Panta Rhei; Revisiting the Black-Scholes Model.

Abstract

The Black-Scholes model was published in 1973 and markets have continued to evolve ever since. This thesis investigates the performance of the model in today's market and it is a continuation of existing research on options. The model is tested for SPX Standardized Options for the period 2004-2012. Besides testing the performance of the model, the effects of the credit crisis are also examined. This thesis can be divided in two separate parts, namely a literature study and an empirical research. The empirical research consists of four simple tests; an implied volatility test, a volatility smile test, a Put-Call Parity test and a test for normality. The results of the research indicate that the average predicted volatility (the implied volatility) systematically deviates from the realised volatility, that a volatility smile is visible in the data, that Put-Call Parity principle does not uphold and that the daily returns of the SPX Index are not distributed normally. The research also indicates there is a significant effect during and after the credit crisis on options and on the performance of the model.

Keywords; SPX-Index, Options, Credit Crisis, Implied Volatility, Put-Call Parity, Normality, Black-Scholes

Bachelor thesis by
P. Verheijen

Thesis Supervisor
Dr. R. Jongen

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Chapter 1: Introduction

Everything changes over time and everything has a beginning. The first evidence found of option contracts being traded is found in Aristotle's book ''Politics''. It provides a reference to Thales of Miletus, the direct quote can be found in Book I, Chapter 11, sections 5-10 and is as follows:

''There is, for example, the story which is told of Thales of Miletus. It is a story about a scheme for making money, which is fathered on Thales owing to his reputation of wisdom; but it involves a principle of general application. He was reproached for his poverty which was supposed to show the usefulness of philosophy; but observing from his knowledge of meteorology (so the story goes) that there was likely to be a heavy crop of olives [next summer], and having a small sum at his command, he paid down earnest-money, early in the year, for the hire of all the olive-presses in Miletus and Chios; and he managed, in the absence of any higher offer, to secure them at a low rate. When the season came, and there was a sudden and simultaneous demand for a number of presses, he let out the stock he had collected at any rate he chose to fix; and making considerable fortune he succeeded in proving that it is easy for philosophers to become rich if they so desire, though it is not the business which they are really about.''

Over time option contracts became better defined and more broadly traded, one of the challenges investors continued to face was the valuation of these contracts. For a theoretical scenario the value of such a contract could be calculated with the use of simple mathematics, however the valuation of such a contract in a real world scenario is much more complex. A strong focus on a valuation model started in the second half of the twentieth century. A lot of time and research led to the famous publication of the Black-Scholes model by Fischer Black and Myron Scholes in the year 1973. This model wasn't only a tool for the valuation of option contracts, it also brought with it insight in the workings of options.

The existence of a valuation model for options led to a rise in popularity of the security. Option contracts nowadays are a big part of the financial industry and it is virtually impossible to think them away. With the growth in option contracts, new types of option contracts were being formed. These options contracts, known as exotic options, are options that are different in several key characteristics such as the payment, exercise rights and underlying asset(s).

The main focus of this thesis, however, is the Black-Scholes model, a valuation model that was published in 1973 and that is being used up until this very day. The Black-Scholes model was only tested for the American financial market. One could wonder how the model performs in different markets, such as markets on different continents or even in different countries. Yet again, this is not the main focus of this thesis. Keeping the location of where the model is being used constant, there is still another variable that greatly influences the performance of the model; simply put that variable is time. Today's market is not the same as it was in 1973, the main question of this thesis is the comparison of the performance of the Black-Scholes model through time. The model performed well in 1973, but since then a lot of time has passed and we passed the second millennium. During the second millennium we went through several financial crises, the most grave of them being the credit crisis.

The aim of this thesis is to investigate the performance of the model after the year 2000, and more specifically; before, during and after the credit crisis.

Chapter 2: Related Literature

The right to buy or sell an asset is a security called an option, it is subject to certain conditions within a specified period of time. There are two styles of options; an ''European'' option can be exercised only on a specified future date, an ''American'' option can be exercised at any time up to the date the option expires. The beforehand agreed price that is paid for the asset when the option is exercised is the ''exercise price'' or the ''strike price''. The last day on which the option may be exercised is called the ''expiration date'' or the ''maturity date''. When an option gives the right to buy an asset, it is called a ''call option'', when an option gives the right to sell an asset, it is called a ''put'' option. (Black & Scholes 1973)

The ''Black-Scholes'' option pricing model developed by Fischer Black and Myron Scholes was published in 1973. Empirical research about the performance continually followed afterwards. Before its publication the model was tested by Black and Scholes in 1972. For their research they used a large body of call option data and the results of this research indicated several things; The actual prices at which options are bought and sold deviate in certain systematic ways from the values predicted by the formula. Option buyers pay prices that are consistently higher than those predicted by the formula. Option writers, however, received prices that are at about the level predicted by the formula. There are large transaction costs in the option market, all of which are effectively paid by option buyers. Also, the difference between the price paid by options buyers and the value given by the formula is greater for options on low-risk stocks than for options on high-risk stocks. The market appears to underestimate the effect of differences in variance rate on the value of an option. Given the magnitude of the transaction costs in this market, however, this systematic misestimating of value does not imply profit opportunities for a speculator in the option market. (Black & Scholes 1973).

The main paper that forms the foundation of this thesis is by P. Fortune (1996), the paper consists out of five different tests on the Black-Scholes model, namely an implied volatility test, a volatility smile, a Put-Call Parity test, a test for option pricing errors and a test for normality. All these tests have been redone with different data, with the exception of the test for option pricing errors, namely since this requires transaction data.

The results of the tests are as follow; For the first test, the implied volatility test, the volatility estimated by the Black-Scholes model is a poor estimate of true volatility. Compared to the observed volatility, the implied volatility is mostly an upwardly biased estimate. Regarding the second test, the volatility smile, the research indicates that there is a smile in the implied volatility; Near-the-money options tend to have lower implied volatilities than moderately out-of-the-money or in-the-money options. For the third test, Put-Call Parity, the study indicates that puts tend to have a higher implied volatility than equivalent calls, indicating that puts are overpriced relative to calls. This overpricing is not random, it is systematic. This suggests the possibility that there are unexploited arbitrage profits. Regarding the fifth test, test for normality, the study indicates that the distribution of changes in the logarithm of stock prices has ‘’fat tails’’ relative to a normal distribution (more extreme changes than the normal distribution would predict). (Fortune 1996)

Another paper (Günther 2012), more recently, evaluates the Black-Scholes option pricing model in a practical manner, it evaluates how well it does when it is used for delta hedging. This is not the main focus of this thesis, but it does evaluate the performance of the Black-Scholes model in a practical manner. It has the following results and conclusion; the assumptions underlying the Black-Scholes model do not hold in the real market, however, the only truly uncertain parameter in the Black-Scholes equation is the volatility. The Black-Scholes model works relatively well for European call options when the right volatility is used. The risk involved with option trading is still reduced when using the Black-Scholes option pricing model with delta hedging.

When going through related literature it seems that the Black-Scholes model is mostly tested on American option data. Since other markets may behave differently, the Black-Scholes model should also be tested for these markets. The next paper is about the performance of the model for the Australian market. This paper looks at the factors of the Black-Scholes model from two different perspectives, namely from a collective level and an individual level.

It has the following results; the results of this paper indicate that the Black Scholes model is relatively accurate. The results based on a method of maximum likelihood indicate that the factors of the Black-Scholes collectively are statistically significant. However at the individual level neither historical volatility nor implied volatility is statistically significant. (Mckenzie, Gerace, and Subedar 2007)

Most of the research on options, and specifically, the Black-Scholes model focus on investigating the performance. One important aspect that may be overlooked because of this is an explanation towards the found performance. The next paper gives us insight in a possible explanation for poor performance of the model.

This paper was published by E.M. Miller, it's about divergence of opinion and its effects on asset pricing. It has the following conclusion; in practice, uncertainty and risk imply divergence of opinion. In a market with little or no short selling the demand for a particular security will come from the minority who hold the most optimistic expectations about it. Since divergence of opinion is likely to increase with risk, it is quite possible that expected returns will be lower for risky securities, rather than higher. Even for risk neutral investors optimal strategies will involve the use of risk premiums in evaluating securities. The presence of a substantial number of well informed investors will prevent there from being substantially undervalued securities, but there may be securities whose prices have been bid up to excessive levels by a badly informed minority, thus contradicting the efficient market hypothesis. (Miller 1977).

