

Master Thesis

To obtain the academic degree of
Master of Science in Economics & Business
(Major in International Economics)

Endogenous Export Taxation under Fixed and Flexible Import Tariff Regimes

Erasmus School of Economics

Department of Economics

Author: Larentzakis Antonis

Exam Number: 368300

Email address: larentzakis.antonis@gmail.com

Supervisor: Professor Jean-Marie Viaene

Study Program: Business Economics

Specialization: International Economics

Date: 5/6/2013

Abstract

The European Union and the U.S.A. propelled an agenda on a free-trade agreement¹, whereas “*The Pacific Alliance*” launched a treaty of 90% obliteration on merchandise trade tariffs². The impact of globalization does not merely impoverish international trade instruments, but also tends to significantly eradicate them. Consequently, as a part of our thesis we developed the theory of endogenous export taxation in order to generate substitute and auxiliary trade policies. The main conclusion is that fiscal policies and specifically the enforcement of a tax on production, can affect an exporting industry, similarly to an export tax. The aforementioned result is accurate under fixed and flexible import tariff regimes. The model examines industries exhibiting increasing, constant and decreasing returns to scale, simultaneously with elastic, inelastic and isoelastic export supply curves. The key result is that if an exporting industry supplies a commodity inelastically, then an endogenous export tax will increase the producer surplus despite the fact that the cost increases. The latter conclusion is examined for perfect and imperfect market conditions, with and without the presence of fixed cost. The model uses a paradox and techniques completely different and innovative compared to the formal trade theory. To conclude, as a part of the microeconomic implications, we proved that the existence of fixed cost does not necessarily lead to increasing returns to scale; a contradictory result according to the fundamental microeconomic theory.

Keywords: Endogenous Export Tax; Import Tariff; Fixed and Flexible Import Tariff Regimes; Economies of Scales; Fixed Cost; Increasing Returns to Scale

¹ <http://www.bbc.co.uk/news/business-21439945>

² <http://www.economist.com/news/americas/21578056-region-falling-behind-two-alternative-blocks-market-led-pacific-alliance-and>

Acknowledgments

I appreciatively recognize the recommendations of Professor Vladimir Karamychev. Correspondingly, my personal friends Ilias Plakocefalos and Vlassis Mantzoukas for the long discussions we had on our ideas and Professors Konstantinos Gatsios and Dimitra Dimakou for the solid microeconomic background they provided during the author's studies as a bachelor student at the Department of Economics of Athens University of Economics and Business. Last but not least, I am grateful to my parents who gave me the opportunity to study in the Erasmus University, despite the appearance of economic difficulties.

Overall, an extremely beneficial year at the Erasmus School of Economics is completed with this thesis. It has been an exhausting yet rewarding year and I am deeply grateful for the experience and knowledge I have obtained. The cooperation with the Professors of the Master in International Economics was more than satisfactory and especially with the Professor Jean-Marie Viaene, without whose stoic patience and academic experience I could have never had the opportunity to broaden my thoughts.

Table of Contents

<i>Chapter 1</i>	4
1.1 Introduction	4
1.2 Literature Review	7
<i>A. Traditional Theory</i>	7
<i>B. Economies of Scale and Imperfect Competition</i>	8
1.3 Problem Statement and Research Objective	10
1.4 Scientific Relevance	11
<i>Chapter 2</i>	13
2.1 Structure of the Model	13
2.2 Residual import demand and export supply	14
2.3 Mathematic and Economic assumptions	16
2.4 Microeconomic Framework	19
2.5 Cost and Price Elasticity	22
2.6 Economies of Scale	26
2.7 The Paradox	28
2.8 Endogenous taxation subject to flexible trade policy regimes	34
2.9 Monopoly, Monopolistic Competition, Free Entry and Implications	38
<i>Chapter 3</i>	42
3.1 Concluding remarks	42
3.2 Policy Implications	44
3.3 Academic Contribution	47
3.4 Limitations and Directions for Future Research	50
3.5 References	52
3.6 Appendix	55
<i>A. Derivation of net (residual) aggregate import demand (MD) and export supply curves (XS)</i>	55
<i>B. Cost/Marginal Cost/Average Cost microeconomic foundation and illustration</i>	57
<i>C. Conditions for positive effect of the indirect tax to the Producer Surplus</i>	64
<i>D. Figures</i>	65
<i>E. Production and cost functions without fixed cost</i>	67
<i>F. Cobb-Douglas Functions with Fixed Cost</i>	68

Chapter 1

1.1 Introduction

We have the privilege to observe the evolution of a new world with significant economic characteristics such as globalization. The world international trade as a percentage of the global GDP is twice higher in 2006 compared to 1960. According to the New Kaldor Facts, and specifically to the first new stylized fact, the unique phenomenon of the extension of the markets to their limits is now a reality. The term of globalization as represented in the first new Kaldor fact, is reflected in the increase of international trade and foreign direct investment (Jones and Romer, 2009).

Furthermore, the EU's introduction of the Euro combined with the remarkable financial growth of China and the BRICS in general, has brought new and competent players to the international trade environment. The USA and Great Britain, wounded from the latest financial crisis are now obliged to rearrange their position in a constantly insecure and unstable international environment. In general, developed countries have to adjust to this new framework of emerging economies.

In the last decade, the evolution of the internet, the fall of dictatorship in many developing countries and unambiguously the worldwide transformation of the technological and political context, has affected the economic procedures and has generated new challenges for the economic theory and science.

Historically speaking, from the Ricardian comparative advantage and the assumption of constant returns to scale more than a century ago; up to the New Trade Theory and the monopolistic competition assumptions, which attempt to incorporate increasing returns to scale, a long era follows in order to capture and explain the international environment as far as trade is concerned.

OECD countries, through various international trade rounds such as Doha and Uruguay rounds, attempt to figure a grid of common international trade rules. These countries need to protect their internal welfare, but at the same time, stimulate international trade in order to attain potential profits.

Trade barriers are one of the 10 most vital problems in the world according to the Copenhagen Consensus 2012. China has a 95 % world market share for rare earths, and started to restrict its supply to the world in 2006³. From 2003 to 2009, 65 out of 128 countries in the WTO Trade Policy Reviews imposed export taxes, mostly in agriculture, minerals, metals and forestry, with developing countries and emerging economies being more active.

Moreover, export restrictions to raw materials generate problems to industrialized importing countries. The main reasons are that the production and the reserves of these materials are geographically concentrated and that export trade policies are characterized by feeble disciplines. Trade diplomacy forced China to change its policies due to the violation of World Trade Organization laws.

The latter, is merely a typical example of the significance of trade policy and its impact on global welfare and economy. Therefore in this thesis we will attempt to generate a new trade model. This model incorporates constant decreasing and increasing returns to scale in order to form a new instrumental variable more suitable to the international trade policy which can help countries to adjust their policies to the new international environment.

Meanwhile, this new international trade policy can help international organizations to be prepared against such policies, since they potentially generate significant profits for the exporters but extensive problems to the importers and generally to the world trade environment as well.

The model is useful for the entire range of scale economies and concerns large countries which can affect their exporting price. It follows a general form which allows it to extend in various dimensions and market formations. We will focus on straightforwardness in perfect competition and monopoly. The exporting country faces a diminishing demand curve and the model concerns large countries with sufficient monopolistic power. On the other hand the exporter chooses a price policy based on the free market conditions, where the demand is equal to the supply.

³ UK Parliamentary Office of Science and Technology, Post Note 368, Jan 2011

The title of the thesis is “Endogenous Export Taxation under Fixed and Flexible Import Tariff Regimes” and we would like to further explain its purpose. Import tariff or export tax trade theory models suggest formulas for the optimum values of the aforementioned through a static and exogenous technique. Primarily, demand and supply derive the equilibrium and afterwards the optimum tariff is calculated. Demand and supply are identified and then the models maximize the importer’s or the exporter’s welfare so as to obtain the optimum. In our framework, the endogenous taxation derives the equilibrium and not the opposite. Thus an indirect taxation changes the supply curve and “determines” the equilibrium.

Therefore, the results are similar to an export tax, but via an endogenous way. It is completely different compared to the way of thinking of export taxation. Indirect export taxation changes the export curve and as a result the equilibrium is determined endogenously. The endogenous export tax determines the equilibrium and the final results, unlike to the formal way where the opposite appears. The difference is that an endogenous tax derives the optimum equilibrium from the exporter’s perspective, whereas an “exogenous” tax is determined according to the equilibrium in order to obtain the optimum equilibrium. We determine the equilibrium, which is optimum through an indirect tax. In formal theory the equilibrium determines the welfare maximizing export tax.

Furthermore, the flexible import tariff regime is an environment where the import tariff is completely flexible and adjustable to the optimum tariff formula. On the other hand, rigidities occur in the short run and the import tariff is fixed. Commonly, fixed and flexible are referred to time, but if regulations characterize the import tariff regime then it is fixed even in the long run. As a matter of fact, in the EU the European law does not allow its members to use a completely independent trade policy among them. Mutual trade rules characterize the European Union and also external rules have to be followed from its members. Therefore, fixed import tariff regime refers to different conditions as time or regulations, which do not allow the import tariff to be flexible in the short run or even in the long run.

To conclude, the title of the thesis refers to the regimes we are going to examine with respect to a tax presented through a different aspect compared to the formal trade theory.

1.2 Literature Review

The Comparative Advantage, which has been analyzed in depth and reached its own limits, was the core of the international trade traditional theory. The reason is that significant yet obsolete models failed to explain the fundamental characteristics of global economy such as economies of scale and imperfect competition. However, the economic science has been endowed and enriched from the classical theory of international trade, despite the observed limitations. Economies of scale and imperfect competition characterize the world economy as Krugman and Helpman mention on “Trade Policy and Market Structure”, and this new approach integrates trade theory with industrial organization.

A. *Traditional Theory*

The core of the traditional trade theory is originally stated in Adam Smith’s “Wealth of Nations” in 1776, but David Ricardo is the main contributor of the comparative advantage concept in trade theory. The H-O and the Ricardian model, which have been extensively used in trade policies until the middle of the last century, are the main guidelines of the comparative advantage framework. So as not to digress from the theoretical framework of the thesis we will focus on the trade policy relative literature review, explicitly concerning import tariffs and export taxes.

Thomas M. Humphrey on “Classical and Neoclassical Roots of the Theory of Optimum Tariffs” summarizes the related topics. Robert Torrens published the first formal optimum tariff model in 1844 and the early idea goes back to 1824 in his *Essays on the Production of Wealth*. N. Kaldor with *the model of the optimum tariff*, Lerner’s *demand and supply curve analysis of the optimum tariff* and Torren’s *tariff model*, derive the effect of a tariff on the imposing country. John Stuard Mill’s *view of the size of Terms-of-Trade improvement and the elasticity of the foreign offer curve* and Alfred Marshall-Henry Sidwick optimum tariff models in the 1870s and 1880s confirm that the economics of export taxation and import tariffs, maintained the interest in trade theory for almost two centuries.

Various Monographs on the pure theory of international trade, including classical and neoclassical trade models, have been published from Alfred Marshall in the 18th century and from Miltiades Chacoliades in the 19th century.

B. Economies of Scale and Imperfect Competition

The models of traditional trade theory assume constant returns to scale for the industry and as Chacoliades (1973) mentions: *...when decreasing or increasing economies of scale exist the international trade theory has been scanty and unsatisfactory to the treatment of them (p.170)*. Moreover, economies of scale did not have the necessary consideration from the formal trade theory and the main reason was that they gave the impression of a demanding situation due to the implications of the IRS for market structure (Krugman, 1979). Economies of scale can be internal to the firm (Krugman, 1979) or external to the firm (Chacoliades, 1970; Melvin, 1969; Kemp, 1964; Negishi, 1969) and as Krugman explains, exploiting the IRS has trade as a result⁴.

Caves and Auquier (1979) stated that a country with an export industry, characterized by monopolistic power in international markets, can impose an export tax (or a cartel), in order to maximize its own welfare. Correspondingly, scale economies in production and market structure imperfections, are forces with explanatory power on the concept of a significant export increase in manufacturing industries (Caves, 1980). Trade boundaries may be beneficial in the presence of imperfectly correlated risk according to Newbery and Stiglitz (1981), when they examined the Pareto inferior trade and optimal trade policy. Lack of thorough information, leads the market away from perfect competition and such trade policies are desirable under asymmetric information (Newbery and Stiglitz, 1981).

Trade policies under oligopolistic market formations differ from the ones considered for perfect competitive markets (Dixit, 1984). The author also indicates that economists have to be aware of these policies, because they can potentially operate in favor of the oligopolistic industries. Moreover, under imperfect competition, the price exceeds the marginal cost of exports and a subsidizing country can increase its welfare, despite the fact that the terms of trade move against the country (Brander and Spencer, 1985). When oligopolistic industries contend for a specific factor of production, then free trade is optimal under symmetric exporting countries with fixed

⁴ Technological differences or differences on factor endowments stimulate trade according to the traditional trade theory.

technological coefficients and inelastic supply of the factor (Dixit and Grossman, 1986).

In 1988, one of the most substantial research papers on trade policy regarding increasing returns to scale and monopolistic competition was written by Markusen and Venables. They commented that trade and industrial organization policies can have different outcomes on imperfectly competitive industries, according to different papers of that period. They constructed a singular yet extended model which incorporates different market structures. The key results are that a small import tariff or an export subsidy improves welfare more if the markets are segmented and if the free entry oligopoly assumption occurs, but segmentation and free entry are independent. However, their conclusions are not concrete, but they are definitely helpful to understand the importance of market structure and entry assumptions for applied trade policies.

To continue, an optimum import tariff or an export tax⁵ equals the inverse supply curve price elasticity for the former and the inverse demand curve price elasticity for the latter⁶. However, the inverse-elasticity formula has to be considered as an upper bound, a situation not always desirable in discussions on optimal trade policies (Rodrik, 1989). Moreover, if countries compete under Nash behaviors, then the revenue maximizing export taxes obtain higher welfare for a country than under welfare maximizing export taxes⁷ (Panagariya and Schiff, 1994). Higher levels of welfare can be reached under Nash revenue maximizing taxes compared to Nash optimum taxes (Yilmaz,⁸ 1999). Market power makes an export tax beneficial, irrespective to the reaction of other importing or exporting countries, but without market power, the necessary equilibrium is lost (Devarajan, 1996).

Moreover, an interesting approach, related to the export tax for the China situation, was suggested by Stiglitz and Lau (2005). Despite the consequences of an export tax

⁵According to the Lerner symmetry theorem, there is equivalence between an export tax and an import tariff in respect to the domestic relative prices. See Vousden (1990, p.45).

⁶Relative proves are derived in Helpman and Krugman (1989, p.17) and Bowen *et al.* (2012, p.143).

⁷...*this result cannot be obtained in the traditional two-country models* (Panagariya and Schiff, 1994).

⁸Lerner Symmetry is not justified in the policy-oriented analysis of optimum export tax (Yilmaz, 1999) Also international investment disturbs the equivalence (Blanchard, 2009).

from China, they explain that it is preferable to an exchange rate evaluation. The welfare effect of the tax depends on the elasticity of the world demand, if the elasticity is lower than one it increases. If it is elastic the results depend on different parameters, such as the import content of exports (Howell-Zee, 2007). This approach proves the significance of trade policies through a different framework. Inelastically supplied goods face relatively higher tariffs from countries that exploit their market power, despite the optimal tariff theory corresponding to the foreign export elasticity. Comparative evidence exists for countries not belonging to the World Trade Organization but also for the USA (Broda *et al.*, 2006).

Finally, Bertrand's and Cournot's duopoly has been extensively used in the trade theory so as to incorporate market imperfections. In Bertrand's duopoly, the differentiated products' maximum-revenue and optimum-welfare tariffs are lower compared to Cournot's duopoly (Clarke and Collie, 2006). Also, in the same framework the welfare in the Bertrand Nash equilibrium is higher than the one under free trade for both involved countries, when an export tax is used due to a positive externality (Eaton and Grossman, 1986; Clarke and Collie, 2008).

1.3 Problem Statement and Research Objective

Import tariffs, export taxes, quotas and restrictions are the main instruments applied in international trade, starting from policy makers. Correspondingly, the exchange rate appreciation and depreciation affect trade, but it is usually internalized to monetary policies through a macroeconomic framework. As we have already seen in the literature review, trade policy instruments have kept trade theory alive for almost two centuries. Thus, optimum trade policies have emerged for different market structures; from perfect competition to monopoly, monopolistic competition and oligopoly.

In the introduction we explained that the volatility of the international environment might lead to new conditions in the international trade. The main question is if it is possible to generate a new trade policy instrument, which can be used complementary or auxiliary to the aforementioned instrumental variables. At the same time, and in order to remain close to the latest literature tendencies, the model has to incorporate increasing returns to scale and market imperfections. Nevertheless, the key question

is: can we incorporate a new policy through a completely different way compared to the relevant models of optimum trade policies? Therefore the objective is to introduce a new trade policy, applicable to imperfect and perfect market structures through a different way to the relevant literature and at the same time valuable to industries characterized from increasing, decreasing and constant returns to scale.

1.4 Scientific Relevance

In order to analyze the concept profoundly, the new trade policy instrument is going to be an endogenous export tax. The reason we call it endogenous is that compared to the traditional trade theory, which implies that the optimum export tax equals the inverse price elasticity of the demand; the endogenous export tax is applied on the revenue maximizing behavior of the producer and not on the market price as is commonly used. According to the microeconomic theory and the production theory, a producer minimizes the production cost under the restriction of the production function, and from the aforementioned procedure the supply curve occurs. The supply curve is represented by the marginal cost which is higher from the minimum average variable cost in the short run and from the minimum average cost in the long run.

An endogenous export tax, as we name it, is internalized to the behavior of the producer. We will derive that such a tax can change the slope and the position of the supply curve. The difference is that in literature, researchers primarily use demand, supply and market structures and then generate models which derive the optimum export tax. The supply is always exogenously predetermined. Rodick, Markusen, Venables, Brander, Spencer, Krugman, Dixit, Caves, Auquier, Yilmaz, Blanchard and every model about optimum export taxes or import tariffs introduces the tax through welfare functions by subtracting the tax from the price derived from a framework where the supply curve is predetermined.

Moreover, a second difference is that while a country imposes an endogenous export tax, the importing country enforces an import tariff. Therefore, we can see the effects of a simultaneous use of an import tariff and an export tax. Additionally, in the model we will analyze the situation and the effects, when rigidities appear with respect to the import tariff. At least in the short run, an increase of the supply price elasticity, due to

an endogenous export tax, creates rigidities for the importing country, because the import tariff has to be adjusted according to the inverse-elasticity formula.

The model can be easily utilized for perfect competition and monopoly, but it can also be extended for monopolistic competition and oligopoly. Furthermore, the same model can be introduced at its simplest form in order to explain the entire range of scale economies. In relevant models, researchers consider market imperfections and analyze various formations but this is done for IRS, DRS, and CRS separately. Obviously, increasing returns to scale is the most challenging issue for trade economists, but this does not mean that the rest of the scale economies should be put aside. The global economy is not only characterized by IRS, but also from DRS and CRS. To summarize, the model introduces a trade policy applied endogenously to the production, subject to rigidities about rival trade policy at least in the short run. It can also explain market imperfections such as monopoly and also entire economies of scale. Therefore, and despite the fact that it is a very simple model, we have generated a context totally different from the relative literature by using results and assumptions widely used in the latest trade models.

To continue, the new trade policy is valuable for large exporting countries, especially for the ones with enough market power. Its contribution becomes more valuable under the assumption that export taxes are going to face disciplinary measures from world organizations. Also, this model can be used in industries with cartel formations in order to prevent such trade policies, something significant for world organizations.

Finally, the model uses a completely general form, with parameters oriented towards every function and equation, as opposed to other relative models where at least for demand or supply functions, researchers assume a specific value for the constant or the slope⁹. The disadvantage of the model is that it is not very technical compared to significant trade models and it has limitations due to its own assumptions. On the other hand, the advantage is that it introduces a completely new way of thinking regarding trade policies. An endogenous export tax on the production through factor price taxation or cost taxation is completely different from an export tax imposed to the output or the price producers receive according to the market structure.

⁹ For example in many papers they assume constant marginal costs (Markusen and Venables, 1988)

Chapter 2

2.1 Structure of the Model

In the beginning, we will analyze the assumptions and the microeconomic foundation of the model. More specifically, we will explain how the import demand and the export supply curves are derived through classical trade theory. Then, we will discuss the microeconomic implications of linear functional forms with respect to the relative curves.

Moreover, the paradox arising from the relevant framework has to be presented and examined in depth. In terms of perfect competition, and without any state intervention, if the production cost increases the producer surplus decreases. It is a common result completely rational with respect to producer behavior. However, the paradox arises when the supply curve is inelastic and a specific tax is introduced. Thus the producer surplus increases despite the fact that the production cost also increases. When the supply curve is elastic the paradox disappears and rational results appear again.

To continue, we have to adjust the model to trade theory. The assumption of a constant specific tax is not completely accurate with respect to trade theory. The reason is that, the importing country that imposes a specific import tariff has incentives to impose the optimum import tariff and not just a fixed one. As we will see, according to the theory, the optimum import tariff equals the inverse price elasticity of the supply and every time the slope of the supply curve changes, the import tariff is also modified.

The latter framework will be examined for constant, decreasing and increasing returns to scale, also under rigidities concerning the import tariff and flexible import tariff regimes. As a final point, monopoly can be easily interpreted through the aforementioned framework since technically no specific adjustments are necessary, and primary results can extend to monopoly market imperfections. If free entry is allowed to the industry, then in the long run equilibrium, we move from monopoly to perfect competition as monopolistic competition theory explains. Therefore apart

from perfect competition and monopoly, monopolistic competition can be theoretically incorporated to the model.

