

# Realized GARCH Option Pricing using the Filtered Historical Simulation Approach

Veditam Bishoen (319230)

Erasmus Universiteit Rotterdam (EUR)

Friday, August 09, 2013

Supervisor: Michel van der Wel, Ph.D.

Co-reader: Wing Wah Tham, Ph.D.



## **ABSTRACT**

In this thesis I combine the filtered historical simulation (FHS) method of Barone-Adesi, Engle and Mancini (2008) with the Realized GARCH model of Hansen, Huang and Shek (2011). I use the RGARCH model instead of the GJR-GARCH model, since realized data contains information on the volatility and contributes to more accurate volatility estimates. Using an empirical analysis, I apply the FHS method on in-the-money and out-of-the-money option data of the S&P 500, for the period January 2<sup>nd</sup> 2002 till December 30<sup>th</sup> 2011. Based on statistical criteria such as the root mean squared error and the mean absolute error, the FHS method in combination with the RGARCH model improves option pricing. The latter method also produces accurate Value-at-Risk forecasts.

*Keywords:* Filtered Historical Simulation, Realized GARCH, Option Pricing, State-Price-Density.

## ACKNOWLEDGMENTS

To obtain the degree of Master in Econometrics and Management Science, specialization Quantitative Finance, I have been working on this master thesis during the past year. While, I take full responsibility for the contents of this thesis, I would like to take this opportunity to thank a number of people whose help was invaluable to me.

First of all, I owe particular thanks to Michel van der Wel<sup>1</sup> for not only supervising my thesis, but also for his inspiration, his valuable advises and guidance through the whole process of this thesis. Furthermore, I would like to thank the co-reader Wing Wah Tham<sup>2</sup>, for his approval and a judgment on this thesis. Also, I am grateful to my family, especially my big brother, for their unceasing encouragement, moral support, comments and advises. Finally, I would like to express my gratitude to one and all who, directly or indirectly, have contributed to this work.

---

<sup>1</sup> Assistant Professor at the Erasmus University Rotterdam (EUR)

<sup>2</sup> Assistant Professor of Financial Econometrics, Econometric Institute, Erasmus School of Economics

# CONTENTS

<b>1</b>	<b>Introduction</b> .....	<b>1</b>
<b>2</b>	<b>Literature</b> .....	<b>4</b>
<b>3</b>	<b>Methods</b> .....	<b>7</b>
3.1	<i>GJR-GARCH with Filtered Historical Simulation</i> .....	7
3.2	<i>Realized GARCH</i> .....	9
3.3	<i>Realized GARCH with Filtered Historical Simulation</i> .....	10
3.4	<i>Evaluation Methods</i> .....	12
3.4.1	Statistical Criteria.....	12
3.4.2	State-Price Density.....	13
3.4.3	Volatility Evaluation.....	14
<b>4</b>	<b>Data</b> .....	<b>19</b>
4.1	<i>S&amp;P 500 index values</i> .....	19
4.2	<i>S&amp;P 500 European Option data</i> .....	20
4.2.1	Filtering the data.....	21
4.3	<i>US Treasury rates</i> .....	23
4.4	<i>The Realized Kernel</i> .....	24
4.5	<i>The VIX Index</i> .....	25
4.6	<i>Sample Periods and Datasets Overview</i> .....	26
<b>5</b>	<b>Empirical Results</b> .....	<b>28</b>
5.1	<i>GJR-GARCH with Filtered Historical Simulation</i> .....	28
5.1.1	OTM Options.....	30
5.1.2	ITM and OTM Options.....	33
5.2	<i>Realized GARCH</i> .....	35
5.3	<i>Realized GARCH with Filtered Historical Simulation</i> .....	36
5.3.1	Statistical Criteria.....	37
5.3.2	State-Price-Density Analysis.....	41
5.3.3	Volatility Evaluation.....	43
<b>6</b>	<b>Conclusion</b> .....	<b>46</b>

<b>Bibliography</b> .....	<b>48</b>
<b>Appendix A    Parameter Transformations</b> .....	<b>A-1</b>
<i>A.1    Parameter Transformations for the GJR Model</i> .....	<i>A-1</i>
<i>A.2    Parameter Transformations for the RGARCH Model</i> .....	<i>A-1</i>
<b>Appendix B    Simulation Analysis</b> .....	<b>B-1</b>
<i>B.1    Simulation Procedure</i> .....	<i>B-1</i>
<i>B.2    Simulation Results</i> .....	<i>B-2</i>
<b>Appendix C    Additional Replication Results of BAEM (2008)</b> .....	<b>C-1</b>
<b>Appendix D    Additional Result of the FHS-RGARCH Method</b> .....	<b>D-1</b>

## 1 INTRODUCTION

Since the 1980s and 1990s a major growth in the use of derivative products has taken place. Among the several types of derivatives, *options* are the most popular. With the increasing popularity of options, option pricing has become a major topic in financial literature (see e.g. Merton (1976), Cox, Ross and Rubinstein (1979), Bakshi, Cao and Chen (1997), Lee (2004), Barone-Adesi, Engle and Mancini (2008)). Regarding the financial pricing theory, an important concept is volatility. Volatility is a quantity which measures the fluctuation in prices, whether it may be asset or derivative prices. Often volatility is also seen as a measure of risk. When it comes to derivative pricing, there are two types of volatility i.e. realized volatility and the implied volatility. The realized volatility is simply a calculation as a function of historical returns e.g. the 1 month standard deviation of daily returns on options. The implied volatility on the other hand refers more to a volatility level based on the market's assessments. It can be implied from the derivative prices using a certain pricing model e.g. the Black-Scholes model. A well-known measure of the implied volatility of option prices on the S&P 500 is the Chicago Board Options Exchange Market Volatility Index, also known as the VIX index. In this thesis, I incorporate a realized measure of the volatility (i.e. the realized kernel) in an option pricing model.

While several option pricing models have become popular during the past 30 years, few models make besides the historical asset prices use of the information contained in option prices to actually price the options (see e.g. Duan (1995) and Heston and Nandi (2000)). However, a recent option pricing method of Barone-Adesi, Engle and Mancini (2008), BAEM hereafter, uses the information of the prices of the option and the underlying while allowing for disturbances with different density parameters by means of the filtered historical simulation (FHS) method. Although the method of BAEM (2008) outperforms other competing option pricing models, it is based on the asymmetric Generalized Auto-Regressive Conditional Heteroskedasticity (GARCH) model of Glosten, Jagannathan and Runkle (1993), which does not take into account information of realized measures<sup>3</sup> such as the realized variances.

Outside of the option pricing literature Hansen, Huang and Shek (2011) show that the GARCH volatility model is outperformed by the Realized GARCH, hereafter RGARCH, model. The key idea of the RGARCH model is to use the information of a realized measure in modelling the volatility.

---

<sup>3</sup> A realized measure is a statistic derived from high-frequency data, e.g. realized variance, realized kernel, intraday range, number of transactions and trading volume.

## 1 Introduction

Moreover, the model not only specifies a relationship for the return series and the conditional volatility, but a measurement equation relating the realized measure to the conditional volatility is specified as well. The model inherits an ARMA structure for the conditional variance and the realized measure. In this way it facilitates coherence between returns and future volatility in a simple manner.

The contribution of this thesis is to merge the approach of BAEM (2008) for option pricing with the innovation of the RGARCH model to use realized information. To implement this, I start from the method of BAEM (2008), the FHS-GJR method hereafter, but I use a RGARCH instead of the Glosten, Jagannathan and Runkle (GJR) model. In this way the key features of the FHS method, such as historical and option pricing returns, flexible change of measures and nonparametric innovation distributions, are combined with a RGARCH model, which takes information on realized measures into account. Thus, in this thesis I study *whether applying the filtered historical simulation method on the RGARCH model of Hansen, Huang and Shek (2011), improves the option pricing method of Barone-Adesi, Engle and Mancini (2008)*.

To answer this question, I perform an empirical analysis. As a starting point, I replicate the results of BAEM (2008) as far as the FHS approach is concerned, by using similar data. Next, I reproduce the results of HHS (2011) for the Realized GARCH model, also using similar data. Thereafter, I combine the FHS approach of BAEM (2008) and the RGARCH model to check whether applying the FHS method on a RGARCH model improves the option pricing method of BAEM (2008). This combination is referred to as the FHS-RGARCH method.

The data I use for the empirical analysis consist of several parts. Firstly, for the replication of BAEM (2008), I use data on European options on the S&P 500 index, for the period January 2<sup>nd</sup> 2002 – December 29<sup>th</sup> 2004. Also, the replication of BAEM requires data on the S&P 500 index values for the period January 1<sup>st</sup> 1985 until December 29<sup>th</sup> 2004. For the replication of the RGARCH model, I use the data on the S&P 500 returns and the realized kernel, for the period January 1<sup>st</sup> 2002 – August 31<sup>st</sup> 2008. Finally, when combining the Realized GARCH model with the FHS method, I use European options on the S&P 500 index for the period January 2<sup>nd</sup> 2002 – December 30<sup>th</sup> 2011 and data on the S&P 500 index values and the realized kernel for the period January 3<sup>rd</sup> 1996 until December 30<sup>th</sup> 2011.

I compare the results of the FHS-RGARCH method with those of the FHS-GJR method in several ways. Firstly, I evaluate these methods on their ability to price options correctly. As evaluation criteria I use the statistical criteria: the root mean squared error, the mean absolute error, the minimum and

## 1 Introduction

maximum pricing errors and the percentage of positive pricing errors. Furthermore, I look at the state price density estimates and their accordance with the characteristics in financial markets. Also, I perform an inference on the volatility estimates resulting from the FHS-RGARCH method. I evaluate these volatility estimates by evaluating the VaR using the correct conditional coverage test and comparing the test results to those of the FHS-GJR method. Finally, the volatility estimates can be compared to the volatility indicated by the VIX index.

The FHS method combined with the RGARCH model is able to price options better compared to the GJR model based on the statistical criteria such as the root mean squared error. This is due to the high information contained by both the FHS method (through historical shocks) and the RGARCH model (through the realized measure). Also, the FHS-RGARCH method produces risk-neutral densities quite similar to pricing densities. Besides, based on the conditional coverage test, the FHS-RGARCH method produces accurate VaR and thus volatility estimates.

The remainder of this paper is organized as follows. Chapter 2 describes previous conducted research on option pricing and GARCH models. In Chapter 3 I describe the methods I use in thesis, together with the evaluation methods. Chapter 4 elaborates on the dataset used for the empirical research. I present the results of the empirical analysis in Chapter 5. Chapter 6, finally, concludes this thesis.



## 2 LITERATURE

In 1973, Black and Scholes developed the first option pricing model (Black and Scholes (1973)). Their option pricing method was based on a replication of a portfolio consisting of the underlying asset and a riskless bond. However, this pricing technique becomes less precise for markets with changing volatilities, which happens to be the case in real markets. Therefore several recent studies have been using models that consider both historical asset and option pricing (i.e., risk neutral) return dynamics (e.g. Duan (1995) and Heston and Nandi (2000)). Among models that focus on return dynamics, GARCH models are the most popular ones. GARCH is often used to capture the changing volatility or risk of asset returns (e.g. Holt and Aradhyula (1990), Ledoit, Santa-Clara and Wolf (2003), Wagner and Marsh (2005)). When it comes to option pricing, recent GARCH-type pricing models focus not only on the dynamics of the historical asset returns, but on the option pricing returns as well. However, until 2008, previous studies assumed the same model parameters for the historical and pricing distributions, leading to poor pricing performances (Chernov and Ghysels (2000) and Christoffersen and Jacobs (2004)). This changed when Barone-Adesi, Engle and Mancini (2008) proposed a new method of option pricing, in which they considered the historical and pricing distributions to be different. Barone-Adesi, Engle and Mancini apply a filtered historical simulation technique on the asymmetric GARCH model of Glosten, Jagannathan and Runkle (1993), the GJR model.

The filtered historical simulation (FHS) method introduced by Barone-Adesi, Giannopoulos, and Vosper (1999) is a scenario generating technique which makes use of a combination of past returns and nonlinear econometric models to simulate possible future price values which are then used to construct a probability distribution. Compared to a simple historical simulation, the FHS method has the major advantage of incorporating the volatility reflected in markets; i.e. in order to make past returns stationary and generate suitable shocks for the simulation process, the FHS method first scales the returns on each day by the volatility on that day. Next, the volatility conditions prevailing in the markets are reflected by multiplying the scaled returns by forecasts of the volatility. Besides the advantage of incorporating volatility conditions, another advantage of the FHS method is the use of a semi-parametric simulation technique. In the FHS method, no assumption is made regarding the probability distribution of the return series. Instead the historical or empirical distribution is used, allowing the data to set its own course. In this way the FHS method allows for fat tails, volatility clusters and changing means. Since risk estimation is highly dependent on good prediction of uncommon events, simulation models drawing innovations from theoretical distributions may discard 'catastrophic risks'. However, the FHS method accounts for these risks and is therefore a

prudent approach for parameter calibration of volatility models. Other advantages of the FHS method are that this simulation technique easily deals with dependencies across large numbers of assets, generates a lot of scenarios for the mean and variance of each risk factor in a multi-period horizon, and is able to capture cross-dependencies while maintaining reasonable computation time.<sup>4</sup> The FHS method appears promising, though, one may need to apply additional refinements to take time-varying correlations into account and to choose the appropriate length of the historical sample period (Pritsker (2001)). By using the FHS method, BAEM (2008) allows for an option-pricing distribution which differs in shape from the historical distribution. With the help of an extensive empirical analysis, BAEM show that their method outperforms other competing GARCH models and ad hoc Black-Scholes option pricing models.

Besides developments in option pricing methods such as that of Barone-Adesi, Engle and Mancini (2008), progress has also been made in the volatility models itself. After the introduction of the autoregressive conditional heteroskedasticity (ARCH) model in 1982 by Robert F. Engle, his student Bollerslev came up with a more general version of this model in 1986, called the Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) model. Since then, several variants of this GARCH model have been developed (e.g. EGARCH by Nelson (1991), NGARCH of Engle and NG (1991), GJR-GARCH (Glosten, Jagannathan and Runkle (1993)), QGARCH by Sentana (1995) and TGARCH by Zakoian (1994)). All these GARCH models have in common that they only use the squared returns to extract information about the latent volatility process. The difference between the models, however, lies in the specified GARCH equations that determine how the squared returns are related to the level of volatility. A disadvantage of only using the squared returns to determine the volatility level is that the conditional variance in GARCH models needs many periods to adjust to its new level (see Andersen et al. (2003)). To tackle this problem a new type of model, the so-called GARCH-X model, is employed (see Engle (2002) and Forsberg and Bollerslev (2002)). The GARCH-X model relates the volatility level in the GARCH equation also to realized measures such as the realized variance, bipower variation, realized kernel etc. Though, in this model, no latent volatility process of the realized measures is specified.

The first model introducing additional latent volatility processes for the realized measures is the multiplicative error model (MEM) of Engle and Gallo (2006). Another well-known model of this type is the HEAVY model of Shephard and Sheppard (2010). Research, among which Christoffersen et al. (2010) and Dobrev and Szerszen (2010), show that the economic and statistical gains of volatility

---

<sup>4</sup> See the official website of the FHS: <http://www.filteredhistoricalsimulation.com/>

models with well specified realized measures are quite large. The reasoning is that good realized measures are much more informative about the current volatility level than the squared returns. Consequently, Hansen, Huang and Shek (2011) introduce the Realized GARCH model (RGARCH) which is a GARCH model with realized measures, in which a '*measurement equation*' relates the conditional variance of the returns to the realized measure. Using an empirical application, Hansen, Huang and Shek (HHS, hereafter) show that the RGARCH model substantially improves the empirical fit compared to the standard GARCH models that only use return series. Compared to the other well performing models that incorporate realized measures (i.e. the MEM and HEAVY model), the reasoning behind the RGARCH model is that it is a simple yet well performing model that directly specifies a relation between the returns and future volatility allowing in this way construction of multiple period forecasts.

Inspired by the success of the filtered historical simulation method applied by BAEM (2008) for option pricing and the success of the RGARCH model in empirical fitting, I introduce a combination of the filtered historical simulation method of BAEM (2008) with the RGARCH model of Hansen, Huang and Shek (2011). The FHS approach which allows for semi-parametrical shocks in the return series is similarly applied on the shocks in the realized kernel series. In this way the benefits of semi-parametric simulation of the FHS method and the extra information of the realized measure in the RGARCH model are combined effectively. Besides applying the FHS method on a RGARCH model instead of a GJR model, compared to BAEM (2008), I not only expand the sample period from 2002 – 2004 to 2002 – 2011, but I also consider in-the-money options and options with very short maturities as well, while BAEM (2008) restrict their empirical analysis to out-of-the money options with certain characteristics (see Section 4.2.1).

### 3 METHODS

In this chapter I describe the methods and models. The first subsection in this chapter gives an overview of the FHS-GJR method and describes the filtered historical simulation steps to be applied on the GJR model (as is done by BAEM (2008)). Next, the second subsection delineates the RGARCH model of HHS (2011). Combining the FHS method for option pricing of BAEM (2008) with the RGARCH model, in the third subsection I describe the FHS-RGARCH method, which is the main innovation of this thesis. Finally, in the last subsection of this chapter I describe three evaluation methods used in this thesis, which are categorized into statistical criteria, state-price-densities and volatility evaluation.

#### 3.1 GJR-GARCH with Filtered Historical Simulation

As mentioned earlier in Chapter 2, Barone-Adesi, Engle and Mancini (2008) use an option pricing method which is based on the application of the filtered historical simulation (FHS) on a GJR volatility model. The FHS method is a semi-parametric simulation technique. The FHS method as applied by BAEM (2008) is given by the following steps:

1. Let  $S_t$  denote the price of the underlying on day  $t$ . For a given day  $t$  estimate then, using  $n$  historical log-returns of the underlying ( $r_j = \log\left(\frac{S_j}{S_{j-1}}\right)$ , for  $j = 1 - n + t, 2 - n + t, \dots, t$ ), the following GJR model:

$$\begin{aligned} r_j &= \mu + \varepsilon_j \\ \sigma_j^2 &= \omega + \beta\sigma_{j-1}^2 + \alpha\varepsilon_{j-1}^2 + \gamma I_{j-1}\varepsilon_{j-1}^2, \end{aligned} \tag{1}$$

in which  $\varepsilon_j = \sigma_j z_j$ ,  $z_j \sim f(0,1)$  and  $I_{j-1} = 1$ , when  $\varepsilon_{j-1} < 0$  and 0 otherwise. Thus, an assumption of zero mean and unit variance for the innovations  $z_j$  is made. For convenience I leave out the index  $t$  on the parameters which are estimated each day  $t$  using a rolling window of length  $n$ . Note that the estimation of the set of parameters  $\theta = \{\omega, \beta, \alpha, \gamma\}$  occurs under the historical measure  $\mathbb{P}$ , since the historical (index) prices are used. The empirical innovations  $z_j$  are also estimated.

2. Initialize a set of parameters  $\theta^*$  equal to  $\hat{\theta}$ , which is the estimate of  $\theta$  obtained in step 1.
3. Given  $\theta^*$ , simulate an asset return path from  $t$  to  $t + \tau$  using Equation (1) for  $j = t + 1, \dots, T$ , where  $T = t + \tau$ . Simulate the return path by randomly drawing one of the estimated past innovations of step 1, say  $z_{[1]}$ , and consequently calculating the conditional variance  $\sigma_{t+1}^2$ . Next, draw a second innovation  $z_{[2]}$  and calculate  $\sigma_{t+2}^2$ . This process continues

until time  $t + \tau$ . The  $\tau$ -period return is then equal to  $S_T/S_t = \exp(\tau\mu^* + \sum_{i=1}^{\tau} \sigma_{(t+i)} Z_{[i]})$ , where  $\mu^*$  is fixed at the short term risk-free rate.

4. Estimate the state price density or martingale measure (Harrison and Kreps (1979)) for  $\tau$  periods by repeating step 3 a large number of times.
5. Calculate the price of a call option with strike  $K$  and maturity  $T$  at time  $t$  as  $\frac{e^{-i\tau}}{L} \sum_{l=1}^L \max(S_T^{(l)} - K, 0)$ , where  $S_T^{(l)}$  is the simulated asset price at time  $T$  in the  $l^{\text{th}}$  sample path and  $L$  is the total amount of simulated paths.  $i$  corresponds to the  $\tau$ -period interest rate. Calculate put prices analogously.
6. Let the parameters  $\theta^*$  vary in such a way that the mean squared pricing error  $\sum_{h=1}^{N_t} d(K_h, T_h)^2$  is minimized, where  $d(K_h, T_h)$  is the difference between the price simulated by the GARCH model and the market price of option  $h$  with strike  $K_h$  and maturity  $T_h$ .  $N_t$  is the total number of options on a given day  $t$ . Stop with the calibration of  $\theta^*$  when the mean squared pricing error is under a predefined threshold.

Step 3 to step 6 can be seen as an optimization procedure. Following BAEM (2008), I perform this minimization process using the `fminsearch` algorithm of Matlab. More specifically, I make use of the Nelder-Mead algorithm. However, the GJR model requires some restrictions. In fact, for the variance series  $\sigma_t^2$  to be positive, it is necessary that  $\omega > 0$ . Also, stationarity in the GARCH equation requires that  $\alpha + \beta + \frac{1}{2}\gamma < 1$ , while  $\alpha, \beta$  and  $\gamma$  should be positive (Villar and Ortega (2010)). Since the GJR model parameters require some stationarity restrictions, I use parameter transformations in order to use unconstrained minimization. See Appendix A, for the exact parameter transformations. To estimate the  $\tau$ -period interest rate  $i$  in step 5, I use the 1-month, 3-month, 6-month, 1-year, 2-year and 5-year US Treasury Constant Maturity rates. In particular, assuming 30 days for each month, the  $\tau$ -period interest rate can be obtained by applying linear interpolation.

In this thesis I differentiate between estimated parameters and calibrated parameters. Estimated parameters are obtained using a certain estimation procedure, such as (quasi) maximum likelihood, and rely on observed data. These are obtained in the first step of the FHS procedure. The calibrated parameters on the other hand, are obtained by setting the parameters in such a way that the difference between the simulated option prices and the observed option prices are minimized. Calibration in this context thus also relies on simulations. Therefore, calibrated values may differ, though insignificantly, each time an empirical analysis is conducted.

### 3.2 Realized GARCH

In the previous subsection I described the FHS-GJR method. However, in this thesis I apply the FHS method of BAEM (2008) on a volatility model that incorporates the information contained in realized measures. Beside MEM and HEAVY, another model that completely specifies the relations between realized measures and the volatility is the Realized GARCH model of Hansen, Huang and Shek (2011). In accordance with HHS (2011), I use the log-linear specification of the RGARCH model. The main reason for this choice is that the logarithmic specification automatically ensures that the variance and the realized kernel are positive. In addition, since the FHS method uses a moving window estimation technique, the RGARCH model may be exposed to large shocks. For the linear specification, this may result in violating the conditions of positiveness of the realized kernel and thus the variance series, requiring extra complications for model estimation. Since the log-linear specification automatically guarantees positiveness of the realized kernel and the variance, this problem cannot occur for the log-linear specification.

The RGARCH(1,1) model in its log-linear specification can be written as:

$$\begin{aligned} r_t &= \mu + \sigma_t z_t \\ \log(\sigma_t^2) &= \omega + \beta \log(\sigma_{t-1}^2) + \psi \log(x_{t-1}) \\ \log(x_t) &= \xi + \varphi \log(\sigma_t^2) + \kappa(z_t) + v_t \end{aligned} \quad (2)$$

in which  $z_t \sim i.i.d.(0,1)$ ,  $v_t \sim i.i.d.(0, \sigma_v^2)$  and  $\sigma_t^2 = var(r_t | \mathcal{J}_t)$  where the information set is defined as  $\mathcal{J}_t = g(r_t, x_t, r_{t-1}, x_{t-1}, \dots)$ , with  $r_t$  and  $x_t$  the return and the realized measure on day  $t$ . The three equations are called the return equation, the GARCH equation and the measurement equation, respectively.  $\kappa(z_t)$  is called the leverage function, since it captures the relation between the shocks in returns and future volatilities (indirectly). This is a crucial characteristic since it is well-known in the financial world that the market tends to be more sensitive to negative shocks than positive shocks. Following HHS, I assume that the leverage function has a simple quadratic form, namely  $\kappa(z_t) = \kappa_1 z_t + \kappa_2 (z_t^2 - 1)$ , such that  $E[\kappa(z_t)] = 0$  for each distribution with  $E[z_t] = 0$  and  $V[z_t] = 1$ . This leverage function  $\kappa(z_t)$  is closely related to the news impact curve which depicts the effect of positive and negative shocks on future volatility. The Realized GARCH model used in HHS (2011) assumes  $\mu = 0$ . However, for completeness and comparability with the return equation of the GJR model, the RGARCH model as presented in Equation (2) allows for an average return.

The RGARCH model of Equation (2) implies some parameter restrictions. For the GARCH equation, no positiveness restrictions are required for  $\omega$  since the log-linear specification automatically allows for positive values of  $\sigma_t^2$ . However, to let  $\log(\sigma_t^2)$  be a stationary time series, it is required that

$\beta \geq 0, \psi \geq 0$  and  $\beta + \psi < 1$ . Regarding the measurement equation, no special restrictions are required for the parameters due to the log-linear specification. The variance of  $v_t$ , though, needs to be positive ( $\sigma_v^2 > 0$ ).

To estimate the RGARCH model I employ the quasi-maximum likelihood (QML) method, which is consistent with literature (see e.g. Lee and Hansen (1994), Harvey et. al (1994), Alizadeh et. al (2002), Hansen, Huang and Shek (2011)). Adopting a Gaussian specification for both  $z_t$  and  $v_t$ , the log-likelihood function to be maximised can be expressed as:

$$\ell(r, x; \eta) = -\frac{1}{2} \sum_{t=1}^n \log(\sigma_t^2) + \frac{(r_t - \mu)^2}{\sigma_t^2} + \log(\sigma_v^2) + \frac{v_t^2}{\sigma_v^2} \quad (3)$$

in which  $\eta = \{\mu, \omega, \beta, \psi, \xi, \varphi, \kappa_1, \kappa_2, \sigma_v^2\}$ . The estimates  $\hat{\eta}$  are obtained by numerical optimization.<sup>5</sup> Although the RGARCH model requires constrained optimization, I perform the QML using the `fminsearch` function of the software package Matlab. To utilize the unconstrained optimisation method, I again use parameter transformations. I refer to Appendix A for these transformations.

