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MARKET BASKET ANALYSIS OF BEAUTY PRODUCTS

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Abstract:

Companies nowadays are rich in vast amounts of data but poor in information extracted from that data. Big data is seen as a valuable resource and although the concept of data mining is still new and developing, companies in a variety of industries are relying on it for making strategic decisions. Facts that otherwise may go unnoticed can be now revealed by the techniques that sift through stored information.

Market basket analysis is a very useful technique for finding out co-occurring items in consumer shopping baskets. Such information can be used as a basis for decisions about marketing activity such as promotional support, inventory control and cross-sale campaigns.

The main objective of the thesis is to see how different products in a beauty shop assortment interrelate and how to exploit these relations by marketing activities. Mining association rules from transactional data will provide us with valuable information about co-occurrences and co-purchases of products. Such information can be used as a basis for decisions about marketing activity such as promotional support, inventory control and cross-sale campaigns.

Keywords: data mining, market basket analysis, association rules, multinomial logit.

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Chapter 1

Introduction

1.1 Overview

The highly technological era that we live in has made it possible for companies to gather enormous quantities of data. Data mining is becoming more and more common for many businesses worldwide. The large amount of data that is being gathered on a daily basis captures useful information across different aspects of every business. The collection of data on a highly disaggregate level is seen as a raw material for extracting knowledge. While some facts can be revealed directly from disaggregate data, often we are interested to find hidden rules and patterns. Non-trivial insights can be generated through data mining. Data mining contains of various statistical analyses that reveal unknown aspects of the data. Mining tools have been found useful in many businesses for uncovering significant information and hence, providing managers with solutions for complicated problems.

Data mining is commonly seen as a single step of a whole process called Knowledge Discovery in Databases (KDD). According to Fayyad et.al, 'KDD is the nontrivial process of identifying valid, novel, potentially useful and ultimately understandable patterns in data.' (Usama Fayyad, Gregory Piatetsky-Shapiro, and Padhraic Smyth, 1996)

Data mining is a technique that encompasses a huge variety of statistical and computational techniques such as: association-rule mining, neural network analysis, clustering, classification, summarising data and of course the traditional regression analyses.

Data mining gained popularity especially in the last two decades when advances in computing power provided us with the possibility to mine voluminous data. Extracting knowledge and hidden information from data using a whole set of techniques found its applications in various contexts. Knowledge discovery is widely used in marketing to identify and analyse customer groups and predict future behaviour. Data mining is an effective way to provide better service to customers and adjust offers according to their needs and motivations.

1.2 Business use of data mining

Companies nowadays are rich in vast amounts of data but poor in information extracted from that data. Big data is seen as a valuable resource and although the concept of data mining is still new and developing, companies in a variety of industries are relying on it for making strategic decisions. Facts that otherwise may go unnoticed can be now revealed by the techniques that sift through stored information. When applying mining tools and techniques we seek to find useful relationships, patterns and anomalies that can help managers make better business decisions.

Data mining tools perform analyses that are very valuable for business strategies, scientific research and getting to know your customers better. Managerial insights are no longer the only factor trusted when it comes to decision-making. Data driven decisions can lead to better firm performance.

Data-based implications are gaining popularity while the gut instinct of managers is remaining in the background. Analysing data not only improves firm performance but gives us accurate insights on different aspects of the business.

Data mining is widely used in marketing for spotting sales trends, developing better marketing campaigns and finding the root cause of specific problems like customer defection or fraudulent transactions, for example. It is also used for prediction of behaviour: which customers are most likely to leave us (customer churns) or what are the things that an individual will be most interested to see in a website.

1.3 Research problem description

In the recent years analysing shopping baskets has become quite appealing to retailers. Advanced technology made it possible for them to gather information on their customers and what they buy. The introduction of electronic point-in sale increased the use and application of transactional data in market basket analysis. In retail business analysing such information is highly useful for understanding buying behaviour. Mining purchasing patterns allows retailers to adjust promotions, store settings and serve customers better.

Identifying buying rules is crucial for every successful business. Transactional data is used for mining useful information on co-purchases and adjusting promotion and advertising accordingly. The well-known set of beer and diapers is just an example of an association rule found by data scientists.

The main objective of the thesis is to see how different products in a beauty shop assortment interrelate and how to exploit these relations by marketing activities. Mining association rules from transactional data will provide us with valuable information about co-occurrences and co-purchases of products. Some shoppers may purchase a single product during a shopping trip, out of curiosity or boredom, while others buy more than one product for efficiency reasons.

1.4 Motivation for the study

The main point of interest for retailers is to understand dependencies among purchases. Consumers buy various combinations of products on a single shopping trip, but choice scenarios do not seem to be random to market analysts. '…These multicategory decisions result in the formation of consumers' "shopping baskets" which comprise the collection of categories that consumers purchase on a specific shopping trip**.'** (Puneet Manchanda, Asim Ansari and Sunil Gupta, 1999).

Motivation objectives

Over the past two decades a lot of attention has been devoted to the subject of data mining. While retailers are involved in this topic because of the absolute utility of market basket data, market analysts are interested because of the research and technical challenges they face while analysing the data.

Increasing amount of data is being generated every second and this allows experts to search for meaningful associations among customer purchases. Customers make purchase decisions in several product categories on a single shopping trip. Interdependencies among products have faced increased attention recently as retailers are trying to improve their businesses by applying quantitative analyses to their data.

It is very important for retailers to get to know what their customers are buying. Some products have higher affinity to be sold together and hence the retailer can benefit from this affinity if special offers and promotions are developed for these products. It is also important to the retailer to cut off products from the assortment which are not generating profits. Deleting loss-making, declining and weak brands may help companies boost their profits and redistribute costs towards aspects of the more profitable brands. (Kumar, 2009) This is yet another reason why data mining is seen as a powerful tool for many businesses to regularly check if they are selling too many brands, identify weak ones and possibly merge them with healthy brands. Data mining techniques are highly valued for the useful information they provide so that the retailer can serve customers better and generate higher profits.

Chris Anderson in his book ‘The long tail: Why the future of business is selling less of more’ explains a concept of the ‘98% rule’, which is quite contrasting to the well-known 80/20 rule. In other words, 2% of the items a retailer sells are frequent, while 98% of the items have very low frequencies, which create a long tail distribution. This is why the presence of this ‘98% rule’ in the retail business created the need for data mining software and made quantitative analysis a must for retailers.(Anderson, 2006)

1. **Find products with affinity to be sold together.**

A lot of research has been done in marketing to show that there are demand interdependencies among certain related products within a single store. Retailers tend to exploit this tendency by adjusting price promotions in a profit-maximising way. They can also exploit these product associations by incorporating them into promotional strategies.
Analysing purchases in multiple categories allows retailers to benefit from promotion and other marketing activities. Incorporation of product interdependencies into a pricing strategy is an effective way of boosting profits.

For example, Mulhern and Leone(1991) study the impact of price promotions on cake mix and cake frosting. Their main objective is to evaluate the overall profitability of implicit price bundling. Reducing the price of cake mix increase purchases of both cake mix and frosting and the overall profit improves. The study shows how promotions have positive impact on the sales of a complementary product.

Finding associations between product purchases is an effective way to adjust price promotions better and make better predictions on the effect of price bundling. Also, it is important to keep product complementarities in mind when making promotions. Complementary products often sell well together but this does not mean that they are a pair and a price increase in one of the set will not affect sales of the other one. Complementarity gives managers control over their customers’ buying behaviour, but co-occurrence of specific product categories in a single shopping basket is less controllable. Market basket analysis reveals all the underlying patterns of buying behaviour that cannot be simply observed. (Puneet Manchanda, Asim Ansari and Sunil Gupta, 1999)

Analysing shopping baskets also shows multi-category dependencies across products which allows retailers to bundle new products that have not been discovered yet as a set.

2. Improve in-store settings and optimise product placement.

Gaining insight on product interdependencies can help retailers optimise store layout. It is an important aspect of retailing business because in-store settings may help increase sales if done right. It also influences buying behaviour, store traffic and the whole shopping atmosphere. If market basket analysis reveals that certain products are often purchased together, it is of great interest for the retailer to put these two items or categories of products close to each other to facilitate the customer. Another option is to place them as far as possible from each other so that customers are exposed to much more products while trying to find the other product. However, the latter option may have negative consequences due to the fact the customers tend to get annoyed if they cannot find fast what they are looking for and need to waste time strolling around the whole store.

Optimisation of in-store settings may help improve shopping experience by reducing congestion and saving time for customers. With the right space planning the store benefits from increased cross product sales and impulse purchases. Moreover, store layout and atmosphere has a very strong impact on customer perceptions. A study made by (Bill Merrilees and Dale Miller, 2001) shows that store layout and atmosphere has a positive effect on customer loyalty. In-store settigns as light, music, layout, applealing stock dispalys and easy to find goods are seen as determinants of pleasant and enjoyable shopping experience.

Various dimensions of store layout have positive effect on customers' purchase intentions and loyalty. This is why it is so crucial to extract knowledge from data so one can adjust store settings in order to improve customers' shopping experience.

3. Improve layout of the catalogue of e-commerce site.

Visual displays of products apply also to the catalogue of the firm online site. E-commerce website interface plays significant part of customers' perceptions. A key success factor for profitable e-commerce site is the layout. In order to be able to determine an optimised layout for website it is important to know the interdependencies among different products.

A lot of research has been done in finding an optimal location, colouring and design for catalogues of e-commerce sites. The last step of successfully implementing a website strategy is to know how to place different products in order to maximise cross-sales. For instance, if we know which products have affinity to be sold together, we have to make sure that they are side by side on the same page on the website. It is also possible to provide discount in the form of shipping benefits for a group of products that have higher probabilities of selling together.

4. **Control inventory based on product demand.**

For the recent years, with more powerful analytical software it is possible to predict almost everything. It is now feasible to predict product demand based on data from past purchases, for example. For this objective it is important to know which products are related in terms of cross-sales.

Being able to find the probability of purchase for each product or a certain set of products is essential for controlling inventory. It has been observed that greater volume of products in the inventory can lead to higher levels of demand. (David R. Bell and Yasemin Boztu˘g, 2007). Many researchers have tried to give explanation for this phenomenon. Recent studies have found the impact of promotion on stockpilling and increased demand.
(Assunc¸ ˜ao, J. L., & Meyer, R. J., 1993) analyse the nature of the relationship which exists between price, promotion, sales and consumption. The authors' main finding is that price promotions encourage stockpilling, while on the other hand stockpilling rationally leads to increase in consumption.

However, the consumption time depends on the type of product that is associated with stockpiling. Foods and drinks are considered to be consumed faster than non-food goods. In this case, most of the beauty products cannot be stockpiled for long time due to extended consumption time. A face cream for example, can be used for 5-6 months before it is over. While a shampoo or toothpaste usually last not more than a month. Here comes the challenge of how many people are there in a single household. If the case is about a whole family stockpiling would be appropriate because families tend to shop more rare but in larger quantities. That is why it is harder to predict consumption time of products in a beauty store, but after examining which ones sell best, it will be very beneficial for the retailer so that he is always prepared with profit generating products available in stock.

Chapter 2

Literature Review

2.1 Background of the study

Data mining has taken an important part of marketing literature for the last several decades. Market basket analysis is one of the oldest areas in the field of data mining and is the best example for mining association rules.

Various algorithms for Association Rule Mining (ARM) and Clustering have been developed by researchers to help users achieve their objectives. Rakesh Agrawal and Usama Fayyad are one of the pioneers in data mining. They account for a number of developed algorithms and procedures.

According to Shapiro, rule generating procedures can be divided into procedures that find quantitative rules and procedures that find qualitative rules. (Rakesh Agrawal, Ramakrishnan Srikant) elaborate on the concept of mining quantitative rules in large relational tables. Quantitative rules are defined in terms of the type of attributes contained in these relational tables. Attributes can be either quantitative (age, income, etc.) or categorical (certain type of a product, make of a car). Boolean attributes are such attributes that can take on one of two options (True or False, 1 or 0). They are considered a special case of categorical attributes. The authors call this mining problem the Quantitative Association Rules problem. An example of a generated quantitative rule is :

If ((Age : [30…39]) + (Married : Yes)) → (Number of cars = 2)

The example combines variables that have quantitative and boolean attributes.

(S. Prakash, R.M.S. Parvathi, 2011) propose a qualitative approach for mining quantitative association rules. The nature of the proposed approach is qualitative because the method converts numerical attributes to binary attributes.

However, finding qualitative rules is of main interest in this analysis. These rules are most commonly represented as decision trees, patterns or dependency tables. (Gregory Piatetsky-Shapiro, William Frawley, 1991) The type of attributes used for mining qualitative rules is categorical.

(Rakesh Agrawal, Tomasz Imielinski, Arun Swami, 1993) is one of the first published papers on association rules that proposes a rule mining algorithm that discovers qualitative rules with no restriction for boolean attributes. The authors test the effectiveness of the algorithm by applying it to data obtained from a large retailing company.

Association rules found application in many research areas such as: market basket analysis, recommendation systems, intrusion detection etc.

In marketing literature market basket analysis has been classified into two models: explanatory and exploratory. First, exploratory models will be thoroughly explained in this paper as they are of higher relevance for the research and after that an explanation of explanatory models will be given. The main idea behind exploratory models is the discovering of purchase patterns from POS (point-of-sale) data. Exploratory approaches do not include information on consumer demographics or marketing mix variables. (Katrin Dippold, Harald Hruschka, 2010) Methods like association rules (Rakesh Agrawal, Sirkant Ramakrishnan, 1994) or collaborative filtering (Andreas Mild, Thomas Reutterer, 2003) summarise a vast amount of data into a fewer meaningful rules or measures. Such methods are quite useful for discovering unknown relationships between the items in the data. Moreover, these methods are computationally simple and can be used for undirected data mining. However, exploratory approaches are not appropriate for forecasting and finding the cause-roots of complex problems. They are just used to uncover distinguished cross-category interdependencies based on some frequency patterns for items or product categories purchased together. A typical application of these exploratory approaches is identifying product category relationships by simple association measures. Pairwise associations are used to compare entities in pairs and judge which entity is prefered or has greater amount of some quantitative property. (Julander, 1992) compares the percentage of shoppers buying a certain product and the percentage of all total sales generated by this product. By making such comparisons, one can easily find out the leading products and what is their share of sales. Examining which the leading products are for consumers is extremely important since a large number of shoppers come into contact with these specific product types every day. As the departments with leading products generate much in-store traffic, it is crucial to use this information for placing other specific products nearby. The paper by Julander also shows how combinatory analysis can be used to study the patterns of cross-buying between certain brands or product groups: for instance, what is the percentage of shoppers that buy products A+C, but not B or what is the percentage of shoppers that buy only A. It also deals with the probabilities that shoppers will purchase from one, two or more departments in a single visit in the store.

Another significant stream of research in the field of exploratory analysis is the process of generating association rules. Substantial amount of algorithms for mining patterns from market basket data have been proposed. From the co-operative work of Rakesh Agrawal and Ramakrishnan Srikant they present two new algorithms for discovering large itemsets in databases, namely Apriori and AprioriTid. These two algorithms are similar with regard to the function that is used to determine the candidate itemsets, but the difference is that the AprioriTID does not use the database for counting support after the first pass (first iteration) while Apriori makes multiple passes over the database (more information on methodology in Chapter 4). The results from the study show that these two new algorithms perform much better than the previously known AIS (R. Agrawal, T. Imielinski, and A. Swami, 1993) and SETM (M. Houtsma and A. Swami, 1993) algorithms. Since the introduction of the Apriori algorithm, it has been considered the most useful and fast algorithm for finding frequent itemsets. Many improvements have been made on the Apriori algorithm in order to increase its efficiency and effectiveness. (M.J.Zaki, M.Ogihara, S. Parthasarathy, 1996). There are few algorithms developed that are not based on the Apriori,but they still address the issue of speed of Apriori. The following papers (Eu-Hong (Sam) Han, George Karypis, Vipin Kumar, 1999) , (Jong Soo Park, Ming-Syan Chen, Philip S. Yu) propose new algorithms which are not based on the Apriori, but all of them are being compared to Apriori in terms of execution time.

(Robert J. Hilderman, Colin L. Carter, Howard J. Hamilton, and Nick Cercone) develop a framework for knowledge discovery from market basket data. Combining Apriori and AOG (D.W. Cheung, A.W. Fu, and J. Han., 1994) algorithms in the methodology, the purpose of the paper is not only to explain how to discover customer purchase patterns, but to find out customer profiles by dividing customers into distinct classes. The authors provide an extensive explanation of the share-confidence framework. Results show that it can give better feedback than the support- confidence framework.

Another use of market basket data is found in the finite mixture model in the paper by (Rick L. Andrews , Imran S. Currim, 2002). The idea of the model is to identify segments of households that have identical behaviour across product categories. The authors use both marketing variables and scanner panel data to answer the research questions. The study shows that household demographic variables are found to be more strongly correlated to price sensitivity compared to results in previous studies.The research divides customers into heavy users and lighter users. Heavy user households are found to be less price sensitive, visiting the store less often, in most cases high income customers.While, on the other hand, lighter users are mainly students or people that visit the store very often and are very price sensitive. The results show that households that have identical behaviour across product categories tend to be lighter users than households that behave independently. Also households with identical behaviours are said to be more price sensitive,less sensitive to store advertising, also showing weaker loyalty in terms of brand names. The topic on distribution of consumer brand preferences is adressed in the paper by (Gary J. Russel, Wagner A. Kamakura, 1997) using long-run market basket data. The authors show how brand preference segmentation can be discovered without the availability of marketing mix data. A number of simplifying assumptions need to be made in order to permit these cross-category preferences to be estimated. However, using knowledge on marketing mix activity gives the researcher greater flexibility to employ more complex techniques in the analysis than simply using scanner data.

Exploratory models are very useful for uncovering cross-category relations, but not for finding their causes. While the main task of exploratory market basket analysis is to reveal and present hidden relationships between product categories, explanatory models aim at explaining effects. Datasets for such models consist of market basket data, customer attributes and marketing mix variables. The purpose of explanatory models is to identify and quantify cross-category choice effects of marketing variables, such as price, promotion and other marketing features. (Andreas Mild, Thomas Reutterer, 2003) Most of the explanatory models rely greatly on regression analysis, logit,probit and multivariate logistic model.

Mining transactional data along with household data gives retailers and managers space for customised target marketing actions. Analysing past purchases makes it possible for supermarkets to price goods intelligently while still serving heterogeneous consumers. (Nanda Kumar and Ram Rao, 2006). For researchers scanner data is seen as a mean to discover the effects of marketing actions on consumer behaviour. Using the shopping basket as a unit of analysis instead of single articles can provide retailers with consumer-oriented information.

Consumer purchase behaviour is a well-studied area in the marketing literature. The topic of price sensitivity and ellasticity is also well-studied through applied data mining techniques. Customers are commonly divided into large-basket shoppers and small-basket shoppers. Large-basket shoppers have higher expected basket attractiveness in EDLP[1] stores , while small-basket shoppers would rather go for HILO[2] format of a store. (David R. Bell and Yasemin Boztu˘g, 2007) . In this case with a beauty store,consumers tend to be small rather than large-basket shoppers.

