Hedging Longevity Risk

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Abstract

Longevity risk, i.e. people living longer than expected, is getting more important for pension funds and insurance companies. Although longevity risk is traditionally viewed as unhedgeable, a market for mortality-linked derivatives is developing since the early 2000s. In this thesis I forecast mortality rates with the well-known Lee-Carter model. With these forecasts I price longevity index swaps using Monte Carlo simulations and the equivalent utility pricing principle. In order to hedge against longevity risk with the longevity index swaps I use a static approach. I analyse in what matter hedging longevity risk influences a stylised market value balance sheet for Nationale-Nederlanden under the Solvency II regulations. I find that the solvency capital requirements and the risk margin on the market value balance sheet decrease if longevity risk is hedged. The hedging costs involved are lower than the decrease in liabilities, i.e. longevity index swaps provide a profitable opportunity for hedging longevity risk.

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1 Introduction

Longevity risk, i.e. the risk that people live longer than expected, is one of the main risks insurance companies encounter and it has become increasingly more important in the early 2000s. Liabilities of insurance companies involved with pensions increase due to increasing longevity. In 1950 Dutch men reached on average an age of 59 years and women 63 years. 60 years later in 2010 men became 78 years and women 82 years on average. For example, in 2006 people retired in the Netherlands at the age of 65 and become on average 77 years old, which means someone receives a pension for 12 years. In 2011 people became 80 on average, which results in a pension of 15 years. For the insurer the implies a 20% increase in its liabilities. Companies exposed to longevity risk need to manage this risk in an appropriate way.

Longevity risk is traditionally viewed as a non-hedgeable risk, since there is no liquid market to hedge this risk. However, in the last few years the number of longevity-linked derivatives has been increasing and the market is growing. In this thesis I consider what financial instruments are available for hedging longevity risk in the situation in which no longevity index swaps are present. A big issue of these (over-the-counter) instruments is that they are not standardised and are therefore expensive and often illiquid. I focus mainly on longevity index swaps, which are in fact standardised. In this thesis I study longevity index swaps, how to price them and if hedging longevity risk with these swaps is profitable.

The longevity index swap is a rather new type of swap and answers the need of standardised and transparent products to hedge longevity risk. Before the index swap insurance companies could enter contracts with a reinsurer specifically suited for their companies to cover their longevity risk. These coverages bring high costs and no liquidity. Index swaps however are calibrated on the national population of a country, data which are publicly available. Because of the transparency and standardisation index swaps can be much cheaper than other longevity products. A disadvantage is that populations may differ resulting in basis risk.

My goal in this thesis is to find an attractive solution to hedge longevity risk with longevity index swaps. In particular, the implications for the market value balance sheet are important. Does hedging longevity risk reduce the solvency capital requirements and risk margin enough to outweigh the costs of hedging? At last I will discuss the implications for Nationale-Nederlanden to hedge longevity risk. Is it profitable for

Nationale-Nederlanden to enter longevity derivatives, given the upcoming Solvency II regulations?

With this thesis I present a complete framework starting from modelling mortality rates, pricing longevity index swaps to hedging longevity risk using index swaps. I provide a practical approach for efficiently exploiting rather new longevity instruments for hedging longevity risk. Additionally, Nationale-Nederlanden will have a better understanding of the implications of hedging longevity risk assuming that this risk is no longer non-hedgeable. I place this framework in a Solvency II setting.

In this thesis Nationale-Nederlanden or any insurance company in general has to pay its client a yearly cash flow as a retirement or pension until the client dies. A client starts receiving his of her retirement from the age of 65 years. This means that if this client lives longer than expected, e.g. the client dies at the age of 80 years instead of 78 years the insurer has two years of extra cash flow to the client and thus more liabilities than expected.

By hedging longevity risk an insurance company reduces its solvency capital requirements, which is capital the insurer has to hold in order to fulfil uncertain future liabilities. According to Solvency II regulations, insurers need to value their liabilities on a market-consistent basis. The market value of the liabilities exists of the best estimate plus a risk margin. The best estimate represents the liabilities which can be hedged. For the unhedgeable part a risk margin is added. In section 5 I explain the Solvency II regulations in more detail. Longevity risk is traditionally accounted for in the risk margin. The Life Market is an upcoming market (see section 2) which would mean that longevity risk could be hedged and therefore the solvency capital requirements and the risk margin could decrease. The question is if this decrease in solvency capital requirements and risk margin is large enough to cover the hedging costs.

Before we hedge with the index swaps we first determine the price. There are no quoted prices for index swaps, nor is it possible to construct a replicating portfolio to determine the price. Therefore, I use a model to price the swaps. I use Monte Carlo simulations to generate a large number of scenarios concerning mortality rates. I estimate and forecast mortality rates according to the Lee-Carter model (Lee and Carter, 1992), a well-known model in the mortality rates area. With the Lee-Carter model I simulate sample paths and calculate the payoff of the longevity index swap. By averaging and discounting the payoff I calculate the price of the swap, using the risk neutral valuation

principle (Hull, 2012). Additionally I use the equivalent utility pricing principle described by Cui (2008) to price longevity index swaps.

I use Dutch mortality rates ranging from 1950 to 2010 for the ages 1 to 99 for the estimation of the mortality rates. These data are provided by the Dutch Central Bureau of Statistics (CBS) and can be downloaded from their website (CBS, 2012). I do this, since in case the longevity swaps will develop to a full grown liquid market, they will most likely be calibrated on mortality data from the CBS.

After I have calculated the price of the swap I apply a hedging strategy proposed by Deutsche Bank to Nationale-Nederlanden. In this hedge Deutsche Bank could be the counterparty of Nationale-Nederlanden, the insurer exposed to longevity risk. Deutsche Bank proposes a corridor hedge, in which the index swap generates a payoff if the floating mortality rate is at least 10% smaller than the predetermined fixed rate, with a maximum of 20%. According to this payoff structure I price the swap and hedge against longevity risk. The goal of hedging longevity risk is to reduce the solvency capital requirements and the risk margin on the market value balance sheet. For Dutch men of old age (75 years or older) longevity risk is not a big matter, however for younger men this hedge seems profitable. For these ages the reduction in solvency capital requirements and the risk margin outweigh the hedging costs.

For Nationale-Nederlanden it is important to deal with longevity risk in a constructive way. The total sum of the liabilities of adds up to over 76 billion Euros in 2011 (Nationale-Nederlanden, 2011). Almost 60 billion is due to technical provisions and about 38 billion due to longevity-linked technical provisions. If we look at its economic capital, market risk takes the largest part for Nationale-Nederlanden's risk exposure with longevity/mortality risk following as the number two. 20% of its market shares and positions is in longevity-linked products.

The remainder of this thesis is organised the following: in section 2 I discuss the different financial instruments available for hedging longevity risk. In section 3 I follow with the description of the Lee-Carter model. In section 4 I continue with pricing of the index swaps. Next, in section 5 I provide a better understanding of the upcoming Solvency II regulations for insurance companies. In section 6 I discuss a hedging strategy with the longevity index swaps in context of Solvency II and I conclude in section 7. In section 8 I propose some additions to this research. Most sections start with the discussion of literature of the specific topic at hand.

2 Longevity Index Swaps

Literature starting from the early 2000s describes the rise of a new market to hedge longevity risk. At the beginning of 2012 this market is still illiquid, but in time it could be very promising. In this new market longevity-linked instruments can be traded to hedge longevity risk. One of such instruments is the longevity index swap, a financial product in which the payoff depends on floating and fixed mortality rates, much like the interest rate swap. In this section I describe the new swap and its structure. In the sections hereafter I discuss pricing and hedging with respect to this longevity swap.

Before we discuss the swap in section 2.2 I first provide some background information from literature concerning longevity/linked instruments. In particular I discuss some literature about the longevity index swap and its applications.

2.1 Literature Review

Blake, Cairns and Dowd (2006a) list a number of solutions how pension funds and insurance companies can handle longevity risk, such as diversifying a part of the longevity risk away across different products or socio-economic groups and entering a form of reinsurance with a reinsurer. However, they focus on longevity-linked securities, including over-the-counter contracts. Stigter et al. (2010) and De Lange and Meijer (2010) discuss longevity risk for the Dutch market. Stigter et al. (2010) address this risk in a general way giving a range of possible solutions. These solutions vary from increasing the premium the active population pays to a pension fund or link the age an employee becomes a pensioner to his or her life expectancy to hedging with longevity swaps. They conclude with the rather obvious notion that it is essential that every implementation considering longevity risk should be applied carefully. De Lange and Meijer (2010) focus more on the indemnity and index swap with respect to the Dutch pension market. Stigter et al. (2010) explain that indemnity swaps are customised instruments based on the population of the specific pension fund and are only traded over-the-counter. Because these swaps are customised, they are highly effective in hedging longevity risk, but also rather expensive. It is important though that the pension fund implements this swap precisely to maintain this high effectiveness. Another more important property of indemnity swaps is that they focus on pensioners. The contract matures when the last person in the population dies. De Lange and Meijer (2010) also discuss that the indemnity swap. This swap is successfully applied for hedging in the UK, however, since indemnity swaps are mainly focused on current pensioners, these swaps are no appropriate hedging instrument against Dutch longevity risk. The Dutch pension market has a relative large young workforce, which brings the most longevity risk. And exactly the risk from this young group is not hedged with indemnity swaps.

De Lange and Meijer (2010) continue to elaborate on index swaps. In section 2.2 I discuss these index swaps in more detail. De Lange and Meijer (2010) say index swaps provide a promising solution for (Dutch) longevity risk. Index swaps are more standardised and transparent and use a widely accepted index. Therefore the chance of a liquid market is more realistic (Stigter et al., 2010; Blake et al., 2008). They note that in order to use these instruments properly, mortality tables should be updated frequently and we should get a better grasp of additional basis risk. Blake, Dowd and Cairns (2008) describe that besides transparency, hedge effectiveness is also an important factor in the success of a new liquid market. In case of index swaps this means that the arising basis risk, due to differences between the reference population and the index population, must be analysed. Blake, Dowd and Cairns (2008) also discuss the failure of the first generation life market, a bond-based market. In particular, they discuss the launch of a longevity bond issued by the European Investment Bank in November 2004 and how pricing and institutional issues and hedging ineffectiveness prevented the attempt of becoming a success. They continue to discuss the second attempt of a life market, a derivative-based market. Blake, Dowd and Cairns (2008) are confident that with the current awareness of longevity risk and the insufficient reinsurance capacity, a new life market will emerge, given the economic importance and the ineffectiveness of already existing instruments. The instruments in this new market have to be transparent and homogeneous, according to them, and there has to be enough investor interest.

The reason why these longevity-linked instruments have so much attention is because insurers are exposed daily to longevity risk and are confronted with problems how to deal with this. People pay premia during their working life and expect a cash flow in return each year after their retirement. The insurer has to fulfil these liabilities to its client with the risk that its client live longer than expected.

Sherris and Wills (2007) describe that longevity risk is getting more attention from insurance companies, since this risk is increasing in many countries and takes significant proportions. However, there is no liquid market for hedging longevity risk. They discuss

the longevity market will develop further in the 2010s now that the activity of the research programs in the longevity area has laid a foundation and given the development of mortality indices for use of financial product design in the 2000s. Blake et al. (2011) discuss the importance of governments hedging longevity risk with longevity bonds. Insurance liabilities are increased by the addition of a market value margin reflecting the cost of capital (Keller, 2006) to cover non-hedgeable risks. If governments issue longevity bonds, they can help private insurers to overcome this longevity risk problem. They go a bit further and say governments can also help by producing a longevity index so that markets for hedging longevity risk can grow. Cairns (2012) reinforces the earlier conclusions of Cairns et al. (2011b) that simple hedging strategies are not robust relative to the inclusion of recalibration risk. He did find hedging strategies to be robust relative to the inclusion of parameter uncertainty and Poisson risk. Furthermore, he introduces the notion of Nuga hedging and demonstrates how the use of Delta-Nuga hedging results in strategies that are robust relative to uncertainties of the calibration of the key drift parameter. With respect to index-based q-forwards Cairns (2012) finds that q-forwards which mature on the target valuation date are less robust than q-forwards which have longer maturities.

Coughlan et al. (2011) describe a framework to analyse basis risk. A good hedge could be qualified by one in which the basis risk is small. Using this analysis they evaluate the hedge effectiveness for a hypothetical pension plan with the same mortality characteristics as the affluent sub-population for a static hedge, based on a longevity index linked to the national population. Cairns et al. (2011b) show correlation hedge effectiveness can be broken down into several contributions. They show that population basis risk has a significant influence on the correlation. Through their case study they find that longevity risk can be substantially hedged using index hedges as a good alternative to customised longevity hedges. The index swap is linked to a relevant longevity index. Since the pension plan member can differ from the index population, a hedge with index swaps could involve basis risk, so this hedge reduces longevity risk but does not eliminate it. On the other hand, index swaps are standardised resulting in being cheaper and less complex. Loeys et al. (2007) discuss that most basis risk arises due to differences in sex and age and that mortality rates should differentiate between these to factors in order for the Life Market to become successful. Since mortality rates are published separately for males and females and differentiate in age, I assume their is no basis risk and will not evaluate my hedging strategy based on basis risk reduction.

To hedge longevity risk we use in this thesis longevity index swaps for the price we determine in section 4. By forecasting mortality rates we can estimate our liabilities to clients in the future. If a client lives longer than we expected, we have to pay more years of retirement and so we underestimated out future cash flows, i.e. we are exposed to longevity risk. If we want to fix out future cash flows we buy index swaps and pay each year a predetermined fixed amount to the party from whom we buy the swaps and receive a floating amount, depending on the realised mortality rate. I discuss this longevity index swap in more detail in the next subsection.

Given the magnitude of the involvement of Nationale-Nederlanden in longevity-linked deals and risk exposure, a standardised longevity index swap could be – as a new and cheaper instrument for establishing more certainty in future cash flows – of real interest to Nationale-Nederlanden. Because of the standardisation and transparency it is easier to take a position in larger deals if you compare it with current more expensive overthe-counter deals.

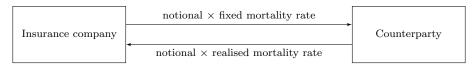
2.2 Index Swaps

An index swap is a deal in which one party pays the second party a predetermined fixed cashflow and receives a floating cashflow. The floating cashflow is based on some index. In our case, the index swaps are calibrated on the Dutch national male population and are designed to hedge longevity risk (recall sections 1 and 2.1). The Dutch Central Bureau of Statistics (CBS) forecasts Dutch mortality rates, although the model they is not known to me. Insurers analyse these numbers to determine their future liabilities. A well-known problem however is that people live longer than anticipated on, with the result that future liabilities are higher than expected. To get more certainty insurers can buy index swaps from a certain counterparty.

Index swaps act a lot like the more common interest rate swap. One party pays a fixed amount and receives a floating amount and the counterparty vice versa. These amounts are usually not actually paid to one another. Instead, only the net cashflow is paid to the rightful party. The amounts are determined by multiplying a predetermined notional with the fixed or realised mortality rate, depending on cash flow direction. The fixed rate is the best estimate of the mortality rate, i.e. the mortality rate based on the prognoses table of the CBS. The floating rate is determined using the forecasted

mortality rate, for example the forecasts made with the Lee-Carter model (see section 3. The insurer can buy a longevity index swap, which means that the company receives the floating amount and pays the fixed. This results in the fact that the insurer has less uncertainty concerning his future liabilities, since he pays a fixed amount. If people live longer and thus have a lower mortality rate, the insurer takes a loss on the longevity swap, but she is hedged against increasing future liabilities. Figure 1 gives a graphical representation of this relation.

Figure 1: This figure shows the payments of an index swap to hedge for longevity risk. An insurer pays a notional multiplied by a predetermined fixed rate to its counterparty, e.g. a life insurance company. The counterparty pays on its term the pension fund the notional multiplied with the realised mortality rate. This way the pension fund is certain of its liabilities and therefore exposed to less risk. In case the realised mortality rates are smaller than expected the insurer takes a loss, since it receives less from the counterparty than its pays. On the other side the insurer is hedged against this longevity risk.



An insurer can use index swaps to hedge longevity risk by entering this contract. The insurer receives a floating amount depending on realised mortality rates from the counterparty and pays a fixed amount. We have to forecast these cashflows and put a fair price on the swaps. In section 3 and 4 respectively I discuss forecasting and pricing.

3 The Lee-Carter Model

In order to price and hedge longevity index swaps we first model mortality rates. In the mortality rate world the Lee-Carter model is a well known model to do this. In this section I explain the model proposed by Lee and Carter (1992) and discuss fitting the model and forecasting. Like the previous section I start of with the review of some literature.

3.1 Background

Lee and Carter (1992) marked the beginning of stochastic mortality trend literature (Cui, 2008) and their model has become the 'leading statistical model of mortality in the demographic literature' (Deaton and Paxson, 2004). They describe a now well-known and commonly used model to forecast mortality rates. The natural logarithms

of age-specific death rates are modelled as a linear function of an unobserved periodspecific intensity index, with parameters depending on age. Using the Singular Value Decomposition they fit their model to the matrix of US death rates from 1933 to 1987 and find life expectancy in 2065 has increased to 86 years and that 46% of the population will survive to age 90. This holds for the male and female combined. Cairns et al. (2011a) aim to produce consistent mortality forecasts for two populations. They propose an Age-Period-Cohort model which incorporates a mean-reverting stochastic spread that allows for different trends in mortality improvement rates in the short run, but parallel improvements in the long run. Instead of a frequentist or classical approach Cairns et al. (2011a) use a Bayesian framework to fit the model to combine estimation of the unobservable state variables and the parameters of the stochastic processes driving them into a single procedure. This single-stage approach results in more consistent estimates of the unobservable period and cohort effects. Age-Period-Cohort model is actually a simplified Renshaw-Haberman model (Renshaw and Haberman, 2006) which, in its turn, is a extension of the Lee-Carter model. Renshaw and Haberman (2006) extend the Lee-Carter model by adding a cohort effect to the already present age and period effects.

Dowd et al. (2010) discuss seven of the eight models described by Cairns et al. (2007) and describe among other things a framework for back-testing the models. There are basically two types of models when it comes to modelling mortality rates, differing in the assumption of smoothness between ages. Contrary to the Lee-Carter model and its extensions and generalisations, the Cairns, Blake and Dowd (CBD) models assume smoothness between ages. All models make the same underlying assumption that the age, period and cohort effects are qualitatively different in nature. Dowd et al. (2010) conclude that the Lee-Carter model, among others, performs well most of the times. They also emphasize that their results are based on a particular data set over a limited sample periods and thus make no claim of how the models perform over other data sets or sample periods.

I will focus on the Lee-Carter model, since this model is the most widely used and simple to understand. That way, the results can be easily compared with existing literature. The data on which I fit the model are Dutch mortality data of the male population provided by the Dutch Central Bureau of Statistics (CBS).

3.2 The Model

Before Lee and Carter (1992) introduced their model, mortality rates were predicted by extrapolating observed data, without using any form of stochastic modelling. Lee and Carter (1992) started with stochastic mortality modelling and their model has become the leading model of mortality.

The Lee-carter model uses only age-specific mortality, no sex-specific factors are included. This implies that you have to calibrate the model on data of male and female populations separately. Let m(x,t) be the central death rate for age x in year t. This rate is defined as

$$m(x,t) = \frac{D(x,t)}{E(x,t)},\tag{3.1}$$

where D(x,t) is the number of deaths aged x in year t and E(x,t) is the number of lives aged x in the *middle* of year t. Now the central death rates are fitted by the model

$$ln m(x,t) = a_x + b_x k_t + \varepsilon_{x,t},$$
(3.2)

where a_x and b_x are age-specific constants and k_t is a time-varying index. In more detail, a_x is the observed natural logarithm of central death rates per age in the past and b_x can be seen as the sensitivity of m(x,t) to the index k_t . $\varepsilon_{x,t} \sim N\left(0, \sigma_{\varepsilon}^2\right)$ are white noises and reflect the variations not captured by the model.

3.2.1 Fitting the Model

The standard Lee-Carter model has an identification problem. Given the following transformations the model is invariant:

$$b_x \to cb_x$$
 $k_t \to \frac{1}{c}k_t$ $\forall c \in \mathbb{R}, c \neq 0$
 $a_x \to a_x - b_x c$ $k_t \to k_y + c$ $\forall c \in \mathbb{R}.$

Therefore, two restriction are added (Cairns et al., 2007; Lee, 2000):

$$\sum_{t} k_t = 0$$

$$\sum_{x} b_x = 1.$$
(3.3)

The first restriction implies that for each age the estimate for a_x will (approximately) be equal to the mean over t of the log death rates. The second restriction has no natural explanation, but is necessary to tackle the identification problem. This constraint had no impact on the quality of the fit or on forecasts of mortality (Cairns et al., 2007).

