Managers confirm stereotypes

Check the robustness of the results Dur, Kamphorst and Swank found in ‘Don’t Demotivate: Discriminate!’

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August 18, 2013

Abstract

The purpose of this paper is to check the robustness of the results Dur, Kamphorst and Swank found in ‘Don’t Demotivate: Discriminate!’. In their paper they consider confidence management with a promotion model. The manager has to promote one of the two employees based on their abilities. They found that it is optimal for the manager to discriminate when employees anticipate one employee to be favored. I check the robustness by adjusting the ability distributions of the employees, making the distributions non continuous. First of all, in this paper it is never optimal for the manager to promote an employee with a lower ability, while Dur, Kamphorst and Swank found the opposite. Second, the nature of discrimination differs. They found that the manager confirms the beliefs the employees have about the preferences of the manager. The results of this paper show that the manager confirms the beliefs the employees have about their own ability distributions. In other words, the manager confirms stereotypes. When employees expect their ability distributions to be equal, it can be optimal for the manager to use a nondiscriminatory promotion strategy.
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1. Introduction

Managers do have to motivate their employees to exert effort. To achieve this, the manager encourages the employees and assess their abilities. The manager uses confidence management when he tries to increase the employees confidence with the actions he takes. Necessary for confidence management is the fact that the manager has private information about the employees abilities (Bénabou & Tirole, 2003). Examples of actions the manager can take to increase the employees confidence are; announcing an employee of the month, assign an important client to an employee or promote an employee. With these actions the manager shows to the worker that he has confidence in him, which could boost the self-confidence of the worker. Dur, Kamphorst and Swank (2013) consider a confidence management setting with a promotion model. They show that all that is needed for discrimination is the anticipation by the workers that one worker will be favored. In their promotion model the manager has to promote one of the two employees. For the promotion decision the manager considers the output after the promotion decision. Beliefs play an important role in their paper. The beliefs of the employees may affect the promotion decision. They show that discrimination becomes self-fulfilling as the promotion decision matters more for the confidence of one employee who expects to be favored rather than an employee who expects to be disfavored (Dur, Kamphorst, & Swank, 2013). Take for example two employees, one male and one female. Once they both believe that the manager favors males, the manager indeed has an incentive to favor males. The reason is the fact that the self-confidence of the male would be ruined when he is not promoted. Meanwhile, the females self-confidence remains relatively intact even if she is not promoted. Therefore the manager has an incentive to confirm the beliefs that imply that the male is favored. There are three equilibria Dur, Kamphorst and Swank establish with their promotion game. The first is one without discrimination where the manager always promotes the more able one. In the second and the third equilibria the manager does discriminate. In these equilibria the favored employee is three times as likely to receive a promotion even though ex ante the employees have the same abilities. Further, they investigate which of these equilibria are stable. They show that only the discriminatory equilibria are stable. Despite the fact that the nondiscriminatory equilibrium is not a stable equilibrium, this is indeed an equilibrium the manager would prefer to commit himself to.

Really interesting about these results is the fact that discrimination is caused by the employees themselves. Despite the fact that the manager prefers to apply a promotion strategy without discrimination, which is also social optimal, this is not a stable equilibrium. The reason for this is that it is an optimal response for the manager to confirm the beliefs of the employees. So it is optimal for the manager to discriminate when the employees believe the manager has a favor for one of them. In this paper I check the robustness of these results by adjusting the ability distributions
of the employees. Dur, Kamphorst and Swank assume that the abilities of the employees are independently drawn from a uniform distribution on the interval $[0,1]$. In this paper I assume that ability is not continuous anymore, the employees can have a high or a low ability. The remaining of the model is exactly the same. The four important features of the promotion game remain intact. First, the promotion decision contains information about the employees’ abilities. Second, an employee’s effort depends positively on his perception of his ability. Third, the manager has better information about the employees’ abilities than the employees themselves. Finally, the manager can only convey information about the employees’ abilities through the promotion decision.

With this adjusted ability distribution, there are still some discriminatory equilibria and there is one equilibrium without discrimination. Dur, Kamphorst and Swank determine that there is discrimination in their paper when the probability that one employee is promoted is higher than the probability that the other employee is promoted, while ex ante the workers are equally qualified. This means that when employees differ in ability, not always the employee with the highest ability is promoted. On this point the results of this paper deviate. In the equilibria of this paper the manager always promotes the higher able employee when abilities differ. There is only discrimination when abilities are equal. Here, the manager discriminates when even though he observes ex post that the abilities are equal, the probability that he promotes one employee is higher than the probability that he promotes the other. The nature of discrimination is also different in this paper. Dur, Kamphorst and Swank show that it is optimal for the manager to discriminate when the employees anticipate one employee to be favored. The manager confirms the beliefs of the employees about the preferences of the manager. In this paper the manager confirms the beliefs of the employees about their ability distributions. If the employees believe that one employee has a higher probability to have a high ability it is optimal for the manager to confirm these beliefs. It turns out that a nondiscriminatory equilibrium is possible if the employees expect their ability distributions to be equal. Suppose employees believe that the probability that males have a high ability is higher than the probability that females have a high ability. In that case it is optimal for the manager to confirm these beliefs, when employees have the same ability the male is more likely to be promoted than the female. In other words, the manager does confirm stereotypes with his promotion strategy. Once employees expect males and females to have the same ability distributions it is optimal for the manager not to discriminate.

This paper is organized as follows. The next section relates this paper to the existing literature. Section 3 presents the promotion game, which is analyzed in section 4. Finally, in section 5 I conclude and discuss. In the appendix the mathematical calculations are represented.
2. Related literature

The model of this paper builds on the confidence management literature. Before I go into detail about confidence management I want to mention that the promotion in this paper is based on cheap talk. Crawford and Sobel (1982) show that cheap talk, which are costless, non-verifiable statements, is more likely to be informative if the goals of the agents are more aligned (Crawford & Sobel, 1982). While Crawford and Sobel studied cheap talk with only one audience, Farrell and Gibbons (1989) studied if the presence of a second receiver may help or instead may hurt prospects for cheap talk. They found that the presence of a second receiver can discipline or subvert the relationship with the other (Farrell & Gibbons, 1989). If there is more than one receiver, a manager with private information can also credibly rank the employees. Chakraborty and Harbaugh (2007) show that simple complementary conditions ensure that an expert with private information about multiple issues can credibly rank the issues for a decision maker (Chakraborty & Harbaugh, 2007). In the model of this paper the manager has to promote one of the employees, which can be seen as ranking the employees.