2.2 Residual import demand and export supply

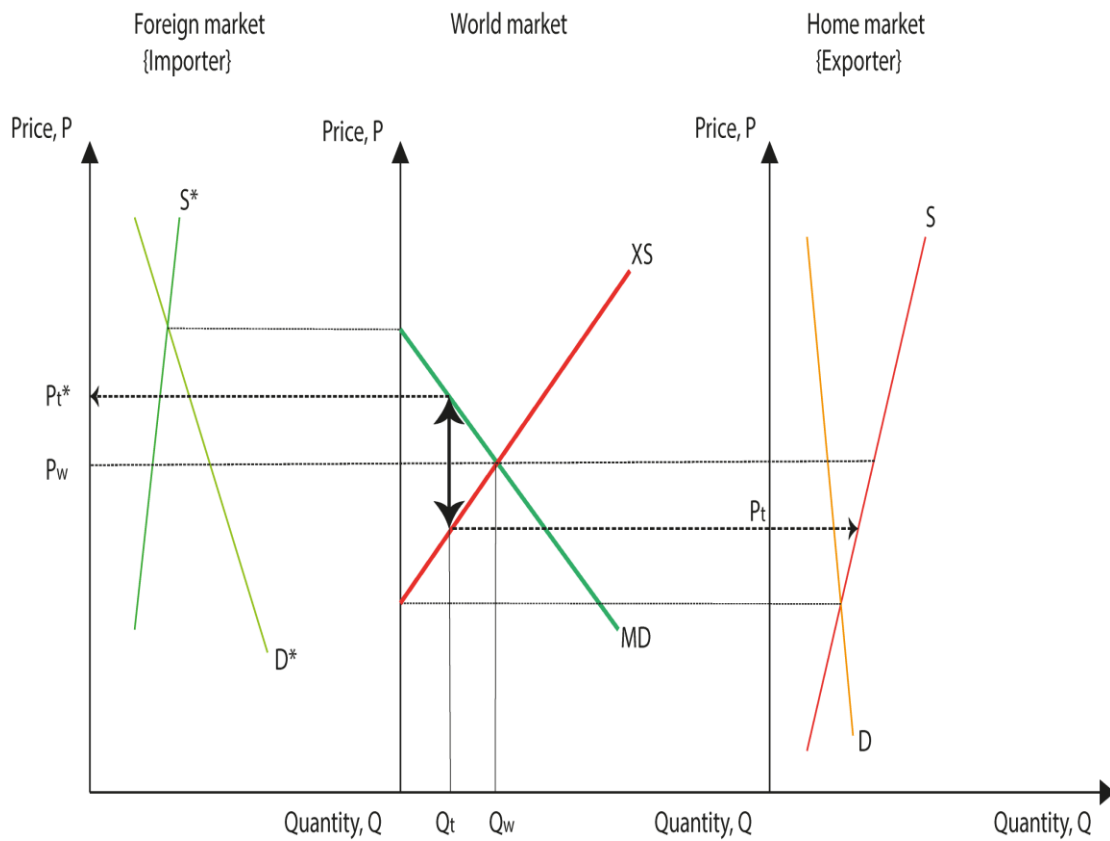
In this model we represent the demand for imports through a linear demand curve and the exports through a linear supply curve. Traditionally, demand curves arise through the utility maximization problem of the consumer, under its budget constraint. Usually, Marshallian demand curves are commonly used instead of Hicksian. The former represents the total effect and the latter the substitution effect. Also, Hicksian demand curves always have a negative slope, but this is not always the situation in Marshallian demand curves where it is possible for curves with a positive slope to occur. The reason for the first is that the substitution effect always leads to a lower demand under a rise of the commodity price, whereas in the second it is possible to come across “Giffen goods”. “Giffen goods” are goods where the income effect has an opposite and stronger influence than the substitution effect and finally the total result leads to demand curves with an upward tendency.

In the international trade the aggregate net demand for imports, or the residual demand, as it is usually named, arises from a different framework. If a good is produced in a country and the domestic demand exceeds domestic supply, then the residual demand has to be covered through imports. For specific details an analytic graph represents the situation in Appendix A. On the other hand, the exports supply curve is derived from a similar concept and not from the cost minimization problem of the producer. Commonly, producers minimize their production cost under the budget constraint, which is the “isoquant” curve. Currently, if a country’s supply exceeds the demand for a specific price, and if prices are different between the importers and exporter’s domestic markets, then trade will arise.

In the model, only residual demand and supply curves will be examined. The first reason is that we can present the effect of the endogenous tax through a more comprehensible way to the reader. The second reason is that if a commodity is not produced in a country, then the domestic and the import demand are the same since the domestic demand is subtracted from zero. Moreover, an unscrupulous assumption is that a commodity is produced in a country and only consumed in another one. In this case also, the domestic supply equals the export supply since the exporter

subtracts the supply curve from zero. Intuitively speaking, an import tariff or an export tax does not complicate the model, if we interpret it through aggregate residual demand and supply curves. However, the difference between the imports demand curve and the exports supply curve, compared to a normal demand and supply curve, should be completely understandable. In Figure 2.1 we present the effect of a specific import tariff or export tax in the three markets, the domestic exporter's market, the foreign importer's market and the world market. In the literature, the importer's market is mentioned as domestic and the exporter's market as foreign. The reason we use an opposite definition is that the model is constructed through the eyes of the exporter and we observe the importer as a "rival" country.

Figure 2.1



A specific import tariff or export tax increases the price abroad, inside the importer's market and decreases the price in the home exporter's country. As a result, trade declines from Q_w to Q_t . At this point, we should mention that the entire analysis refers to large countries, which can affect world price through their tariffs or taxes.

Obviously, they have significant demand and supply shares and through their aggregate demand and supply they affect global prices and the world trade.

2.3 Mathematic and Economic assumptions

In the model we use linear functions in general form. The demand function is given and predefined. The slope and the location are always the same.

$$D(q) \equiv P(q) = c - d \cdot q \rightarrow q = \frac{c}{d} - \frac{1}{d} \cdot p \quad , \quad \{c, d\} \in \mathbb{R}^+ \cup (0) \quad (1.1)$$

The supply function which is the main representative of our endogenous tax trade policy is also in general form, but the parameters can change.

$$S(q) \equiv MC(q) = a + b \cdot q \rightarrow q = -\frac{a}{b} + \frac{1}{b} \cdot p \quad , \quad \{b\} \in \mathbb{R}^+ \cup (0) , a \in \mathbb{R} \quad (1.2)$$

At this point, we have to mention one fundamental characteristic of the model. In order to prove the paradox and eventually the effects of trade policy on producer surplus, a specific assumption must be used in order to attain comparative statics and also for simplicity reasons. As it is going to be clear in the end, this assumption does not change the results qualitatively but quantitatively. The assumption is that without any state intervention the free trade equilibrium point is always the same. Therefore demand and supply are equal and they always cross over a specific point. The most important reason as we will see is that we can mathematically express the model through one specific parameter which is the only endogenous variable and through which, we can interpret the endogenous export tax. From (1.1) and (1.2) we obtain:

$$D(q) = S(q) \rightarrow \{p', q'\} \quad (1.3)$$

the price $\{p'\}$ and quantity $\{q'\}$, are the free trade equilibrium outcomes. Moreover, a specific tariff is introduced. The import tariff is constant in order to analyze the paradox, but in further parts this tax is going to be represented through a function of the supply curve price elasticity¹⁰. As we extend the model, every time the import tariff is constant we assume there are rigidities and for different reasons it cannot adjust to the optimum import tariff. Therefore if the import tariff regime is not

¹⁰ We have already mentioned that the optimum import tariff equals to the inverse supply curve price elasticity.

flexible, then the specific import tariff is constant. The use of an ad valorem tax does not also change the results qualitatively but quantitatively.

With this assumption (1.4) we define $\{t\}$ as a specific tax per quantity consumed. Correspondingly, this kind of specific tax (tariff) has the mathematical characteristic that it does not change the results if it imposed on consumption or production. For the latter the supply curve adjusts and shifts to the left and for the former the demand curve moves down at the amount of the tariff. According to the algebra it is not crucial where it is imposed, but in our framework we will impose the tariff inside the demand curve.

To continue, the supply curve has a linear general form and at the same time always crosses the free trade equilibrium point. As a result, the constant (α) of the supply function and the slope (b), depend on a linear function. From (1.2) and (1.3) we obtain the following.

$$p' = a + b \cdot q' \rightarrow a = p' - q' \cdot b \rightarrow b = (p'/q') - (a/q') \quad (1.5)$$

For example: $a = \overline{10} - \overline{10} \cdot b$. Therefore, due to (1.5); if the parameter $\{\alpha\}$ changes, then the parameter $\{b\}$ is also going to change. In the model we change the slope of the supply curve and ceteris paribus we can prove the paradox. Parameter $\{\alpha\}$ is the only endogenous variable so far, because an increase on this parameter will decrease the slope of the curve and if the demand curve and the import tariff do not change, a paradox arises. We will, precisely prove the paradox with a graphic and algebraic analysis in the following steps.

As it is mentioned on (1.4), we need a new function which will provide us with the equilibrium points for the producer. We will use a straight line, parallel to the demand curve and displaced downwards to the amount of the tax. From (1.1) and (1.4) we obtain:

$$q = \left(\frac{c}{d}\right) - \left(\frac{1}{d}\right) \cdot p \rightarrow q_t = q - t \rightarrow q_t = \left(\frac{c}{d} - t\right) - \left(\frac{1}{d}\right) \cdot p \quad (1.6)$$

Equation (1.6), represents a new linear curve which is the predetermined demand curve displaced downwards to the amount of the tax. Mention that $q_t(p)$, is the most important factor of this analysis, because the coordinates of q_t , determine the necessary elements, required to calculate the producer surplus \equiv PS. From now on, the

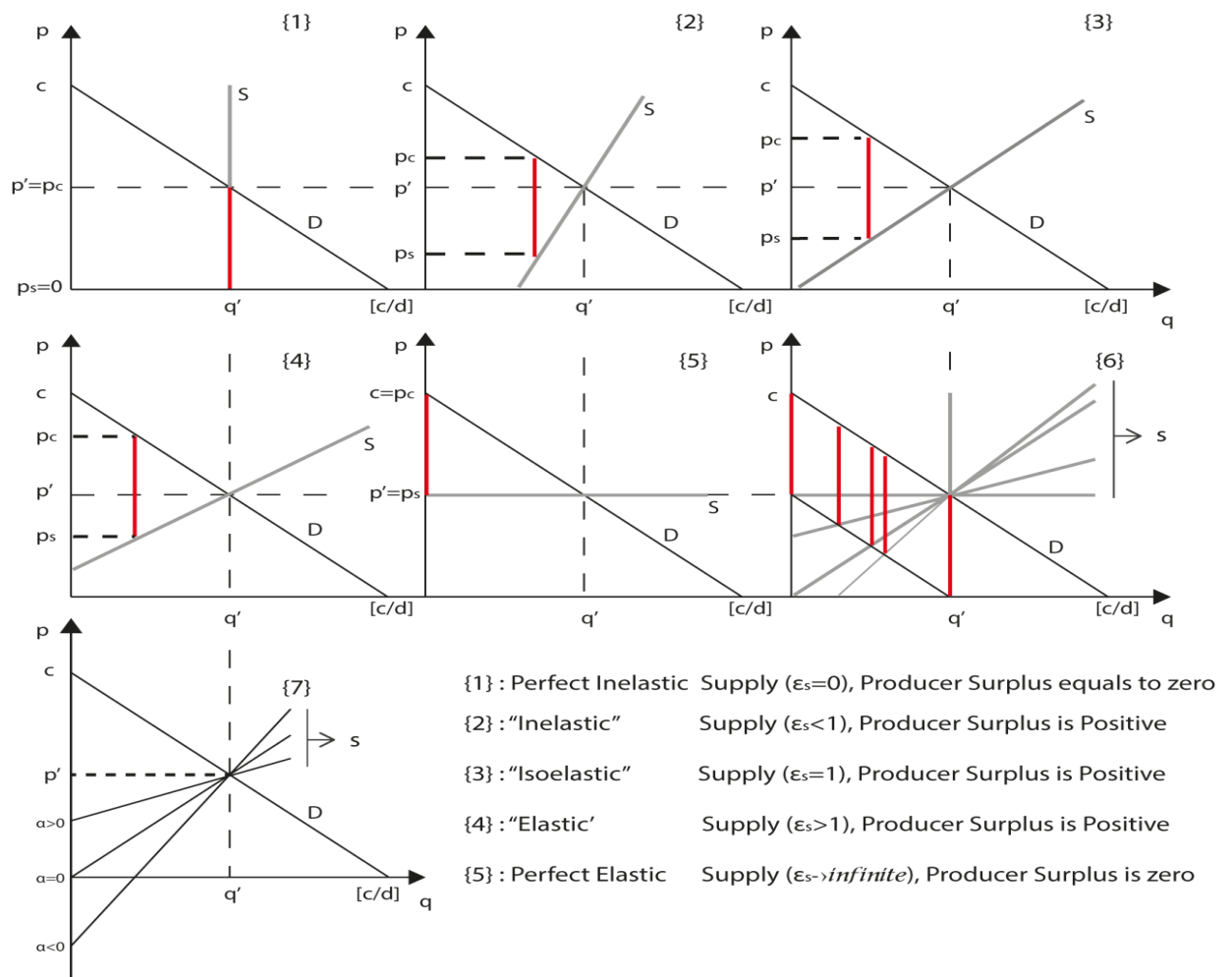
price derived from $q_t(p)$ will be referred to the price producer sells (receives) because of the tax $p_t \equiv p_s$.

Finally, we always operate on the first quadrant, under a continuous range of values.

$$\begin{aligned}
 & \text{Price received from producers} \equiv p_s \in (0, p') \\
 & \text{Price received from consumers} \equiv p_c \in (p', c) \\
 & \text{Quantity} \equiv q \in (0, q') ; \text{ Price} \equiv p \in (0, c) \\
 & c \equiv \text{willingness to pay}
 \end{aligned}
 \tag{1.7}$$

On the next board of diagrams we can graphically observe the results of a supply curve shift. We will extensively explain the framework, but this grid of figures provides a very vibrant idea of the primary concept.

Figure 2.2



Starting from graph {1} and moving to graph {5}, the slope of the supply curve decreases from infinite to zero. On this path the import tariff (red line on the graph), is constant as is the demand. We will prove in further parts the effect of this change with respect to the producer surplus. Graph {6} summarizes the previous graphs in order to help us comprehend the modeling framework. Finally, graph {7} will be a beneficial assistance, with respect to the effect of the parameter $\{\alpha\}$ to the supply curve price elasticity and slope. When the aforementioned parameter is negative the supply curve has always elasticity lower than one, and consequently when $\{a\}$ is zero the elasticity equals to one and if it is positive the elasticity is higher than one. After the microeconomic implications in the next part we will specifically explain the elasticity concept.

2.4 Microeconomic Framework

The examination of the market through linear demand and supply curves generates comfortable formations in the use of algebra, but at the same time technical restrictions regarding the economic background arise. On Appendix B, an extensive algebraic and geometric analysis is conducted. A perfectly competitive market generates the conditions, where the industry produces according to the rule where the price equals the marginal cost. On the specific appendix, six graphs have been designed. The first three of them are under fixed cost and the last three of them are without fixed cost. Without the fixed cost the average cost is always larger than the marginal cost and as we proved on Appendix B, decreasing returns to scale characterize the production for every positive production point. However, if production faces fixed cost, then depending on the production output, decreasing, increasing and constant returns to scale can be analyzed under linear supply curves.

At the first graph on Appendix B, the supply curve represented through the marginal cost curve is relatively elastic since the price elasticity is larger than unity. Based on the assumption that the producers impose a price according to the marginal cost and not according to the average cost, the examination can proceed only for decreasing and constant returns to scale. The reason is that despite the fact the average cost is

declining and it is higher than the marginal cost¹¹, a price according to the marginal cost linear curve, does not cover the average cost. The same situation occurs in the second and the third graph where the supply curve is isoelastic and inelastic. Therefore, linear supply curves are convenient for constant and decreasing scale economies, but not for increasing scale economies. Moreover, it has already been mentioned that the model will attempt to incorporate DRS, CRS, IRS, and the first two of them can be easily interpreted. As far as the third situation of increasing returns to scale is concerned, we will profoundly analyze the way we can introduce them to the model on the specific section. Nevertheless, if the producers choose a price policy according to the average cost, then increasing returns to scale can be also easily explained under the specific assumptions of linear marginal cost supply curves.

To continue, in the diagrams we derive three equilibrium points depending on the position of the demand curve. The first, demonstrates a situation where the demand is relatively larger to the other two points. At point one the marginal cost is higher than the average cost and the industry faces decreasing returns to scale. In Figure 2.2 we introduce the primary idea of a supply slope change, and at the same time if the price the producer receives is larger than the average cost, then graphs {1} till {5} in Figure 2.2 always can be interpreted for decreasing returns to scale. The reason is that if the free trade equilibrium point is combined with an import tariff, so that the production is derived under a DRS framework, then shifting the supply curve will also shift the average cost curve because marginal, average and total cost are interrelated through the same parameters of the model. Therefore, if the starting point of graph {1} in Figure 2.2 is combined with the first point on the first three graphs of Appendix A, then the entire framework can be interpreted for decreasing returns to scale.

The second equilibrium point, demonstrates a situation where demand is relatively smaller to the first point but larger than the third. In the first three graphs on Appendix A, the second point demonstrates equilibrium where the marginal cost is always the same to the minimum average cost and therefore constant returns to scale characterize the production. The third equilibrium point demonstrates a situation where the

¹¹ In respect to the cost theory, falling long run average cost curve represents increasing returns to scale. At the minimum where $LAC=LMC$, constant returns to scale characterize the production. Also if the long run average cost increases, decreasing returns to scale characterize the production. Finally, the marginal cost is higher than the average cost from the point where the average cost has a positive slope.

marginal cost is lower than the average cost but the average cost is decreasing. In order to be accurate with the microeconomic theory, we are forced at this point to further analyze the concept. Formally, if the long run average cost is falling then increasing returns to scale characterize the production, but if the price received from the producers is lower than this falling average cost then the producer faces losses. The complexity of the problem is that in formal theory, while the average cost is decreasing, the marginal cost is initially decreasing and then rising before it crosses the minimum long run average cost. If the price is lower than this minimum, the firm has damages in the long run and therefore with linear supply curves and a price rule according to the marginal cost, cannot be easily interpreted for increasing returns to scale. Consequently we will use linear supply curves but through a different framework in order to incorporate increasing returns to scale in the model in further parts. On the other hand, we can examine elastic, isoeastic and inelastic supply curves for industries characterized by constant and decreasing returns to scale.

Furthermore, at the minimum average cost point where the average cost equals the marginal cost, the production output depends on two parameters as we derived on Appendix B. If the parameter $\{b\}$ increases then the slope of the supply curve decreases according to the equation (1.2) and the minimum average cost point moves to the left and the production output moves also to the left because it depends negatively on this parameter (see Appendix B). The reason we mention it, is that a change to the endogenous variable $\{\alpha\}$ changes the slope according to the equation (1.5) and transposes both marginal and average costs, while the minimum average cost is also shifted since it strictly depends on the endogenous variable $\{\alpha\}$.

To conclude, depending on the position of the demand curve and the value of the import tariff, the entire framework can explain the effects of a supply curve shift under constant and decreasing returns to scale. If the producers use the average cost instead of the marginal cost curve in order to price the commodity, then increasing returns to scale can also be examined through the model¹².

In the next part, we will further explain the concept of the price elasticity and the connection between the cost elasticity and the price elasticity. The reason is that a change of the endogenous variable $\{\alpha\}$, changes the price elasticity of the supply

¹² In further sections, it will be clear why we also mention the specific possibility.

curve which is represented through the marginal cost curve. As we will see, a change to the cost function can change the cost elasticity and the cost elasticity affects indirectly the price elasticity, through which the parameter $\{\alpha\}$ can change. The main technical concept of the model is that by increasing the production cost, we can change the slope of the supply exports curve, and observe the different effects of such a policy to the producer surplus. The price elasticity is a very significant concept, because if we are able to analyze the effects for every possible value of the elasticity, then we can make the model beneficial for export supply curves of any possible range of values regarding price elasticity. Also, the cost elasticity will be our fundamental device in order to introduce a trade policy through a different way. Therefore, it must be clear when we use such definitions with respect to elasticity, in order to attain a beneficial way to introduce an endogenous export tax through an increase in production cost.

2.5 Cost and Price Elasticity

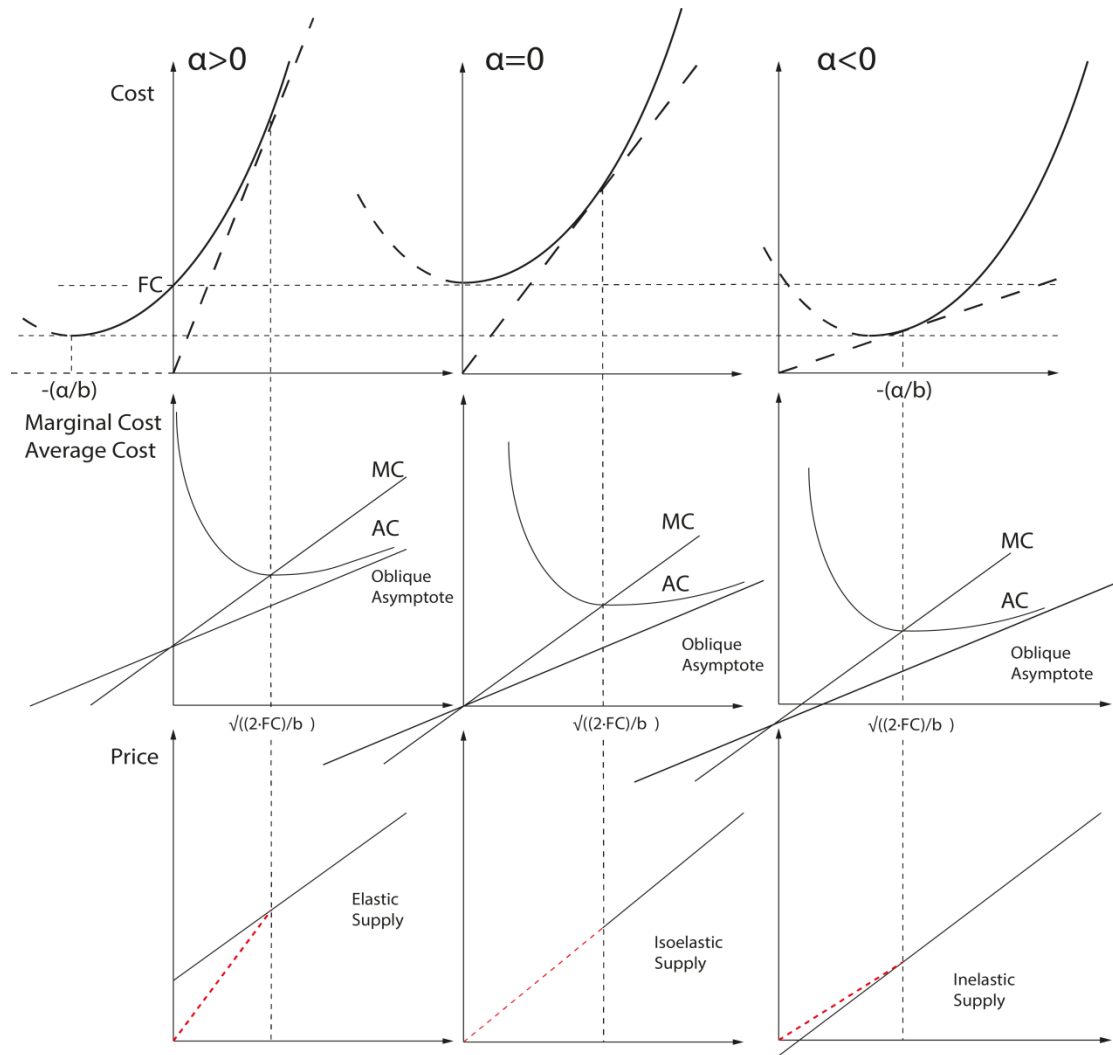
Elasticity is a measure of response. It provides beneficial information of a dependent variable response due to a change to the independent variable. Precisely, a percentage change on the independent variable will cause a percentage change on the dependent variable. If the price increases by 2% and the quantity decreases by 3% then the price elasticity is minus three over two. Many types of elasticities exist and are widely used in economics. In our framework we will focus on supply and demand curve price elasticity, as on cost elasticity also.