The right side of the Lee-Carter model (see equation (3.2) consists only of parameters we have to estimate and an unknown index k_t . Since k_t is unknown we cannot fit the model by ordinary regression methods. To find a least squares solution the Singular Value Decomposition (SVD) method is applied to the matrix of the logarithms of the death rates after the averages over time of the age-specific rates have been subtracted (SVD) (ln $m(x,t) - \alpha$). SVD is a factorization of a matrix an has the form:

$$M = U\Sigma V^T, (3.4)$$

where M is an $m \times n$ matrix, U a unitary matrix of dimensions $m \times n$, Σ a rectangular matrix with the singular values of the matrix M and V also a unitary matrix of dimensions $n \times n$. Lee and Carter (1992) use the first column of U as an estimate for the vector b_x in (3.2) and estimate k_t of that same equation by multiplying the first element of Σ by the first column of V (Cui, 2008; Lee and Carter, 1992).

From this point on k could directly be used as estimated in (3.4). Instead, k is estimated in a second stage to produce exactly the observed number of total deaths for the year in question, given a certain population age distribution and the previously estimated a and b. k is estimated as through (Lee and Carter, 1992; Lee, 2000):

$$D_t = \sum \left[\exp\left(a_x + b_x k_t\right) N_{x,t} \right], \tag{3.5}$$

where D_t is the total deaths in year t and $N_{x,t}$ is the population age x in year t. The parameters a_x and b_x stay as estimated.

This second stage estimation guarantees that the life tables fitted over the sample years will fit the total number of deaths and the population age distribution (Lee, 2000). Another advantage is that the empirical time series k can include years for which age-specific data on mortality are not available. Next we fit the model in equation (3.2) on the data with our obtained estimates. In section 3.3 I discuss the results of the estimations.

3.2.2 Forecasting Mortality Rates

Now we can forecast the log mortality rates, starting by projecting the parameter k_t into the future. This parameter is modelled as a random walk with drift (Lee and Carter, 1992 and Cui, 2008):

$$k_t = c + k_{t-1} + \eta_t, (3.6)$$

where permanent shocks $\eta_t \sim N\left(0, \sigma_{\eta}^2\right)$ are white noises. Shocks $\varepsilon_{x,t}$ and η_t are independent. The drift parameter c is known and its maximum likelihood estimate is calculated by (Cui, 2008 and Girosi and King, 2007):

$$\hat{c} = \frac{k_T - k_1}{T - 1},\tag{3.7}$$

where k_T and k_1 are respectively the last and first estimates of the time series k_t from equation (3.5). T is the number of elements in the time series k_t . This drift parameter has a variance equal to:

$$\sigma_{\hat{c}}^2 = \frac{\sigma_{\eta}^2}{T - 1},\tag{3.8}$$

where σ_{η}^2 is the variance of the random walk with drift process given in (3.6) and is calculated as:

$$\sigma_{\eta}^{2} = \frac{1}{T-1} \sum_{t=1}^{T-1} (k_{t+1} - k_{t} - \hat{c})^{2}.$$
 (3.9)

If we would make, for example, a two-step-ahead forecast of k_t we have:

$$k_{t} = c + k_{t-1} + \eta_{t}$$

$$= c + (c + k_{t-2} + \eta_{t-1}) + \eta_{t}$$

$$= 2c + k_{t-2} + (\eta_{t-1} + \eta_{t})$$

To forecast $k_{t+\tau}$ we can generalise this by following the same procedure iteratively τ times:

$$\hat{k}_{T+\tau} = k_T + \tau c + \sum_{i=1}^{\tau} \eta_{T+i-1}.$$
(3.10)

Since we assume that the increments are i.i.d., we can simplify (3.10) to:

$$\hat{k}_{T+\tau} = k_T + \tau c + \sqrt{\tau} \eta_T. \tag{3.11}$$

Given the expression in (3.11) we forecast $\hat{k}_{T+\tau}$ for one year, $\tau = 1$, up until 50 years, $\tau = 30$. We use equation (3.11) use Monte Carlo simulation to generate 1,000 sample paths for $\hat{k}_{T+1}, \ldots, \hat{k}_{T+30}$. We assume the parameters a_x and b_x to be constant over time, an assumption often made (Lee and Carter, 1992; Cui, 2008; Cairns et al., 2007). We use the estimates of a and b as described in equation (3.2) to forecast the mortality rates. We calculate the forecasted mortality rates as:

$$m(x, T + \tau) = a_x + b_x \hat{k}_{T+\tau},$$
 (3.12)

which is equivalent to filling in the Lee-Carter model in equation (3.2). Since we simulate 1,000 times for $\hat{k}_{T+\tau}$ we get also 1,000 possible forecasts of the mortality rates. By averaging the forecasts over the simulations we get the final mortality rate forecasts.

3.3 Results

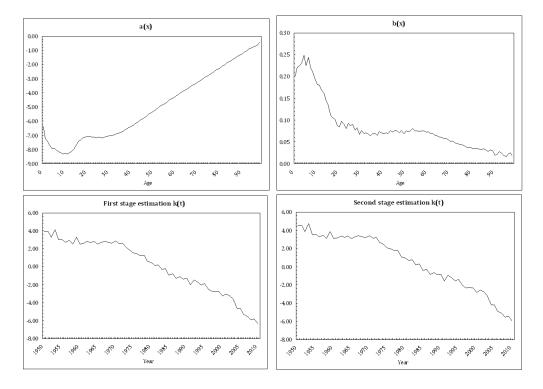
In this section I present the results of the estimation and forecasting of the Lee-Carter model using Dutch male mortality date ranging from 1950 until 2010. The data cover the ages from 1 year until 99 year. The data are provided by the Dutch Central Bureau of Statistics and can be downloaded from their website (CBS, 2012). You can also find the mortality rate tables in appendix A.3.

Figure 2 shows estimates of a_x , b_x and k_t for the first stage and second stage estimation. In appendix A.1 you find the numerical values belonging to a_x , b_x and k_t . We want to solve the linear regression model in (3.2) and find the solution, given the restriction in equation (3.3), using singular value decomposition. Like Lee and Carter (1992) we take the first column of the matrix U in equation (3.4) as the estimate for b_x . To estimate a_x we calculate the average of the mortality rates for each age. At last we estimate k_t we solve equation (3.5).

From figure 2 we see that the parameter a_x rises quite linear, which means that the average logarithmic mortality rate increases linear with respect to age. The parameter b_x , which describes the sensitivity of age-effects, is rather constant, given some fluctuations, until the age of 60. From this point b_x decreases almost linear with respect to age. From the age of 60 mortality rates become every year less sensitive to mortality improvements than the year before. We expected this, since the older people are the greater the probability of dying is. A mortality improvement for someone of e.g. the age of 80

years, implies perhaps a few months extra to live, for which for someone of 40 years old this could imply a few years extra to live. Since we estimate k_t in two steps we make sure that the life tables fitted over the sample years will fit the total number of deaths and the population age. We also see that the first and second stage estimates are not very different from one another.

Figure 2: This figure gives a graphical presentation of the parameter estimates of the regression model in equation (3.2), for all ages only for males. The parameters a_x and b_x are assumed constant over time and vary only with age. The parameter k_t varies over time. We see that the the average death rate a_x increases with age almost linearly. We also see that the sensitivity to mortality improvements, b_x is rather constant between the ages of 20 and 60 years old. From 60 years old mortality improvements have every year less effect on mortality rates. The parameter k_t is estimated in two stages. The second stage makes sure that the life tables fitted over the sample years will fit the total number of deaths and the population age distribution.



In figure 3 I show the fitted mortality rates together with the real mortality rates for 1950 to 2010. Due to the second stage estimation of k_t we expected the fit to be good. We also see that k_t declines over the years (as expected), which indicates that life expectancy increases. Assuming the estimates of a_x and b_x to be constant we forecast mortality rates using the regression model in equation (3.2). For k_T we simulate 1,000 sample paths using equation (3.11). If we take the average over all 1,000 sample paths we

obtain the forecasts. Figure 4 shows the forecasted mortality rates for the ages 25 years to 85 years with 95% confidence intervals. We see that mortality rates keep decreasing over the years. This means for insurance companies that liabilities will increase in time and that extra capital needs to be held to deal with these increased liabilities. Based on the forecasted mortality rates we can price mortality-linked derivatives. In section 4 we discuss pricing of longevity index swaps using these forecasted mortality rates.

Figure 3: This figure shows the fitted mortality rates according to the Lee-Carter model, see equation (3.2), estimated on data from 1950 until 2010 only for males. The dotted lines are the actual mortality rates and the solid red lines are the fitted ones. For the estimation the second stage estimates of k are used.

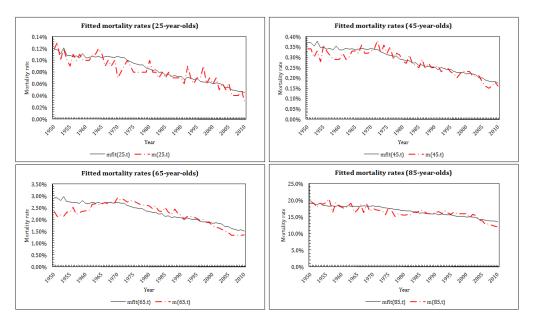
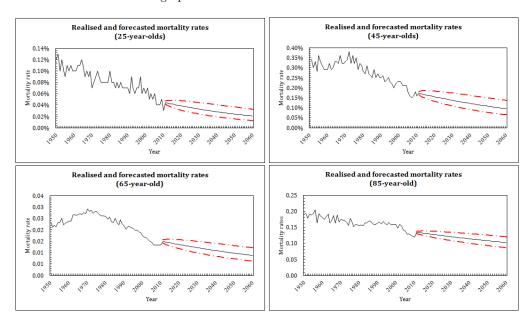


Figure 4: This figure shows fitted mortality rates based on available data until 2010 only for males. From 2011 we forecast mortality rates 50 years ahead and use Monte Carlo simulations to do so. We use the regression model in equation (3.2) and simulate k_T with equation (3.11). To come to the forecasted mortality rates we simulate 1,000 sample paths. By averaging over the 1,000 simulations, we obtained the final forecasts shown in the graphs below.



4 Pricing Index Swaps

In this section I price index swaps using two different methods. Since there is no liquid market for index swaps yet, there are no quoted prices nor is it possible to construct a replicating portfolio to value the swaps. I discuss pricing index swaps by means of simulations and I discuss the Equivalent Utility Pricing Principle proposed by Cui (2008) as an addition to the first pricing method. Before I discuss actual pricing methods I start with discussing some literature about the pricing of longevity products.

4.1 Background

As a recap for longevity index swaps you can read section 2.2 again and look back at figure 1. The floating or realised mortality rate is based on a national population, in our case the male Dutch population, and is for every insurer, reinsurer and pension fund the same in that certain country. Given a set of simulations of the realised mortality rates we could calculate the payoff of a longevity index swap for these simulations. In a nutshell we can discount and average these payoffs to achieve the price of the swap. In section 4.2 and 4.3 I discuss the pricing methodology in more detail.

It is difficult to price longevity index swaps. Bauer, Börger and Ruß (2008) compare different methods to price longevity derivatives, since no consensus has been reached in the literature. They note that mortality risk premia should be priced by the market, but that there are no liquidly traded securities. Therefore, it is not possible to rely on market data for pricing purposes. Furthermore, Bauer, Börger and Ruß (2008) conclude that it is very plausible for life annuities to include a risk premia for longevity risk. And that there is evidence that this risk premium accounts for a significant part of the amount exceeding the actuarially fair price. Bauer, Börger and Ruß (2008) derive a time series for the market price of risk within annuity quotes based on UK data. They analyse the relationship to interest rates and the stock market and find considerable correlations indicating that the independence assumption of the risk-adjusted mortality evolution and the development of the financial market may not be adequate. Furthermore they propose an option-type longevity derivative, which allows an insurer to keep the "equity tranche" of the longevity risk in the company's own books.

Cairns, Blake and Dowd (2006) introduce mortality market models equivalent to the LIBOR and swap market models in the interest rate literature. They calibrate their 2-

factor model to published pricing data of the EIB/BNP-Bond. However, Bauer, Börger and Ruß (2008) believe that relying on these data is problematic since pricing may have been an issue for its failure.

Instead of using EIB/BNP-Bond data Lin and Cox (2005) use observed annuity prices to estimate the market price of risk for annuity mortality and use the same distribution of these data to price mortality bonds. They use the Wang Transform to price longevity bonds. This method is used earlier by Blake et al. (2006b). The Wang transform appears fairly unmotivated though and does not provide a universal framework for pricing financial and insurance risks however (Pelsser, 2007). Besides, Bauer, Börger and Ruß (2008) describe that it is not clear whether annuity prices offer an adequate starting point when pricing derivatives.

Loeys, Panigirtzoglou and Ribeiro (2007) use the Sharpe ratio method to price longevity forwards and explain the method as how JP Morgan intends to use it in pricing its q-forwards. This method is based on parallels with the capital market. They note that the longevity market is net short longevity meaning that there are more agents financially hurt if longevity rises unexpectedly than agents who benefit from these unexpected rises. As a result, the mortality forward that will attract investors into the longevity market must lie below the expected mortality rate.

4.2 Monte Carlo Simulation

The first method we use to price index swaps from section 2 is plain Monte Carlo simulation. I assume there is a complete market free of arbitrage, so investors do not face extra market risk. Therefore I simulate under the risk neutral measure. Before we start with pricing the swaps, I first discuss the payoff function. The payoff of longevity index swaps depends on the realisations of the morality rates. The predetermined notional sets the magnitude of the payoff:

$$V[m(x, \text{fixed}), t] = \text{notional} \times (E[m(x, t)] - m(x, \text{fixed}))_{+}$$

$$= \text{notional} \times (\hat{m}(x, t) - BE)_{+}$$

$$= \text{notional} \times (\text{floating rate} - \text{fixed rate})_{+},$$

$$(4.1)$$

where E[m(x,t)] is the expected value, or forecasted value, of the mortality rate and m(x, fixed) the predetermined fixed mortality rate, i.e. the best estimate. The factor

 $(a)_{+}$ is the maximum of a and 0. I calculate the forecasts of the mortality rates with the Lee-Carter model as I described in section 3.2.2 and determine the best estimate by taking the x-year-old cohorts from mortality forecasts done by the Dutch Central Bureau of Statistics (CBS). This seems logical given the fact that the longevity index swaps are standardised and need to be transparent. This way the best estimate is for every Dutch party who is involved with longevity index swaps the same. Using the best estimate as the fixed rate and the forecasted mortality rates as the floating rate, we calculate the payoff for each age cohort as described above.

Now we know the payoff function of the index swaps we can go back to pricing the swaps. In section 3.2.2 we simulate the factor $\hat{k}_{T+\tau}$ from equation (3.11) 1,000 times. This way we create 1,000 sample paths of mortality rates. For each sample path we calculate the payoff for each moment in time as in equation (4.1). If we buy an index swap now at time t=0 the market value of the swap should be equal to 0. Mathematically this means:

$$MV(x) = \sum_{t=1}^{T} (p_{x,t} - c_x) \cdot DF_t = 0,$$
(4.2)

where T is the time of maturity of the swap. I set T equal to 20. $p_{x,t}$ is the payoff at time t for a person of age x according to equation (4.1) and c_x is the coupon. This coupon is constant over time. The factor DF_t is the discount factor. I use the European swap curve of April 30th, 2012 for the discount factors, shown in figure 7. If we solve this equation for the coupon, we get the coupon for which the market value of the swap is 0 at time t = 0. This coupon is the price we should pay for the swap in a certain scenario. If we calculate this coupon for all 1,000 simulation and take the average we get the price of the swap. I use the risk neutral valuation principle (Hull, 2012) to price the longevity index swap.

Later on in section 6 I do not use the difference between the forecasted and the reference mortality rate, but the change is mortality rate with respect to the best estimate. So from here on I determine the payoff of the swap not by the usual way described in equation (4.1), but like the payoff shown in equation (4.3).

$$V[m(x, \text{fixed}), t] = \text{notional} \times \left(\frac{E[m(x, t)] - m(x, \text{fixed})}{m(x, \text{fixed})}\right)_{+}$$

$$= \text{notional} \times \left(\frac{\text{floating rate - fixed rate}}{\text{fixed rate}}\right)_{+}.$$
(4.3)

In section 4.4 I discuss the results of these simulations.

4.3 Equivalent Utility Pricing Principle

Cui (2008) proposes a new method to price longevity risk premia to tackle the pricing obstacle which prevents trading of longevity-linked securities in financial markets. She considers an incomplete market setting and therefore investors have to be compensated for there additional market risk. She uses the real world measure in her method. She models the longevity risk with the Lee-Carter model (Lee and Carter, 1992) and estimates it according to UK and Dutch mortality data. The size of the risk premium depends on the payoff structure of the security due to this incompleteness. Furthermore, Cui (2008) shows that these risk premia are consistent with the limited market observations and other financial risk premia, given certain assumptions.

To price longevity-linked swaps Cui (2008) combines the equivalent utility pricing principal with the martingale approach. First, she assumes a complete financial market, with constant risk free rate r. The pricing problem is optimized from the seller's point of view. The optimization problem is as follows:

$$\max_{\{x_t, D_t\}_{t=0}^T, W_T} V_0 = E\left[\int_0^T \exp(-\delta t)u(D_t)dt + \exp(-\delta t)u(W_T)\right] \text{ s.t.}$$
 (4.4)

$$E\left[\int_0^T M_t D_t dt + M_T W_T\right] = W_0, \tag{4.5}$$

where δ is the subjective discount rate of the shareholders. The shareholder of the company derives her utility from dividends D_t and final wealth W_T . The company want to maximize the shareholder's utility by optimizing the asset allocation x_t and dividend decisions. The initial equity capital of the company is given by W_0 . The per-period

utility u(.) is defined as:

$$u(S) = -\frac{1}{\alpha(W_0)} \exp\left(-\alpha(W_0)S\right),\tag{4.6}$$

where S is the survival index and α the parameter of risk aversion dependent on the initial wealth, $\alpha(W_0) = \bar{\alpha}W_0^{-b}, b \in [0, 1]$. In equitation (4.5) M_t is the pricing kernel for the complete financial market. Cui (2008) uses for this kernel:

$$\frac{dM_t}{M_t} = -rdt - \lambda dZ_t. \tag{4.7}$$

The problem in equation (4.4) concerns a situation without longevity risk. Cui (2008) describes that since longevity risk is unhedgeable from financial markets at the time of writing, which is 2008, the company derives her utility from dividends and the residual claim from the longevity risk, $E(S_t) - S_t$. Therefore, a risk loading π is added to the initial wealth. The optimization problem changes to:

$$\max_{\{x_t, D_t\}_{t=0}^T, W_T} V_0^{\pi} = E \left[\int_0^T \exp(-\delta t) u \left(D_t^{\pi} + E[S_t] - S_t \right) dt + \exp(-\delta t) u(W_T^{\pi}) \right] \text{ s.t.}$$

$$E \left[\int_0^T M_t D_t^{\pi} dt + M_T W_T^{\pi} \right] = W_0 + \pi.$$
(4.8)

Now, Cui (2008) applies the equivalent utility pricing principle and determines the minimum value for π such that the company is indifferent from bearing longevity risk or not:

$$V_0 = V_0^{\pi}. (4.10)$$

Before Cui (2008) comes to this set-up she first discusses a few pricing principles with respect to longevity risk and some notations. If N denotes the initial size of the x-year-old cohort at time zero and tp_x the survival probability of the x-year-old over t years then the number of survivors in this cohort in year t is given by:

$$S_t = N_t p_x, (4.11)$$

with mean $E[S_t]$. S_t is calculated by means of simulation, for the most part the same way I described in section 3.2.2. In this previous section I forecast and simulate 1,000

sample paths for \hat{k}_t . With these simulations the force of mortality is calculated by:

$$\ln m(x+i,t+i) = a_{x+i} + b_{x+i}\hat{k}_{t+i}, \text{ for } i = 1,\dots,T.$$
(4.12)

The next step is to compute the survival probability of the x-year-old cohort:

$$_{t}p_{x} = \exp\left(-\sum_{i=1}^{\tau} m(x+i, t+i)\right), \text{ for } \tau = 1, \dots, T.$$
 (4.13)

Cui (2008) discusses that the longevity market is certainly no perfect market. Because of market imperfections, a risk premium should be added to the price of longevity swaps to compensate for the risk of the issuer. Let P^- denote the minimum risk loading which should compensate an investor for bearing longevity risk $S_t - E[S_t]$. And if U(.) is the utility function of the investor we have:

$$U(W_t) = E \left[U(W_t + E[S_t] + P^- - S_T) \right], \tag{4.14}$$

with W_t the wealth invested by the investor. Cui (2008) assumes the risk free rate to be zero, therefore $W_0 = W_T$. Now we assume that the investor has preferences $U(w) = -\frac{1}{\alpha} \exp(-\alpha w)$, with α the coefficient of absolute risk aversion. We can now rewrite equation (4.14) to:

$$E\left[U(W_t + E[S_t] + P^- - S_T)\right] = -\frac{1}{\alpha}\exp(-\alpha W_t)$$
(4.15)

and we can write the minimum risk loading as:

$$P^{-} = \frac{1}{\alpha} \ln E \left[\exp \left(\alpha (E[S_t] - S_t) \right) \right]$$

= 1 + \frac{1}{\alpha} \ln E \left[\exp \left(E[S_t] - S_t \right) \right]. (4.16)

The risk premium, denoted by R_p that has to be paid to the investor on top of the risk free rate can now be expressed in terms of the minimum risk loading:

$$(1 + R_a + R_p)^{-t} = \frac{E[S_t] + P^-}{N}, \tag{4.17}$$

where R_a is the actuarial discount rate defined by:

$$(1+R_a)^{-t} = \frac{E[S_t]}{N}. (4.18)$$

I calculate the payoff again using equation (4.1). However, instead of using the forecasted mortality rates from the Lee-Carter model, Cui (2008) dictates to use S_t , which is in fact calculated through the Lee-Carter model, as the floating rate in the payoff function. She also writes to use $E[S_t]$ as the fixed rate. Cui (2008) does not further concern about pricing longevity swaps. I will now use the same method as described in section 4.2, but with adapted floating rates. I use equation (4.2) to solve for $p_{x,t}$ to get the price of each sample path. Furthermore, before discounting I add the risk premium R_p to compensate the investor for bearing longevity risk. By discounting the payoff and the risk premium I calculate the price of the swaps. We get the price by taking the average over all 1,000 sample paths. For discounting I use besides the risk premium again the European swap curve shown in figure 7 as discount factors.

