in an ex-post situation, the problem of moral hazard arrises (Cohen 1978).

Empirical evidence for Stiglitz's principal-agent problem was found in various industries and lines of work by Lazear (1996), Paarsch and Schearer (1996), Fernie and Metcalf (1996), McMillan, Whalley and Zhu (1989), Groves et al. (1994), Kahn and Sherer (1990) and Nikkinen and Sahlstrom (2004). However Jensen and Murphy (1990) did not find empirical evidence for top-management incentives.

Temporal resolution of uncertainty is defined by Kreps and Porteus (1978) as a preference for an income stream, where uncertainty about that income stream is resolved in an earlier or later stage due to ambiguity aversion. Here, ambiguity aversion is a synonym for uncertainty aversion however not to confuse with risk aversion, Epstein (1999) defines this difference in the following way: *"Risk aversion comes from a situation where a probability can be assigned to each possible outcome of a situation. Ambiguity aversion applies to a situation when the probabilities of outcomes are unknown or uncertain"*. Kreps and Porteus elaborate further on older two-period models by Sandmo (1970), Sandmo (1971), Rothshield and Stiglitz (1971) and Turnovsky (1973). Here, Kreps and Porteus create a model wherein an individual perceives the resolution of uncertainty at different times as having a different utility, instead of the payoff vector approach. Kreps and Porteus explain the theory using a two period coin game, where heads will generate a cash flow for period one and two of respectively 5,10 and tails will generate a cash flow of 5,0. If the coin is tossed before receiving the payment of 5 or after the payment of 5 should mathematically not matter when it comes to risk aversion, however Kreps and Porteus say it does matter based on uncertainty aversion and thereby also based on temporal resolution of uncertainty. The relation between the ambiguity attitude and preference for the timing of the resolution of uncertainty already advanced from the coin game to long-run risk models by Duffie and Epstein (1992) and the use of nonlinear aggregators for time-separability by Weil (1990) and Tallarini (2000). These are respectively long term temporal resolution of uncertainty models and scenario's with non constant duration lengths per period. However for this paper the coin game gives an easier starting point and further studies can be done on the implementation of moral hazard in long-run risk models or models using nonlinear time-separability.

Empirical evidence for the temporal resolution of uncertainty principle is found by Weber and Ahlbrecht (1996), where they found that the timing preference depended on the source of utility and on the probabilities involved.

First, the basic moral hazard principle is explained using Kreps (1990). This is done by stating and elaborating the model from Kreps using a numerical example of a practical case, namely a one period agent-principal contract at a financial services company.
Second, temporal resolution of uncertainty is explained using Kreps and Porteus' approach (1978). This is done by elaborating the coin game model using a numerical example of a practical case.
Third, a combination of the models of moral hazard and temporal resolution of uncertainty is made. This model is elaborated using a numerical example of a practical case. Here the same numbers, assumptions and principles are used, so that moral hazard and temporal resolution of uncertainty can be identified individually easily.
Last, because of the restrictive nature of Kreps' model of moral hazard, the model is generalized using Holmstrom's model of moral hazard (1979) in combination with the temporal resolution of uncertainty model.

II Moral Hazard

The basic moral hazard model used in this paper is explained by Kreps in *"A course in micro-economic theory"* (1990). This example of moral hazard comprises of a one-period, three stage model. First a contract is made between the principal and the agent. Second the agent chooses his effort based on his utility function and the contract in place. With this effort the agent influences the output. Third the principal gets the output and pays the agent, accordingly. The model shows a risk averse agent with a von Neumann-Morgenstern utility function *U* based on wage *w* and a vector of cost of effort ***a***:

$$U\{w,a\} = \sqrt{w} - a \tag{2.1}$$

The choice for a risk averse agent lies in the fact that there will be no solution for moral hazard if the agent would be risk neutral. Harris and Raviv (1976) state: *"no gains are to be derived from monitoring the agent's action when the agent is risk neutral. There are also no gains when there is no uncertainty ex post about the relationship between the agent's action and the payoff".* Therefore in these cases, the optimal risk sharing arrangement resolves the moral hazard problem.

