Dual Sourcing: an Uncertain Regular Supplier and an Emergency Supplier

Bachelor Thesis

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Abstract

We study a dual sourcing inventory control problem under periodic review. Both demand and supply are uncertain. The suppliers differ in lead times and costs. The cheaper supplier has a larger lead time and has with binomially distributed yield. Two heuristics are introduced; both are a combination of an order-up-to policy for the expedited supplier and ordering based on current demand for the regular supplier. The two heuristics differ in that one incorporates virtual inventory levels and the other only current inventory levels. The heuristic based on virtual inventory levels performs best. Using dual sourcing is preferred over using only the expedited supplier given the uncertainty in supply is not too large. We also compare the performance of our heuristics with that of a dual-index order-up-to policy found in Ju (2012). Our heuristics do not outperform this DIP. There is also a comparison made with a single sourcing heuristic; our dual sourcing heuristics do outperform this single sourcing heuristic.

Contents

1. Introduction

These days some products you find in the stores have traveled half the globe to get there. Companies outsource production abroad to reduce costs. However the supply from far away is not always that certain; some goods may get damaged on the way. Therefore companies may also be supplied by someone closer by; this supplier could be an emergency supplier for the company. Especially a combination of these two types of suppliers can be beneficial. This is called dual sourcing.

The problem we would like to investigate is one concerning dual sourcing with uncertainty in demand and yield. We consider a retailer with two sources of supply. One of these sources is cheap but is located far away and has uncertain yield. The other is more expensive but is closer to the supplier and has certain yield. We would like to find an ordering policy for this company that minimizes the costs.

2. Literature review

Most papers on supply uncertainty deal with single sourcing; where only one supplier is utilized. Inderfurth & Vogelgesang (2013) for example introduce uncertain yield in the case of a single supplier. They study three types of uncertain yields among which binomial yield and determine formulas for dynamic and static safety stock. They use a yield inflation factor in order to deal with the uncertain yield. In case of binomially distributed yield this yield inflation factor is equal to $1/q$; where q is the chance that an order arrives safe and sound. Their orders are based on previous demand minus previous excess supply.

The articles we studied concerning dual sourcing can be divided in three groups based on the strategies they used to order the goods. Arts et al. (2011), Veeraraghavan & Scheller-Wolf (2008) and Ju (2012) used the dual-index policy (DIP). Under the conditions set in their paper Veeraraghavan and Scheller-Wolf (2008) showed that dual-sourcing outperforms single-sourcing when using DIP. Some of these conditions were capacity limitations and a desired service-level. They come to their solution by reducing the problem to a newsvendor problem which they solve by simulation. Arts et al. (2011) use the modified fill rate to ensure a certain level of customer service. They find their costs using an approximation based on Markov chains.

The second strategy is designed by Boute & Van Mieghem (2012); they used average sourcing allocation. The decision here is what fraction of the supply will be met by supplier A (or B). The orders are based on a linear combination of the demand of previous periods. Their model performs quite well in the case of capacitated suppliers.

The third strategy we found in articles was first discussed by Rosenshine & Obee (1976). At the regular supplier they had a standing order. The second supplier was introduced in the form of an emergency supplier. This emergency supplier was introduced to mitigate the effect of increasing lead times; it delivered immediately in the same period. Chiang (2007) did roughly the same thing though they handled not only backlogged demand but also lost-sales. They found the optimal ordering policy by dynamic programming.

3. Problem description

The model we use is a multiple-period, periodic review model. We have different costs for the two suppliers. The lead time of the regular supplier is l_r periods; the lead time of the expedited supplier is l_e with l_e smaller than l_r. For the regular supplier the costs are c_r per unit ordered; for the faster but more expensive supplier the costs are c_e with c_e larger than c_r. We assume there are no fixed costs concerning the orders. If the demand in a certain period cannot be satisfied it will be backlogged. It will be charged a penalty p per unit of backlogged demand at the beginning of the next period. Holding costs h per unit apply to positive inventory minus backlogged orders at the beginning of every period. There are no bounds applied to the size of the orders. The products from the regular supplier are not all usable. A single unit is usable with the chance q causing the amount of product supplied to be binomially distributed. In Table 1 below you can find an explanation of all the notations that will be used.

Notation	Explanation	Notation	Explanation
t	Period index	λ	Average demand
c_r	Regular ordering cost per unit	l_e	Lead time expedited supplier
c_e	Expedited ordering cost per unit	l_r	Lead time regular supplier
h	Holding cost per unit	q	Probability of a unit of regular supply to be usable
p	Penalty cost per unit for backlogged demand	X_t^e	Amount ordered from expedited supplier to arrive in period t
X_t^r	Amount ordered from expedited supplier to arrive in period t	$Y(X_t^r)$	Yield from regular supplier in period t
z^e	Order-up-to level for expedited supplier	I_t	Inventory level in period t
$\tilde{I}_t$	Virtual inventory level in period t	D_t	Demand in period t
S_t	Sample standard deviation in period t		

Table 1: Explanation of notations used

Each period is signified by a sequence of events. Firstly the on-hand inventory is observed (1). Then the demand of that period is given and either fulfilled or backlogged (2). After that the necessary orders are placed (3). Finally previously made orders arrive (4).

