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MSc Economics & Business
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Tactical Asset Allocation with VIX Futures

The implications of adding a long VIX Futures position to an investment portfolio

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ABSTRACT

The study starts with an extensive description of the characteristics of the VIX, VIX Futures and their relation to the S&P 500. This study examines the impact that the addition of VIX Futures to the opportunity set has for an average US investor. The ex-post results indicate that adding VIX Futures to various types of portfolios yield significant diversification benefits for a risk-averse US investor. Besides using traditional performance measures, the study includes an investor's preference for positive skewness and low kurtosis. It is shown that it is ex-post optimal to include a small long VIX Futures position during 2005-2011. Also the prospective analysis suggests including a long VIX Futures position for the near future. Several dynamic trading strategies are examined which indicate that the VIX index can be used as a stock market indicator under certain circumstances. A 'timing with VIX Futures' trading strategy that uses the VIX index as a crisis identifier yields a very favourable risk-return relation. Overall, the unique risk & return characteristics of VIX Futures combined with the strong negative correlation between VIX Futures and the traditional asset classes indicate that VIX Futures function as a very interesting diversification vehicle. The goal of the study is not to provide a specific investment strategy recommendation for the average US investor; the goal of the study is to encourage investors to consider having VIX exposure. It is shown that a long VIX Futures position creates significant diversification opportunities, especially during bearish market environments.

Keywords:
VIX, VIX Futures, MV Analysis, Portfolio Evaluation, Optimal Portfolio Allocation

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Chapter 1: Introduction

From 2004 until early 2007 the global financial markets have been relatively calm. The market volatility, as measured by the S&P 500 volatility index has been below long term averages. Diversification basically means risk reduction through investing in a variety of assets. Diversification opportunities are abundant; it is possible to invest in stocks, bonds, high yield bonds, hedge funds, commodities, private equity and real estate. Correlations between these different asset classes are considered to be relatively low, ensuring a limited volatility as the various asset classes do not fluctuate in value at the same time or rate. From September 2008 to March 2009 the S&P 500 drops almost 47%. Other asset classes that are considered effective equity diversifiers face substantial losses. Traditional diversification instruments prove to be a relatively ineffective hedge against the 2008 stock market crash. Even hedge fund strategies and commodity indices are not immune from declining.

One asset class performs well during the crash; volatility. While the stock market plummets with almost 50% during 2008-2009, the CBOE Volatility Index rockets with more than 125% during the same period. The negative correlation to the S&P 500 leads many investors to explore the VIX as a potential way to protect their portfolios from another collapse. It seems that the VIX index enables managers to grasp the portfolio diversification that they are looking for. The on-going globalization and integration of the worldwide financial markets combined with the uncertainty in these financial markets are reasons for the search for an effective diversification instrument.

This study examines the impact of adding VIX Futures to the opportunity set of an average US investor. All indices and portfolios are taken from the viewpoint of an American investor, and are quoted in US dollars. The study starts with assessing the correlations between various asset classes; thereafter the study demonstrates that the addition of VIX Futures to 3 different traditional portfolios result in significant diversification benefits based on a mean-variance analysis. To continue, the study investigates whether holding a long VIX Futures position is efficient for a risk-averse investor with a preference for positive skewness and low kurtosis. All the various test results are in line with the expectation that adding VIX Futures to a traditional portfolio yield diversification benefits. Besides testing for the traditional portfolios, 2 optimal portfolios are constructed based on historical data ranging from 2005 – 2011; one optimal portfolio has the opportunity to include VIX exposure, while the other does not have this opportunity. Both portfolios are tested and evaluated extensively; the results indicate that it would be optimal to include a small long allocation to VIX Futures.

The VIX Futures are introduced in March 2004, due to liquidity reasons the data sample in this study starts from January 2005. The data set spans from January 2005 until December 2011; the results only have limited forecasting power as the data is heavily influenced by the 2008 financial crisis. It is not expected that the financial markets will behave in the same way in the near future. In order to account for this limited forecasting power this study constructs 2 prospective portfolios based on a prospective financial outlook for 2012-2016. The results indicate that due to the inverse relationship between VIX Futures and the other classes it is optimal to include a small long VIX Futures exposure for the coming years, even though the VIX Futures are expected to yield negative average returns. Additionally, the study examines various dynamic trading strategies based on the level of VIX, VIX Futures prices and the inverse relationship between the VIX, VIX Futures and the S&P 500.

The goal of this study is to point out the various opportunities an investor creates by adopting VIX Futures in his opportunity set. The study shows that ex-post diversification benefits are realized by considering a long VIX Futures allocation during this period. The study does not provide a specific strategy recommendation for the average US investor. However, the study tries to encourage investors to consider having VIX exposure in their portfolio, by the use of VIX Futures.

This research is original for several reasons, the empirical evidence in favour of using VIX Futures as an effective diversification instrument is scarce. This paper therefore is very relevant and aims to fill various gaps in the existing literature. First of all, this paper can be seen as an extension of Szado (2009) and Briere, Burgues & Signori (2009) as their framework for analysing VIX Futures portfolios is extended to a more relevant and extended time frame. Second, this study is unique as it employs a mean-variance analysis that creates ex-post optimal portfolios during 2005-2011. The effects of adding a long VIX Futures position to a portfolio hereby become apparent. To continue, the study fills a gap in the existing literature as an ex-ante optimal portfolio is constructed for 2012-2016, that indicates that a long VIX Futures allocation is optimal. The study employs a sensitivity analysis that provides a clear understanding of the relation between VIX Futures and a pure equity portfolio. Finally, the study denotes three sample trading strategies based on the VIX and VIX Futures, the sample strategies demonstrate to investors the potential of including VIX Futures to their opportunity set.

The study proceeds as follows: Chapter 2 comprehends the main motivation behind this paper; the correlations between the various asset classes are investigated, while also the relevant literature is examined. Chapter 3 focuses on the relationships between the VIX, VIX Futures and the S&P 500. Chapter 4 summarizes the data set that is used; Chapter 5 encloses the methodology that is used in order to perform the various performance tests. Chapter 6 discusses the study's ex-post results as well as the ex-ante optimal portfolio analysis for 2012-2016. Chapter 7 simulates dynamic ex-post trading strategies, while Chapter 8 discusses the implications of the study's findings and concludes.

Chapter 2: Motivation

This section entails a brief introduction to the VIX, as it is important to understand what the VIX represents and implies. Also the various asset class correlations are analysed together with relevant previous literature in order to fully gauge the scope of this paper.

2.1 VIX

"Officially the VIX is known as the CBOE Volatility Index, the VIX is often symbolized as the "fear index". The VIX measures the implied volatility that is being priced into S&P 500 index options. The VIX offers a rightful indication of 30-day implied volatility priced by a wide variety of options from the S&P 500 Index option market. It is important to fully understand what this means; The VIX is forward-looking as it measures the volatility that investors expect to see in the market. It is implied by the current prices of S&P 500 index options and represents the expected future volatility over the next 30 days.

Dr. Robert Whaley of the Vanderbilt University introduced the CBOE Volatility Index in 1993. The original VIX is constructed from the expected volatility priced in the contracts of 8 S&P 100 (OEX) put- and call-options. The intention behind the VIX is to provide a benchmark of expected short-term market volatility. The methodology of the VIX is revised by the CBOE in combination with Goldman Sachs in 2003. The underlying options changed from the S&P 100 to the S&P 500. The S&P 500 has become the most active index option market in the US. Furthermore the S&P 500 is considered to be the benchmark index for the performance of the United States stock market. The S&P 500 Total Return Index is therefore also used in this study as the benchmark index for the American stock market. The S&P 500 consists of 500 of the largest US companies that meet specific requirements based on market capitalization, liquidity, industry sector, and more.

The VIX functions as an indicator of the 30-day implied volatility based on various S&P 500 Index option prices. The option price used to estimate the VIX is the mid-point between the Bid/Ask spread of the option, both at-the-money and out-of-the-money put and call option contracts are considered. The expiration of the options ranges from second month expiration until the front month contract has only 8 days remaining until expiration. The front month option contract is rolled over to the next month in order to eliminate some of the end-of-the-contract volatility that can occur in the market.

The VIX level thus represents the market's expectation of 30-day volatility. The VIX is expressed annually; a VIX level of 40 would results in an expected move of 11.54% over the next 30 days. The formula for determining the expected 30-day volatility movement is simple:

$$30 - \text{day movement} = \frac{\text{VIX}}{\sqrt{12}}$$

Assuming that there are on average 252 trading days in a year, the single-day implied volatility can be determined:

$$1 - \text{day movement} = \frac{\text{VIX}}{\sqrt{252}}$$

Hence a VIX of 40 can be interpreted as the S&P 500 option market anticipating a daily price move of 2.5%.

2.2 Asset Class Correlations

At the beginning of the 21st century the correlations between equities, bonds and alternative assets are considered to be relatively low. Correlations between assets measure the degree to which the prices of the individual assets move together. A high correlation indicates that both assets have a large common risk factor. The magnitude of this common risk factor can be driven by changes in the market structure as well as by changes in the macro-economic outlook. It is assumed that economies become more integrated, resulting in more efficient and global financial markets. Many researchers argue that traditionally perceived diversifiers failed to do so in the recent economic crisis. The search for a new effective diversifier intensified.

Korovilas and Alexander (2012) conclude that international equities have become more correlated as have international bonds and even the equity- bond correlation has risen. Chow, Jacquier, Kritzman and Lowry (1999) examine the correlations between various asset classes during market stress. They find that the returns of all asset classes become more volatile during market turmoil. They conclude that the asset class correlations rise in turbulent times.

The portfolios prove to be less diversified during these times as is assumed under ‘normal’ conditions; Market stress limits the diversification properties of the financial assets. They suggest including commodities, as commodities provide diversification benefits when markets experience stress. Daskalai & Skiadopoulos (2011) on the contrary find that the alleged diversification benefits of commodities barely offer diversification since 2008.

Markwat (2012) examines the effects of an increased interdependence between international stock markets on the probability of global crashes. Markwat finds that global crashes are very catastrophic for the average investor as diversification opportunities evaporate during such crises.

Exhibit 2.2 summarizes the various asset class correlations for the entire sample period. Based on previous literature the study expects to find an increased correlation between the assets during the peak of the financial crisis (Aug 2008 – Dec 2008). Investors who think that their portfolio is well diversified based on historical data are proven wrong; their portfolio is effectively barely diversified. It is expected that the correlations between the assets remained high since the peak of the financial crisis, due to the ongoing uncertainty in the global financial markets. The asset class correlations during the financial crisis are denoted in *Italic* and between brackets ( ).

The correlation matrix shows that the negative correlation of Bonds with Equity is lower during the crisis than during the entire sample. Besides Bonds, also the correlation of High Yield bonds, Hedge Funds, Commodities, Private Equity and Real Estate with Equity is higher during the crisis period. This is in line with the literature that traditional diversifiers offer less diversification opportunities during a financial crisis. From exhibit 2.2 it can also be concluded that the negative correlation between Equity and the VIX / VIX Futures increases during the crisis. This prompts that equity volatility, represented by the VIX, may serve as natural diversifier, as its negative correlation with Equity and the other asset classes increases during the crisis period. It seems as the VIX and VIX Futures provide diversification benefits exactly when they are needed most.

Exhibit 2.2: Asset Class Correlation

2005 - 2011 *(Aug - Dec 2008)*	EQUITY	BONDS	HIGH YIELD	HEDGE FUNDS	COMMODITY	PRIVATE EQUITY	REAL ESTATE	VIX	VIX FUTURES
EQUITY	1.00	*(-0.18)*	*(0.17)*	*(0.62)*	*(0.42)*	*(0.83)*	*(0.83)*	*(-0.87)*	*(-0.75)*
BONDS	-0.29	1.00	*(0.2)*	*(-0.05)*	*(-0.18)*	*(-0.07)*	*(-0.17)*	*(0.11)*	*(-0.01)*
HIGH YIELD	0.31	0.07	1.00	*(0.45)*	*(0.37)*	*(0.38)*	*(-0.00)*	*(-0.20)*	*(-0.22)*
HEDGE FUNDS	0.56	-0.16	0.52	1.00	*(0.48)*	*(0.68)*	*(0.33)*	*(-0.67)*	*(-0.50)*
COMMODITY	0.35	-0.18	0.27	0.42	1.00	*(0.47)*	*(0.22)*	*(-0.44)*	*(-0.30)*
PRIVATE EQUITY	0.81	-0.23	0.48	0.59	0.42	1.00	*(0.69)*	*(-0.73)*	*(-0.66)*
REAL ESTATE	0.81	-0.20	0.14	0.31	0.23	0.67	1.00	*(-0.65)*	*(-0.62)*
VIX	-0.76	0.26	-0.23	-0.55	-0.25	-0.60	-0.55	1.00	*(0.81)*
VIX FUTURES	-0.69	0.22	-0.23	-0.49	-0.24	-0.59	-0.55	0.79	1.00

Note: Exhibit shows correlation of daily return 2005-2011, versus Aug 2008- Dec 2008 between brackets. The indices used to represent the various asset classes are described in Chapter 4; all indices are taken from the viewpoint of an American investor and are quoted in US Dollars.

2.3 Previous Literature

Exhibit 2.2 illustrates that equity volatility arises as a natural diversifier for a portfolio. Multiple studies are performed investigating the relationship between equities, other asset classes and equity volatility. Most of these studies start with a reference to a paper by Black (1976) where he introduces the "leverage effect" in his study of stock market volatility. This "leverage effect" theory advocates a negative correlation between the changes in stock prices with changes in stock volatility.

Christie (1982) examines the relationship between the variance of equity returns and several explanatory variables. He concludes that equity variance has a strong positive association with financial leverage. The strong negative elasticity of variance relative to the value of equity is due to this financial leverage. Schwert (1989) examines the relation of stock market volatility with general economic activity and stock trading. He finds that the volatility of the stock market increases by a factor 2 or 3 during the Great Depression crisis period. He also finds that when stock prices drop relative to the prices of bonds, the volatility of stocks increases. Moreover, Glosten et al. (1993) investigate the trade-off between risk and return among different securities within a given time period. They find support for the negative correlation between conditional return and conditional variance of return. Additionally, Turner et al. (1989) investigate the relation between movements in volatility and excess returns. They find that that there is a negative correlation between movements in volatility and in excess returns.

All these studies find that equity downturns are accompanied by an increasing leverage and a higher volatility. Besides the negative correlation between equity and equity volatility, another interesting aspect regarding this relation is found by Haugen (1991).

Haugen (1991) estimates volatility changes in daily returns to the Dow Jones Industrial Average. He examines the reaction of the level of stock prices and the expected returns to these changes in volatility. He finds that stock prices and the expected returns are heavily affected by changes in volatility. More interesting, he notices an asymmetry in the market's reaction to a rise in volatility as opposed to a decrease. This finding is further supported by Munenzon (2010), who suggests that equity volatility arises as a natural diversifier as its negative correlation with equities increases exactly when diversification is needed most.

Whaley (2000) prompts that the negative asymmetric correlation between the S&P 500 returns and changes in the VIX index can be explained as follows: when expected market volatility increases, investors demand higher rates of return on stocks; stock prices therefore fall. The real relation between an increase in expected volatility and the decrease of stock market prices is more complicated. Whaley documents that the change in VIX rises at a higher absolute rate when the stock market falls than when stock markets rise. The VIX presents itself much more as a barometer of investor's fear than as a barometer of investor's greed.

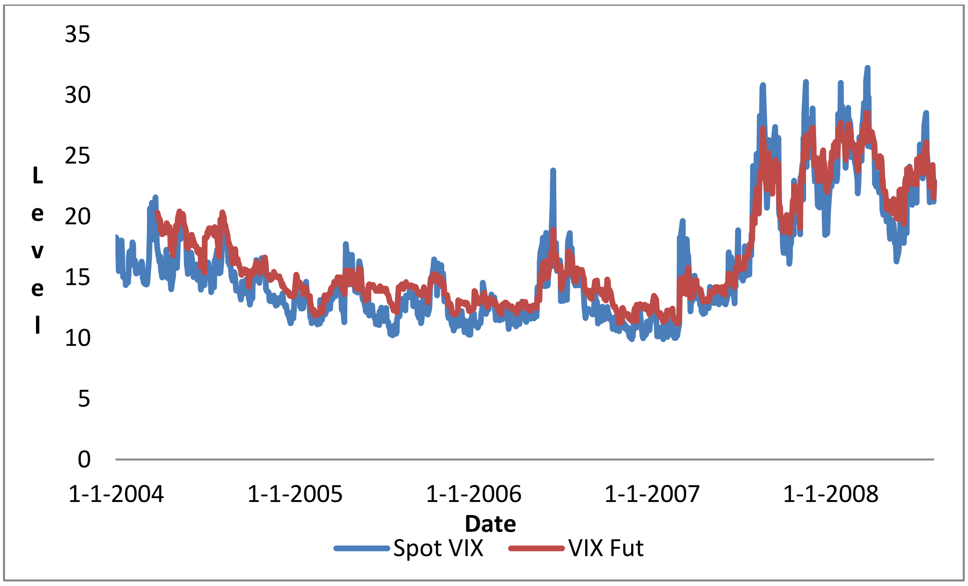
Daigler & Rossi (2006) perform a very interesting study; they study the consequences of including volatility as a separate asset class. They show that risk-return benefits can be realized by including volatility as an asset. The addition of volatility to an S&P 500 stock portfolio reduces risk substantially without having much effect on the portfolio's return. They suggest that the strong negative correlation between spot VIX and the S&P 500 implies that there are significant benefits to adding volatility to a stock portfolio.

The paper of Daigler & Rossi (2006) is in line with the scope of this research, as they examine the diversification opportunities from adding VIX to a portfolio. One limitation of their study is that they explore the effects of adding spot VIX to an equity portfolio. Spot VIX is not an investable product, so their research is completely hypothetical. Others also investigate whether the addition of spot VIX to a portfolio provides diversification benefits:

Dash & Moran (2005) research whether diversification benefits can be realized by adding volatility (VIX) to a hedge fund portfolio. They find that the VIX is not only negatively correlated to hedge fund returns; the correlation profile is asymmetric with the correlation being more negative in negative months for hedge funds. Hence, when hedge funds are performing the worst, the diversification benefit of adding volatility is best. More surprisingly, they find that correlation between hedge fund returns and VIX is positive when the diversification or protection is least needed. They suggest a small allocation of 0% to 10% portion of the portfolio to volatility.

Brière, Burgues, and Signori (2010) employ an ex-ante analysis in order to find diversification benefits for long equity investors based on mean-variance optimization. They conclude that it is possible to reduce the risk profile of a portfolio significantly through including a pure volatility investment. One limitation of their study is the time period that they analyse. Their data starts in 1990 and ends in August 2008; before the start of the banking crisis. VIX Futures prices have been available since 2004; they approximate the VIX Futures prices prior to 2004 based on the average relation between the VIX Futures and the VIX during 2004 – 2008. The VIX Futures prices of 1990 until 2004 are therefore based on the unrealistic assumption that the relation between the VIX and VIX Futures is stable over time; there is no direct financial relationship between the VIX and VIX Futures, as is explained later. Furthermore due to the exponentially increased liquidity of the VIX Futures market the relation to the spot VIX changed significantly during 2004-2008. As can be seen in exhibit 2.3 the relation becomes more accurate with the increased liquidity of VIX Futures.