Moreover, this model is about confidence management. As Benabou and Tirole (2003) explain, there is only a confidence management motive if the principal has private information on the agent’s type. They show that performance incentives offered by an informed principal may have an effect on an agent’s perception of his own abilities (Bénabou & Tirole, 2003). The implication is that managerial decisions may affect the motivation of the employees. Ishida (2006) applies this idea to optimal promotion policies. The principal has an incentive to use promotions strategically to boost the agent’s self-confidence if the principal has superior knowledge about the agent’s productivity compared to the agent himself (Ishida, 2006). Unlike my model, Ishida assumes that the productivity of the promoted job depends more on the agent’s innate ability. Crutzen, Swank and Visser (2007) also study confidence management. Like this paper, they examine a tournament model à la Lazear and Rosen (1980) in which two employees compete for a promotion. In the tournament theory, the focus is on the effect of the promotion decision on the employees’ behavior before the promotion decision (Lazear & Rosen, 1980). Crutzen, Swank and Visser (2007) analyze, as this paper, how the promotion decision affects employees’ behavior after the promotion. They consider confidence management on interpersonal comparisons in teams. They show that it may hurt an organization to compare employees by using ordinary cheap talk when the degree of synergies between employees is high (Crutzen, Swank, & Visser, 2007). The model of this paper resembles their model formed in the benchmark case, where synergies are absent. Both studies analyze under which conditions the

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1 The results of this paper would not change if a financial reward is associated with the promotion, because I only consider the effort of the employees after the promotion.
manager differentiates between employees; say that one employee is better than the other. The two main differences are the ability distributions and the number of possible messages. They assume that abilities are drawn from iid random variables, with continuous density functions \( f(\cdot) \) on \([0,1]\) while in the model of this paper ability can only take two values, high or low. Moreover, in the model of this paper the manager must promote one employee, while in their model the manager also has the option to abstain from promoting. In the benchmark case, they show that the manager publicly compares the two employees as soon as he observes a difference in ability and that he weakly prefers to abstain from promoting one of the employees if abilities are equal. They argue that this strategy is an equilibrium for any continuous density function (Crutzen, Swank, & Visser, 2007). In this paper I show how results differ if ability is not continuous anymore.

Last, but certainly not least, this paper is about discrimination. As mentioned in section 1, Dur, Kamphorst and Swank offer a new theory of discrimination. They show that in a confidence management setting where the manager has to promote one employee, self-fulfilling discriminatory equilibria exist. Even though the manager does not favor one of the employees, he has an incentive to favor the employee who the employees expect to be favored. They show that there are two discriminatory equilibria in which the probability that the favored employee is promoted is three times higher than the probability that the other employee is promoted, even though \textit{ex ante} the employees have the same abilities. There is also one equilibrium without discrimination, but this equilibrium is unstable. If the manager could commit to a promotion strategy he would prefer to commit to one without discrimination, because total expected output is maximized without discrimination. The model Dur, Kamphorst and Swank use is a promotion game in which the manager has to promote one of the employees based on their abilities. In their model abilities are independently drawn from a uniform distribution on the interval \([0,1]\), while in the promotion game of this paper ability can only be high or low. The remaining of the model is exactly the same in this paper.

The explanation for discrimination in this paper is not taste-based discrimination. Becker (1957) explains that an individual has a ‘taste for discrimination’ if he acts as if he is willing to pay something, either directly or in the form of a reduced income, to be associated with some persons instead of others (Becker, 1957). Taste-based discrimination models include in the utility functions of the manager or the employees a desire to avoid promoting a certain employee. Likewise, the explanation for discrimination is not based on the enforcement of some social norm. Akerlof (1985) represents a robust model of discrimination in which some trading partners will boycott any firm that violates a discriminatory social custom. This results in the fact that no entrant can profit by violating the discriminatory custom (Akerlof, 1985). Peski and Szentes (2011) also show discrimination based
on social pressure. They consider a dynamic economy in which agents are repeatedly matched with one another and decide whether to enter into profitable partnerships. They show that in some environments, every stable equilibrium involves discrimination. Individuals discriminate, because they do not want to be associated with the other race (Peski & Szentes, 2013 (forthcoming)). Dur, Kamphorst and Swank emphasize that the discrimination in their model is caused by the presence of discriminatory preferences in the society itself. In the first instance, the manager does not want to discriminate, if he could he would commit to a promotion strategy without discrimination. Unfortunately, when the employees anticipate discrimination it is optimal for the manager to confirm these beliefs.

Although the discrimination in this paper is likewise based on the fact that the manager does confirm the beliefs of the employees, the nature of discrimination differs. Dur, Kamphorst and Swank show that the manager confirms the beliefs the employees form about the promotion strategy of the manager. When employees expect one employee to be favored the manager will indeed favor that employee. Thus the manager confirms the beliefs the employees form about the preferences of the manager. While the results of this paper show that the manager discriminates as soon as the employees themselves expect ex ante a higher ability for one employee compared to the other. Once the employees anticipate that the ability distributions differ, it is optimal for the manager to confirm these beliefs. Accordingly, the manager confirms the beliefs the employees form about their ability distributions. As soon as the employees believe the ability distributions of both employees are equal, the manager can use a nondiscriminatory promotion strategy.

Other papers of discrimination where beliefs play an important role are written by Arrow (1973) and Coate and Loury (1993). In both papers employees must make an investment to be qualified for a skilled job or a demanding task. Arrow shows that employers’ prejudicial beliefs can be self-fulfilling when employee productivity is endogenous (Arrow, 1973). In Arrows model different groups receive different wages for the same work. Coate and Loury modified this setup so that different groups receive the same wage for the same work, but one group has a lower probability of being assigned to the higher-paid jobs (Coate & Loury, 1993). As the employer does not perfectly observe the investment the employees take, he must form an expectation about the investment. If the employer believes that the employees of one group are less likely to invest, this leads to the possibility of self-fulfilling equilibria. The manager is less likely to assign the employees of this group to the higher-paid jobs, with the consequence that the employees of this group have less incentives to make an investment. This in turn leads to a lower probability that the employees of this group make an investment, which confirms the ex ante beliefs of the employer (Coate & Loury, 1993).
3. The promotion game

In this paper I use the promotion game formed by Dur, Kamphorst and Swank. They used this model to show that self-fulfilling discriminatory equilibria exist, where workers expect the manager to favor one worker (Dur, Kamphorst, & Swank, 2013). In their promotion game the abilities of the employees are independently drawn from a uniform distribution on the interval [0,1]. In this paper I adjust this assumption, here there are two types of abilities, low and high. Below, I describe this slightly adapted promotion game.

I consider a manager who supervises two employees $i \in \{1,2\}$. Each employee $i$ chooses effort $e_i, e_i \geq 0$ to produce output, $y_i = a_i e_i$, where $a_i$ denotes $i$’s ability. An important feature of this model is that the manager has superior information about $a_i$. Specifically, the manager observes $a_i$, while the employees only know that there are two types of abilities, $a_i \in \{a_l, a_h\}$ with $a_h > a_l$, and they know the distribution of these abilities. Suppose that the distribution of these abilities can differ per employee. Specifically we assume that,

\[
\Pr(a_1 = a_h) = \mu \quad \text{and} \quad \Pr(a_1 = a_l) = 1 - \mu
\]

\[
\Pr(a_2 = a_h) = \nu \quad \text{and} \quad \Pr(a_2 = a_l) = 1 - \nu
\]

With $1 > \mu$ and $1 > \nu$. Effort is costly. For simplicity, we assume that the costs of effort are quadratic.

