On the Perseverance of Discrimination and the Effectiveness of Affirmative Action

Master Thesis - Yara van Hal

1 Introduction

> Give a girl an education and introduce her properly into the world, and ten to one but she has the means of settling well, without further expense to anybody (Jane Austen in *Mansfield Park*, ch. 1, 1814).

Although nowadays women do have access to education, the odds are not as favourable to women as Ms. Austen probably would have hoped for, 200 years after she wrote *Mansfield Park*. In 2012 a little over one percent of the CEOs of the Fortune 500 Companies was black and merely four percent was female[1]. In that same year in the European Union, females accounted for only two and a half percent of the CEO positions[2]. In the UK currently one out of twelve members of the supreme court is female[3]. Discrimination is prohibited by law and frowned upon by most. Then why does this clear segregation at the top exist? Following Milgrom and Oster (1987), a labour market with persistent job discrimination cannot hold. Basic economic theory predicts that if a certain group of people is discriminated by some (or even most) employers, the underpaid and underemployed group will be hired by the other non-discriminating employers. Those employers seek the highest profits and are attracted by the low wages of the discriminated group. The labour demand for the discriminated group will grow and by consequence their wages will rise. However, empirical evidence shows different results. Although many have shed their light on this complex subject, no explanation has been accepted unanimously by economists. This paper addresses this well discussed issue from a new point of view in order to seek the answer to the question why segregation in the labour market is a persistent phenomenon.

[1] http://money.cnn.com/2012/03/22/news/companies/black-ceo/index.htm
http://postcards.blogs.fortune.cnn.com/2012/07/18/fortune-500-women-ceos-2/ (Last visited on March 11th 2013)

[2] http://www.europarl.europa.eu/news/en/headlines/content/20130301STO06143/html/Wonderful-to-be-a-woman-It-still-pays-more-to-be-a-man (Last visited on March 11th 2013)

[3] http://www.guardian.co.uk/law/2013/feb/15/supreme-court-denies-government-pressure(Last visited on March 11th 2013)

With a simple model it is shown that the existence of discrimination is more likely to happen than not. Not because of prejudiced employers but due to the existence of prejudice among employees. A rational manager who knows the abilities and the existence of prejudice among the employees will bear both in mind when deciding who to promote. It shows that even if the employees believe to have the same expected ability ex ante, one employee could have a far better chance of being promoted despite their actual ability. This will happen if a promotion affects the beliefs about the ability ex post of one employee more intensively than those of the other. Being promoted or not will affect the level of effort of the employees differently ex post. The results stated above will apply to many situations where both employees feel that one of them is (even slightly) favoured over the other.

To illustrate the above with an example, imagine a situation where a female and a male employee compete for the same promotion. Both have the same prejudices about their abilities. They believe that ex ante the manager shares this view. However, they both also believe that the man has a slight advantage and thus a higher probability to be promoted. If the manager, after observing the actual abilities, decides that the man will not get the promotion, the male employee will be disappointed. More disappointed than the female would be if she were to be passed over. He will be demotivated and his effort level will drop. Worse than her level would. The manager, who aims at the highest output, would then perhaps be better off promoting the man, even though the female employee has a better ability.

In the model described in this paper, two employees also compete for the same promotion. They are uncertain about their abilities, but they do know the range in which the level of their abilities lie. They form expectations accordingly. The manager decides who gets the promotion. Subsequently, the employees update their beliefs about their expected abilities. Ability and effort are assumed to be complements. Therefore, getting the promotion enhances the effort level of the promoted employee. On the other hand, the effort level of his counterpart is diminished. An equilibrium exists where one employee is three times more likely to be promoted, even though ex ante the employees have the same expected ability.

The setting of the model in this paper is based on the tournament model of Lazear and Rosen (1981). They show that incentives through promotions are desirable when monitoring output is costly. Inducing effort through a competitive environment will result in optimal levels of effort. Employees are motivated by the prospect of a promotion. The results found by Lazear and Rosen are confirmed in several studies. Ehrenberg and Bognanno (1990) conclude that the prize structure of a tournament influences performance. Promotion-based incentives are sometimes believed to be even more powerful than bonus-based incentives (Takahashi, 2006). Ferral (1996) states that the compensation for the cost of effort induced by the promotion depicts 30 percent of the annual salary of associates in major U.S. law firms. Nevertheless, there is also a downside to using tournaments as an incentive. Baker, Jensen and Murphy (1988) describe how an employee with excellent perspectives diminishes the incentives of his competing coworkers.

One aspect of promotions has not been addressed by the studies listed above: what will happen after the promotion? What happens to the promoted employee and to the former peers who did not get the promotion? Being passed over for promotion reduces the promotion incentive if their chances of being promoted in the future are also slim (Baker, Jensen and Murphy, 1988). The effect of the promotion, deception or motivation, is addressed in this study. The second part of the model discusses what happens to the effort levels of the employees if the manager commits to a promotion rule with affirmative action. It turns out that a perfectly fair promotion rule, where only ability matters, is preferred solely by the manager and surprisingly not by the employees. The employees prefer some, but not total, advantage. These preferences occur even if the promotion does not alter utility because of accompanying raise or prestige.

A few important assumptions are made in the model. First, ability and effort are assumed to be complements. This assumption is consistent with other literature, primarily in the field of social-psychology. Bandura (1977) argues that the level of effort depends on self-efficacy. Bandura (1997, p. 2) defines self-efficacy as

> "beliefs in one's capabilities to organize and execute the courses of action required to manage prospective situations."