The vector for the cost of effort is *[0 5]*. Which are the possibilities of effort the agent can choose. Next to this utility function the agent has a reservation price *(U=9)*. The reservation price is the utility the agent can get when he does not accept the contract. The choice of effort leads to three possible output scenario's *(n=0,1,2)*. Scenario *n=0* leads to a low output for the principal, *n=1* leads to a moderate output for the principal and *n=2* leads to a high output for the principal. This effort influences the distribution of possibilities ($\boldsymbol{p_n}$) for the scenario's to occur. These distributions for effort are both known by the agent and the principal. A practical case for this model can be an agent working in financial services with a stock portfolio. The effort of the agent is not directly observable, but the output is observable and the probability of a low, moderate of high return of the stock portfolio depends on the effort of the agent. When an effort of 0 (***a***=*0*) is chosen, the probabilities of the outcomes (*n=0,1,2*) are:

$$p_n = [0.6; 0.3; 0.1]$$

When an effort of 5 (***a***=*5*) is chosen, the probabilities of the outcomes change to:

$$p_n = [0.1; 0.3; 0.6]$$

It can thus been seen that the principal prefers the agent to put effort in his work (in this case the management of a stock portfolio), since then there is a higher chance for the high output scenario's. The wages, which depend on the outcome need therefore be distributed in the correct way, to ensure the agent putting in effort. Introducing $x_n^2 = w$ for simplicity, leads to a utility function for the agent of:

$$U_I = 0.6x_0 + 0.3x_1 + 0.1x_2 \qquad \textit{for } (\boldsymbol{a}{=}0) \qquad (2.2)$$

$$U_{II} = 0.1x_0 + 0.3x_1 + 0.6x_2 - 5 \qquad \textit{for } (\boldsymbol{a}{=}5) \qquad (2.3)$$

Figure 2.1 shows a schematic sequence of actions and events of the model.

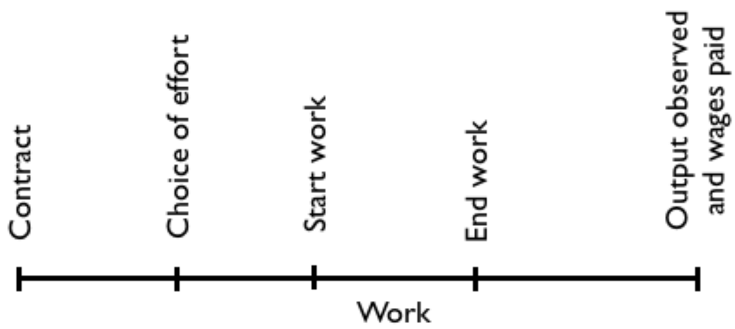

Figure 2.1: Sequence of actions and events

As mentioned, for the principal it is best that the agent will have high effort in this situation. Therefore, according to moral hazard theory, the principal needs to pay wages (x_0, x_1, x_2) that will ensure $U_{II}>U_I$, while also achieving a utility higher than the reservation price, resulting in $U_{II} \geq 9$ to avoid the agent not accepting the contract in the first place. These two constraints are also know as the individual rationality and incentive constraint respectively. This leads to the following Pareto-optimal wages for the scenario's:

$$x_n = [5.428; 14.000; 15.428]$$

$$w_n = [29.463; 196.000; 238.023]$$

The iterative calculations for these results are not discussed in this paper, since they are seen as irrelevant, because of the simplicity. Further elaboration on the calculation behind the numbers can be found in Kreps (1990).

III Temporal Resolution of Uncertainty

The temporal resolution of uncertainty model, found in Kreps and Porteus (1978), is the coin game example. In this model there are two periods. In the first period there is a guaranteed payoff of 5, whereas for the second period a fair coin is flipped to receive a payoff of either 0 or 10 (50%-50% chance). Since there is a guaranteed payoff in the first period of 5, the coin can be flipped either before receiving the first payoff or after receiving the first payoff, without changing the outcome of the payoffs. This means that the von Neumann-Morgenstern utility function would be indifferent for both options for risk-averse, risk neutral and risk seeking agents. However, the individual perceives the timing of the resolution of uncertainty on the payoffs as having different utility, because of the principle of uncertainty aversion explained by Epstein (1999). Despite the different utility based on the timing of the resolution of uncertainty, time discounting is not incorporated in this model for simplicity reasons. This is in alignment with Kreps and Porteus, who also neglect the time discounting principle[1]. The specific utility functions for the 'early' and 'late' resolution of uncertainty on the payoffs are chosen arbitrarily for this paper, however are in accordance with the four axioms found in Kreps and Porteus. These four axioms are not mentioned in detail in this paper, but can be found under axiom 2.1, 2.2, 2.3 and 3.1. This gives the following system of equations for this paper:

$$U_{early} = \sum_{i=m}^{n} p_i \varphi\left(\sqrt{w_i}\right) \tag{3.1}$$

$$U_{late} = \varphi\left(\sum_{i=m}^{n} p_i \sqrt{w_i}\right) \tag{3.2}$$

$$\varphi = f(x)^2 \tag{3.3}$$

[1] *An example where both time discounting and temporal resolution of uncertainty is used is Westerfield and Percival (2007). They show a method of integrating time discounting by investigating uncertainty resolution for multi-period investment decisions.*