The price elasticity of demand explains the quantity sensitivity due to price volatility and usually has a negative sign. The reason is that if the price of the product increases the consumer will demand less. On the other hand, firms will produce an extra unit of output only if the price increases, since the production cost increases. Cost elasticity measures the responsiveness of the cost due to a change in the production.

Figure 2.3 represents cost, marginal cost, average cost and the supply curve in our framework. On the first three graphs the cost curve is designed for different values of the endogenous variable $\{\alpha\}$. When the tangent equals the radius crossing from the origin the cost elasticity is one, and the marginal cost is equal to the minimum average

cost. On the left of this point the cost elasticity is lower than unity and the average cost is higher than the marginal cost and at the same time it is decreasing. On the right of this point the average cost is rising and the marginal cost is higher than the average cost, at the same time the cost elasticity is higher than unity.

Figure 2.3



$$\epsilon_{cq} \equiv \frac{MC}{AC} = \frac{dC}{dq} \cdot \frac{q}{C} \equiv \frac{\frac{dC}{dq}}{\frac{C}{q}} \equiv \frac{\text{tangent}}{\text{radius}} \rightarrow \text{Cost Elasticity}$$

$$\epsilon_{pq} = \frac{dp}{dq} \cdot \frac{q}{p} \equiv \frac{\frac{dp}{dq}}{\frac{p}{q}} \rightarrow \text{Supply price elasticity}$$

$$\epsilon_{qp} = \frac{dq}{dp} \cdot \frac{p}{q} \equiv \frac{\frac{dq}{dp}}{\frac{q}{p}} \rightarrow \text{Demand price elasticity}$$

(1.8)

Moreover, on the second panel of Figure 2.3 we have designed again the average and the marginal cost for different values of the endogenous variable $\{\alpha\}$, for details and analytical graphs examine the concept on Appendix B. On the third panel, the supply curves are also represented for different values of the endogenous variable $\{\alpha\}$. As we can observe, when the endogenous variable is positive the price elasticity is higher than unity, when $\{\alpha\}$ equals to zero the price elasticity always is equal to unity and when $\{\alpha\}$ is negative the supply curve price elasticity is always lower than unity. Graphically, we can see it since the red dashed line on the third panel which represents the slope of the radius crossing from the origins is, lower, higher, and equal to the slope of the linear supply curve, depending on the graph. At the beginning it is larger and the supply is always elastic, moreover it is equal to the slope and the curve is always isoelastic, and finally the radius has a lower slope from the linear supply curve and therefore the supply is always inelastic.

The fact that the linear supply curves face all economies of scale if sufficient fixed costs exist is remarkably interesting. The problem is that despite the fact the producer faces increasing returns to scale in the beginning; he cannot take advantage of them, because a price according to the marginal cost is always lower than the average cost. As we have already mentioned, only if the market conditions allow the producer to charge a price according to the average cost, then the firm can exploit IRS for its own advantage. Another remarkable point is, that despite the fact cost curves are strictly convex since the second derivative with respect to the output is positive, in a situation defined as decreasing returns to scale in the long run, the firm also faces increasing returns to scale at the first stage. The reason is the existence of fixed costs. Formally, since the second derivative is positive then the cost increases (at an increasing rate), but the fixed cost allows the existence of IRS despite the fact the cost increases (at an increasing rate).

To continue, we will use some transformations to the cost and price elasticity of the supply, in order to provide a connection between them. The reason is that an endogenous taxation on the production will change the cost curve and therefore the cost elasticity. Thus, this change will affect the price elasticity and at the same time it will give us the opportunity to change the endogenous variable $\{\alpha\}$, in order to examine the effects of this policy on the producer surplus.

Initially, in equation (1.8) we defined supply elasticity according to the official definition. As we can see it is the inverse formula compared to demand elasticity. The reason is that quantity is the independent variable for supply, and respectively, price is for demand. Economics use the inverse origins, by imposing price on the vertical axis despite the fact it is the independent variable for the demand function. From the same point of view, quantity is the dependent variable for supply and it is imposed on the horizontal axis. According to the mathematical definition equation (1.8) it is accurate, but when we discuss about economics the opposite definition is commonly used in order to use the same formulas for demand and supply price elasticity. Therefore, and hence, we will define supply curve price elasticity according to the economic formula, otherwise the mirror results occur.

$$\varepsilon_s = \frac{dq}{dp} \cdot \frac{p}{q} = \frac{1}{b} \cdot \left(\frac{p}{\left(-\frac{a}{b} + \frac{1}{b} \cdot p\right)} \right) = \frac{p}{-a+pb} \equiv \frac{\text{radius}}{\text{tangent}} \quad (1.9)$$

$$\varepsilon_{cq} = \frac{dC}{dq} \cdot \frac{q}{C} = \frac{MC \cdot q}{C} \rightarrow q = \frac{\varepsilon_{cq} \cdot C}{MC} \xrightarrow{(1.9)} \varepsilon_s = \frac{1}{b} \cdot \frac{p^2}{\varepsilon_{cq} \cdot C} \quad (1.10)$$

$$\varepsilon_s = \frac{1}{b} \cdot \frac{MC^2}{\varepsilon_{cq} \cdot C} = \frac{1}{b} \cdot \frac{p^2}{\varepsilon_{cq} \cdot C}$$

If a central planner decides to increase the fix cost through taxation or different ways, then the cost elasticity decreases. When we move to the price elasticity, on the denominator we can see that the cost elasticity decreases but the cost increases. Therefore we will use derivatives to derive the necessary conditions allowing such a policy to change the price elasticity of the supply. Otherwise, an increase in the cost through the fixed cost for example may have the opposite or no results with respect to the slope of the supply curve. $\left\{ \frac{d\varepsilon_{cq}}{dC} = -\frac{MC \cdot q}{C^2} \right\}$, if the fraction equals to unity, then an increase to the cost will decrease equally the cost elasticity and then the supply price elasticity will not change, since the two opposite forces will be cancelled out. If the fraction is higher than one, then the supply price elasticity will increase and the supply will become more elastic regarding taxation. Finally if the fraction is lower than one then this policy will decrease the elasticity of the supply curve.

Finally, we will express the endogenous variable $\{\alpha\}$ as a function of the supply price elasticity in order to use the function later in the next part where the first results are going to appear. From (1.5) and (1.9) we obtain:

$$\alpha = -\frac{p'}{q' \cdot \varepsilon_s} + \frac{p'}{q'} \quad (1.11)$$

as we can see if the price elasticity increases the fraction decreases and the endogenous parameter increases because of the negative sign of the fraction. The idea is that a tax will affect the cost and then, as a chain rule the cost will affect the cost elasticity and thus the cost elasticity will affect the price elasticity and the price elasticity will change parameter $\{\alpha\}$ and the supply curve slope will change as well. Therefore, when we will try to derive the results on the final part, we will use partial derivatives and the chain rule in order to calculate the final result.

2.6 Economies of Scale

We have frequently used the term economies of scale in our script so far. We have also explained that, we will try to incorporate decreasing, constant and increasing returns to scale in our model. The definition we have used until now is based to the cost theory, and precisely to the convexity and concavity of the cost function. If the cost function is convex then the economy is characterized by decreasing economies to scale and the opposite happens if it is concave. On the other hand, another very frequent definition of increasing returns to scale is that the long run average cost is falling. In our framework, the cost function is convex since the second derivative is positive, but simultaneously the long run average cost is falling at least until it reaches the marginal cost. The reason these definitions contradict, is the existence of fixed costs which are going to help us explain the concept further.

Economies of scale characterize production and exist in it and not in the cost function. The reason that the cost function is profoundly used to define economies of scale is that it a mathematical connection exists between them. This connection allows the expression of scale effects through cost function instead of production function. Nevertheless, when the fixed cost appears in the production, this connection is totally disturbed. Basically, when we operate with linear supply curves and a fixed cost

exists, then non-decreasing returns to scale characterize the production (increasing returns to scale). Without fixed cost and under linear supply curves the production is always characterized by decreasing returns to scale and the only way to represent production without fixed cost and constant returns to scale is the use of linear constant marginal cost with a slope equal to zero¹³.

Furthermore, since we use fixed costs in the cost function, the effects and results in our framework are robust and accurate only for industries which face increasing returns to scale. It is a controversial statement, especially at this point, but we will further explain why this distinction is binding for our framework and results. Constant returns to scale have been deeply analyzed so far in trade theory, whereas decreasing returns to scale are important but increasing returns is the challenging issue for any applicable model in the trade theory.

According to the fundamental definition, if a firm increases its inputs by a number higher than one and the production increases more than this number, then IRS characterizes the production. If the production increases less than this number, then DRS characterizes the production and, finally, if it increases equally then CRS characterizes the production. Scale economies characterize the production and more precisely the production function. When the producer minimizes the cost under the budget constraint, the marginal cost, or the supply curve is derived. A Cobb-Douglas function is commonly used to describe the production function; and the parameters of this function define the convexity or concavity of the cost function (Appendix E).

The definition becomes more complex when we discuss about the transformation function which represents the production set and the transformation frontier. The production function is derived from the transformation function, and the cost function is derived from the production function. Without fixed costs, the characteristics regarding scale effects are the same in the aforementioned functions. Therefore, we can correctly characterize a production regarding economies of scale through the cost function. With fixed costs, the transformation function is not anymore a differentiable and as a result the transformation set and the production function is not strictly convex. This result, arising from inconsistencies due to fixed cost, does not allow the cost convexity-concavity criterion to be accurate anymore. Therefore, the use of this

¹³ For technical details see Andreu *et al.* (1995, pp.127-147).

criterion is a common mistake in economics, and in this thesis we want to explain the significance of this reference in trade theory.

To conclude, and after a deeper understanding of the microeconomic theory, the utilization of linear supply curves and the existence of fixed cost, characterize the model and our framework with increasing returns to scale. In order to achieve the challenging issue of incorporating IRS, we are going to maintain fixed costs and therefore the model will also be applicable for CRS and DRS. Meanwhile, in Appendix D Figure 1 we also want to show that without fixed cost in our framework, the derivation of the paradox is not possible, and generally speaking, it is not possible to use inelastic linear supply curves without fixed cost because the cost is negative for a specific range of values. Precisely, it is possible under inelastic supply curves to observe negative cost, depending on the equilibrium points. But for simplicity reasons, we will assume for DRS that the equilibrium points are always at a point where the average cost is positive and not negative; as a result we can also explain DRS for inelastic supply curves without a fixed cost. Therefore, we advise any relevant studies or researches to be very careful with the use of linear algebra, and that our analysis made its contribution to understand the microeconomic foundation deeper than before.

2.7 The Paradox

In Figure 2.4 we can observe a paradox. In the beginning when the supply curve is completely inelastic the producer surplus is zero because the price he receives equals zero and there is no trade. When the cost increases and the slope of the supply curve decreases the supply curve shifts from S_1 to S_2 and the producer surplus is positive and higher than before, despite the fact that the cost increased. The producer surplus is the trapezoid represented in the area {0123}. Also, the import tariff did not change since we assume there are some rigidities. As we will prove with algebra, the paradox which is observed geometrically here, we will try to explain further the microeconomic implications of the paradox. The more elastic the supply curve is, the less the contribution of the producer to the tax revenue becomes. Therefore this kind of movement has a positive effect on its surplus because every time the elasticity increases the producer contributes less and his surplus decreases less than before.

Moreover, every time the elasticity increases the dead weight loss increases, and an opposite force arises. This force decreases the producer's surplus, every time the elasticity increases. These two opposite forces¹⁴, as we will see in the concluding remarks will increase the producer's surplus until the elasticity becomes one and then the surplus will decrease. The reason is that the first force is stronger at the beginning and after a critical point the other force dominates. Finally, the negative force overcomes the positive after a critical point and for this reason the paradox and these specific results occur.

The producer's surplus with inelastic supply is a trapezoid. We will use the geometrical principle in order to calculate the producer surplus and specifically the effect of the indirect endogenous tax on the area. In the following equations, we will substitute the functions we defined, and then we will calculate the derivative with respect to the endogenous tax. The endogenous tax is defined as t_s and it is imposed to the cost function and more precisely to the fixed cost.

Therefore the cost function will become:

$$C \equiv g(P) = \int_0^q [a + b \cdot q] \cdot dq = a \cdot q + \left(\frac{b}{2}\right) \cdot q^2 + FC + t_s$$

And under the assumption that:

$$\left| \frac{d\varepsilon_{cq}}{dC} = -\frac{MC \cdot q}{C^2} \right| > 1 ,$$

then $\frac{\partial \varepsilon_s}{\partial t_s} > 0$, tax affects positively supply curve price elasticity.

The equilibrium price and quantity for the producer are:

$$p_s = \frac{b \cdot (c - d \cdot t) + a \cdot d}{b + d}, \quad q_s = \left(\frac{c}{d} - t\right) - \left(\frac{1}{d}\right) \cdot \left(\frac{b \cdot (c - d \cdot t) + a \cdot d}{b + d}\right)$$

Then the producer surplus and the derivative in respect to the endogenous tax are:

$$PS = \left(\frac{1}{2}\right) \cdot \left(q_s - \frac{a}{b}\right) \cdot p_s, \text{ but } a < 0, \text{ so } PS = \left(\frac{1}{2}\right) \cdot \left(q_s + \frac{a}{b}\right) \cdot p_s, \text{ with } |\alpha|$$

¹⁴ See: Appendix D Figure 2 the graphic analysis of the two opposite forces.

$$\frac{\partial PS}{\partial t_s} = \frac{1}{2} \cdot \left[\frac{\partial p_s}{\partial t_s} \cdot q_s + \frac{\partial q_s}{\partial t_s} \cdot p_s + \frac{\partial (\alpha/b)}{\partial t_s} \cdot p_s + \frac{\partial p_s}{\partial t_s} \cdot \frac{a}{b} \right]$$

Since $\{p_s, q_s, b\}$ depend on $\{\alpha\}$, and $\{\alpha\}$ depends on ε_s , and finally ε_s depends on t_s :

$$\frac{\partial PS}{\partial t_s} = \frac{1}{2} \cdot \left[p_s \cdot \left(\frac{\partial q_s}{\partial a} \cdot \frac{\partial a}{\partial \varepsilon_s} \cdot \frac{\partial \varepsilon_s}{\partial t_s} \right) + \left(q_s + \frac{a}{b} \right) \cdot \left(\frac{\partial p_s}{\partial a} \cdot \frac{\partial a}{\partial \varepsilon_s} \cdot \frac{\partial \varepsilon_s}{\partial t_s} \right) + p_s \cdot \left(\frac{p' \cdot q'}{(p' - \alpha)^2} \cdot \left(\frac{\partial a}{\partial \varepsilon_s} \cdot \frac{\partial \varepsilon_s}{\partial t_s} \right) \right) \right]$$

where:

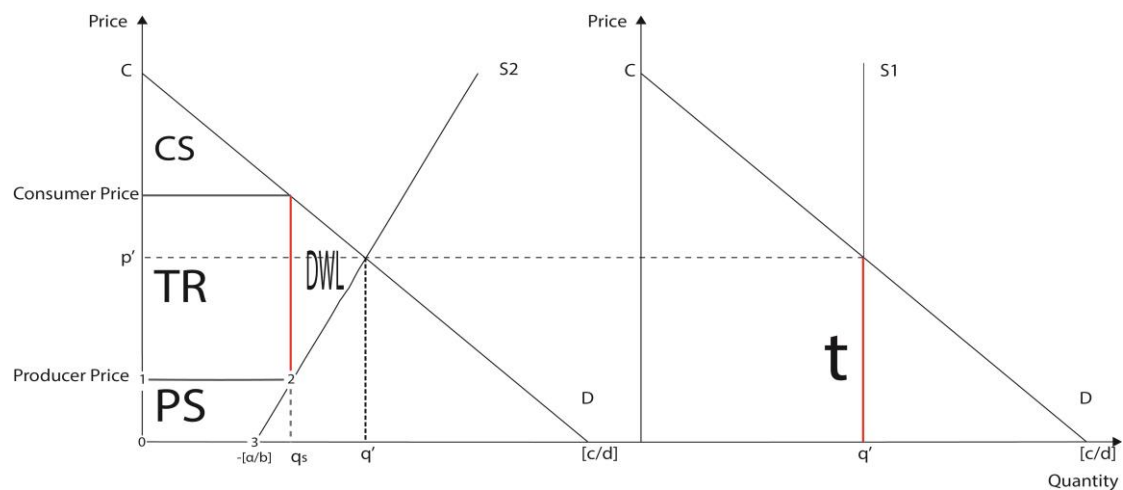
$$\frac{\partial p_s}{\partial a} = \frac{(\gamma - \frac{\alpha}{\delta} + d) \cdot \left[d - \frac{c}{\delta} + \frac{d}{\delta} - \frac{2 \cdot a \cdot d}{\delta \cdot p'} \right] + \frac{1}{\delta} \left[(\gamma - \frac{\alpha}{\delta} + d) \cdot (c - d \cdot (1 - \frac{a}{p'})) + a \cdot d \right]}{(\gamma - \frac{\alpha}{\delta} + d)^2} > 0$$

$$\frac{\partial q_s}{\partial a} = -\frac{1}{d} \cdot \frac{\partial p_s}{\partial a} < 0, \quad \frac{\partial a}{\partial \varepsilon_s} = \frac{p'}{q' \cdot (\varepsilon_s)^2} > 0, \quad \frac{\partial \varepsilon_s}{\partial t_s} > 0, \quad \gamma = \frac{p'}{q'}, \delta = q'$$

Finally, the effect of the endogenous taxation has a positive effect if:

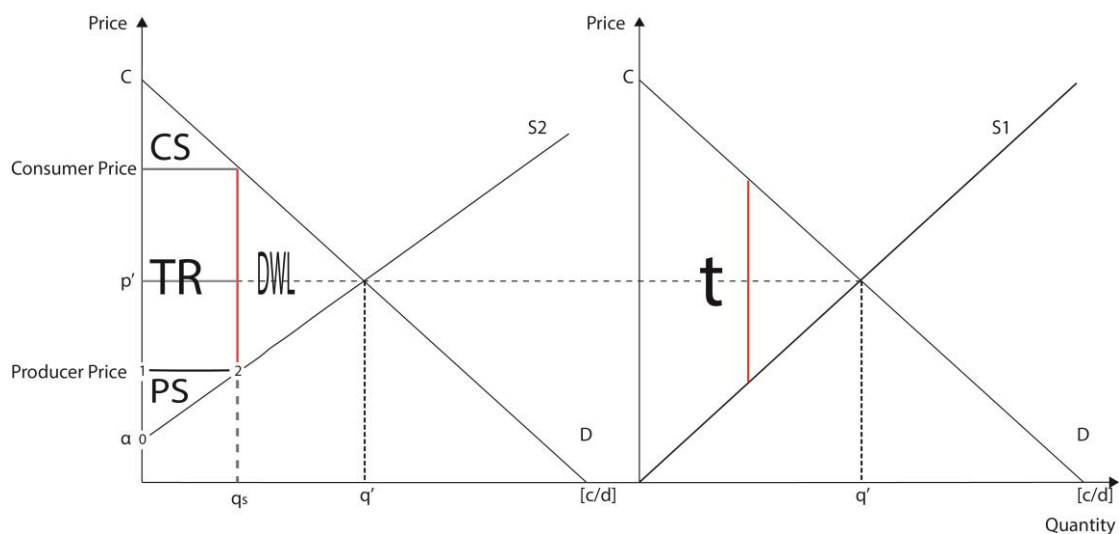
$$d < \left[\left(\frac{\frac{\partial p_s}{\partial a}}{\frac{p' \cdot q'}{(p' - \alpha)^2}} \right) + \left(\frac{b \cdot p_s}{b \cdot q_s + \alpha} \right) \right]$$

Figure 2.4



To conclude, since the derivative is positive the indirect endogenous tax on the cost function will increase the producer surplus if the supply curve is inelastic. The aforementioned result is accurate if the import tariff is subject to rigidities.

Figure 2.5



We will use the same procedure to calculate the effect when the supply curve is elastic. The producer surplus is the triangle {012}, and as we will see it decreases if the elasticity increases.

