The impact of a shortage of credit on price-regulated and unregulated markets:

an extended Cournot model

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THE IMPACT OF A SHORTAGE OF CREDIT ON PRICE-REGULATED AND UNREGULATED MARKETS:

AN EXTENDED COURNOT MODEL

By Marjolein H. M. Benschop

I. Introduction

New Basel III regulations are said to encourage a credit crunch because of the increase in the required equity capital for banks (The Economist, 28th of Sep 2012). Since banks have higher costs to acquire equity or start deleveraging, banks raise their interest rates to cover the cost. In the real economy the impact on firms can be found in the increase in the costs of bank loans. Less firms will invest and firms might decrease their production because of the increasing costs. This paper will examine the influence of a decrease in credit supply on price-regulated and unregulated markets. One might ask whether the effect of a capital shortfall on price and quantity is larger in price-regulated markets than in unregulated markets.

The recent crisis had a major influence on the economy and questions have been raised about which policies worsen the effects of the crisis on markets. It is important for authorities to know if price-policies will increase the effect of a capital shortfall on production. While normally the prices are used as a mechanism to solve the demand and supply problem, in price regulated markets this is not allowed so a larger decrease in production may arise. Several studies have researched the effects of price regulation (e.g. Averch and Johnson, 1962; Sheshinksi, 1976; Crew and Kleindorfer, 1996), but there is still insufficient theory to explain the effect of a capital shock on unregulated and price-regulated markets. The result of this paper is relevant for the analysis of the effects of the recent crisis, but also for future crises that may result in a larger credit crunch.

In this paper, markets are seen as a Cournot duopoly (Cournot, 1838) with homogeneous goods. First, the Cournot model will be analysed for identical firms on the market. Second, this model will be analysed with heterogeneous firms that have different cost functions. Both models will be analysed in three steps: first the equilibrium quantities, price, and revenue will be calculated, then the effect of a capital crunch on the equilibrium values will be determined and finally the condition under which participation will occur will be determined.

The remainder of this paper is organized as follows. Section II reviews the existing literature that is relevant for this paper, including the different forms of price regulation. Section III deals with the unregulated and regulated models for a market with homogeneous firms and section IV will do the same for a market with heterogeneous firms. Finally, section V concludes the findings from this research.

II. Existing Literature

We will analyze the literature with regard to price regulation first. Regulation can be viewed in several ways: quantity, quality, price regulation or even capital regulation. Focusing on price regulation, four main forms exist: rate of return regulation, price cap regulation, revenue cap regulation and benchmarking/yardstick regulation. All these forms have the goal to lower prices, but the way the regulation is executed and the consequences on production and efficiency differ. Price regulation that is meant to stimulate firms to improve their efficiency is called incentive regulation.

Firstly, looking at the rate of return and its possible effects. The rate of return regulation limits the rate on cost of capital. Prices and costs are not fixed, but the rate of return is constrained. Averch and Johnson (1962) find a relationship between rate of return regulation and the capital stock of a firm. Averch and Johnson (1962) conclude that regulated monopolies have a bigger capital stock than the unregulated monopoly. This is called the Averch-Johnson effect: firms accumulate more capital to increase their profit. Since the prices that monopolies are allowed to charge are based on its costs, the regulation does not encourage firms to become more efficient or to lower their costs. In the case of Averch and Johnson (1962), firms accelerate more capital to increase its (depreciation) costs, and thus to be allowed to increase its prices. The approach of the rate of return regulation is criticised, because a regulated monopoly has no incentive to minimize costs. An empirical paper of the Averch and Johnson effect has been written by Spann (1974). Rate of return regulation was common in the telephone and electricity power market in almost every state of the US. Spann (1974) describes the behaviour of regulated electricity power markets and concludes that the Averch Johnson thesis does explain some of the behaviour in these regulated markets.

After criticism on the rate of return regulation, price-cap regulation started to take form. Instead of a constraint on the rate of return, firms are now constrained by a fixed price. Price cap regulation requires firms to adjust their prices to the price cap index that is calculated by a standard index (e.g. Consumer Price Index) and a proxy for the average firm in the industry. Braeutigam and Panzar (1993) say that price-caps are often used in practice in combination with rate of return regulation, but that theoretically and empirically this method has not really been examined. In theory, price cap regulation could minimize costs of monopolies since the maximum prices are based on an index that is begged on external factors of the firm (e.g. factor prices or regional incomes) instead of its costs. However, inefficiencies might still occur because of external reasons and if the price-cap is set below average cost, then it might be impossible for a firm to cover all the costs. Price cap regulation can be seen as limiting the average revenue of a product and is often referred to as a good instrument to make a market more competitive instead of monopolistic (Crew & Kleindorfer, 1996).

When papers do not define the exact form of price-regulation but just mention a maximum price, then this can be viewed as a price-cap. Papers on the effect of price regulation consist of on quantity and quality analyses, such as in Sheshinksi (1976) and Mougeot and Naegelen (2005). Sheshinksi (1976) has examined a price-regulated monopoly and finds that in case of a negative profit, the regulated price will always lead to an increased output and a decreased quality. He focuses on welfare and shows that a regulated price might increase welfare. Mougeot and Naegelen (2005) examined price-regulated hospitals, where price regulation consists of a fixed price. When there is incomplete information, fixed-price policy is shown not to be efficient since it leads to a lower output, quality and expenses.

Thirdly, revenue-cap regulation is a way to decrease prices via limiting the total revenue for a monopoly. However, it could also influence the quantity produced negatively. In practice, a revenue-cap gives firms the incentive to lower its costs by producing less and to increase prices till the binding revenue cap is reached (Crew and Kleindorfer, 1996). In this sense, revenue-cap regulation stimulates monopolies and does not benefit efficiency. In Norway, revenue-cap regulation has been applied to the electricity market in 1997 to promote efficiency (Bjørndal and Jørnsten, 2002).

Finally, Benchmarking means comparing the performance of a company to companies that are similar. Often those that are the most efficient will receive a bonus and the least efficient a fine (Jamasb and Pollitt, 2001).

Benchmarking is a broad term for several methods such as data envelope analysis (DEA), which uses regression analysis to determine the companies that almost reach the efficient frontier and deserve extra profits. Benchmarking and rate on return regulation are often used with price cap regulation. Jamasb and Pollitt (2001) discuss the experience with benchmarking in countries such as the OECD countries. In Spain, distribution utility companies are compared with model firms (dependent on geographical location) to redistribute the total revenues of the market to the utility companies (Jamasb and Pollitt, 2001).

An overview of the different forms of price regulation and examples is shown in the following table.

Forms	Explanation	Example	Source
Price cap	$P = \overline{P}$	Telecommunication in OECD countries.	OECD. Retrieved on June, 6th 2013 from http://www.oecd.org/sti/ieconomy/1909801.pdf
Revenue cap	$Pq_i = \overline{Pq_i}$	Norwegian transmission and distribution companies in 1997.	Bjørndal, M. and K. Jørnsten. 2002. "Revenue Cap Regulation in a Deregulated Electricity Market – Effects on a Grid Company,''*Norwegian School of Economics and Business Administration.*
Rate of return	$\pi_i = \overline{\pi_i}$	Telephone and electric power companies U.S.	Spann, R.M. 1974. "Rate of return regulation and efficiency in production: an empirical test of the Averch-Johnson Thesis,'' *The Bell Journal of Economics and Management Science* 5(1): 38-52
Benchmarking	Different forms exist	Electricity markets in most OECD and other countries.	Jamasb, T. and M. Pollitt. 2001. "Benchmarking and regulation: international electricity experience,'' *Elsevier* 9:107-130

Cournot (1838) discusses the Cournot duopoly: a model in which two identical firms compete with homogeneous products. Cournot (1838) analyses the behaviour of the two firms and finds an equilibrium in which both firms produce the same quantity. Given a price function of $P = a - Q$ and marginal costs of c, the optimal quantities are

$$q_1^* = q_2^* = \frac{a-c}{3}$$

Cournot (1838) expanded the view of market structures, since it is not focused on extremes as monopolies and perfect competition. Also relevant for this paper is the theory about the Cournot model with heterogeneous firms. Dutta (1999) discusses the Cournot model while taking into account the efficiency of the players, so a game with heterogeneous firms .In a situation of complete information, both firms know the type of the other firm: efficient or inefficient. A low-cost firm, firm 1, will produce more than the Cournot duopoly quantity in the simple model so that the optimal quantity of a low-cost firm is

$$q^* > \frac{a-c}{3}$$

The other firm, firm 2, has higher costs and knows that firm 1 is efficient. It will decrease its output since it knows that firm 1 will produce more than the Cournot duopoly quantity. Its optimal quantity will then be

$$q^* < \frac{a-c}{3}$$

When a firm has lower costs than the other firm, it is more efficient and will produce more than the other. The higher the costs of the inefficient firm are, the higher the equilibrium profit is for the efficient firm. In a graph, the reaction function of firm 2 will move to the right if firm 1 is inefficient and will move to the left is firm 1 is efficient. The quantity pair represents a larger share in total quantity for the efficient firm.