The level of self-efficacy affects motivation. People with low self-efficacy for a task want to avoid that task. The other way around, with a high self-esteem regarding a task or job, one puts in a high level of effort (Bandura, 1977). Moreover, it also works the other way around. The more effort put in, the more a positive result contributes to self-efficacy. This was concluded by Salomon (1984) who found a positive correlation between self-efficacy and mental effort.

Second, employees are assumed not to have perfect information about their own ability. Although standard economic theory assumes perfect knowledge of one's ability, it is more realistic that a person only has certain beliefs about their own ability. The assumption of imperfect knowledge of one's own ability is already common among social-psychologists. Schunk (1991) states that people update their beliefs by acquiring information from, among other factors, their accomplishments and experiences. Dominguez-Martinez and Swank (2009) describe several papers in the field of social-psychology on this specific topic and integrate those findings in an economic model of self-assesment. Also, the ubiquitous existence of overconfidence is showed in several papers, which also proves that accurate self-assesment is rare (Barber and Odean, 2001; Croson and Gneezy, 2009; Niederle and Vesterlund, 2005).[4] In the model of this paper it is assumed that the employees update their beliefs about their ability when they receive a message (being promoted or not) from the manager.

The paper is organized as follows. The next Section discusses related literature. The third Section presents the model in which the existence of discrimination in equilibrium is explained. The fourth Section discusses the model with affirmative action. In the fifth Section concluding remarks are made.

2 Related Literature

The reason why discrimination in the workplace persists is a thoroughly researched topic. Becker (1957, 1971, 1993) and Arrow (1972, 1998) wrote various papers and books on the topic of why and how discrimination is as persistent as it is. Becker states that disappearance of discrimination in the long-run due to the competition of non-discriminating employers could happen (1957, 1971). However, it depends

[4] Men tend to be more overconfident than women (Among others, Barber and Odean, 2001). The existence of prejudice among the employees, such as overconfidence, is therefore reasonable. Furthermore, Guimond et al. (2006) concludes that men and women rely on gender stereotypes to define themselves. The effect of commonly shared prejudice on the model is explained in the appendix.

on the distribution of tastes for discrimination among employers and the production functions of the firms (1993). Moreover, discrimination in the long-run depends also on customers and employees, whose discrimination cannot be competed away. Arrow (1998) states that the explanation of discrimination due to discriminating employees instead of employers explains segregation within industries but not by occupation. The least prejudiced workers will work with the discriminated workers under the least prejudiced employers. Discrimination will thus lead to segregation but not to a significant large wage gap.

Both Milgrom and Oster (1987) and Bjerk (2008) explain segregation at the top positions in firms due to the smaller probability of being promoted for the minority workers in the lower levels of a firm. Milgrom and Oster (1987) give the *Invisibility Hypothesis* as an explanation. Promotion signals ability to other firms. Victims of discrimination are more likely to be *Invisible* when they enter the labour market. Employers have an incentive to hide able *Invisibles* from other firms by not promoting them. Bjerk (2008) states that "sticky floors" rather than glass ceilings are the reason for the lack of diversity at the higher levels of firms. Victims of discrimination have less chance to be promoted in lower levels of the firm since they are for instance less able to signal their ability in the beginning stages of their career. He concludes that discrimination at the top level does not truly exist.

Another explanation comes from Akerlof (1985). If there are more prejudiced customers than victims of discrimination, then the cost for a non-discriminating firm of conforming to the discriminating opinion of customers is less than maintaining a non-discriminating profile (Akerlof, 1985). This paper confirms that study by stating that the managers maximize profit by acting conform the prejudice of their employees.

Affirmative action policies are installed to help promote the opportunities of women and minorities within for instance the workplace. Coate and Loury (1993) describe a model how self-fulfilling prophecies exist when it comes to affirmative action. They argue that affirmative action policies do not eliminate stereotypes. First, in the situation where the workers belonging to the majority are chosen more often for skills acquired jobs, the minority workers do not want to acquire the skills. Since the chance that they will be promoted is slim, skills are not likely to be necessary. Then as a result, the probability that a minority workers is skilled decreases.

When the policy is implemented, the minority workers are still not motivated to acquire skills since they will be promoted because of the group they belong to, no matter the skills. Then the ex ante beliefs of the employer are confirmed. Fryer and Loury (2005) confirm that effort deteriorates when certain affirmative action policies are applied. On the other hand, Holzer and Neumark (1999) show that when affirmative action is used in hiring it lessens the credentials of the minority worker somewhat, but does not weakens his performance. Kalev, Kelly and Dobbin (2006) show through a large data analysis that many types of affirmative action policies only have moderate effects. To sum up, the effects of such policies is controversial and not unanimously accepted to generate the effects aimed for. This paper shows that only the manager prefers a perfectly fair promoting rule where only ability matters. The employees do not and they resist such a rule. In the appendix it is shown that when prejudice exist, even the manager does not want to commit to a complete fair promoting rule.