In this model, a specific functions are used for simplicity, however as long as $\varphi'' > 0$, any function for 3.1, 3.2 and 3.3 will work. The choice for these specific risk-averse utility functions is chosen out of simplicity and makes it able to easily combine this model in a later stage with the specific moral hazard model described in part II, where risk-aversion is needed to get a solution. The choice of function 3.1, 3.2 and 3.3 is also exactly the same as the model from Kreps and Porteus, therefore resulting in a the same numerical utility outcome. The implementation of general risk averse functions with temporal resolution of uncertainty is already described and found accurate by Etner (2006). Etner 2006 writes that there exists a correct relation between both concepts and that it is possible under the assumptions to include risk aversion together with temporal resolution of uncertainty. Figure 3.1 gives a schematic drawing of the coin game. This scheme is used to calculate the utility outcome of both the early and late resolution of uncertainty. In this scheme the top side (a) is the late resolution of uncertainty and the bottom side (b,c) is the early resolution of uncertainty. On the left side (above and below the square) the implementation of the general system of equations can be seen.

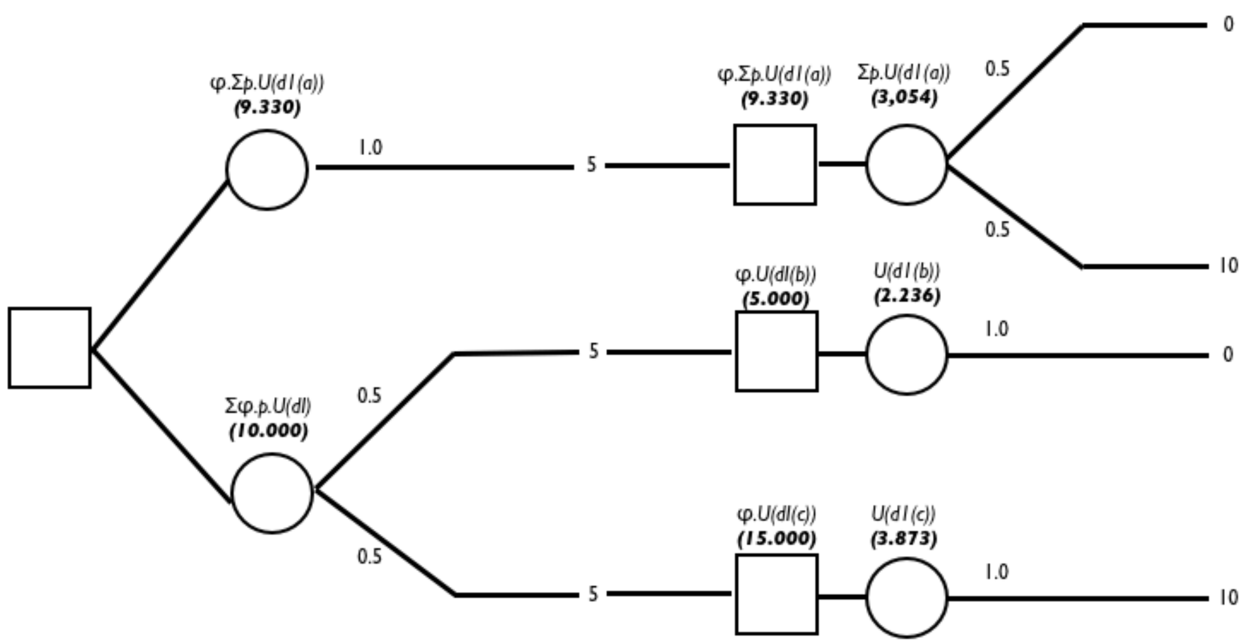

Figure 3.1: Schematic drawing of the coin game example.

The system of equations leads to a utility for flipping the coin after (function 3.4) and before (function 3.5) the first period of:

$$U = \left(\frac{1}{2}\sqrt{5} + \frac{1}{2}\sqrt{15}\right)^2 = 9.330 \qquad (3.4)$$

$$U = \frac{1}{2}\left(\sqrt{5}\right)^2 + \frac{1}{2}\left(\sqrt{15}\right)^2 = 10.000 \qquad (3.5)$$

This therefore states a preference for relieving the uncertainty in an early stage to gain a higher utility, namely the bottom part of the schematic drawing.