III. Model for homogeneous firms

IIIa. Unregulated market

Before considering the different types of price regulation in the model, we start with analysing the unregulated market with a Cournot duopoly. To keep the static Cournot model simple, there is complete information and there are only two firms on the market. These firms are identical, the products are homogenous and the marginal costs are equal in both firms.

The firms are faced with the same price (demand) function because of homogeneous products. The equilibrium price is given as $P = a - (Q)$, whereby a is a constant variable, aggregate quantity is $Q = q_1 + q_2 \geq 0$ and $P \geq 0$. In the Cournot model, total costs are given by $C_1(q_1) = cq_1$ and constant marginal costs at c. However, in this model total costs are divided into capital costs K and labour costs L. By doing this, the effect of capital on production can be analysed. Fixed costs are also added to the model to have a more realistic cost function and to analyse the participation constraint. Fixed capital costs are not separately shown in this model since an increase in fixed capital costs does not affect the optimal quantity q_j^* produced by firms. This leads to the cost function $C_1(q_1) = (k + l)q_1 + F$ and constant marginal costs of $k + l$. In the simple model of Cournot, we assume that the costs for both companies on the market are equal. Another assumption is that, the participation constraints are fulfilled in the unregulated market. A participation constraint is the criterion for a firm to enter or stay in the market. The participation constraint that has to be considered in the unregulated model, is the break-even point in which fixed costs are covered

$$q_{1} > \frac{F}{(P(q_1, q_2) - k - l)}$$

With both the equilibrium price and the costs given, the profit function is as follows

$$\pi_1(q_1, q_2) = (P(q_1, q_2) - k - l)q_1 - F = q_1[a - (q_1 + q_2) - k - l] - F$$

Profit is assumed to be equal to the utility for a firm, so a firm will maximise its profit. To calculate the optimal quantity for a firm, the profit needs to be maximised given the optimal quantity q^* of the other firm. Nash equilibrium is when both firms reach their optimal quantities(q_1^*, q_2^*). Since $q_i \geq 0$ it is also assumed that $q_j^* < a - c$ holds in the optimization. Maximising the profit function for firm 1, leads to

$$\max\ q_1[a - (q_1 + q_2^*) - k - l] - F$$

$$\frac{\delta \pi_1}{\delta q_1} = -(k + l) + a - q_2^* - 2q_1 = 0$$

The same is true for firm 2

$$\max\ q_2[a - (q_2 + q_1^*) - k - l] - F$$

$$\frac{\delta \pi_2}{\delta q_2} = -(k + l) + a - q_1^* - 2q_2 = 0$$

When looking at the two-player game, Nash equilibrium exists when the firms choose the following quantity

$$q_1^* = \frac{1}{2}\,(a - k - \ l \ - q_2^*) \qquad (1)$$

$$q_2^* = \frac{1}{2}\,(a - k - \ l \ - q_1^*) \qquad (2)$$

Substitution of equation (1) into equation (2) gives the following optimal quantities

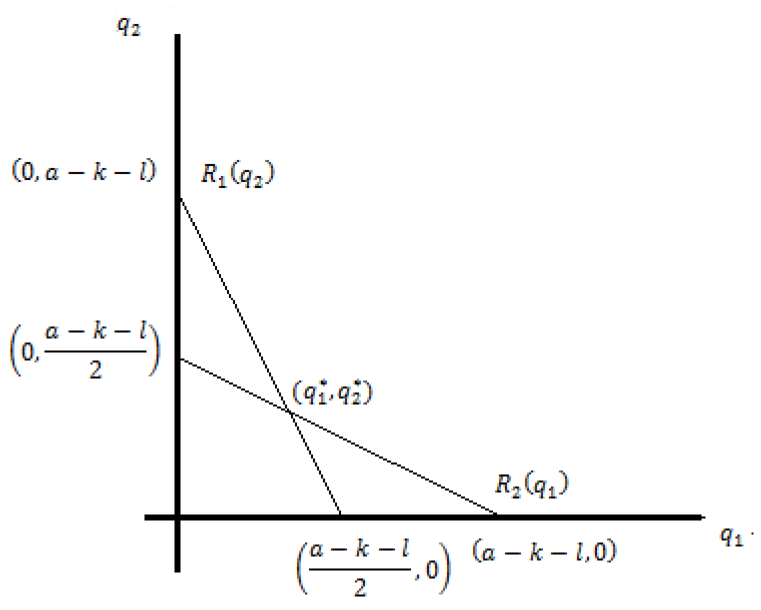
$$q_1^* = q_2^* = \frac{a - k - l}{3}$$

The same result is obtained when looking at the Cournot model graphically. The firms can be viewed as players that choose a strategy in a game. The firms have complete information, so they will anticipate the strategy of the other firm. Firms will find the best response to every possible strategy of the other firm, so these functions are called reaction or best-response functions.

$$R_2(q_1) = \frac{1}{2}(a - q_1 - k - \ l)$$

$$R_1(q_2) = \frac{1}{2}(a - q_2 - k - \ l)$$

In the Nash equilibrium, the reaction functions intersect as shown by the graph. The reaction functions are identical, since the firms have identical cost functions. The equilibrium is represented by the intersection of both functions, which is on a 45° line because of symmetry.



To analyse the overall effect on increase of capital costs on the market, also equilibrium price and revenue have to be analysed. The total quantity produced in this equilibrium is as follows

$$Q^* = 2\left(\frac{a-k-l}{3}\right) = \frac{2a-2k-2l}{3}$$

The price in this equilibrium is then

$$P(q_1^*, q_2^*) = a - \frac{2a-2k-2l}{3} = \frac{1}{3}a + \frac{2}{3}k + \frac{2}{3}l = \frac{a+2k+2l}{3}$$

Revenue for firm 1 (equal for firm 2) is as follows

$$P(q_1^*, q_2^*)\, q_1^* = (\frac{a+2k+2l}{3})\,(\frac{a-k-l}{3})$$

$$= \frac{a^2 + ak + la - 2k^2 - 4kl - 2l^2}{9}$$

Now, we know the quantities, price and revenue in the unregulated model. We use derivatives to explain the effect of a capital crunch on the different variables on the market. Firstly, we look at the effect of an increase in capital costs on the optimal quantities. Quantity should be differentiated to capital costs. This results in

$$\frac{\delta q_1^*}{\delta k} = \frac{\delta q_2^*}{\delta k} = -\frac{1}{3}$$

Both firms experience the same shock in capital costs, since the firms are identical. Quantity and capital costs are negatively related, so when capital costs increase with 1% the optimal quantity per firm decreases with $-\frac{1}{3}$ %. Furthermore, price will be influenced by an increase in capital costs. This because quantity has a negative relationship with price, so when quantity decreases then automatically price will go up. Profit is directly influenced by an increase in capital costs, which will influence the optimal quantity of the firms. Consequently, there will be different best response functions than before the capital shock. A change in k for both firms thus leads to an equilibrium with a different quantity pair and different values of the other equilibrium variables. The effect of an increase in capital costs on price in equilibrium is shown by the first-order difference of equilibrium price to capital costs

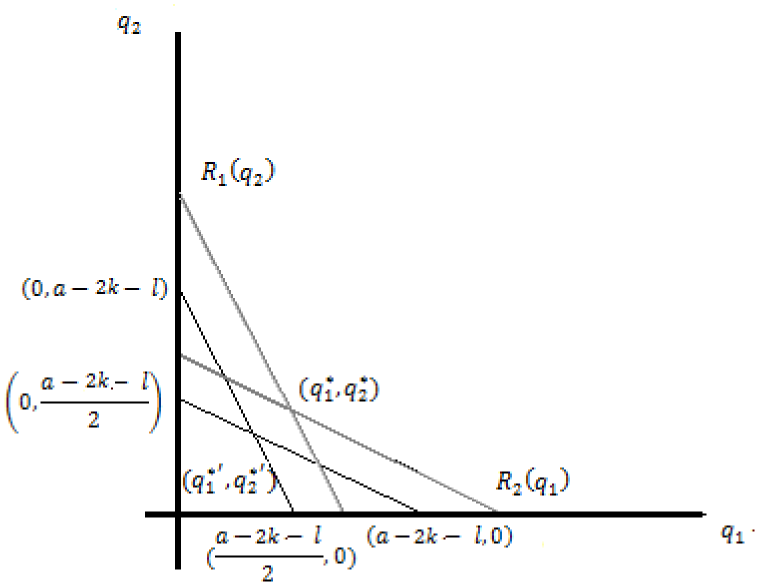
$$\frac{\delta P(q_1^*, q_2^*)}{\delta k} = \frac{2}{3}$$

Price will increase when capital costs increase, since both firms will produce less in this situation. The effect of an increase in capital costs on revenue in equilibrium is solved by taking the first-order difference of the equilibrium revenue to capital costs

$$\frac{\delta P(q_1^*, q_2^*)\, q_1^*}{\delta k} = \frac{\delta P(q_1^*, q_2^*)\, q_2^*}{\delta k} = \frac{a - 4k - 4l}{9}$$

The effect of an increase in capital costs on revenue depends on the unknown variables *a, k* and *l.*

The following is a graphical illustration of what happens to the equilibrium in the market when the marginal costs of capital move from *k* to *2k.* The grey lines represent the reaction functions before the shock and the black lines after the shock.