3 The Model

Consider two employees supervised by a manager. The manager has superior information about the ability of the employees. Each employee chooses a level of effort e_i. The output of effort y_i is both determined by effort and an employee's ability: $y_i = a_i e_i$. Ability and effort are thus assumed to be complements. The manager obtains perfect information about the ability of his subordinates. However, an employee only has certain beliefs about his own ability and the ability of his fellow co-worker. The employees share the perception of the distribution of a_1 en a_2; a_1 is uniformly distributed between 0 and β, as a_2 is uniformly distributed between 0 and 1. In this section $\beta = 1$. The distribution of a_1 and a_2 are equal. If $\beta > 1$, $(\beta - 1)$ reflects prejudices about the ability of employee 1. Important to notice here is that the employees believe that the manager shares their perception ex ante as well. Thus, the ex ante believed probability of employee 1 having a higher ability than his fellow employee is the probability that $a_1 \geq \frac{1}{2}$. It turns out that, even when $\beta = 1$, the situation is much more likely where one employee has a much larger probability to be promoted than the other. The situation $\beta > 1$ is discussed in the appendix.

The preferences of the employee are determined by the expected output and the cost of effort.[5]

$$U_{A_i(e_i)} = E\left(a_i e_i\right) - \frac{1}{2}e_i^2 \tag{1}$$

The manager wants to maximize the expected output:

$$U_M = \sum_{i=1}^{2} y_i = a_1 e_1 + a_2 e_2 \tag{2}$$

The timing of the game is as follows. First, nature draws a_1 and a_2. The manager observes a_1 and a_2. The employees, however, do not. After observing the types (i.e. ability) of the employees, the manager makes a promotion decision m ($m = \{1, 2\}$). He will promote either one of the employees. The decision affects the effort level chosen by the employees, since the effort level equals the expected ability: $e^* = (E\left(a_i \mid m\right))$. The promotion decision does not affect the employees' work, but it will affect the level of effort chosen when the decision depends on the ability of the employees. Beliefs are updated according to Bayes' rule. Once the promotion decision has been made and the beliefs have been updated, the employees choose their effort level. Finally, payoffs are realized.

In the next section perfect Bayesian-Nash equilibria are identified. In those equilibria the effort choices of the employees are optimal given their beliefs about their abilities. The employees choose effort levels that according to their expected ability given the promotion decision ($e_i = E(a_i \mid m)$. Given the employees' effort choices, the manager's promotion decision is also optimal. We will identify equilibrium strategies in which the promotion decision depends on a_1 and a_2. Babbling equilibria are disregarded.

3.1 *Equilibrium Promotion Strategies*

The manager sends message $m = 1$ if the expected output of this action exceeds the expected output of sending $m = 2$. This is the case if the equation below holds:

$$a_1 E(a_1 \mid m = 1) + a_2 E(a_2 \mid m = 1) \geq a_1 E(a_1 \mid m = 2) + a_2 E(a_2 \mid m = 2),$$

[5] Recall the literature of Bandura mentioned in the introduction which supports the assumption that effort and ability are complements.

which leads to:

$$a_1 \geq a_2 * \frac{E(a_2 \mid m=2) - E(a_2 \mid m=1)}{E(a_1 \mid m=1) - E(a_1 \mid m=2)} \rightarrow a_1 \geq a_2 * t \tag{3}$$

Equation 3 can be regarded as the promotion rule. If $a_1 = a_2 * t$, the manager is indifferent between promoting employee 1 and 2. As can be seen in the equation above, t denotes the relative sensitivity of the employees to the promotion decision. If one examines the fraction in equation 3, it becomes clear that for $t < 1$, the reaction of employee 1 to the promotion decision is relatively stronger than the reaction of employee 2. Namely, if $t < 1$ the difference in response of employee 1, between whether he is promoted or not, is greater than the difference in response of employee 2:

$$E(a_1 \mid m=1) - E(a_1 \mid m=2) > E(a_2 \mid m=2) - E(a_2 \mid m=1)$$

It appears that, as described later, this difference is mainly due to the different responses of the employees if they do not get the promotion. Employee 1 is more disappointed when he is passed up than employee 2:

$$E(a_1 \mid m=2) < E(a_2 \mid m=1)$$

For $t > 1$, it is obviously the opposite.

The message strategy of the manager is characterized by a line straight through the origin. We are going to determine this line and describe the different equilibria.

Proposition 1 *Assume that $\beta = 1$. In this situation three equilibria exist where the promotion decision of the manager, depends on $a_1$and a_2. These strategies are characterized by $t = 1$, $t = \frac{1}{2}$ and $t = 2$, as is shown graphically in figure 1.*

Proof. First, assume that $t = 1$. Then, one can verify that

$$E(a_i \quad | \quad m = i) = \frac{\int_0^1 \int_{a_{-i}}^1 a_i da_i da_{-i}}{\int_0^1 \int_{a_{-i}}^1 da_i da_{-i}} = \frac{2}{3}$$

$$\text{and } E(a_i \quad | \quad m = -i) = \frac{\int_0^1 \int_{a_{-i}}^1 a_{-i} da_i da_{-i}}{\int_0^1 \int_{a_{-i}}^1 da_i da_{-i}} = \frac{1}{3}.$$