To obtain the standard deviations of the QML estimations, I use the inverse of the Fisher information matrix as follows:

$$\hat{\Omega}_\eta = \left( \frac{\partial^2 \ell(r, x; \eta)}{\partial \eta \partial \eta'} \right)^{-1} \Bigg|_{\eta = \hat{\eta}} \quad (4)$$

where  $\hat{\Omega}_\eta$  is the estimated variance matrix of the QML estimates. So, the standard deviations can be obtained with the hessian matrix of the log-likelihood function. I compute the hessian matrix numerically with the `fminunc` optimization function in Matlab, which is able to produce the hessian of the optimized function for the optimum values.

### 3.3 Realized GARCH with Filtered Historical Simulation

In the previous two subsections I described the FHS-GJR method, which is the FHS method applied on a GJR model and the RGARCH(1,1) model of HHS (2011). In this subsection I describe the FHS-RGARCH method. As I pointed out earlier, the FHS-RGARCH method is a combination of the previously described simulation steps of the FHS method but with a RGARCH model instead.

<sup>5</sup> Although the RGARCH model requires constrained optimization, I perform the QML using the `fminsearch` function of the software package Matlab. The reason for this is that the Nelder-Mead algorithm produces robust optima compared to the Sequential Quadratic Programming algorithm used by HHS (2011). Applying the unconstrained optimisation method requires parameter transformations. These parameter transformations can be found in Appendix A.

Moreover, the steps are similar to those of the FHS-GJR method but Equation (1) should be replaced by Equation (2). Beside the realized measures, another dissimilitude of the RGARCH and the GJR-GARCH model is the manner in which the models allow for a leverage effect. The GJR model, on the one hand, incorporates the leverage effect directly in the GARCH equation. The RGARCH model, on the other hand, includes the leverage effect indirectly through the measurement equation. The latter model may thus tend to neglect leverage effects as the realized measure tend to become less informative to the volatility.

Combining the filtered historical simulation method with the RGARCH model, I apply the FHS-RGARCH method as follows:

1. For a given day  $t$ , using  $n$  historical log-returns of the underlying asset ( $r_j = \log\left(\frac{S_j}{S_{j-1}}\right)$ , for  $j = 1 - n + t, 2 - n + t, \dots, t$ ) I estimate the following RGARCH(1,1) model:

$$\begin{aligned} r_j &= \mu + \sigma_j z_j \\ \log(\sigma_j^2) &= \omega + \beta \log(\sigma_{j-1}^2) + \psi \log(x_{j-1}) \\ \log(x_j) &= \xi + \varphi \log(\sigma_j^2) + \kappa(z_j) + v_j \end{aligned} \quad (5)$$

in which  $z_j \sim i.i.d. (0,1)$ ,  $v_j \sim i.i.d. (0, \sigma_v^2)$  and  $\sigma_j^2 = var(r_j | \mathcal{J}_j)$  where the information set is defined as  $\mathcal{J}_j = g(r_j, x_j, r_{j-1}, x_{j-1}, \dots)$ . Thus, the model assumes zero mean and unit variance for the innovations  $z_j$ . Regarding the shocks in the log realized kernel the model only assumes zero mean. Again, I leave out the index  $t$  on the parameters that are estimated using a moving window. Note that similar to the FHS-GJR method, the estimation of the set of parameters  $\theta_{RGARCH} = \{\omega, \beta, \psi, \xi, \varphi, \kappa_1, \kappa_2\}$  occurs under the  $\mathbb{P}$  measure, since the historical (index) prices are used. The innovations  $z_j$  and  $v_j$  are also estimated.

2. I initialize a set of parameters  $\theta_{RGARCH}^*$  equal to  $\hat{\theta}_{RGARCH}$ , which is the QML estimate of  $\theta_{RGARCH}$  obtained in step 1.
3. Given  $\theta_{RGARCH}^*$ , I simulate an asset return path from  $t$  to  $t + \tau$  using Equation (5) for  $j = t + 1, \dots, T$ , where  $T = t + \tau$ . Similar to the FHS-GJR method, I simulate a return path by drawing randomly one of the estimated past innovations with the corresponding kernel shock of step 1, say  $z_{[1]}$  and  $v_{[1]}$ , and consequently calculating the conditional variance  $\sigma_{t+1}^2$ .<sup>6</sup> Next, I draw a second pair of a return innovation and a kernel shock  $z_{[2]}$  and  $v_{[1]}$ , and

<sup>6</sup> Thus the past innovations and kernel shocks are bootstrapped simultaneously.



calculate  $\sigma_{t+2}^2$ . I continue this process until time  $t + \tau$ . The  $\tau$ -period return then equals  $S_T/S_t = \exp(\tau\mu^* + \sum_{i=1}^{\tau} \sigma_{(t+i)} z_{[i]})$ , where I fix  $\mu^*$  at the short term risk-free rate.

4. I estimate the state price density or martingale measure for  $\tau$  periods by repeating step 3 a large number of times.
5. I calculate the price of a call option with strike  $K$  and maturity  $T$  at time  $t$  as  $\frac{e^{-i\tau}}{L} \sum_{l=1}^L \max(S_T^{(l)} - K, 0)$ , where  $S_T^{(l)}$  is the simulated asset price at time  $T$  and  $L$  is the total amount of simulated paths, while  $i$  corresponds to the  $\tau$ -period interest rate. I calculate put prices in a similar way.
6. I vary the parameters  $\theta_{RGARCH}^*$  in such a way that the mean squared pricing error  $\sum_{h=1}^{N_t} d(K_h, T_h)^2$  is minimized, where  $d(K_h, T_h)$  is the difference between the price simulated by the model and the market price of option  $h$  with strike  $K_h$  and maturity  $T_h$ .  $N_t$  is the total number of options on a given day  $t$ . The calibration of  $\theta_{RGARCH}^*$  is finished when the mean squared pricing error is under a predefined threshold.<sup>7</sup>

As in the case of the FHS-GJR method, the FHS procedure for this RGARCH model also uses linear interpolation for constructing the interest rates assuming 30 days a month. Similar to the FHS-GJR method and the QML estimation of the RGARCH model, I use the Nelder-Mead algorithm in combination with the parameter transformations for the optimizations. In addition, in the estimation step (step 1), I use the estimates of the previous day as start values, while for the first day the results of HHS (2011) are given as start values.

### 3.4 Evaluation Methods

In this section I describe the methods I use to evaluate the different models. The different methods can be roughly divided in to three groups, namely evaluation based on statistical criteria, evaluation based on the state-price-density and evaluation of the estimated volatility. These methods are described in the following subsections.

#### 3.4.1 Statistical Criteria

To evaluate the FHS-RGARCH method, I use several statistical criteria. First of all I compare the results using the root mean square error (RMSE) which is the square root of the averaged squared deviations between the model and actual prices. To compare the results of the different models I

---

<sup>7</sup> To implement a robust method I perform the minimization process using the fminsearch algorithm of Matlab. More specifically, I make use of the Nelder-Mead algorithm. Since the RGARCH model requires some parameter restrictions, I use parameter transformations in order to use unconstrained minimization. See Appendix A, for the exact parameter transformations.

also use the mean absolute error (MAE), the mean outside error, which is the average error outside the bid-ask spread, the minimum and maximum pricing errors and the percentage of positive pricing errors. During the calculation of the pricing errors, the actual price of each option is calculated as the average of the best bid and best ask price.

### 3.4.2 State-Price Density

When it comes to derivative pricing, the price of a derivative is calculated as the net present value of the future cash-flows, using a risk-free rate  $i$ . In this process, the expected value is calculated using a probability density function (PDF) of the future cash-flows. Cox and Ross (1976) call this probability function the state-price-density (SPD) or the risk-neutral PDF, while Harrison and Kreps (1976) names it the martingale measure. In formula form the price of a derivative on time  $t$ , say  $D_t$ , with a payoff on the future date  $T = t + \tau$  can be expressed as:

$$D_t = E_{\mathbb{Q}}[D_T e^{-i\tau} | \mathcal{J}_t] = e^{-i\tau} \int_0^{\infty} D_T(S_T) q_{t,T}(S_T) dS_T \quad (6)$$

where  $D_T(S_T)$  is the payoff of the derivative on time  $T$  as a function of the price of the underlying  $S_T$  on time  $T$ ,  $q_{t,T}(S_T)$  is the martingale measure on time  $t$  under the risk-neutral measure  $\mathbb{Q}$  for payoffs received at time  $T$ .  $E_{\mathbb{Q}}[\cdot]$  is the expectation function under the  $\mathbb{Q}$  measure and  $\mathcal{J}_t$  is the set containing all information up to time  $t$ .

Determining the SPD can be quite a difficult task, since the risk-neutral PDF describes the probabilities of future prices. An approach to tackle this problem is to price the derivative using a historical measure, and apply a change of measure in the form of a SPD per unit probability. The derivative price can then be written as:

$$D_t = E_{\mathbb{P}}[D_T M_{t,T} | \mathcal{J}_t] = \int_0^{\infty} D_T(S_T) M_{t,T}(S_T) p_{t,T}(S_T) dS_T \quad (7)$$

where  $\mathbb{P}$  is the historical measure,  $p_{t,T}$  is the PDF under the historical measure at time  $t$  for payoffs received at time  $T$  and  $M_{t,T}$  is the SPD per unit probability.  $M_{t,T}$  is also called the asset pricing kernel (Rosenberg and Engle (2002)) or stochastic discount factor (Campbell, Lo and MacKinlay (1997)). Mathematically,  $M_{t,T}$  is also known as a Radon-Nikodym derivative. An advantage of the SPD per unit probability is the high information content of it or as Aït-Sahalia and Lo (1998) puts it: "From a pricing perspective, SPDs are "sufficient statistics" in an economic sense; they summarize all relevant information about preferences and business conditions for purposes of pricing financial securities." Using the SPD per unit probability and historical asset or index prices, the equilibrium or market prices of the derivatives can be approximated. On the other hand, using the market prices of

the derivatives and the historical distribution, the SPD containing information about market characteristics can be obtained. While several option pricing models assume a special form for  $M_{t,T}$  (see e.g. Aït-Sahalia and Lo (2000), Jackwerth (2000) and Rosenberg and Engle (2002)), the option pricing method of BAEM using the FHS approach makes no certain assumption on the change of measure. Beside statistical measures such as the RMSE or the MAE, one could also evaluate the FHS-GJR and FHS-RGARCH option pricing methods by verifying to what extent the estimated SPDs correspond to market characteristics.

### 3.4.3 Volatility Evaluation

Besides the correctness of option pricing, another way to evaluate the different methods is the estimation accuracy of the volatility. Therefore, in this section I describe the volatility evaluation of the FHS-GJR and the FHS-RGARCH method. Since proxies for volatility are usually quite noisy, I evaluate the methods indirectly by means of the Value-at-Risk (VaR). Hence, the first subsection focuses on the construction of the VaR measures. To evaluate the VaR forecast, I use the correct (un)conditional coverage tests and the independence test. These are described in the second subsection. I compare the VaR results of the FHS-GJR and the FHS-RGARCH method to the VaR resulting from the VIX index.

#### 3.4.3.1 Constructing the Value-at-Risk (VaR)

The VaR is the minimum return that could occur over a given holding period with a specified confidence level. To construct the daily VaR measures, the volatilities are required. After constructing the volatilities series using the FHS method, I calculate the VaRs. Mathematically the  $h$ -day VaR on day  $t$  can be defined as:

$$VaR_t(q) = x: P[r_{t+h} \leq x | \mathcal{J}_t] = F_{t+h|t}(x) = q \quad (8)$$

where  $P[\cdot]$  is the probability function,  $\mathcal{J}_t$  is the information set up to time  $t$  and  $F_{t+h|t}(\cdot)$  is the cumulative distribution function of  $r_{t+h}$  conditional on  $\mathcal{J}_t$ . In this thesis I limit the VaR analysis only to the one-day VaR.

Assuming  $r_t \sim N(\mu, \sigma_t)$ , the  $(100 \times q)\%$  - VaR can then be calculated as:

$$VaR_t(q) = \mu + z_q \sigma_{t+1} \quad (9)$$

#### 3.4.3.2 Tests on the VaR

One could say that a good VaR forecast should be one for which the fraction of the returns observations lower than the VaR is about the same as the VaR probability. So for a 5% VaR, a good

VaR is when approximately 5% of the returns sample is lower than the VaR. To evaluate the Value-at-Risk forecasts formally, I use the tests of Christoffersen (1998). The idea behind these tests is following:

Starting from a general  $(100 \times q)\%$  interval forecast with the interval  $(L_{t+1|t}(q), U_{t+1|t}(q))$  constructed such that

$$P[L_{t+1|t}(q) \leq r_{t+1} \leq U_{t+1|t}(q) | \mathcal{J}_t] = q,$$

the VaR can then obtained by setting

$$L_{t+1|t}(q) = -\infty \text{ and } U_{t+1|t}(q) = VaR_t(q).$$

Thus the idea for these tests is to make use of the techniques developed for evaluating interval forecasts. Thus a good VaR forecast should meet the following properties:

1. The fraction of the observations lower than the VaR must be equal to the nominal coverage probability. This is called the “correct unconditional coverage”.
2. Return observations lower than the VaR should be spread out over the whole sample rather than occur in clusters. This is called “independence”, which is relevant for “volatility clustering”.

The combination of both properties together is called the correct conditional coverage.

#### 3.4.3.2.1 Correct Unconditional Coverage

To test for correct unconditional coverage, I use the likelihood ratio (LR) test of Christoffersen (1998). For a one-day  $(100 \times q)\%$  VaR the test is as follows:

Let  $I_{t+1}$  be the indicator function such that

$$I_{t+1} = \begin{cases} 1, & \text{if } r_{t+1} < VaR_t(q) \\ 0, & \text{otherwise.} \end{cases}$$

Thus,  $I_{t+1}$  gives the violations of the VaR. Since a good VaR should have correct unconditional coverage, the expected number of violations should be equal to the nominal coverage probability. This implies the following for the null-hypothesis:

$$H_0: P[I_{t+1} = 1] = E[I_{t+1}] = q$$

Assuming independence of  $I_{t+1}, I_t, \dots, I_1$  the likelihood function for the forecasts with coverage probability  $p = P[I_{t+1} = 1]$ , is given by:

$$\begin{aligned}
 \mathcal{L}(p; I_T, I_{T-1}, \dots, I_1) &= P[I_T = i_T, I_{T-1} = i_{T-1}, \dots, I_1 = i_1] \\
 &= P[I_T = i_T]P[I_{T-1} = i_{T-1}] \dots P[I_1 = i_1] \\
 &= (1-p)^{T_0} p^{T_1},
 \end{aligned}$$

where  $T_1 = \sum_{t=1}^T i_t$  and  $T_0 = T - T_1$ . The idea of the LR test is then compare the likelihood under the null hypothesis that  $p = q$  to the alternative  $p = \pi$ , where  $\pi$  is obtained using maximum likelihood (ML) estimation. The ML estimator of  $\pi$  is given by:

$$\hat{\pi} = \hat{P}[I_{t+1} = 1] = \frac{T_1}{T_0 + T_1}.$$

The likelihood ratio value for the unconditional coverage test is then given by:

$$LR_{uc} = -2 \log \left( \frac{(1-q)^{T_0} q^{T_1}}{(1-\hat{\pi})^{T_0} \hat{\pi}^{T_1}} \right) \stackrel{asy}{\sim} \chi_1^2, \quad (10)$$

where  $\chi_1^2$  is the chi-square distribution with 1 degree of freedom.

#### 3.4.3.2.2 Independence

Besides correct unconditional coverage, good VaR estimates should have independence too. This implies that the VaR violations should not occur in clusters, but instead they should be spread out over the whole sample period. To test for independence in the violations, the null hypothesis can be defined as:

$$H_0: P[I_{t+1} = 1 | \mathcal{J}_t] = P[I_{t+1} = 1], \quad \text{for all } t.$$

The alternative hypothesis is a first-order Markov chain. Using the theory of conditional probabilities that  $P[A, B] = P[A|B] \times P[B]$ , the likelihood can be written as:

$$\begin{aligned}
 \mathcal{L}(p; I_T, I_{T-1}, \dots, I_1) &= P[I_T = i_T, I_{T-1} = i_{T-1}, \dots, I_1 = i_1] \\
 &= P[I_T = i_T | I_{T-1} = i_{T-1}, \dots, I_1 = i_1] \times P[I_{T-1} = i_{T-1}, \dots, I_1 = i_1] \\
 &\vdots \\
 &= \prod_{t=2}^{T-1} P[I_{t+1} = i_{t+1} | I_t = i_t] \times P[I_1 = i_1].
 \end{aligned}$$

Suppose that the transition probability matrix of the first-order Markov chain is given by:

$$\Pi_1 = \begin{pmatrix} 1 - \pi_{01} & \pi_{01} \\ 1 - \pi_{11} & \pi_{11} \end{pmatrix},$$

where  $\pi_{ij} = P[I_{t+1} = j | I_t = i]$ , the likelihood is then as follows:

$$\mathcal{L}(\Pi_1; I_T, I_{T-1}, \dots, I_1) = (1 - \pi_{01})^{T_{00}} (\pi_{01})^{T_{01}} (1 - \pi_{11})^{T_{10}} (\pi_{11})^{T_{11}}.$$

$T_{ij}$  represents the number of observations in such a way that  $I_{t+1} = j$  and  $I_t = i$ . The maximum likelihood estimate of  $\Pi_1$  is then given by:

$$\hat{\Pi}_1 = \begin{pmatrix} 1 - \hat{\pi}_{01} & \hat{\pi}_{01} \\ 1 - \hat{\pi}_{11} & \hat{\pi}_{11} \end{pmatrix} = \begin{pmatrix} \frac{T_{00}}{T_{00} + T_{01}} & \frac{T_{01}}{T_{00} + T_{01}} \\ \frac{T_{10}}{T_{10} + T_{11}} & \frac{T_{11}}{T_{10} + T_{11}} \end{pmatrix}.$$

Under the null-hypothesis of independence, it holds that  $\pi_{01} = \pi_{11} \equiv \pi_2$ . The likelihood then becomes

$$\mathcal{L}(\pi_2; I_T, I_{T-1}, \dots, I_1) = (1 - \pi_2)^{(T_{00}+T_{10})} (\pi_2)^{(T_{01}+T_{11})}.$$

$\pi_2$  can be estimated using maximum likelihood which yields:

$$\hat{\pi}_2 = \frac{T_{01} + T_{11}}{T_{00} + T_{01} + T_{10} + T_{11}}.$$

Using the maximum likelihood estimates of  $\hat{\pi}_{01}$ ,  $\hat{\pi}_{11}$  and  $\hat{\pi}_2$  the likelihood ratio for the test on independence is then equal to:

$$LR_{ind} = -2 \log \left( \frac{(1 - \hat{\pi}_2)^{(T_{00}+T_{10})} (\hat{\pi}_2)^{(T_{01}+T_{11})}}{(1 - \hat{\pi}_{01})^{T_{00}} (\hat{\pi}_{01})^{T_{01}} (1 - \hat{\pi}_{11})^{T_{10}} (\hat{\pi}_{11})^{T_{11}}} \right) \stackrel{asy}{\sim} \chi_1^2, \quad (11)$$

where  $\chi_1^2$  is the chi-square distribution with 1 degree of freedom.

### 3.4.3.2.3 Correct Conditional Coverage

While the previous two test focus on one of the properties of a good VaR estimate, in the evaluation of VaR measures it is necessary that both requirements are fulfilled. Hence, one could use the test for correct conditional coverage which is a joint test on both correct unconditional coverage and independence. The likelihood ratio value of the latter is a combination of the likelihood ratios of the correct unconditional coverage and independence test. The null-hypothesis to be tested for a  $(100 \times q)\%$  VaR is defined as:

$$H_0: P[I_{t+1} = 1 | \mathcal{J}_t] = P[I_{t+1} = 1] = q$$

Under the alternative hypothesis,  $I_{t+1}$  is a first-order Markov chain, with transition probability matrix  $\Pi_1$ . This leads to the following likelihood ratio statistic:

$$LR_{cc} = -2 \log \left( \frac{\mathcal{L}(q; I_T, I_{T-1}, \dots, I_1)}{\mathcal{L}(\hat{\Pi}_1; I_T, I_{T-1}, \dots, I_1)} \right) \stackrel{asy}{\sim} \chi_2^2,$$

where  $\chi_2^2$  is the chi-square distribution with 2 degrees of freedom. Reasoning the same as before and conditioning on the first observation in test for unconditional coverage, it follows that  $\hat{\pi} = \hat{\pi}_2$ . The likelihood ratio value for the conditional coverage test can then easily be computed as:

$$\begin{aligned} LR_{cc} &= -2 \log \left( \frac{\mathcal{L}(q; I_T, I_{T-1}, \dots, I_1)}{\mathcal{L}(\hat{\pi}; I_T, I_{T-1}, \dots, I_1)} \right) - 2 \log \left( \frac{\mathcal{L}(\hat{\pi}_2; I_T, I_{T-1}, \dots, I_1)}{\mathcal{L}(\hat{\Pi}_1; I_T, I_{T-1}, \dots, I_1)} \right) \\ &= LR_{uc} + LR_{ind} \end{aligned} \quad (12)$$

where  $LR_{uc}$  and  $LR_{ind}$  are the likelihood ratio values of the correct unconditional coverage and independence test, respectively.

## 4 DATA

In this previous chapter, I described the methods and models I use in this thesis. In this chapter I describe the data that I use for the empirical analysis in this thesis. This chapter consists of 6 subsections. The first five subsections each describe a different type of data. The several types of data I use are the S&P 500 index data, S&P 500 Europeans-style option data, US Treasury rates, data on the realized kernel of the S&P 500 and the VIX index. Finally, in the last subsection I describe the different sample periods I consider and present an overview of the datasets.

### 4.1 S&P 500 index values

In this research I use several datasets for the empirical analysis. Firstly, I use the S&P 500 index values which I downloaded from the Economic Research website of the Federal Reserve Bank of St. Louis<sup>8</sup>. I use this dataset to estimate the model in the FHS method using a rolling window (step 1 of the FHS method). The S&P 500 index data covers the period January 2<sup>nd</sup> 1985 until December 30<sup>th</sup> 2011. An overview of the evolution of the S&P 500 series during the time period January 2<sup>nd</sup> 1985 until December 30<sup>th</sup> 2011 is given in Figure 1. This period incorporates two well-known crises: the burst of the tech and dotcom bubble in 2000 and the credit crunch in 2007. Especially the recent financial crisis resulted in a large negative shock at the end of 2008, which becomes clear if we look at Figure 1. Further investigation points out that this shock occurs on Friday, October 10<sup>th</sup> 2008, after the collapse of the Icelandic banks. From this figure, we can also see that the S&P is dominated by three booming periods (1985-2000, 2002-2007 and 2009-2011), which corresponds to the cycle of booming periods after the occurrence of crises.

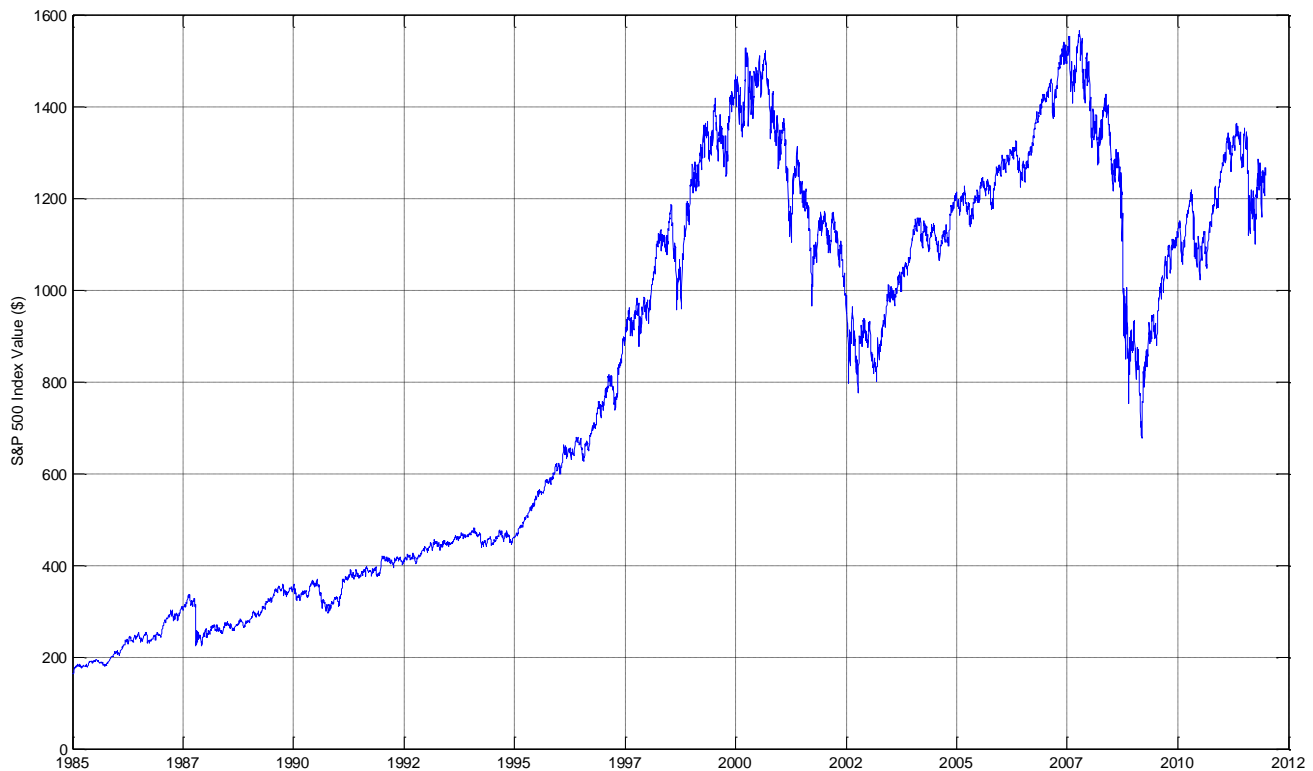
Estimates based on the historical returns may act as a good simulation basis for option pricing. However, the generally highly increasing values of the S&P 500 during 1985-1997 may impact the estimated average return ( $\hat{\mu}$ ). During the FHS procedure it is more appropriate to calculate the options prices using a fixed short term risk-free rate  $\mu^*$  (see step 3 of the FHS calibration procedure), rather than an approximate average of the historical return,  $\hat{\mu}$ , since the former occurs under the risk free measure  $\mathbb{Q}$ , while  $\hat{\mu}$  is estimated under the historical measure  $\mathbb{P}$ . Obviously it is also possible to let  $\mu$  be calibrated along with the GARCH parameters of the GJR model. I present the results regarding these calibrations in Section 5.1.