The reaction functions are symmetrical, which means that both firms will still produce the same quantity. However, now the aggregate quantity is smaller than before the shock: the reaction functions shift to the left and are closer to the origin. The closer to the origin, the smaller the offered quantities. To analyse if both firms still want to participate on the market, we have to look at the effect of the increase in capital costs on the participation constraints. Now, we fill in the price in the participation constraint to analyse under which condition participation of both firms will occur. The participation constraint in the unregulated model before the capital shock is

$$q_1 = q_2 > \frac{F}{\left(\frac{a+2k+2l}{3} - k - l\right)}$$

This means that both firms will participate on the market if the fixed costs are covered by the contribution margin. Because of a capital shock, capital costs and price will increase while optimal quantity will decrease. When capital costs will increase 1%, then price will increase $\frac{2}{3}$%. This means that the participation constraint will be bigger so both firms will have to produce more to break-even when capital costs increase. We assume firms will produce the optimal quantity or not participate at all, so we also fill the optimal quantity in the constraint.

$$\frac{a-k-l}{3} > \frac{F}{\left(\frac{a+2k+2l}{3} - k - l\right)}$$

It follows that both firms will participate when the fixed costs are small enough to be covered

$$F < \left(\frac{a-k-l}{3}\right)^2$$

IIIb. Regulated market

While a variety of definitions of the term price-regulation have been suggested, this model will use the definition price-cap since it is the most commonly used form of price-regulation. When we analyse the regulated market, prices are fixed in the Cournot model by using a price cap $P = \dot{\mathrm{P}}$. A price cap that is lower than the equilibrium price is called effective, because it affects the prices that firms will ask on the market. When the price cap is higher than the equilibrium price, than firms will keep producing their optimal quantity and the equilibrium price will not change. In the unregulated model, firms had all the freedom to determine prices. Thus, when the maximum price is higher than the equilibrium price, the regulation is not effective. Authorities have to determine the price cap in a way that the price cap is lower than the equilibrium price. However, participation constraints can be violated when the price cap is lower. It follows from the participation constraint that price should be high enough to keep producing

$$q_1 > \frac{F}{(P(q_1, q_2) - k - l)}$$

$$P(q_1, q_2) > \frac{F}{q_1} + k + l$$

The participation constraints are identical for both firms. Authorities have to be aware that the participation constraints stay fulfilled, because otherwise the product will not be supplied on the market anymore and welfare could decrease. As mentioned before, the price in the unregulated market equilibrium is

$$P(q_1^*, q_2^*) = a - (q_1^* + q_2^*) = \frac{a + 2k + 2l}{3}$$

So effective regulation should install a price cap

$$\dot{\mathrm{P}} < \frac{a + 2k + 2l}{3}$$

However, to fulfil the constraint

$$\dot{\mathrm{P}} > \frac{F}{q_i} + k + l$$

So the price cap has a very limited range

$$\frac{F}{q_i} + k + l < \dot{\mathrm{P}} < \frac{a + 2k + 2l}{3}$$

Price cap regulation focuses on a maximum price, not on a quota of production. Thus, a is the only variable in the price function $P(Q) = a - Q$ that the government can change without regulating quantity. However, by changing a also the offered quantities change and these quantities affect the price level again. We have to look at first-order differentiations to see in which direction aggregate quantity and price will move after a change in the constant variable *a*. It is important for authorities to know whether their policy, increasing or decreasing *a*, achieves a lower price. First, the direct effect of *a* on price is shown by the first-order difference

$$\frac{\delta P}{\delta a} = 1$$

When authorities change *a*, this adjustment is fully reflected in the price. Now, it is important to know what happens to quantity when authorities change *a*. We take the first-order difference

$$\frac{\delta q_i^*}{\delta a} = \frac{1}{3}$$

This relationship between quantity and *a* is positive, because *a* increases price. Firms want to produce more when the price is higher. However, price is influenced by the new quantity pair via aggregate quantity. So, when q_i^* and q_j^* both change, this will have effect on the price function via *Q*. *Q* is affected both by q_i^* and q_j^* for $\frac{1}{3}$%, so in total by $\frac{2}{3}$%. To see the effect of aggregate quantity on price, we take the first-order difference

$$\frac{\delta P}{\delta Q} = -1$$

Aggregate quantity has a negative relationship with price and the effect of aggregate quantity of $\frac{2}{3}$% is fully reflected in the price. It is clear then, that a change in variable *a* has consequences for price directly and indirectly. However, the direct effect of *a* on price is bigger than the indirect effect *a* on price via quantity. Thus, overall a positive relationship exists between constant variable *a* and price. Even though, the differentials already show the direction, it is still important to see how the Nash equilibrium is exactly determined by *a*.

Assume the government installs βa , where $0 < \beta < 1$.

$$\pi_i\left(q_i, q_j\right) = (P(q_1, q_2) - k - l\,)q_i - F_i = q_i[\beta a - (q_1 + q_2) - k - l\,] - F_i$$

$$\max\ q_1[\beta\ a - (q_1 + q_2^*) - k - l\,] - F_1$$

$$\frac{\delta \pi_1}{\delta q_1} = -(k + l) + \beta a - q_2 - 2q_1 = 0$$

When looking at the two-player game, Nash equilibrium exists when the firms choose the following quantity

$$q_1^* = \frac{1}{2}\,(\beta a - k - l - q_2^*) \qquad (1)$$

$$q_2^* = \frac{1}{2}\,(\beta a - k - l - q_1^*) \qquad (2)$$

Substituting equation (2) into equation (1) and with symmetry, the optimal quantities are

$$q_1^* = q_2^* = \frac{\beta}{3}a - \frac{1}{3}k - \frac{1}{3}l$$

It is clear that, a decrease in *a* leads to a decrease in produced quantity by both firms which will lead to an increase in the price.

$$Q^* = q_1^* + q_2^* = 2\left(\frac{\beta}{3}a - \frac{1}{3}k - \frac{1}{3}l\right)$$

The price in this equilibrium is then

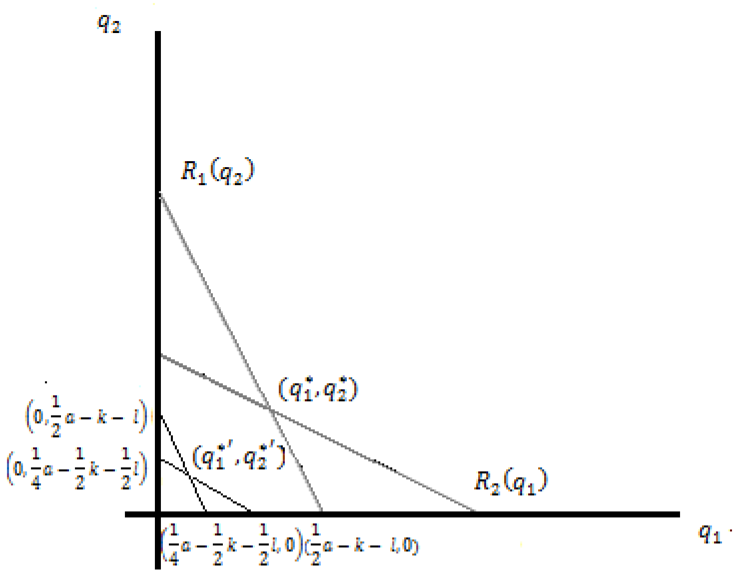
$$\dot{\mathrm{P}}(q_1^*, q_2^*) = \beta a - Q = \frac{\beta}{3}a + \frac{2}{3}k + \frac{2}{3}l$$

The direct effect of a decrease in *a* on price is bigger than the effect via Q, which leads to a lower equilibrium price than before. Thus, decreasing *a* will lead to effective price regulation. Revenue for both firms is

$$\dot{\mathrm{P}}(q_1^*, q_2^*)q_1^* = \dot{\mathrm{P}}(q_1^*, q_2^*)q_2^* = (\frac{\beta}{3}a + \frac{2}{3}k + \frac{2}{3}l)\,(\frac{\beta}{3}a - \frac{1}{3}k - \frac{1}{3}l)$$

$$= \frac{\beta a^2 + \beta ak + l\beta a - 2k^2 - 4kl - 2l^2}{9}$$

Because $0 < \beta < 1$ this revenue is lower than in the unregulated model with homogenous producers, because the price and quantity produced have both decreased. We will refer to price cap regulation as achieved with βa in the following. The following graph shows the effect of a price-cap on the market of $\beta = \frac{1}{2}$



The grey curves represent the old equilibrium in the unregulated model with homogeneous firms without a capital shock and the black curves represent the new equilibrium in the regulated model with homogeneous firms without a capital shock. When a decreases the best –response functions move to the left towards the origin, just like when capital costs increase. The effect of diminishing a to $\frac{1}{2}a$ leads to a decrease of more than 50% in output, since costs do not change because of ceteris paribus. The exact change of the curves is not known because of the unknown variables. However, the ratio's between movements is clear: both curves will move as much and stay symmetrically.