Exhibit 2.3: Relation spot VIX – VIX Futures 2004-2008



All these researches are in line with the results found in Exhibit 2.2, the study can conclude that there is a strong negative correlation between most asset classes and the level of VIX. Bonds show a small negative correlation with the Equity, but this correlation is lower during the crisis than during the entire sample. Hence holding a portfolio with bonds did not provide the diversification benefits one expected during the crisis. The VIX has a strong negative correlation with most asset classes when diversification opportunities are needed most; during market turmoil. The VIX appears as a very interesting diversification vehicle.

There are also a couple of studies that investigate the addition of investable VIX products to a portfolio. Wien (2010) revises trading strategies based on the VIX. He questions the profitability of hedging with VIX options as there are many traps and mistaken assumptions about the VIX. Alexander and Korovilas (2012) perform multiple studies in order to investigate the functioning and possible opportunities of VIX ETN's. They investigate if equity investors can benefit from including an exposure to VIX Futures and / or their exchange-traded notes in their portfolio. They find that volatility diversification through the use of VIX Futures is only optimal during the first few months of a great crisis of similar magnitude to the banking crisis in late 2008

Manda (2010) studies the stock market volatility and the behaviour of various measures of volatility before, during and after the 2008 financial crisis. He also checks whether the leverage effect is observed during this period. He finds that volatility increases significantly during the 2008 crisis period compared to the pre-crisis period. He also finds the leverage effect during the sample period; a negative correlation between changes in stock prices and changes in stock volatility.

Szado (2009) assesses the impact of a long VIX investment as a diversifier for a traditional portfolio during the 2008 credit crisis. The paper by Szado (2009) forms the basis of this study. He investigates whether VIX Futures (or VIX call options) serve as a useful diversification instrument during the financial crisis of 2008. Literature suggest that a long volatility position may result in negative excess returns over the long run, but that a long volatility position may also provide significant diversification benefits in large downward moves of the market. Szado concludes that VIX Futures can be used in order to provide diversification during 2008 credit crisis. He analyses the impact of small VIX Futures allocation on the performance of three traditional portfolios. These traditional portfolios are: **A:** a 100% long-only equity portfolio, **B:** a mixed portfolio with 60% stocks and 40% bonds, and **C:** a fully diversified portfolio with multiple asset classes. He tests the impact of including a 2.5% or 10% long VIX Futures position. The addition of VIX Futures to the three portfolios both increases returns and reduces the standard deviation of the returns. In periods of extreme market losses all financial assets tend to lose value, but in such a case VIX Futures position serves as protection. This paper follows the methodology and portfolios developed by Szado (2009). Szado concludes that during a period of market downturn, significant diversification benefits can be realized by including VIX Futures in the portfolio. The study employs an ex-post analysis based on a very tendentious time period as his data ranges only from 2006 until December 2008.

The characteristics of the VIX, the correlation matrix between the asset classes as well as the previous literature lead to the question that motivates this study: Will a long VIX futures position provide diversification benefits to a portfolio?

H_0 = Diversification benefits can be realized by adding VIX Futures to a portfolio.

H_a = No diversification benefits can be realized by adding VIX Futures to a portfolio.

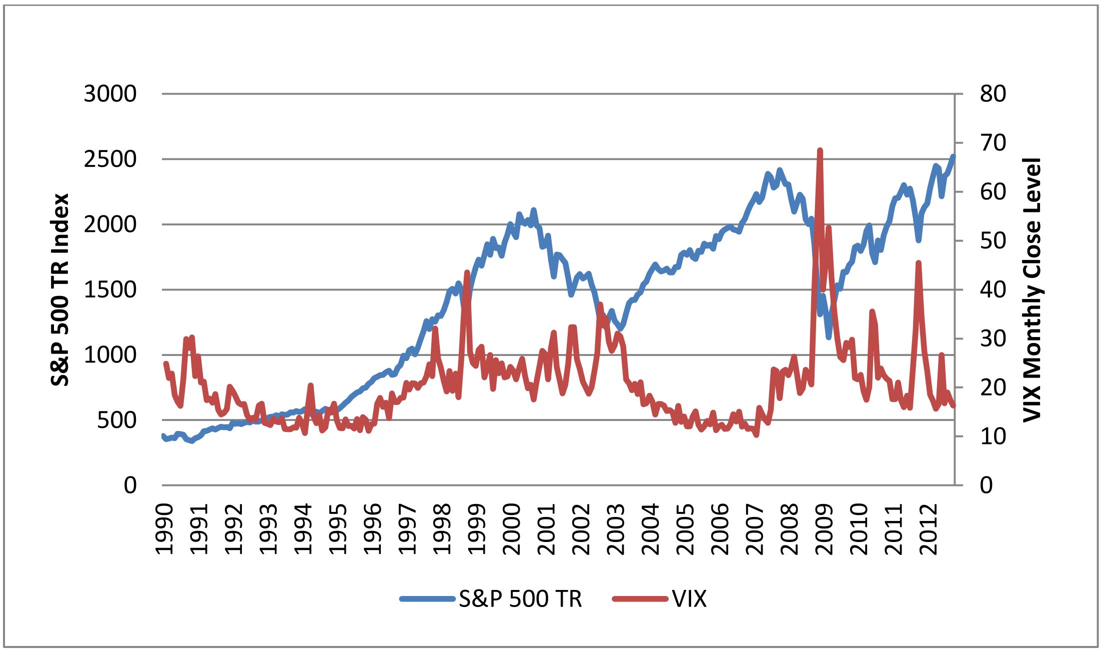
Chapter 3: Relationships VIX & VIX Futures

Before an investor can add a long VIX Futures position to his portfolio it is important to understand the exact relationship between the VIX and the S&P 500. Additionally, the relationship between the VIX Futures and spot VIX is examined in order to gauge the implications of having a long VIX Futures position.

3.1 VIX Relation to S&P 500

The VIX is an indicator of 30-day implied volatility determined by the prices of S&P 500 index options. In order to understand the VIX and its normal behaviour with respect to the S&P 500, the history of the VIX relation to the S&P 500 is examined. Exhibit 3.1 shows the month-ending levels of the S&P 500 and the spot VIX from the beginning of January 1990 through September 2012.

Exhibit 3.1: Historic Relation S&P 500 & VIX 1990-2011



From this figure it can be seen that the VIX frequently spikes. For example, the jump in the middle of 1990 occurred when Iraq attacked Kuwait, the jump in October 1997 occurred following a stock market panic attack in which the Dow Jones fell 555 points. The October 1998 spike resembles a period of general nervousness in the stock market. After each of these spikes in the level of the VIX, the VIX returns to normal. This figure suggests a strong negative relationship between the VIX and the S&P 500, but this is not always the case. There are also times when a run-up in stock prices is accompanied by a run-up in volatility. In January 1999, the VIX was rising while the level of the S&P 500 index was also rising. Hence, while investors became more nervous, the market was still rising.

This can be further emphasized when we examine the relationship between the VIX and the S&P 500 during the 2008 crisis. As can be seen in Exhibit 3 the VIX fell sharply in early 2009, while the S&P 500 continued to fall.

Exhibit 3.1.2: S&P500 TR & VIX, 2004-2012

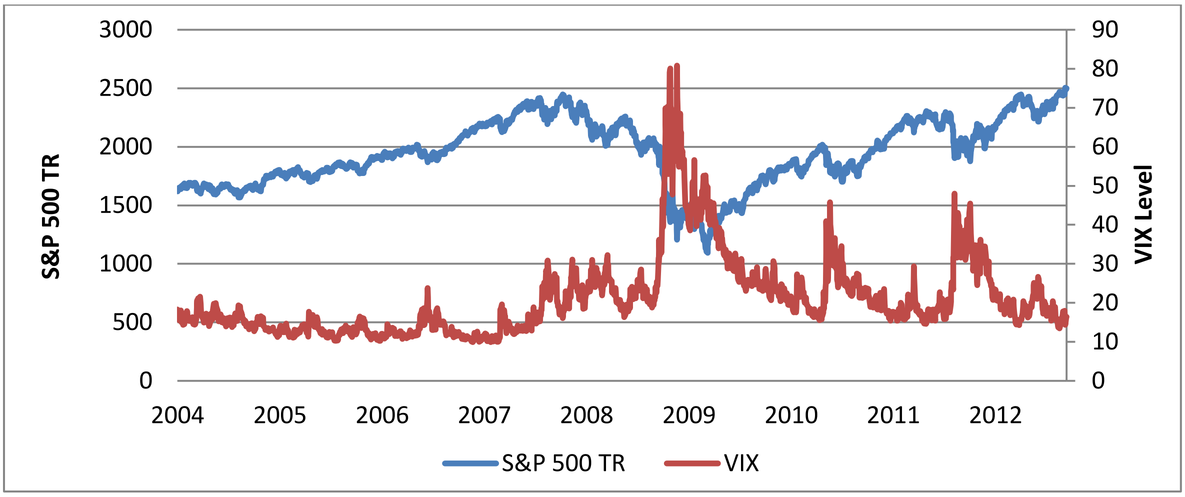


Exhibit 2.2 indicates that there is a strong negative correlation between the VIX and the S&P 500, this inverse relation seems to be extremely strong when the S&P 500 plummets. Exhibit 3.1 & 3.1.2 suggest that the strong negative correlation between the VIX and S&P 500 is time varying and conditional; the VIX and S&P 500 tend to move in opposite directions, however this is not always the case as can be seen in early 2009.

Exhibit 3.1 and 3.1.2 show that the VIX is basically flat over the long-term, while the S&P 500 has risen tremendously since 1990. Spikes in volatility happen quickly, and then the market returns to normal. The short term inverse relationship between the S&P 500 and the VIX is very interesting. This inverse relationship can be further emphasized by the negative correlation of -0.76 between the VIX and the S&P 500. As stated before the VIX is a measure of implied volatility priced by the S&P 500 index option market through the use of a wide variety of options. Implied volatility is estimated by the current prices of option contracts and displays the market's estimation of volatility over the next 30 days. The inverse relationship can be explained as follows: When investors fear the direction of where the S&P 500 is going, they want to protect themselves. Protection can be found in multiple ways, for example through buying S&P 500 index put options. When a lot of investors want these index options, this results in a rising demand for index put options. This increase in demand results in a higher price for put option contracts. The higher price of the option contracts lead to a higher implied volatility. Note that it is also true that a higher implied volatility results in a higher option price, as implied volatility is the forward-looking volatility. The implied volatility is an indicator of the movement that is expected from the underlying over the life of the option. The VIX is a measure of implied volatility of the S&P 500 option market, and thus rises when the market is under pressure. As the demand for portfolio insurance (protection) is higher with plummeting markets than with rising markets the relation between the rates of change in the VIX and the S&P 500 is asymmetric. The VIX rises more when the S&P moves down, than when the S&P moves up. Therefore the VIX is more of a 'fear' indicator than a 'greed' indicator.

Rolling correlations can be used to track the historic relation between the VIX, VIX Futures and the S&P 500. Exhibit 3.1.3 shows the time varying 12-month rolling correlations of the S&P 500 with the VIX from 2004 – 2011. A 12-month time frame is chosen as it is long enough to observe a stable pattern and short enough to be timely. Exhibit 3.1.3 concludes that there is a strong negative correlation over time between the VIX, VIX futures and the S&P 500. The negative correlation between VIX Futures with the S&P 500 increases in time, this is due to the illiquidity of the VIX Futures market at the beginning.

Exhibit 3.1.3: 12 Month Rolling Correlation S&P 500 & VIX

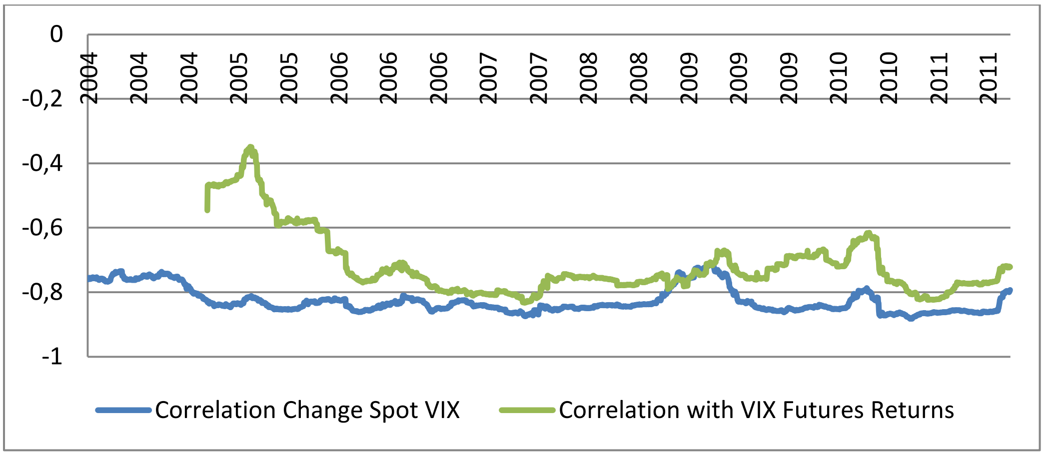
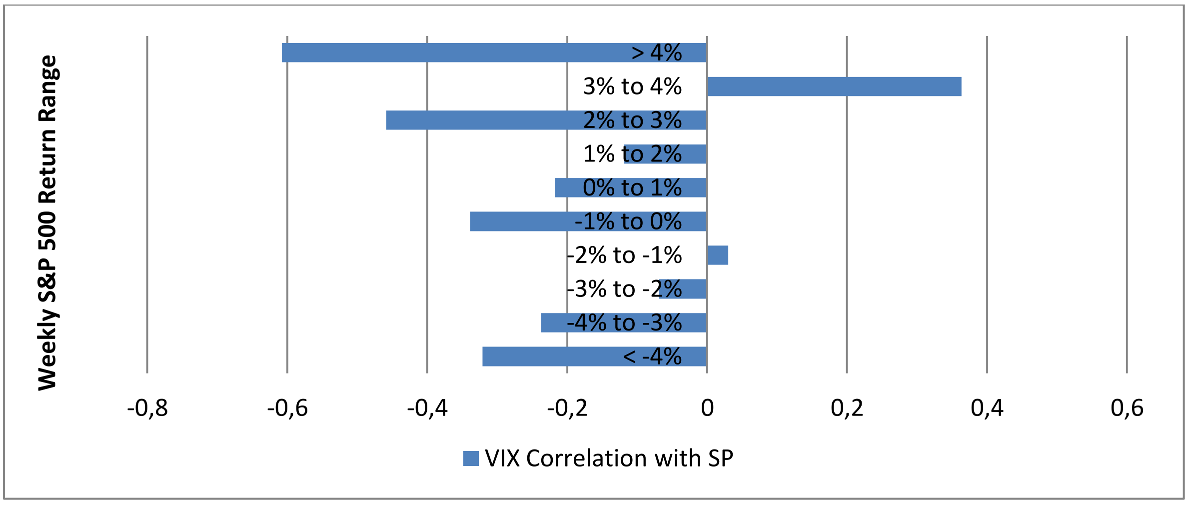


Exhibit 3.1.4 shows the conditional correlation between the VIX and S&P 500 during 2004 – 2011. The conditional correlation diagram suggests that the negative correlation is particularly strong when the S&P 500 experiences large changes, positive or negative. Hence, exhibits 3.1.3 & 3.1.4 provide further evidence that the relation between the VIX and S&P 500 is strongly negative but time varying. This is in line with the scope of this research.

Exhibit 3.1.4: Conditional Correlation S&P 500 & VIX 2004 - 2011



3.2 VIX Futures & Relationship to VIX

Due to the inverse relationship between the VIX and the S&P 500 many people are looking for ways to trade the VIX. Since March 26, 2004 it is possible to trade a product that is directly related to the level of the VIX. The CBOE Futures Exchange began trading futures contracts based on the VIX. VIX Futures can be listed for up to 9 consecutive months; they expire on the Wednesday that is 30 days before the following month's standard option expiration date. Standard option expiration is always on the third Friday of a month; backing up 30 days automatically is a Wednesday. For example the October 2010 futures contract; the standard option expiration date was November 19th 2010, hence the October VIX Futures contract expired on Wednesday October 20th 2010. The symbol for a VIX futures contract consists of the letters VX, a letter indicates the expiration month, and a number represents the expiration year. The October 2010 contract is written down as: VX-V-10. The contract multiplier for each VIX Futures contract is $1000. This means that for example when the October 2010 contract is trading at 20.00, the value of the contract is $20.000. The minimum amount a VIX Future contract can move is 0.05; hence the minimum price movement is $50 for a contract.

VIX Futures are cash settled based on the final settlement price. Returns are estimated based on the difference between the sales price and the settlement price. The final settlement price is determined by a Special Opening Quotation process, which occurs the day after expiration and is based on the opening prices of the relevant S&P 500 index option contracts.

After the introduction of the VIX Futures the market was relatively illiquid, the market for offsetting a contract was small. The VIX futures market has experienced tremendous volume & open interest growth over the past few years. Volatility is now considered to be a separate asset class. VIX futures are widely used for hedging against a drop in stock market prices. The liquidity of the VIX Futures market is indicated through expressing the trading volume as well as the open interest of VIX Futures contracts. A large open interest indicates a lot of trading activity, as open interest represents the number of contracts outstanding that have not yet been settled. Exhibit 3.2 and 3.2.2 denotes the tremendous volume & open interest growth the VIX Futures market has shown over the past few years.