IV Moral Hazard and Temporal Resolution of Uncertainty

The combination of coin game and principal-agent moral hazard theory leads to a model wherein exactly the same principle is used as the basic moral hazard model, only here there is a two period contract, wherein the agent is told in the beginning (t=0) or in the middle of his work (t=1) what the outcome of his work will be after two periods of work without intermediate payment, if he keeps up the same effort he has putting in so far. This last assumption is really important, otherwise the agent will stop putting in any effort, since the output is already guaranteed. This can be related to the 'real world' by for instance an agent working in the financial services sector, where his effort cannot be observed directly, but the return on his investment portfolio can. The resolution of uncertainty can be seen as an intermediate evaluation of his work, where the principal only has data about the stock portfolio of the agent, not the effort. The agent has to stay under contract for both periods and is not allowed to work for the reservation price in the second period because he signed a contract which prevents him for working at a competitor for at least one period after he leaves the company. The reservation price is again equal to 9 per period, so will be 18 for the whole contract. Also the effort comprises still of the vector $\boldsymbol{a}$ = *[0 5]* and the probabilities of different investment scenario's on the vector $\boldsymbol{p_n}$ *[0.6 0.3 0.1]* for $\boldsymbol{a}=0$ and $\boldsymbol{p_n}$ *[0.1 0.3 0.6]* for $\boldsymbol{a}=5$. If there was full certainty that the agent will be told in the beginning after choosing his effort, which scenario would occur, then the wages would be equal to the basic one period moral hazard model. However, by implementing a resolution uncertainty in the form of an evaluation meeting, temporal uncertainty is introduced into the moral hazard model. This creates again an early resolution of uncertainty and a late resolution of uncertainty utility function, as with the coin game temporal resolution of uncertainty model. In this case however there are more constraints and variables. There are three scenario's with two different possible probability scenario's depending on the effort of the agent, instead of two scenario's with a fixed probability distribution. The effort constraint gives the fact that the highest utility for the principal is achieved if there is more chance for a higher output scenario, hence the wage distribution over the output needs to be such that the agent will chose to put effort in, however these wages cannot exceed the gain in output for the principal, since then it would be better for the principal to led the agent put in no effort. The reservation price constraint leads to the fact that the utility of the wage distribution cannot go under the utility of the reservation price. Since the agent will then choose to work outside the company for the reservation price. This leads to the following system of equations:

$$U_{early} = \sum_{i=m}^{n} p_i \varphi\left(\sqrt{w_i} - a\right) \tag{4.1}$$

$$U_{late} = \varphi\left(\sum_{i=m}^{n} p_i \left(\sqrt{w_i} - a\right)\right) \tag{4.2}$$

$$\varphi = f(x)^2 \tag{4.3}$$

Figure 4.1 gives a schematic drawing and the timeline for the sequence of events in the combined model. As can be seen, both figures resemble the figures of the separate coin game and moral hazard problem stated in the previous chapters, only with a few adaptations. So is the schematic drawing extended from two possibilities to three possibilities and is the time sequence of the moral hazard problem extended to a two period model with only one payment at the end, just like the basic model.