At the beginning of the game, the manager makes a promotion decision $m \in \{1,2\}$, where $m = i$ denotes that employee $i$ is promoted. Being promoted (or not) does not affect the employee’s work or pay. However, as we show below, if the promotion decision depends on the employees’ abilities, the promotion decision affects effort levels and in turn output. In this model, the sole role of the promotion decision is to convey information to the employees about their abilities.\(^2\)

The employees want to contribute to the organization, but are effort averse. More specifically, employee $i$’s preferences are represented by

\[
U_i(e_i) = E(a_i|m)e_i - \frac{1}{2} e_i^2
\]

where $E(a_i|m)$ is $i$’s expectation of his ability, conditional on the promotion decision $m$. The manager aims at maximizing output:

\[
U_m(m, a_1, a_2) = \sum_{i=1}^2 y_i = a_1 e_1 + a_2 e_2
\]

\(^2\) Important in this model is that the manager can commit to the number of promotions but not to a promotion strategy. One can think of other means of conveying information, for instance money.
The timing of the model is as follows. At the beginning of the game nature draws $a_1$ and $a_2$. The manager observes $a_1$ and $a_2$, but the employees do not. Next, the manager makes the promotion decision. The employees update their beliefs about their abilities and choose an effort level. Finally, payoffs are realized.

The effort strategies of the employees are simple in this game. Each employee chooses an effort level that is equal to the expected value of his ability, conditional on the promotion decision

$$e_i = E(a_i|m).$$

The promotion strategy of the manager, $s^M(a_1,a_2)$ stipulates for all combinations of $a_1$ and $a_2$ her promotion decision, $m$. I identify perfect Bayesian-Nash equilibria in which (i) employees effort choices are optimal, given their beliefs about their abilities; (ii) the manager's promotion strategy is optimal, given employees' effort choices; and (iii) beliefs are updated according Bayes' rule.

This is a cheap-talk game. It is well-known that in cheap talk games there always exist babbling equilibria. A babbling equilibrium is an equilibrium in which the sender's strategy is independent of type and the receiver's strategy is independent of signal (Sobel, 2012). In this model, the manager's promotion decision is independent of $a_1$ and $a_2$ and the employees beliefs are identical to the beliefs before the promotion, $E(a_1 = a_H|m) = \mu$ and $E(a_2 = a_H|m) = \nu$. In the remainder of this paper I ignore these babbling equilibria. Instead, I identify the equilibrium promotion strategies in which the promotion decision does depend on $a_1$ and $a_2$.

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\(^3\) How the promotion decision affects the expected value of $a_1$ depends on the manager's promotion strategy. To keep notation simple, ignore the latter is ignored and this expectation is simply written as $E(a_1|m)$. 

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4. The equilibrium promotion strategies

To find the equilibrium promotion strategies I need to examine the optimal effort strategies for the employees and the optimal promotion strategy for the manager. As mentioned before, the optimal effort strategy for each employee is given by $e_i = E(a_i|m)$. According to this optimal effort strategy of the employees, the promotion decision affects the behavior of the employees through their beliefs. The manager prefers to send $m = 1$ to $m = 2$, if $m = 1$ yields a higher total output than $m = 2$, which is the case if:

$$E(a_1|m = 1)a_1 + E(a_2|m = 1)a_2 > E(a_1|m = 1)a_1 + E(a_2|m = 1)a_2$$

If the reverse holds, the manager would like to promote employee 2. Moreover, if both equations are equal to each other the manager would like to use a mixed strategy.

There are three cases in which the manager has to choose which employee to promote; case 1: $a_i = a_{-i}$, case 2: $a_1 = a_h$ and $a_2 = a_l$, and case 3 $a_1 = a_l$ and $a_2 = a_h$. The general strategy for these three cases is represented by:

I. $a_i = a_{-i}$ → $\alpha = \text{probability employee 1 is promoted}$
   $$(1 - \alpha) = \text{probability employee 2 is promoted}$$

II. $a_1 = a_h \& a_2 = a_l$ → $\pi = \text{probability employee 1 is promoted}$
   $$(1 - \pi) = \text{probability employee 2 is promoted}$$

III. $a_1 = a_l \& a_2 = a_h$ → $\delta = \text{probability employee 1 is promoted}$
   $$(1 - \delta) = \text{probability employee 2 is promoted}$$

The manager uses a mixed strategy in the cases I-III if:

I. $E(a_1|m = 1) + E(a_2|m = 1) = E(a_1|m = 2) + E(a_2|m = 2)$

II. $E(a_1|m = 1)a_h + E(a_2|m = 1)a_l = E(a_1|m = 2)a_h + E(a_2|m = 2)a_l$

III. $E(a_1|m = 1)a_l + E(a_2|m = 1)a_h = E(a_1|m = 2)a_l + E(a_2|m = 2)a_h$

The expectations are fixed by promotion strategy. Given this, it is not possible that there is a promotion strategy with more than one mixed strategy. A mixed strategy in case I as well as in case II or case III requires $E(a_1|m = 1) = E(a_1|m = 2)$ and $E(a_2|m = 1) = E(a_2|m = 2)$. This is only the case in babbling equilibria. A mixed strategy simultaneously in cases II and III is only possible if $a_h = a_l$, which is in contradiction to the earlier assumption $a_h > a_l$. From this I can conclude that an equilibrium with more than one mixed strategy is not possible.
I examine if the promotion strategies listed in table 1 are an optimal response for the manager.

<table>
<thead>
<tr>
<th>Promotion strategy</th>
<th>( s^M(a_1, a_2) )</th>
<th>( a_i = a_{-i} )</th>
<th>( a_1 = a_h ) &amp; ( a_2 = a_l )</th>
<th>( a_1 = a_l ) &amp; ( a_2 = a_h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (B)</td>
<td>1 (0)</td>
<td>1 (0)</td>
<td>0 (1)</td>
<td></td>
</tr>
<tr>
<td>C (D)</td>
<td>0 (1)</td>
<td>1 (0)</td>
<td>0 (1)</td>
<td></td>
</tr>
<tr>
<td>E (F)</td>
<td>1 (0)</td>
<td>0 (1)</td>
<td>0 (1)</td>
<td></td>
</tr>
<tr>
<td>G (H)</td>
<td>( \alpha (1 - \alpha) )</td>
<td>1 (0)</td>
<td>0 (1)</td>
<td></td>
</tr>
<tr>
<td>I (J)</td>
<td>( \alpha (1 - \alpha) )</td>
<td>1 (0)</td>
<td>1 (0)</td>
<td></td>
</tr>
<tr>
<td>K (L)</td>
<td>1 (0)</td>
<td>( \pi (1 - \pi) )</td>
<td>0 (1)</td>
<td></td>
</tr>
<tr>
<td>M (N)</td>
<td>0 (1)</td>
<td>( \pi (1 - \pi) )</td>
<td>0 (1)</td>
<td></td>
</tr>
<tr>
<td>O (P)</td>
<td>0 (1)</td>
<td>1 (0)</td>
<td>( \delta (1 - \delta) )</td>
<td></td>
</tr>
<tr>
<td>Q (R)</td>
<td>1 (0)</td>
<td>1 (0)</td>
<td>( \delta (1 - \delta) )</td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Promotion strategies, with \( \alpha, \pi \) and \( \delta \) denote a mixed strategy. The strategies between the brackets are the reverse language strategies.