Exhibit 3.2: Total Volume Traded VIX Futures Contracts

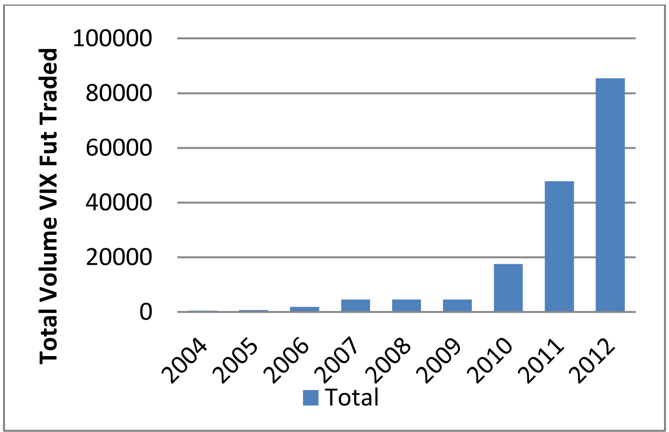
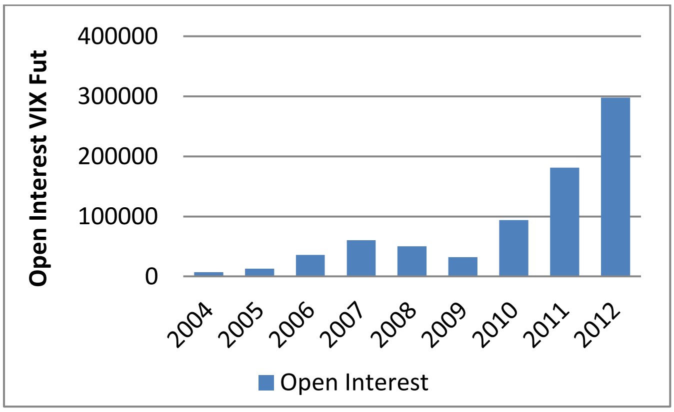


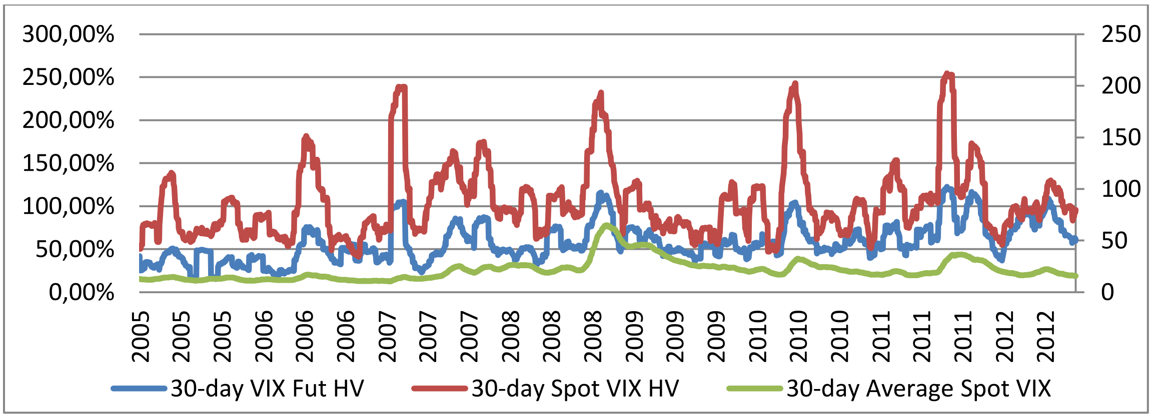
Exhibit 3.2.2: Open Interest VIX Futures Contracts



Another interesting aspect regarding the VIX futures contracts is its pricing relationship with the VIX index. The value of a VIX Futures contract exhibits the markets expectation of where the VIX index will trade as of the expiration date of the futures contract. With respect to VIX futures, there is no underlying basket of securities that may replicate owning the VIX index. Therefore, no financial relationship exists that holds the VIX futures in a fair value range around the VIX index based on arbitrage trades. As stated above, the VIX mean reverts over the long term, while VIX Futures are expected to indicate the direction of the VIX. When the VIX is above its long-term mean, the futures are likely to trade at a discount versus the index (backwardation). If the VIX is below its long-term mean the futures trade above the index (contango). Uncertainty is reflected in a higher implied volatility; long dated contracts have more uncertainty than short dated contracts, as it is harder to predict a price in 2 months than in 1 month. This higher uncertainty is priced in the Futures contract, as it generally trades at a premium to the VIX index. The extra uncertainty can be seen as a sort of risk premium that is included in the VIX Futures price. VIX Futures are generally in contango, which means that there is a high roll-over cost for VIX Futures; the expected volatility in 2 months is higher than in 1 month, thus the 1 month VIX Future is more expensive than the expiring VIX Future contract. In order to have a long allocation to VIX Futures, the contracts are rolled over into the next front-month contract at the close on the day prior to expiration. This indicates that having a consistent long VIX Futures allocation in a portfolio is relatively expensive. It seems that a portfolio with a long allocation to VIX Futures performs well during bearish market environments, during bullish market environments a portfolio consisting of VIX Futures is expected to underperform a pure long stock portfolio.

Besides the pricing relationship between VIX Futures and the VIX level, they both exhibit a different historical volatility. The VIX Futures tend to have a drastic lower volatility than spot VIX, as the VIX is mean reverting and the VIX Futures represent the market's expectation of where the index will be as of expiration of the contract. The rolling 30-day historical volatilities of spot VIX and VIX Futures are shown in exhibit 3.2.3. The difference between the volatilities is quite large; especially the volatility of the spot VIX is very high at times.

Exhibit 3.2.3: 30-day Rolling Historical Volatility of VIX, 2005-2012



Chapter 4: Data

This chapter sets out the definitions of the variables that will be used in the research as well as the data used for the analyses. Additionally the descriptive statistics for the main variables are summarized and discussed.

This study utilizes daily data for a range of assets from January 2005 to December 2011, resulting in over 1750 daily returns for the different portfolios. This is done because of the availability of the VIX Futures. As stated before VIX Futures contracts began trading on March 26, 2004. However, for liquidity reasons the study starts the analysis from 1st January 2005. Datastream is used to gather all the daily prices and returns for the various asset classes, the specific code is stated in parenthesis. All indices are taken from the viewpoint of an American investor; all the indices are American based indices and hence prices are quoted in US dollars. "From all indices the Total Return Index is used as the TR measures the performance of the various assets by assuming that all cash distributions are reinvested, in addition to tracking the components' price movements." The S&P 500 Index (S&PCOMP) is used to represent returns to stocks. Literature suggest to use the S&P 500 index as a benchmark index for equity, as it represents 500 of the largest US companies from various industries. Furthermore a number of alternative asset classes are included in order to provide a well-diversified benchmark portfolio to which a long VIX exposure is added and subsequently tested. Hedge funds are represented by the investable HFRX Global Hedge Fund Index (HFRXHF$). Real estate returns are represented by the S&P US REIT Index (SBBRUSL). Private Equity is represented by the S&P Listed Private Equity Index (SPLPEI$). Commodity returns are represented by the S&P GSCI Index (GSCITOT). The CBOE VIX level is also retrieved from DataStream (CBOEVIX). The VIX Futures contracts prices (CVXCS00) are downloaded from DataStream, as DataStream continuously rolls the future position into the next-front month contract at the close on the day prior to expiration. This data is in line with the data retrievable from the CBOE website.

The total return indices for Bonds and High Yield Bonds are not available through Datastream, and were subsequently downloaded from Barclays Bank PLC. Bonds are represented by the Barclays Capital US Aggregate Index (LHAGGBD). High Yield bonds are represented by the Barclays Capital US High Yield Index (LHYIELD).

The data sample used in this research spans from January 2005 until December 2011, as the VIX Futures were only introduced in March 2004. From January 2005 until 2007 the financial markets have been relatively calm. From early 2008 onwards the uncertainty in the financial markets rises significantly, resulting in the financial crisis of late 2008. During the financial crisis of 2008-2009 the S&P 500 fell by almost 47% of peak to trough. Other asset classes that were considered effective diversifiers faced substantial losses. Volatility performs well in the face of the crash; the VIX index rockets with more than 125% during the same period. From 2009 the financial markets continue to behave very uncertain, it is generally perceived that due to the US debt-ceiling crisis as well as the Euro debt crisis, the financial markets are on the brink of a relapse into another recession. The VIX and also VIX Futures perform a lot better during these years than is normally expected. The results found in this study are therefore heavily affected by these turbulent times. Great care should be taken in interpreting and applying the results of this analysis.

In the following section the descriptive statistics for the main variables in the research are stated. Based on the descriptive statistics, expectations can be formulated about possible outcomes for the research.

Exhibit 4: Descriptive Statistics 2005 - 2011

	Equities	Bonds	High Yield	Hedge Fund	Commodities	Private Equity	Real Estate	VIX Spot	VIX Fut
Average Ann Return	2,60%	5,53%	7,36%	-0,35%	-1,11%	-7,52%	0,24%	8,08%	9,72%
Max Ann Return	26%	8%	58%	13%	33%	52%	31%	95%	88%
Min Ann Return	-37%	3%	-26%	-23%	-46%	-67%	-42%	-46%	-45%
Average Ann StDev	21%	4%	5%	4%	26%	27%	37%	110%	59%
Skew	-0,37	-0,04	-1,59	-1,07	-0,30	-0,23	-0,13	0,76	0,79
Kurtosis	11,83	4,67	27,48	11,44	5,37	11,36	12,55	7,38	7,07
% Up Days	55%	55%	62%	56%	51%	52%	52%	45%	44%
% Down Days	45%	45%	38%	44%	49%	48%	48%	55%	56%
Return Total Period	20%	47%	67%	-2%	-7%	-41%	2%	76%	97%

Note: All average returns are geometric averages and the returns are continuous compounded returns (Log Returns)

Exhibit 4 summarizes the performance of the individual asset classes during 2005-2011. Note that the average annual returns and annual standard deviations differ greatly between the various asset classes. Different returns and different standard deviations indicate that significant diversification benefits can be realized by combining various assets in a portfolio. From exhibit 4 it can be seen that Bonds and High Yield bonds show a very favourable risk return relation compared to Equities, Hedge Funds, Commodities, Private Equity and Real Estate. Exhibit 2.2 illustrates that the correlation between Bonds & High Yield bonds and the other assets is very low and slightly negative. Bonds and High Yield bonds are not as heavily influenced by the 2008 financial crisis as the other assets. The maximum annual drawdowns of the other assets are a lot higher than for Bonds & High Yield bonds.

The volatility of spot VIX is very high compared to the other assets even compared to VIX Futures, as is explained earlier. The VIX Futures yield an annual average return of 9.72% which is really high as the monthly roll-over costs of the VIX Futures are already incorporated in the returns. Another interesting aspect is that all traditional asset classes show a negative skewness and a high kurtosis. Skewness is a measure of asymmetry in the return distribution, while kurtosis measures the "peakedness" or fat-tail degree of a distribution. A distribution with excess kurtosis has a peak in the middle and fat tails compared to a normal distribution. A distribution with negative skewness has more returns far away to the left of its mean return. Most investors do not like negative skewness and prefer a distribution with low kurtosis. The asset classes High Yield and Hedge Fund have a return distribution with skewness lower than -1 and an excess kurtosis higher than 1 (normal kurtosis =3), which means that the probability of having large unexpected negative returns is large. The VIX and VIX Futures yield a positive skewness, which prompts that these are very interesting diversification vehicles for most investors.

Chapter 5: Methodology

In order to assess the diversification impact of having VIX exposure, the addition of VIX exposure to three different portfolios is considered during 2005 – 2012. Portfolio **A:** a 100% long-only equity portfolio, **B:** a mixed portfolio with 60% stocks and 40% bonds, and **C:** a 60.5% stocks, 30.5% bonds, 1.3% high yield bonds, 1.2% hedge funds, 0.3% commodity, 1.6 % private equity, 4.5% real estate portfolio. The construction of these traditional portfolios is retrieved from a study by Szado (2009) and is based on the 'Pensions and Investments 2007 Average Allocation for US Institutional Tax-Exempt Assets'. Furthermore the portfolio allocations of these traditional portfolios show strong resemblance with the 'World Market Cap' constructed by Doeswijk, Swinkels and Lam (2012). To these portfolios a 2.5% and 10% long allocation to near-month VIX Futures is added. Each month, on the day before expiration of the futures the positions are rolled over to the next one-month contract.

Besides measuring the diversification impact of adding VIX exposure to the three traditional portfolios named above, the study will also perform a Mean Variance analysis in order to estimate the optimal portfolios during 2005-2012. As stated before the data sample is heavily affected by the recent turbulent markets, therefore also a prospective analysis is developed based on hypothetical returns and standard deviations. The last section simulates dynamic trading strategies based on various assumptions regarding the VIX and VIX Futures.

Note: All prices are quoted in US dollars, as this research is from the viewpoint of an American investor.

5.1 Mean Variance Optimization & CAPM

Markowitz (1952) introduced the mean-variance analysis; the concept of this theory is that a portfolio is constructed from different assets. The assets in the investment portfolio are not selected based on their individual performance; they are selected based on their performance relative to the other assets in the portfolio. Diversification can be realized by combining different assets. For selecting the right investments an investor should consider both expected return and the variability of these returns. Assets that minimize the portfolio's variance for a given expected return are demonstrated to be the most efficient. Expected return of the portfolio R_p is measured by the return on asset i; R_i and w_i is the weighting of component asset i:

$$E(R_p) = \sum_i \omega_i E(R_i)$$

Portfolio return variance is measured as:

$$\sigma_p^2 = \sum_i \omega_i^2 \sigma_i^2 + \sum_i \sum_{j \neq i} \omega_i \omega_j \sigma_i \sigma_j \rho_{ij}$$

Where ρ_{ij} is the correlation coefficient between the returns on assets I and J, σ is the standard deviation of the asset. An investor can reduce its portfolio risk simply by holding a combination of assets which are not perfectly positively correlated.

Portfolio Expected Return is denoted as:

$$E(R_p) = \omega^T \mu$$

And Portfolio Variance is denoted as:

$$\sigma_p = \omega^T \Sigma \omega$$

∑ is the covariance matrix for the returns on the assets in the portfolio; q is the risk tolerance factor.

$$\omega = \begin{matrix} \omega_1 \\ \vdots \\ \omega_n \end{matrix} \quad \mu = \begin{matrix} E(R_1) \\ \vdots \\ E(R_n) \end{matrix} \quad C = \begin{matrix} \sigma_{11} & \sigma_{1n} \\ \vdots & \vdots \\ \sigma_{n1} & \sigma_{nn} \end{matrix}$$

Matrices are used in order to calculate the efficient frontier, which minimizes:

$$\omega^T \Sigma \omega - q * \omega^T \mu$$

The Mean-Variance Efficient frontier represents the optimal asset allocation in the portfolio that when these assets are combined they yield the best possible risk / returns relationship.

The efficient frontier projects the optimal asset allocation of the opportunity set that yield the maximum return given a level of risk. By definition, no mean-variance investor would choose to hold a portfolio not located on the efficient frontier. To conclude, the mean-variance theory assumes investors preferences depend only on the mean and variance of a return;

- At a given mean, lower variance is preferred
- At a given variance, a higher mean is preferred

Exhibit 5.1: Mean Variance Efficient Frontier

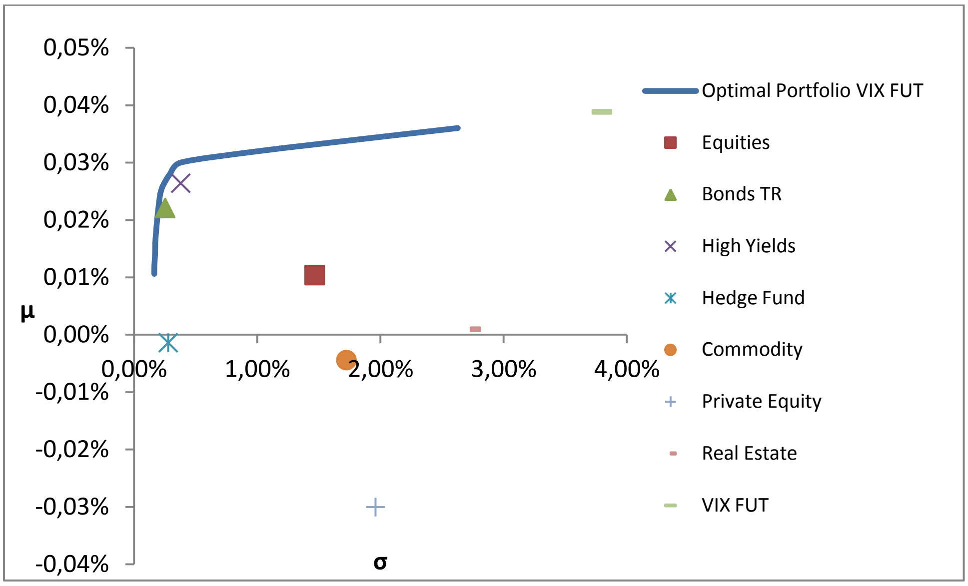


Exhibit 5.1 is a graphical illustration of the efficient frontier of the Optimal Portfolio with VIX Futures during 2005-2011. As can be seen in the graph all assets lie either below or to the right of the efficient frontier. This indicates that the risk – return relation of an individual asset is worse than the risk-return of the optimal portfolio. Combining the different individual assets results in a better risk – return performance, illustrated by the efficient frontier.

5.2 CAPM & Leland's Alpha and Beta

The Capital Asset Pricing Model can be seen as an extension of the mean-variance analysis. The CAPM holds the same assumptions as the Mean Variance analysis, as well as 2 additional assumptions:

- There is no information asymmetry between various investors
- There is a competitive equilibrium

The risk of a portfolio in CAPM consists solely of systematic risk, a risk common to all securities. Unsystematic risk is the risk of individual assets and can be diversified away. The CAPM also assumes that the risk / return relation of a portfolio can be optimized by estimating the efficient frontier. The CAPM assumes that each additional asset introduced to a portfolio further diversifies the portfolio; hence the optimal portfolio consists of all assets. As unsystematic risk can be diversified away, this implies that the total risk of a portfolio consists solely of market risk (β). The SML represents the efficient frontier in the CAPM and is similar to the efficient frontier in MV-analysis except for the x-axis. The y-axis still represent the expected return, the x-axis for the SML represents the risk (β) of the portfolio. Since this risk (β) reflects the portfolio's sensitivity to systematic risk or market risk. The market as a whole is assumed to have a β = 1.

The CAPM can be computed as:

$$E(R_i) = R_f + \beta_i\left[E\left(R_m - R_f\right)\right]$$

Where, $E(R_i)$ is the expected return on the capital asset, R_f is the risk-free rate, β_i is the sensitivity of asset i in relation to the volatility of the market. It is a measure that relates the sensitivity of an asset return to the return of the market, and can be computed as:

$$\beta_p = \frac{cov\left(r_p, r_{mkt}\right)}{var(r_{mkt})}$$

In order to account for the risk-averse investor the study also performs a test similar to the CAPM; Leland's alpha & beta. Leland's measures reflect the preference of an investor for low kurtosis and positive skewness. The CAPM assumes either that all asset returns are normally distributed or that investors solely have mean-variance preferences. Both assumptions are suspect according to Leland (1999). Leland argues that strategies with low kurtosis and positive skewness will be incorrectly underrated. Portfolios with a low kurtosis and positive skewness have a limited downside risk which is not identified by the CAPM. Leland introduces a model similar to the CAPM; the models only differ in estimating the risk factor (β). Leland adjusts the CAPM beta in order to produce a more correct risk measurement for portfolios that have non-normal return distributions.

