A Nonlinear Approach to the Factor Augmented Model: The FASTR Model

Master's Thesis

B.J. Spruijt - 320624
Erasmus University Rotterdam

August 2012

This research seeks to combine Factor Augmentation with Smooth Transition Regression, in order to be able to distinguish between regimes. Nine FASTR models are examined in the prediction of five stock excess returns and realized volatility. Statistical performance measures, such as the Directional Accuracy test, conclude positive significant accuracy for most time series. Excess returns achieved in portfolio optimization are up to 25.225%, with a Sharpe Ratio of 0.553. Expansions are added to the model, including the soft-thresholding method LARS, as well as factor selection. Results conclude the model with expansions performs even better on the Mean Squared Error and Correctly Predicted Signs tests.

Keywords: Factor Augmentation; Smooth Transition Regression Model; Portfolio Allocation; Factor Selection; Least Angle Regressions.

Great thanks go to prof.dr. D.J.C. van Dijk and dr. C. Çakmakli of the Econometric Institute of the Erasmus University of Rotterdam, for providing most of the data for this research, as well as giving advice throughout the research.

1. Introduction

Predicting excess returns of stocks has been a central problem for many investors throughout the years. New strategies have been adapted based on various distinct older models in order to forecast the movements of assets and to speculate on changes in the market or hedge against the possible risks. Although there are multiple methods that significantly outperform the random walk, up to this date, there is no model containing the proper methods to accurately predict the excess returns of an asset class, not to mention multiple asset classes. This paper takes another attempt by focusing on the combination of two popular methods.

The first is commonly known as the Factor Augmented model, as discussed by Stock & Watson (2002a, 2002b, 2005), Çakmakli & Van Dijk (2010) and Bai (2010), among others. The central point of this model is the large set of variables – for example macroeconomic predictors – used to predict excess stock returns. Welch & Goyal (2008) state in their article that excess stock returns cannot be predicted by any macroeconomic variable. However, the content of the tests in, for example, Çakmakli & Van Dijk (2010) concludes that multiple factors built from these macroeconomic factors, using principal component analysis, do contain significant information. They examine the performance on both a statistical as well as an economic perspective, reaching the conclusion that the Factor Augmented model is able to outperform other models which use only a small set of exogenous variables.

The second method adds a nonlinear component to the model. This component has the ability to enhance switching regimes, depending on the state of the economy. This state may for example be either a bull or a bear regime. Many models with switching regimes have been tested for the prediction of the business cycle in previous studies, since Hamilton (1989) proposed to use Markov-Switching models. Chauvet & Potter (2000) for instance seek leading indicators of the stock market in order to predict whether the economy is in a bull or bear regime. They show that using a two-state Markov model helps to correctly forecast the regime. Also, many have shown that adding nonlinearity to the model enhances the profitability in portfolio management (see, for example, Ang & Bekaert (2002, 2004) and Guidolin & Timmermann (2005, 2006a, 2008a, 2008b, 2008c), among many others). For these purposes, Lin & Teräsvirta (1993) propose the use of a Smooth Transition Regression (STR) model, which they use to test the constancy of the parameters. This model is commonly extended to the Smooth Transition AutoRegressive (STAR) model (examples of this model can be found in Teräsvirta & Anderson (1992), Teräsvirta (1994) and Van Dijk, Teräsvirta & Franses (2000), among others). The Smooth Transition models allow, by means of a logistic function, to add weights depending on exogenous or lagged endogenous variables, instead of a single threshold value.

This paper combines the previous two methods into a Factor Augmented Smooth Transition Regression model, hereafter referred to as the FASTR model. The option to combine Factor Augmented models with a nonlinear component is discussed before, by Giovannetti (2011), who uses an adaptive nonparametric model. This method linearly combines unknown nonlinear functions of the factors and lags of the dependent variable. He cites that "Combining factor-augmented models and nonlinear estimation should be a natural forecasting strategy, given the dimensionality reduction provided by the factor approach". The unknown functions do not extend to the regime switching, however, which distinguishes this research.

The FASTR model in this research predicts the excess returns and realized volatilities of five return series. The first three asset options are a Small Cap, Medium Cap and Big Cap portfolio, where the division is based on the Market Equity of the included stocks. The last two options are the S&P500 Index and the Gold commodity. The data consists of monthly observations and is predicted over the sample of June 1978 until November 2011. A large set of macroeconomic predictors, adapted from the research of Stock & Watson (2005), is used in the factor augmentation, as well as some common financial indicators obtained from the research of Çakmakli & Van Dijk (2010).

For the purpose of estimating the regime, the nonlinear component focuses on both endogenous as exogenous variables. A version of the STAR model and the combination of the leading indicators along with the STR model, following Chauvet & Potter (2000), are considered. In total, nine variants of the FASTR model are tested for statistical and economic value. The benchmark in this paper is the linear Factor Augmented model, as discussed in Çakmakli & Van Dijk (2010). The statistical performance is measured by means of five tests: the Relative Mean Squared Error and the test of Diebold & Mariano (2002) examine whether the errors of the FASTR model are significantly smaller than the benchmark; the Correctly Predicted Signs test and the Directional Accuracy test of Pesaran & Timmermann (1992) are used to determine the accuracy; and finally, the Excess Predictability test of Anatolyev & Gerko (2005) values the outcomes of the models relative to taking random long and short positions in the respective assets.

The economic performance focuses on portfolio management. A broad selection of the previously mentioned papers (for instance, Ang & Bekaert (2002), Van Dijk & Franses (1999) and Guidolin & Timmermann (2008b), among others) discuss the profitability of considering multiple regimes, and show that average returns raise significantly compared to the linear model. Furthermore, Guidolin & Timmermann (2006b) conclude that correlations between stocks and bonds change completely during the switch of regimes, which indicates reallocating the portfolio may lead to a higher return. This paper takes a closer look at the allocation between the regimes. The procedure for the optimal allocation follows Campbell & Viceira (2002), whom discuss the use of a myopic portfolio strategy, and Brandt (2010), who offers common techniques for portfolio optimization. The profits for each of the FASTR models, based on these optimal trading strategies, are compared to three Buy-and-Hold strategies. The performance indicators are the annualized excess returns and volatility, along with the Sharpe Ratio. The latter is subjected to a test of significance, proposed by Ledoit & Wolf (2008). They state that the common technique of Jobson & Korkie (1981), which is later corrected by Memmel (2003), is not optimal in the use of time series. Instead, they propose the use of a bootstrapping method in order to test whether the Sharpe Ratio differs significantly from the ratio of the benchmark.

The methods described above are executed in order to test the hypothesis that nonlinearity adds significant value to the Factor Augmented model. The main research question of this study therefore is

'To what extent are the predictions of excess stock returns affected when Factor Augmentation and Nonlinearity are combined?'

When the two methods are combined, there is the possibility that different regimes generated by the STR component have influence on the explained variance in the principal component analysis. For example, a recession may explain more/less of the variance in the principal component analysis. Therefore, the sub-question of this research regarding this hypothesis is

'Do different regimes in the model affect the total amount of variance explained in the factor augmentation?'

Results obtained after the prediction contained a very high Correctly Predicted Signs statistic for the realized volatilities, and the Excess Predictability test shows that multiple FASTR models are able to profit more than taking random actions. The economic performance shows excess returns up to 25.255% on an annual basis, with a Sharpe ratio of 0.553. The bootstrap of Ledoit & Wolf (2008) is able to obtain some significant positive values when the Sharpe Ratios are compared against the benchmarks.

In order to try and improve the performance of the model, the research adds three expansions to the FASTR models. At first, the algorithms of 'Hard-Thresholding' and 'Soft-Thresholding' are taken into consideration. Instead of selecting all the variables in the large set of macroeconomic predictors, these algorithms only include the variable whenever it has a significant value on the dependent variable. Tibshirani (1996) was one of the first to propose a method, but the methods are used and optimized in a variety of financial papers, for example Efron, Hastie, Johnstone & Tibshirani (2004), Zou & Hastie (2005) and Bai & Ng (2008). Efron et al. propose a fast-working algorithm, based on the height of the correlations of the exogenous variables and the dependent variables, named Least Angle Regressions (LARS). This method is used in the selection of the macroeconomic variables. Other expansions include the use of factor selection and changes in the logistic function. The results often contain better performances of the RMSE, DM, CPS and DA statistics. The EP and portfolio optimization show mixed results, where the models that perform less in the standard models now result in more profit.

The set-up of this research is as follows. Chapter 2 contains details on the dependent variables, which consist of the five series to be forecasted, as well as the risk-free rate considered in this research. Furthermore, more information is given on the large dataset of macroeconomic variables used in the factor augmentation and the financial variables. The explanation and implementation of the latter sets continues in Chapter 3. This latter chapter also discusses the general settings of the FASTR models in more detail, and constructs the performance indicators used for comparison. Chapter 4 contains the results of the FASTR models and both the statistical and economic performance. Furthermore, it measures the added value of the different regimes to the factor augmentation: a different regime might contain a larger variance explained in the factor analysis. Chapter 5 discusses extensions to the basic idea of the FASTR models. The expansions are discussed in full detail and the results of the added features follow in this chapter as well. Chapter 6 concludes this research.

2. Data and implementations

The data is split in two parts, the dependent variables and the exogenous variables. The dependent variables consist of the excess returns and realized volatilities of six asset options. These options include three portfolios – consisting of respectively small, medium and big stocks – obtained from the website of K. French, who divides a large number of stocks in five quantiles, depending on their market equity.[1] The Small, Medium and Big Cap portfolios are considered as the 2nd, 4th and 5th quantile of this division respectively. Two other asset options are the Gold commodity and the S&P500 Index. The last asset class is considered the risk-free rate. For this purpose, the 1-month U.S. Treasury T-bill is chosen. It is assumed the investor knows the risk-free return of the next month. The risk-free rate is therefore not included in the prediction of the return series, but it is used in the economic performance later on. The data consists of daily returns and ranges from April 1968 to November 2011. Section 2.1 gives more detail in these dependent variables, and shows how to compute the realized volatility of the given assets.

The exogenous variables are divided in macroeconomic and financial variables, which contribute differently to the model. The former are obtained from the research of

[1] For more information about the data, refer to http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/.

Stock & Watson (2005). Some of the variables are excluded, to enhance the distinction between the macroeconomic and financial variables. This follows the findings of Çakmakli & Van Dijk (2010), whom state that the omitted series contain information in a financial matter. The variables included in this research are summarized in Table B.1 in Appendix B. Overall, the series can be classified in different categories, namely Output & Income; Employment & Hours; Sales; Consumption; Housing Starts & Sales; Orders; Inventories; Money and Credit Quantity Aggregates; Exchange Rates; Price Indexes; and Average Hourly Earnings. In order to ensure the stationarity, the variables are subject to a transformation, which is also found in Table B.1. Furthermore, the table consists of a column which determines whenever the variable is called 'slow' – indicating the variable does not react to shocks of monetary policy or shocks in the financial market within one month – or 'fast' – shocks to monetary policy or to the financial markets are directly influencing the respective variable.

The financial variables, adapted from the research of Çakmakli & Van Dijk (2010), consist of nine series and are summarized in Table B.2 in Appendix B. These series include, for example, [changes in] the interest rate, the dividend yield etcetera. Both the financial as well as the macroeconomic data consist of monthly observations, ranging again from April 1968 to November 2011. More information about the exogenous variables and the implementations is provided in the next chapter.

2.1. The dependent variables

The returns of the dependent variables need to be converted to monthly excess returns and monthly realized volatility in order to be able to forecast using the macroeconomic and financial variables. The excess returns are computed by taking the cumulative product of the daily returns of the corresponding month. That is, the excess returns are established at the end of each month. The monthly returns are subtracted by the 1-month U.S. Treasury Bill in order to obtain the excess returns.

The realized volatility is computed by means of the daily returns of the respective month, as shown in Equation (2.1).

$$RV_t = \sqrt{\sum_{j=1}^{T} (r_j - \bar{r})^2 \left[1 + \frac{2}{T} \sum_{k=1}^{T} (T-k) \hat{\phi}_t^k \right]} \tag{2.1}$$

Here, r_j is the return at day j; $\bar{r}$ is the average of the returns in month t; T is the total number of trading days in month t; and $\hat{\phi}_t$ is the first order autocorrelation in month t. The computation of the realized volatility holds a correction term. Scholes & Williams (1977) state that daily closing prices of returns exhibit non-synchronous information, as the price is mostly referred to as the last trade occurred on the specific date. The time of this trade may be inconsistent throughout the days in the same month. To account for this error, following French, Schwert & Stambauch (1987) and Çakmakli & Van Dijk (2010), a term should be added to the computation of the realized volatility. According to French et al. (1987), the subtraction of the mean in the first part of the equation is not necessary, as it gives neglecting differences. However, the return series used in this research – as shown later – indicate that the mean may deviate from zero enough to contain influence.

The descriptive statistics of the dependent variables are given in Table 2.1. Both the excess returns and realized volatility for the five series are measured in annualized percentages. The minimum and maximum are not scaled to annual values. Instead, these values are captured within one month. The risk-free rate is not present in the table, but is graphed in Figure A.1 in Appendix A. The most important note for the risk-free rate is that the return equals zero at the end of the sample. This may have consequences for the economic performance during that period.

The table shows the highest average excess return for the small cap portfolio, which in turn also brings the highest volatility, as to be expected. The larger the cap, the 'safer' the investments become, in the sense that it yields a lower average excess return, along with a lower standard deviation. An exception is the Gold option, which shows a relatively high standard deviation for the excess returns, along with a lower excess return than the Big Cap portfolio.

The (auto)correlations of the realized volatility in Table 2.1 are computed by subtracting the median. The first-order autocorrelations for the realized volatilities are all around 0.5, which may result in the fact that the first lags of the volatility may bring some information in the model, when included. The correlations between Gold and the portfolio returns are almost equal to zero, indicating no correspondence between the two different investment options. This may extend the options of portfolio allocation in the economic performance later on, thanks to the availability of an extra option in regimes such as recessions, in which the assets may lack a good performance.

Descriptive Statistics		Mean	Standard Deviation	Min	Max			
Small Cap	r_e	8.958%	21.647%	-27.971%	27.465%			
	RV	16.122%	10.416%	0.611%	27.215%			
Medium Cap	r_e	7.818%	18.963%	-24.262%	22.532%			
	RV	15.769%	9.937%	0.921%	27.819%			
Big Cap	r_e	6.356%	16.938%	-21.194%	19.503%			
	RV	15.366%	9.274%	1.165%	26.334%			
S&P 500	r_e	5.301%	15.639%	-22.075%	16.294%			
	RV	14.784%	8.248%	1.107%	26.108%			
Gold	r_e	5.592%	20.283%	-23.581%	28.378%			
	RV	15.427%	11.357%	0.152%	32.524%			
		Auto Corr.		Correlations				
		ρ_1	ρ_{12}	Small	Medium	Big	S&P	Gold
Small Cap	r_e	0.164	0.054	1				
	RV	0.496	0.158	1				
Medium Cap	r_e	0.127	0.024	0.947	1			
	RV	0.558	0.130	0.938	1			
Big Cap	r_e	0.081	0.025	0.876	0.966	1		
	RV	0.556	0.113	0.869	0.970	1		
S&P 500	r_e	0.047	0.048	0.838	0.927	0.973	1	
	RV	0.541	0.113	0.827	0.923	0.968	1	
Gold	r_e	0.065	0.090	0.006	0.008	-0.002	0.001	1
	RV	0.506	0.246	0.219	0.248	0.254	0.268	1

Table 2.1. Descriptive statistics of five of the six asset options. Both the excess returns and the realized volatility are measured in annual percentages. The minimum and maximum percentages are captured within one month. The (auto)correlations of the realized volatility are computed by subtracting the median of the respective series.

3. Methods

This section first discusses the main method, the Factor Augmented Smooth Transition (FASTR) model. The characteristics in this Chapter are maintained general. The specification of the models follows in Chapter 4, where the results of the FASTR models are discussed. Section 3.1 starts with the explanation of the two components in the FASTR models, the linear factor augmentation and the nonlinear smooth transition regression. Later on, in Section 3.2, the performance indicators are discussed. A total of five statistical performance measures are expressed in Section 3.2.1, while the portfolio optimization and corresponding significance test of the Sharpe Ratio follow in Section 3.2.2.

3.1. The FASTR Models

Multiple versions of the FASTR model are examined and discussed in this paper. The aim is to predict the excess returns of the stocks in question, in advance defined as $r_{e,t+1}$, and the corresponding realized volatilities, defined as σ_{t+1}. The return and volatility of the risk-free rate is not examined by the FASTR models, as it is assumed the investor knows the return in one month. All versions of the FASTR model use a two-regimes switching approach. One step before the full version of the FASTR model is reached, the model can be written in a STR form as in Equation (3.1), which mainly follows Teräsvirta & Anderson (1992) for the nonlinear switching-regime.

$$\begin{aligned} y_{t+1} = \{\beta_{0,1} + \boldsymbol{\beta}'_{x,1}\boldsymbol{x}_t + \boldsymbol{\beta}'_{z,1}\boldsymbol{z}_t\}[1 - G(s_t;\gamma,c)] \\ + \{\beta_{0,2} + \boldsymbol{\beta}'_{x,2}\boldsymbol{x}_t + \boldsymbol{\beta}'_{z,2}\boldsymbol{z}_t\}[G(s_t;\gamma,c)] + \varepsilon_{t+1} \end{aligned} \tag{3.1}$$

Here, y_{t+1} could be both $r_{e,t+1}$ as well as σ_{t+1}; $\boldsymbol{x}_t = [x_{1,t}\ x_{2,t} \ldots x_{n,t}]'$ is a *1 x n* vector including various macroeconomic variables at time *t;* $\boldsymbol{z}_t = [z_{1,t}\ z_{2,t} \ldots z_{n,t}]'$ is a *1 x m* vector of financial variables; $G(s_t;\gamma,c)$ is the logistic function defined as

$$G(s_t;\gamma,c) = \frac{1}{1 + exp[-\gamma(s_t - c)]} \tag{3.2}$$

With γ the sensitivity of the logistic function, c the threshold value and s_t an exogenous variable to estimate the regime. Furthermore, it is assumed in Equation (3.1) that ε_{t+1} is an idiosyncratic error. For convenience, the above equation can be written differently, as in Equation (3.3).

$$y_{t+1} = \{\beta_{0,1} + \boldsymbol{\beta}'_{x,1}\boldsymbol{x}_t + \boldsymbol{\beta}'_{z,1}\boldsymbol{z}_t\} \\ + \{\beta_0^* + \boldsymbol{\beta}_x^{*\prime}\boldsymbol{x}_t + \boldsymbol{\beta}_z^{*\prime}\boldsymbol{z}_t\}[G(s_t;\gamma,c)] + \varepsilon_{t+1} \tag{3.3}$$

Where the fact has been used that $\beta_0^* = \beta_{0,2} - \beta_{0,1}$, $\boldsymbol{\beta}_x^{*\prime} = \boldsymbol{\beta}'_{x,2} - \boldsymbol{\beta}'_{x,1}$ and $\boldsymbol{\beta}_z^{*\prime} = \boldsymbol{\beta}'_{z,2} - \boldsymbol{\beta}'_{z,1}$. To arrive at the FASTR model, another transformation needs to be made, with respect to the factor augmentation. This is explained in Section 3.1.1. The characteristics of the logistic function are discussed in Section 3.1.2.

3.1.1. The Factor Augmentation

The linear part of Equation (3.2) deals with the macroeconomic and financial inputs for the excess return series. The remaining set of macroeconomic variables, adapted from Stock & Watson (2005) contains a total of 101 variables. In order to account for stationarity, most variables are subjected to a transformation, which can be found in Appendix B.1. After the transformation, the time series are accounted for outliers. Similar to the research of Stock & Watson (2005), outliers are defined as observations that, in absolute value, deviate more than 6 interquartile ranges from the median value. To prevent look-ahead bias, a moving window of the previous 120 observations – equaling the past 10 years – is used to compute the median and interquartile ranges up to the specific observation. Whenever an outlier is present, it is replaced by the median value of the past five periods.

To reach the expression of the FASTR model, the factor augmentation has to be implemented in Equation (3.3). Especially the set of macroeconomic variables is large in number and, to reduce the risk of parameter estimation, is captured in a factor structure. That is, factors are used in the model, composed as Equation (3.4).

$$\boldsymbol{x}_t = \Lambda \boldsymbol{f}_t + e_t \tag{3.4}$$

Here, Λ is the n *x* k matrix of eigenvectors and $\boldsymbol{f}_t = [f_{1,t}\ f_{2,t} \dots f_{k,t}]'$ is the k *x 1* vector of factors, where $k \ll n$. These factors can be estimated by Principal Component Analysis. The purpose is to reduce the number of parameters, while still accounting for explaining most of the variance in the complete set of variables $\boldsymbol{x}_t$. Before the Principal Component Analysis can be used, the variables have to be scaled. The transformations of the variables mentioned in Appendix B.1 are capable of adding stationarity to the time series. However, due to the different approximations of the variables, the factors may be centralized on a couple specific variables. Therefore, the variables are standardized, where the mean and standard deviation are computed in the moving window. After the scaling and principal components are completed, the factors can be substituted in Equation (3.2). Hence, we obtain the complete version of the FASTR model in Equation (3.5).

$$\begin{aligned} y_{t+1} = &\left\{\beta_{0,1} + \boldsymbol{\beta}'_{f,1}\boldsymbol{f}_t + \boldsymbol{\beta}'_{z,1}\boldsymbol{z}_t\right\} \\ &+ \left\{\beta^*_{0,2} + \boldsymbol{\beta}^{*\prime}_{f,2}\boldsymbol{f}_t + \boldsymbol{\beta}^{*\prime}_{z,2}\boldsymbol{z}_t\right\}[G(s_t;\gamma,c)] + \varepsilon_{t+1} \end{aligned} \tag{3.5}$$

To determine the number of factors taken into account, the negative log likelihood in combination with the BIC criteria is used. At least one factor and at most six factors are taken in the model, in correspondence to Çakmakli & Van Dijk (2010). Following the research of Bai (2010), who finds that the 2$^{\text{nd}}$ and 5$^{\text{th}}$ principal component contain most significant information, this amount of factors should be well enough to capture most of the variations. In addition, lags of the factors are considered. In order to keep the computational burden limited, all factors up to the last significant factor are added. That is, the 5$^{\text{th}}$ factor can only be included in the model whenever the factors 1 through 4 are included as well. Lags of the factors are only considered whenever the original factor is in the best model, and the same rule applies here as is the case for the original factors.

The financial variables, described in Appendix B.2, are adapted from the research of Çakmakli & Van Dijk (2010) and contain indicators such as the dividend yield, interest rate and default spread. Some remarks should be taken into account. Three versions of the monthly interest rate are captured in the financial variables. However, altogether these variables lead to perfect multicollinearity between the combination of the monthly rate and lag, and the first differences of the interest rate. When regressing both series on any dependent variable, the equation reaches a near singular matrix. For this sake, the first differences of the monthly interest rate are omitted for this research. Second, the assets mentioned earlier may not respond to shocks in every single financial variable. The variables that contain useful information for each of the dependent variables – both excess returns and realized volatility – are selected by means of backwards elimination based on the in-sample observations. This is done only at the start of the out-of-sample for each dependent variable, and it is assumed that the significance over the out-of-sample does not change or that the chosen variables do not lose significance on the dependent variable. The backwards elimination uses all variables in a regression on the dependent variable. The explanatory variable that is least significant will be deleted. The process is repeated until all variables are significant or only one financial variable remains.

The result of this backwards elimination is summarized in Table 3.1. An 'X' defines that the variable is taken into the prediction of the dependent variable later on. The annual interest rate shows to be valuable for almost every prediction series, except for the realized volatility of Gold. The log Implied Volatility Index contains significance for each of the realized volatilities, and the Dividend Yield responds to most of the excess returns. The monthly interest rate, along with its first lag, and the default spread are not included in most predictions.

		PE	**DY**	**I1**	**I1(-1)**	**ΔI1**	**I12**	**I12(-1)**	**VOL**	**DS**
SC	ER		X		X		X			
	RV	X					X		X	
MC	ER		X				X	X		
	RV						X		X	
BC	ER		X				X	X		
	RV						X		X	
S&P	ER		X				X	X		
	RV						X		X	
GOLD	ER						X		X	X
	RV	X	X						X	

Table 3.1. Test of the significance of explanatory financial variables on the excess returns and realized volatility of the assets. PE = Price/Earnings ratio; DY = Dividend Yield; I1 = monthly interest rate; I1(-1) = lag of the monthly interest rate; ΔI1 = first difference of the monthly interest rate; I12 = annual interest rate; I12(-1) = lag of the annual interest rate; VOL = log implied Volatility Index; and DS = Default Spread. The used method is backwards elimination. ΔI1 is not taken into consideration as it leads to multicollinearity combined with I1 and I1(-1). An 'X' indicates a significant value, hence a valuable addition to the prediction of the dependent variable.

The financial variables are subjected to another check. Çakmakli & Van Dijk (2010) purposely separated the financial influence from the macroeconomic set of variables, where Bai (2010) used all variables in the factor analysis. The importance of the financial variables seems to differ between the papers, and are therefore used in different sections of the model. Alternative to Equation (3.5), where the financial variables are considered to have a nonlinear movement in time, two other models are discussed throughout this paper. The first considers the financial variables in a linear way. That is, as in Equation (3.6).

$$
\begin{aligned}
y_{t+1} = &\{\beta_{0,1} + \boldsymbol{\beta}'_{f,1}\boldsymbol{f}_t\}[1 - G(s_t; \gamma, c)] \\
&+ \{\beta_{0,2} + \boldsymbol{\beta}'_{f,2}\boldsymbol{f}_t\}[G(s_t; \gamma, c)] + \boldsymbol{\beta}'_z\boldsymbol{z}_t + \varepsilon_{t+1}
\end{aligned}
\tag{3.6}
$$

The factors in the model are again estimated as in Equation (3.4). The last model states that the influences of the financial variables are not significant at all, and therefore omits the values from the equation, that is

$$
\begin{aligned}
y_{t+1} = &\{\beta_{0,1} + \boldsymbol{\beta}'_{f,1}\boldsymbol{f}_t\}[1 - G(s_t; \gamma, c)] \\
&+ \{\beta_{0,2} + \boldsymbol{\beta}'_{f,2}\boldsymbol{f}_t\}[G(s_t; \gamma, c)] + \varepsilon_{t+1}
\end{aligned}
\tag{3.7}
$$

3.1.2. The Smooth Transition Regression

The nonlinear part in Equations (3.5), (3.6) and (3.7), the function $G(s_t; \gamma, c)$, determines the weight for the scenario of the market being in either a bull or bear regime, based on the exogenous variable s_t. This variable can both be endogenous and exogenous. In this research, a logistic function is chosen, as defined in Equation (3.2). The ease of the function is that it compresses the values in the range of [0, 1], indicating that it can easily assign weights in the FASTR models.[2] The threshold value c is usually set to the mean or median value of the time series, to distinguish the different regimes. The coefficient γ is the sensitivity of the logistic function. If γ is set to a high value, a small deviation from the threshold already assigns a very small weight and hence the logistic function transforms into a threshold function. On the other hand, a value close to zero leads to weights that are always equal to 0.5.

Three series are used to obtain weights, in order to obtain estimates of the bull and bear regimes. The first is driven by the average return of a historical horizon. Tu (2010) stated that, according to the peaks and troughs acknowledged by the NBER, the

[2] For more information on the possible transformation functions, refer to Van Dijk, Teräsvirta & Franses (2000).

average length of an expansion [recession] equals 13.1 [6.1] months.[3] However, due to the fact that, according to the NBER, recessions need to be at least 2 quarters long, short bear regimes are often overlooked. On the other hand, using a single value of the lagged time series may be inaccurate due to shocks that may occur. Keeping this in mind, the horizon is set on the past 4 months.

The second case considers the use of the log version of the implied volatility index. Many found that the volatility is higher for bear markets compared to bull markets (see, for example Ang & Bekaert (2002, 2004)). An example can also be found in Figure 3.2, which shows the excess returns and realized volatility of the S&P 500 Index in panels 1 and 2 respectively. The red dots in the first panel show returns smaller than -7%. The dots in the second panel show the corresponding volatilities. This shows that large negative excess returns are often enhanced with larger than average realized volatility. Combined with the findings of Section 2.1, where the realized volatilities show a high positive first-order autocorrelation, the log Implied Volatility Index looks able to provide reasonable estimates for the regimes.

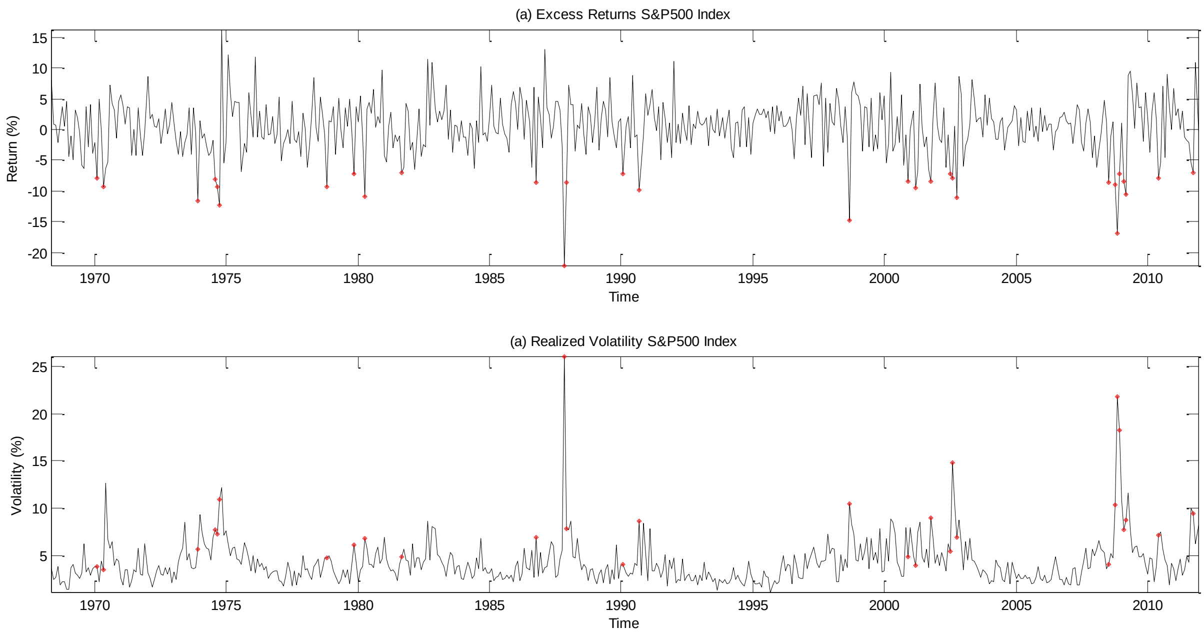

Figure 3.2. Excess returns (panel a) and realized volatility (panel b) of the S&P 500. The red dots from panel a determine the returns that are smaller than -7%. Panel b indicates that these large losses are commonly accompanied by high volatilities.

[3] For an overview of the dates of the peaks and troughs, go to http://www.nber.org/cycles/cyclesmain.html.

The last option follows Perez-Quiros & Timmermann (2000), whom find leading indicators for stock returns. Examples that they discuss are the price-earnings ratio, the M1 base of monetary aggregates, and the default spread. The last one is used as an exogenous variable in this research. The Default Spread is computed by subtracting Moody's Aaa rated bond yield from the Baa rated bond yield. Figure A.2 in Appendix A shows the Default Spread for the complete sample, along with the log Implied Volatility Index.

The three exogenous variables are standardized over the moving window, used to predict the current observation. The Default Spread is close to zero for every value, and the Implied Volatility Index on the other hand includes values between -2 and -9. The range of the variables partly determines the sensitivity of the logistic function as well. In order to give both a fair chance of being able to attain all weights, along with the parameters of the logistic function, the variables are standardized.

In total, the prediction of the excess returns and the realized volatility by the FASTR models might include a lot of parameters needed to be estimated. In order to minimize the chance of overestimating the parameters, a genetic algorithm is used in order to optimize the values of the parameters in the logistic function. Whenever these values are known, the rest of the parameters in the FASTR model can be solved by an OLS regression. The genetic algorithm uses multiple function iterations in order to minimize the chance of ending up in a local maximum. Given this procedure, the genetic algorithm is able to concentrate on searching for the optimal values of γ and c, while OLS computes the optimal values given the optimized nonlinear parameters. The range of possible values for γ is set to [0, 10], while the optimized value for c lies between $[median\left(s_{\{t-Window:t-1\}}\right) \pm std(s_{\{t-Window:t-1\}})]$, where s_t is the exogenous variable and the median and standard deviation are computed over the window sample.