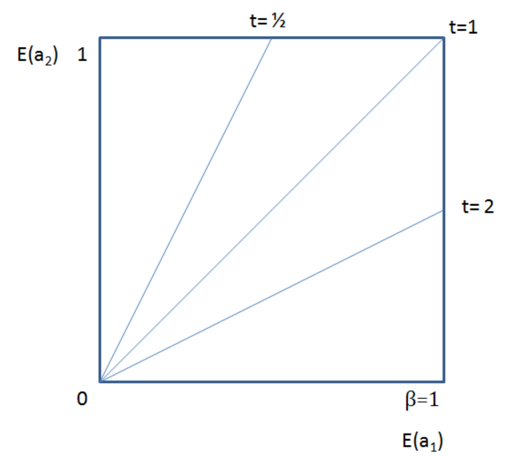

Figure 1

Substituting these values in equation 3, provides $t = 1$. Next, consider $t = \frac{1}{2}$. In this case the expected abilities are as follows:

$$E(a_1 \mid m = 1) = \frac{\int_0^1 \int_{\frac{1}{2}a_2}^1 a_1 da_1 da_2}{\int_0^1 \int_{\frac{1}{2}a_2}^1 da_1 da_2} = \frac{11}{18}$$

$$E(a_2 \mid m = 1) = \frac{\int_0^1 \int_{\frac{1}{2}a_2}^1 a_2 da_1 da_2}{\int_0^1 \int_{\frac{1}{2}a_2}^1 da_1 da_2} = \frac{4}{9}$$

$$E(a_1 \mid m = 2) = \frac{\int_0^1 \int_0^{\frac{1}{2}a_2} a_1 da_1 da_2}{\int_0^1 \int_0^{\frac{1}{2}a_2} da_1 da_2} = \frac{1}{6}$$

$$E(a_2 \mid m = 2) = \frac{\int_0^1 \int_0^{\frac{1}{2}a_2} a_2 da_1 da_2}{\int_0^1 \int_0^{\frac{1}{2}a_2} da_1 da_2} = \frac{2}{3}$$

Again, if one substitutes these values in equation 3, the value of $t = \frac{1}{2}$ is found. The opposite case emerges when $t = 2$. ■

In case $t = 1$, the employee with the higher ability will be promoted. This appeals to one's sense of fairness. The difference in reaction to the promotion decision is utterly important when it comes to the promotion strategy. If employee 1 is more disappointed when he is passed over for the promotion, if $t = \frac{1}{2}$, he is three times more likely than employee 2 to get the promotion, even though the abilities of both the employees are equal ex ante. To translate this to a real life example like in the introduction, imagine a male and a female employee competing for the same promotion. In this case, employee 1 is the male employee. Although ex ante they believe to have the same expected ability, the male employee's reaction to the promotion decision is much stronger ($t = \frac{1}{2}$) and therefore the effect on his effort is larger. The manager will promote the male employee when the actual ability of the female is twice the ability of the male or less ($a_{male} \geq \frac{1}{2}a_{female}$).

If one examines the three equilibria, it becomes clear that only the equilibria $t = \frac{1}{2}$ and $t = 2$ are stable. Assume that the manager commits to a promotion rule where only ability matters ($m = 1 \; if \; a_1 > a_2$). In other words, the value of $t = 1$. However, the employees happen to feel that employee 1 is slightly favoured. Suppose they believe that $t = \frac{9}{10}$. If one substitutes these values in equation 3, a value of $t = \frac{5}{6} \sim 0.833$ is found. This is the optimal response of the manager to the beliefs of the employees. Thereafter the employees will respond to the new promotion rule. The smallest doubt concerning the commitment of the manager to the "fair" promotion rule will result in a lower effort level of the employees. This process repeats itself until the value of $t = \frac{1}{2}$. Costs occur for the manager when he decides to stick to the honest promotion rule $t = 1$.

The process as described above will not exceed $t = \frac{1}{2}$. Beyond that point the employees will not learn enough about the level of their ability. The probability that employee 1 is promoted, if $t < \frac{1}{2}$, is that large that the message sent by the manager does not contain much information. In that case the employees are not able to match the level of effort to the ability precise enough. This deteriorates the utility of the employees. The utility of the employees depends, as equation 1 shows, on their own output minus the cost of effort. If they cannot properly match the level of effort to their ability, their level of utility will drop.

This result is remarkable. If two people aim for the same promotion and they both feel that one of them has a slight advantage, that employee has a higher probability to be promoted. This means that severe discrimination, where one employee has three times the probability to be promoted than his counterpart, is much more likely to happen than not.

4 The effect of affirmative action on effort

Let us now include affirmative action in the model. As described before in Section 2, affirmative action policies are applied to promote the opportunities of minority workers. A situation is considered where the manager commits to a promotion rule where only ability matters. The promotion rule entails that employee 1 is promoted if $a_1 \geq \eta a_2$, where η denotes the affirmative action. If $\eta = 1$ only ability matters. The optimal values of η ex ante will be calculated below, both for the employees and the manager. First, the situation where $\eta \geq 1$ will be described, in which employee 2 benefits from the rule. The utility function of the employees remains

$$U_{A_i(e_i)} = E\left(a_i\right)^2 - \frac{1}{2}E\left(a_i\right)^2$$

Given the promotion decision the expected utilities for employee 1 and 2 are:

$$E\left(a_1 \mid m=1\right) = \frac{\int_0^{\frac{1}{\eta}}\int_{\eta a_2}^1 a_1 da_1 da_2}{\int_0^{\frac{1}{\eta}}\int_{\eta a_2}^1 da_1 da_2} = \frac{2}{3}$$