Market basket data combined with household panel data is commonly used by researchers to investigate brand choice and price elasticities (Nanda Kumar and Ram Rao, 2006). Marketing researchers aim to go beyond the trivial correlation approach by finding out the source of cross-category dependence in shopping basket data. Explanatory models are used in this case when the purpose is to explain and predict certain effects. Data sets for such models consist of marketing mix variables and customer attributes. Logit and probit models are commonly used for estimating cross-category effects and predicting brand choice (Gary J Russell, Ann Petersen, 2000).

(Katrin Dippold, Harald Hruschka, 2010) use multivariate logit model to meausre dependencies and sales promotion effects across different categories in a retail assortment and how these effects influence purchase probabilities. As most approaches identify association rules across categories, this multivariate binomial logit model allows for examining main and interaction effects between categories which provides beneficial information on consumer behaviour in terms of predicting the effects of promotion.

Moreover, sensitivity to marketing mix variables is a very common consumer trait, which has been very well studied with the availability of scanner data and household observable variables. There is a strong relationship between household demographic variables and price sensitivity. (Andrew Ainslie, Peter E. Rossi, 1998) measure the covariance of observed and unobserved heterogeneity in marketing mix sensitivity across various categories. Household variables as well as shopping behaviour variables play an important role in explaining price sensitivity.

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[1]EDLP – Every Day Low Price – a pricing strategy that promises consumers low prices without the need to wait for sale events..

[2]HILO – High-Low Pricing – a pricing strategy where goods are regularly priced higher than competitors, but through promotions or coupons, key items are offered on lower prices.

A common practice for researchers when using explanatory models is to investigate a limited number of cross-category effects. (Gary J Russell, Ann Petersen, 2000) examine brand choice process in four paper goods categories. Brand choice among categories can be easily calculated with a conditional probability formula*, but as the number of categories increases, the level of complexity jumps exponentially. Expanding this general approach to a multivariate logistic model by adding household data gives us the possibility to explore more thoroughly consumer purchase behaviour within a specific store. The authors propose a market basket model based on the idea that choice in one category has impact on choices in all other categories.

Not only because of computational simplicity, but many studies limit included categories to those that are most commonly purchased. However, there has been quite some controversy that results on cross-category objects can be biased because of the small subset of retail assortment that is used in explanatory analysis.Taking into account fewer number of categories can lead to under or overestimation of the values of interaction effects so that some values can even take opposite incorrect signs. Although a research by (Siddhartha Chib, P. B. Seetharaman and Andrei Strijnev) confirms that there is a bias when using a small subset of categories, no such proof is found that there are extreme switches to positive or negative signs of coefficients. However, techniques for mining association rules can easily cope with very large number of categories (or items).

There are some drawbacks and areas of controversy with the exploratory analysis as well. Despite the usefulness of discovering meaningful cross-category interdependencies, the managerial value of exploratory models is somewhat limited. It provides only limited number of recommendations regarding decision-making since there are no apriori assumptions about 'response' and 'effect' and no marketing variables are incorporated into the analysis. Neglecting both consumer hererogeneity and marketing mix effects may also lead to biases.

(Yasemin Boztuğ , Thomas Reutterer, 2006) propose a model that link both explanatory and exploratory approaches in an attempt to overcome limitations from both approaches. The proposed models employs data compression first and then estimates cross-category purchase effects in order to reduce the complexity of the model and to select only meaningful categories that are relevant to a specific segment of households. This two-stage procedure that combines feature from both exploratory and explanatory models can be used as a guideline for selecting categories to be included for estimating cross-category effects.

In the book by (Michael J.A.Berry, Gordon Linoff, 1997), the authors suggest an approach of including all kinds of items in the categories. More frequent items do not need to be aggregated at all, while less frequent items need to be rolled up to a higher level of the taxonomy. The term taxonomy refers to a classification of products in a hierarchical fashion.
All the single items of a store assortment are on the lowest level of the taxonomy. Based on some shared characteristics, items can be grouped into a category that climbs up the taxonomy. For example, there are five different aromas of a cream soap. They can be all grouped into a category 'Cream Soap', which is a subcategory of 'Soaps'. (See Table 3.3)

Transaction-level data that reflects individual purchases is used in the standard rule mining procedures. However, a lot of models have been proposed for analysis of market basket data at the aggregate level. Data is most commonly aggregated by measures of time so that the base unit is no longer individual transaction, but daily sales in a store for example. It is also possible to roll up transaction-level data by more than one attributes. Here comes the problem of multi-dimensionality discussed in the paper by (Svetlozar Nestorov, Nenad Jukić, 2003). Information on several dimensions – product, location, customer and calendar exists for each transaction. The usual single dimension question – What items are frequently bought together in a transaction? – is now extended to – What products are boughts together in a particular region in a particular month?. When multiple dimensions are involved some associations might be hidden so a new model that captures these dimensions is proposed by the authors. The concept of extended association rules has several advantages in terms of the generated rules: they are easy to explain, providing more accurate predictions for certain variables and the number of discovered rules is likely to be much less for the same threshold support.

Significant amount of papers also contribute to the filed by comparing different mining techniques. Such an example is a recent paper by (A. M. Khattak, A. M. Khan, Sungyoung Lee and Young-Koo Lee, 2010). The authors make comparative analysis of two data mining techniques : ARM (association-rule mining) and Clustering. They use transaction data from a supermarket (Sales Day) to extract important information. Apriori algorithm is used for association rule mining. Its main objective is to find associated products and place them close to each other so that they can benefit from increased sales. When it comes to classification, Clustering is a very preferred technique. The authors apply K-means clustering to classify different classes of products sold together, customers based on their behaviour and purchasing power. The main advantage behind the clustering technique is that in this case there is data available on the customers' profile like age, purchasing power, also customer traffic. Extracting and analysing information from it gives retailer the advantage of improving their business by adopting and implementing new strategies to facilitate customers and maximise sales.

However, a lot of attention has been paid to the problem of generating too many association rules. The problem is addressed in a paper by (Szymon Jaroszewicz, Dan A. Simovici). Hundreds or thousands of association rules can be generated when the minimum support is low (see p. 28 for definition of minimum support). This is why a measure for judging the interestingness of a rule is proposed by the authors. They present an algorithm that computes the interestingness of itemsets with respect to Baysean networks. Interestingness of an itemset is said to be ' the absolute difference between its support estimated from the data and from the Baysean network'.

Given the quantitative nature of the field of data mining, most of the literature on that topic proposes different algorithms and techniques for optimised mining and generation of association rules. Different techniques are needed for different objectives so here is a table-overview of already established knowledge and ideas. (Table 2.2)

2.2 Table overview of existing literature and methodology on market basket analysis

Method and selected references	Characteristics of the analysis	Primary task of the analysis	Level of Aggregation	Marketing mix
1. Pairwise Associations (Julander, 1992)	Exploratory	Represent relationships	Aggregate	No
2. Association Rules (Robert J. Hilderman, Colin L. Carter, Howard J. Hamilton, and Nick Cercone), (Rakesh Agrawal, Sirkant Ramakrishnan, 1994)	Exploratory	Discovery of association rules	Aggregate	No
3. Finite Mixture Model (Rick L. Andrews , Imran S. Currim, 2002) (Garry J.Russel, Wagner A. Kamakura, 1997)	Exploratory	Identification of customer preference segments.	Disaggregate	Possible
4. Multivariate Logistic Model (Gary J Russell, Ann Petersen, 2000), (Harald Hruschka, Martin Lukanowicz, Christian Buchta, 1999)	Explanatory	Estimate and predict cross-category effects.	Aggregate	Possible
5. Regression Analysis (Francis J. Mulhern and Robert P. Leone, 1991), (Walters, 1991)	Explanatory	Analysing the impact of price on product and category choice	Aggregate	Yes
6. Intercategory Choice Dynamics (Pradeep K. Chintagunta and Sudeep Haldar, 1998) (Bari A. Harlam and Leonard M. Lodish, 1995)	Explanatory	Analysing purchase timing across categories.	Aggregate	Yes
7. Logit / Probit Models. (Andrew Ainslie, Peter E. Rossi, 1998), (Puneet Manchanda, Asim Ansari and Sunil Gupta, 1999), (P. B. Seetharaman, Andrew Ainslie and Pradeep K. Chintagunta, 1999), (Byung-Do Kim, Kannan Srinivasan, Ronald T. Wilcox, 1999)	Explanatory	Modelling multicategory choice decisions.	Disaggregate (Individual level)	Yes

Chapter 3

Data

3.1 Data description

The given dataset is a collection of sales records in a large transactional database. The study is based on data from a cosmetic chain in Sofia, Bulgaria. The stores represent products from a local cosmetic company and brands from three other international make-up companies. In the dataset we have information for the four stores of the company:
(Store 1, Store 2, Store 3, Store 4).

Description of stores:

Store 1 - located in Sofia. This is the first shop of the cosmetic chain. The data available is yearly data from 02.03.2012 to 17.02.2013.
Sales data graph available (see Table 3, appendix A)
Store 2 – located in Sofia. It has worked from April 2012 until October 2012. It was closed because of a low turnover.
Sales data graph available (see Table 4, appendix A)
Store 3 – located in Sofia. It was opened in July 2012. It is still functioning but it is slowly increasing turnovers due to non-central location.
Sales data graph available (see Table 5, appendix A)
Store 4 – located in Sandanski, tourist city. It has been opened in November 2012. It is still functioning but is slowly increasing turnovers.
Sales data graph available (see Table 6, appendix A)

The transaction database consists of the following information:

- Date – date of purchase;
- Time – time of purchase;
- Bon number – number of transaction;
- Item number :
    - Products starting with 400 in the column Item Number are products of the Bulgarian cosmetic company.

- Products starting with 100,200,300,500,600 are the make-up products by different companies.

- Product quantity – quantity purchased;
- Product price;
- Revenue = Price x Quantity;

Table 3.1.

Store №	№ of transactions	Average № of items per single transaction	Monthly Sales (in BGN)	Period
Store 1	12 893	1.04615	73 903	02.03.2012 - 17.02.2013
Store 2	976	1.87705	6 058	15.05.2012 - 09.10.2012
Store 3	3752	1.20579	21 550	27.07.2012 - 17.02.2013
Store 4	1 447	1.31503	12 161	24.10.2012 - 17.02.2013

Monthly sales in units and monthly revenue for each store have been represented graphically. (Appendix A, Table 3-10)

The X axis accounts for the months for which data is available, while the Y axis tells us how much products have been purchased in that period and what is the monthly revenue for each store.

3.2 Aggregation approach

Crucial point of shopping baskets analysis is the decision how to aggregate the data. Choosing the right level of detail is a critical point for the researcher. Depending on the research question, there are different levels of aggregation possible – aggregation over product categories, over brands, over brand extensions and so on.

In the given dataset, individual items were aggregated over product categories. This type of aggregation leads to generalisation of items so that a single product category will account for several distinct items. Generalised items have the advantage of extracting between-categories relationships.

Products with their SKU[3] codes fall into hierarchical categories, called taxonomies. According to Berry and Linoff, if we want to mine actionable results, it is better to specify items at a more detailed level.

SKU[3] – Stock-keeping unit. The term is used to identify each distinct product in the assortment.

The following considerations have been taken into account while aggregating the data:

- First phase – start with more generalised items.
- Second phase - aggregate items to higher levels of the taxonomy.
- More common items – no need to be aggregated at all.
- Less common items – aggregate at a higher level of the taxonomy.

Table 3.2 below provides information on the number of unique items before and after aggregation.

Table 3.2

Type of aggregation	Number of unique items before aggregation **	Number of unique items after aggregation **
Category aggregation	393*	67

*** Some products may have a wide variety of descriptors (such as type of colour for hair dye) so the number of the unique products before aggregation might be larger due to this fact.**

****The list of items before aggregation and the list of product categories after aggregation is on Table 1 and Table 2 in Appendix A.**

Table 3.3 below provides a sample example of how items were aggregated to product categories.

Table 3.3

Aggregated product category	Disaggregate data
Shower cream	Shower cream Greenline Yoghurt Banana and Strawberry
	Shower cream Greenline Yoghurt Camu Camu
	Shower cream Greenline Yoghurt Vanilla & Fig
	Shower cream Aroma Greenline Bamboo Milk Extract
	Shower cream Aroma Greenline Grapefruit
	Shower cream Aroma Greenline Aloe extract
	Shower cream Aroma Greenline Cotton Milk Extract
Body Lotion	Body lotion Aroma Greenline Calming Guarana
	Body lotion Aroma Greenline Nourishing Blueberry
	Body lotion Aroma Greenline Hydrating Vanilla & Fig
Hair accessories	Headband
	Hairclips
	Barrette
	Claw clip
Bar soap	Toilet soap Aroma Fresh Pink Orchid
	Toilet soap Aroma Fresh Water Lily

	Toilet soap Aroma Fresh Lilac
	Toilet soap Aroma Fresh Aloe
	Toilet soap Aroma Luxury Oils Relaxing
	Toilet soap Aroma Luxury Oils Energising
	Toilet soap Aroma Luxury Oils Stimulating
	Toilet soap Aroma Luxury Oils Balancing
	Toilet soap Aroma Luxury Oils Massaging
Hair conditioner	Aroma Greenline Conditioner Hair Repair / Olive Oil
	Aroma Greenline Conditioner Colored Hair / Pomegranate
	Aroma Greenline Conditioner Nourishing / Q10 and Bamboo
	Hair Conditioner Aroma Fresh Honey Milk
	Hair Conditioner Aroma Fresh Avocado Milk
	Hair Conditioner Aroma Fresh Aloe Milk
	Hair Conditioner Aroma Fresh Calendula

3.3 Considerations and assumptions prior to analysis

1. **Quantities**

Quite often, large and small packages of yoghurt, for example, are not the same product for customers in their perceptions. However, in the given dataset, most of the products have only one packaging size available. For that reason, sizes will be removed for greater ease in the analysis.

2. **Language**

The initial dataset is in Bulgarian and therefore all the products with their names and specifications have been translated to English.

3. **Brands**

In the dataset, we have data on products of a particular cosmetic company, which excludes the availability of different brands from another companies. Since the cosmetic company does not produce make up, the only different brands in the cosmetic chain are three make up brands. The main purpose of the study is not related in any aspect to brands so in the translation procedure brands have been removed for ease in use and analysis.

4. **Correlation**

A general assumption in the analysis is that sales in different product categories of the shopping basket are correlated.

3.4 Research questions and hypotheses

Research questions:

1. **RQ1: What type of beauty care product categories are frequently purchased together?**

   Items from the cosmetic stores, aggregated to product categories will be analysed for finding co-purchases and frequently purchased product categories. This analysis will provide the retailer with valuable information that aids in adjusting promotions and offerings accordingly.

   The association rule analysis is an undirected approach, which means that no a-priori hypotheses are needed to conduct the analysis. The whole idea of the approach is to mine patterns from the data and let the user decide which ones are important for managerial decisions. However, we do have some expectations on the data and will test if following hypotheses are true.

Hypotheses:

- H_1: Hair product categories (Shampoo, Hair dye, Hair conditioner, Hair mask) have high affinity to be purchased together.
- H_2: Cleansing product categories (Shower cream, Soaps, Shower gel, Toothpaste) have high affinity to be purchased together.
- H_3: Make-up products (Make-up for lips, eyes and skin) have high affinity to be purchased together.

2. **RQ2: Do season and time of the day have a significant effect on the likelihood of purchase of beauty care products?**

Time of purchase and season of purchase will be examined whether they have a significant effect on the likelihood of purchase of some of the most frequently purchased product categories, found in RQ1.

Statistical significance refers to the likelihood that an event or result is caused by defined predictor variables. Multinomial logistic regression will be used to predict the probability that a consumer chooses to purchase a specific product category out of three , given the season and time as independent variables. The term significance will be used to test whether the estimated regression coefficients will be significantly different from zero and what is the impact of time and season on the probability of purchase. Four hypotheses will be tested for 3 sets of possible product category choices:

- [Make-up eyes],[Make-up lips],[Make-up skin]
- [Face cream day],[Hand cream],[Medical shampoo]
- [Shower cream],[Nail polish],[Body lotion]

Hypotheses:

- H_1: Season has a significant effect on the likelihood of purchase of beauty care products.
- H_2: Time of the day has a significant effect on the likelihood of purchase of beauty care products.
- H_3: There is a significant difference between the likelihood of purchasing certain products in the morning and in the evening.
- H_4: There is a significant difference between the likelihood of purchasing certain products in different seasons.

Chapter 4

Research methodology

4.1 Market basket analysis

Given the availability of transaction data, market basket analysis is a perfect starting point for the research. Undirected data mining is useful in cases when the researcher is unaware of specific patterns prior to analysis. (Berry and Linoff). However, in this dataset we already have some knowledge of the data. The a-priori assumption that items sold in different product categories are correlated provides a foundation for executing market basket analysis that will lead to more concrete conclusions about the data.

Berry and Linoff divide rules produced by market basket analysis into three most common types: the useful, the trivial and the inexplicable.

Quality information that can suggest a course of action can be derived from the useful rule. Such a rule can be found in the classic example of beer and diapers on Thursdays. Different explanations of this rule have been proposed, though it is widely believed that young couples tend to prepare for the weekend by purchasing diapers for the baby and beer for the dad. Locating the diapers next to the isle with beer is a wonderful opportunity for every supermarket to increase sales in both products.

Trivial rules reproduce facts that can be simply derived from common knowledge. For example, it is logical that someone who is purchasing paint will also purchase paint brushes. Therefore, trivial rules may not always provide valuable information on a possible course of action.

Another problem that may arise with trivial rules is when an interesting rule turns out to be the result from a special marketing campaign or a product bundle. It is useful to have detailed information on previous marketing campaigns before running the analysis because it will show us which rules are the results from a certain campaign or promotion and which from consumer preferences. For that reason, market basket analysis is extremely useful in measuring the success and impact on sales of a previous marketing campaign.

Worst-case scenario is mining inexplicable rules. Not only they do not provide a suggested course of action, they are also difficult to understand and explain. Such rules still provide us with information, but useless and inapplicable. For example, a study made in USA shows that after the opening of a hardware store, toilet rings were the most sold product. (Berry and Linoff). To be able to extract valuable information, details on store settings or discounts are needed, but it simply does not give us insight on consumer behaviour.

4.2 Strengths and weaknesses of Market Basket Analysis

One of the main advantages of market basket analysis is that it is perfect for undirected data mining. This technique is used when we do not know where to begin with a large dataset. The majority of data mining techniques are not used for undirected data mining, while market basket analysis can be easily applied to analyse big data and provide the user with an appropriate start.

First, we have to start with the way data is recorded- the data format. Each variable has a related data type – the type of data that an object can hold. Data recorded in a variable-length format is useful because it saves space. The difference between fixed format and variable-length data is the number of characters that a record can hold. For fixed- length format, each field has to be predefined to be long enough to hold the longest name. This can be seen as a waste of space for records that have short names. While with variable-length data, each field can be as long as its record's length. When it comes to transactional data, the most natural way to represent the items is having them recorded in a variable-length data type. While many techniques operate with data records in a fixed format, market basket analysis can handle variable-length data without losing important information.