---

<sup>8</sup> <http://research.stlouisfed.org/fred2/series/SP500/downloaddata?cid=32255>



Figure 1: S&amp;P 500 Price Index Values starting from 2 January 1985 until 30 December 2011



Note: This figure depicts the S&P 500 Price Index Values over the period January 2, 1985 till December 30, 2011.

## 4.2 S&P 500 European Option data

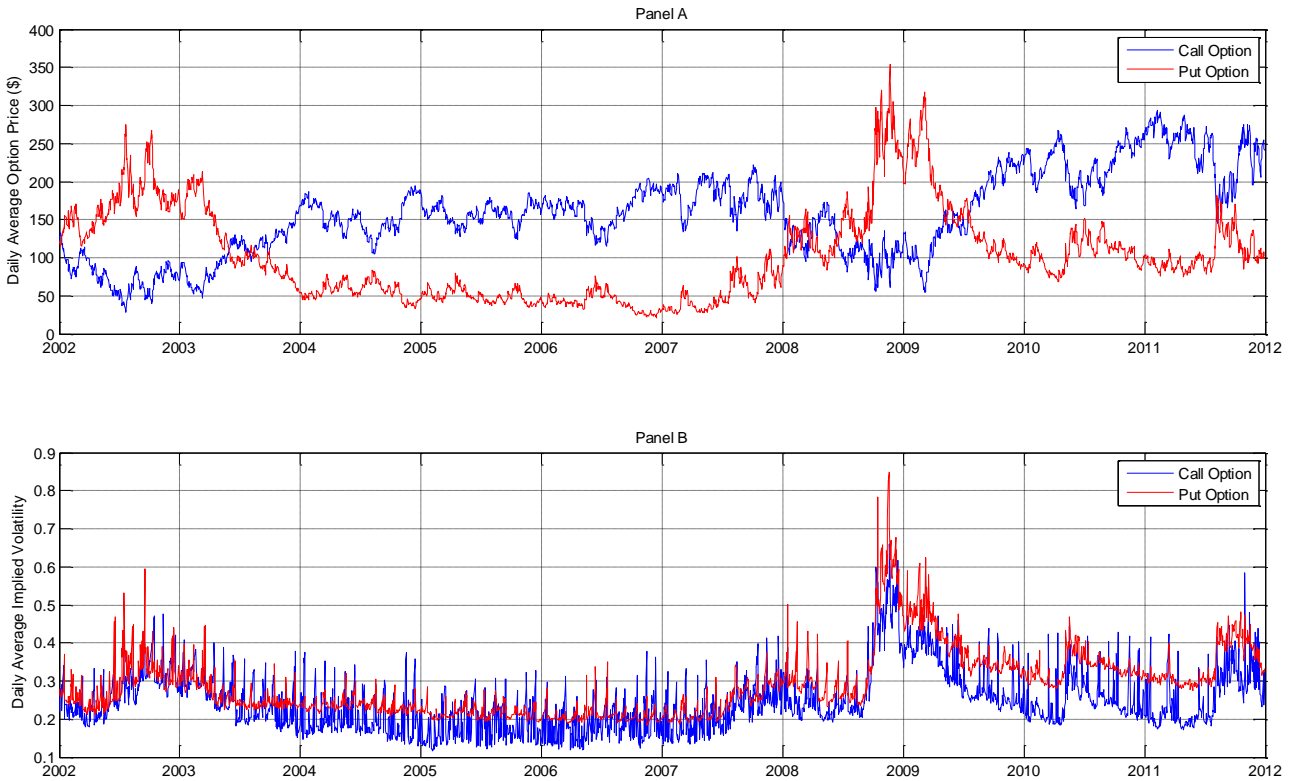
Besides the S&P 500 European option data, I use daily data of European-style S&P 500 options. Unlike the index values, the option data ranges from January 2<sup>nd</sup> 2002 until December 30<sup>th</sup> 2011. I downloaded the option data from the OptionMetrics database. This dataset includes for each day, for each option (distinguished by a unique option ID) the strike price ( $K$ ), the maturity ( $\tau$ ), the best bid and ask price on that day for that specific option, the implied volatility<sup>9</sup> and the option type (call or put).

Figure 2 presents the daily average option price for call and puts, together with the implied volatility. We notice two large shocks in the put prices (panel A) and in the implied volatilities (panel B), during 2002 and around the end of 2008, 10<sup>th</sup> October to be more specifically. These shocks typically represent the effects of the burst of the dotcom bubble and the recent financial crisis, respectively. Especially for the recent financial crisis period, the shocks in the put prices correspond to quickly decreasing S&P 500 index values and US Treasury rates (see Figure 3 later on), both negatively related to the value of put options. This explains the large positive shock in the put prices during the

<sup>9</sup> The implied volatility is calculated using the Black-Scholes model (see RiskMetrics Group (2009) for the implied volatility calculation description).

recent financial crisis. The shocks in the call prices, though, remain moderate during this period, while the implied volatility of call-options closely follows the implied volatility of puts. The implied volatilities in panel B also exhibit a spiky pattern. The reason for this pattern is the incorporation of in-the-money (ITM) options in the dataset. Compared to out-of-the money (OTM) options, ITM options have higher values, which correspond to higher levels of implied volatility.

**Figure 2: Daily average option prices and implied volatilities for January 2<sup>nd</sup> 2002 – December 30<sup>th</sup> 2011**



*Note: This figure shows the daily average prices (panel A) and implied volatilities (panel B) of S&P 500 call and put options for the period January 2<sup>nd</sup> 2002 to December 30<sup>th</sup> 2011. The daily-average call-price is calculated as the average option price of all call options on a certain day. Each option price is calculated as the average of the best bid and ask price. The average put prices are calculated similarly. The daily average implied volatility is calculated analogously to the daily average option price.*

### 4.2.1 Filtering the data

Following the research of Barone-Adesi, Engle and Mancini (2008), I filter the option data for only Wednesdays. I apply this filter to reduce the total number days in the sample period 2002 – 2011 from 2519 to 516, leading to a reduction of the option data observations from 2,674,422 to 435,455. In this way the required computation time decreases significantly. I present an overview of the some key statistics of this filtered data in Table 1. The data inhibits the volatility smile property, i.e. the Black-Scholes volatility decreases as options tend to become at-the-money (ATM). The same holds for the option prices itself. Regarding the option prices, an important point to note here is that option prices at a certain moneyness level are influenced by both in-the-money and out-of-the-money options e.g. at a moneyness level of 0.80, call-options are OTM, but put options on the other

hand are ITM. Since ITM prices of options are quite higher than the prices of OTM options, the average option price at a certain moneyness level is dominated by the prices of ITM options. Since ITM options decrease in prices as the option moves towards an ATM position, the average option price of both call and puts (ITM and OTM) show a smile-shape when depicted against the moneyness, measured by the ratio of the spot price ( $S$ ) and the strike price ( $K$ ). From Table 1 another basic property of options becomes clear i.e. as the maturity of options increases, the value of the option increases while the volatility and the bid – ask spread decrease.

**Table 1: Data Description for January 2<sup>nd</sup> 2002 – December 30<sup>th</sup> 2011**

Moneyness $S/K$		Maturity					
		Less than 60		60 to 160		More than 160	
		Mean	Std.	Mean	Std.	Mean	Std.
<0.85	Option Price \$	127.46	199.95	138.68	227.62	178.64	269.13
	$\sigma^{bs}$ (%)	52.87	40.68	28.34	11.08	21.52	6.86
	Bid-Ask (%)	114.31	93.64	102.19	90.55	48.21	69.23
	Observations	16213		16104		39230	
0.85-1.00	Option Price \$	36.20	47.47	56.77	53.00	100.37	64.90
	$\sigma^{bs}$ (%)	23.70	17.37	19.25	7.13	19.72	5.73
	Bid-Ask (%)	40.86	60.83	16.15	31.61	5.40	5.24
	Observations	48431		30697		38384	
1.00-1.15	Option Price \$	43.18	46.87	64.07	47.01	108.65	59.00
	$\sigma^{bs}$ (%)	28.96	17.83	23.81	7.69	22.41	6.01
	Bid-Ask (%)	18.04	29.94	6.35	5.15	4.12	3.02
	Observations	51257		27113		34770	
>1.15	Option Price \$	71.76	130.04	78.58	129.28	126.49	162.14
	$\sigma^{bs}$ (%)	55.38	35.12	39.91	15.75	32.06	11.46
	Bid-Ask (%)	76.58	74.20	50.52	64.34	21.94	37.89
	Observations	39938		33090		60228	

*Note: This table reports the mean, standard deviation (Std.) and the number of observations for each moneyness/maturity category of out-of-the-money S&P 500 options observed on Wednesdays from January 2, 2002 until December 30, 2011.  $\sigma^{bs}$  represents the Black-Scholes implied volatility. The bid ask spread (Bid-Ask) is calculated as  $100 \times (\text{ask price} - \text{bid price}) / \text{market price}$ , where the market price is assumed to be equal to the average of the bid and ask prices. The moneyness is the spot price divided by the strike price ( $S/K$ ). The maturity is measured in calendar days. This table is inspired by Table 1 of BAEM (2008).*

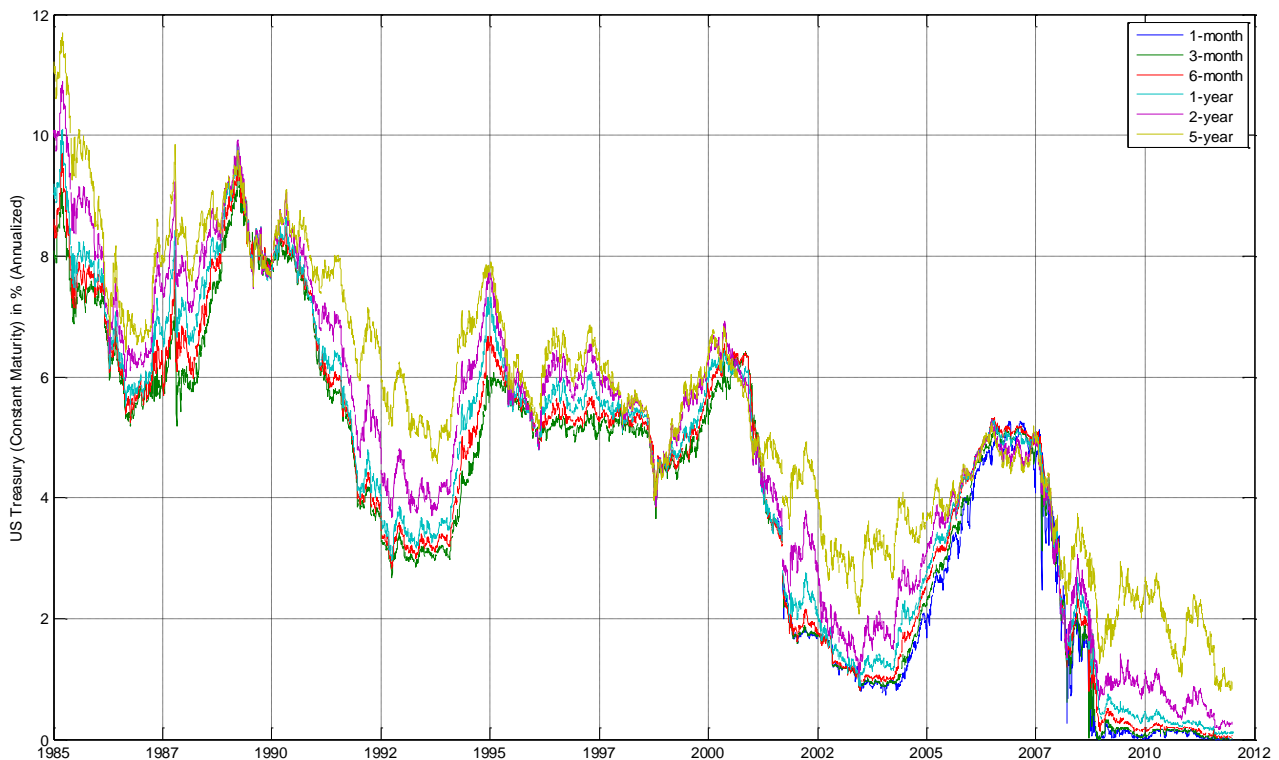
For the replication data to be similar to the data of BAEM (2008) who consider for the option data only the time-period of January 2<sup>nd</sup> 2002 to December 29<sup>th</sup> 2004, I filter the data even more. Moreover, in the data regarding the replication of BAEM (2008) I filter the data for only OTM options. Also, options with prices lower than 0.05\$, implied volatilities higher than 0.7 and time to maturities outside the interval of 10-360 days are discarded. This yields a total of 25371 observations spread over 155 Wednesdays. Compared to the data of BAEM (2008) there are some minor differences

which is because the filtered data of BAEM has two extra days as they incorporate for two unavailable Wednesdays the subsequent days, while I simply discard them in my data filtration.

### 4.3 US Treasury rates

To approximate relevant interest rates used for the option pricing, I use the 1-month, 3-month, 6-month, 1-year, 2-year and 5-year US Treasury Constant Maturity yields. These rates are also downloaded from the Economic Research website of the Federal Reserve Bank of St. Louis. I present the movements of the yields through time in Figure 3. Throughout the years there is a general decreasing trend in the US Treasury rates. Since interest rates are positively related to the value of calls, this would imply a decreasing trend in the call prices. However, this seems not to be the case if we look back at the graph of the option prices given in panel A of Figure 2. In contrast, there is a slightly increasing trend in the value of the call options which may be explained by the increasing value of the underlying in the same period (see Figure 1). For put options, one could argue its development similarly. For the recent financial crisis period, we also see a sudden decrease in the interest rate values, corresponding to the positive shocks in the put prices discussed before.

Figure 3: US Constant Treasury Yields

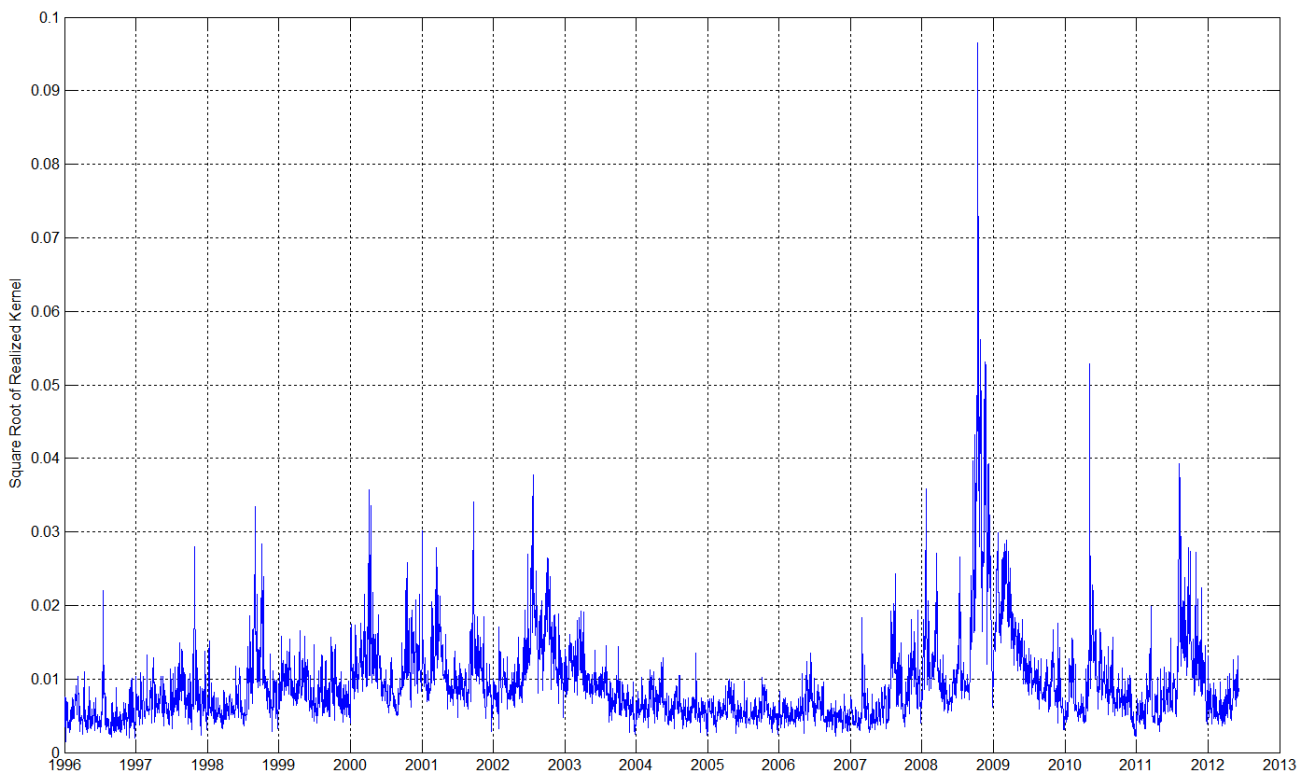


Note: This figure shows the US constant treasury yields in percentage for the 1-month, 3-month, 6-month, 1-year and 2-year bonds.

#### 4.4 The Realized Kernel

The implementation of the Realized GARCH model requires a realized measure. For this purpose, I use S&P 500 realized kernel data. Although the realized kernel data of HHS (2011) is available at the Journal of Applied Econometrics Data Archive<sup>10</sup>, I use realized kernel data ranging from January 3<sup>rd</sup> 1996 until December 30<sup>th</sup> 2011 similar to that of Shephard and Sheppard (2010), which I downloaded from the Oxford-Man Institute's realized library.<sup>11</sup> The reasoning behind this choice is that the data of HHS (2011) only covers the period January 1<sup>st</sup> 2002 until August 29<sup>th</sup> 2008. In order to apply the FHS-RGARCH method on option prices starting from 2002, realized kernel data prior to the year 2002 is needed for the estimation in the FHS procedure (step 1). Therefore, the realized kernel data of HHS is not enough to apply the FHS method on the RGARCH model starting from 2002. However, for the replication of HHS (2011) I use the S&P index returns (open-to-close) and the realized kernel data from the Journal of Applied Econometrics Data Archive. An overview of the square root S&P 500

**Figure 4: Square root of realized kernel for the S&P 500 for January 3<sup>rd</sup> 1996 to December 30<sup>th</sup> 2011**



*Note: This figure depicts the square root of the realized kernel for the period January 3<sup>rd</sup> 1996 to December 30<sup>th</sup> 2011. This kernel data is downloaded from the Oxford-Man Institute and is similar to the realized kernel as used in Shephard and Sheppard (2010).*

<sup>10</sup> <http://qed.econ.queensu.ca/jae/2012-v27.6/hansen-huang-shek/>

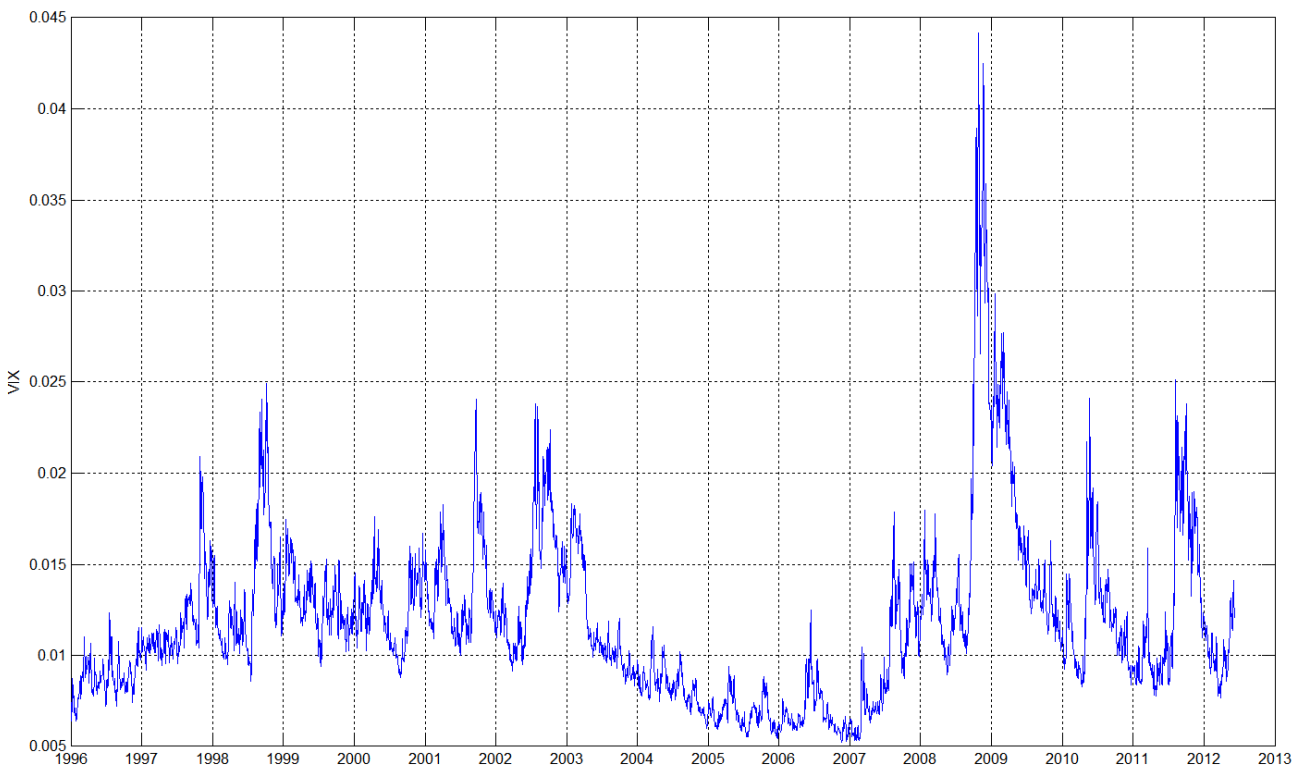
<sup>11</sup> Heber, Gerd, Asger Lunde, Neil Shephard and Kevin Sheppard (2009) "Oxford-Man Institute's realized library", Oxford-Man Institute, University of Oxford, Library Version 0.2.

realized kernel data from the Oxford-Man Institute is graphed in Figure 4. An eye catcher is the enormous shock at the end of 2008. This corresponds to the previously discussed shocks in the option prices and the S&P 500 index during the recent financial crises. Furthermore we see somewhat higher squared realized kernel values after the burst of the tech and dotcom bubble in 2000.

#### 4.5 The VIX Index

As described in section 3.4.3, I also look at the estimated volatility to evaluate the FHS-GJR and the FHS-RGARCH model. I compare the VaR models with VIX index. The VIX index data is downloaded from the official CBOE website.<sup>12</sup> The VIX as downloaded from the CBOE website is an indication of the expected volatility of the S&P 500 index. In the beginning the VIX as introduced by Whaley (1993) was based on eight at-the-money call and put options on the S&P 100 index. As the demand for OTM and ATM options and S&P 500 options increased during the 90s, the calculation of the VIX index was changed in 2003. The new VIX index is based on ATM and OTM S&P 500 options. The CBOE has made

Figure 5: Daily VIX index for the period 1996 - 2011



*Note: This figure shows the daily values of the not-annualized VIX index for the period 1996 – 2011. The data is downloaded from the official CBOE website. The index value is calculated using the new methodology, thus using ATM and OTM S&P 500 options.*

<sup>12</sup> <http://www.cboe.com/micro/vix/historical.aspx>

for the period 1990 to present datasets of both calculation methods available on their website. The VIX index I use in this thesis is based on the new methodology.

The daily VIX index is given in Figure 5. The VIX index looks quite similar to the square root of the realized kernel in Figure 4. However, the extremes of the VIX index appear to be smaller. The enormous shock in 2008 is clearly visible. Also, the high volatility values during the dotcom crisis appear from Figure 5.

#### 4.6 Sample Periods and Datasets Overview

As described in the previous subsections, there are several types of data used in this research. Also, depending on the purpose of the data, the sample periods differ. The latter partly depends on the purpose of the datasets. For the replication of BAEM (2008), the sample period of the option data ranges from January 2nd 2002 till December 29th 2004. Besides, I use the US Treasury rates for the period 2002 – 2004 for the replication of BAEM (2008). Replication of the results of BAEM (2008) also requires the use of S&P 500 index values of the period 1985 – 2004. The latter is of importance for the first step of the FHS procedure. For the replication of Hansen, Huang and Shek sample period simply covers the period 2002 – 2008. To analyse the FHS-RGARCH method empirically, I use the sample period 2002 – 2011 for the option data and the US Treasury rates and the period 1996 – 2011 for the S&P 500 index values and the realized kernel. The latter sample period is quite short compared to the S&P 500 index sample used for the replication of BAEM (2008) due to the fact that the available realized kernel data starts on January 3rd 1996 at its earliest. Since there are quite some datasets and different sample periods I use in this research, I present for the sake of clarity in a summarizing overview of the main purpose of the each dataset and the relevant sample-period in Table 2 on the next page.