3.1 Research goal

The goal of our research is to design and study the performance of a periodic review model concerning dual sourcing.

4. Heuristics

We developed two different heuristics for our dual sourcing problem; these are discussed in section 4.1 and 4.2 respectively. In order to see whether these two outperform a single sourcing heuristic we introduce on in section 4.3. The performance of our heuristics will be analyzed in section 5.

4.1 Heuristic 1: emergency order-up-to level based on current inventory level (CIL)

The first heuristic is a combination of the one used in Inderfurth & Vogelgesang (2013) and an order-up-to policy as can be found in Arts et al. (2011). Here we use the expedited supplier as a safety net in case the inventory gets to low by implementing an order-up-to policy; the expedited supplier is used as an emergency supplier. We find the optimal order-up-to level for each set of parameters by checking several levels for the associated costs. In period t we order in the following way:

$$X^e_{t+l_e} = z^e - I_t \; if \; I_t < z^e$$

$$X^r_{t+l_r} = \left\lfloor \frac{D_t - X^e_{t+l_e}}{q} \right\rfloor \; if \; D_t - X^e_{t+l_e} \geq 0 \; and \; \frac{1}{q} < \frac{c_e}{c_r}$$

In this heuristic we replenish the stock in each period, and apply a yield inflator factor to correct for the uncertain yield. In case of a low inventory (when the inventory falls below z^e) we have our emergency supplier. The orders are essentially based on the following idea:

$$E\left(Y(X^r_{t+l_r})\right) \cong D_t - X^e_{t+l_e}$$

The expected yield of the regular order will roughly replace the demand minus the expedited order. Any orders placed should be integer, therefore the regular order is rounded down.

For smaller q when $1/q > c_e/c_r$ no orders are placed at the regular supplier. In those cases it is cheaper to order from the expedited supplier only. A simple example can illustrate this. If q = 0.7, c_r = 100 and c_e = 200 ordering one unit from the regular supplier only will cost 143 (using our heuristic, 100/0.7). Therefore it is beneficial to use both suppliers. However in the case of q = 0.4 one unit from the regular supplier will cost 250 (100/0.4). In that case it is best to order from the expedited supplier only.

4.2 Heuristic 2: emergency order-up-to level based on virtual inventory level (VIL)

The second heuristic is a slight alteration of the first one. We use virtual inventory level to determine whether or not we place an emergency order. This virtual inventory level is loosely based on the expected inventory level. This is how we find the virtual inventory level:

$$\tilde{I}_{l_e} = -1 \times \lambda \times l_e$$

$$\tilde{I}_{t+l_e+1} = E(I_{t+l_e}) - D_t + \lceil E(X^r_{t+l_e}) \rceil + X^e_{t+l_e} - \lceil E(X^r_t) \rceil + Y(X^r_t)$$

The first formula is the one used in the first period to determine the order at the expedited supplier. For the successive periods the virtual inventory level is calculated using the second formula. The demand and orders made are incorporated in this formula. As the virtual inventory is loosely based on the expected inventory level, the expected demand is incorporated by adding $-1 \times \lambda \times l_e$. We update the expected demand each period with the current demand ($-D_t$). The orders expected are added each period ($\lceil E(X^r_{t+l_e}) \rceil + X^e_{t+l_e}$) and the orders that have just arrived are integrated as well ($-\lceil E(X^r_t) \rceil + Y(X^r_t)$).

Using this virtual inventory level we order in the following way in period t:

$$X^e_{t+l_e} = z^e - \tilde{I}_{t+l_e} \; if \; \tilde{I}_{t+l_e} < z^e$$

$$X^r_{t+l_r} = \left\lfloor \frac{D_t - X^e_{t+l_e}}{q} \right\rfloor \; if \; D_t - X^e_{t+l_e} \geq 0 \; and \; \frac{1}{q} < \frac{c_e}{c_r}$$

The virtual inventory level used to determine the expedited order is calculated in the previous period. This is done because previously placed orders arrive after orders are placed; not all information needed to determine the VIL is available at the time of ordering. Therefore we use the virtual inventory level from the previous period. As you can see the only difference between this heuristic and the previous one is the use of virtual inventory instead of current inventory. This could lead to some cost reduction though.

4.3 Single sourcing heuristic

We choose a single sourcing heuristic that resembles our dual sourcing heuristics closely in order for it to be a vital alternative. Using only the regular supplier in the same manner as our two heuristics would not give the desired results; severe backlogging would occur. Therefore we only use the expedited supplier in the following way:

$X^{e}_{t+l_e} = z^e - I_t \ if \ I_t < z^e$

Again we determine the optimal order-up-to level; this will probably be higher than the order-up-to levels found for the two heuristics.

The first heuristic (CIL) is exactly the same as this single sourcing heuristic in the case that $1/q > c_e/c_r$. They will produce similar results in those cases.