3.2. Performance Testing

The alternatives of the FASTR model mentioned in the previous section lead to nine models: three ways to define the financial variables, times three exogenous variables. The performance of all models is tested in both statistical and economic value. Five performance measures are used for the statistical value: the Relative Mean Squared

Error (RMSE) and Diebold-Mariano (DM) test provide statistics for the performance relative to the benchmark of linear factor augmentation; the Correctly Predicted Signs (CPS) test and Directional Accuracy (DA) test of Pesaran & Timmermann (1992) measure the accuracy of the predictions; and the profitability on a single excess return series is checked by means of the Excess Predictability (EP) test of Anatolyev & Gerko (2005). The procedures of these tests are explained in Section 3.2.1. The economic value is captured in Section 3.2.2. The usage of portfolio optimization is explained in more detail, and the way to determine the weights for the optimization is expressed. The returns are valued by means of the Sharpe Ratio, and the bootstrap proposed by Ledoit & Wolf (2008).

3.2.1. Statistical performance tests

The models explained in the earlier section are checked on value according to a benchmark, which is obtained through the research of Çakmakli & Van Dijk (2010). Comparing against this linear factor augmented model reveals the value of adding nonlinearity regarding the forecasts of excess returns and realized volatility. The benchmark model is written similar to the factor augmentation of the models considered in the previous section.

$$y_{B,t+1} = \beta_0 + \boldsymbol{\beta}_f \boldsymbol{f}_t + \boldsymbol{\beta}_z \boldsymbol{z}_t + \varepsilon_{t+1} \tag{3.8}$$

Here, $y_{B,t+1}$ can again be either the excess return or the realized volatility for the benchmark. The definitions and assumptions of the factors and errors are equal to the FASTR models. The factors in the model are estimated as was the case for the FASTR models. The first performance measure is the Relative Mean Squared Error (RMSE), as proposed by Bai & Ng (2008). The standard is the linear factor augmented model mentioned in Equation (3.8). That is, the RMSE is computed as

$$RMSE_k = \frac{\frac{1}{N-T}\sum_{t=T+1}^{N}\left(\hat{y}_{k,t} - y_t\right)^2}{\frac{1}{N-T}\sum_{t=T+1}^{N}\left(\hat{y}_{B,t} - y_t\right)^2} \tag{3.9}$$

In this equation, $\hat{y}_{k,t}$ stands for the forecasted series of model k; $\hat{y}_{B,t}$ are the predictions of the benchmark; y_t are the real observations at time t; N is the total number of observations; and T is the last observation of the in-sample period.

In advance, to check whether the mean squared error is significantly lower than the benchmark, the test of Diebold & Mariano (2002) is used. The DM test statistic is given in Equation (3.10).

$$DM = \frac{\frac{1}{N-T}\sum_{j=T+1}^{N}\left(e_{B,j}^2 - e_{F,j}^2\right)}{\sqrt{\frac{1}{N-T}Var(\boldsymbol{e}_B^2 - \boldsymbol{e}_F^2)}} \sim N(0,1) \tag{3.10}$$

Where $\boldsymbol{e}_B^2$ is the *1 x (N – T)* vector of squared errors of the benchmark model and $\boldsymbol{e}_F^2$ is the *1 x (N – T)* vector of squared errors of the FASTR model. A value exceeding the critical value indicates the errors of the FASTR model are significantly lower compared to the benchmark.[4]

The next two tests measure the accuracy of the predictions. First is the Correctly Predicted Signs test, which can be computed as in Equation (3.11).

$$CPS_k = \frac{\sum_{t=T+1}^{N} h_{k,t}}{N-T} \tag{3.11}$$

In this equation, $h_{k,t}$ is defined as the hit for model k.[5] This is different for the excess returns and the realized volatility. For the returns we can define the threshold value of zero, separating positive and negative values. For the realized volatility, the historical median is used. That is, for the excess returns the hits follow Equation (3.12).

$$h_{k,t} = \begin{cases} 1, & if\ \hat{y}_{k,t}y_t > 0 \\ 0, & otherwise \end{cases} \tag{3.12}$$

The notation is kept the same. The equation states that, whenever the forecasted excess return and the real excess return at time t are both positive or negative, the hit equals one. For the realized volatility, it can be computed as

[4] A DM value lower than the negative critical value states that the FASTR model produces significantly larger errors compared to the benchmark.
[5] From this point, the k models also include the benchmark.

$$h_{k,t} = \begin{cases} 1, & if \left[\hat{y}_{k,t} - median(y_{1:t-1})\right] * [y_t - median(y_{1:t-1})] > 0 \\ 0, & otherwise \end{cases} \tag{3.13}$$

That is, the hit equals one if the sign of the forecasted realized volatility, subtracted by the median of the real observation up to time t, is equal to the sign of the real value. For the computation of the median, an expanding window is used, which starts at the first observation of the in-sample period.

The CPS test is standard, and does not give a precise value for the performance of the model. Pesaran & Timmermann (1992) propose a test to measure the predictability of the dependent series, the so-called Directional Accuracy (DA) test. The null hypothesis accompanying the test states that the model cannot accurately predict the directions of the return series. A value exceeding the critical value indicates that the model does predict the return series more accurately than random actions. First, define the hits by

$$\begin{aligned} Y_t &= 1 \quad if\ y_t > 0, \\ \hat{Y}_{t,k} &= 1 \quad if\ \hat{y}_{t,k} > 0 \end{aligned} \tag{3.14}$$

In the equations, y_t is again the time series of real returns and $\hat{y}_{t,k}$ is the prediction at time t for model k. For all hits, define the probabilities by

$$P_y = \frac{1}{N-T} \sum_{t=T+1}^{N} Y_t, \quad P_{\hat{y}} = \frac{1}{N-T} \sum_{t=T+1}^{N} \hat{Y}_{t,k} \tag{3.15}$$

The DA test statistic can be written as

$$DA_k = \frac{(CPS_k - P^*)}{\sqrt{var(CPS_k) - var(P^*)}} \sim N(0,1) \tag{3.16}$$

Where CPS_k is the result of the Correctly Predicted Signs test given above. This result holds asymptotically, according to Pesaran & Timmermann (1992). The individual sections of the equation can be defined as in Equation (3.17).

$$
\begin{aligned}
P^* &= P_y P_{\hat{y}} + (1 - P_y)(1 - P_{\hat{y}}) \\
var(CPS_k) &= \frac{1}{N-T} P^*(1 - P^*) \\
var(P^*) &= \frac{1}{N-T}(2P_y - 1)^2 P_{\hat{y}}(1 - P_{\hat{y}}) \\
&\quad + \frac{1}{N-T}(2P_{\hat{y}} - 1)^2 P_y(1 - P_y) \\
&\quad + \frac{4}{(N-T)^2} P_y P_{\hat{y}}(1 - P_y)(1 - P_{\hat{y}})
\end{aligned}
\tag{3.17}
$$

Continuing on the findings of Pesaran & Timmermann (1992), Anatolyev & Gerko (2005) construct an accuracy test for a trading strategy. The test is known as the Excess Predictability (EP) test, and computes the value of the model relative to a benchmark, with the same chance of predicting a positive/negative sign as the model to be tested. The null hypothesis of the test states that the model does not significantly outperform the benchmark. Define again the predicted excess return or realized volatility at time t by $\hat{y}_{t,k}$ and the real return or volatility as y_t. Following the definitions of Anatolyev & Gerko (2005), the EP test can be computed by

$$
EP = \frac{A_T - B_T}{\sqrt{\hat{V}_{EP}}} \sim N(0,1) \tag{3.18}
$$

The result holds asymptotically. The individual parts are computed by means of Equation (3.19).

$$
\begin{aligned}
A_T &= \frac{1}{N-T} \sum_{t=T+1}^{N} sign(\hat{y}_{t,k}) * y_t \\
B_T &= \left(\frac{1}{N-T} \sum_{t=T+1}^{N} sign(\hat{y}_{t,k}) \right) \left(\frac{1}{N-T} \sum_{t=T+1}^{N} y_t \right) \\
\hat{V}_{EP} &= \frac{4}{(N-T)^2} \hat{p}_{\hat{y}}(1 - \hat{p}_{\hat{y}}) \sum_{t=T+1}^{N} (y_t - \bar{y})^2
\end{aligned}
\tag{3.19}
$$

In the last equality, $\bar{y}$ stands for the mean of the real return series. A_T is the total return of the sample, obtained by taking a long position when the model predicts a positive return, and going short for a negative prediction; B_T computes the same statistic for a benchmark that has similar chances of going long and short, but does so on random occasions. The variance represents the variance of A_T – B_T, and uses the probability $p_{\hat{y}} = P\left[sign\left(\hat{y}_{t,k}\right) = 1\right]$. The computation of $\hat{p}_{\hat{y}}$ follows Equation (3.20).

$$\hat{p}_{\hat{y}} = \frac{1}{2}\left(1 + \frac{1}{N-T}\sum_{t=T+1}^{N} sign\left(\hat{y}_{t,k}\right)\right) \tag{3.20}$$

3.2.2. Economic performance tests

After testing individual return series, the series are combined in the portfolio optimization. For this research, a mean-variance portfolio is used, based on the quadratic utility of an investor. For this purpose, two separate limitations are submitted to the possibilities of the investor. First, the investor is not allowed to go short in the asset options. That is, the weights should be in the interval of [0, 1]. Second, the investor is allowed to go short in the asset options, but this is limited to [-1, 2]. At all times, the sum of the weights equals 1.

Steps of the derivation can be found in Campbell & Viceira (2002), and Brandt (2010) explains more about the characteristics of portfolio maximization. The latter provides an analytical solution to the problem. The mean-variance portfolio is written as Equation (3.21).[6]

$$\max_{\boldsymbol{\omega_t}} W_{t+1} = E_t\left[\hat{r}_{p,t+1}\right] - \frac{1}{2}\gamma\hat{\sigma}^2_{p,t+1} \tag{3.21}$$

That is, the maximum wealth in the next period W_{t+1} is a trade-off between the expected returns and volatility. The variable aimed to optimize the wealth is the *q x 1* vector $\boldsymbol{\omega_t}$, which corresponds to the weights given to the asset options; q is the number of asset options in the portfolio optimization. Furthermore, γ defines the risk aversion of the investor, where $\gamma > 0$. The higher the risk aversion, the more the investor cares about

[6] The equation differs from Campbell & Viceira (2002), in the sense that they use the assumption that the returns are log-normally distributed. By rewriting the formula to the logs, a term equal to half the variance is added to the maximization problem (the so-called Jensen Inequality), which is excluded in this formula.

minimizing the risk in the next period. In Equation (3.21), the expected returns and the estimate of the volatility are defined as

$$\begin{aligned} E_t\left[\hat{r}_{p,t+1}\right] &= r_{f,t+1} + \boldsymbol{\omega}_t' \hat{\boldsymbol{r}}_{e,t+1} \\ \hat{\sigma}^2_{p,t+1} &= \boldsymbol{\omega}_t' \hat{\Sigma}_{t+1} \boldsymbol{\omega}_t \end{aligned} \tag{3.22}$$

In the first equality of Equation (3.22), $\boldsymbol{r}_{e,t+1}$ stands for the *q x 1* vector of excess returns of the risky assets and $\boldsymbol{\omega}_t$ is the *q x 1* vector of the weights. The risk-free rate is not included in the asset options. Note that this vector does not always sum up to one. The remainder is invested in the risk-free rate, or borrowed whenever the sum of $\boldsymbol{\omega}_t$ exceeds 1. As before, it is assumed that the investor knows the return of the risk-free rate in period *t + 1*, at the beginning of month *t*. The second equality computes the variance out of the predicted realized volatilities at time *t*. The covariance matrix $\hat{\Sigma}_{t+1}$ is computed by means of

$$\hat{\Sigma}_{\mathrm{t+1}} = \hat{D}_{t+1} \hat{R}_{t+1} \hat{D}_{t+1} \tag{3.23}$$

In this equation, $\hat{D}_{t+1}$ stands for the *q x q* matrix with the realized volatilities of the assets on the diagonal. The matrix R_{t+1} stands for the *q x q* correlation matrix between the asset options at time *t + 1*. The assumption is made that the correlations do not change quickly over time. Hence, the estimates of the correlations at time *t + 1* are assumed to be equal to the correlation matrix at time *t*. A moving window of the past 10 years is used to compute the correlation matrix.

The analytical solution for the weights in Campbell & Viceira (2002) and Brandt (2010) cannot be used in this matter, due to the restrictions proposed earlier. Another method should be found to optimize the weights given in Equation (3.21). The chosen solution is the use of Monte Carlo simulation, as proposed by Brandt, Goyal, Santa-Clara & Stroud (2005). In the research, they use simulated portfolio weights in order to estimate a portfolio of multiple assets in discrete time, and subjected to restrictions on the weights, similar to this research. The allocation in their paper is based on a dynamic portfolio, indicating that the utility is maximized over multiple periods at the same time rather than the myopic strategy used in this paper. They find that the difference between the standard errors of the weights by using this simulation method and other optimization techniques can be neglected whenever the amount of samples is high.

The Monte Carlo method starts by drawing S samples of weight vectors, which are *(q+1) x 1* in length. All individual weights should be in the interval respective to the limitations. Hereafter, the weights have to be scaled so the total weight equals 1. For each draw, q values are used to determine the weights of the asset options and add the risk-free rate to the maximization problem by using the last weight. That is,

$$\max_{\omega_t}\left(r_{f,t+1} + \boldsymbol{\omega}'_{\{1:q\},t}\hat{\boldsymbol{r}}_{e,t+1}\right) - \frac{1}{2}\gamma\left(\boldsymbol{\omega}'_{\{1:q\},t}\hat{\Sigma}_{t+1}\boldsymbol{\omega}_{\{1:q\},t}\right) + \left(1 - \iota\boldsymbol{\omega}'_{\{q+1\},t}\right)r_{f,t+1} \tag{3.24}$$

The returns made by the models are computed by multiplying the obtained weights by the real returns. The Sharpe Ratio is computed by dividing the annualized excess returns by the annual standard deviation of the returns.

To test for significance, the Sharpe Ratios of the FASTR models are compared to the Sharpe Ratio of the benchmark. Jobson & Korkie (1981) proposed to test between two Sharpe ratios, which was corrected by Memmel (2003). However, Ledoit & Wolf (2008) state that using the method proposed in these two papers is not accurate in the evaluation of time series, and propose to use a bootstrap method to test the difference between the Sharpe Ratios. The null hypothesis states that the difference is zero. That is $H_0\colon \Delta = 0$, where Δ is equal to the difference between Sharpe Ratios. The notations of Ledoit & Wolf are followed in this research. Start by defining the estimate of the difference between the Sharpe Ratios as

$$\hat{\Delta} = \widehat{Sh}_B - \widehat{Sh}_F = \frac{\hat{\mu}_B}{\hat{\sigma}_B} - \frac{\hat{\mu}_F}{\hat{\sigma}_F} \tag{3.25}$$

In the equation, $\hat{\mu}_i$ stands for the mean excess return of the benchmark (B) or the FASTR model (F) and $\hat{\sigma}_i$ is the annualized volatility of the benchmark or FASTR model. The bootstrap consists of a few steps. The first is to fit a semi-parametric model to the return series $\boldsymbol{r}_{e,B}$ and $\boldsymbol{r}_{e,F}$. Using the bootstrap, M pseudo return series are created using this semi-parametric model. The $1-\alpha\%$ confidence intervals for the pseudo series are computed and it is checked whether the value to be tested, in this case 0, is present in the intervals. In order to estimate the covariance matrix, which is needed to compute the confidence intervals, Ledoit & Wolf propose the use of the circular block bootstrap of Politis & Romano (1992). This, along with the use of the Delta method, provides a good estimate. Refer to Ledoit & Wolf (2008) for further information regarding the estimation of the covariance matrix. By applying the optimization of Ledoit & Wolf, the optimal block size is shown to be six, and is therefore used in this research.

A quick way to compute the p-value of the bootstrap is by means of Equation (3.26).

$$Pvalue = \frac{\left(\tilde{d}^{*,m} \geq d\right) + 1}{M+1} \tag{3.26}$$

Where M is the total number of bootstrap iterations, $\tilde{d}^{*,m}$ is the estimate of the m^{th} iteration, and d is the estimate of the original data. That is,

$$d = \frac{|\hat{\Delta}|}{s(\hat{\Delta})}, \qquad \tilde{d}^{*,m} = \frac{|\hat{\Delta}^{*,m} - \hat{\Delta}|}{s(\hat{\Delta}^{*,m})} \tag{3.27}$$

In this equation, $s(\hat{\Delta})$ is the standard deviation of the original return series, and $s(\hat{\Delta}^{*,m})$ is the standard deviation of the m^{th} iteration of the bootstrap.

4. Results

This chapter examines the results of the FASTR models, compared to the benchmark given in the previous chapter. In order to forecast, a moving window is used. This moving window consists of the last 120 observations, which correspond to the past 10 years. Due to this set-up, the in-sample is set to April 1968 until May 1978. The out-of-sample, containing a total of 402 observations, starts at June 1978 and ends at November 2011.

In order to check for stability throughout the complete out-of-sample, the observations are divided in three sub-periods. The first subset is June 1978 to December 1991, which contains the crash at October 1987 and the recession in the US in the early '90s.[7] The second sub-sample ranges from January 1992 to December 2004, which starts relatively flat, but becomes more volatile around 1998. The last sub-sample starts at January 2005 and mostly reflects the performance in the credit crunch.

Some assumptions are made in advance to the results. First, the investor accounts for compounding returns. That is, the profit of the current month is reinvested in the next month. Second, the transaction costs are not taken into account when computing the average annual returns. The main reason is due to the Small, Medium and Big Cap portfolios. The stocks included in these portfolios may switch over time, but no information is available on whether or when this happens. Therefore, transaction costs cannot be computed.

A side-note should be made on the notation. Due to the amount of models, each version is given a code, consisting of two letters. The first letter determines the influence of the financial variables, which could be Nonlinear (N), Linear, (L), or excluded (E). The second letter shows the value of the variable in the STR component, shown in Equation (3.2). The possible options are the Lagged versions of the dependent variable (L); the implied Volatility index (V); or an exogenous variable, in this case the Default spread (E). The benchmark, the factor augmented model, is defined as FA.

The chapter is split up in three parts. Section 4.1 starts with the statistical performance, revealing the strong and less strong characteristics of the FASTR models relative to the benchmark model. The RMSE, CPS and EP test explained in Section 3.2.1 can generally be found in the section itself, while results of the significance tests of Diebold & Mariano (2002) and Pesaran & Timmermann (1992) are found in Appendix A. Section 4.2 contains the average weights and annualized returns and volatility of the economic performance. The significance test of the Sharpe Ratio follows these results. Last, Section 4.3 discusses the sub-question of dependencies between the principal component analysis and the switching regimes.

[7] For a check on the volatility in the periods, refer to Figure 3.2 for the excess returns and realized volatilities of the S&P500 Index.

4.1. Statistical performance

The section starts with the evaluation of the errors, by means of the RMSE and Diebold-Mariano test. Table 4.1 contains the Relative Mean Squared Error of the FASTR models, compared to the FA model. The first line shows the Mean Squared Error (MSE) of this benchmark, followed by the performance of the nonlinear augmented models. The RMSE is computed over the complete sample, see panel (a), as well as three sub-samples, to test for stability in the predictions of the models. All forecasts – that is, excess returns and realized volatilities – are made in percentages. The bold values indicate the lowest value for the given predicted series.

Overall, the FASTR models often do not beat the FA model. For the cap portfolios and the S&P500, the lowest values of the RMSE are close to, but often not far under the MSE of the benchmark, indicating the models have trouble to obtain better predictions. This is different for the realized volatility of Gold, as the FASTR.EL model only contains 80.2% of the squared errors of the FA model. The 'weak' performance can be subscribed to the last sub-sample, as the errors are in general higher for this set relative to the other samples. The models that exclude the financial variables are closest to the MSE of the FA. The models in which the financial variables are considered to have nonlinear information do not contain any of the lowest values, and often contain mean squared errors that are at least 30% larger. Also, the models which use the default spread do not contain any of the lowest values. The models with the lowest values are FASTR.EL and FASTR.EV (both 4).

Table A.3 in Appendix A contains the Diebold-Mariano test statistics for the given squared errors. The chosen significance level is 5% on each side, leading to a critical value of 1.645. A positive value indicates that a model works significantly better than the FA model, and are printed bold for convenience. Throughout the results for the complete sample, only three models are positively significant for one time series, where the FASTR.EL model captures 2, for the excess return and realized volatility of Gold. The first two sub-samples both contain 3 significant positive values, the sample of the credit crunch shows only one positive sign of significance. Overall, it can be concluded from these two tests that, over the complete sample, the FASTR models do not beat the FA model in most cases. This is mainly due to the last, volatile sub-sample.

RMSE	Small Cap		Medium Cap		Big Cap		S&P 500		Gold	
(a) Complete Sample	$r_{e,t+1}$	RV_{t+1}	$r_{e,t+1}$	RV_{t+1}	$r_{e,t+1}$	RV_{t+1}	$r_{e,t+1}$	RV_{t+1}	$r_{e,t+1}$	RV_{t+1}
FA	41.053	7.350	30.782	6.652	25.278	6.053	22.003	4.913	35.442	8.603
FASTR.NL	1.396	1.973	1.601	1.089	1.460	1.085	1.363	1.083	1.402	0.973
FASTR.NV	1.306	1.178	1.289	1.142	1.322	1.107	1.388	1.084	1.172	1.769
FASTR.NE	1.220	1.679	1.381	1.424	1.448	1.442	1.410	1.428	6.566	1.149
FASTR.LL	1.113	1.201	1.152	1.038	1.140	**0.994**	1.085	**0.984**	0.989	0.823
FASTR.LV	1.084	1.208	1.128	1.051	1.139	1.062	1.101	0.988	1.092	1.138
FASTR.LE	1.073	1.410	1.043	1.537	1.045	1.237	1.112	1.213	5.672	1.044
FASTR.EL	**0.977**	1.259	1.054	1.102	**0.998**	1.005	1.059	1.100	**0.917**	**0.802**
FASTR.EV	1.012	**1.014**	**1.033**	**1.045**	1.035	1.029	**1.011**	1.031	1.002	1.172
FASTR.EE	1.029	1.592	1.051	1.796	1.053	1.251	1.041	1.349	0.992	1.072
(b) 1978/06 – 1991/12										
FA	35.649	7.551	27.883	6.403	24.948	6.182	24.109	5.481	56.788	14.196
FASTR.NL	1.143	2.345	1.098	1.059	1.035	1.063	1.028	1.085	1.482	0.962
FASTR.NV	1.336	1.134	1.277	1.118	1.307	1.074	1.309	1.070	1.140	2.052
FASTR.NE	1.217	1.201	1.246	1.098	1.231	1.057	1.212	1.050	1.248	1.070
FASTR.LL	1.048	1.284	1.022	1.006	1.017	0.956	1.019	1.026	0.990	0.816
FASTR.LV	1.042	1.027	1.098	1.039	1.082	1.023	1.041	1.010	1.082	1.188
FASTR.LE	1.000	1.020	1.045	1.051	1.024	1.017	1.004	1.035	1.080	1.043
FASTR.EL	0.992	1.351	**1.007**	**0.968**	**0.972**	**0.923**	0.957	0.977	**0.895**	**0.787**
FASTR.EV	**0.980**	**0.942**	1.024	0.980	0.986	0.940	**0.953**	**0.971**	0.993	1.253
FASTR.EE	0.997	1.028	1.061	1.063	1.033	1.063	0.986	1.123	0.921	1.089
(c) 1992/01 – 2004/12										
FA	38.537	5.633	28.312	5.000	22.633	4.445	18.758	3.688	14.253	3.383
FASTR.NL	1.287	1.667	1.174	1.077	1.238	1.011	1.167	0.946	1.068	0.928
FASTR.NV	1.185	1.103	1.176	1.099	1.163	1.078	1.163	1.033	1.133	1.180
FASTR.NE	1.091	1.235	1.155	1.219	1.229	1.204	1.229	1.333	1.199	0.984
FASTR.LL	1.007	1.038	1.049	**0.998**	1.041	**0.945**	1.037	**0.882**	0.977	0.845
FASTR.LV	1.065	1.039	1.103	1.042	1.141	1.088	1.108	0.990	1.030	0.964
FASTR.LE	1.111	1.053	1.002	1.066	1.037	1.070	1.092	1.079	1.099	1.102
FASTR.EL	**0.934**	1.012	0.989	1.024	1.007	0.971	1.017	0.896	**0.938**	**0.821**
FASTR.EV	0.959	**1.000**	0.973	1.081	1.060	1.089	1.006	1.064	0.940	1.070
FASTR.EE	0.944	1.040	**0.945**	1.213	**0.962**	1.241	**0.964**	1.211	1.008	0.928
(d) 2005/01 – 2011/11										
FA	56.392	10.183	41.120	10.246	31.378	8.823	23.967	6.096	33.346	7.430
FASTR.NL	1.851	1.748	2.825	1.136	2.427	1.186	2.315	1.234	1.400	1.050
FASTR.NV	1.424	1.321	1.450	1.211	1.516	1.182	1.877	1.166	1.311	1.213
FASTR.NE	1.390	2.836	1.852	2.013	2.093	2.197	2.070	2.203	28.660	1.588
FASTR.LL	1.331	1.251	1.459	1.113	1.466	1.094	1.285	1.027	0.992	**0.830**
FASTR.LV	1.162	1.649	1.200	**1.073**	1.227	**1.092**	1.209	**0.949**	1.177	1.098
FASTR.LE	1.113	2.350	**1.094**	2.565	1.089	1.699	1.357	1.679	24.706	0.994
FASTR.EL	**1.012**	1.381	1.200	1.339	**1.028**	1.151	1.325	1.547	**0.974**	0.843
FASTR.EV	1.121	**1.133**	1.123	1.092	1.077	1.092	**1.135**	1.101	1.082	0.954
FASTR.EE	1.177	2.987	1.176	3.231	1.208	1.519	1.262	1.904	1.217	1.130

Table 4.1. Relative Mean Squared Error of the excess returns and realized volatility for the FASTR models, relative to the factor augmented model used in Çakmakli & Van Dijk (2010). The FA shows the Mean Squared Error (MSE), and the FASTR models are a comparison of this MSE. The last two letters of the FASTR models contain the influence of the financial variables and the usage of the STR variable respectively. Computed over the complete sample (panel a), as well as for three sub-periods (panels b to d).

Figure 4.2 contains a graph of the real excess returns of the S&P500 against the predicted excess returns by the FASTR.NL model, in order to see what results in the high RMSE in the previous table. Panel (a) shows the real excess returns and panel (b) exhibits the predicted returns. The first impression shows that the predictions are less volatile than the real returns. The black lines indicate where the real returns are less than -10% for the given month, and the two red lines indicate the extremes of the predicted returns. These lines are reflected in the other panel as well. The FASTR model seems to predict a shock directly after a black line in some cases, especially at the end. The red lines can be found in the end of the sample, during the credit crunch, but the real returns do not show any special value for the predictions to react to.

The reason for this behavior is (partly) found in the parameters of the logistic function, which are optimized by the genetic algorithm. The outcomes of the logistic function tend to go to 0 or 1 quite often, as γ goes to the maximum value possible. In the event of a value close to [but not equal to] 0, the weight is almost completely on the first parts of the FASTR models in Equations (3.5), (3.6) and (3.7). OLS enhances the parameters in the second part of the equations with high values. Whenever, in the previous period, a low value is present in for example the average of the lagged dependent variable, the current prediction obtains a weight higher than 0, the high values of the parameters get more weight, and the predictions are off target.

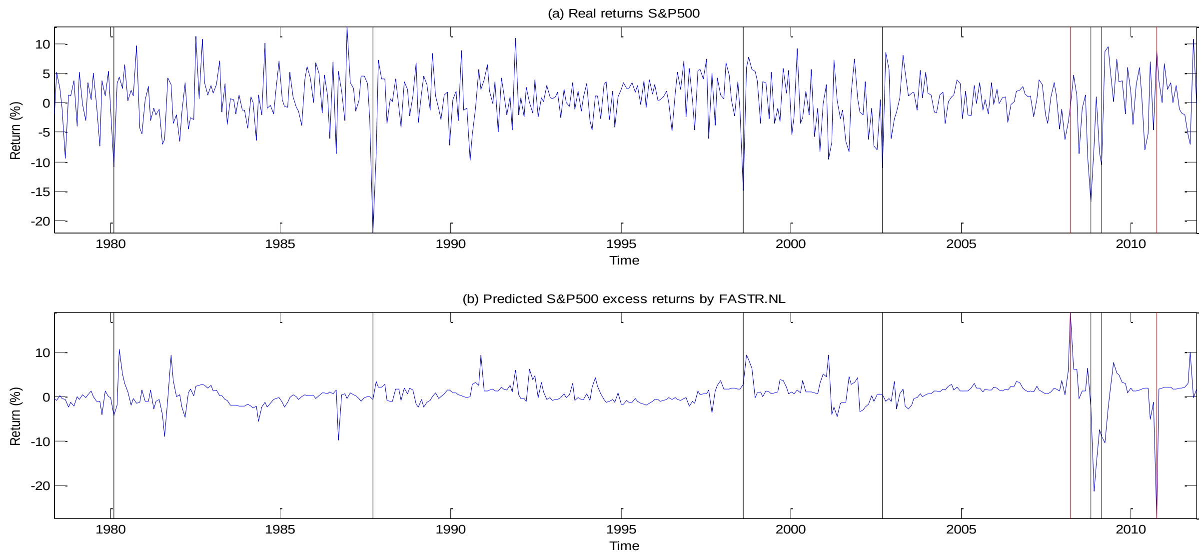

Figure 4.2. Real excess returns (panel a) and predicted excess returns by the FASTR.NL model (panel b) for the S&P500. The black lines indicate a real return lower than -10%. The red lines can be found at the maximum and minimum of the predicted returns.

Table 4.3 contains the outcomes of the Correctly Predicted Signs (CPS) test. The volatilities are tested against the median of the real observations, as explained in Section 3.2.2. In contrast to the results of the RMSE and DM, the CPS test contains a larger value for at least one of the FASTR models compared to the FA. The largest values are often found for the FASTR.EL model, and the correctly predicted signs can be up to 71.6%.

Looking at the sub-samples, even higher percentages can be found. In the first time interval, the models work especially well for the excess returns, but the realized volatility remains low. This is the other way around for the last two sub-samples, where FASTR.LL reaches a CPS of 81.9% during the credit crunch. The FASTR.NL model, which shows very high values for the RMSE, also contains some of the highest values in each of the sub-samples. This does, however, not mean that it works best over the complete sample.

Table A.4 in Appendix A contains the results of the DA test of Pesaran and Timmermann (1992). The same critical value is used as before, that is, a positive value larger than 1.645 indicates that the model is significantly more accurate than taking random actions. Over the complete sample, the realized volatilities for each model, including the FA model, contain strong positive values. The values for the excess returns are, however, often not significant. Only 4 values (2 for Small Cap, 1 for Medium Cap and 1 for Gold) are larger than 1.645. The FASTR.EL model performs best, with 7 significant values. Looking at the sub-sets, it can be seen that the good performance is mainly because of the predictions in the second period.