Leland models portfolio returns the same way as the Capital Asset Pricing Model:

$$E\left(r_p\right) = r_f + B_{p*}\left[E(r_{mkt}) - r_f\right]$$

Leland's beta is given by:

$$B_{p*} = \frac{cov\left[r_p, -(1 + r_{mkt})^{-b}\right]}{cov\left[r_{mkt}, -(1 + r_{mkt})^{-b}\right]}$$

This is different from the Capital Asset Pricing Model where beta is given by:

$$\beta_p = \frac{cov(r_p, r_{mkt})}{var(r_{mkt})}$$

Leland's alpha follows:

$$\alpha_p = E(r_p) - B_{p*}[E(r_{mkt}) - r_f] - r_f$$

Leland's alpha estimation is the same as the alpha estimation in the CAPM model; only the beta estimation is different as the CAPM does not capture the preference of an investor for positive skewness and low kurtosis.

5.3 Performance Evaluation

Multiple performance evaluation tests are used in this study in order to reliably evaluate and compare all the different portfolios. In this chapter the basic methodology for the various performance-evaluation tests is explained.

5.3.1 Sharpe Ratio

Sharpe (1966) introduced the Sharpe ratio as an economic tool to model the risk / return performance of a portfolio. The Sharpe ratio measures the risk-adjusted performance of a portfolio; it evaluates whether an excess return is realized through wise investing decisions or that the excess return is because the portfolios bears additional risk. The greater a portfolio's Sharpe ratio the better the risk / return performance. A negative Sharpe ratio implies that it is better to simply hold the risk-free asset compared to the risky assets.

The main advantage of the Sharpe ratio is its applicability, as it can be computed from any observed series of returns. A disadvantage of the Sharpe ratio is that it assumes that returns are normally distributed; the standard deviation of the returns is the only measure of risk.

The Sharpe Ratio can be computed as follows:

$$SR = \frac{(R_p - R_f)}{\sigma}$$

And:

$$\sigma = \sqrt{VAR(R_p - R_f)}$$

Where R_p - R_f is the excess return of a portfolio over the risk free rate and σ is the standard deviation of the excess return.

5.3.2 Modified Sharpe Ratio

When returns are not normally distributed the portfolio cannot be explained only by its mean and volatility, the Sharpe ratio is not valid anymore. The Modified Sharpe Ratio (MSR) adjusts for the risk of a portfolio to come from not only volatility but also from higher moments like skewness and kurtosis. The MSR can be computed as:

$$MSR = \frac{(R_p - R_f)}{MVAR}$$

And:

$$MVAR = R_p - \left[\left(z_c + \frac{1}{6}(z_c^2 - 1)S + \frac{1}{24}(z_c^3 - 3z_c)K - \frac{1}{36}(2z_c^3 - 5z_c)S^2\right)\sigma_p\right]$$

Where (R_p – R_f) is the excess return of a portfolio, MVAR is the modified Value-At-Risk, which adjusts the standard deviation for skewness (S) and excess kurtosis (K) of the return distribution. z_c is the critical z-score at a certain confidence interval, σ_p is the standard deviation of the portfolio.

5.3.3 Stutzer Index

Stutzer (2000) introduced a portfolio performance index that incorporates higher moments. The index is a measure that compares the performance of a portfolio to a benchmark. Portfolios are favoured when they are expected to earn a higher average return than the average return of this benchmark over an unknown length of time. When a portfolio is expected to earn a higher average return than the benchmark, the probability that the manager's average return will underperform the benchmark decays to zero in time. The Stutzer index assumes that investors are 'loss averse'; loss aversion refers to the tendency of individuals for being more sensitive to losses than to gains in their level of wealth. This loss aversion is reflected in the decay rate; the Stutzer index maximizes the rate at which the probability of underperforming the benchmark decays to zero. When returns are normally distributed, the Stutzer index is the same as the Sharpe ratio. The results of the two performance indices differ when the returns are not normally distributed. The Stutzer index penalizes excess kurtosis (kurtosis > 3) and negative skewness.

Stutzer's information statistic I_p is given as:

$$I_p = \max_{\theta}\left[-\log\left(\frac{1}{T}\sum_{t=1}^{T} e^{\theta rt}\right)\right]$$

Where r_t is the excess return of the portfolio over the chosen benchmark, θ is a number less than zero and chosen in order to maximize I_p.

From Stutzer's information statistic, the Stutzer index can be derived:

$$Stutzer\ Index = \frac{Abs(\overline{r})}{\overline{r}} \sqrt{2I_p},$$

Where $\overline{r}$ is the mean excess return and $Abs(\overline{r})$ is the absolute value of the mean excess return. When returns are normally distributed, the performance ratio can be rewritten as:

$$I_p = \frac{1}{2}\lambda_p^2$$

Where λ is the Sharpe ratio; if returns are normally distributed the expected values of the Sharpe ratio and Stutzer index are the same. When returns are non-normal distributed the Stutzer index penalizes high kurtosis and negative skewness.

5.4 Mean Variance Spanning

Mean-Variance Spanning tests the impact of introducing an additional asset to the opportunity set. If the mean-variance frontiers of the benchmark portfolio and the portfolio with the additional asset have exactly 1 point in common, this is known as intersection. If the mean-variance frontiers of both portfolios coincide, there is spanning and no benefit is made from adding the additional asset to the opportunity set. Huberman & Kandel (1987) introduce the concept of mean-variance spanning. The hypothesis of spanning can be depicted as:

H_0: There is no net benefit of adding VIX Futures to a portfolio

H_a: There is a net benefit of adding VIX Futures to a portfolio

The goal of the spanning test is to examine whether the investor's portfolio improves by adding VIX Futures to the opportunity set. The effect of adding VIX Futures on the mean-variance frontier is analysed by the use of the likelihood ratio test. The likelihood ratio test compares the unconstrained maximum likelihood estimator with the constrained maximum likelihood estimator of the null hypothesis. The Likelihood ratio can be stated as:

$$LR = -T\ln(U) \sim \chi_2^2$$

In this study the addition of 1 new asset to the investment portfolio is considered, therefore the correct F-test should be:

$$\left(\frac{1}{U} - 1\right)\left(\frac{T - K - 1}{2}\right) \sim F_{2,T-K-1}$$

Where U is the likelihood ratio test, which is equal to:

$$U = \left(\frac{\widehat{c_1}}{\hat{c}}\right)\left(\frac{\sqrt{1+\frac{\widehat{d_1}}{\widehat{c_1}}}}{\sqrt{1+\frac{\hat{d}}{\hat{c}}}}\right)$$

Where $\left(\frac{\widehat{c_1}}{\hat{c}}\right)$ and $\left(\frac{\sqrt{1+\frac{\widehat{d_1}}{\widehat{c_1}}}}{\sqrt{1+\frac{\hat{d}}{\hat{c}}}}\right)$ should be close to one if the benchmark span the larger set. Also, $\hat{a}$, $\hat{b}$, $\hat{c}$ and $\hat{d}$ are constants for the K+1 assets and $\widehat{a_1}$, $\widehat{b_1}$, $\widehat{c_1}$ and $\hat{d}_1$ are constant for the K benchmark asset, all constants are estimated based on the variance/covariance matrix as well as the returns.

Chapter 6: Results

In this section the study presents the results of the proposed research and the implications that these results have. In the first part the thesis shows and discusses the performance of 3 traditional portfolios, without VIX exposure and with a 2.5% or 10% long allocation to near-month VIX Futures. With the addition of VIX Futures exposure the thesis expects to find both an increase in return and a reduction of the standard deviation. The standard deviation of a portfolio depends on the volatility and correlations between the various assets.

The second part will start by computing a Mean Variance Analysis based on the average historical returns of the various assets. The goal is to estimate two optimal portfolios for the entire sample one with and one without the possibility to include VIX Futures. The study expects to find an optimal portfolio with a small VIX Futures allocation when possible. The performance of the various portfolios are evaluated using a traditional performance measure; the Sharpe ratio, as well as performance measures that are robust to non-normality such as the Stutzer index, Modified Sharpe ratio and Leland's Alpha & Beta.

Additionally, the Efficient Frontier of the optimal portfolio with VIX Futures is estimated and evaluated. The specific relation between the VIX Futures and the stock market is evaluated by applying a sensitivity analysis. All test performed are ex-post analyses based on a time period that is heavily influenced by the 2008 financial crisis, the forecasting power of these portfolios and tests is limited. Two additional portfolios are constructed in Chapter 6.4; these portfolios are based on prospective data for the period 2012-2016.

6.1 Traditional Portfolio Results

The diversification opportunities provided by traditionally perceived diversifiers diminished in recent years, this study tests a new diversification instrument; the addition of a long near-month VIX Futures position is added to three traditional portfolios during 2005-2011.

Portfolio A is a 100% long-only equity portfolio, **Portfolio B** is a mixed portfolio of Equity and Bonds (60%/40%), **Portfolio C** is considered to be a traditionally diversified portfolio that consists of Equity (60,5%), Bonds (30.5%), High Yield bonds (1.3%), Hedge Funds (1.2%), Commodities (0.4%), Private Equity (1.6%) and Real Estate (4.5%). The construction of these traditional portfolios is retrieved from a study by Szado (2009) and is based on the 'Pensions and Investments 2007 Average Allocation for US Institutional Tax-Exempt Assets'. Furthermore the portfolio allocations of these traditional portfolios show strong resemblance with the 'World Market Cap' constructed by Doeswijk, Swinkels and Lam (2012).

Exhibit 6.1: Summary Statistics for portfolios A, B & C with VIX Futures 2005 – 2011

	A	A + 2.5% VIX FUT	A + 10% VIX FUT	B	B + 2.5% VIX FUT	B + 10% VIX FUT	C	C +2.5% VIX FUT	C + 10% VIX FUT	100% VIX FUT
Average Ann Return	2.60%	2.78%	3.31%	3.77%	3.92%	4.37%	3.24%	3.40%	3.89%	9.72%
Max Ann Return	26%	25%	20%	19%	17%	15%	22%	20%	15%	88%
Min Ann Return	-37%	-34%	-27%	-21%	-18%	-11%	-25%	-23%	-15%	-45%
Average Ann StDev	21%	19%	15%	12%	11%	8%	14%	13%	10%	59%
Mean daily return	0.010%	0.011%	0.013%	0.015%	0.016%	0.017%	0.013%	0.014%	0.016%	0.039%
daily StDev	1.47%	1.37%	1.09%	0.86%	0.77%	0.58%	1.00%	0.91%	0.70%	3.80%
Skew	-0.37	-0.33	-0.15	-0.39	-0.33	0.08	-0.44	-0.41	-0.18	0.79
Kurtosis	12	13	16	12	14	17	12	13	16	7
% Up Days	55%	55%	52%	56%	55%	51%	56%	55%	51%	44%
% Down Days	45%	45%	48%	44%	45%	49%	45%	45%	49%	56%
Return Total Period	20%	21%	26%	30%	32%	36%	25%	27%	31%	97%
Annual SR	0.025	0.035	0.075	0.130	0.156	0.257	0.079	0.098	0.174	0.129
Annual Stutzer	0.000	0.016	0.032	0.016	0.039	0.079	0.000	0.016	0.028	0.049

Notes: Summary statistics for the traditional portfolios with a 2.5% or 10% allocation to VIX Futures from Jan. 2005 to Dec. 2011. .

Exhibit 6.1 provides the summary statistics for the three traditional portfolios with the addition of 2.5% or 10% VIX Futures to the portfolios in separate columns over the entire period. The returns are calculated geometrically, in order to get an accurate measurement of what the actual average annual return over the seven-year period is. As expected the results show that an allocation to VIX Futures increases the return of the various portfolios. For example, the 100% long-only Equity portfolio's average annual return improves from 2.60% to 2.78% and then to 3.31%. The benefits of increasing returns can be easily offset with an increasing standard deviation. Exhibit 6.1 shows that also the Average Annual Standard

Deviation of the returns improve through including VIX Futures. For the 100% Equity portfolio it is shown that the Standard Deviation is reduced from 21% to 19% and 15%. These results are in line with the proposed research, that there are diversification benefits realizable from adding VIX Futures to the traditional portfolios. These findings are also in line with the research performed by Szado (2009): He finds that adding 2.5% and 10% VIX exposure to a traditional portfolio during 2006-2008 results in increased returns and reduced standard deviations.

The addition of VIX Futures also improves the maximum annual drawdown for the portfolios significantly. For example, for the Equity/Bond portfolio maximum annual drawdown is reduced from -21% to -18% with a 2.5% VIX Futures allocation and then to -11% with a 10% VIX Futures allocation. It seems that a long VIX Futures position provides diversification when it is needed most; during large downward moves of the market.

When the performance of the base portfolios are compared to each other, another interesting aspect regarding diversification is shown. In various literature and economic models it is assumed that diversification benefits are realized by including different asset classes. Chapter 2.2 concludes that the correlation between the various asset classes is relatively high, because of the increasing globalization and integration of the financial markets as well as the on-going uncertainty in these markets as a result of the global financial crisis of 2008. When only the base portfolios with no VIX allocation are revised, it is obvious that the addition of alternative assets to the equity portfolio provides some diversification benefits; as the Average Annual Return is 3.24% for the diversified portfolio compared to 2.60% for the equity portfolio, combined with a lower standard deviation. (21% for portfolio **A** vs. 14% for portfolio **C)** Adding alternative assets to the Equity / Bond portfolio on the contrary result in a decrease in return and an increase in the standard deviation. (Portfolio **B**'s return equals 3.77% with a standard deviation of 12%) These findings further support the notion that traditional perceived diversifiers failed to do so in recent times. However, this conclusion is only valid for the specific combination of these alternative assets in the above specified proportions.

6.1.2 CAPM & Leland's Alpha, Beta Results Base Portfolios

The risk of a portfolio in the CAPM model consists solely of market risk that is common to all securities. The alpha is the measure of excess return with regard to the market portfolio.

Leland's performance measurement test can be seen as an extension of the CAPM. As stated earlier the CAPM Beta is adjusted in order to create a more reliable and correct risk measurement for portfolios with non-normal return distributions. The returns are evaluated against the market, which is represented by the S&P 500 Total Return index.

Portfolio **A** is an all equity portfolio represented by the S&P 500 total return index and is also the market portfolio. Therefore the beta of portfolio **A** is equal to 1 and alpha is 0. As stated before beta measures the correlated volatility of the portfolio in relation with the volatility of the market. A positive beta implies that the portfolio follows the benchmark; if the market goes down, the portfolio will follow. A negative beta, on the contrary means that the portfolio moves in the opposite direction of the market; if the market goes down, the portfolio goes up. A portfolio has a beta greater than 1 when the returns of the portfolio vary more than the returns of the market. If a portfolio has returns which vary less than the market's returns the beta is smaller than 1.00.

Beta is the only risk factor in the CAPM model; hence a portfolio with a low beta yields lower risk than a portfolio with a high beta. Based on this the VIX & VIX Futures should have a negative beta to the market and greater than -1.00. Alpha is the coefficient that represents the excess return of the portfolio compared to the market. Alpha is often described as the coefficient that resembles the active managers' performance; it can be seen as the risk-adjusted return of the portfolio.

Exhibit 6.1.2: CAPM & Leland's α & β Results Base Portfolios

	Portfolio A	Portfolio A + 2.5 VIX FUT	Portfolio A + 10% VIX FUT	Portfolio B	Portfolio B + 2.5% VIX FUT	Portfolio B + 10% VIX FUT	Portfolio C	Portfolio C + 2.5% VIX FUT	Portfolio C + 10% VIX FUT	100% VIX
Ann Return	2.60%	2.78%	3.31%	3.77%	3.92%	4.37%	3.24%	3.40%	3.89%	9.72%
CAPM Beta	1	0.93	0.72	0.58	0.52	0.34	0.68	0.62	0.43	-1.80
CAPM Alpha	0	0.35%*	1.44%*	2.26%*	2.56%*	3.47%*	1.47%*	1.79%*	2.76%*	14.41%*
Leland Beta	1.00	0.85	0.41	0.55	0.41	0.00	0.66	0.52	0.11	-4.90
Leland Alpha	0.00%	0.56%*	2.25%*	2.34%*	2.85%*	4.35%*	1.51%*	2.04%*	3.61%*	22.46%*

Note: Portfolio A is also the market portfolio, as it represents the market for an US investor. Portfolio B is a traditional Equity/bond portfolio, while Portfolio C is the traditional diversified portfolio. The figures above are annualized estimates from daily data; the risk adjusted performance (alpha) is the average annual outperformance of a portfolio with respect to the market. α* = alpha is significantly greater than 0 for a 1% significance level.

All the alphas are tested if they are significantly greater than 0. This is done by the use of simple T-test. The hypothesis can be formulated as follows:

H_0: $\alpha = 0$

H_a: $\alpha > 0$

The T-test values can be tested against a 1 % significance level, with a T-test statistic of 2.34. Based on the results the study can reject H_0 for all alphas except for the market portfolio.

Exhibit 6.1.2 shows that the addition of a VIX Futures to the all equity portfolio results in a lower CAPM beta, which indicates that the portfolio is becoming less sensitive to changes in the market; hence having a VIX Futures exposure limits the risk of the portfolio. The CAPM alpha (risk-adjusted return) is significantly greater than 0, which indicates superior performance of the portfolio relative to the market. Leland's beta also drops when VIX Futures are included in the equity portfolio. The difference between the two betas may be substantial when the portfolio's returns are distinctly skewed. The alphas improve when VIX Futures are included. This suggests an outperformance of the portfolio to the market, while the lower beta also indicates that the portfolio is less risky than the market.

When besides equity other asset classes are included in the portfolio, the beta and alpha change. Bonds are seen as a traditional diversifier and are expected to be relatively stable in turbulent markets; bonds have a very low slightly negative beta to the stock market. Portfolio B is a traditional Equity / Bond portfolio with a beta of 0.58 (CAPM) or 0.55 (Leland). The portfolio generates excess returns in relation to the market as is measured by CAPM alpha & Leland's alpha. The incorporation of VIX Futures again has a substantial impact on the portfolios' betas and alphas. The relation between the VIX Futures and the market prompts that the beta drops when a VIX Futures allocation is included. The combination of a lower beta with a significant positive alpha indicates that the portfolio outperforms the market based on excess return and lower risk.

Portfolio C is the traditionally diversified portfolio; based on the asset correlations & individual asset performance a lower beta is expected than for portfolio B, this is supported by the results in exhibit 6.1.2. Adding VIX Futures again results in a large drop for the betas. The alphas suggest a slight outperformance of the portfolio to the market when a small VIX Futures allocation is included. The combination of a lower beta with a significant positive alpha indicates that the portfolio outperforms the market based on excess return and lower risk.

Exhibit 6.1.2 shows that the CAPM beta is higher than Leland's beta for all portfolios. This advocates that according to the CAPM the portfolios are more sensitive to changes in the market. The difference is due to the estimation of beta, whereas the CAPM beta does not capture the preference of an investor for positive skewness and low kurtosis. In this case the CAPM undervalues the portfolios as it does not capture the negative skewness with the market returns. Leland's beta incorporates the investor's preference for low kurtosis and positive skewness. The addition of VIX Futures result in a more positive skewness as can be seen in exhibit 6.1, transpiring in a lower Leland's beta than the CAPM beta.