$$E\left(a_2 \mid m=1\right) = \frac{\int_0^{\frac{1}{\eta}}\int_{\eta a_2}^1 a_2 da_1 da_2}{\int_0^{\frac{1}{\eta}}\int_{\eta a_2}^1 da_1 da_2} = \frac{1}{3\eta}$$

$$E\left(a_1 \mid m=2\right) = \frac{\int_0^{\frac{1}{\eta}}\int_0^{\eta a_2} a_1 da_1 da_2 + \int_{\frac{1}{\eta}}^1\int_0^1 a_1 da_1 da_2}{\int_0^{\frac{1}{\eta}}\int_0^{\eta a_2} da_1 da_2 + \int_{\frac{1}{\eta}}^1\int_0^1 da_1 da_2} = \frac{3\eta-2}{6\eta-3}$$

$$E\left(a_2 \mid m=2\right) = \frac{\int_0^{\frac{1}{\eta}}\int_0^{\eta a_2} a_2 da_1 da_2 + \int_{\frac{1}{\eta}}^1\int_0^1 a_2 da_1 da_2}{\int_0^{\frac{1}{\eta}}\int_0^{\eta a_2} da_1 da_2 + \int_{\frac{1}{\eta}}^1\int_0^1 da_1 da_2} = -\frac{3\eta^2-1}{3\eta-6\eta^2}$$

The ex ante probability that employee 2 will be promoted is $(1-\frac{1}{2\eta})$. This is shown graphically in figure 2.

Inserting the values found above in equation 1 yields the expected utility for employee 2

$$\begin{aligned} E(U_{A_2}) &= (1-\frac{1}{2\eta})(\frac{1}{2}\left(-\frac{3\eta^2-1}{3\eta-6\eta^2}\right)^2) + \left(\frac{1}{2\eta}\right)\left(\frac{1}{2}\left(\frac{1}{3\eta}\right)^2\right) \\ &= \frac{1}{36\eta^2\left(2\eta-1\right)}\left(9\eta^3-6\eta+2\right) \qquad (4) \end{aligned}$$

Keeping in mind that $\eta \geq 1$, then the optimal value of η is $\frac{1}{3}\sqrt{3}+1 \sim 1.577.$[6]

[6] $\frac{d}{d\eta}\left(\frac{1}{36\eta^2(2\eta-1)}\left(9\eta^3-6\eta+2\right)\right) = -\frac{1}{36\eta^3(2\eta-1)^2}\left(9\eta^3-24\eta^2+18\eta-4\right)$. Solution is: $\frac{2}{3}, 1-$

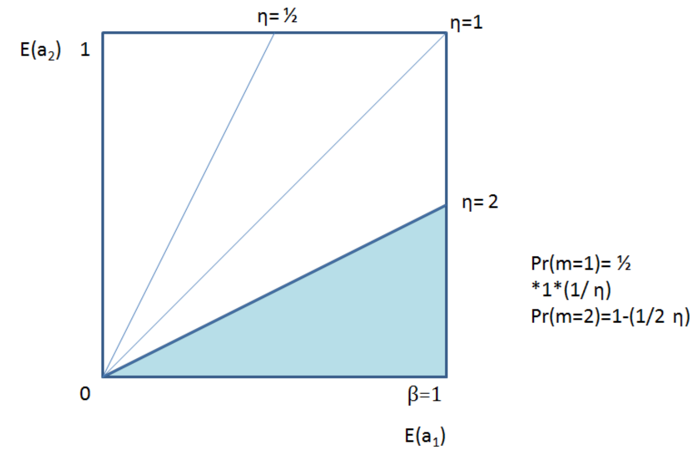

Figure 2

The optimal value of η can also be calculated for employee 1. However, for employee 1, an optimal value of η can only be found when $\eta \leq 1$. The following expected utilities can be found if one makes similar calculations as above (only now the probability that $m = 1$ is $\left(1 - \frac{1}{2}\eta\right)$):

$$E\left(a_1 \mid m = 1\right) = \frac{\int_0^1 \int_{\eta a_2}^1 a_1 da_1 da_2}{\int_0^1 \int_{\eta a_2}^1 da_1 da_2} = \frac{1}{3\eta - 6}\left(\eta^2 - 3\right)$$

$$E\left(a_1 \mid m = 2\right) = \frac{\int_0^1 \int_0^{\eta a_2} a_1 da_1 da_2}{\int_0^1 \int_0^{\eta a_2} da_1 da_2} = \frac{1}{3}\eta$$

Inserting these values in equation 1 gives the following expected utility for employee 1

$$\begin{aligned} E(U_{A_1}) &= \frac{1}{2}(1 - \frac{1}{2}\eta)\left(\frac{1}{3\eta - 6}\left(\eta^2 - 3\right)\right)^2 + \frac{1}{4}\eta\left(\frac{1}{3}\eta\right)^2 \\ &= -\frac{1}{36\left(\eta - 2\right)}\left(2\eta^3 - 6\eta^2 + 9\right) \end{aligned} \quad (5)$$

Optimizing equation 5 given the restriction $\eta \leq 1$ yields an optimal value of $\eta = \frac{3}{2} - \frac{1}{2}\sqrt{3} \sim 0.634$.