The strategies between the brackets in table 1 are the reverse language strategies of the strategies in the same column. For example, strategy B is the reverse language strategy of strategy A. In a reverse world, where reverse language is used, being promoted means the same as not being promoted in the normal world. This is the reason that for example at strategy H, \((1 - \alpha)\) the probability is that employee 1 is promoted (which was in a normal world ‘not being promoted’). The beliefs of the employees about their abilities are also the reverse. Specifically, the beliefs about the abilities of the employees if employee 1 is promoted are equal to the beliefs about the abilities of the employees if employee 2 is promoted in the reverse world.\(^4\) In the end, this means that a strategy and its reverse language strategy are the same. They both have the same conditions under which they are optimal. For this reason I do not discuss these strategies further in this paper. Remember, if a strategy is an optimal promotion strategy for the manager, this also counts for its reverse language strategy.

Strategies E (F) and I (J) – Q (R) can never be optimal promotion strategies for the manager. It is quite intuitive that these promotion strategies are not optimal, they all contain an inconsistency. For instance, with promotion strategy K the manager always promotes employee 1 in case three. This means that he promotes employee 1 when employee 1 has a low ability and employee 2 has a high ability. At the same time the manager prefers to promote sometimes employee 2 in case I.

\(^4\) See appendix (1) for an example: the beliefs of the employees about their abilities in strategies A & B
specifically with a probability \((1 - \alpha)\). This is inconsistent, because it costs more for the manager not to promote employee 2 in case III than in case I. The same kind of reasoning holds for the other strategies.

For the remaining promotion strategies A (B), C (D) and G (H) I consider if, and under which conditions, they are optimal for the manager. I begin with the promotion strategies where no mixed strategy is used, A (B) and C (D).

**Strategy A (B)**

<table>
<thead>
<tr>
<th>Promotion strategy (s^M(a_1, a_2))</th>
<th>(\alpha = a_{-i})</th>
<th>(\pi = a_h\ &amp; a_2 = a_i)</th>
<th>(\delta = a_i &amp; a_2 = a_h)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A (B)</strong></td>
<td>(1) (0)</td>
<td>(1) (0)</td>
<td>(0) (1)</td>
</tr>
</tbody>
</table>

Recall from section 3, in equilibrium the beliefs of the employees about their abilities are updated according to Bayes’ rule. If employee 1 is promoted the beliefs are given by

\[
\Pr(a_1 = a_h|m = 1) = \frac{\mu}{1 - \nu + \mu\nu} \\
\Pr(a_2 = a_h|m = 1) = \frac{\mu\nu}{1 - \nu + \mu\nu}
\]

Moreover, if employee 2 is promoted

\[
\Pr(a_1 = a_h|m = 2) = 0 \\
\Pr(a_2 = a_h|m = 2) = 1
\]

Using these beliefs and the optimal effort strategy of the employees, this promotion strategy can be optimal in cases I-III if:

I. \(a_i = a_{-i} \Rightarrow \alpha = 1\) is optimal for the manager if: \(\mu > (1 - \nu)\& \nu > (1 - \mu)\)

II. \(a_1 = a_h \& a_2 = a_i \Rightarrow \pi = 1\) is optimal for the manager if: \(\mu > (1 - \nu)\frac{a_1}{a_h} \& \nu > \frac{a_1 - \mu a_h}{a_i}\)

III. \(a_1 = a_i \& a_2 = a_h \Rightarrow \delta = 0\) is optimal for the manager if: \(\mu < (1 - \nu)\frac{a_h}{a_i} \& \nu < \frac{a_1 - \mu a_h}{a_i}\)

\(^5\) See appendix (2) for mathematical calculations
This promotion strategy maximizes output if: 

\[ (1 - \nu) < \mu < (1 - \nu) \frac{a_h}{a_l} \land (1 - \mu) < \nu < \frac{a_l - \mu a_h}{a_l}. \]

These boundaries show that the values of \( \mu \) as well as \( \nu \) are not allowed to be too low or too high, figured in figure 1 and 2.

![Figure 1](image1.png)

**Figure 1. Boundaries of \( \mu \) for which strategy A (B) is an optimal promotion strategy.**

![Figure 2](image2.png)

**Figure 2. Boundaries of \( \nu \) for which strategy A (B) is an optimal promotion strategy.**

If \( \mu \) or \( \nu \) is too low, the conditions for the first case are not satisfied. The manager has an incentive to promote the other employee than set by its promotion strategy when \( a_l = a_{-i} \); promote employee 2 at promotion strategy A. The reason for this incentive is the fact that by deviating from its promotion strategy, the manager can show that one employee has \( a_h \) with certainty. The employees hold low beliefs about their abilities when employee 1 is promoted if \( \mu \) and \( \nu \) are low. Therefore it is attractive for the manager to promote employee 2, because this has the benefits that employee 2 believes then that he has \( a_h \) with certainty. This comes at the costs that employee 1 knows he has \( a_l \). The benefits are higher than the costs, because the employees’ beliefs about their abilities are low when employee 1 is promoted.

The conditions for the second case are less restrictive compared to the conditions for the first case. Due this, the conditions for the second case are automatically satisfied if the conditions for the first case are satisfied (\( a_h > a_l \)).

The conditions for the third case are not met if \( \mu \) and \( \nu \) are too high. If one of these parameters exceeds this boundary, the manager has an incentive to deviate from its promotion strategy in the third case; promote employee 1 at promotion strategy A. The beliefs the employees have about their own abilities when employee 1 is promoted are high when \( \mu \) and \( \nu \) are high. The manager ruins these beliefs when he promotes employee 2. The fact that the manager reveals that one employee has \( a_h \) may be good, but at the same time he cannot prevent that the other employee finds out that he has \( a_l \). To prevent this, the manager may deviate from its promotion strategy.
Therefore, the manager deviates from its promotion strategy if $a_1 = a_i$ & $a_2 = a_h$, because the costs of deviating are low and the benefits are high.

This is a discriminatory equilibrium, because the manager always promotes employee 1 when \textit{ex post} he observes that abilities are equal. This discrimination is present when the employees have the same ability distributions \textit{ex ante} as well as when the abilities differ \textit{ex ante}. The manager does not discriminate in this equilibrium when abilities differ, then the employee with the highest ability is always promoted.