4.4 Results

In this section I describe the pricing results of the simulation method described in section 4.2 and the equivalent utility pricing principle of section 4.3.

Figure 5 shows the best estimates of mortality rates for a few age cohorts of Dutch men. In our data set we set the maximum attainable age to 99 years old there, implying that the mortality rate of a 80-years-old over 20 years is 100%. We simulated and forecasted mortality rates with the linear regression model in equation (3.2), in which the parameters were estimated with mortality rates of the Dutch male population and you can find these in figure 2 and appendix A.1. The best estimates of mortality rates serve as the fixed rates in the payoff function (4.3).

We see that for every age cohort mortality rates keep increasing with time and that the rates increase faster the older we get. But we can also note that mortality rates for younger men increase slower relative to men who were of the same age a couple of years ago. To show this more clearly we plot in figure 6 the best estimates of 20 year old men and 40 year old men together. The graph of the 20 year old cohort starts for men from this cohort who have turned 40 already. So the mortality rates at year zero both represent 40-year-old men, for which the dotted line represents men from the 40-year-old

cohort and the other line men from the 20-year-old cohort. You can see that the dotted line is higher than the fluent line, which means that a man of 40 years old has a higher probability of dying than a 20 year old man will have in 20 years. This shows that life expectancy is increasing.

Figure 5: The figure shows the best estimates for the 20, 40, 60 and 80-years-old cohort, respectively. The first plot gives the probabilities for a 20-year-old of dying the next year until the probability that he dies in 50 years from now. In our data set 99 years old is the maximum attainable age. Therefore, after 20 years the probability of dying for an 80-year-old is 100%. We use the linear regression model in equation (3.2) to simulate and forecast mortality rates, in which the parameters of the model are estimates with mortality rates of the Dutch male population. From the forecasted mortality rates we calculate the best estimate.

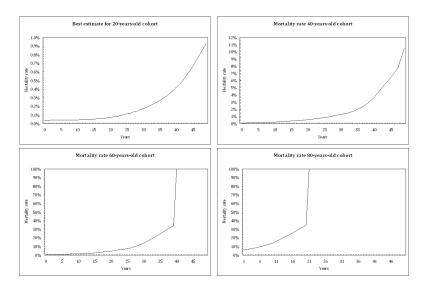


Figure 7 shows the European swap curve of April 30th, 2012, which I use for the discount factors for calculating the market value of the swap in equation (4.2). I assume this curve to be constant over time, which means that I use the same discount factors at time t as I do at time $t + \tau$. The data for the swap curve are provided by Nationale-Nederlanden.

For each age I calculate the coupon by solving equation (4.2) so the market value of the swap at time t=0 is equal to 0 given the payoff structure of the swap and do this for all 1,000 sample paths I simulated in section 3.2.2. By taking the average of the 1,000 coupons I calculate the price of the swap. Figure 8 shows these prices for some age cohorts. The left plot shows the prices calculated with the simulation method described in section 4.2. The right panel the prices calculated with pricing principle of Cui (2008). Both methods use equation (4.1) as the payoff function, setting the notional equal to 1

Figure 6: This figure shows the 20 and 40 years-old cohort. The 20-years curve shows the mortality rates for 20-year-olds who reach the age 40. You can see that a man who is 40 years now has a higher probability of dying than a 20 year old man will have in 20 years from now. This also shows that life expectancy is increasing.

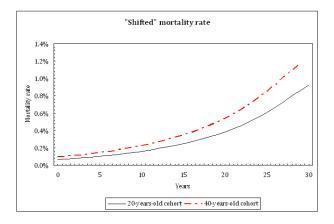
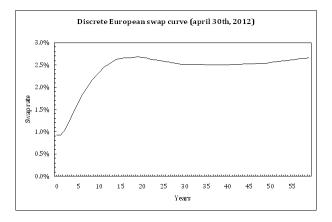
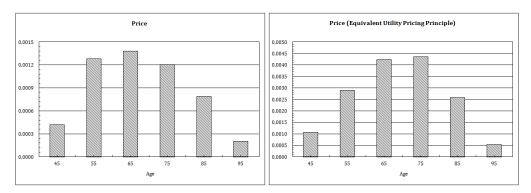


Figure 7: The graph below shows the European swap curve of April 30th, 2012. I use these swap rates as discount factors in my calculations, e.g. in equation (4.2).



and are based on the mortality rates of the Dutch male population.

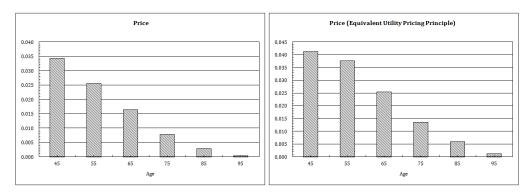
Figure 8: This figure shows the prices of a longevity index swap, indexed on forecasted mortality rates using the regression model in equation (3.2). The model is estimated on Dutch mortality data ranging from 1950 to 2010 provided by the Dutch Central Bureau of Statistics. The time of maturity of this longevity swap is 20 years. The left plot gives the prices calculated using Monte Carlo simulations as described in section 4.2 for wicht the payoff is calculated according to equation (4.1). The prices shown in the left plot are calculated as discribed in section 4.3, also using the payoff function in equation (4.1). For the calculation of the prices I use a notional of one hundred units and a cohort size of one person. To calculate the price I take the average of 1,000 coupons calculated by solving equation (4.2) so the market value of the swap is 0 in each sample path. Both methods show the same pattern for the prices with respect to age. The price calculated with the equivalent utility pricing principle is higher due to the added risk premium for the incomplete market which is considered.



In section 4.2 I "converted" the prices in such a way that the payoff was determined by taking the change of the floating rate with respect to the fixed rate. Figure 9 shows the same plots as in figure 8, but now with the payoffs calculated as in equation (4.3). The left plot shows the prices of section 4.2 for which right plot presents again the results in case I use the equivalent utility pricing principle. Like before, we see the same patterns in both methods and see clearly higher prices for younger men. After men get older longevity swap become cheaper. This makes sense since the risk that of people living longer than expected is lower for older people than for younger people. For insurance companies the need to hedge longevity risk for an 80-year-old man is of much less importance than hedging for a 40-year-old men. The prices of the equivalent utility pricing principle from section 4.3 are higher than those of the Monte Carlo simulation method from section 4.2. This makes sence, since in this method we added a risk premium for the risk investors bear. From here on I continue with the first method for the rest of the research.

In the next section I continue to describe the Solvency II framework for more perspective on the matter of longevity risk. After that I continue with the discussion of

Figure 9: This figure shows the prices of a longevity index swap, indexed on forecasted mortality rates using the Lee-Carter model. The model is estimated on Dutch mortality data ranging from 1950 to 2010 provided by the Dutch Central Bureau of Statistics. The time of maturity of this longevity swap is 20 years. The left plot shows the prices calculated using Monte Carlo simulations as described in section 4.2, the right plot shows the prices calculated using the equivalent utility pricing principle described in section 4.3. The payoff is calculated according to equation (4.3). For the calculation of the prices I use a notional of one and a cohort size of one person. Hedging longevity risk for 40-year-old men is more expensive due to higher longevity risk. The prices in the right plot are higher due to the addition of a risk premium as a compensation for risk investors bear.



hedging longevity risk in section 6 using the pricing method from section 4.2.

5 Solvency II

Before we go to the topic hedging I first discuss the Solvency II regulations briefly. It is not necessary to have a full understanding of the Solvency II regulations and thus I discuss only the most relevant topics. In the section 6 I construct the balance sheets according to Solvency II without and with hedging longevity risk. Based on the differences in these balance sheets I conclude if the hedge is effective. For those who are not interested in the Solvency II regulations at all, could only read sections 5.2.1 and 5.2.3, skipping the rest in this section, without losing the concept of the next section and conclusion.

Solvency II is developed by the European authorities to regulate pension funds and (re)insurance companies. In a nutshell Solvency II dictates insurers to value their liabilities at market value. For insurers it is important to improve their risk position with respect to Solvency II. One way to do so is to search for better ways of hedging. Longevity risk is traditionally seen as a non-hedgeable risk. According to Solvency II insurers have to hold a certain amount of cash to cover for longevity risk. Longevity index swaps make non-hedgeable longevity risk hedgeable and provide possibly a better

alternative for longevity risk than the capital to hold dictated by Solvency II.

In section 5.1 I discuss the Solvency II regulations in more detail but I still stay at the surface. In section 5.2 I discuss the most important aspects of the market value balance sheet containing the solvency capital requirements, best estimate and the risk margin respectively in sections 5.2.1, 5.2.2 and 5.2.3. Once we have a clear understanding of these topics we continue with the hedging part in the next section.

5.1 General introduction: Three Pillars

The Solvency II regulations, or short Solvency II, form a supervisory framework for insurers, reinsurers and pension funds in Europe (PWC, 2012). The reason for this new framework is to make supervisory requirements more realistic with respect to the risks faced by these companies. Solvency II requires from insurers to satisfy stricter standards concerning capital, risk management and reporting. These supervisory requirements are classified into three pillars. Table 1 shows the three pillars from Solvency II. The first pillar concerns the quantitative requirements of a company and pillar two the qualitative requirements. The last pillar captures the disclosure requirements and is important for reporting to the authorities and providing transparency of the company.

Table 1: Solvency II regulations can be classified into three pillars. European insurers must satisfy this new supervisory framework in the beginning of 2014. (source: PWC (2012)).

	Solvency II		
Pillar 1: Quantitative Requirements	Pillar 2: Supervisory Review	Pillar 3: Disclosure Requirements	
 Market-consistent valuation of assets and liabilities Solvency requirements through Standard model, or (Partial) Intern model 	 Own Risk and Solvency Assessment (ORSA) Corporate governance 	 Transparency of solvency and financial conditions of the corporation Solvency and Financial Condition Report Regular Supervisory Report Quantitative Reporting Template 	
		- ORSA Report	

We are interested in pillar one, the quantitative requirements. This pillar dictates the amount of capital an insurance company has to hold to stay solvent in the future. In Table 1 you see the requirements for the assets and liabilities are noted. Contrary to former regulations assets and liabilities need to be valued market-consistently. In this pillar Solvency II also provides a standard model which companies can use to estimate how much capital they should hold to stay solvent. This model is the same for every company and is therefore very general, which results in very strict assumptions. Every insurer is allowed to construct its own model more specific to its situation and therefore probably more efficient. For which longevity risk is unhedgeable in the standard model it could be hedgeable in a company's specific model.

Due to hedging longevity risk we expect the solvency capital requirement and the risk margin to decrease (see sections 5.2.1 and 5.2.3 for more details) and the hedging costs to increase. In section 6 we apply a hedging strategy and look if it is beneficial.

5.2 Market Value Balance Sheet

One of the main aspects for insurers is the new method to value their assets and liabilities and the calculation of the solvency capital requirements (see section 5.2.1). Up until now insurers discounted their liabilities using a constant discount factor of roughly 3% or 4%. According to Solvency II regulations insurers have to value their liabilities on a market-consistent basis. This implies that instead of a predetermined constant discount factor insurers must use e.g. the triple A European swap curve to discount their liabilities. Of course, this also holds for the assets, but since I focus on pricing and hedging longevity risk I will only pay attention to the liability side of the balance sheet. Table 2 gives a stylised presentation of the balance sheet of a insurance company, in which the assets and liabilities are valued on a market-consistent basis. The two parts on the liability side represent the non-hedgeable and the hedgeable risks. In sections 5.2.1, 5.2.2 and 5.2.3 I explain the valuation of the liabilities in more detail.

5.2.1 Solvency Capital Requirements

The required capital in table 2 consists of the solvency capital requirements. In the directive of the European Parliament and the Council (2009), i.e. the binding framework for Solvency II, the following definition of the solvency capital requirements is stated:

...the Solvency Capital Requirements "shall correspond to the Value-at-

Risk of the basis own funds of an insurance or reinsurance undertaking subject to a confidence level of 99.5% over a one-year period".

Put differently, the solvency capital requirements should reflect all quantifiable risks a firm might face. The solvency capital requirements are:

$$SCR = VaR_{\alpha}(h), \tag{5.1}$$

where α is the significance level of 0.5% and h the horizon of one year. Or, we want to determine the smallest amount of capital in such a way that the following holds:

$$Pr[AC(t+1) > 0|AC(t) = x] \ge 0.995,$$
 (5.2)

where AC(t) is the available capital at time t. Insurers should calculate their solvency capital requirements at least once a year.

5.2.2 Best Estimate

On the stylised balance sheet showed in table 2 the best estimate and the risk margin form together the market value of the liabilities. The best estimate should be equal to the probability weighted average of the present value of future cash flows for current obligations taking into account all up-to-date financial market and actuarial information. The best estimate represents only the hedgeable market risks though. For example, by constructing a replicating portfolio to replicate the cash flows of the insurance liabilities the best estimate can be estimated.

Table 2: Stylized balance sheet for an insurance company, in which assets and liabilities are valued on a market-consistent basis. The liabilities consist of to parts: the technical provisions represent all hedgeable risks, the available capital represents the non-hedgeable risks.

Assets	Liabilities
Investments	Available Capital
	• Required capital
	• Free surplus
	Technical provisions
	• Best Estimate
	• Risk Margin

5.2.3 Risk Margin and Cost-of-Capital

For the non-hedgeable risks insurers have to add a risk margin to the technical provisions on the balance sheet. Solvency II dictates that insurers can calculate this risk margin using the Cost-of-Capital approach. According to the Solvency II Directive (European Parliament and the Council, 2009) the cost of holding the solvency capital requirements is assumed to attract a premium over the risk-free interest rate, which is called the Cost-of-Capital. Keller (2006) defines the Cost-of-Capital as the cost of future regulatory capital for the entire run-off of a portfolio. Solvency II states that the Cost-of-Capital is equal to 6%, although there are no scientific methods or models on which this percentage is based.

The risk margin is now calculated by summing the discounted product of the net present values of the expected solvency capital requirements at time t=0 to infinity and the Cost-of-Capital:

$$RM = \sum_{t=0}^{\infty} CoC \cdot SCR_t \cdot DF_t. \tag{5.3}$$

In this equation the SCR are the solvency capital requirements, CoC is the Cost-of-Capital and DF_t are the discount factors. We take the sum to infinity, in other words until everyone in the portfolio has died. We calculate the solvency capital requirements for every year, for which the cashflows of the years before disappear. We use the European swap curve of figure 7 as discount factors.

6 Hedging Longevity Risk

In this section I discuss a hedging strategy in which I use the longevity index swaps I discussed in section 2 with the prices from section 4. The strategy is proposed by Deutsche Bank in which Deutsche Bank could be the counterparty. In recent literature only static hedging is considered with respect to longevity risk. Due to the illiquidity of the life market it is difficult to buy or sell longevity related products. If an insurer wants to hedge its longevity risk it applies a static hedge to avoid the difficulty of rebalancing every time period. Therefore I will also focus on static hedging. Longevity risk is traditionally seen as a non-hedgeable risk. In section 5 I discussed that this implies that the insurer needs to include longevity risk in the solvency capital requirements

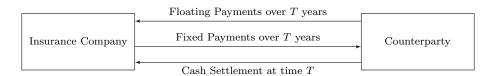
and indirectly in the risk margin on the market value balance sheet. If we assume that longevity risk is hedgeable with longevity index swaps though, then we expect the solvency capital requirements and the risk margin to become lower. We do want of course that the reduction in the risk margin and the solvency capital requirements compensates for the hedging costs.

6.1 Corridor Hedge

AEGON closed in 2011 a longevity risk deal with Deutsche Bank, a so-called corridor hedge. In this section I analyse this hedging strategy and look if it is beneficial. In section 4 I discussed the pricing of a longevity index swap. In this section I continue with this swap in the corridor hedge.

The swap deal is for the most part like the swap described in section 2, minding a few differences. Figure 10 gives a graphical representation of the longevity index swap presented by Deutsche Bank. The hedge consists of two parts: yearly cashflows to hedge the longevity risk from years t=1 until T. And a cash settlement from the counterparty to the insurer so that the insurer is hedged against longevity risk after year T. The cash settlement is paid depending on the remaining liabilities projected by the longevity risk at the end of the contract. The floating payment periodically from Deutsche Bank, from now on named the counterparty, to the insurance company is associated with the realised mortality rate of the reference index. In this case the reference index are the mortality rates of the Dutch male population, provided by the Dutch Central Bureau of Statistics. The insurer pays periodically a predetermined fixed amount to the counterparty.

Figure 10: This figure gives a graphical representation of the longevity index swap deal presented by Deutsche Bank. During the time of the contract the insurance company pays the counterparty a predetermined fixed amount in order to secure its future outgoing cash flows. The counterparty pays on its term a floating amount to the insurer, depending on the movement of the mortality rates. At the end of the contract a cash settlement is paid depending on the mortality rates in te future.



The corridor structure of this hedge is established by a protection floor and ceiling. The counterparty only pays the insurer in case the mortality rates decrease more than 10% with respect to the initial best estimate in that year. If the mortality rates decrease

more than 20% the counterparty pays a maximum up to that 20%. So the payments of this deal are capped and floored at different levels of mortality stresses. Figure 11 shows a stylised example of a payoff profile in which the payoff is capped and floored.

Figure 11: This figure shows an example of a payoff profile of the corridor hedge presented by Deutsche Bank.



6.1.1 Pricing according to the Corridor Hedge

In section 4 we calculated the price of a longevity index swap using the change in mortality rate with respect to the best estimate as the payoff function (recall equations (4.1) and (4.3)). Deutsche Bank proposed a corridor structure in which stresses in the mortality rate should be a minimum of 10% for the swap to pay out. Additionally, the swap pays out up to a maximum of a 20% stress. Therefore the payoff function changes to:

$$V[m(x, \text{fixed}), t] = \text{notional} \times \min[X, 20\%] \cdot I_{X > 10\%}, \tag{6.1}$$

where

$$X = -\frac{E[m(x,t)] - m(x, \text{fixed})}{m(x, \text{fixed})}$$
$$= -\frac{\text{floating rate - fixed rate}}{\text{fixed rate}}$$

and I_A is an indicator function with the value 1 if A is true and 0 otherwise.

One other aspect of the pricing of the swap changes in this new setting. In section 4.2 I calculated the prices by solving equation (4.2) for the coupon. I summed there from t = 1 until T. Since the counterparty of the longevity index swap deal we are now dealing with pays a cash settlement at the end of the contract to the insurer, we calculate

the price the same way as we did in equation (4.2), but now we sum until infinity.

Using these new prices for the longevity index swaps we can hedge against longevity risk and research if this is profitable according to Solvency II.