CPS	**Small Cap**		**Medium Cap**		**Big Cap**		**S&P 500**		**Gold**	
(a) Complete Sample	$r_{e,t+1}$	RV_{t+1}	$r_{e,t+1}$	RV_{t+1}	$r_{e,t+1}$	RV_{t+1}	$r_{e,t+1}$	RV_{t+1}	$r_{e,t+1}$	RV_{t+1}
FA	0.550	0.649	0.542	0.637	0.535	0.652	0.540	0.677	0.493	0.657
FASTR.NL	0.537	0.627	0.535	0.602	0.530	0.667	0.540	0.707	0.525	0.659
FASTR.NV	0.522	0.654	0.537	0.637	0.542	0.667	0.550	0.664	0.537	0.654
FASTR.NE	0.503	0.637	0.498	0.659	0.530	0.654	0.532	0.642	0.480	0.659
FASTR.LL	**0.552**	0.634	0.560	0.629	0.555	0.692	0.540	0.702	0.510	**0.716**
FASTR.LV	0.550	**0.679**	0.530	**0.682**	0.510	0.662	0.503	0.667	0.532	0.637
FASTR.LE	0.525	0.672	0.527	0.679	0.537	0.667	0.522	0.637	0.473	0.644
FASTR.EL	**0.552**	0.595	**0.582**	0.629	0.552	**0.699**	**0.567**	**0.716**	**0.545**	0.647
FASTR.EV	0.540	0.674	0.550	0.647	**0.557**	0.664	0.560	0.689	0.540	0.592
FASTR.EE	0.535	0.662	0.527	0.632	0.552	0.600	0.555	0.577	0.522	0.659
(b) 1978/06 – 1991/12										
FA	**0.601**	0.522	0.577	0.552	0.540	0.546	0.540	0.595	0.442	0.632
FASTR.NL	**0.601**	0.509	**0.607**	0.522	**0.583**	0.558	**0.571**	0.620	0.503	0.614
FASTR.NV	0.540	0.509	0.528	0.571	0.522	0.571	0.534	0.589	0.491	0.632
FASTR.NE	0.552	0.509	0.509	0.577	0.528	0.564	0.546	0.607	0.405	0.614
FASTR.LL	0.564	0.497	0.577	0.528	0.558	0.577	0.534	0.577	0.472	**0.706**
FASTR.LV	0.558	0.558	0.540	0.583	0.509	0.564	0.503	0.589	0.466	0.607
FASTR.LE	0.540	**0.564**	0.522	**0.607**	0.534	0.601	0.528	0.601	0.411	0.614
FASTR.EL	0.571	0.466	0.564	0.577	0.528	**0.638**	0.546	0.607	**0.534**	0.620
FASTR.EV	0.571	**0.564**	0.515	0.571	0.515	0.601	0.534	**0.663**	0.528	0.515
FASTR.EE	0.546	0.509	0.534	0.540	0.558	0.522	0.564	0.571	0.472	0.540
(c) 1992/01 – 2004/12										
FA	0.494	0.673	0.474	0.635	0.494	0.686	0.526	0.705	0.494	0.667
FASTR.NL	0.506	**0.724**	0.474	0.641	0.462	0.731	0.494	0.782	0.506	0.635
FASTR.NV	0.519	0.712	0.519	0.712	0.545	0.712	0.583	0.705	0.564	0.583
FASTR.NE	0.449	0.692	0.481	0.673	0.532	0.660	0.519	0.590	0.500	0.635
FASTR.LL	0.526	0.692	0.539	0.647	0.532	**0.756**	0.539	**0.789**	0.506	**0.699**
FASTR.LV	0.532	0.718	0.506	**0.750**	0.462	0.705	0.500	0.718	**0.583**	0.571
FASTR.LE	0.494	0.699	0.506	0.731	0.494	0.660	0.481	0.609	0.462	0.635
FASTR.EL	**0.558**	0.647	**0.622**	0.628	0.577	0.724	**0.609**	0.776	0.532	0.654
FASTR.EV	0.526	0.699	0.596	0.712	**0.583**	0.622	0.590	0.628	0.526	0.487
FASTR.EE	0.513	0.718	0.506	0.718	0.526	0.628	0.539	0.590	0.539	0.583
(d) 2005/01 – 2011/11										
FA	0.554	0.711	0.602	0.771	0.602	0.747	0.566	0.783	0.590	0.663
FASTR.NL	0.470	0.663	0.506	0.723	0.554	0.687	0.566	0.723	0.602	**0.711**
FASTR.NV	0.494	0.675	**0.590**	0.687	0.578	0.723	0.518	0.747	0.578	0.627
FASTR.NE	0.506	0.699	0.506	0.783	0.530	**0.807**	0.530	0.807	0.590	0.663
FASTR.LL	**0.578**	**0.723**	0.566	**0.807**	0.590	0.771	0.554	**0.819**	0.590	0.663
FASTR.LV	0.566	0.699	0.554	0.723	0.602	0.735	0.506	0.771	0.566	0.615
FASTR.LE	0.554	0.675	0.578	0.735	**0.627**	0.747	**0.590**	0.735	**0.615**	0.602
FASTR.EL	0.506	0.711	0.542	0.675	0.554	0.759	0.530	0.807	0.590	0.663
FASTR.EV	0.506	0.711	0.530	0.602	0.590	0.783	0.554	0.783	0.590	0.590
FASTR.EE	0.554	0.711	0.554	0.627	0.590	0.602	0.566	0.590	0.590	0.675

Table 4.3. The Correctly Predicted Signs of the predicted excess returns and realized volatility, computed as in Equation (3.10). The last two letters of the FASTR models contain the influence of the financial variables and the usage of the STR variable respectively. Computed over the complete sample (panel a), as well as for three sub-periods (panels b to d).

The large changes during the sub-samples for both the RMSE and CPS reflect a negative result on the stability. It is preferred that the models work generally equally throughout the complete time interval. Figure 4.4 contains the RMSE and CPS for the FASTR.EL model, over a moving window of 60 months (five years). Panel (a) and (b) contain the RMSE of the excess returns and realized volatility respectively, and panel (c) and (d) contain the CPS for the same series. In the graphs there are large changes over time. The predictions of the models therefore show to be very unstable over the complete sample. The RMSE of the realized volatility seems to be most stable, except for the large increase at the end. Overall, the predictions show to be better than the FA model, but the end raises the value considerably, concluding the FA model still predicts better, or at least more stable.

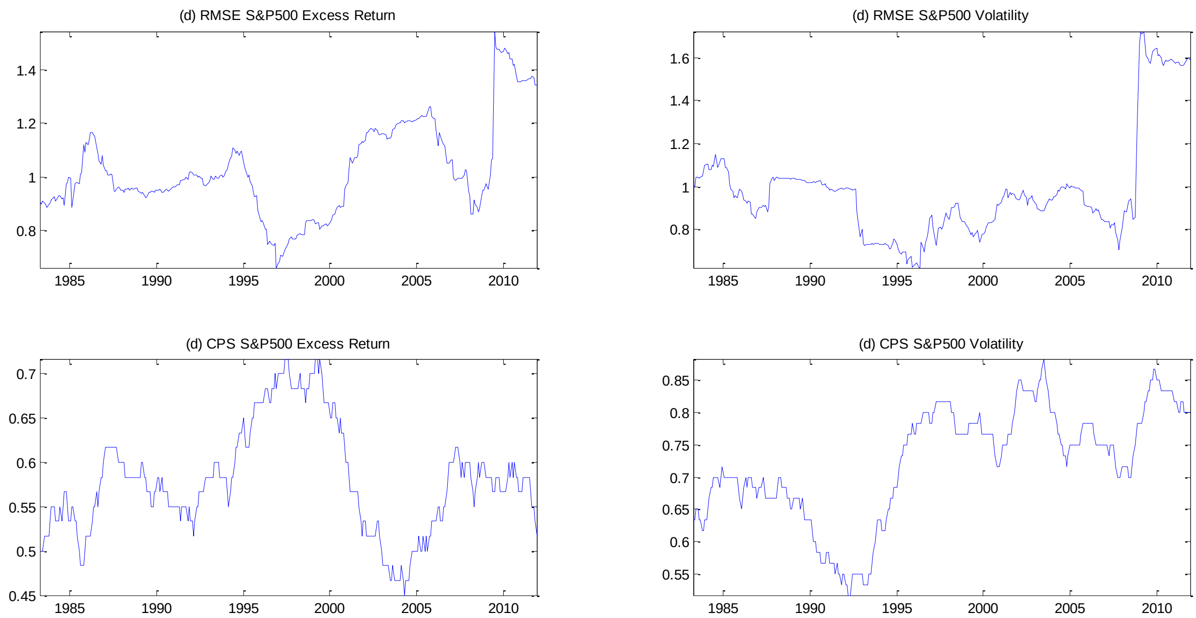

Figure 4.4. RMSE and CPS over a moving window of 60 months, for the excess returns (panel a and c) and realized volatility (panel b and d) predicted by FASTR.EL.

The last statistical performance test was the Excess Predictability test of Anatolyev & Gerko (2005). The null hypothesis states that the model does not perform better than an investor taking random actions. A positive value larger than 1.645 indicates that the null hypothesis is rejected and the model does perform better. A negative value smaller than -1.645 also rejects the null hypothesis, but concludes that random actions are performing better than the model.

EP (a) Complete Sample	**Small Cap**	**Medium Cap**	**Big Cap**	**S&P 500**	**Gold**
FA	**5.482**	1.965	4.339	5.301	-0.646
FASTR.NL	4.837	3.924	0.599	2.956	5.972
FASTR.NV	1.492	5.577	**4.956**	3.756	6.164
FASTR.NE	0.051	-3.387	-3.182	0.283	-3.763
FASTR.LL	5.166	2.474	4.847	2.777	1.949
FASTR.LV	5.462	-0.369	-2.540	-6.250	1.745
FASTR.LE	5.086	**5.592**	4.572	3.654	-0.412
FASTR.EL	2.944	3.574	0.570	**5.724**	5.317
FASTR.EV	-3.363	0.457	-0.677	2.459	**7.634**
FASTR.EE	1.679	0.099	0.554	5.636	5.866
(b) 1978/06 – 1991/12					
FA	6.322	4.796	3.018	4.030	-1.101
FASTR.NL	**6.706**	**5.706**	3.278	3.507	**2.569**
FASTR.NV	4.203	2.729	2.843	2.874	1.312
FASTR.NE	4.472	1.090	3.096	2.693	-3.176
FASTR.LL	4.291	3.685	**3.403**	1.923	-0.153
FASTR.LV	5.085	1.714	-1.237	-2.060	-0.596
FASTR.LE	4.521	2.232	2.649	3.778	-1.362
FASTR.EL	2.164	2.982	-1.035	1.804	2.236
FASTR.EV	1.099	0.202	-1.394	-0.053	1.871
FASTR.EE	2.007	-0.219	0.110	**4.668**	1.971
(c) 1992/01 – 2004/12					
FA	-1.812	-4.411	-0.037	0.369	-0.576
FASTR.NL	0.494	-0.718	-1.442	-1.272	0.331
FASTR.NV	-2.172	0.207	-0.806	-0.023	1.422
FASTR.NE	-3.616	-1.074	-2.589	-0.677	-2.302
FASTR.LL	-0.038	0.096	**0.653**	-0.414	3.117
FASTR.LV	**1.152**	-1.789	-3.175	-3.758	2.407
FASTR.LE	-0.882	**1.721**	-1.820	-4.728	-4.263
FASTR.EL	1.146	1.532	-1.220	**3.590**	0.246
FASTR.EV	-3.736	-0.201	-2.591	0.320	**4.484**
FASTR.EE	-1.395	0.163	-0.022	-1.503	0.458
(d) 2005/01 – 2011/11					
FA	1.568	1.876	1.516	1.423	-2.102
FASTR.NL	-1.227	-0.621	-1.232	2.171	0.583
FASTR.NV	-0.370	2.437	3.095	0.906	1.916
FASTR.NE	-0.534	-4.212	-5.697	-2.134	-0.878
FASTR.LL	1.558	-1.210	1.660	3.162	-1.546
FASTR.LV	0.081	-0.067	1.941	-0.347	0.231
FASTR.LE	**2.063**	**2.475**	**5.801**	**6.778**	**2.684**
FASTR.EL	-0.040	-0.645	3.098	1.167	-0.525
FASTR.EV	-1.228	-0.072	1.620	1.757	1.415
FASTR.EE	1.275	0.632	0.970	2.165	-0.131

Table 4.5. Excess Predictability test of Anatolyev & Gerko (2005). The significance level is set to 5%, which equals a critical value of 1.96. An EP larger than this critical value indicates a model that performs better than taking random actions. The null hypothesis equals that the model does not significantly outperform the benchmark. The highest values are bold, while significant negative values are given in red. Computed over the complete sample (panel a), as well as for three sub-periods (panels b to d).

Table 4.5 shows that for most models, the random actions are outperformed by the forecasts of the model. In six cases, the random actions seem to be more profitable. The FASTR.NE model shows to have most trouble beating the random actions, as 3 out of 5 results are significantly negative. The highest values are spread over the models, where the FA model contains the highest value for the Small Cap portfolio. Furthermore, for the Big Cap and S&P500 Index, the FA model is still close to the FASTR models. The FASTR.LL model shows all significant values and therefore seems to perform best.

The sub-samples show different results. Where the first interval only contains two significantly negative values, the second sample contains 13. The last sample has 4 results in which the random actions significantly outperform the models, out of which 3 are again for the FASTR.NE model. The FASTR.LE model works best during the credit crunch, having all of the five best values in that sample. Thanks to these results, the same conclusions for the stability can be obtained. The results include large changes between the intervals, and therefore the models are inaccurate in the overall performance.

Concluding this section, it can be stated that the models work reasonably well overall, based on the CPS and DA tests. However, the benchmark of the Factor Augmentation cannot be beaten easily, as shown by the RMSE, DM and EP tests. Stability is lacking for most of the models, indicated by the three sub-samples and Figure 4.4.

Another conclusion that can be drawn from the tests above, is that models can give reasonable predictions for one series, but end up performing badly for other series. For example, the FASTR.EL model contains the highest CPS and EP over the sample for the S&P500, for both excess returns and realized volatility, but the predictions for the Small Cap portfolio almost do not exceed the benchmark. This shows there is certain danger in predicting only one time series and drawing conclusions on the performance of that specific series.

4.2. Economic performance

The economic performance of the FASTR models is compared against the benchmark model of the factor augmentation, as well as three buy-and-hold strategies. The latter invest in respectively the risk-free rate, the Small Cap portfolio and the Big Cap portfolio at the start of the out-of-sample period, and do not change the investment. The S&P500

is left out of the performance check. The main reason is the high correlation between the Big Cap portfolio and the S&P500, shown in Section 2.1. The assets in both options are also in general of the same type. Therefore, a switch in the weights between these two options does not necessarily lead to a lower utility, and may therefore give inaccurate estimates of the weights.

The weights are computed for three different risk aversions, following the choices of Brandt (2010). The aversion of 5 is used throughout this section. The results of the risk aversions 2 and 10 are found in Appendix A. A lower risk aversion indicates a higher risk that is taken to obtain a higher return. Furthermore, the weights are computed with two separate limitations. The first form does not take short-selling into account. That is, the weights for every asset option should be in the interval [0, 1]. The second option does allow for short-selling, which expands the interval of the possible weights to [-1, 2]. The tables in this section also contain a subdivision for the predicted regimes, determined by taking the optimized parameters of the Big Cap predictions. That is, for each model, the estimates of γ and c are used to determine the value of the logistic function, and with that the chance the model gives of being in a good or bad regime in the next period. This is somewhat different for the exogenous variables. For the lagged dependent variables, a weight above 0.6 defines a 'good regime', below 0.4 is called a 'bad regime' and in between is considered neutral. For the log Implied Volatility Index and Default Spread it is the other way around, as a high value means a bad state.

An extra assumption is made regarding large shocks in the predictions. The mean and standard deviation are computed over a window equal to 10 years. Whenever at least one of the standardized predictions (either in excess returns or realized volatility) in a certain month exceeds 6, the investor invests everything in the risk-free rate. This may happen when a flaw occurs in predicting the time series, or during a shock.

Table 4.6 contains the average of the weights with a risk aversion equal to 5, for the restrictions that the weights should be in the interval of [0, 1] on the left and [-1, 2] on the right. For the complete sample, the average weights seem similar between the models for the first limitation. The right-hand side of the table does show differences between the models, with the most important change the negative weight of the FASTR.LL in the Small Cap portfolio. All other models give a considerable weight to this option, even up to 0.64 for the FASTR.EV model.

Weights $\gamma = 5$	No Short-Selling					Short-Selling Allowed				
	R_f	Small	Medium	Big	Gold	R_f	Small	Medium	Big	Gold
FA	0.21	0.21	0.14	0.12	0.32	0.13	0.22	0.18	-0.00	0.47
Complete Sample										
FASTR.NL	0.22	0.20	0.15	0.18	0.26	0.22	0.05	0.14	0.32	0.28
FASTR.NV	0.20	0.24	0.16	0.15	0.24	0.25	0.28	0.18	0.14	0.15
FASTR.NE	0.14	0.29	0.14	0.17	0.27	0.02	0.36	0.13	0.26	0.22
FASTR.LL	0.20	0.18	0.18	0.16	0.28	0.15	-0.02	0.16	0.31	0.40
FASTR.LV	0.26	0.23	0.12	0.13	0.27	0.33	0.15	0.10	0.10	0.32
FASTR.LE	0.19	0.28	0.12	0.18	0.23	0.14	0.28	0.29	0.13	0.17
FASTR.EL	0.18	0.29	0.19	0.16	0.18	0.24	0.49	0.33	-0.00	-0.06
FASTR.EV	0.16	0.28	0.10	0.22	0.23	0.21	0.64	0.09	0.02	0.04
FASTR.EE	0.17	0.25	0.09	0.22	0.27	0.10	0.29	0.07	0.36	0.18
Good Regime										
FASTR.NL	0.21	0.19	0.17	0.17	0.26	0.20	-0.02	0.19	0.37	0.26
FASTR.NV	0.21	0.21	0.17	0.15	0.26	0.23	0.14	0.25	0.16	0.23
FASTR.NE	0.09	0.30	0.13	0.21	0.26	-0.09	0.38	0.09	0.39	0.23
FASTR.LL	0.22	0.20	0.19	0.15	0.23	0.22	0.03	0.16	0.27	0.31
FASTR.LV	0.21	0.24	0.11	0.15	0.28	0.16	0.16	0.16	0.13	0.39
FASTR.LE	0.22	0.23	0.10	0.19	0.26	0.27	0.09	0.14	0.17	0.34
FASTR.EL	0.20	0.21	0.23	0.18	0.18	0.32	0.21	0.45	0.09	-0.07
FASTR.EV	0.13	0.28	0.09	0.23	0.27	0.09	0.66	0.10	-0.02	0.16
FASTR.EE	0.26	0.07	0.04	0.27	0.36	0.41	-0.34	-0.21	0.80	0.34
Bad Regime										
FASTR.NL	0.25	0.20	0.09	0.22	0.25	0.35	0.30	-0.06	0.15	0.26
FASTR.NV	0.24	0.25	0.16	0.15	0.21	0.44	0.36	0.02	0.22	-0.04
FASTR.NE	0.18	0.30	0.14	0.10	0.27	0.08	0.43	0.23	0.03	0.24
FASTR.LL	0.15	0.14	0.17	0.19	0.35	0.04	-0.12	0.18	0.32	0.58
FASTR.LV	0.30	0.19	0.13	0.13	0.25	0.40	0.10	0.04	0.28	0.17
FASTR.LE	0.17	0.36	0.17	0.09	0.21	-0.01	0.61	0.56	-0.21	0.05
FASTR.EL	0.14	0.42	0.15	0.14	0.16	0.09	0.87	0.20	-0.11	-0.06
FASTR.EV	0.26	0.29	0.08	0.18	0.19	0.52	0.60	0.01	0.00	-0.14
FASTR.EE	0.11	0.45	0.10	0.09	0.24	-0.26	1.04	0.27	-0.27	0.21
Neutral Regime										
FASTR.NL	0.13	0.20	0.18	0.23	0.26	-0.26	0.12	0.11	0.36	0.67
FASTR.NV	0.12	0.47	0.16	0.15	0.11	0.05	1.03	0.10	-0.14	-0.04
FASTR.NE	0.24	0.17	0.10	0.24	0.25	0.39	-0.11	-0.17	0.83	0.05
FASTR.LL	0.18	0.17	0.18	0.12	0.35	0.01	-0.06	0.14	0.51	0.40
FASTR.LV	0.36	0.23	0.11	0.07	0.23	0.78	0.19	-0.05	-0.24	0.32
FASTR.LE	0.16	0.19	0.08	0.37	0.21	0.16	-0.10	-0.02	0.93	0.03
FASTR.EL	0.24	0.25	0.12	0.15	0.24	0.38	0.55	0.18	-0.11	-0.01
FASTR.EV	0.09	0.19	0.24	0.37	0.11	0.05	0.57	0.31	0.34	-0.27
FASTR.EE	0.07	0.14	0.17	0.49	0.13	0.31	-0.17	0.25	1.02	-0.41

Table 4.6. Average of the weights, for two forms of determining the weights. In the first form the investor is not allowed for go short, in the second form the investor can go short, which enlarges the interval of possible weights to [-1, 2]. The S&P500 is left out, because of the high similarities between the Big Cap and S&P. The good, bad and neutral regimes are computed by taking the optimized parameters of the Big Cap excess returns in the logistic function. A good regime captures the best 40% of the values, a bad regime the least 40%, and the neutral regime the remaining 20% in the middle. The risk aversion coefficient is taken to be 5.

There are also large differences between the regimes itself. However, a note should be made regarding the number of periods for each regime. Table A.5 in Appendix A contains the number of occurrences in each regime, for each of the models. Here can be found that, for example, the FASTR.NL model only contains 13 periods in the neutral regime, which is low compared to the other models. This may indicate another lack of performance of the logistic function. Nevertheless, most models invest an amount in the risk-free rate on average, to hold some safety, along with the hedging options of gold.

Tables A.6 and A.7 in Appendix A contain the weights for risk aversions equal to 2 and 10 respectively. The differences compared to the risk aversion of 5 are mostly found in the weights of the risk-free rate, which diminish for the risk-loving investor and rise for the trader with aversion equal to 10. For the latter, this seems to be at the cost of the Medium Cap investment, which is now closer to zero. For the risk-loving investor, a larger percentage goes to the Small Cap portfolio, especially if short-selling is allowed.

Table 4.7 contains the average annual excess returns, computed according to the compounding returns over a year. The annualized volatility is computed by means of the square-root-of-time rule. The Sharpe Ratio is computed by dividing the annualized return by the annualized volatility. Over the complete sample, the highest annual percentage is reached by the FASTR.EL model, which earns 12.007% on average each year. The corresponding Sharpe Ratio equals 0.722, which is higher than the FA model, as well as the Buy-and-Hold strategies. Looking at the returns in the different regimes, this model shows to be able to maintain a high return throughout all different states, with the lowest return of 10.469% in excess of the risk-free rate. Other models that seem stable between the regimes are FASTR.NV, which shows a very large Sharpe Ratio during the neutral periods, and FASTR.EV.

Whenever short-selling is allowed, the volatility raises quickly and the values for the Sharpe Ratio decrease. The highest return obtained overall is 16.515%, by the FASTR.NL model, but it seems this is almost completely achieved in the good states. The FASTR.EL and FASTR.EV show the best performance between the different regimes.

Economic Performance $\gamma = 5$	**No Short-Selling Allowed**			**Short-Selling Allowed**		
	Average Annualized Excess Return	Annualized Volatility	Sharpe Ratio	Average Annualized Excess Return	Annualized Volatility	Sharpe Ratio
Buy-and-Hold R_f	0.000%	0.980%	0.000			
Buy-and-Hold Small	10.573%	21.077%	0.502			
Buy-and-Hold Big	8.644%	16.722%	0.517			
Complete Sample						
FA	6.726%	**12.948%**	0.520	9.815%	**29.939%**	0.328
FASTR.NL	7.550%	14.498%	0.521	**16.515%**	34.468%	**0.479**
FASTR.NV	10.421%	15.532%	0.717	15.324%	33.133%	0.463
FASTR.NE	3.706%	16.274%	0.228	1.847%	36.545%	0.051
FASTR.LL	7.733%	14.300%	0.541	11.833%	32.708%	0.362
FASTR.LV	6.736%	14.647%	0.460	9.242%	32.986%	0.280
FASTR.LE	6.302%	14.125%	0.446	9.532%	33.815%	0.282
FASTR.EL	**12.007%**	16.639%	**0.722**	14.041%	33.697%	0.417
FASTR.EV	9.997%	16.644%	0.601	15.789%	33.645%	0.469
FASTR.EE	8.649%	16.214%	0.533	14.366%	34.366%	0.418
Good Regimes						
FASTR.NL	9.160%	14.607%	0.627	**21.109%**	35.706%	**0.591**
FASTR.NV	9.656%	14.676%	0.658	18.209%	32.231%	0.565
FASTR.NE	-0.128%	15.877%	-0.008	-5.990%	33.794%	-0.177
FASTR.LL	6.004%	13.624%	0.441	7.697%	32.562%	0.236
FASTR.LV	5.269%	14.646%	0.360	14.307%	29.799%	0.480
FASTR.LE	2.743%	14.078%	0.195	-5.818%	30.593%	-0.190
FASTR.EL	**11.829%**	17.101%	**0.692**	9.256%	35.118%	0.264
FASTR.EV	8.526%	16.224%	0.526	15.958%	34.504%	0.463
FASTR.EE	5.144%	**13.523%**	0.380	-0.647%	**28.998%**	-0.022
Bad Regimes						
FASTR.NL	3.668%	**14.045%**	0.261	6.133%	**31.253%**	0.196
FASTR.NV	11.841%	16.377%	0.723	12.589%	39.259%	0.321
FASTR.NE	6.630%	17.099%	0.388	15.436%	38.044%	0.406
FASTR.LL	12.357%	15.780%	0.783	21.620%	32.464%	0.666
FASTR.LV	**15.599%**	16.511%	**0.945**	6.107%	39.824%	0.153
FASTR.LE	11.349%	14.221%	0.798	30.640%	38.030%	0.806
FASTR.EL	12.794%	16.650%	0.768	16.017%	33.609%	0.477
FASTR.EV	13.513%	18.630%	0.725	15.930%	34.295%	0.465
FASTR.EE	12.426%	18.034%	0.689	**34.151%**	39.102%	**0.873**
Neutral Regimes						
FASTR.NL	-1.169%	15.360%	-0.076	-8.595%	25.411%	-0.338
FASTR.NV	**12.678%**	**9.437%**	**1.343**	3.077%	26.399%	0.117
FASTR.NE	10.710%	13.583%	0.789	-18.961%	42.202%	-0.449
FASTR.LL	5.841%	14.346%	0.407	11.905%	34.954%	0.341
FASTR.LV	-0.201%	11.318%	-0.018	-1.656%	32.469%	-0.051
FASTR.LE	3.337%	13.889%	0.240	3.699%	28.727%	0.129
FASTR.EL	10.469%	14.787%	0.708	**30.135%**	27.423%	**1.099**
FASTR.EV	9.241%	12.528%	0.738	14.127%	**24.862%**	0.568
FASTR.EE	7.645%	17.390%	0.440	4.991%	31.799%	0.157

Table 4.7. Annualized excess returns, volatility and Sharpe Ratio for the average weights given in Table 4.6, with a risk aversion of 10. The excess returns are compounded, which means that earned profits are reinvested. In the first form the investor is not allowed for go short, in the second form the investor can go short, which enlarges the interval of possible weights to [-1, 2]. The S&P500 is left out, because of the high similarities between the Big Cap and S&P. The good, bad and neutral regimes are computed by taking the optimized parameters of the Big Cap excess returns in the logistic function. A good regime captures the best 40% of the values, a bad regime the least 40%, and the neutral regime the remaining 20% in the middle.

Table A.8 in Appendix A contains the returns for the different sub-samples discussed in the previous section. It is shown here that, for no short-selling, quite some models outperform the Buy-and-Hold strategies as well as the FA model in the first and last period, but that the performance in the second time interval is lacking. This also holds when the ability to go short is available. However, due to the high volatilities obtained, the Sharpe Ratios are often far beneath the benchmarks. The FASTR.LE and FASTR.EV seem to be most stable throughout the complete sample.

Table 4.8 contains the test results of the bootstrap proposed by Ledoit & Wolf (2008). The number of bootstraps is chosen to be 5,000, following recommendations of Ledoit & Wolf. The table shows the p-values, indicating the rejection of the null hypothesis that the difference between the Sharpe Ratios of the tested model and benchmark is not significantly different from 0. A rejection of the null hypothesis can therefore indicate that the model is significantly better or worse than the benchmark. The benchmarks tested are the Buy-and-Hold strategies of the Small and Big Caps, as well as the FA model. With no short-selling, and based on a significance level of 5%, there is one significant p-value, but it belongs to the FASTR.NE model and is significantly worse than the FA benchmark. On a 10% significance level, the FASTR.EL model seems to outperform the Buy-and-Hold Small Cap strategy. When short-selling is allowed, this strategy is also insignificant.

	No Short-Selling			**Short-Selling Allowed**		
	Buy-and-Hold Small	Buy-and-Hold Big	FA	Buy-and-Hold Small	Buy-and-Hold Big	FA
FA	0.341	0.632	-			-
FASTR.NL	0.460	0.769	0.736	0.597	0.413	0.595
FASTR.NV	0.069	0.164	0.410	0.612	0.416	0.604
FASTR.NE	0.316	0.155	0.039	0.030	0.015	0.136
FASTR.LL	0.352	0.670	0.830	0.392	0.237	0.929
FASTR.LV	0.623	0.976	0.417	0.216	0.099	0.709
FASTR.LE	0.658	0.988	0.326	0.230	0.139	0.627
FASTR.EL	0.059	0.195	0.577	0.341	0.204	0.781
FASTR.EV	0.267	0.593	0.887	0.469	0.232	0.657
FASTR.EE	0.508	0.883	0.634	0.376	0.241	0.812

Table 4.8. P-values of the Bootstrap on the Sharpe Ratio, proposed by Ledoit & Wolf (2008). The null hypothesis states that the difference between the Sharpe Ratios does not differ significantly from zero. The risk aversion used to compute the weights is equal to 5. Positive significant values are bold (based on significance level of 5%), negative significant values are shown in red.

The annual excess returns, volatilities and Sharpe Ratios of the risk aversions 2 and 10 can be found in Tables A.9 and A.10 in Appendix A respectively. For the aversion of 2, the FASTR.EL shows the highest returns, with 25.225% in excess of the risk-free rate when short-selling is allowed. For the aversion of 10, the FASTR.NV seems to perform best, with the two highest Sharpe Ratios over the complete sample. Furthermore, the model seems to perform well for every regime, with the highest return and Sharpe Ratio in the neutral regime. The outcomes of the bootstrap for the risk aversions of 2 and 10 can be found in Tables A.11 and A.12 respectively. However, the same conclusions can be drawn as for the risk aversions of 5. The FASTR.EL model for the aversion of 2 and FASTR.NV model for aversion 10 beat the Small Cap portfolio, but all other values are insignificant or, in the case of the FASTR.NE model, significantly worse than the benchmarks. The option to go short shows to be more risky based on the test.

From this section, it can again be concluded that there are models that perform well, with higher returns and Sharpe Ratios compared to the Buy-and-Hold strategies as well as the FA model. However, the bootstrap method of Ledoit & Wolf (2008) indicates that these values are mostly insignificant, except for one of the outcomes of the FASTR.EL model with a significance level of 10% and, for aversions other than 5, the FASTR.NV and FASTR.EL model with a significance level of 5%.

4.3. Dependence of regimes on the factors

The FASTR models combine the power of nonlinearity to the factor augmentation. However, one question that arises is whether the two regimes determined by the logistic function have different influence on the factor augmentation. That is, does the expectation of the logistic function affect the number of factors taken into account, or does the explained variance in the principal component analysis increase/decrease while the number of factors remains the same?

For this matter, the S&P500 Index is again taken into account. Figure 4.9 contains the explained variance (panel a and b) and the number of factors (panel c and d) adapted in the model over time, for the predictions of the FASTR.NL model and the FA model respectively. The red dot in the lower figures corresponds to the use of lagged factors in the model. This only happens a few times for both models, 9 times for the

FASTR.NL model and 11 times for the FA model. The FASTR.LE model uses lags most often, in total 18 times. Furthermore, the model mostly considers one factor throughout the sample. The factor explains roughly 45% of the variance at the start of the out-of-sample, but decreases to just above 30% at the end of the sample. Around the end, the number of factors reaches its maximum of four, and the total explained variance rises. Overall it can be concluded that the BIC criteria shows to be skeptical in the use of more than one factor for the prediction of the returns, there are no large changes between the FASTR models and FA benchmark.

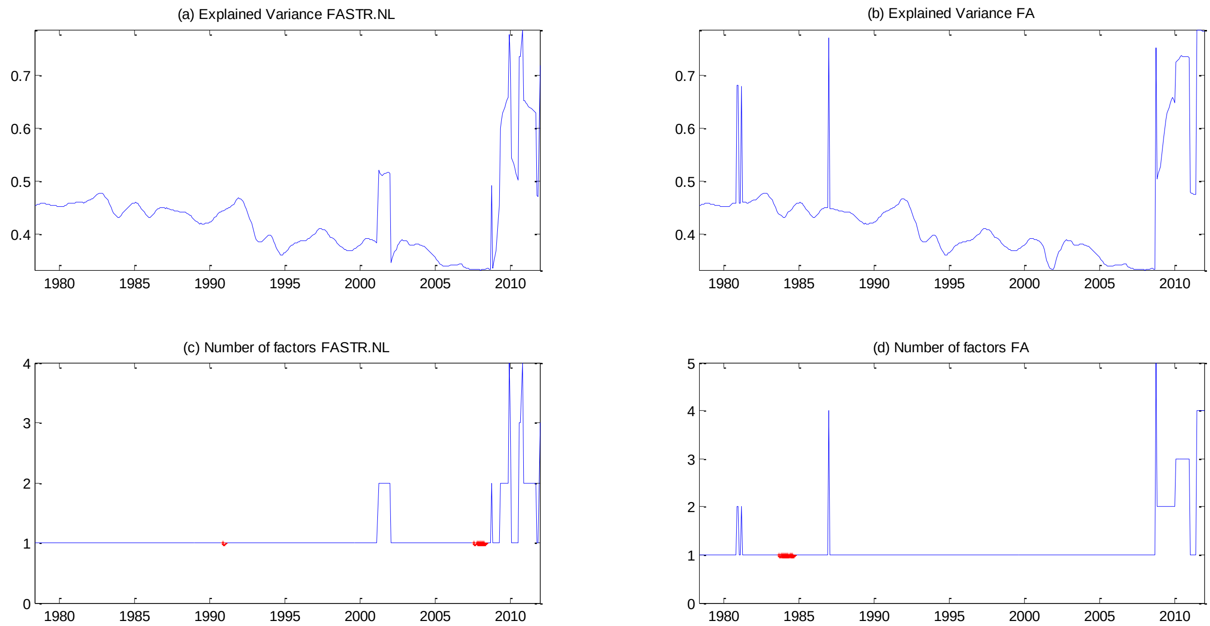

Figure 4.9. Explained variance (a) and number of factors (b) for predictions of the S&P500 excess returns, according to the FASTR.NL model. The red dot in panel (b) indicates the use of lagged factors.