6.1.3 Sharpe Ratio Base Portfolios

The Sharpe ratio illustrates the return one receives for bearing an additional risk by investing in a risky asset versus investing in the risk free asset. Exhibit 6.1 concludes that the daily Sharpe ratio increases when VIX Futures are added to the portfolio. This means that a portfolio with VIX exposure provides a better risk / return performance than a portfolio without VIX exposure. For example, for Portfolio **B** without 2.5% VIX exposure yield an annualized Sharpe ratio of 0.13 versus an annualized Sharpe ratio of 0.156 with 2.5% VIX exposure. One disadvantage of the Sharpe ratio is that it assumes that the returns are normally distributed or that investors only care about the return and standard deviation of the returns. The Sharpe ratio can be upwardly biased when returns are not distributed normally, for example when a portfolio has negative skewness and excess kurtosis the Sharpe ratio will overstate the portfolios performance.

6.1.4 Testing for Normality

Negative skewness symbolizes more negative returns far to the left (single large losses) of its mean return. Kurtosis is the measure of peakedness of a return distribution and has a value of 3 when normal. Excess kurtosis results in a higher probability for extreme returns than is assigned under 'normal' conditions. It appears from exhibit 6.1 that all the portfolios have returns that are non-normal distributed; the summary statistics indicate a negative skewness for almost all portfolios, only VIX Futures have a positive skewness. Furthermore all portfolios combine this skewness with a level of kurtosis that is significantly higher than 3. This proposed non normality notion can be tested by applying the Jarque-Bera test. Jarque-Bera tests whether the skewness and kurtosis match the level of a normal distribution:

H_0: Returns are normally distributed, skewness is zero and kurtosis = 3

H_a: Returns are non-normally distributed.

This test statistic can be compared with a Chi-Square test distribution with 2 degrees of freedom. The null hypothesis can be rejected if the value for the Jarque-bera test exceeds the appropriate critical value. The Jarque-Bera statistic is computed as:

$$JB = \frac{n}{6}\left(s^2 + \frac{(k-3)^2}{4}\right)$$

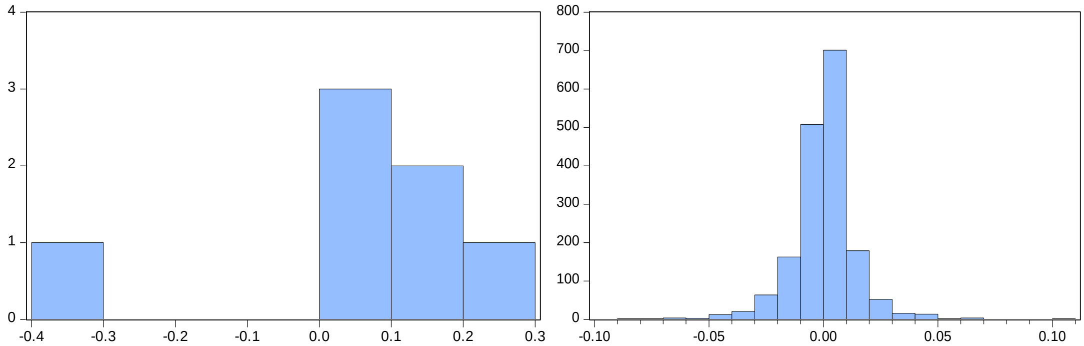
Where n is the number of observations, s is the skewness of the portfolio, and k is the portfolio's kurtosis. Exhibit 6.1.4 summarizes the results for the Jarque-Bera test; this study tested annual data and daily data.

Exhibit 6.1.4: Jarque-Bera Test Results Base Portfolios

	Portfolio A	Portfolio A + 2.5 VIX FUT	Portfolio A + 10% VIX FUT	Portfolio B	Portfolio B + 2.5% VIX FUT	Portfolio B + 10% VIX FUT	Portfolio C	Portfolio C + 2.5% VIX FUT	Portfolio C + 10% VIX FUT	OBS
JB Daily	5728	6788	11442	6119	8133	15172	5613	7078	12463	1752
JB Annualized	2,17	2,61	3,78	2,29	2,87	2,76	1,86	2,38	3,46	7
Sign Level α	0,1	0,05	0,01							
Critical Value	4,61	5,99	9,21							

As can be seen from exhibit 6.1.4, this table provides conflicting results. Based on daily data the null hypothesis is rejected, based on annual data the null hypothesis is not rejected.[1] This difference is due to the impact of the sample size on the JB test statistic. It is very difficult to reject a null hypothesis based on only 7 observations. The non-normality of the portfolios is supported by the graphical histograms of the various return distributions. For example portfolio A without VIX futures:

Exhibit 6.1.4.2: Graphical Return Distribution Portfolio A



Note: The first graph shows the return distribution for annualized returns, the 2nd graph shows the return distribution for daily returns for Portfolio A.

[1] The analysis also calculated the JB-test statistic for Monthly and Quarterly data, resulting in a JB test statistic > critical value, hence for monthly & quarterly data the null hypothesis is also rejected.

6.1.5 Modified Sharpe Ratio

The Modified Sharpe Ratio (MSR) is similar to the Sharpe ratio; the Sharpe ratio can be summarized solely by the mean and variance of portfolio returns, the MSR uses the Modified Value at Risk as the risk measure which incorporates the skewness and excess kurtosis of the portfolio's returns distribution.

Exhibit 6.1.5: SR & MSR Base Portfolios with VIX Futures

	Portfolio A	Portfolio A + 2.5 VIX FUT	Portfolio A + 10% VIX FUT	Portfolio B	Portfolio B + 2.5% VIX FUT	Portfolio B + 10% VIX FUT	Portfolio C	Portfolio C + 2.5% VIX FUT	Portfolio C + 10% VIX FUT	100% VIX FUT
Annual SR	0.025	0.035	0.075	0.130	0.156	0.257	0.079	0.098	0.174	0.129
Annual MSR	0.0055	0.0074	0.0140	0.0279	0.0316	0.0456	0.0166	0.0198	0.0309	0.0523

Note: the ratios are annualized by multiplying the daily data with √(252)

The annual SR's for all the portfolios are positive which implies that the portfolios outperform the risk free rate. However the returns are not normally distributed as was concluded in the section above. When analysing the MSR, it appears that the portfolios still outperform the risk free rate on average during the sample period but with a lesser magnitude. This points out that it is profitable for an investor to invest in the risky asset instead of holding the risk free asset. The smaller magnitude of outperformance of the MSR (difference MSR & SR) is due to the excess kurtosis and negative skewness of the return distributions. The excess kurtosis and negative skewness denote that there is a relatively serious potential for extreme losses. The additions of VIX Futures to the base portfolios improve the MSR.

6.1.6 Stutzer Index

The Stutzer index reflects the desire to avoid underperformance to a designated benchmark and to realize performance in excess of the benchmark. A strong feature of the Stutzer index is that the index does not assume normal distributed returns. As stated before the Stutzer index is in line with the notion of risk aversion as it penalizes negative skewness and excess kurtosis; the chance of incurring large losses.

Exhibit 6.1.6: Stutzer Index Results Base Portfolios

	Portfolio A	Portfolio A + 2.5 VIX FUT	Portfolio A + 10% VIX FUT	Portfolio B	Portfolio B + 2.5% VIX FUT	Portfolio B + 10% VIX FUT	Portfolio C	Portfolio C + 2.5% VIX FUT	Portfolio C + 10% VIX FUT	100% VIX
Stutzer Index	0.0000	0.0159	0.0317	0.0163	0.0386	0.0794	0.0000	0.0159	0.0285	0.0495

Note: The Stutzer index is a performance evaluation measure of a portfolio against a pre-specified benchmark. The benchmark used in this research is the Risk Free Rate, as is advised by Stutzer (2000) and Feldman & Roy (2004).

Exhibit 6.1.6 shows the Stutzer index for all the portfolios, note that a higher Stutzer index indicates a better performance. The addition of VIX Futures results in a higher Stutzer index for all the portfolios. These findings suggest that including VIX Futures in a portfolio yield a better risk / return performance, while controlling for the loss aversion of the typical investor. This is in line with the hypothesis that VIX Futures provide diversification benefits during 2005 – 2011. These findings are further supported by Szado (2009) who performs a similar research; he tests whether the addition of VIX Futures provide diversification benefits during the financial crisis of 2008 and finds that this is true, as the Stutzer index increases when VIX Futures are added to a portfolio.

6.1.7 Performance Tests Evaluation

Multiple performance evaluation tests are conducted in order to measure and compare the various portfolios. The Sharpe ratio is the only test that assumes the portfolios' returns are normally distributed or that investor's only care about the portfolios' return and variance. The other tests assume non-normal return distributions, combined with an investor preference for low kurtosis and positive skewness. All the various test results are in line with the scope of this research; adding VIX Futures to a traditional portfolio yield diversification benefits. The non-normality analysis therefore does not imply different conclusions than the 'normal' performance evaluation analysis.

6.2 Mean Variance Optimal Portfolio's

In this section similar tests are conducted in order to determine the optimal portfolio during 2005-2011. The time period chosen is a time period dominated by the financial crisis of 2008. From 2005 until early 2007 the markets are relatively calm. During 2008 the S&P 500 plummets with almost 47%, other asset classes also suffer substantial losses during this period. From late 2008 until 2011 the market are still very uncertain. The performance of the various asset classes during these years are summarized in exhibit 6.2.

Exhibit 6.2: Summary Statistics Asset Classes 2005-2011

	Equities	Bonds	High Yield	Hedge Fund	Commodities	Private Equity	Real Estate	VIX Spot	VIX Fut
Average Ann Return	2,60%	5,53%	7,36%	-0,35%	-1,11%	-7,52%	0,24%	8,08%	9,72%
Max Ann Return	26%	8%	58%	13%	33%	52%	31%	95%	88%
Min Ann Return	-37%	3%	-26%	-23%	-46%	-67%	-42%	-46%	-45%
Average Ann StDev	21%	4%	5%	4%	26%	27%	37%	110%	59%
Skew	-0,37	-0,04	-1,59	-1,07	-0,30	-0,23	-0,13	0,76	0,79
Kurtosis	11,83	4,67	27,48	11,44	5,37	11,36	12,55	7,38	7,07
% Up Days	55%	55%	62%	56%	51%	52%	52%	45%	44%
% Down Days	45%	45%	38%	44%	49%	48%	48%	55%	56%
Return Total Period	20%	47%	67%	-2%	-7%	-41%	2%	76%	97%

When comparing the various returns it becomes obvious that Bonds & High Yield bonds show a very favourable average annual return compared to Equities, Hedge Funds, Commodities, Private Equity and Real Estate. The returns of these asset classes are more heavily influenced by the financial crisis of 2008, as the correlations between these asset classes are positive and relatively high. The large drop in Equity during late 2008 and early 2009 thus had a significant impact on the returns & standard deviation for the various asset classes. Bonds are considered to be a traditional diversifier that performs well when Equity is not. Another interesting note is the performance of High Yield bonds during 2005-2011. High Yield bonds have a high average annual return with a low standard deviation compared to the other asset classes. The average returns for Equity, Hedge Funds, Commodities, Private Equity and Real Estate are low, while the standard deviation is high compared to the average returns and standard deviations of Bonds, High Yield bonds and the VIX Futures.

Based on the risk / return relation of the various asset classes a prediction for the optimal portfolio can be made. The optimal portfolio assigns weights to different asset classes to utilize all possible diversification benefits by maximizing its return while controlling for risk. Based on the figures in Exhibit 6.2 this study predicts the optimal portfolio to be dominated by an allocation to Bonds, High Yield bonds and, when included, VIX Futures. Exhibit 6.2.1 summarizes the Portfolio allocation of the optimal portfolios with and without the addition of VIX Futures.

Exhibit 6.2.1: Optimal Portfolio Allocation

Portfolio Allocation	Rp with VIX	Rp NO VIX
Equities	3,83%	1,45%
Bonds	61,63%	64,43%
High Yield	33,16%	34,12%
Hedge Funds	0,00%	0,00%
Commodities	0,00%	0,00%
Private Equity	0,00%	0,00%
Real Estate	0,00%	0,00%
VIX Futures	1,38%	n/a
Σw	100,00%	100,00%
Average Ann Return	6,49%	6,77%
Average Ann St Dev	3,10%	3,16%

Note: Σw is the sum of the weights and must be equal to 1. Short selling is not allowed.

The optimal portfolio without the possibility of VIX Futures consists almost entirely of Bonds & High Yield bonds, as was expected. A small allocation to Equity provides the portfolio with some diversification benefits. Adding VIX Futures to the opportunity set has a small impact on the portfolio weights. The portfolio weights for Bonds and High Yield bonds diminish, while now a larger portion of Equity & VIX Futures are included in the optimal portfolio. From exhibit 6.2.1 it appears that a 1.38% allocation to VIX Futures is optimal during 2005-2012. These findings indicate that it is optimal to include a very small VIX Futures allocation, which is in line with the scope of the research.

Exhibit 6.2.1.2: Optimal Portfolio Summary Statistics 2005-2011

Portfolio Statistics:	Rp with VIX	Rp NO VIX
Average Ann Return	6,77%	6,49%
Max Ann Return	21%	22%
Min Ann Return	-7%	-7%
Average Ann StDev	3,10%	3,16%
Mean daily return	0,024%	0,024%
daily StDev	0,21%	0,21%
Skew	-0,72	-0,75
Kurtosis	11	12
% Up Days	57%	56%
% Down Days	43%	44%
Return Total Period	53%	53%

Exhibit 6.2.1.2 denotes the summary statistics for the optimal portfolios during 2005-2012. The average annual return is slightly higher for the optimal portfolio with VIX Futures than for the portfolio without VIX Futures. This seems to be in line with the scope of the research. When analysing the standard deviations of the returns, the portfolio with VIX Futures shows a lower risk than the portfolio without VIX Futures. This is in line with the scope of the research, as it suggests that through incorporating a very risky asset (VIX Futures) the risk of the total portfolio diminishes. This suggestion indicates that diversification benefits can be realized by adding VIX Futures to the portfolio. Another interesting note is that the portfolio without VIX Futures has a higher negative skewness and excess kurtosis. This implies that the chance of incurring unexpected extreme losses is higher for the portfolio without VIX exposure than for the portfolio with VIX exposure.

6.2.2. CAPM & Leland's Alpha, Beta Results Optimal Portfolios

The alphas and betas for CAPM & Leland's can be found in exhibit 6.2.3. As mentioned earlier the VIX has a strong negative beta to the market. Therefore this study expects to find a negative beta for portfolios that incorporate a significant allocation to VIX Futures. Bonds and High Yield bonds have a low beta to the market.

The CAPM & Leland's alpha measure the excess return of a portfolio compared to the market's return, which is called the risk-adjusted return. Positive alphas suggest excess returns in relation to the market, while negative alphas suggest an underperformance with respect to the market. All the alpha's are tested if they are significantly greater than 0. This is done by the use of simple T-test. The hypothesis can be formulated as follows:

H_0: $\alpha = 0$

H_a: $\alpha > 0$

The T-test values can be tested against a 1 % significance level, with a T-test statistic of 2.34. Based on the results the study can reject H_0 for all alphas except for the market portfolio.

The expectations are supported by the results found in exhibit 6.2.3. The optimal portfolio with VIX exposure has a Leland's beta of 0.03, while the optimal portfolio without VIX Futures exposure has a Leland's beta of 0.07. The CAPM beta's for both portfolios are also in line with these expectations, as the CAPM beta for the portfolio with VIX exposure is lower than the beta for the portfolio without VIX exposure. Leland's statistics incorporate the investor's preference for low kurtosis and positive skewness. The CAPM does not, hence it overvalues the portfolios sensitivity relative to the market risk as the CAPM beta > Leland's beta. The findings indicate that both portfolios exhibit low market risk sensitivity. All the alphas are significantly greater than 0, which points out that both portfolios generate excess return relative to the market. Based on the CAPM and Leland's test statistics it can be concluded that both portfolios have low market sensitivity (risk), combined with a positive risk adjusted return (excess return). The results suggest a small outperformance of the VIX Futures Optimal Portfolio relative to the Optimal Portfolio without VIX Futures as the sensitivity to market risk is lower and the excess return is slightly higher for the VIX Futures portfolio.

6.2.3. Sharpe Ratio Optimal Portfolio

The Sharpe ratio is the most commonly used performance measurement tool. It can be biased when returns are not distributed normally; hence the study needs to test for normality through the use of the Jarque Bera test. When the portfolios' returns proof to be non-normally distributed their performance will be tested by the Modified Sharpe ratio and the Stutzer Index. The results of the various tests are summarized in exhibit 6.2.3.

Exhibit 6.2.3: Optimal Portfolio Performance Test Results

Test Statistics:	Rp with VIX	Rp NO VIX
SR	1.23	1.18
JB	4700	4697
MSR	0.279	0.274
Rp	6.77%	6.49%
Rm	2.60%	2.60%
CAPM β	0.088	0.092
CAPM α	6.54%	6.26%
Leland β	0.03	0.07
Leland α	6.70%	6.30%
Stutzer Index	0.265	0.237

CAPM / Lelenad α* = alpha is significantly greater than 0 for a 1% significance level. The Sharpe ratio & MSR are annualized by multiplying the daily ratios with √(252).

Exhibit 6.2.3 illustrates that the Sharpe ratio of the portfolio with VIX Futures is higher than for the portfolio without VIX Futures. For example, the annual Sharpe Ratio for the portfolio with VIX Exposure is 1.23 while the Sharpe Ratio for the portfolio without VIX exposure is 1.18. This suggests that the portfolio with VIX Futures outperforms the portfolio without VIX allocation. This is in line with the test results found in the previous chapter as well as with the test results found by Szado (2009).

This suggestion can be tested with a simple t-test for significant outperformance:

H_0: Sharpe ratio Rp with VIX = Sharpe ratio Rp no VIX

H_a: SR Rp with VIX ≠ SR Rp no VIX

The T-test values can be tested against a 5% significance level, with a T-test statistic of 1.96. Based on the results the study cannot reject H_0. The Sharpe ratio of the VIX Futures portfolio is not significantly higher than the Sharpe ratio of the portfolio without VIX Futures.

6.2.4. Testing for Normality

The Jarque-Bera test is used to test whether the portfolios exhibit normal return distributions. As shown in exhibit 6.2.1.2 both portfolios have a negative skewness and excess kurtosis. A negative skewness combined with excess kurtosis implies that the chance of incurring unexpected extreme losses is large; this is something investors do not prefer. The results of the Jaque-Bera test in exhibit 6.2.3 indicate that both portfolios have non-normal return distributions.

6.2.5. Modified Sharpe Ratio

As mentioned earlier the Modified Sharpe Ratio is almost identical to the Sharpe ratio; however the MSR uses the Modified Value at Risk as the risk measure. The MSR test results can be observed from exhibit 6.2.3; the optimal portfolio with VIX Futures has a MSR of 0.279 and the portfolio without VIX Futures has a MSR of 0.274. Both test results are significantly positive against a 1% significant level, which indicates that both portfolios outperform the risk free rate. The MSR is higher for the portfolio with VIX Futures than for the portfolio without VIX Futures this means that the addition of VIX Futures leads to a better risk / return performance. This suggestion can be tested with a simple t-test for significant outperformance.