**Strategy C (D)**

<table>
<thead>
<tr>
<th>Promotion strategy $s^M(a_1, a_2)$</th>
<th>$a_i = a_{-i}$</th>
<th>$a_1 = a_h$ &amp; $a_2 = a_l$</th>
<th>$a_1 = a_l$ &amp; $a_2 = a_h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C (D)</td>
<td>0 (1)</td>
<td>1 (0)</td>
<td>0 (1)</td>
</tr>
</tbody>
</table>

When employee 1 is promoted beliefs are given by,

$$\Pr(a_1 = a_h|m = 1) = 1$$

$$\Pr(a_2 = a_h|m = 1) = 0$$

If employee 2 is promoted,

$$\Pr(a_1 = a_h|m = 2) = \frac{\mu \nu}{1 - \mu + \mu \nu}$$

$$\Pr(a_2 = a_h|m = 2) = \frac{\nu}{1 - \mu + \mu \nu}$$

This promotion strategy can be optimal for the manager to use in cases I-III if:

1. $a_i = a_{-i} \to \alpha = 0$ is optimal for the manager if: $\mu > (1 - \nu)$ & $\nu > (1 - \mu)$
2. $a_1 = a_h$ & $a_2 = a_l \to \pi = 0$ is optimal for the manager if: $\mu < \frac{a_h - \nu a_l}{a_h}$ & $\nu < (1 - \mu) \frac{a_h}{a_l}$
3. $a_1 = a_l$ & $a_2 = a_h \to \delta = 0$ is optimal for the manager if: $\mu > \frac{a_l - \nu a_h}{a_l}$ & $\nu > (1 - \mu) \frac{a_l}{a_h}$

These promotion strategies are optimal for the manager to use if: $(1 - \nu) < \mu < \frac{a_h - \nu a_l}{a_h}$ & $(1 - \mu) < \nu < (1 - \mu) \frac{a_h}{a_l}$. Similar to strategy A, the values of $\mu$ as well as $\nu$ are not allowed to be too low or too high. This is represented in figures 3 and 4.

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ootnote{See appendix (3) for mathematical calculations}
Figure 3. Boundaries of $\mu$ for which strategy C (D) is an optimal promotion strategy.

Figure 4. Boundaries of $\nu$ for which strategy C (D) is an optimal promotion strategy.

For the first case the same reasoning as before holds. If $\mu$ and $\nu$ are too low it is optimal for the manager not to stick to its promotion strategy if abilities are equal. By deviating the manager can reveal that one employee has for sure $a_h$. The benefits of deviating are higher than the costs, because the employees’ beliefs about their abilities are low when employee 2 is promoted.

In the second case $\mu$ and $\nu$ are not allowed to be too high. The manager has an incentive to deviate from its promotion strategy when $a_1 = a_h$ & $a_2 = a_i$; promote employee 1 at promotion strategy C. The same reasoning as in the third case for strategy A (B) can be applied. If the manager sticks to its promotion strategy, he reveals that one employee has $a_h$ and one has $a_i$. If $\mu$ and $\nu$ are high, the employees have high beliefs about their abilities when employee 2 is promoted. The manager does not want to ruin these beliefs, thus he prefers to deviate from its promotion strategy in case II. The benefits are higher than the costs, because the employee’s beliefs of their abilities are high when employee 2 is promoted.

The conditions for the third case are already satisfied if the conditions for the first case are satisfied, because these are less restrictive ($a_h > a_i$).

Likewise promotion strategy A, the manager discriminates in promotion strategy C. This equilibrium is the mirror of the first equilibrium, here employee 2 is favored. The manager always promotes employee 2, even thought ex post he observes that the employees have the same abilities. This discrimination takes place when ability distributions are equal ex ante and when these are unequal. The manager does promote the employee with a high ability when abilities differ. This means that no discrimination takes place when abilities differ.

From previous results we learn that pure strategies A-D can be optimal discriminatory strategies for the manager to use. These promotion strategies are only optimal response for the
manager under certain conditions. If the probabilities that employee 1 and 2 have a high ability are too high or low, the strategies are no equilibrium. Reason for this is the fact that the manager wants to show that one employee certainly has a high ability or he want to avoid showing that one employee has a low ability. This problem may be solved if the manager uses a mixed strategy in the first case, then the manager can never show with his promotion decision that one employee has with certainty a high or low ability. If a mixed strategy in case 1 where $\alpha = 0.5$ is possible, this is an equilibrium without discrimination.

**Strategy G (H)**

<table>
<thead>
<tr>
<th>Promotion strategy $s^H(a_1, a_2)$</th>
<th>$a_i = a_{-i}$ $\alpha$</th>
<th>$a_1 = a_h &amp; a_2 = a_l$ $\pi$</th>
<th>$a_1 = a_l &amp; a_2 = a_h$ $\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>G (H)</td>
<td>$\alpha (1 - \alpha)$</td>
<td>1 (0)</td>
<td>0 (1)</td>
</tr>
</tbody>
</table>

The beliefs under this mixed strategy are given by,

\[
\Pr(a_1 = a_h|m = 1) = \frac{\mu - \mu \nu + a \mu \nu}{\mu - \mu \nu + 2 a \mu \nu + \alpha - a \mu - a \nu}
\]

\[
\Pr(a_2 = a_h|m = 1) = \frac{a \mu \nu}{\mu - \mu \nu + 2 a \mu \nu + \alpha - a \mu - a \nu}
\]

\[
\Pr(a_1 = a_h|m = 2) = \frac{\mu \nu - a \mu \nu}{1 - 2 a \mu \nu - \mu + \mu \nu - a + a \mu + a \nu}
\]

\[
\Pr(a_2 = a_h|m = 2) = \frac{\nu - a \mu \nu}{1 - 2 a \mu \nu - \mu + \mu \nu - a + a \mu + a \nu}
\]

This promotion strategy does maximize output for the manager for all values of $\mu$ and $\nu$, because the conditions for all cases are always satisfied in this game:

I. $a_i = a_{-i} \rightarrow \alpha$, it is optimal for the manager to mix with $\alpha = \frac{\mu - \mu \nu}{\mu + \nu - 2 \mu \nu}$

II. $a_1 = a_h \& a_2 = a_l \rightarrow \pi = 1$ is optimal for the manager if: $a_h > a_l$

This condition is always satisfied.

III. $a_1 = a_l \& a_2 = a_h \rightarrow \delta = 0$ optimal for the manger if: $a_l < a_h$

This condition is always satisfied.

From these results, I conclude that promotion strategy G is optimal for the manager to use for all values of $\mu$ and $\nu$. When abilities differ the manager promotes the employee with the highest ability. When abilities are equal the manager uses a mixed strategy. He promotes employee 1 with

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7 See appendix (4) for mathematical calculations
probability $\alpha$ and employee 2 with probability $(1 - \alpha)$, with $\alpha = \frac{\mu - \nu}{\mu + \nu - 2\mu\nu}$. What values can $\alpha$ take? When $\mu = \nu$ it turns out that $\alpha = 0.5$. This is an equilibrium without discrimination. First of all, the manager does not discriminate when abilities differ, because the manager always promotes the more able one in that case. Moreover, both employees have the same probability to be promoted when their abilities are equal, given that ex ante they both have the same abilities. This also indicates no discrimination. The probability that employee 1 gets the promotion when abilities are equal increases if $\mu > \nu$, $\alpha > 0.5$. This probability decreases if $\mu < \nu$, $\alpha < 0.5$. Promotion strategy $G$ contains discrimination when $\alpha > 0.5$, likewise when $\alpha < 0.5$. In the first (second) case the probability that the manager promotes employee 1 (2) is higher than the probability that he promotes employee 2 (1), even though he observes ex post equal abilities. If there is no chance that employee 1 has a high ability or if employee 2 certainly has a high ability, $\alpha = 0$. Meanwhile, the probability that employee 1 is promoted when abilities are equal is one when employee 1 certainly has a high ability or employee two certainly has a low ability.