**Table 2: Overview of the different datasets and sample periods**

<b>Purpose of use</b>	<b>Type of Data</b>	<b>Sample-period</b>
Replication of the results of BAEM (2008) as far as the FHS method is concerned (see Section 5.1)	S&P 500 index values from OptionMetrics	1985 – 2004
	S&P 500 option data filtered for Wednesdays and OTM options with certain properties (see section 4.2.1)	2002 – 2004
	US Treasury Constant Maturity rates from the Federal Reserve Bank of St. Louis	2002 – 2004
Replication of the results of HHS (2008) for the RGARCH (1,1) model (see Section 5.2)	S&P 500 open-to-close and close-to-close returns from the Journal of Applied Econometrics Data Archive	2002 – 2008
	Realized Kernel data of HHS (2011) from the Journal of Applied Econometrics Data Archive	2002 – 2008
Application of the FHS-RGARCH method and the FHS-GJR method for comparison (see Section 5.3)	S&P 500 index values from OptionMetrics	1996 – 2011
	S&P 500 option data filtered for Wednesdays only	2002 – 2011
	Realized kernel data as used by Shephard and Sheppard (2010) from the Oxford-Man Institute	1996 – 2011
	US Treasury Constant Maturity rates from the Federal Reserve Bank of St. Louis	2002 – 2011
	VIX data from the official CBOE website, based on the new calculation methodology.	2002 – 2011

*Note: This table gives an overview of the several datasets used in this research. For each main purpose, the used data type and the relevant sample-period are given.*



## 5 EMPIRICAL RESULTS

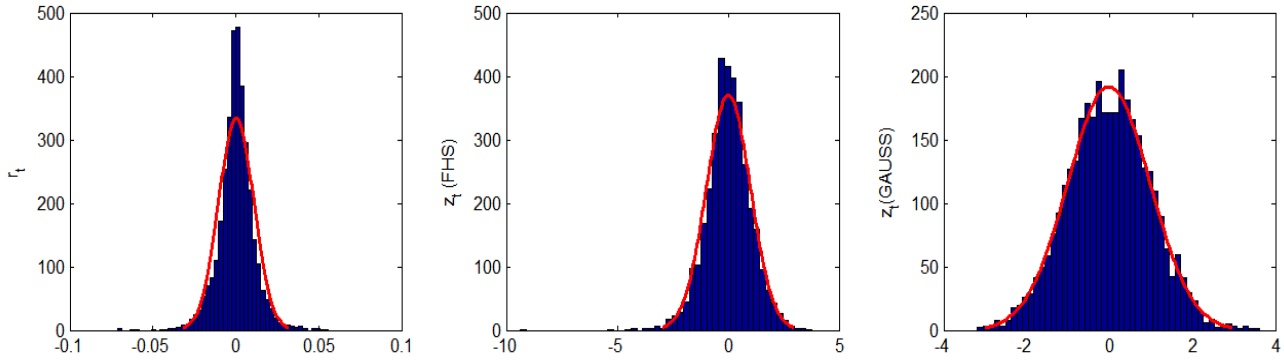
Up till now, I have described the methods and the data I use to study whether applying the filtered historical simulation method with the RGARCH improves the option pricing method of BAEM (2008). In this section I report the results of the empirical analysis for the methods described in Chapter 3. This section consists of three subsections. The first subsection reports the results of the replication of BAEM (2008) as far as the FHS approach is concerned. Also, this subsection reports the results for applying the FHS-GJR method on both OTM and ITM options for the period 2002 – 2004. The results are compared using the statistical criteria and the state-price-densities. The second subsections presents estimation results of the RGARCH model using the data of HHS (2011) and the realized kernel from the Oxford-Man Institute. Finally, the last subsection reports the results of the FHS-GJR and the FHS-RGARCH applied on the period 2002 – 2011 for both OTM and ITM options. These results are compared not only using statistical criteria and the state-price-densities, but VaR estimates as well.

### 5.1 GJR-GARCH with Filtered Historical Simulation

This section presents the results of the FHS-GJR method applied on the data similar to BAEM (2008), for each Wednesday in the period 2002 – 2004. I report not only the results for the OTM option data but also for the ITM and OTM option data together. The latter results provide insight on the goodness of the FHS-GJR method for both OTM and ITM option pricing. To obtain the results, the length of the estimation window  $n$  is set equal to 3500 days (13 years approximately), while the number of simulation paths  $L$  for step 5 of the FHS procedure is set to 1000 (similar to BAEM).

I present the results for the FHS-GJR method applied using (i) FHS innovations and (ii) Gaussian innovations. The FHS innovations are obtained by randomly drawing estimated past innovations (see step 3 of the FHS procedure in Section 3.1), while the Gaussian innovations are randomly drawn from the standard normal distribution. Histograms of these innovations together with a histogram of the returns are presented for an arbitrary day, July 9<sup>th</sup> 2003, in Figure 6. The returns are clearly not normally distributed, which corresponds to the general characteristics of return series. This characteristic is closely followed by the FHS innovations, which is indeed one of the advantages of the FHS method. Moreover, it seems that the histograms of the FHS innovations and the return series indicate similar distributions, apart from the fact that the FHS innovations are standardized. This is one of the advantages of the semi-parametrical property of the FHS method. The normally distributed GAUSS innovations on the other hand, closely follow the standard normal distribution, which is obvious since I assume standard normality for the simulated GAUSS innovations.

Figure 6: Histograms of the returns, the FHS and Gaussian innovations for July 9<sup>th</sup> 2003



Note: This figure presents histograms together with the theoretical distributions. For the return series, the FHS and the GAUSS innovations, this is assumed to be the normal distribution. Although the FHS method does not require any distribution assumption, for comparison purposes, the normal distribution is plotted.

Table 3 presents the estimations and calibrations for  $\mu$  together with the short term treasury rates. The calibration yields negative, though insignificant, values for  $\mu$ , which are obtained under the risk-neutral measure  $\mathbb{Q}$ . However, compared to the treasury rates the calibrated  $\mu$  values differ from the significant fixed treasury rates, which could be due to the fact that the latter are also influenced by regulation. From this table we can also see that the GJR estimation of  $\mu$  is quite higher than the calibrated values and the short term treasury rates. As argued previously in Section 4.1, this is due to the fact that the GJR estimation value is based on historical data ( $\mathbb{P}$  measure), while the calibrated average return ( $\mu^*$ ) and the short term treasury rates represent the risk-neutral world. For the calibration of the GJR parameters of the GARCH equation ( $\theta$ ) it is appropriate to either use short term treasury rate or the calibrated values for  $\mu$ . Therefore, in further analyses, the average return is, in accordance with BAEM (2008), fixed at the 1-month treasury rate during the calibration of the GARCH equation parameters.

Table 3: Estimated and calibrated values for  $\mu$  and the short term interest rate for January 2nd 2002 – December 29th 2004

	$\hat{\mu}_{GJR}$ (%)		$\mu_{GAUSS}^*$ (%)		$\mu_{FHS}^*$ (%)		$r_{1\text{ month}}$ (%)		$r_{3\text{ month}}$ (%)	
	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.
<b>2002</b>	12.87	0.74	-0.60	1.24	-1.18	1.55	1.63	0.18	1.64	0.19
<b>2003</b>	10.87	0.38	-1.36	0.94	-1.39	1.06	1.02	0.13	1.03	0.11
<b>2004</b>	11.48	0.42	-0.54	1.83	-0.74	1.93	1.27	0.40	1.40	0.45

Note: This table reports the average values of  $\mu$  for the different methods and the 1-month and 3-month rates for US T-Bills with constant maturities. The rates are annualized assuming 365 days each year and are given in percentages.  $\mu_{FHS}^*$  and  $\mu_{GAUSS}^*$  are calibrated using the GJR model on only OTM options.

### 5.1.1 OTM Options

#### 5.1.1.1 Statistical Criteria

Table 4 presents several pricing error criteria for the FHS-GJR method when applied on only OTM options for the period 2002 – 2004. The pricing errors for the calibration using Gaussian innovations (GAUSS) are generally a bit smaller than the pricing errors of the calibration using FHS innovations. For example, the FHS innovations yield a mean absolute error of 1.77, while the Gaussian innovations result in a mean absolute error of 1.60. A possible reason for a slightly better performance of the Gaussian innovations may be the fact that the past does not guarantee for the future. Therefore, the historical distribution of the FHS method may fall short to describe future possible shocks, while the Gaussian distribution assumes equal probability of positive and negative shocks. For both methods, though, we see that the minimum and maximum pricing errors are more or less similar. Also, a bit less than a half of the pricing errors are positive, indicating that the options are slightly underpriced during the replication. Compared to the results of BAEM (2008), the pricing errors in the replication analysis are a bit higher. However, further investigation on the pricing errors of both calibration methods still indicate correctness of the FHS-GJR method, since the absolute pricing error series have a standard deviation of 2.11 dollars for the FHS method and 1.91 dollars for the GAUSS method, implying insignificance of the absolute pricing error series. For some additional results on the replication of BAEM (2008), I refer to Appendix C.

**Table 4: Pricing error statistics for the calibration of OTM options for the years 2002 – 2004**

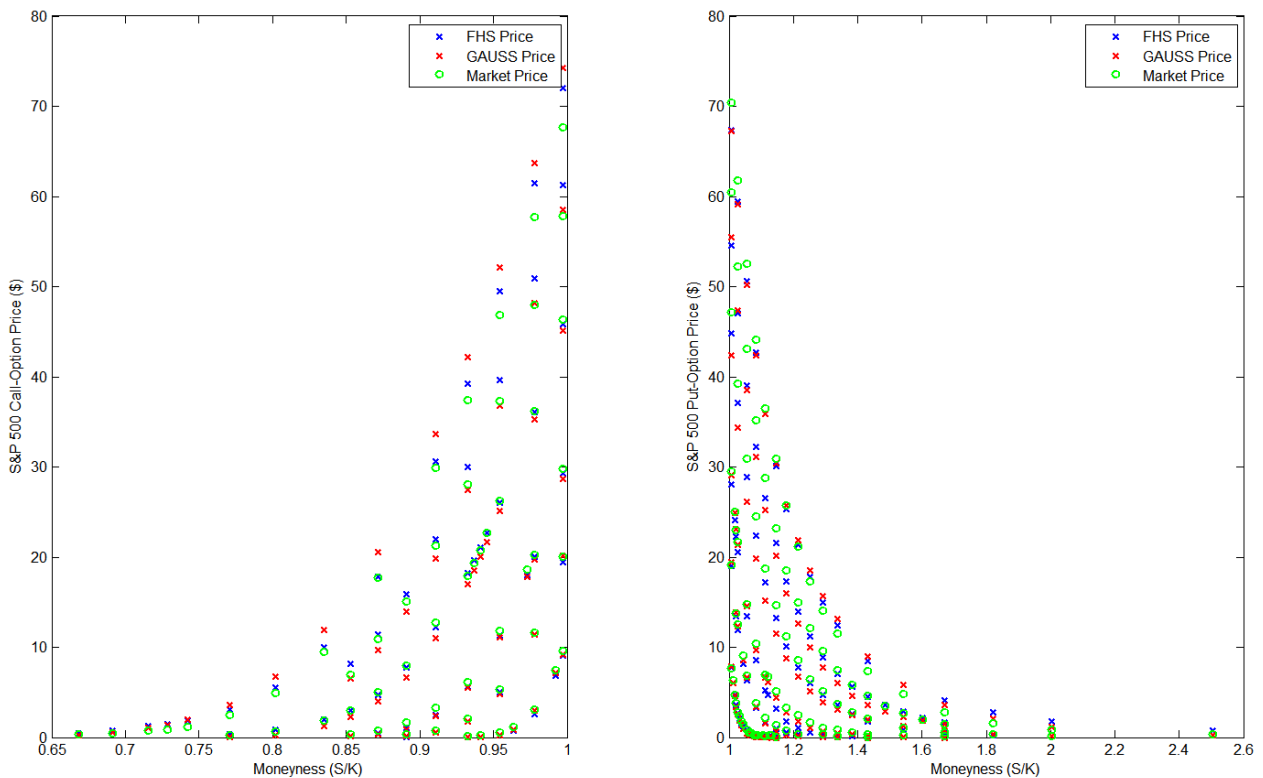
	RMSE	MAE	MOE	Min	Max	Err>0%	ErrBD%	MAE%	MOE%
<i>FHS</i>	2.75	1.77	1.28	-19.73	15.80	46.28	178.23	35.01	23.40
<i>GAUSS</i>	2.49	1.60	1.12	-19.16	12.94	42.67	159.02	29.44	17.88
<i>FHS (BAEM (2008))<sup>13</sup></i>	0.87	0.44	0.08	-6.02	4.64	50.50	23.69	21.16	10.75

*Note: This table reports the aggregate pricing errors statistics for the calibrations applied on out-of-the-money S&P 500 options. RMSE stands for the root mean square error of the dollar pricing error (model price–market price); MAE is the dollar average absolute pricing error and MOE is the dollar average pricing error outside the bid-ask spread (MAE% and MOE% are in relative terms as a percentage of the market price); Min (Max) is the minimum (maximum) pricing error; Err>0% is the percentage of positive pricing errors and ErrBD% is the average absolute pricing error as a percentage of the bid-ask spread. Inspired by Table 5 of BAEM (2008).*

We can also observe the correctness of the calibration methods when we look at Figure 7. In this figure we can see an example of how closely the model prices and market prices are from each other for the arbitrary day, July 9<sup>th</sup> 2003. It seems that the prices according to the model do not differ much from the market prices.

<sup>13</sup> Retrieved from Barone-Adesi, Engle and Mancini (2008), Table 5.

Figure 7: Call and put prices as determined by the FHS-GJR method for OTM options on 9 July 2003



Note: This figure shows the option prices for the FHS-GJR method based on FHS and Gaussian innovations, together with the market price for out-of-the-money options on July 9<sup>th</sup> 2003. The price of calls (puts) is plotted on the left (right) side. Moneyness is the ratio of the S&P 500 index value  $S$  over the strike price  $K$ . Inspired by figure 2 of BAEM (2008).

### 5.1.1.2 State-Price-Density Analysis

Another way to evaluate the models is by looking at the state-price-density per unit probability ( $M_{t,T}$ ). The numerical values of  $M_{t,T}$ , with  $T = t + \tau$  for 4 different maturities for July 9<sup>th</sup> 2003 are given in Table 5. It appears that  $M_{t,T}$  is decreasing as the state-price of the S&P 500 is increasing. Since the change of measure may also be interpreted as the shadow price of risk per unit probability (see Persson (1994)), for OTM options the decreasing trend of  $M_{t,T}$  implies that for higher state prices agents are less willing to take risk and therefore require higher risk premiums, which is in accordance with economic theory. The reasoning is as follows. Because  $M_{t,T}$  is the change of measure from  $\mathbb{P}$  to  $\mathbb{Q}$ , low values of  $M_{t,T}$  imply higher probabilities under the historical measure compared to the risk-neutral measure. Therefore, when switching from a risk-neutral price to a risk containing price, the agent requires a higher risk-premium.

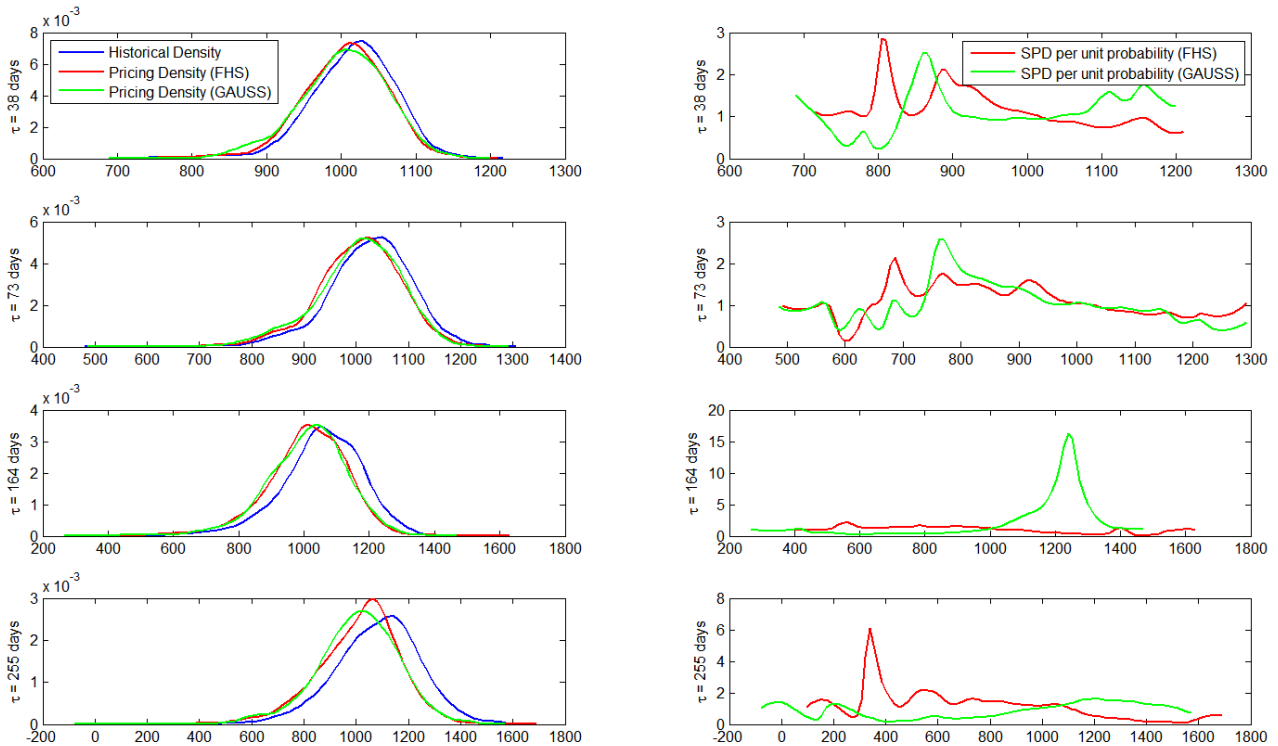
The generally decreasing trend of  $M_{t,T}$  also becomes clear from Figure 8 (except for the GAUSS innovations for a time to maturity of 164 days). From this figure, it also seems that the SPDs are slightly negatively skewed, implying overall small positive returns, while less frequent but more extreme negative returns. The negative skewness characteristic is in accordance with financial markets.

**Table 5: State-price-densities per unit probability on July 9<sup>th</sup> 2003 for different maturities using OTM options**

S&P 500		$M_{t,t+38}$	$M_{t,t+73}$	$M_{t,t+164}$	$M_{t,t+255}$
800	FHS	1.99	1.18	1.57	1.59
	GAUSS	2.08	1.54	1.59	1.55
	FHS (BAEM (2008))	1.72	1.37	1.56	1.69
900	FHS	1.48	1.41	1.71	1.4
	GAUSS	1.65	1.18	1.69	1.67
	FHS (BAEM (2008))	1.26	1.27	1.48	1.64
1000	FHS	1.06	1.15	1.32	1.53
	GAUSS	1.03	1.22	1.29	1.36
	FHS (BAEM (2008))	1.08	1.15	1.22	1.26
1100	FHS	0.7	0.67	0.8	1.01
	GAUSS	0.68	0.7	0.82	1.02
	FHS (BAEM (2008))	0.64	0.71	0.84	0.9
1200	FHS	0.29	0.56	0.44	0.55
	GAUSS	0.44	0.39	0.45	0.57
	FHS (BAEM (2008))	0.2	0.36	0.48	0.59

Note: This table reports the state price densities per unit probability of  $\tau = 38, 73, 164$  and  $255$  days, estimated on July 9th 2003.  $M_{t,t+\tau}$  is the discounted ratio of the pricing over the historical densities of the calibrated GJR model. The FHS(BAEM(2008)) values are retrieved from Barone-Adesi, Engle and Mancini (2008), Table 10

**Figure 8: SPDs and SPD per unit probability estimates for different maturities**



Note: This figure depicts the pricing and historical density estimates for different time horizons in the left graphs. The pricing densities are calibrated on the cross-section of OTM options using both FHS and Gaussian innovations for the GJR model. The historical densities are obtained by fitting the GJR model using  $n=3500$  historical returns. The right graphs show the SPD per unit probability calculated as the discounted value of the pricing density over the historical densities.

5.1.2 ITM and OTM Options

5.1.2.1 Statistical Criteria

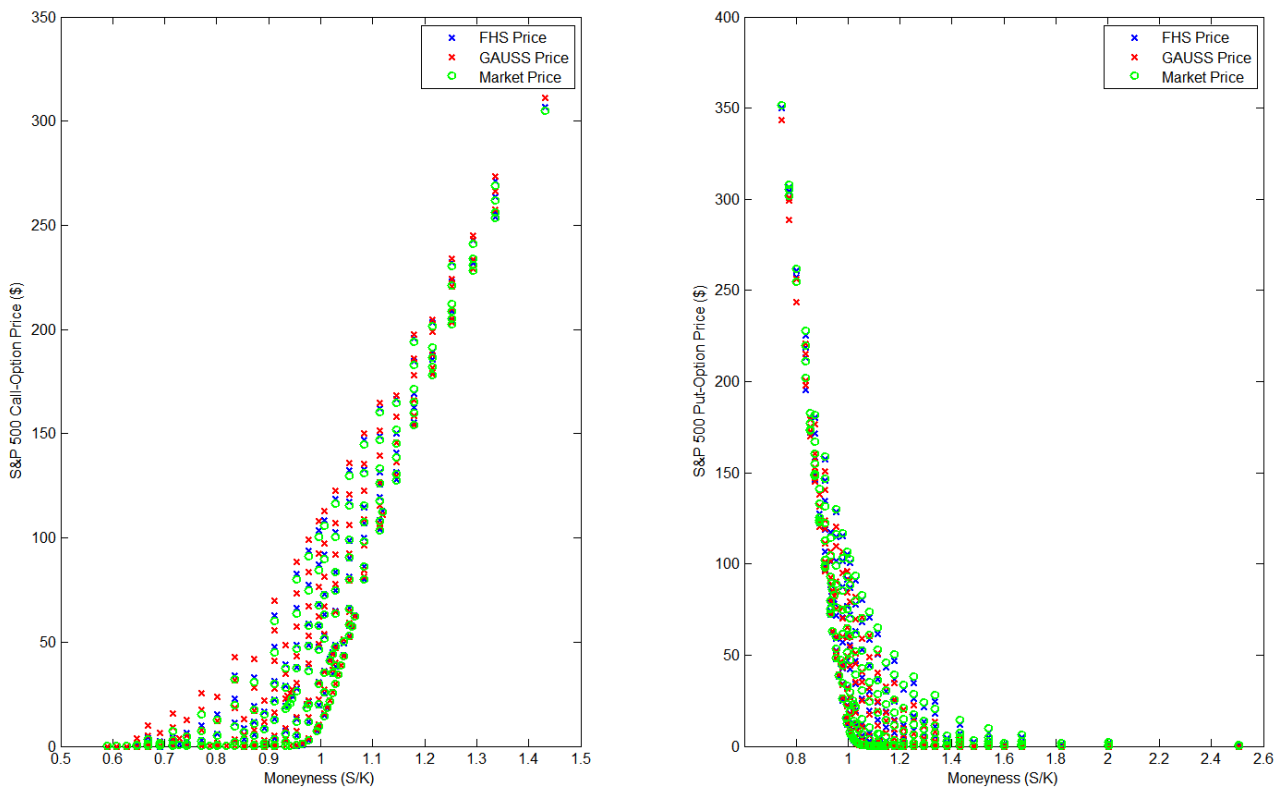
Since I use both ITM and OTM options for the FHS-RGARCH method, I also present some results of the FHS-GJR method applied on both ITM and OTM options for the period 2002 – 2004. In this way I can analyse whether incorporating all options in the dataset may significantly influence the results of the FHS-GJR method. The pricing error criteria for this period for both ITM and OTM options are given in Table 6.

Table 6: Pricing error statistics for the calibration of ITM and OTM options for the years 2002 – 2004

	RMSE	MAE	MOE	Min	Max	Err>0%	ErrBD%	MAE%	MOE%
FHS	4.53	3.11	2.37	-35.01	26.97	41.16	205.08	32.32	17.97
GAUSS	4.41	3.06	2.31	-35.15	27.17	40.52	198.24	31.51	17.05

Note: This table reports the aggregate pricing errors statistics for the calibrations applied on both OTM and ITM S&P 500 options for the period 2002 – 2004. See Table 4 for legend.

Figure 9: Call and put prices as determined by the FHS-GJR method for ITM and OTM options on 9 July 2003



Note: This figure shows the option prices for the FHS-GJR method based on FHS and Gaussian innovations, together with the market price for in-the-money and out-of-the-money options on July 9<sup>th</sup> 2003. The price of calls (puts) is plotted on the left (right) side. Moneyness is the ratio of the S&P 500 index value S over the strike price K.