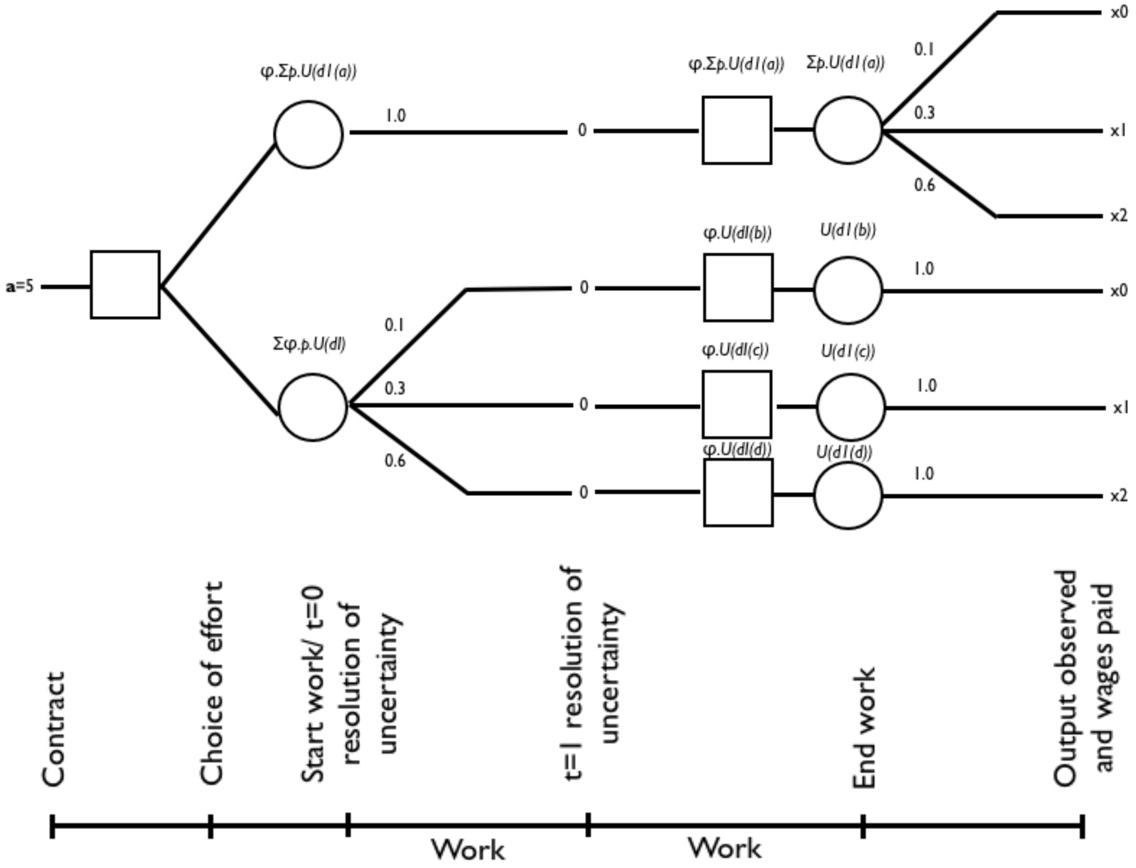

Figure 4.1: Schematic drawing of the combined model and sequence of actions and events

This model is harder to solve than the earlier basic moral hazard model or the coin toss model. Therefore the calculations for this were done with Matlab. First a system of equations for both situations were set up, just like the moral hazard model. This gives the following indirect equations:

$$U_{early\_I} = 0.6 \cdot (x_0)^2 + 0.3 \cdot (x_1)^2 + 0.1 \cdot (x_2)^2 \tag{4.4}$$

$$U_{early\_II} = 0.1 \cdot (x_0 - 10)^2 + 0.3 \cdot (x_1 - 10)^2 + 0.6 \cdot (x_2 - 10)^2 \tag{4.5}$$

$$U_{late\_I} = (0.6 \cdot x_0 + 0.3 \cdot x_1 + 0.1 \cdot x_2)^2 \tag{4.6}$$

$$U_{late\_II} = (0.1 \cdot x_0 + 0.3 \cdot x_1 + 0.6 \cdot x_2 - 10)^2 \tag{4.7}$$

$$U_{res} = 18 \tag{4.8}$$

These indirect equations were then solved into direct equations using Matlab. Then, by using an iterative process the program automatically solved the 'cheapest' wage solution for the principal. Therefore creating the Pareto optimal solution for the principal. All the specific program code can be found in the appendix.

The Matlab calculations lead to the following Pareto-optimal wages for the scenarios when the evaluation meeting happens at t=0, also here $x_n^2 = w$:

$$x_n = [3.447; 7.211; 4.502]$$

$$w_n = [11.882; 51.999; 20.268]$$

This leads to an expected payment to the agent of 28.95. This is the wage paid for every scenario multiplied with the probability of that scenario. Since the rational agent is persuaded to work with effort, the probabilities of scenario II (with effort) are taken.

And the following Pareto-optimal wages for the scenario when the evaluation meeting happens at t=1:

$$x_n = [3.162; 5.754; 6.191]$$

$$w_n = [9.998; 33.109; 38.328]$$

This leads to an expected payment to the agent of 33.93. It can be seen that the principal will prefer the early resolution of uncertainty. Therefore, if the principal would need to pay to get an early resolution (for example costs to arrange the evaluation meetings), the principal is prepared to pay the difference in expected payment: 4.98.

The difference in wages for the evaluation meeting at t=0 compared to the basic moral hazard model are influenced by the fact that the reservation price and effort is doubled because of the introduction of two periods and the function phi is introduced. Here the doubling of the reservation price and effort leads also to a doubling of the wages, however the function phi creates overall lower wages than the moral hazard model. The utility function in both periods is the same as with Kreps' moral hazard model, extended with the coin game formula's.

When the meeting happens at t=1, the temporal resolution of uncertainty principle leads to higher wages, because of the fact that the function phi includes a bigger part of the function, which is also seen at the coin game. All the general formulas can also be found in the schematic drawing, where the first period does not generate any income and the second period will.