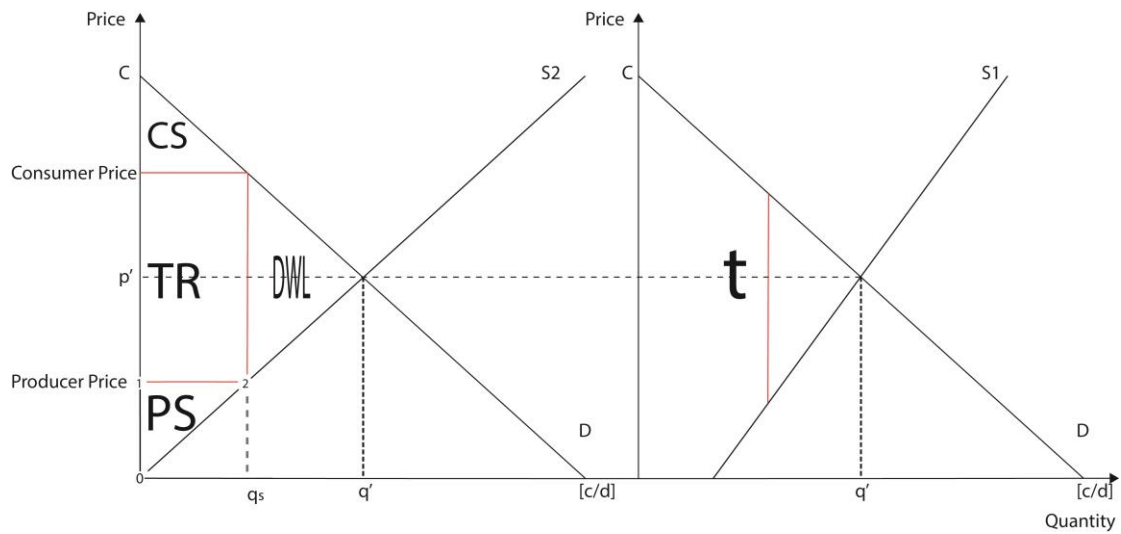
The producer surplus and the derivative in respect to the endogenous tax are:

$$PS = \left(\frac{1}{2}\right) \cdot (p_s - \alpha) \cdot q_s, \quad \alpha > 0$$

$$\frac{\partial PS}{\partial t_s} = \frac{1}{2} \cdot \left[\frac{\partial p_s}{\partial t_s} \cdot q_s + \frac{\partial q_s}{\partial t_s} \cdot p_s - \frac{\partial \alpha}{\partial t_s} \cdot q_s - \alpha \cdot \frac{\partial q_s}{\partial t_s} \right] \leq 0 \quad \forall \alpha \geq 0$$

To conclude, since the derivative is negative the indirect endogenous tax on the cost function will diminish the producer surplus if the supply curve is elastic. The aforementioned result is accurate if the import tariff is subject to rigidities.

Figure 2.6



The supply curve on this graph is isoelastic and the producer surplus is represented by the triangle {012}. The situation here is that, since the parameter $\{\alpha\}$ is zero, the specific derivative is also zero. That means that the slope of our function, which is a function depending on the parameter t_s is zero and we have a turning point. While the supply was inelastic, the tax increased the profits and the opposite for elastic supply. Therefore, from the time when the slope of the function, represented through the prime derivative is positive, until the supply curve is isoelastic and then the slope decreases since the derivative is negative, we can conclude that the tax maximizes the producer surplus when the supply curve is isoelastic and the endogenous parameter $\{\alpha\}$ is zero.

The disadvantage of this technique is that we have to use three different functions to calculate the producer surplus, and not a unique function. However, the utilization of linear functions makes the calculations very easy, and if anyone uses a non-parametric example with specific values of the parameters, he will observe the effect we try to derive in general form.

The producer surplus and the derivative in respect to the endogenous tax are:

$$PS = \left(\frac{1}{2}\right) \cdot (p_s) \cdot q_s ,$$

$$\frac{\partial PS}{\partial t_s} = \frac{1}{2} \cdot \left[\frac{\partial p_s}{\partial t_s} \cdot q_s + \frac{\partial q_s}{\partial t_s} \cdot p_s \right] = 0 , \forall a = 0$$

In this part of our analysis, we tried to explain the effect of an endogenous taxation on the producer surplus. We used graphical analysis and derivatives to secure our conclusions. Obviously, the linear assumptions of the model facilitate the observation of the results, even with a simple numeric example. Nevertheless, a general form is more useful, since we can observe the parameters of the model and derive beneficial conclusions.

Moreover, we assumed that the import tariff is constant and it does not change when the elasticity of the supply is modified. This rigidity generates motivating results and the reason is that such an assumption is not commonly adopted in trade theory. Meanwhile, as we will see in the final part, the assumption makes the derivation of our result harder, compared to a flexible regime where the tariff is volatile and adjustable. The latter remark is going to be helpful so as to avoid calculations since we can explain why this paradox can arise through feebler assumptions.

To continue, the paradox can be explained through different ways. The first, concerning the two opposite forces, has already been analyzed on the specific section, although another reason exists. If a single firm increases its own price as a result of the taxation, and the other firms of the industry do not follow the same pattern, then it is not possible to achieve potential profits due to the paradox. On the other hand, if the industry increases the price then externalities arise for the industry and they can exploit the paradox. If all the firms raise the price of the commodity together, then the externality is positive for all of them and due to this behavior the rising cost spreads out to the industry. We could say a cartel behavior appears in the industry and despite the fact that the taxation forces the industry to increase the price, the profits increase despite the fact that the exports decrease.

The latter perceptive creates questions about the negative effects in the importing country and its welfare. If a country can indirectly tax the industry under the specific framework, then the industry can increase its own profits and the country receives tax revenues through a legal way according to the world trade regulations. Obviously, this policy is not straightforwardly applicable, but the model opens a discussion on the effects of internal taxation to the international trade environment. Finally, on the conclusions and policy implications we will explain the situation further.

2.8 Endogenous taxation subject to flexible trade policy regimes

In the previous part, rigidities characterized the tariff faced from the exporter's perspective. In this part, the import tariff will be the optimum tariff, which equals the inverse price elasticity of the exports, represented by the supply curve of the price elasticity.

$$t = \frac{1}{\varepsilon_s} \quad (1.12)$$

When the slope of the supply curve increases the tariff increases and vice versa. Therefore when the elasticity of the supply curve increases the slope decreases and the tariff decreases. At the extreme where the supply curve is completely elastic and the slope is zero the optimum import tariff equals zero and the free trade equilibrium is the optimum solution from the importer's perspective.

In the previous part we saw that $\{\varepsilon_s\}$ depends on the new form of the indirect tax, which transforms the production of the industry. Consequently, while calculating the effect of this new policy on the producer surplus we also have to use the derivative of the tax with respect to t_s , since the tariff depends now on the supply price elasticity.

When the elasticity is lower than unity we proved that the profits increase with the imposition of $\{t_s\}$ and the indirect tax increases the profits until the exports' supply curve becomes isoelastic. Currently, if the importers use the optimum tariff, every time the exporters increase the elasticity through the indirect tax $\{t_s\}$, the importers are forced to use a lower import tariff instead of a constant tariff as assumed before. Obviously, if a constant tariff can increase the profits when the elasticity is lower than unity, a lower tariff due to the use of the optimum tariff formula will increase the exporter's surplus even more than before. Thus, what we proved in the previous part is that the indirect tax has a positive effect under inelastic supply curves even with stronger assumptions (constant and higher tariff). On condition that the producer surplus can increase with a constant tariff, it can also increase even more under a lower tax rate due to the use of the optimum tariff inverse elasticity formula.

To continue, a supply curve characterized by an elasticity which is higher than unity needs a subsidy instead of an indirect tax in order to increase the exporters' profits. The reason is that a decrease on the elasticity will increase profits; consequently a

negative tax (subsidy) is now necessary in order to decrease the cost and shift the curve back to the opposite direction. At present, the optimum tariff as we will further prove, arises as an opposite force because it decreases. When the supply is relatively elastic the price the producer receives increases and the quantity exported decreases. We proved that an increase on the elasticity will decrease the profits when the elasticity is higher than unity, but now due to the optimum tariff a new effect arises.

The optimum import tariff decreases every time the supply elasticity increases, and as a result, the price the exporters receive, increases relatively to a constant tariff and the exports also relatively increase. As we will prove with the algebra, the introduction of the optimum tariff extends the paradox even to the elastic supply curve, but until a critical point.

To conclude, the optimum import tariff enforces the effect of the indirect tax on the production (t_s), when the elasticity is lower than unity and extends the positive effect of the (t_s) even when the elasticity is higher than unity. Therefore, in this final part we will focus on the effect of the indirect tax when the supply is elastic. We also explained the effect when the supply is relatively inelastic and it is not necessary to explain it again mathematically because we proved based on stronger assumptions that the effect is positive.

On the following graphs we represent the same concept but now the tariff is modified. The first graph concerns inelastic supply, and the rest two of them isoelastic and elastic supply curves. The situation and the analysis is exactly the same as in our previous analysis, and the only adjustment with respect to the derivatives is, that equilibrium price and quantity for the producer are not anymore a function of the endogenous parameter $\{\alpha\}$ but also a function of the import tariff. The reason is that the import tariff is a function of the supply elasticity and the supply elasticity is a function of the indirect tax.

Figure 2.7 (inelastic supply)

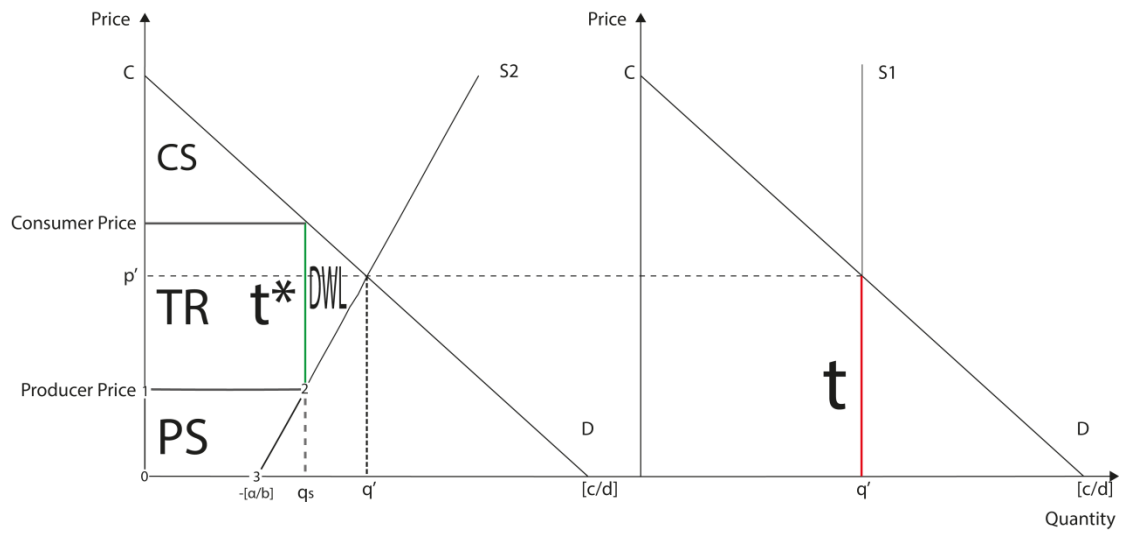


Figure 2.8 (isoelastic supply)

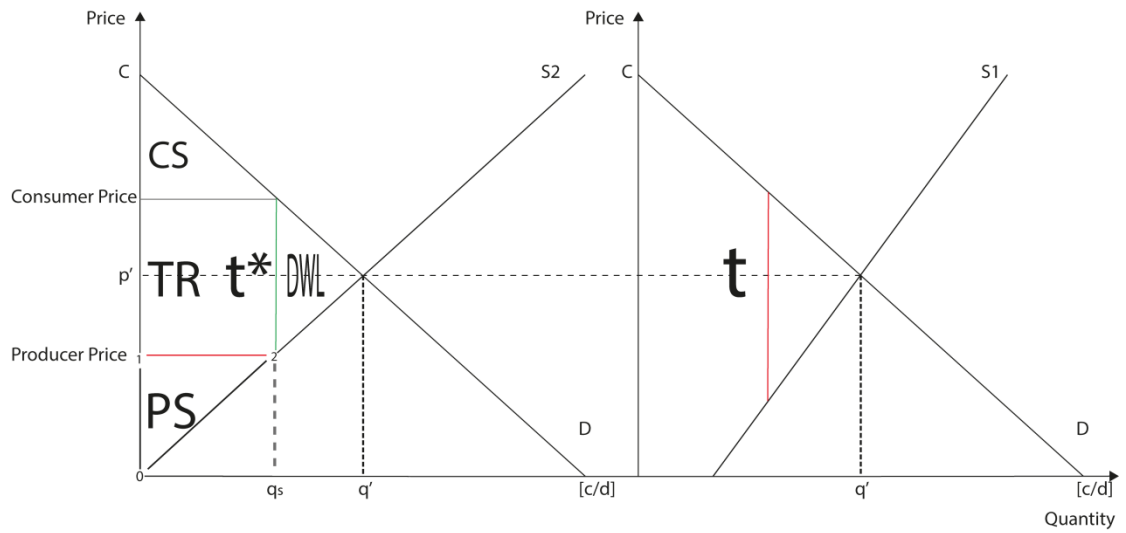
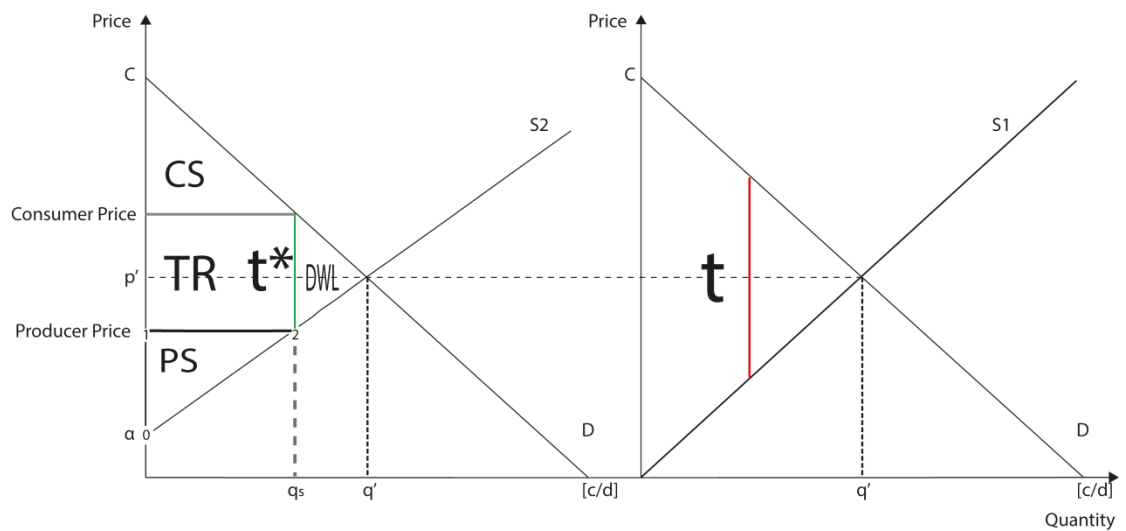


Figure 2.9 (elastic supply)



The producer surplus and the derivative in respect to the endogenous tax are:

$$PS = \left(\frac{1}{2}\right) \cdot (p_s - a) \cdot q_s$$

$$\frac{\partial PS}{\partial t_s} = \frac{1}{2} \cdot \left[\frac{\partial p_s}{\partial t_s} \cdot q_s + \frac{\partial q_s}{\partial t_s} \cdot p_s - \frac{\partial a}{\partial t_s} \cdot q_s - \frac{\partial q_s}{\partial t_s} \cdot a \right]$$

$$\frac{\partial PS}{\partial t_s} = \frac{1}{2} \cdot \left[q_s \cdot \left[\frac{\partial p_s}{\partial a} \cdot \frac{\partial a}{\partial \varepsilon_s} \cdot \frac{\partial \varepsilon_s}{\partial t_s} + \frac{\partial p_s}{\partial t} \cdot \frac{\partial t}{\partial \varepsilon_s} \cdot \frac{\partial \varepsilon_s}{\partial t_s} - \frac{\partial a}{\partial \varepsilon_s} \cdot \frac{\partial \varepsilon_s}{\partial t_s} \right] + (p_s - a) \cdot \left[\frac{\partial q_s}{\partial a} \cdot \frac{\partial a}{\partial \varepsilon_s} \cdot \frac{\partial \varepsilon_s}{\partial t_s} + \frac{\partial q_s}{\partial t} \cdot \frac{\partial t}{\partial \varepsilon_s} \cdot \frac{\partial \varepsilon_s}{\partial t_s} \right] \right]$$

$$\frac{\partial t}{\partial \varepsilon_s} = -\frac{1}{\varepsilon_s^2} < 0$$

The partial derivatives are the same as before, the only difference is that we have included the partial derivative of the import tariff with respect to the supply elasticity. On Appendix C we derive the necessary conditions which allow a positive effect even if the supply is elastic. The condition is feasible and it derives in general parametric form the maximum value of $\{\alpha\}$ allowing a positive effect. On condition that $\{\alpha\}$ is higher than this value, then the latter derivative is negative and the indirect endogenous tax has a negative effect, as also under fixed trade policy regimes.

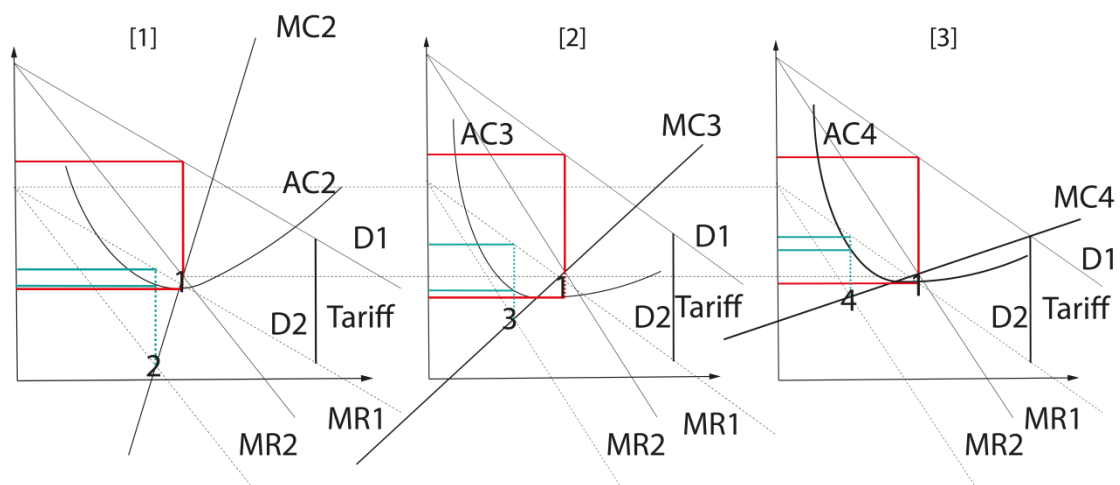
In the next section, we will extend the model frontiers for monopoly, monopolistic competition and we will discuss the effects of free entry. The conditions and assumptions are now stronger because new parameters induce our results. Precisely, the same framework appears again, but now the producer surplus also depends on different conditions due to monopoly and free entry, which operate as an opposite force to the trade policy. The positive is that the same areas appear and we can easily explain them without algebraic calculations.

To conclude, we observed the effect of the new trade policy on the producer surplus under competitive price rule and fixed and flexible regimes with respect to the import tariff. Under fixed regime, the endogenous taxation has a positive effect when the supply is inelastic and the opposite when the commodity is elastically produced. Under flexible regime, where the importer can modify the import tariff according to the inverse-elasticity optimum formula, the endogenous trade policy is beneficial for inelastic supply and also for elastic supply export curves, but until a critical value of the parameter $\{\alpha\}$.

2.9 Monopoly, Monopolistic Competition, Free Entry and Implications

In Figure 2.10, we present a monopoly including and excluding an import tariff, for elastic and inelastic marginal costs. In monopoly, the firm or industry does not have a supply curve and decides to produce at the point where the marginal cost is equal to the marginal revenue. The equilibrium point 1 is the same for the three graphs in order to maintain free trade equilibrium the same as before. At the second graph, the marginal cost is less inelastic and on the third it is elastic. The red square represents the producer surplus without tariff where the equilibrium is at point 1. Moreover, an import tariff displaces demand and marginal revenue curves downwards to the amount of the import tariff. The equilibrium moves to points {2, 3, and 4} for each graph. At the new equilibrium points the marginal cost equals the new marginal revenue. The producer surplus is now the blue smaller tetragonal and we can observe that at the beginning it increases and then it decreases similarly to the competitive concept.

Figure 2.10



The monopolist produces according to the monopoly formula and then he receives a price according to the demand curve. Meanwhile in this specific production he faces an average cost. The difference from the price received, minus the average cost of the specific production, multiplied with the production, derives the producer surplus. Every time the marginal cost becomes more elastic the quantity produced decreases and the price received also increases. Eventually, the same framework as before occurs, but now another parameter influences the result. The average cost at the equilibrium production increases every time the marginal cost becomes more elastic.

If the difference between the prices derived from demand, minus the average cost, increases relatively more than the decrease of the production, then the monopolistic profit increases like the competitive framework. If the specific difference, increases relatively less than the decrease in the production, then the monopolistic profits decrease. The same paradox and framework arises if we use linear marginal cost and demand curves.

To continue, we will proceed to discuss free entry. Under the competitive analysis the industry enjoys noncompetitive profits if the marginal cost curve is higher than the average cost curve. We have thoroughly explained the situation and considered why the marginal cost must always be higher than the average cost. We also explained why this is always accurate, even if we change the slope of the marginal cost curve. Precisely, if the demand is large enough, the equilibrium points under a competitive framework will always lead to a situation where the marginal cost is higher than the average. The latter result raises a new concept. If the sector has noncompetitive profits, and despite the fact that the price is always equal to the marginal cost, then other firms have incentives to enter to the industry. The latter, leads to opposite forces because the supply curve of the industry increases and moves on the right. It is exactly the opposite direction of our trade policy. Therefore, if conditions in the industry allow free entry, then in the long run the beneficial effects of the endogenous trade policy recant.

Moreover, there are different possible scenarios. If the supply is elastic and free entry is possible, then the profits will increase until the supply becomes isoelastic. On the other hand, if the supply is inelastic the profits will decrease until the indirect trade policy cancels out itself completely. Therefore, we must mention the possibility of free entry because it can potentially modify our ideas. The reasons we do not consider such an assumption are the following. Firstly, the existence of fixed costs glides the incentives of free entry, especially if we take significant fixed costs into consideration. For example, in the car industry noncompetitive profits appear, but it is not an easy decision for a firm to take due to high fixed costs. Also for high technological products like mobile phones or computers, it is not easy to compete with Apple, Samsung, Toshiba and Dell even if free entry is allowed.