Now, we know the quantities, price and revenue in the price-regulated model. Firstly, we analyse the effect of capital costs on optimal quantities so the first-order difference of quantity to capital costs should be taken

$$\frac{\delta q_1^*}{\delta k} = -\frac{1}{3}$$

It is clear then, that effect of capital costs on optimal quantity is not influenced by price regulation. The negative relationship is still the same as in the unregulated model: quantity decreases by an increase in capital costs. The effect of an increase in capital costs on price in equilibrium is shown by the first-order difference of equilibrium price to capital costs

$$\frac{\delta \dot{\mathrm{P}}(q_1^*, q_2^*)}{\delta k} = \frac{2}{3}$$

Price will increase when capital costs increase, since both firms will produce less in this situation. The effect of an increase in capital costs on revenue in equilibrium is solved by taking the first-order difference of the equilibrium revenue to capital costs

$$\frac{\delta \dot{\mathrm{P}}(q_1^*, q_2^*)q_1^*}{\delta k} = \frac{\delta \dot{\mathrm{P}}(q_1^*, q_2^*)q_2^*}{\delta k} = \frac{\beta a - 4k - 4l}{9}$$

The effect of an increase in capital costs on revenue depends on the unknown variables βa, k and l. The effect of an increase in capital costs on optimal quantity and equilibrium price is not influenced by a price-cap, while the effect of an increase in capital costs on equilibrium revenue is influenced by a price-cap.

The following table summarizes all the results from the model with homogeneous producers mentioned above, when price regulation is referred to replacing a by βa and a capital shock. The effect of the capital shock is shown for both firms and is referred to as the first-order differences to capital costs.

	$P(q_1^*, q_2^*)$	$q_1^* = q_2^*$	$P(q_1^*, q_2^*)\ q_1^* = P(q_1^*, q_2^*)\ q_2^*$
Unregulated	$\frac{a + 2k + 2l}{3}$	$\frac{a - k - l}{3}$	$\frac{a^2 + ak + la - 2k^2 - 4kl - 2l^2}{9}$
Effect capital shock	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{a - 4k - 4l}{9}$
Regulated	$\frac{\beta a + 2k + 2l}{3}$	$\frac{\beta a - k - l}{3}$	$\frac{\beta a^2 + \beta ak + l\beta a - 2k^2 - 4kl - 2l^2}{9}$
Effect capital shock	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{\beta a - 4k - 4l}{9}$

It is clear, that a strict price-cap policy (small β) decreases price and optimal quantity more than a looser price-cap policy. This is important for authorities to keep an eye on, because when the regulation is too strict the price may be too low to fulfil the participation constraint. To analyse if both firms still want to participate on the price-regulated market, we have to fill the equilibrium price in the participation constraint. The participation constraint in the price-regulated model before the capital shock is

$$q_1 = q_2 > \frac{F}{\left(\frac{\beta a + 2k + 2l}{3} - k - l\right)}$$

Since $0 < \beta < 1$, the price will be decreased in the regulated model and this increases the participation constraint. It is clear that in the regulated market the condition under which participation will occur is bigger so firms might leave the market. Because of a capital shock, capital costs and price will increase while optimal quantity will decrease. When capital costs will increase 1%, then price will increase $\frac{2}{3}$%. This means that the participation constraint will be bigger so both firms will have to produce more to break-even. Firms will produce the optimal quantity or not participate at all, so we also fill the optimal quantity in the constraint.

$$\frac{\beta a - k - l}{3} > \frac{F}{\left(\frac{\beta a + 2k + 2l}{3} - k - l\right)}$$

It follows that both firms will participate when the fixed costs are small enough to be covered

$$F < \left(\frac{\beta a - k - l}{3}\right)^2$$

Comparing this constraint to the constraint of the unregulated model, it is clear that firms are more likely not to participate in a price-regulated market than in an unregulated market.

IV. Model for heterogeneous firms

IVa. Unregulated market

In our model, we add an unknown variable ϵ that represents efficiency to give a more realistic view of the influence of efficiency on production. The ratio variable ϵ has a range of $0 < \epsilon < 1$ and the more efficient a firm is compared to the other, the closer the variable is to zero[1]. The cost functions are as follows, given that firm 2 is the efficient firm

$$C_1(q_1) = (k + l)q_1 + F$$

$$C_2(q_2) = (\epsilon k + \epsilon l)q_2 + F$$

The more efficient firm 2 is compared to firm 1, the smaller ϵ and firm 2's costs are. Since the cost functions differ from each other, also the participation constraints differ from each other. Firstly, the participation constraint of firm 1 is

$$q_1 > \frac{F}{(P(q_1, q_2) - k - l)}$$

Secondly, the participation constraint of firm 2 is

$$q_2 > \frac{F}{(P(q_1, q_2) - \epsilon k - \epsilon l)}$$

Firm 1 has a larger participation constraint than firm 2, which means that because of firm's 1 higher costs it might not break-even and leave the market earlier than firm 2. Price is for both firms equal, since it depends on aggregate quantity $P = a - (q_1 + q_2)$. However, since the costs of the firms differ the profit functions also differ

$$\pi_1(q_1, q_2) = q_1[\, a - (q_1 + q_2) - k - l\,] - F$$
$$\pi_2(q_1, q_2) = q_2[\, a - (q_1 + q_2) - \epsilon k - \epsilon l] - F$$

Both firms will maximise their profit function. Firm 1's optimal quantity is the same as the model with the homogenous producers

$$\max\ q_1[\, a - (q_1 + q_2^*) - k - l\,] - F_1$$
$$\frac{\delta \pi_1}{\delta q_1} = -(k + l) + a - q_2^* - 2q_1 = 0$$
$$q_1^* = \frac{1}{2}\,(a - k - l - q_2^*) \qquad (1)$$

Firm 2's optimal quantity is solved by taking the first-order difference from profit to q and equalising this to zero

$$\max\ q_2[\, a - (q_1^* + q_2) - \epsilon k - \epsilon l] - F_2$$
$$\frac{\delta \pi_2}{\delta q_2} = a - q_1^* - 2q_2 - \epsilon k - \epsilon l = 0$$

[1] The appendix shows the example where firm 2 is twice as efficient as firm 1 and ϵ therefore is $\frac{1}{2}$.

$$q_2^* = \frac{1}{2}\ (a - \epsilon k - \epsilon l\ - q_1^*) \qquad (2)$$

Substituting equation (2) in equation (1) leads to

$$q_1^* = \frac{1}{2}\ (a - k - l - \left[\frac{1}{2}(a - \epsilon k - \epsilon l\ - q_1^*)\right])$$

$$= \frac{1}{3}a + \left(\frac{1}{3}\epsilon - \frac{2}{3}\right)\mathrm{k} + \left(\frac{1}{3}\epsilon - \frac{2}{3}\right)\mathrm{l}$$

The more efficient firm 2 becomes, the smaller ϵ is and the smaller firm 1's optimal quantity will be. It is interesting to see that efficiency not only affects the optimal quantities but it also influences the effect of capital costs and labour costs. Substituting equation (1) in equation (2) leads to

$$q_2^* = \frac{1}{2}\left(a - \epsilon k - \epsilon l - \left[\frac{1}{2}(a - k - l\ - q_2^*)\right]\right)$$

$$= \frac{1}{3}a + \left(\frac{1}{3} - \frac{2}{3}\epsilon\right)\mathrm{k} + \left(\frac{1}{3} - \frac{2}{3}\epsilon\right)\mathrm{l}$$

The more efficient firm 2 becomes, the smaller ϵ is and the larger the optimal quantity of firm 2 will be. Graphically, the best response functions of both firms have to intersect. It is optimal for firm 2 to produce more than firm 1 when firm 2 is more efficient ($\epsilon < 1$).

$$R_1(q_2) = \frac{1}{2}\ (a - k - l\ - q_2^*)$$

$$R_2(q_1) = \frac{1}{2}(a - \epsilon k - \epsilon l\ - q_1^*)$$

Before considering the capital shock, we also have to look at the price and revenue in this equilibrium. To calculate price, the aggregate quantity has to be calculated

$$Q^* = \frac{1}{3}a + \left(\frac{1}{3}\epsilon - \frac{2}{3}\right)\mathrm{k} + \left(\frac{1}{3}\epsilon - \frac{2}{3}\right)\mathrm{l} + \frac{1}{3}a + \left(\frac{1}{3} - \frac{2}{3}\epsilon\right)\mathrm{k} + \left(\frac{1}{3} - \frac{2}{3}\epsilon\right)\mathrm{l}$$

$$= \frac{2}{3}a - \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\mathrm{k} - \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\mathrm{l}$$

Remarkable is that the total quantity produced is negatively influenced by capital costs and when efficiency of firm 2 increases, then total quantity decreases. This means that efficiency of one of the firms has a negative influence on aggregate quantity. The price in equilibrium is then

$$P(q_1^*, q_2^*) = \frac{1}{3}a + \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\mathrm{k} + \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\mathrm{l}$$

This leads to the following revenue for firm 1

$$P(q_1^*, q_2^*)q_1^* = \left[\frac{1}{3}a + \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\mathrm{k} + \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\mathrm{l}\right]\left[\frac{1}{3}a + \left(\frac{1}{3}\epsilon - \frac{2}{3}\right)\mathrm{k} + \left(\frac{1}{3}\epsilon - \frac{2}{3}\right)\mathrm{l}\right]$$

Since the firms' optimal quantities are not equal, revenue for firm 2 is

$$P(q_1^*, q_2^*)q_2^* = \left[\frac{1}{3}a + \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\mathrm{k} + \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\mathrm{l}\right]\left[\frac{1}{3}a + \left(\frac{1}{3} - \frac{2}{3}\epsilon\right)\mathrm{k} + \left(\frac{1}{3} - \frac{2}{3}\epsilon\right)\mathrm{l}\right]$$

Firm 2 has higher revenue because it produces more than firm 1 in equilibrium.