5. Performance heuristics

The performance of the heuristic can be assessed by reviewing several characteristics. We not only look into the total costs but also the inventory on hand, the amount of backlogging and the ratio of the amount ordered from the expedited supplier to the amount ordered from the regular supplier. We also compare the two heuristics with other heuristics: a single sourcing heuristic and a heuristic found in literature. By changing parameters such as the different costs, the lead times of the suppliers, the demand rate or the yield fraction q, we can compare the heuristics. The standard setting of the parameters is as follows: $\lambda = 2$, $l_e = 2$, $l_r = 6$ and $q = 0.7$. The standard settings for the costs are $h = 5$, $p = 495$, $c_e = 130$ and $c_r = 100$. When varying one parameter we keep the others constant. We assume the demand to be Poisson distributed.

We used simulation to determine the costs from using one of the heuristics for given parameters. After a warm-up period of 100 periods the simulation runs until the sample standard deviation is adequately small or 400000 periods are run. The pseudocode for the simulation for the heuristics can be found in the appendix. The only parameter not set is the order-up-to level z_e; we conduct a search for the optimal z_e for every set of parameters.

We study the effects of different parameters on both our heuristics; in each comparison we will also include the single sourcing heuristic. In section 5.1 we look at the effect of the average demand λ on the costs. In section 5.2 the effects of the yield factor q will be shown. The effects of the lead time will be given in section 5.3 and the effects of the expedited ordering costs will be given in section 5.4. Finally in section 5.5 we will make a comparison with a heuristic found in literature.

5.1 Effect of Demand

The demand is Poisson distributed; therefore increasing the mean demand λ also means the variance of the demand increases. Both obviously cause a rise in average costs per period as can be seen in Figure 1. The average total costs for the first heuristic (CIL) are larger than those for the second heuristic (VIL). The rise in average total costs can be mostly contributed to the increase in ordering costs to support the rise in demand.

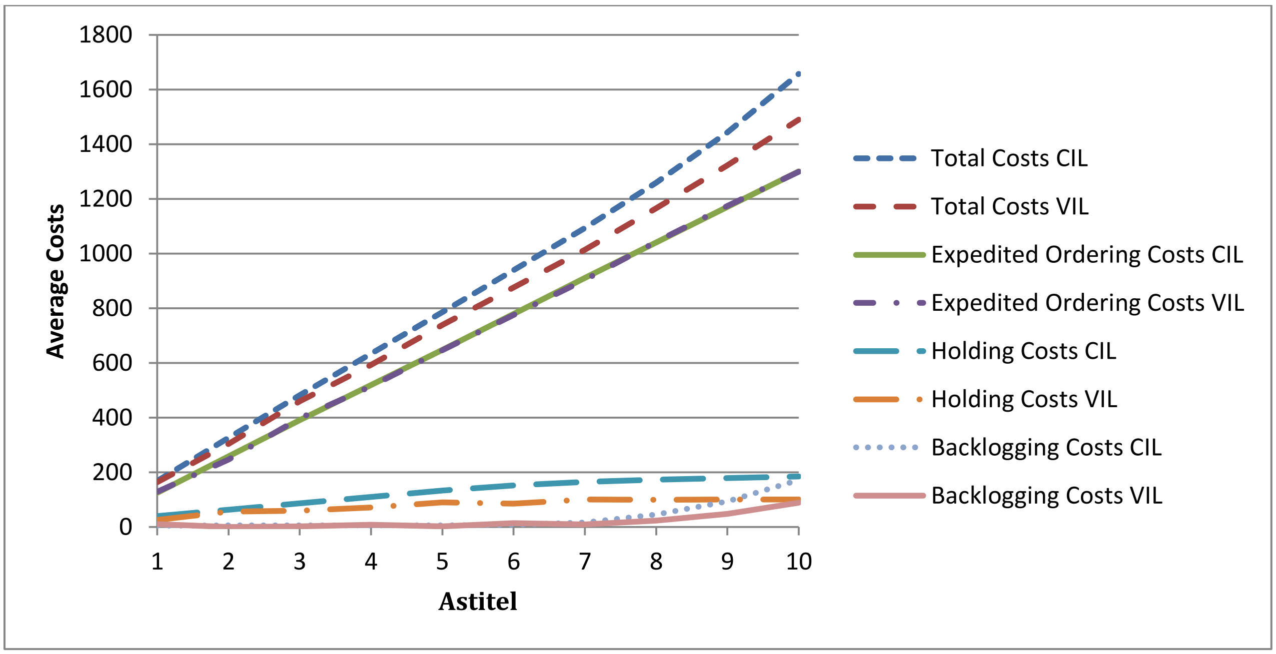

Figure 1: Effects of Demand on Costs (q = 0.7)

There is also a slight rise in holding costs which is possibly caused by the increasing variance in demand. The inventory increases more for the CIL heuristic than for the VIL heuristic. The use of more information in the VIL heuristic may allow it to be feasible to have a lower inventory level. For higher average demand λ the backlogging costs also increase. This is probably also due to the higher variance of the demand.