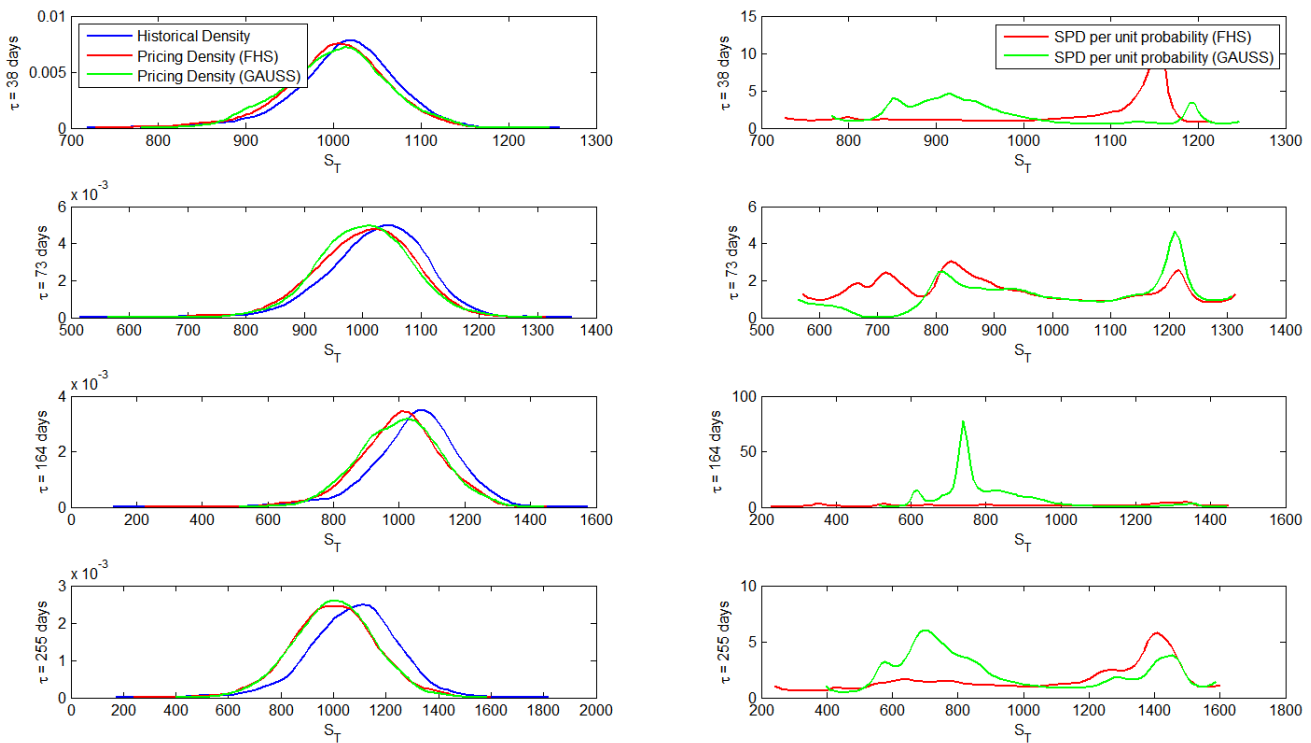
If we compare Table 6 with Table 4, it seems that the pricing errors increase as I incorporate ITM options in the dataset. A possible explanation for this is that the prices of ITM options are quite different from OTM options. In this way the calibrated model has to accommodate both ITM and

OTM options in correct option pricing. This makes it more difficult to find one set of parameters that fits all. However, further investigation of these pricing errors point out again that the FHS-GJR method generally performs well, i.e. the standard deviations of the absolute pricing errors of the FHS and GAUSS calibration methods are 3.29 and 3.17 respectively. This indicates insignificant absolute pricing error series for the FHS-GJR method, even when ITM options are incorporated. I present an overview of the model performance for July 9<sup>th</sup> 2003 in Figure 9. We can see from this figure that prices determined by the FHS-GJR methods are quite close to the market prices. In short, the FHS-GJR method is also able to determine the prices of in-the-money options quite well.

### 5.1.2.2 State-Price-Density Analysis

For ITM and OTM options, the interpretation of  $M_{t,T}$  becomes a bit ambiguous, since lower state prices influence both ITM puts and OTM call options. Figure 10 gives the pricing and historical densities, for OTM and ITM options together with the change of measure. It appears from this figure that similar to the situation of OTM options, the SPDs are a bit negatively skewed and the pricing

Figure 10: SPDs and SPD per unit probability estimates for different maturities for ITM and OTM options



Note: This figure depicts the pricing and historical density estimates for different time horizons in the left graphs. The pricing densities are calibrated on the cross-section of OTM options using both FHS and Gaussian innovations for the GJR model. The historical densities are obtained by fitting the GJR model using  $n=3500$  historical returns. The right graphs show the SPD per unit probability calculated as the discounted value of the pricing density over the historical densities.

densities of the FHS and GAUSS are quite close. For the change in measure, there is no overall trend. A possible reason for this is that since both OTM and ITM options are included, the information contained by the SPD per unit probability can be hardly allocated to a certain option type.

## 5.2 Realized GARCH

In the previous subsection I presented some results relating to the replication of BAEM (2008). In this subsection I report the results regarding the estimation of the RGARCH model given in Equation (2). I present the results for the replication of HHS (2011) as described in Section 3.2. Also, I show the results obtained from estimation of the RGARCH model using realized kernel data of the Oxford-Man institute. Finally, to gain some insight in the effect of incorporating an average return parameter ( $\mu$ ), I report the results of the parameter estimation without any restrictions on  $\mu$ .

The results for the estimation of the RGARCH model using the data of HHS (2011) are presented in Table 7. This table presents different estimation results, namely: (i) estimation using the data of HHS (2011), (ii) the results of HHS (2011), (iii) the results when the RGARCH model is estimated using the realized kernel data of the Oxford-Man institute when  $\mu$  is restricted to zero and (iv) estimation results using the Oxford-Man data when  $\mu$  is unrestricted. The results are based on returns scaled in percentages. When we compare the results for the replication of HHS (2011), the estimated RGARCH parameters are quite similar to the results of HHS (2011), while the standard deviations of the estimates are a bit smaller. Though, there is much resemblance between the replicative results and that of HHS (2011).

To investigate the effect of the different datasets, we can compare the estimation results of the HHS replication to that of the Oxford-Man Institute with  $\mu$  fixed, for the period 2002 – 2008. It seems that using the kernel data of the Oxford Man Institute produces in general similar results as that of HHS (2011). Besides the difference between the use of realized kernel data from Hansen, Huang and Shek and the realized kernel data from the Oxford Man Institute, we can also conclude from Table 7 what the effect is of incorporating the years 2009 – 2011. It seems that there is no major difference in the estimated parameters when the latter period is added to the sample. This could probably be due to the fact that a part of the recent financial crises already is incorporated in the period 2002 – 2008.

Finally, from Table 7 it also becomes clear that containing an average return parameter ( $\mu$ ) does not lead to major changes in the estimated values of the other parameters, which corresponds to the insignificance of the latter parameter when we look at the mean and the standard deviation of  $\mu$ . Concluding, the good performance results of the RGARCH model, does not fall short, when



estimating the RGARCH model with a parameter for the average return for the period 2002 – 2011 using the realized kernel data of the Oxford-Man Institute.

**Table 7: Estimation results for the RGARCH model using the realized kernel data of HHS (2011) and the data from the Oxford-Man Institute**

	Replication of HHS (2011)				Oxford-Man Institute $\mu$ restricted				Oxford-Man Institute $\mu$ unrestricted			
	2002 – 2008		Results of HHS <sup>1</sup>		2002 – 2008		2002 – 2011		2002 – 2008		2002 – 2011	
	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.
$\hat{\mu}$	0		0		0		0		0.00	0.02	0.01	0.02
$\hat{\omega}$	0.07	0.02	0.06	0.02	0.15	0.02	0.17	0.01	0.15	0.02	0.17	0.01
$\hat{\beta}$	0.53	0.03	0.55	0.03	0.64	0.02	0.61	0.02	0.64	0.02	0.61	0.02
$\hat{\psi}$	0.43	0.03	0.41	0.03	0.35	0.02	0.38	0.02	0.35	0.02	0.37	0.02
$\hat{\xi}$	-0.19	0.04	-0.18	0.05	-0.43	0.03	-0.45	0.03	-0.43	0.03	-0.45	0.03
$\hat{\phi}$	1.03	0.04	1.04	0.06	0.98	0.03	0.98	0.03	0.98	0.03	0.98	0.03
$\hat{\kappa}_1$	-0.06	0.01	-0.07	0.01	0.04	0.01	0.03	0.01	0.04	0.01	0.03	0.01
$\hat{\kappa}_2$	0.07	0.01	0.07	0.01	0.03	0.01	0.03	0.01	0.03	0.01	0.03	0.01
$\sigma_v$	0.38	0.01	0.38	0.08	0.51	0.01	0.52	0.01	0.51	0.01	0.52	0.01

*Note: This table reports the estimated values of the parameters of the Realized GARCH model. The table reports the estimates using the data from HHS (2011) together with the results of HHS (2011) and the parameter estimates when the realized kernel data of the Oxford-Man Institute is used. Since the Realized GARCH model as estimated in HHS (2011) does not allow for an average in the return series, the parameter  $\mu$  is fixed equal to 0 and therefore has no standard deviation.*

### 5.3 Realized GARCH with Filtered Historical Simulation

In this section I present the empirical results for the application of the FHS-RGARCH method.<sup>14</sup> The results I present are based on ITM and OTM options for the sample January 2<sup>nd</sup> 2002 – December 30<sup>th</sup> 2011. Besides the results of the FHS-RGARCH method, for comparability, I also present the results of the FHS-GJR method applied to ITM and OTM options for the latter sample period. The results are obtained using an estimation window length of  $n = 1472$  days (approximately 4 years) for the first step of the FHS procedure. The reason for the smaller window length compared to previous results in Section 5.1 is that the realized kernel data is available from 1996. In order to decrease the computation time, the number of simulation paths used in the FHS procedure ( $L$ ) is set equal to 100. From a conducted simulation study it appears that the reduction in the number of simulation paths used in the FHS procedure does not have a major influence on the results. See Appendix B, for the simulation study.

<sup>14</sup> Some additional results for the FHS-RGARCH method are presented in Appendix D.

### 5.3.1 Statistical Criteria

Table 8 presents some price error statistics for the BAEM and the FHS-RGARCH method for 2002 – 2011. Except for the results of the calibrations, this table also present the pricing errors resulting from option prices determined by parameter estimation instead of calibration (see the GJR and RGARCH rows). For completeness, the results for the FHS-GJR method applied on the period 2002 – 2004 are presented again.

Looking at Table 8, it seems that for the FHS-GJR method, the calibrated parameters with the Gaussian innovations perform the best. This is in line with results for the period 2002 – 2004 already presented in Table 6. Also, it seems that comparing the FHS-GJR method for 2002 – 2008 to the results of the period 2002 – 2004, the pricing errors seem to increase. This is because the former sample period includes the financial crisis of 2008 which is a period with higher volatilities. Therefore, it becomes more difficult to price options more accurately which leads to higher pricing errors. However, looking at the mean and the standard deviation of the absolute pricing errors, the BEAM method is still able to price the options appropriately, even for the period 2002 – 2011.

**Table 8: Pricing error statistics for the calibration of ITM and OTM options for the period 2002 – 2011**

	RMSE	MAE	MOE	Min	Max	Err>0%	ErrBD%	MAE%	MOE%	SAE
<b>FHS-GJR (2002 – 2004)</b>										
<i>GJR</i>	51.22	29.52	28.70	-176.32	280.42	44.50	1438.69	93.43	78.57	41.86
<i>FHS</i>	4.53	3.11	2.37	-35.01	26.97	41.16	205.08	32.32	17.97	3.29
<i>GAUSS</i>	4.41	3.06	2.31	-35.15	27.17	40.52	198.24	31.51	17.05	3.17
<b>FHS-GJR (2002 – 2011)</b>										
<i>GJR</i>	35.30	18.72	17.63	-348.02	342.74	42.26	691.62	83.73	66.73	29.93
<i>FHS</i>	22.98	12.46	11.37	-172.78	244.16	40.18	482.97	72.72	55.78	19.31
<i>GAUSS</i>	19.69	11.10	10.02	-192.06	193.36	38.09	443.28	64.71	47.74	16.26
<b>FHS-RGARCH (2002 – 2011)</b>										
<i>RGARCH</i>	39.60	19.68	18.59	-416.52	374.20	43.06	886.80	115.46	98.42	34.36
<i>FHS</i>	8.14	5.34	4.29	-129.37	65.29	42.31	285.24	80.22	63.27	6.14
<i>GAUSS</i>	8.41	5.62	4.57	-106.87	64.05	42.18	303.91	95.49	78.44	6.25

*Note: This table reports the aggregate pricing errors statistics for the FHS-GJR method and the FHS-RGARCH method, when the calibrations are applied on OTM and ITM S&P 500 options for the period January 2<sup>nd</sup> 2002 till 30<sup>th</sup> December 2011. GJR and RGARCH indicate the pricing errors resulting from option prices determined by the estimated model parameters. FHS and GAUSS indicate the pricing errors resulting from the option prices determined by the calibrated parameters using FHS and Gaussian innovations respectively. RMSE stands for the root mean square error of the dollar pricing error (model price–market price); MAE is the dollar average absolute pricing error and MOE is the dollar average pricing error outside the bid-ask spread (MAE% and MOE% are in relative terms as a percentage of the market price); Min (Max) is the minimum (maximum) pricing error; Err>0% is the percentage of positive pricing errors and ErrBD% is the average pricing error as a percentage of the bid-ask spread. SAE is the standard absolute error which is the standard deviation of the absolute pricing errors.*

If we look at the results for the FHS-RGARCH method in Table 8, it seems that the calibration methods are much better than option pricing based on the QML estimated parameters of the RGARCH model. Moreover, the FHS method seems to perform the best, though, the Gaussian innovations yield smaller extreme pricing errors. Compared to the FHS-GJR method, the FHS-RGARCH method is performing much better, whether using FHS or GAUSS innovations. However, this does not seem to be the case if the options are priced using estimated parameters instead of calibrated parameters.

It is interesting to examine how close the model prices are to the market price. Therefore, for both the FHS-GJR method and the FHS-RGARCH method, I present the model prices and the market prices for July 9<sup>th</sup> 2003 in Figure 11. It seems that the GJR model prices are closer to the market prices when the estimated parameters are used. Surprisingly, the estimated RGARCH model seems to perform very bad in determining the option prices, although it incorporates extra information through the realized measure. However, the prices based on the calibrated parameters of the RGARCH model (the FHS-RGARCH method) seem to lie closer to the market prices compared to the GJR model. This is in line with the results observed in Table 8.

Since the BAEM and the FHS-RGARCH methods are option pricing techniques, it may be interesting to observe how well the model prices are compared to the market prices for a financial crises period. As mentioned before, the enormous drop in the S&P 500 value occurs on October 10<sup>th</sup> 2008. Therefore, I have plotted for both methods the model and market prices for the next Wednesday, which is October 15<sup>th</sup> 2008, in Figure 12. From this figure it clearly appears that the GJR model is unable to produce accurate option prices on such a volatile day, whether it uses estimated or calibrated parameters. On the other hand, the RGARCH model performs quite well on this highly volatile day.

To get an impression of the pricing errors during the whole period of 2002 – 2011, I present in Figure 13 for call (panel A) and put (panel B) options for 2002 – 2011 the pricing errors of the FHS method applied on a GJR model (BAEM) and the pricing errors of the FHS method applied on a RGARCH model (FHS-RGARCH). It is remarkable that the FHS-RGARCH performs consistently well, though we observe a slight increase of the pricing errors during the recent financial crises period. The FHS-GJR method on the other hand is submitted to the state of the financial markets, since the pricing errors for the period 2007 – 2009 are relatively high (especially for put options). Another point that becomes clear from Figure 13 is that for the FHS-GJR method the pricing errors are generally positive (negative) for call (put) options, implying overpricing (underpricing) of the call (put) options. The FHS-RGARCH method on the other hand does not exhibit this problem.

5 Empirical Results

Figure 11a: Option prices for the FHS-GJR method on July 9<sup>th</sup> 2003

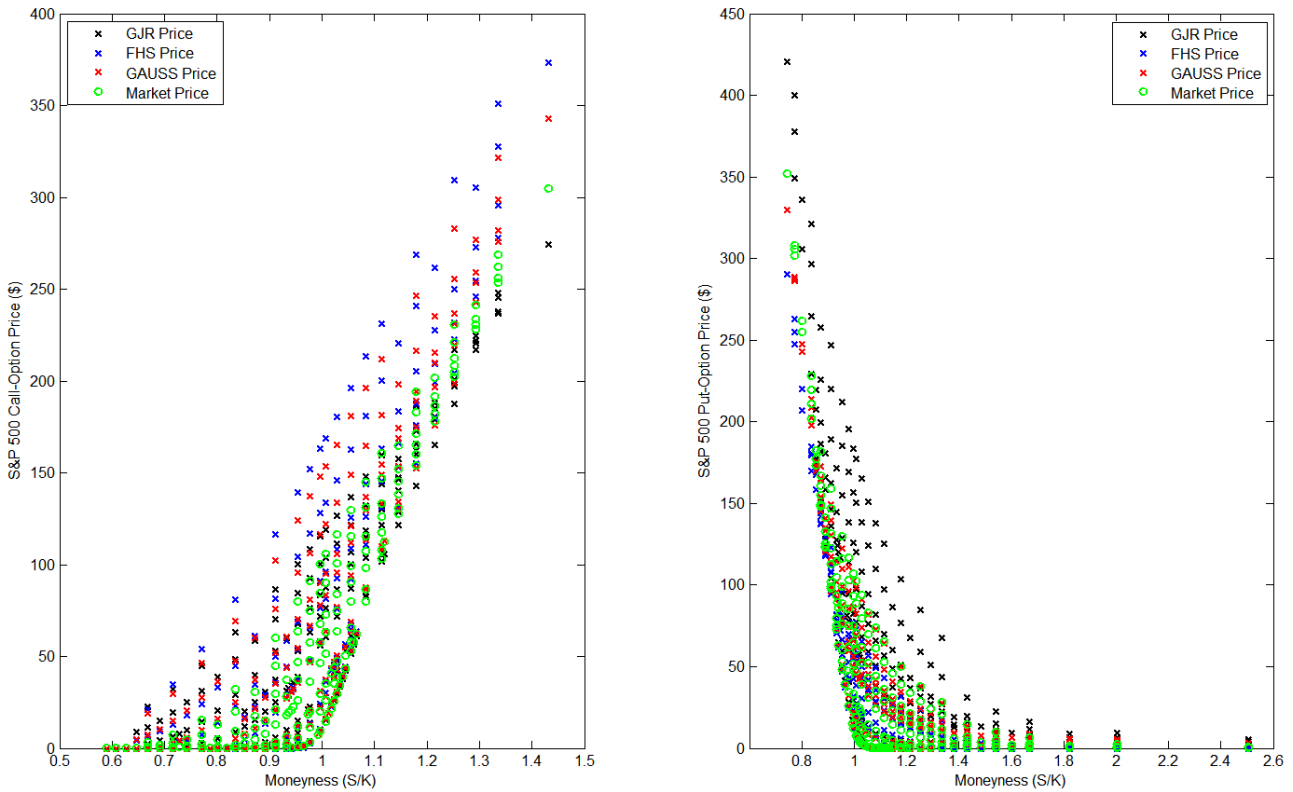
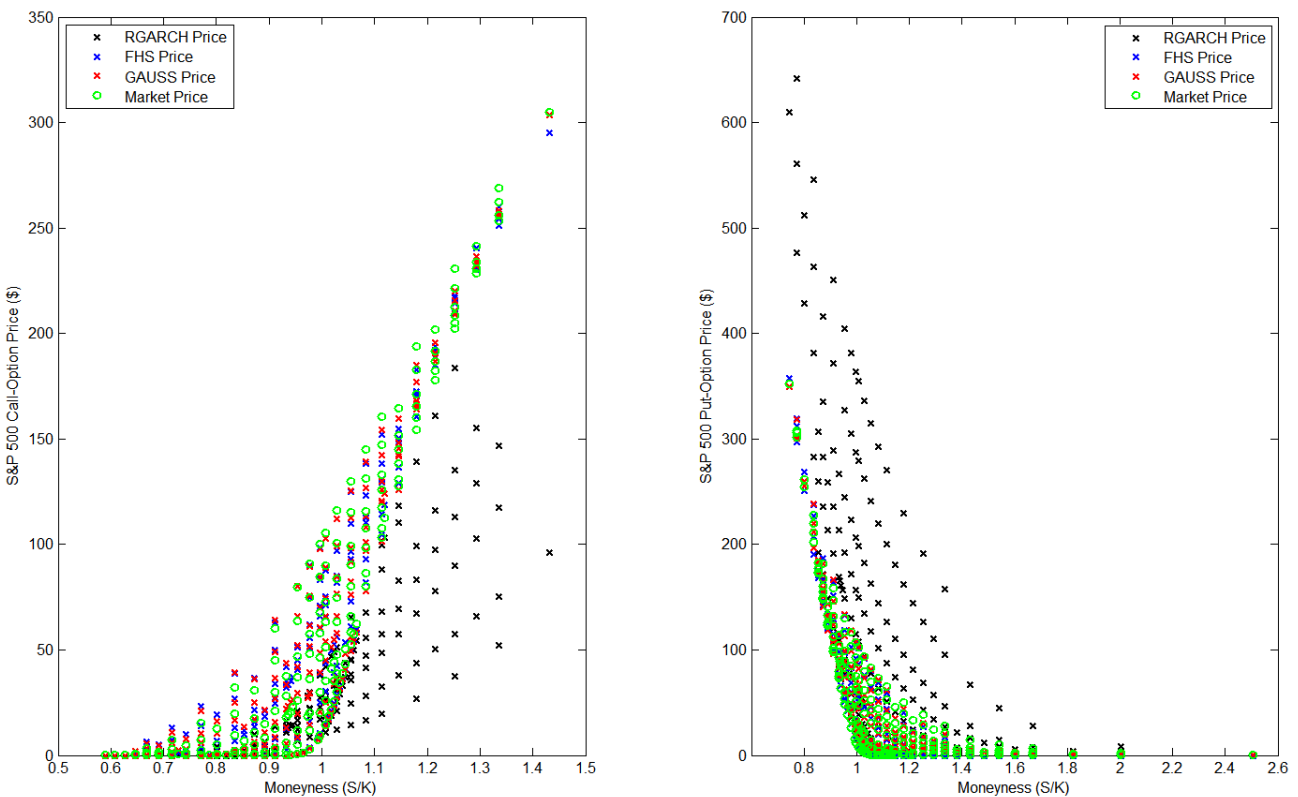


Figure 11b: Option prices for the FHS-RGARCH method on July 9<sup>th</sup> 2003



Note: This figure shows the option prices for the FHS-GJR method (Figure 11a) and the FHS-RGARCH method (Figure 11b), based on the estimated parameters, the FHS and the Gaussian innovations, together with the market price for in-the-money and out-of-the-money options on July 9<sup>th</sup> 2003. The price of calls (puts) is plotted on the left (right) side. Moneyness is the ratio of the S&P 500 index value  $S$  over the strike price  $K$ .

5 Empirical Results

Figure 12a: Option prices for the FHS-GJR method on October 15<sup>th</sup> 2008

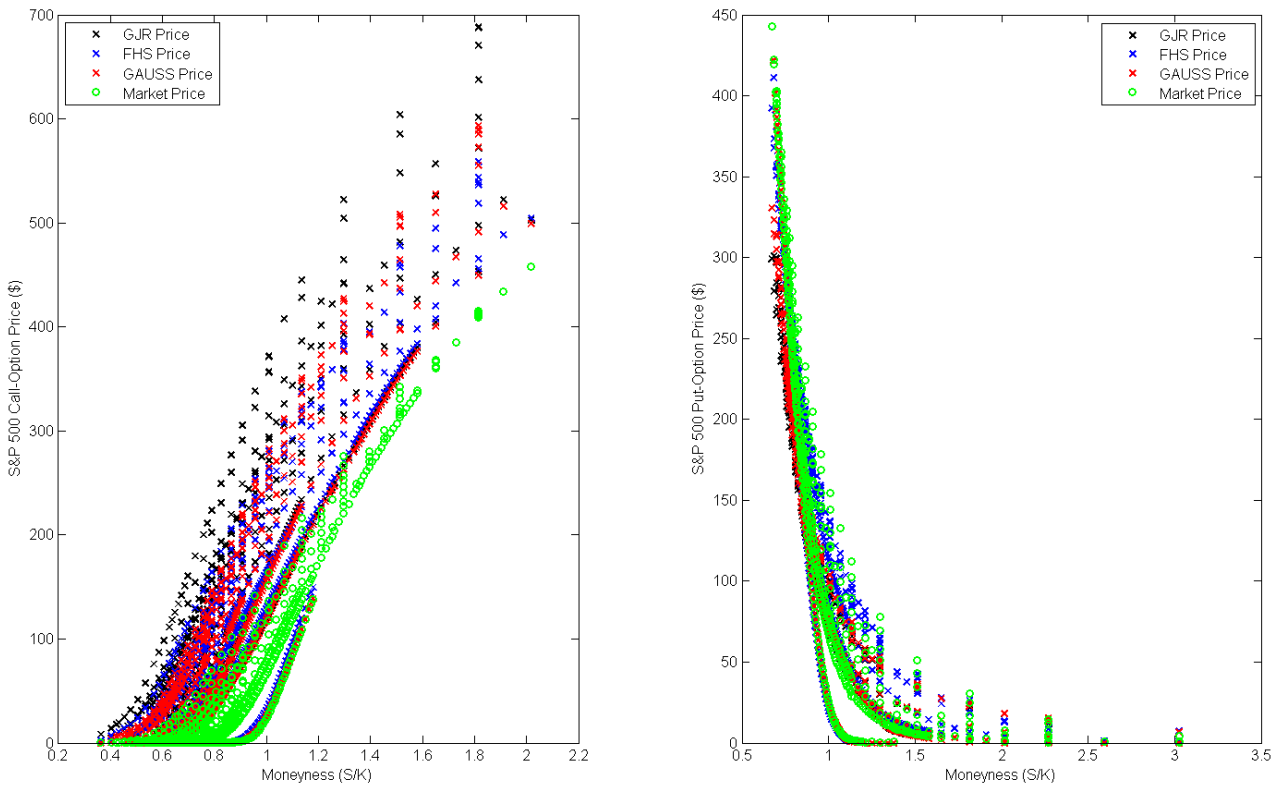
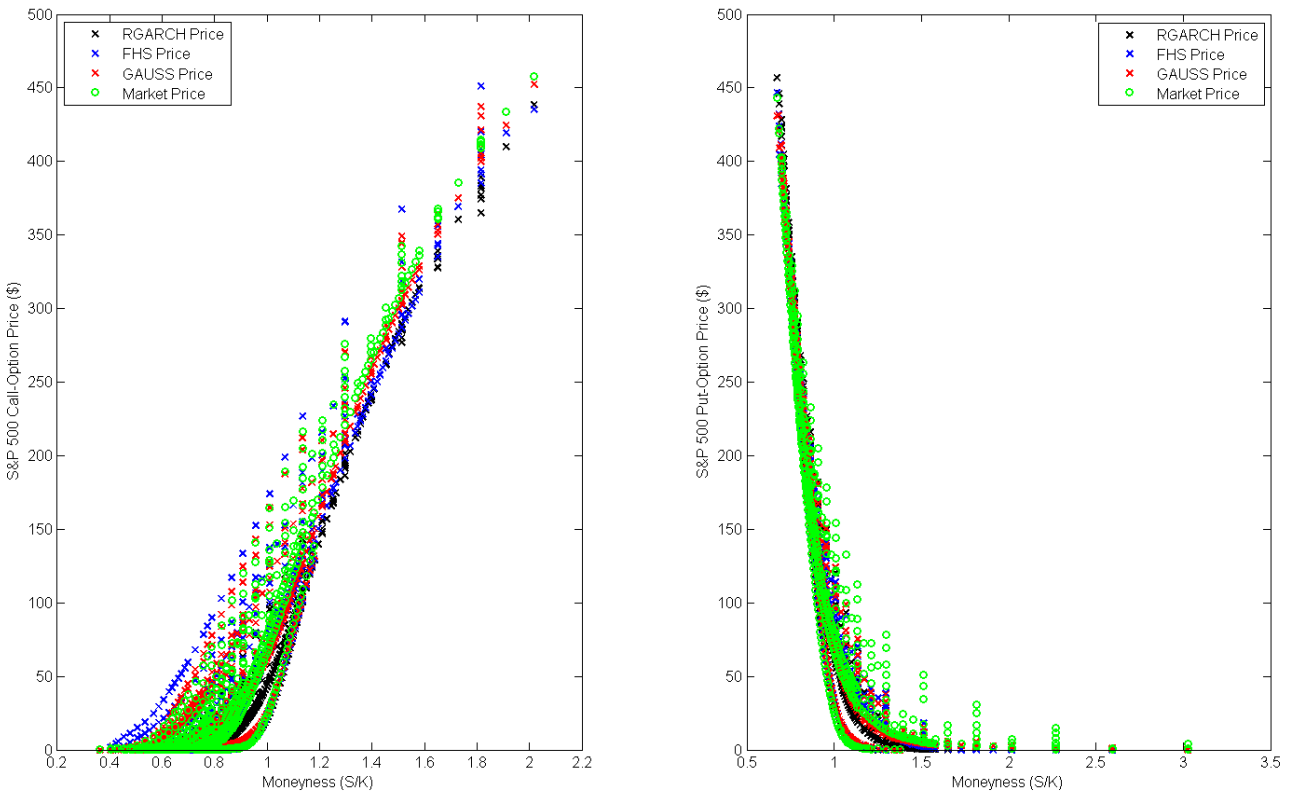