V Generalizing the moral hazard model using Holmstrom

Due to the restrictive nature of Kreps' moral hazard model, a generalization is made using Holmstrom (1979). Holmstrom describes in his paper "Moral Hazard and Observability" a more general model for the principal-agent contract. In the basis this is the same model as Kreps', however instead of using a numerical example, only general functions exist. Here a single agent creates a monetary outcome on the basis of a privately chosen effort a and a normally distributed random nature θ, giving a function of output of $x=f(a,\theta)$. The principal's utility function is only dependent on income $(r(x))$, giving $G(r(x))$, while the agent's utility function depends on income and effort $H(w,a)$. Holmstrom restricts the model further by stating $H(w,a)=U(w)-V(a)$, where $V'(a) > 0$ and $U'' < 0$. This ensures that a is a productive input with a direct disutility for the agent and that the worker is risk averse, since moral hazard can be avoided if the worker is risk neutral (Harris and Raviv 1976). The principal may be either risk neutral or risk averse $G'' \leq 0$. Only the monetary outcome is observed by the principal, making the sharing rules only dependent on the monetary outcome, where $s(x)$ and $r(x)=x-s(x)$ are the shares of respectively agent and principal. Furthermore Holmstrom states that the probability distribution for θ is agreed upon by principal and agent and that the effort of the agent is chosen before θ is known, this is also in accordance to the previous moral hazard model. The Pareto efficient sharing outcome between principal and agent is created by the maximization of $E\{G(x-s(x))\}$, where the minimal $H(w,a)$ is constrained with a reservation price of H^* and where a is chosen based on the argument of maximization of *E{H(s(x),a')}*. Holmstrom states that, since *a* can't be observed, it is impossible to fix effort based on a forcing contract, therefore *a* becomes an argument of utility maximization. Holmstrom describes that the system of equations can be solved when assuming that all relations can be written as a function of θ, as found in Spence and Zeckhauser (1971), Ross (1973) and Harris and Raviv (1976). This gives $x=f(a,\theta)$. However in accordance to Mirrlees (1974), Holmstrom sees the output *x* as a randomly distributed function of the variables *a* and θ, giving $f(x,a)$. Here $f(x,a)$ follows out of $f(a,\theta)$, where the output itself is now randomly distributed instead of the now suppressed theta. Holmstrom also fixes $s(x) \in [c,d+x]$ to ensure the existence of the solution. This fixes the solution of *s(x)* in a domain ensuring an appropriate outcome. In this case the agents wealth puts a lower boundary condition on the solution and the principals wealth an upper bound, which is in accordance to the 'real world'. Mirrlees does this, because he states an optimal Pareto-efficient sharing rule is not possible without a restriction of the domain of the sharing outcome. Changing the system of equations to accommodate this gives a maximization of the integral:

$$\int G\left(x-s(x)\right)\cdot f(x,a)dx \tag{5.1}$$

Here again the minimal utility for the agent is constrained with a reservation price (H^*). Which leads to the integration of:

$$\int\left(U\left(s(x)\right)-V(a)\right)\cdot f(x,a)dx \geq H^* \tag{5.2}$$

The effort is chosen with the following constraint, where f_a resembles the derivative of the output function to *a:*

$$\int U\left(s(x)\right)\cdot f_a(x,a)dx = V'(a) \tag{5.3}$$

Using a Lagrangian to solve the equations, Holmstrom gives the following optimal sharing rule solution, where λ describes the multiplier for the minimal utility function for the agent and μ the multiplier for the constrained effort function:

$$\frac{G^{'}\left(x-s(x)\right)}{U^{'}\left(s(x)\right)}=\lambda+\mu\cdot\frac{f_a(x,a)}{f(x,a)} \tag{5.4}$$

Furthermore, in order to create a disutility for the agent for effort *V'(a) > 0*. Also more effort leads a a more positive probability distribution of the monetary outcome, giving that the function of output should be $f_a \leq 0$. This leads to a multiplier for the constrained effort function of $\mu > 0$ in the Lagrangian formula, because this multiplier is constrained by the following function[2]:

$$\mu=-\frac{\int G\left(x-s(x)\right)f_a(x,a)dx}{\int U\left(s(x)\right)f_{aa}(x,a)dx-V^{"}(a)} \tag{5.5}$$

Holmstrom illustrates this general model with an example, here he takes *G(r(x))=w, U(w) = 2*(w^0.5), V(a)=a*^{2} and the output *x* has a probability function of *exp(1/a).* This example is easily solved since no boundary conditions are needed, creating a direct solution for the sharing rule instead of an iterative solution, which needs to be solved with Matlab, as was seen with the combined specific model in this paper. The share of the agent (*s(x)*) and the share of the principal *(r(x))* are therefore respectively:

$$s(x)=\left[\lambda+\mu\cdot\frac{(x-a)}{a^2}\right]^2 \tag{5.6}$$

$$r(x)=x-\left[\lambda+\mu\frac{(x-a)}{a^2}\right]^2 \tag{5.7}$$

[2] *An extensive version of the proof for this can be found in the appendix of Holmstrom (1979) "Moral Hazard and Observability" The Bell Journal of Economics. Vol.10, No.1, pp. 74-91.*