To continue, firms inside the industry frequently use managerial techniques to prevent potential entry. For example, huge investments, advertising, or strategic entry barriers. The industrial organization explains the situation further and examines contestable markets and the conditions for free entry¹⁵. In many industries, entry is not allowed due to regulations or due to the existence of administrative malfunctions like bureaucracy. In Greece or other countries with such problems, even Greek firms cannot join an industry for the aforementioned reasons. Therefore, free entry complicates our model and potentially can eliminate the indirect endogenous taxation and its results, but under significant fixed costs and other assumptions, it is not easy.

To conclude, our model is more accurate for exporting industries where free entry cannot easily appear. Nevertheless, if we assume that there is such an industry, and free entry is possible despite the existence of fixed costs and the lack of any entry barriers, then we must consider this assumption. On the other hand, even if free entry is possible, then at least in the short run, our policy is applicable and even if, in the long run, free entry operates as a significant opposite force, then the supply must be inelastic and not elastic.

Competitive markets and monopoly are included in our agenda, and after the discussion concerning free entry we will also try to explain the implications of monopolistic competition. More specifically, the theory of monopolistic competition includes different models depending on miscellaneous assumptions. In our framework we will focus on the long run equilibrium of the competitive monopolistic firm with free entry. The same equilibrium model exists for price competition and for a combination of price competition and free entry. The difference now is that, we define free entry as another third country or a vector of different countries which can produce close substitutes of our exports. For example, a country exports cars and another country also decides to invest in such an industry, or we can use a simpler example. Tourism is an export commodity for south European countries, and the political crisis in Egypt increased the Greek tourism that year. The specific model of monopolistic competition can explain this particular concept. Due to the political crisis, the demand for Greek tourism increased and the opposite can happen if a developing country decides to invest on tourism. Based on the aforementioned

¹⁵ See Belleflamme and Peitz (2010, ch. 16).

assumption, the demand for the country with some kind of monopolistic power is going to decrease and in the long run equilibrium, the entry will stop until the point where the price received is equal to the average cost.

The marginal cost represents the curve according to which a country exports a vector of similar commodities, which define an industry like tourism. Then, another country which exports a close substitute will decrease the demand for the first country. Even if the first country receives monopolistic profits, these are going to vanish, until the demand is such that the profits become from monopolistic to competitive, and then entry to the industry will stop. We used a simple example, in order to make clear how it is possible for a monopolistic industry, facing an always smaller demand, become competitive, if monopolistic competition characterizes the industry worldwide.

Now let's assume that demand curve 1 is a demand curve already displayed downwards to the amount of a common import tariff. If free entry is allowed worldwide, then demand curve 1 is going to decrease more, compared, for example, with demand curve 2. What we want to explain here is that under monopolistic competition and worldwide free entry, the effect of an import tariff which moves downwards the demand curve is similar to the effect according to worldwide free entry. Therefore, if this reduction allows for smaller but noncompetitive profits, then our concept is applicable like monopoly.

To conclude, we explained the effects of free entry to the industry due to monopolistic profits. Free entry can have positive or harmful results in our model, but we also explained why free entry is not easily applicable under our assumptions.

Moreover, we discussed about monopoly and the potential effects of an indirect endogenous taxation. We explained why the monopoly is characterized through a similar way compared to competitive assumptions.

Finally, we introduced the term worldwide free entry, which can bring our model close to monopolistic competition models. The general idea arises from a paradox, as we profoundly proved on the first competitive part and can be extended under specific assumptions in monopoly and monopolistic competition.

Chapter 3

3.1 Concluding remarks

A plethora of profoundly analyzed results and conclusions can be derived from our theoretical model. We examined fixed and flexible import tariff regimes under the existence and also the absence of fixed costs. Correspondingly, elastic, isoelastic and inelastic export curves can be explained and examined, for industries which exhibit increasing, decreasing and constant returns to scale. The aforementioned cases and combinations of them can be interpreted for competitive, monopolistic and competitive monopolistic markets. Nevertheless, the possibility of free entry due to noncompetitive profits can potentially reverse our results. The latter assumption does not downgrade our model as we have explained in the relative section, especially under the existence of significant fixed costs. On the other hand, our model is not advantageous, at least in the long run, if free entry is possible in specific industries. Obviously, theory and practice differ and perfect competition is not encountered in real world, but we have to mention the effects of free entry to our concluding remarks. Overall, free entry does not constrain the theory of endogenous export taxation under fixed and flexible import tariff regimes.

The theory and the results arise from a simple microeconomic paradox. The model is based on this paradox and we tried to extend it for different market and production conditions. The paradox proves that if a supply curve is inelastic and a specific tariff is imposed, then the producer surplus increases despite the fact that the cost increases. The paradox is accurate until the supply curve becomes isoelastic and disappears when the price elasticity of the supply is elastic. Various reasons explain the paradox, but the core is that when the elasticity increases, two opposite forces compete and create the paradox. The deadweight loss increases and has a negative effect on the producer surplus while the elasticity increases, although the tax share contributed from the producer to the tax revenue decreases. The result is that the later effect is stronger than the former when the supply is inelastic and this is the reason why the producer surplus increases. When the supply is elastic the first effect is stronger and the paradox disappears since the producer surplus decreases.

Therefore, the producer surplus will increase if a central planner taxes the production, under the assumption that the supply curve is inelastic. When the supply is elastic the tax has a negative effect and when the supply is isoelastic the tax has its maximum effect. The aforementioned result is accurate under a fixed import tariff regime. When the regime is flexible the positive effect is extended also for elastic supply curves but up to a critical point and then it reverses. We derived these results under linear marginal costs with an upward tendency and by assuming that the demand and the free trade equilibrium are constant. To continue, without fixed costs the results are accurate for industries which are characterized by diseconomies of scale. Under fixed costs the results are also applicable for industries exhibiting increasing returns to scale. We explained why free entry does not significantly affect our results under fixed costs, but theoretically if firms can potentially enter the industry then the outcome is going to be also perfect competitive profits. Without free entry, the model is accurate for the economies and diseconomies of scale, but under free entry constant returns to scale characterize the production in the long run and the model is not beneficial. The reason why this happens is that a perfect competitive market structure will drive the equilibrium to a point where profits are zero and the production is characterized by constant returns to scale.

Moreover, if the marginal cost is linear the same results are accurate for a monopoly. The difference is that the results can change quantitatively but not qualitatively. Depending on the parameter values, the paradox can be extended or shrink. Precisely, if the difference between the price and the average cost increases relatively more than the decrease in the output, every time the elasticity increases, then the paradox appears again. Furthermore, if free entry is allowed then in monopolistic competition models with free entry the paradox and the suggested trade policy are also applicable. The only difference is that, worldwide free entry will lower the demand faced from our exporter's perspective. As a result, an opposite effect appears to the endogenous export tax and conflicts with the results. In the long run, monopoly will be converted to a perfect competitive framework with zero profits.

To conclude, the main finding is that a production tax can be substitute or auxiliary to an export tax and obtain similar results. Depending on the market structure and the production capacities, the new trade policy can always be beneficial under specific assumptions and the quantitative results depend on the aforementioned factors.

3.2 Policy Implications

Imposing a tax on an entire industry in order to increase the production cost is not the easiest decision for a politician. We will start our discussion with an example. China has a natural monopoly on rare earths and decided to impose export taxes in order to obtain beneficial tax revenues through its exports. Then, the European Union, the U.S.A, and other countries, complained to the World Trade Organization for discriminatory competition, because China supplied these materials inside the country at lower prices. As a result the Chinese industries had an advantage in the global competition. The World Trade Organization, through diplomacy forced China to decrease the export taxes. The question is, if China taxes the industry as a part of its internal fiscal policy, can the rest of world intervene? The answer is that trade organizations can force a country with respect to its trade policy, but not as far as its fiscal policy is concerned. Therefore, if China taxes the raw material industry, the price will increase and they will attain significant profits through a completely legal way. We proved that under specific conditions the taxation will increase the profits, despite the fact that the cost will increase. On the other hand, the price is also going to increase for the local firms but the tax revenue can be used to support them. If directly amplifying the profits is restricted due to trade regulations, the central planner can, for example, decrease the insurance contribution for the local firms and indirectly decrease the cost. Basically, there are many ways to legally support local firms in order to minimize the loss due to the increase in the local price.

The aforementioned example proves that another player is on the game, regarding trade policy instruments. If, under certain conditions, the fiscal policy can be supplementary to the trade policy, then the international trade environment is going to be much more complex. The reason is that there is no global law which allows an external organization to intervene in internal economic decisions concerning the fiscal policy. The policy implications due to our result become now more interesting. The key results of our research become beneficial not only for a country, but also for international organizations. Preventing such policies, as we have suggested, is a difficult and challenging issue. As we will discuss further on this section about the extensions of the model, if China, for example, decides to follow our key results then “rival” countries or unions can do the same for other commodities. Also, we should

bear in mind that our policy improves local welfare up to a critical point, but deteriorates the importer's welfare and the global welfare as well.

Moreover, we have to discuss how taxation can be imposed. We introduced the indirect tax through a lump sum tax but there are also different ways. Improving the employment conditions for the workers can be costly. The environment under which the commodity is produced is another approach. The quality of the product and the environmental pollution is a significant matter and improving the conditions of production according to an international organization for standardization is costly. At this point we have two remarks. The first is that improving the conditions of production in order to improve safety and quality can increase the fixed cost even in the long run. An analogous choice, aiming to prevent the possibility of environmental pollution, will have similar effects. A convenient example is gold mining. Thirty years ago, a Canadian Company, which specialized in gold mining, invested in Greece. At the beginning, the company followed a more rapidly and less costly production, but the result was environmental pollution. As a result, the local citizens complained and the government decided to change the mining conditions. One year later the company bankrupted and left the region with enormous pollution and many people unemployed. The aforementioned example attempts to explain how improving conditions concerning the employers, production or quality and environment, can increase the cost of production.

Furthermore, a different approach can be undertaken through an increase to the labor or capital cost. Increasing the rents, in our opinion, is not easily applicable, and therefore we will focus on labor cost. The central planner can adopt such a policy for a particular industry through an increase of the minimum wage or the insurance contributions from the employer's perspective. If the labor cost increases then the total cost increases and the indirect policy will obtain our key results. Basically, if the labor cost increases for the specific industry and the revenues can be used to decrease the labor cost for local industries which need the exporting commodity then we can achieve our results. The violation of international laws is not always easy and this is the reason why we always try to discuss about legal ways of increasing the cost and redistribute the tax revenue. On the other hand, many countries don't follow the rules and it is very hard to apply an international regulation. Even in the European Union many rules are violated by the majority of its own members. The illegal framework,

allows us to suggest many different policies which can enforce the theory of endogenous taxation, but this is not the contribution we want to obtain. Therefore, we only discussed fragile yet legal assumptions about applied trade policies convenient to our theory.

To continue, time is necessary in order to achieve our key results through taxation or different ways. In the thesis we avoid to derive a formula of optimum endogenous tax for this reason. If an industry supplies a commodity inelastically, time is needed to change the production in order to become more elastic or even isoelastic or elastic. An industry with a very inelastic supply curve needs time to adjust the production to the new higher cost function. Also, firms have different shares in an industry. Maybe some of them cannot afford the higher cost, or other may attain relatively much more profits compared to smaller firms of the industry. Obviously, a theoretical model is commonly used as a benchmark and not as an applied model. Perfect competition is a model with unrealistic assumptions but the economy and the welfare is healthier when we come closer to this benchmark. Therefore, the theory and key results we obtained are beneficial under circumstances, but from theory to practice things differ.

Additionally, we will discuss the concept of pass-through. With a commonly applied export tax or import tariff, firms can choose if they will internalize a part of the cost due to the tariff. Depending on their own marketing plan and administrative choices, firms decide if a complete pass-through is going to be imposed to the price or not. In our framework this is not convenient because the tax changes the production cost and it is not imposed on the commodity's final price.

To conclude, our ideas bring new topics on the agenda of the international trade policy. From one point of view, such a policy looks very promising under certain conditions. However, it is not very easy to apply such a policy in real economy. If a central planner decides to follow our suggestions through a continuous plan, then it can be beneficial for specific industries at least in the short run. Besides, international organizations should be aware of the specific framework because these kinds of policies can be harmful to global welfare and especially for the importers. Higher prices and lower consumption always minimizes welfare, and from our point of view our results are more helpful for international organizations. The reason is that a "legal" cartel can be introduced in order to apply our key results and as economists

we want to improve global welfare and not the opposite. A paradox, as we found out, makes the policy implications very interesting and we should always keep in mind the general equilibrium and not the specific situations. Trade policies always are implicated with macroeconomic policies and these implications are always more complex than the simple ones.

3.3 Academic Contribution

In this section we will discuss the contribution of the model on trade theory, and also some microeconomic implications we observed while we operated on the model. The main paradox was the core idea and we constructed the model based on that. Through a very simple microeconomic framework, we proved that it is possible for an industry which faces a rising linear marginal cost curve, to obtain higher profits even if the cost increases. Almost in every handbook of economics the authors describe separately the two opposite forces, as we named them, and never tried, or were interested to observe the final result. The paradox, which appears when the supply curve is inelastic, has never been observed before, at least under our framework. Maybe it is not something essential but it is something innovative. When professors teach to bachelor students that “the most elastic the demand or the supply is, compared to each other, the less the contribution to the tax revenue is” and “the most elastic the demand or the supply is, the higher the deadweight loss is”, they never explain the tradeoff between these two opposite forces. Obviously, our paradox is applicable not only in trade theory but in every framework where supply, demand and a specific tax derive the equilibrium.

Moreover, the core idea helped us construct a model which is very simple and applicable in many situations. Firstly, we explained all the possible range for the supply price elasticities and secondly for all the economies of scale. In the meanwhile we applied our results for perfect competition monopoly and monopolistic competition. These primary characteristics of our theory make the model very flexible and useful, compared to models which are more difficult and complicated but also very limited. Furthermore, the latter framework analyses the effects, under fixed and flexible import tariff regimes. Fixed import tariff regimes, incorporated to a model with flexible import tariff regimes, have never appeared before. Also, the entire

framework has been evaluated, with and without fixed costs and for homogenous and heterogeneous commodities.

Therefore, we constructed a theory which created a model with the following characteristics. Firstly, it is very simple because of the existence of linear functions. Secondly, it explains all elasticities and scale economies. Thirdly, it is applicable to perfect and imperfect market structures and finally it is based on a paradox never mentioned before. Obviously, our contribution is not so important if we do not refer to the following. To begin with, the introduction of a new trade policy instrument is our main contribution in the trade theory. For centuries, the key instruments were: import tariffs, export taxes quotas, restrictions, and exchange rate appreciations and depreciations. Also, the derivation of our results was not based on the usual path, followed from trade economists, but we followed a completely different way compared to the formal trade theory. The application of the new trade policy, determines the equilibrium and not the opposite, as trade economists do. It is a more aggressive and composite policy, but also much more engrossing. The simultaneous combination of an export tax and import tariff is also something innovative. The reason is that it is not rational to impose an export tax on a commodity facing an import tariff, but we can see that if we impose it indirectly through an “endogenous tax” then we can improve our position.

Additionally, the imposition of the fixed cost in the cost curve, even in the long run raises exciting remarks. The most challenging issue was the incorporation of increasing returns to scale in our model. We could do this through different ways but we preferred to keep the model’s solidity and this is the reason we introduced fixed costs. We have profoundly explained why fixed cost characterizes our model with IRS and the theory also explains the issue further. The theory explicitly indicates that if we impose a fixed cost then IRS characterizes our framework. The main reason is that the connection between the cost function and the production function is not robust anymore but distorted due to fixed costs. The main question is that if IRS characterizes the production, then how is it possible to have convex cost curves and increasing long run average costs under IRS? The aforementioned conflict raised new thoughts to us and we wanted to expand on it despite the fact that the microeconomic theory is very well established.

As we can see, under fixed cost the cost function is convex but the average cost is falling and after it reaches its minimum it raises again, something which is really contradictory. From our point of view there is only one explanation. The theory supports that if we have a cost function with fixed cost, we do not know the exact formation of the production function and this is the reason we cannot describe scale economies through the convexity and concavity of the cost function anymore. At this point the conflict arises again since it is not possible to have IRS under convex cost curves. If a production is characterized by diseconomies of scale and finally we solve the minimization cost problem and add the fixed cost, we can have the following results. Even if the production exhibits DRS, the existence of fixed costs forces the producer to increase its output in order to eliminate the fixed cost. Fixed cost is part of the average cost and up to a critical point the aforementioned existence leads to a diminishing average cost until a critical point. We could say that fixed cost creates an externality, due to the incentives to produce more. Even under DRS, it has a positive effect since the average cost is falling even if the cost function is convex. Until now the microeconomic theory is accurate but not complete. The result is IRS even if the production exhibits DRS due to the existence of this externality. Basically, it completely depends on the value of the fixed cost and the slope of the supply curve¹⁶. If the fixed cost is significant, the output which leads back again to DRS is far away from the equilibrium production and the producer is never going to face DRS. If the fixed cost is not relatively high then the producer is going to cross over this critical point by producing much less than before. Therefore, if the fixed cost is significant, the producer faces IRS even if the production function exhibits DRS. On the other hand, if the fixed cost is not significant, the externality disappears in the primal stages of production and the production and cost function can both be interpreted through DRS.

One other remark is that, when the IRS characterizes the production with or without fixed cost, then the marginal cost is always lower than the average cost. In competitive formation, where the marginal cost curve represents the supply curve, it is impossible to have positive profits since the average cost is never covered. Obviously under perfect competition in the long run, the firm will produce only under constant returns to scale but only if free entry is possible. Besides, the marginal cost is always

¹⁶ See Appendix F

higher than the average cost if DRS characterizes the production and the profits are positive. In the real world, free entry and perfect competition are not easily applied and if a firm produces under DRS then the profits are positive. Under the assumption that the aggregate demand increases, then the specific firm will increase the production and the profits even if DRS characterizes the production.

The question is how is it possible to take advantage of IRS, if by increasing the production the profits are going to be negative and also how is it possible to have incentives to decrease production under DRS if profits are positive and always increasing under specific assumptions? A contradictory result occurs at this point and we just have to mention these questions. Microeconomic implications are not the purpose of this thesis, but these remarks raised through our research contribute a little bit more.

To conclude, the model is a combination of different parameters and assumptions which makes it innovative. The main contribution is that it is based on a new paradox and uses techniques completely different to the formal trade theory. Obviously, entirely innovative approaches and attempts are always very precarious, but on the other hand procedures like these, can bring interesting and significant changes. Out of the box thinking is challenging, innovative, but also harmful sometimes, and we really hope that our ideas might contribute to the trade theory even to the minimum.

3.4 Limitations and Directions for Future Research

Each theory has its own advantages and disadvantages. Especially when it depends on assumptions and how heroic they are. A static or semi-static analysis is always helpful for comparative statics but the dynamic framework is always closer to reality. The main limitation of our model and generally speaking of static models is that our world is not static. If a central planner decides to follow our suggestions, then the importer is going to react. If close substitutes exist for our export commodity, then the importer will prefer them. The assumptions of perfect competition are always useful in theory, but restrict models in real world. Our framework is limited under perfect competition, but enforced under significant monopoly power. The assumption that the demand is constant is also a heroic assumption at least in the long run. Countries create bonds

between them and aggressive trade policies are not always acceptable from the international environment. We do not live in a peaceful world, but this does not mean that we always have to suggest always policies which can create conflicts. Despite the fact that our model is quite general and widespread, the assumptions of perfect competition and constant demand from the importer's perspective are the key limitations.

From our point of view the theory of endogenous export taxation under fixed and flexible import tariff regimes is just the beginning. One main direction is the quantitative. Without empirical results the application of our policies is not very accurate. Therefore an empirical analysis must be conducted. To continue, it would be very interesting to observe a model with two countries. Depending on the supply elasticity of each country, they could choose different reaction policies. Therefore reaction functions could be designed, incorporating our new trade policy instrument. A dynamic framework is always more challenging and our future research plan is the introduction of a dynamic trade budget constrain. A vector of possible trade instruments, incorporated under a duopoly could interestingly extend the results of our analysis.

Moreover, a more precise analysis of the total welfare, as also of the internal and external welfare is compulsory. Obviously the exporter is always better off, but the global welfare and the effects on the importer's welfare are also significant. Solving a model with linear functions is easier with respect to the algebra but it is limited to the economic intuition. The same analysis could derive more robust results for different functional forms.

To conclude, a new theory is always promising since it can be extended in every direction. Also the core idea of our thesis is a paradox which can also be used in different areas of economics. Our opinion is that the first step must be a duopoly and then the incorporation of the new trade policy instrument in a dynamic stochastic general equilibrium model. Precisely we mentioned that a dynamic international trade policy including a vector of different trade policies sounds very promising.

3.5 References

- Andreu, M.-C., Whinston, M.D., and Green, J.R. (1995), *Microeconomic Theory* (New York, Oxford: Oxford University Press).
- Auquier, A.A. and Caves, R.E. (1979), "Monopolistic Export Industries, Trade Taxes and Optimal Competition Policy," *The Economic Journal*, 89 (355), 559-581.
- Belleflamme, P. and Peitz, M. (2010), *Industrial Organization Markets and Strategies* (New York: Cambridge University Press).
- Blanchard, E.J. (2009), "Trade Taxes and International Investment," *The Canadian Journal of Economics*, 42 (3), 882-899.
- Bowen, H.P., Hollander, A., and Viaene, J.-M. (2012), *Applied International Trade* (Basingstoke, U.K.: Palgrave Macmillan).
- Brander, J.A. and Spencer, B.J. (1985), "Export Subsidies and International Market Share Rivalry," *Journal of International Economics*, 18, 83-100.
- Broda, C., Limão, N., and Weinstein, D.E. (2006), "Optimal Tariffs: The Evidence," NBER Working Paper No. 12033 (Cambridge, MA: NBER).
- Caves, R.E. (1980), "International Trade and Industrial Organization: Introduction," *The Journal of Industrial Economics*, 29 (2), 113-117.
- Chacholiades, M. (1973), *The Pure Theory of International Trade* (London: MacMillan).
- Clarke, R. and Collie D.R. (2006), "Optimum-Welfare and Maximum-Revenue Tariffs under Bertrand Duopoly," *Scottish Journal of Political Economy*, 53 (3), 398-408.
- Clarke, R. and Collie D.R. (2008), "Welfare in the Nash Equilibrium in Export Taxes under Bertrand Duopoly," *Bulletin of Economic Research*, 60 (2), 183-189.
- Devarajan, S. *et al.* (1996), "The Whys and Why Nots of Export Taxation," Policy Research Working Paper No. 1684 (Washington, DC: World Bank).