Another major advantage and strength of the analysis is its operational simplicity. Unlike neural networks, computations in MBA are rather simple and the technique is quite comfortable for smaller problems.

However, as the number of items and transactions increases, the computations needed to generate association rules grow very quickly, even exponentially. A possible solution for this problem is to reduce the number of items. This can be easily done by generalising the items and aggregating them at a higher level of taxonomy. Although, generalised items are not always very actionable, there are some methods to control the process of rule generation. Minimum support pruning is such an example. More detailed information on minimum support with formulas and examples is given later in this chapter.

The main problem of this analysis is determining the right level of aggregation. During the process of generalisation, some information may be lost and frequencies of items may differ from the original levels. Possible solution is to insert virtual items that can capture lost information from the generalised items. This is the case with rare items.

4.3 Association rule mining

Using data mining techniques on transactional data leads to the generation of association rules and finding correlations between products in the records. The main concept of association rules is to examine all possible rules between items and turn them into 'if-then' statements.

4.3.1 Definition

Let I = { $i_1,i_2,i_3,. . . , i_m$} is the set of all items available at the store.
By T = {$t_1,t_2,t_3, . . . , t_n$} we define the set of all transactions in the store.
Each transaction t_i = {i_2,i_4,i_9} contains a subset of items from the whole market basket dataset.
An itemset is every collection of zero or more items from the transaction database.
The number of items that occur in a transaction is called a transaction width.
Let's suppose X is a set of items, e.g. X = {beer, diapers, bread}
Transaction t_j contains an itemset X if X is a subset of t_j ($X \subseteq t_j$).

An association rule can be expressed in the form of X →Y, where X and Y are two disjoint itemsets (do not have any items in common).
X is an antecedent and Y is a consequent, in other words, X implies Y.
The main concept of association rules is to examine all possible rules between items and turn them into 'if-then' statements. In this case the 'if' part is X or the antecedent, while the 'then' part is Y or the consequent.

Antecedent → consequent [support, confidence]

The antecedent and consequent are often called rule body and rule head accordingly. The generated association rule relates the rule body with the rule head. There are several important criteria of an association rule: the frequency of occurrence, the importance of the relation and the reliability of the rule.

Table 4.1
Revised table for functions of association rules (P.D. McNicholas, T.B. Murphy, M. O'Regan)

Function	Definition
Support	S (X→Y) = P (X,Y) and S(X) = P(X)
Confidence	C (X→Y) = P(Y | X)
Expected Confidence	EC (X→Y) = P (Y)
Lift	L(X→Y) = c(X→Y) / P(Y) = P (X,Y) / (P(X)P(Y))
Importance	I (X→Y) = log (P(X|Y) / P(Y| not X)

Example 1: X {[Toothbrushes] + [Toothpaste]} → Y {[Shampoo]}

Support = 20%

Confidence = 50%

Lift = 1.5

There are two basic parameters of Association Rule Mining (ARM): support and confidence. (Qiankun Zhao, Sourav S. Bhowmick, 2003) They both measure the strength of an association rule**.** Since the database is quite large, there is a risk of generating too many unimportant and obvious rules, which may not be of our interest. In that case a common practice is to define thresholds of support and confidence prior to analysis if we want to generate only useful and interesting rules.

Support of an association rule is the percentage of records that contain X U Y to the total number of records in the database. In other words, the support measures how often a rule is applicable to the given dataset. In this measure of strength, quantity is not taken into account. The support count increases by one for each time the item is encountered in a different transaction T from the database D. For example, if a customer buys three tubes of toothpaste in a single transaction, the support count number of [Toothpaste] increases by one.

In other words, the support measures whether an item is present in the transaction or not, ignoring the quantity purchased. If X consists of two items, for example [Toothpaste] and [Toothbrushes], again the support count number increases by one for every distinct item that is present in the transaction. A high support value means that the rule involves a big part of the database.

Support can be derived from the following formula:

$$\text{Support (XY)} = \frac{\text{Support count of XY}}{\text{Total number of transactions in D}}$$

If the support of X and Y (a set of items) is 10%, it means that X and Y appear together in 10% of the transactions. Retailers will not be interested in items with such low support, as they appear to be purchased together quite rarely. An exception might be when the items of interest are expensive and or generate high profits. Even though such items are rarely purchased, they will be even more profitable if the retailer knows how to exploit the relation between them. In the case with a beauty products store we need higher support in order to mine useful and interesting association rules. It is advisable to define minimum support before the mining process. Specifying the needed minimum support as a threshold prior to analysis generates only itemsets whose supports exceed that given threshold. However, still there may be some items of interest that are not purchased frequently but give us insightful information. This is the case with expensive and luxury goods in a supermarket, for example. They are not purchased quite often, but the value of the purchase is what matters most. This is why in the aggregation process of the data, more expensive items are rolled up at higher levels of the taxonomy as they do not appear that often in the transactions.
In the given example above, the support of the rule is 20%, which means that the combination of the 3 products occurs in 20% of all transactions.

Confidence of an association rule is defined as the percentage of the number of transactions that contain XUY to the total number of records that contain X. In other words, confidence is a measure of the strength of association rules and is used to determine how frequently items from itemset Y appear in transactions that contain itemset X. Let's suppose we have a rule X →Y. Confidence tells us how likely it is to find Y in a transaction that contains X.

Formula

Confidence(X/Y) = $\frac{\text{Support(XY)}}{\text{Support(X)}}$

In example 1, the confidence is 50%. This means that 50% of all transactions that contain [Toothbrushes] and [Toothpaste] also contain [Shampoo]

[Shampoo] occurs in at least 50% of the transactions in which {[Toothbrushes] and [Toothpaste]} occur.

Lift measures the importance of a rule. The lift value is represented as the ratio of the confidence and the expected confidence of a rule. The lift can take over values between zero and infinity. In every association rule we have an antecedent and a consequent, also called rule body and rule head accordingly.

Rule body [toothbrushes] + [toothpaste] → Rule head [shampoo]

If the value of the lift is greater than 1 this means that both the rule body and the rule head appear more often together than expected. The occurrence of the rule body positively affects the occurrence of the rule head. The other way around, if the lift value is lower than 1, this means that both the rule body and rule head appear less often together than expected and the occurrence of the rule body negatively affects the occurrence of the rule head. However, if the lift value is near 1, the rule body and rule head appear together as often as expected. (Lift in an association rule)

Lift can be derived from the following formula:

$$L(X\to Y) = c(X\to Y) / P(Y) = P(X,Y) / (P(X)P(Y))$$

From the given example 1, the lift value is 1.5 which means that the combination of [toothbrushes],[toothpaste] and [shampoo] is found about 1.5 time more often than expected. However, there is an assumption under which the expected number of occurrences is determined.(See formula for expected confidence in Table 1). The assumption states that the existence of [toothbrushes] and [toothpaste] in a group does not influence the probability to find [shampoo] in the same group and vice versa.

Importance

There is no association between X and Y if the importance is 0. If the importance score is positive, this means that the probability of Y increases when X is true. A negative importance score says the opposite: the probability of Y decreases when X is true.

It is also known as a Weight-of-Evidence (WOE). The importance is derived by the following formula:

$$I (X \rightarrow Y) = \log (P(X|Y) / P(Y| \text{not } X)$$

Generated rules can be grouped into rules that have direct and rules that have indirect relationships. If two rules, say R1 and R2, share at least one item (no matter if it is in the rule body or rule head), they belong to the same rule group and they are directly related. Indirectly related rules are such rules that do not contain the same item in both the rule body and rule head.

The association rules problem can be easily defined as it follows:

Given a threshold S (the minimum support) and a threshold c (the minimum confidence), we are interested to find all rules in the form of X → Y, where X and Y are sets of items, such that:

1. X and Y appear together in at least s% of the transactions.
2. Y occurs in at least c% of the transactions, in which X occurs.

A given association rule is supported in the database, if it meets both the minimum support and minimum confidence criteria.

The main purpose of Association rule mining is to find items that satisfy the prerequisite conditions for minimum support and minimum confidence. These conditions can be formally expressed as follows:

4.3.2 Definition:

T is a set of transactions in a given D database. We are interested to find rules with

$$\text{Support} \geq \text{minsup}$$
$$\text{Confidence} \geq \text{minconf},$$

where minsup and minconf are predefined thresholds of support and confidence, respectively.

The process of association rule mining can be described in two consequent steps:

1. Generating Frequent Itemsets

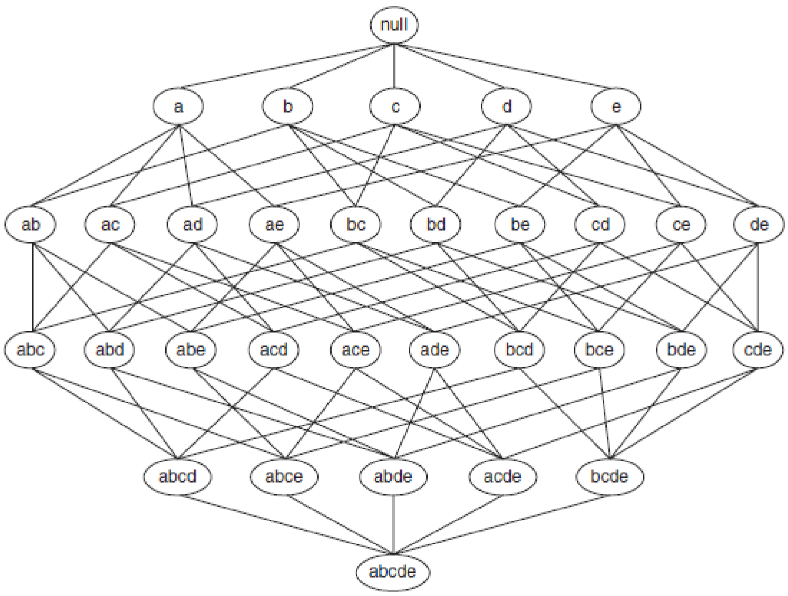
All itemsets that exceed the minsup threshold are generated and are called frequent itemsets.

2. Generating association rules.

The objective is to generate high-confidence rules from the already generated in the previous step frequent itemsets.

Frequent Itemset Generation

This first step in affinity analysis consists of generating all the rules that would be candidates for indicating association between the items. In other words, the idea is to find all possible combinations of single items, pairs of items, triplets of items and so on in the transactional database. However, as already mentioned, as the number of items and therefore possible combinations increases, the level of complexity rises exponentially. A dataset with k items can potentially generate up to 2^k-1 frequent itemsets (excluding the null set). A lattice structure is usually used to visualise all possible combinations of items in frequent itemsets. (Figure 1)



In order to determine the support count for every candidate itemset we need an efficient technique that can find an optimal solution. A brute-force search is a very straightforward technique that is used to systematically enumerate all possible candidates for the solution and check whether each candidate satisfies the problem's statement.

The biggest advantage of this approach is that the brute-force approach always finds the best solution if it exists and is very simple to implement. However, as the size of the problem increases with the number of candidate solutions, the brute-force method may not always terminate in reasonable time. This is why the method is preferred when the
dataset is not very large.

The classic approach for generating frequent itemsets is using the Apriori algorithm. (Rakesh Agrawal, Sirkant Ramakrishnan, 1994). According to the Apriori property: ‘All subsets of a frequent itemset must also be frequent’. If it has been verified that an itemset X is infrequent, there is no need for further investigating its subsets as they must be infrequent too. For example, in the given dataset, if a transaction that contains of { Hair conditioner, shampoo, hair dye} is frequent, a transaction containing {hair conditioner, shampoo} is also frequent.

Generating Association Rules

Each frequent itemset Y can produce up to 2^k-2 association rules, ignoring rules that have empty antecedents or consequents (0 →Y or Y → 0). An association rule can be extracted by partitioning a single itemset Y into two non-empty subsets, X and Y –X, such that X → Y- X satisfies the confidence threshold. It is a prerequisite and necessary condition for all such rule to have met the support threshold because they were generated from a frequent itemset. (Association analysis: Basic concepts and rules.)

For example, if we have a frequent itemset X= {a, b, c}, there are 2^k-2= 6 candidate association rules that can be generated from X:

{a, b}→ {c}
{a, c}→ {b}
{b, c}→ {a}
{a}→ {b, c}
{b}→ {a, c}
{c}→ {a, b}

Rule generation in Apriori algorithm

In order to generate association rules, the Apriori algorithm uses a level-wise approach, where each level corresponds to the number of items that belong to the rule consequent. At first, all the high confidence rules that have only one item in the rule consequent are extracted. Then new rules are generated from these ones.

For example, if:

$\{a, c, d\} \rightarrow \{b\}$

$\{a, b, d\} \rightarrow \{c\}$

are high-confidence rules, then the candidate rule $\{a, d\} \rightarrow \{b, c\}$ is generated by merging the consequents of both rules. (Association analysis: Basic concepts and rules.)

In other words, candidate rule is generated by merging two rules that share the same prefix in the rule consequent.

The Apriori algorithm

General idea

The Apriori is the most commonly used algorithm for frequent item set mining. It starts with identifying the frequent individual items in the transactional database and proceeds with extending them to larger and larger itemsets until they appear often enough in the database.

The algorithm is terminated when no further extensions that satisfy the minimum support condition are found.

The main idea of the algorithm is scanning the database for frequent itemsets, while on each following step pruning those items that are found to be infrequent. There are two very important steps in the candidate generation – the join and the prune step. In the first step, joining L_k with itself results in the generation of C_{k+1}. While in the prune step, if there is any k-itemsets that is infrequent it is pruned because it cannot be a subset of the frequent (k+1) itemset.

C_k – candidate itemsets with size k.

L_k – frequent itemsets with size k.

The Apriori algorithm can be represented in the following steps:

1. 1.Find frequent items and put the to L_k (k=1).
2. Use L_k to generate a collection of candidate itemsets C_{k+1} with size (k+1).
3. Scan the database to find which items in C_{k+1} are frequent and put them into L_{k+1}.
4. If L_{k+1} is not empty:
    - K:=k+1
    - Go to step №2.

The example below shows how the Apriori works in a few simple steps. Let's suppose that a sample database of transactions consists of the following sets: {a, c, d}, {b, c, e}, {a, b, c, e}, {b, e}. Each letter corresponds to a certain product from the assortment. For example {a} is shampoo, {b} is hair conditioner.

On the first step, the algorithm counts up the frequencies of each item separately, also called supports. If we want to be sure that an item is frequent, we can predefine the minimum support level. In this case, the minimum support is 2. Therefore, four of the items are found to be frequent.

In the next step a list of all the 2-pairs of frequent items is generated. The already found infrequent items are excluded for further analysis. In order to find all possible two-item pairs, the Apriori algorithm prunes the of all possible combinations .

At the last step, by connecting a frequent pair to a frequent single item a list ot all the three-triplets of frequent items is generated. The algorithm ends at this step, because the pair of four items generated at the next step doesn't meet the required minimum support.

Sample example:

Database D

TID	Items
1	a c d
2	b c e
3	a b c e
4	b e

Scan D

C_1

Itemset	support
{a}	2
{b}	3
{c}	3
{d}	1
{e}	3

L_1

Itemset	support
{a}	2
{b}	3
{c}	3
{e}	3

C_2

Itemset
{a b}
{a c}
{a e}
{b c}
{b e}
{c e}

C_2

Itemset	Support
{a b}	1
{a c}	2
{a e}	1
{b c}	2
{b e}	3
{c e}	2

L_2

Itemset	support
{a c}	2
{b c}	2
{b e}	3
{c e}	2

C_3

Itemset
{b c e}

L_3

Itemset	support
{b c e}	2

Levels of taxonomies:

It was already mentioned that taxonomies are used for classification of items in a hierarchical fashion. Taxonomies are also dealing with levels of complexity. The process of rule generation will take longer time if there are more items in a single transaction. This is the case with any big supermarket. The average transaction is larger compared to convenience stores, for example.

In the given dataset, the size of transactions tend to be somewhere in between. Customers purchase relatively few items on a single shopping trip and looking for rules that contain four and more items may apply to only few transactions. This is why no quite complicated computations are expected in the analysis as the average number of items that a customer buys in a single transaction is around 2.

4.4 Multinomial Logistic Regression

In order to gain better insights from the data, Market Basket Analysis alone is not enough. Comparing purchases made in different seasons or in different times of the day would ensure more information that could possibly suggest action.

This model-based approach comes as a consequence of the market basket analysis. As it is now clear which items are frequently purchased together, it will be beneficial for the analysis to find out what is the likelihood of purchasing certain items at different times of the day and in different seasons. This analysis will provide managers with information on which is the best season for promotion of specific product categories and at what time of the day consumers are more likely to purchase items from a specific product category. The items aggregated into product categories are used here to model the choice of one or more frequently purchased product categories.

Consumers' motivation for purchase might be influenced by external factors like the season of the year or the time of the day. The objective of this model-based approach is to study the relationship between a dependent variable, which is the choice of one among several frequently purchased product categories and independent variables, which represent the factors that influence purchases. The main purpose for this study is to show that the category choice is a function of the time of purchase and the current season. We are interested to evaluate the probability of a category membership.

Logistic regression can be extended to handle responses that are taking more than two possible outcomes – this is the multinomial logistic regression (also called multinomial logit). It is used to predict the probability that the dependent variable is a member of a certain category based on multiple independent variables. Multinomial logit models are used to model the relationship between a polytomous response variable and a set of independent variables. The models for polytomous data are extensions of the models for binary data – the individual can belong to more than two classes. These models can be classified into two distinct types, depending on whether the dependent variable has an ordered or unordered structure . In studying consumer behaviour, an individual can choose among several options, so the response variable does not have an ordered structure.

Predicting the probability of different possible outcomes of the dependent variable is commonly used for choice modelling. We are also interested to identify variables that play an important role in the prediction of the outcome of interest.

The multinomial logit compares multiple groups through a combination of binary logistic regressions. It allows each category of the dependent variable to be compared to a reference category. The group comparisons are equivalent to the comparisons for a dummy-coded dependent variable, with the group with the highest numeric score used as the reference group. When there are n possible outcomes, the model consists of n-1 logit equations. We can say that the model fits n-1 separate binary logistic models, where we compare category 1 to the reference category, then category 2 to the reference category and so on.

If the dependent variable is category choice and we are interested to find the probability of a consumer purchasing one of the following options:

1. [Shampoo]
2. [Toothpaste]
3. [Toothbrush]

The analysis would then compare customers that bought [Shampoo] relative to [Toothpaste] and customers that bought [Shampoo] relative to [Toothbrush] (see equation (1) and (2)).

The model of choice behaviours between three options for purchase can therefore be represented by using two logistic models. Multinomial logit provides a set of coefficients for each of the two comparisons. These coefficients are of highest interest for the researcher because they allow us to build equations that will help calculating the probability of a category membership. We can assume that a certain case will belong to the group that has the highest estimated probability. The effect of explanatory variables can be assessed for each logit model and for the model as a whole.

$$\text{Log } \frac{\Pr(Y = Shampoo)}{\Pr(Y = Toothpaste)} = \beta_{10} + \beta_{11}X_1 + \beta_{12}X_2 + \ldots + \beta_{1k}X_k \qquad (1)$$

$$\text{Log } \frac{\Pr(Y = Shampoo)}{\Pr(Y = Toothbrush)} = \beta_{20} + \beta_{21}X_1 + \beta_{22}X_2 + \ldots + \beta_{2k}X_k \qquad (2)$$

Where β's are the regression coefficients.
Estimation of the unknown parameters β_1 in each vector can be done by various techniques depending on the researcher's goals. However, quite often parameters are estimated using maximum likelihood techniques.