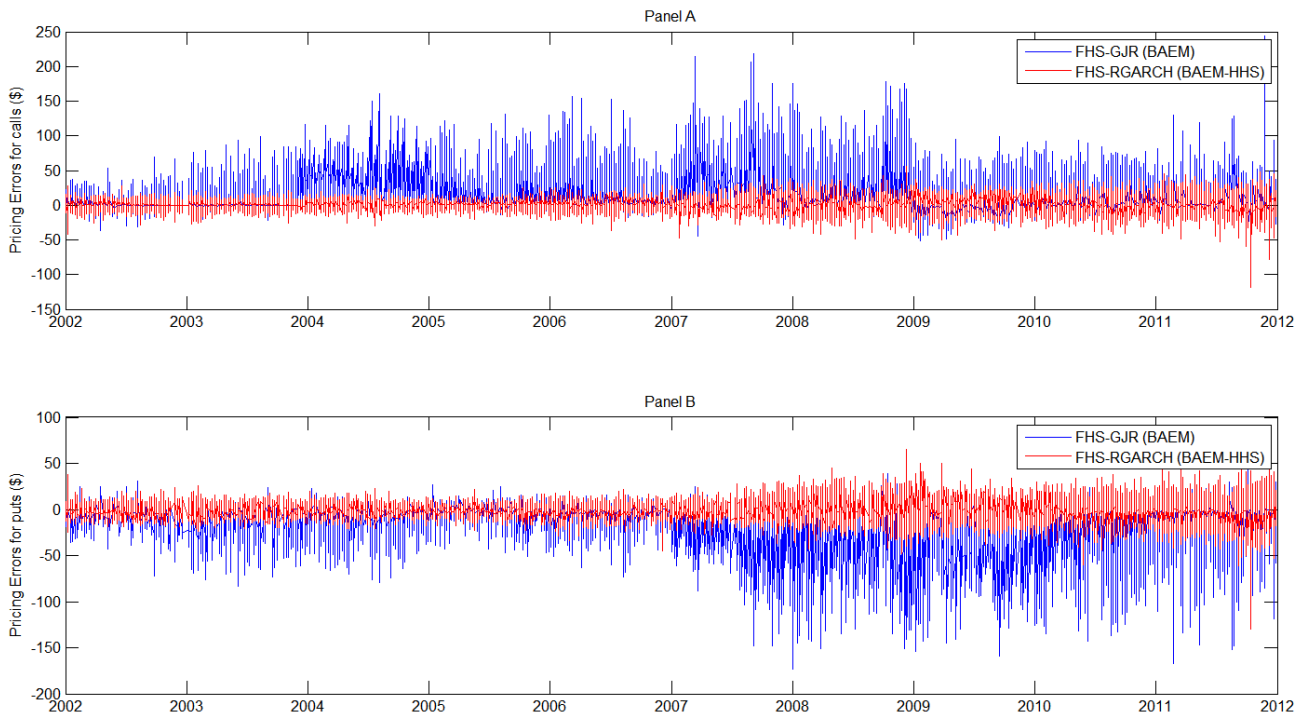
Figure 12b: Option prices for the FHS-RGARCH method on October 15<sup>th</sup> 2008



Note: This figure shows the option prices for the FHS-GJR method (Figure 12a) and the FHS-RGARCH method (Figure 12b), based on the estimated parameters, the FHS and the Gaussian innovations, together with the market price for in-the-money and out-of-the-money options on October 15<sup>th</sup> 2008. The price of calls (puts) is plotted on the left (right) side. Moneyness is the ratio of the S&P 500 index value  $S$  over the strike price  $K$ .

5 Empirical Results

Figure 13: Pricing errors for the FHS-GJR and the FHS-RGARCH method for the period 2002 – 2011.



Note: This figure depicts for the period 2002 – 2011 for call (panel A) and put (panel B) options the pricing errors of the BAEM and the FHS-RGARCH method when using FHS innovations.

Concluding, one could say that applying the FHS method, surely improves option pricing, whether taking innovations of the historical distribution or the Gaussian distribution. Furthermore, using a RGARCH model instead of the GJR model does not lead to better option pricing results. However, when applied in combination with the FHS method, the RGARCH model performs a lot better than the GJR model, whether it concerns turbulent periods in the financial markets or not.

### 5.3.2 State-Price-Density Analysis

Figure 14 presents the SPDs and  $M_{t,T}$  for FHS-GJR and the FHS-RGARCH method, for different maturities, on October 15<sup>th</sup> 2008. Since these methods are applied on both ITM and OTM options, the interpretation of  $M_{t,T}$  becomes a bit more difficult, as explained before. It appears from this figure that similar to the previous results that the SPDs are slightly negatively skewed. However, unlike the previous results, the pricing densities of the FHS innovations are closer to the historical densities rather than the pricing density using Gaussian innovations. This is expressed by a very constant SPD per unit probability (close to 1). Especially for the FHS-RGARCH method, there is hardly any difference between the historical and the pricing density, which could probably be due to the information taken into account by the FHS and the RGARCH model together. In fact, while the FHS method incorporates historical information on the return series, the RGARCH model contributes by incorporating the historical information of the realized measure. This results in very good PDF

5 Empirical Results

Figure 14a: SPDs and  $M_{t,T}$  estimates for different maturities on October 15<sup>th</sup> 2008 for the FHS-GJR method

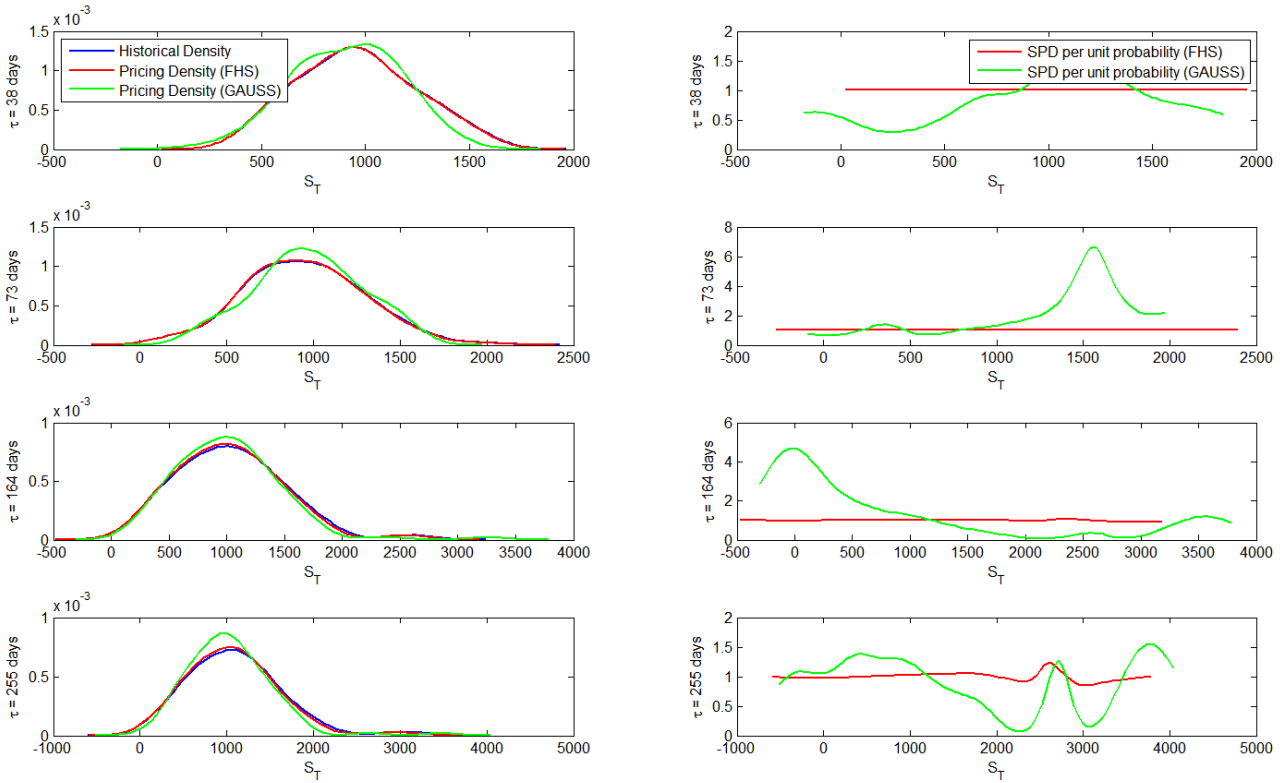
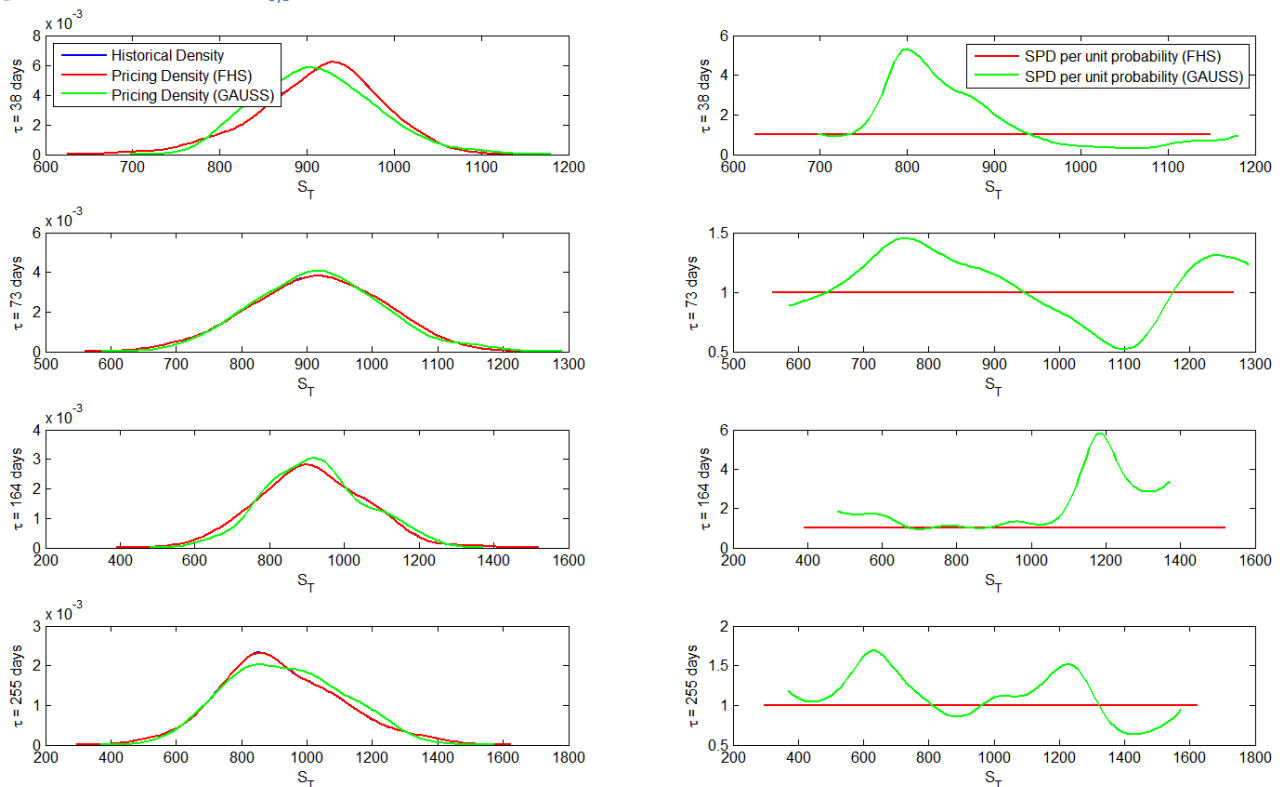


Figure 14b: SPDs and  $M_{t,T}$  estimates for different maturities on October 15<sup>th</sup> 2008 for the FHS-RGARCH method



Note: This figure depicts the pricing and historical density estimates for different time horizons on October 15<sup>th</sup> 2008 in the left graphs. The pricing densities are calibrated on the cross-section of OTM options using both FHS and Gaussian innovations for the GJR model (Figure 14a) and the RGARCH model (Figure 14b). The historical densities are obtained by fitting the GJR, respectively RGARCH, model using  $n=1472$  historical returns. The right graphs show the SPD per unit probability calculated as the discounted value of the pricing density over the historical densities.

estimation. The SPDs using Gaussian innovations on the other hand differ slightly more from the historical distribution. This indicates the importance of the ability of the FHS method to incorporate historical information.

### 5.3.3 Volatility Evaluation

In the previous sections I have presented the results of the FHS-RGARCH model as far as the statistical criteria and the state-price-densities is concerned, for the sample period 2002 – 2011. As described in section 3.4.3, another way to evaluate the FHS-RGARCH method is to look at the estimated volatilities. Therefore, this section compares the VaRs of the FHS-GJR and the FHS-RGARCH model to the VaRs estimated using the VIX index.

Figure 15 presents 1%, 5% and 10% VaR estimates for the FHS-GJR (Figure 15a), the FHS-RGARCH (Figure 15b) method and the VIX index (Figure 15c). For the sake of clarity I plotted the VaR estimates of the three methods in separate charts. The VaR estimates for the FHS calibration methods yield slightly higher volatilities compared to the VIX index. In fact, in absolute terms, the smallest VaR estimates are obtained from the VIX index, followed by the FHS-GJR and last by the FHS-RGARCH method. However, higher absolute VaR estimates do not necessarily imply bad volatility estimates. When we look at the conditional coverage test results in Table 9, the VaR estimates seem to be performing quite good based on a 5% significance level. For all the VaR estimates except for the 5% VaR for the FHS-GJR model, the null-hypotheses of approximately 1%, 5% and 10% independent violations cannot be rejected, indicating proper VaR estimates. Based on the p-values, the VIX index produces the best VaR estimates, followed by the FHS-GJR model and finally by the FHS-RGARCH model.

**Table 9: Results of correct conditional coverage test for VaR estimates**

	1% VaR		5% VaR		10% VaR	
	$LR_{cc}$	P-value	$LR_{cc}$	P-value	$LR_{cc}$	P-value
<i>FHS-GJR</i>	1.62	0.45	7.36	0.03	1.66	0.44
<i>FHS-RGARCH</i>	4.01	0.13	4.35	0.11	2.13	0.35
<i>VIX</i>	0.34	0.84	1.13	0.57	0.85	0.65

*Note: This table reports the likelihood ratio and the corresponding p-values for the correct conditional coverage test applied on the 1%, 5% and 10% VaR estimates constructed using the FHS-GJR, the FHS-RGARCH method and the VIX index, for the period 2002 - 2011.*



5 Empirical Results

Figure 15a: S&P 500 returns together with the VaR-values for the FHS-GJR method for 2002 – 2011

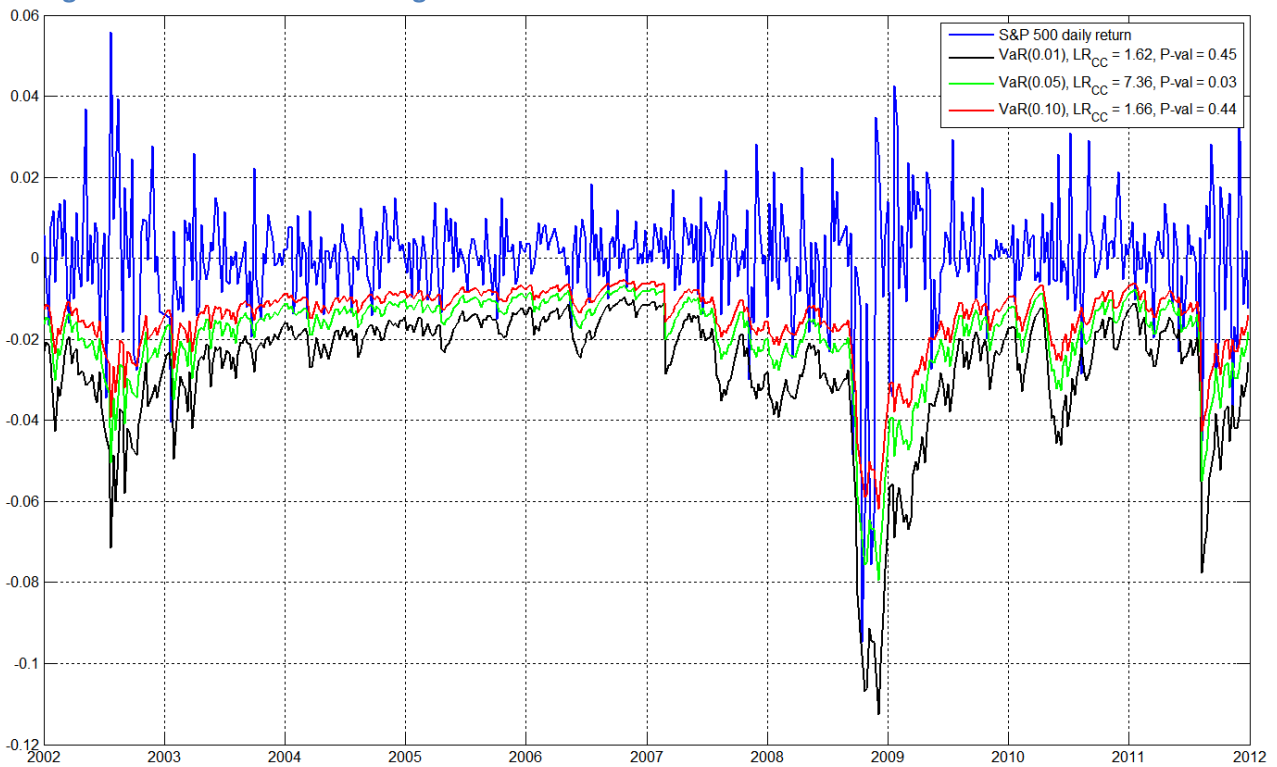
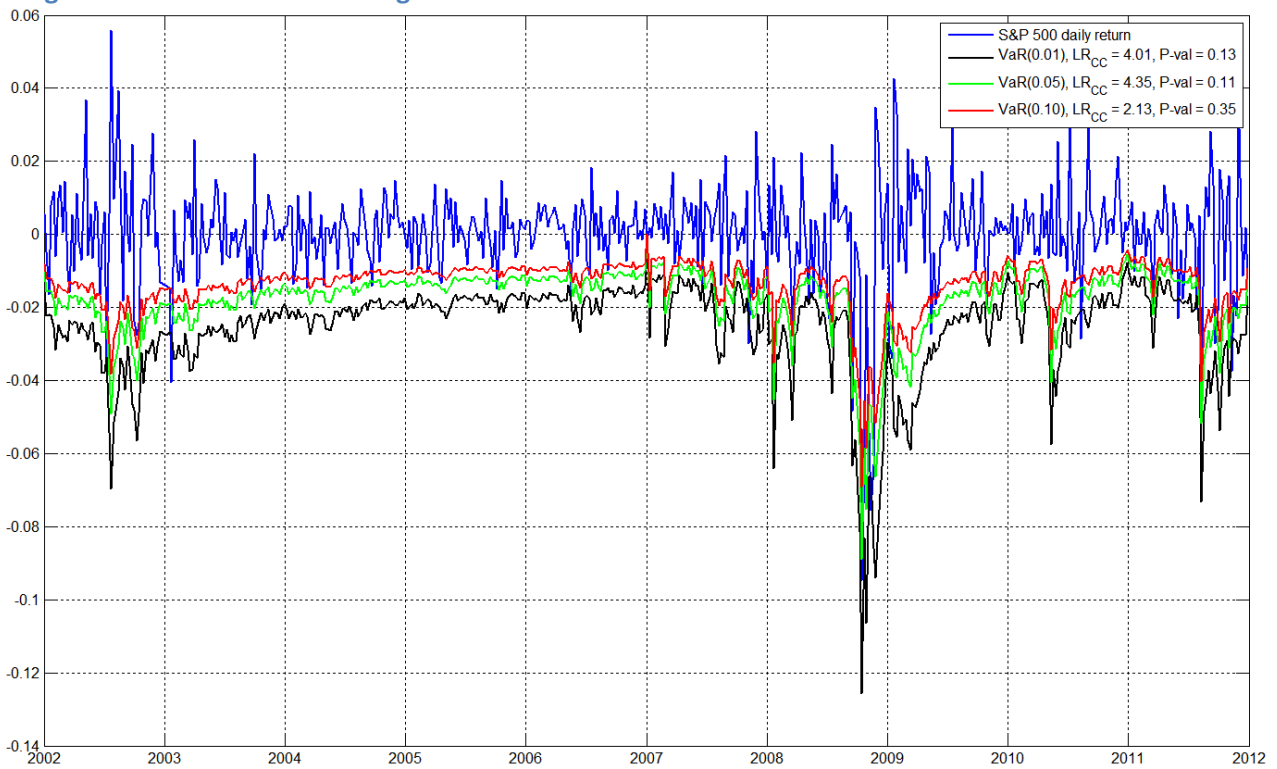
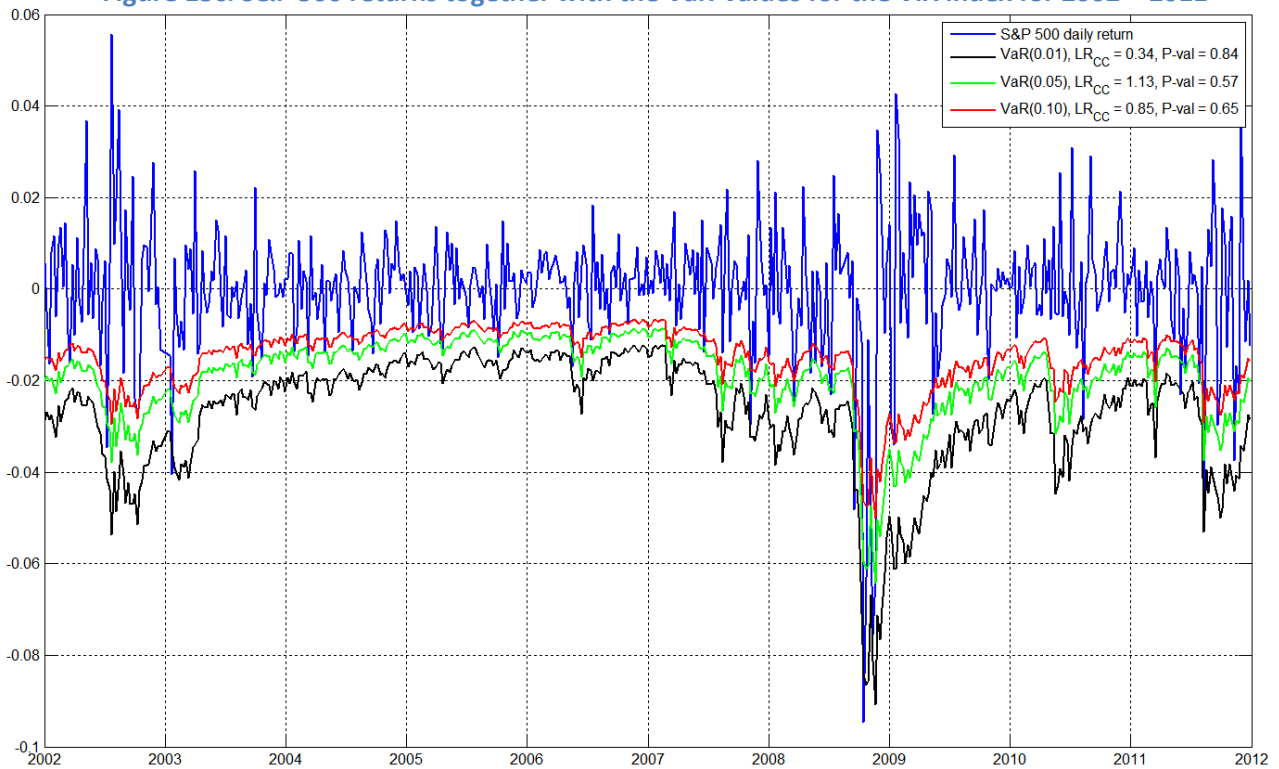


Figure 15b: S&P 500 returns together with the VaR-values for the FHS-RGARCH method for 2002 – 2011



5 Empirical Results

Figure 15c: S&P 500 returns together with the VaR-values for the VIX index for 2002 – 2011



Note: This figure depicts the return series of the S&P 500 index together with the 1%, 5% and 10% VaR-measures constructed using the FHS-GJR (Figure 15a), FHS-RGARCH method (Figure 15b) and the VIX index (Figure 15c), for each Wednesday during the period January 2<sup>nd</sup> 2002 to December 30<sup>th</sup> 2011. The legends also show the likelihood ratio and corresponding p-values of the correct conditional coverage test.