6.1.2 Initial Market Value of Liabilities

Now we want to know what the solvency capital requirements and risk margin are before we hedge in order to see if the situation improves in case we do hedge longevity risk. To calculate the solvency capital requirements in the situation in which we do not buy any swap I discount the forecasted mortality rates for each age multiplied by the cashflows the insurer needs to pay to her clients until the day they die. The mortality rates are estimated and forecasted with the regression model in equation (3.2) as I described in section 3. And just as in section 4 I use the European swap curve of April 30th, 2012 for discount factors. I set the cashflows equal to 1. To obtain the net present value I sum up all these discounted payoffs:

$$NPV(x) = \sum_{t=1}^{\infty} \left[CF \cdot \prod_{i=1}^{t} (1 - \hat{m}(x+i-1,i)) \cdot DF(t) \right],$$
 (6.2)

where NPV(x) is the net present value for the cohort of age x, CF is the insured cash flow at time t, $\hat{m}(x,t)$ is the forecasted mortality rate for the cohort of age x at time t and DF_t is the discount factor at time t. The cash flow has no subscript since I assume it to be constant over time. The summation goes to infinity, which means that the insurers pays until everyone in a certain age cohort has died. Obviously, the cash flows need to be paid to clients who are still alive, therefore we calculate the probability of the client being alive by subtracting 1 by the mortality rate. In appendix A.2 you can read a more detailed explanation of the calculation of the net present value.

By calculating the net present value for all simulations for a certain age I obtain a distribution for the net present value for this age. The 99.5% quantile ($\alpha = 0.005$) is the capital an insurer should hold in order to fulfil its future liabilities. Part of this sum is the best estimate (see section 5.2.2), the remaining part are the solvency capital requirements (see section 5.2.1). I already described the calculation of the risk margin in section 5.2.3 (see equation (5.3)). In section 6.2 I discuss the results of the prices and the initial solvency capital requirements and risk margin.

6.1.3 Hedged Market Value of Liabilities

Now we want to hedge longevity risk by buying longevity index swaps. By hedging longevity risk an extra term is added to the solvency capital requirements because of the swap. As I describe in equation (6.2) the value of the solvency capital requirements are calculated by summing the discounted future cash flows the insurer has to pay to the client until the client dies. The extra term that is added to this summation represents the swap. If we buy the swap now at time t=0 the extra term is added for the next 20 years, since the swap we consider has a time to maturity of 20 years. We modify equation (6.2) to

$$NPV_{\text{swap}}(x) = \sum_{t=1}^{\infty} \left[MV_{\text{swap}}(x,t) + CF \cdot \prod_{i=1}^{t} (1 - \hat{m}(x+i-1,i)) \cdot DF(t) \right],$$
 (6.3)

where $MV_{\text{swap}}(x,t)$ is the market value of the swap at time t for age x. Like before, if we calculate the net present value for all simulations take the 99.5% quantile we get the solvency capital requirements.

In section 6.1.1 we calculated the price, which includes also the cash settlement, with payoff function (6.1) and equation (4.2). The hedging costs can now simply be calculated by discounting the price and sum the discounted prices for the time of the swap:

Hedging Costs
$$(x) = \sum_{t=T+1}^{\infty} c_x \cdot DF_t,$$
 (6.4)

where c_x is the price of the swap for hedging the longevity risk of a person of age x. If we sum up the hedging costs with the solvency capital requirements and risk margin after hedging, we want that this sum is lower than the initial solvency capital requirements and risk margin in order for the hedge to be profitable.

6.2 Results

In this section I cover the hedging results. Before I start with these results I come back to some pricing results. In section 4 I used the payoff function given in equation (4.3). In section 6 I analyse a swap proposed by Deutche Bank, in which I change the payoff function to equation (6.1). For pricing the corridor swap I still use the mortality rates of the Dutch male population provided by the Dutch Central Bureau of Statistics. We

simulate mortality rates with the regression model in equation (3.2) in which we draw sample path for k_T using equation (3.11). To calculate the price we want the market value in equation (4.2) to be 0 at time t = 0. So for every sample path we calculate the coupon by solving the sum in (4.2), for which we take the sum from 1 to infinity, and take the average over all 1,000 coupons to get the price.

Figure 12 shows the prices for several ages calculated using the corridor payoff function (6.1). Due to the corridor structure of the payoff function, the longevity index swap only generates a payoff if the change of the mortality rate with respect to the best estimate is between 10% and 20%. This results often in no payoff. Therefore prices should be lower than we calculated in section 4. In the figure we see roughly the same pattern as in figure 9. The younger we are the more longevity risk we bring and thus the more expensive the swaps are. Besides the pattern we also see the prices are lower, as we expected. In fact, the prices for swaps for higher ages are (almost) 0. This means that the mortality rates hardly deviate from the best estimate, at least not more than 10%. For insurers the implies longevity risk for high ages, in this case for Dutch men, is not enough to hedge. For ages under 65 years it is interesting for insurers to hedge longevity risk.

Figure 12: This figure shows the prices of longevity index swaps calculated with Monte Carlo simulation using the regression model in equation (3.2). We simulate mortality rates according to the model in which we draw sample paths using equation (3.11). For actual pricing we solve for the coupon c in equation (4.2) and want the market value at time t=0 to be 0. If we take the average over all coupons we get the price. We use equation (6.1) as the payoff function of the swap. This means that we only get a payoff from the swap if mortality rates drop more than 10% relative to the best estimate up to a maximum of 20%. I calculate the price with a notional of 1 Euro.

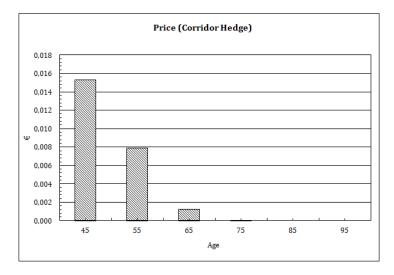
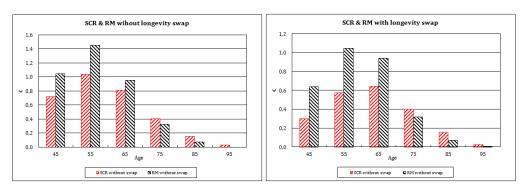


Figure 13 shows the solvency capital requirements and risk margins for stylised portfolios of middle-aged and older men. I calculate the solvency capital requirements by taking the 99.5% quantile of the net present values of the liabilities. The net present values I calculate with equations (6.2) and (6.3) for all 1,000 sample paths of the simulations from section 3. In the each plot the left histogram shows the solvency capital requirements, for which the right histogram shows the resulting Risk Margins. Both the hedged case and the non-hedged case are displayed.

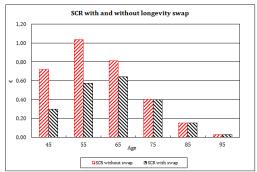
Figure 13: This figure shows the solvency capital requirements for a few ages. With the simulated mortality rates from section 3 I calculate for each sample path the net present value of the liabilities for each age cohort as in equations (6.2) and (6.3). The solvency capital requirements is calculated as the 99.5% quantile of the 1,000 net present values of each cohort. I calculate the risk margin with equation (5.3). In the left plot the left histogram represents the solvency capital requirements, the right histogram is shows the resulting risk margin. The left panel shows these statistics in case no longevity index swaps are bought to hedge longevity risk. The right panel shows the same statistics in the hedged case.



To make the numbers more easy to compare, the left panel in figure 14 displays both solvency capital requirements next to each other. We see that the solvency capital requirements decrease due to our hedging strategy. And as a result the risk margins decrease as well (see the right panel in the same figure). The differences are bigger the younger men are. The makes sense, since a man of 45 years old has a higher probability of living longer than expected compared to a man of 85 years old. Additionally, we tend to underestimate mortality rate forecasts, which increases the risks on the long term further. You can see in 14 that the capital requirements and the risk margins do not change for men of 85 and 95 years old and hardly for 75-year-olds. Table 3 shows the decreases due to the hedge in percentage. The only question that remains is if the reduction in solvency capital requirements and risk margin enough to outweigh the hedging costs.

Figure 15 shows the solvency capital requirements and risk margin in case of hedging

Figure 14: This figure shows the solvency capital requirements with and without swap next to each other and the same for the risk margin. The numbers are the same as in figure 13 For all ages hold that the solvency capital requirements and the resulting risk margin do not increase if we hedge longevity risk.



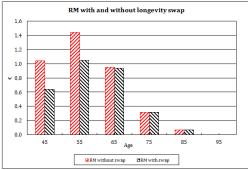
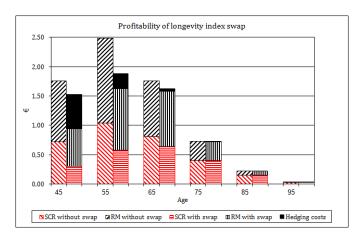


Table 3: This table shows the change in solvency capital requirements and risk margin in case we hedge longevity risk with respect to the case in which we do not hedge. For all ages hold that the solvency capital requirements and risk margin decreases or at least stay the same due to the hedge.

Age	Change in SCR	Change in Risk Margin
45	-58.2%	-38.7%
55	-44.7%	-27.5%
65	-20.8%	-1.3%
75	-1.5%	-0.8%
85	0.0%	0.0%
95	0.0%	0.0%

Figure 15: This figure show the sum of the solvency capital requirements and the risk margin in the left histogram and the sum of the solvency capital requirements, risk margin and hedging costs in the right histogram. The capital requirements and the risk margins are the same as in figure 13, for which the hedging costs are calculated using equation (6.4). The left histogram showes the situation in case no index swaps are bought in order to hedge against longevity risk.



and not hedging, including the hedging costs. The capital requirements, risk margin and hedging costs are summed. We calculate the costs with equation (6.4). You can see that the total liabilities are higher if we do not hedge longevity risk, which means it is beneficial for a insurer to hedge against longevity risk with index swap, given the payoff structure from equation (6.1). Table 4 presents the exact numbers which belong to the figure. We see also in this table that the corridor hedge does not make a difference for ages higher than 75 years. For younger ages however, the hedging strategy reduces the solvency capital requirements and risk margin enough to outweigh the costs.

Table 4: This table gives the numbers of the solvency capital requirements, risk margins and hedging costs on which figure 15 is based. The capital requirements are calculated with equation (6.2) or (6.3), depending on hedging or not. The risk margins are calculated with equation (5.3) and the hedging costs with (6.4).

		No Swap			With Swap					
Age	SCR	Risk Margin	Sum	Age	SCR	Risk Margin	Costs	Sum		
45	0.720	1.043	1.763	45	0.301	0.639	0.584	1.525		
55	1.039	1.446	2.485	55	0.575	1.048	0.262	1.884		
65	0.811	0.952	1.763	65	0.643	0.940	0.041	1.648		
75	0.405	0.321	0.726	75	0.399	0.318	0.002	0.719		
85	0.153	0.071	0.223	85	0.153	0.071	0.000	0.223		
95	0.026	0.005	0.032	95	0.026	0.005	0.000	0.032		

So, given the payoff function in (6.1), the longevity index swap proposed by Deutsche Bank provides a beneficial opportunity for insurance companies to hedge longevity risk.

By buying this swap they can protect themselves against the risk that people get older than expected. In context of Solvency II this means insurers need to hold less capital as a buffer for non-hedgeable risks. The reduction in solvency capital requirements and risk margin outweighted in our case the costs of hedging longevity risk. Prices of longevity index swaps are higher the younger people are, however the benefits of this corridor hedge decrease if the age of the concerning cohort become higher.

7 Conclusion

The main focus in this thesis is to provide a complete framework of pricing and hedging longevity risk with longevity index swaps. These index swaps are hardly traded in the early 2010s and there is no liquid market whatsoever. Therefore I work with a stylised portfolio and simulated data.

I start to model longevity with Dutch male mortality rates from 1950 until 2010. With the Lee-Carter model model I forecast the mortality rates and construct 1,000 sample paths by means of Monte Carlo simulations. For each sample path I continue by calculating the payoff of the longevity index swap by looking at the change in mortality rate with respect to a best estimate. This best estimate is a the average prognosis of the mortality rates over a certain horizon specified for each age. The index swap pays out in case the forecasted mortality rate becomes smaller than the best estimate. In this case the clients of an insurer live longer than expected, which means that the liabilities of the insurer are higher than anticipated on beforehand. Since the market value of the swap is supposed to be 0 at time t=0 we can calculate the fixed coupon that we need to pay at time t=0 given the discounted payoffs. We calculate the price by taking the average of the coupons of the 1,000 sample paths.

Next, I analyse a corridor hedge, for which the payoff structure of the index swap is slightly changed. The payoff is still determined by the change in mortality rate with respect to the best estimate. However, this change is floored at 10% and capped at 20%, meaning that the payoff is 0 if the change is smaller than 10% and the index swap pays only up to 20% if the change is larger than 20%.

Given this payoff function I hedge against longevity risk in order to lower the solvency capital requirements and the resulting risk margin. For this hedge to be profitable the decrease in solvency capital requirements and risk margin needs to be larger than the involving hedging costs.

I find that the solvency capital requirements in and risk margin decrease if we hedge longevity risk and that the costs which are involved with the longevity index swaps are lower than this decrease. Using this payoff structure it is not possible to hedge against longevity risk for higher ages (85 years or older). The mortality rates never drop more than 10%, which means that the index swap never generates any payoff.

For younger men holds that the benefits of the hedge outweigh the hedging costs, though the magnitude of the benefits still decreases as age increases. If a liquid Life Market would arise and longevity risk will become a hedgeable risk, insurers will be able to hedge longevity risk using standardised longevity index swaps and be able to hold less capital for future liabilities and risk.

8 Further Research

This research can be extended in a number of directions and details to perfect. A disadvantage, for example, of index swaps is the arising basis risk. Basis risk arises due to differences in the calibration population and the reference population. An insurer could have a population with different characteristics from the national population, e.g. relatively less smokers. Smokers tend to live shorter on average than non-smokers, so an insurer with less smokers could be exposed to more longevity risk than anticipated on when she hedges against longevity risk with index swaps. In case of index swaps it is important for insurers and pension funds to be able to quantify and manage this basis risk and preferably reduce it to a minimum.

Several studies indicate that the two most important drivers of variations in mortality experience over time are age and gender. Loeys, Panigirtzoglou and Ribeiro (2007) say that a market in longevity must at least differentiate by these two drivers in order to be successful. Mortality rates are actually published separately for men and women and differentiate by age. Due to these differentiations differences in index and reference populations are already small and basis risk is already greatly reduced. On the other hand Coughlan et al. (2011) discuss the significance of basis risk analysis. Basis risk is something that could be taken into account in future research.

Furthermore, I use a stylised portfolio for my research, while it may be more concrete and informative to include a portfolio of an insurer actually used in practice. However, since my focus was to provide a framework and more understanding of the product, it was did not suit my needs. In the future the implementation of a more realistic portfolio could be taken into account.

Another point of attention is the forecasting model. I use the standard Lee-Carter model, while there are several additions made to this model. The Lee-Carter model is however still the most widely used by far and therefore more suited to compare with other researches. To perfect this thesis a more sophisticated forecast model could be implemented. Furthermore, in this context, I could extrapolate my data on which I fit my model. The data from the Central Bureau of Statistics ranges from ages 1 to 99 years old. It is not uncommon to extrapolate these data to the age of 120, since every year more people pass the 100 years of age.

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Appendix

A.1 Estimates of a_x , b_x and k_t

Table 5: This table shows the esimates for a_x , b_x and k_t from the regression model in equation (3.2). The values are estimates with a singular value decomposition I described in section 3. The estimates of k_t are the second stage estimated according to the Lee-Carter model from section 3.

Age	ax	bx	Age	ax	bx	Year	kt	Year	kt
1	-6.371	0.198	51	-5.274	0.075	1950	4.114	1981	0.397
2	-7.184	0.214	52	-5.171	0.073	1951	4.151	1982	0.356
3	-7.448	0.220	53	-5.065	0.075	1952	3.496	1983	0.021
4	-7.700	0.230	54	-4.957	0.080	1953	4.371	1984	0.006
5	-7.914	0.248	55	-4.852	0.076	1954	3.211	1985	-0.633
6	-7.950	0.238	56	-4.751	0.075	1955	3.202	1986	-0.501
7	-8.047	0.236	57	-4.643	0.074	1956	2.986	1987	-1.121
8	-8.149	0.223	58	-4.533	0.075	1957	3.170	1988	-0.859
9	-8.240	0.228	59	-4.444	0.075	1958	2.861	1989	-1.039
10	-8.305	0.212	60	-4.334	0.072	1959	3.470	1990	-0.945
11	-8.342	0.201	61	-4.239	0.072	1960	2.819	1991	-1.574
12	-8.274	0.163	62	-4.137	0.069	1961	2.822	1992	-1.297
13	-8.183	0.158	63	-4.034	0.068	1962	3.072	1993	-1.491
14	-8.086	0.144	64	-3.940	0.063	1963	2.922	1994	-1.970
15	-7.990	0.157	65	-3.839	0.062	1964	3.075	1995	-1.725
16	-7.622	0.127	66	-3.739	0.061	1965	2.801	1996	-2.161
17	-7.420	0.112	67	-3.646	0.060	1966	3.061	1997	-2.440
18	-7.301	0.104	68	-3.546	0.057	1967	3.154	1998	-2.680
19	-7.166	0.105	69	-3.451	0.056	1968	2.982	1999	-2.580
20	-7.132	0.087	70	-3.365	0.053	1969	2.833	2000	-2.886
21	-7.115	0.086	71	-3.269	0.049	1970	3.090	2001	-2.848
22	-7.107	0.097	72	-3.166	0.050	1971	2.815	2002	-3.241
23	-7.121	0.093	73	-3.066	0.045	1972	2.915	2003	-3.148
24	-7.120	0.084	74	-2.974	0.044	1973	2.422	2004	-4.135
25	-7.163	0.095	75	-2.877	0.043	1974	2.093	2005	-4.557
26	-7.130	0.086	76	-2.778	0.041	1975	1.774	2006	-4.920
27	-7.165	0.092	77	-2.687	0.039	1976	1.579	2007	-5.797
28	-7.159	0.077	78	-2.593	0.036	1977	1.461	2008	-5.607
29	-7.129	0.081	79	-2.497	0.035	1978	1.483	2009	-5.853
30	-7.066	0.068	80	-2.408	0.034	1979	0.838	2010	-5.987
31	-7.049	0.075	81	-2.317	0.032	1980	0.731		
32	-7.016	0.070	82	-2.226	0.032				
33	-6.976	0.069	83	-2.140	0.030				
34	-6.902	0.070	84	-2.040	0.029				
35	-6.863	0.066	85	-1.962	0.028				
36	-6.764	0.069	86	-1.871	0.029				
37	-6.714	0.069	87	-1.790	0.026				
38	-6.632	0.064	88	-1.703	0.024				
39	-6.528	0.071	89	-1.623	0.025				
40	-6.431	0.071	90	-1.551	0.023				
41	-6.349	0.069	91	-1.480	0.015				
42	-6.259	0.071	92	-1.405	0.016				
43	-6.119	0.069	93	-1.330	0.020				
44	-6.036	0.076	94	-1.255	0.017				
45	-5.919	0.073	95	-1.204	0.012				
46	-5.819	0.076	96	-1.149	0.010				
47	-5.711	0.075	97	-1.101	0.014				
48	-5.605	0.070	98	-1.048	0.013				
49	-5.493	0.076	99	-0.932	0.009				
50	-5.391	0.068							

A.2 Calculation Net Present Value

In section 6.1.2 the following definition was given for the Net Present Value (NPV):

$$NPV(x) = \sum_{t=1}^{\infty} \left[CF \cdot \prod_{i=1}^{t} (1 - \hat{m}(x+i-1,i)) \cdot DF(t) \right].$$
 (A.1)

The first term within the square brackets is just the cash flow that needs to be paid to the clients who are alive. The second term represents the probability of a client of age x still being alive. $\hat{m}(x,t)$ gives the probability of dying of an x-year-old at time t. This implies that the probability of this person is alive at time t is $1 - \hat{m}(x,t)$.

If we set t equal to 2 and we set x equal to 45, then there is a difference between the probability of an 45-year-old being alive in two years and the probability that a 45-year-old now is still alive in two years. In the first case we talk about the probability $1 - \hat{m}(45, 2)$. In the second case we see the 45-year-old person as a cohort and talk about the probability that this same person is still alive in two years: $(1 - \hat{m}(45, 1)) \cdot (1 - \hat{m}(46, 2))$.