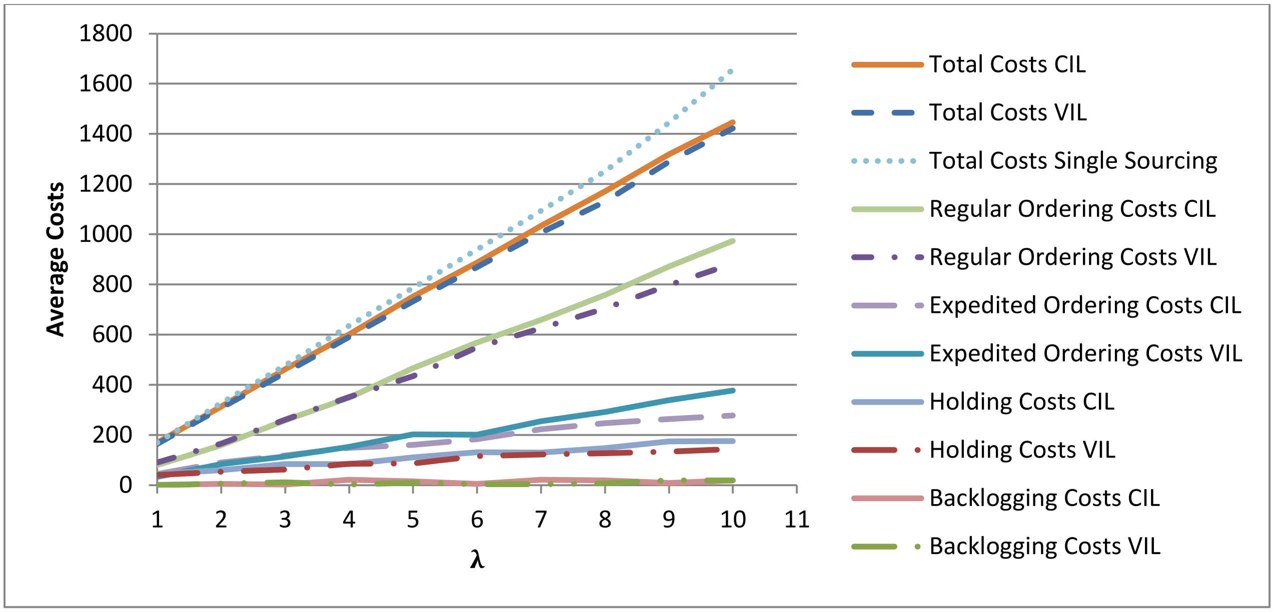

Figure 2: Effects of Demand on Costs (q = 0.8)

For q = 0.7, c_r = 100 and c_e =130 no orders are placed at the regular supplier, therefore we also examine the effects of λ in the case of q = 0.8. In Figure 2 you can see that the VIL heuristic again outperforms the CIL heuristic. This is mostly due to the higher holding costs for the CIL heuristic. You can also see that the CIL heuristic orders more from the regular supplier and less from the expedited supplier than the VIL heuristic. This can also be seen in Figure 3. For higher λ CIL orders more from the regular supplier and less from the expedited supplier. For the VIL heuristic the percentages remain roughly the same for higher λ.

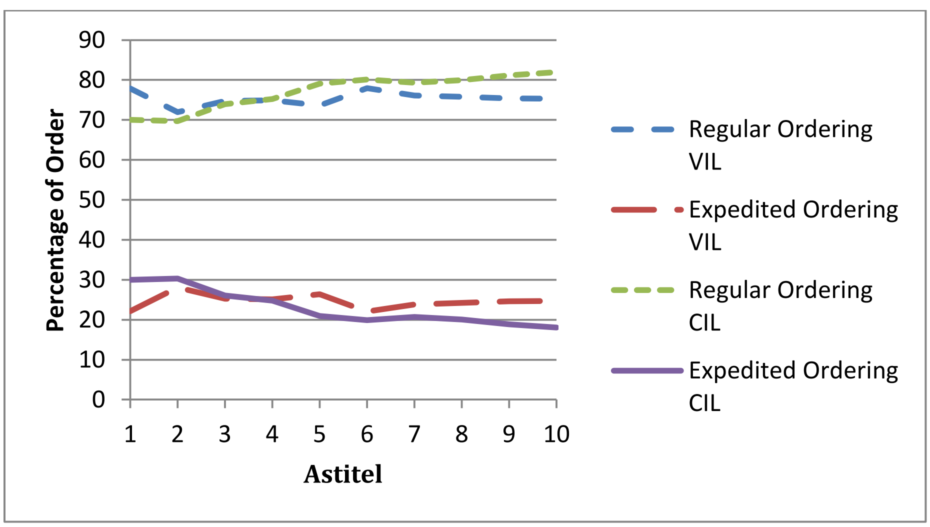

Figure 3: Impact of λ on ordering percentages (q = 0.8)

Our heuristics perform equally well or better than a single sourcing heuristic. For q = 0.7 the first heuristic yields the same costs as the single sourcing heuristic because both order only from the expedited supplier using the current inventory level. For q = 0.8 CIL outperforms single sourcing (Figure 2). In both cases VIL results in lower costs than the single sourcing heuristic. In the case of q = 0.8 the cost difference is caused by higher ordering costs for the single sourcing heuristic. The three heuristics differ in average inventory level (Figure 4). Using a dual-sourcing heuristic results in a lower average inventory level.