The multinomial logit is an attractive technique for researchers because it does not assume normality, linearity and homogeneity of variance for the independent variables. Another very important assumption regarding the multinomial logit is related to the independence among dependent variable choices. The membership or simply the choice of one category is not related to the choice of another category of the response variable.

Last but not least, the multinomial logistic regression assumes that the outcomes may not be perfectly separated by the predictors and therefore there are no unrealistic estimated coefficients or exaggerated effect sizes.

However, in this model it is of high importance to check for multicollinearity among the predictors. When two or more of the independent variables are strongly correlated , they provide redundant information about the response and bias the coefficient estimates. Multicollinearity increases the standard errors of the coefficients, which means that coefficients for some independent variables may be found to be significantly different from zero, whereas without multicollinearity and with lower standard errors, these same coefficients might have been found to be significant and the researcher may not have come to null findings in the first place. Although multicollinearity does not decrease reliability of the model, it affects calculations regarding individual predictors. As multicollinearity can inflate standard errors, this may result in misleading and confusing results
Variance Inflation Factor (VIF) is a statistical measure used to detect multicollinearity among independent variables. VIF provides an index that tells us how much the variance of an estimated regression coefficient is increased because of collinearity. If there is no correlation between any two variables, the VIF measure will be 1. Using the rule of the thumb, if the VIF value is 5 or greater than 5, we can speak of high multicollinearity.
The square root of the Variance Inflation Factor indicated how much larger the standard error is compared to what it would be if that predictor variable were uncorrelated with the other predictor variables.

A customer can choose among j alternatives in a choice set.

Let's denote Y to be the dependent (response) variable that can take three possible options: $Y_j = \{1,2,3\}$.

The outcomes are coded as follows:

[Toothpaste] – 1;

[Shampoo] – 2;

[Nail polish] – 3;

Let $X_i = (X_1 \ldots X_k)$ be the independent (predictor) variables, where:

- X_1 = time of the day;

X_1 is a predictor categorical variable that can take on three possible values: $X_1 = \{1, 2, 3\}$

Time of the day means the exact time when the purchase was made. Time of purchase has been aggregated into three categories:

- Morning (1) – [09:00 am – 13:00 am]
- Afternoon (2) – [13:00 am – 17:00 pm]
- Evening (3) – [17:00 pm – 20:00 pm]

- X_2 = season;

X_2 is a predictor categorical variable that can take on four possible values: $X_2 = \{1, 2, 3, 4\}$

The values 1 to 4 are the months of the year aggregated to seasons as it follows:

- Spring (1) – [March, April, May]
- Summer (2) – [June, July, August]
- Autumn (3) – [September, October, November]
- Winter (4) – [December, January, February]

Let $\beta_j = (\beta_1, \beta_2, \ldots, \beta_j)$ be the regression coefficients.

The linear equation is expressed in (3):

$$Y_j = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_i \qquad (3)$$

The probability of the response variable taking a certain value (a certain event occurring) can also be expressed as in (4) and (5):

$$\Pr(Y = j) = \frac{e^{y}}{1 + e^{-y}} \qquad (4)$$

$$\Pr(Y_i = j) = \frac{e^{(x_i \beta_j)}}{\sum e^{(x_i \beta_j)}} \qquad (5)$$

Pr (Y_i= j) is the probability of belonging to group j;

X_i is a vector of explanatory variables;

β_j are the regression coefficients estimated using the maximum likelihood estimation.

Chapter 5
Data analysis and results

5.1 Market Basket Analysis

Shopping basket analysis is an efficient tool that helps in analysing purchase transactions and identifying sales opportunities. The execution of shopping basket analysis was done using SQL Server Data Mining Add-in for Excel. For the rule generating procedure was used the basic support-confidence framework with the Apriori algorithm. The criteria for support and confidence for executing market basket analysis were pre-determined by the researcher. The minimum support was set to 0.1 (10%) and the minimum confidence was set to 0.4 (40%). The scanner data from the stores was aggregated and market basket analysis was run individually for each store.

The employed model analyses data to find out which items frequently appear together in transactions. This procedure requires limited amount of variables –one for the Transaction ID and one for the Product ID (Product Name). This is why this model is highly preferred in the case of market basket data.

Theoretically, every association rule is characterised by [support, confidence]. Nevertheless, in SQL Server Data Mining Add-in for Excel, each rule has a [probability, importance] measure. Association rules table shows the percentages of association between various items in the itemsets. The generated association rules in this table are characterised by two measures: probability and importance.

Probabilities are the chance that a consumer will purchase the consequent (Shampoo) if he has purchased certain products (Hair mask, Hair conditioner). In other words, probability is simply the confidence of a rule.

The importance of a rule is a measure of the likelihood that a rule head will appear together with a rule body (for the formula see Table 1, Chapter 4.1). If the importance is a positive number, then the rule head is more likely to appear with the rule body together in a transaction than without. A positive importance score means that the probability of the rule head goes up when the rule body is true.

The dependency network provides a graphical representation of the relations between items. It simply shows how frequently purchased items are linked together and which product depends on which other.

Results for Store №1

Table 5.1
Association rules for Store №1

Probability	Importance	Rule
92 %	1.40	Face cream night, Hand cream -> Face cream day
79 %	0.75	Shower cream, Hair conditioner -> Shampoo
73 %	0.71	Shower gel, Hair conditioner -> Shampoo
67 %	0.68	Hair mask, Hair conditioner -> Shampoo
67 %	0.67	Face cream day, Hair conditioner -> Shampoo
61 %	0.65	Hair conditioner, Toothpaste -> Shampoo
57 %	0.68	Mouthwash, Toothbrushes -> Toothpaste
57 %	0.61	Hair conditioner, Universal face cream -> Shampoo
53 %	1.23	Brushes -> Make-up eyes
48 %	0.61	Cream soap, Toothbrushes -> Toothpaste
44 %	0.56	Hair conditioner -> Shampoo
44 %	0.51	Hair conditioner, Hair Dye -> Shampoo
41 %	0.55	Bar soap, Shampoo -> Toothpaste
40 %	0.54	Hand cream, Toothbrushes -> Toothpaste
40 %	0.53	Wet wipes, Toothbrushes -> Toothpaste

Table 5.1 provides association rules generated from the aggregated data for Store №1.

Products with high affinity to be sold together.

- It can be seen that if a customer buys a product from category [Face cream night] and [Hand cream], the probability that he will also buy [Face cream day] in the same visit is 92%.
- If a customer purchases [Shower cream] and [Hair conditioner] together, there is 79% probability that he will also purchase [Shampoo].
- If a consumer buys [Shower gel] and [Hair conditioner], the observed probability that he will purchase [Shampoo] as well is 73%.
- If a customer purchases [Hair mask] and [Hair conditioner] together, there is 67% probability that he will also purchase [Shampoo].
- In more than half of the cases (53%) when consumers bought [Brushes], they also bought [Make-up eyes].

Retailers can exploit these associations by incorporating them into promotional strategies. These rules can be also used as a guideline for product recommendations in the e-commerce site. Every time a consumer buys a product from category [Face cream night] the system will automatically suggest that he may want to buy a product from category [Hand

cream] and [Face cream day] as well. Analogically, when a customer buy products from category [Brushes], he may also want to buy products from category [Make-up eyes].
The extracted association rules will also help to improve the in-store settings. Placing hand creams next to face creams will ease consumers in their choice while reminding them what else they need to buy. Placing brushes next to make-up for eyes will also provide benefits in terms of increased sales of products from both categories.

Most frequently purchased products in bundles in Store 1: (see more in Table 1, App. B)

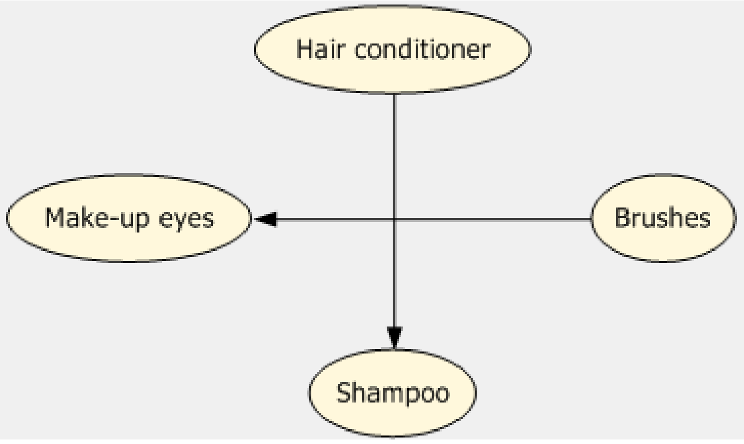
[Hair conditioner] and [Shampoo] – 253 bundles
[Toothbrush] and [Toothpaste] – 246 bundles
[Toothpaste] and [Shampoo] – 205 bundles
[Shampoo] and [Hair Dye] – 166 bundles
[Face cream night] and [Face cream Day] – 160 bundles
[Universal face cream] and [Shampoo] – 134 bundles
[Universal face cream] and [Toothpaste] – 108 bundles
[Toothbrush] and [Shampoo] – 108 bundles
[Baby care] and [Cream Zdrave] – 102 bundles
[Mouthwash] and [Toothpaste] – 100 bundles

Table 5.2
Shopping basket recommendations for Store №1

Selected Item	Recommendation	Sales of Selected Items	Linked Sales	% of linked sales	Average value of recommendation	Overall value of linked sales
Hair conditioner	Shampoo	611	253	41.41 %	1.09	664
Brushes	Make-up eyes	34	18	52.94 %	2.85	97

The shopping basket recommendations report shows how items are related and provides recommendations that would be beneficial for the retailer. Each association rule has supporting statistics that help evaluate its potential strength so that if a rule exceeds certain probability threshold then it can be taken into account. In this case, the recommendation report suggests that selling products from categories{ [Hair conditioner] and [Shampoo]} and {[Brushes] and [Make-up eyes]} together will increase sales of both items.

Table 5.3
Dependency network for Store №1.

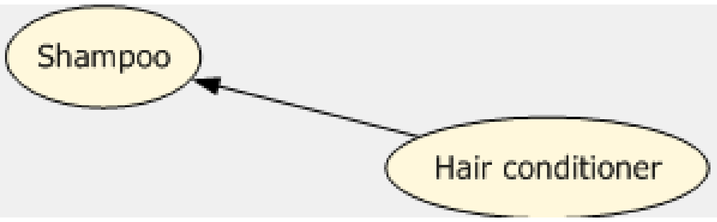


Results for Store №2

Table 5.4
Association rules for Store №2

Probability	Importance	Rule
43 %	0.52	Hair conditioner -> Shampoo

Table 5.5
Dependency network for Store №2



- The data mining algorithm generated only one rule. If a consumer buys [Hair conditioner], the probability that he will buy [Shampoo] as well is 43%. This is quite a trivial rule which does not provide us with insightful and actionable information. It is quite obvious that the store was highly unprofitable. It worked for a total of 5 months and the number of transactions available is the lowest. Table 4 and 8 (Appendix A) graphically represent the sales data.

- The results for Store №2 are not surprising due to the fact that the store was closed because of a low turnover. The reason for generating only one association rule is because of the minimum support threshold. In all the analyses the minimum support condition was set to 0.4. Rules generated below this threshold are not considered common. As the average items per sale are around one, we are only interested with rules with high supports and confidences. Table 5 (Appendix B) shows that the highest number of bundled sales is 20 which is nothing compared to the bundled sales in Store 1.

Table 5.6
Shopping basket recommendations for Store №2

Selected Item	Recommendation	Sales of Selected Items	Linked Sales	% of linked sales	Average value of recommendation	Overall value of linked sales
Hair conditioner	Shampoo	38	20	52.63 %	1.72	65

The only recommendation that could be generated for **Store №2** is again selling [Shampoo] and [Hair conditioner] together. However, the store is already closed and this is not an actionable information for the retailer.

Results for Store №3

No rules found -> No Dependency Network

- The purpose of the analysis is to find rules that satisfy the conditions generated by the researcher. In this case the minimum support threshold is 10% and the minimum confidence threshold is 40%. However, no association rules could be generated with aggregate scanner data for Store №3. Although there were a decent number of transactions available, no specific rules were found. This is mainly due to the predefined support and confidence thresholds. Although there are enough transactions for market basket analysis, customers probably do not purchase many items together and those purchased together, do not appear as often together as we are interested.

- Table 6 (Appendix B) shows most popular bundles of products. They are quite similar to the bundles in Store 1, but the number of sales here is significantly lower. This is the reason why for the given levels of support and confidence no association rules could be found.

- Another reason for not finding any rules is that the store is newly opened and is slowly increasing turnovers due to the non-central location of the store.

- Obviously, the trivial products that often sell together are purchased frequently in bundles of 2.

  - [Hair conditioner] and [Shampoo]
  - [Toothpaste] and [Shampoo]
  - [Toothbrushes] and [Toothpaste]
  - [Hair Dye] and [Shampoo]
  - [Hair mask] and [Shampoo]
  - [Make-up lips] and [Make-up eyes]
  - [Shower cream] and [Shampoo]

  See more on Table 6 (Appendix B).

Table 5.7
Shopping basket recommendations for Store №3

Selected Item	Recommendation	Sales of Selected Items	Linked Sales	% of linked sales	Average value of recommendation	Overall value of linked sales
Brushes	Make-up eyes	25	10	40.00 %	1.88	47

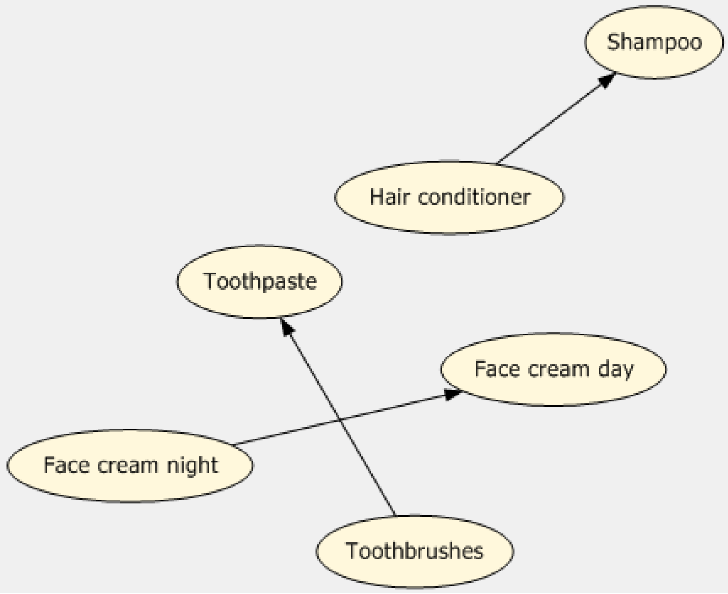
- The recommendation for the retailer is to bundle [Brushes] and [Make-up eyes] since they sell together in 40% of the cases.

Results for Store №4

Table 5.8
Association rules for Store №4

Probability	Importance	Rule
67 %	0.94	Hair conditioner -> Shampoo
56 %	0.95	Face cream night -> Face cream day
46 %	0.82	Toothbrushes -> Toothpaste

Table 5.9
Dependency network for Store №4



Results for Store №4

- Store №4 is the last one opened from the cosmetic chain, but the results from recommendations are surprisingly actionable. The interestingness of the results is due to the fact that the store is located in a city, which is famous for tourism and relaxation activities.

The generated rules do not differ much from the rules generated for Stores 1 and 2. (Table 5.8)

- If a consumer purchases [Hair conditioner] he will purchase [Shampoo] as well with 67% probability.
- If a consumer purchases [Face cream night], there is 56% probability that he will purchase [Face cream day] as well.
- If a consumer purchases [Toothbrushes], the chance that he will also purchase [Toothpaste] is 46%.

However, in this case, pure sales numbers tell us more information that can be quite actionable.

- Every time (100%), a consumer bought [Aftershave lotion], he also bought [Shampoo]. In 83% of the cases when a consumer bought [Aftershave lotion] he also bought [Toothpaste]. This is a good opportunity for the retailer to promote products from these categories.

- In around 80-90% of the cases when [Shampoo] was purchased, products from one of the following categories were also purchased:
    - [Anti-age care]
    - [Hair dye]
    - [Hair conditioner]
    - [Shower gel]
    - [Body lotion]
    - [Anti-wrinkle set]
- This is the only store where [Jellewery] was bought quite often. It is associated with almost every other cosmetic product. It is apparent that tourists in the resort buy [Jewellery] almost every time they go to the store. It is commonly associated with: [Make-up lips],[Nail polish],[Wet wipes], [Face cream day],[Shampoo],[Hair dye], [Toothpaste].

  The retailer can use this information for promoting [Jellewery] with items from some of these product categories. [Jellewery] is one of the most expensive product categories in the store, so it would be a good idea for the retailer to increase prices of jellewery and start selling more and different varieties of it. As consumers will have more options to choose from , they will stay longer in the store, which increases the likelihood that they will buy something else.

  A perfect location for [Jellewery] would be nearby [Nail Polish] and all kinds of [Make-up]. Women tend to match jellewery with nail polish and make-up, so this in-store placement would be a reminder for ladies that they need to buy the same colour of nail polish and lipstick for the new necklace, for example.

5.2 Multinomial logistic regression.

The purpose of this model-based approach is to test how different seasons and times of the day affect the probabilities of purchasing items from certain product categories. It is important to note that in order to investigate seasonal effects, we need yearly sales data. The only store that has data available for one whole year is Store №1. This is why multinomial logistic regression will be run only for Store №1. Multinomial logit analysis was executed in IBM SPSS Statistics software.

Model №1

The purpose of this model is to investigate how different seasons and times of the day affect purchases of different kinds of make up. In other words we want to see when consumers are more likely to purchase lipstick than font de teint. The analysis will provide valuable information to the retailer so he can adjust promotional activities accordingly.

The response variable can take on three possible outcomes:
Y = {make-up for eyes, make-up for lips, make-up for skin}
Independent variables are time of purchase and season.

Time of purchase has been aggregated to three categories:

- Morning (1) – [09:00 am – 13:00 am]
- Afternoon (2) – [13:00 am – 17:00 pm]
- Evening (3) – [17:00 pm – 20:00 pm]

Months have been aggregated to seasons:

- Spring (1) – [March, April, May]
- Summer (2) – [June, July, August]
- Autumn (3) – [September, October, November]
- Winter (4) – [December, January, February]

The baseline (reference) category was set to be [Make-up for skin].

- Make-up for eyes consists of all the brands and types of eye shadows, eye liners, eye pencils, eyebrow pencils etc.
- Make-up for lips consists of all the brands and types of lip liners, lipsticks, lip-glosses and lip-balms.
- Make-up for skin consists of all the brands and types of fond de teint, concealers, mattifying powders, bronzing powders, blushes etc.