## 6 CONCLUSION

In this thesis I have combined the option pricing method of Barone-Adesi, Engle and Mancini (2008) with the Realized GARCH volatility model of Hansen, Huang and Shek (2011). In fact, this thesis verified whether applying the filtered historical simulation method on the RGARCH model of Hansen, Huang and Shek (2011), improves the option pricing method of Barone-Adesi, Engle and Mancini (2008). At the same time, while BAEM (2008) is restricted to option pricing of out-of-the-money options, this thesis included also in-the-money options. Using an empirical analysis for the period 2002 – 2011, I verified the option pricing method of a FHS approach on a RGARCH model.

First I started with the replication of the FHS-GJR method to assure proper implementation and understanding of the applied method. Using similar data to BAEM (2008), I applied the FHS method on a GJR-GARCH model. The results of the replication generally matched those of BAEM (2008). After that, I extended the replication of the FHS-GJR method to a dataset including ITM options, while keeping the same sample period as BAEM (2008). This led to poorer, but still good option pricing results. Obviously, it is harder for a model to produce results that fit both types of options (ITM and OTM) than to produce results that fit only one option type (OTM only).

Next, after replicating the FHS-GJR method, I continued with replicating the results of HHS (2011). Using the same dataset as Hansen, Huang and Shek (2011), I confirmed the implementation of the RGARCH(1,1) model. Thereafter, I estimated the RGARCH model using the realized kernel similar to Shephard and Sheppard (2010) to verify whether using different realized kernel data could lead to major differences. It appeared that using different realized kernel data has no major influence on the RGARCH estimation results. Even extending the period from 2002 – 2008 to 2002 – 2011 or allowing for an average return parameter in the model do not lead to major differences in the results, indicating the robustness of the RGARCH model as already praised by HHS (2011).

Subsequently, proper implementation of the FHS method of BAEM (2008) and the RGARCH model of HHS (2011), gave rise to the main innovation of this thesis which is the combination the FHS method with the RGARCH model (the FHS-RGARCH method). To compare the result to the FHS method combined with a GJR model (FHS-GJR method) I applied both methods on a dataset containing OTM and ITM options for the period 2002 – 2011. Based on statistical criteria, an estimated RGARCH model is unable to price options well, while the FHS method combined with the RGARCH model is strikingly better in option pricing, when compared to the FHS approach applied on a GJR model. Analysis of the pricing error series pointed out that where the FHS-GJR method is still delicate for turbulent periods, the FHS-RGARCH method deals with turbulent periods very well. Also, for the FHS-

## 6 Conclusion

RGARCH model, the estimated state-price-densities under the risk-neutral measure are very similar to the historical distribution. Finally, the VaR estimates constructed from the FHS-GJR, the FHS-RGARCH or the VIX index perform quite well based on the correct conditional coverage test.

In this thesis I showed that the FHS method in combination with the RGARCH model improves option pricing when we look at measures such as the root mean squared error, the mean absolute error, state-price densities and estimated volatilities. The FHS method, which employs historical information of returns, combined with the RGARCH model, which uses the information of realized measures, contribute very well to the pricing of both out-of-the-money and in-the-money options. Even for the crises periods, this pricing method seems quite stable.

For further research I have several suggestions. A first possibility is to apply another volatility model which incorporates a realized measure such as the MEM or HEAVY model. Also the use of jump-robust realized measures could improve option pricing using the combination of the FHS method and the RGARCH model. Finally, further research could focus on the performance of option pricing methods based on a combination of the FHS method and a stochastic volatility model.

**BIBLIOGRAPHY**

**Aït-Sahalia, Y., and A. W. Lo, 1998**, “Nonparametric Estimation of State-price Densities Implicit in Financial Assets Prices”, *Journal of Finance*, Vol. 53, p. 499–548.

**Aït-Sahalia, Y., and A. W. Lo, 2000**, “Nonparametric Risk Management and Implied Risk Aversion”, *Journal of Econometrics*, Vol. 94, p. 9–51.

**Alizadeh, S., M.W. Brandt and F.X. Diebold, 2002**, “Range-based estimation of stochastic volatility models”, *Journal of Finance*, Vol. 57, p. 1047– 1091.

**Andersen T.G., T. Bollerslev, F.X. Diebold, P. Labys, 2003**, “Modeling and forecasting realized volatility”, *Econometrica*, Vol. 71, No. 2, p. 579–625.

**Bakshi, G., C. Cao and Z. Chen, 1997**, “Empirical Performance of Alternative Option Pricing Models”, *Journal of Finance*, Vol. 52, p. 2003–2049.

**Barone-Adesi, G., R. F. Engle and L. Mancini, 2008**, “GARCH Option Pricing Model with Filtered Historical Simulation”, *Review of Financial Studies*, Vol. 21, p. 1223–1258.

**Barone-Adesi, G., Giannopoulos, K., and L. Vosper, 1999**, "VaR without Correlations for Nonlinear Portfolios," *Journal of Futures Markets*, Vol. 19, p. 583 - 602.

**Black, F., and M. Scholes, 1973**, “The Valuation of Options and Corporate Liabilities”, *Journal of Political Economy*, Vol. 81, p. 637–654.

**Campbell, J. Y., A. W. Lo, and C. MacKinlay, 1997**, *The Econometrics of Financial Markets*, Princeton, NJ: Princeton University Press.

**Chernov, M., and E. Ghysels, 2000**, “A Study Towards a Unified Approach to the Joint Estimation of Objective and Risk Neutral Measures for the Purpose of Options Valuation”, *Journal of Financial Economics*, Vol. 56, p. 407–458.

**Christoffersen, P.F., 1998**, “Evaluating Interval Forecasts”, *International Economic Review*, Vol. 39, p. 841-862.

**Christoffersen P., B. Feunou, K. Jacobs, N. Meddahi, 2010**, “The economic value of realized volatility”, Working paper McGill University.

Bibliography

- Christoffersen, P., and K. Jacobs, 2004**, "Which GARCH Model for Option Valuation?", *Management Science*, Vol. 50, p. 1204–1221.
- Cox, J. C., and S. A. Ross, 1976**, "The Valuation of Options for Alternative Stochastic Processes", *Journal of Financial Economics*, Vol. 3, p. 145–166.
- Cox, J. C., S. A. Ross and M. Rubinstein, 1979**, "Option pricing: A simplified approach", *Journal of Financial Economics*, Vol. 7, p. 229–263.
- Duan, J. C., 1995**, "The GARCH Option Pricing Model", *Mathematical Finance*, Vol. 5, p. 13–32.
- Dobrev D., P. Szerszen, 2010**, "The information content of high-frequency data for estimating equity return models and forecasting risk", Working paper, Board of Governors of the Federal Reserve System, International Finance Discussion Papers, nr. 1005.
- Engle, R. F., 1982**, "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation", *Econometrica*, Vol. 50, p. 987–1007.
- Engle, R. F., 2002**, "New frontiers of ARCH models", *Journal of Applied Econometrics*, Vol. 17, p. 425–446.
- Engle, R. F., G. Gallo, 2006**, "A multiple indicators model for volatility using intra-daily data", *Journal of Econometrics*, Vol. 131, p. 3–27.
- Engle, R.F. and V. K. Ng, 1991**, "Measuring and testing the impact of news on volatility", *Journal of Finance*, Vol. 48, No. 5, p. 1749–1778.
- Forsberg, L., T. Bollerslev, 2002**, "Bridging the gap between the distribution of realized (ECU) volatility and ARCH modeling (of the EURO): the GARCH-NIG model", *Journal of Applied Econometrics*, Vol. 17, No. 5, p. 535–548.
- Glosten, L. R., R. Jagannathan, and D. E. Runkle, 1993**, "On the Relation Between the Expected Value and the Volatility of the Nominal Excess Return on Stocks", *Journal of Finance*, Vol. 48, p. 1779–1801.
- Hansen, P. R., Z. Huang, and H. H. Shek, 2011**, "Realized GARCH: A Joint Model for Returns and Realized Measures of Volatility", *Journal of Applied Econometrics*.
- Harrison, M. and D. Kreps, 1979**, "Martingales and Arbitrage in Multiperiod Securities Markets", *Journal of Economic Theory*, Vol. 20, p. 381–408.

Bibliography

**Harvey, A., E. Ruiz and N. Shephard, 1994**, “Multivariate stochastic variance models”, *Review of Economic Studies*, Vol. 61, p. 247-264.

**Heston, S., and S. Nandi, 2000**, “A Closed-form GARCH Option Valuation Model”, *Review of Financial Studies*, Vol. 13, p. 585 – 625.

**Holt, M. T., and S. V. Aradhyula, 1990**, “Price Risk in Supply Equations: An Application of GARCH Time-Series Models to the U.S. Broiler Market”, *Southern Economic Journal*, Vol. 57, No. 1, p. 230 – 242.

**Jackwerth, J. C., 2000**, “Recovering Risk Aversion from Option Prices and Realized Returns”, *Review of Financial Studies*, Vol. 13, p. 433–451.

**Ledoit O., P. Santa-Clara and M. Wolf, 2003**, “Flexible Multivariate GARCH Modeling With an Application to International Stock Markets”, Vol. 85, No. 3, p. 735 – 747.

**Lee, R. W., 2004**, “Option Pricing by Transform Methods: Extensions, Unification, and Error Control”, *Journal of Computational Finance*, Vol. 7, No. 3, p. 51 – 86.

**Lee, S. and B.E. Hansen, 1994**, “Asymptotic Theory for the GARCH (1,1) Quasi-Maximum Likelihood Estimator”, *Journal of Econometric Theory*, Vol. 10, p. 29 – 52.

**Merton, R. C., 1976**, “Option pricing when underlying stock returns are discontinuous”, *Journal of Financial Economics*, Vol. 3, p. 125 – 144 .

**Nelson, D. B., 1991**, "Conditional heteroskedasticity in asset returns: A new approach", *Econometrica*, Vol. 59, p. 347-370.

**Persson S., 1994**, “Pricing Life Insurance Contracts under Financial Uncertainty”, Norwegian School of Economics and Business Administration.

**Pritsker, M., 2001**, “The Hidden Dangers of Historical Simulation”, Federal Reserve Bank of Boston, *FEDS*, Discussion Paper No. 2001-27.

**RiskMetrics Group, 2009**, “An introduction to OptionMetrics implied volatility data”, *Research Technical Note*.

**Rosenberg, J. V., and R. F. Engle, 2002**, “Empirical Pricing Kernels”, *Journal of Financial Economics*, Vol. 64, p.341–372.

**Sentana, E., 1995**, “Quadratic ARCH models”, *Review of Economic Studies*, Vol. 62, p. 639–61.

Bibliography

**Shephard N., K. Sheppard, 2010**, "Realising the future: forecasting with high frequency based volatility (HEAVY) models", *Journal of Applied Econometrics*, Vol. 25, p. 197–231.

**Teräsvirta T., 2009**, "An introduction to univariate GARCH models", *Handbook of Financial Time Series*, T. Mikosch, T. Kreiss, JP. Davis, R.A. Andersen and T. Gustav (eds), Springer, Berlin, p. 17–42.

**Villar, J. R. and E. R. Ortega, 2010**, "Volatility Models with Leverage Effect", Universidad Carlos III de Madrid.

**Wagner N. and T. A. Marsh, 2005**, "Measuring Tail Thickness under GARCH and an Application to Extremal Exchange Rate Changes", *Journal of Empirical Finance*, Vol. 12, p. 165 – 185.

**Whaley, R.E., 2008**, "Understanding VIX", *Vanderbilt Owen Graduate School of Management*, working paper,

**Zakoian, JM., 1994**, "Threshold heteroskedastic models", *Journal of Economic Dynamics and Control*, Vol. 18, No. 5, p. 931-955.



## APPENDIX A PARAMETER TRANSFORMATIONS

In this appendix I present the parameter transformations I used to perform the FHS method by means of an unconstrained optimization algorithm (Nelder-Mead) instead of constrained optimization.

### A.1 Parameter Transformations for the GJR Model

The parameter restrictions for the GJR model in Equation (1) are as follows<sup>15</sup>:

$$\begin{aligned}\omega &> 0 \\ \alpha, \beta, \gamma &\geq 0 \\ \alpha + \beta + \frac{1}{2}\gamma &< 1\end{aligned}\tag{A1}$$

To imply the restrictions one can use the unrestricted parameters  $\tilde{\beta}$ ,  $\tilde{\alpha}$  and  $\tilde{\gamma}$  and utilise the following relations:

$$\begin{aligned}\omega &= \exp(\tilde{\omega}) \\ \beta &= \frac{\exp(\tilde{\beta})}{1 + \exp(\tilde{\beta}) + \exp(\tilde{\alpha}) + \frac{1}{2} * \exp(\tilde{\gamma})} \\ \alpha &= \frac{\exp(\tilde{\alpha})}{1 + \exp(\tilde{\beta}) + \exp(\tilde{\alpha}) + \frac{1}{2} * \exp(\tilde{\gamma})} \\ \gamma &= \frac{\exp(\tilde{\gamma})}{1 + \exp(\tilde{\beta}) + \exp(\tilde{\alpha}) + \frac{1}{2} * \exp(\tilde{\gamma})}\end{aligned}\tag{A2}$$

By substituting the relations in Equation (A2), one could verify whether the conditions of Equation (A1) are met.

### A.2 Parameter Transformations for the RGARCH Model

Though already mentioned in Section 3.2, the parameter restrictions for the RGARCH model in Equation (2) are:

$$\begin{aligned}\beta &\geq 0 \\ \psi &\geq 0 \\ \beta + \psi &< 1 \\ \sigma_v^2 &> 0\end{aligned}\tag{A3}$$

<sup>15</sup> Villar and Ortega (2010)

## Appendix A Parameter Transformations

The application of parameters transformations for the RGARCH model is almost similar to the GJR model. Therefore, one could use the following transformations:

$$\begin{aligned}\beta &= \frac{\exp(\tilde{\beta})}{1 + \exp(\tilde{\beta}) + \exp(\tilde{\psi})} \\ \psi &= \frac{\exp(\tilde{\psi})}{1 + \exp(\tilde{\beta}) + \exp(\tilde{\psi})} \\ \sigma_v^2 &= \exp(\tilde{\sigma}_v^2)\end{aligned}\tag{A4}$$

in which  $\tilde{\beta}$ ,  $\tilde{\psi}$  and  $\tilde{\sigma}_v^2$  are unrestricted. Using these relations, the conditions of Equation (A3) can be verified.

## APPENDIX B SIMULATION ANALYSIS

In this appendix I perform a simulation analysis on the filtered historical simulation method applied on the GJR model. Using a simulation study, one can extend the knowledge about this method regarding its sensitivity to different settings. Also, a simulation study can be used to provide directions in what to expect from an empirical analysis.

### B.1 Simulation Procedure

The simulation procedure consists mainly of three parts. The first part encompasses simulating the S&P 500, the second focuses on generating the option data and the third part consists of applying the FHS method using the simulated data from the first two parts.

To simulate the S&P 500 index, the GJR model of Equation (1) is assumed as the DGP. For convenience this model is presented again as:

$$\begin{aligned} r_t &= \mu + \varepsilon_t \\ \sigma_t^2 &= \omega + \beta\sigma_{t-1}^2 + \alpha\varepsilon_{t-1}^2 + \gamma I_{t-1}\varepsilon_{t-1}^2, \end{aligned} \quad (\text{B1})$$

The simulation procedure is then as follows. First, for each simulation  $m$  I randomly draw  $T$  standard normally distributed innovations given by the set  $Z^{(m)} = \{z_1, z_2, \dots, z_T\}$ . Next I set the parameter values  $\theta = \{\omega, \beta, \alpha, \gamma\}$  equal to  $\theta_0$ . The parameters under the  $\mathbb{P}$  measure are thus set equal to  $\theta_0$ . Subsequently, assuming the model of Equation (B1), I simulate  $M$  return paths of length  $T$  with  $r_t^{(m)}$  representing the simulated S&P 500 return of the  $m^{\text{th}}$  path at time  $t$ . For each simulation, I then construct the simulated S&P 500 index series  $(S_1^{(m)}, S_2^{(m)}, \dots, S_T^{(m)})$  assuming the first S&P 500 value equal to the real index value, that is  $\forall m: S_1^{(m)} = S_1$ , where  $S_1$  represents the real index value at time  $t = 1$ .

After generating the index series, I simulate the option data. For each simulation  $m$  and each point in time for which the calibration method will be applied (that is  $t = T - N_{\text{calibration}} + 1, \dots, T$ , where  $T > N_{\text{calibration}}$ ), I perform the following simulation steps.

1. For options with predefined maturities and strike prices, I simulate  $L_1$  return paths with the length of the largest maturity ( $\tau_{\text{max}}$ ) using the parameter values  $\theta_0$  and bootstrapped innovations. The bootstrapped innovations are randomly drawn from  $Z^{(m)}$  with replacement. An important point to note here is that since I simulate the  $L_1$  return paths using the  $\theta_0$ , I assume that the process of the index values is similar under the  $\mathbb{P}$  and  $\mathbb{Q}$  measures in the data generating process.

2. I calculate the call-option price at time  $t$  as  $C_{K,\tau}^{(m)} = \frac{\exp(-i\tau)}{L_1} \sum_{l=1}^{L_1} \max(S_\tau^{(m,l)} - K, 0)$ . The prices of for puts are calculated in a similar way.

After determining the option price, I apply the previously described six-steps FHS procedure which yields the calibrated values  $\theta_t^{*(m)}$ . The number of simulation paths I use during the FHS procedure is denoted by  $L_2$ .

## B.2 Simulation Results

For the simulation, I use the following parameter settings:<sup>16</sup>

- Number of simulations ( $M$ ) = 50
- Number of points in time for which I apply the FHS method ( $N_{calibration}$ ) = 25
- Number of points in time for which I simulate the S&P 500 path ( $T$ ) = 3500
- Number of different maturities ( $N_\tau$ ) = 5, ranging from 10 to 260
- Number of different strike prices ( $N_K$ ) = 10, ranging from 50 to 400
- Number of return paths simulated in the DGP of the option prices ( $L_1$ ) = 100

Table B1 reports the average value of the calibrated parameters for different simulation settings. I calculate the calibrated values of the parameters by averaging over all simulations (which yields  $\theta_t^*$ ), followed by averaging over the time-dimension, yielding  $\theta_{mean}^*$ . To compute the standard deviations, I calculate for each point in time the sample standard deviation over all simulations, after which I average over time.

If we look at Table B1 it appears that as the window length ( $n$ ) of the estimation step (step 1 of the FHS procedure) increases, the standard deviations of the calibrated parameters generally decrease. Also, the standard deviations of the parameters increase as the number of simulation paths in the FHS procedure ( $L_2$ ) increases. A possible explanation for this not so intuitive relation is that as the number of simulation paths during the FHS procedure increases, the possible scenarios taken into account by this method increase. In this way the parameters are calibrated based on a wider range of possible outcomes of the S&P 500 values, leading to an increment of the standard deviations of the

<sup>16</sup> Due to the enormous computation time required to perform the simulations, the total number of simulations and the number of calibration points in time is kept relatively small. Also, I restrict the simulation study to the 12 scenarios in Table B1 in order to limit the computation time. Since the simulation study serves as a basis for parameters in the empirical analysis to be conducted, in the choice of scenarios, I vary only the parameters that may determine the empirical results. The required computation time for each scenario is approximately  $M \times N_{calibration}$  minutes, yielding for the given parameter settings about two weeks of computation time for the 12 different simulation scenarios in Table B1.

Appendix B Simulation Analysis

calibrated parameters. A point to be noted here is that an increase of the number of simulations in a standard simulation study leads to more accurate results. However, since  $L_2$  represents the number of simulation paths used in the FHS procedure (see step 5), increasing  $L_2$  does not necessarily produce more accurate calibrations, while the calibrated parameters may be more robust to shocks in the S&P 500 index. In order to increase the accuracy of the calibrated parameters, one could increase the number of simulations ( $M$ ). Though, one has to keep eye for the consumed computation time. Another notable result from Table B1 is the effect of the interest rate on the calibrated parameters. It seems that if I incorporate an interest rate in the simulation and FHS-calibration procedure, the calibrated parameters are closer to the parameters of the DGP which are represented by the results in the first row. This supports the use of interest rates in the FHS method.

**Table B1: FHS-calibrated parameter values for simulations**

$n$	$L_2$	$i$ (%)	$\omega^* \times 10^6$		$\beta^*$		$\alpha^*$		$\gamma^*$	
			Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.
			1.390		0.930		0.007		0.100	
1000	100	0	9.119	25.726	0.717	0.328	0.059	0.172	0.371	0.538
1000	100	6	1.788	0.778	0.924	0.013	0.007	0.008	0.104	0.018
1000	500	0	6.442	16.056	0.771	0.301	0.038	0.134	0.281	0.442
1000	500	6	1.965	1.222	0.920	0.023	0.011	0.013	0.103	0.028
2000	100	0	3.809	9.140	0.748	0.301	0.063	0.153	0.309	0.455
2000	100	6	1.522	0.428	0.929	0.010	0.006	0.007	0.102	0.013
2000	500	0	5.107	12.437	0.754	0.310	0.042	0.124	0.317	0.480
2000	500	6	1.562	0.523	0.928	0.012	0.008	0.008	0.100	0.017
3000	100	0	3.739	8.753	0.743	0.306	0.071	0.163	0.291	0.452
3000	100	6	1.431	0.377	0.930	0.009	0.006	0.006	0.102	0.012
3000	500	0	5.555	14.318	0.753	0.300	0.053	0.141	0.312	0.458
3000	500	6	1.436	0.340	0.929	0.010	0.009	0.007	0.099	0.014

*Note: This table reports the values of the FHS calibrated GJR parameters averaged over time, for different settings of  $n, L_1, L_2$  and  $i$ . The results in the first row represent the values of the parameters used in the data generating process. The interest rate is represented as annualized. The mean is calculated by averaging first over all simulations ( $M$ ) followed by averaging over the points in time for which the FHS method is applied, thus for  $t = T - N_{calibration} + 1, \dots, T$ . The standard deviation is calculated over all simulations and then averaged over the number of calibration points.*

To give an overview of the parameters over time resulting from the simulation study, Table B2 presents the calibrated parameter values for 25 days. The mean and standard deviation of the calibrated parameters are given by:

$$\theta^* = \{5.40 * 10^{-6} \quad 0.75 \quad 0.06 \quad 0.30\}$$

Appendix B Simulation Analysis

$$\sigma_{\theta^*} = \{1.54 * 10^{-5} \quad 0.32 \quad 0.17 \quad 0.48\}$$

where  $\theta^*$  indicates the average value of  $\theta_t^*$  over time and  $\sigma_{\theta^*}$  indicates the standard deviation of  $\theta_t^*$  averaged over time. For clarification, the values of  $\theta_t^*$  and their respective standard deviations are given below. The values are calculated by averaging over all simulations ( $M$ ).

**Table B2: FHS-calibrated parameter values over time for  $L_1 = 120, L_2 = 110, n = 1000$  and  $r = 0\%$**

$T$	$\omega^* \times 10^6$		$\beta^*$		$\alpha^*$		$\gamma^*$	
	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.
<b>3476</b>	6.44	15.30	0.72	0.35	0.07	0.20	0.30	0.51
<b>3477</b>	4.74	13.39	0.75	0.31	0.07	0.18	0.28	0.45
<b>3478</b>	4.78	11.44	0.75	0.33	0.05	0.16	0.32	0.50
<b>3479</b>	4.80	10.42	0.70	0.36	0.06	0.17	0.37	0.58
<b>3480</b>	5.29	12.76	0.72	0.33	0.06	0.17	0.34	0.53
<b>3481</b>	5.41	12.51	0.72	0.33	0.04	0.11	0.34	0.52
<b>3482</b>	5.50	17.97	0.77	0.30	0.06	0.16	0.27	0.45
<b>3483</b>	4.51	11.64	0.75	0.32	0.04	0.14	0.37	0.54
<b>3484</b>	4.36	14.00	0.76	0.32	0.05	0.15	0.31	0.50
<b>3485</b>	5.29	16.11	0.73	0.34	0.07	0.20	0.34	0.53
<b>3486</b>	6.32	17.74	0.73	0.35	0.05	0.16	0.31	0.54
<b>3487</b>	3.44	10.69	0.78	0.29	0.05	0.13	0.23	0.37
<b>3488</b>	5.88	16.18	0.71	0.34	0.07	0.18	0.40	0.56
<b>3489</b>	4.25	10.75	0.75	0.32	0.04	0.13	0.33	0.52
<b>3490</b>	6.27	16.57	0.76	0.28	0.07	0.20	0.30	0.42
<b>3491</b>	5.82	14.45	0.75	0.31	0.08	0.22	0.29	0.44
<b>3492</b>	4.50	11.92	0.77	0.29	0.05	0.14	0.25	0.36
<b>3493</b>	3.90	13.96	0.77	0.30	0.07	0.21	0.24	0.39
<b>3494</b>	6.31	24.35	0.75	0.33	0.07	0.21	0.27	0.44
<b>3495</b>	6.73	19.11	0.72	0.35	0.05	0.17	0.36	0.55
<b>3496</b>	5.43	17.35	0.81	0.27	0.06	0.16	0.23	0.40
<b>3497</b>	6.11	20.68	0.74	0.34	0.05	0.16	0.26	0.46
<b>3498</b>	6.87	24.19	0.79	0.27	0.04	0.14	0.30	0.47
<b>3499</b>	7.10	18.49	0.75	0.32	0.06	0.18	0.29	0.47
<b>3500</b>	5.00	13.25	0.77	0.30	0.03	0.13	0.24	0.38

*Note: This table reports for the calibrated parameter values over time ( $\theta_t^*$ ), for the number of calibration points ( $N_{calibration} = 25$ ). The length of the window for the GJR-GARCH estimation is equal to 1000 observations ( $n=1000$ ).*

## APPENDIX C ADDITIONAL REPLICATION RESULTS OF BAEM (2008)

This appendix describes some additional results for the replication of BAEM (2008), using only OTM option data for the period 2002 – 2004. Table C1 presents the estimation and calibration results of the GJR parameters using the filtered data similar to BAEM (2008). The length of the estimation window  $n$  is equal to 1000 and the number of simulation paths  $L$  is set to 3500 (similar to BAEM). From this table it is clear that the standard deviations of the calibrated parameters (whether with Gaussian or FHS innovations) are higher compared to the standard deviations of the GJR estimates (compare panel A and B to each other). For the annualized volatility on the other hand, it seems that the calibration methods lead to smaller annualized volatilities with smaller standard deviations.