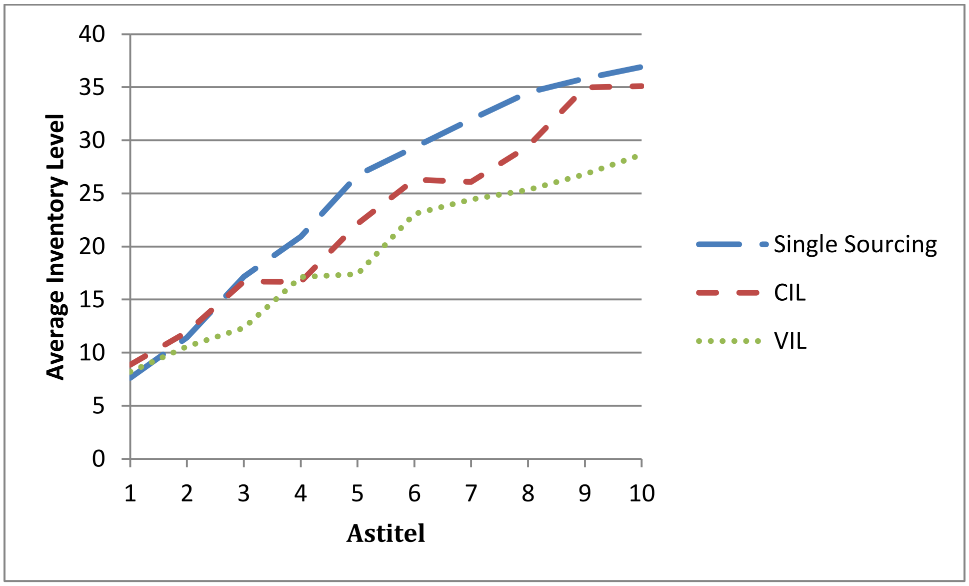

Figure 4: Influence of demand on inventory levels (q = 0.8)

5.2 Effect of the yield fraction q

A decrease of the yield factor q causes an increase in costs as the yield of the regular supplier is binomially distributed with parameters X_t^r and q; the smaller q gets the smaller the expected yield will be. This decreased yield will have to be offset by ordering more, which raises the costs. This can be seen in Figure 5 where you can also see that for small enough q the ordering costs are not influenced by q. In those cases $1/q > c_e/c_r$ only the expedited supplier will be used. For c_r = 100 and c_e = 130 this will be when q is smaller than 0.77.

In Figure 5 we can also see that the second heuristic outperforms the first heuristic again. This is mostly due to higher holding costs for CIL but in some cases CIL accumulates higher backlogging and ordering costs as well. In Figure 5 it is clear that fluctuations in the average total costs are caused by the average total ordering costs.

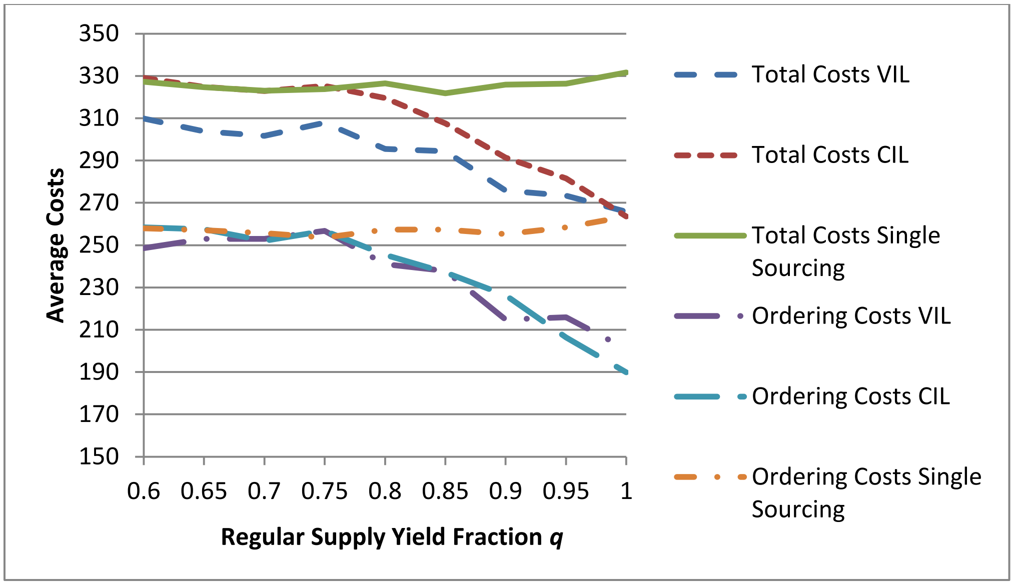

Figure 5: Effect of yield fraction q on costs

For smaller q more is ordered from the expedited supplier as this is cheaper (Figure 6). As mentioned before for $q < 0.77$ only the expedited supplier is used. In those cases the first heuristic CIL is the same as our single sourcing heuristics which also only uses the expedited supplier. Therefore the CIL heuristic does not outperform the single sourcing heuristic in those cases (Figure 5). The second heuristic (VIL) does result in lower costs in all cases.