Chapter 3: Data and Methodology

The empirical data has been obtained from two different sources. The closing prices of the SPX-index and SPX Volatility-Index for the period 2004-2012 are obtained from the Federal Reserve Bank of St. Louis. Data about the standardized SPX options is obtained from Optionmetrics, this data includes the daily closing prices of both the call and put options, the implied volatility and the characteristics (put or call, expiration date, strike price). The implied volatility is already given; it is calculated according to the Black-Scholes model. The daily interest curve used by Optionmetrics is calculated from a collection of continuously-compounded zero-coupon interest rates at various maturities for various currencies, collectively referred to as the zero curve. The SPX-index was chosen since the options for this index are European-style and the Black-Scholes model is designed for European-style options.

The methodology of the paper consists of four separate tests; namely an implied volatility test, a volatility smile, a put-call parity test, and a test for normality. The implied volatility test is done by comparing the implied volatility of the standardized options to the closing price of the SPX Volatility Index. The volatility smile is done by comparing the implied volatility to the moneyness, the moneyness is a variable constructed to see how far an option is in/at/out-of-the-money. The variable is calculated as 1 + (S-X)/S, S being the stock price and X being the exercise price. The put-call parity test is done by calculating the relative difference of the implied volatility of put and call options with the same maturity. The relative difference of the implied volatility is calculated as the implied volatility of the call minus the implied volatility of the corresponding put, and this difference is divided by the implied volatility of the call. Lastly, for the test for normality, we calculate the SPX-Index daily returns. These returns are plotted in a Q-Q plot and a histogram and the returns are tested for normality by using the Jarque-Bera test.

Chapter 4: Results

Test1: Implied Volatility

For the implied volatility test, five different graphs have been made. The first two graphs compare the average implied volatility and the spread of the implied volatility with the SPX Volatility Index for the entire period of 2004-2012. The last three graphs are compares the average implied volatility with the SPX Volatility Index, only for three different time periods, namely 2004, 2007-2009 and 2012. These periods were chosen to analyze the effect of the credit crisis; the year 2004 being a year before the credit crisis, the years 2007-2009 being the start and peak of the credit crisis and the year 2012 being a year where the credit crisis is over.

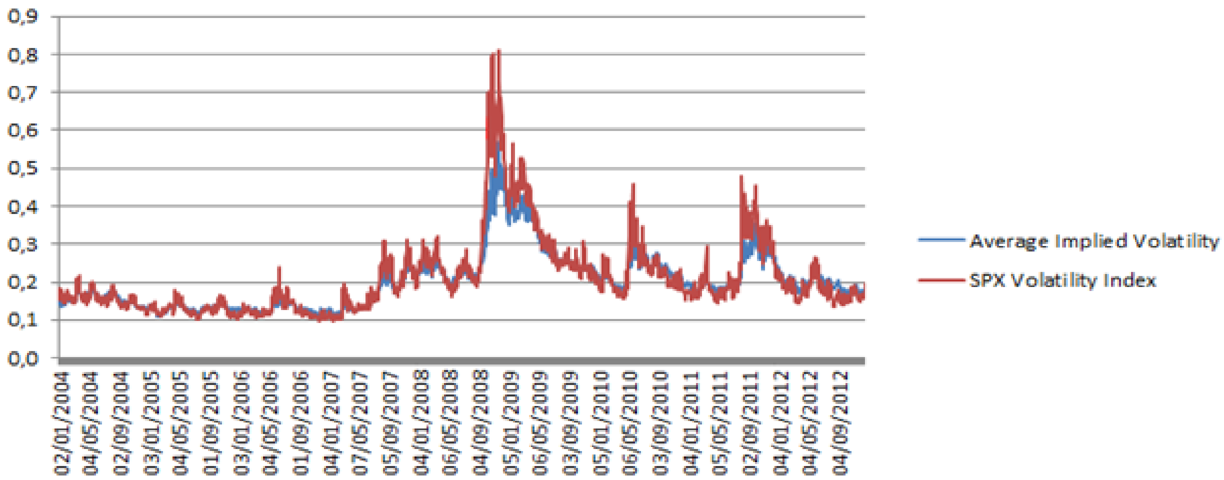

Figure 1.a *All implied volatilities (in %) of the standardized SPX options compared to the SPX Volatility Index for the period 02/01/2004-16/08/2012.*

In the first graph (Figure 1.a) we can see that the implied average volatility and SPX Volatility Index do not systematically differ. The volatility ranges from roughly 0,1 to 0,8. On average the Black-Scholes model predicts the realized volatility quite well. It does however seem to struggle with periods of high volatility such as the credit crisis in late 2008.

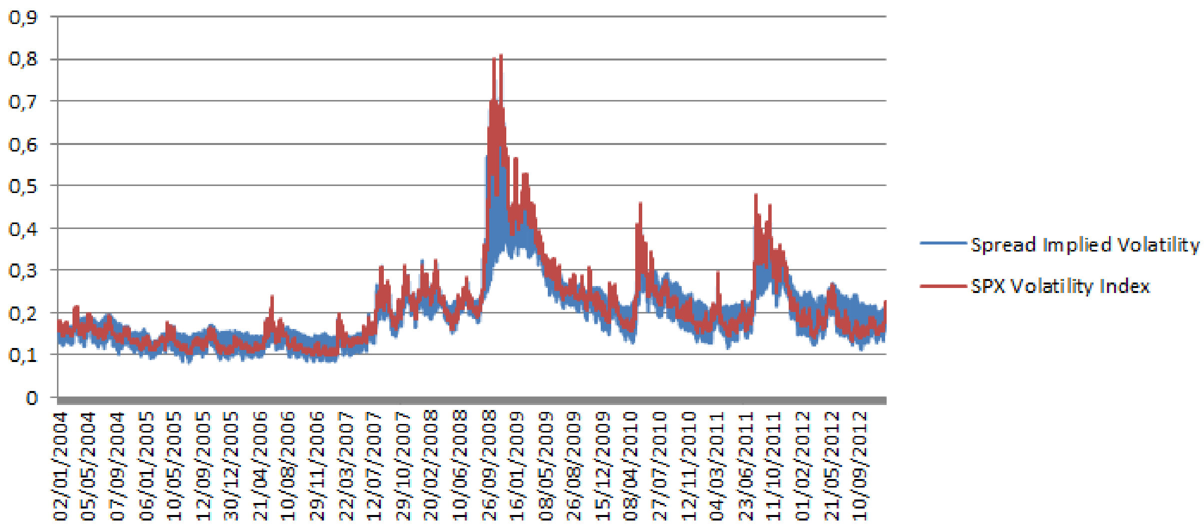

Figure 1.b *The average implied volatility (in %) of the standardized SPX options compared to the SPX Volatility Index for the period02/01/2004-16/08/2012*

Averaging the implied volatility can cause a misrepresentation of the facts; therefore another graph (Figure 1.b) has been made. In this graph all the implied volatilities (from the different maturities, both calls and puts) are visible as the ''spread implied volatility''. In this graph we can see that the spread is roughly 0,05 to 0,10, with a spread of >0,2 during the credit crisis. Systematically the spread does seem to differ upwards or downwards, however, during the credit crisis and other periods in time with high volatility it seems to differ mostly downwards. Looking at this graph it seems that the Black-Scholes model does not accurately predict the realized volatility well.

The next three graphs aim to analyze the effects of the credit crisis on the performance of the Black-Scholes model. For this section, three different time periods have been chosen. A period before the credit crisis started, a period during the credit crisis and a period afterwards. Subsequently these periods are 2004, 2007-2009 and 2012.