The logit equation is : $\log \frac{\Pr(Yi)}{\Pr(Yj)} = \beta_0 + \beta_1\text{*Time} + \beta_2\text{* Season},$

where i= {make-up for eyes, make-up for lips}.

j= the reference category (make-up for skin).

Table 5.2.1 Coefficients[a]

Model	Collinearity Statistics	
	Tolerance	VIF
1 Time	1.000	1.000

a. Dependent Variable: Season

Before conducting the multinomial logit, it is of high importance to check for multicollinearity between the independent variables. As we have only two independent variables, it would be more appropriate to use the term collinearity. Running a linear regression between the two independent variables will provide us with the Variance Inflation Factor. On Table 5.2.1 we can see that the VIF value is 1.000 which means that there is no collinearity between the two variables.

Multinomial logit analysis results for Model №1

Table 5.2.2. Model Fitting Information

Model	Model Fitting Criteria	Likelihood Ratio Tests		
	-2 Log Likelihood	Chi-Square	df	Sig.
Intercept Only	154.273			
Final	125.717	28.556	10	.001

For a good model fit the -2 Log Likelihood (-2LL) value should be lower for the full model than it is for the null model with intercept only. From table 5.2.2 we can see that this is exactly the case, which indicates a good model fit. Moreover, the model fit is significant (χ^2 (10))= 28.556, $p < 0.05$) which means that the full model is outperforming the null one in predictive power.

Table 5.2.3 Goodness-of-Fit

	Chi-Square	df	Sig.
Pearson	19.118	12	.086
Deviance	19.892	12	.069

The Goodness-of-Fit table (Table 5.2.3) provides further evidence of good fit for the model. Here, both the Pearson and Deviance statistics are chi-square based methods. However, this time lack of significance is interpreted as a good fit. Both p-values are greater than the established cut-off (0.05) which shows another evidence of good model fit.

Table 5.2.4 Likelihood Ratio Tests

Effect	Model Fitting Criteria	Likelihood Ratio Tests		
	-2 Log Likelihood of Reduced Model	Chi-Square	df	Sig.
Intercept	125.717[a]	.000	0	.
Time	141.119	15.403	4	.004
Season	139.091	13.375	6	.037

The chi-square statistic is the difference in -2 log-likelihoods between the final model and a reduced model. The reduced model is formed by omitting an effect from the final model. The null hypothesis is that all parameters of that effect are 0.

a. This reduced model is equivalent to the final model because omitting the effect does not increase the degrees of freedom.

Results from table 5.2.4 tell us if each of the predictors contributes meaningfully to the full model. The statistics in this table are similar to those in the Model Fitting Information table. However, here, each element of the model is being compared to the full model. Insignificant variables can be dropped off the analysis as they do not bring any value. Luckily, in this case both independent variables are significant (p-values < 0.05).

Table 5.2.5 **Parameter Estimates**

Category aggregation[a]		B	Std. Error	Wald	df	Sig.	Exp(B)	95% Confidence Interval for Exp(B)	
								Lower Bound	Upper Bound
Make-up eyes	Intercept	1.421	.263	29.175	1	.000			
	[Time=1]	-.491	.238	4.271	1	.039	.612	.384	.975
	[Time=2]	-.458	.228	4.049	1	.044	.632	.405	.988
	[Time=3]	0[b]	.	.	0	.	.	.	.
	[Season=1]	.350	.267	1.710	1	.191	1.418	.840	2.395
	[Season=2]	.054	.302	.032	1	.857	1.056	.584	1.909
	[Season=3]	-.182	.291	.393	1	.531	.833	.471	1.474
	[Season=4]	0[b]	.	.	0	.	.	.	.
Make-up lips	Intercept	.627	.281	4.980	1	.026			
	[Time=1]	-.213	.244	.762	1	.383	.808	.500	1.304
	[Time=2]	.086	.232	.138	1	.710	1.090	.692	1.716
	[Time=3]	0[b]	.	.	0	.	.	.	.
	[Season=1]	.783	.281	7.768	1	.005	2.187	1.261	3.792
	[Season=2]	.502	.316	2.531	1	.112	1.652	.890	3.067
	[Season=3]	.401	.301	1.771	1	.183	1.493	.827	2.696
	[Season=4]	0[b]	.	.	0	.	.	.	.

a. The reference category is: Make-up skin.

b. This parameter is set to zero because it is redundant.

Table 5.2.5 provides us the estimated log-odds for choosing Make-up for eyes versus Make-up for skin and Make-up for lips versus Make-up for skin in different times of the day and seasons.

The unit of analysis is the Exp(B).It represents the change in the odds ratio associated with a one unit change in the predictor variable. If the value of Exp(B) is greater than 1, then it indicates that as the predictor increases the odds of the outcome occurring also increase. The other way around, if the Exp(B) is less than 1, as the predictor increases the odds of the outcome occurring decrease.

I. The intercept is 1.421. This is the multinomial logit estimate for the possible outcome [Make-up eyes] relative to [Make-up skin], when all the other variables are set to zero. For [Make-up lips] relative to [Make-up skin] the intercept is 0.627

II. Four of the regression coefficients in the model are insignificant:
[Make-up eyes] relative to [Make-up skin]
   - For Season=1 the p-value is 0.191
   - For Season=2 the p-value is 0.857.
   - For Season=3 the p-value is 0.531.

   [Make-up lips] relative to [Make-up skin]
   - For Time=1 the p-value is 0.383.
   - For Time=2 the p-value is 0.710.
   - For Season=1 the p-value is 0.112.
   - For Season=2 the p-value is 0.183.

The following conclusions can be inferred from the table:

- Consumers are 0.612 times less likely to purchase [Make-up for eyes] in the morning than in the evening relative to [Make-up for skin].
- Consumers are 0.632 times less likely to purchase [Make-up eyes] in the afternoon than in the evening relative to [Make-up skin]
- Consumers are 2.187 times more likely to purchase [Make-up lips] in the spring than in winter relative to [Make-up skin]

General conclusions:

- Time of the day has significant effect on the likelihood that a consumer will prefer to buy [Make-up eyes] in the morning and afternoon than in the evening relative to [Make-up skin]
- Season has a significant effect on the likelihood that a consumer will prefer to buy [Make-up lips] in spring than in winter relative to [Make-up skin]

Managerial implications

- Special offers and promotions on products from product category [Make-up lips] will have bigger effect in spring than in winter as consumers are more likely to purchase lipsticks and lip-glosses in spring than in winter compared to make-up for skin.

Model №2

The purpose of this second model is to investigate the effect of seasons and different times of the day on the likelihood that a consumer purchases products from categories [Medical Shampoo],[Hand Cream] and [Face Cream Day].

The response variable can take on three possible outcomes:

Y = {[Medical Shampoo],[Hand Cream], [Face Cream Day]}

The logit equation is : $\log \frac{\Pr(Yi)}{\Pr(Yj)} = \beta_0 + \beta_1 * \text{Time} + \beta_2 * \text{Season}$,

where i= {face cream day, hand cream}.

j= the reference category (medical shampoo).

Independent variables are time of purchase and season. Both variables are aggregated and coded as in Model №1.

The baseline (reference) category was set to be [Medical Shampoo]

Table 5.2.6 Coefficients[a]

Model		Collinearity Statistics	
		Tolerance	VIF
1	Time	1.000	1.000

a. Dependent Variable: Season

Analogically with Model №1, linear regression between the independent variables was run to check for collinearity. The VIF value is 1.000, so there is no correlation between time and season.

Table 5.2.7 Model Fitting Information

Model	Model Fitting Criteria	Likelihood Ratio Tests		
	-2 Log Likelihood	Chi-Square	df	Sig.
Intercept Only	247.068			
Final	149.395	97.673	10	.000

Table provides evidence that the full model predicts better than the null model. The model fit is significant (χ^2 (10))=97.673, $p <0.05$) which indicates a good model.

Table 5.2.8 **Goodness-of-Fit**

	Chi-Square	df	Sig.
Pearson	29.309	12	.094
Deviance	29.161	12	.089

The lack of significance (p>0.05) for both the Pearson and Deviance statistics indicates a good model fit.

Table 5.2.9 **Likelihood Ratio Tests**

Effect	Model Fitting Criteria	Likelihood Ratio Tests		
	-2 Log Likelihood of Reduced Model	Chi-Square	df	Sig.
Intercept	149.395[a]	.000	0	.
Time	173.010	23.615	4	.000
Season	222.105	72.711	6	.000

The chi-square statistic is the difference in -2 log-likelihoods between the final model and a reduced model. The reduced model is formed by omitting an effect from the final model. The null hypothesis is that all parameters of that effect are 0.

a. This reduced model is equivalent to the final model because omitting the effect does not increase the degrees of freedom.

The results from table 5.2.9 indicate whether we need to drop off some of the independent variables. However, in this case, both Time and Season are significant and therefore bring meaningful information to the analysis.

Table 5.2.10 **Parameter Estimates**

Category aggregation[a]		B	Std. Error	Wald	df	Sig.	Exp(B)	95% Confidence Interval for Exp(B)	
								Lower Bound	Upper Bound
Face cream day	Intercept	-.837	.167	25.201	1	.000			
	[Time=1]	.707	.175	16.305	1	.000	2.027	1.439	2.857
	[Time=2]	.411	.164	6.252	1	.012	1.508	1.093	2.081
	[Time=3]	0[b]	.	.	0	.	.	.	.
	[Season=1]	.431	.188	5.262	1	.022	1.538	1.065	2.222
	[Season=2]	1.053	.179	34.789	1	.000	2.867	2.021	4.069
	[Season=3]	.125	.173	.521	1	.470	1.133	.808	1.589
	[Season=4]	0[b]	.	.	0	.	.	.	.
Hand cream	Intercept	.034	.140	.059	1	.809			
	[Time=1]	.487	.157	9.655	1	.002	1.627	1.197	2.212
	[Time=2]	.021	.147	.020	1	.887	1.021	.766	1.361
	[Time=3]	0[b]	.	.	0	.	.	.	.
	[Season=1]	.313	.165	3.607	1	.058	1.367	.990	1.888
	[Season=2]	-.133	.179	.549	1	.459	.876	.616	1.244
	[Season=3]	-.114	.151	.575	1	.448	.892	.664	1.199
	[Season=4]	0[b]	.	.	0	.	.	.	.

a. The reference category is: Medical shampoo.

Table 5.2.10 provides the needed coefficients that tell us how much more or less likely a customer is to buy a certain product at a certain time.

The intercept is -0.837. This is the multinomial logit estimate for the possible outcome [Face cream day] relative to [Medical shampoo], when all the other variables are set to zero. For [Hand cream] relative to [Medical shampoo] the intercept is 0.034 and is insignificant (p-value=0.809)

Four of the regression coefficients in the model are insignificant:

[Face cream day] relative to [Medical shampoo]

- For Season=3 the p-value is 0.470.

[Hand cream] relative to [Medical shampoo]

- For Time=2 the p-value is 0.887.
- For Season=2 the p-value is 0.459
- For Season=3 the p-value is 0.448.

Looking at the Exp(B), the following statements can be inferred from table 5.2.10:

- Consumers are 2.027 times more likely to purchase [Face cream day] in the morning than in the evening relative to [Medical shampoo].
- Consumers are 1.508 times more likely to purchase [Face cream day] in the afternoon than in the evening relative to [Medical shampoo].
- Consumers are 1.538 times more likely to purchase [Face cream day] in spring than in winter relative to [Medical shampoo].
- Consumers are 2.867 times more likely to purchase [Face cream day] in summer than in winter relative to [Medical shampoo].
- Consumers are 1.627 times more likely to purchase [Hand cream] in the morning than in the evening relative to [Medical shampoo].
- Consumers are 1.367 times more likely to purchase [Face cream day] in spring than in winter relative to [Medical shampoo].

General conclusions:

1. Consumers are most likely to purchase [Face cream day] and [Hand cream] in the morning than in other times of the day relative to [Medical shampoo].
2. Consumers are most likely to purchase [Face cream day] and [Hand cream] in spring than in other seasons relative to [Medical shampoo]

Managerial implications:

- Spring is the best season for promotional activities of products from product categories [Face cream day] and [Hand cream] as customers are most likely to purchase them in spring than winter.

Model №3

In the third model purchase probabilities will be investigated for products from categories [Body lotion],[Nail polish] and [Shower cream].

The response variable can take on three possible outcomes:

Y = {[Body lotion],[Nail polish], [Shower cream].}

The logit equation is : $\log \frac{\Pr(Yi)}{\Pr(Yj)} = \beta_0 + \beta_1 * \text{Time} + \beta_2 * \text{Season}$,

where i= {body lotion, nail polish}.

j= the reference category (shower cream).

Independent variables are time of purchase and season. Both independent variables are aggregated and coded as in Model №1.

The baseline (reference) category was set to be [Shower cream]

Table 5.2.11 **Coefficients[a]**

Model	Collinearity Statistics	
	Tolerance	VIF
1 time	1.000	1.000

a. Dependent Variable: season

After running a linear regression between the independent variables, Table 5.2.11 provides the VIF value. It is again 1.000, so there is no collinearity between the predictors.

Table 5.2.12 **Model Fitting Information**

Model	Model Fitting Criteria	Likelihood Ratio Tests		
	-2 Log Likelihood	Chi-Square	df	Sig.
Intercept Only	174.087			
Final	125.236	48.851	10	.000

The likelihood ratio test is significant ($\chi^2 (10) = 48.851$, $p < 0.005$) , which means that the model perfectly fits the data.

Table 5.2.13 Goodness-of-Fit

	Chi-Square	df	Sig.
Pearson	16.479	12	.170
Deviance	16.769	12	.158

Both the Pearson and Deviance statistics on Table 5.2.13 are greater than 0.05, which in this case means that the model adequately fits the data.

Table 5.2.14 Likelihood Ratio Tests

Effect	Model Fitting Criteria	Likelihood Ratio Tests		
	-2 Log Likelihood of Reduced Model	Chi-Square	df	Sig.
Intercept	125.236[a]	.000	0	.
season	170.580	45.344	6	.000
time	128.433	3.196	4	.086

The chi-square statistic is the difference in -2 log-likelihoods between the final model and a reduced model. The reduced model is formed by omitting an effect from the final model. The null hypothesis is that all parameters of that effect are 0.

a. This reduced model is equivalent to the final model because omitting the effect does not increase the degrees of freedom.

The information on Table 5.2.14 tells us if we need to drop off any of the predictors. In this case, the p-value of time is greater than the established cut-off (0.05) so it needs to be ignored. Season is a significant variable (p-value = 0.000) and provides meaningful information to the analysis.

Table 5.2.15 **Parameter Estimates**

Category agg[a]		B	Std. Error	Wald	df	Sig.	Exp(B)	95% Confidence Interval for Exp(B)	
								Lower Bound	Upper Bound
Body lotion	Intercept	-.580	.230	6.379	1	.012			
	[season=1]	.220	.244	.810	1	.368	1.246	.772	2.010
	[season=2]	.416	.230	3.268	1	.071	1.516	.966	2.381
	[season=3]	.151	.257	.345	1	.557	1.163	.703	1.924
	[season=4]	0[b]	.	.	0	.	.	.	.
	[time=1]	.292	.210	1.936	1	.164	1.339	.887	2.022
	[time=2]	.112	.199	.317	1	.573	1.119	.757	1.653
	[time=3]	0[b]	.	.	0	.	.	.	.
Nail polish	Intercept	-.665	.231	8.306	1	.004			
	[season=1]	1.314	.237	30.796	1	.000	3.720	2.339	5.916
	[season=2]	.935	.235	15.838	1	.000	2.548	1.608	4.040
	[season=3]	.483	.262	3.382	1	.066	1.620	.969	2.710
	[season=4]	0[b]	.	.	0	.	.	.	.
	[time=1]	-.027	.194	.020	1	.888	.973	.665	1.424
	[time=2]	.037	.178	.042	1	.837	1.037	.731	1.471
	[time=3]	0[b]	.	.	0	.	.	.	.

a. The reference category is: Shower cream.

b. This parameter is set to zero because it is redundant.

Purchase likelihoods can be inferred from the coefficients on Table 5.2.15.
The intercept is -0.580. This is the multinomial logit estimate for the possible outcome [Body lotion] relative to [Shower cream], when all the other variables are set to zero. For [Nail polish] relative to [Shower cream] the intercept is – 0.665. According to the results in table 5.2.14 time is an insignificant predictor in this model so the coefficients for time need to be neglected.

Four of the rest of the regression coefficients in the model are insignificant:
[Body lotion] relative to [Shower cream]

- For Season=1,2 and 3, the p-value > 0.05

[Nail polish] relative to [Shower cream]

- For Season=3, the p-value is 0.066.
- For Season=2 the p-value is 0.459

- For Season=3 the p-value is 0.448.

Looking at the Exp(B), the following statements can be inferred from table 5.2.15:

- Consumers are 3.720 times more likely to purchase [Nail polish] in spring than in winter relative to [Shower cream].
- Consumers are 2.548 times more likely to purchase [Nail polish] in summer than in winter relative to [Shower cream].

General conclusions:

- Season has significant effect on the likelihood of purchasing products from product category [Nail polish] relative to [Shower cream]
- In spring customers are most likely to purchase products from product category [Nail polish] than in any other season relative to [Shower cream]

Managerial implications:

- Promoting [Nail polish] in spring may have positive effect on sales as customers have highest likelihood to purchase products from this category in spring.

Chapter 6

Conclusions

6.1 General Discussion

Market basket analysis is a very useful technique for finding out co-occurring items in consumers shopping baskets. Such information can be used as a basis for decisions about marketing activity such as promotional support, inventory control and cross-sale campaigns. Tracking not so apparent product affinities and leveraging on them is often seen as a real challenge in the retail business.

Even though most of the generated rules are somewhat predictable for a cosmetic store, they still provide value to the retailer. The problem with trivial rules is often found in the marketing literature, but solely depends on the size and type of store. In this research, the stores of the cosmetic chain are rather small ones and the number of transactions is not as big as the number in a big hypermarket, for example. Moreover, the assortment is somewhat limited due to the fact that the stores represent and sell mainly products of a certain cosmetic company. Therefore, it is a bit difficult to mine unusual and interesting rules. However, it is important for the retailer to know exactly which products are purchased together and in what time of the year. The generated rules may not be unusual and interesting, but they are useful and actionable.

6.2. Academic contribution

The market basket problem can be seen as the best example of mining association rules. Discovering association rules has been a well-studied area for the past decade. Building up on previous researches by using established methods for mining association rules allowed for discovering useful information for the retailer. After aggregating the data and finding product affinities, the multinomial logistic regression extends the analysis by adding up some probabilities of a consumer purchasing certain products in different seasons and in certain times of the day. Evaluating probabilities of a category membership depending on the two factors – season and time of the day provides the retailer with better understanding of consumers' needs and suggests action for advertising.