This is an equilibrium where the manager does not discriminate when ex ante both workers are equally qualified. Once one employee is qualified higher ex ante, the manager has an incentive to favor this employee. There is discrimination because even though the manager observes that both employees have the same abilities ex post, the probability that he promotes the employee who has a higher ability ex ante is higher than the probability that the other employee gets the promotion. The employees expect the manager to favor one of the employees when the ex ante ability distributions differ per employee. If employee 1 has a higher probability to have a high ability than employee 2 ($\mu > \nu$), the employees expect the probability that employee 1 is promoted to be higher than the probability that the manager promotes employee 2. The manager confirms these beliefs, as $\alpha > 0.5$ when $\mu > \nu$. The reverse holds if $\mu > \nu$.

Important to notice is the nature of the discrimination. The manager does discriminate as soon as the employees believe their ability distributions differ, regardless of the real ability distributions. Thus, it is optimal for the manager to discriminate if the employees believe $\mu \neq \nu$. In other words, the manager confirms stereotypes with this promotion strategy. This discrimination differs from the one Dur, Kamphorst and Swank found. According to their results, the manager does discriminate in order to confirm the employees beliefs about the preferences of the manager.
5. Conclusion and Discussion

The purpose of this paper is to check the robustness of the results Dur, Kamphorst and Swank found in ‘Don’t Demotivate: Discriminate!’ in which they offer a new theory of discrimination. They consider a confidence management setting with a promotion model. In this model the manager has to promote one of the two employees. The role of the promotion decision is to convey information to the employees about their abilities, which are independently drawn from a uniform distribution on the interval \([0,1]\). For this promotion decision the manager considers the employees’ behavior after the promotion decision. Therefore, beliefs play an important role. They show that it is optimal for managers to discriminate when employees anticipate one employee to be favored by the manager. The only stable equilibria are discriminatory equilibria, in which the probability that the favored employee is promoted is three times higher than the probability that the other employee gets the promotion, even though \textit{ex ante} the employees have the same abilities. In their paper it is optimal for the manager to confirm the beliefs the employees have about the preferences of the manager.

To check the robustness of these results I adjust the ability distributions of the employees. I assume that ability is not continuous anymore, the employees can only have a high or a low ability. The remaining of the model is exactly the same. This results in three discriminatory equilibrium promotion strategies if employees believe the ability distributions to be different. Two promotion strategies are discriminatory because the manager always promotes the same employee when he observes \textit{ex post} that the employees have the same abilities (promotion strategies A (B) and C (D)). These promotion strategies are only optimal for the manager to use under specific conditions. The probability that the employees have a high ability are not allowed to be too high or too low. With the third strategy the manager also discriminates when he observes that both employees have the same ability. The manager discriminates because he promotes the employee who is expected to have a higher ability \textit{ex ante} with a higher probability than the other even though he observes \textit{ex post} that abilities are equal (strategy G (H)). There are no specific conditions for this strategy to be optimal. Further I consider the equilibrium promotion strategies when the employees believe that both employees have the same ability distributions. In that case the first two discriminatory equilibria still hold in which the manager always promotes the same employee when he observes that both employees have the same ability. These are still only equilibrium promotion strategies when the probabilities that the employees have a high ability are not too high or too low. Moreover, if employees believe ability distributions to be the same, there is one equilibrium promotion strategy without discrimination. The manager always promotes the employee with the highest ability when
abilities differ and when abilities are equal both employees have a probability to be promoted of one half.

These results differ from the results Dur, Kamphorst and Swank found. First of all, they found that it may be optimal for a manager to promote the employee with the lowest ability. Meanwhile, the results of this paper show that it is always optimal for the manager to promote the employee with the highest ability when abilities differ. Second, the nature of discrimination differs. Dur, Kamphorst and Swank found that it is optimal for the manager to confirm the beliefs the employees have about the promotion strategy of the manager. Once the employees anticipate one employee to be favored it is optimal to promote this employee with a higher probability than the other employee. The results of this paper show that it is optimal for the manager to confirm the beliefs the employees have about their ability distributions. When the employees believe that one employee has a higher probability to have a high ability compared to the other employee, it is optimal for the manager to promote that employee with a higher probability once he observes abilities to be equal. Concluding, in this paper I show that it is optimal for the manager to confirm stereotypes.

As usual, the results are derived from a model based on many assumptions. These assumptions are made to keep simple results, but at the same time they can be restrictive. In this model the manager has for example perfect private information about the employee’s abilities. In practice this may not be completely reasonable. It would be more credible if the manager as well as the employees get a signal about the employee’s abilities. These signals must contain different information, because information asymmetry is necessary for confidence management. It would be reasonable that the worker gets a signal based on earlier performance appraisals. The manager at the same time gets a signal by using his experience of comparing the performance of an employee to the performance of other employees. This is a possible extension for further research, where the manager gets a more informative signal about the employee’s abilities than the employees themselves.
6. Appendix

(1)

Reverse language: the beliefs about the abilities of the employees if employee 1 is promoted are equal to the beliefs about the abilities of the employees if employee 2 is promoted in the reverse world:

\text{Strategy A (B)}

\[\Pr(a_1 = a_h | m = 1) = \frac{\mu}{1 - \nu + \mu\nu} (= 0)\]

\[\Pr(a_2 = a_h | m = 1) = \frac{\mu\nu}{1 - \nu + \mu\nu} (= 1)\]

\[\Pr(a_1 = a_h | m = 2) = 0 (= \frac{\mu}{1 - \nu + \mu\nu})\]

\[\Pr(a_2 = a_h | m = 2) = 1 (= \frac{\mu\nu}{1 - \nu + \mu\nu})\]

(2)

\text{Strategy A (B)}

\[a_i = a_{-i} \to \alpha = 1 \text{ optimal for the manager if: } \mu > (1 - \nu) \& \nu > (1 - \mu)\]

\[E(a_1 | m = 1)a_1 + E(a_2 | m = 1)a_2 > E(a_1 | m = 2)a_1 + E(a_2 | m = 2)a_2\]

\[\frac{\mu}{1 - \nu + \mu\nu}a_h + \left(1 - \frac{\mu}{1 - \nu + \mu\nu}\right)a_i + \frac{\mu\nu}{1 - \nu + \mu\nu}a_h + \left(1 - \frac{\mu\nu}{1 - \nu + \mu\nu}\right)a_i > a_h + a_i\]

\[\frac{\mu(1 + \nu)}{1 - \nu + \mu\nu}(a_h - a_i) + 2a_i > a_h + a_i\]

\[\frac{\mu(1 + \nu)}{1 - \nu + \mu\nu}(a_h - a_i) > a_h - a_i\]

\[\mu > (1 - \nu) = \nu > (1 - \mu)\]

\[a_1 = a_h \& a_2 = a_i \to \pi = 1 \text{ optimal for the manger if: } \mu > (1 - \nu) \frac{a_i}{a_h} \& \nu > \frac{a_i - a_h}{a_i}\]

\[E(a_1 | m = 1)a_1 + E(a_2 | m = 1)a_2 \frac{\mu}{1 - \nu + \mu\nu}a_h + \left(1 - \frac{\mu}{1 - \nu + \mu\nu}\right)a_i + \frac{\mu\nu}{1 - \nu + \mu\nu}a_h + \left(1 - \frac{\mu\nu}{1 - \nu + \mu\nu}\right)a_i > 0a_h + 1a_i a_h + 1a_h + 0a_i a_i\]
\[ \frac{\mu}{1 - \nu + \mu \nu} (a_h - a_l) a_h + a_h a_l + \frac{\mu \nu}{1 - \nu + \mu \nu} (a_h - a_l) a_l + a_l^2 > a_h a_l + a_h a_l \]

\[ \frac{\mu a_h + \mu a_h a_l}{1 - \nu + \mu \nu} (a_h - a_l) > a_l (a_h - a_l) \]

\[ \mu > (1 - \nu) \frac{a_l}{a_h} = \nu > \frac{a_l - a_h \mu}{a_l} \]