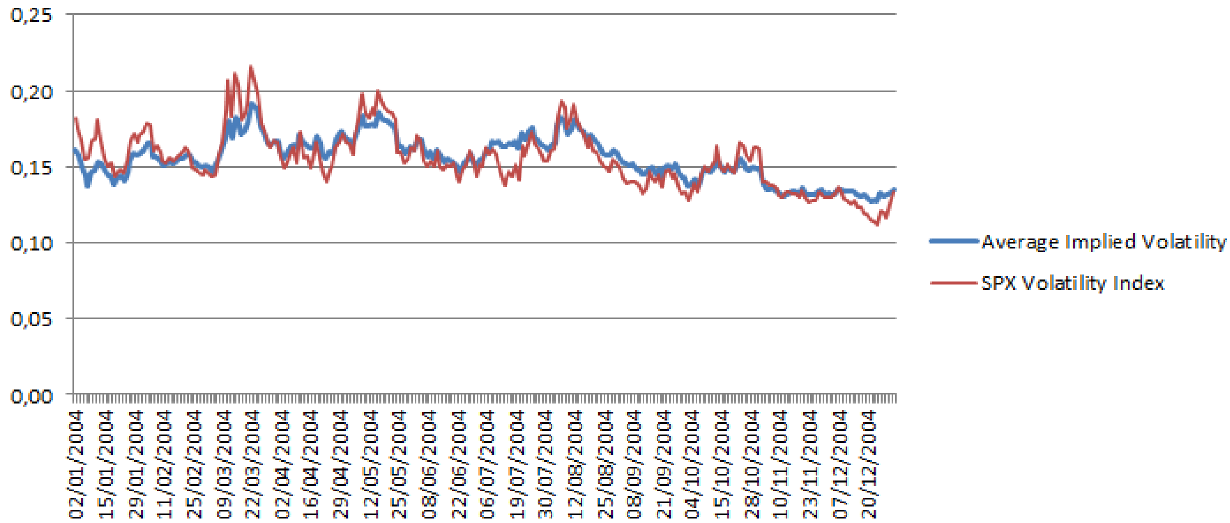

Figure 1.c *The average implied volatility (in %) of the standardized SPX options compared to the SPX Volatility Index for the period 2004 (Pre-Credit crisis).*

The first graph (Figure 1.c) is for the period before the credit crisis, the average implied volatility and the SPX Volatility Index stay within the range of 0,225-0,100. The average implied volatility does not systematically differ upward or downward compared to the SPX Volatility Index and seems to fit quite well.

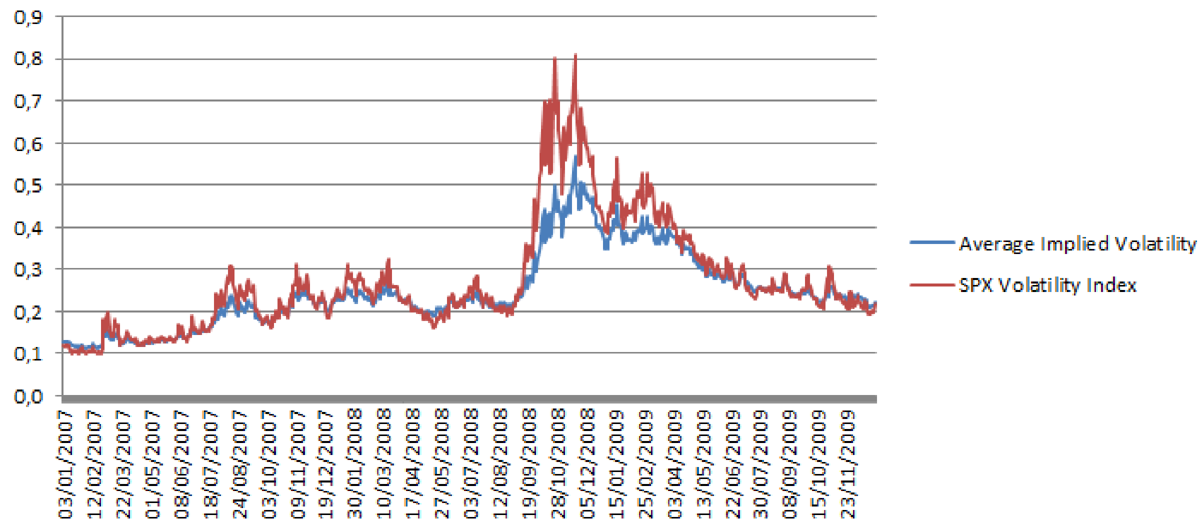

Figure 1.d *The average implied volatility (in %) of the standardized SPX options compared to the SPX Volatility Index for the period 2007-2009 (During the Credit crisis).*

The second graph (Figure 1.d) is for the period during the credit crisis, it contains the start the peak and the decline in volatility during the credit crisis. During the first period, roughly up to the first half of 2008, the volatility stays within the range of 0,1-0,3. This is already higher what is visible in the year 2004. The second period, starting with the second half of 2008 and ending with a decline in volatility, roughly around the end of 2008, the volatility stays within a range of 0,2-0,8. The last period, start of 2009 up until the end of 2012, the volatility starts of at around 0,6 and declines to roughly 0,3-0,2. The effects of the credit crisis are very clear in the different periods of this graph. The average implied volatility seems to fit the SPX Volatility quite well; it only differs systematically downwards in during periods of high volatility. The model seems to misestimate lower during periods of high volatility.

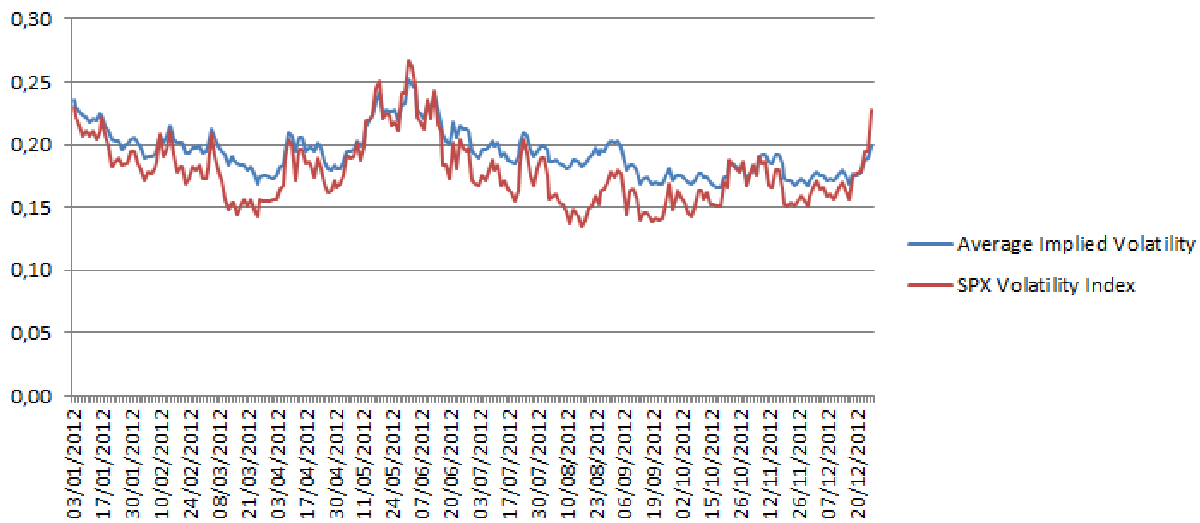

Figure 1.e *The average implied volatility (in %) of the standardized SPX options compared to the SPX Volatility Index for the period 2012 (Post-Credit crisis).*

The last graph (Figure 1.e) of the implied volatility tests shows the average implied volatility and SPX Volatility Index for the period 2012. The volatility seems roughly stay within the range of 0,25-0,15. The volatility is higher in comparison to the volatility in the year 2004; the effects of the credit crisis may seem to persist even after its ending. The average implied volatility now seems to systematically differ upwards compared to the SPX Volatility Index.

All the graphs combined lead to the following insights; the average implied volatility of the Black-Scholes model corresponds well with the SPX Volatility Index, the spread of implied volatility, however, does not. The credit crisis does have a significant effect on the performance of the model; during times of high volatility the implied volatility calculated with the model seems to differ mostly downwards compared to the SPX Volatility Index, and after the credit crisis the average implied volatility seems to systematically differ upwards compared to the SPX Volatility Index.