H_0: MSR ratio Rp with VIX = MSR ratio Rp no VIX

H_a: MSR Rp with VIX ≠ MSR Rp no VIX

The T-test values can be tested against a 5% significance level, with a T-test statistic of 1.96. Based on the results the study cannot reject H_0. The MSR ratio of the VIX Futures portfolio is not significantly higher than the MSR ratio of the portfolio without VIX Futures.

6.2.6. Stutzer Index

The diversification benefits provided by including VIX Futures can be further illustrated by comparing the Stutzer index for both optimal portfolios. The Stutzer index for the portfolio with VIX Futures is 0.265 and for the portfolio without VIX Futures the index is 0.237. The portfolio with a small allocation to VIX Futures outperforms the portfolio without VIX Futures based on the Stutzer index. This is in line with the other performance measurement tests as well as the results found in the previous chapter and the results found by Szado (2009). This suggestion can be tested with a simple t-test for significant outperformance.

H_0: Stutzer Index Rp with VIX = Stutzer Index Rp no VIX

H_a: Stutzer Index Rp with VIX ≠ Stutzer Index Rp no VIX

The T-test values can be tested against a 5% significance level, with a T-test statistic of 1.96. Based on the results the study cannot reject H_0. The Stutzer of the VIX Futures portfolio is not significantly higher than the Stutzer Index of the portfolio without VIX Futures.

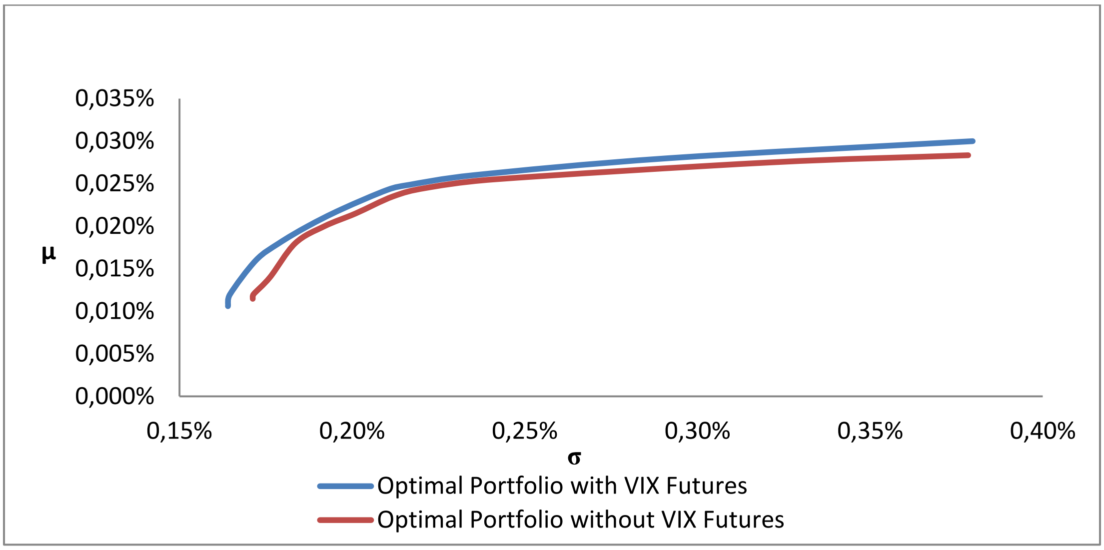
6.2.7 Mean Variance Spanning Test

This study evaluates whether a long VIX Futures position provide diversification benefits. Besides investing in Equity and VIX Futures, it is assumed that it is possible to invest in Bonds, High Yield Bonds, Hedge Funds, Commodities, Real Estate & Private Equity. The spanning test suggested by Huberman & Kandel (1987) is used in order to test whether the addition of VIX Futures as an asset class provide diversification benefits. The test analyses the mean-variance frontier of a benchmark portfolio with a portfolio with the additional VIX Futures; if they have 1 point in common this is known as intersection. If the mean-variance frontiers coincide, there is spanning and no benefit is made from adding the additional asset. The relevant critical value for the F-test at a 1% and 5% significance are:

$$\mathbf{F_{2,\,225,\,0.05} = 2.64} \quad \text{and} \quad \mathbf{F_{2,\,225,\,0.01} = 3.87}$$

The spanning test used is the likelihood ratio test; it rejects the null hypothesis of spanning as it generates a F-value of 53.74. The investor's opportunity set is improved by adding VIX Futures to its investment universe. Economically this implies that significant diversification benefits can be realized by adding VIX Futures to the investment opportunity set.

Exhibit 6.2.7: MV-Spanning Frontier Optimal Portfolios



The test results are supported by exhibit 6.2.7; it is shown that the efficient frontier of the optimal portfolio without VIX Futures does not intersect or coincide the efficient frontier of the optimal portfolio with VIX Futures. There is no spanning, and hence diversification benefits are realized by adding VIX Futures to the opportunity set.

6.2.8. Optimal Portfolio Performance Test Evaluation

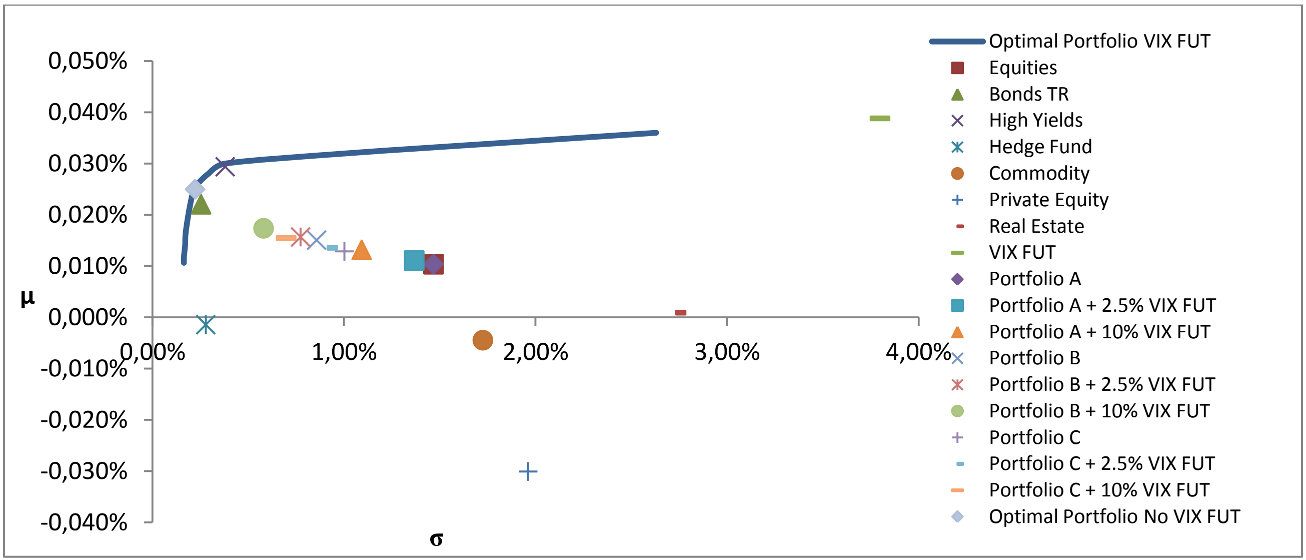
Multiple performance evaluation tests are conducted in order to measure and compare the 2 optimal portfolios. The Sharpe ratio is the only test that assumes the portfolios' returns are normally distributed or that investors only care about the portfolios' return and variance. At first, the results of the various performance tests indicate a slight outperformance of the optimal portfolio with VIX Futures compared to the portfolio without VIX Futures exposure. The MV-spanning test indicates that the opportunity set of an investor significantly improves by including VIX Futures. Chapter 6.2 indicates that it is optimal to allocate 1.38% of the portfolio to VIX Futures. The impact that the small allocation of VIX Futures has on the portfolio results is quite small. The Sharpe ratio, MSR and Stutzer index statistics of the portfolio with VIX Futures are not significantly greater than their counterparts of the portfolio without VIX Futures. All test statistics are significantly greater than 0, hence the portfolios do outperform the risk free rate. Based on these test statistics the study cannot conclude that the addition of VIX Futures significantly improve the optimal portfolio. The MV spanning results indicate that the opportunity set improves, the optimal portfolio for 2005-2011 on the contrary does not significantly improve by including a small allocation to VIX Futures. These results are not in line with the results from Chapter 6.1 and are in conflict with the hypothesis of the thesis.

6.3 Efficient frontier & Portfolio

Exhibit 6.3 illustrates the efficient frontier of the optimal portfolio with VIX Futures. Furthermore all other portfolios as well as all separate asset classes are denoted in order to provide a clear illustration of the performance of each asset. It is shown that diversification benefits can be realized by including multiple asset classes in the portfolio. Most separate asset classes do not have a very favourable individual risk – return performance. For example Commodity; that yields a lower return while exhibiting a higher variance than the optimal portfolio. This notion is expressed graphically as Commodity lies below and to the right of the bullet. The traditionally perceived portfolios also lie below and to the right of the bullet, which implies that they do not yield a favourable risk – return payoff. This can be explained by the performance of Equity as an asset class; Equity is also located far to the right and below the bullet. All the traditional portfolios have a drastic portion allocated to Equity.

The most interesting feature of this graph is the risk – return feature of the VIX Futures compared to all other investment opportunities. It can be seen that 'VIX Futures' is in the upper right corner of the graph, which implies that VIX Futures yield a relatively high expected return with a very high level of risk. This would suggest that diversification benefits can be realized by allocation a small portion of the portfolio to VIX Futures.

Exhibit 6.3: Efficient Frontier & Portfolios



From the efficient frontier it can be concluded that the portfolio improves by including multiple asset classes. The efficient frontier represents the combination of assets that together yield the best risk-return profile. From the graph it can be concluded that the optimal portfolio during 2005-2011 consists mainly out of Bonds / High Yield bonds as these assets lie the closest to the efficient frontier. The portfolio of Bonds / High Yield bonds is improved by allocating a small portion of the portfolio to other asset classes, in order to diversify the portfolio. This is graphically illustrated as the efficient frontier lies above and to the left of the Bonds / High Yield bonds. These findings are in line with the results of Chapter 6.2, which concludes that the optimal portfolio consists mainly out of Bonds & High Yield bonds. Equity and VIX futures are included in the optimal portfolio and thus provide some diversification benefits.

6.4 Prospective Optimal Portfolios

The financial crisis of 2008 and the on-going uncertainty in the financial markets have a strong impact on the performance of the various asset classes during the sample period. The results in this study are heavily affected by the turbulent financial markets. The realized returns, the portfolios' performance as well as the performance of the VIX and VIX Futures provide only limited forecasting power. The VIX Futures are only introduced in 2004 making it impossible to conduct a similar research based on data that is not heavily influenced by the

2008 financial crisis. The financial markets are not expected to behave in the same way as they have done in in recent years. In order to test whether a small long allocation to VIX Futures provide diversification benefits in the future, various assumption need to be made; the returns, standard deviations and correlations of all the different asset classes are based on historical data from 2005-2011; for the coming years only the correlations will be assumed to be the same. For estimating the returns and standard deviations this study will follow the annual forecast vision made by Robeco. These prospective figures are hypothetical estimates based on the fact that recent times are tough and improvements are not yet in sight. The on-going debate about the US debt ceiling as well as the continuous threat of a Euro crisis has brought the financial markets back on the brink of a relapse into another recession. The expected returns and expected standard deviations are thus prospective, while the correlations between the various assets are historical.

Exhibit 6.4: Prospective Returns & Standard Deviations with Historical Correlations

Historical Correlations 2005 - 2011									Historical	Historical	2012-2016	2012-2016
Correlations	Equities	Bonds	High Yield	Hedge Funds	Commodities	Private Equity	Real Estate	VIX Futures	Mean Returns	StDev	Mean Returns	StDev
Equities	1,00								2,60%	21%	7%	18%
Bonds	-0,29	1,00							5,53%	4%	1,25%	5%
High Yield	0,30	0,07	1,00						7,36%	5%	4,25%	12%
Hedge Funds	0,56	-0,16	0,52	1,00					-0,35%	4%	4,75%	15%
Commodities	0,35	-0,17	0,27	0,42	1,00				-1,11%	26%	4,75%	25%
Private Equity	0,81	-0,23	0,48	0,59	0,42	1,00			-7,52%	27%	7%	25%
Real Estate	0,80	-0,20	0,13	0,30	0,22	0,67	1,00		0,24%	37%	6%	20%
VIX Futures A	-0,69	0,22	-0,23	-0,49	-0,24	-0,58	-0,54	1,00	9,72%	59%	1,79%	40%
VIX Futures B	-0,69	0,22	-0,23	-0,49	-0,24	-0,58	-0,54	1,00	9,72%	59%	-2%	40%

Note: This study considers 2 prospective values for the VIX Futures returns; 7% & -2%, the standard deviation is expected to be 40%

The prospective values for the various asset classes differ greatly from the historical values. In general it can be concluded that the prospective volatility is expected to be lower than the realized volatility over 2005-2011 for all the assets. The expected return for Bonds and High Yield bonds is lower combined with a higher standard deviation. Hedge Funds and Equity are expected to yield a higher expected return accompanied with a lower standard deviation. With respect to the VIX Futures, 2 assumptions are made; the first is a positive outlook for VIX Futures based on the continuous uncertainty in the financial markets; a high average annual return (1.79%) with a high standard deviation (40%). These estimates are based on the CAPM model for estimating expected returns:

$$R_p = \alpha + \beta(R_m - R_f) - R_f$$

The CAPM α & β are estimated based on historical data in Chapter 6. The VIX Futures β = -1.80, α = 14.41%. The expected return for the VIX Futures can be calculated with the equation above. This results in an average annual expected return of 1.79% for VIX Futures.

The second assumption is based on the expectation for VIX Futures to perform during 'normal' times; a small negative average annual return (-2%) and a high standard deviation (40%). The small negative return is due to the costs associated with carrying a long VIX Futures position; the contracts need to be rolled over monthly. Long dated VIX Futures contracts trade at a premium to short dated contracts as the uncertainty is greater. The roll-over cost of VIX Futures indicates an underperformance of VIX Futures relative to the VIX. The standard deviation is set to 40%, this is based on the same notion that recent time are tough and that improvement are not yet in sight. Spikes in volatility happen quickly, resulting in a relatively high volatility for the VIX and VIX Futures during turbulent times. The prospective portfolio statistics assume that the stock market will be a lot more stable as the expected return for Equity is a lot higher while the expected standard deviation is lower. The VIX Futures are therefore expected to yield a lower standard deviation for the coming years.

Exhibit 6.4.2: Prospective versus Historical Portfolios

			VIX FUT μ	*2%*	*VIX FUT μ*	*-2%*
			VIX FUT σ	*40%*	*VIX FUT σ*	*40%*
	Historical Data	**Historical Data**	**2012 - 2016**	**2012 - 2016**	**2012 - 2016**	**2012 - 2016**
Portfolio Allocation	**Rp with VIX**	**Rp NO VIX**	**Rp with VIX**	**Rp NO VIX**	**Rp with VIX**	**Rp NO VIX**
Equities	3,83%	1,45%	27,51%	19,26%	26,53%	19,26%
Bonds	61,63%	64,43%	46,44%	64,21%	51,55%	64,21%
High Yield	33,16%	34,12%	8,20%	12,33%	9,30%	12,33%
Hedge Funds	0,00%	0,00%	6,49%	2,14%	5,70%	2,14%
Commodities	0,00%	0,00%	0,64%	1,81%	0,93%	1,81%
Private Equity	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
Real Estate	0,00%	0,00%	0,67%	0,24%	0,54%	0,24%
VIX Futures	1,38%	n/a	10,07%	n/a	5,45%	n/a
Σw	100,00%	100%	100,00%	100%	100,00%	100%
Average Annual Return	6,49%	6,77%	3,43%	2,88%	3,14%	2,88%
Average Annual StDev	3,10%	3,16%	4,70%	4,98%	4,71%	4,98%

Note: short selling is not allowed, it is assumed that there are no transaction costs. The prospective portfolios performance is not representative, as all information used is completely hypothetical.

Exhibit 6.4.2 shows the optimal portfolios during 2005-2011 and 2012-2016 with- and without the possibility to include VIX Futures. Short selling is not allowed and there is an assumption that transaction costs are not involved. As stated before, the prospective portfolios are based on the prognosis by Robeco and these estimates are very uncertain. This study can be used as an example of how to interpret and to use prospective estimates. It is not possible to analyse all different prospective portfolio estimates, therefore this study uses only Robeco's prospective figures. No rights can be retrieved from the results, as the study only provides a hypothetical illustration.

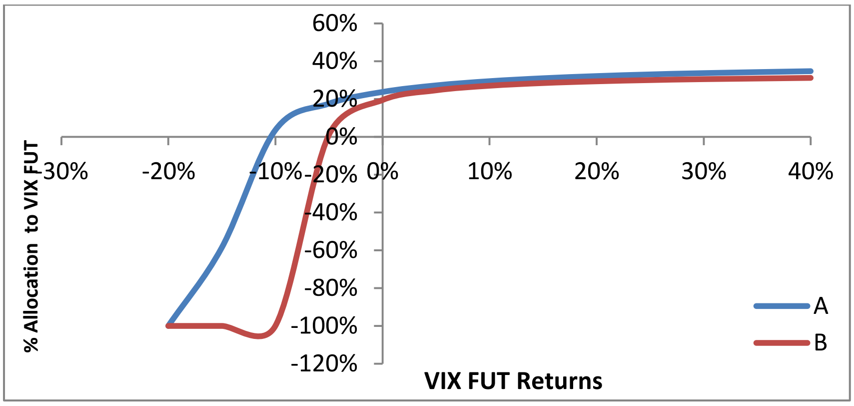
When comparing the historical optimal portfolios with the hypothetical prospective portfolios a couple of interesting changes can be denoted. Based on historical data the optimal portfolios consist mainly of Bonds and High Yield bonds, as the risk return relation of these asset classes dominate all the other asset classes. By allocating a small portion of the portfolio to Equity and VIX Futures diversification benefits are realized. Because of the continuous financial turmoil, the low correlations to the other asset classes and the low standard deviation still a relatively large portion of the portfolio is allocated to Bonds. Equities are expected to perform a lot better than during 2005-2011; hence a large allocation to Equity is included. The prospective optimal portfolio consists of almost all asset classes. Based on the prospective returns and variances, diversification benefits can be realized by utilizing the entire investment opportunity set. The optimal portfolio based on prospective data shows a strong resemblance with the Market Cap Portfolio estimated by Doeswijk, Swinkels and Lam (2012).