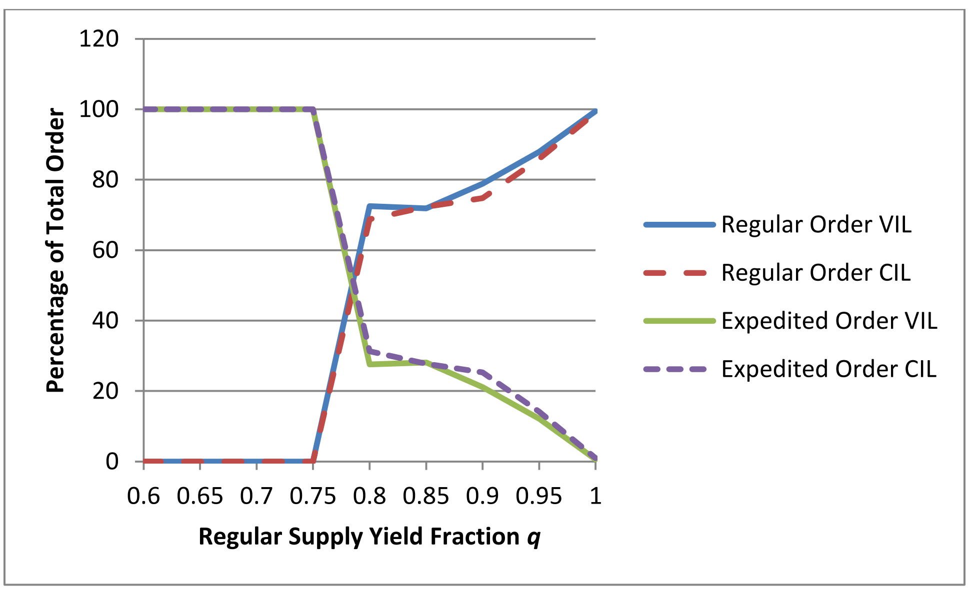

Figure 6: Influence of yield fraction q on ordering patterns

5.3 Effect of lead time

The lead time of a supplier can influence the costs in many ways: backlogging costs may occur, on hand inventory is necessary and ordering patterns may differ for different lead times. As can be seen in Table 2 and Table 3 the average total costs do not differ much for different lead times. What is interesting though is the fact that the first heuristic CIL finally outperforms the VIL heuristic (for l_e = 0). The reason for this is quite simple: the expedited order for the VIL heuristic is based on the virtual inventory level. This virtual inventory does not take the demand in the current period into account. The order for the CIL heuristic does take this demand into account. In the case of l_e = 0 the information of the current demand is very useful to determine the expedited order. Therefore the CIL heuristic performs better. In the case of q = 0.8 it does so by almost exclusively ordering from the expedited supplier. This ordering behavior is why the VIL heuristic does not consider the current demand; it would reduce the regular orders to zero. The VIL heuristic does outperform CIL for larger expedited lead times.

l_e	0		2	
l_r	CIL	VIL	CIL	VIL
2	291	299	331	308
6	286	295	328	308
8	292	302	327	308

Table 2: Average total costs for different lead times (q = 0.7)

l_e	0			2		
l_r	CIL	VIL	Single Sourcing	CIL	VIL	Single Sourcing
2	290	302	290	300	300	328
6	293	303	296	318	305	329
8	294	299	287	325	310	325

Table 3: Average total costs for different lead times (q = 0.8)

The single sourcing heuristic performs just as well as the CIL heuristic in the cases that $1/q > c_e/c_r$ as these two heuristics are the same for those cases. For larger q the single sourcing heuristic performs just as well or slightly worse than the least successful of the two heuristics.

5.4 Effect of costs of expedited ordering

In the case of rising expedited ordering costs it would be beneficial to decrease the percentage ordered from the expedited supplier. However this is not the case for our two heuristics; the ordering pattern remains roughly the same for increasing c_e (if $1/q < c_e/c_r$). As you can see in Figure 7 the expedited ordering costs drive the increase in average total costs. We traced this pattern up to an expedited ordering cost of 500. This means that the amount ordered from the expedited supplier remains constant even for increasing costs.

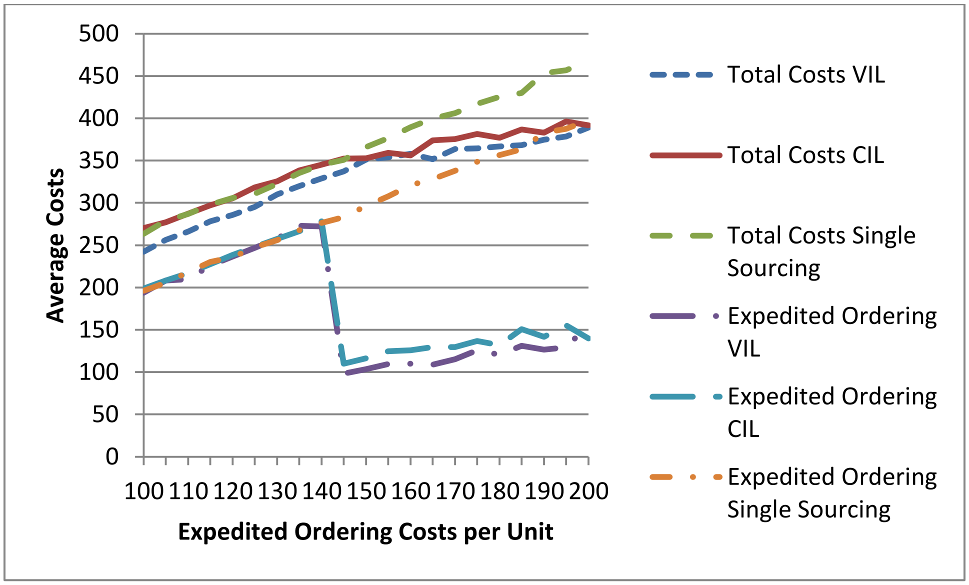

Figure 7: Effect of expedited ordering costs on average costs (λ=2, q=0.7, l_e=2, l_r=4, c_e=130)

The dual sourcing heuristics outperform the single sourcing heuristic in the case of large enough expedited ordering costs. For smaller c_e (c_e < 143) CIL is the same as the single sourcing heuristic.