The intermediate steps to get to equation 5.6 and 5.7 are based on equation 5.4. As a start, the probability function can be rewritten into the output function, giving:

$$f(x,a)=\frac{1}{a}e^{-\frac{x}{a}} \tag{5.8}$$

The derivative with respect to effort therefore gives:

$$f(x,a)=\frac{e^{-\frac{x}{a}}(x-a)}{a^3} \tag{5.9}$$

Dividing equations 5.8 and 5.9 and filling these into equation 5.4 will therefore give respectively:

$$\frac{f_a(x,a)}{f(x,a)}=\frac{x-a}{a^2} \tag{5.10}$$

$$\frac{G'(x-s(x))}{U'(s(x))}=\lambda+\mu\cdot\frac{x-a}{a^2} \tag{5.11}$$

Now the derivative of the general equation for utility of the agent and the principal can be filled in with the example conditions, respectively:

$$U'(s(x))=\frac{d}{dx}2\sqrt{s(x)}=\frac{s'(x)}{\sqrt{s(x)}} \tag{5.12}$$

$$G'(x-s(x))=\frac{d}{dx}(x-s(x))=1-s'(x) \tag{5.13}$$

This then gives the basis of equation 5.6 and thus also 5.7, which is only its opposite with respect to x.

$$\sqrt{s(x)}=\lambda+\mu\cdot\frac{x-a}{a^2} \tag{5.14}$$

Using the Lagrangian and formula of 5.5 for the multiplier μ and Matlab iteration for the multiplier λ, the multipliers in this example are determined to be: $\mu=a^3$ and $\lambda=\frac{1}{2a}-2a^2$.

The next step is to introduce temporal resolution of uncertainty into Holmstroms model by again creating a two period model with the same equations, where a evaluation meeting occurs either in the beginning or in the middle of the contract. When the evaluation meeting is at the beginning of the first period, the model will resemble a repetition of Holmstroms model with a new reservation price H^{**} of twice the amount of the old reservation price H^{*}, just as with Kreps' model of moral hazard. When the evaluation meeting is after the first period the second derivative of the *phi function* in utility function of the agent will need to be smaller than zero, just as the already stated combined numerical model of Kreps and Kreps and Porteus. This leads to a new utility function U^{*} for the early and late resolution of respectively $U(\varphi w)$ and $U^{*}=\varphi U(w)$ where $\varphi'' \geq 0$. Therefore the utility is 'discounted' more when the resolution of uncertainty occurs at t=1 instead of t=0, hence a higher $s(x)$ or lower $V'(a)$ is needed for the agent to be able to stay under his reservation price of H^{**}. Furthermore Borch's (1962) explains that $s(x)$ only will be Pareto optimal if the right hand side of equation 5.4 remains constant. Therefore if $\varphi'' \geq 0$, this results in that only a higher $s(x)$ is possible, since the variable of effort is found on the right hand side of equation 5.4, therefore making it impossible to change also $V'(a)$. This will automatically lead to a less optimal risk sharing situation due to a more concave utility function for the agent in the second period and a consecutive decrease in share for the principal. Phi can also be chosen as concave instead of convex, which leads to a preference of late resolution instead of early resolution. No further assumptions are needed when Holmstrom's model is altered to accommodate temporal resolution of uncertainty. Again the change in wages due an evaluation meeting before the first period can be completely explained by the change of the reservation price and effort because of the two period model, instead of the one period model and the introduction of the function phi. When the formulas are changed, the equations of 5.4 and 5.5 for the early resolution of uncertainty will now look as follows:

$$\frac{G'(x-s(x))}{U'(\varphi(s(x)))}=\lambda+\mu\cdot\frac{f_a(x,a)}{f(x,a)} \tag{5.15}$$

$$\mu = \frac{\int G(x - s(x)) \cdot f_a(x,a)dx}{\int U(\varphi(s(x))) \cdot f_{aa}(x,a)dx - V''(\varphi(a))} \tag{5.16}$$

The equations of the late resolution of uncertainty will be:

$$\frac{G'(x - s(x))}{U^{*'}} = \lambda + \mu \cdot \frac{f_a(x,a)}{f(x,a)} \tag{5.17}$$

$$\mu = \frac{\int G(x - s(x)) \cdot f_a(x,a)dx}{\int U(\varphi(s(x))) \cdot f_{aa}(x,a)dx - V^{*''}} \tag{5.18}$$

As a check, filling in the specific formulas from Chapter IV in the general Holmstrom model: *G(w)=w, U(w)=√w, V(a)=a, x~exp(1/a)* and φ''=2, will need to give a similar result of preference for early resolution of uncertainty. The only difference will be that instead of using three scenario's of output dependent on effort, a distribution of output is used dependent on effort. The optimal sharing rule for early and late resolution of uncertainty will become respectively:

$$s(x)_{early} = \frac{a^2 \ln\left(\frac{(x-a)}{a^2} + \lambda + 1\right)}{\mu} \tag{5.19}$$

$$s(x)_{late} = \frac{2a^2 \ln\left(\frac{(x-a)}{a^2} + \lambda + 1\right)}{\mu} \tag{5.20}$$

As can be seen from equations 5.12 and 5.13, early resolution will lead to a lower share of the output for the agent, therefore a higher utility for the principal.