Second, Figure 4.10 contains the explained variance for both the excess returns and the realized volatility of the S&P500 Index, for FASTR.NV; FASTR.LE; and FASTR.EL models. The observations are split in the three regimes, explained in the previous section. However, in this case the optimized parameters of the excess returns and realized volatility itself are used to get the value of the logistic function, rather than the Big Cap portfolio parameters used previously. The blue dots in the graphs indicate a good regime, the red dots represent bad regimes, and black is a neutral regime. It immediately shows that the number of neutral regimes is very limited, and centered at one place over the complete sample. This gives another reason to believe that optimizing the parameters of the logistic function is not efficient in combination with the factor models.

The graphs also do not show clear differences between the good and bad regimes, in the sense that good regimes capture more of the explained variance or vice versa. Hence, to answer the first sub-question for the standard model, *'Do different regimes in the model affect the total amount of variance explained in the factor augmentation?'*, the answer is 'no'.

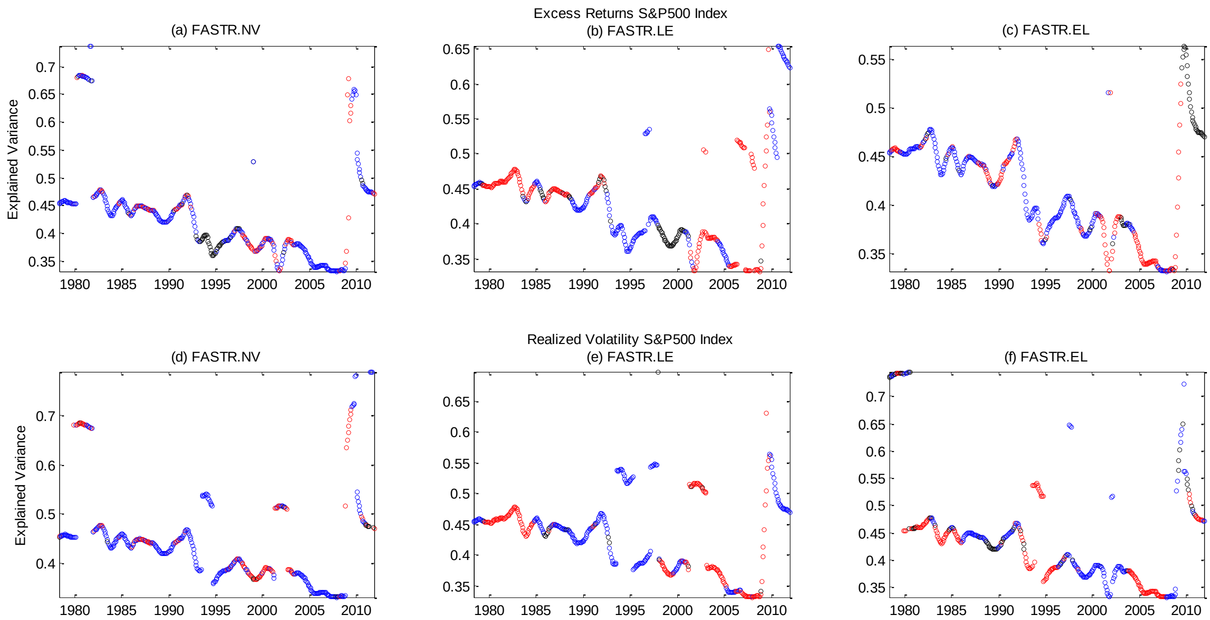

Figure 4.10. Explained variance for three models, for the S&P500 excess returns and realized volatility. The blue dots present a good state, while red dots indicate a bad state.

5. Extensions

This section covers some extensions to the basic method of the FASTR model, explained in Chapter 3. The extensions all try to cover a solution to the flaws in the methods. Section 5.1 starts off with the previously mentioned 'hard-thresholding' and 'soft-thresholding'. The methods are discussed, along with a short literature review, and the implications of both methods are provided in this section. The second expansion considers the logistic function, and in detail the parameters of this function. Section 5.2 considers whether the models perform better whenever these are fixed, or if a threshold model suffices for the factor augmentation to give effective predictions. The last expansion, mentioned in Section 5.3, discusses the possibility of extensive factor selection. In the standard model, the assumption is made that if factor j is significant, factors 1 to $j - 1$ are also taken in the model. What happens if this assumption is dropped? Section 5.4 contains the results, as well as the statistical and economic performances of the models with the extensions. As an added feature, the expansion of factor selection is looked upon, to see if a different conclusion can be attained for the sub-question.

5.1. Hard- and Soft-Thresholding

Thresholding methods are able to select the predictors that include significant information on the dependent variable, rather than using a whole set of predictors. This way, uninformative predictors are left out of the principal component analysis and the factors included in the model may be more useful in the prediction of the dependent variable. There are two categories of thresholding, namely hard-thresholding and soft-thresholding techniques.

The hard-thresholding method simply looks at the significance of one variable when regressed on the dependent variable, in this case the excess returns or realized volatility. For each of the predictors, the variable is regressed on the dependent variable, including a constant, that is

$$y_{t+1} = \beta_0 + \beta_1 z_{i,t} + \varepsilon_{t+1}, \quad for\ i = 1, \dots, 9 \tag{5.1}$$

In Equation (5.1), $z_{i,t}$ stands for one of the predictors. The variable is selected whenever the p-value indicating the significance on the dependent variable is lower than the threshold level α. However, the method lacks a certain amount of performance. As only one predictor is tested at a time, the common information in the predictors is overlooked.

The financial variables were tested by means of backwards elimination at the start of the sample in the standard model. The difference between hard-thresholding and backwards elimination is that the latter starts with the complete sample and stepwise omits the variable that is least significant, given the other predictors in the model. Therefore, it can be considered a special case of the hard-thresholding technique. The backwards elimination was assumed to select the same variables throughout the out-of-sample period. In the expansion, this technique is executed for each observation, to select only the financial variables that are significant for the current window. The significance level is set to 0.05. At all times, at least one financial variable should be included to maintain the differences in the FASTR models.

The macroeconomic variables are subjected to soft-thresholding. There are some common techniques in this category, such as the ridge estimator and the LASSO method. The LASSO, short for '*Least Absolute Shrinkage and Selection Operator*', is proposed by Tibshirani (1996) and is able to select the most valuable variables, omitting the other predictors, to decrease the number of variables in the model. It penalizes the model based on the L_1 norm, which corresponds to the absolute value of the parameters, other than the L_2 norm of the Ridge estimator, which penalizes the squared parameters. This has shown to be a reasonable competitor to the ridge estimator, however it has its limitations, as pointed out by Zou & Hastie (2005). First, when the number of predictor variables m is larger than the number of observations t in the sample, the LASSO method can only include t predictors in total. Second, in the event of 'grouping' in the predictor variables, the LASSO method only includes one of the variables in the model and neglects the others. For these shortcomings, Zou & Hastie (2005) develop the elastic net, which is a combination of both the ridge estimator and the LASSO method, that is, the elastic net solves

$$\begin{aligned} \widehat{\boldsymbol{\beta}} &= argmin_{\beta}\ [L(\lambda_1, \lambda_2, \boldsymbol{\beta})] \\ L(\lambda_1, \lambda_2, \boldsymbol{\beta}) &= \|\boldsymbol{y} - X\boldsymbol{\beta}\|^2 + \lambda_2 \|\boldsymbol{\beta}\|^2 + \lambda_1 |\boldsymbol{\beta}|_1 \end{aligned} \tag{5.2}$$

In Equation (5.2), $|\boldsymbol{\beta}|_1 = \sum_{i=1}^{n} |\beta_i|$ and $\|\boldsymbol{\beta}\|^2 = \sum_{i=1}^{n} \beta_i^2$. The elastic net clearly takes a fraction of both penalties mentioned earlier. Furthermore, Zou & Hastie (2005) show that this problem can be solved in the same matter as the LASSO method, but can give more accurate estimates by an extra penalty. However, the computation of the betas, in order to check which predictors should be included, is computationally very intensive.

Efron, Hastie, Johnstone & Tibshirani (2004) propose a version that not only shows which predictors are most correlated with the dependent series, but takes about the same time to compute as an ordinary least squares. The method is commonly known as '*Least Angle Regressions*', or LARS, and shows similarities to the LASSO method. It gradually adopts the predictors one by one, with the selection based on the correlation with the dependent variable. This is done by evaluating the correlations with the errors step by step. The predictor that shows the highest correlation, is added to the active set and omitted from the remaining set. The parameter of that predictor is increased, until another predictor in the remaining set shows an equal correlation to the errors. This variable is then selected in the active set. By increasing the parameter, the correlation with the error drops, and therefore the correlation with the dependent variable rises.

Before the algorithm starts, it is necessary to standardize the macroeconomic predictor variables and to center – that is, subtract the mean of the series – the dependent variable. Furthermore, it should be assumed that the predictor variables are linearly independent. The algorithm begins by computing the correlations between the predictor variables and the dependent variables. That is, compute the correlations in step *i* as in Equation (5.3).

$$\hat{\boldsymbol{c}}_{(i)} = X'(\boldsymbol{y} - \hat{\boldsymbol{\mu}}_{(i-1)}) \tag{5.3}$$

Where X is the *n x p* matrix of remaining predictor variables; $\boldsymbol{y}$ is the *n x 1* vector of the dependent variable; and $\hat{\boldsymbol{\mu}}_{(i-1)}$ stands for the estimate $X\boldsymbol{\beta}$ at step *i – 1*. $\hat{\boldsymbol{\mu}}_{(0)}$ is taken to be $\mathbf{0}$. Altogether, the equation checks which predictor contains most influence on the errors. Define $\hat{C}_{(i)} \equiv \max_j(|\hat{\boldsymbol{c}}_{(i)}|)$ and $\mathcal{R}_{(i)} \equiv \{j : |\hat{\boldsymbol{c}}_{(i)}| = \hat{C}_{(i)}\}$, where $\mathcal{R}_{(i)}$ can be defined as the indices of predictors that are in the active set. Add the j^{th} predictor to the active set of factors $\mathcal{X}$, and make sure the correlation is positive. This can be done as in Equation (5.4).

$$\mathcal{X}_{(i)} = [\mathcal{X}_{(i-1)} \;\; s_j\boldsymbol{X}_j], \qquad s_j = sign(\hat{\boldsymbol{c}}_{(i),j}) \tag{5.4}$$

Afterwards, compute $G_{(i)} = \mathcal{X}'_{(i)}\mathcal{X}_{(i)}$ and $A_{(i)} = \left(\boldsymbol{\iota}' G_{(i)}^{-1}\boldsymbol{\iota}\right)^{-\frac{1}{2}}$, where $\boldsymbol{\iota}$ represents a vector of ones, with the size equal to the number of columns in the active vector. Then define

$$\boldsymbol{u}_k = \mathcal{X}_{(i)}\boldsymbol{w}_{(i)}, \qquad \boldsymbol{w}_{(i)} = A_{(i)}G_{(i)}^{-1}\boldsymbol{\iota}, \qquad \boldsymbol{a}_{(i)} = X'\boldsymbol{u}_{(i)} \tag{5.5}$$

The estimate $\widehat{\boldsymbol{\mu}}_{(i)}$ can be updated by means of $\widehat{\boldsymbol{\mu}}_{(i)} = \widehat{\boldsymbol{\mu}}_{(i-1)} + \hat{\gamma}\boldsymbol{u}_{(i)}$, where

$$\hat{\gamma} = \min_{j \in \mathcal{R}_{(i)}^{c}}{}^{+}\left\{\frac{\hat{C}_{(i)} - \hat{c}_{(i),j}}{A_{(i)} - a_{(i),j}}, \frac{\hat{C}_{(i)} + \hat{c}_{(i),j}}{A_{(i)} + a_{(i),j}}\right\} \tag{5.6}$$

The plus sign indicates that the minimum is only taken over the values that are strictly positive. The interval over which j can search for the minimum value, given by $j \in \mathcal{R}_{(i)}^{c}$, contains the indices that are still in the remaining set, hence the complement of $\mathcal{R}_{(i)}$. The given value for j is taken into the active set in the next iteration.[8]

Each iteration adds a predictor to the active set. In order to decide the best amount of predictors taken into account for estimation, Bai & Ng (2008) propose the use of the BIC criterion. That is, in each iteration, compute the BIC, based on the log-likelihood. Select the amount of predictors for which the BIC criterion gives the smallest value. However, as indicated by Çakmakli & van Dijk (2010), following the BIC often only includes a few predictors into the model. Therefore they implemented an early stopping that always includes the p predictors that are most correlated. In this research, p is set to 50.

5.2. The logistic function

In the previous chapter, there were signs that optimizing the parameters of the logistic function leads to inaccurate predictions of the dependent variables and implausible divisions of the regimes (see Table A.5). This gives reason to change the settings of the logistic function. A possible solution to this problem is to set a fixed value to γ, and to fix

[8] This last step can be explained by the fact that the column in X with index j now also contains the maximum correlation, along with the factors that already were in the active set.

c to the median of the window sample of the exogenous variable. In this case, the smooth transition of the logistic function stays intact, and the exogenous input maintains the ability to assign weights. Another advantage of fixing γ and c is the decrease in parameters, which may lead to less overestimation. Another solution is to transform the logistic function into a threshold function. If the weights tend to reach the extremes in most cases, a threshold model may suffice in combination with the factor augmentation.

To decide between the two options, the outcomes of the logistic function with the optimized parameters, found in the previous chapter, is used. The weights are divided in three parts: the extremes, which cover the weights lower than 5% or higher than 95%; the middle, which captures the weights between 45% and 55%; and the rest, which covers the weights outside the ranges of the first two. Table 5.1 contains the number of occurrences for each FASTR model and each time series.

		Small Cap		**Medium Cap**		**Big Cap**		**S&P 500**		**Gold**	
		$r_{e,t+1}$	RV_{t+1}	$r_{e,t+1}$	RV_{t+1}	$r_{e,t+1}$	RV_{t+1}	$r_{e,t+1}$	RV_{t+1}	$r_{e,t+1}$	RV_{t+1}
NL	Extreme	289	277	296	257	318	275	230	211	263	274
	Mid	8	9	6	17	8	17	72	7	60	21
	Rest	105	116	100	128	76	110	100	184	79	107
NV	Extreme	315	269	287	313	270	312	270	304	299	276
	Mid	7	7	6	8	37	8	40	11	10	9
	Rest	80	126	109	81	95	82	92	87	93	117
NE	Extreme	268	280	269	267	296	296	300	292	343	301
	Mid	30	17	50	27	29	16	23	18	6	11
	Rest	104	105	83	108	77	90	79	92	53	90
LL	Extreme	252	308	232	248	203	293	276	226	261	263
	Mid	7	9	32	18	27	10	28	18	60	13
	Rest	143	85	138	136	172	99	98	158	81	126
LV	Extreme	303	206	276	231	270	252	257	199	194	244
	Mid	6	84	52	61	62	38	82	88	123	23
	Rest	93	112	74	110	70	112	63	115	85	135
LE	Extreme	354	304	334	361	301	340	297	308	316	308
	Mid	4	6	4	4	32	7	36	13	9	4
	Rest	44	92	64	37	69	55	69	81	77	90
EL	Extreme	262	278	219	179	184	156	286	150	299	229
	Mid	6	8	54	29	36	17	38	33	25	10
	Rest	134	116	129	194	182	229	78	219	78	163
EV	Extreme	295	234	302	221	284	174	289	186	335	223
	Mid	2	16	6	22	30	59	36	84	7	18
	Rest	105	152	94	159	88	169	77	132	60	161
EE	Extreme	290	327	301	306	292	320	292	336	252	268
	Mid	43	13	46	13	53	10	49	11	32	39
	Rest	69	62	55	83	57	72	61	55	118	95

Table 5.1. Occurrences of the weights of the logistic function. Extreme indicates the number of occurrences that are less than 5% from the extremes 0 and 1; Mid represents the occurrences in the vicinity of 0.5 (between 0.45 and 0.55) and Rest defines the number of occurrences in between. The total number equals 402, the amount of predictions.

The extremes overall contain most observations, but a large part of the observations is spread over the other two columns as well. The Mid range does, in general, not capture a lot of observations. Based on these results, and considering that the smooth transition model can also reach the extreme values, the smooth transition model is chosen over the threshold model. Following the examples of Van Dijk, Teräsvirta & Franses (2000), the sensitivity γ is set to 2.5, and the threshold parameter c to the median of the time series.

5.3. Factor selection

The last expansion concerns the selection of the factors in the optimal model. In the basic model, when factor j still contains a significant influence, given by the BIC, all factors up to this factor are included in the model. However, Bai (2010) considers the factors separately and finds that the second and fifth factor contain most valuable information most of the time. Thanks to the fixing of the parameters discussed just now, predicting the time series takes a considerable less amount of time, which makes it possible to consider the factors separately.

The procedure is as follows: at first, each of the 12 factors – that is, for predicting the time series at time $t+1$, use six factors of periods $t-1$ and t each – is added separately to the model that is estimated. By means of the log-likelihood and the BIC criteria, as was the case in the basic model, the best model out of these twelve is computed. The factor in the best model is added to the active set. The algorithm starts over, but adds the active set, plus one of the remaining factors to the model that should be estimated. These steps are repeated until the minimum of the BIC criteria does not exceed the BIC of the last step, or the number of remaining factors equals zero.

5.4. Results of the extensions

The results are split up in the statistical and economic performance. The assumptions and characteristics, such as the size of the moving window and the number of maximum factors included, are maintained from Chapter 4. In advance, the model without expansions covered in the previous chapter is referred to as the standard model. Section 5.4.1 contains the statistical tests. In these tests, the benchmark is the factor augmented model of Chapter 4. This way, consequences of the expansions on the FA model can be checked as well. Section 5.4.2 covers the economic performance. The main risk aversion is 5, and results of the aversions of 2 and 10 are found in Appendix A. Last, Section 5.4.3 contains a short evaluation of the factor selection expansion.

5.4.1. Statistical performance

The results of the first test, the RMSE, are compared to the results of the basic model, referred to as the Old FA. The outcomes are given in Table 5.2. The expansions have been considered all together. Furthermore, the fixed parameters of the logistic function as well as the thresholding methods are solely tested to see the added performance of the expansions. The factor selection cannot be tested solely due to the computational expense. In order to account for the added value of the factor selection, the first two expansions are tested together. By comparing these results to the top of the table, the changes accounted by the factor selection are found.

The values of the RMSE that have decreased compared to the standard model are shown in blue. Throughout the top of the panel, most values are highlighted. The RMSE of the benchmark factor itself, however, only decreases two times. The FASTR.NE and FASTR.EL models contain the best performance compared to the standard model, as all values have decreased. The FASTR.LL model seems less affected by the expansions.

The decrease in the RMSE can be accounted for by the fixed logistic parameters and the factor selection. The top panel of the table often shows smaller values compared to the second panel. The thresholding does not seem to have much effect. On the contrary, for the FASTR.NE and FASTR.LE models, the RMSE has increased by a very large amount for some time series. This may still be due to the optimization of the logistic parameters, as these large errors have disappeared in panel b.

RMSE All Expansions	Small Cap		Medium Cap		Big Cap		S&P 500		Gold	
	$r_{e,t+1}$	RV_{t+1}	$r_{e,t+1}$	RV_{t+1}	$r_{e,t+1}$	RV_{t+1}	$r_{e,t+1}$	RV_{t+1}	$r_{e,t+1}$	RV_{t+1}
Old FA	41.053	7.350	30.782	6.652	25.278	6.053	22.003	4.913	35.442	8.603
FA	0.986	1.038	1.043	1.065	1.050	1.050	1.080	1.036	0.981	1.048
FASTR.NL	1.055	1.067	1.142	1.067	1.134	1.088	1.136	1.081	1.042	0.916
FASTR.NV	1.074	1.059	1.136	1.131	1.154	1.149	1.164	1.085	1.086	1.088
FASTR.NE	1.082	1.058	1.135	1.266	1.087	1.247	1.144	1.045	1.043	1.004
FASTR.LL	1.001	1.026	1.061	1.053	1.078	1.021	1.095	1.024	0.998	0.896
FASTR.LV	0.998	**1.005**	1.060	**1.044**	1.072	1.017	1.100	1.018	1.008	0.996
FASTR.LE	1.050	1.031	1.130	1.319	1.133	1.326	1.150	1.064	0.981	1.059
FASTR.EL	**0.932**	1.072	0.961	1.077	**0.935**	**1.003**	0.938	**0.985**	**0.898**	**0.798**
FASTR.EV	0.939	1.067	**0.954**	1.087	0.951	1.041	**0.933**	1.016	0.916	1.095
FASTR.EE	0.934	1.136	0.964	1.277	0.949	1.296	0.947	1.235	0.912	1.049
Thresholding & Fixed Logistic										
FA	1.031	1.050	1.048	1.083	1.082	1.055	1.042	1.028	1.028	0.984
FASTR.NL	1.125	1.128	1.180	1.172	1.213	1.117	1.142	1.159	1.137	0.924
FASTR.NV	1.168	1.093	1.162	1.148	1.185	1.064	1.115	1.079	1.188	1.109
FASTR.NE	1.132	1.160	1.161	1.196	1.188	1.215	1.134	1.056	1.118	0.988
FASTR.LL	1.054	1.073	1.064	1.082	1.105	1.078	1.070	1.074	1.048	0.901
FASTR.LV	1.068	1.052	1.081	1.070	1.107	**1.033**	1.068	**1.028**	1.077	1.042
FASTR.LE	1.125	1.127	1.152	1.291	1.198	1.254	1.192	1.066	1.059	1.029
FASTR.EL	**0.953**	1.057	**0.967**	1.107	**0.961**	1.091	**0.956**	1.086	**0.950**	**0.811**
FASTR.EV	0.971	**1.022**	1.000	**1.069**	0.971	1.037	0.969	1.039	0.988	1.087
FASTR.EE	0.988	1.185	1.011	1.225	0.997	1.193	0.994	1.241	0.960	0.945
Thresholding										
FA	1.031	1.050	1.048	1.083	1.082	1.055	1.042	1.028	1.028	0.984
FASTR.NL	1.265	1.600	1.312	1.225	1.513	1.119	1.289	1.308	1.134	0.926
FASTR.NV	1.242	1.252	1.230	1.280	1.264	1.165	1.225	1.279	1.399	3.144
FASTR.NE	3.400	1.572	1.301	37.081	1.299	15.268	1.316	2.014	1.683	1.078
FASTR.LL	1.096	1.226	1.142	1.108	1.191	1.059	1.277	1.096	1.070	0.938
FASTR.LV	1.086	1.340	1.137	1.073	1.141	1.175	1.115	**1.017**	1.105	1.118
FASTR.LE	3.283	1.191	1.177	34.700	1.196	13.834	1.207	1.062	1.252	0.983
FASTR.EL	**0.979**	1.275	1.048	1.105	**0.996**	1.122	**0.993**	1.076	**0.940**	**0.816**
FASTR.EV	0.996	**1.049**	**1.045**	**1.051**	1.020	**1.028**	1.008	1.022	1.037	1.122
FASTR.EE	1.038	1.148	1.069	1.282	1.058	1.223	1.071	1.322	1.038	1.117
Fixed Logistic Parameters										
FA	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
FASTR.NL	1.101	1.129	1.132	1.015	1.123	**1.000**	1.103	1.004	1.130	0.914
FASTR.NV	1.100	1.083	1.135	1.047	1.139	1.036	1.130	1.010	1.212	1.119
FASTR.NE	1.078	1.105	1.074	1.074	1.055	1.050	1.047	1.037	1.167	1.048
FASTR.LL	1.007	1.042	1.027	1.025	1.029	**1.000**	1.018	**0.991**	1.021	0.866
FASTR.LV	1.026	**1.015**	1.048	**1.023**	1.055	1.011	1.054	1.000	1.154	1.030
FASTR.LE	1.013	1.032	1.047	1.045	1.058	1.034	1.047	1.020	1.134	1.062
FASTR.EL	**0.946**	1.100	**0.967**	1.100	**0.960**	1.039	**0.948**	1.043	**0.925**	**0.819**
FASTR.EV	0.962	1.041	0.988	1.084	0.970	1.060	0.959	1.045	0.941	1.136
FASTR.EE	0.976	1.135	1.004	1.258	0.991	1.279	0.980	1.259	0.927	1.088

Table 5.2. Relative Mean Squared Error of the excess returns and realized volatility for the FASTR models, relative to the factor augmented model used in Çakmakli & Van Dijk (2010). The Old FA shows the Mean Squared Error (MSE) from the standard model of the previous chapters, and the FA and FASTR models are a comparison of this MSE. The last two letters of the FASTR models contain the influence of the financial variables and the usage of the STR variable respectively. Panel b shows the statistic when thresholding and fixed logistic parameters are used, and panel c and d contain the single expansions.

The DM statistic, given in Table A.13 in Appendix A shows similar results to the outcomes of the RMSE. Four values in total are positive significant, and all are found in the prediction of Gold. However, many values have increased compared to the standard model, which gives less negative significant statistics.

The results of the CPS test are found in Table 5.3**.** Compared to the standard model, most of the values have again increased, as well as the highest CPS values for each time series. The excess returns do not show a value below 0.525, which was the case in the standard model. The highest value for the excess returns is achieved by the FASTR.NE model, the worst model in the previous Chapter, and equals 0.597.

The best CPS for the realized volatilities are all above 0.67, where FASTR.EL attains most of the highest values. The highest value of all is also for the FASTR.EL model and equals 0.736, which is for the Gold asset option. This time, the thresholding seems to account for most increases in the CPS, while the fixed logistic parameters show fewer improvements. The factor selection does not add much direction compared to the standard model, and usually shows a decrease in the CPS. With this, it is concluded that, so far, the thresholding and factor selection show a trade-off, where both increase the performance in one test, and have a negative influence on the other.

Table A.14 in Appendix A shows again the DA test statistic. Indicated again by the blue values, most models perform better relative to the standard model. All predictions of the realized volatility are significant, as well as some of the excess returns, which is an improvement. The only series that does not have any significant values is the excess return of the Mid Cap portfolio. The DA of the models excluding the financial variables also seems to have lost power in most cases.

CPS	**Small Cap**		**Medium Cap**		**Big Cap**		**S&P 500**		**Gold**	
(a) All Expansions	$r_{e,t+1}$	RV_{t+1}	$r_{e,t+1}$	RV_{t+1}	$r_{e,t+1}$	RV_{t+1}	$r_{e,t+1}$	RV_{t+1}	$r_{e,t+1}$	RV_{t+1}
FA	0.555	0.622	0.542	0.657	0.575	0.667	0.550	0.677	0.565	0.684
FASTR.NL	0.562	0.642	0.530	0.629	0.557	0.689	0.525	0.697	0.570	0.721
FASTR.NV	0.545	0.664	0.537	0.667	0.565	0.667	0.560	0.697	**0.575**	0.677
FASTR.NE	0.555	0.667	0.560	0.672	**0.597**	0.682	**0.592**	0.682	0.537	0.692
FASTR.LL	**0.577**	0.629	0.535	0.667	0.580	0.692	0.552	0.687	0.560	0.726
FASTR.LV	0.540	0.662	0.530	0.672	0.565	0.684	0.545	0.694	0.555	0.674
FASTR.LE	0.565	**0.679**	0.545	**0.674**	0.567	0.677	0.547	0.677	0.560	0.674
FASTR.EL	0.560	0.612	0.570	0.634	0.577	**0.707**	0.537	**0.704**	0.560	**0.736**
FASTR.EV	0.550	0.644	**0.580**	0.622	0.570	0.674	0.587	0.684	0.535	0.580
FASTR.EE	0.550	0.634	0.540	0.614	0.567	0.577	0.540	0.552	0.545	0.677
(b) Thresholding & Fixed Logistic										
FA	0.565	0.657	0.550	0.639	**0.595**	0.692	**0.592**	0.692	0.515	0.674
FASTR.NL	0.552	0.654	0.552	0.627	0.577	0.659	0.580	0.716	0.537	0.689
FASTR.NV	0.527	**0.684**	0.520	0.692	0.540	0.687	0.565	**0.719**	0.535	0.662
FASTR.NE	0.555	0.669	0.537	0.657	0.585	0.659	0.587	0.704	0.485	0.649
FASTR.LL	**0.572**	0.659	0.560	0.637	0.592	0.677	0.587	0.692	0.527	**0.721**
FASTR.LV	0.542	0.669	0.517	0.657	0.555	0.697	0.570	0.684	0.530	0.667
FASTR.LE	0.562	0.679	0.527	**0.697**	0.552	**0.711**	0.560	**0.719**	**0.537**	0.639
FASTR.EL	0.552	0.577	0.567	0.597	0.572	0.639	0.567	0.682	0.520	**0.721**
FASTR.EV	0.557	0.639	**0.577**	0.602	0.567	0.637	0.545	0.679	0.522	0.607
FASTR.EE	0.532	0.652	0.540	0.614	0.555	0.577	0.517	0.610	0.532	0.657
(c) Thresholding										
FA	0.565	0.657	0.550	0.639	0.595	0.692	0.592	0.692	0.515	0.674
FASTR.NL	0.570	0.632	0.557	0.647	0.562	0.649	0.575	0.699	0.547	0.692
FASTR.NV	0.547	0.697	0.555	0.654	0.572	0.667	0.602	0.697	0.550	0.664
FASTR.NE	0.535	0.642	0.547	0.632	0.597	0.649	0.570	0.684	0.488	0.639
FASTR.LL	0.562	0.654	0.545	0.639	0.570	0.687	0.570	0.692	0.527	**0.721**
FASTR.LV	0.515	0.672	0.517	0.684	0.550	0.684	0.567	0.697	0.530	0.642
FASTR.LE	0.560	0.679	0.535	0.669	0.562	0.714	0.545	0.716	0.510	0.634
FASTR.EL	0.537	0.590	0.585	0.627	0.565	0.679	0.560	0.714	0.505	0.659
FASTR.EV	0.557	0.672	0.572	0.644	0.557	0.674	0.555	0.679	0.525	0.627
FASTR.EE	0.540	0.672	0.560	0.592	0.577	0.575	0.547	0.582	0.515	0.659
(d) Fixed Logistic Parameters										
FA	0.550	0.649	0.542	0.637	0.535	0.652	0.540	0.677	0.493	0.657
FASTR.NL	0.565	0.644	0.552	0.619	0.540	0.652	0.547	0.689	0.530	0.702
FASTR.NV	0.510	0.659	0.527	0.664	0.517	0.649	0.535	0.667	0.517	0.649
FASTR.NE	0.512	0.672	0.532	0.662	0.522	0.654	0.510	0.632	0.493	0.729
FASTR.LL	**0.567**	0.607	0.577	0.629	0.565	0.674	0.552	0.687	0.515	0.724
FASTR.LV	0.535	0.639	0.545	0.642	0.505	0.649	0.515	0.654	0.527	0.647
FASTR.LE	0.517	**0.687**	0.540	**0.677**	0.515	0.677	0.515	0.644	0.490	0.652
FASTR.EL	**0.567**	0.592	**0.582**	0.610	0.560	**0.682**	0.547	0.689	**0.552**	**0.736**
FASTR.EV	0.547	0.642	0.570	0.612	**0.577**	0.674	**0.567**	**0.699**	0.532	0.590
FASTR.EE	0.552	0.647	0.547	0.634	0.560	0.587	0.532	0.555	0.542	0.657

Table 5.3. The Correctly Predicted Signs of the predicted excess returns and realized volatility, computed as in Equation (3.10). The last two letters of the FASTR models contain the influence of the financial variables and the usage of the STR variable respectively. Panel b shows the statistic when thresholding and fixed logistic parameters are used, and panel c and d contain the single expansions.

Do the expansions have influence on the stability? Figure 5.4 shows graphs of the stability of the RMSE and CPS for both the excess returns and realized volatility of the S&P500. The predictions are made by the FASTR.EL model. The graphs indicate that the stability is not present, just like in the previous chapter. Especially the estimates of the realized volatility are instable, ranging between 0.301 and 1.291 for the RMSE and 0.500 and 0.883 for the CPS.