Overall overview of contributions

The contributions of this thesis are as follows:

1. Products purchased in bundles of 2 and 3 were found for all the four stores of the cosmetic chain.

2. Association rules were generated with the supporting probabilities and importance.

3. Dependency networks are used to visually represent the product interrelationships.

4. Average values per sale and overall value of bundle were estimated for every store of the chain (see Appendix B).

5. The multinomial logistic regression provides a model for predicting the likelihood of a consumer purchasing an item from a certain product category at a specific time of the day and in a specific season.

6. The multinomial logit also compares the likelihood of choosing a product out of several options.

6.3. Managerial Implications

In the recent years, more and more retailers are seeking competitive edge through advanced and innovative technology. Market basket analysis is the next step in the retail evolution. Applications of association rule mining are growing rapidly in different sectors – from analysing debit and credit card purchases to fraud detections.

Mining into big data provides managers with a unique window into what is happening with ones business so that they can implement strategies efficiently. Obscure patterns can be discovered using market basket analysis which can help for planning more effective marketing efforts. It can be used not only for cross-sale and up-sale campaigns, but for managing better inventory control and satisfying shoppers' needs. Almost all departments of a company can benefit from a single analysis – not only the high levels of Management but also Store operations, Merchandising and Advertising and Promotion departments.

6.4 Limitations of the study and directions for future research.

Although market basket analysis is computationally simple and very efficient, there are several important limitations. Since there is not available data on households and individual consumers, interdependencies across purchases of individual consumers or households are neglected. In other words, due to data restrictions, homogeneity across purchases is assumed.

Individual household level data would be very beneficial for the analysis. A combination of insights on product dependencies and household level data could help retailers in better pricing and promotion decisions for different customer segments.

An area of interest for the researcher might be to investigate sequences of purchases and events concerning the customer. Although sequential time series analysis would be an appropriate technique to use, anonymous transactions do not unveil information on consumer behaviour.

Availability of household data would be very beneficial for the second research question. A model-based approach can be used for prediction and forecasting. Running a multinomial logistic regression with more independent variables would be very effective to better predict product choice.

Having non-anonymous household data also allows for applying techniques like clustering, decision trees or artificial neural networks that could provide more insightful information on the consumers and their preferences.

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Appendix A: Data description

Table 1: Items before aggregation

List of items before aggregation		
Fond de teint long stay	Fond de teint mattifying	Fond de teint mousse
Fond de teint natural	After sun milk	After sun spray
Aftershave balm active	Aftershave balm sensitive	Aftershave lotion active
Aftershave lotion sensitive	Almond oil	Anti-age oil for dry skin
Anti-age oil for mixed skin	Anti-age oil for sensitive skin	Anti-cellulite massage oil
Anti-wrinkles oil	Apricot oil	Argan oil
Avocado oil	Baby Oil	Barrette
Bath foam baby	Body lotion calming	Body lotion moisturising
Body lotion nourishing	Body massage oil relaxing	Body milk organic olive oil
Body milk Q10+	Bouquet Lux 80ml	Bracelet
Bronzing Powder	Brush	Brush liquid eyeliner
Brush smoky eyes	Claw clip	Cleansing face gel anti-acne
Cleansing face lotion	Cleansing face milk	Cleansing tonic
Coconut oil	Colourful rouge	Colourful Rouge
Compact powder	Concealer 3 in 1	Concealer 5 in 1
Concealer stick	Concealer with brush	Cosmetic bag
Cream against skin irritation baby	Cream Aroma A+E 75 ml.	Cream Aroma Q10+ around eyes
Cream Dunavski Vulni 45 ml.	Cream eye-contour Collagen + Omega-3	Cream Greenline Calming Calendula
Cream Greenline Healing Aloe	Cream Greenline Universal Jojoba	Cream Greenline Universal Rice
Cream Happy Baby Protective	Cream Hyaluron + Retinol eye-contour	Cream Medico Ideal
Cream Zdrave	Cream Zdrave Baby	Cream Zdrave Light
Cream Zdrave universal	Creamy rouge	Cream Zdrave Forte for atopic skin
Cream Zdrave Forte for heels and elbows	Day cream age control olive oil	Day Cream Aroma Q10+
Day cream Aroma Q10+ Very Dry Skin	Day Cream Collagen + Omega-3	Day cream Hyaluron + Retinol
Dental floss	Deodorant men earth	Deodorant men fire
Deodorant men ocean	Deodorant men wind	Depilatory cream
Earrings	Eye pencil	Eye pencil automatic
Eye pencil long staying	Eye pencil smoky eyes	Eyebrow pencil
Eyebrow set	Eyelashes	Eyeliner
Eyeliner gel	Eye shadow	Eye shadow applicators
Eye shadow base	Eye shadow brush	Eye shadow long staying
Face cream almond	Face cream calendula	Face cream cucumber
Face cream honey	Face cream lemon	Face gel anti-acne
Face lotion acne stop	Fake nails	Feet cream menthol
Font de teint	Font de teint last finish	Font de teint long staying
Font de teint mattifying	Font de teint mousse	Font de teint wake me up
French manicure pencil	French manicure strips	Grape seed oil
Hair band	Hair clip	Hair clips
Hair conditioner aloe milk all	Hair conditioner calendula	Hair conditioner coloured hair

hair types	normal hair	pomegranate
Hair conditioner damaged hair olive oil	Hair conditioner milk almond damager hair	Hair conditioner milk avocado dry hair
Hair conditioner milk honey thin weak hair	Hair conditioner restructuring Q10 and bamboo	Hair Dye
Hair dye professional colour	Hair elastics	Hair growth stimulant
Hair mask coloured hair pomegranate	Hair mask damaged hair olive oil	Hair mask restructuring Q10 and bamboo
Hairspray mega strong	Hairspray ultra strong	Hand cream cherry
Hand cream melon	Hand cream Q10	Hand cream water lily
Headband	Henna	Intimate gel aloe
Intimate lotion calendula	Intimate lotion camomile	Jasmine Lily
Jojoba oil	Kids set	Lip balm
Lip balm caring	Lip balm cherry	Lip balm melon
Lip balm raspberry	Lip balm strawberry	Lip gloss
Lip gloss colour changing	Lip gloss long staying	Lip gloss shine
Lip liner	Lip liner automatic	Lipstick
Lipstick Ultimate Colour	Lipstick Ultimate Shine	Liquid eyeliner
Liquid eyeliner waterproof	Liquid hand soap antibacterial	Liquid hand soap juicy fruits
Liquid hand soap lemon grass	Liquid hand soap moisturising	Liquid hand soap sensitive
Liquid hand soap Shea butter	Liquid hand soap softening	Liquid nail quick dry
Local gel anti-acne	Macadamia oil	Make-up base
Mascara eyelashes eyebrows	Mascara multifunctional	Mascara volume
Mascara waterproof	Massaging oil rose	Mattifying day cream
Moisturising face cream cucumber	Mosaic powder	Mouthwash active+total
Mouthwash extreme power white	Mouthwash kids	Mouthwash parodont active
Mouthwash total 12 night repair	Nail art paint	Nail hardener
Nail polish colour	Nail polish fast dry	Nail polish french manicure
Nail polish magnet	Nail polish nudes	Nail polish remover
Nail polish top coat	Nail polish top quick dry	Nail polish whitening
Nail strengthener	Nail strengthening butter	Nail tattoo stickers
Necklace	Night cream age control olive oil	Night cream collagen + omega-3
Night cream hyaluron + retinol	Night cream Q10+ very dry skin	Nourishing day cream
Nourishing face cream honey	Olive oil	Orange Jasmine
Palette eye shadows	Peach oil	Pencil sharpener
Powder	Powder bronzing	Powder brush
Powder mattifying	Powder shimmering	Protective cream Zdrave baby
Protective face cream lemon	Regenerating face cream almond	Regenerating night cream
Revitalising face cream avocado	Ring	Rose Water
Rouge	Rouge brush	Serum collagen + omega-3
Sesame oil	Set	Set After shave lotion Viking Active + Deodorant Earth
Set After shave lotion Viking Active + Deodorant Fire	Set After shave lotion Viking Active + Deodorant Wind	Set Aroma Collagen+ Omega3 + gift
Set Aroma Hyaluron + Retinol	Set Aroma Organic cream	Set liquid eyeliner eye shadow

	night+day+milk	
Set smoky eyes	Set soaps Aroma Vital 2+1 - Antibacterial, Nourishing, Exfoliating	Set soaps Aroma Vital 2+1 - Moisturising, Softening, Protective
Set Toothpaste Astera Active + Propolis Herbal	Set Toothpaste Astera Total 12 Night Repair 75 ml. + Toothbrush Astera 3	Set Toothpaste Astera Total 12 Renamel 75 ml. + Toothbrush Astera 3
Set Toothpaste Astera Total 12 Whitening 75 ml. + Toothbrush Astera 3	Set Toothpaste Astera+ Herbal + Toothbrush Astera 3 medium	Set Viking mix - Deodorant Wind + Shampoo/ Shower Gel
Shampoo 2 in 1	Shampoo aloe milk all hair types	Shampoo aloe natural
Shampoo anti-dandruff all hair types	Shampoo anti-dandruff energising	Shampoo anti-dandruff greasy hair
Shampoo anti-dandruff sensitive scalp	Shampoo anti-dandruff strengthening	Shampoo anti-dandruff tea tree
Shampoo anti-dandruff thin hair	Shampoo baby	Shampoo baby no tears
Shampoo calendula normal hair	Shampoo coloured hair pomegranate	Shampoo damaged hair olive oil
Shampoo egg damaged hair	Shampoo egg natural	Shampoo forte Anti hair loss
Shampoo forte seborrhoea	Shampoo forte anti-dandruff	Shampoo green apple natural
Shampoo herbal natural	Shampoo melon	Shampoo men 2 in 1
Shampoo men anti-dandruff	Shampoo men anti-hair loss	Shampoo milk almond damaged hair
Shampoo milk avocado dry hair	Shampoo milk honey thin weak hair	Shampoo nettle greasy hair
Shampoo nettle natural	Shampoo normal hair citrus	Shampoo restructuring Q10 and bamboo
Shaving cream active	Shaving cream sensitive	Shaving foam active
Shaving foam sensitive	Shea butter	Short necklace
Shower cream banana strawberry	Shower cream camu-camu	Shower cream vanilla fig
Shower gel aqua	Shower gel black orchid	Shower gel calming aloe
Shower gel edelweiss	Shower gel exfoliating bamboo	Shower gel freesia
Shower gel green apple	Shower gel men energising	Shower gel men moisturising
Shower gel men nourishing	Shower gel men regenerating	Shower gel moisturising cotton
Shower gel peach	Shower gel pomegranate mango	Shower gel raspberry
Shower gel refreshing mint	Shower gel revitalising grapefruit	Shower gel silk proteins
Smokey eyes set	Soap aloe	Soap antibacterial
Soap aqua natural	Soap baby	Soap balancing coconut oil
Soap cherry natural	Soap energising bergamot oil	Soap exfoliating
Soap healthy baby	Soap lilac	Soap massaging Shea butter
Soap melon natural	Soap moisturising	Soap nourishing
Soap pink orchid	Soap protective	Soap red fruits natural
Soap relaxing lavender oil	Soap softening	Soap stimulating cacao butter
Soap water lily	Sphere	Strengthening nail oil
Styling foam mega strong	Styling foam ultra strong	Styling gel extra strong hold
Styling gel wet look	Sun milk kids SPF 30	Sun milk SPF 10
Sun spray SPF 15	Sun spray SPF 25	Teeth floss Astera
Toothbrush Astera Active 3 Hard Mix	Toothbrush Astera Active 3 Medium Mix	Toothbrush Astera Active 3 Soft Mix

Toothbrush Astera Active Clean 1+1	Toothbrush Astera Excel 6 Mix	Toothbrush Astera Flex Active 1+1 with hanger
Toothbrush Astera Kids Mix	Toothbrush Astera Parodont Control Mix	Toothbrush Astera Power White Mix
Toothbrush Astera Twister Mix	Toothbrush Time Index 2/1 Mix	Toothbrush Time Index Mix
Toothpaste arctic fresh	Toothpaste baking soda	Toothpaste calcium
Toothpaste caries protection	Toothpaste cavity protection	Toothpaste deep clean
Toothpaste extra fresh	Toothpaste family care	Toothpaste for smokers
Toothpaste fructo micro granules	Toothpaste fructo whitening	Toothpaste herbal care
Toothpaste kids apple	Toothpaste kids ice cream	Toothpaste kids strawberry
Toothpaste micro granules	Toothpaste parodont active	Toothpaste parodont active herbal
Toothpaste parodont active sensitive	Toothpaste parodont protection	Toothpaste parodont white
Toothpaste plague removal	Toothpaste power white	Toothpaste propolis gold
Toothpaste re-white now	Toothpaste re-white sensitive	Toothpaste re-white white and bright
Toothpaste sensitive	Toothpaste strong enamel	Toothpaste total
Toothpaste vitamin 3	Toothpaste white and fresh	Toothpaste whitening
Wet wipes auto care	Wet wipes baby care	Wet wipes blueberry
Wet wipes body	Wet wipes cherry	Wet wipes face
Wet wipes hands	Wet wipes kids	Wet wipes kitchen and bathroom
Wet wipes orchid	Wet wipes white tea	Wheat germ oil

Table 2 List of items after aggregation

Aftershave balm	Aftershave lotion	Anti-acne
Anti-age face care	Anti-age face cream	Anti-age oil
Anti-age set	Anti-wrinkle face care	Anti-wrinkle set
Baby care	Bar soap	Body lotion
Body milk	Brushes	Cleansing face
Cosmetic bag	Cream soap	Cream Zdrave
Dental floss	Dental Floss	Deo men
Face cream day	Face cream night	Face set
Foot care	Glycerine soap	Hair accessories
Hair conditioner	Hair Dye	Hair Dye Prof.
Hair mask	Hair styling	Hand cream
Henna	Intimate care	Jewellery
Kids set	Lip care	Liquid soap
Make-up blush	Make-up eyes	Make-up lips
Make-up skin	Massaging oil	Medical shampoo
Men set	Mouthwash	Nail care
Nail polish	Natural oils	Other
Rose water	Shampoo	Shampoo men
Shaving cream	Shaving foam	Shaving women
Shower cream	Shower gel	Shower gel men
Soap set	Sun care	Toothbrushes
Toothpaste	Toothpaste set	Universal face cream