5.5 Comparison with a heuristic found in Ju (2012)

We have found that in most cases the second heuristic outperforms the first one. However it is uncertain whether companies would truly achieve low ordering costs using these heuristics. Therefore we compare our heuristics with another heuristic found in literature. In Ju (2012) orders are placed according to a dual-index order-up-to policy. The difference with our heuristics is that an order-up-to level is used for both the expedited (z_e) and the regular supplier (z_r).

We compared the average total costs that were found using both heuristics. The parameters differed slightly in order to make a comparison on several fronts. The regular supply yield fraction q was 0.5, 0.7 or 0.9. The average demand was either 2 or 4. The penalty p applied to backlogged demand was either 95 or 495. And finally the expedited ordering costs per unit varied, being 120, 150 or 180.

Using these variation of parameters we found that our first heuristic led to average total costs that were on average 15 percent higher than those found with the heuristic in Ju (2012). The second heuristic performed better but still led to costs 11 percent higher on average. In the appendix a table containing the comparisons can be found.

Our heuristics perform best for lower q and smaller expedited ordering costs. For larger q our heuristics probably order more from the expedited supplier than Ju's heuristic. The problem with increasing expedited ordering costs is the same as we found in 5.4; for larger c_e the amount ordered from the expedited supplier does not decrease.

Overall Ju's heuristic performs better than our heuristics. The major differences are explained above but Ju performs better in all cases. The likely cause for this is that Ju's heuristic results in lower inventory levels. Unfortunately we do not have data concerning the inventory levels Ju's heuristic generates. However it is plausible that using two order-up-to levels would allow for lower average inventory levels. Ordering in this fashion is definitely more flexible than our heuristics.

6. Conclusion

The second heuristic based on virtual inventory levels outperforms the first heuristic based on current inventory levels in almost all cases. The one exception is the case of an expedited lead time of zero. The heuristics result in lower costs than a single sourcing heuristic in the case that the yield fraction q is not too small. In the case that $1/q > c_e/c_r$ the CIL heuristic is the same as the single sourcing heuristic. The heuristics do not perform as well as the dual-index order up to policy in Ju (2012). Especially in the case of a large yield fraction q and large expedited ordering costs c_e the heuristics perform poorly. The reason for this is that the amount ordered from the expedited supplier remains constant though it would be beneficial to decrease it.

7. Discussion

It might seem beneficial to use these heuristics for companies that are required to place an order at their regular supplier every period. However using the parameter settings in this paper the regular order was often zero. In the case of $1/q > c_e/c_r$ only the expedited supplier is used.

It would be interesting to investigate the influence of the distribution of the demand on the performance of the heuristics. Here we have used Poisson distributed demand. Changing to uniformly distributed demand may affect the performance significantly. Using a different distribution may allow us to research the influence of the variance of the demand. Now this is not really possible because the variance is equal to the mean for the Poisson distributed demand.

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Appendix

Heuristic 1: emergency order-up-to level based on current inventory level (CIL)

Initiate all necessary parameters and variables

Run in period
while t < 100
    t++
    Determine demand in period t (Poisson random)
    Fulfill and/or backlog demand and backlogged demand
    Order goods from emergency supplier in the following way:

$$X^e_{t+l_e} = z^e - I_t \; if \; I_t < z^e$$

Order goods from regular supplier in the following way:

$$X_{t+l_r}^r = \left\lfloor \frac{D_t - X_{t+l_e}^e}{q} \right\rfloor \; if \; D_t - X_{t+l_e}^e \geq 0 \; and \; 1/q < c_e/c_r$$

Reveal yield from regular supplier in period t (Binomial random)
Update inventory level with yield from regular and emergency supplier

$$I_{t+1} = I_t + Y(X_t^r) + X_t^e$$

end

while $\frac{S_t}{\sqrt{t-100}} > 5 \; or \; t < 400000$
t++
Update holding costs (+ inventory level * holding cost per unit)
Update backlogging costs (+ backlogged demand * backlogging costs per unit)
Determine demand in period t (Poisson random)
Fulfill and/or backlog demand and backlogged demand
Order goods from emergency supplier in the following way:

$$X_{t+l_e}^e = z^e - I_t \; if \; I_t < z^e \; and \; 1/q < c_e/c_r$$

Order goods from regular supplier in the following way:

$$X_{t+l_r}^r = \left\lfloor \frac{D_t - X_{t+l_e}^e}{q} \right\rfloor \; if \; D_t - X_{t+l_e}^e \geq 0$$