Test 2: Volatility Smile

The volatility smiles made for this test are a comparison of the implied volatility to the ''moneyness''. Moneyness being a variable that is an indication of how far in/at/out-of-the money an option is.

For this test six different graphs have been made; the first three graphs (Figure 2.A-C) are during the credit crisis and the second three graphs (Figure 2.D-F) are after the credit crisis has ended. For both periods the graphs represent different times till maturity, subsequently these are one month, six months and a year. Comparing the first three graphs with the second three graphs should show the effects of the credit crisis. Comparing the three graphs within one period should show the effect that time to maturity has.

In the first three graphs (Figure 3.a-c) the volatility seems to roughly stay within the range of 0,75 to 0,2. The longer the time to maturity, the lower the peak in volatility becomes, for one year till maturity the peak only goes as high as 0,5 instead of 0,75 (Figure 3.a) or 0,55 (Figure 3.b). A smile pattern seems to be visible in these three graphs.

In the second three graphs (Figure 3.d-f) the volatility seems to roughly stay within the range of 0,25 to 0,125. The lowest visible volatilities seem to increase for options with a longer time to maturity; for options with six months till maturity the lowest volatilities are around the value 0,175 and for options with a year till maturity around the value 0,18. In the first graph a frown is visible (opposite to what is expected), the second graph nor a frown nor a smile and in the last graph we see a small smile. A longer time to maturity could increase the ''smile'' effect.

When comparing the first three graphs (Figure 3.a-c) with the second three graphs, it is visible that in periods of high volatility, such as the credit crisis, volatility smiles are more predominant in the data.

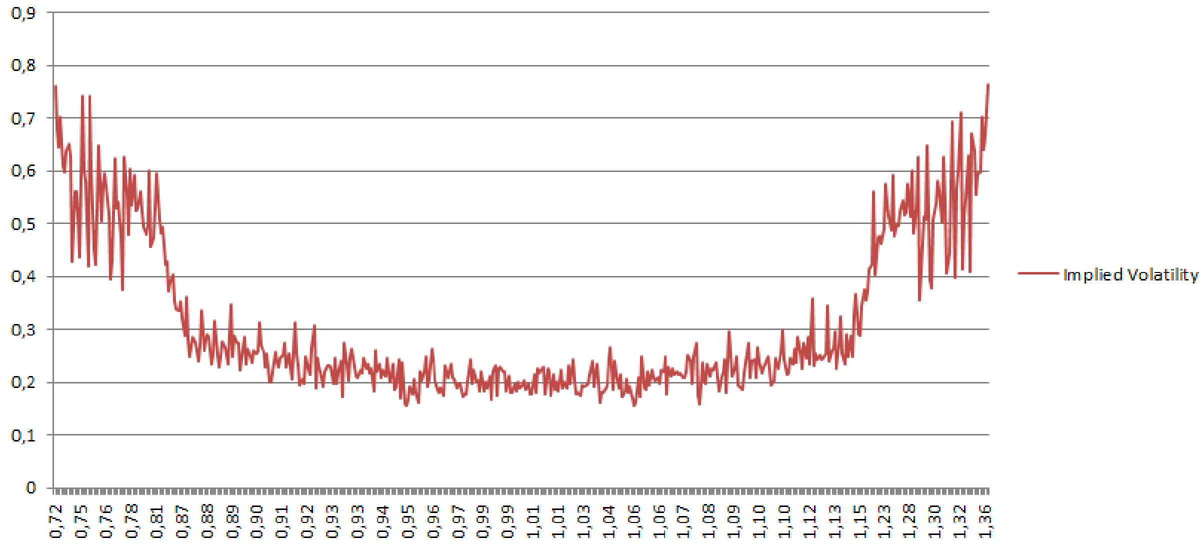

Figure 2.a *The Implied volatility depending on the ''moneyness'' of the option. Moneyness is defined as (S-X)/x +1. Period is 2008 (During the Credit crisis), . Time tol maturity for the options is one month.*

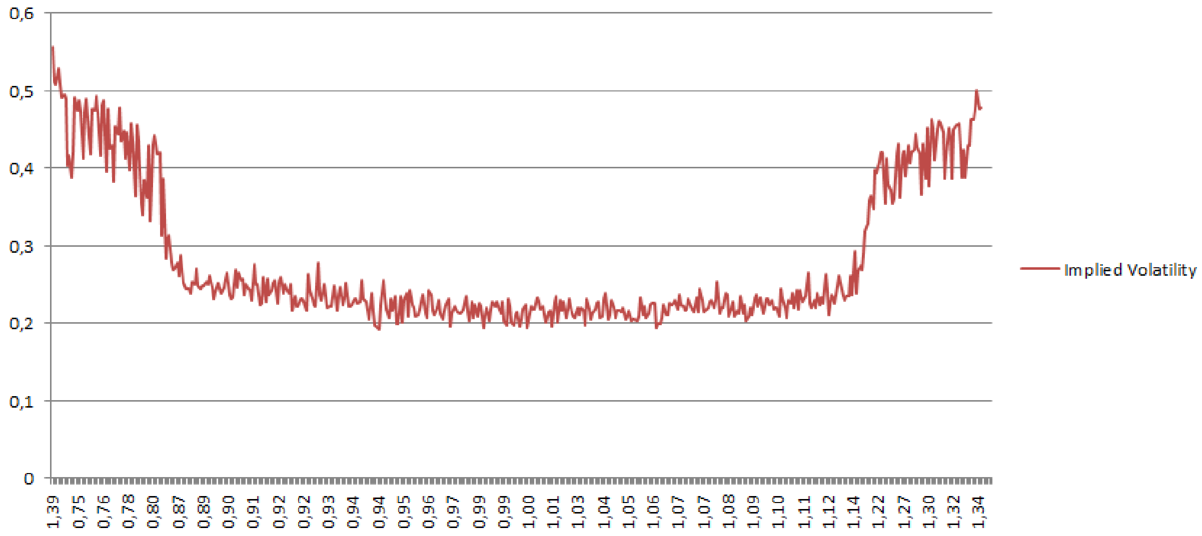

Figure 2.b *The Implied volatility depending on the ''moneyness'' of the option. Moneyness is defined as (S-X)/x +1. Period is 2008 (During the Credit crisis), . Time to maturity for the options is six months.*

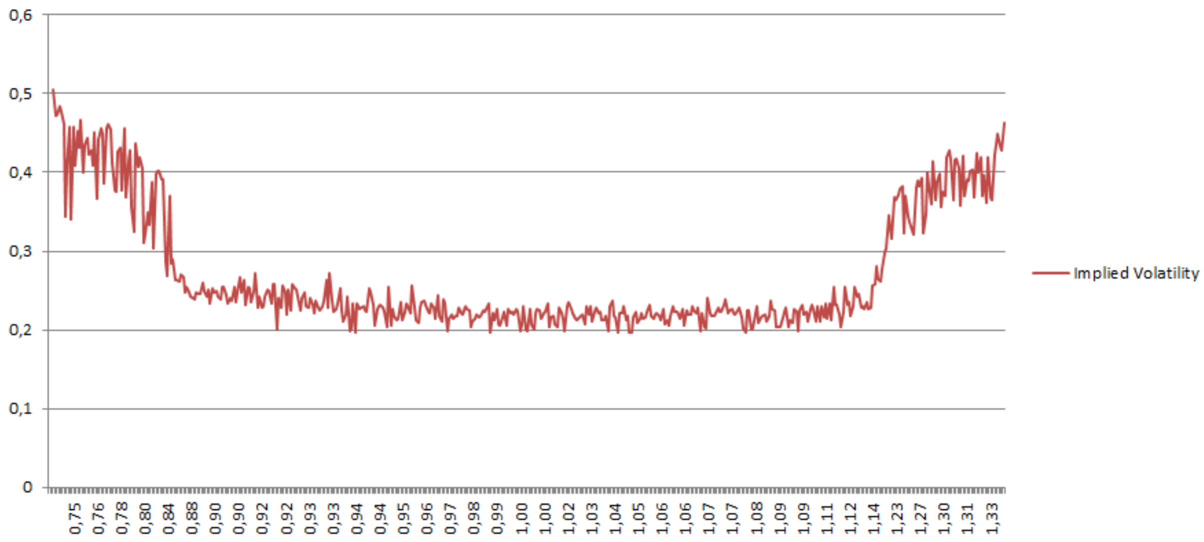

Figure 2.c *The Implied volatility depending on the ''moneyness'' of the option. Moneyness is defined as (S-X)/x +1. Period is 2008 (During the Credit crisis), . Time to maturity for the options is a year.*