Wet wipes		

Table 3
Store № 1 – Monthly sales in units for Store №1

Table 4
Store № 2 – Monthly sales in units for Store №2

Table 5
Store № 3 – Monthly sales in units for Store №3

Table 6
Store № 4 – Monthly sales in units for Store №4

Table 7
Monthly revenue for Store №1

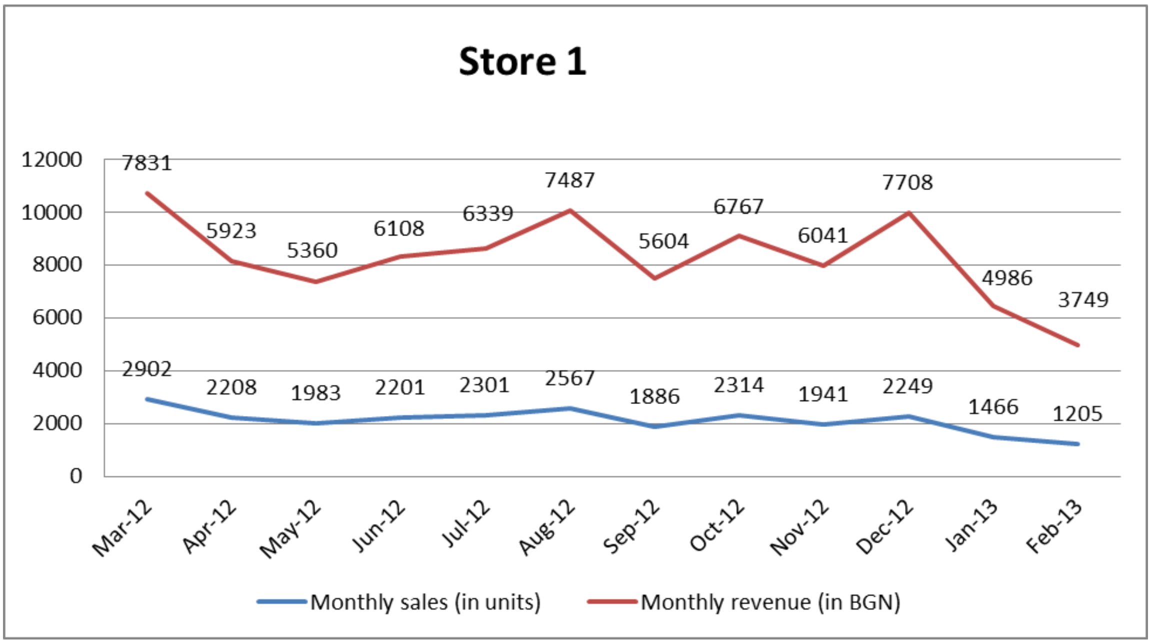


Table 8
Monthly revenue for Store №2

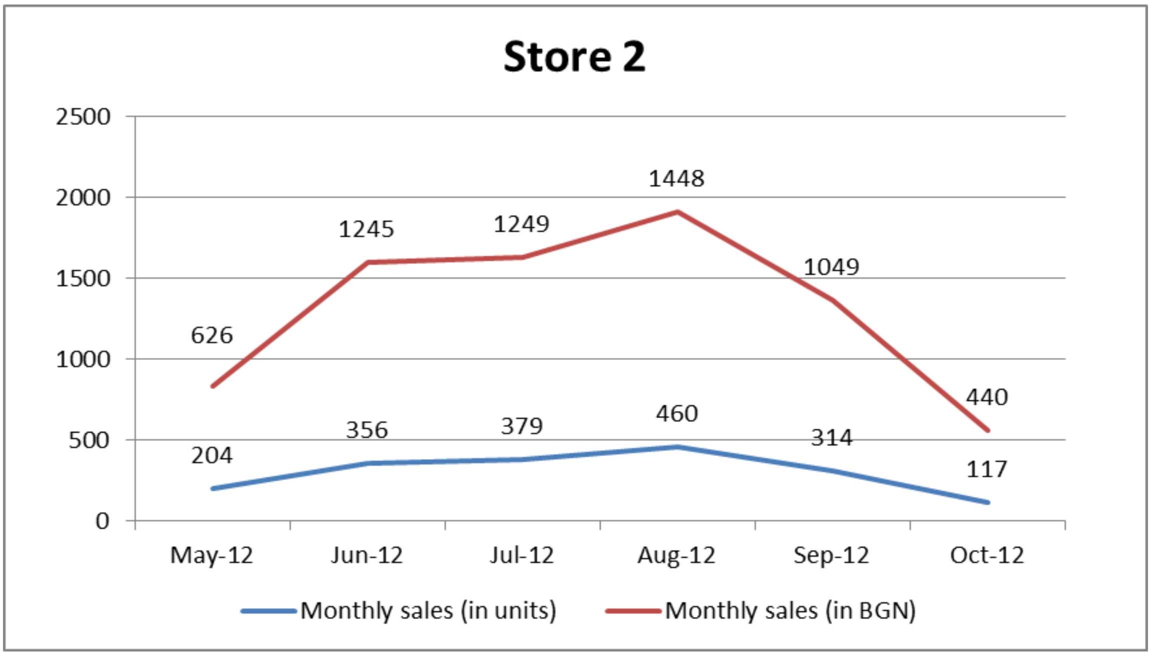


Table 9
Monthly revenue for Store №3

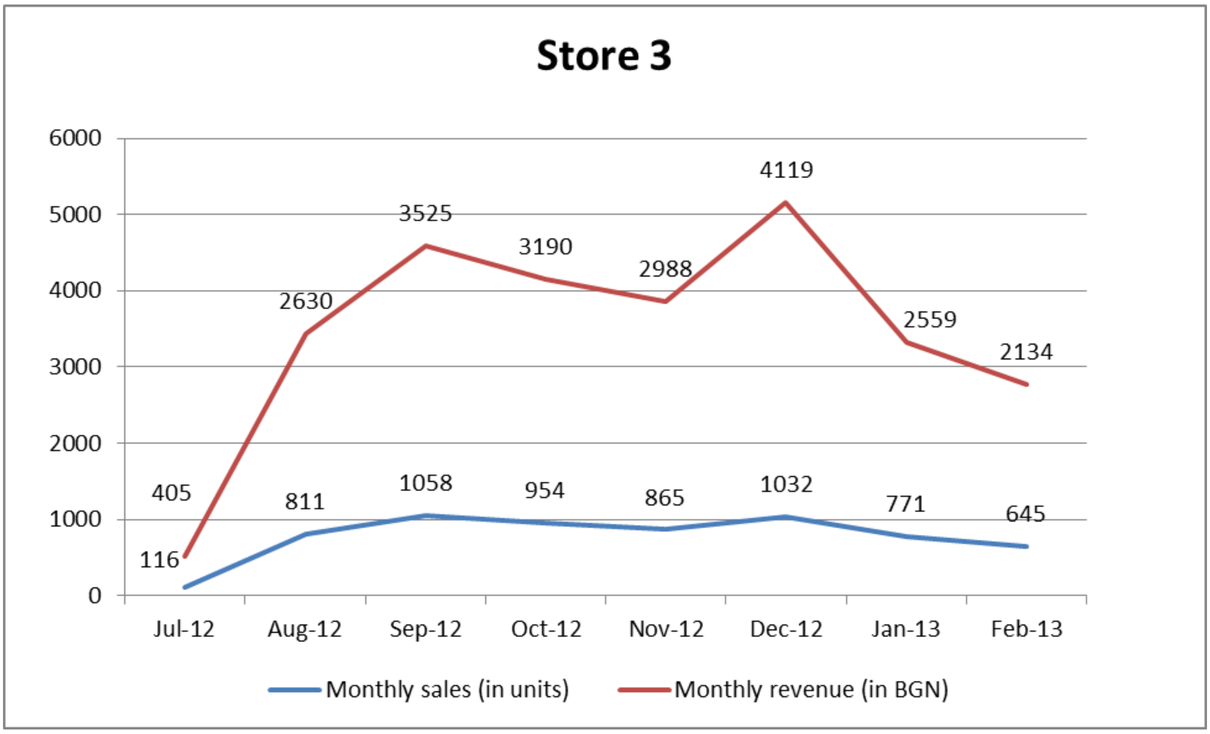
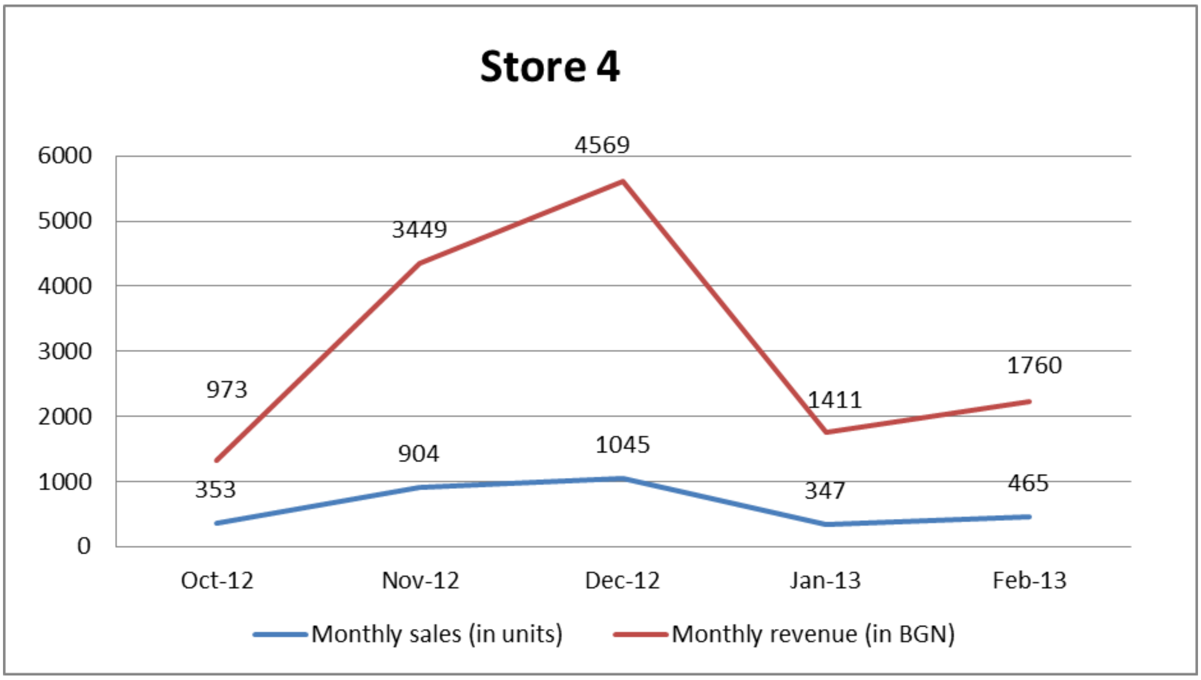


Table 10
Monthly revenue for Store №4



Appendix B Market Basket Analysis results

Table 1

Store 1 – Shopping basket bundled items – bundle size 2

Bundle of items	Bundle size	Number of sales	Average Value Per Sale	Overall value of Bundle
Hair conditioner, Shampoo	2	253	5.15	1302
Toothbrushes, Toothpaste	2	246	4.17	1027
Toothpaste, Shampoo	2	205	5.06	1037
Shampoo, Hair Dye	2	166	5.76	957
Toothpaste, Hair Dye	2	160	5.24	838
Universal face cream, Hair Dye	2	134	5.26	705
Face cream night, Face cream day	2	108	12.48	1348
Universal face cream, Shampoo	2	108	5.06	547
Universal face cream, Toothpaste	2	102	4.62	471
Toothbrushes, Shampoo	2	100	4.56	456
Baby care, Cream Zdrave	2	97	7.24	702
Mouthwash, Toothpaste	2	95	5.60	532
Baby care, Shampoo	2	95	5.43	516
Cream Zdrave, Shampoo	2	88	6.77	596
Baby care, Toothpaste	2	87	4.24	369
Wet wipes, Toothpaste	2	87	3.48	303
Cream soap, Toothpaste	2	87	3.27	284
Hair conditioner, Hair Dye	2	84	5.42	456
Hair mask, Shampoo	2	82	6.96	571
Cream Zdrave, Universal face cream	2	82	5.74	471
Bar soap, Toothpaste	2	82	2.62	215
Toothbrushes, Hair Dye	2	76	4.81	366
Cream Zdrave, Toothpaste	2	74	6.20	459
Cream soap, Shampoo	2	74	3.80	281
Hand cream, Shampoo	2	71	4.94	351
Shower cream, Shampoo	2	70	6.33	443
Hand cream, Universal face cream	2	69	4.80	331
Glycerine soap, Toothpaste	2	68	3.18	216
Cream Zdrave, Hair Dye	2	66	6.17	407
Make-up lips, Make-up eyes	2	64	8.70	557
Toothbrushes, Universal face cream	2	61	4.21	257
Shower gel, Shampoo	2	57	4.79	273
Glycerine soap, Cream soap	2	56	2.02	113
Wet wipes, Shampoo	2	53	3.93	208
Bar soap, Shampoo	2	53	3.16	168
Hand cream, Toothpaste	2	52	4.60	239

Nail polish, Make-up eyes	2	49	7.45	365
Nail polish, Make-up lips	2	48	7.43	357
Toothbrushes, Cream Zdrave	2	48	6.19	297
Baby care, Universal face cream	2	48	4.44	213
Shaving cream, Toothpaste	2	48	3.77	181
Wet wipes, Toothbrushes	2	48	3.22	155
Hand cream, Hair Dye	2	47	5.32	250
Baby care, Hair Dye	2	47	4.57	215
Mouthwash, Toothbrushes	2	46	5.18	238
Baby care, Toothbrushes	2	46	4.46	205
Bar soap, Wet wipes	2	46	1.64	75
Glycerine soap, Bar soap	2	45	1.48	66
Bar soap, Hair Dye	2	43	3.54	152
Shower cream, Toothpaste	2	42	5.61	236
Hair conditioner, Toothpaste	2	42	4.44	187
Medical shampoo, Shampoo	2	41	11.69	479
Glycerine soap, Shampoo	2	41	3.81	156
Cream soap, Hair Dye	2	41	3.75	154
Face cream day, Hand cream	2	40	8.06	323

Table 3
Store 1 – Shopping basket bundled items – bundle size 3

Bundle of items	Bundle size	Number of sales	Average Value Per Sale	Overall value of Bundle
Hair conditioner, Shampoo, Hair Dye	3	36	8.08	291
Toothbrushes, Toothpaste, Shampoo	3	36	6.53	235
Toothpaste, Shampoo, Hair Dye	3	27	7.95	215
Cream soap, Toothpaste, Shampoo	3	27	6.16	166
Universal face cream, Toothpaste, Hair Dye	3	26	7.47	194
Hair conditioner, Toothpaste, Shampoo	3	26	6.74	175
Toothbrushes, Toothpaste, Hair Dye	3	25	7.42	185
Baby care, Toothpaste, Shampoo	3	25	6.81	170
Mouthwash, Toothbrushes, Toothpaste	3	23	7.80	179
Bar soap, Toothpaste, Shampoo	3	23	5.18	119
Hair mask, Hair conditioner, Shampoo	3	21	9.47	199
Universal face cream, Shampoo, Hair Dye	3	21	8.25	173
Wet wipes, Toothbrushes, Toothpaste	3	19	6.15	117
Shower cream, Hair conditioner, Shampoo	3	18	9.20	166
Universal face cream, Toothpaste, Shampoo	3	18	6.59	119
Cream soap, Toothbrushes, Toothpaste	3	18	5.28	95

Hair conditioner, Universal face cream, Shampoo	3	17	7.66	130
Hand cream, Universal face cream, Shampoo	3	17	7.29	124
Cream Zdrave, Toothpaste, Hair Dye	3	16	7.76	124
Hand cream, Toothbrushes, Toothpaste	3	16	7.00	112
Wet wipes, Toothpaste, Shampoo	3	16	6.54	105
Toothbrushes, Universal face cream, Toothpaste	3	16	6.27	100
Baby care, Cream Zdrave, Toothpaste	3	15	12.60	189
Baby care, Cream Zdrave, Shampoo	3	15	11.41	171
Cream Zdrave, Universal face cream, Hair Dye	3	15	8.75	131
Hair conditioner, Toothbrushes, Shampoo	3	15	6.94	104
Glycerine soap, Cream soap, Toothpaste	3	15	4.58	69
Glycerine soap, Bar soap, Toothpaste	3	15	3.48	52
Face cream night, Face cream day, Hand cream	3	14	14.57	204
Nail polish, Make-up lips, Make-up eyes	3	14	12.22	171
Toothbrushes, Cream Zdrave, Toothpaste	3	14	10.15	142
Hand cream, Shampoo, Hair Dye	3	14	8.55	120
Toothbrushes, Shampoo, Hair Dye	3	14	7.77	109
Baby care, Toothbrushes, Toothpaste	3	14	7.42	104
Shower gel, Hair conditioner, Shampoo	3	14	7.05	99
Cream soap, Shampoo, Hair Dye	3	14	6.38	89
Glycerine soap, Toothbrushes, Toothpaste	3	13	4.89	64
Face cream day, Hair conditioner, Shampoo	3	12	11.45	137
Mouthwash, Toothpaste, Shampoo	3	12	8.59	103
Baby care, Cream Zdrave, Universal face cream	3	12	8.52	102
Hand cream, Universal face cream, Hair Dye	3	12	8.17	98
Baby care, Universal face cream, Shampoo	3	12	7.35	88
Hair conditioner, Baby care, Shampoo	3	12	7.24	87
Glycerine soap, Toothpaste, Shampoo	3	12	6.01	72
Glycerine soap, Cream soap, Shampoo	3	12	4.65	56
Cream Zdrave, Universal face cream, Shampoo	3	11	8.83	97
Hair conditioner, Toothpaste, Hair Dye	3	11	7.45	82
Baby care, Toothbrushes, Shampoo	3	11	7.37	81

Wet wipes, Hair conditioner, Shampoo	3	11	6.28	69
Glycerine soap, Toothpaste, Hair Dye	3	11	5.96	66
Cream soap, Universal face cream, Shampoo	3	11	5.90	65
Glycerine soap, Bar soap, Hair Dye	3	11	4.58	50
Bar soap, Wet wipes, Toothpaste	3	11	3.83	42
Hair conditioner, Cream Zdrave, Shampoo	3	10	10.80	108
Cream Zdrave, Toothpaste, Shampoo	3	10	10.19	102
Hair mask, Shampoo, Hair Dye	3	10	10.17	102
Cream Zdrave, Shampoo, Hair Dye	3	10	9.37	94
Hair mask, Universal face cream, Shampoo	3	10	9.18	92
Baby care, Shampoo, Hair Dye	3	10	8.07	81
Shower cream, Toothbrushes, Toothpaste	3	10	7.89	79
Hand cream, Toothpaste, Shampoo	3	10	7.82	78
Mouthwash, Wet wipes, Toothpaste	3	10	7.02	70
Shaving cream, Toothpaste, Shampoo	3	10	6.33	63
Cream soap, Toothpaste, Hair Dye	3	10	6.33	63
Cream soap, Hair conditioner, Shampoo	3	10	6.13	61
Wet wipes, Hair conditioner, Toothpaste	3	10	5.75	57
Cream soap, Toothbrushes, Shampoo	3	10	5.26	53
Cream soap, Wet wipes, Toothpaste	3	10	4.23	42
Glycerine soap, Bar soap, Cream soap	3	10	2.41	24

Table 5

Store 2 – Shopping basket bundled items – bundle size 2

Bundle of items	Bundle size	Number of sales	Average Value Per Sale	Overall value of Bundle
Hair conditioner, Shampoo	2	20	6.29	126
Make-up lips, Nail polish	2	19	6.71	127
Make-up eyes, Nail polish	2	17	6.96	118
Toothpaste, Shampoo	2	16	5.39	86
Make-up lips, Make-up eyes	2	15	9.83	147
Shower cream, Shampoo	2	12	6.56	79
Nail care, Nail polish	2	11	7.75	85
Cream Zdrave, Shampoo	2	10	7.47	75

Store 3 Itemsets
No Rules -> No Dependency Network

Table 6

Store 3 – Shopping basket bundled items – bundle size 2

Bundle of items	Bundle size	Number of sales	Average Value Per Sale	Overall value of Bundle
Hair conditioner, Shampoo	2	68	5.89	401
Toothpaste, Shampoo	2	34	5.63	191
Toothbrushes, Toothpaste	2	33	4.36	144
Universal face cream, Shampoo	2	31	5.65	175
Hair Dye, Shampoo	2	29	6.42	186
Hair mask, Shampoo	2	28	6.96	195
Make-up lips, Make-up eyes	2	25	10.94	273
Shower cream, Shampoo	2	24	6.49	156
Nail polish, Make-up eyes	2	23	8.63	198
Cream Zdrave, Shampoo	2	23	6.11	141
Baby care, Shampoo	2	22	4.81	106
Toothpaste, Hair Dye	2	21	5.22	110
Toothbrushes, Shampoo	2	19	5.04	96
Universal face cream, Hair Dye	2	17	5.37	91
Bar soap, Toothpaste	2	17	2.83	48
Hand cream, Shampoo	2	16	5.11	82
Wet wipes, Shampoo	2	16	4.66	75
Mouthwash, Toothpaste	2	15	5.47	82
Baby care, Toothpaste	2	15	3.63	54
Hair conditioner, Hair Dye	2	14	6.09	85
Toothpaste, Universal face cream	2	14	4.63	65
Make-up blush, Make-up eyes	2	13	12.33	160
Make-up eyes, Shampoo	2	13	8.42	109
Shower cream, Hair mask	2	13	7.20	94
Cream Zdrave, Universal face cream	2	13	5.60	73
Jewellery, Make-up eyes	2	12	11.03	132
Jewellery, Nail polish	2	12	9.60	115
Nail care, Nail polish	2	12	8.66	104
Cream Zdrave, Toothpaste	2	12	5.61	67
Mouthwash, Toothbrushes	2	12	5.12	61
Toothbrushes, Hair Dye	2	12	4.86	58
Hand cream, Toothpaste	2	12	4.82	58
Hair mask, Hair conditioner	2	11	6.97	77
Hair mask, Universal face cream	2	11	6.38	70
Hair mask, Hand cream	2	11	5.63	62
Baby care, Cream Zdrave	2	11	4.69	52
Wet wipes, Universal face cream	2	11	3.57	39

Bar soap, Shampoo	2	11	3.49	38
Make-up skin, Make-up eyes	2	10	20.62	206
Anti-acne, Cleansing face	2	10	15.61	156
Medical shampoo, Cream Zdrave	2	10	13.80	138
Face cream night, Face cream day	2	10	10.65	107
Make-up lips, Nail polish	2	10	8.68	87
Brushes, Make-up eyes	2	10	8.00	80
Deo men, Toothpaste	2	10	5.31	53
Shower gel, Shampoo	2	10	4.71	47
Bar soap, Universal face cream	2	10	3.25	33