Table 6 gives an abstract presentation of a part of a mortality table. Given this table the following holds:

$$1 - \hat{m}(45, 2) = 1 - b$$

and

$$(1 - \hat{m}(45, 1)) \cdot (1 - \hat{m}(46, 2)) = (1 - a) \cdot (1 - e).$$

Table 6: This table shows a part of an abstract mortality table.

Now back to the matter at hand, the second term in equation (A.1) represents the probability of a person of age x still being alive at time t.

The third term in for the NPV stands for the discount factors to take the time value of money into account.

A.3 Mortality Rates of the Dutch Male Population

Table 7: This table shows the mortality rates of the Dutch male population from 1950 until 2010 for the ages of 1 until 99 years old. The data are provided by the Dutch Central Bureau of Statistics.

	1950	1951	1952	1953	1954	1955	1956	1957	1958	1959
1	0.0059	0.0057	0.0051	0.0050	0.0045	0.0036	0.0041	0.0038	0.0038	0.0037
2	0.0024	0.0022	0.0020	0.0022	0.0018	0.0018	0.0016	0.0017	0.0018	0.0020
3	0.0015	0.0017	0.0015	0.0017	0.0013	0.0012	0.0012	0.0012	0.0014	0.0014
4	0.0013	0.0013	0.0013	0.0012	0.0013	0.0012	0.0010	0.0009	0.0010	0.0012
5	0.0012	0.0011	0.0011	0.0011	0.0008	0.0007	0.0007	0.0010	0.0007	0.0010
6	0.0009	0.0010	0.0008	0.0011	0.0008	0.0007	0.0008	0.0006	0.0007	0.0009
7	0.0008	0.0008	0.0009	0.0011	0.0007	0.0008	0.0006	0.0006	0.0007	0.0008
8	0.0007	0.0008	0.0006	0.0007	0.0006	0.0007	0.0007	0.0008	0.0006	0.0006
9	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005	0.0006
10	0.0006	0.0004	0.0006	0.0007	0.0005	0.0005	0.0005	0.0006	0.0004	0.0006
11	0.0004	0.0005	0.0004	0.0007	0.0005	0.0004	0.0004	0.0004	0.0004	0.0005
12	0.0006	0.0006	0.0004	0.0006	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005
13	0.0005	0.0006	0.0005	0.0006	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004
14	0.0006	0.0006	0.0006	0.0008	0.0004	0.0007	0.0003	0.0007	0.0004	0.0006
15	0.0008	0.0006	0.0005	0.0009	0.0005	0.0007	0.0005	0.0006	0.0005	0.0006
16	0.0007	0.0006	0.0007	0.0009	0.0007	0.0006	0.0006	0.0007	0.0006	0.0007
17	0.0010	0.0008	0.0005	0.0007	0.0006	0.0006	0.0007	0.0006	0.0006	0.0007
18	0.0008	0.0008	0.0008	0.0009	0.0007	0.0008	0.0009	0.0008	0.0008	0.0010
19	0.0010	0.0009	0.0008	0.0010	0.0008	0.0009	0.0008	0.0008	0.0009	0.0010
20	0.0010	0.0009	0.0008	0.0009	0.0006	0.0009	0.0008	0.0009	0.0008	0.0010
21	0.0011	0.0010	0.0011	0.0009	0.0008	0.0010	0.0009	0.0011	0.0009	0.0008
22	0.0012	0.0011	0.0010	0.0012	0.0010	0.0008	0.0011	0.0010	0.0011	0.0012
23	0.0013	0.0014	0.0009	0.0010	0.0009	0.0010	0.0010	0.0011	0.0011	0.0010
${\bf 24}$	0.0014	0.0015	0.0010	0.0013	0.0008	0.0011	0.0010	0.0011	0.0009	0.0009
25	0.0012	0.0013	0.0010	0.0013	0.0010	0.0009	0.0011	0.0010	0.0011	0.0010
26	0.0011	0.0014	0.0011	0.0012	0.0012	0.0009	0.0011	0.0012	0.0011	0.0010
27	0.0013	0.0014	0.0013	0.0012	0.0011	0.0011	0.0010	0.0009	0.0010	0.0009
28	0.0012	0.0014	0.0010	0.0012	0.0009	0.0010	0.0010	0.0009	0.0008	0.0008
29	0.0013	0.0014	0.0012	0.0009	0.0010	0.0009	0.0010	0.0010	0.0010	0.0011
30	0.0013	0.0011	0.0011	0.0014	0.0011	0.0011	0.0011	0.0010	0.0010	0.0012
31	0.0012	0.0011	0.0010	0.0011	0.0013	0.0011	0.0011	0.0012	0.0011	0.0012
32	0.0013	0.0012	0.0013	0.0014	0.0013	0.0011	0.0011	0.0010	0.0013	0.0012
33	0.0012	0.0013	0.0014	0.0014	0.0012	0.0011	0.0011	0.0012	0.0011	0.0011
34	0.0012	0.0017	0.0015	0.0012	0.0014	0.0013	0.0010	0.0014	0.0011	0.0012
35	0.0016	0.0015	0.0014	0.0016	0.0012	0.0012	0.0014	0.0010	0.0013	0.0013
36	0.0013	0.0015	0.0016	0.0016	0.0015	0.0011	0.0011	0.0013	0.0012	0.0014
37	0.0019	0.0016	0.0014	0.0016	0.0015	0.0016	0.0017	0.0013	0.0012	0.0016
38	0.0016	0.0016	0.0017	0.0016	0.0017	0.0015	0.0017	0.0016	0.0013	0.0013
39	0.0022	0.0020	0.0019	0.0019	0.0019	0.0018	0.0016	0.0017	0.0016	0.0020
40	0.0021	0.0023	0.0019	0.0022	0.0018	0.0020	0.0018	0.0018	0.0018	0.0021
41	0.0023	0.0022	0.0020	0.0024	0.0023	0.0020	0.0022	0.0019	0.0019	0.0020
42	0.0025	0.0026	0.0021	0.0026	0.0023	0.0020	0.0022	0.0023	0.0021	0.0024
43	0.0026	0.0027	0.0026	0.0028	0.0028	0.0027	0.0023	0.0026	0.0030	0.0024
44	0.0031	0.0032	0.0031	0.0029	0.0027	0.0026	0.0027	0.0029	0.0031	0.0029
45 46	0.0034	0.0034	0.0030	0.0033	0.0028	0.0036	0.0033	0.0031	0.0029	0.0029
$\begin{array}{c} 46 \\ 47 \end{array}$	0.0035 0.0040	0.0039 0.0042	0.0034 0.0039	0.0037 0.0039	0.0034 0.0040	0.0038	0.0034 0.0033	0.0036	0.0032 0.0033	0.0035
47	0.0040 0.0042	0.0042 0.0044	0.0039	0.0039 0.0045	0.0040 0.0044	0.0044	0.0033	0.0038 0.0043	0.0033 0.0044	0.0040 0.0038
						0.0043				
49 50	0.0045	0.0048	0.0046	0.0045	$0.0051 \\ 0.0050$	0.0050	0.0048 0.0053	0.0049	0.0043	0.0047
90	0.0054	0.0057	0.0058	0.0054	0.0050	0.0049	0.0053	0.0052	0.0048	0.0043

Continuation of table 7

	1070			4070			1070		1070	1070
	1950	1951	1952	1953	1954	1955	1956	1957	1958	1959
51	0.0068	0.0057	0.0064	0.0062	0.0063	0.0065	0.0060	0.0058	0.0062	0.0060
$\bf 52$	0.0064	0.0071	0.0065	0.0067	0.0066	0.0065	0.0065	0.0067	0.0064	0.0062
53	0.0074	0.0069	0.0066	0.0068	0.0071	0.0077	0.0074	0.0077	0.0077	0.0075
54	0.0085	0.0084	0.0083	0.0076	0.0084	0.0083	0.0078	0.0083	0.0077	0.0084
55	0.0084	0.0082	0.0085	0.0088	0.0094	0.0093	0.0086	0.0089	0.0088	0.0090
56	0.0086	0.0092	0.0098	0.0105	0.0101	0.0087	0.0106	0.0094	0.0098	0.0105
57	0.0104	0.0109	0.0102	0.0102	0.0112	0.0107	0.0110	0.0106	0.0104	0.0117
58	0.0114	0.0115	0.0118	0.0115	0.0123	0.0111	0.0130	0.0121	0.0124	0.0124
59	0.0121	0.0119	0.0121	0.0129	0.0140	0.0142	0.0139	0.0119	0.0133	0.0136
60	0.0136	0.0130	0.0134	0.0143	0.0148	0.0154	0.0150	0.0142	0.0148	0.0153
61	0.0146	0.0147	0.0143	0.0152	0.0159	0.0160	0.0160	0.0162	0.0163	0.0160
62	0.0157	0.0163	0.0164	0.0167	0.0165	0.0174	0.0194	0.0167	0.0179	0.0185
63	0.0180	0.0189	0.0182	0.0179	0.0182	0.0182	0.0186	0.0192	0.0192	0.0190
64	0.0198	0.0187	0.0182	0.0198	0.0202	0.0202	0.0215	0.0202	0.0208	0.0213
65	0.0228	0.0208	0.0216	0.0209	0.0226	0.0226	0.0245	0.0217	0.0224	0.0229
66	0.0245	0.0240	0.0240	0.0235	0.0230	0.0224	0.0253	0.0245	0.0243	0.0242
67	0.0260	0.0264	0.0250	0.0261	0.0267	0.0287	0.0264	0.0283	0.0268	0.0269
68	0.0288	0.0284	0.0287	0.0297	0.0298	0.0292	0.0284	0.0287	0.0272	0.0301
69	0.0325	0.0334	0.0331	0.0311	0.0324	0.0316	0.0317	0.0320	0.0316	0.0332
70	0.0365	0.0351	0.0364	0.0378	0.0334	0.0355	0.0351	0.0350	0.0348	0.0350
71	0.0419	0.0363	0.0374	0.0402	0.0387	0.0393	0.0375	0.0387	0.0393	0.0372
72	0.0435	0.0440	0.0417	0.0426	0.0434	0.0446	0.0464	0.0445	0.0419	0.0424
73	0.0475	0.0469	0.0460	0.0481	0.0469	0.0489	0.0510	0.0482	0.0486	0.0496
74	0.0515	0.0541	0.0537	0.0532	0.0532	0.0538	0.0550	0.0505	0.0525	0.0542
75 7 5	0.0603	0.0564	0.0573	0.0622	0.0603	0.0594	0.0625	0.0615	0.0590	0.0583
76	0.0673	0.0680	0.0619	0.0699	0.0657	0.0705	0.0688	0.0633	0.0655	0.0628
77	0.0718	0.0770	0.0684	0.0754	0.0708	0.0752	0.0716	0.0704	0.0700	0.0739
78 70	0.0834	0.0836	0.0772	0.0817	0.0772	0.0816	0.0817	0.0775	0.0747	0.0768
79	0.0905	0.0947	0.0883	0.0903	0.0880	0.0881	0.0903	0.0881	0.0877	0.0869
80	0.0952	0.1037	0.0978	0.1012	0.1013	0.0961	0.1023	0.0974	0.1001	0.0976
$\begin{array}{c} 81 \\ 82 \end{array}$	0.1091 0.1230	$0.1106 \\ 0.1256$	$0.1105 \\ 0.1175$	0.1052 0.1282	0.1029 0.1194	$0.1080 \\ 0.1222$	0.1133 0.1269	$0.1038 \\ 0.1166$	0.1103 0.1197	$0.1007 \\ 0.1126$
83	0.1250 0.1357	0.1280 0.1283	$0.1175 \\ 0.1256$	0.1282 0.1350	0.1194 0.1334	0.1222 0.1289	0.1269 0.1389	0.1100 0.1327	0.1197 0.1294	0.1120 0.1238
84	0.1537 0.1543	0.1263 0.1590	0.1250 0.1358	0.1330 0.1440	0.1334 0.1442	0.1269 0.1527	0.1589 0.1580	0.1327 0.1410	0.1294 0.1460	0.1238 0.1422
85	0.1545 0.1636	0.1590 0.1606	0.1538 0.1513	0.1440 0.1603	0.1442 0.1586	0.1527 0.1618	0.1692	0.1410 0.1408	0.1400 0.1612	0.1422 0.1549
86	0.1650 0.1677	0.1000 0.1912	0.1615	0.1005 0.1765	0.1723	0.1013 0.1872	0.1092 0.1702	0.1408 0.1726	0.1612 0.1695	0.1749 0.1722
87	0.1907	0.1801	0.1829	0.1873	0.1949	0.1937	0.2019	0.1914	0.1892	0.1773
88	0.2076	0.2059	0.1923	0.2029	0.2149	0.2188	0.2219	0.2023	0.1970	0.1773
89	0.2365	0.2039 0.2240	0.1331 0.2298	0.2269	0.2143 0.2387	0.2356	0.2352	0.2023 0.2162	0.1370	0.1363 0.2167
90	0.2545	0.2495	0.2183	0.2525	0.2437	0.2591	0.2592 0.2598	0.2659	0.2331	0.2440
91	0.2421	0.2349	0.2638	0.2618	0.2359	0.2499	0.2615	0.2406	0.2572	0.2531
92	0.2733	0.3094	0.2817	0.2679	0.2479	0.2568	0.2921	0.2973	0.2430	0.2656
93	0.3077	0.2960	0.3051	0.3018	0.2537	0.2650	0.3290	0.3099	0.2430 0.3129	0.2000 0.3117
94	0.3085	0.3590	0.3599	0.3450	0.2997	0.3181	0.2740	0.2890	0.3741	0.3414
95	0.2634	0.2950	0.3345	0.2722	0.2926	0.3243	0.3305	0.3311	0.3870	0.3555
96	0.4044	0.3672	0.3897	0.3232	0.2846	0.3609	0.3179	0.3603	0.3163	0.3412
97	0.3609	0.4954	0.3359	0.4237	0.3881	0.3636	0.3041	0.2967	0.3768	0.3158
98	0.4054	0.5060	0.4727	0.3721	0.5882	0.5366	0.3393	0.5167	0.3265	0.4308
99	0.4034	0.3238	0.5510	0.4722	0.5000	0.5634	0.5135	0.3390	0.5147	0.4024

Continuation of table 7

	1960	1961	1962	1963	1964	1965	1966	1967	1968	1969
1	0.0028	0.0030	0.0031	0.0028	0.0023	0.0021	0.0027	0.0022	0.0022	0.0022
2	0.0016	0.0015	0.0012	0.0012	0.0014	0.0012	0.0010	0.0012	0.0012	0.0012
3	0.0013	0.0011	0.0012	0.0011	0.0012	0.0012	0.0011	0.0010	0.0010	0.0010
4	0.0011	0.0009	0.0011	0.0010	0.0008	0.0009	0.0009	0.0009	0.0007	0.0007
5	0.0011	0.0007	0.0008	0.0008	0.0008	0.0008	0.0007	0.0006	0.0008	0.0007
6	0.0007	0.0009	0.0007	0.0008	0.0007	0.0006	0.0008	0.0007	0.0007	0.0006
7	0.0006	0.0007	0.0006	0.0008	0.0007	0.0007	0.0006	0.0007	0.0006	0.0006
8	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005	0.0005	0.0005	0.0006	0.0006
9	0.0005	0.0005	0.0005	0.0005	0.0006	0.0004	0.0006	0.0006	0.0005	0.0004
10	0.0005	0.0003	0.0004	0.0004	0.0005	0.0005	0.0004	0.0005	0.0005	0.0006
11	0.0004	0.0004	0.0004	0.0004	0.0005	0.0004	0.0005	0.0005	0.0003	0.0004
12	0.0003	0.0004	0.0004	0.0004	0.0005	0.0003	0.0004	0.0004	0.0004	0.0004
13	0.0004	0.0004	0.0004	0.0005	0.0005	0.0003	0.0004	0.0005	0.0005	0.0004
14	0.0003	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0004	0.0004
15	0.0005	0.0005	0.0005	0.0004	0.0005	0.0004	0.0006	0.0006	0.0005	0.0004
16	0.0006	0.0005	0.0006	0.0006	0.0005	0.0005	0.0008	0.0008	0.0008	0.0008
17	0.0007	0.0008	0.0006	0.0007	0.0008	0.0008	0.0009	0.0010	0.0009	0.0010
18	0.0008	0.0009	0.0008	0.0007	0.0011	0.0009	0.0008	0.0011	0.0011	0.0007
19	0.0008	0.0008	0.0010	0.0008	0.0011	0.0010	0.0012	0.0010	0.0011	0.0013
20	0.0009	0.0009	0.0011	0.0011	0.0012	0.0011	0.0012	0.0010	0.0012	0.0011
21	0.0009	0.0012	0.0010	0.0009	0.0010	0.0011	0.0009	0.0011	0.0010	0.0012
22	0.0009	0.0010	0.0011	0.0009	0.0011	0.0011	0.0011	0.0011	0.0011	0.0011
23	0.0012	0.0010	0.0010	0.0011	0.0011	0.0010	0.0010	0.0013	0.0010	0.0010
${\bf 24}$	0.0011	0.0011	0.0009	0.0010	0.0007	0.0010	0.0011	0.0009	0.0009	0.0009
25	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0009	0.0010	0.0009	0.0010
26	0.0010	0.0011	0.0012	0.0009	0.0010	0.0009	0.0010	0.0010	0.0009	0.0010
27	0.0011	0.0010	0.0011	0.0012	0.0011	0.0010	0.0009	0.0012	0.0010	0.0008
28	0.0010	0.0010	0.0011	0.0010	0.0008	0.0009	0.0007	0.0010	0.0009	0.0010
29	0.0011	0.0010	0.0011	0.0009	0.0011	0.0010	0.0010	0.0009	0.0012	0.0008
30	0.0009	0.0010	0.0010	0.0010	0.0010	0.0010	0.0011	0.0010	0.0012	0.0011
31	0.0011	0.0010	0.0013	0.0010	0.0011	0.0009	0.0010	0.0012	0.0011	0.0009
32	0.0010	0.0012	0.0010	0.0010	0.0011	0.0010	0.0010	0.0009	0.0010	0.0011
33	0.0009	0.0010	0.0013	0.0011	0.0012	0.0010	0.0010	0.0012	0.0011	0.0011
34	0.0011	0.0010	0.0012	0.0014	0.0012	0.0014	0.0011	0.0016	0.0015	0.0011
35	0.0014	0.0013	0.0015	0.0010	0.0013	0.0013	0.0013	0.0012	0.0012	0.0012
36	0.0013	0.0012	0.0014	0.0015	0.0015	0.0013	0.0015	0.0014	0.0017	0.0014
37	0.0015	0.0015	0.0015	0.0015	0.0012	0.0016	0.0013	0.0014	0.0015	0.0015
38	0.0014	0.0015	0.0016	0.0017	0.0018	0.0018	0.0016	0.0015	0.0015	0.0014
39	0.0014	0.0017	0.0019	0.0017	0.0018	0.0018	0.0019	0.0017	0.0020	0.0019
40	0.0021	0.0018	0.0016	0.0020	0.0021	0.0021	0.0020	0.0019	0.0022	0.0022
41	0.0018	0.0021	0.0021	0.0023	0.0017	0.0025	0.0024	0.0024	0.0024	0.0021
42	0.0022	0.0023	0.0023	0.0024	0.0023	0.0022	0.0024	0.0023	0.0022	0.0025
43	0.0027	0.0028	0.0023	0.0024	0.0027	0.0027	0.0025	0.0028	0.0028	0.0027
44	0.0029	0.0028	0.0027	0.0028	0.0029	0.0030	0.0030	0.0030	0.0030	0.0031
45	0.0029	0.0032	0.0029	0.0030	0.0033	0.0033	0.0032	0.0036	0.0032	0.0032
46	0.0032	0.0034	0.0037	0.0037	0.0034	0.0039	0.0034	0.0033	0.0037	0.0041
47	0.0038	0.0040	0.0043	0.0040	0.0042	0.0038	0.0041	0.0039	0.0043	0.0042
48	0.0040	0.0044	0.0043	0.0052	0.0050	0.0043	0.0047	0.0044	0.0043	0.0051
49	0.0050	0.0049	0.0047	0.0050	0.0059	0.0052	0.0051	0.0058	0.0056	0.0053
50	0.0052	0.0052	0.0055	0.0057	0.0054	0.0055	0.0055	0.0057	0.0059	0.0059