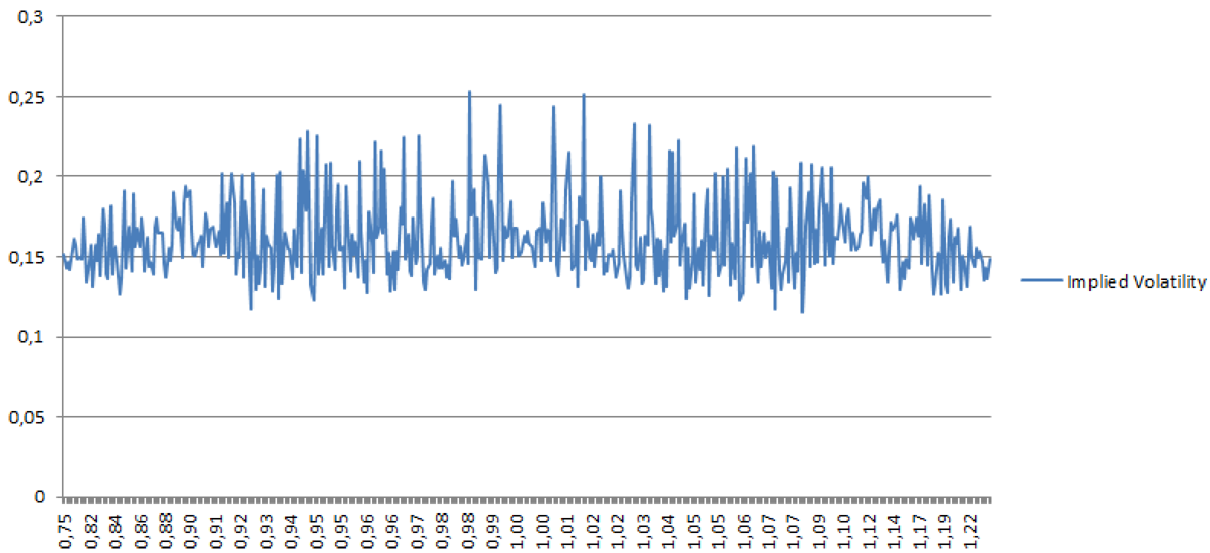

Figure 2.d *The Implied volatility depending on the ''moneyness'' of the option. Moneyness is defined as (S-X)/x +1. Period is 2012 (Post-Credit crisis), . Time to maturity for the options is one month.*

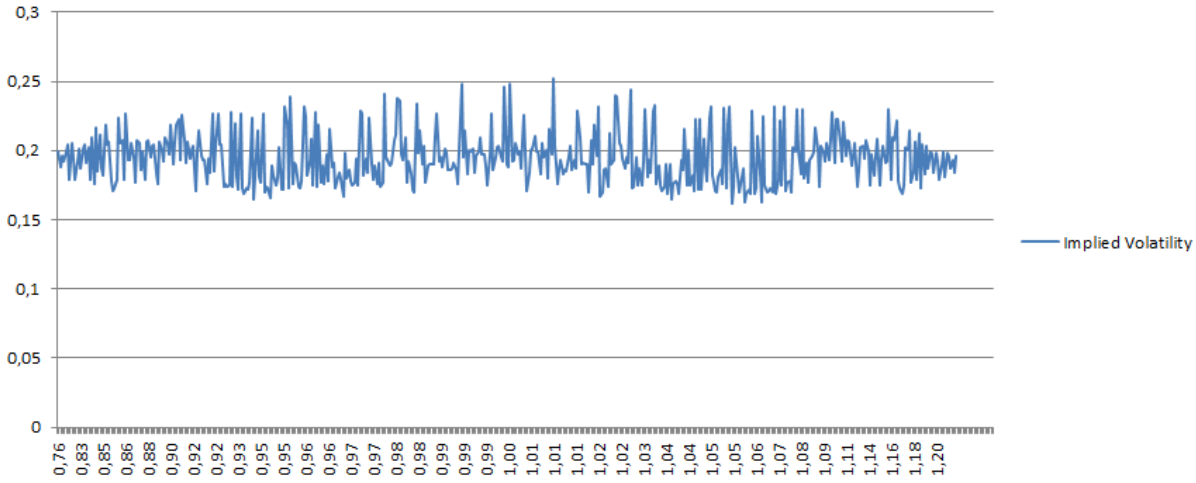

Figure 2.e *The Implied volatility depending on the ''moneyness'' of the option. Moneyness is defined as (S-X)/x +1. Period is 2012 (Post-Credit crisis), . Time to maturity for the options is six months.*

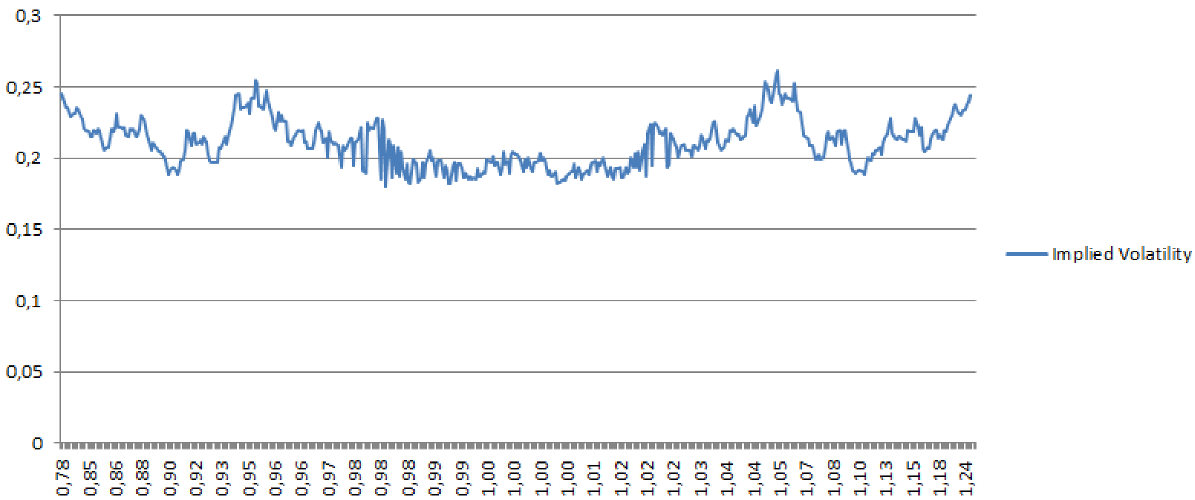

Figure 2.f *The Implied volatility depending on the ''moneyness'' of the option. Moneyness is defined as (S-X)/x +1. Period is 2012 (Post-Credit crisis), . Time to maturity for the options is a year.*

Test 3: Put-Call Parity

For this test the relative difference in implied volatility has been calculated in order to compare the implied volatility of call options with the implied volatility of put options, this was done for options with different times till maturity, namely subsequently one month, six months and a year. These graphs (Figure 3.a-c) should show if Put-Call Parity upholds across time and for different times till maturity.

When looking at the three graphs at the same time, three different time periods are visible for all maturities. The first period being roughly before the credit crisis, the second period contains a peak in late 2008 and the last period being at the credit crisis. The relative difference in implied volatility also seems to decrease when the time to maturity increases therefore; Put-Call Parity seems to uphold better after the credit crisis than before, and when the time to maturity is longer.