- Dixit, A. (1984), "International Trade Policy for Oligopolistic Industries," *Economic Journal Conference Papers*, 94, 1-16.
- Dixit, A. and Grossman, G.M. (1986), "Targeted Export Promotion with Several Oligopolistic Industries," *Journal of International Economics*, 21, 233-250.
- Eaton, J. and Grossman, G.M. (1986), "Optimal Trade and Industrial Policy under Oligopoly," *Quarterly Journal of Economics*, 101, 383-406.
- Helpman, E. and Krugman, P.R. (1989), *Trade Policy and Market Structure* (Cambridge, MA, and London: MIT press).
- Howell-Zee, H. (2007), "Export Taxes in Times of Trade Surpluses," *Journal of International Trade & Economic Development*, 16 (2), 137-157.
- Humphrey, T.M. (1987), "Classical and Neoclassical Roots of the Theory of Optimum Tariffs," *Federal Reserve Bank of Richmond Economic Review*, 73 (4), 17-28.
- Jones, C.I. and Romer, P.M. (2009) "The new Kaldor Facts: Ideas, Institutions, Population, and Human Capital," NBER Working Paper No. 15094 (Cambridge, MA: NBER).
- Krugman, P.R. (1979), "Increasing Returns, Monopolistic Competition and International Trade," *Journal of International Economics*, 9 (4), 469-479.
- Lawrence, L. and Stiglitz, J. (2005), "China's Alternative to Revaluation," *Financial Times*, (Apr. 25), 21.
- Markusen, J.R. and Venables, A.J. (1988), "Trade Policy with Increasing Returns and Imperfect Competition: Contradictory Results from Competing Assumptions," *Journal of International Economics*, 24, 299-316.
- Newbery D.M.G. and Stiglitz, J.E. (1981), "Pareto Inferior Trade and Optimal Trade Policy," *DRD discussion papers*, 50, 39.
- Panagariya, A. and Schiff, M. (1994), "Can Revenue Maximizing Export Taxes Yield higher Welfare than Welfare Maximizing Export Taxes?," *Economics Letters*, 45, 79-84.

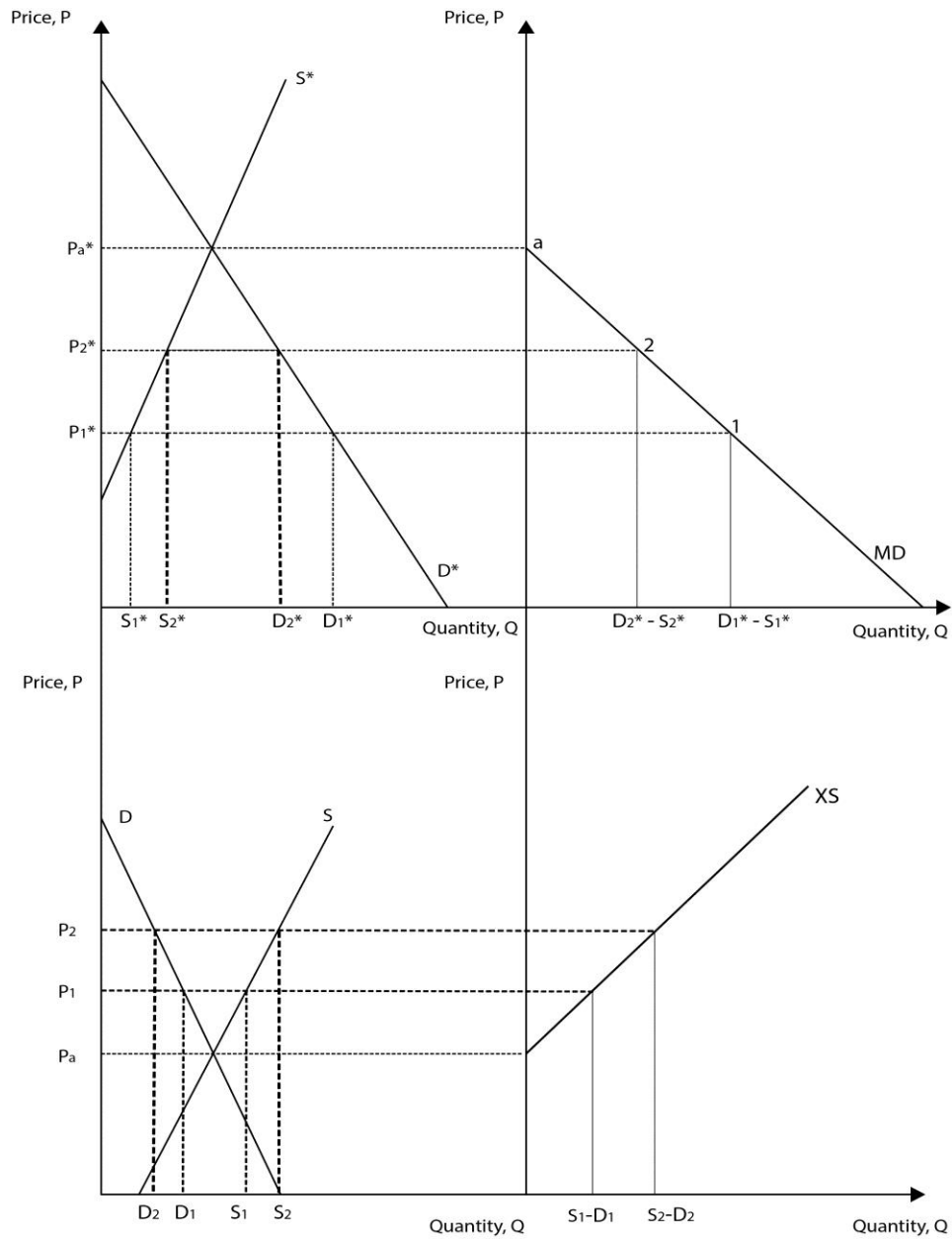
Rodrik, D. (1989), "Optimal Trade Taxes for a Large Country with Non-atomistic Firms," *Journal of International Economics*, 26 (1), 157-167.

Vousden, N. (1990), *The Economics of Trade Protection* (Cambridge: Cambridge University Press).

Yilmaz, K. (1999), "Optimal Export Taxes in a Multicountry Framework," *Journal of Development Economics*, 60, 439-465.

3.6 Appendix

A. Derivation of net (residual) aggregate import demand (MD) and export supply curves (XS)



D^* : Importer's demand for home commodity

S^* : Importer's supply for home commodity

D : Exporter's demand for home commodity

S : Exporter's supply for home commodity

Derivation of MD: Home importer's consumers demand less while the price increases from importer's producers and the demand for imports declines.

Derivation of XS: Home exporters producers supply more while demand from exporter's home consumers' decreases and the available supply for exports increases.

When the export supply curve (XS) equals the import demand curve (MD) there is equilibrium. If the importing country does not produce the good then D and MD are the same. If the exporting country doesn't consume the good then S and XS are also the same. In the model we assume that MD is predetermined and constant. Also exporters home demand does not change and exporters supply curve and therefore XS are the only curves who change in the model.

B. Cost/Marginal Cost/Average Cost microeconomic foundation and illustration

a) Positive Fixed Cost, $FC > 0$.

1. *Supply function* : $MC \equiv f(P) = a + b \cdot q$

2. *Cost function* : $C \equiv g(P) = \int_0^q [a + b \cdot q] \cdot dq = a \cdot q + \left(\frac{b}{2}\right) \cdot q^2 + FC$

3. *Average Cost function* : $AC \equiv h(P) = \frac{C}{q} = a + \left(\frac{b}{2}\right) \cdot q + \frac{FC}{q}$

The AC function is a rational function, since it can be represented as a fraction of two polynomial functions where the degree of the nominator is higher than the degree of the denominator. In order to graph the AC function we need to calculate the polynomial division to draw the oblique asymptotes.

4. *Oblique Asymptote* : $P = a + \left(\frac{b}{2}\right) \cdot q$

Since q is positive we will draw and interpret the asymptote and the hyperbola in the 1st quadrant instead of the 1st and third quadrant.

Monotonicity of AC function :

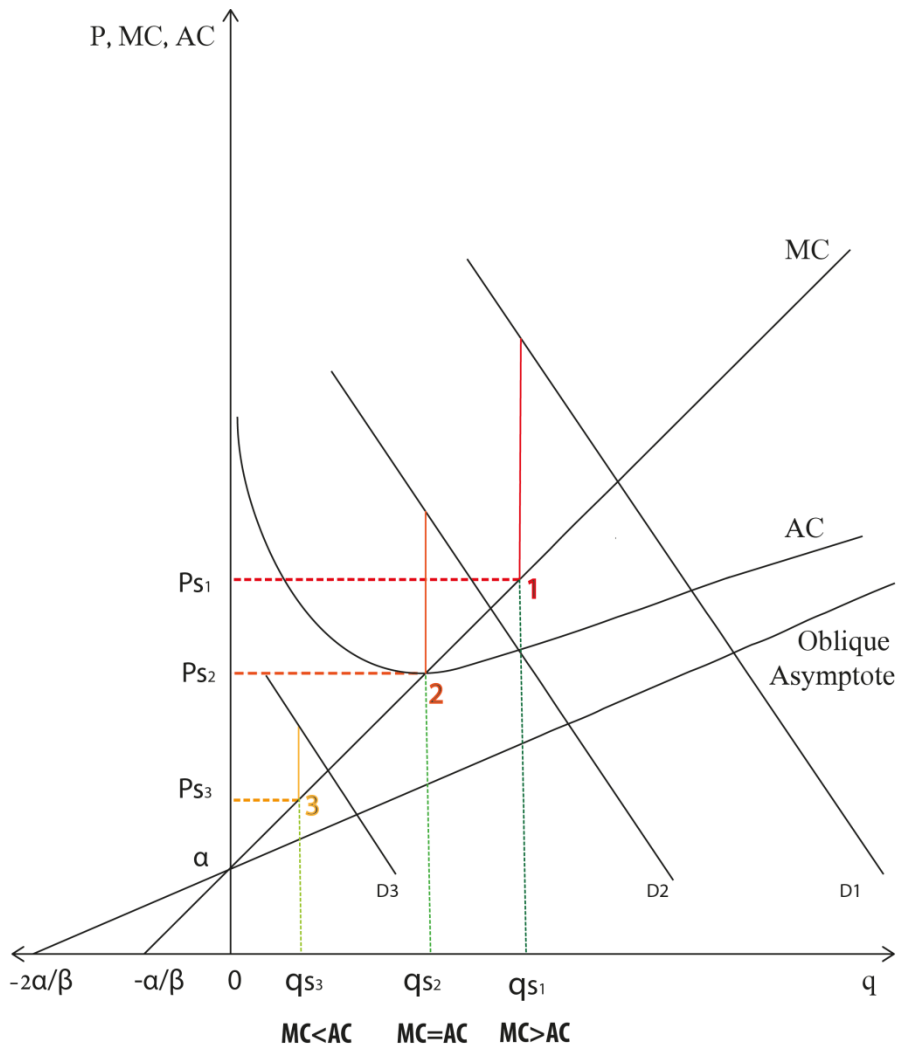
5. $\frac{dAC}{dq} = \frac{b}{2} - \frac{FC}{q^2}$, $\frac{d^2AC}{dq^2} = \frac{FC}{q^3} > 0, \forall q \geq 0$

6. $\frac{dAC}{dq} = \frac{b}{2} - \frac{FC}{q^2} = 0 \rightarrow AC_{min}$ when, $q = \sqrt{\frac{2 \cdot FC}{b}}, \forall q \geq 0$

7. In the same time $AC = MC, \forall q \geq 0$ when $q = \sqrt{\frac{2 \cdot FC}{b}}, \forall \{a, b, FC\}$

As we will see on the graph, AC diminishes and in the same time is higher than MC, $\forall q < \sqrt{\frac{2 \cdot FC}{b}}$, and the opposite $\forall q > \sqrt{\frac{2 \cdot FC}{b}}$. Finally $AC=MC, \forall q = \sqrt{\frac{2 \cdot FC}{b}}$.

i. $\varepsilon_s > 1, a > 0$

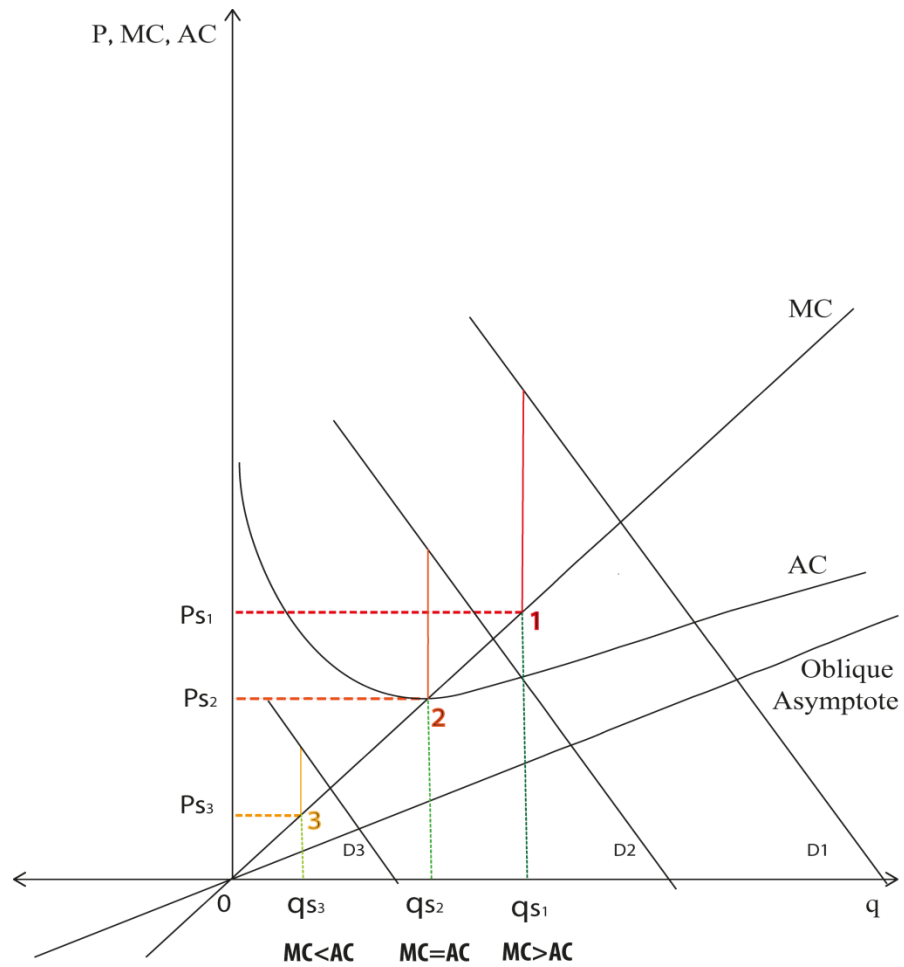


a. Point 1 : $q_s > \sqrt{\frac{2 \cdot FC}{b}} \rightarrow MC > AC \rightarrow \varepsilon_{cq} \equiv \frac{MC}{AC} = \frac{dC}{dq} \cdot \frac{q}{C} > 1 \rightarrow DRS, AC \uparrow$

b. Point 2 : $q_s = \sqrt{\frac{2 \cdot FC}{b}} \rightarrow MC = AC \rightarrow \varepsilon_{cq} \equiv \frac{MC}{AC} = \frac{dC}{dq} \cdot \frac{q}{C} = 1 \rightarrow CRS, AC_{min}$

c. Point 3 : $q_s < \sqrt{\frac{2 \cdot FC}{b}} \rightarrow MC < AC \rightarrow \varepsilon_{cq} \equiv \frac{MC}{AC} = \frac{dC}{dq} \cdot \frac{q}{C} < 1 \rightarrow IRS, AC \downarrow$

ii. $\varepsilon_s = 1, a = 0$

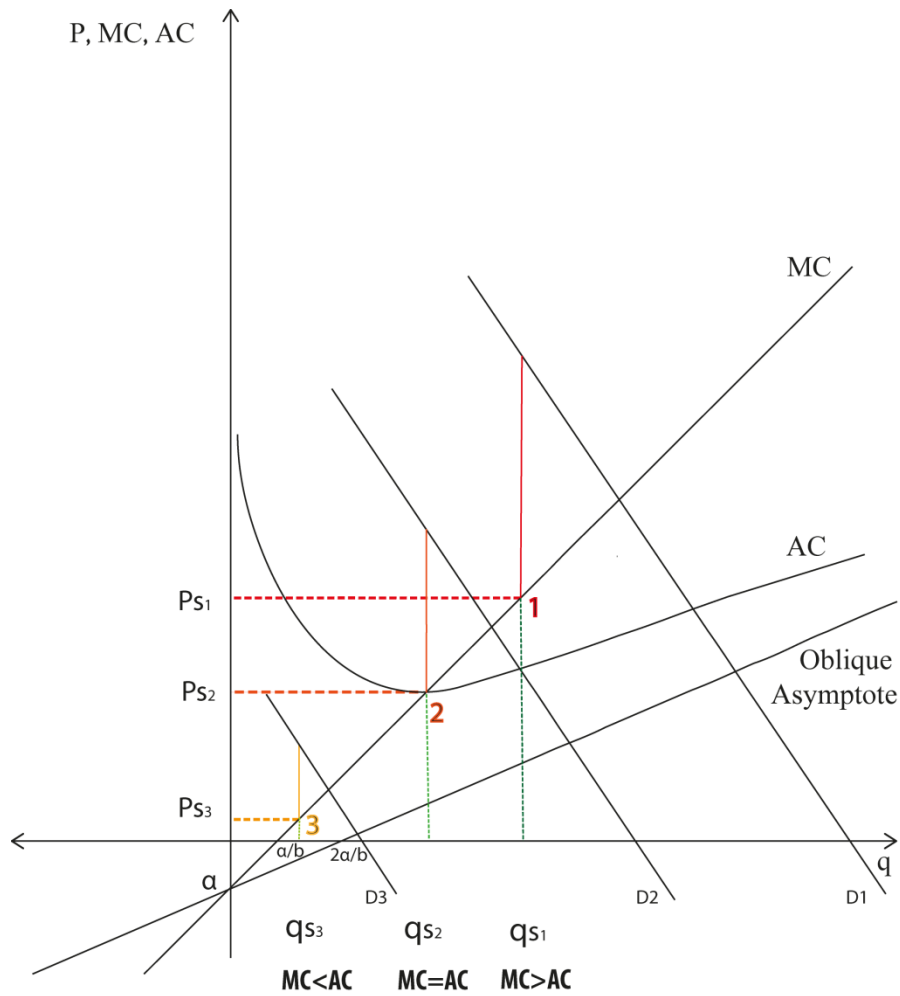


a. Point 1 : $q_s > \sqrt{\frac{2 \cdot FC}{b}} \rightarrow MC > AC \rightarrow \varepsilon_{cq} \equiv \frac{MC}{AC} = \frac{dC}{dq} \cdot \frac{q}{C} > 1 \rightarrow DRS, AC \uparrow$

b. Point 2 : $q_s = \sqrt{\frac{2 \cdot FC}{b}} \rightarrow MC = AC \rightarrow \varepsilon_{cq} \equiv \frac{MC}{AC} = \frac{dC}{dq} \cdot \frac{q}{C} = 1 \rightarrow CRS, AC_{min}$

c. Point 3 : $q_s < \sqrt{\frac{2 \cdot FC}{b}} \rightarrow MC < AC \rightarrow \varepsilon_{cq} \equiv \frac{MC}{AC} = \frac{dC}{dq} \cdot \frac{q}{C} < 1 \rightarrow IRS, AC \downarrow$

iii. $\epsilon_s < 1, a < 0$



a. Point 1 : $q_s > \sqrt{\frac{2 \cdot FC}{b}} \rightarrow MC > AC \rightarrow \epsilon_{cq} \equiv \frac{MC}{AC} = \frac{dC}{dq} \cdot \frac{q}{C} > 1 \rightarrow DRS, AC \uparrow$

b. Point 2 : $q_s = \sqrt{\frac{2 \cdot FC}{b}} \rightarrow MC = AC \rightarrow \epsilon_{cq} \equiv \frac{MC}{AC} = \frac{dC}{dq} \cdot \frac{q}{C} = 1 \rightarrow CRS, AC_{min}$

c. Point 3 : $q_s < \sqrt{\frac{2 \cdot FC}{b}} \rightarrow MC < AC \rightarrow \epsilon_{cq} \equiv \frac{MC}{AC} = \frac{dC}{dq} \cdot \frac{q}{C} < 1 \rightarrow IRS, AC \downarrow$

b) Cost Function with no Fixed Cost, FC=0.