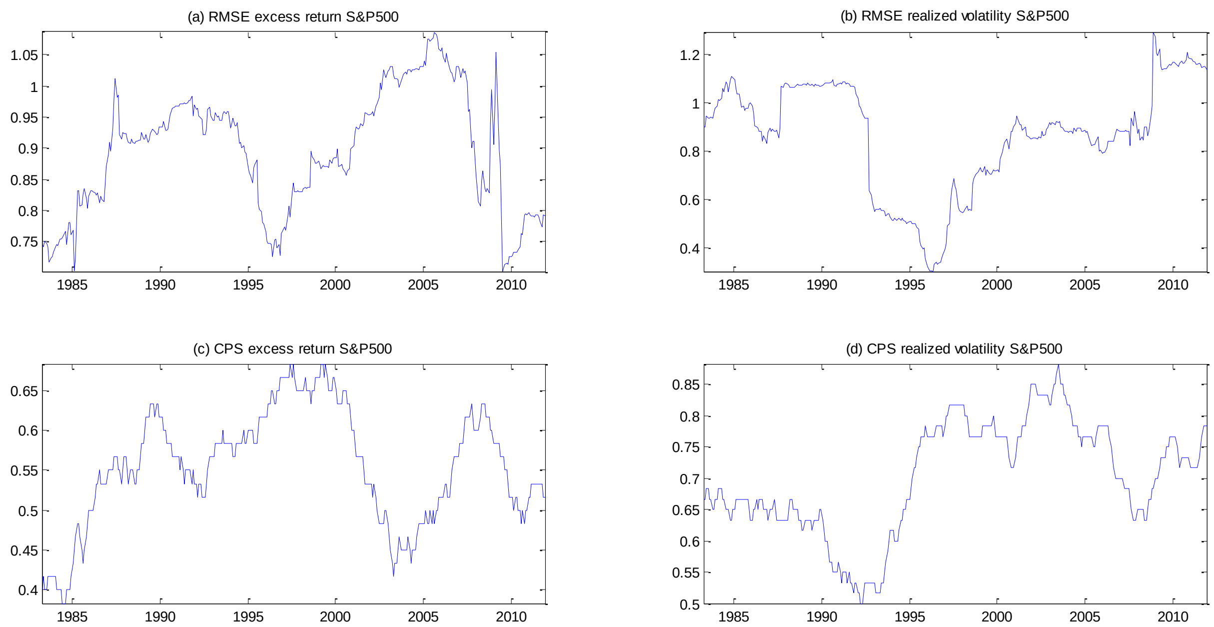

Figure 5.4. RMSE and CPS over a moving window of 60 months, for the excess returns (panel a and c) and realized volatility (panel b and d) predicted by FASTR.EL.

The results of the last test, the EP, are summarized in Table 5.5. There are more significant negative values compared to the standard model in Section 4.1. These values are often for the models excluding the financial variables. Especially the FASTR.EL model, which often performs best, shows 4 results that are far below 0. The FASTR.NE on the other hand shows all significant positive values. Compared to the previous tests, the number of improvements also decreased.

This decrease, in particular that of the FASTR.E models, can be accounted to the factor selection. Comparing panel (a) of the table with panel (b), there are large losses, also for the FA model. The other two expansions seem to provide mixed results.

EP (a) All Expansions	Small Cap	Medium Cap	Big Cap	S&P 500	Gold
FA	2.778	0.539	4.340	-1.523	7.364
FASTR.NL	2.501	0.110	2.219	-3.524	5.748
FASTR.NV	2.081	2.060	1.221	0.375	6.564
FASTR.NE	4.425	**4.847**	**5.637**	**8.059**	4.950
FASTR.LL	6.179	-1.668	3.695	0.356	4.161
FASTR.LV	1.700	-0.136	3.526	-2.142	7.856
FASTR.LE	**6.362**	3.058	0.848	0.309	6.430
FASTR.EL	-5.571	-12.770	-4.243	-13.696	**9.643**
FASTR.EV	-6.342	2.132	-10.354	0.207	5.224
FASTR.EE	0.817	-1.001	2.932	-0.955	7.685
(b) Thresholding & Fixed Logistic					
FA	4.793	1.920	7.684	**10.554**	4.148
FASTR.NL	3.755	0.963	4.103	6.267	1.310
FASTR.NV	0.749	-1.939	-2.974	1.883	4.670
FASTR.NE	4.657	0.298	4.255	8.295	-2.281
FASTR.LL	**5.876**	**2.965**	**7.859**	9.210	-0.661
FASTR.LV	1.266	-2.998	-1.860	2.252	3.900
FASTR.LE	5.809	0.297	-1.542	5.312	4.099
FASTR.EL	-4.462	2.205	4.269	8.738	1.609
FASTR.EV	-0.607	0.454	-1.560	-3.855	3.154
FASTR.EE	-4.645	-2.852	-1.260	-1.970	**6.097**
(c) Thresholding					
FA	4.793	1.920	7.684	**10.554**	4.148
FASTR.NL	3.214	4.196	5.698	6.721	4.813
FASTR.NV	4.150	2.357	2.115	8.624	**7.716**
FASTR.NE	5.322	3.050	**9.390**	7.108	-3.556
FASTR.LL	1.892	0.890	6.128	5.619	1.368
FASTR.LV	-0.306	-2.869	-1.222	2.717	5.035
FASTR.LE	**5.446**	0.441	4.104	-1.804	0.321
FASTR.EL	-0.781	3.216	5.868	6.455	2.605
FASTR.EV	-0.432	2.537	-0.687	-1.045	2.960
FASTR.EE	0.136	**4.673**	6.281	1.597	5.574
(d) Fixed Logistic Parameters					
FA	5.482	1.965	4.339	**5.301**	-0.646
FASTR.NL	6.035	2.132	2.946	3.389	0.009
FASTR.NV	-0.682	0.330	0.866	2.026	1.639
FASTR.NE	0.528	1.713	0.206	0.505	1.024
FASTR.LL	**7.156**	**4.829**	**5.424**	5.205	1.319
FASTR.LV	2.449	1.761	-2.689	-3.010	-1.654
FASTR.LE	2.589	3.625	0.168	2.454	-1.241
FASTR.EL	-2.286	4.531	3.505	3.425	3.898
FASTR.EV	-4.351	-1.886	-0.953	3.886	3.484
FASTR.EE	-1.102	-1.987	0.696	-0.701	**6.098**

Table 5.5. Excess Predictability test of Anatolyev & Gerko (2005). The significance level is set to 5%, which equals a critical value of 1.96. An EP larger than this critical value indicates a model that performs better than taking random actions. The null hypothesis equals that the model does not significantly outperform the benchmark. Panel b shows the statistic when thresholding and fixed logistic parameters are used, and panel c and d contain the single expansions.

The section shows mixed results regarding the expansions. Although most values for the RMSE, DM, CPS and DA tests have increased in value, the EP shows less promising results, and the stability over a moving window of 60 months is still lacking. The thresholding mainly takes care of the improvements of the CPS and DA and partly of the EP, where the factor selection improves the quality of the RMSE and DM. The fixed logistic parameters expansion shows improvements over all tests, where only the EP results are limited.

5.4.2. Economic performance

The EP in the previous section showed, even though the expansions are considered good, less promising outcomes in the predictability of the models that exclude financial predictors. The question is whether these negative outcomes are also reflected in the economic performances of the models. The same rules apply as in Section 4.2. That is, transaction costs are left out of the research and the interval of the weights is split up in two options: the intervals [0, 1] and [-1, 2]. The averages of the weights with a risk aversion coefficient of 5 are given in Table 5.6**.** The good, bad and neutral regimes are constructed in the same way as in the previous Chapter. There are large differences between the models now. The FASTR.E models only invest a small amount in the risk-free rate compared to the other models, and invest a larger amount in the Small Cap portfolio, on average. This idea is expanded when short-selling is allowed. In this case, the FASTR.E models invest on average over 0.50 in the Small Cap portfolios. Furthermore, the option to go short in the asset is for most models the reason to go short in the Gold option and invest a substantial amount in the Small and Medium cap portfolios. The highest value is attained by the FASTR.EV model, which invests on average 0.68 in the Small cap portfolio.

Weights $\gamma = 5$	No Short-Selling					Short-Selling Allowed				
	R_f	Small	Medium	Big	Gold	R_f	Small	Medium	Big	Gold
FA	0.22	0.24	0.19	0.19	0.17	0.24	0.24	0.44	0.09	-0.02
Complete Sample										
FASTR.NL	0.20	0.24	0.20	0.21	0.15	0.23	0.22	0.32	0.26	-0.03
FASTR.NV	0.18	0.25	0.15	0.20	0.21	0.08	0.25	0.31	0.24	0.11
FASTR.NE	0.16	0.25	0.22	0.19	0.18	0.11	0.25	0.49	0.18	-0.03
FASTR.LL	0.21	0.24	0.20	0.20	0.16	0.22	0.21	0.37	0.23	-0.03
FASTR.LV	0.22	0.25	0.17	0.17	0.19	0.30	0.22	0.40	0.06	0.02
FASTR.LE	0.22	0.24	0.21	0.18	0.15	0.30	0.16	0.43	0.13	-0.03
FASTR.EL	0.09	0.34	0.21	0.20	0.16	0.14	0.62	0.28	0.03	-0.08
FASTR.EV	0.07	0.33	0.15	0.21	0.23	0.04	0.68	0.18	0.05	0.04
FASTR.EE	0.13	0.33	0.12	0.17	0.25	0.03	0.56	0.16	0.13	0.11
Good Regime										
FASTR.NL	0.17	0.33	0.14	0.16	0.20	-0.06	0.48	0.36	0.02	0.21
FASTR.NV	0.14	0.24	0.14	0.22	0.26	-0.14	0.31	0.30	0.32	0.20
FASTR.NE	0.20	0.25	0.16	0.23	0.16	0.25	0.29	0.28	0.35	-0.17
FASTR.LL	0.20	0.24	0.23	0.13	0.19	0.09	0.09	0.66	-0.05	0.21
FASTR.LV	0.20	0.26	0.15	0.19	0.21	0.18	0.28	0.33	0.13	0.08
FASTR.LE	0.30	0.22	0.16	0.20	0.11	0.63	0.08	0.29	0.25	-0.25
FASTR.EL	0.11	0.35	0.20	0.19	0.14	0.33	0.54	0.30	-0.04	-0.13
FASTR.EV	0.05	0.30	0.12	0.27	0.26	-0.19	0.63	-0.00	0.37	0.19
FASTR.EE	0.23	0.16	0.08	0.28	0.25	0.41	-0.03	-0.24	0.85	0.02
Bad Regime										
FASTR.NL	0.12	0.23	0.26	0.23	0.15	-0.08	0.09	0.51	0.48	0.00
FASTR.NV	0.20	0.27	0.18	0.18	0.17	0.26	0.22	0.30	0.18	0.03
FASTR.NE	0.14	0.25	0.30	0.11	0.20	0.00	0.17	0.75	-0.03	0.11
FASTR.LL	0.14	0.26	0.22	0.23	0.16	-0.05	0.19	0.40	0.44	0.01
FASTR.LV	0.22	0.27	0.20	0.14	0.18	0.40	0.20	0.47	-0.03	-0.04
FASTR.LE	0.16	0.25	0.27	0.13	0.20	-0.01	0.17	0.65	-0.03	0.22
FASTR.EL	0.10	0.26	0.25	0.30	0.09	0.22	0.28	0.40	0.39	-0.29
FASTR.EV	0.09	0.36	0.18	0.15	0.21	0.28	0.78	0.30	-0.23	-0.12
FASTR.EE	0.05	0.45	0.16	0.07	0.26	-0.28	1.02	0.54	-0.43	0.16
Neutral Regime										
FASTR.NL	0.28	0.19	0.19	0.22	0.12	0.70	0.15	0.14	0.24	-0.23
FASTR.NV	0.24	0.23	0.11	0.22	0.20	0.27	0.12	0.33	0.16	0.12
FASTR.NE	0.12	0.25	0.17	0.30	0.16	0.01	0.40	0.28	0.34	-0.04
FASTR.LL	0.28	0.22	0.16	0.22	0.13	0.55	0.32	0.12	0.25	-0.24
FASTR.LV	0.29	0.19	0.14	0.19	0.19	0.36	0.08	0.44	0.12	-0.00
FASTR.LE	0.14	0.31	0.14	0.29	0.13	0.25	0.45	0.12	0.29	-0.11
FASTR.EL	0.06	0.41	0.18	0.12	0.23	-0.06	0.97	0.17	-0.23	0.14
FASTR.EV	0.09	0.32	0.16	0.22	0.22	0.04	0.49	0.46	-0.11	0.12
FASTR.EE	0.05	0.46	0.12	0.10	0.27	-0.22	1.04	0.28	-0.38	0.28

Table 5.6. Average of the weights, for two forms of determining the weights. In the first form the investor is not allowed for go short, in the second form the investor can go short, which enlarges the interval of possible weights to [-1, 2]. The S&P500 is left out, because of the high similarities between the Big Cap and S&P. The good, bad and neutral regimes are computed by taking the optimized parameters of the Big Cap excess returns in the logistic function. A good regime captures the best 40% of the values, a bad regime the least 40%, and the neutral regime the remaining 20% in the middle. The risk aversion coefficient is taken to be 5.

The number of occurrences in the regimes is given in Table A.15 in Appendix A. It can now be seen that the division of the number of observations is more even between the good and bad regimes, and a reasonable amount is displayed as neutral. The numbers are similar for models containing the same exogenous variable, thanks to the fact that the logistic parameters are fixed. Looking back at the weights, the FASTR.EE reveals the most different results compared to the other models. It invests a reasonable amount in the risk-free rate and goes short in the Small cap portfolio when it is able in the good regimes. In the other two regimes, the amount invested in the Small Cap portfolio is very high, and the risk-free rate is low. It also takes a large short position in the Big Cap portfolio.

The strategy of the FASTR.EE model pays of, as shown in Table 5.7**.** The annualized excess returns and volatility are again given, as well as the Sharpe Ratio. The return in the good states is lower than in the other two regimes, but overall it obtains the highest returns for both restrictions and best Sharpe Ratio when short-selling is allowed. Overall, however, the Sharpe Ratios have decreased compared to the Buy-and-Hold benchmarks. The highest value on the left-hand side is 0.573 for the FASTR.NE model. The Sharpe Ratios on the right are somewhat lower, but contain large excess returns. The models with the default spread seem to work best in this case, which obtained the least promising results in the previous chapter. Especially the neutral regimes give high returns and Sharpe Ratios for each of these models.

Table A.18 in Appendix A contains the results based on the three sub-samples. This table shows that stability over the complete sample is hard to obtain. The models are able to obtain higher excess returns than the benchmarks, in particular in the last sub-sample, but are often lacking in the second interval.

Economic Performance $\gamma = 5$	No Short-Selling Allowed			Short-Selling Allowed		
	Average Annualized Excess Return	Annualized Volatility	Sharpe Ratio	Average Annualized Excess Return	Annualized Volatility	Sharpe Ratio
Buy-and-Hold R_f	0.000%	0.980%	0.000			
Buy-and-Hold Small	10.573%	21.077%	0.502			
Buy-and-Hold Big	8.644%	16.722%	0.517			
Complete Sample						
FA	6.806%	**14.001%**	0.486	9.096%	**26.340%**	0.345
FASTR.NL	7.970%	15.034%	0.530	12.726%	29.314%	0.434
FASTR.NV	6.122%	14.379%	0.426	11.835%	29.812%	0.397
FASTR.NE	8.584%	14.993%	**0.573**	15.671%	30.875%	0.508
FASTR.LL	6.982%	14.748%	0.473	9.410%	27.865%	0.338
FASTR.LV	6.095%	14.446%	0.422	6.779%	27.9.39%	0.243
FASTR.LE	7.846%	14.752%	0.532	11.456%	27.889%	0.411
FASTR.EL	9.097%	17.807%	0.511	14.291%	30.128%	0.474
FASTR.EV	6.814%	17.753%	0.384	9.527%	29.809%	0.320
FASTR.EE	**9.188%**	16.236%	0.566	**16.737%**	32.811%	**0.510**
Good Regimes						
FASTR.NL	2.935%	12.595%	0.233	-5.040%	29.117%	-0.173
FASTR.NV	5.362%	12.104%	0.443	**21.271%**	24.944%	**0.853**
FASTR.NE	2.939%	11.391%	0.258	8.218%	28.720%	0.286
FASTR.LL	2.167%	12.177%	0.178	-6.287%	25.381%	-0.248
FASTR.LV	5.729%	11.753%	0.487	17.567%	25.046%	0.701
FASTR.LE	1.185%	**11.383%**	0.104	-1.873%	23.499%	-0.080
FASTR.EL	**8.909%**	14.509%	**0.614**	8.425%	**23.473%**	0.359
FASTR.EV	3.690%	14.133%	0.261	6.376%	26.820%	0.238
FASTR.EE	4.848%	14.008%	0.346	7.463%	30.076%	0.248
Bad Regimes						
FASTR.NL	11.885%	**15.425%**	0.771	21.591%	29.141%	0.741
FASTR.NV	5.635%	16.825%	0.335	6.537%	34.614%	0.189
FASTR.NE	**12.245%**	17.823%	0.687	19.177%	32.820%	0.584
FASTR.LL	11.752%	16.062%	**0.732**	17.908%	28.862%	0.621
FASTR.LV	5.315%	17.359%	0.306	-0.887%	30.941%	-0.029
FASTR.LE	12.196%	17.698%	0.689	21.555%	32.609%	0.661
FASTR.EL	11.819%	17.093%	0.692	**30.088%**	**27.511%**	**1.094**
FASTR.EV	7.317%	21.416%	0.342	11.819%	34.285%	0.345
FASTR.EE	11.309%	18.535%	0.610	22.740%	36.846%	0.617
Neutral Regimes						
FASTR.NL	8.381%	16.276%	0.515	19.591%	29.344%	0.668
FASTR.NV	11.923%	11.996%	0.994	-4.066%	26.434%	-0.154
FASTR.NE	16.688%	14.886%	1.121	**32.116%**	31.217%	1.029
FASTR.LL	6.484%	15.266%	0.425	14.766%	28.497%	0.518
FASTR.LV	11.406%	**10.914%**	1.045	-3.999%	25.615%	-0.156
FASTR.LE	17.510%	13.058%	1.341	28.056%	21.923%	**1.280**
FASTR.EL	6.848%	20.497%	0.334	5.767%	35.990%	0.160
FASTR.EV	**19.558%**	13.983%	**1.399**	13.863%	**19.944%**	0.695
FASTR.EE	17.983%	14.555%	1.236	31.017%	25.534%	1.215

Table 5.7. Annualized excess returns, volatility and Sharpe Ratio for the average weights given in Table 5.6, with a risk aversion of 5. The excess returns are compounded, which means that earned profits are reinvested. In the first form the investor is not allowed for go short, in the second form the investor can go short, which enlarges the interval of possible weights to [-1, 2]. The good, bad and neutral regimes are computed by taking the optimized parameters of the Big Cap excess returns in the logistic function. A good regime captures the best 40% of the values, a bad regime the least 40%, and the neutral regime the remaining 20% in the middle.

The bootstrap of Ledoit & Wolf (2008), again computed over $M = 5{,}000$ bootstraps, results in similar conclusions, as shown in Table 5.8. None of the Sharpe Ratios significantly outperforms any of the benchmarks on a 5% or 10% significance level. The FASTR.EV model contains one negative significant value on a 10% level, against the Buy-and-Hold Big portfolio, but other than this all values are insignificant.

	No Short-Selling			**Short-Selling Allowed**		
	Buy-and-Hold Small	Buy-and-Hold Big	FA	Buy-and-Hold Small	Buy-and-Hold Big	FA
FA	0.417	0.785	-	0.411	0.218	-
FASTR.NL	0.320	0.692	0.891	0.522	0.310	0.688
FASTR.NV	0.705	0.829	0.517	0.451	0.248	0.873
FASTR.NE	0.278	0.593	0.660	0.699	0.460	0.378
FASTR.LL	0.543	0.969	0.693	0.354	0.162	0.825
FASTR.LV	0.734	0.800	0.322	0.190	0.100	0.283
FASTR.LE	0.349	0.673	0.725	0.566	0.331	0.592
FASTR.EL	0.623	0.827	0.660	0.479	0.253	0.723
FASTR.EV	0.648	0.259	0.212	0.127	0.068	0.822
FASTR.EE	0.340	0.713	0.907	0.575	0.332	0.635

Table 5.8. P-values of the Bootstrap on the Sharpe Ratio, proposed by Ledoit & Wolf (2008). The null hypothesis states that the difference between the Sharpe Ratios does not differ significantly from zero. The risk aversion used to compute the weights is equal to 5. Positive significant values are bold (based on significance level of 5%), negative significant values are shown in red.

The results for the risk aversion coefficient of 2 and 10 are similar to the outcomes above. The averages of the weights are shown in Tables A.16 and A.17 in Appendix A, while the annualized returns and volatility are given in Tables A.19 and A.20**.** The annualized returns have decreased for most of the models, while the volatilities increased somewhat relative to the standard models. The FASTR.NE shows the highest excess return and Sharpe Ratio for an aversion of 10, while the FASTR.EL model contains a return of 25.109% in excess of the risk-free rate over the complete sample for an aversion of 2. This is mainly obtained in the bad regimes, in which it is able to provide a return over 50% annually. The large returns come with a large volatility, which is roughly 47% in the same regime. However, according to the p-values computed by the bootstrap of Ledoit & Wolf (2008), found in Tables A.21 and A.22, no significant higher Sharpe Ratios are obtained over the complete sample.

Concluding the economic performance of the models, the overall economic performance of the models lacks power relative to the standard model. While models using the default spread have increased in value, other models seem to decrease in performance. The expansions used in this chapter decrease the differences in the models of Chapter 4, instead of increasing the value of all. Overall, the FASTR.NE and FASTR.EL model show the most promising results in this chapter. The Ledoit & Wolf (2008) bootstrap concludes that none of the Sharpe Ratios is significant at the 5% level, but there are still models that are able to obtain high excess returns.

5.4.3. Examining the factor selection

Figure 5.9 contains results of the factor selection. The excess returns of the S&P500 Index are used, and the predictions are obtained from the FASTR.EL model. In panel (a) and (c), the appearances of the current and lagged factors are given respectively. Panel (b) contains the total explained variance over the complete sample, and panel (d) shows a histogram of the number of appearances for each factor. For the latter it should be noted that the lagged factors are mentioned as factors 7 until 12. The number of factors taken into account at each month is not taken into consideration, because after examining the results, this turns out to be one for each month. This is also the reason for the low explained variance. Furthermore, looking at panel (d), it can be seen that the 5th and 6th factor – for current and lagged factors – are used most. The first current factor is used the least, and the first lagged factor is ranked 8th, while these two contain the most explained variance.

Table A.23 in Appendix A contains the same graphs, but now uses the excess returns of the Gold commodity. This series does use the first current factor most often, but this does not lead to an explained variance that is much higher than the previously tested series. Conclusions from Bai (2010), who concludes that the 2nd and 5th factor contain most valuable information, are therefore not supported. This pattern differs across the return series, indicated by these two figures.

The panels (a) and (c) also cover the differences between the good, bad and neutral regimes, in the same manner as in Section 4.3. The good regimes, again pointed out by the blue dots, do not show a different pattern when compared to the bad regimes (red dots). In order for the regimes to differ in the total explained variance, one of the regimes needs to use larger values more often than the other regimes. This statement can be supported by the fact that over the complete sample, only one factor is invited each month. With this, the sub-question of this research can again be answered with a 'no': the different regimes in the model do not affect the total amount of explained variance in the predictions.

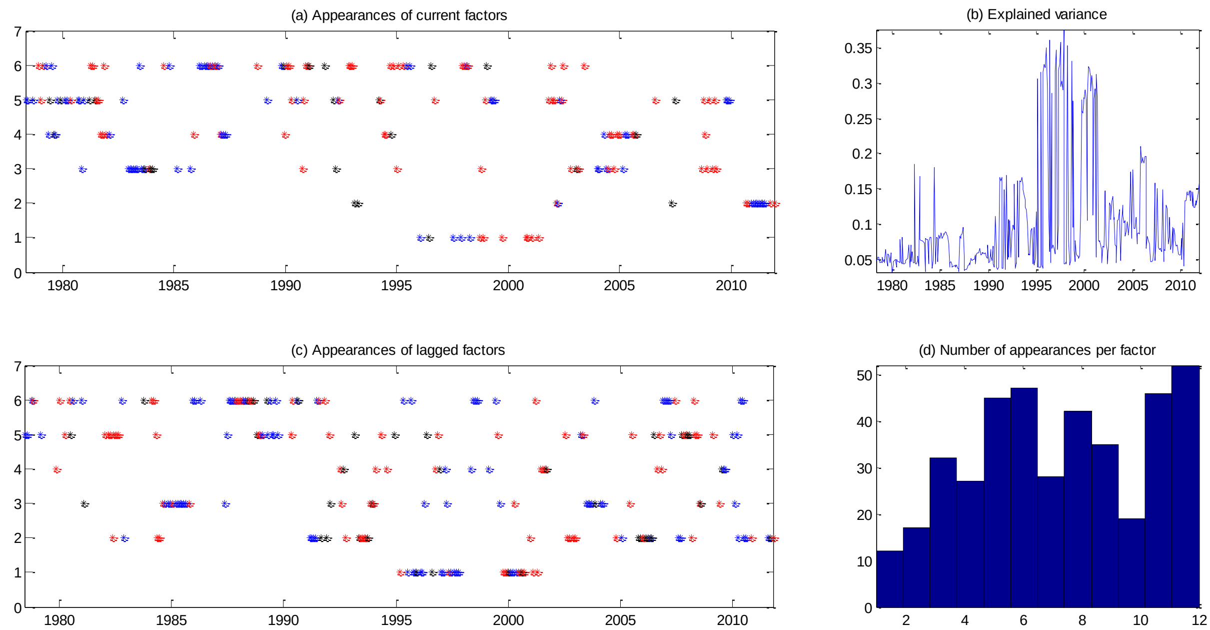

Table 5.9. Panel (a) and (c) show the appearances of the current and lagged factors, panel (b) the total explained variance and panel (d) the number of appearances per factor. The time series is the excess returns of the S&P500 Index, and the predictions are from the FASTR.EL model. The lagged factors in panel (d) are indicated by the integers 7 to 12. The good, bad and neutral regimes in panels (a) and (c) are computed by taking the optimized parameters of the Big Cap excess returns in the logistic function. A good regime captures the best 40% of the values, a bad regime the least 40%, and the neutral regime the remaining 20% in the middle.

6. Conclusions

The research proposes the use of the combination of factor augmentation and nonlinearity by means of a smooth transition model. The model, in advance called the FASTR model, uses three different exogenous variables in the logistic function – the lagged endogenous variable; the log Implied Volatility Index; and the Default Spread – and changes the influence of financial variables between nonlinear, linear and no influence at all. In the factor augmentation, 101 macroeconomic variables are used. For the purpose of testing the performance of the FASTR models, five time series are examined, namely three Cap portfolios, as well as the S&P500 Index and the Gold commodity. The excess returns and realized volatilities are predicted on a monthly basis, over the sample of June 1978 until November 2011. The predictions are compared against a linear factor augmentation model. The main research question is *'To what extent are the predictions of excess stock returns affected when Factor Augmentation and Nonlinearity are combined?'*.

The statistical tests conclude that the FASTR model performs well in the accuracy, shown by the Correctly Predicted Signs test and Directional Accuracy test, but it cannot beat the benchmark, according to the Relative Mean Squared Error and Diebold-Mariano test. The Excess Predictability test indicates that some FASTR models, as well as the benchmark, are capable of outperforming the random investments. Based on the economic performance by means of portfolio optimization, high returns are obtained. The height of the returns varies with the risk aversion. The maximum return achieved by the most risky strategy equals 25.225% in excess of the risk-free rate, with a corresponding Sharpe Ratio of 0.553. The bootstrap method, on the difference of the Sharpe Ratios between the FASTR models and the Buy-and-Hold benchmarks, concludes that the difference is, however, insignificant. Based on a 10% significance level, the FASTR.EL does outperform the Buy-and-Hold Small Cap significantly, according to the Sharpe Ratios.

Three expansions are added to the model, of which the first is the thresholding of the macroeconomic variables by the Least Angle Regressions and the financial variables by backwards elimination throughout the complete sample. The second fixes the logistic parameters, to reduce the number of parameters to be estimated and to add accuracy for predicting the next regime. The last is the use of factor selection. The RMSE, DM, CPS and DA show improvements in the predictions for most models, but the results of the EP and portfolio optimization were lacking relative to the standard model.

Overall, the research concludes that the FASTR model excluding the financial variables and using the lags of the dependent variable as exogenous variable contains the most promising results. But, in general, the improvements of the FASTR model seem insignificant over the linear FA model. Adding nonlinearity to the factor augmented models therefore does *not* affect the predictions enough so the model performs significantly better relative to factor augmentation alone.

The sub research question of the research, *'Do different regimes in the model affect the total amount of variance explained in the factor augmentation?'*, can be answered by a simple 'no'. The changes between the regimes indicated by the FASTR models did not show large changes in the explanation of the factors on the dependent variables.

A side conclusion is made regarding the performance of a model based on only one time series. For instance, after the expansions, the FASTR.EL shows positive significant results on all five tests for the Gold option, but leads to different conclusions for the other time series. Therefore, this research shows that it is dangerous to conclude on the value of a model, based on only one time series.

Notes for further investigation may include the use of the threshold model, as indicated in this research. This may suffice for the prediction of excess returns and realized volatility, rather than the use of the smooth transition model. Furthermore, there may be room for improvements in the factor selection, which, as indicated here, works rather unsatisfying as only one factor is taken into consideration, and the total explained variance is lacking.

References

[1] Anatolyev, S. & Gerko, A. (2005). A Trading Approach to Testing for Predictability. *Journal of Business & Economic Statistics*, Vol. 23, No. 4, 455-461.

[2] Ang, A. & Bekaert, G. (2002). International Asset Allocation with Regime Shifts. The *Review of Financial Studies*, Vol. 15, No. 4, 1137-1187.

[3] Ang, A. & Bekaert, G. (2004). How Regimes Affect Asset Allocation. *Financial Analysts Journal*, Vol. 60, No. 2, 86-99.

[4] Bai, J. (2010). Equity Premium Predictions with Adaptive Macro Indexes. *Federal Reserve Bank of New York Staff Reports,* No. 475.

[5] Bai, J. & Ng, S. (2008). Forecasting Economic Time Series using Targeted Predictors. *Journal of Econometrics,* Vol. 146, 304-317.

[6] Boivin, J. & Ng, S. (2006). Are more Data Always Better for Factor Analysis? *Journal of Econometrics*, Vol. 132, 169-194.

[7] Brandt, M.W. (2010). Portfolio Choice Problems. Chapter 5 from 'Handbook in Financial Econometrics, Volume I: Tools and Techniques'.

[8] Brandt, M.W., Goyal, A., Santa-Clara, P. & Stroud, J.R. (2005). A Simulation Approach to Dynamic Portfolio Choice with an Application to Learning About Return Predictability. *The Review of Financial Studies*, Vol. 18, No. 3, 831-873.

[9] Çakmakli, C. & van Dijk, D. (2010). Getting the Most out of Macroeconomic Information for Predicting Stock Returns and Volatility. *Tinbergen Institute Discussion Paper*, 2010-115/4.

[10] Campbell, J.Y. & Viceira, L.M. (2002). Strategic Asset Allocation: Portfolio Choice for Long-Term Investors. *Oxford University Press*.

[11] Chauvet, M. & Potter, S. (2000). Coincident and Leading Indicators of the Stock Market. *Journal of Empirical Finance*, Vol. 7, No. 1, 87-111.

[12] Chelley-Steeley, P.L. (2005). Modeling Equity Market Integration using Smooth Transition Analysis: A Study of Eastern European Stock Markets. *Journal of International Money and Finance*, Vol. 24, 818-831.

[13] Diebold, F.X. & Mariano, R.S. (2002). Comparing Predictive Ability. *Journal of Business & Economic Statistics*, Vol. 20, No. 1, 134-144.

[14] Efron, B., Hastie, T., Johnstone, I. & Tibshirani, R. (2004). Least Angle Regression. *The Annals of Statistics*, Vol. 32, No. 2, 407-451.

[15] Fama, E. & French, K. (1993). Common Risk Factors in the Returns of Stocks and Bonds. *Journal of Financial Economics*, Vol. 33, 3-56.

[16] French, K., Schwert, G. & Stambaugh, R. (1987). Expected Stock Returns and Volatility. *Journal of Financial Econometrics*, Vol. 19, 3-29.

[17] Giovannetti, B.C. (2011). Nonlinear Forecasting using Factor-Augmented Models. *Journal of Forecasting*, doi 10.1002/for. 1248.