The only point of view that has not been covered yet is that of the manager. If

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$\frac{1}{3}\sqrt{3}, \frac{1}{3}\sqrt{3} + 1$

$\eta \geq 1$, then

$$
\begin{aligned}
E(U_M) &= \int_0^{\frac{1}{\eta}} \int_{\eta a_2}^{1} \left(a_1 \frac{2}{3} + a_2 \frac{1}{3\eta} \right) da_1 da_2 \\
&\quad + \int_0^{\frac{1}{\eta}} \int_0^{\eta a_2} (a_1 \frac{3\eta - 2}{6\eta - 3} + a_2 (-\frac{3\eta^2 - 1}{3\eta - 6\eta^2})) da_1 da_2 \\
&\quad + \int_{\frac{1}{\eta}}^{1} \int_0^{1} (a_1 \frac{3\eta - 2}{6\eta - 3} + a_2 (-\frac{3\eta^2 - 1}{3\eta - 6\eta^2})) da_1 da_2 \\
&= -\frac{1}{9\eta^2 (2\eta - 1)} \left(-9\eta^3 + 2\eta^2 + 3\eta - 1 \right) \qquad (6)
\end{aligned}
$$

This leads also to the optimal value of $\eta = 1$ for $\eta \in [1, \rightarrow]$.

If $\eta \leq 1$ then

$$
\begin{aligned}
E(U_M) &= \int_0^{1} \int_{\eta a_2}^{1} \left(a_1 \frac{1}{3\eta - 6} \left(\eta^2 - 3 \right) + a_2 \frac{2\eta - 3}{3\eta - 6} \right) da_1 da_2 \\
&\quad + \int_0^{1} \int_0^{\eta a_2} \left(a_1 \frac{1}{3} \eta + a_2 \frac{2}{3} \right) da_1 da_2 \\
&= \frac{1}{9 (\eta - 2)} \left(-\eta^3 + 3\eta^2 + 2\eta - 9 \right) \qquad (7)
\end{aligned}
$$

Equation 7 leads to an optimal value of $\eta = 1$ for $\eta \in [0, 1]$.

Proposition 2 *Assume a promotion rule where employee 1 is promoted iff $a_1 \geq \eta a_2$. In this case the manager prefers a value of $\eta = 1$. However, the employee 1 and 2 optimize their utility with values of $\eta = \frac{3}{2} - \frac{1}{2}\sqrt{3}$, respectively $\eta = 1 + \frac{1}{3}\sqrt{3}$.*

Remarkable is the outcome that the promotion rule $a_i = \eta a_{-i}$, where $\eta = 1$, which seems to be the most fair, finds resistance among the employees. The optimal promotion rule for the manager is to promote the employee with the best ability. Surprisingly however, both employees have objections against this rule. Why do employees want to have an unfair promoting rule? Although they do not want to be promoted completely regardless the level of their ability, a skewed promotion-rule is preferred. To answer this question the effects of the value of η will be explained. What happens to employee 1 when η drops below the value of 1? The utility function of the employee is his contribution to the firm less the cost of effort (equation 1). The lower the value of η, the higher the probability of being promoted. Being promoted increases the expected ability. This, in turn, raises the level of effort and thus the expected utility of the employee. However, the lower the value of η, the less the promotion decision truly says something about the level of ability. This affects the expected ability and hence the expected utility, negatively. It is therefore optimal for the employee to balance out these two effects. In other words, the employee wants the level of effort exerted to match the level of ability. Learning the level of ability is therefore important. A value of $\eta = 0$ where employee 1 will always be promoted is not optimal for the employee, since this yields no information about his ability. The employees derive utility of knowing their ability level, so that they will not exert too much or not enough effort. The results found above propose an interesting outlook on affirmative action. For the manager a promotion decision is optimal when it is based solely on ability. The employees, however, do not feel the same way. This is due to the fear of not exerting enough effort or exerting too much effort. People tend to avoid tasks they believe to be too difficult for them.

Often promotion is accompanied by a raise or bonus. Furthermore, an employee might derive utility because of the honor of being promoted. Both effects amplify the wish of the employees of a deviation of $\eta = 1$. The optimal value of η for for instance employee 1 will be lower.

5 Conclusion

In this paper a model is developed in order to explain the perseverance of discrimination in the workplace. In the model two employees compete for a promotion. The promotion decision is made by a manager. The manager is assumed to have perfect information regarding the ability of the employees. The employees, on the other hand, only know the range in which their abilities lie. The employees update their beliefs about their abilities according to Bayes' rule, once they know the promotion decision. The utility of the employees depends on their contribution to the firm and the cost of effort. Since ability and effort are assumed to be complements, a positive decision increases the utility of the employee. It turns out, that even if the ex ante expected utility of the employees is the same, the probability to be promoted can be vastly different for the two employees. This happens if they both feel that one is favoured over the other. The disappointment of the 'favoured' employee when he is not promoted, deteriorates his effort level in a way that it might be optimal for the manager to promote him. An equilibrium is found where one employee is three times more likely to be promoted than his peer. It is concluded that this situation is much more likely to happen than a situation with equal opportunities for both employees.

Furthermore, the effects are described of an affirmative action policy to promote equality for the employees. It is concluded that a perfectly fair promotion rule, where only ability matters, is only preferred by the manager. The employees derive higher utility when being favoured, even without a bonus or the honour of being promoted. However, they also derive utility of learning about their ability in order to match the level of effort to the level of ability.

This paper contributes to the existing literature by using a new approach. The (de)motivation effect of a promotion is presented as the reason why discrimination in the workplace persists. It contributes to for instance the work of Arrow (1972), where the prejudice is assumed to exist among employees rather than employers. The findings of this paper have an important implication. Addressing the segregation problem by affirmative action policies might not even be optimal, not for the employers, nor for the employees.