Also here the solution of the early and late sharing rules follows the same steps as in the example. From formula 5.4 the output function is filled in. Since the distribution of x is the same in this example as in the Holmstrom example, the formula's 5.8 up to and including 5.10 for early and late resolution are similar to the example. Now for early resolution and late resolution, formula 5.11 is changed in respectively:

$$\frac{G'\left(x-s(x)\right)}{U'\left(\varphi\left(s(x)\right)\right)}=\lambda+\mu\cdot\frac{x-a}{a^2} \tag{5.21}$$

$$\frac{G'\left(x-s(x)\right)}{U^{*}{}'}=\lambda+\mu\cdot\frac{x-a}{a^2} \tag{5.22}$$

Filling in the utility functions with the specific formula's and again taking the derivative of these utility functions, just as in the Holmstrom example will give formula's 19 and 20.

VII Conclusion

When the outcome of the combined model of moral hazard and temporal resolution is compared with the outcomes of the separate models, it can be seen that the early resolution of uncertainty gives completely the same results as the basic moral hazard model. The increase in wages for the evaluation meeting at t=0 compared to the basic moral hazard model are only influenced by the fact that the reservation price and effort is doubled because of the introduction of two periods. Therefore, if the periods double but the wage is only paid once, then the wages double as well. The utility function in both periods is the same as with Kreps' moral hazard model. While when the meeting happens at t=1, the temporal resolution of uncertainty principle leads to higher wages, because of a higher 'discount factor' for utility on the wage in the second period also seen at the coin game, but also because the wages need to be in equilibrium with the reservation price and the two scenario's of effort from the basic moral hazard model.

The same result can be seen in the generalization of the Holmstrom model. Here the same principle is seen of the increase in wages. The amplitude of the wage change now only depends on which formula's are chosen in the model. Here also a preference for early resolution can be found when phi is chosen to be convex instead of concave.

VIII Discussion

Chapter six already showed the generalization of phi, therefore this give extra insight in the problem. In the introduction it was shown that Arrow (1985) gives critique on the moral hazard principle, because of the low realism of the model due to the assumption of narrow self-interest and ignoring non-monetary preferences, such as job esteem, fairness and ethics. Another point of critique can therefore be added to this for the combined model about the choice of phi in relation to real life practice.

Furthermore it can be interesting to try to remove some of the assumptions in the combined model stated in this paper to attain an even more real life or practical resemblance to contract theory in practice.

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Appendix

MATLAB code Moral Hazard and Temporal Resolution of Uncertainty (Ch. 4)

Resolving uncertainty in early stage

```
% system of equations (indirect notation)

0.1*((x0-10)^2) + 0.3*((x1-10)^2) + 0.6*((x2-10)^2) = 18

0.1*((x0-10)^2) + 0.3*((x1-10)^2) + 0.6*((x2-10)^2) = 0.6*(x0^2) +
0.3*(x1^2) + 0.1*(x2^2)


% rewriting system of equations (direct notation)

x0=3.44744

x2=12-sqrt((x0^2)+4*x0+12*x1-56)


% MATLAB program

old=10000

for x1=3:0.001:200
    x0=3.44744
    x2=12-sqrt((x0^2)+4*x0+12*x1-56)
    agentpay=0.1*(x0^2)+0.3*(x1^2)+0.6*(x2^2)
    if x2<0
       return
    else
       if agentpay>old
          return
       else
          old=agentpay
       end
    end
end
```

Resolving uncertainty in late stage

```
% system of equations (indirect notation)

(0.1*x0 + 0.3*x1 + 0.6*x2 - 10)^2 = 18

(0.1*x0 + 0.3*x1 + 0.6*x2 - 10)^2 = (0.6*x0 + 0.3*x1 + 0.1x2)^2
```

```
% rewriting system of equations (direct notation)

x1=(-7*x0+30*sqrt(2)-20)/3

x2=x0+20
```

```
% MATLAB program

old=10000

for x0=0:0.001:200
    x1=(-7*x0+42*sqrt(2)-20)/3
    x2=x0-12*sqrt(2)+20
    agentpay=0.1*(x0^2)+0.3*(x1^2)+0.6*(x2^2)
    if x1<0
       return
    else
       if agentpay>old
          return
       else
          old=agentpay
       end
    end
end
```