[18] Guido, R., Pearl, J. & Walsh, K. (2011). Market Timing under Multiple Economic Regimes. *Accounting & Finance*, Vol. 51, 501-515.

[19] Guidolin, M. & Timmermann, A. (2005). Strategic Asset Allocation and Consumption Decisions under Multivariate Regime Switching. *Federal Reserve Bank of St. Louis*, working paper, No. 2005-002A.

[20] Guidolin, M. & Timmermann, A. (2006a). An Econometric Model of Nonlinear Dynamics in the Joint Distribution of Stock and Bond Returns. *Journal of Applied Econometrics*, Vol. 21, No. 1, 1-22.

[21] Guidolin, M. & Timmermann, A. (2006b). Term Structure of Risk under Alternative Econometric Specifications. *Journal of Econometrics*, Vol. 131, No. 1, 285-308.

[22] Guidolin, M. & Timmermann, A. (2008a). Size and Value Anomalies under Regime Switching. *Journal of Financial Econometrics*, Vol. 6, No. 1, 1-48.

[23] Guidolin, M. & Timmermann, A. (2008b). Asset Allocation under Multivariate Regime Switching. *Journal of Economic Dynamics and Control,* Vol. 31, No. 11, 3503-3544.

[24] Guidolin, M. & Timmermann, A. (2008c). International Asset Allocation under Regime Switching, Skew and Kurtosis Preferences. *The Review of Financial Studies*, Vol. 21, No. 2, 889-935.

[25] Hamilton, J. D. (1989). A new Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle. *Econometrica*, Vol. 57, No. 2, 357-384.

[26] Jobson, J.D. & Korkie, B.M. (1981). Performance Hypothesis Testing with the Sharpe and Treynor Measures. *Journal of Finance*, Vol. 36, 889-908.

[27] Ledoit, O. & Wolf, M. (2008). Robust Performance Hypothesis Testing with the Sharpe Ratio. *Journal of Empirical Finance*, Vol. 15, 850-859.

[28] Lin, C.-F. J. & Teräsvirta, T. (1993). Testing the Constancy of Regression Parameters against Continuous Structural Change. *Journal of Econometrics*, Vol. 62, 211-228.

[29] Ludvigson, S.C. & Ng, S. (2007). The Empirical Risk-Return Relation: A Factor Analysis Approach. *Journal of Financial Economics*, Vol. 83, 171-222.

[30] Memmel, C. (2003). Performance Hypothesis Testing with the Sharpe Ratio. *Financial Letters*, Vol. 1, 21-23.

[31] Perez-Quiros, G. & Timmermann, A. (2000). Firm Size and Cyclical Variations in Stock Returns. *The Journal of Finance*, Vol. 55, No. 3, 1229-1262.

[32] Pesaran, M.H. & Timmermann, A. (1992). A Simple Nonparametric Test for Predictive Performance. *Journal of Business & Economic Statistics*, Vol. 10, No. 4, 461-465.

[33] Politis, D.N. & Romano, J.P. (1992). A Circular Block-Resampling Procedure for Stationary Data. *Exploring the Limits of Bootstrap* (R. Lepage and L. Billard, eds.), 263-270.

[34] Scholes, M. & Williams, J. (1977). Estimating Betas from Nonsynchronous Data. *Journal of Financial Economics*, Vol. 5, 309-327.

[35] Stock, J.H. & Watson, M.W. (2002a). Macroeconomic Forecasting using Diffusion Indexes. *Journal of Business & Economic Statistics*, Vol. 20, No. 2, 147-162.

[36] Stock, J.H. & Watson, M.W. (2002b). Forecasting using Principal Components from a Large Number of Predictors. *Journal of the American Statistical Association*, Vol. 97, No. 460, 1167-1179.

[37] Stock, J.H. & Watson, M.W. (2005). Implications of Dynamic Factor Models for VAR Analysis. *NBER Working Paper,* No. 11467.

[38] Teräsvirta, T. & Anderson, H.M. (1992). Characterizing Nonlinearities in Business Cycles using Smooth Transition Autoregressive Models. *Journal of Applied Econometrics*, Vol. 7, S119-S136.

[39] Teräsvirta, T. (1994). Specification, Estimation, and Evaluation of Smooth Transition Autoregressive Models. *Journal of the American Statistical Association*, Vol. 89, No. 425, 208-218.

[40] Tibshirani, R. (1996). Regression Shrinkage and Selection via the Lasso. *Journal of the Royal Statisctical Society. Series B (Methodological)*, Vol. 58, No. 1, 267-288.

[41] Tu, J. (2010). Is Regime Switching in Stock Returns Important in Portfolio Decisions? *Management Science*, Vol. 56, No. 7, 1198-1215.

[42] Welch, I. & Goyal, A. (2008). A Comprehensive Look at the Empirical Performance of Equity Premium Prediction. *The Review of Financial Studies*, Vol. 21, No. 4, 1455-1508.

[43] Van Dijk, D. & Franses, P.H. (1999). Modeling Multiple Regimes in the Business Cycle. *Macroeconomic Dynamics*, Vol. 3, 311-340.

[44] Van Dijk, D., Teräsvirta, T. & Franses, P.H. (2000). Smooth Transition Autoregressive Models: A Survey of Recent Developments. *Econometric Reviews*, Vol. 21, 1-47.

[45] Zou, H. & Hastie, T. (2005). Regularization and Variable Selection via the Elastic Net. *Journal of Royal Statistical Society*, Series B, Vol. 67, No. 2, 301-320.

Appendix A. Extra Figures and Tables

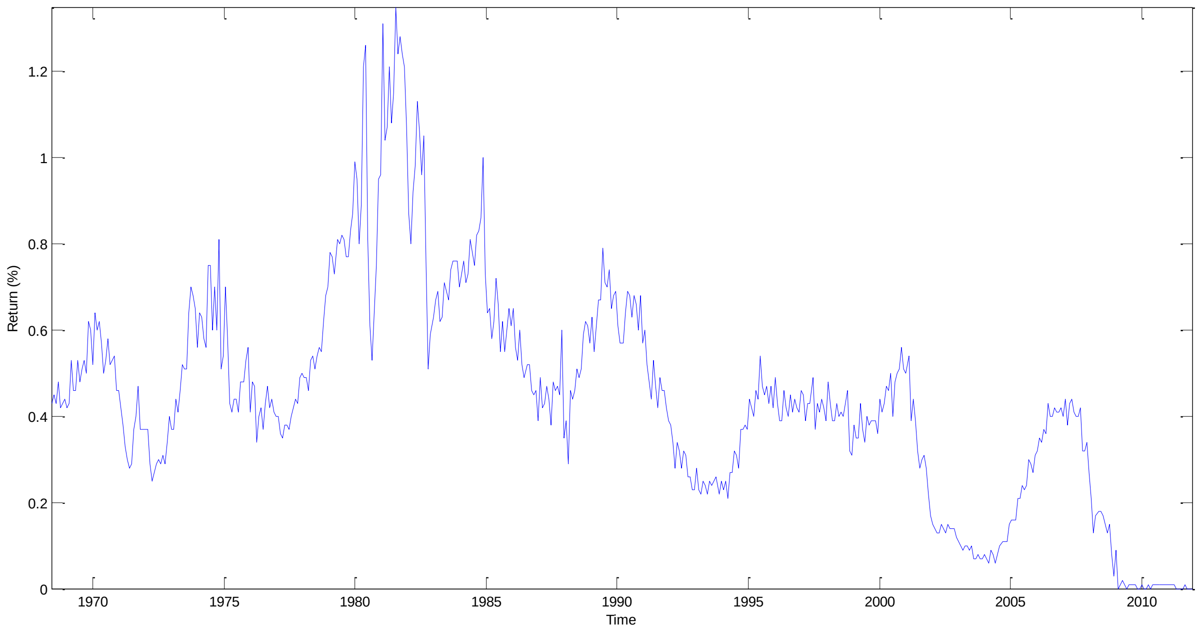

Figure A.1. The 1-month U.S. Treasury Bill, used as the risk free rate.

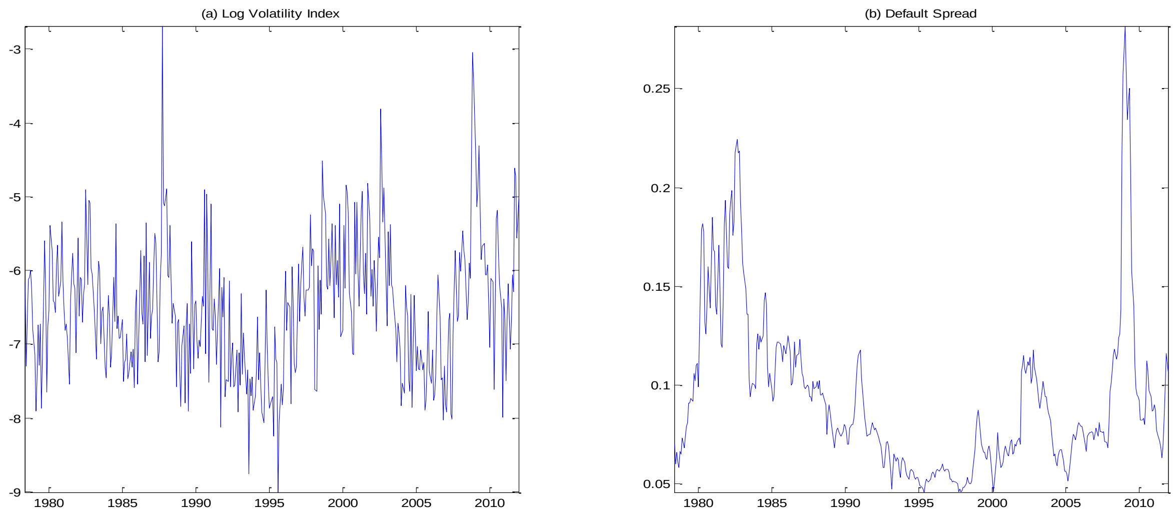

Figure A.2. The log Volatility Index and the Default Spread. Both are used as exogenous variables in the logistic function.

DM Complete Sample	Small Cap		Medium Cap		Big Cap		S&P 500		Gold	
	$r_{e,t+1}$	RV_{t+1}	$r_{e,t+1}$	RV_{t+1}	$r_{e,t+1}$	RV_{t+1}	$r_{e,t+1}$	RV_{t+1}	$r_{e,t+1}$	RV_{t+1}
FASTR.NL	-3.252	-2.269	-2.156	-1.245	-2.932	-1.156	-2.305	-1.602	-1.350	0.430
FASTR.NV	-2.806	-1.770	-2.684	-1.491	-2.955	-1.219	-3.143	-1.638	-1.742	-1.557
FASTR.NE	-3.437	-2.182	-2.394	-1.918	-2.402	-1.684	-2.589	-1.589	-1.100	-1.280
FASTR.LL	-1.817	-3.013	-2.310	-0.709	-2.261	0.103	-2.012	0.394	0.381	**3.540**
FASTR.LV	-1.957	-1.830	-2.575	-0.642	-2.839	0.813	-2.679	0.336	-1.890	-3.282
FASTR.LE	-2.348	-1.566	-1.501	-1.503	-1.335	-1.310	-2.198	-1.815	-1.031	-0.993
FASTR.EL	0.505	-3.124	-1.068	-1.279	0.044	-0.090	-0.743	-1.183	**1.812**	**2.416**
FASTR.EV	-0.222	-0.187	-0.533	-0.633	-0.582	-0.362	-0.227	-0.740	-0.042	-2.918
FASTR.EE	-0.584	-1.596	-0.943	-1.634	-0.930	-2.391	-0.725	-2.787	0.094	-0.893
1978/06 – 1991/12										
FASTR.NL	-1.716	-1.461	-1.620	-1.379	-0.575	-0.817	-0.407	-1.463	-1.069	0.476
FASTR.NV	-1.985	-1.995	-1.768	-1.782	-1.823	-1.059	-1.830	-0.897	-1.292	-1.429
FASTR.NE	-1.834	-2.228	-2.928	-1.219	-2.543	-0.884	-1.966	-0.889	-1.847	-0.551
FASTR.LL	-1.214	-2.873	-0.656	-0.237	-0.615	**2.016**	-0.598	-0.682	0.245	**2.717**
FASTR.LV	-1.434	-0.893	-2.872	-0.960	-2.570	-0.762	-1.234	-0.319	-1.322	-3.517
FASTR.LE	-0.008	-0.456	-1.044	-1.032	-0.457	-0.410	-0.071	-0.786	-1.319	-0.744
FASTR.EL	0.098	-2.465	-0.085	0.706	0.431	0.562	0.606	0.562	1.596	**1.819**
FASTR.EV	0.251	1.179	-0.317	0.454	0.206	0.715	0.691	0.715	0.090	-3.189
FASTR.EE	0.029	-0.411	-0.688	-1.037	-0.436	-1.896	0.173	-1.896	0.779	-0.926
1992/01 – 2004/12										
FASTR.NL	-1.842	-1.482	-1.565	-0.628	-2.184	-0.114	-1.651	0.628	-1.467	0.843
FASTR.NV	-1.198	-0.868	-2.180	-0.849	-2.041	-0.852	-1.763	-0.460	-1.590	-1.629
FASTR.NE	-1.208	-2.139	-1.967	-2.463	-2.329	-2.423	-2.168	-3.134	-2.653	0.120
FASTR.LL	-0.143	-0.644	-0.961	0.034	-0.714	0.962	-0.753	**1.844**	0.601	**2.310**
FASTR.LV	-0.960	-0.519	-1.632	-0.489	-2.363	-1.348	-2.170	0.158	-0.706	0.404
FASTR.LE	-1.930	-1.200	-0.045	-1.238	-1.199	-1.259	-2.634	-1.084	-2.787	-1.279
FASTR.EL	1.056	-0.175	0.159	-0.380	-0.096	0.559	-0.298	1.338	1.360	**2.258**
FASTR.EV	0.588	0.003	0.356	-1.375	-0.787	-1.773	-0.096	-0.974	1.223	-0.623
FASTR.EE	1.161	-0.507	1.056	-3.517	0.730	-3.254	0.746	-2.862	-0.239	0.892
2005/01 – 2011/11										
FASTR.NL	-2.358	-1.551	-1.848	-0.726	-2.470	-0.923	-1.981	-1.643	-1.537	-0.270
FASTR.NV	-1.670	-1.016	-1.438	-0.785	-1.663	-0.688	-2.128	-1.387	-0.882	-1.707
FASTR.NE	-2.793	-1.723	-1.537	-1.489	-1.555	-1.391	-1.642	-1.158	-1.063	-1.364
FASTR.LL	-1.626	-1.450	-2.068	-0.731	-2.095	-0.524	-1.783	-0.215	0.108	**1.678**
FASTR.LV	-1.360	-1.685	-1.297	-0.316	-1.357	-0.377	-1.541	0.513	-1.278	-1.112
FASTR.LE	-2.079	-1.488	-1.268	-1.403	-0.934	-1.173	-1.908	-1.541	-1.016	0.066
FASTR.EL	-0.127	-2.046	-1.676	-1.432	-0.326	-0.831	-1.034	-1.793	0.343	1.385
FASTR.EV	-0.831	-0.600	-0.696	-0.433	-0.416	-0.369	-0.916	0.771	-0.921	0.499
FASTR.EE	-1.524	-1.550	-1.236	-1.466	-1.185	-1.570	-1.436	-1.953	-0.884	-0.497

Table A.3. Diebold-Mariano statistic, based on the mean squared errors computed in Section 4.1, computed over the complete sample, as well as three sub-periods. A positive value indicates a positive performance of the tested model relative to the benchmark, which is the linear Factor Augmentation model. The critical level is 1.645, corresponding to a significance level of 5%. The significant positive values are bold.

DA	Small Cap		Medium Cap		Big Cap		S&P 500		Gold	
Complete Sample	$r_{e,t+1}$	RV_{t+1}	$r_{e,t+1}$	RV_{t+1}	$r_{e,t+1}$	RV_{t+1}	$r_{e,t+1}$	RV_{t+1}	$r_{e,t+1}$	RV_{t+1}
FA	1.302	5.254	1.162	5.375	1.099	6.067	**1.317**	7.040	-0.313	6.381
FASTR.NL	1.422	4.521	0.944	3.930	0.648	6.694	0.933	8.235	1.002	6.401
FASTR.NV	0.174	5.595	0.552	5.371	0.696	6.671	0.930	6.507	1.498	6.186
FASTR.NE	-0.692	4.905	-1.437	6.313	-0.294	6.173	0.083	5.578	-0.803	6.502
FASTR.LL	1.720	4.648	1.621	5.145	**1.362**	7.715	0.662	8.059	0.405	**8.735**
FASTR.LV	**1.725**	**6.612**	0.813	**7.212**	-0.008	6.465	-0.235	6.613	1.305	5.506
FASTR.LE	0.359	6.463	-0.064	7.166	0.144	6.680	-0.476	5.410	-1.117	5.959
FASTR.EL	0.723	2.597	**1.725**	5.168	0.234	**8.016**	0.911	**8.730**	**1.823**	6.345
FASTR.EV	-0.327	6.394	0.144	5.851	0.929	6.665	1.244	7.564	1.607	3.970
FASTR.EE	0.358	6.049	-0.521	5.167	0.188	3.938	0.475	2.899	0.901	6.599
1978/06 – 1991/12										
FA	2.958	1.564	2.153	1.621	1.348	1.318	1.412	2.529	-1.337	3.419
FASTR.NL	**3.324**	0.121	**2.723**	1.063	**2.199**	1.740	**1.846**	3.188	0.265	2.946
FASTR.NV	0.718	0.681	0.487	2.075	0.375	2.012	0.630	2.493	-0.306	3.419
FASTR.NE	0.987	0.609	-0.117	2.050	0.340	1.709	0.751	2.780	-2.413	2.992
FASTR.LL	1.867	0.574	2.000	1.506	1.677	2.161	1.163	2.077	-0.618	**5.264**
FASTR.LV	2.133	2.399	1.196	2.184	0.632	1.863	0.555	2.438	-0.788	2.805
FASTR.LE	0.996	1.604	0.255	2.677	0.362	2.592	0.235	2.593	-2.244	3.088
FASTR.EL	0.737	0.548	0.851	**2.785**	-0.067	**3.907**	0.550	3.175	**0.912**	3.940
FASTR.EV	0.571	**2.706**	-0.520	2.440	0.073	2.876	0.681	**4.395**	0.741	0.585
FASTR.EE	0.853	0.259	0.192	1.145	0.593	0.627	0.889	1.837	-0.547	1.140
1992/01 – 2004/12										
FA	-0.957	3.799	-1.204	2.739	-0.730	4.543	0.069	5.014	0.013	4.219
FASTR.NL	-0.013	5.410	-0.703	3.011	-0.997	5.746	-0.466	7.010	-0.042	3.453
FASTR.NV	0.013	5.305	-0.586	5.341	0.192	5.272	**0.996**	5.029	1.401	2.006
FASTR.NE	-1.678	4.788	-1.540	4.300	-0.436	4.096	-0.231	2.113	-0.161	3.453
FASTR.LL	0.404	4.388	0.152	3.056	-0.156	**6.376**	-0.402	**7.152**	0.190	**5.220**
FASTR.LV	0.348	5.227	-0.158	**6.593**	-1.620	5.147	-0.788	5.408	**2.042**	1.732
FASTR.LE	-0.548	4.710	-0.844	6.138	-1.006	4.032	-1.447	2.668	-0.843	3.426
FASTR.EL	**0.611**	3.671	**1.910**	2.363	**0.338**	5.531	0.679	6.823	0.142	4.292
FASTR.EV	-1.078	5.092	0.323	5.005	-0.017	2.858	0.255	2.969	0.125	-0.002
FASTR.EE	-0.402	**5.817**	-1.033	5.735	-0.766	3.379	-0.502	2.198	0.615	2.073
2005/01 – 2011/11										
FA	0.222	0.746	**1.450**	4.220	1.510	4.160	0.828	4.878	-0.986	0.270
FASTR.NL	-1.283	0.482	-0.658	3.178	-0.038	2.923	0.504	3.603	0.797	**2.551**
FASTR.NV	-0.614	-1.394	1.273	2.133	0.819	3.640	-0.274	4.160	0.126	-0.580
FASTR.NE	-0.568	0.130	-1.005	4.587	-0.674	**5.437**	-0.392	5.369	-0.451	0.270
FASTR.LL	**0.933**	**2.552**	0.599	**5.102**	0.973	4.645	0.222	**5.625**	-0.041	0.788
FASTR.LV	0.736	-0.342	0.457	3.047	1.380	3.937	-0.410	4.630	-0.050	-0.742
FASTR.LE	0.346	1.401	0.865	3.481	**1.869**	4.301	**1.138**	3.902	0.526	-1.631
FASTR.EL	-0.270	0.746	0.034	2.439	0.143	4.803	0.012	5.549	**0.914**	1.652
FASTR.EV	-0.270	-0.865	0.255	-1.144	1.409	4.897	0.821	4.878	0.768	-1.383
FASTR.EE	0.286	-0.865	0.222	-0.674	0.934	0.813	0.599	0.101	-0.451	-0.181

Table A.4. Directional Accuracy (DA) test statistics, based on the CPS of Section 4.1. Computed over the complete sample, as well as for three sub-periods. A positive value indicates a positive performance on the accuracy of the time series. The critical level is 1.645, corresponding to a significance level of 5%.

# Occurrences	Good Regime	Bad Regime	Neutral Regime
FASTR.NL	299	90	13
FASTR.NV	278	80	44
FASTR.NE	189	180	33
FASTR.LL	253	112	37
FASTR.LV	230	100	72
FASTR.LE	173	166	63
FASTR.EL	208	145	49
FASTR.EV	252	115	35
FASTR.EE	166	177	59

Table A.5. Number of occurrences for three regimes. The regimes are computed by taking the optimized parameters of the Big Cap excess returns in the logistic function. A good regime captures the best 40% of the values, a bad regime the least 40%, and the neutral regime the remaining 20% in the middle.

Weights	**No Short-Selling**					**Short-Selling Allowed**				
$\gamma = 2$	R_f	Small	Medium	Big	Gold	R_f	Small	Medium	Big	Gold
FA	0.17	0.25	0.15	0.13	0.30	0.00	0.34	0.21	-0.02	0.47
Complete Sample										
FASTR.NL	0.17	0.21	0.16	0.18	0.27	0.11	0.10	0.23	0.24	0.32
FASTR.NV	0.17	0.26	0.17	0.16	0.24	0.04	0.32	0.32	0.19	0.14
FASTR.NE	0.11	0.30	0.15	0.17	0.27	-0.12	0.44	0.23	0.23	0.23
FASTR.LL	0.17	0.19	0.19	0.17	0.27	0.02	0.07	0.28	0.30	0.33
FASTR.LV	0.22	0.24	0.12	0.13	0.28	0.21	0.18	0.16	0.08	0.37
FASTR.LE	0.15	0.29	0.14	0.18	0.23	-0.00	0.34	0.37	0.16	0.13
FASTR.EL	0.11	0.30	0.22	0.17	0.19	-0.15	0.52	0.52	0.18	-0.07
FASTR.EV	0.10	0.28	0.12	0.25	0.26	-0.23	0.73	0.30	0.15	0.05
FASTR.EE	0.13	0.25	0.10	0.23	0.29	-0.17	0.37	0.24	0.33	0.22
Good Regime										
FASTR.NL	0.17	0.21	0.18	0.17	0.27	0.10	0.06	0.30	0.27	0.27
FASTR.NV	0.17	0.23	0.18	0.16	0.27	0.02	0.19	0.36	0.21	0.22
FASTR.NE	0.07	0.30	0.14	0.21	0.27	-0.25	0.49	0.22	0.38	0.16
FASTR.LL	0.20	0.22	0.20	0.16	0.23	0.11	0.17	0.27	0.27	0.18
FASTR.LV	0.18	0.25	0.12	0.15	0.30	0.06	0.19	0.28	0.11	0.37
FASTR.LE	0.18	0.25	0.12	0.19	0.27	0.23	0.16	0.16	0.15	0.31
FASTR.EL	0.12	0.23	0.27	0.19	0.19	-0.06	0.25	0.57	0.34	-0.10
FASTR.EV	0.07	0.27	0.11	0.25	0.29	-0.34	0.71	0.31	0.15	0.17
FASTR.EE	0.20	0.07	0.04	0.30	0.39	0.30	-0.21	-0.26	0.68	0.50
Bad Regime										
FASTR.NL	0.19	0.21	0.09	0.22	0.28	0.24	0.22	0.02	0.13	0.39
FASTR.NV	0.19	0.26	0.17	0.17	0.21	0.21	0.40	0.12	0.31	-0.04
FASTR.NE	0.15	0.30	0.16	0.10	0.28	-0.05	0.48	0.30	-0.06	0.33
FASTR.LL	0.11	0.15	0.18	0.21	0.36	-0.11	-0.13	0.33	0.27	0.64
FASTR.LV	0.24	0.20	0.15	0.13	0.28	0.26	0.17	0.07	0.28	0.21
FASTR.LE	0.15	0.37	0.18	0.09	0.21	-0.20	0.66	0.69	-0.16	0.01
FASTR.EL	0.08	0.42	0.18	0.15	0.17	-0.33	0.91	0.52	-0.01	-0.09
FASTR.EV	0.16	0.31	0.10	0.20	0.24	0.11	0.67	0.24	0.02	-0.04
FASTR.EE	0.08	0.45	0.13	0.08	0.25	-0.52	1.10	0.53	-0.29	0.18
Neutral Regime										
FASTR.NL	0.12	0.20	0.19	0.23	0.26	-0.47	0.20	0.13	0.31	0.83
FASTR.NV	0.10	0.47	0.17	0.15	0.11	-0.19	1.01	0.41	-0.14	-0.08
FASTR.NE	0.13	0.17	0.13	0.31	0.26	0.23	-0.07	-0.18	0.97	0.05
FASTR.LL	0.13	0.20	0.19	0.14	0.34	-0.16	-0.01	0.23	0.57	0.37
FASTR.LV	0.32	0.26	0.11	0.07	0.24	0.64	0.15	-0.10	-0.30	0.61
FASTR.LE	0.09	0.22	0.08	0.41	0.19	-0.12	0.01	0.10	1.06	-0.05
FASTR.EL	0.15	0.25	0.16	0.16	0.27	-0.01	0.52	0.34	0.02	0.14
FASTR.EV	0.07	0.19	0.28	0.37	0.09	-0.57	1.01	0.44	0.592	-0.48
FASTR.EE	0.05	0.15	0.17	0.48	0.15	-0.41	-0.16	0.76	1.23	-0.42

Table A.6. Average of the weights, for two forms of determining the weights. In the first form the investor is not allowed for go short, in the second form the investor can go short, which enlarges the interval of possible weights to [-1, 2]. The S&P500 is left out, because of the high similarities between the Big Cap and S&P. The good, bad and neutral regimes are computed by taking the optimized parameters of the Big Cap excess returns in the logistic function. A good regime captures the best 40% of the values, a bad regime the least 40%, and the neutral regime the remaining 20% in the middle. The risk aversion coefficient is taken to be 2.

Weights $\gamma = 10$	No Short-Selling					Short-Selling Allowed				
	R_f	Small	Medium	Big	Gold	R_f	Small	Medium	Big	Gold
FA	0.28	0.17	0.13	0.11	0.31	0.33	0.11	0.20	-0.03	0.39
Complete Sample										
FASTR.NL	0.28	0.18	0.13	0.17	0.25	0.39	0.01	0.10	0.28	0.21
FASTR.NV	0.27	0.22	0.14	0.14	0.23	0.46	0.20	0.13	0.07	0.13
FASTR.NE	0.21	0.26	0.12	0.16	0.25	0.25	0.27	0.06	0.21	0.21
FASTR.LL	0.27	0.15	0.16	0.15	0.27	0.32	-0.10	0.14	0.28	0.36
FASTR.LV	0.32	0.20	0.10	0.13	0.25	0.49	0.11	0.08	0.08	0.24
FASTR.LE	0.25	0.25	0.12	0.16	0.22	0.28	0.19	0.22	0.12	0.18
FASTR.EL	0.30	0.27	0.14	0.13	0.16	0.50	0.41	0.24	-0.12	-0.03
FASTR.EV	0.30	0.25	0.07	0.18	0.20	0.53	0.50	-0.02	-0.01	-0.00
FASTR.EE	0.26	0.23	0.07	0.20	0.24	0.38	0.20	-0.03	0.33	0.13
Good Regime										
FASTR.NL	0.27	0.17	0.15	0.16	0.25	0.38	-0.07	0.15	0.33	0.20
FASTR.NV	0.28	0.19	0.14	0.15	0.25	0.43	0.07	0.20	0.09	0.21
FASTR.NE	0.16	0.25	0.12	0.21	0.25	0.15	0.26	0.02	0.31	0.27
FASTR.LL	0.28	0.16	0.17	0.14	0.24	0.36	-0.07	0.14	0.26	0.31
FASTR.LV	0.27	0.21	0.09	0.15	0.28	0.34	0.15	0.09	0.11	0.32
FASTR.LE	0.28	0.20	0.09	0.19	0.23	0.37	0.01	0.11	0.24	0.27
FASTR.EL	0.33	0.19	0.16	0.14	0.17	0.55	0.15	0.39	-0.06	-0.04
FASTR.EV	0.27	0.25	0.06	0.18	0.24	0.42	0.55	-0.05	-0.02	0.10
FASTR.EE	0.36	0.06	0.04	0.23	0.31	0.62	-0.37	-0.22	0.77	0.20
Bad Regime										
FASTR.NL	0.33	0.19	0.08	0.18	0.23	0.48	0.26	-0.08	0.14	0.21
FASTR.NV	0.32	0.24	0.14	0.11	0.19	0.63	0.27	-0.02	0.15	-0.03
FASTR.NE	0.23	0.29	0.13	0.11	0.25	0.29	0.36	0.13	0.02	0.20
FASTR.LL	0.24	0.13	0.14	0.17	0.32	0.28	-0.17	0.19	0.28	0.43
FASTR.LV	0.36	0.19	0.11	0.12	0.22	0.55	-0.00	0.06	0.25	0.15
FASTR.LE	0.22	0.34	0.16	0.07	0.21	0.16	0.51	0.44	-0.22	0.12
FASTR.EL	0.25	0.38	0.13	0.11	0.14	0.39	0.76	0.06	-0.20	-0.01
FASTR.EV	0.41	0.25	0.06	0.14	0.15	0.78	0.46	-0.03	-0.04	-0.17
FASTR.EE	0.18	0.43	0.08	0.09	0.22	0.05	0.89	0.09	-0.24	0.22
Neutral Regime										
FASTR.NL	0.18	0.16	0.16	0.24	0.26	-0.00	0.06	0.15	0.29	0.49
FASTR.NV	0.14	0.43	0.15	0.14	0.13	0.37	0.95	-0.08	-0.18	-0.06
FASTR.NE	0.36	0.16	0.09	0.18	0.21	0.60	-0.11	-0.11	0.63	-0.01
FASTR.LL	0.25	0.17	0.13	0.11	0.34	0.14	-0.07	0.05	0.41	0.47
FASTR.LV	0.43	0.19	0.11	0.05	0.22	0.92	0.15	0.07	-0.24	0.10
FASTR.LE	0.25	0.16	0.06	0.33	0.20	0.36	-0.13	-0.03	0.72	0.07
FASTR.EL	0.35	0.25	0.08	0.12	0.19	0.59	0.49	0.08	-0.12	-0.04
FASTR.EV	0.19	0.20	0.20	0.29	0.12	0.46	0.27	0.27	0.18	-0.18
FASTR.EE	0.21	0.11	0.13	0.46	0.10	0.69	-0.29	0.12	0.80	-0.33

Table A.7. Average of the weights, for two forms of determining the weights. In the first form the investor is not allowed for go short, in the second form the investor can go short, which enlarges the interval of possible weights to [-1, 2]. The S&P500 is left out, because of the high similarities between the Big Cap and S&P. The good, bad and neutral regimes are computed by taking the optimized parameters of the Big Cap excess returns in the logistic function. A good regime captures the best 40% of the values, a bad regime the least 40%, and the neutral regime the remaining 20% in the middle. The risk aversion coefficient is taken to be 10.