Strategy A (B): \( a_1 = a_l \) & \( a_2 = a_h \) → \( \delta = 0 \) optimal for the manager if: \( \mu < (1 - \nu) \frac{a_h}{a_l} \) & \( \nu < \frac{a_l - a_h \mu}{a_l} \)

\[ [E(a_1|m = 1)a_1]a_l + [E(a_2|m = 1)a_2]a_h < [E(a_1|m = 2)a_1]a_l + [E(a_2|m = 2)a_2]a_l \]

\[ \left[ \frac{\mu}{1 - \nu + \mu \nu} a_h + \left(1 - \frac{\mu}{1 - \nu + \mu \nu} a_l\right) a_l + \left[\frac{\mu \nu}{1 - \nu + \mu \nu} a_h + \left(1 - \frac{\mu \nu}{1 - \nu + \mu \nu} a_l\right) a_l \right] a_h \right] \]

\[ < [0 a_h + 1 a_l] a_l + [1 a_h + 0 a_l] a_h \]

\[ \frac{\mu a_l + \mu a_h}{1 - \nu + \mu \nu} (a_h - a_l) a_l + a_l^2 + \frac{\mu \nu}{1 - \nu + \mu \nu} (a_h - a_l) a_h + a_h a_l < a_l^2 + a_h^2 \]

\[ \mu < (1 - \nu) \frac{a_h}{a_l} = \nu < \frac{a_l - \mu a_h}{a_l} \]

(3)

Strategy C (D)

\( a_i = a_{a_i} \) → \( \alpha = 0 \) optimal for the manager if: \( \mu > (1 - \nu) = \nu > (1 - \mu) \)

\[ E(a_1|m = 1)a_1 + E(a_2|m = 1)a_2 < E(a_1|m = 2)a_1 + E(a_2|m = 2)a_2 \]

\[ a_h + a_l < \frac{\mu \nu}{1 - \nu + \mu \nu} a_h + \left(1 - \frac{\mu \nu}{1 - \nu + \mu \nu} a_l\right) a_l + \frac{\mu}{1 - \nu + \mu \nu} a_h + \left(1 - \frac{\mu}{1 - \nu + \mu \nu} a_l\right) a_l \]

\[ a_h + a_l < \frac{\mu \nu + \mu}{1 - \nu + \mu \nu} (a_h - a_l) + 2 a_l \]

\[ a_h - a_l < \frac{\mu \nu + \mu}{1 - \nu + \mu \nu} (a_h - a_l) \]

\[ \mu > (1 - \nu) = \nu > (1 - \mu) \]

\( a_1 = a_h \) & \( a_2 = a_l \) → \( \pi = 1 \) optimal for the manager if: \( \mu < \frac{a_h - a_l}{a_h} \) & \( \nu < (1 - \mu) \frac{a_h}{a_l} \)

\[ [E(a_1|m = 1)a_1]a_h + [E(a_2|m = 1)a_2]a_l > [E(a_1|m = 2)a_1]a_h + [E(a_2|m = 2)a_2]a_l \]
\[ [a_h + 0 a_l] a_h + [0 a_h + 1 a_l] a_l > \left[ \frac{\mu \nu}{1 - \nu + \mu \nu} a_h + \left(1 - \frac{\mu \nu}{1 - \nu + \mu \nu}\right) a_l \right] a_h + \left[ \frac{\mu}{1 - \nu + \mu \nu} a_h + \left(1 - \frac{\mu}{1 - \nu + \mu \nu}\right) a_l \right] a_l \]

\[ a_h^2 + a_l^2 > \frac{\mu \nu}{1 - \nu + \mu \nu} (a_h - a_l) a_h + a_l a_h + \frac{\mu}{1 - \nu + \mu \nu} (a_h - a_l) a_l + a_l^2 \]

\[ a_h (a_h - a_l) > \frac{\mu \nu a_h + \mu a_l}{1 - \nu + \mu \nu} (a_h - a_l) \]

\[ \mu < (1 - \nu) \frac{a_h}{a_l} = \nu < (1 - \mu) \frac{a_h}{a_l} \]

\[ a_s = a_i \text{ and } a_\delta = a_h \rightarrow \delta = 0 \text{ optimal for manager if: } \mu > \frac{a_i - \nu a_h}{a_i} \text{ and } \nu > (1 - \mu) \frac{a_i}{a_h} \]

\[ [E(a_1|m = 1)a_1] a_l + [E(a_2|m = 1)a_2] a_h < [E(a_1|m = 2)a_1] a_l + [E(a_2|m = 2)a_2] a_l \]

\[ [1a_h + 0a_l] a_l + [0a_h + 1a_l] \frac{\mu \nu}{1 - \nu + \mu \nu} + \left(1 - \frac{\mu \nu}{1 - \nu + \mu \nu}\right) a_l \]

\[ + \left(1 - \frac{\mu}{1 - \nu + \mu \nu}\right) a_l \]

\[ a_h a_l + a_h a_l < \frac{\mu \nu}{1 - \nu + \mu \nu} (a_h - a_l) a_l + a_l^2 + \frac{\mu}{1 - \nu + \mu \nu} (a_h - a_l) a_h + a_h a_l \]

\[ a_l (a_h - a_l) < \frac{\mu \nu a_l + \mu a_h}{1 - \nu + \mu \nu} (a_h - a_l) \]

\[ \mu > \frac{a_i - \nu a_h}{a_i} = \nu > (1 - \mu) \frac{a_i}{a_h} \]

\[ (4) \]