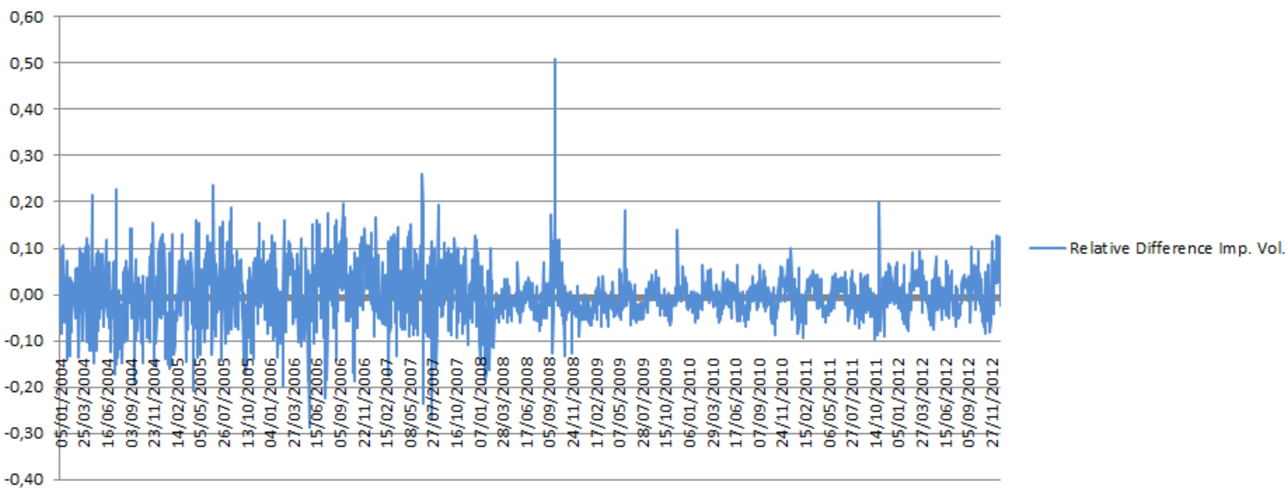

Figure 3.a. *Relative difference in implied volatility for the period 2004-2012. . Time to maturity for the options is one month. Relative difference is calculated as (C-P)/C.*

In the first graph (Figure 3.a) the relative difference in implied volatility roughly stays within the range of 0,25 – minus 0,20, with a spike of 0,50 during the credit crisis.

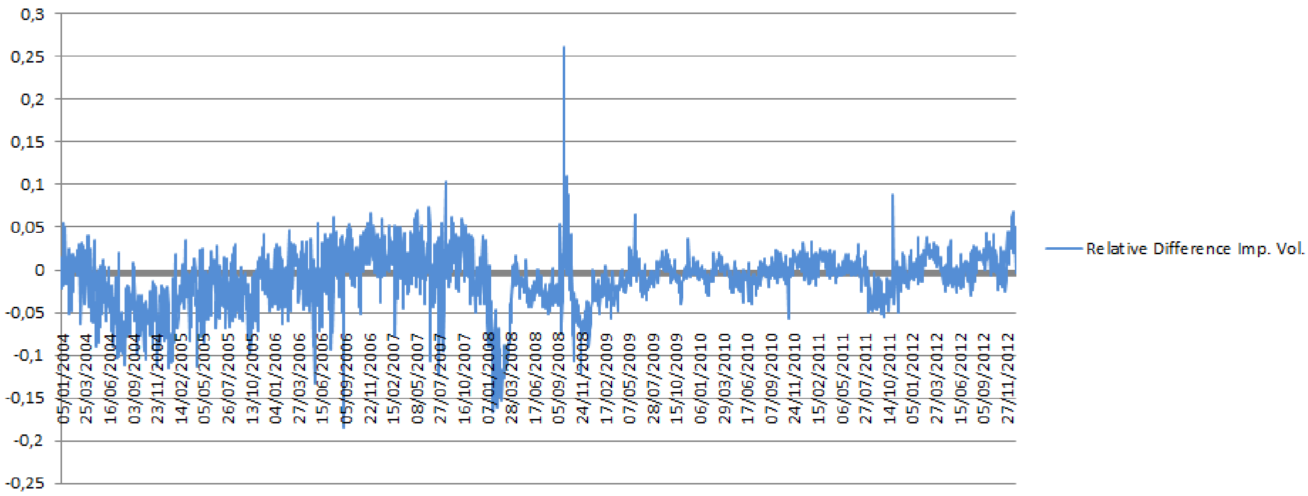

Figure 3.b. *Relative difference in implied volatility for the period 2004-2012. Time to maturity for the options is six months. Relative difference is calculated as (C-P)/C.*

In the second graph (Figure 3.b) the relative difference in implied volatility roughly stays within the range of 0,1 – minus 0,15, with a spike of 0,25 during the credit crisis. Compared to the first graph (Figure 3.a) this graph seems to have periods where the relative difference in volatility substantially swings downwards. This would indicate that for some periods in time the implied volatility of put options is higher than that of call options; this was not visibly present in the previous graph (3.a.)

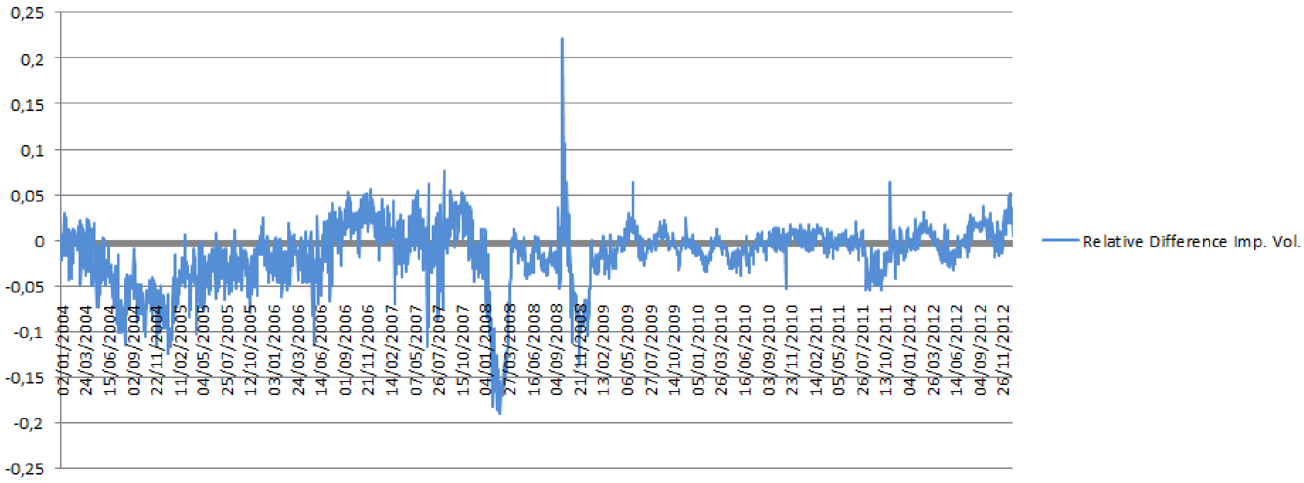

Figure 3.C. *Relative difference in implied volatility for the period 2004-2012. Time to maturity for the options is six months. Relative difference is calculated as (C-P)/C.*

In the third graph (Figure 3.c) the relative difference in implied volatility roughly stays within the range of 0,05 – minus 0,10, with a spike of 0,225 during the credit crisis. Compared to the previous graph (Figure 3.b) this graph also seems to have periods where the relative difference in volatility substantially swings downwards. This would indicate that for some periods in time the implied volatility of put options is higher than that of call options.

Taking everything all together the graphs lead to the following insights; Put-Call Parity does not uphold, it does however seem to hold up better when the time to maturity increases and over time the Put-Call Parity also seems to hold up better. Significant changes in volatility all seeable throughout options with different maturities, for example the spike in the financial crisis and the dips in the period before the financial crisis.

Test 4: Normality Test

For the last test we analyze the distribution of the returns of the closing prices of the SPX Index. The first graph shows the closing prices of the SPX Index, the second graph shows the returns of the daily closing prices. These daily returns are analyzed and are used to plot a histogram and a Q-Q Plot.