The findings have yet to be confirmed in an empirical setting. However, monitoring output or motivation is as difficult as it is costly. Future research towards the motivational effects of promotions could contribute to further confirm these findings.

6 Appendix

6.1 Equilibria with the existence of prejudice

As described in the introduction, the existence of shared prejudice about the abilities is a logical assumption. An example is the phenomenon of underconfidence of women when it comes to science (Ehrlinger and Dunning, 2003). Below it is shown that if $\beta > 1$ the effect of the message sent by the manager on the expected ability is more pronounced than is shown in Section 3. Recall that a_1 is uniformly distributed between 0 and β, a_2 is uniformly distributed between 0 and 1 and that the employees believe that the manager shares their perception ex ante. First, the situation where $t > \beta$ will be described, subsequently where $t < \beta$. It is shown when $t > \beta$ two

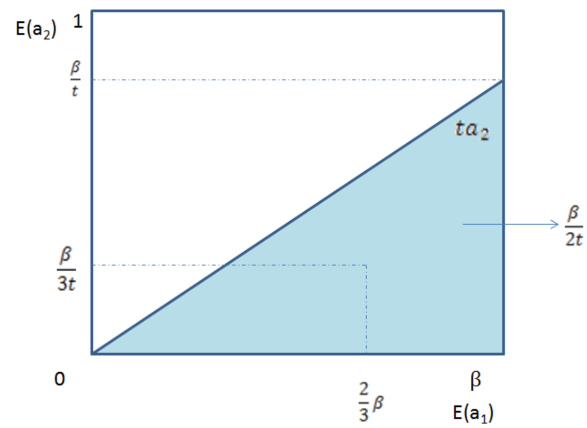

Figure 3

equilibria exist where employee 2 has a higher probability to be promoted than employee 1 for $\beta \leq \sqrt{\frac{9}{8}}$. Then, when $t < \beta$ and $\beta > \sqrt{\frac{9}{8}}$ a unique equilibrium exists where employee 1 has a far higher probability to be promoted than his co-worker.

Suppose $t > \beta$, the expected ability of the employees following the promotion decision are determined below:

$$E\left(a_1 \mid m=1\right) = \frac{\int_0^{\frac{\beta}{t}} \int_{ta_2}^{\beta} \frac{1}{\beta} a_1 da_1 da_2}{\int_0^{\frac{\beta}{t}} \int_{ta_2}^{\beta} \frac{1}{\beta} da_1 da_2} = \frac{2}{3}\beta$$

$$E\left(a_2 \mid m=1\right) = \frac{\int_0^{\frac{\beta}{t}} \int_{ta_2}^{\beta} \frac{1}{\beta} a_2 da_1 da_2}{\int_0^{\frac{\beta}{t}} \int_{ta_2}^{\beta} \frac{1}{\beta} da_1 da_2} = \frac{1}{3t}\beta$$

$$E\left(a_1 \mid m=2\right) = \frac{\int_0^{\frac{\beta}{t}} \int_0^{ta_2} \frac{1}{\beta} a_1 da_1 da_2 + \int_{\frac{\beta}{t}}^{1} \int_0^{\beta} a_1 \frac{1}{\beta} da_1 da_2}{\int_0^{\frac{\beta}{t}} \int_0^{ta_2} \frac{1}{\beta} da_1 da_2 + \int_{\frac{\beta}{t}}^{1} \int_0^{\beta} \frac{1}{\beta} da_1 da_2} = \beta \frac{2\beta - 3t}{3\beta - 6t}$$

$$E\left(a_2 \mid m=2\right) = \frac{\int_0^{\frac{\beta}{t}} \int_0^{ta_2} \frac{1}{\beta} a_2 da_1 da_2 + \int_{\frac{\beta}{t}}^{1} \int_0^{\beta} a_2 \frac{1}{\beta} da_1 da_2}{\int_0^{\frac{\beta}{t}} \int_0^{ta_2} \frac{1}{\beta} da_1 da_2 + \int_{\frac{\beta}{t}}^{1} \int_0^{\beta} \frac{1}{\beta} da_1 da_2} = -\frac{1}{6t^2 - 3t\beta}\left(\beta^2 - 3t^2\right)$$

This is shown graphically in figure 3.

Similar to what has been done before, the conditional expectations found above, are inserted in equation 3. Simplified, this leads to $(3t - 2\beta) = t^2\beta$ Solving for t yields two possible values:

$$t_{1,2} = \frac{3 \pm \sqrt{9 - 8\beta^2}}{2\beta} \tag{8}$$

Equation 8 shows that for a value of $\beta > \sqrt{\frac{9}{8}}$ no real value of t exists. In this situation, which requires that $t > \beta$, $\beta \leq \sqrt{\frac{9}{8}}$. Furthermore, if $\beta \rightarrow 1$, then equation 8 reduces to two of the equilibria as described in Lemma 1($t = 1$ and $t = 2$). Finally, the larger β, the more t_1 and t_2 converge, until at $\beta = \sqrt{\frac{9}{8}}$, $t_1 = t_2 = \sqrt{2}$.