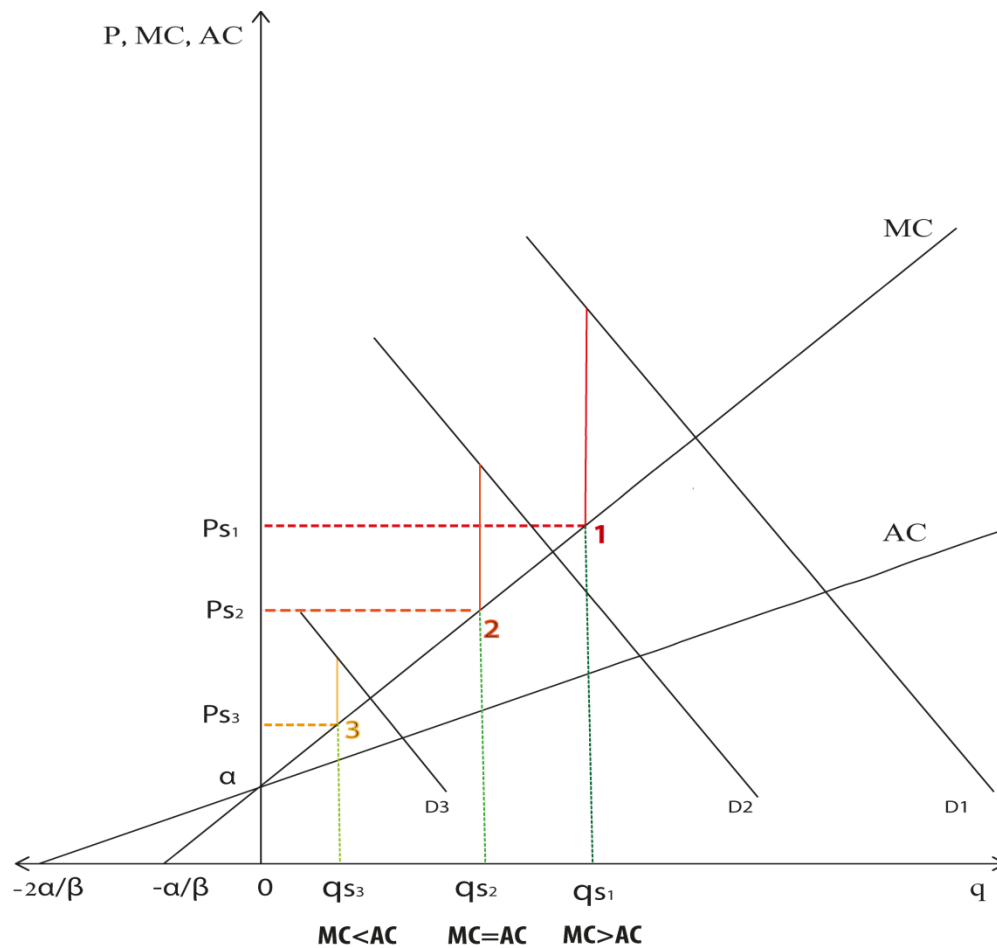
1. Supply function : $MC \equiv f(P) = a + b \cdot q$

2. Cost function : $C \equiv g(P) = \int_0^q [a + b \cdot q] \cdot dq = a \cdot q + \left(\frac{b}{2}\right) \cdot q^2$

3. Average Cost function : $AC \equiv h(P) = \frac{C}{q} = a + \left(\frac{b}{2}\right) \cdot q$

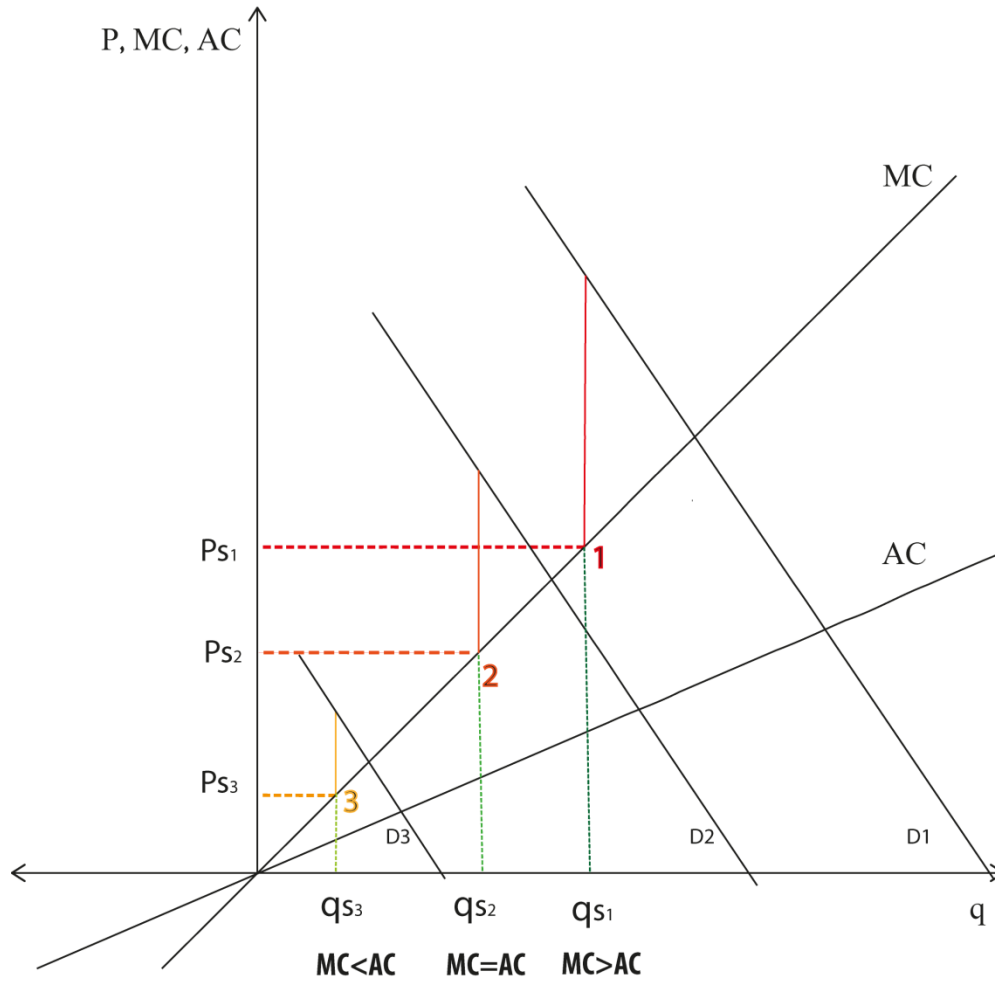
i. $\epsilon_s > 1, a > 0$

Point 1,2,3 : $MC > AC \rightarrow \epsilon_{cq} \equiv \frac{MC}{AC} = \frac{dC}{dq} \cdot \frac{q}{C} > 1 \rightarrow DRS, AC \uparrow$



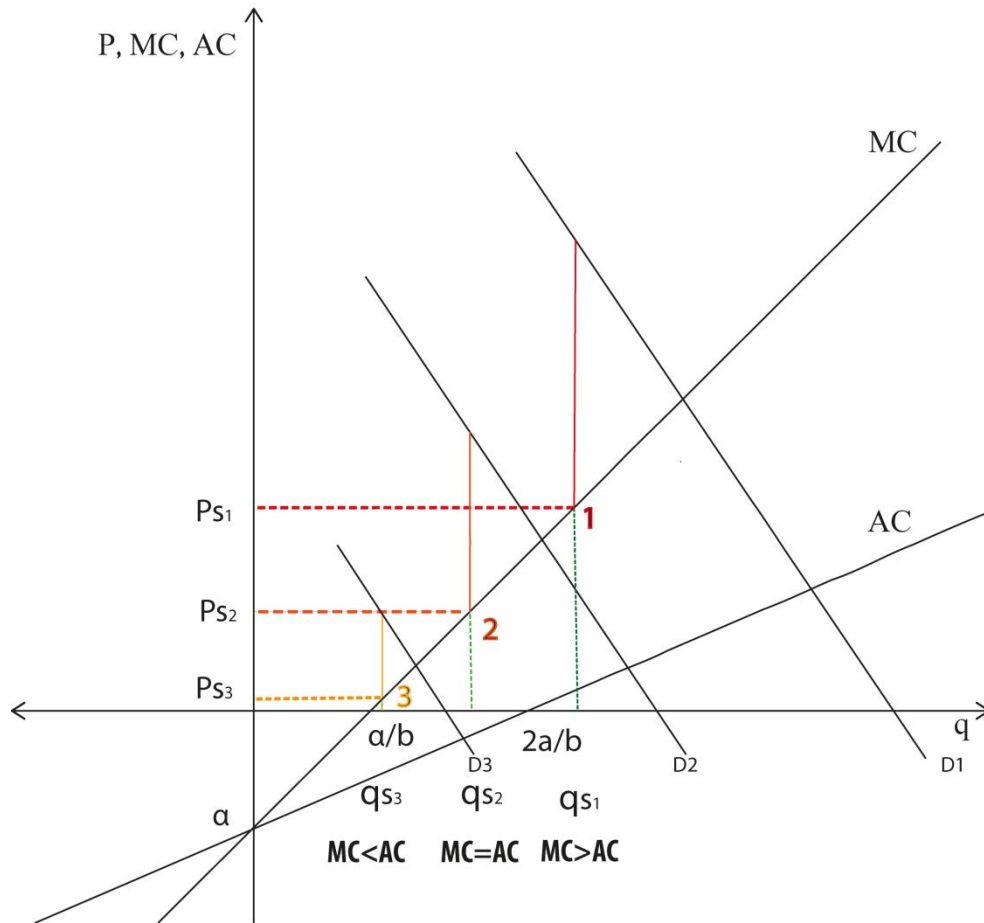
ii. $\varepsilon_s = 1, a = 0$

Point 1,2,3 : $MC > AC \rightarrow \varepsilon_{cq} \equiv \frac{MC}{AC} = \frac{dC}{dq} \cdot \frac{q}{C} > 1 \rightarrow DRS, AC \uparrow$



iii. $\epsilon_s < 1, a < 0$

$$\text{Point 1,2,3 : } MC > AC \rightarrow \epsilon_{cq} \equiv \frac{MC}{AC} = \frac{dC}{dq} \cdot \frac{q}{C} > 1 \rightarrow \text{DRS, } AC \uparrow$$



C. Conditions for positive effect of the indirect tax to the Producer Surplus

$$1. \quad \forall a < \left[\frac{\left\{ \frac{\partial p_s}{\partial t_s} \cdot \frac{\partial a}{\partial t_s} \right\} \cdot q_s}{\frac{\partial q_s}{\partial t_s}} + p_s \right] \text{ and } a \in (0, p'), \text{ then}$$

$$\frac{\partial PS}{\partial t_s} = \frac{1}{2} \cdot \left[\frac{\partial p_s}{\partial t_s} \cdot q_s + \frac{\partial q_s}{\partial t_s} \cdot p_s - \frac{\partial a}{\partial t_s} \cdot q_s - \frac{\partial q_s}{\partial t_s} \cdot a \right] > 0$$

$$2. \quad \frac{\partial q_s}{\partial t_s} \equiv \frac{\partial q_s}{\partial a} \cdot \frac{\partial a}{\partial \varepsilon_s} \cdot \frac{\partial \varepsilon_s}{\partial t_s} + \frac{\partial q_s}{\partial t} \cdot \frac{\partial t}{\partial \varepsilon_s} \cdot \frac{\partial \varepsilon_s}{\partial t_s} > 0$$

after the substitution of the specific derivatives we have the following result(condition)

$$3. \quad \frac{\partial p_s}{\partial a} < \frac{d^2 \cdot q'}{(b+d) \cdot p'} \rightarrow \frac{\partial q_s}{\partial t_s} > 0$$

$$4. \quad \frac{\partial p_s}{\partial a} \cdot \frac{\partial a}{\partial \varepsilon_s} \cdot \frac{\partial \varepsilon_s}{\partial t_s} + \frac{\partial p_s}{\partial t} \cdot \frac{\partial t}{\partial \varepsilon_s} \cdot \frac{\partial \varepsilon_s}{\partial t_s} - \frac{\partial a}{\partial \varepsilon_s} \cdot \frac{\partial \varepsilon_s}{\partial t_s} > 0 \rightarrow$$

$$\rightarrow \frac{\partial a}{\partial \varepsilon_s} \left(\frac{\partial p_s}{\partial a} - 1 \right) + \frac{\partial p_s}{\partial t} \cdot \frac{\partial t}{\partial \varepsilon_s} > 0 \rightarrow \frac{\partial p_s}{\partial a} > 1 \{strong\} \text{ or } > \frac{1}{2} \{weak\}$$

$$5. \quad \text{Therefore } \xrightarrow{1,2,3,4} \frac{1}{2} < 1 < \frac{\partial p_s}{\partial a} < \frac{d^2 \cdot q'}{(b+d) \cdot p'}$$

$$6. \quad \text{Also : } \frac{d^2 \cdot q'}{(b+d) \cdot p'} > 1 \rightarrow d^2 \cdot q' - d \cdot p' - b \cdot p' > 0$$

Concluding Conditions

$$A. \quad \frac{\partial q_s}{\partial t_s} > 0 \quad \forall \quad \frac{\partial p_s}{\partial a} < \frac{d^2 \cdot q'}{(b+d) \cdot p'}$$

$$B. \quad \left\{ \frac{\partial p_s}{\partial t_s} - \frac{\partial a}{\partial t_s} \right\} > 0 \quad \forall \quad \left\{ 1 < \frac{\partial p_s}{\partial a}, strong \right\}, \left\{ \frac{1}{2} < \frac{\partial p_s}{\partial a}, weak \right\}$$

C. *Therefore there are four possible conditions*

$$\bullet \quad \frac{\begin{matrix} (+) \\ (+) \end{matrix}}{\begin{matrix} (-) \\ (-) \end{matrix}}, \frac{\begin{matrix} (-) \\ (-) \end{matrix}}{\begin{matrix} (+) \\ (+) \end{matrix}} < p_s, \frac{\begin{matrix} (+) \cdot q_s \\ (-) \end{matrix}}{\begin{matrix} (-) \end{matrix}} < p_s$$

• *The strongest is $\{d > \frac{p'}{2 \cdot q'}\}$, since*

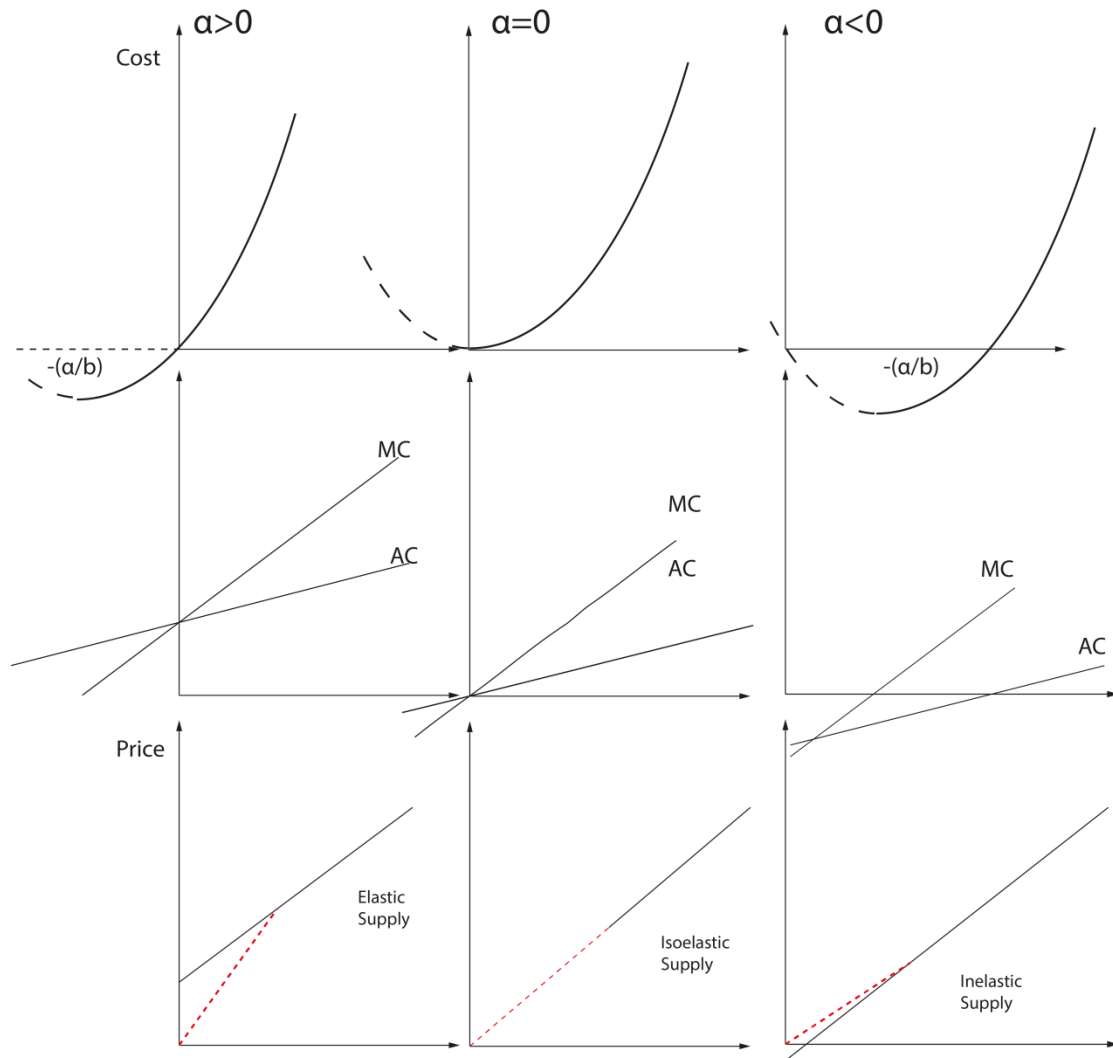
$$\frac{d[d^2 \cdot q' - d \cdot p' - b \cdot p']}{d[d]} = 0 \rightarrow d > \frac{p'}{2 \cdot q'} > 0 \text{ and}$$

$$\frac{d^2[d^2 \cdot q' - d \cdot p' - b \cdot p']}{d[d]^2} = 2 \cdot q' > 0$$

Therefore at the free trade equilibrium point the demand price elasticity including the strong condition must be (-2) , which leads to the concluding result that in order to have a positive effect on the exporters' profits by using the new export policy the importers demand must be relatively elastic.

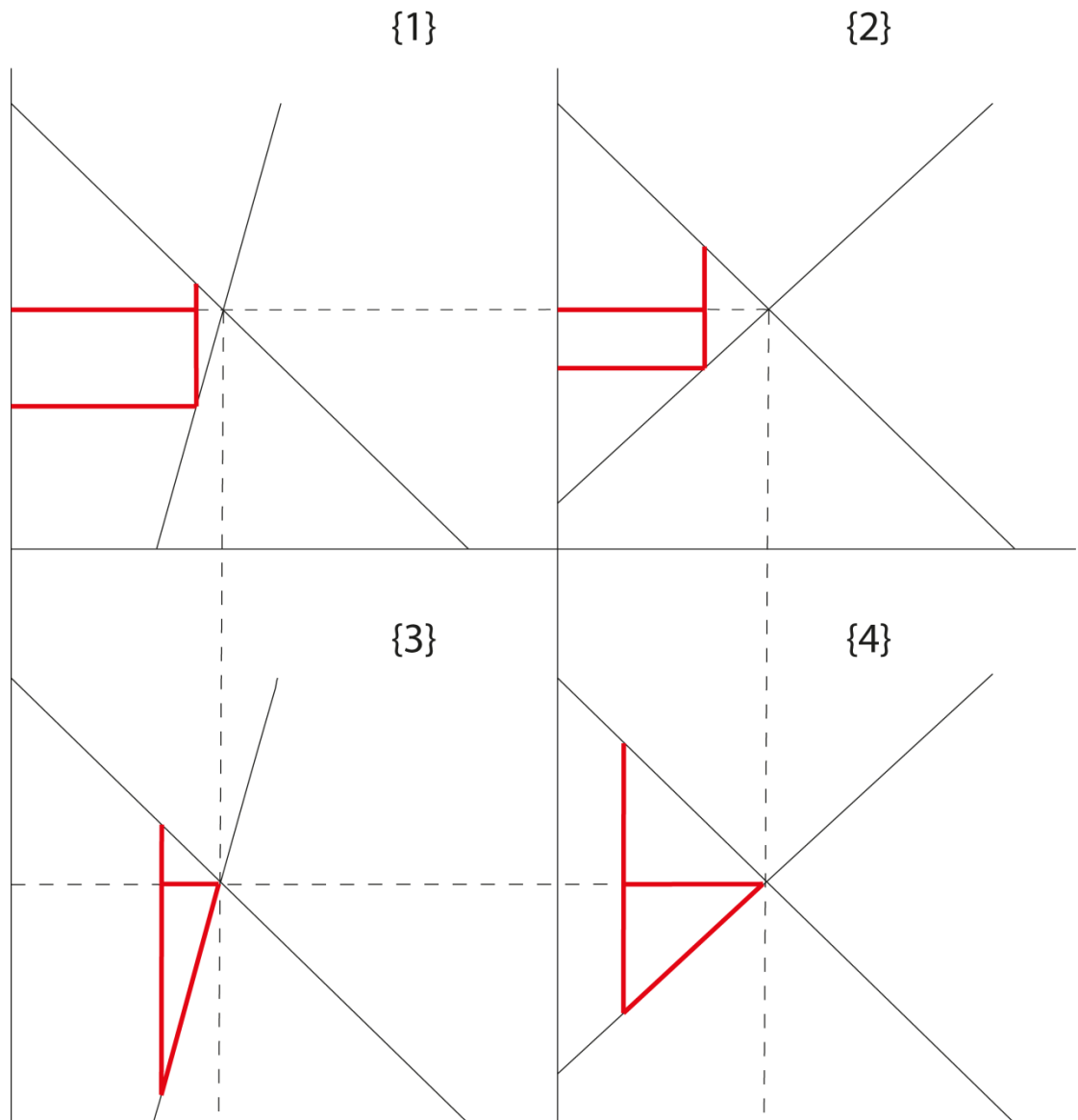
D. Figures

Figure 1



As we can see, when the marginal cost is inelastic it is possible to face negative cost if the equilibrium is on a point where the average cost is negative. The question is if we can use linear curves without fixed cost for a general analysis.

Figure 2



From the first graph to the second, the contribution of the producer to the tax revenue is lower while the elasticity increases. Therefore, every time the elasticity increases the specific effect decreases.

From the third graph to the fourth the deadweight loss increases, the share of the deadweight loss on the producer surplus is higher while the elasticity increases. Therefore, every time the elasticity increases the specific effect increases.

Finally, the first effect (graphs, 1-2) is positive to the producer surplus and the second negative (graphs, 3-4).