Continuation of table 7

	1960	1961	1962	1963	1964	1965	1966	1967	1968	1969
51	0.0063	0.0062	0.0063	0.0063	0.0059	0.0068	0.0063	0.0066	0.0058	0.0063
$\bf 52$	0.0064	0.0073	0.0072	0.0065	0.0063	0.0078	0.0068	0.0068	0.0081	0.0068
53	0.0075	0.0078	0.0075	0.0077	0.0075	0.0080	0.0083	0.0081	0.0078	0.0081
$\bf 54$	0.0088	0.0084	0.0088	0.0087	0.0087	0.0095	0.0096	0.0088	0.0083	0.0097
55	0.0092	0.0088	0.0098	0.0103	0.0101	0.0100	0.0099	0.0098	0.0104	0.0102
56	0.0105	0.0104	0.0108	0.0104	0.0114	0.0107	0.0106	0.0106	0.0110	0.0108
57	0.0111	0.0108	0.0132	0.0117	0.0119	0.0123	0.0121	0.0127	0.0120	0.0120
58	0.0125	0.0130	0.0130	0.0130	0.0131	0.0136	0.0142	0.0139	0.0132	0.0131
59	0.0143	0.0142	0.0150	0.0150	0.0147	0.0152	0.0154	0.0148	0.0158	0.0157
60	0.0146	0.0143	0.0164	0.0168	0.0159	0.0169	0.0166	0.0170	0.0164	0.0166
61	0.0168	0.0167	0.0183	0.0173	0.0177	0.0178	0.0181	0.0187	0.0189	0.0192
62	0.0177	0.0183	0.0194	0.0188	0.0194	0.0195	0.0198	0.0202	0.0205	0.0206
63	0.0203	0.0207	0.0221	0.0213	0.0230	0.0212	0.0222	0.0213	0.0229	0.0235
64	0.0205	0.0219	0.0229	0.0235	0.0236	0.0248	0.0247	0.0234	0.0224	0.0254
65	0.0232	0.0233	0.0258	0.0260	0.0257	0.0261	0.0265	0.0261	0.0267	0.0264
66	0.0256	0.0256	0.0286	0.0291	0.0295	0.0278	0.0289	0.0299	0.0295	0.0319
67	0.0287	0.0288	0.0281	0.0303	0.0309	0.0308	0.0303	0.0312	0.0314	0.0312
68	0.0306	0.0309	0.0337	0.0336	0.0333	0.0340	0.0326	0.0344	0.0354	0.0353
69	0.0339	0.0336	0.0352	0.0351	0.0368	0.0356	0.0371	0.0384	0.0378	0.0377
70	0.0347	0.0367	0.0370	0.0389	0.0389	0.0406	0.0392	0.0398	0.0415	0.0426
71	0.0390	0.0382	0.0394	0.0420	0.0410	0.0428	0.0427	0.0448	0.0454	0.0431
72	0.0434	0.0420	0.0418	0.0472	0.0445	0.0463	0.0485	0.0468	0.0494	0.0542
73	0.0477	0.0450	0.0485	0.0496	0.0480	0.0512	0.0537	0.0516	0.0506	0.0546
74	0.0499	0.0538	0.0516	0.0542	0.0539	0.0567	0.0531	0.0555	0.0580	0.0602
75	0.0557	0.0592	0.0599	0.0620	0.0548	0.0586	0.0600	0.0622	0.0638	0.0650
76	0.0622	0.0615	0.0664	0.0668	0.0632	0.0653	0.0668	0.0677	0.0690	0.0663
77	0.0706	0.0698	0.0697	0.0704	0.0682	0.0715	0.0746	0.0728	0.0737	0.0754
78	0.0766	0.0742	0.0833	0.0824	0.0799	0.0783	0.0799	0.0778	0.0812	0.0811
7 9	0.0875	0.0821	0.0901	0.0896	0.0849	0.0895	0.0813	0.0810	0.0874	0.0892
80	0.0917	0.0917	0.0985	0.0976	0.0948	0.0925	0.0964	0.0873	0.0954	0.0927
81	0.1067	0.1005	0.1130	0.1106	0.0994	0.1051	0.1020	0.0995	0.1031	0.1012
82	0.1159	0.1135	0.1155	0.1220	0.1054	0.1160	0.1111	0.1116	0.1152	0.1151
83	0.1317	0.1254	0.1397	0.1312	0.1171	0.1218	0.1197	0.1157	0.1264	0.1278
84	0.1441	0.1324	0.1495	0.1412	0.1263	0.1357	0.1400	0.1239	0.1396	0.1324
85	0.1520	0.1485	0.1556	0.1605	0.1399	0.1458	0.1575	0.1404	0.1581	0.1428
86	0.1696	0.1631	0.1727	0.1685	0.1536	0.1648	0.1605	0.1551	0.1636	0.1670
87	0.1845	0.1807	0.1767	0.1897	0.1665	0.1831	0.1796	0.1627	0.1833	0.1647
88	0.2027	0.1824	0.2051	0.2127	0.1825	0.1919	0.1907	0.1757	0.1901	0.1866
89	0.2127	0.2111	0.2297	0.2322	0.1916	0.2061	0.2054	0.2039	0.2149	0.1966
90	0.2134	0.2282	0.2313	0.2248	0.2241	0.2342	0.2204	0.2025	0.2298	0.2204
91	0.2598	0.2383	0.2752	0.2357	0.2147	0.2259	0.2335	0.2437	0.2502	0.2311
92	0.2726	0.2334	0.2508	0.3198	0.2379	0.2697	0.2643	0.2410	0.2689	0.2513
93	0.3305	0.2990	0.2897	0.2927	0.2786	0.2560	0.2857	0.2701	0.2617	0.2921
94	0.3026	0.3029	0.2918	0.3446	0.2685	0.3436	0.2965	0.2609	0.3213	0.3167
95	0.4000	0.3448	0.4026	0.3702	0.2824	0.2756	0.3529	0.2783	0.3264	0.2989
96	0.3567	0.3228	0.3923	0.3262	0.3850	0.3447	0.2797	0.2955	0.3003	0.3127
97	0.4215	0.3077	0.4046	0.4567	0.3016	0.3684	0.3210	0.2973	0.3871	0.2795
98	0.2143	0.2819	0.4024	0.3974	0.3188	0.3977	0.2771	0.2791	0.3013	0.4947
99	0.3949	0.3587	0.3487	0.4455	0.3889	0.4404	0.4424	0.4085	0.4315	0.4323

Continuation of table 7

	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979
1	0.0023	0.0020	0.0019	0.0020	0.0021	0.0018	0.0019	0.0016	0.0021	0.0017
2	0.0010	0.0010	0.0010	0.0012	0.0012	0.0010	0.0008	0.0007	0.0008	0.0008
3	0.0009	0.0011	0.0009	0.0008	0.0007	0.0008	0.0006	0.0007	0.0006	0.0007
4	0.0009	0.0009	0.0007	0.0006	0.0007	0.0006	0.0005	0.0005	0.0006	0.0004
5	0.0007	0.0007	0.0006	0.0005	0.0005	0.0004	0.0005	0.0005	0.0005	0.0004
6	0.0006	0.0006	0.0006	0.0005	0.0006	0.0005	0.0003	0.0004	0.0005	0.0004
7	0.0006	0.0006	0.0005	0.0005	0.0005	0.0003	0.0005	0.0004	0.0004	0.0003
8	0.0006	0.0004	0.0005	0.0005	0.0004	0.0004	0.0005	0.0004	0.0005	0.0003
9	0.0005	0.0005	0.0005	0.0005	0.0004	0.0004	0.0003	0.0004	0.0003	0.0004
10	0.0006	0.0004	0.0004	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002	0.0003
11	0.0004	0.0004	0.0004	0.0004	0.0003	0.0004	0.0003	0.0003	0.0003	0.0003
12	0.0004	0.0005	0.0004	0.0004	0.0003	0.0004	0.0004	0.0003	0.0003	0.0003
13	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003	0.0003	0.0004	0.0003
14	0.0004	0.0003	0.0005	0.0003	0.0003	0.0004	0.0004	0.0004	0.0005	0.0003
15	0.0007	0.0004	0.0005	0.0004	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004
16	0.0008	0.0009	0.0008	0.0006	0.0009	0.0006	0.0006	0.0007	0.0006	0.0005
17	0.0011	0.0010	0.0012	0.0010	0.0010	0.0009	0.0008	0.0009	0.0010	0.0009
18	0.0011	0.0011	0.0011	0.0011	0.0013	0.0009	0.0009	0.0010	0.0009	0.0008
19	0.0011	0.0014	0.0013	0.0013	0.0014	0.0010	0.0012	0.0013	0.0011	0.0011
20	0.0012	0.0012	0.0012	0.0011	0.0011	0.0012	0.0008	0.0012	0.0011	0.0011
21	0.0012	0.0012	0.0011	0.0012	0.0011	0.0011	0.0011	0.0011	0.0010	0.0008
22	0.0013	0.0010	0.0011	0.0011	0.0012	0.0008	0.0010	0.0011	0.0010	0.0010
23	0.0011	0.0011	0.0011	0.0012	0.0009	0.0008	0.0010	0.0010	0.0009	0.0007
24	0.0010	0.0010	0.0009	0.0011	0.0009	0.0009	0.0010	0.0009	0.0011	0.0008
25	0.0007	0.0008	0.0009	0.0010	0.0009	0.0008	0.0008	0.0008	0.0008	0.0008
26	0.0010	0.0009	0.0010	0.0009	0.0009	0.0009	0.0009	0.0010	0.0008	0.0009
27	0.0009	0.0011	0.0009	0.0008	0.0008	0.0008	0.0007	0.0009	0.0007	0.0007
28	0.0009	0.0008	0.0010	0.0009	0.0008	0.0008	0.0008	0.0009	0.0008	0.0008
29	0.0011	0.0008	0.0010	0.0008	0.0007	0.0008	0.0009	0.0009	0.0008	0.0008
30	0.0010	0.0011	0.0011	0.0012	0.0007	0.0010	0.0008	0.0007	0.0006	0.0009
31	0.0010	0.0011	0.0010	0.0009	0.0008	0.0010	0.0010	0.0009	0.0009	0.0009
32	0.0012	0.0012	0.0010	0.0010	0.0008	0.0008	0.0009	0.0009	0.0008	0.0008
33	0.0013	0.0010	0.0013	0.0012	0.0011	0.0010	0.0011	0.0010	0.0010	0.0010
34	0.0011	0.0011	0.0011	0.0014	0.0010	0.0012	0.0011	0.0010	0.0011	0.0009
$\frac{35}{36}$	$0.0010 \\ 0.0014$	$0.0010 \\ 0.0015$	0.0012 0.0015	0.0012 0.0013	0.0011	$0.0010 \\ 0.0012$	0.0011 0.0013	0.0011 0.0012	0.0011 0.0012	0.0011 0.0011
					0.0016					
37	0.0015	0.0014	0.0015	0.0013	0.0011	0.0012	0.0015	0.0013	0.0012	0.0013
38	0.0016 0.0017	0.0016 0.0018	0.0016	0.0016 0.0017	0.0016 0.0017	0.0015	0.0014 0.0015	0.0013 0.0016	0.0013 0.0018	0.0014
$\begin{array}{c} 39 \\ 40 \end{array}$			0.0020 0.0018		0.0017 0.0016	0.0015	0.0015 0.0016		0.0018 0.0017	0.0014
	0.0020	0.0020		0.0022		0.0018		0.0019	0.0017 0.0019	0.0015 0.0017
$\begin{array}{c} 41 \\ 42 \end{array}$	0.0021 0.0024	0.0022 0.0021	0.0020 0.0026	0.0019 0.0024	0.0021 0.0023	0.0018 0.0019	0.0021 0.0021	0.0022 0.0022	0.0019 0.0022	0.0017 0.0021
42	0.0024 0.0028	0.0021 0.0032	0.0026 0.0029	0.0024 0.0024	0.0023 0.0027	0.0019 0.0024	0.0021 0.0025	0.0022 0.0025	0.0022 0.0024	0.0021 0.0022
$\frac{43}{44}$	0.0028 0.0029	0.0032 0.0033	0.0029 0.0031	0.0024 0.0028	0.0027 0.0026	0.0024 0.0029	0.0025 0.0028	0.0025 0.0030	0.0024 0.0025	0.0022 0.0025
45	0.0029 0.0033	0.0033 0.0034	0.0031 0.0038	0.0028 0.0032	0.0026	0.0029 0.0032	0.0028 0.0035	0.0030	0.0025 0.0032	0.0025 0.0031
$\frac{45}{46}$	0.0033 0.0037	0.0034 0.0035	0.0038 0.0042	0.0032 0.0039	0.0036 0.0037	0.0032 0.0038	0.0038	0.0029 0.0036	0.0032 0.0038	0.0031 0.0034
47	0.0037 0.0040	0.0035 0.0040	0.0042 0.0041	0.0039 0.0044	0.0037 0.0039	0.0038 0.0047	0.0038 0.0041	0.0030 0.0038	0.0038 0.0043	0.0034 0.0038
48	0.0040 0.0045	0.0040 0.0045	0.0041 0.0046	0.0044 0.0047	0.0039 0.0043	0.0047 0.0045	0.0041 0.0047	0.0038 0.0044	0.0043 0.0041	0.0038 0.0043
49	0.0043 0.0053	0.0045 0.0051	0.0040 0.0051	0.0047 0.0053	0.0045 0.0055	0.0049	0.0047 0.0052	0.0044 0.0050	0.0041 0.0046	0.0043 0.0048
50	0.0062	0.0051	0.0051	0.0050	0.0056	0.0043 0.0057	0.0052	0.0050	0.0040 0.0051	0.0048 0.0051

Continuation of table 7

	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979
51	0.0065	0.0065	0.0069	0.0060	0.0063	0.0066	0.0059	0.0056	0.0060	0.0056
52	0.0075	0.0070	0.0072	0.0074	0.0062	0.0076	0.0070	0.0065	0.0065	0.0064
53	0.0086	0.0085	0.0082	0.0080	0.0080	0.0075	0.0078	0.0073	0.0076	0.0068
54	0.0093	0.0099	0.0095	0.0089	0.0080	0.0087	0.0087	0.0083	0.0078	0.0082
55	0.0102	0.0104	0.0098	0.0102	0.0097	0.0098	0.0097	0.0090	0.0093	0.0091
56	0.0115	0.0114	0.0118	0.0116	0.0107	0.0113	0.0103	0.0097	0.0104	0.0092
57	0.0127	0.0122	0.0131	0.0122	0.0125	0.0123	0.0113	0.0112	0.0111	0.0108
58	0.0146	0.0134	0.0138	0.0123	0.0144	0.0130	0.0136	0.0136	0.0129	0.0126
59	0.0158	0.0147	0.0150	0.0136	0.0140	0.0151	0.0139	0.0143	0.0136	0.0132
60	0.0173	0.0172	0.0172	0.0166	0.0162	0.0163	0.0156	0.0158	0.0152	0.0144
61	0.0187	0.0182	0.0191	0.0178	0.0177	0.0181	0.0192	0.0180	0.0174	0.0177
62	0.0217	0.0209	0.0198	0.0202	0.0197	0.0202	0.0198	0.0192	0.0196	0.0178
63	0.0239	0.0228	0.0232	0.0223	0.0209	0.0222	0.0234	0.0199	0.0207	0.0212
64	0.0256	0.0246	0.0259	0.0239	0.0230	0.0243	0.0240	0.0226	0.0238	0.0224
65	0.0284	0.0274	0.0278	0.0266	0.0272	0.0271	0.0265	0.0257	0.0254	0.0255
66	0.0296	0.0302	0.0304	0.0303	0.0287	0.0295	0.0302	0.0281	0.0289	0.0275
67	0.0335	0.0315	0.0345	0.0314	0.0335	0.0330	0.0332	0.0311	0.0311	0.0306
68	0.0363	0.0369	0.0363	0.0349	0.0352	0.0370	0.0368	0.0351	0.0349	0.0331
69	0.0407	0.0392	0.0385	0.0404	0.0377	0.0409	0.0388	0.0390	0.0390	0.0353
70	0.0416	0.0410	0.0436	0.0403	0.0405	0.0424	0.0438	0.0408	0.0416	0.0401
71	0.0463	0.0436	0.0478	0.0468	0.0433	0.0472	0.0469	0.0447	0.0478	0.0451
72	0.0495	0.0498	0.0529	0.0495	0.0497	0.0521	0.0511	0.0484	0.0494	0.0513
73	0.0571	0.0543	0.0559	0.0533	0.0533	0.0546	0.0552	0.0541	0.0558	0.0555
74	0.0581	0.0628	0.0617	0.0576	0.0562	0.0585	0.0593	0.0595	0.0590	0.0583
75	0.0655	0.0637	0.0655	0.0656	0.0621	0.0627	0.0663	0.0620	0.0636	0.0637
76	0.0698	0.0731	0.0718	0.0708	0.0658	0.0718	0.0708	0.0684	0.0704	0.0671
77	0.0764	0.0749	0.0799	0.0762	0.0774	0.0761	0.0798	0.0726	0.0767	0.0746
78	0.0805	0.0832	0.0878	0.0820	0.0801	0.0823	0.0808	0.0787	0.0826	0.0800
79	0.0899	0.0858	0.0893	0.0869	0.0895	0.0920	0.0908	0.0850	0.0873	0.0870
80	0.0921	0.0999	0.0985	0.0969	0.0909	0.1003	0.0958	0.0940	0.0950	0.0917
81	0.1069	0.1023	0.1076	0.1025	0.1005	0.1085	0.1077	0.0982	0.1048	0.1038
82	0.1137	0.1128	0.1132	0.1128	0.1074	0.1172	0.1148	0.1040	0.1108	0.1081
83	0.1236	0.1225	0.1236	0.1239	0.1148	0.1188	0.1246	0.1172	0.1199	0.1183
84	0.1303	0.1336	0.1418	0.1338	0.1245	0.1357	0.1388	0.1236	0.1300	0.1208
85	0.1487	0.1464	0.1454	0.1428	0.1354	0.1510	0.1440	0.1311	0.1372	0.1358
86	0.1613	0.1699	0.1619	0.1552	0.1565	0.1617	0.1584	0.1494	0.1486	0.1444
87	0.1779	0.1689	0.1788	0.1694	0.1636	0.1691	0.1669	0.1559	0.1629	0.1510
88	0.1826	0.1843	0.1889	0.1936	0.1871	0.1892	0.1780	0.1718	0.1833	0.1655
89	0.2094	0.1893	0.2170	0.1973	0.2048	0.2025	0.2065	0.1723	0.1876	0.1731
90	0.2062	0.2170	0.2269	0.2200	0.2032	0.2155	0.2074	0.1867	0.1926	0.1980
91	0.2362	0.2271	0.2402	0.2310	0.2253	0.2295	0.2133	0.2079	0.2175	0.2127
92	0.2555	0.2506	0.2325	0.2403	0.2225	0.2595	0.2602	0.2297	0.2379	0.2160
93	0.2721	0.2924	0.2615	0.2692	0.2898	0.2743	0.2679	0.2353	0.2494	0.2519
94	0.2956	0.2800	0.3228	0.3167	0.2397	0.2953	0.2750	0.2399	0.2538	0.2515
95	0.3084	0.3560	0.3256	0.3529	0.3066	0.2592	0.2903	0.2274	0.2959	0.2431
96	0.3819	0.3629	0.3912	0.3253	0.3571	0.3384	0.2355	0.2759	0.3120	0.2926
97	0.3320	0.3553	0.3968	0.4420	0.4102	0.4054	0.3207	0.1922	0.3645	0.2950
98	0.2912	0.4465	0.3720	0.3410	0.5318	0.4105	0.4716	0.3447	0.1745	0.2583
99	0.4241	0.3229	0.4055	0.4340	0.4231	0.4245	0.4805	0.4327	0.4117	0.2191