**Strategy G (H)**

\[ a_i = a_{-i} \rightarrow a, \text{ it is optimal for the manager to mix with } \alpha = \frac{\mu - \mu \nu}{\mu + \nu - 2 \mu \nu} \]

\[ E(a_1|m = 1)a_1 + E(a_2|m = 1)a_2 = E(a_1|m = 2)a_1 + E(a_2|m = 2)a_2 \]

\[ \left(\frac{\mu - \mu \nu + a \mu \nu}{\mu - \mu \nu + 2a \mu \nu + \alpha - \mu \alpha - \alpha \nu}\right)a_h + \left(1 - \frac{\mu - \mu \nu + a \mu \nu}{\mu - \mu \nu + 2a \mu \nu + \alpha - \mu \alpha - \alpha \nu}\right)a_l + \left(\frac{a \mu \nu}{\mu - \mu \nu + 2a \mu \nu + \alpha - \mu \alpha - \alpha \nu}\right)a_h + \left(\frac{\mu \nu - \mu \alpha}{\mu - \mu \nu + 2a \mu \nu + \alpha - \mu \alpha - \alpha \nu}\right)a_l = \left(\frac{\mu \nu - \mu \alpha}{1 - 2a \mu \nu - \mu + \nu \alpha + \alpha \mu + \alpha \nu}\right)a_h + \left(1 - \frac{\mu \nu - \mu \alpha}{1 - 2a \mu \nu - \mu + \nu \alpha + \alpha \mu + \alpha \nu}\right)a_l \]

\[ \frac{\mu - \mu \nu + 2a \mu \nu}{\mu - \mu \nu + 2a \mu \nu + \alpha - \mu \alpha - \alpha \nu} \]

\[ = \frac{\nu + \mu \nu - 2a \mu \nu}{1 - 2a \mu \nu - \mu + \nu \alpha + \alpha \mu + \alpha \nu} \]
\[ \mu - 2\alpha \mu^2 \nu - \mu^2 + \mu^2 \nu - \alpha \mu + \alpha \mu^2 + 4\alpha \mu \nu - 2\mu \nu - 2\alpha \mu \nu^2 + \mu \nu^2 - \alpha \nu + \alpha \nu^2 = 0 \]

\[ \alpha = \frac{-\mu + \mu^2 - \mu^2 \nu + 2\mu \nu - \mu \nu^2}{-2\mu^2 \nu - \mu + \mu^2 + 4\mu \nu - 2\mu \nu^2 - \nu + \nu^2} \]

\[ \alpha = \frac{\alpha - \mu \nu (\mu + \nu - 1)}{(\mu + \nu - 2\mu \nu)(\mu + \nu - 1)} \]

\[ \alpha = \frac{\mu - \mu \nu}{\mu + \nu - 2\mu \nu} \]

\( a_1 = a_h \& a_2 = a_l \rightarrow \pi = 1 \) is optimal for the manager if: \( a_h > a_l \)

\[ [E(a_1|m = 1) a_h + [E(a_2|m = 1) a_2] a_l > [E(a_1|m = 2) a_1] a_h + [E(a_2|m = 2) a_2] a_l \]

\[ \mu a_h - \mu \nu a_h + \alpha \mu a_h + \alpha \mu \nu a_h > \frac{\mu a_h + \mu \nu a_l - \alpha a_l}{\mu - \mu \nu + 2\alpha \mu \nu + \alpha - \alpha \mu - \alpha \nu} \]

\[ a_h (1 - \mu + \alpha \mu + \mu \nu - \alpha - \nu - 2\alpha \mu \nu + 2\alpha \nu) > -\alpha a_l (\alpha \nu + \mu \nu - \alpha - \mu - 2\alpha \mu \nu + 2\alpha \mu) \]

When I fill in \( \alpha = \frac{\mu - \mu \nu}{\mu + \nu - 2\mu \nu} \) in this equation I find the following:

\[ \mu (1 - \mu + \alpha \mu + \mu \nu - \alpha - \nu - 2\alpha \mu \nu + 2\alpha \nu) = \frac{\mu^2 \nu^2 - \mu^2 \nu^2 \nu + \mu \nu}{\mu + \nu - 2\mu \nu} \]

\[ \nu a_l (\alpha \nu + \mu \nu - \alpha - \mu - 2\alpha \mu \nu + 2\alpha \mu) = -\frac{\mu^2 \nu^2 - \mu^2 \nu^2 \nu + \mu \nu}{\mu + \nu - 2\mu \nu} \]

It turns out that

\[ \mu (1 - \mu + \alpha \mu + \mu \nu - \alpha - \nu - 2\alpha \mu \nu + 2\alpha \nu) = -\alpha a_l (\alpha \nu + \mu \nu - \alpha - \mu - 2\alpha \mu \nu + 2\alpha \mu), \] with

\[ \alpha = \frac{\mu - \mu \nu}{\mu + \nu - 2\mu \nu}. \]

The necessary condition was:

\[ \mu a_h (1 - \mu + \alpha \mu + \mu \nu - \alpha - \nu - 2\alpha \mu \nu + 2\alpha \nu) > -\alpha a_l (\alpha \nu + \mu \nu - \alpha - \mu - 2\alpha \mu \nu + 2\alpha \mu) \]

What remains is:

\( a_h > a_l \)

\( a_1 = a_l & a_2 = a_h \rightarrow \delta = 0 \) optimal for the manager if: \( a_l < a_h \)

\[ [E(a_1|m = 1) a_l + [E(a_2|m = 1) a_2] a_h < [E(a_1|m = 2) a_1] a_l + [E(a_2|m = 2) a_2] a_l \]

\[ \frac{\mu a_l - \mu \nu a_l + \alpha \mu a_l + \alpha \mu \nu a_l}{\mu - \mu \nu + 2\alpha \mu \nu + \alpha - \alpha \mu - \alpha \nu} < \frac{\mu a_l - \mu \nu a_l + \nu a_h - \alpha \mu \nu a_h}{1 - 2\alpha \mu \nu - \mu + \mu \nu - \alpha + \alpha \mu + \alpha \nu} \]
\[
\mu a_l - \mu^2 a_l + \alpha \nu^2 a_h + \mu \nu^2 a_h + \alpha \mu^2 a_l + \mu^2 \nu a_l - \alpha \nu a_h - \mu \nu a_h - \alpha \mu a_l - \mu \nu a_l - 2 \alpha \mu \nu^2 a_h - 2 \alpha \mu^2 \nu a_l + 2 \alpha \mu \nu a_h + 2 \alpha \mu \nu a_l < 0
\]

\[
\mu a_l (1 - \mu + \alpha \mu + \mu \nu - \alpha - \nu - 2 \alpha \mu \nu + 2 \alpha \nu) + \nu a_h (\alpha \nu + \mu \nu - \alpha - \mu - 2 \alpha \mu \nu + 2 \alpha \mu) < 0
\]

\[
\mu a_l (1 - \mu + \alpha \mu + \mu \nu - \alpha - \nu - 2 \alpha \mu \nu + 2 \alpha \nu) < \nu a_h (\alpha \nu + \mu \nu - \alpha - \mu - 2 \alpha \mu \nu + 2 \alpha \mu)
\]

As in case II:

\[
\mu (1 - \mu + \alpha \mu + \mu \nu - \alpha - \nu - 2 \alpha \mu \nu + 2 \alpha \nu) = -\nu a_l (\alpha \nu + \mu \nu - \alpha - \mu - 2 \alpha \mu \nu + 2 \alpha \mu), \text{ with } \alpha = \frac{\mu - \mu \nu}{\mu + \nu - 2 \mu \nu}.
\]

The necessary condition was:

\[
\mu a_l (1 - \mu + \alpha \mu + \mu \nu - \alpha - \nu - 2 \alpha \mu \nu + 2 \alpha \nu) < -\nu a_h (\alpha \nu + \mu \nu - \alpha - \mu - 2 \alpha \mu \nu + 2 \alpha \mu)
\]

What remains is:

\[a_l < a_h\]
7. References