To analyse the effect of capital costs on production, quantity should be differentiated to capital costs. Since the firms do not have identical capital costs, the effect of a capital shock on firm 1's quantity differs from the effect on firm 2's quantity. The first-order difference from the optimal quantity of firm 1 to capital costs is

$$\frac{\delta q_1^*}{\delta k} = \frac{1}{3}\epsilon - \frac{2}{3}$$

Since $0 < \epsilon < 1$, $\frac{1}{3}\epsilon$ will always be smaller than $\frac{2}{3}$ thus capital costs will always have a negative effect on the optimal quantity of firm 1. Since an inefficient firm needs more capital and has higher capital costs, a capital shock will have a bigger (negative) impact on firm 1 than on firm 2. The first-order difference from the optimal quantity of firm 2 to capital costs is

$$\frac{\delta q_2^*}{\delta k} = \frac{1}{3} - \frac{2}{3}\epsilon$$

It is clear that the effect of an increase in capital costs on the optimal quantity depends on the efficiency of firm 2. The influence of an increase in capital costs is positive if $\frac{1}{3} - \frac{2}{3}\epsilon > 0$, thus $\epsilon < \frac{1}{2}$. The influence is negative if $\epsilon > \frac{1}{2}$ and neutral if $\epsilon = \frac{1}{2}$. Notice that the more efficient firm 2 is, the closer ϵ is to zero and the bigger the difference between both firm's costs. This means that when firm 2 becomes more efficient, then ϵ becomes smaller and the negative influence of an increase in capital costs decreases. At one point, an increase in capital costs even starts to have a positive influence on the optimal quantity of firm 2. So when a big difference in efficiency between the two firms exists ($\epsilon < \frac{1}{2}$), a capital crunch will induce the second, the more efficient, firm to produce more. The effect of an increase in capital costs on price in equilibrium is shown by the first-order difference of equilibrium price to capital costs

$$\frac{\delta P(q_1^*, q_2^*)}{\delta k} = \frac{1}{3}\epsilon + \frac{1}{3}$$

The effect of an increase in capital costs on price is always positive but the effect will be bigger when the difference between firm 1 and firm 2's efficiency (ϵ big).

$$\frac{\delta P(q_1^*, q_2^*)q_1^*}{\delta k} = \left[\frac{1}{3}a + 4k + 2l\right]\left[\left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\left(\frac{1}{3}\epsilon - \frac{2}{3}\right)\right]$$

Now it is clear that, the effect of an increase in capital costs on the equilibrium quantities, price and revenue depend on the efficiency of firm 2. However, the effect on revenue also depends on the variables a, k and l. The effect of an increase in capital costs on firm's 1 revenue will always be negative since $0 < \epsilon < 1$. The more efficient firm 2 is, the smaller ϵ is and the more firm 1's revenue will decrease when a capital shortfall arises.

$$\frac{\delta P(q_1^*, q_2^*)q_2^*}{\delta k} = \left[\frac{1}{3}a + 4k + 2l\right]\left[\left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\left(\frac{1}{3} - \frac{2}{3}\epsilon\right)\right]$$

The effect of an increase in capital costs on the revenue of firm 2 is positive when $\epsilon < \frac{1}{2}$, neutral when $\epsilon = \frac{1}{2}$ and negative when $\epsilon > \frac{1}{2}$. This means that if firm 2 is really efficient ($\epsilon < \frac{1}{2}$), its revenue will increase when a capital shortfall arises. This because firm 2 will produce more and price will increase because of an increase in capital costs.

To analyse if both firms still want to participate on the market with price-regulation, we have to look at the effect of the increase in capital costs on the participation constraints. Now, we fill in the price in the participation constraint to analyse under which condition participation of both firms will occur. Before the capital shock, the participation constraint in the unregulated model of firm 1 is

$$q_1 > \frac{F}{\left(\frac{1}{3}a + \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)k + \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)l - k - l\right)}$$

$$\frac{1}{3}a + \left(\frac{1}{3}\epsilon - \frac{2}{3}\right)k + \left(\frac{1}{3}\epsilon - \frac{2}{3}\right)l > \frac{F}{\left(\frac{1}{3}a + \left(\frac{1}{3}\epsilon - \frac{2}{3}\right)k + \left(\frac{1}{3}\epsilon - \frac{2}{3}\right)l\right)}$$

$$F < \left[\frac{1}{3}a + \left(\frac{1}{3}\epsilon - \frac{2}{3}\right)k + \left(\frac{1}{3}\epsilon - \frac{2}{3}\right)l\right]^2$$

When a capital shock arises, capital costs will increase but the effect on the participation constraint depends on the efficiency of firm 2. The more efficient firm 2 is, the smaller ϵ is and the smaller firm 1's optimal quantity. In the case of a really efficient firm 2, it will be harder for firm 1 to cover its fixed costs and it might not participate on the market. Before the capital shock, the participation constraint in the unregulated model of firm 2 is

$$q_2 > \frac{F}{\left(\frac{1}{3}a + \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)k + \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)l \; - \; \epsilon k - \epsilon l\right)}$$

$$\frac{1}{3}a + \left(\frac{1}{3} - \frac{2}{3}\epsilon\right)k + \left(\frac{1}{3} - \frac{2}{3}\epsilon\right)l \; > \; \frac{F}{\left(\frac{1}{3}a + \left(\frac{1}{3} - \frac{2}{3}\epsilon\right)k + \left(\frac{1}{3} - \frac{2}{3}\epsilon\right)l\right)}$$

$$F < \left[\frac{1}{3}a + \left(\frac{1}{3} - \frac{2}{3}\epsilon\right)k + \left(\frac{1}{3} - \frac{2}{3}\epsilon\right)l\right]^2$$

When a capital shock arises, capital costs will increase but the effect on the participation constraint depends on the efficiency of firm 2. The more efficient firm 2 is, the smaller ϵ is and the bigger firm 2's optimal quantity. In this case, it would be easier for firm 2 to satisfy its participation constraint.

IVb. Regulated market

Authorities should again keep in mind the participation constraints of both firms to see whether the regulated price is high enough for producers to stay in the market. Firstly, the participation constraint of firm 1 is

$$q_1 > \frac{F}{(P(q_1, q_2) - \; k - l)}$$

$$\dot{P}(q_1, q_2) > \frac{F}{q_1} + \; k + l$$

Secondly, the participation constraint of firm 2 is

$$q_2 > \frac{F}{(P(q_1, q_2) - \epsilon k - \; \epsilon l)}$$

$$\dot{P}(q_1, q_2) > \frac{F}{q_2} + \; \epsilon k + \; \epsilon l$$

Effective price regulation is when the price cap is higher than the equilibrium price. To keep the producers in the market, the price cap should be higher than the participation constraints. However, it is also possible than only firm 2's participation constraint is satisfied. For both firms to stay in the market, the price-cap should be in between

$$\frac{1}{3}a + \left(\frac{2}{3}\epsilon + \frac{2}{3}\right)\mathrm{k} + \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\mathrm{l} > \dot{\mathrm{P}}(q_1, q_2) > \frac{F}{q_1} + \ k + l$$

We discuss the general model with the regulation variable β and efficiency variable ϵ.The profit functions in the concrete model are as follows

$$\pi_1(q_1, q_2) = q_1[\ \beta a - (q_1 + q_2)\ -\ k - l\ ] -\ F$$
$$\pi_2(q_1, q_2) = q_2[\beta a - (q_1 + q_2) -\ \epsilon k -\ \epsilon l] -\ F$$

Again profits should be maximised to find firm 1's optimal quantity

$$\max\ q_1[\beta a - (q_1 + q_2^*)\ -\ k - l\ ] -\ F$$
$$\frac{\delta\pi_1}{\delta q_1} = \ -(k +\ l) +\ \beta a - q_2 - 2q_1 =\ 0$$
$$q_1^* =\ \frac{1}{2}\ (\beta a - k -\ l\ - q_2^*) \qquad (1)$$

Firm 2's optimal quantity is then

$$\max\ q_2[\beta a - (q_1 + q_2) -\ \epsilon k -\ \epsilon l] -\ F_2$$
$$\frac{\delta\pi_2}{\delta q_2} = \beta a - q_1 - 2q_2 -\ \epsilon k -\ \epsilon l\ =\ 0$$
$$q_2^* =\ \frac{1}{2}\ (\beta a -\ \epsilon k -\ \epsilon l\ - q_1^*) \qquad (2)$$

In equilibrium, both firms will produce their optimal quantity which is higher for firm 2 than for firm 1.