The most interesting feature of the prospective optimal portfolios is their allocation to VIX Futures. As stated earlier, VIX Futures are expected to be very volatile (σ=40%), and to yield either a positive return (+1.79%) or a negative return (-2%). These VIX Futures values are prospective estimates and are used in order to illustrate the diversification benefits realizable by adding VIX Futures to the opportunity set. As can be seen in exhibit 6.4.2 it is optimal to allocate over 10% to VIX Futures when the expectation is that VIX Futures yield a positive return. When the markets are stable the VIX Futures return is expected to be slightly negative, due to the roll-over cost and the consequent underperformance relative to the VIX index. Regardless of the negative expected return for VIX Futures it is still optimal to include a 5% allocation to VIX Futures. This indicates that diversification benefits can be realized with adopting VIX Futures in the opportunity set; even though VIX Futures are expected to yield negative average returns. The high variance of VIX Futures and the negative correlation to the other asset classes are the characteristics of VIX Futures that lead to significant diversification benefits. These findings are in line with the scope of the research and a can be seen as an illustration of the benefits realizable by incorporating VIX exposure.

6.5 Sensitivity Analysis VIX Futures

Based on historical data starting January 2005 until December 2011 the average annual return for the VIX Futures is 9.72% with an average annual standard deviation of 60%. In the optimal portfolio estimation based on historical data the study concluded that a 1.4% allocation to VIX Futures is optimal. For the prospective portfolios the study estimates the average annual return for the VIX Futures to be 1.79% or -2%, with an average annual standard deviation of 40%. The optimal portfolio based on prospective data includes either a 10% or a 5% allocation to VIX Futures depending on the expected average annual return for VIX Futures. The prospective values are based on the notion that times are tough with improvements not yet in sight. As concluded earlier VIX Futures thrive when the markets are uncertain, and moving down. It is interesting to see how the optimal portfolio allocation changes when the VIX Futures characteristics alter. The sensitivity is measured by monitoring the changes in the allocation output caused by altering the characteristics of the VIX Futures, while keeping the other variables constant. Exhibit 6.5 depicts the results found for the sensitivity analysis for adding VIX Futures to an all-equity portfolio. The percentage allocation to VIX Futures depends on the expected return for the VIX Futures, the variance of the VIX Futures, the expected return on equity and the variance of equity. Scenario A is based upon the prospective variance for the VIX Futures, the prospective annual average return of equity and the prospective average annual standard deviation of equity. Scenario B is based upon the historically estimated variance for the VIX Futures, the historical average annual return of equity, and the historical average annual standard deviation of equity. The expected return range for the VIX Futures is from -20% to +40%. Short- selling is allowed; under the constraint that it is not possible to short sell more than 100%. This assumption is based on liquidity reasons. The return range for VIX Futures is very large; this is done in order to emphasize the impact the VIX Futures characteristics have on the portfolio.

Exhibit 6.5: Sensitivity Analysis VIX Futures



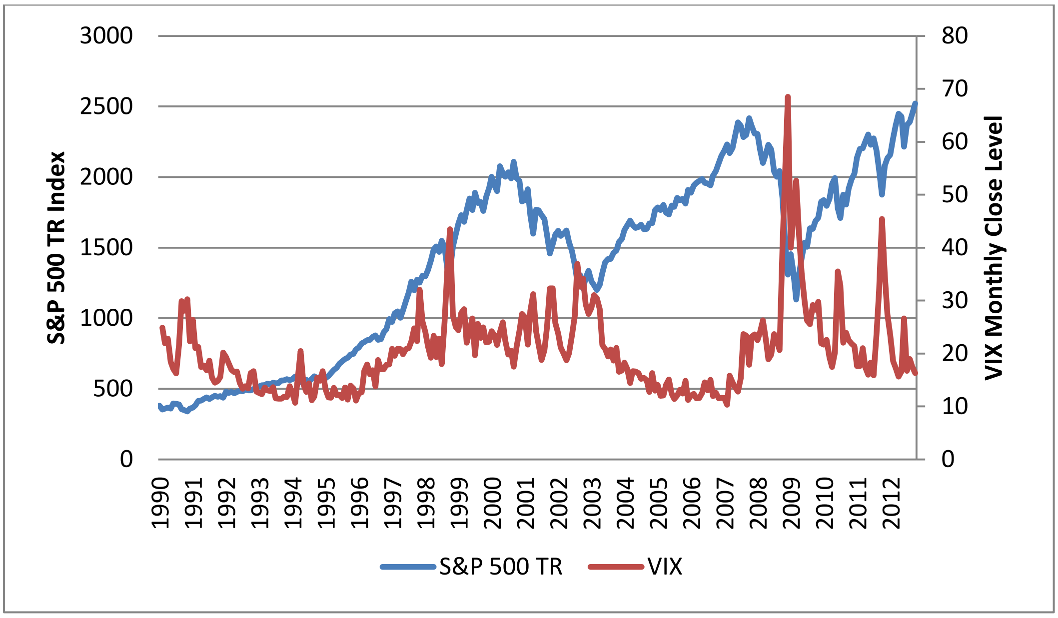
Portfolio	Equity μ	Equity σ	VIX σ
A (prospective)	7%	18%	40%
B (historical)	2,60%	21%	60%

From exhibit 6.5 the study can conclude that the percentage allocation to VIX Futures is very sensitive to changes in the expected return of VIX Futures. The slopes of the various sensitivity curves are concave. It is shown that when the VIX Futures are expected to yield large negative returns, short selling the VIX Futures seems optimal. For medium negative expected returns (± - 10%), it appears as if diversification benefits can be realized by allocating a small portion of the portfolio to shorting VIX Futures. When the VIX Futures are expected to yield small negative returns (-5% to - 1%), an allocation of ± 10% to VIX Futures seems to be optimal. Positive expected returns for the VIX Futures results in an allocation of 20% to VIX Futures. An allocation of > 30% to VIX Futures does not seem optimal, regardless of the expected returns of VIX Futures. The large return range for VIX Futures is chosen in order to emphasize these findings. An expected return for VIX Futures greater than +10% is highly unlikely, the sensitivity analysis indicates that the allocation to VIX Futures does not increase when the VIX Futures yield extraordinary returns. This implies that the VIX Futures can provide significant diversification benefits, but that it is not optimal to allocate a drastic portion of a portfolio to VIX Futures due to the high volatility.

Chapter 7: Dynamic Trading Strategies

A high level of VIX is often highlighted on the news, as it coincides with a sharp fall in the stock market. A commonly used nickname for the VIX index therefore is the Fear & Greed index. The reason of why the VIX tends to rise when the stock market declines is often neglected. As mentioned earlier, the VIX measures the implied volatility priced in the S&P 500 Index Options market through the use of a wide variety of options. When investors fear a stock market decline, they are most likely to seek protection by the use of (put) option contracts. The implied volatility of option prices rises when there is an increasing demand for option contracts. Therefore, the VIX rises when the stock market is under pressure. Before the study discusses the value of the VIX as a stock market indicator, it necessary to take another look at the graph of the VIX versus the S&P500 from 1990 – 2012. Besides the strong negative correlation another interesting aspect can be denoted. From the graph it shows that the VIX seems to be much more of a short term than a long term crisis indicator, as spikes in volatility happen quickly and the VIX returns to ‘normal’ in time.

Exhibit 7: S&P 500 TR & VIX, 1990 -2012



7.1 Dynamic Trading Literature

There are several researchers and trading blogs that investigate the VIX as a stock market indicator: Wealth Daily Blogspot: VIX as a stock market indicator. They conclude that the VIX can be used as a contrarian indicator; they suggest betting against the 'crowd'. If the 'crowd' believes the markets are bullish, it is time to get bearish. They test the VIX as a stock market indicator; a level of the VIX below 20 indicates that the 'crowd' is bullish, everybody want to buy stocks, resulting in very high prices. Consequently this strategy indicates that it is time to sell (bearish). A level of VIX above 30 means that the 'crowd' is scared; everybody is willing to sell stocks, resulting in low prices. The strategy indicates that this is the moment to buy stocks, as stocks trade at bargain prices. Jenkins (2012) suggests buying the S&P 500 when the VIX is at a high. A high level of the VIX corresponds to a low level for the S&P 500 due to the negative correlation between the VIX and the S&P500. His strategy recommends selling the S&P 500 when the VIX is very low; this indicates that the stock market is at or near a high. He concludes that the VIX does not provide the exact market's high or low, but applying this strategy puts you in the neighborhood of both.

Bolotowsky (2007) also investigates the VIX as a thermometer; he estimates a normal range for the VIX at 15 – 30. When the VIX drops below 15, this means that the market is in the steady state. When the VIX rises above 30, this indicates that stock markets are nervous. Whaley (2000) introduces the VIX index and estimates a normal range for the VIX. According to Whaley the mean daily closing level of the VIX is 18.88. He estimates that 75%

of the time the VIX closes within the range 12 – 29. He uses this interval to predict the rate of return for the S&P 500. He concludes that the VIX works reasonably well as a stock market indicator.

Copeland & Copeland (1999) investigate whether a change in the level of the VIX can be used as a stock market indicator. They examine the predictive power that changes in the VIX have for large capitalization stocks' daily market returns. They find that an increase in the level of VIX compared to its previous close is a significant indicator of positive daily market returns for large-capitalization stocks.

Stokes (2012) develops a strategy that is based on the level of VIX compared to its moving average. He suggests that large cap stocks tend to outperform small cap stocks when the VIX is relatively high. He finds that if the VIX closes at least 5% higher than its 3-month moving average, the large caps (S&P500) outperform the small caps (Russel 2000). If the VIX closes 5% lower than its 3-month moving average the small cap index outperforms the large cap index. When the VIX index closes within this 10% range, no trades are made as the VIX has no predictive power. Lestikow & Yu test whether the VIX level can function as a signal for switching between large and small cap indexes. They conclude that a high level of VIX indicates that large cap stocks (S&P500) outperform small cap stocks. A high level of VIX is identified as being 10% above its 75-day moving average. They also find that the S&P500 performed well for a low spot value of VIX <18.

Rhoads (2011) examines a moving average trading strategy; he uses the VIX as a filter for being long the index only in times when the VIX is lower than its moving average. He applies 3 filters of different magnitudes; a 20-day, 50- day, and 200 –day moving average. He concludes that the VIX as a long term indicator does not appear to be very useful. Using the VIX as a short term market indicator at the other hand does yield positive returns. However, he concludes that simply buying and holding the S&P 500 index over the same period results in a greater positive return than any of the strategies.

Rhoads (2011) also researches the predictive power of combining VIX Futures and the index. He concludes that the VIX Futures price relative to the index can be used as an indicator of panic in the stock market. When the VIX Futures often trade at a discount compared to the index, the S&P 500 loses. He finds that when the VIX Futures trade at a premium compared to the index the S&P 500 is most likely to perform positive.

Simon & Campasano (2012) investigate trading strategies based on VIX Futures. They find that the VIX Futures basis does not have significant forecasting power regarding the change in spot VIX. For example, when the 1-month VIX Futures contract trades at 25 and the VIX is 20. If the VIX Futures would have forecasting power, the VIX is expected to trade at 25 in 1 month. They find that this is not the case, and hence conclude that the VIX Futures price falls to 20.

7.2 The VIX as a stock market indicator

The strong inverse relationship between the VIX and the S&P 500 is shown in exhibit 7. This relationship can be further illustrated with the correlation of -0.76 between the VIX and the S&P 500. The VIX is estimated based on the implied volatility priced by the S&P 500 Index option market through the use of a wide variety of option prices. When investors are concerned with the direction of the S&P 500, they want to be protected against a crash. This can be done in multiple ways, for example through the acquisition of S&P 500 index put options. When there is big increase in demand for these put options, this result in an increasing implied volatility of option prices. The VIX is a measure of implied volatility of the S&P 500 option market and so the VIX rises when the market is under pressure. The VIX is *implied* by the current prices of S&P 500 index options and represents the expected *future* market volatility over the next 30 days.

As the VIX represents the expected future volatility combined with the inverse relationship between the VIX and the S&P 500, it appears as the VIX may be used to predict the future direction of the stock market:

- Dynamic A: “When the VIX is low it’s time to GO, when the VIX is high it’s time to BUY”

A high level of VIX indicates that there is a lot of uncertainty in the market. In bearish markets there is no one to sell stocks to as no one wants to have stocks; demand is low, supply is high; time to buy! For bullish markets on the contrary, everyone in the market wants to have stocks; demand is high, supply is low: time to go! This strategy results in buying stocks when all investors are fearful (high VIX) and sell stocks when investors are greedy (low VIX).

Dynamic A is a strategy where the VIX functions as a stock market indicator; Sell the S&P 500 when the value of the VIX drops below a certain threshold, buy the index when the VIX is above a certain threshold. Exhibit 7 shows the mean revering characteristic of the VIX; the VIX index trades most of the time within a certain range (15-30) when this is not the case, the VIX spikes. Based on the data for the level of VIX, the mean closing level is calculated; the average level of VIX for 2005-2011 is 21.97. This average is a lot higher than the average estimated by Whaley (2000). Based on this higher average and the previous literature of Bolotowsky and Jenkins, the normal VIX trading range is estimated to be 15-30.

Multiple adjustments can be made to this strategy in order to test its functionality and robustness; for example the Buy signal can be based on the VIX being higher than 35, or the Sell signal could be raised to a drop in the level of VIX below 20.

- Dynamic A1: Sell S&P 500 if VIX <15, Buy S&P 500 if VIX > 30
- Dynamic A2: Sell S&P 500 if VIX < 15, Buy S&P 500 if VIX > 35
- Dynamic A3: Sell S&P 500 if VIX < 15, Buy S&P 500 if VIX > 40
- Dynamic A4: Sell S&P 500 if VIX < 20, Buy S&P 500 if VIX > 30
- Dynamic A5: Sell S&P 500 if VIX < 20, Buy S&P 500 if VIX > 35
- Dynamic A6: Sell S&P 500 if VIX < 20, Buy S&P 500 if VIX > 40

Exhibit 7.2: The VIX as a stock market indicator "I"

	Dynamic A1	Dynamic A2	Dynamic A3	Dynamic A4	Dynamic A5	Dynamic A6	100% Equity
Average Ann Return	-16.84%	-15.29%	-17.46%	-20.76%	-19.27%	-21.34%	2.60%
Average Ann StDev	16%	18%	17%	20%	19%	18%	21%
Mean daily return	-0.074%	-0.07%	-0.08%	-0.09%	-0.09%	-0.10%	0.010%
daily StDev	1.22%	1.13%	1.06%	1.27%	1.19%	1.12%	1.47%
Skew	-0.47	-0.59	-0.81	-0.35	-0.44	-0.60	-0.37
Kurtosis	23	30	36	20	25	30	12
% Up Days	20%	18%	16%	28%	26%	24%	55%
% Down Days	28%	25%	23%	40%	37%	35%	45%
Return Total Period	-73%	-69%	-74%	-80%	-78%	-81%	20%

Dynamic A does not perform well during 2005-2011; the ill performance of Dynamic A can be explained by the performance of the VIX during the same period. The first filter of Dynamic A is based on the notion that when the VIX is low, it is wise to sell stocks as markets are complacent. This strategy seems to be just, but the VIX barely drops below 15. Based on exhibit 7 it can be concluded that the second part of this strategy; buying stocks when the VIX is above 30/35/40, is not very favourable. The VIX index continues to rise over these levels and seems to peak at levels over 45. Buying stocks when the VIX moves higher and while the stock market continues to move down is not very favourable. This is supported by exhibit 7, which shows that a VIX level of > 30 does not automatically imply a peak in the level of VIX.

An adjustment can be made in order to improve Dynamic A; a filter is added that identifies a peak in the level of VIX.

- Dynamic A4-1: Sell S&P 500 if VIX < 20, Buy S&P 500 if 30 < VIX >3-day MA VIX
- Dynamic A4-2: Sell S&P 500 if VIX < 20, Buy S&P 500 if 30 < VIX >10-day MA VIX
- Dynamic A4-3: Sell S&P 500 if VIX < 20, Buy S&P 500 if 30 < VIX >25-day MA VIX

When the VIX is within the 'normal' range, no position is taken.

Exhibit 7.2.2: The VIX as a Stock Market Indicator "II"

	Dynamic A4-1	Dynamic A4-2	Dynamic A4-3	100% Equity
Average Ann Return	14.47%	0.49%	-11.87%	2.60%
Average Ann StDev	13.01%	12.82%	15.45%	21%
Mean daily return	0.054%	0.002%	-0.051%	0.010%
daily StDev	0.89%	0.81%	0.98%	1.47%
Skew	3.50	1.79	0.01	-0.37
Kurtosis	32	18	29	12
% Up Days	26.94%	25.74%	24.66%	55%
% Down Days	33.45%	34.25%	34.70%	45%
Return Total Period	157.54%	3.48%	-58.74%	20%

Dynamic A2 is a strategy that waits for a peak in the VIX above 30 and let the VIX start to decline before buying the S&P 500. With this strategy a VIX level of below 20 is considered to be bearish; indicating that investors have become complacent and that it is time to sell the S&P500. Dynamic A4-1/2/3 perform a lot better than Dynamic A4, therefore it appears that identifying a peak results in a better performing strategy. The length of the moving average has a substantial impact on the performance of the strategy. From exhibit 7.2.2 it can be concluded that comparing the VIX with a 3-day moving average is optimal. This indicates that the VIX is much more of a short term tool, as is supported by Rhoads (2011). This is supported by exhibit 7.1 where it is shown that spikes in volatility happen quickly.

The average annual return of Dynamic A4-1 is higher than the annual return of being long the S&P 500 index, while the standard deviation is lower. This indicates that the dynamic trading strategy has a better risk / return relation than the market. This is the only strategy that implies that the VIX can be used as a stock market indicator. Because only 1 of all the tested strategies outperforms being only long the S&P 500 index, the study cannot conclude that the VIX can be used as a reliable stock market indicator. This is in conflict with previous academic work on the functionality of the VIX as a stock market indicator. Most studies simply use the exact level of the VIX as a thermometer, these strategies yield negative average returns. This study shows that only a strategy based on the level of VIX, combined with the VIX being below its 3.day moving average results in a better performance.

7.3 Combining VIX Futures and the VIX index as a stock market indicator

The value of a VIX Futures contract is in general expected to symbolize the VIX level as of the expiration date of the contract. Based on this, it appears that the VIX Futures price relative to the VIX level can be used as an indicator of the direction of volatility. The expected movement in the direction of volatility can be used as a stock market indicator. This is in contrast with the findings by Simon & Campasano, who conclude that the VIX Futures have no forecasting power for the VIX.

A possible limitation of using VIX Futures versus the VIX index level as a stock market indicator is the pricing relationship of VIX Futures with the VIX index. When the VIX is above its long-term mean, the futures are likely to trade at a discount (backwardation). On the other hand, if the VIX is low and investors expect the VIX to rise towards its long-term mean, the futures will trade above the index (contango).