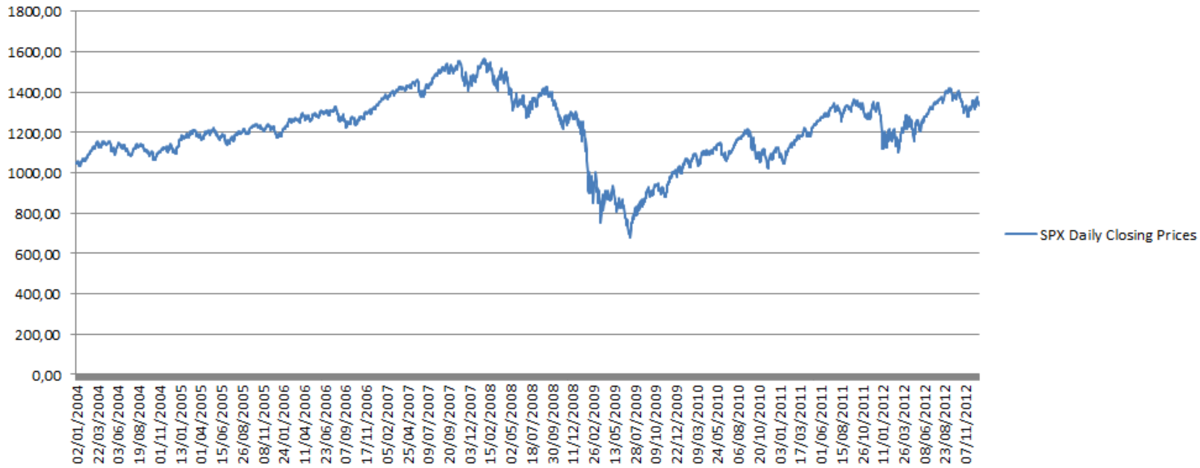

Figure 4.A. *The closing prices of the SPX index for the period 2004-2012.*

In the first graph (Figure 4.a) we see the closing prices of the SPX Index for the period 2004-2012. During the credit crisis we can see a rapid decrease in price, the rest of the graph mostly shows an increase in price with a much lower volatility. The index price ranges from roughly 1600 (just before the crisis) and 700 (at the end of the crisis).

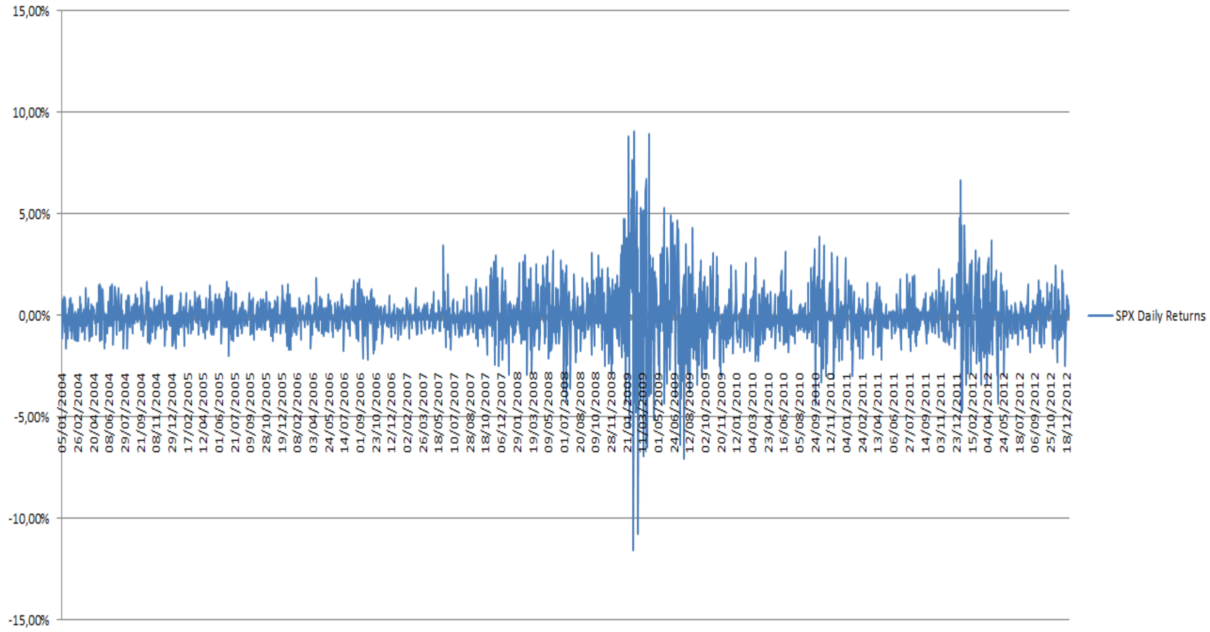

Figure 4.B. *The daily returns for the SPX index for the period 2004-2012.*

To better analyze the returns, a graph (Figure 4.b) has been made of the returns. Times of high volatility are better visible now within the graph, during the financial crisis the daily returns range from roughly 10,00% to minus 10.00%, For the rest of the graph the returns range from roughly 2.00% and minus 2.00%, with some periods having higher returns.

To get a first impression if the daily returns follow a normal distribution a histogram has been made. Within the histogram the frequency of the daily returns is plotted against the normal distribution for the dataset. This plot indicates that the peak is higher and the tails are smaller, compared to the normal distribution that is.

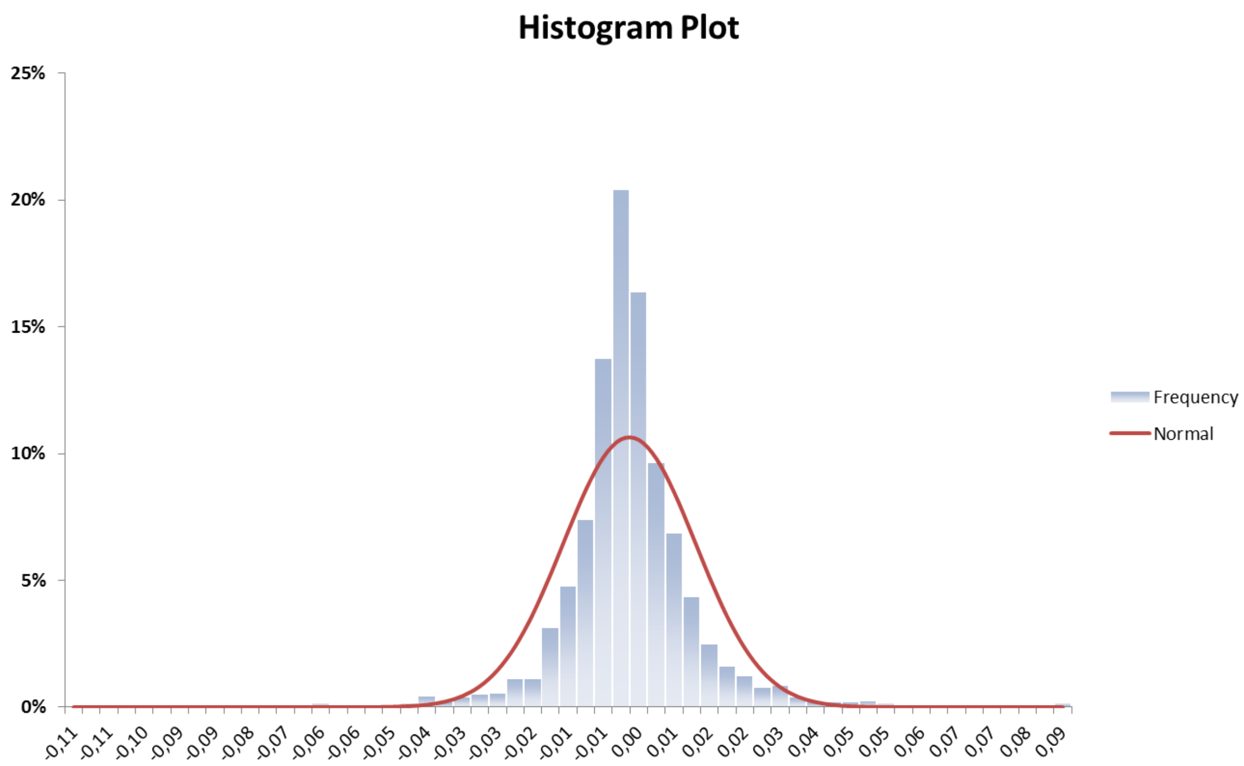

Figure 4.D. A histogram of the daily returns plotted against a normal distribution.