Substituting equation (2) into equation (1) leads to

$$q_1^* =\ \frac{1}{2}\left(\beta a - k -\ l - \left[\frac{1}{2}(\beta a -\ \epsilon k -\ \epsilon l\ - q_1^*)\right]\right)$$
$$= \frac{\beta}{3}a + \left(\frac{1}{3}\epsilon - \frac{2}{3}\right)\mathrm{k} + \left(\frac{1}{3}\epsilon - \frac{2}{3}\right)\mathrm{l}$$

Substituting equation (1) into equation (2) leads to

$$q_2^* =\ \frac{1}{2}\left(\beta a -\ \epsilon k -\ \epsilon l - \left[\frac{1}{2}(\beta a - k -\ l\ - q_2^*)\right]\right)$$
$$= \frac{\beta}{3}a + \left(\frac{1}{3} - \frac{2}{3}\epsilon\right)\mathrm{k} + \left(\frac{1}{3} - \frac{2}{3}\epsilon\right)\mathrm{l}$$

The efficient firm will produce more than the inefficient firm because the costs are lower for firm 1 and thus the contribution margin too. To calculate price the aggregate quantity has to be calculated first

$$Q^* = \frac{2\beta}{3}a + \left(\frac{1}{3}\epsilon - \frac{2}{3}\right)\mathrm{k} + \left(\frac{1}{3}\epsilon - \frac{2}{3}\right)\mathrm{l} + \left(\frac{1}{3} - \frac{2}{3}\epsilon\right)\mathrm{k} + \left(\frac{1}{3} - \frac{2}{3}\epsilon\right)\mathrm{l}$$
$$= \frac{2\beta}{3}a \ - \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\mathrm{k} - \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\mathrm{l}$$

The price in equilibrium is then

$$\dot{\mathrm{P}}(q_1^*, q_2^*) = \ \frac{\beta}{3}a + \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\mathrm{k} + \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\mathrm{l}$$

It is clear, that the price in this regulated model is lower than the unregulated model with heterogeneous producers. This means that also in the heterogeneous model the price can be reduced by decreasing a. Aggregate quantity has reduced which increases the price, but the negative direct effect of a on price is bigger so that price is lower in this model. By using this price and the optimal quantity of firm 1, the revenue of firm 1 is

$$\dot{\mathrm{P}}(q_1^*, q_2^*)q_1^* = \left[\frac{\beta}{3}a + \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\mathrm{k} + \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\mathrm{l}\right]\left[\frac{\beta}{3}a + \left(\frac{1}{3}\epsilon - \frac{2}{3}\right)\mathrm{k} + \left(\frac{1}{3}\epsilon - \frac{2}{3}\right)\mathrm{l}\right]$$

This revenue is smaller than in the unregulated model, since both price and optimal quantity have decreased. Solving revenue of firm 2 with optimal quantity of firm 2 and price leads to

$$\dot{\mathrm{P}}(q_1^*, q_2^*)q_2^* = \left[\frac{\beta}{3}a + \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\mathrm{k} + \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\mathrm{l}\right]\left[\frac{\beta}{3}a + \left(\frac{1}{3} - \frac{2}{3}\epsilon\right)\mathrm{k} + \left(\frac{1}{3} - \frac{2}{3}\epsilon\right)\mathrm{l}\right]$$

Firm 2 still has higher revenue because it produces more than firm 1 in equilibrium, but the revenue of firm 2 has also decreased compared to the unregulated model. To analyse the effect of capital costs on production, optimal quantity should be differentiated to capital costs. This results in

$$\frac{\delta q_1^*}{\delta k} = \frac{1}{3}\epsilon - \frac{2}{3}$$
$$\frac{\delta q_2^*}{\delta k} = \frac{1}{3} - \frac{2}{3}\epsilon$$

It is clear then, that effect of capital costs on optimal quantity is not influenced by price regulation. The relationship is still the same as in the unregulated model with heterogeneous producers: optimal quantity of firm 1 will decrease by an increase in capital costs, while the optimal quantity of firm 2 stays the same. The effect of an increase in capital costs on price is given by the first-order difference from price to capital costs

$$\frac{\delta \dot{\mathrm{P}}(q_1^*, q_2^*)}{\delta k} = \frac{1}{3}\epsilon + \frac{1}{3}$$

This first-order difference, the effect of a capital shock on price is still the same as in the unregulated model before. It is always positive but the effect will be bigger when the difference between firm 1 and firm 2's efficiency (large ϵ). The effect of an increase in capital costs on the revenue of firm 1 is as follows

$$\frac{\delta \dot{P}(q_1^*, q_2^*)q_1^*}{\delta k} = \left[\frac{\beta}{3}a + 4k + 2l\right]\left[\left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\left(\frac{1}{3}\epsilon - \frac{2}{3}\right)\right]$$

The effect of an increase in capital costs on revenue depends on the variables $\beta,\ a, k,$ and l, so the strictness of the policy depends how large the effect of capital is on revenue. The stricter the price-regulation, the smaller β is and the smaller the (negative) effect of an increase in capital costs on the revenue of firm 1. The effect of an increase in capital costs on firm's 1 revenue will always be negative since $0 < \epsilon < 1$. The more efficient firm 2 is, the smaller ϵ is and the more firm 1's revenue will decrease when a capital shortfall arises. The effect of an increase in capital costs on the revenue of firm 2 is as follows

$$\frac{\delta \dot{P}(q_1^*, q_2^*)q_2^*}{\delta k} = \left[\frac{\beta}{3}a + 4k + 2l\right]\left[\left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\left(\frac{1}{3} - \frac{2}{3}\epsilon\right)\right]$$

Also this effect first-order difference depends on the price-regulation variable β. The effect of an increase in capital costs on the revenue of firm 2 is positive when $\epsilon < \frac{1}{2}$, neutral when $\epsilon = \frac{1}{2}$ and negative when $\epsilon > \frac{1}{2}$. When the effect is already positive because of ϵ, then a looser price-policy (large β) will be preferred instead of a strict price-policy, because it decreases the positive effect of a capital shortfall on firm 2's revenue less. While a strict price-policy (small β) is better for firm's 2 revenue when the effect of a capital shortfall was already negative.

We analyse the participation constraints now to determine the condition under which firms to participate. Both firms have a different constraint and optimal quantity. The participation constraint in the price-regulated model of firm 1 is

$$q_1 > \frac{F}{\left(\frac{\beta}{3}a + \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)k + \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)l - k - l\right)}$$

$$\frac{\beta}{3}a + \left(\frac{1}{3}\epsilon - \frac{2}{3}\right)k + \left(\frac{1}{3}\epsilon - \frac{2}{3}\right)l > \frac{F}{\left(\frac{\beta}{3}a + \left(\frac{1}{3}\epsilon - \frac{2}{3}\right)k + \left(\frac{1}{3}\epsilon - \frac{2}{3}\right)l\right)}$$

The participation constraint is similar to the constraint in the unregulated model. However, the participation constraints in the regulated model are higher $(0 < \beta < 1)$ so firms will less likely participate than in the unregulated model. When a capital shock arises, capital costs will increase but the effect on the participation constraint depends on the efficiency of firm 2. The more efficient firm 2 is, the smaller ϵ is and the smaller firm 1's optimal quantity. In the case of a really efficient firm 2, it will be harder for firm 1 to cover its fixed costs and it might not participate on the market.

$$F < \left[\frac{\beta}{3}a + \left(\frac{1}{3}\epsilon - \frac{2}{3}\right)k + \left(\frac{1}{3}\epsilon - \frac{2}{3}\right)l\ \right]^2$$

Before the capital shock, the participation constraint in the price-regulated model of firm 1 is

$$q_2 > \frac{F}{\left(\frac{\beta}{3}a + \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)k + \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)l\ -\ \epsilon k - \epsilon l\ \right)}$$

$$\frac{\beta}{3}a + \left(\frac{1}{3} - \frac{2}{3}\epsilon\right)k + \left(\frac{1}{3} - \frac{2}{3}\epsilon\right)l\ > \frac{F}{\left(\frac{\beta}{3}a + \left(\frac{1}{3} - \frac{2}{3}\epsilon\right)k + \left(\frac{1}{3} - \frac{2}{3}\epsilon\right)l\ \right)}$$

The participation constraint is similar to the constraint in the unregulated model. However, the participation constraints in the regulated model are higher ($0 < \beta < 1$) so firms will less likely participate than in the unregulated model. When a capital shock arises, capital costs will increase but the effect on the constraint depends on the efficiency of firm 2. The more efficient firm 2 is, the smaller ϵ is and the bigger firm 2's optimal quantity. In this case, it would be easier for firm 2 to satisfy its participation constraint. For firm 2 to participate, fixed costs need to be smaller than the following

$$F < \left[\frac{\beta}{3}a + \left(\frac{1}{3} - \frac{2}{3}\epsilon\right)k + \left(\frac{1}{3} - \frac{2}{3}\epsilon\right)l\right]^2$$

The next tables summarize the results from the model with heterogeneous producers. Firm 2 is efficient relative to firm 1 and its costs depend on the efficiency factor ϵ. The price-cap consists of the regulation factor β that diminishes the constant variable a.