Since the employees believe that the manager shares their perception ex ante, the ex ante believed probability of employee 1 having a higher ability than his fellow employee is the probability that $a_1 \geq \frac{1}{2}$ or $1 - \frac{\frac{1}{2}}{\frac{3}{4}\sqrt{2}} \sim 0.529$. However, ex post, the probability that employee 2 will be promoted is $\frac{5}{8} \sim 0.625$.[7]

The final equilibrium that will be described occurs when $t < \beta$. Again, the expected abilities according to the employees are calculated:

$$E(a_1 \mid m = 1) = \frac{\int_0^1 \int_{ta_2}^{\beta} \frac{1}{\beta} a_1 da_1 da_2}{\int_0^1 \int_{ta_2}^{\beta} \frac{1}{\beta} da_1 da_2} = -\frac{1}{3(t - 2\beta)}\left(3\beta^2 - t^2\right)$$

$$E(a_2 \mid m = 1) = \frac{\int_0^1 \int_{ta_2}^{\beta} \frac{1}{\beta} a_2 da_1 da_2}{\int_0^1 \int_{ta_2}^{\beta} \frac{1}{\beta} da_1 da_2} = -\frac{1}{3}\frac{3\beta - 2t}{t - 2\beta}$$

$$E(a_1 \mid m = 2) = \frac{\int_0^1 \int_0^{ta_2} \frac{1}{\beta} a_1 da_1 da_2}{\int_0^1 \int_0^{ta_2} \frac{1}{\beta} da_1 da_2} = \frac{1}{3}t$$

$$E(a_2 \mid m = 2) = \frac{\int_0^1 \int_0^{ta_2} \frac{1}{\beta} a_2 da_1 da_2}{\int_0^1 \int_0^{ta_2} \frac{1}{\beta} da_1 da_2} = \frac{2}{3}$$

If one substitutes these values into equation 3, the following equation will be found: $t = \frac{1}{3\beta - 2t}$, and solving this yields:

$$t = \frac{3}{4}\beta - \frac{1}{4}\sqrt{9\beta^2 - 8}^{8} \tag{9}$$

Equation 9, as 8, leads to a few conclusions. First, if $\beta \rightarrow 1$, the equilibrium of $t = \frac{1}{2}$, as described in Lemma 1 occurs. Second, the larger β, the larger the probability that employee 1 is promoted. Which makes sense, since the expected ability of employee 1 will be higher. Surprisingly, even for low values, the manager is inclined to promote employee 1. For instance, at again $\beta = \sqrt{\frac{9}{8}}, t \sim 0.27$. There is a

[7] $\frac{\frac{3}{8}\beta + \frac{1}{4}\beta}{\beta} = \frac{\frac{15}{32}\sqrt{2}}{\frac{3}{4}\sqrt{2}} = \frac{5}{8}$

[8] Note that $t = \frac{3}{4}\beta + \frac{1}{4}\sqrt{9\beta^2 - 8}$ would contradict the assumption of $t < \beta$.

vast difference between the probability that employee 1 is more able than employee 2 ex ante (approximately 0.53) and the probability that employee 1 will be promoted (approximately 0.8).

To sum all the above up:

Proposition 3 *Suppose the game as described above. If* $1 \leq \beta \leq \sqrt{\frac{9}{8}}$*, then the following three equilibria exist, in which the promotion decision depends on* a_1 *and* a_2*. First,* $t = \frac{3+\sqrt{9-8\beta^2}}{2\beta}$*. Second,* $t = \frac{3-\sqrt{9-8\beta^2}}{2\beta}$*. And last,* $t = \frac{3}{4}\beta + \frac{1}{4}\sqrt{9\beta^2 - 8}$*. If* $\beta > \sqrt{\frac{9}{8}}$*, a unique equilibrium exists in which the promotion decision depends on* a_1 *and* a_2*:* $t = \frac{3}{4}\beta + \frac{1}{4}\sqrt{9\beta^2 - 8}$*.*

6.2 Applying affirmative action when prejudice exists

If one adds the affirmative action policy as described before (promote employee 1 iff $a_1 \geq \eta a_2$), it shows that if $\beta > 1$,the manager also rejects the fair promoting rule of $\eta = 1$. For instance, if $\beta = \frac{21}{20}$,the expected utility of the manager is

$$\begin{aligned} E\left(U_M \mid \beta = \frac{21}{20}\right) &= \int_0^1 \int_{\eta a_2}^{\frac{101}{100}} \left(a_1 \left(\frac{10\,000\eta^2 - 30\,603}{30\,000\eta - 60\,600}\right) + a_2 \left(\frac{200\eta - 303}{300\eta - 606}\right)\right) da_1 da_2 \\ &\quad + \int_0^1 \int_0^{\eta a_2} \left(a_1 \frac{\eta}{3} + \frac{2}{3} a_2\right) da_1 da_2 \\ &= \frac{(-202\,000\,000\eta^3 + 612\,060\,000\eta^2 + 404\,000\,000\eta - 1854\,633\,609)}{1800\,000\,000\eta - 3636\,000\,000} \end{aligned}$$

Which leads to optimal values of η of approximately 1.069, 0.953 and 2.523. A small deviation of $\beta = 1$, only a five percent rise, leads to a deviation of the optimal value of η for the manager. If any prejudice exists among the employees, nobody wants a fair promotion rule of $\eta = 1$. The results found in section 4 are now amplified and are remarkable to say the least. Programs of to promote equality are that case not optimal, not for the employees, nor for the employers.

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