Exhibit 7.3: VIX Futures prices versus VIX Index level

	Trading Days	FUT > VIX	% Trading Days above Index	S&P 500 Returns
2005	250	218	87%	5%
2006	250	207	83%	16%
2007	250	179	72%	5%
2008	251	145	58%	-37%
2009	250	208	83%	26%
2010	251	215	86%	15%
2011	250	172	69%	2%

From analysing exhibit 7.3 it shows that the VIX Futures generally trade at a premium to the VIX index. This is in line with the research performed by Rhoads (2011). The reason for this pricing relation is that uncertainty results in a higher implied volatility. A long dated contract has more uncertainty than a short dated contract; because it is harder to predict a price in two months than in one. This uncertainty is reflected in a higher implied volatility, which leads to higher prices for VIX Futures contracts versus the VIX level.

To conclude, the study develops two dynamic trading strategies based on the VIX Futures price relative to the VIX index, in order to predict the movement of the stock market. When VIX Futures are at a discount to the VIX index, this indicates that investors believe that the VIX index will decline and the S&P 500 will rise. On the other hand, when VIX Futures trade at a premium to the VIX index, investors expect the VIX to rise and they anticipate a stock market collapse.

- Dynamic B1: Sell S&P 500 if VIX FUT > VIX index, Buy S&P 500 if VIX FUT < VIX
- Dynamic B2: Sell S&P 500 if VIX FUT > 1.05*VIX index, Buy S&P 500 if VIX FUT < VIX

Dynamic B2 differs from Dynamic B1 as Dynamic B2 only goes long the index when the VIX Futures trade at a drastic premium with respect to the index.

Exhibit 7.3.2: Performance Statistics Dynamic B

	Dynamic B1	Dynamic B2	100% Equity
Average Ann Return	-45%	-42%	2.60%
Average Ann StDev	20%	18%	21%
Mean daily return	-0.236%	-0.221%	0.010%
daily StDev	1.45%	1.31%	1.47%
Skew	-0.22	-0.33	-0.37
Kurtosis	12	17	12
% Up Days	39%	28%	55%
% Down Days	60%	49%	45%
Return Total Period	-99%	-97%	20%

The performance statistics of the two Dynamic trading strategies are summarized in exhibit 7.3.2. From these results it shows that both strategies failed as a stock market indicator. Both strategies yield very large negative returns relative to being long the index during the entire sample set. In estimating the results for both dynamic trading strategies, transaction costs are neglected. Using the VIX Futures relative to the VIX index does not improve the performance of a portfolio versus just holding the S&P 500 index. Combining the VIX Futures price with the VIX index level fails as a reliable stock market indicator. These findings are in line with the study by Simon & Campasano (2012).

7.4 Timing with VIX Futures

The value of a particular VIX futures contract represents the expectation in the market of where the VIX index level will be as of the expiration date of the contract. There is no underlying basket of securities that can be acquired in order to replicate owning the VIX index; hence there is no actual financial relationship between the spot VIX and the VIX Futures contract prices.

Exhibit 7.3 shows that the VIX Futures contracts generally trade at a premium to the VIX index, due to a risk premium that is included in the VIX Futures price as a result of a greater uncertainty. Hence, VIX Futures contracts are far more likely to be in contango with respect to the index than in backwardation. This implies that expected volatility in 2 months is higher than the expected volatility in 1 month, resulting in a negative roll over yield for VIX Futures. In order to keep a long allocation to VIX Futures the position needs to be rolled over to the next month. Having a consistent long VIX Futures allocation in a portfolio is relatively costly. A portfolio with a long allocation to VIX Futures is expected to perform well during bearish market environments, during bullish market environments a portfolio consisting of VIX Futures is expected to underperform a pure long stock portfolio. In Chapter 6.4 the study concludes that possessing a small long allocation of VIX Futures is optimal, even though the expected returns of VIX Futures are negative. The Chapter concludes that VIX Futures provide significant diversification benefits during bearish market environments. The VIX Futures sensitivity analysis conducted in Chapter 6.5 indicates that allocating a large portion of the portfolio to VIX Futures is never optimal.

Holding VIX Futures in a portfolio is very beneficial during periods of equity market bearishness; although it is costly to have a long VIX Futures allocation. A VIX Futures trading strategy that only incorporates a long VIX Futures position when markets are bearish seems to be optimal. The level of VIX can be used in order to determine whether the stock market is bearish or bullish, as the VIX moves up when the stock market plummets. Various assumptions can be made in order to establish a VIX based crisis identifier:

- Dynamic C1: VIX level > 30, 10% allocation to VIX Futures
- Dynamic C2: VIX_{t+1} > 105% VIX_t, 10% allocation to VIX Futures
- Dynamic C3: VIX level > 15-day Moving Average VIX, 10% allocation to VIX Futures
- Dynamic C4: VIX level > 30-day Moving Average VIX, 10% allocation to VIX Futures
- Dynamic C5: VIX level > 100-day Moving Average VIX, 10% allocation to VIX Futures

The first strategy identifies a crisis period when the spot VIX level is greater than 30. In Chapter 7 the study concludes that the VIX has a mean reverting pattern, with a ‘normal’ level of 16-30. A spike in volatility often coincides with a stock market crash; a spike is considered a spike when the VIX trades above 30. Chapter 6 concludes that it is not optimal to include a large allocation to VIX Futures for a portfolio, therefore when a crisis is identified ‘only’ a 10% allocation to VIX Futures is included.

The second strategy identifies a crisis period when the spot VIX rises drastically compared to the previous close. This strategy is based on the characteristic of the VIX that spikes in volatility happen quickly. This is supported by exhibit 7.1 and the strong negative correlation between the VIX and the stock market.

The third, fourth and fifth strategies identify a crisis period based on the level of VIX compared to its moving average over a specified time period. Using the level of VIX compared to its moving average seems to be a more reliable crisis identifying tool than simply comparing the level of VIX to its previous close. A rise in the level of VIX can occur without being a crisis, due to the mean reverting characteristic of the VIX. Three different windows are incorporated in order to illustrate the effectiveness of using the VIX as a crisis identifier. The notion that the VIX is more of a short term market tool than a long term market tool, as is concluded earlier, is tested. Transaction costs are included while estimating the strategies' results and are set to be 0.1% per trade. The turnover represents the number of times a 10% allocation to VIX Futures is traded.

Exhibit 7.4: Performance Statistics Dynamic C

	Dynamic C1	Dynamic C2	Dynamic C3	Dynamic C4	Dynamic C5	100% Equity
Average Ann Return	7%	18%	25%	21%	14%	2.60%
Average Ann StDev	18%	18%	17%	17%	17%	21%
Mean daily return	0.026%	0.067%	0.090%	0.077%	0.051%	0.010%
daily StDev	1.26%	1.28%	1.24%	1.23%	1.21%	1.47%
Skew	-0.17	0.05	0.07	0.06	-0.09	-0.37
Kurtosis	10	13	12	12	12	12
% Up Days	55%	56%	55%	55%	55%	55%
% Down Days	44%	44%	45%	45%	45%	45%
Return Total Period	58%	221%	380%	283%	145%	20%
Turnover	44	536	294	233	145	0

The results of the dynamic trading strategies based on the VIX as a crisis identifier, indicate that significant diversification benefits can be realized by including a 10% VIX Futures allocation during 'crisis' periods.

The first strategy, with VIX as a crisis identifier if the value of the VIX exceeds 30, has a very favourable risk / return performance compared to a pure long equity portfolio; the expected return is higher and the standard deviation is lower. The turnover is also relatively low which indicates that crises periods are rare, and thus transaction costs are kept at a minimum.

The second strategy has a favourable return and standard deviation, however using this method of identifying a crisis by the level of VIX is less credible. This strategy denotes a very large turnover, resulting in relatively high transaction costs. A trade is made at the start and at the end of a ‘crisis’ period, therefore a large number of trades indicates that there are a lot of crisis periods.

The third, fourth and fifth strategies all perform significantly better than the pure equity portfolio. Both the return and standard deviations of these strategies outperform the equity portfolio. Another interesting note is that identifying a crisis by comparing the spot VIX with a short selected moving average result in a better performance. This suggests that the VIX is more of a short term market tool compared to a long term market tool, which is supported by Rhoads (2011).

Based on the results of these strategies, it can be concluded that the VIX can be used as a reliable crisis identifier. When a crisis is identified a 10% long allocation to VIX Futures should be included in the portfolio as it improves the portfolios’ performance. The study expects that the first timing strategy (C1) keeps performing well, as it identifies a stock market crash reliably. The turnover & transaction costs are kept at a minimum, which appears to be credible as stock market crashes do not occur on a daily basis. The third strategy (C3) is also expected to perform well, as it yields a very favourable risk – return relation.

Chapter 8: Conclusion

The tremendous volume & open interest growth of the VIX Futures market in recent years prove that volatility trading is becoming increasingly popular. The goal of this study is to point out the various opportunities an investor has by adopting the VIX Futures in his opportunity set. The study shows that ex-post diversification benefits are realized by considering a long VIX Futures allocation during 2005-2011.

The study starts by making an extensive analysis for the various asset class correlations during 2005 – 2011. The study finds that due to the high positive correlations between the traditional asset classes, it is hard for an investor to diversify its portfolio in times of market turmoil. One asset class performs well in the face of the financial crisis; volatility. Volatility trading is highly complex as the exact relation between the VIX, the VIX Futures and the stock market is hard to understand. This study provides a detailed explanation of the VIX, the VIX Futures and their relationships with the stock market.

This study examines the addition of a small long VIX Futures position to three traditional portfolios. The findings indicate that a long VIX Futures allocation results in a better risk-return performance, these findings are in line with previous studies by Szado (2009) and Briere, Burgues and Signori (2009). This study extends their findings, as this study is based on a longer, credible and reliable time window. Additionally this study tests for significant outperformance by the portfolios with VIX Futures relative to the portfolios without VIX Futures. The Sharpe ratio is the most commonly used performance measurement tool. The portfolios' returns proof to be non-normally distributed and therefore various performance evaluation tests are used that are robust against non-normality, such as the MSR, Stutzer index and Leland's alpha. The study shows that holding a long VIX Futures position is efficient for a risk-averse investor with a preference for positive skewness and low kurtosis.

The empirical evidence in favour of using VIX Futures as an effective diversification instrument is scarce. This paper tries to fill this gap in the literature by employing a mean variance analysis that creates an ex-post optimal portfolio during 2005 – 2011. The optimal portfolio weights indicate that it is optimal to include a small long VIX Futures position.

The data set ranges from January 2005 until December 2011, and is heavily affected by the financial crisis of 2008. It is not expected that the financial markets will behave in the same way in the near future; the ex-post results only have limited forecasting power.

The prospective analysis shows that even though the VIX Futures are expected to yield negative returns, it is still optimal to include VIX Futures in a portfolio as diversification

benefits can be realized. The sensitivity analysis concludes that VIX Futures provide significant diversification benefits, but it is never optimal to allocate a drastic portion of a portfolio to VIX Futures. A consistent long VIX Futures position is expected to lose money in the long run, due to the roll over costs of maintaining a front-month VIX Futures position.
The study also simulates various dynamic trading strategies and concludes that the VIX index can be used only in specific occasions as a reliable stock market indicator. Using the VIX index as a crisis identifier instrument in order to diversify an equity portfolio with a 10% VIX Futures allocation during crises periods seems to be very profitable. Note that the VIX proves to be much more of a short term indicator than a long term indicator.

The unique risk and return characteristics of VIX Futures make them a very interesting diversification vehicle, especially during times when traditional perceived diversifying instruments prove to be less effective. A portfolio with a small VIX Futures allocation benefits greatly in bearish market environments. During bullish markets at the other hand the same portfolio is not expected to outperform a portfolio without VIX Futures. The strong negative correlation between VIX Futures and equity as well as the positive skewness of VIX Futures justify their addition to many portfolios; even though a long VIX Futures position yield small negative returns.

8.2: Limitations & Further Research

The data sample used in this research spans from January 2005 until December 2011. The VIX Futures are introduced in March 2004, because of liquidity reasons the data starts only in January 2005. From January 2005 until 2007 the financial markets are relatively calm. From early 2008 the uncertainty in the financial markets raises significantly, this result in the financial crises of late 2008. During the financial crisis of 2008-2009 the S&P 500 loses almost 47%. Other asset classes that are considered effective diversifiers face substantial losses. Volatility performs well in the face of the crash; the VIX index rockets with more than 125% during the same period. From 2009 the financial market continue to behave very uncertain, due to the US debt-ceiling crisis as well as the Euro debt crisis, the financial markets are on the brink of a relapse into another recession. The VIX and also VIX Futures perform a lot better during these years than is normally expected. Studies that investigate diversification benefits by including VIX exposure in a portfolio therefore result in finding positive outcomes, partly because of the biased data sample. All the tests performed in this study are ex-post, based on data that starts in 2005 until 2011. Even the prospective portfolios are partially biased, as the historical correlation matrix is used in order to determine the optimal portfolios. It is not possible to perform a similar analysis based on data that is not affected by the financial crisis, as there is no such data readily available. It is possible to perform an ex-ante analysis, but various assumptions need to be made which can be extremely subjective.

There is not a lot of empirical evidence that suggest the use of VIX Futures as an effective diversification instrument. There are some studies that examine using the spot VIX or VIX Futures as an effective equity diversifier, all of these studies are based on data starting from 2005 and are therefore limited in their forecasting power.

The various dynamic trading strategies that are revised in this study are created in order to show the opportunities an investor has with the VIX. Besides considering a long VIX Futures position or using the spot VIX as a crisis identifier, various alternative trading strategies can be developed and examined based on the inverse relationship between the VIX and the stock market.

References:

Russel Rhoads, "Trading VIX Derivatives: Trading and hedging strategies using VIX Futures, Options, and Exchange-Traded Notes", CFA
ISBN 978-0-470-93308-4

Szado, E., 2009. VIX Futures and Options – A Case Study of Portfolio Diversification During the 2008 Financial Crisis, CFA. The CBOE provided research support for this paper. The Journal of Alternative Investments 12, pp 65-85.

Alexander, C., Korovilas, D., 2012. Diversification of Equity with VIX Futures: Personal Views and Skewness Preference, ICMA Centre, Henley Business School at Reading, March 2012.

Black, F., 1976. Studies of Stock Price Volatility Changes, In Proceedings of the 1976 American Statistical Association, Business and Economical Statistics Section, pp 171-181.

Briere, M., Burgues, A., Signori, O., 2009. Volatility Exposure for Strategic Asset Allocation, The Journal of Portfolio Management 36 (3), pp 105 – 116.

Copeland, M., Copeland, T., 1999. Market Timing: Style and Size Rotation Using the VIX, Financial Analysts Journal Vol. 55, No. 2, March 1999, pp 73-81.

Chow, G., Jacquier, E., Kritzman, M., and Lowry, K., 1999. Portfolio Optimization in Good Times and Bad, Financial Analysts Journal, Vol. 55, No. 3 May/June 1999, pp 65 - 73

Christie, A., 1982. The Stochastis Behavior of Common Stock Variances: Value, Leverage and Interest rate Effects, Journal of Financial Economics, Vol. 10, Issue 4, December 1982, pp 407 – 432.

Daigler, R., Rossi, L., 2006. A Portfolio of Stocks and Volatility, The Journal of Investing 15 (2), pp 99 – 106

Dash, S., Moran, M., 2005. VIX as a Companion for Hedge Fund Portfolio. The Journal of Alternative Investments 8 (3), pp 75 – 80.

Daskalai, C., Skiadopoulos, G., 2011. Should investors include commodities in their portfolios after all? Journal of Banking and Finance 35, pp 2606-2626

DeRoon, F., Nijman, T., 2001. Testing for Mean-Variance Spanning: A Survey, Journal of Empirical Finance Vol. 8, Februari 2001, pp 111-155

Doeswijk, R., Lam, T., Swinkels, L., Strategic Asset Allocation: The Global Multi-Asset Market Portfolio 1959-2011 (November 2, 2012)

Fledman, B., Roy, D., 2004. Passive Options-based Investment Strategies: The Case of the CBOE S&P 500 Buy Write Index. Supported by the Chicago Board Options Exchange.

Glosten, L., Jaganathan, R., Runkle, D., On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks, The Journal of Finance, Vol. 48, Issue 5, Dec. 1993, pp 1779-1801

Haugen, R., 1991. The Effect of Volatility Changes on the Level of Stock Prices and Subsequent Expected Returns, The Journal of Finance, Vol. 46, Issue 3, July 1991, pp 985 – 1007

Huberman, G., Kandel, S,. 1987. Mean-Variance Spanning. The journal of Finance, Vol 42, No. 4. September 1987, pp 873-888

Jenkins, J., 2012, Understanding the VIX indicator, Investment U Research, March 2012.

Kan, R., Zhou, G., 2012 Tests of Mean-Variance Spanning, Journal Annals of Economics and Finance, Vol. 13, No. 1, May 2012, pp 139-187

Leland, H., 1999. Beyond Mean – Variance: Performance Measurement in a Nonsymmetrical World, Financial Analysts Journal, Jan / Feb 1999, pp 27 – 36

Leistikow, D., Yu, S., 2006. VIX Signaled Switching for Style-Differential and Size-Differential Short-Term Stock Investing, July 2010

Levinston, M., 2009. Guide to Financial Markets, The Economist, Fifth Edition, 2009

Manda, K., 2010. Stock Market Volatility during the 2008 Financial Crisis. April 2010.

Markowitz, H., 1952. Portfolio Selection, The Journal of Finance, Vol. 7, No. 1, March 1952, pp. 77 - 91

Markwat, T., 2012. The Rise of Global Stock Market Crash Probabilities, Robeco Asset Management

Munenzon, M., 2010. 20 Years of VIX: Fear, Greed and Implications for Traditional Asset Classes, Febrary 2010, CFA, CAIA

Munenzon, M., 2010. 20 Years of VIX: Fear, Greed and Implications for Alternative Investment Strategies, Risk Management eJournal, April 2010

Schwert, G., 1989. Why Does Stock Market Volatility Change Over Time? The Journal of Finance, Vol. 44, No 5. Dec. 1989, pp 115-1153

Stokes, M., VIX- based Pairs Trading (Market Neutral) Strategy, January 2010

Stutzer, M., 2000. A Portfolio Performance Index, Financial Analysts Journal, May/June 2000, Vol. 56, No. 3, pp 52 – 61

Turner, C., Startz, R,. Nelson, C., 1989. A Markov model of Heteroskedasticity, Risk and Learning in the Stock Market, Journal of Financial Economics 25, 1989, pp 3 – 22.

Wealth Daily Blogspot: VIX as a stock market indicator

Whaley, R. E., 2009. Understanding VIX, Vanderbilt University – Owen Graduate School of Management, in The Journal of Portfolio Management, Spring 2009, Vol. 35, No.3: pp 98-105

Whaley, R., 2000, The Investor Fear Gauge, Journal of Portfolio Management, 26, pp 12-17.

Wien, J., 2010. The VIX Reality Check, Active Trader Magazine, May 2010