	$P(q_1^*, q_2^*)$	q_1^*	q_2^*	$P(q_1^*, q_2^*)q_1$	$P(q_1^*, q_2^*)q_2$
Unregulated	$\left[\frac{1}{3}a + \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\mathrm{k} + \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\mathrm{l}\right]$	$\left[\frac{1}{3}a + \left(\frac{1}{3}\epsilon - \frac{2}{3}\right)\mathrm{k} + \left(\frac{1}{3}\epsilon - \frac{2}{3}\right)\mathrm{l}\right]$	$\left[\frac{1}{3}a + \left(\frac{1}{3} - \frac{2}{3}\epsilon\right)\mathrm{k} + \left(\frac{1}{3} - \frac{2}{3}\epsilon\right)\mathrm{l}\right]$	$\left[\frac{1}{3}a + \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\mathrm{k} + \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\mathrm{l}\right]\left[\frac{1}{3}a + \left(\frac{1}{3}\epsilon - \frac{2}{3}\right)\mathrm{k} + \left(\frac{1}{3}\epsilon - \frac{2}{3}\right)\mathrm{l}\right]$	$\left[\frac{1}{3}a + \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\mathrm{k} + \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\mathrm{l}\right]\left[\frac{1}{3}a + \left(\frac{1}{3} - \frac{2}{3}\epsilon\right)\mathrm{k} + \left(\frac{1}{3} - \frac{2}{3}\epsilon\right)\mathrm{l}\right]$
Effect of capital	$\frac{1}{3}\epsilon + \frac{1}{3}$	$\frac{1}{3}\epsilon - \frac{2}{3}$	$\frac{1}{3} - \frac{2}{3}\epsilon$	$\left[\frac{1}{3}a + 4k + 2l\right]\left[\left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\left(\frac{1}{3}\epsilon - \frac{2}{3}\right)\right]$	$\left[\frac{1}{3}a + 4k + 2l\right]\left[\left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\left(\frac{1}{3} - \frac{2}{3}\epsilon\right)\right]$
Regulated	$\left[\frac{\beta}{3}a + \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\mathrm{k} + \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\mathrm{l}\right]$	$\left[\frac{\beta}{3}a + \left(\frac{1}{3}\epsilon - \frac{2}{3}\right)\mathrm{k} + \left(\frac{1}{3}\epsilon - \frac{2}{3}\right)\mathrm{l}\right]$	$\left[\frac{\beta}{3}a + \left(\frac{1}{3} - \frac{2}{3}\epsilon\right)\mathrm{k} + \left(\frac{1}{3} - \frac{2}{3}\epsilon\right)\mathrm{l}\right]$	$\left[\frac{\beta}{3}a + \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\mathrm{k} + \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\mathrm{l}\right]\left[\frac{\beta}{3}a + \left(\frac{1}{3}\epsilon - \frac{2}{3}\right)\mathrm{k} + \left(\frac{1}{3}\epsilon - \frac{2}{3}\right)\mathrm{l}\right]$	$\left[\frac{\beta}{3}a + \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\mathrm{k} + \left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\mathrm{l}\right]\left[\frac{\beta}{3}a + \left(\frac{1}{3} - \frac{2}{3}\epsilon\right)\mathrm{k} + \left(\frac{1}{3} - \frac{2}{3}\epsilon\right)\mathrm{l}\right]$
Effect of capital	$\frac{1}{3}\epsilon + \frac{1}{3}$	$\frac{1}{3}\epsilon - \frac{2}{3}$	$\frac{1}{3} - \frac{2}{3}\epsilon$	$\left[\frac{\beta}{3}a + 4k + 2l\right]\left[\left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\left(\frac{1}{3}\epsilon - \frac{2}{3}\right)\right]$	$\left[\frac{\beta}{3}a + 4k + 2l\right]\left[\left(\frac{1}{3}\epsilon + \frac{1}{3}\right)\left(\frac{1}{3} - \frac{2}{3}\epsilon\right)\right]$

V. Conclusion

This paper has explained the effect of a capital shortfall on regulated and unregulated markets. Firstly, in the unregulated market with two homogeneous producers, both firms produce the same quantity also when a capital shock arises. A capital shortfall will increase capital costs and decrease the optimal quantity of both firms. Aggregate quantity will decrease and price will increase as a consequence. When price is regulated, firms also cut down their production. Lower aggregate quantity will lead to an increase in price and this is the indirect effect of regulation on price. However, the direct effect of regulation is bigger thus after regulation (diminishing a) price is lower than before. Interesting is that, firms do not become more competitive because of this price-regulation. Firms produce less than before the price-regulation and thus their optimal quantity in the regulated model becomes more like the monopoly quantity in the unregulated model. A capital shock also decreases the optimal quantities of the firms in the regulated market. In this model it is possible that firms will not satisfy their participation constraint since price is lower than in the unregulated model and costs are higher because of a capital shock. In this case authorities might worsen the welfare of customers when firms will not produce anymore at all.

Secondly, in the unregulated market with two heterogeneous firms, the effect of capital costs on quantity is not the same as in the homogeneous model. Both firms know each other's costs due to complete information, thus whether they are efficient or inefficient. Now, the best response functions are not symmetrical and thus the optimal quantities are not equal. Both firms know the other firm's best response functions, so the most efficient firm will produce more than the inefficient firm. This because the efficient firm has a higher contribution margin. A capital shortfall will increase both firms' cost functions, but since the inefficient firm has more (capital) costs, its cost function will increase relatively more. The share of the efficient firm in the aggregate quantity will become bigger than before the shock. Besides, this increase in the share of the total quantity, the quantity of the efficient firm might actually increase in absolute terms. In this model, a really interesting relationship between capital costs and optimal quantity exits because the relationship depends on how efficient a firm is compared to the other firm. The more efficient a firm is, the smaller the negative effect is of capital costs and at one point the effect even becomes positive. So when a firm is twice as efficient as the other, it does not react on changes in capital costs but when the firm is more than twice as efficient than it will even produce more than before the capital shock. This model reacts in a similar way on price regulation: optimal quantities decrease. However, it is not known whether the market share of the efficient firm increases or decreases since this depends on the unknown variables. In the regulated model, the capital costs decrease the optimal quantity of the inefficient firm and the effect on the efficient firm depends on its efficiency. The inefficient firm might leave the market since it has a competitive disadvantage, has a regulated price and is faced with a capital shock. Again, the efficient firm might even increase its output due to a capital shock.

This study has shown that, price-regulation does not worsen the effect of an increase in capital costs on output (optimal quantities). It is true that both price regulation and a shortfall in credit decrease the aggregate quantity, so the combination of both makes the aggregate quantity small. In the heterogeneous model, price regulation and a capital shock might lead to a more efficient market since it becomes harder for an inefficient firm to break-even. When the inefficient firm leaves, the efficient firm will be in a monopoly position but its price will still be constrained due to the regulation. However, price-regulation does worsen the effect of an increase in capital costs on revenue.

This work contributes to existing knowledge of price regulation and capital shortfall by providing an extended Cournot model. However, the findings are limited by the use of assumptions and unknown variables and have implications for future practice. In practice, authorities do not have complete information so they do not know exactly what will happen when they decrease price. Regarding further research, it would be interesting to assess the effects of a capital shortfall on markets with heterogeneous products and to compare it to these results. Besides, empirical research would also be helpful in determining the practical use of the theoretical models in this paper.

References

- Averch, H. and L. Johnson. 1962. “Behavior of the Firm under Regulatory Constraint,’’ *American Economic Review* 52: 1053-69

- Bjørndal, M. and K. Jørnsten. 2002. “Revenue Cap Regulation in a Deregulated Electricity Market – Effects on a Grid Company,’’ *Norwegian School of Economics and Business Administration.*

- Braeutigam, R.R. and J.C. Panzar. 1993. ‘’Effects of the Change from Rate-of-Return to Price-Cap Regulation,’’ *American Economic Review* 83(2): 191-198

- Cournot, A.A. 1838. Recherches sur les principes mathématiques de la théorie des richesses. Hachette, Paris.

- Crew, M.A. and P.R. Kleindorfer. 1996. "Price caps and revenue caps: incentive and disincentives for efficiency**,**’’ *Springer*

- Dutta, P.K. 1999. Strategies and Games: Theory and Practice. Massachusetts Institute of Techonology.