**Table C1: Estimated and calibrated GJR parameters for OTM options for 2002 –2004**

Panel A:

GJR	$\hat{\omega} \times 10^6$		$\hat{\beta}$		$\hat{\alpha} \times 10^3$		$\hat{\gamma}$		Persistency		Ann. Vol.	
	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.
Year												
2002	1.356	0.125	0.930	0.005	7.294	0.759	0.096	0.009	0.986	0.001	0.283	0.085
2003	1.130	0.189	0.932	0.006	6.422	1.527	0.101	0.007	0.989	0.002	0.194	0.051
2004	0.994	0.045	0.932	0.002	7.025	1.736	0.103	0.003	0.991	0.000	0.143	0.023

Panel B:

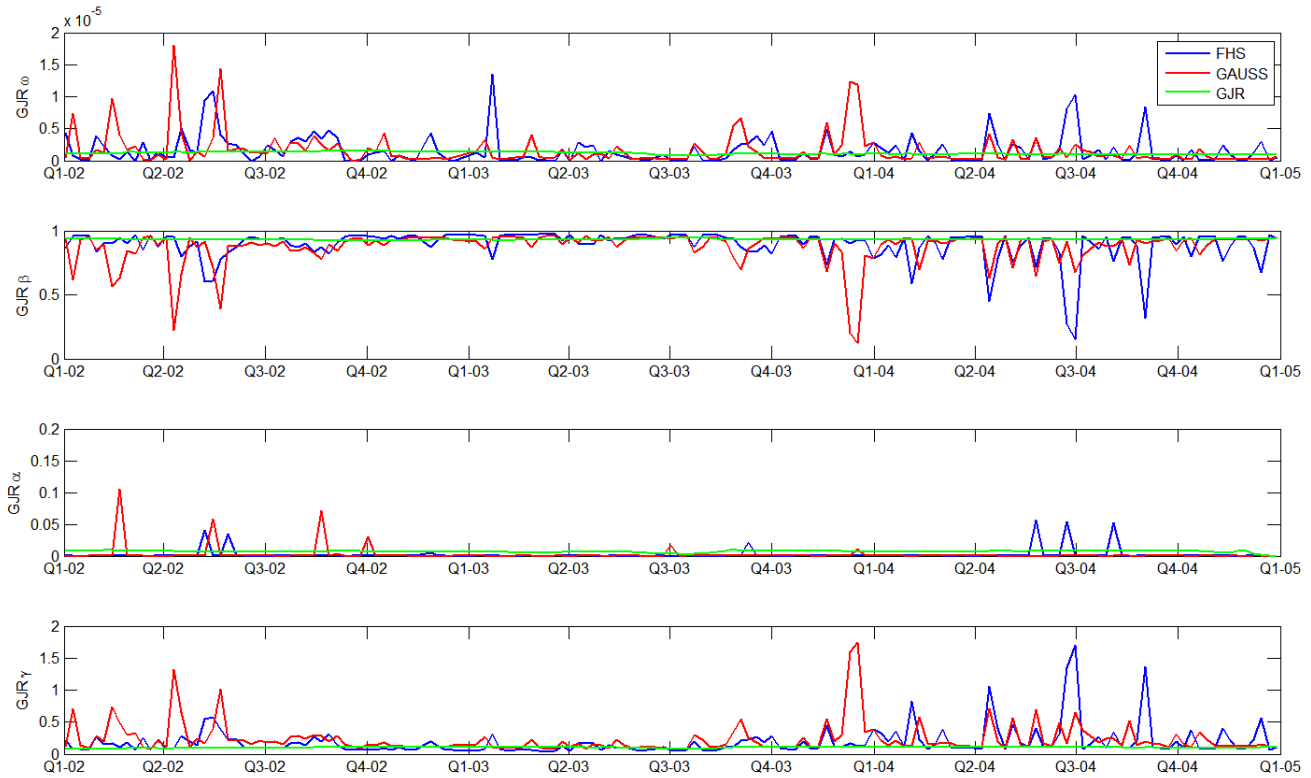
GAUSS	$\omega^* \times 10^6$		$\beta^*$		$\alpha^* \times 10^3$		$\gamma^*$		Persistency		Ann. Vol.	
	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.
Year												
2002	2.347	3.432	0.852	0.146	5.387	19.661	0.259	0.240	0.987	0.023	0.290	0.094
2003	1.718	2.603	0.878	0.160	0.585	2.904	0.230	0.313	0.993	0.011	0.195	0.053
2004	0.834	0.956	0.885	0.084	0.000	0.001	0.225	0.163	0.998	0.004	0.143	0.029

FHS

Year	Mean		Std.		Mean		Std.		Mean		Std.	
2002	2.001	2.290	0.902	0.080	1.649	7.433	0.158	0.115	0.983	0.020	0.285	0.089
2003	1.287	2.141	0.928	0.057	0.431	3.076	0.125	0.090	0.991	0.013	0.196	0.052
2004	1.565	2.310	0.843	0.184	3.213	12.791	0.290	0.355	0.992	0.011	0.143	0.036

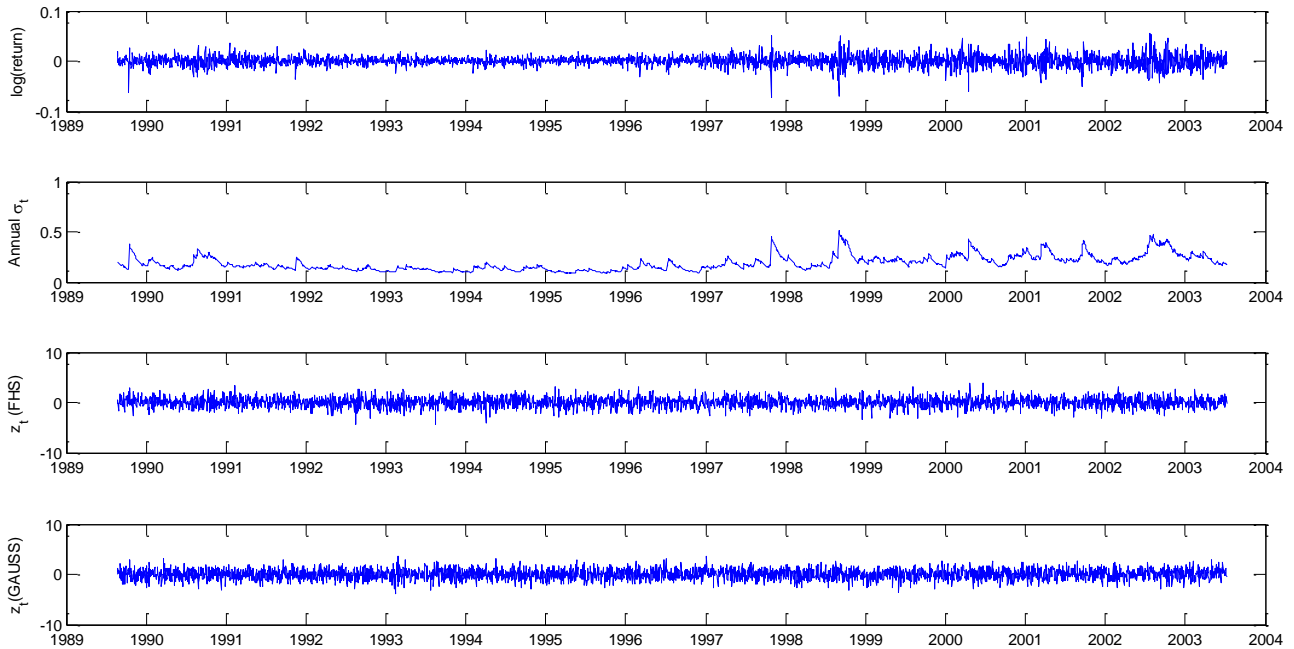
Note: This table reports the mean and standard deviation (Std.) of each of the parameters of the GJR-GARCH model estimated (Panel A) and calibrated (Panel B) on OTM S&P 500 options on each Wednesday from January 2, 2002 to December 29, 2004. Estimation occurs using maximum likelihood and 3500 historical log-returns while the model is calibrated using both the Filtered Historical Simulation (FHS) method and a calibration method using GAUSS innovations. The GJR model under the historical measure  $\mathbb{P}$  is specified by  $r_t = \mu + \varepsilon_t$ ,  $\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \alpha\varepsilon_{t-1}^2 + \gamma I_{t-1}\varepsilon_{t-1}^2$ , and under the risk-neutral measure  $\mathbb{Q}$  by  $r_t = \mu + \varepsilon_t$ ,  $\sigma_t^2 = \omega^* + \beta^*\sigma_{t-1}^2 + \alpha^*\varepsilon_{t-1}^2 + \gamma^* I_{t-1}\varepsilon_{t-1}^2$ , with  $I_{t-1} = 1$ , when  $\varepsilon_{t-1} < 0$ , and  $I_{t-1} = 0$  otherwise. The persistency is calculated as  $\alpha + \beta + \frac{\gamma}{2}$  under the historical measure  $\mathbb{P}$  and  $\alpha^* + \beta^* + \frac{\gamma^*}{2}$  under the risk-neutral measure  $\mathbb{Q}$ . Ann. Vol. stands for the annualized volatility.

Figure C1: Estimated and calibrated GJR parameters for OTM options for 2002 –2004



Note: This figure presents the parameter values of the GJR GARCH model based on the ML estimation (GJR), the FHS method and the calibration with Gaussian innovations. The GJR model under the historical measure  $\mathbb{P}$  is specified by  $r_t = \mu + \varepsilon_t$ ,  $\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \alpha\varepsilon_{t-1}^2 + \gamma I_{t-1}\varepsilon_{t-1}^2$ , with  $I_{t-1} = 1$ , when  $\varepsilon_{t-1} < 0$ , and  $I_{t-1} = 0$  otherwise.

Figure C2: The daily log returns, the conditional GARCH volatility and the FHS innovations of the S&P 500 index from August 23<sup>rd</sup> 1989 until July 9<sup>th</sup> 2003



Note: This figure depicts the daily log-returns of the S&P 500 Index from August 23<sup>rd</sup> 1989 until July 9<sup>th</sup> 2003 (3500 observations), the conditional GARCH volatility  $\sigma_t$  (annualized), the FHS and GAUSS innovations  $z_t$ . The GJR model is estimated using the ML method under the assumption of Gaussian innovations. The GAUSS innovations are assumed to be normally distributed. Comparable to Figure 1 of BAEM (2008).



**Table C2: GJR Pricing errors for OTM options for 2002 – 2004 disaggregated by moneyness and maturities**

Moneyness		Maturity								
		Less than 60			60 to 160			More than 160		
<i>S/K</i>		RMSE	MOE	MAE%	RMSE	MOE	MAE%	RMSE	MOE	MAE%
<0.85	<i>FHS</i>	0.40	0.12	59.51	0.72	0.28	53.26	1.82	0.99	67.74
	<i>GAUSS</i>	0.42	0.13	60.96	0.52	0.17	44.28	1.13	0.46	36.48
0.85-1.00	<i>FHS</i>	1.52	0.71	33.72	1.87	0.81	18.10	3.86	2.23	11.59
	<i>GAUSS</i>	1.57	0.74	34.55	1.67	0.68	15.83	3.75	2.14	11.12
1.00-1.15	<i>FHS</i>	1.55	0.74	30.79	3.32	1.84	13.79	6.20	4.22	12.85
	<i>GAUSS</i>	1.50	0.70	27.32	3.03	1.65	13.06	5.52	3.68	11.63
>1.15	<i>FHS</i>	0.78	0.35	84.82	1.36	0.66	55.93	2.50	1.21	32.81
	<i>GAUSS</i>	0.77	0.34	73.54	1.21	0.56	45.04	2.05	0.95	29.05

*Note: This table reports the pricing errors of the GJR model calibrated using FHS and Gaussian innovations disaggregated by moneyness and maturities. RMSE stands for the root mean square error of the dollar pricing error (model price–market price); MAE is the dollar average absolute pricing error and MOE is the dollar average pricing error outside the bid-ask spread (MAE% and MOE% are in relative terms as a percentage of the market price); Min (Max) is the minimum (maximum) pricing error; Err>0% is the percentage of positive pricing errors and ErrBD% is the average absolute pricing error as a percentage of the bid-ask spread. Inspired by Table 6 of BAEM (2008).*

## APPENDIX D ADDITIONAL RESULT OF THE FHS-RGARCH METHOD

In this appendix I present some additional results for the FHS-RGARCH method applied on both ITM and OTM option data for the period 2002 – 2011. The results are obtained using a window length  $n$  of 1472 days and 100 simulation paths ( $L = 100$ ). The estimated and calibrated parameters for the GARCH equation of the RGARCH model are given in Table D1. The estimated and calibrated parameters for the measurement equation are presented in Table D2.

**Table D1: Estimated and calibrated parameters of the GARCH equation of the RGARCH model for 2002 – 2011**

Panel A:

RGARCH	$\hat{\omega}$		$\hat{\beta}$		$\hat{\psi}$		Persistency		Ann. Vol.	
	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.
Year										
2002 – 2006	0.259	0.113	0.464	0.035	0.532	0.034	0.724	0.109	0.198	0.111
2007 – 2009	0.230	0.041	0.470	0.035	0.530	0.035	0.700	0.033	0.292	0.200
2010 – 2011	0.312	0.023	0.389	0.015	0.611	0.015	0.701	0.016	0.229	0.122

Panel B:

GAUSS	$\omega^*$		$\beta^*$		$\psi^*$		Persistency		Ann. Vol.	
	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.
Year										
2002 – 2006	0.205	0.933	0.614	0.340	0.373	0.340	0.818	0.975	0.225	0.138
2007 – 2009	-0.268	3.030	0.508	0.393	0.478	0.390	0.240	3.100	0.340	0.407
2010 – 2011	0.413	1.974	0.632	0.347	0.364	0.344	1.044	1.916	0.127	1.289

FHS

Year	Mean		Std.		Mean		Std.		Mean		Std.	
2002 – 2006	0.289	1.137	0.651	0.317	0.328	0.305	0.940	1.100	0.035	0.490		
2007 – 2009	0.133	0.593	0.610	0.343	0.380	0.340	0.743	0.583	0.312	0.232		
2010 – 2011	0.096	0.831	0.779	0.235	0.212	0.232	0.875	0.850	0.259	0.226		

Note: This table reports the mean and standard deviation (Std.) of each of the parameters of the GARCH equation of the RGARCH model estimated (Panel A) and calibrated (Panel B) on each Wednesday from January 2, 2002 to December 29, 2004. Estimation is done using maximum likelihood and 1000 historical log-returns while the model is calibrated using both the Filtered Historical Simulation (FHS) method and a calibration method using GAUSS innovations. The return and GARCH equation of the RGARCH model under the historical measure  $\mathbb{P}$  are specified by  $r_t = \mu + \sigma_t z_t$ ,  $\log(\sigma_t^2) = \omega + \beta \log(\sigma_{t-1}^2) + \psi \log(x_{t-1})$  and under the risk-neutral measure  $\mathbb{Q}$  by  $r_t = \mu^* + \sigma_t z_t$ ,  $\log(\sigma_t^2) = \omega^* + \beta^* \log(\sigma_{t-1}^2) + \psi^* \log(x_{t-1})$ . (See Table D2 for results regarding the parameters of the measurement equation). The persistency is calculated as  $\beta + \psi$  under the historical measure  $\mathbb{P}$  and  $\beta^* + \psi^*$  under the risk-neutral measure  $\mathbb{Q}$ . Ann. Vol. stands for the annualized volatility.

**Table D2: Estimated and calibrated parameters of the measurement equation of the RGARCH model for January 2<sup>nd</sup> 2002 – December 30<sup>th</sup> 2011**

Panel A:

RGARCH	$\hat{\xi}$		$\hat{\varphi}$		$\hat{\kappa}_1$		$\hat{\kappa}_2$	
	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.
Year								
2002 – 2006	-2.357	0.295	0.796	0.030	-0.124	0.036	0.078	0.015
2007 – 2009	-1.876	0.210	0.848	0.024	-0.064	0.004	0.101	0.009
2010 – 2011	-1.627	0.047	0.878	0.004	-0.082	0.008	0.117	0.003

Panel B:

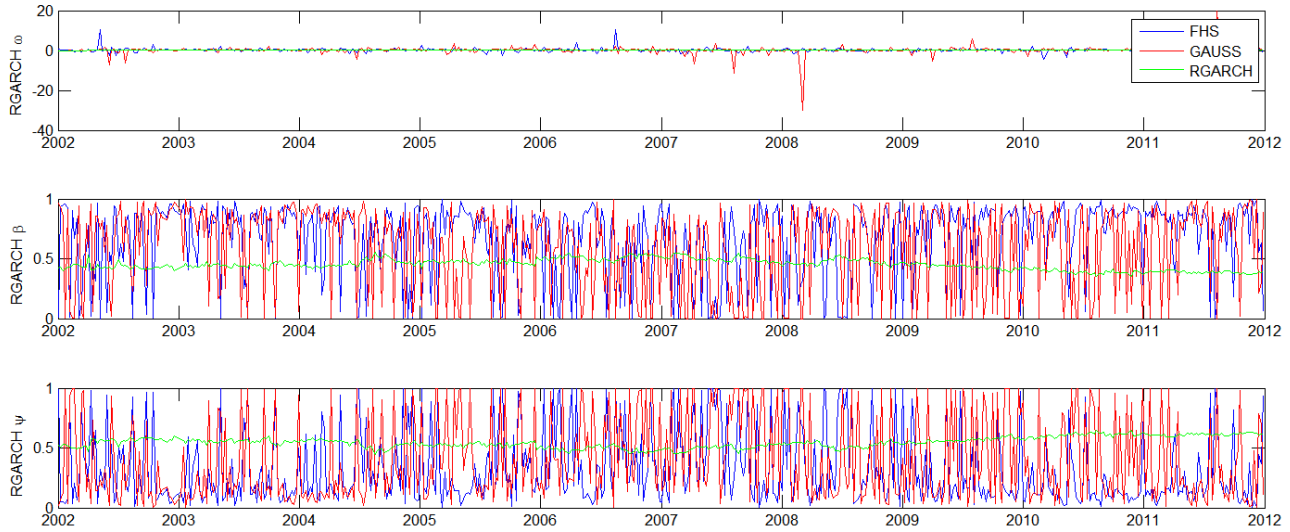
GAUSS	$\xi^*$		$\varphi^*$		$\kappa_1^*$		$\kappa_2^*$	
	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.
Year								
2002 – 2006	-2.928	5.357	0.896	0.391	-0.343	0.773	-0.050	0.959
2007 – 2009	-2.207	8.885	0.788	0.273	-0.225	0.317	-0.413	2.107
2010 – 2011	-2.845	4.540	0.853	0.205	-0.275	0.619	-0.144	0.994

FHS

Year	Mean		Std.		Mean		Std.	
2002 – 2006	-3.752	7.569	0.836	0.292	-0.360	1.194	-0.122	1.445
2007 – 2009	-2.224	3.031	0.825	0.235	-0.197	0.858	0.024	1.215
2010 – 2011	-2.001	2.870	0.912	0.220	-0.145	0.407	0.156	0.552

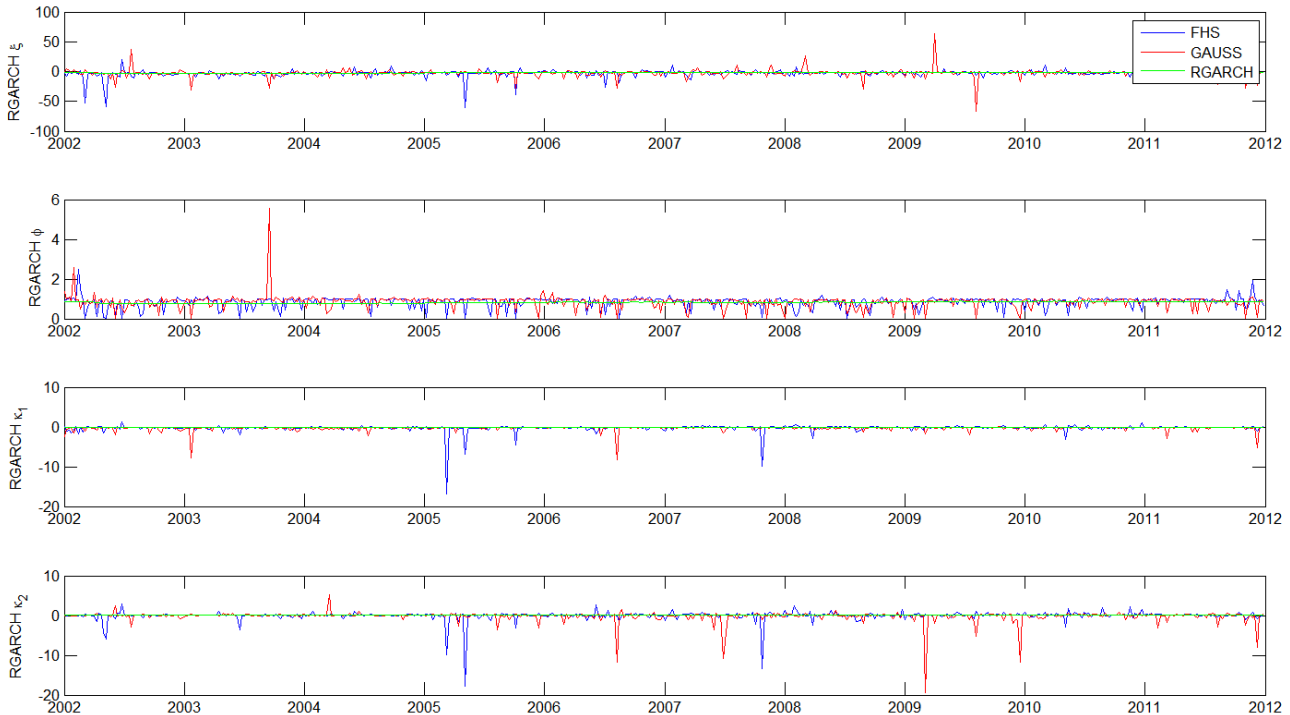
Note: This table reports the mean and standard deviation (Std.) of each of the parameters of the measurement equation of the RGARCH model estimated (Panel A) and calibrated (Panel B) on each Wednesday from January 2, 2002 to December 29, 2004. Estimation is done using maximum likelihood and 1000 historical log-returns while the model is calibrated using both the Filtered Historical Simulation (FHS) method and a calibration method using GAUSS innovations. The measurement equation of the RGARCH model under the historical measure  $\mathbb{P}$  is specified by  $x_t = \xi + \varphi\sigma^2 + \kappa_1 z_t + \kappa_2(z_t^2 - 1) + v_t$  and under the risk-neutral measure  $\mathbb{Q}$  by  $x_t = \xi^* + \varphi^*\sigma^2 + \kappa_1^* z_t + \kappa_2^*(z_t^2 - 1) + v_t$ .

Figure D1: Estimated and calibrated parameters of the GARCH equation of the RGARCH model for 2002 – 2011



Note: This figure presents the parameter values of the GARCH equation of the RGARCH model based on the QML estimation (RGARCH), the FHS method and the calibration with Gaussian innovations. The return and GARCH equation of the RGARCH model under the historical measure  $\mathbb{P}$  are specified by  $r_t = \mu + \sigma_t z_t$ ,  $\log(\sigma_t^2) = \omega + \beta \log(\sigma_{t-1}^2) + \psi \log(x_{t-1})$  and under the risk-neutral measure  $\mathbb{Q}$  by  $r_t = \mu^* + \sigma_t z_t$ ,  $\log(\sigma_t^2) = \omega^* + \beta^* \log(\sigma_{t-1}^2) + \psi^* \log(x_{t-1})$ .

Figure D2: Estimated and calibrated parameters of the measurement equation of the RGARCH model for January 2<sup>nd</sup> 2002 – December 30<sup>th</sup> 2011



Note: This figure presents the parameter values of the measurement equation of the RGARCH model based on the QML estimation (RGARCH), the FHS method and the calibration with Gaussian innovations. The measurement equation of the RGARCH model under the historical measure  $\mathbb{P}$  is specified by  $x_t = \xi + \varphi \sigma^2 + \kappa_1 z_t + \kappa_2 (z_t^2 - 1) + v_t$  and under the risk-neutral measure  $\mathbb{Q}$  by  $x_t = \xi^* + \varphi^* \sigma^2 + \kappa_1^* z_t + \kappa_2^* (z_t^2 - 1) + v_t$ .