E. Production and cost functions without fixed cost

$$\min [w_1 \cdot x_1 + w_2 \cdot x_2] \quad s.t \quad f(x_1, x_2) = x_1^a \cdot x_2^b = y, (a, b) > 0 \quad [E.1]$$

$$L(x_1, x_2, \lambda) = w_1 \cdot x_1 + w_2 \cdot x_2 - \lambda \cdot (f(x_1, x_2) - y) \quad [E.2]$$

$$F.O.C. : \left[\frac{\partial L}{\partial x_1} = 0, \frac{\partial L}{\partial x_2} = 0 \right], \quad S.O.C. : [w_1^2 \cdot f_{22} - f_1 \cdot f_2 \cdot f_{12} + w_1^2 \cdot f_{11}] < 0$$

Production function is concave on 3 dimensions and convex on 2 dimensions.

$$x_1^* = x_1(w_1, w_2, y), \quad x_2^* = x_2(w_1, w_2, y) \rightarrow \text{optimum choice for inputs}$$

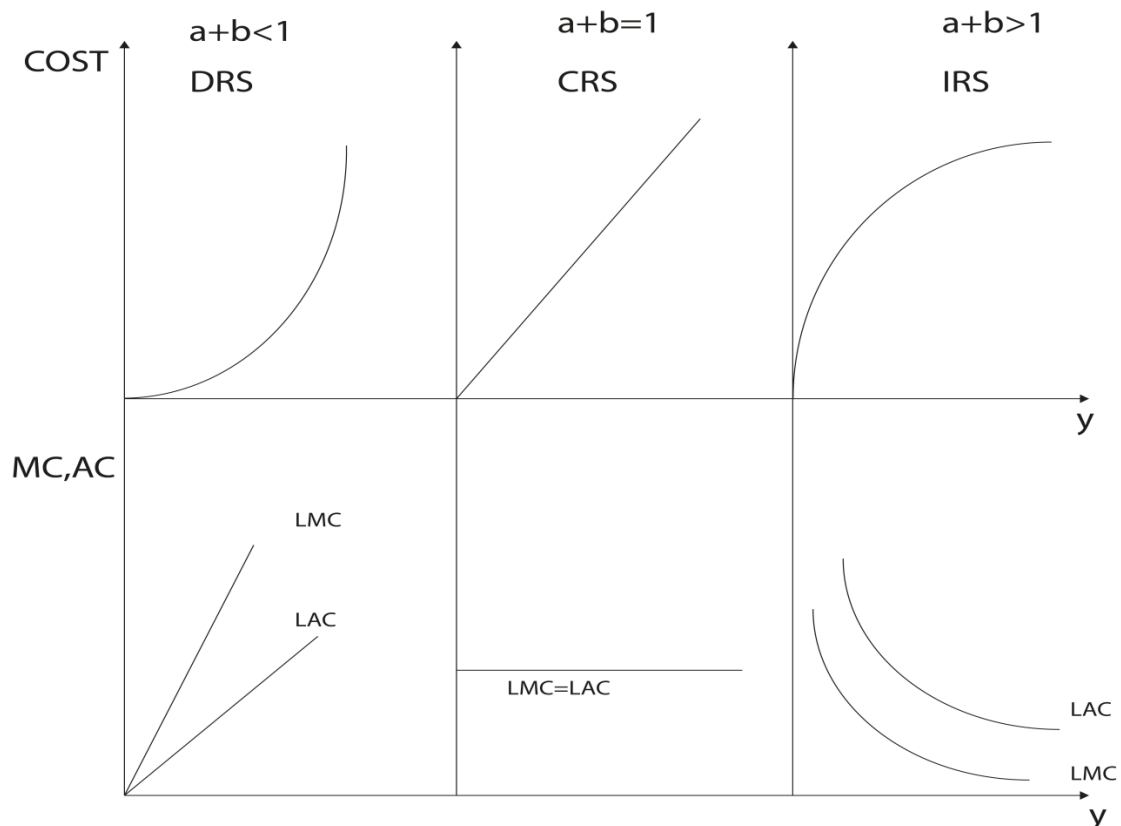
$$C^* = C(w_1, w_2, y) = w_1 \cdot x_1^* + w_2 \cdot x_2^* \quad [E.3]$$

$$LTC \equiv C = (a + b) \cdot y^{\frac{1}{a+b}} \cdot \left(\frac{w_1}{a}\right)^{\frac{a}{a+b}} \cdot \left(\frac{w_2}{b}\right)^{\frac{b}{a+b}} \quad [E.4]$$

$$1. a = b = \frac{1}{2} \rightarrow a + b = 1 = 1 \rightarrow C(1,1, y) = 2y^1 \rightarrow CRS$$

$$2. a = b = \frac{1}{4} \rightarrow a + b = \frac{1}{2} < 1 \rightarrow C(1,1, y) = 2y^2 \rightarrow DRS$$

$$3. a = b = 1 \rightarrow a + b = 2 > 1 \rightarrow C(1,1, y) = 2y^{\frac{1}{2}} \rightarrow IRS$$



F. Cobb-Douglas Functions with Fixed Cost.

We will assume that the inputs capital and labor are contained from two parts. The first part is always flexible in the long run and the second part is fixed even in the long run. Therefore the inputs depend on one fixed and one flexible part. We also assume that the fixed part is a fraction or percentage of the flexible part. This percentage can be the same or different for capital and labor.

$$FC_{\text{capital}} = k \cdot \gamma, \quad \text{Capital} \equiv K = k + k \cdot \gamma = k \cdot (1 + \gamma)$$

$$FC_{\text{labor}} = l \cdot \delta, \quad \text{Labor} \equiv L = l + l \cdot \delta = l \cdot (1 + \delta)$$

If, $\{\gamma, \delta\}$ are zero then capital and labor equal to the flexible part. If $\{\gamma, \delta\}$ equal to unity then 100% of the flexible part is fixed. We assume that:

$\{\gamma, \delta\} \in [0,1]$, if $\{\gamma, \delta\} = 0$, then fixed cost is zero,

if $\{\gamma, \delta\} = 1$, then fixed cost is maximum in our framework.

Without any loss of generality the aforementioned parameters can have values larger than unity but as we will see at the end there is no difference on the results.

Now we will assume that we have a Cobb-Douglas production function.

$$f(k, l) = [K(k)]^a \cdot [L(l)]^b = [k \cdot (1 + \gamma)]^a \cdot [l \cdot (1 + \delta)]^b = (1 + \gamma)^a \cdot (1 + \delta)^b \cdot k^a \cdot l^b$$

Now we will assume different formations and as we will see at the end it is indifferent which of them we will use to explain our thoughts.

$$1. f(k, l) = (1 + \gamma)^a \cdot (1 + \delta)^b \cdot k^a \cdot l^b$$

$$2. f(k, l) = (1 + \gamma)^{a+b} \cdot k^a \cdot l^b, \quad \text{if } \gamma=\delta$$

$$3. f(k, l) = (1 + \gamma)^a \cdot k^a \cdot l^b, \quad \text{if } \delta=0$$

$$4. f(k, l) = (1 + \delta)^b \cdot k^a \cdot l^b, \quad \text{if } \gamma=0$$

Finally, if $f(k, l)$ is homogenous of degree k , then we can use Euler's theorem and also if Euler's theorem is accurate then $f(k, l)$ is homogenous of degree k .

$$1. f(\lambda k, \lambda l) = \lambda^k \cdot f(k, l) = \lambda^{\alpha+b} \cdot (1 + \gamma)^a \cdot (1 + \delta)^b \cdot k^a \cdot l^b = \lambda^{\alpha+b} \cdot q$$

$$2. f(\lambda k, \lambda l) = \lambda^k \cdot f(k, l) = \lambda^{\alpha+b} \cdot (1 + \gamma)^{a+b} \cdot k^a \cdot l^b = \lambda^{\alpha+b} \cdot q$$

$$3. f(\lambda k, \lambda l) = \lambda^k \cdot f(k, l) = \lambda^{\alpha+b} \cdot (1 + \gamma)^a \cdot k^a \cdot l^b = \lambda^{\alpha+b} \cdot q$$

$$4. f(\lambda k, \lambda l) = \lambda^k \cdot f(k, l) = \lambda^{\alpha+b} \cdot (1 + \delta)^b \cdot k^a \cdot l^b = \lambda^{\alpha+b} \cdot q$$

We observe here that despite the fact fixed cost exists; the production function is characterized from DRS, CRS, and IRS as we have seen on Appendix E. From our point of view the conflict of the theory is that it does not internalize the externalities appeared from parameters $\{\gamma, \delta\}$. Precisely, if we keep in mind the ordinary function obviously the theory is accurate since:

$$1. [\lambda^{\alpha+b} \cdot (1 + \gamma)^a \cdot (1 + \delta)^b] > 1, \forall (a + b) > 0, \text{ if } \lambda > 1, \{\gamma, \delta\} > 0.$$

$$2. [\lambda^{\alpha+b} \cdot (1 + \gamma)^a \cdot (1 + \delta)^b] \cdot k^a \cdot l^b > \lambda^{\alpha+b} \cdot k^a \cdot l^b$$

Now we have a serious problem to solve. If the exhibitors of the function are equal or higher than one, then the new function including fixed costs always exhibits IRS. It is not very hard to prove this, but we will focus on the challenging scenario where the exhibitors are lower than unity. As we will see, the later assumption restricts the values of $\{\lambda\}$, which allow us to take advantage of IRS. In other words, if we increase capital and labor more than a specific value, the producer will again face DRS even under fixed cost in the new transformed function. At this point we must be very careful, because we prove that microeconomic theory needs to be seriously completed. Thus we will prove that under fixed cost, the production function can exhibit DRS.

$$1. [\lambda^{\alpha+b-1} \cdot (1 + \gamma)^a \cdot (1 + \delta)^b] > 1, a + b < 1$$

If the latter inequality does not hold, then the production function will not exhibit increasing returns to scale even if $\{\gamma, \delta\}$ reach the maximum possible value.

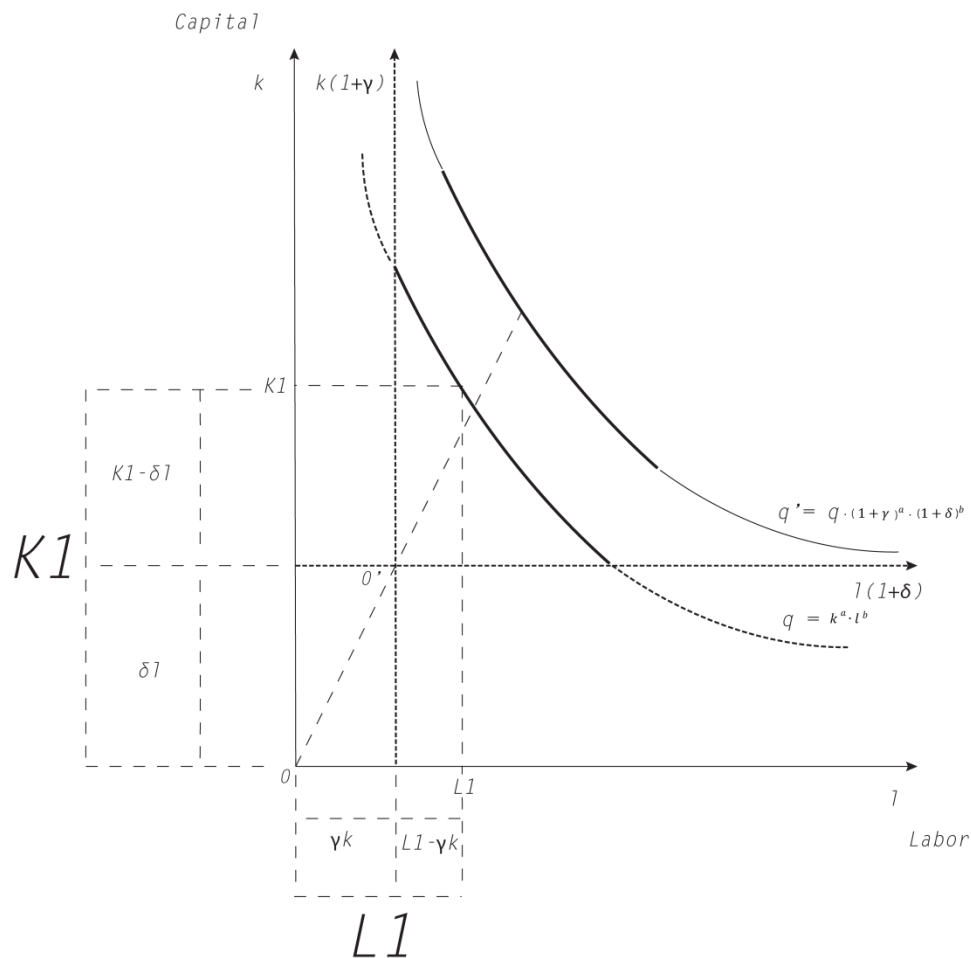
$$2. \lambda < [(1 + \gamma)^a \cdot (1 + \delta)^b]^{\frac{1}{1-(a+b)}}$$

$$\mathbf{A.} \ a + b = \frac{1}{2} \rightarrow \lambda = 2^{\frac{1}{2}} < 2 \qquad \mathbf{B.} \ a + b < \frac{1}{2} \rightarrow \lambda < 2^{\frac{a+b}{1-(a+b)}} < 2^{\frac{1}{2}}$$

$$\mathbf{C.} \ a + b > \frac{1}{2} \rightarrow 2 < \lambda < 2^{\frac{a+b}{1-(a+b)}}, \text{ and if } a + b \rightarrow \infty, \text{ then } 2 < \lambda < \infty$$

The results based on the assumption that the parameters of the fixed cost have the maximum value ($\gamma=\delta=1$). What we proved here is that, even with significant fixed costs and depending on the exhibitors, the firm can face IRS for specific values of $\{\lambda\}$. In cases A, B and C if $\{\lambda\}$ is higher than the values, the equalities demonstrate, then an increase of the inputs to a higher level than these values, will increase the output relatively less. That is, diseconomies of scale appear to the production function even with fixed cost.

To conclude, if the primary production function exhibits DRS, then the positive externality raised from the fixed cost, disappears if the level of the DRS is significant $\{(a + b), \text{ very low}\}$ or the amount of the fixed cost is also not significant. Therefore, depending on the significance of the fixed cost and the degree of the diseconomies of scale, the breaking point which brings back diseconomies under fixed cost, comes closer if the aforementioned are low and moves far away if they are not significant. Precisely, if $(a+b)$ is close to one and the fixed cost is very significant the breaking point goes to infinite. On the other hand, if the opposite appears and $(a+b)$ is lower than half, then if the producer doubles its inputs he will face diseconomies of scale. The result can explain the graph representing marginal and average cost under fixed cost where the long run average cost is falling at the beginning, even with fixed costs. Furthermore, the breaking point on the graph depends on the fixed cost and the slope. The slope as we have seen on Appendix E depends on the exhibitors of the production function as also on the input prices. In practice, maybe this breaking point is far away but we prove that in theory the phenomenon may appear under specific conditions.



The latter mathematical foundation and illustration, is based on the assumption that the fixed cost represented through fixed capital is a constant percentage of the variable capital, as also for labor. We will use an example in order to make our thoughts clear to the reader. The reason is that, if variable capital changes, then in order to keep the parameters $\{\gamma, \delta\}$ constant, the fixed capital or labor must change. Obviously, that means that the fixed inputs are not fixed and the fixed cost is not fixed since it changes. What we are going to show is that the fixed cost changes but it remains fixed cost, as a result it always remains the same percentage of the variable inputs and we can use our analysis. Fixed cost is a cost we have to pay independently if we produce or not. As we will see in our example, it does not necessary mean that the fixed cost must be always constant; it can remain fixed cost and also modified. The situations where it must change and as a result the percentage is constant are when it reaches its own limits.

A firm organizes concerts and different events. The firm needs to rent a stadium and also equipment in order to organize the concert. The stadium and the equipment is the total capital. Every time a concert is organized, the firm has to rent equipment and a stadium in order to make one concert. Therefore, up to this point the total capital is always flexible and the stadium is a fraction or percentage of the equipment.

$$\text{capital} = \text{equipment} + \text{stadium} \rightarrow K = 1 = \text{equipment} + \text{stadium}$$

$$1. \text{equipment} = g \cdot K, \quad \text{stadium} = (1 - g) \cdot K \rightarrow g \cdot K + (1 - g) \cdot K = K$$

$$2. \text{stadium} = \frac{(1 - g)}{g} \cdot \text{equipment} = \gamma \cdot \text{equipment}$$

Similarly, they need to pay a band to play music and also rent labor for security, organization and other administrative duties. We can derive a formula for labor but it is exactly the same and we will focus on capital. When the firm organizes one concert, they rent the stadium for one day and the equipment also for one day and they produce one unit of output, which is the concert. If they want to increase the output and make more concerts they have to follow the same procedure and rent capital for one more day. While the output is produced, the firm needs one unit of capital and inside the capital the proportion is constant for the stadium. Now the firm decides to buy a contract and rent Wembley Arena for 100 years. That is, a fixed cost appears since even if the production is zero the firm has a positive cost. We also assume that if the firm wants, it can rent the stadium to someone else and we make this distinction in order to avoid the characteristic of sunk cost. We can also say that they bought the contract but they have to pay a specific amount every month for 100 years, but even without this assumption the contract is a fixed cost even if they paid once at the beginning for the entire period.

Moreover, one part of the capital is now fixed because the firm does not have to rent a stadium every time. Now the externality appears. When the firm organizes the first concert they rent only equipment, but the necessary one unit of capital is less than the daily equipment plus the total stadium. The proportion of the stadium and the equipment remains the same due to the externality. Precisely, despite the fact we own the stadium we only need just a proportion in order to organize a concert and this is the reason the proportion is always the same even if the capital increased.

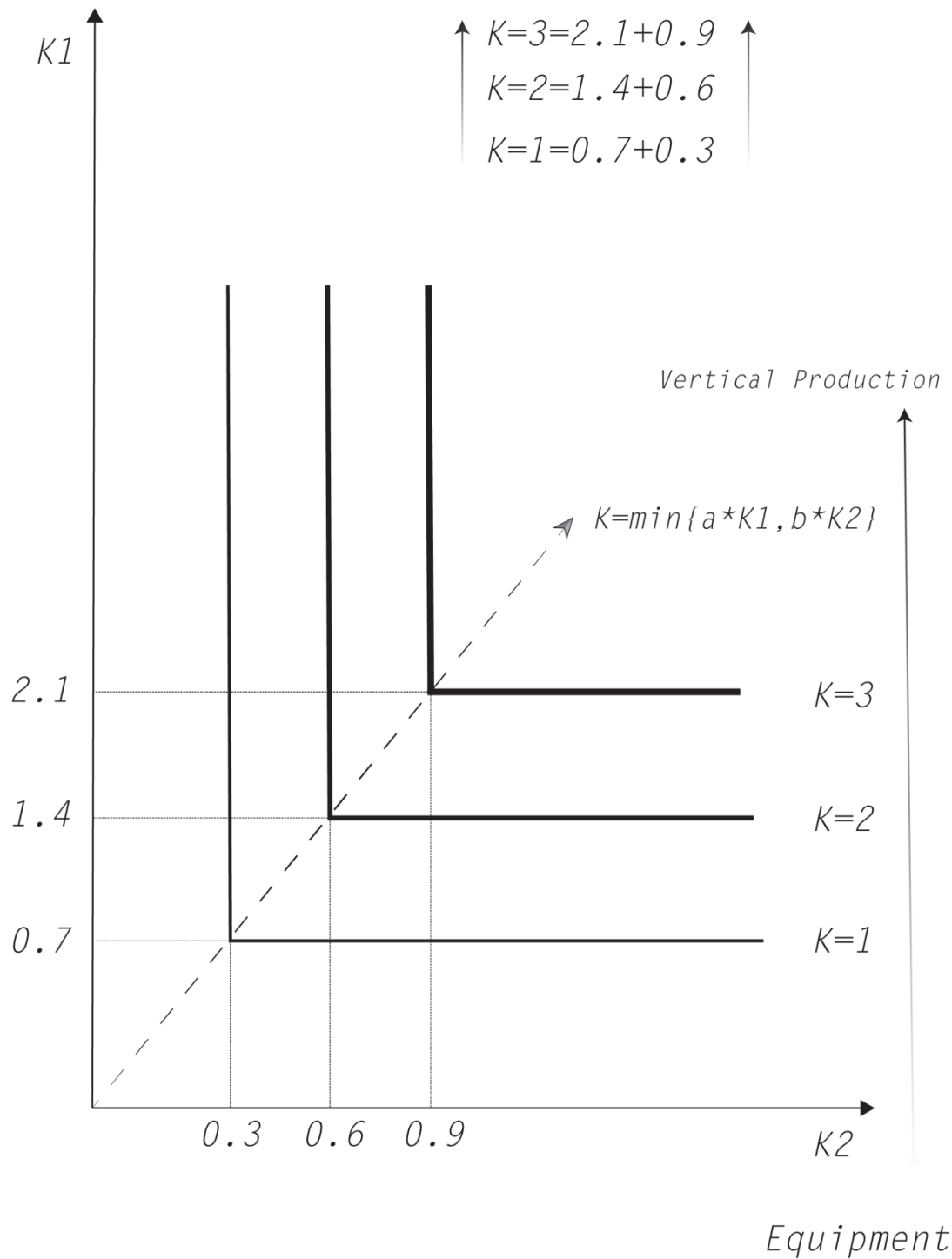
To continue, the second day the firm organizes one extra event and they rent equipment for one extra day but not the stadium. While one unit of output is produced the firm needs the same proportion of equipment and stadium and therefore one unit of capital to produce one unit of output. The firm continues this for the entire year and makes concerts every day. Obviously, the average fixed cost is falling while the number of the concerts increases but the proportion needed to produce one unit of these outputs remains the same. One unit of output means that the isoquant curve represents the optimum combinations of capital and labor which produce one unit of output.

Now things become more complex. It is different when the firm increases the output by organizing events different days and by organizing events the same day. Vertical and horizontal productions are different ways to increase production. If the firm wants to increase output from one unit to two units the same day, they also need to rent again another stadium or increase the fixed cost with another long-run contract. The transposition of the isoquant curve to higher levels of production need more capital and labor but the proportion remains the same between fixed and flexible capital. Even if the firm decides for another contract, the total capital increases because the fixed and flexible part of the total capital increased, but by the same proportion.

Moreover, if the fraction between stadium and equipment is constant, every time we produce a specific amount of output (different isoquant), we can represent the production function through a convenient way. The aforementioned can happen through fixed proportions between the parts of the capital but not necessarily between capital and labor. In the graph of Appendix F we show a simple example how the isoquant curve moves on the right with fixed cost on labor and capital. Also, the origins move due to the same reason and that's why the Inada conditions do not appear to hold for the isoquant curve without fixed cost.

To conclude, we understand that our idea is not completely accurate and has some disadvantages, but it is the only theory which can explain the conflict between the production function and the cost function. Indeed, only in our theory the production function can face DRS, the transformed production function IRS until a critical point, the cost function can be convex and the same time the average cost can diminish and arise under convex cost curves.

Stadium



With vertical production our theory is accurate, with horizontal production it is not, since the proportion changes if the capital increases horizontally.

Stadium