- Jamasb, T. and M. Pollitt. 2001. “Benchmarking and regulation: international electricity experience,’’ *Elsevier* 9:107-130

- Mougeot, M. and F. Naegelen. 2005. “Hospital price regulation and expenditure cap policy,’’ *Journal of Health Economics* 24: 55–72

- Sheshinski, E. 1976. “Price, Quality and Quantity Regulation in Monopoly Situations,” *Economica* 43(170) :127-137’’

- Spann, R.M. 1974. ‘’ Rate of return regulation and efficiency in production: an empirical test of the Averch-Johnson Thesis,’’ *The Bell Journal of Economics and Management Science* 5(1): 38-52

Appendix

This is an example in which we assume firm 2 to be twice as efficient as firm 1. This results in higher marginal costs of firm 1 compared to firm 2. The fixed costs are still identical. The cost functions are as follows

$$C_1(q_1) = (k + l)q_1 + F$$

$$C_2(q_2) = \left(\frac{1}{2}k + \frac{1}{2}l\right)q_2 + F$$

Important is also that the participation constraints of the firms differ from each other. In this unregulated model, we again assume that both firms satisfy their participation constraint. However, these constraints can be violated when capital costs increase or regulation is added. Firstly, the participation constraint of firm 1 is

$$q_1 > \frac{F}{(P(q_1, q_2) - k - l)}$$

$$P(q_1, q_2) > \frac{F}{q_1} + k + l$$

Secondly, the participation constraint of firm 2 is

$$q_2 > \frac{F}{\left(P(q_1, q_2) - \frac{1}{2}k - \frac{1}{2}l\right)}$$

$$P(q_1, q_2) > \frac{F}{q_2} + \frac{1}{2}k + \frac{1}{2}l$$

It is clear then, that the firm 1's break-even volume is higher than the break-even volume of firm 2. Price is for both firms equal, since it depends on aggregate quantity $P = a - (q_1 + q_2)$. However since the costs of the firms differ, the profit functions also differ

$$\pi_1(q_1, q_2) = q_1[\, a - (q_1 + q_2) - k - l\,] - F$$

$$\pi_2(q_1, q_2) = q_2[\, a - (q_1 + q_2) - \frac{1}{2}k - \frac{1}{2}l] - F$$

Both firms will maximise their profit function. Firm 1's optimal quantity is the same as the model with the homogenous producers

$$\max\ q_1[\, a - (q_1 + q_2^*) - k - l\,] - F_1$$

$$\frac{\delta\pi_1}{\delta q_1} = -(k + l) + a - q_2 - 2q_1 = 0$$

$$q_1^* = \frac{1}{2}(a - k - l - q_2^*) \qquad (1)$$

Firm 2's optimal quantity is solved by taking the first-order difference from profit to q and equalising this to zero

$$\max\ q_2[a - (q_1 + q_2) - \frac{1}{2}k - \frac{1}{2}l] - F_2$$

$$\frac{\delta\pi_2}{\delta q_2} = a - q_1 - 2q_2 - \frac{1}{2}k - \frac{1}{2}l = 0$$

$$q_2^* = \frac{1}{2}(a - \frac{1}{2}k - \frac{1}{2}l - q_1^*) \qquad (2)$$

It is possible that both firms produce the same quantity, but then firm 2 has a higher profit. In equilibrium, both firms will produce their optimal quantity. We use substitution to solve the optimal quantities. Substituting equation (2) in (1) leads to

$$q_1^* = \frac{1}{2}(a - k - l - \left[\frac{1}{2}\left(a - \frac{1}{2}k - \frac{1}{2}l - q_1^*\right)\right])$$

$$q_1^* = \frac{1}{3}a - \frac{1}{2}k - \frac{1}{2}l$$

Substituting q_1^* in $q_2^* = \frac{1}{2}(a - \frac{1}{2}k - \frac{1}{2}l - q_1^*)$ leads to

$$q_2^* = \frac{1}{2}\left(a - \frac{1}{2}k - \frac{1}{2}l - \left[\frac{1}{2}(a - k - l - q_2^*)\right]\right)$$

$$q_2^* = \frac{1}{3}a$$

The efficient firm (firm 2) will produce more than the inefficient firm because the costs are lower. Firm 1 will produce less than firm 2 because it has higher costs, and thus a lower contribution margin. However, it is remarkable that firm 2's optimal quantity is not influenced by capital costs. When looking clearly at the formulas, it turns out that it is a coincidence that firm 2's optimal quantity is not influenced by k. It is clear that when the costs of firm 2 were three times as small, the optimal quantity is influenced by capital costs. To calculate price the aggregate quantity has to be calculated

$$Q^* = \frac{1}{3}a - \frac{1}{2}(k+l) + \frac{1}{3}a = \frac{2}{3}a - \frac{1}{2}k - \frac{1}{2}l$$

The price in equilibrium is then

$$P(q_1^*, q_2^*) = a - Q^* = a - \left(\frac{2}{3}a - \frac{1}{2}k - \frac{1}{2}l\right) = \frac{1}{3}a + \frac{1}{2}k + \frac{1}{2}l$$

This leads to the following revenue for firm 1

$$P(q_1^*, q_2^*)q_1 = \left(\frac{1}{3}a + \frac{1}{2}k + \frac{1}{2}l\right)\left(\frac{1}{3}a - \frac{1}{2}k - \frac{1}{2}l\right)$$

Since the firms' optimal quantities are not equal, revenue for firm 2 is

$$P(q_1^*, q_2^*)q_2 = \left(\frac{1}{3}a + \frac{1}{2}k + \frac{1}{2}l\right)\frac{1}{3}a$$

Firm 2's revenue is bigger than firm 1's revenue because it produces more than firm 1 in equilibrium.

To analyse the effect of capital costs on production, quantity should be differentiated to capital costs. Since the firms do not have identical capital costs, the effect of a capital shock on firm 1's quantity differs from the effect on firm 2's quantity.

$$\frac{\delta q_1^*}{\delta k} = -\frac{1}{2}$$

$$\frac{\delta q_2^*}{\delta k} = 0$$

Since the inefficient firm needs more capital and has higher capital costs, a capital shock will have a bigger impact on firm 1 than on firm 2. A capital shock replaces *k* into *k'* for which $k' > k$. More concrete would be when capital costs rise from *k* to *2k*. What would happen to the equilibrium price and quantity? First the profit functions of both firms when *k* is replaced by *2k*

$$\pi_1(q_1, q_2) = q_1[\, a - (q_1 + q_2) - 2k - l\,] - F$$

$$\pi_2(q_1, q_2) = q_2[\, a - (q_1 + q_2) - k - \frac{1}{2}l] - F$$

Both firms will maximise their profit function. Firm 1's optimal quantity is the same as the model with the homogenous producers

$$\max\ q_1[\, a - (q_1 + q_2^*) - 2k - l\,] - F_1$$

$$\frac{\delta \pi_1}{\delta q_1} = a - 2k - l - q_2 - 2q_1 = 0$$

$$q_1^* = \frac{1}{2}(a - 2k - l - q_2^*) \qquad (1)$$

Firm 2's optimal quantity is solved by taking the first-order difference from profit to q and equalising this to zero

$$\max\ q_2[\,a-(q_1+q_2)-\ k-\ \frac{1}{2}l]-\ F_2$$

$$\frac{\delta\pi_2}{\delta q_2}=\ a-q_1-2q_2-\ k-\ \frac{1}{2}l\ =\ 0$$

$$q_2^*=\ \frac{1}{2}\left(a-\ k-\frac{1}{2}l\ -q_1^*\right)\qquad(2)$$

We use substitution to solve the optimal quantities. Substituting equation (2) in (1) leads to

$$q_1^*=\ \frac{1}{2}\ (a-2k-\ l-\left[\frac{1}{2}\left(a-\ k-\ \frac{1}{2}l\ -q_1^*\right)\right])$$

$$q_1^*=\frac{1}{3}a-k-\frac{1}{2}l$$

Substituting q_1^* in $q_2^*=\ \frac{1}{2}\ (a-\ k-\ \frac{1}{2}l\ -q_1^*)$ leads to

$$q_2^*=\frac{1}{2}\left(a-\ k-\ \frac{1}{2}l-\left[\frac{1}{2}(a-2k-\ l\ -q_2^*)\right]\right)$$

$$q_2^*=\frac{1}{3}a$$

As predicted by the first-order conditions, the optimal quantity of firm 1 decreases while the optimal quantity of firm 2 stays the same. To calculate price, the aggregate quantity has to be calculated

$$Q^*=\ \frac{1}{3}a-k-\frac{1}{2}l+\ \frac{1}{3}a=\ \frac{2}{3}a-k-\frac{1}{2}l$$

The price in equilibrium is then

$$P(q_1^*,q_2^*)=a-Q^*=a-\left(\frac{2}{3}a-k-\frac{1}{2}l\right)=\frac{1}{3}a+k+\frac{1}{2}l$$

This leads to the following revenue for firm 1

$$P(q_1^*,q_2^*)q_1=\left(\frac{1}{3}a+k+\frac{1}{2}l\right)(\frac{1}{3}a-k-\frac{1}{2}l)$$

Again price has increased due to an increase in capital costs and quantity has decreased. The firms' optimal quantities are not equal, revenue for firm 2 is

$$P(q_1^*,q_2^*)q_2=\left(\frac{1}{3}a+k+\frac{1}{2}l\right)\frac{1}{3}a$$