Update emergency ordering costs (+ amount ordered * cost per unit)
Update regular ordering costs (+ amount ordered * cost per unit)
Reveal yield from regular supplier in period t (Binomial random)
Update inventory level with yield from regular and emergency supplier

$$I_{t+1} = I_t + Y(X_t^r) + X_t^e$$

Calculate sample standard deviation S_t
end

Return the costs

Heuristic 2: emergency order-up-to level based on virtual inventory level (VIL)

Initiate all necessary parameters and variables

Run in period
while t < 100
t++
Determine demand in period t (Poisson random)
Fulfill and/or backlog demand and backlogged demand
Order goods from emergency supplier in the following way:

$$X_{t+l_e}^e = z^e - \tilde{I}_{t+l_e} \; if \; \tilde{I}_{t+l_e} < z^e \; and \; 1/q < c_e/c_r$$

Order goods from regular supplier in the following way:

$$X^r_{t+l_r} = \left\lfloor \frac{D_t - X^e_{t+l_e}}{q} \right\rfloor \; if \; D_t - X^e_{t+l_e} \geq 0$$

Reveal yield from regular supplier in period t (Binomial random)
Update inventory level with yield from regular and emergency supplier

$$I_{t+1} = I_t + Y(X^r_t) + X^e_t$$

Update virtual inventory level with demand in period t
Update virtual inventory level with yield from regular supplier
Update virtual inventory level with virtual yield in period t+l_r
end

while $\frac{S_t}{\sqrt{t-100}} > 5 \; or \; t < 400000$
t++
Update holding costs (+ inventory level * holding cost per unit)
Update backlogging costs (+ backlogged demand * backlogging costs per unit)
Determine demand in period t (Poisson random)
Fulfill and/or backlog demand and backlogged demand
Order goods from emergency supplier in the following way:

$$X^e_{t+l_e} = z^e - \tilde{I}_{t+l_e} \; if \; \tilde{I}_{t+l_e} < z^e \; and \; 1/q < c_e/c_r$$

Order goods from regular supplier in the following way:

$$X^r_{t+l_r} = \left\lfloor \frac{D_t - X^e_{t+l_e}}{q} \right\rfloor \; if \; D_t - X^e_{t+l_e} \geq 0$$

Update emergency ordering costs (+ amount ordered * cost per unit)
Update regular ordering costs (+ amount ordered * cost per unit)
Reveal yield from regular supplier in period t (Binomial random)
Update inventory level with yield from regular and emergency supplier

$$I_{t+1} = I_t + Y(X^r_t) + X^e_t$$

Update virtual inventory level with demand in period t
Update virtual inventory level with yield from regular supplier
Update virtual inventory level with virtual yield in period t+l_r
Calculate sample standard deviation S_t
end

Return the costs

Comparison of the heuristics with the heuristic found in Ju (2012)

λ	c_e	p	q	Average total Cost (Ju)	Average total cost (VIL)	Difference from Ju % (VIL)	Average total cost (CIL)	Difference from Ju % (CIL)
2	120	95	0.5	258	276	6.9	293	13.6
2	120	95	0.7	258	280	8.5	293	13.7
2	120	95	0.9	246	263	7.1	275	11.6
2	120	495	0.5	264	285	7.8	307	16.5
2	120	495	0.7	264	290	9.9	307	16.3

2	120	495	0.9	255	283	10.9	291	14.1
2	150	95	0.5	317	338	6.7	356	12.4
2	150	95	0.7	313	336	7.5	346	10.5
2	150	95	0.9	247	287	16.0	287	16.2
2	150	495	0.5	323	351	8.7	364	12.8
2	150	495	0.7	322	344	6.9	354	10.1
2	150	495	0.9	254	294	15.7	302	19.1
2	180	95	0.5	377	399	5.9	414	9.9
2	180	95	0.7	313	356	13.8	366	16.9
2	180	95	0.9	247	295	19.6	311	26.0
2	180	495	0.5	383	406	5.9	427	11.4
2	180	495	0.7	323	371	14.8	380	17.7
2	180	495	0.9	255	318	24.6	317	24.4
4	120	95	0.5	505	539	6.7	574	13.7
4	120	95	0.7	505	537	6.4	577	14.3
4	120	95	0.9	477	524	9.8	542	13.6
4	120	495	0.5	512	547	6.8	592	15.6
4	120	495	0.7	512	547	6.9	595	16.2
4	120	495	0.9	488	544	11.4	554	13.5
4	150	95	0.5	624	660	5.8	694	11.2
4	150	95	0.7	609	643	5.6	665	9.3
4	150	95	0.9	478	555	16.0	571	19.4
4	150	495	0.5	631	672	6.5	711	12.7
4	150	495	0.7	621	669	7.7	686	10.4
4	150	495	0.9	488	584	19.7	595	21.9
4	180	95	0.5	743	780	5.0	815	9.7
4	180	95	0.7	610	685	12.3	705	15.6
4	180	95	0.9	477	600	25.9	621	30.1
4	180	495	0.5	750	793	5.8	835	11.3
4	180	495	0.7	623	709	13.8	720	15.6
4	180	495	0.9	489	619	26.6	629	28.7
					Minimum	5.0		9.3
					Maximum	26.6		30.1
					Average	11.0		15.4