Economic Performance $\gamma = 5$	**No Short-Selling Allowed**			**Short-Selling Allowed**		
	Average Annualized Excess Return	Annualized Volatility	Sharpe Ratio	Average Annualized Excess Return	Annualized Volatility	Sharpe Ratio
1978/06 – 1991/12						
Buy-and-Hold R_f	0.000%	0.764%	0.000			
Buy-and-Hold Small	10.666%	20.195%	0.528			
Buy-and-Hold Big	9.878%	16.737%	0.590			
FA	8.520%	14.362%	0.593	15.833%	34.453%	0.460
FASTR.NL	**15.229%**	15.258%	0.998	**38.351%**	36.746%	**1.044**
FASTR.NV	15.038%	14.877%	**1.011**	29.364%	35.047%	0.838
FASTR.NE	4.832%	17.857%	0.271	9.390%	42.127%	0.223
FASTR.LL	8.309%	14.124%	0.588	17.798%	34.199%	0.520
FASTR.LV	6.804%	**13.516%**	0.503	14.073%	**32.048%**	0.439
FASTR.LE	7.496%	15.014%	0.499	17.606%	37.592%	0.468
FASTR.EL	14.005%	18.517%	0.756	14.257%	38.259%	0.373
FASTR.EV	11.726%	18.972%	0.618	21.110%	33.307%	0.634
FASTR.EE	11.540%	17.972%	0.642	22.751%	39.231%	0.580
1992/01 – 2004/12						
Buy-and-Hold R_f	0.000%	0.478%	0.000			
Buy-and-Hold Small	12.440%	20.423%	0.609			
Buy-and-Hold Big	9.526%	15.673%	0.608			
FA	4.146%	**11.700%**	0.354	2.265%	**26.904%**	0.084
FASTR.NL	2.610%	14.108%	0.185	1.669%	33.330%	0.050
FASTR.NV	5.884%	13.501%	0.436	-1.462%	33.290%	-0.044
FASTR.NE	2.271%	13.182%	0.172	-5.464%	32.225%	-0.170
FASTR.LL	6.113%	14.930%	0.409	7.360%	33.683%	0.219
FASTR.LV	7.212%	15.286%	0.472	7.072%	35.706%	0.198
FASTR.LE	3.755%	12.643%	0.297	0.591%	28.456%	0.021
FASTR.EL	8.513%	15.457%	0.551	14.720%	32.916%	0.447
FASTR.EV	**10.695%**	15.619%	**0.685**	**19.881%**	36.275%	**0.548**
FASTR.EE	4.711%	15.758%	0.299	-0.697%	29.551%	-0.024
2005/01 – 2011/11						
Buy-and-Hold R_f	0.000%	0.574%	0.000			
Buy-and-Hold Small	7.005%	23.950%	0.293			
Buy-and-Hold Big	4.780%	18.532%	0.258			
FA	8.244%	**12.156%**	0.678	13.325%	25.469%	0.523
FASTR.NL	3.088%	12.930%	0.239	8.133%	30.639%	0.265
FASTR.NV	10.513%	15.431%	0.681	23.962%	27.754%	0.863
FASTR.NE	4.294%	18.228%	0.236	2.247%	32.156%	0.070
FASTR.LL	9.690%	13.460%	0.720	9.400%	27.566%	0.341
FASTR.LV	5.724%	15.608%	0.367	4.544%	29.454%	0.154
FASTR.LE	8.878%	14.943%	0.594	12.155%	35.202%	0.345
FASTR.EL	**14.890%**	14.779%	**1.008**	12.374%	**24.694%**	0.501
FASTR.EV	5.573%	13.234%	0.421	-0.343%	28.663%	-0.012
FASTR.EE	10.748%	13.014%	0.826	**29.724%**	32.019%	**0.928**

Table A.8. Annualized excess returns, volatility and Sharpe Ratio for three sub-periods, with a risk aversion of 5. The excess returns are compounded, which means that earned profits are reinvested. In the first form the investor is not allowed for go short, in the second form the investor can go short, which enlarges the interval of possible weights to [-1, 2]. The S&P500 is left out, because of the high similarities between the Big Cap and S&P.

Economic Performance $\gamma = 2$	**No Short-Selling Allowed**			**Short-Selling Allowed**		
	Average Annualized Excess Return	Annualized Volatility	Sharpe Ratio	Average Annualized Excess Return	Annualized Volatility	Sharpe Ratio
Buy-and-Hold R_f	0.000%	0.980%	0.000			
Buy-and-Hold Small	10.573%	21.077%	0.502			
Buy-and-Hold Big	8.644%	16.722%	0.517			
Complete Sample						
FA	6.580%	**14.381%**	0.458	12.035%	**37.965%**	0.317
FASTR.NL	7.964%	15.201%	0.524	16.753%	40.696%	0.412
FASTR.NV	10.318%	15.184%	0.680	18.662%	42.064%	0.444
FASTR.NE	3.867%	16.809%	0.230	-1.705%	44.901%	-0.038
FASTR.LL	8.419%	15.353%	0.548	9.011%	41.880%	0.215
FASTR.LV	6.783%	15.224%	0.446	5.043%	41.332%	0.122
FASTR.LE	6.294%	14.995%	0.420	6.950%	42.734%	0.163
FASTR.EL	**14.054%**	17.954%	**0.783**	**25.225%**	45.617%	**0.553**
FASTR.EV	11.197%	17.906%	0.625	23.175%	45.763%	0.506
FASTR.EE	9.287%	17.393%	0.534	22.898%	46.025%	0.498
Good Regimes						
FASTR.NL	9.102%	15.236%	0.597	23.688%	41.095%	0.576
FASTR.NV	8.955%	15.163%	0.591	**24.022%**	40.213%	**0.597**
FASTR.NE	0.376%	16.210%	0.023	-10.889%	42.646%	-0.255
FASTR.LL	6.488%	14.609%	0.444	4.591%	40.391%	0.114
FASTR.LV	4.997%	15.142%	0.330	12.191%	38.812%	0.314
FASTR.LE	2.524%	15.023%	0.168	-13.328%	39.523%	-0.337
FASTR.EL	**12.948%**	18.711%	**0.692**	16.066%	46.236%	0.348
FASTR.EV	8.822%	17.305%	0.510	23.895%	43.615%	0.548
FASTR.EE	5.537%	**14.135%**	0.392	6.707%	**37.423%**	0.179
Bad Regimes						
FASTR.NL	5.586%	15.148%	0.369	1.136%	**40.349%**	0.028
FASTR.NV	13.153%	17.572%	0.749	10.279%	51.610%	0.192
FASTR.NE	5.754%	17.745%	0.324	11.993%	46.644%	0.257
FASTR.LL	13.603%	17.245%	0.789	20.447%	44.985%	0.455
FASTR.LV	16.475%	17.035%	**0.967**	3.867%	49.405%	0.078
FASTR.LE	12.229%	**14.682%**	0.833	33.049%	46.255%	0.715
FASTR.EL	15.757%	17.462%	0.902	31.113%	48.231%	0.645
FASTR.EV	**16.550%**	20.426%	0.810	19.736%	52.104%	0.379
FASTR.EE	13.323%	19.884%	0.670	**42.817%**	50.452%	**0.849**
Neutral Regimes						
FASTR.NL	-0.849%	15.412%	-0.055	-16.775%	**31.894%**	-0.526
FASTR.NV	13.857%	**10.004%**	**1.385**	2.500%	33.922%	0.074
FASTR.NE	**14.586%**	14.744%	0.989	-15.610%	47.547%	-0.328
FASTR.LL	6.291%	14.449%	0.435	16.051%	42.925%	0.374
FASTR.LV	-0.187%	12.316%	-0.015	-13.759%	36.669%	-0.375
FASTR.LE	1.744%	15.554%	0.223	6.133%	39.737%	0.154
FASTR.EL	13.850%	16.292%	0.850	**50.958%**	33.516%	**1.520**
FASTR.EV	11.191%	12.801%	0.874	29.596%	39.359%	0.752
FASTR.EE	8.244%	17.734%	0.465	16.260%	53.139%	0.306

Table A.9. Annualized excess returns, volatility and Sharpe Ratio for the average weights given in Table A.6, with a risk aversion of 2. The excess returns are compounded, which means that earned profits are reinvested. In the first form the investor is not allowed for go short, in the second form the investor can go short, which enlarges the interval of possible weights to [-1, 2]. The S&P500 is left out, because of the high similarities between the Big Cap and S&P. The good, bad and neutral regimes are computed by taking the optimized parameters of the Big Cap excess returns in the logistic function. A good regime captures the best 40% of the values, a bad regime the least 40%, and the neutral regime the remaining 20% in the middle.

Economic Performance $\gamma = 10$	**No Short-Selling Allowed**			**Short-Selling Allowed**		
	Average Annualized Excess Return	Annualized Volatility	Sharpe Ratio	Average Annualized Excess Return	Annualized Volatility	Sharpe Ratio
Buy-and-Hold R_f	0.000%	0.980%	0.000			
Buy-and-Hold Small	10.573%	21.077%	0.502			
Buy-and-Hold Big	8.644%	16.722%	0.517			
Complete Sample						
FA	6.782%	**11.750%**	0.577	8.547%	23.137%	0.369
FASTR.NL	7.985%	13.473%	0.593	**14.254%**	29.272%	0.487
FASTR.NV	**9.696%**	13.117%	**0.739**	13.736%	26.736%	**0.497**
FASTR.NE	4.196%	15.127%	0.277	0.832%	29.693%	0.028
FASTR.LL	7.702%	12.942%	0.595	10.883%	25.114%	0.433
FASTR.LV	6.085%	13.732%	0.443	8.495%	25.157%	0.338
FASTR.LE	5.449%	13.039%	0.418	7.808%	26.640%	0.293
FASTR.EL	8.671%	14.692%	0.590	7.815%	24.483%	0.319
FASTR.EV	8.533%	13.922%	0.613	10.041%	**22.549%**	0.445
FASTR.EE	8.022%	14.303%	0.561	10.852%	26.586%	0.408
Good Regimes						
FASTR.NL	**9.419%**	13.788%	**0.683**	**17.157%**	31.146%	**0.551**
FASTR.NV	8.686%	13.261%	0.655	15.376%	26.666%	0.577
FASTR.NE	0.105%	14.366%	0.007	-7.670%	27.930%	-0.275
FASTR.LL	6.376%	12.500%	0.510	8.197%	25.845%	0.317
FASTR.LV	4.203%	13.569%	0.310	12.216%	22.815%	0.535
FASTR.LE	1.722%	12.885%	0.134	-3.067%	23.916%	-0.128
FASTR.EL	9.033%	15.194%	0.595	5.013%	24.968%	0.201
FASTR.EV	6.795%	13.934%	0.488	9.676%	23.492%	0.412
FASTR.EE	3.170%	**12.159%**	0.261	-3.190%	**22.031%**	-0.145
Bad Regimes						
FASTR.NL	3.981%	**12.494%**	0.319	5.784%	23.837%	0.243
FASTR.NV	11.944%	14.569%	0.820	12.148%	29.984%	0.405
FASTR.NE	7.958%	16.311%	0.488	15.418%	30.532%	0.505
FASTR.LL	11.283%	13.641%	0.827	15.587%	22.362%	0.697
FASTR.LV	**14.425%**	15.648%	**0.922**	7.517%	30.152%	0.249
FASTR.LE	10.540%	13.644%	0.773	24.807%	30.131%	0.823
FASTR.EL	9.339%	14.836%	0.630	9.042%	24.385%	0.371
FASTR.EV	12.683%	14.723%	0.861	10.573%	**22.240%**	0.475
FASTR.EE	13.457%	15.966%	0.843	**27.575%**	31.183%	**0.884**
Neutral Regimes						
FASTR.NL	3.836%	12.890%	0.298	**9.699%**	16.289%	**0.595**
FASTR.NV	**12.020%**	**9.009%**	**1.334**	2.915%	20.551%	0.142
FASTR.NE	8.167%	12.157%	0.672	-20.479%	32.865%	-0.623
FASTR.LL	6.084%	13.982%	0.435	15.492%	28.283%	0.548
FASTR.LV	1.067%	10.978%	0.097	-1.396%	24.842%	-0.056
FASTR.LE	2.847%	11.562%	0.246	-2.141%	22.476%	-0.095
FASTR.EL	5.207%	12.079%	0.431	16.716%	22.881%	0.731
FASTR.EV	7.705%	11.004%	0.700	10.926%	**16.261%**	0.672
FASTR.EE	6.220%	14.343%	0.434	6.297%	21.354%	0.295

Table A.10. Annualized excess returns, volatility and Sharpe Ratio for the average weights given in Table A.7, with a risk aversion of 10. The excess returns are compounded, which means that earned profits are reinvested. In the first form the investor is not allowed for go short, in the second form the investor can go short, which enlarges the interval of possible weights to [-1, 2]. The S&P500 is left out, because of the high similarities between the Big Cap and S&P. The good, bad and neutral regimes are computed by taking the optimized parameters of the Big Cap excess returns in the logistic function. A good regime captures the best 40% of the values, a bad regime the least 40%, and the neutral regime the remaining 20% in the middle.

	No Short-Selling			Short-Selling Allowed		
	Buy-and-Hold Small	Buy-and-Hold Big	FA	Buy-and-Hold Small	Buy-and-Hold Big	FA
FA	0.638	1.000	-	0.303	0.189	-
FASTR.NL	0.522	0.840	0.722	0.412	0.265	0.696
FASTR.NV	0.115	0.286	0.269	0.494	0.294	0.597
FASTR.NE	0.283	0.144	0.108	0.012	0.003	0.103
FASTR.LL	0.395	0.741	0.540	0.126	0.064	0.562
FASTR.LV	0.755	0.853	0.811	0.054	0.021	0.273
FASTR.LE	0.837	0.784	0.624	0.083	0.047	0.285
FASTR.EL	**0.040**	0.144	0.162	0.555	0.338	0.417
FASTR.EV	0.305	0.614	0.637	0.370	0.191	0.556
FASTR.EE	0.621	0.989	0.988	0.429	0.273	0.573

Table A.11. P-values of the Bootstrap on the Sharpe Ratio, proposed by Ledoit & Wolf (2008). The null hypothesis states that the difference between the Sharpe Ratios does not differ significantly from zero. The risk aversion used to compute the weights is equal to 2. Positive significant values are bold (based on significance level of 5%), negative significant values are shown in red.

	No Short-Selling			Short-Selling Allowed		
	Buy-and-Hold Small	Buy-and-Hold Big	FA	Buy-and-Hold Small	Buy-and-Hold Big	FA
FA	0.181	0.370	-	0.559	0.386	-
FASTR.NL	0.220	0.460	0.740	0.717	0.517	0.781
FASTR.NV	**0.030**	0.088	0.543	0.808	0.576	0.657
FASTR.NE	0.561	0.319	0.047	0.042	0.017	0.050
FASTR.LL	0.210	0.428	0.814	0.691	0.464	0.797
FASTR.LV	0.590	0.977	0.172	0.410	0.248	0.757
FASTR.LE	0.649	0.990	0.167	0.350	0.218	0.433
FASTR.EL	0.186	0.457	0.594	0.288	0.167	0.800
FASTR.EV	0.124	0.298	0.823	0.727	0.441	0.743
FASTR.EE	0.242	0.559	0.516	0.530	0.333	0.986

Table A.12. P-values of the Bootstrap on the Sharpe Ratio, proposed by Ledoit & Wolf (2008). The null hypothesis states that the difference between the Sharpe Ratios does not differ significantly from zero. The risk aversion used to compute the weights is equal to 10. Positive significant values are bold (based on significance level of 5%), negative significant values are shown in red.

DM All Expansions	Small Cap $r_{e,t+1}$	Small Cap RV_{t+1}	Medium Cap $r_{e,t+1}$	Medium Cap RV_{t+1}	Big Cap $r_{e,t+1}$	Big Cap RV_{t+1}	S&P 500 $r_{e,t+1}$	S&P 500 RV_{t+1}	Gold $r_{e,t+1}$	Gold RV_{t+1}
FASTR.NL	-1.208	-1.228	-2.544	-0.941	-2.228	-0.879	-2.094	-1.023	-0.447	1.185
FASTR.NV	-1.672	-0.846	-2.894	-1.470	-2.305	-1.363	-2.355	-1.190	-1.382	-1.068
FASTR.NE	-1.507	-0.923	-1.834	-1.444	-1.043	-1.576	-1.431	-0.569	-0.806	-0.042
FASTR.LL	-0.013	-0.876	-1.853	-1.239	-1.861	-0.467	-1.608	-0.494	0.050	1.326
FASTR.LV	0.068	-0.101	-1.939	-0.684	-1.784	-0.256	-1.859	-0.323	-0.186	0.052
FASTR.LE	-0.862	-0.876	-1.506	-1.640	-1.392	-1.695	-1.539	-1.157	0.413	-0.819
FASTR.EL	1.222	-0.960	0.789	-0.861	1.337	-0.036	1.358	0.241	**1.940**	**2.565**
FASTR.EV	1.136	-0.911	0.938	-1.022	1.108	-0.514	1.485	-0.262	**1.790**	-2.052
FASTR.EE	1.236	-1.593	0.780	-2.371	1.169	-2.358	1.200	-2.630	**2.118**	-1.065
Thresholding & Fixed Logistic										
FASTR.NL	-1.571	-1.611	-2.083	-1.632	-2.308	-1.010	-1.770	-1.147	-1.135	1.043
FASTR.NV	-2.649	-1.191	-2.860	-1.703	-2.267	-0.638	-1.456	-0.790	-2.350	-1.844
FASTR.NE	-2.191	-1.478	-2.432	-1.880	-1.753	-1.329	-1.561	-0.583	-2.190	0.134
FASTR.LL	-1.068	-1.699	-1.605	-1.242	-1.667	-1.002	-1.076	-1.171	-0.918	1.308
FASTR.LV	-1.366	-0.735	-2.057	-0.918	-1.810	-0.473	-1.146	-0.454	-1.542	-0.700
FASTR.LE	-1.845	-1.464	-2.281	-1.571	-1.884	-1.428	-1.472	-0.834	-1.235	-0.423
FASTR.EL	1.224	-1.164	0.931	-2.288	1.119	-1.485	1.179	-1.409	0.915	**2.588**
FASTR.EV	0.633	-0.392	-0.006	-1.430	0.778	-0.905	0.850	-0.997	0.268	-2.102
FASTR.EE	0.332	-2.015	-0.282	-3.793	0.096	-3.663	0.146	-3.885	0.880	0.701
Thresholding										
FASTR.NL	-2.821	-2.533	-2.811	-1.913	-2.923	-0.896	-1.953	-1.883	-1.112	0.963
FASTR.NV	-2.569	-2.459	-2.731	-2.768	-2.655	-0.889	-2.218	-1.621	-1.352	-1.054
FASTR.NE	-1.133	-1.987	-3.630	-1.008	-2.705	-1.015	-2.845	-1.581	-2.116	-0.700
FASTR.LL	-1.707	-2.876	-2.729	-1.354	-2.532	-0.725	-1.375	-1.312	-1.203	0.769
FASTR.LV	-1.425	-1.890	-1.972	-0.767	-1.896	-1.417	-1.791	-0.311	-1.676	-1.446
FASTR.LE	-1.083	-1.761	-2.407	-1.012	-2.199	-1.020	-2.503	-0.830	-1.964	0.244
FASTR.EL	0.458	-3.295	-1.018	-1.473	0.089	-1.632	0.163	-1.177	0.986	**2.237**
FASTR.EV	0.066	-0.562	-0.687	-0.501	-0.307	-0.372	-0.180	-0.378	-0.623	-2.159
FASTR.EE	-0.715	-1.690	-1.086	-2.780	-0.904	-2.642	-1.014	-2.289	-0.395	-1.062
Fixed Logistic Parameters										
FASTR.NL	-2.066	-1.641	-2.837	-0.534	-2.776	-0.004	-2.859	-0.156	-1.921	**2.007**
FASTR.NV	-2.745	-1.171	-3.533	-0.827	-3.349	-0.683	-3.046	-0.281	-1.851	-2.527
FASTR.NE	-2.259	-1.301	-1.824	-1.417	-1.268	-0.904	-1.084	-0.849	-1.308	-0.480
FASTR.LL	-0.307	-1.524	-1.053	-0.820	-0.953	0.019	-0.683	0.378	-0.443	**3.141**
FASTR.LV	-1.136	-0.309	-2.375	-0.470	-2.437	-0.217	-2.539	0.001	-1.264	-0.754
FASTR.LE	-0.595	-0.637	-1.673	-1.359	-1.933	-1.030	-1.493	-0.634	-1.272	-1.281
FASTR.EL	1.426	-1.906	0.917	-1.558	1.135	-0.650	1.432	-0.826	1.390	**2.483**
FASTR.EV	0.831	-0.864	0.304	-2.002	0.803	-1.571	1.128	-1.170	1.248	-2.393
FASTR.EE	0.685	-2.566	-0.106	-4.193	0.256	-4.401	0.501	-4.631	1.626	-1.735

Table A.13. Diebold-Mariano statistic, based on the mean squared errors computed in Section 5.4.1. A blue value shows the performance is better than the standard model. Panel (a) contains all expansions, panel (b) only thresholding and fixed logistic parameters, and panels (c) and (d) contain the last two expansions solely. A positive value indicates a positive performance of the tested model relative to the benchmark, which is the linear Factor Augmentation model. The critical level is 1.645, corresponding to a significance level of 5%. The significant positive values are bold.

DA All Expansions	**Small Cap**		**Medium Cap**		**Big Cap**		**S&P 500**		**Gold**	
	$r_{e,t+1}$	RV_{t+1}	$r_{e,t+1}$	RV_{t+1}	$r_{e,t+1}$	RV_{t+1}	$r_{e,t+1}$	RV_{t+1}	$r_{e,t+1}$	RV_{t+1}
FA	1.629	4.054	0.389	6.222	1.522	6.804	0.497	7.033	2.597	7.419
FASTR.NL	1.923	4.961	0.140	5.071	1.170	7.617	-0.389	7.882	2.798	8.960
FASTR.NV	1.155	5.967	0.326	6.608	1.117	6.709	0.902	7.818	**2.997**	7.145
FASTR.NE	1.334	**6.111**	**0.770**	6.806	**2.238**	7.335	**2.177**	7.209	1.498	7.771
FASTR.LL	**2.598**	4.362	0.160	6.608	2.002	7.772	0.747	7.465	2.397	9.124
FASTR.LV	1.195	5.854	0.102	6.880	1.549	7.548	0.756	7.772	2.198	7.063
FASTR.LE	1.927	6.650	0.554	**6.915**	1.326	7.165	0.528	7.008	2.410	7.063
FASTR.EL	-0.161	3.396	-0.178	5.314	0.152	**8.377**	-1.467	**8.201**	2.416	**9.606**
FASTR.EV	-1.029	5.059	0.135	4.778	-0.264	7.111	1.248	7.377	1.398	3.362
FASTR.EE	0.319	4.714	-1.118	4.500	0.144	3.164	-0.703	1.734	1.798	7.244
Thresholding & Fixed Logistic										
FA	1.842	5.673	0.990	5.476	**2.754**	7.772	**2.624**	7.638	0.600	7.169
FASTR.NL	1.451	5.634	1.220	4.961	1.987	6.363	1.863	8.629	**1.503**	7.756
FASTR.NV	0.318	**6.898**	-0.205	7.611	0.402	7.488	1.422	8.723	1.400	6.534
FASTR.NE	1.216	6.266	0.051	6.196	1.448	6.391	2.013	8.116	-0.600	6.251
FASTR.LL	**2.198**	5.785	**1.483**	5.371	2.490	7.102	2.079	7.646	1.102	**9.004**
FASTR.LV	1.006	6.191	0.020	6.204	1.084	7.952	1.674	7.326	1.200	6.841
FASTR.LE	1.637	6.658	-0.184	**7.826**	0.585	**8.531**	0.863	**8.729**	1.499	5.855
FASTR.EL	0.239	1.581	0.951	3.720	1.073	5.554	1.255	7.216	0.803	8.988
FASTR.EV	0.344	4.820	1.074	3.923	0.965	5.469	0.317	7.167	0.900	4.439
FASTR.EE	-0.818	5.672	-0.157	4.517	-0.315	3.070	-1.018	4.263	1.298	6.610
Thresholding										
FA	1.842	5.673	0.990	5.476	**2.754**	7.772	2.624	7.638	0.600	7.169
FASTR.NL	**2.333**	4.731	1.534	5.786	1.617	5.956	1.993	7.926	1.899	7.848
FASTR.NV	1.093	**7.413**	1.024	6.086	1.468	6.662	**2.842**	7.824	**1.998**	6.733
FASTR.NE	0.866	5.202	0.755	5.165	2.573	5.982	1.311	7.307	-0.500	5.949
FASTR.LL	1.694	5.634	1.139	5.473	1.761	7.510	1.566	7.674	1.101	**9.056**
FASTR.LV	-0.022	6.334	0.058	**7.333**	1.073	7.404	1.548	7.826	1.200	5.956
FASTR.LE	1.488	6.766	0.081	6.697	0.945	**8.608**	0.317	**8.626**	0.401	5.790
FASTR.EL	0.147	2.408	**1.858**	5.005	0.955	7.181	0.704	8.553	0.201	6.772
FASTR.EV	0.630	6.291	1.188	5.715	0.888	7.022	1.105	7.121	1.000	5.492
FASTR.EE	0.241	6.496	0.770	3.600	1.306	2.952	0.324	3.165	0.600	6.724
Fixed Logistic Parameters										
FA	1.302	5.254	1.162	5.375	1.099	6.067	1.317	7.040	-0.313	6.381
FASTR.NL	**2.519**	5.032	1.608	4.719	0.775	6.129	0.997	7.641	1.201	8.076
FASTR.NV	-0.063	5.711	0.433	6.518	-0.099	5.982	0.648	6.705	0.699	5.962
FASTR.NE	-0.626	6.366	-0.472	6.402	-0.939	6.158	-1.279	5.197	-0.308	9.281
FASTR.LL	2.303	3.243	**2.368**	5.083	**1.703**	7.022	1.207	7.527	0.606	8.966
FASTR.LV	1.084	4.789	1.426	5.621	-0.046	6.105	0.217	6.203	1.117	5.852
FASTR.LE	-0.134	**6.951**	0.338	**7.053**	-0.576	7.075	-0.347	5.693	-0.417	6.083
FASTR.EL	0.933	2.279	1.597	4.252	0.601	**7.335**	0.153	7.611	**2.110**	**9.561**
FASTR.EV	-0.138	4.902	0.827	4.367	1.423	7.145	**1.439**	**8.103**	1.298	3.767
FASTR.EE	0.277	5.662	-0.468	5.345	0.092	3.437	-0.485	1.990	1.699	6.546

Table A.14. Directional Accuracy (DA) statistic, based on the Correctly Predicted Signs computed in Section 5.4.1. A blue value shows the performance is better than the standard model. Panel (a) contains all expansions, panel (b) only thresholding and fixed logistic parameters, and panels (c) and (d) contain the last two expansions solely. A positive value indicates a positive performance on the accuracy of the time series. The critical level is 1.645, corresponding to a significance level of 5%.

	Good Regime	Bad Regime	Neutral Regime
FASTR.NL	165	175	62
FASTR.NV	182	180	40
FASTR.NE	176	179	47
FASTR.LL	165	175	62
FASTR.LV	182	180	40
FASTR.LE	176	179	47
FASTR.EL	165	175	62
FASTR.EV	182	180	40
FASTR.EE	176	179	47

Table A.15. Number of occurrences for three regimes. The regimes are computed by taking the optimized parameters of the Big Cap excess returns in the logistic function. A good regime captures the best 40% of the values, a bad regime the least 40%, and the neutral regime the remaining 20% in the middle.

Weights $\gamma = 2$	**No Short-Selling**					**Short-Selling Allowed**				
	R_f	Small	Medium	Big	Gold	R_f	Small	Medium	Big	Gold
FA	0.17	0.26	0.21	0.20	0.16	-0.00	0.32	0.54	0.19	-0.04
Complete Sample										
FASTR.NL	0.16	0.25	0.22	0.22	0.15	-0.01	0.29	0.43	0.36	-0.06
FASTR.NV	0.14	0.26	0.18	0.21	0.21	-0.13	0.30	0.41	0.28	0.14
FASTR.NE	0.12	0.25	0.24	0.20	0.19	-0.09	0.33	0.57	0.23	-0.04
FASTR.LL	0.17	0.24	0.21	0.22	0.16	-0.02	0.29	0.48	0.30	-0.04
FASTR.LV	0.18	0.27	0.19	0.18	0.18	0.05	0.32	0.45	0.11	0.07
FASTR.LE	0.17	0.25	0.22	0.20	0.16	0.06	0.26	0.53	0.22	-0.07
FASTR.EL	0.04	0.37	0.21	0.22	0.15	-0.50	0.88	0.54	0.19	-0.10
FASTR.EV	0.02	0.32	0.17	0.24	0.25	-0.64	0.94	0.37	0.17	0.16
FASTR.EE	0.08	0.34	0.15	0.18	0.25	-0.38	0.71	0.39	0.16	0.11
Good Regime										
FASTR.NL	0.13	0.34	0.17	0.17	0.19	-0.23	0.53	0.49	0.07	0.13
FASTR.NV	0.11	0.24	0.15	0.24	0.26	-0.31	0.33	0.40	0.34	0.23
FASTR.NE	0.16	0.25	0.17	0.25	0.17	0.10	0.37	0.30	0.41	-0.18
FASTR.LL	0.15	0.24	0.27	0.14	0.20	-0.11	0.22	0.72	-0.07	0.24
FASTR.LV	0.17	0.27	0.16	0.20	0.21	-0.05	0.37	0.36	0.17	0.14
FASTR.LE	0.26	0.23	0.17	0.22	0.12	0.44	0.13	0.41	0.31	-0.29
FASTR.EL	0.05	0.38	0.21	0.23	0.13	-0.30	0.73	0.57	0.15	-0.15
FASTR.EV	0.02	0.28	0.15	0.28	0.27	-0.74	0.80	0.18	0.44	0.31
FASTR.EE	0.17	0.15	0.10	0.32	0.26	0.14	0.05	-0.10	0.89	0.02
Bad Regime										
FASTR.NL	0.09	0.24	0.28	0.25	0.15	-0.32	0.16	0.63	0.61	-0.08
FASTR.NV	0.16	0.28	0.22	0.18	0.16	-0.01	0.30	0.43	0.24	0.04
FASTR.NE	0.10	0.25	0.32	0.12	0.22	-0.24	0.23	0.87	0.02	0.13
FASTR.LL	0.09	0.28	0.23	0.26	0.14	-0.35	0.28	0.58	0.53	-0.05
FASTR.LV	0.17	0.27	0.23	0.16	0.16	0.11	0.28	0.58	0.03	-0.00
FASTR.LE	0.11	0.26	0.29	0.14	0.21	-0.25	0.27	0.74	0.07	0.17
FASTR.EL	0.04	0.30	0.26	0.32	0.07	-0.53	0.58	0.74	0.63	-0.42
FASTR.EV	0.02	0.37	0.19	0.19	0.23	-0.55	1.12	0.49	-0.07	0.01
FASTR.EE	0.02	0.49	0.19	0.08	0.23	-0.78	1.25	0.83	-0.44	0.14
Neutral Regime										
FASTR.NL	0.25	0.19	0.20	0.23	0.13	0.41	0.22	0.22	0.34	-0.20
FASTR.NV	0.19	0.22	0.13	0.24	0.23	0.20	0.16	0.30	0.14	0.20
FASTR.NE	0.08	0.27	0.19	0.31	0.16	-0.25	0.58	0.45	0.41	-0.18
FASTR.LL	0.25	0.22	0.16	0.24	0.14	0.32	0.35	0.20	0.36	-0.24
FASTR.LV	0.25	0.22	0.14	0.20	0.19	0.28	0.21	0.28	0.12	0.11
FASTR.LE	0.09	0.31	0.16	0.33	0.11	-0.15	0.65	0.17	0.50	-0.18
FASTR.EL	0.03	0.42	0.18	0.14	0.24	-0.63	1.25	0.34	-0.18	0.21
FASTR.EV	0.02	0.32	0.18	0.25	0.23	-0.62	0.70	0.72	0.02	0.17
FASTR.EE	0.02	0.45	0.16	0.09	0.28	-0.79	1.16	0.56	-0.29	0.36

Table A.16. Average of the weights, for two forms of determining the weights. In the first form the investor is not allowed for go short, in the second form the investor can go short, which enlarges the interval of possible weights to [-1, 2]. The good, bad and neutral regimes are computed by taking the optimized parameters of the Big Cap excess returns in the logistic function. A good regime captures the best 40% of the values, a bad regime the least 40%, and the neutral regime the remaining 20% in the middle. The risk aversion coefficient is taken to be 2.