Continuation of table 7

	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989
1	0.0019	0.0016	0.0018	0.0017	0.0020	0.0015	0.0020	0.0016	0.0013	0.0015
2	0.0007	0.0007	0.0007	0.0008	0.0007	0.0005	0.0006	0.0007	0.0007	0.0005
3	0.0005	0.0006	0.0006	0.0005	0.0004	0.0006	0.0005	0.0004	0.0004	0.0003
4	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003	0.0004	0.0003	0.0003	0.0003
5	0.0004	0.0005	0.0003	0.0003	0.0003	0.0002	0.0002	0.0002	0.0002	0.0003
6	0.0004	0.0003	0.0003	0.0004	0.0004	0.0003	0.0003	0.0003	0.0003	0.0002
7	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002	0.0003	0.0002	0.0002	0.0002
8	0.0004	0.0003	0.0003	0.0002	0.0002	0.0002	0.0001	0.0002	0.0003	0.0002
9	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002	0.0002	0.0001	0.0002	0.0002
10	0.0003	0.0001	0.0003	0.0002	0.0002	0.0002	0.0002	0.0001	0.0002	0.0003
11	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002	0.0002	0.0002	0.0002	0.0002
12	0.0003	0.0003	0.0003	0.0002	0.0002	0.0002	0.0003	0.0002	0.0002	0.0001
13	0.0002	0.0004	0.0003	0.0002	0.0003	0.0002	0.0002	0.0003	0.0002	0.0002
14	0.0003	0.0003	0.0004	0.0004	0.0003	0.0002	0.0003	0.0002	0.0003	0.0002
15	0.0003	0.0003	0.0003	0.0004	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
16	0.0005	0.0005	0.0005	0.0005	0.0005	0.0004	0.0004	0.0005	0.0004	0.0005
17	0.0007	0.0007	0.0007	0.0005	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005
18	0.0008	0.0007	0.0010	0.0008	0.0006	0.0007	0.0005	0.0006	0.0005	0.0005
19	0.0009	0.0009	0.0007	0.0008	0.0007	0.0008	0.0007	0.0006	0.0007	0.0008
20	0.0010	0.0007	0.0008	0.0008	0.0008	0.0008	0.0006	0.0008	0.0007	0.0006
21	0.0010	0.0009	0.0007	0.0008	0.0008	0.0008	0.0007	0.0007	0.0007	0.0008
22	0.0011	0.0011	0.0009	0.0009	0.0007	0.0007	0.0007	0.0008	0.0008	0.0007
23	0.0009	0.0008	0.0007	0.0007	0.0007	0.0009	0.0008	0.0008	0.0008	0.0007
$\bf 24$	0.0009	0.0008	0.0007	0.0009	0.0008	0.0008	0.0008	0.0007	0.0007	0.0008
25	0.0010	0.0008	0.0008	0.0007	0.0008	0.0007	0.0008	0.0007	0.0007	0.0007
26	0.0008	0.0008	0.0007	0.0008	0.0008	0.0006	0.0008	0.0006	0.0007	0.0007
27	0.0008	0.0008	0.0008	0.0006	0.0008	0.0007	0.0009	0.0005	0.0009	0.0006
28	0.0010	0.0008	0.0007	0.0008	0.0008	0.0008	0.0008	0.0007	0.0008	0.0008
29	0.0008	0.0008	0.0010	0.0008	0.0009	0.0009	0.0007	0.0007	0.0007	0.0008
30	0.0009	0.0009	0.0008	0.0008	0.0010	0.0009	0.0008	0.0008	0.0008	0.0008
31	0.0010	0.0010	0.0008	0.0008	0.0007	0.0008	0.0008	0.0007	0.0008	0.0007
32	0.0008	0.0010	0.0008	0.0009	0.0010	0.0008	0.0009	0.0008	0.0009	0.0009
33	0.0009	0.0008	0.0008	0.0010	0.0009	0.0009	0.0008	0.0010	0.0009	0.0010
34	0.0009	0.0010	0.0008	0.0010	0.0009	0.0010	0.0009	0.0010	0.0010	0.0008
35	0.0009	0.0009	0.0011	0.0011	0.0011	0.0011	0.0009	0.0010	0.0010	0.0010
36	0.0012	0.0013	0.0013	0.0009	0.0012	0.0011	0.0010	0.0011	0.0011	0.0011
37	0.0013	0.0012	0.0013	0.0012	0.0012	0.0012	0.0012	0.0012	0.0011	0.0011
38	0.0014	0.0013	0.0014	0.0013	0.0013	0.0011	0.0013	0.0013	0.0013	0.0013
39	0.0013	0.0014	0.0015	0.0015	0.0016	0.0014	0.0013	0.0013	0.0012	0.0013
40	0.0017	0.0016	0.0016	0.0017	0.0017	0.0014	0.0016	0.0015	0.0013	0.0015
41	0.0016	0.0018	0.0019	0.0017	0.0018	0.0018	0.0016	0.0017	0.0016	0.0018
42	0.0020	0.0018	0.0019	0.0018	0.0018	0.0020	0.0021	0.0018	0.0017	0.0016
43	0.0024	0.0020	0.0021	0.0022	0.0022	0.0020	0.0021	0.0020	0.0022	0.0019
44	0.0026	0.0024	0.0025	0.0022	0.0020	0.0025	0.0024	0.0025	0.0023	0.0023
45	0.0028	0.0027	0.0031	0.0027	0.0026	0.0025	0.0029	0.0025	0.0027	0.0025
$\begin{array}{c} 46 \\ 47 \end{array}$	0.0034 0.0038	0.0026 0.0038	0.0031 0.0035	0.0032 0.0032	0.0030 0.0034	0.0028	0.0027 0.0035	0.0028 0.0031	0.0029 0.0027	0.0026 0.0030
48	0.0038 0.0042	0.0038 0.0040	0.0035 0.0039	0.0032 0.0036	0.0034 0.0035	0.0029 0.0035	0.0035 0.0038	0.0031 0.0034	0.0027 0.0031	0.0030 0.0036
48 49	0.0042 0.0048	0.0040 0.0039	0.0039 0.0045	0.0036 0.0048	0.0035 0.0042	0.0035 0.0041	0.0038 0.0042	0.0034 0.0042		
49 50	0.0048 0.0053	0.0039 0.0056	0.0045 0.0050	0.0048 0.0047	0.0042 0.0046	0.0041 0.0045	0.0042 0.0050	0.0042 0.0044	0.0034 0.0045	0.0035 0.0041
30	0.0055	0.0050	0.0050	0.0047	0.0040	0.0040	0.0050	0.0044	0.0045	0.0041

Continuation of table 7

	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989
51	0.0059	0.0058	0.0054	0.0050	0.0051	0.0051	0.0048	0.0048	0.0048	0.0046
$\bf 52$	0.0068	0.0064	0.0066	0.0058	0.0065	0.0058	0.0060	0.0055	0.0055	0.0055
53	0.0075	0.0066	0.0065	0.0064	0.0066	0.0064	0.0068	0.0062	0.0062	0.0059
54	0.0071	0.0074	0.0076	0.0077	0.0079	0.0074	0.0077	0.0070	0.0068	0.0071
55	0.0093	0.0087	0.0087	0.0087	0.0083	0.0089	0.0082	0.0078	0.0070	0.0076
56	0.0102	0.0089	0.0096	0.0091	0.0099	0.0092	0.0094	0.0091	0.0079	0.0087
57	0.0105	0.0106	0.0104	0.0102	0.0108	0.0104	0.0104	0.0102	0.0093	0.0094
58	0.0121	0.0123	0.0119	0.0120	0.0114	0.0116	0.0115	0.0111	0.0111	0.0118
59	0.0128	0.0138	0.0132	0.0133	0.0118	0.0127	0.0119	0.0127	0.0117	0.0120
60	0.0152	0.0141	0.0150	0.0140	0.0144	0.0149	0.0143	0.0144	0.0136	0.0135
61	0.0162	0.0160	0.0166	0.0154	0.0157	0.0148	0.0155	0.0143	0.0146	0.0144
62	0.0178	0.0176	0.0188	0.0175	0.0173	0.0168	0.0176	0.0160	0.0162	0.0164
63	0.0207	0.0210	0.0208	0.0196	0.0185	0.0186	0.0191	0.0192	0.0181	0.0181
64	0.0237	0.0220	0.0221	0.0212	0.0213	0.0227	0.0214	0.0202	0.0196	0.0205
65	0.0252	0.0240	0.0251	0.0231	0.0227	0.0245	0.0227	0.0218	0.0238	0.0222
66	0.0274	0.0272	0.0269	0.0268	0.0276	0.0261	0.0253	0.0260	0.0245	0.0252
67	0.0300	0.0306	0.0298	0.0299	0.0283	0.0293	0.0292	0.0284	0.0273	0.0265
68	0.0343	0.0320	0.0329	0.0326	0.0323	0.0330	0.0345	0.0305	0.0310	0.0291
69	0.0363	0.0368	0.0371	0.0350	0.0359	0.0357	0.0361	0.0350	0.0337	0.0321
70	0.0396	0.0400	0.0382	0.0403	0.0393	0.0382	0.0391	0.0373	0.0361	0.0357
71	0.0442	0.0444	0.0420	0.0419	0.0417	0.0429	0.0410	0.0422	0.0396	0.0416
72	0.0472	0.0491	0.0464	0.0479	0.0471	0.0457	0.0458	0.0451	0.0454	0.0443
73	0.0543	0.0522	0.0519	0.0512	0.0517	0.0503	0.0524	0.0492	0.0506	0.0501
74	0.0605	0.0563	0.0574	0.0581	0.0573	0.0558	0.0569	0.0504	0.0549	0.0534
75	0.0631	0.0649	0.0600	0.0622	0.0643	0.0643	0.0623	0.0566	0.0589	0.0593
76	0.0692	0.0686	0.0686	0.0711	0.0691	0.0681	0.0702	0.0656	0.0623	0.0665
77	0.0761	0.0737	0.0781	0.0757	0.0724	0.0749	0.0730	0.0721	0.0707	0.0700
78	0.0786	0.0806	0.0804	0.0783	0.0790	0.0806	0.0806	0.0788	0.0791	0.0793
7 9	0.0868	0.0890	0.0865	0.0892	0.0871	0.0872	0.0875	0.0871	0.0858	0.0887
80	0.0961	0.0928	0.0956	0.0954	0.0945	0.0943	0.0962	0.0917	0.0912	0.0968
81	0.0989	0.1018	0.0987	0.1026	0.1035	0.1029	0.1031	0.0959	0.1035	0.1016
82	0.1099	0.1108	0.1110	0.1065	0.1063	0.1079	0.1105	0.1105	0.1105	0.1121
83	0.1137	0.1194	0.1164	0.1170	0.1165	0.1223	0.1182	0.1162	0.1170	0.1213
84	0.1287	0.1282	0.1287	0.1273	0.1312	0.1292	0.1298	0.1293	0.1291	0.1344
85	0.1343	0.1356	0.1342	0.1409	0.1403	0.1450	0.1450	0.1392	0.1369	0.1370
86	0.1479	0.1483	0.1529	0.1526	0.1494	0.1567	0.1502	0.1491	0.1493	0.1560
87	0.1569	0.1656	0.1584	0.1630	0.1609	0.1630	0.1649	0.1628	0.1636	0.1699
88	0.1753	0.1670	0.1771	0.1717	0.1708	0.1826	0.1789	0.1685	0.1719	0.1802
89	0.1838	0.1818	0.1819	0.1909	0.1854	0.1900	0.1881	0.1814	0.1695	0.1891
90	0.2016	0.1902	0.2005	0.1942	0.1948	0.2119	0.2038	0.1856	0.1900	0.2085
91	0.1999	0.2112	0.2099	0.2045	0.2108	0.2268	0.2225	0.2081	0.2131	0.2387
92	0.2323	0.2157	0.2163	0.2169	0.2343	0.2307	0.2373	0.2326	0.2241	0.2352
93	0.2345	0.2362	0.2458	0.2536	0.2319	0.2247	0.2342	0.2508	0.2386	0.2449
94	0.2385	0.2355	0.2554	0.2784	0.2779	0.2643	0.2556	0.2659	0.2631	0.2534
95	0.2453	0.2537	0.2522	0.2890	0.3084	0.2836	0.3171	0.2608	0.2697	0.2703
96	0.2460	0.2249	0.2315	0.3203	0.2751	0.3000	0.2855	0.2968	0.2862	0.2958
97	0.2459	0.2748	0.2141	0.3395	0.2948	0.3018	0.3048	0.3089	0.3333	0.2896
98	0.2956	0.2986	0.2024	0.3284	0.2917	0.2773	0.3329	0.2647	0.3162	0.3429
99	0.2649	0.3012	0.2888	0.3850	0.3855	0.3548	0.3537	0.3336	0.3167	0.2983

Continuation of table 7

	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
1	0.0015	0.0013	0.0013	0.0013	0.0012	0.0011	0.0012	0.0009	0.0008	0.0007
2	0.0005	0.0006	0.0005	0.0005	0.0004	0.0005	0.0004	0.0004	0.0004	0.0004
3	0.0004	0.0004	0.0004	0.0003	0.0003	0.0005	0.0003	0.0003	0.0004	0.0003
4	0.0003	0.0003	0.0003	0.0002	0.0003	0.0002	0.0002	0.0002	0.0002	0.0002
5	0.0003	0.0002	0.0002	0.0001	0.0002	0.0003	0.0003	0.0002	0.0002	0.0002
6	0.0003	0.0002	0.0003	0.0003	0.0002	0.0002	0.0002	0.0003	0.0002	0.0002
7	0.0003	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001	0.0002	0.0001	0.0002
8	0.0001	0.0002	0.0002	0.0001	0.0002	0.0002	0.0001	0.0001	0.0002	0.0002
9	0.0002	0.0001	0.0003	0.0002	0.0001	0.0001	0.0002	0.0001	0.0002	0.0001
10	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001	0.0002	0.0001	0.0001
11	0.0002	0.0001	0.0002	0.0002	0.0002	0.0002	0.0001	0.0001	0.0001	0.0002
12	0.0002	0.0001	0.0002	0.0002	0.0001	0.0002	0.0001	0.0002	0.0002	0.0001
13	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001
14	0.0002	0.0002	0.0003	0.0003	0.0002	0.0002	0.0002	0.0003	0.0002	0.0003
15	0.0004	0.0002	0.0003	0.0004	0.0003	0.0002	0.0003	0.0002	0.0003	0.0003
16	0.0005	0.0004	0.0004	0.0004	0.0004	0.0006	0.0005	0.0003	0.0004	0.0004
17	0.0006	0.0007	0.0004	0.0005	0.0004	0.0006	0.0005	0.0003	0.0005	0.0005
18	0.0005	0.0006	0.0005	0.0006	0.0005	0.0006	0.0005	0.0006	0.0005	0.0005
19	0.0006	0.0006	0.0007	0.0007	0.0006	0.0005	0.0006	0.0005	0.0007	0.0006
20	0.0007	0.0006	0.0007	0.0008	0.0008	0.0007	0.0006	0.0007	0.0007	0.0006
21	0.0008	0.0008	0.0006	0.0007	0.0007	0.0006	0.0007	0.0007	0.0008	0.0005
22	0.0006	0.0006	0.0007	0.0007	0.0008	0.0007	0.0007	0.0006	0.0007	0.0007
23	0.0006	0.0007	0.0007	0.0007	0.0007	0.0007	0.0008	0.0007	0.0007	0.0006
$\bf 24$	0.0007	0.0007	0.0008	0.0008	0.0007	0.0008	0.0007	0.0007	0.0007	0.0008
25	0.0007	0.0006	0.0009	0.0007	0.0006	0.0007	0.0007	0.0009	0.0006	0.0007
26	0.0007	0.0008	0.0007	0.0007	0.0008	0.0006	0.0009	0.0007	0.0007	0.0006
27	0.0008	0.0006	0.0007	0.0006	0.0006	0.0005	0.0006	0.0006	0.0007	0.0008
28	0.0008	0.0009	0.0009	0.0007	0.0006	0.0005	0.0008	0.0007	0.0008	0.0008
29	0.0007	0.0008	0.0008	0.0009	0.0008	0.0007	0.0007	0.0006	0.0006	0.0006
30	0.0009	0.0008	0.0008	0.0009	0.0008	0.0008	0.0008	0.0008	0.0006	0.0007
31	0.0009	0.0009	0.0010	0.0008	0.0007	0.0008	0.0008	0.0008	0.0008	0.0007
32	0.0008	0.0010	0.0009	0.0008	0.0010	0.0008	0.0008	0.0008	0.0007	0.0008
33	0.0009	0.0007	0.0009	0.0010	0.0009	0.0010	0.0008	0.0009	0.0008	0.0007
34	0.0011	0.0010	0.0009	0.0012	0.0011	0.0011	0.0007	0.0008	0.0010	0.0008
35	0.0010	0.0010	0.0011	0.0010	0.0010	0.0012	0.0012	0.0009	0.0007	0.0009
36	0.0012	0.0012	0.0012	0.0011	0.0012	0.0012	0.0009	0.0010	0.0007	0.0010
37	0.0012	0.0014	0.0012	0.0011	0.0012	0.0011	0.0012	0.0011	0.0010	0.0010
38	0.0013	0.0013	0.0012	0.0012	0.0013	0.0014	0.0012	0.0013	0.0011	0.0012
39	0.0013	0.0014	0.0013	0.0017	0.0012	0.0011	0.0012	0.0012	0.0011	0.0012
40	0.0015	0.0015	0.0015	0.0017	0.0015	0.0016	0.0013	0.0014	0.0013	0.0015
41	0.0015	0.0015	0.0017	0.0014	0.0016	0.0016	0.0017	0.0015	0.0015	0.0015
42	0.0019	0.0018	0.0019	0.0017	0.0020	0.0018	0.0018	0.0015	0.0017	0.0017
43	0.0020	0.0019	0.0021	0.0021	0.0020	0.0022	0.0019	0.0019	0.0020	0.0020
44	0.0022	0.0023	0.0022	0.0019	0.0021	0.0021	0.0021	0.0021	0.0021	0.0019
45	0.0025	0.0026	0.0023	0.0024	0.0025	0.0023	0.0022	0.0020	0.0022	0.0023
46	0.0030	0.0026	0.0027	0.0027	0.0027	0.0027	0.0025	0.0026	0.0024	0.0023
47	0.0031	0.0028	0.0029	0.0029	0.0030	0.0030	0.0030	0.0027	0.0026	0.0029
48	0.0033	0.0030	0.0033	0.0035	0.0032	0.0028	0.0037	0.0030	0.0029	0.0028
49	0.0035	0.0036	0.0039	0.0039	0.0034	0.0034	0.0036	0.0031	0.0033	0.0033
50	0.0039	0.0038	0.0040	0.0046	0.0038	0.0038	0.0038	0.0036	0.0039	0.0038