Descriptive Statistics

AVERAGE:	-0,02%
STD DEV:	1,35%
SKEW:	0,06
EXCESS-KURTOSIS:	10,50
MEDIAN:	-0,08%
MIN:	-11,58%
MAX:	9,03%
Q 1:	-0,58%
Q 3:	0,49%

To analyze the returns further a Q-Q Plot has been made and this is also plotted in a graph (Figure 4.c) Together these strongly indicate that the daily returns are not distributed normally.

	Mean	STDEV
QQ-Plot	-0,02%	1,35%
Q	**Normal**	**Empirical**
3,8%	-1,8	-1,6
7,7%	-1,4	-1,1
11,5%	-1,2	-0,8
15,4%	-1,0	-0,7
19,2%	-0,9	-0,6
23,1%	-0,7	-0,5
26,9%	-0,6	-0,4
30,8%	-0,5	-0,3
34,6%	-0,4	-0,2
38,5%	-0,3	-0,2
42,3%	-0,2	-0,1
46,2%	-0,1	-0,1
50,0%	0,0	0,0
53,8%	0,1	0,0
57,7%	0,2	0,1
61,5%	0,3	0,1
65,4%	0,4	0,2
69,2%	0,5	0,3
73,1%	0,6	0,3
76,9%	0,7	0,4
80,8%	0,9	0,6
84,6%	1,0	0,7
88,5%	1,2	0,9
92,3%	1,4	1,2
96,2%	1,8	1,7

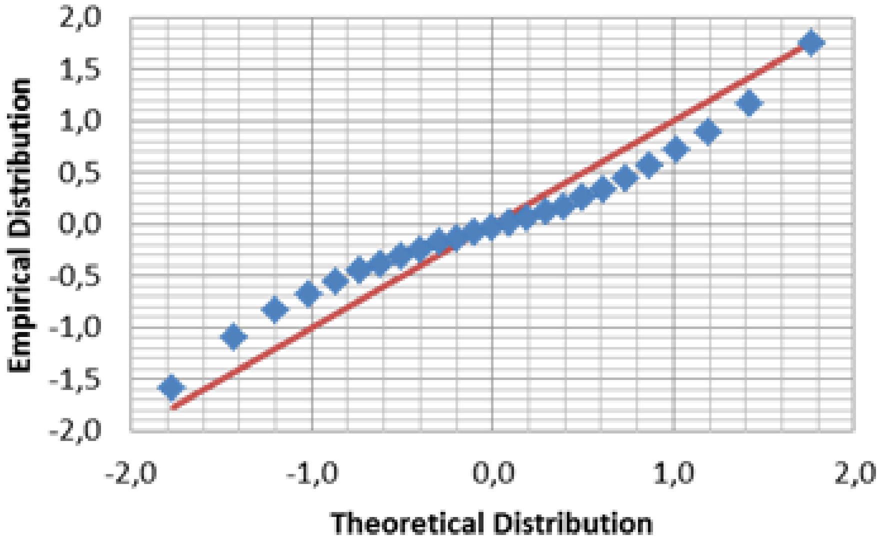

Figure 4.C, A visual representation of the Q-Q Plot.

Lastly to mathematically check if the distributions are distributed normally, a Jarque-Bera test has been performed and the daily returns did not pass the test. The Jargque-Bera score was 9975,58 with a P-value very close to 0,0%, which is well below the required 5,0% to reject the null-hypothesis of normal distribution.

Normality Test	**Score**	**C.V.**	**P-Value**	**Pass?**	**5,0%**
Jarque-Bera	9975,58	5,99	0,0%	FALSE	

Chapter 5: Conclusion

Test 1: Implied Volatility

On an individual level, the implied volatility of the Black-Scholes model does not accurately predict the realized volatility. On a collective level, however, it does quite accurately predict it. The credit crisis does have a significant effect on the credit crisis; before the credit crisis the implied volatility did not systematically differ upwards or downwards compared to the realized volatility, during the credit crisis it differed mostly downwards (implied volatility being lower) and after the credit crisis it differed mostly upwards.

Test 2: Volatility Smile

The moneyness of an option does have a significant effect on an option; options deep in- or out-of-the-money seem to have higher volatilities than options at-the-money. This mostly seems to apply to options during periods with high volatility, such as the credit crisis. Time to maturity seems to decrease the implied volatility of in-the-money options and seems to increase the implied volatility of out-of-the-money options.

Test 3: Put-Call Parity

Taking everything all together the graphs lead to the following insights; Put-Call Parity does not uphold, it does however seem to hold up better when the time to maturity increases and over time the Put-Call Parity also seems to hold up better. Significant changes in volatility all seeable throughout options with different maturities, for example the spike in the financial crisis and the dips in the period before the financial crisis.

Test 4: Normality Test

The daily returns of the SPX Index do not seem to be normally distributed when analyzed visually, nor did it pass the Jarque-Bera test for normality.

Chapter 6: Discussion

In the implied volatility test there is a pattern visible, the average implied volatility relatively to the SPX Volatility Index seems to become higher. It first differed mostly downwards, after the credit crisis it seems to mostly differ upwards. This could be an indication in option prices becoming higher after the credit crisis, as a result of different pricing models being used by market makers, or increased popularity leading to increased market participants that could inflate the price, due to divergence of opinion for example (Miller 1977).

The volatility smile tests for the first three graphs only contained call options-in-the-money and at-the-money, and only put options at-the-money and out-of-the-money. For the second three graphs exactly the opposite. Anything said about the effects of the credit crisis could therefore also be caused by this fact, the real cause is unclear.

The Put-Call Parity seems to uphold better after the credit crisis. This could be an indication of better or more efficient pricing than before the crisis. It could be caused by market makers/participants using a different option valuation model. Participants could theoretically arbitrarily profit from these inefficiencies and make the market more efficient.

The daily returns of the SPX Index also contain the credit crisis; this period could influence the expected normal distribution and harm the results of the Jarque-Bera test. The results found are also the opposite of previous research (Fortune 1996).

This research unfortunately does not contain any transaction data. That data could be used for different tests such as an option pricing error test. (Fortune 1996) The results of this test could then be compared to previous time periods.

A continuation of research on this subject could investigate if the changes caused by the credit crisis seem to persist or if the effects of the credit crisis are non-permanent. And could shed light on the effects of the credit crisis, or periods of high volatility, on volatility smiles.

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Appendix

For deriving a formula of an option in terms of the price of a stock, Black and Scholes assumed ''ideal conditions'' in the market for the stock and for the option, they are as follow:

A) The short-term interest rate is known and is constant through time.

B) The stock price follows a random walk in continuous time with a variance rate proportional to the square of the stock price. Thus the distribution of possible stock prices at the end of any finite interval is lognormal. The variance rate of the return on the stock is constant.

C) The stock pays no dividend or other distributions.

D) The option is ''European,'' that is, it can only be exercised at maturity.

E) There are no transaction costs in buying or selling the stock or the option.

F) It is possible to borrow any fraction of the price of a security to buy it or to hold it, at the short-term interest rate.

G) There are no penalties to short selling. A seller who does not own a security will simply accept the price of the security from a buyer, and will agree to settle with the buyer on some future date by paying him an amount equal to the price of the security on that date.

The formulae derived with this set of assumptions:

(1) The formula to calculate the value of a call option:

$$
\begin{aligned}
C(S,t) &= N(d_1)S - N(d_2)Ke^{-r(T-t)} \\
d_1 &= \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)\right] \\
d_2 &= \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{S}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)(T-t)\right] \\
&= d_1 - \sigma\sqrt{T-t}
\end{aligned}
\tag{1}
$$

(2) The formula to calculate the value of a corresponding put option, based on Put-Call Parity.

$$
\begin{aligned}
P(S,t) &= Ke^{-r(T-t)} - S + C(S,t) \\
&= N(-d_2)Ke^{-r(T-t)} - N(-d_1)S
\end{aligned}
\tag{2}
$$