Weights $\gamma = 10$	**No Short-Selling**					**Short-Selling Allowed**				
	R_f	Small	Medium	Big	Gold	R_f	Small	Medium	Big	Gold
FA	0.31	0.21	0.16	0.16	0.16	0.52	0.10	0.41	0.00	-0.03
Complete Sample										
FASTR.NL	0.27	0.22	0.18	0.18	0.15	0.44	0.16	0.25	0.17	-0.02
FASTR.NV	0.26	0.24	0.13	0.18	0.20	0.40	0.16	0.21	0.16	0.07
FASTR.NE	0.24	0.23	0.20	0.17	0.16	0.35	0.14	0.41	0.08	0.01
FASTR.LL	0.29	0.21	0.17	0.18	0.15	0.48	0.11	0.32	0.11	-0.01
FASTR.LV	0.31	0.23	0.14	0.15	0.18	0.53	0.14	0.33	0.01	-0.01
FASTR.LE	0.30	0.22	0.18	0.15	0.15	0.55	0.07	0.37	0.04	-0.02
FASTR.EL	0.26	0.27	0.16	0.16	0.14	0.54	0.43	0.16	-0.05	-0.08
FASTR.EV	0.27	0.27	0.10	0.17	0.19	0.51	0.42	0.09	-0.01	-0.01
FASTR.EE	0.25	0.29	0.09	0.15	0.21	0.44	0.36	0.05	0.08	0.06
Good Regime										
FASTR.NL	0.23	0.30	0.13	0.15	0.20	0.13	0.41	0.32	-0.04	0.17
FASTR.NV	0.20	0.23	0.12	0.21	0.24	0.21	0.23	0.17	0.24	0.15
FASTR.NE	0.25	0.24	0.15	0.20	0.16	0.42	0.18	0.22	0.26	-0.07
FASTR.LL	0.29	0.20	0.20	0.12	0.18	0.36	-0.02	0.62	-0.11	0.15
FASTR.LV	0.26	0.24	0.13	0.18	0.19	0.40	0.21	0.26	0.09	0.04
FASTR.LE	0.37	0.21	0.15	0.16	0.11	0.80	-0.01	0.24	0.16	-0.19
FASTR.EL	0.30	0.27	0.16	0.14	0.13	0.66	0.38	0.21	-0.11	-0.13
FASTR.EV	0.20	0.27	0.08	0.23	0.22	0.33	0.40	-0.13	0.30	0.10
FASTR.EE	0.35	0.14	0.06	0.25	0.20	0.67	-0.12	-0.32	0.76	0.01
Bad Regime										
FASTR.NL	0.21	0.21	0.23	0.20	0.15	0.25	0.00	0.41	0.32	0.02
FASTR.NV	0.31	0.25	0.14	0.15	0.16	0.55	0.11	0.23	0.10	0.02
FASTR.NE	0.25	0.22	0.26	0.11	0.16	0.30	0.07	0.67	-0.13	0.10
FASTR.LL	0.22	0.23	0.18	0.21	0.17	0.26	0.08	0.36	0.26	0.04
FASTR.LV	0.34	0.23	0.15	0.11	0.16	0.64	0.10	0.39	-0.08	-0.05
FASTR.LE	0.25	0.21	0.23	0.12	0.19	0.28	0.06	0.58	-0.09	0.16
FASTR.EL	0.29	0.18	0.18	0.26	0.09	0.65	0.08	0.24	0.25	-0.22
FASTR.EV	0.33	0.27	0.12	0.12	0.16	0.70	0.48	0.25	-0.28	-0.14
FASTR.EE	0.18	0.41	0.12	0.06	0.23	0.24	0.74	0.37	-0.45	0.09
Neutral Regime										
FASTR.NL	0.36	0.17	0.17	0.19	0.10	0.84	0.11	0.07	0.19	-0.20
FASTR.NV	0.29	0.22	0.10	0.20	0.19	0.62	0.05	0.30	0.07	-0.05
FASTR.NE	0.18	0.24	0.15	0.26	0.17	0.29	0.31	0.18	0.20	0.01
FASTR.LL	0.35	0.20	0.15	0.19	0.11	0.76	0.23	0.06	0.13	-0.17
FASTR.LV	0.35	0.18	0.11	0.17	0.19	0.66	0.01	0.36	0.03	-0.06
FASTR.LE	0.24	0.27	0.12	0.25	0.13	0.61	0.37	0.01	0.11	-0.09
FASTR.EL	0.22	0.36	0.13	0.10	0.19	0.37	0.76	0.06	-0.27	0.07
FASTR.EV	0.31	0.25	0.10	0.17	0.17	0.50	0.24	0.35	-0.13	0.04
FASTR.EE	0.20	0.40	0.11	0.08	0.22	0.37	0.74	0.20	-0.43	0.11

Table A.17. Average of the weights, for two forms of determining the weights. In the first form the investor is not allowed for go short, in the second form the investor can go short, which enlarges the interval of possible weights to [-1, 2]. The good, bad and neutral regimes are computed by taking the optimized parameters of the Big Cap excess returns in the logistic function. A good regime captures the best 40% of the values, a bad regime the least 40%, and the neutral regime the remaining 20% in the middle. The risk aversion coefficient is taken to be 10.

Economic Performance $\gamma = 5$	**No Short-Selling Allowed**			**Short-Selling Allowed**		
	Average Annualized Excess Return	Annualized Volatility	Sharpe Ratio	Average Annualized Excess Return	Annualized Volatility	Sharpe Ratio
1978/06 – 1991/12						
Buy-and-Hold R_f	0.000%	0.764%	0.000			
Buy-and-Hold Small	10.666%	20.195%	0.528			
Buy-and-Hold Big	9.878%	16.737%	0.590			
FA	9.305%	13.861%	0.671	16.094%	**28.237%**	0.570
FASTR.NL	**12.559%**	15.182%	**0.827**	**30.743%**	28.736%	**1.070**
FASTR.NV	6.736%	**13.360%**	0.504	21.627%	31.604%	0.684
FASTR.NE	7.322%	15.241%	0.480	17.278%	33.878%	0.510
FASTR.LL	9.950%	14.470%	0.688	17.925%	28.614%	0.626
FASTR.LV	7.695%	13.865%	0.555	13.629%	28.643%	0.476
FASTR.LE	10.028%	14.634%	0.685	19.863%	28.836%	0.689
FASTR.EL	11.302%	20.067%	0.563	13.410%	35.096%	0.382
FASTR.EV	5.314%	20.392%	0.261	6.279%	35.048%	0.179
FASTR.EE	7.727%	16.707%	0.463	18.580%	35.838%	0.519
1992/01 – 2004/12						
Buy-and-Hold R_f	0.000%	0.478%	0.000			
Buy-and-Hold Small	12.440%	20.423%	0.609			
Buy-and-Hold Big	9.526%	15.673%	0.608			
FA	4.393%	**13.655%**	0.322	-2.036%	**25.184%**	-0.081
FASTR.NL	4.306%	15.335%	0.281	-3.198%	32.276%	-0.099
FASTR.NV	6.066%	14.602%	0.416	5.089%	29.932%	0.170
FASTR.NE	8.172%	15.413%	0.530	13.258%	29.766%	0.445
FASTR.LL	5.023%	14.774%	0.340	-1.214%	30.052%	-0.040
FASTR.LV	4.389%	14.842%	0.296	-0.483%	29.266%	-0.017
FASTR.LE	5.357%	14.212%	0.377	0.514%	27.855%	0.019
FASTR.EL	**8.964%**	16.658%	**0.538**	**13.664%**	27.841%	**0.491**
FASTR.EV	7.987%	16.421%	0.486	10.347%	28.369%	0.365
FASTR.EE	8.140%	16.283%	0.500	9.651%	32.753%	0.295
2005/01 – 2011/11						
Buy-and-Hold R_f	0.000%	0.574%	0.000			
Buy-and-Hold Small	7.005%	23.950%	0.293			
Buy-and-Hold Big	4.780%	18.532%	0.258			
FA	6.706%	14.813%	0.453	18.348%	24.121%	0.761
FASTR.NL	6.465%	13.877%	0.466	12.730%	22.659%	0.562
FASTR.NV	5.081%	15.865%	0.320	7.143%	25.441%	0.281
FASTR.NE	11.742%	**13.794%**	0.851	17.194%	26.735%	0.643
FASTR.LL	5.185%	15.130%	0.343	14.732%	20.868%	0.706
FASTR.LV	6.310%	14.802%	0.426	8.241%	23.531%	0.350
FASTR.LE	8.466%	15.904%	0.532	17.611%	25.642%	0.687
FASTR.EL	5.319%	14.936%	0.356	17.120%	23.218%	0.737
FASTR.EV	7.454%	14.461%	0.515	14.220%	**19.929%**	0.714
FASTR.EE	**13.955%**	15.332%	**0.910**	**27.203%**	26.170%	**1.040**

Table A.18. Annualized excess returns, volatility and Sharpe Ratio for three sub-periods, with a risk aversion of 5. The excess returns are compounded, which means that earned profits are reinvested. In the first form the investor is not allowed for go short, in the second form the investor can go short, which enlarges the interval of possible weights to [-1, 2].

Economic Performance $\gamma = 2$	**No Short-Selling Allowed**			**Short-Selling Allowed**		
	Average Annualized Return	Annualized Volatility	Sharpe Ratio	Average Annualized Return	Annualized Volatility	Sharpe Ratio
Buy-and-Hold R_f	0.000%	0.980%	0.000			
Buy-and-Hold Small	10.573%	21.077%	0.502			
Buy-and-Hold S&P	8.644%	16.722%	0.517			
Complete Sample						
FA	6.554%	**15.425%**	0.425	9.362%	**37.404%**	0.250
FASTR.NL	8.100%	16.076%	0.504	13.205%	38.577%	0.342
FASTR.NV	6.151%	15.858%	0.388	9.498%	38.457%	0.247
FASTR.NE	8.095%	16.148%	0.501	22.012%	41.154%	0.535
FASTR.LL	6.666%	15.748%	0.423	14.317%	38.683%	0.370
FASTR.LV	5.872%	16.182%	0.363	6.330%	38.371%	0.165
FASTR.LE	8.104%	15.953%	0.508	13.466%	38.266%	0.352
FASTR.EL	9.484%	19.220%	0.494	**25.109%**	46.073%	**0.545**
FASTR.EV	8.387%	19.428%	0.432	15.166%	44.690%	0.339
FASTR.EE	**9.505%**	16.814%	**0.565**	23.155%	45.065%	0.514
Good Regimes						
FASTR.NL	3.505%	13.024%	0.269	-5.239%	35.451%	-0.148
FASTR.NV	5.382%	13.023%	0.413	18.225%	**30.713%**	0.593
FASTR.NE	1.485%	12.997%	0.114	14.712%	36.115%	0.407
FASTR.LL	2.013%	12.875%	0.156	-10.634%	33.947%	-0.313
FASTR.LV	5.755%	13.362%	0.431	**18.975%**	30.880%	**0.615**
FASTR.LE	0.055%	**12.468%**	0.004	-4.330%	31.877%	-0.136
FASTR.EL	**10.341%**	15.245%	**0.678**	11.802%	38.450%	0.307
FASTR.EV	5.592%	14.876%	0.376	7.278%	36.284%	0.201
FASTR.EE	5.353%	14.447%	0.371	11.378%	41.230%	0.276
Bad Regimes						
FASTR.NL	12.124%	**16.865%**	**0.719**	30.078%	**40.340%**	0.746
FASTR.NV	5.469%	18.844%	0.290	4.202%	46.047%	0.091
FASTR.NE	12.288%	18.588%	0.661	26.683%	45.661%	0.584
FASTR.LL	11.524%	17.163%	0.671	41.810%	42.115%	0.993
FASTR.LV	4.982%	19.219%	0.259	-3,389%	46.102%	-0.074
FASTR.LE	**13.037%**	18.854%	0.692	27.480%	44.562%	0.617
FASTR.EL	11.985%	18.879%	0.635	**54.135%**	46.999%	**1.152**
FASTR.EV	7.747%	23.663%	0.327	15.442%	53.503%	0.289
FASTR.EE	11.873%	19.186%	0.619	31.254%	50.426%	0.620
Neutral Regimes						
FASTR.NL	8.073%	17.367%	0.465	13.986%	39.038%	0.358
FASTR.NV	12.932%	**12.848%**	1.007	-3.701%	32.435%	-0.114
FASTR.NE	18.168%	16.494%	1.102	32.955%	41.391%	0.796
FASTR.LL	5.966%	16.379%	0.364	12.856%	38.212%	0.336
FASTR.LV	10.523%	12.902%	0.816	-2.339%	**29.518%**	-0.079
FASTR.LE	21.461%	14.773%	1.453	36.116%	32.600%	1.108
FASTR.EL	6.659%	22.023%	0.302	12.623%	50.024%	0.252
FASTR.EV	**25.176%**	16.271%	**1.547**	**56.183%**	34.524%	**1.627**
FASTR.EE	16.544%	15.420%	1.073	40.103%	36.630%	1.095

Table A.19. Annualized excess returns, volatility and Sharpe Ratio for the average weights given in Table A.16, with a risk aversion of 2. The excess returns are compounded, which means that earned profits are reinvested. In the first form the investor is not allowed for go short, in the second form the investor can go short, which enlarges the interval of possible weights to [-1, 2]. The S&P500 is left out, because of the high similarities between the Big Cap and S&P. The good, bad and neutral regimes are computed by taking the optimized parameters of the Big Cap excess returns in the logistic function. A good regime captures the best 40% of the values, a bad regime the least 40%, and the neutral regime the remaining 20% in the middle.

Economic Performance $\gamma = 10$	No Short-Selling Allowed			Short-Selling Allowed		
	Average Annualized Excess Return	Annualized Volatility	Sharpe Ratio	Average Annualized Excess Return	Annualized Volatility	Sharpe Ratio
Buy-and-Hold R_f	0.000%	0.980%	0.000			
Buy-and-Hold Small	10.573%	21.077%	0.502			
Buy-and-Hold S&P	8.644%	16.722%	0.517			
Complete Sample						
FA	6.169%	**11.540%**	0.535	4.850%	19.595%	0.248
FASTR.NL	6.847%	13.362%	0.512	11.403%	23.225%	0.491
FASTR.NV	5.657%	12.421%	0.456	8.724%	21.625%	0.403
FASTR.NE	**8.002%**	12.853%	**0.623**	**13.421%**	23.701%	**0.566**
FASTR.LL	6.662%	12.303%	0.542	8.400%	20.092%	0.418
FASTR.LV	5.401%	12.174%	0.444	5.306%	20.460%	0.259
FASTR.LE	7.227%	12.143%	0.595	9.985%	21.571%	0.463
FASTR.EL	6.698%	14.385%	0.466	8.086%	**19.318%**	0.419
FASTR.EV	5.094%	13.833%	0.368	6.452%	20.196%	0.320
FASTR.EE	7.604%	14.397%	0.528	9.180%	22.908%	0.401
Good Regimes						
FASTR.NL	2.146%	11.584%	0.185	-3.999%	24.578%	-0.163
FASTR.NV	6.038%	10.409%	**0.580**	**17.493%**	19.868%	**0.881**
FASTR.NE	2.485%	10.489%	0.237	5.856%	23.103%	0.254
FASTR.LL	2.591%	10.607%	0.244	-3.267%	19.910%	-0.164
FASTR.LV	5.822%	10.145%	0.574	14.841%	19.320%	0.768
FASTR.LE	1.264%	**9.265%**	0.136	-0.213%	19.743%	-0.011
FASTR.EL	**6.201%**	11.048%	0.561	5.822%	**16.554%**	0.352
FASTR.EV	3.067%	12.456%	0.246	5.279%	18.840%	0.280
FASTR.EE	0.241%	12.689%	0.334	3.739%	19.937%	0.188
Bad Regimes						
FASTR.NL	9.887%	13.750%	0.719	17.790%	20.084%	0.886
FASTR.NV	4.153%	14.513%	0.286	3.025%	23.424%	0.129
FASTR.NE	**11.515%**	14.620%	0.788	**17.791%**	24.510%	0.726
FASTR.LL	9.377%	13.151%	0.713	11.284%	20.083%	0.562
FASTR.LV	3.951%	14.346%	0.275	-1.307%	21.262%	-0.062
FASTR.LE	11.034%	14.583%	0.757	17.806%	24.388%	0.730
FASTR.EL	9.995%	**12.556%**	**0.796**	16.246%	**16.803%**	**0.967**
FASTR.EV	6.032%	15.807%	0.382	7.297%	22.782%	0.320
FASTR.EE	9.007%	16.392%	0.550	12.562%	26.834%	0.468
Neutral Regimes						
FASTR.NL	7.763%	14.192%	0.547	18.333%	23.412%	0.751
FASTR.NV	10.868%	10.668%	1.019	-2.903%	20.350%	-0.143
FASTR.NE	**16.164%**	13.407%	1.206	**26.839%**	22.669%	1.184
FASTR.LL	7.369%	12.681%	0.581	15.228%	20.009%	0.761
FASTR.LV	10.166%	9.949%	1.022	-5.247%	21.082%	-0.249
FASTR.LE	16.099%	10.882%	**1.480**	21.311%	14.939%	**1.427**
FASTR.EL	4.186%	17.708%	0.236	2.847%	22.917%	0.124
FASTR.EV	10.284%	**9.876%**	1.041	8.021%	**12.793%**	0.627
FASTR.EE	15.267%	12.154%	1.256	17.537%	16.178%	1.084

Table A.20. Annualized excess returns, volatility and Sharpe Ratio for the average weights given in Table A.17, with a risk aversion of 2. The excess returns are compounded, which means that earned profits are reinvested. In the first form the investor is not allowed for go short, in the second form the investor can go short, which enlarges the interval of possible weights to [-1, 2]. The S&P500 is left out, because of the high similarities between the Big Cap and S&P. The good, bad and neutral regimes are computed by taking the optimized parameters of the Big Cap excess returns in the logistic function. A good regime captures the best 40% of the values, a bad regime the least 40%, and the neutral regime the remaining 20% in the middle.

	No Short-Selling			Short-Selling Allowed		
	Buy-and-Hold Small	Buy-and-Hold Big	FA	Buy-and-Hold Small	Buy-and-Hold Big	FA
FA	0.839	0.669	-	0.150	0.058	-
FASTR.NL	0.504	0.990	0.469	0.190	0.076	0.570
FASTR.NV	0.904	0.431	0.644	0.121	0.044	0.959
FASTR.NE	0.636	0.991	0.635	0.614	0.383	0.086
FASTR.LL	0.888	0.605	0.916	0.256	0.113	0.367
FASTR.LV	0.725	0.292	0.302	0.041	0.021	0.419
FASTR.LE	0.542	0.949	0.339	0.272	0.127	0.302
FASTR.EL	0.865	0.552	0.943	0.377	0.162	0.320
FASTR.EV	0.738	0.328	0.633	0.049	0.028	0.810
FASTR.EE	0.417	0.790	0.531	0.390	0.213	0.334

Table A.21. P-values of the Bootstrap on the Sharpe Ratio, proposed by Ledoit & Wolf (2008). The null hypothesis states that the difference between the Sharpe Ratios does not differ significantly from zero. The risk aversion used to compute the weights is equal to 2. Positive significant values are bold (based on significance level of 5%), negative significant values are shown in red.

	No Short-Selling			Short-Selling Allowed		
	Buy-and-Hold Small	Buy-and-Hold Big	FA	Buy-and-Hold Small	Buy-and-Hold Big	FA
FA	0.139	0.269	-	0.419	0.226	-
FASTR.NL	0.252	0.540	0.427	0.882	0.605	0.254
FASTR.NV	0.358	0.676	0.283	0.696	0.451	0.335
FASTR.NE	0.083	0.225	0.769	0.917	0.803	0.085
FASTR.LL	0.134	0.290	0.787	0.794	0.527	0.175
FASTR.LV	0.384	0.717	0.213	0.368	0.201	0.995
FASTR.LE	0.089	0.194	0.691	0.877	0.625	0.142
FASTR.EL	0.504	0.960	0.338	0.750	0.459	0.512
FASTR.EV	0.907	0.660	0.162	0.363	0.199	0.798
FASTR.EE	0.310	0.640	0.526	0.561	0.317	0.694

Table A.22. P-values of the Bootstrap on the Sharpe Ratio, proposed by Ledoit & Wolf (2008). The null hypothesis states that the difference between the Sharpe Ratios does not differ significantly from zero. The risk aversion used to compute the weights is equal to 10. Positive significant values are bold (based on significance level of 5%), negative significant values are shown in red.

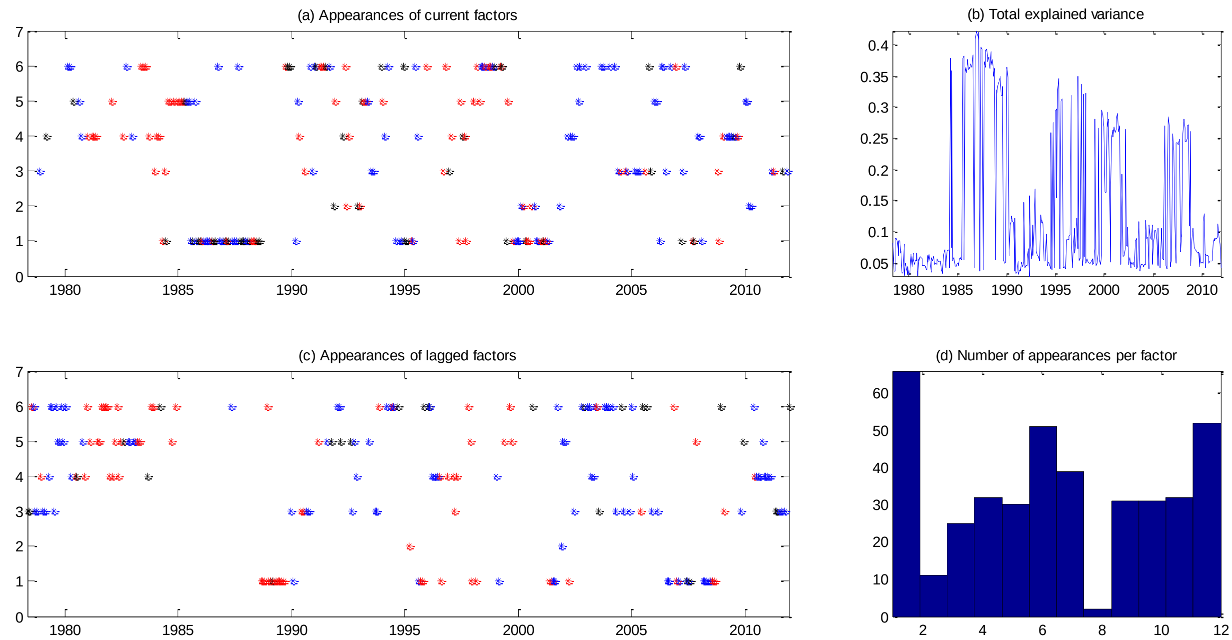

Table A.23. Panel (a) and (c) show the appearances of the current and lagged factors, panel (b) the total explained variance and panel (d) the number of appearances per factor. The time series is the excess returns of Gold, and the predictions are from the FASTR.EL model. The lagged factors in panel (d) are indicated by the integers 7 to 12. The good, bad and neutral regimes in panels (a) and (c) are computed by taking the optimized parameters of the Big Cap excess returns in the logistic function. A good regime captures the best 40% of the values, a bad regime the least 40%, and the neutral regime the remaining 20% in the middle.

Appendix B. Data Descriptions

B.1. Macroeconomic variables

Variable Name	Transf.	Slow	Description
Output and Income			
PI	Δln	Y	Personal income
PI less Transfers	Δln	Y	Personal income excluding current transfer receipts (billions of chained 2005 $)
IP: Total	Δln	Y	Industrial production index – total index
IP: Products	Δln	Y	Industrial production index – products, total
IP: Final Prod	Δln	Y	Industrial production index – final products
IP: Cons Gds	Δln	Y	Industrial production index – consumer goods
IP: Cons Dble	Δln	Y	Industrial production index – durable consumer goods
IP: Cons Nondble	Δln	Y	Industrial production index – nondurable consumer goods
IP: Bus Eqpt	Δln	Y	Industrial production index – business equipment
IP: Mats	Δln	Y	Industrial production index – materials
IP: Dble Mats	Δln	Y	Industrial production index – durable goods materials
IP: Nondble Mats	Δln	Y	Industrial production index – nondurable goods materials
IP: Mfg	Δln	Y	Industrial production index – manufacturing (SIC)
IP: Res util	Δln	Y	Industrial production index – residential utilities
IP: Fuels	Δln	Y	Industrial production index – fuels
NAPM Prodn	lv	Y	NAPM production index (%)
Cap Util	Δlv	Y	Capacity Utilization (Mfg)
Employment and Hours			
Emp CPS Total	Δln	Y	Civilian labor force: employed, total (thousands, SA)
Emp CPS Nonag	Δln	Y	Civilian labor force: employed, nonagricultural industries (thousands, SA)
U: All	Δlv	Y	Unemployment rate: all workers, 16 years and older (%, SA)
U: Mean Duration	Δlv	Y	Unemployment by duration: average duration in weeks (SA)
U < 5 wks	Δln	Y	Unemployment by duration: persons unemployed less than 5 weeks (thousands, SA)
U 5-14 wks	Δln	Y	Unemployment by duration: persons unemployed 5 to 14 weeks (thousands, SA)
U 15+ wks	Δln	Y	Unemployment by duration: persons unemployed over 15 weeks (thousands, SA)
U 15-26 wks	Δln	Y	Unemployment by duration: persons unemployed 15 to 26 weeks (thousands, SA)
U 27+ wks	Δln	Y	Unemployment by duration: persons unemployed over 27 weeks (thousands, SA)
Emp: Total	Δln	Y	Employees on nonfarm payrolls – total private
Emp: Gds Prod	Δln	Y	Employees on nonfarm payrolls – goods producing
Emp: Mining	Δln	Y	Employees on nonfarm payrolls – mining
Emp: Const	Δln	Y	Employees on nonfarm payrolls – construction
Emp: Mfg	Δln	Y	Employees on nonfarm payrolls – manufacturing
Emp: Dble Gds	Δln	Y	Employees on nonfarm payrolls – durable goods
Emp: Nondbles	Δln	Y	Employees on nonfarm payrolls – nondurable goods
Emp: Services	Δln	Y	Employees on nonfarm payrolls – service-providing
Emp: TTU	Δln	Y	Employees on nonfarm payrolls – trade, transportation and utilities
Emp: Wholesale	Δln	Y	Employees on nonfarm payrolls – wholesale trade
Emp: Retail	Δln	Y	Employees on nonfarm payrolls – retail trade
Emp: FIRE	Δln	Y	Employees on nonfarm payrolls – financial activities
Emp: Govt	Δln	Y	Employees on nonfarm payrolls – government
Avg hrs: Gds Prod	lv	Y	Average weekly hours on private nonfarm payrolls – goods-

			producing
Overtime: Mfg	Δlv	Y	Average weekly hours on private nonfarm payrolls – mfg overtime
Avg hrs: Mfg	lv	Y	Average weekly hours, mfg (hours)
NAPM Empl	lv	Y	NAPM employment index (%)
Sales			
M&T Sales	Δln	Y	Manufacturing and trade sales (millions of chained 1996 $)
Retail Sales	Δln	Y	Sales of retail stores (millions of chained 2000 $)
Consumption			
Consumption	Δln	Y	Real consumption
Housing Starts & Sales			
Hstarts: Total	ln	N	Housing starts: nonfarm(1947-58); total farm & nonfarm (1959-) (thousands, SA)
Hstarts: NE	ln	N	Housing starts: northeast (thousands, SA)
Hstarts: MW	ln	N	Housing starts: midwest (thousands, SA)
Hstarts: South	ln	N	Housing starts: south (thousands, SA)
Hstarts: West	ln	N	Housing starts: west (thousands, SA)
BP: total	ln	N	Housing authorized: total new private housing units (thousands, SAAR)
BP: NE	ln	N	Houses authorized by building permits: northeast (thousands, SA)
BP: MW	ln	N	Houses authorized by building permits: midwest (thousands, SA)
BP: South	ln	N	Houses authorized by building permits: south (thousands, SA)
BP: West	ln	N	Houses authorized by building permits: west (thousands, SA)
Orders			
PMI	lv	N	Purchasing managers' index (SA)
NAPM New Orders	lv	N	NAPM new orders index (%)
NAPM Vendor Del	lv	N	NAPM vendor deliveries index (%)
Orders: Total	Δln	N	Mfr's new orders, total manufacturing
Orders: dble gds	Δln	N	Mfr's new orders, durable goods
Unf Orders: dble	Δln	N	Mfr's unfilled orders, durable goods
Inventories			
M&T Invent	Δln	N	Manufacturing and trade inventories (billions of chained 2000 $)
M&T Invent/Sales	Δlv	N	Ratio, mfg and trade inventories to sales (chained 2000 $)
NAPM Invent	lv	N	NAPM inventories index (%)
Money and Credit Quantity Aggregates			
M1	Δ^2ln	N	Money stock: M1 (billions of $, SA)
M2	Δln	N	Money stock: M2 (billions of $, SA)
M2 (real)	Δ^2ln	N	Money supply: M2 (1996 $)
MB	Δ^2ln	N	Monetary base, adjusted for reserve requirement changes (millions of $, SA)
Reserves Tot	Δ^2ln	N	Depository instant reserves: total, adjusted for reserve requirement changes (millions of $, SA)
C&I Loans	Δ^2ln	N	Commercial and industrial loans at all commercials banks
Cons Credit	Δ^2ln	N	Consumer credit outstanding – non-revolving (G19)
Inst Cred/PI	Δlv	N	Ratio, non-revolving consumer credit owned and securitized (seasonally adjusted) to personal income (%)
Exchange Rates			
Ex Rate: Avg	Δln	N	U.S. effective exchange rate (index)
Ex Rate: Switz	Δln	N	Foreign exchange rate: Switzerland (Swiss franc per U.S. $)
Ex Rate: Japan	Δln	N	Foreign exchange rate: Japan (Yen per U.S. $)
Ex Rate: UK	Δln	N	Foreign exchange rate: United Kingdom (Cents per Pound)

Ex Rate: Canada	Δln	N	Foreign exchange rate: Canada (Canadian $ per U.S. $)
Price Indexes			
PPI: Fin Gds	Δ^2ln	N	Producer price index: finished goods (1982=100, SA)
PPI: Cons Gds	Δ^2ln	N	Producer price index: finished consumer goods (1982=100, SA)
PPI: Int Mat'ls	Δ^2ln	N	Producer price index: Intermed material supplies & components (1982=100, SA)
PPI: Crude Mat'ls	Δ^2ln	N	Producer price index: materials (1982=100, SA)
Spot Oil Price	Δ^2ln	N	Spot oil price: West Texas Intermediate (WTI)
NAPM Com Price	lv	N	NAPM commodity prices index (%)
CPI-U: All	Δ^2ln	Y	CPI-U: all items (1982-1984=100, SA)
CPI-U: Apparel	Δ^2ln	Y	CPI-U: apparel & upkeep (1982-1984=100, SA)
CPI-U: Transp	Δ^2ln	Y	CPI-U: transportation (1982-1984=100, SA)
CPI-U: Medical	Δ^2ln	Y	CPI-U: medical care (1982-1984=100, SA)
CPI-U: Comm	Δ^2ln	Y	CPI-U: commodities (1982-1984=100, SA)
CPI-U: dbles	Δ^2ln	Y	CPI-U: durables (1982-1984=100, SA)
CPI-U: Services	Δ^2ln	Y	CPI-U: services (1982-1984=100, SA)
CPI-U: Ex Food	Δ^2ln	Y	CPI-U: all items less food (1982-1984=100, SA)
CPI-U: Ex Shelter	Δ^2ln	Y	CPI-U: all items less shelter (1982-1984=100, SA)
CPI-U: Ex Med	Δ^2ln	Y	CPI-U: all items less medical care (1982-1984=100, SA)
PCE Defl	Δ^2ln	Y	PCE, implied price deflation
PCE Defl: dbles	Δ^2ln	Y	PCE: durables (1987=100)
PCE Defl: nondble	Δ^2ln	Y	PCE: nondurables (1996=100)
PCE Defl: Services	Δ^2ln	Y	PCE: services (1987=100)
Average Hourly Earnings			
AHE: gds	Δ^2ln	Y	Average hourly earnings of production or nonsupervisory workers on private nonfarm payrolls – goods-producing.
AHE: const	Δ^2ln	Y	Average hourly earnings of production or nonsupervisory workers on private nonfarm payrolls –construction.
AHE: mfg	Δ^2ln	Y	Average hourly earnings of production or nonsupervisory workers on private nonfarm payrolls –manufacturing.

Table B.1. Descriptions of the data, adapted from Stock & Watson (2005) and Çakmakli & Van Dijk (2010). The column of transf. determines the transformation needed in order to obtain a stationary variable. Lv = the level (no transformation needed); Δlv = first differences of the level; ln = logarithm of the variable; Δln = first differences of the logarithm; Δ^2ln = second differences of the logarithm. A slow value states the variable does not react to changes based on monetary policy or financial shocks within one month, a fast variable does react within one month.

B.2. Financial variables

Name	Description
PE	Price-Earnings Ratio
DY	Dividend yield
I1	Monthly Interest Rate
I1(-1)	One-period Lagged Monthly Interest Rate
ΔI1	First Differences of the Monthly Interest Rate
I12	Annual Interest Rate
I12(-1)	One-period Lagged Annual Interest Rate
VOL	Volatility Index
DS	Default Spread

Table B.2. Descriptions of the financial variables, used by Çakmakli & Van Dijk (2010).