Continuation of table 7

	1000	1001	1000	1000	1004	1005	1000	1997	1998	1999
	1990	1991	1992	1993	1994	1995	1996			
51	0.0045	0.0046	0.0043	0.0044	0.0043	0.0044	0.0044	0.0040	0.0044	0.0041
52	0.0049	0.0049	0.0049	0.0051	0.0046	0.0050	0.0048	0.0043	0.0045	0.0047
53	0.0059	0.0058	0.0054	0.0052	0.0056	0.0049	0.0055	0.0053	0.0051	0.0047
54	0.0061	0.0068	0.0062	0.0059	0.0056	0.0056	0.0058	0.0056	0.0057	0.0056
55	0.0072	0.0071	0.0063	0.0064	0.0063	0.0061	0.0059	0.0064	0.0065	0.0064
56	0.0085	0.0077	0.0076	0.0079	0.0073	0.0072	0.0069	0.0067	0.0066	0.0067
57	0.0094	0.0085	0.0091	0.0086	0.0084	0.0083	0.0079	0.0079	0.0068	0.0077
58	0.0106	0.0096	0.0095	0.0101	0.0096	0.0087	0.0089	0.0086	0.0079	0.0078
59	0.0120	0.0113	0.0102	0.0111	0.0100	0.0099	0.0096	0.0093	0.0091	0.0090
60	0.0126	0.0122	0.0123	0.0127	0.0114	0.0113	0.0117	0.0105	0.0104	0.0103
61	0.0134	0.0138	0.0130	0.0134	0.0132	0.0133	0.0128	0.0120	0.0123	0.0116
62	0.0157	0.0164	0.0148	0.0150	0.0147	0.0140	0.0133	0.0132	0.0137	0.0128
63	0.0168	0.0174	0.0172	0.0174	0.0166	0.0160	0.0161	0.0146	0.0147	0.0134
64	0.0189	0.0200	0.0186	0.0194	0.0187	0.0181	0.0180	0.0164	0.0171	0.0164
65	0.0214	0.0199	0.0209	0.0209	0.0205	0.0202	0.0193	0.0196	0.0186	0.0183
66	0.0242	0.0236	0.0222	0.0235	0.0215	0.0229	0.0213	0.0208	0.0206	0.0204
67	0.0266	0.0254	0.0245	0.0260	0.0244	0.0247	0.0251	0.0236	0.0245	0.0223
68	0.0283	0.0283	0.0282	0.0298	0.0280	0.0268	0.0270	0.0261	0.0266	0.0250
69	0.0345	0.0314	0.0306	0.0308	0.0306	0.0299	0.0298	0.0289	0.0288	0.0285
70	0.0353	0.0348	0.0350	0.0343	0.0342	0.0338	0.0313	0.0315	0.0297	0.0308
71	0.0392	0.0392	0.0385	0.0386	0.0363	0.0370	0.0375	0.0344	0.0349	0.0335
72	0.0462	0.0433	0.0424	0.0442	0.0408	0.0411	0.0403	0.0392	0.0376	0.0369
73	0.0471	0.0482	0.0465	0.0478	0.0449	0.0442	0.0461	0.0436	0.0429	0.0418
74	0.0556	0.0528	0.0537	0.0529	0.0493	0.0499	0.0496	0.0487	0.0489	0.0471
75	0.0594	0.0557	0.0544	0.0585	0.0558	0.0553	0.0542	0.0525	0.0512	0.0534
76	0.0630	0.0650	0.0654	0.0641	0.0618	0.0614	0.0616	0.0586	0.0570	0.0556
77	0.0675	0.0702	0.0669	0.0716	0.0659	0.0683	0.0681	0.0616	0.0635	0.0630
78	0.0777	0.0757	0.0742	0.0775	0.0728	0.0758	0.0753	0.0736	0.0684	0.0707
79	0.0850	0.0828	0.0813	0.0864	0.0826	0.0818	0.0838	0.0789	0.0786	0.0775
80	0.0938	0.0935	0.0907	0.0924	0.0871	0.0885	0.0920	0.0870	0.0875	0.0823
81	0.1021	0.0985	0.0971	0.1045	0.0962	0.0969	0.0944	0.0986	0.0940	0.0936
82	0.1087	0.1091	0.1035	0.1123	0.1070	0.1097	0.1056	0.1031	0.1043	0.1027
$\begin{array}{c} \bf 83 \\ \bf 84 \end{array}$	$0.1166 \\ 0.1310$	0.1162 0.1313	0.1165 0.1259	0.1192 0.1379	0.1155 0.1312	0.1227 0.1277	0.1201 0.1309	0.1126 0.1258	$0.1150 \\ 0.1244$	0.1134 0.1281
85	0.1310 0.1403	0.1313 0.1422	0.1259 0.1387	0.1379 0.1438	0.1312 0.1390	0.1277 0.1361	0.1309 0.1410	0.1258 0.1374	0.1244 0.1375	0.1281 0.1374
86	0.1403 0.1494	0.1422 0.1512	0.1548	0.1438 0.1618	0.1390 0.1456	0.1501 0.1515	0.1410 0.1511	0.1374 0.1407	0.1575 0.1512	0.1374 0.1453
87	0.1494 0.1669	0.1512 0.1645	0.1548 0.1618	0.1618 0.1649	0.1430 0.1615	0.1515 0.1654	0.1511 0.1694	0.1407 0.1609	0.1512 0.1597	0.1453 0.1644
88	0.1826	0.1045 0.1780	0.1016 0.1766	0.1049 0.1847	0.1015 0.1760	0.1034 0.1848	0.1094 0.1798	0.1009 0.1798	0.1357 0.1755	0.1044 0.1747
89	0.1820 0.1891	0.1780 0.1939	0.1760 0.1830	0.1847 0.2098	0.1760	0.1948 0.1923	0.1798 0.1935	0.1798 0.1952	0.1735 0.1936	0.1747 0.1985
90	0.1091 0.2036	0.1939 0.1979	0.1030 0.2036	0.2307	0.1668 0.2163	0.1925 0.2060	0.1935 0.2097	0.1932 0.2038	0.1930 0.2103	0.1365 0.2159
91	0.2030 0.2149	0.1979 0.2201	0.2030 0.2112	0.2409	0.2103 0.2182	0.2000 0.2283	0.2097 0.2287	0.2038 0.2221	0.2103 0.2440	0.2139 0.2329
92	0.2143 0.2463	0.2400	0.2112 0.2317	0.2527	0.2132 0.2434	0.2263 0.2378	0.2513	0.2333	0.2384	0.2620
93	0.2463 0.2456	0.2400 0.2518	0.2517 0.2539	0.2527 0.2784	0.2434 0.2584	0.2378 0.2458	0.2313 0.2420	0.2333 0.2442	0.2564 0.2644	0.2020 0.2716
94	0.2450 0.2661	0.2518 0.2783	0.2823	0.2784 0.2847	0.2364 0.2748	0.2436 0.2736	0.2420 0.2870	0.2442 0.2725	0.2544 0.2541	0.2710 0.2956
9 5	0.2001 0.2937	0.2765 0.3235	0.2823 0.2901	0.2347	0.2748 0.2964	0.2730	0.3069	0.2125 0.2926	0.2804	0.2950 0.2953
96	0.2902	0.3255 0.3165	0.2969	0.3133	0.2364 0.3360	0.3025 0.3125	0.3003 0.2915	0.3403	0.2304 0.3194	0.2353 0.3157
97	0.2850	0.3103 0.3210	0.2909 0.3071	0.3135 0.3206	0.3724	0.3125 0.3764	0.2315 0.3327	0.3433	0.3194 0.3166	0.3137 0.3721
98	0.3390	0.3210 0.3707	0.3589	0.3200 0.3413	0.3724 0.3583	0.3704 0.3918	0.3321 0.3421	0.3544	0.3553	0.3619
99	0.4090	0.3167	0.3872	0.4016	0.3522	0.4335	0.4304	0.4052	0.4274	0.4306
	0.1000	0.0101	0.0012	0.1010	0.0022	0.1000	U. 100 I	0.1002	U. 12 1	0.1000

Continuation of table 7

	2000	2001	2002	2002	2004	2005	2006	2007	2008	2000	2010
	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
1	0.0008	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0006	0.0006	0.0005	0.0005
2	0.0004	0.0004	0.0004	0.0003	0.0003	0.0003	0.0003	0.0002	0.0002	0.0002	0.0002
3	0.0003	0.0003	0.0003	0.0002	0.0002	0.0003	0.0002	0.0002	0.0002	0.0002	0.0002
4	0.0002	0.0003	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001	0.0002	0.0002	0.0002
5 6	0.0001 0.0002	0.0002 0.0002	0.0001	0.0002 0.0002	0.0001 0.0001	0.0001 0.0002	$0.0001 \\ 0.0001$	0.0002 0.0001	0.0001	0.0001	0.0001 0.0001
7	0.0002 0.0001	0.0002 0.0001	$0.0001 \\ 0.0002$	0.0002 0.0001	0.0001	0.0002 0.0001	0.0001	0.0001	$0.0001 \\ 0.0001$	$0.0001 \\ 0.0002$	0.0001 0.0001
8	0.0001 0.0002	0.0001	0.0002 0.0002	0.0001 0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002 0.0002	0.0001
9	0.0002 0.0002	0.0001	0.0002 0.0001	0.0002 0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0001
10	0.0002 0.0001	0.0001 0.0002	0.0001 0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
11	0.0001	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
12	0.0001	0.0002	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
13	0.0002	0.0002	0.0002	0.0001	0.0001	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001
14	0.0002	0.0002	0.0002	0.0001	0.0001	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001
15	0.0001	0.0003	0.0003	0.0002	0.0003	0.0002	0.0002	0.0002	0.0001	0.0001	0.0001
16	0.0004	0.0004	0.0003	0.0003	0.0003	0.0002	0.0003	0.0002	0.0002	0.0002	0.0001
17	0.0004	0.0003	0.0004	0.0004	0.0003	0.0004	0.0004	0.0003	0.0002	0.0003	0.0003
18	0.0006	0.0005	0.0005	0.0003	0.0004	0.0004	0.0004	0.0005	0.0003	0.0004	0.0003
19	0.0008	0.0006	0.0007	0.0006	0.0005	0.0004	0.0003	0.0005	0.0003	0.0004	0.0004
20	0.0006	0.0007	0.0008	0.0006	0.0005	0.0005	0.0004	0.0005	0.0005	0.0004	0.0004
21	0.0008	0.0005	0.0006	0.0005	0.0005	0.0005	0.0006	0.0006	0.0005	0.0005	0.0004
22	0.0007	0.0005	0.0006	0.0005	0.0005	0.0004	0.0005	0.0005	0.0006	0.0004	0.0005
23	0.0007	0.0008	0.0006	0.0007	0.0004	0.0006	0.0005	0.0004	0.0004	0.0005	0.0005
24	0.0007	0.0005	0.0007	0.0005	0.0006	0.0004	0.0005	0.0006	0.0005	0.0004	0.0005
25	0.0006	0.0007	0.0005	0.0006	0.0005	0.0006	0.0004	0.0004	0.0004	0.0005	0.0003
26	0.0007	0.0006	0.0006	0.0005	0.0006	0.0005	0.0005	0.0004	0.0004	0.0006	0.0005
27	0.0006	0.0007	0.0006	0.0006	0.0005	0.0005	0.0006	0.0004	0.0005	0.0006	0.0004
28	0.0006	0.0006	0.0005	0.0007	0.0006	0.0005	0.0004	0.0005	0.0006	0.0003	0.0004
29	0.0006	0.0007	0.0006	0.0007	0.0007	0.0006	0.0005	0.0005	0.0005	0.0005	0.0003
30	0.0007	0.0007	0.0008	0.0007	0.0006	0.0006	0.0006	0.0007	0.0007	0.0005	0.0005
31	0.0007	0.0008	0.0007	0.0007	0.0007	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
32	0.0007	0.0008	0.0007	0.0007	0.0007	0.0006	0.0006	0.0005	0.0007	0.0005	0.0005
33	0.0008	0.0007	0.0009	0.0005	0.0006	0.0006	0.0007	0.0007	0.0006	0.0006	0.0006
34	0.0008	0.0008	0.0007	0.0010	0.0007	0.0007	0.0007	0.0006	0.0007	0.0006	0.0007
35	0.0008	0.0009	0.0009	0.0008	0.0008	0.0008	0.0006	0.0008	0.0007	0.0006	0.0007
36	0.0010	0.0009	0.0009	0.0008	0.0008	0.0010	0.0008	0.0009	0.0007	0.0007	0.0006
37	0.0010	0.0010	0.0010	0.0009	0.0010	0.0007	0.0009	0.0008	0.0008	0.0007	0.0006
38	0.0009	0.0011	0.0011	0.0009	0.0010	0.0010	0.0009	0.0008	0.0009	0.0009	0.0007
39	0.0013	0.0012	0.0012	0.0012	0.0010	0.0009	0.0010	0.0011	0.0010	0.0008	0.0011
40	0.0012	0.0013	0.0013	0.0013	0.0010	0.0011	0.0010	0.0011	0.0012	0.0009	0.0011
$\begin{array}{c} 41 \\ 42 \end{array}$	0.0015	0.0013	0.0012	0.0014	0.0014	0.0012	$0.0012 \\ 0.0012$	0.0012	0.0012 0.0012	0.0011 0.0012	0.0010
$\begin{array}{c} 42 \\ 43 \end{array}$	0.0016 0.0021	$0.0016 \\ 0.0018$	$0.0015 \\ 0.0018$	0.0016 0.0018	$0.0015 \\ 0.0016$	0.0013 0.0016	0.0012 0.0015	0.0010 0.0014	0.0012 0.0015	0.0012 0.0012	0.0011 0.0013
$\frac{43}{44}$	0.0021 0.0020	0.0018 0.0019	0.0018 0.0019	0.0018 0.0020	0.0018	0.0010 0.0017	0.0013 0.0014	0.0014 0.0012	0.0015 0.0015	0.0012 0.0016	0.0013 0.0012
45	0.0020	0.0013 0.0023	0.0013 0.0021	0.0020 0.0021	0.0013 0.0021	0.0017	0.0014 0.0016	0.0012 0.0015	0.0015 0.0016	0.0010	0.0012
$\frac{45}{46}$	0.0023 0.0024	0.0023 0.0024	0.0021 0.0024	0.0021 0.0026	0.0021 0.0019	0.0018	0.0010 0.0020	0.0013 0.0019	0.0010 0.0019	0.0018 0.0017	0.0016 0.0015
47	0.0024 0.0026	0.0024 0.0029	0.0024 0.0027	0.0020 0.0024	0.0013	0.0013 0.0021	0.0020	0.0013	0.0013 0.0021	0.0017	0.0013
48	0.0020	0.0023	0.0021	0.0024	0.0020	0.0025	0.0025	0.0015	0.0021	0.0020	0.0022
49	0.0034	0.0028	0.0032	0.0032	0.0029	0.0030	0.0028	0.0025	0.0026	0.0024	0.0023
50	0.0037	0.0039	0.0035	0.0036	0.0033	0.0035	0.0031	0.0029	0.0028	0.0027	0.0029
		5.5000	5.5000	5.5000	5.5000	5.5000	5.5001	J.J.J.	2.2020	J.J.J.	0.0020

Continuation of table 7

	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
51	0.0039	0.0041	0.0038	0.0039	0.0038	0.0035	0.0035	0.0030	0.0034	0.0032	0.0031
52	0.0040	0.0042	0.0042	0.0044	0.0040	0.0040	0.0039	0.0035	0.0040	0.0036	0.0033
53	0.0052	0.0049	0.0044	0.0048	0.0051	0.0039	0.0041	0.0039	0.0039	0.0038	0.0036
54	0.0054	0.0052	0.0050	0.0051	0.0048	0.0048	0.0043	0.0043	0.0043	0.0042	0.0040
55 50	0.0063	0.0057	0.0062	0.0055	0.0056	0.0053	0.0049	0.0046	0.0048	0.0046	0.0049
56	0.0070	0.0066	0.0062	0.0062	0.0058	0.0057	0.0056	0.0051	0.0055	0.0056	0.0051
57 58	0.0079 0.0080	0.0071 0.0085	0.0072 0.0077	0.0069 0.0085	0.0064 0.0078	$0.0058 \\ 0.0067$	0.0061 0.0066	0.0063 0.0066	0.0058 0.0062	$0.0060 \\ 0.0062$	0.0057 0.0062
58 59	0.0080 0.0097	0.0085 0.0088	0.0077	0.0085 0.0084					0.0062 0.0072	0.0062 0.0067	0.0062 0.0070
					0.0082	0.0073	0.0071	0.0074			
60	0.0093 0.0108	0.0100	0.0098	0.0091	0.0096	0.0085	0.0081	0.0082 0.0085	$0.0080 \\ 0.0090$	0.0073 0.0087	0.0077
$\begin{array}{c} 61 \\ 62 \end{array}$	0.0108 0.0120	0.0102 0.0117	$0.0101 \\ 0.0115$	$0.0106 \\ 0.0116$	0.0097 0.0108	0.0091 0.0116	0.0093 0.0106	0.0089	0.0090 0.0095	0.0087	0.0080 0.0089
63	0.0120 0.0148	0.0117 0.0119	0.0113 0.0132	0.0116 0.0125	0.0108 0.0120	0.0116 0.0115	0.0106	0.0099 0.0113	0.0095 0.0102	0.0098 0.0105	0.0089 0.0104
64	0.0148 0.0153	0.0119 0.0159	0.0152 0.0151	0.0125 0.0135	0.0120 0.0132	0.0115 0.0126	0.0110 0.0124	0.0113 0.0118	0.0102 0.0114	0.0105 0.0116	0.0104 0.0113
$\begin{array}{c} 64 \\ 65 \end{array}$	0.0155 0.0171	0.0159 0.0164	0.0131 0.0163	0.0155 0.0155	0.0132 0.0148	0.0120 0.0143	0.0124 0.0131	0.0118 0.0132	0.0114 0.0131	0.0110 0.0132	0.0113 0.0133
66	0.0171 0.0202	0.0104 0.0180	0.0103 0.0173	0.0133 0.0176	0.0148 0.0163	0.0143 0.0170	0.0131 0.0145	0.0152 0.0153	0.0131 0.0143	0.0132 0.0138	0.0133 0.0144
67	0.0202 0.0223	0.0130 0.0215	0.0173	0.0176	0.0103 0.0178	0.0170	0.0145 0.0165	0.0160	0.0145 0.0150	0.0158 0.0152	0.0144 0.0154
68	0.0225 0.0246	0.0215 0.0236	0.0204 0.0227	0.0193 0.0212	0.0205	0.0170	0.0103 0.0192	0.0100 0.0174	0.0130 0.0174	0.0132 0.0174	0.0154 0.0166
69	0.0240 0.0254	0.0250 0.0254	0.0227 0.0257	0.0212 0.0244	0.0203 0.0234	0.0193 0.0213	0.0192 0.0206	0.0174	0.0174	0.0174	0.0186
70	0.0294	0.0294 0.0293	0.0237 0.0289	0.0244 0.0250	0.0254 0.0263	0.0213 0.0239	0.0200 0.0237	0.0198 0.0225	0.0133 0.0210	0.0104 0.0202	0.0180 0.0205
71	0.0234 0.0333	0.0233 0.0328	0.0304	0.0306	0.0284	0.0239 0.0279	0.0263	0.0249	0.0210 0.0234	0.0202 0.0232	0.0203 0.0219
72	0.0365	0.0326 0.0356	0.0354	0.0345	0.0204 0.0313	0.0213	0.0203 0.0291	0.0249 0.0272	0.0260	0.0232 0.0247	0.0213 0.0247
73	0.0303 0.0410	0.0350 0.0405	0.0398	0.0345 0.0396	0.0313 0.0364	0.0343	0.0291 0.0320	0.0212	0.0200 0.0293	0.0247 0.0298	0.0247 0.0281
74	0.0410	0.0450	0.0336	0.0409	0.0304 0.0415	0.0345 0.0375	0.0320	0.0346	0.0233 0.0317	0.0238	0.0295
75	0.0509	0.0495	0.0446	0.0447	0.0431	0.0436	0.0395	0.0345	0.0370	0.0325	0.0250 0.0362
76	0.0559	0.0544	0.0527	0.0531	0.0486	0.0464	0.0436	0.0444	0.0414	0.0421	0.0392
77	0.0620	0.0603	0.0527	0.0564	0.0400 0.0518	0.0404 0.0545	0.0430 0.0512	0.0444	0.0414 0.0470	0.0421 0.0457	0.0352 0.0457
78	0.0620 0.0681	0.0660	0.0679	0.0645	0.0611	0.0549	0.0572	0.0549	0.0540	0.0483	0.0522
79	0.0764	0.0724	0.0733	0.0720	0.0658	0.0671	0.0646	0.0623	0.0593	0.0576	0.0546
80	0.0833	0.0794	0.0824	0.0787	0.0746	0.0718	0.0720	0.0660	0.0654	0.0649	0.0622
81	0.0941	0.0908	0.0918	0.0901	0.0856	0.0854	0.0778	0.0741	0.0707	0.0730	0.0680
82	0.1030	0.0975	0.0965	0.0987	0.0905	0.0926	0.0871	0.0845	0.0837	0.0789	0.0771
83	0.1113	0.1133	0.1068	0.1069	0.0986	0.1014	0.0974	0.0932	0.0902	0.0895	0.0848
84	0.1303	0.1235	0.1251	0.1194	0.1107	0.1115	0.1049	0.1074	0.1053	0.0972	0.0931
85	0.1372	0.1287	0.1368	0.1338	0.1238	0.1213	0.1133	0.1147	0.1113	0.1092	0.1068
86	0.1445	0.1408	0.1451	0.1437	0.1355	0.1302	0.1374	0.1250	0.1247	0.1217	0.1204
87	0.1631	0.1603	0.1616	0.1660	0.1522	0.1523	0.1477	0.1404	0.1318	0.1316	0.1266
88	0.1737	0.1706	0.1806	0.1759	0.1673	0.1614	0.1618	0.1608	0.1550	0.1485	0.1499
89	0.1891	0.1941	0.1878	0.1898	0.1866	0.1844	0.1742	0.1681	0.1702	0.1611	0.1619
90	0.2011	0.2048	0.2023	0.2165	0.1934	0.1974	0.1828	0.1994	0.1857	0.1792	0.1756
91	0.2359	0.2334	0.2268	0.2247	0.2140	0.2120	0.2165	0.2110	0.2036	0.1942	0.2035
92	0.2450	0.2408	0.2299	0.2499	0.2309	0.2324	0.2279	0.2358	0.2276	0.2227	0.2191
93	0.2540	0.2553	0.2523	0.2717	0.2355	0.2653	0.2579	0.2302	0.2524	0.2425	0.2339
94	0.2785	0.2922	0.2869	0.2858	0.2710	0.2886	0.2739	0.2585	0.2651	0.2729	0.2675
95	0.3023	0.2798	0.2984	0.3114	0.2888	0.2890	0.3027	0.2881	0.2904	0.2773	0.2787
96	0.3245	0.3502	0.3082	0.3262	0.3148	0.3377	0.3173	0.3118	0.3255	0.2924	0.3143
97	0.3056	0.3333	0.3568	0.3373	0.3520	0.3074	0.3059	0.3320	0.3339	0.3183	0.2968
98	0.3789	0.3632	0.3780	0.3631	0.3595	0.4040	0.3325	0.3039	0.3310	0.3232	0.3539
99	0.4206	0.4068	0.3839	0.4155	0.3947	0.4041	0.4142	0.3930	0.4031	0.3657	0.4057