Store 3 – Shopping basket bundled items – bundle size 3

Table 7

Bundle of items	Bundle size	Number of sales	Average Value Per Sale	Overall value of Bundle
Hair Dye, Shampoo, Toothpaste	3	35	9.96	349
Hand cream, Hair Dye, Toothpaste	3	35	7.95	278
Face cream day, Hair Dye, Toothpaste	3	32	12.00	384
Nail polish, Hair Dye, Shampoo	3	32	11.14	356
Nail polish, Hair Dye, Toothpaste	3	32	10.32	330
Nail polish, Universal face cream, Hair Dye	3	32	9.99	320
Universal face cream, Hair Dye, Shampoo	3	32	9.66	309
Universal face cream, Hair Dye, Toothpaste	3	32	8.83	282
Toothbrushes, Hair Dye, Toothpaste	3	32	8.26	264
Hand cream, Universal face cream, Hair Dye	3	32	7.81	250
Jewellery, Hair Dye, Toothpaste	3	31	17.62	546
Make-up eyes, Nail polish, Universal face cream	3	31	11.96	371
Face cream day, Universal face cream, Shampoo	3	31	11.89	369
Universal face cream, Shampoo, Toothpaste	3	31	9.25	287
Hand cream, Nail polish, Hair Dye	3	31	9.17	284
Hand cream, Universal face cream, Toothpaste	3	31	7.17	222
Jewellery, Hair Dye, Shampoo	3	30	19.22	577
Jewellery, Nail polish, Hair Dye	3	30	18.71	561
Make-up eyes, Hair Dye, Toothpaste	3	30	11.90	357
Face cream day, Hand cream, Hair Dye	3	30	11.05	331
Face cream day, Hand cream, Toothpaste	3	30	10.40	312
Hand cream, Hair Dye, Shampoo	3	30	8.95	269
Toothbrushes, Hand cream, Toothpaste	3	30	6.56	197
Jewellery, Shampoo, Toothpaste	3	29	17.96	521
Hand cream, Jewellery, Toothpaste	3	29	14.74	427
Face cream day, Nail polish, Hair Dye	3	29	13.06	379
Make-up eyes, Nail polish, Hair Dye	3	29	12.72	369
Face cream day, Hair Dye, Shampoo	3	29	12.58	365
Make-up eyes, Nail polish, Toothpaste	3	29	11.81	343

Face cream day, Universal face cream, Toothpaste	3	29	11.28	327
Face cream day, Hand cream, Shampoo	3	29	11.17	324
Make-up eyes, Universal face cream, Toothpaste	3	29	11.13	323
Make-up eyes, Hand cream, Nail polish	3	29	10.94	317
Cream Zdrave, Shampoo, Toothpaste	3	29	9.54	277
Hand cream, Nail polish, Universal face cream	3	29	8.51	247
Hand cream, Shampoo, Toothpaste	3	29	8.27	240
Hand cream, Universal face cream, Shampoo	3	29	8.24	239
Toothbrushes, Hand cream, Hair Dye	3	29	7.24	210
Face cream day, Shampoo, Toothpaste	3	28	12.19	341
Make-up eyes, Universal face cream, Hair Dye	3	28	11.96	335
Face cream day, Universal face cream, Hair Dye	3	28	11.79	330
Face cream day, Hand cream, Nail polish	3	28	11.63	326
Make-up eyes, Hand cream, Hair Dye	3	28	10.70	300
Nail polish, Universal face cream, Shampoo	3	28	10.45	293
Face cream day, Hand cream, Universal face cream	3	28	10.32	289
Make-up eyes, Hand cream, Toothpaste	3	28	10.00	280
Nail polish, Universal face cream, Toothpaste	3	28	9.46	265
Wet wipes, Nail polish, Hair Dye	3	28	8.63	242
Make-up lips, Jewellery, Nail polish	3	27	20.68	558
Make-up eyes, Jewellery, Toothpaste	3	27	18.70	505
Make-up lips, Hand cream, Jewellery	3	27	18.35	495
Jewellery, Universal face cream, Hair Dye	3	27	17.12	462
Jewellery, Nail polish, Toothpaste	3	27	16.93	457
Jewellery, Nail polish, Universal face cream	3	27	16.83	454
Hand cream, Jewellery, Hair Dye	3	27	15.88	429
Toothbrushes, Jewellery, Toothpaste	3	27	15.54	420
Make-up lips, Make-up eyes, Toothpaste	3	27	15.05	406
Wet wipes, Jewellery, Toothpaste	3	27	14.93	403
Face cream day, Nail polish, Shampoo	3	27	13.22	357
Make-up eyes, Nail polish, Shampoo	3	27	12.68	342
Face cream day, Nail polish, Toothpaste	3	27	12.64	341
Face cream day, Nail polish, Universal face cream	3	27	12.39	335
Make-up eyes, Hand cream, Universal face cream	3	27	9.82	265
Hand cream, Nail polish, Toothpaste	3	27	8.71	235
Wet wipes, Hair Dye, Shampoo	3	27	8.51	230
Toothbrushes, Shampoo, Toothpaste	3	27	8.38	226
Wet wipes, Hair Dye, Toothpaste	3	27	8.00	216
Toothbrushes, Universal face cream, Hair Dye	3	27	7.86	212
Toothbrushes, Universal face cream, Toothpaste	3	27	7.15	193
Make-up lips, Jewellery, Hair Dye	3	26	20.90	544
Face cream day, Jewellery, Nail polish	3	26	20.53	534
Make-up lips, Jewellery, Toothpaste	3	26	19.37	504
Face cream day, Jewellery, Toothpaste	3	26	19.32	502
Jewellery, Nail polish, Shampoo	3	26	19.10	497

Jewellery, Universal face cream, Shampoo	3	26	17.52	455
Wet wipes, Jewellery, Hair Dye	3	26	16.29	424
Hand cream, Jewellery, Shampoo	3	26	16.17	421
Jewellery, Universal face cream, Toothpaste	3	26	15.74	409
Hand cream, Jewellery, Nail polish	3	26	15.60	406
Make-up eyes, Face cream day, Nail polish	3	26	14.78	384
Make-up eyes, Face cream day, Toothpaste	3	26	14.14	368
Make-up lips, Nail polish, Hair Dye	3	26	13.61	354
Make-up eyes, Face cream day, Hand cream	3	26	12.77	332
Make-up eyes, Hair Dye, Shampoo	3	26	12.38	322
Make-up lips, Hand cream, Toothpaste	3	26	10.89	283
Toothbrushes, Face cream day, Toothpaste	3	26	10.60	276
Cream Zdrave, Hair Dye, Shampoo	3	26	10.08	262
Hand cream, Nail polish, Shampoo	3	26	9.78	254
Wet wipes, Hand cream, Nail polish	3	26	6.91	180
Face cream day, Jewellery, Shampoo	3	25	20.80	520
Make-up eyes, Jewellery, Nail polish	3	25	19.08	477
Wet wipes, Jewellery, Shampoo	3	25	16.44	411
Make-up lips, Make-up eyes, Nail polish	3	25	16.08	402
Hand cream, Jewellery, Universal face cream	3	25	14.48	362
Make-up lips, Face cream day, Toothpaste	3	25	14.26	357
Wet wipes, Hand cream, Jewellery	3	25	13.57	339
Make-up eyes, Shampoo, Toothpaste	3	25	11.87	297
Make-up lips, Universal face cream, Toothpaste	3	25	11.54	289
Make-up eyes, Universal face cream, Shampoo	3	25	11.52	288
Make-up lips, Hand cream, Hair Dye	3	25	11.31	283
Nail polish, Shampoo, Toothpaste	3	25	10.86	272
Toothbrushes, Make-up eyes, Toothpaste	3	25	10.56	264
Make-up eyes, Hand cream, Shampoo	3	25	10.51	263
Toothbrushes, Face cream day, Hand cream	3	25	9.72	243
Cream Zdrave, Universal face cream, Shampoo	3	25	9.32	233
Toothbrushes, Nail polish, Hair Dye	3	25	9.32	233
Cream Zdrave, Hair Dye, Toothpaste	3	25	9.30	233
Wet wipes, Nail polish, Shampoo	3	25	8.97	224
Toothbrushes, Hair Dye, Shampoo	3	25	8.91	223
Wet wipes, Shampoo, Toothpaste	3	25	8.13	203
Wet wipes, Hand cream, Hair Dye	3	25	6.67	167
Toothbrushes, Hand cream, Universal face cream	3	25	6.35	159
Make-up lips, Make-up eyes, Jewellery	3	24	22.27	534
Face cream day, Jewellery, Hair Dye	3	24	20.93	502
Make-up eyes, Jewellery, Universal face cream	3	24	18.68	448
Anti-wrinkle face care, Hair Dye, Shampoo	3	24	14.98	360
Make-up eyes, Face cream day, Hair Dye	3	24	14.82	356
Make-up eyes, Face cream day, Shampoo	3	24	14.27	342
Make-up lips, Make-up eyes, Hand cream	3	24	13.44	323
Make-up lips, Hair Dye, Shampoo	3	24	12.59	302
Make-up lips, Hair Dye, Toothpaste	3	24	12.24	294
Face cream night, Hair Dye, Toothpaste	3	24	11.84	284

Cream Zdrave, Face cream day, Toothpaste	3	24	11.69	281
Toothbrushes, Face cream day, Hair Dye	3	24	11.19	268
Toothbrushes, Make-up eyes, Hair Dye	3	24	10.94	263
Wet wipes, Make-up eyes, Nail polish	3	24	10.31	247
Toothbrushes, Nail polish, Toothpaste	3	24	8.72	209
Cream Zdrave, Hand cream, Shampoo	3	24	8.58	206
Wet wipes, Nail polish, Toothpaste	3	24	8.29	199
Toothbrushes, Hand cream, Nail polish	3	24	7.77	187
Bar soap, Hair Dye, Shampoo	3	24	7.55	181
Toothbrushes, Hand cream, Shampoo	3	24	7.53	181
Cream Zdrave, Hand cream, Toothpaste	3	24	7.53	181
Wet wipes, Universal face cream, Hair Dye	3	24	7.45	179
Toothbrushes, Wet wipes, Toothpaste	3	24	6.29	151
Make-up lips, Face cream day, Jewellery	3	23	22.40	515
Make-up lips, Jewellery, Universal face cream	3	23	19.11	440
Wet wipes, Make-up lips, Jewellery	3	23	18.36	422
Face cream day, Hand cream, Jewellery	3	23	18.13	417
Make-up eyes, Hand cream, Jewellery	3	23	16.74	385
Wet wipes, Jewellery, Nail polish	3	23	16.03	369
Make-up lips, Face cream day, Nail polish	3	23	15.86	365
Make-up lips, Make-up eyes, Hair Dye	3	23	15.55	358
Make-up lips, Make-up eyes, Universal face cream	3	23	15.24	350
Make-up lips, Face cream day, Shampoo	3	23	15.18	349
Make-up lips, Face cream day, Hair Dye	3	23	14.71	338
Make-up eyes, Face cream day, Universal face cream	3	23	13.98	321
Make-up lips, Face cream day, Hand cream	3	23	13.86	319
Toothbrushes, Hand cream, Jewellery	3	23	13.72	315
Anti-wrinkle face care, Hand cream, Hair Dye	3	23	13.14	302
Make-up lips, Hand cream, Nail polish	3	23	12.53	288
Make-up lips, Universal face cream, Shampoo	3	23	11.96	275
Make-up lips, Universal face cream, Hair Dye	3	23	11.68	269
Face cream night, Hand cream, Hair Dye	3	23	11.09	255
Make-up lips, Hand cream, Universal face cream	3	23	10.85	250
Wet wipes, Face cream day, Nail polish	3	23	10.68	246
Wet wipes, Make-up eyes, Hair Dye	3	23	10.51	242
Toothbrushes, Face cream day, Universal face cream	3	23	10.36	238
Wet wipes, Make-up eyes, Shampoo	3	23	10.18	234
Wet wipes, Face cream day, Toothpaste	3	23	10.04	231
Wet wipes, Make-up eyes, Toothpaste	3	23	9.92	228
Toothbrushes, Make-up eyes, Universal face cream	3	23	9.74	224
Toothbrushes, Make-up eyes, Hand cream	3	23	9.15	211
Toothbrushes, Nail polish, Universal face cream	3	23	8.51	196
Wet wipes, Make-up eyes, Hand cream	3	23	8.32	191
Cream Zdrave, Universal face cream, Toothpaste	3	23	8.29	191
Toothbrushes, Universal face cream,	3	23	8.18	188

Shampoo				
Wet wipes, Nail polish, Universal face cream	3	23	7.57	174
Wet wipes, Hand cream, Shampoo	3	23	6.91	159
Wet wipes, Hand cream, Toothpaste	3	23	5.94	137
Anti-wrinkle face care, Jewellery, Hair Dye	3	22	22.96	505
Make-up lips, Jewellery, Shampoo	3	22	20.53	452
Make-up eyes, Jewellery, Hair Dye	3	22	20.43	449
Face cream day, Jewellery, Universal face cream	3	22	19.32	425
Wet wipes, Face cream day, Jewellery	3	22	18.26	402
Toothbrushes, Jewellery, Hair Dye	3	22	16.25	358
Anti-wrinkle face care, Nail polish, Hair Dye	3	22	15.30	337
Face cream night, Face cream day, Hair Dye	3	22	15.10	332
Anti-wrinkle face care, Hair Dye, Toothpaste	3	22	14.59	321
Face cream night, Face cream day, Toothpaste	3	22	14.30	315
Anti-wrinkle face care, Universal face cream, Hair Dye	3	22	13.86	305
Make-up lips, Nail polish, Universal face cream	3	22	13.57	298
Make-up lips, Nail polish, Toothpaste	3	22	13.47	296
Face cream night, Face cream day, Hand cream	3	22	13.46	296
Anti-wrinkle face care, Wet wipes, Hair Dye	3	22	12.95	285
Cream Zdrave, Face cream day, Shampoo	3	22	12.45	274
Make-up lips, Shampoo, Toothpaste	3	22	12.34	271
Face cream night, Universal face cream, Hair Dye	3	22	11.83	260
Make-up lips, Hand cream, Shampoo	3	22	11.77	259
Toothbrushes, Face cream day, Shampoo	3	22	11.32	249
Hair mask, Hair Dye, Shampoo	3	22	11.28	248
Face cream night, Universal face cream, Toothpaste	3	22	10.95	241
Wet wipes, Make-up lips, Hair Dye	3	22	10.76	237
Hair mask, Hair Dye, Toothpaste	3	22	10.71	236
Wet wipes, Face cream day, Hair Dye	3	22	10.57	233
Wet wipes, Face cream day, Shampoo	3	22	10.48	231
Face cream night, Hand cream, Toothpaste	3	22	10.14	223
Toothbrushes, Make-up lips, Hand cream	3	22	10.12	223
Wet wipes, Make-up eyes, Universal face cream	3	22	9.67	213
Wet wipes, Make-up lips, Hand cream	3	22	9.51	209
Wet wipes, Face cream day, Hand cream	3	22	8.91	196
Toothbrushes, Cream Zdrave, Toothpaste	3	22	7.78	171
Wet wipes, Universal face cream, Shampoo	3	22	7.60	167
Bar soap, Hair Dye, Toothpaste	3	22	6.94	153
Wet wipes, Universal face cream, Toothpaste	3	22	6.89	152
Anti-wrinkle face care, Jewellery, Toothpaste	3	21	21.72	456
Cream Zdrave, Jewellery, Shampoo	3	21	17.65	371
Bar soap, Make-up lips, Jewellery	3	21	17.57	369
Toothbrushes, Jewellery, Shampoo	3	21	17.08	359
Anti-wrinkle face care, Make-up eyes, Hair Dye	3	21	16.91	355
Cream Zdrave, Jewellery, Toothpaste	3	21	16.40	344
Bar soap, Jewellery, Hair Dye	3	21	15.59	327

Cream Zdrave, Make-up lips, Face cream day	3	21	14.76	310
Wet wipes, Jewellery, Universal face cream	3	21	14.73	309
Toothbrushes, Wet wipes, Jewellery	3	21	14.33	301
Make-up lips, Nail polish, Shampoo	3	21	14.17	298
Anti-wrinkle face care, Universal face cream, Shampoo	3	21	14.05	295
Cream Zdrave, Make-up lips, Shampoo	3	21	12.82	269
Wet wipes, Make-up eyes, Face cream day	3	21	12.56	264
Cream Zdrave, Make-up lips, Toothpaste	3	21	12.27	258
Cream Zdrave, Make-up eyes, Nail polish	3	21	12.04	253
Toothbrushes, Make-up lips, Toothpaste	3	21	11.46	241
Hair mask, Shampoo, Toothpaste	3	21	11.22	236
Cream Zdrave, Make-up eyes, Toothpaste	3	21	11.17	235
Toothbrushes, Make-up eyes, Nail polish	3	21	10.99	231
Wet wipes, Make-up lips, Shampoo	3	21	10.93	230
Natural oils, Universal face cream, Toothpaste	3	21	10.87	228
Cream Zdrave, Face cream day, Hand cream	3	21	10.78	226
Bar soap, Make-up lips, Hair Dye	3	21	9.81	206
Cream Zdrave, Universal face cream, Hair Dye	3	21	9.02	190
Toothbrushes, Cream Zdrave, Shampoo	3	21	8.54	179
Cream Zdrave, Hand cream, Hair Dye	3	21	8.35	175
Bar soap, Shampoo, Toothpaste	3	21	7.11	149
Toothbrushes, Wet wipes, Hair Dye	3	21	6.85	144
Wet wipes, Hand cream, Universal face cream	3	21	5.93	124
Anti-wrinkle face care, Wet wipes, Jewellery	3	20	20.69	414
Cream Zdrave, Make-up lips, Jewellery	3	20	20.57	411
Make-up eyes, Jewellery, Shampoo	3	20	19.96	399
Toothbrushes, Make-up lips, Jewellery	3	20	18.86	377
Toothbrushes, Face cream day, Jewellery	3	20	18.47	369
Wet wipes, Make-up eyes, Jewellery	3	20	17.69	354
Anti-wrinkle face care, Make-up lips, Hair Dye	3	20	17.40	348
Anti-wrinkle face care, Make-up eyes, Nail polish	3	20	17.10	342
Make-up lips, Make-up eyes, Face cream day	3	20	17.05	341
Cream Zdrave, Hand cream, Jewellery	3	20	15.27	305
Toothbrushes, Jewellery, Universal face cream	3	20	14.93	299
Bar soap, Jewellery, Toothpaste	3	20	14.56	291
Make-up lips, Make-up eyes, Shampoo	3	20	14.51	290
Face cream night, Face cream day, Universal face cream	3	20	14.23	285
Anti-wrinkle face care, Hand cream, Nail polish	3	20	13.94	279
Hair mask, Face cream day, Toothpaste	3	20	13.40	268
Anti-wrinkle face care, Hand cream, Shampoo	3	20	13.38	268
Toothbrushes, Make-up eyes, Face cream day	3	20	13.16	263
Anti-wrinkle face care, Wet wipes, Nail polish	3	20	13.05	261
Anti-wrinkle face care, Hand cream, Toothpaste	3	20	12.93	259
Face cream night, Hair Dye, Shampoo	3	20	12.75	255
Anti-wrinkle face care, Hand cream, Universal face cream	3	20	12.40	248
Cream Zdrave, Make-up lips, Universal face cream	3	20	12.28	246

Face cream night, Shampoo, Toothpaste	3	20	12.27	245
Hair mask, Nail polish, Hair Dye	3	20	11.98	240
Toothbrushes, Face cream day, Nail polish	3	20	11.66	233
Wet wipes, Make-up lips, Nail polish	3	20	11.53	231
Natural oils, Hair Dye, Toothpaste	3	20	11.52	230
Toothbrushes, Make-up lips, Hair Dye	3	20	11.24	225
Face cream night, Toothbrushes, Hair Dye	3	20	11.20	224
Wet wipes, Make-up lips, Toothpaste	3	20	10.86	217
Cream Zdrave, Nail polish, Shampoo	3	20	10.56	211
Face cream night, Hand cream, Universal face cream	3	20	10.24	205
Natural oils, Hand cream, Hair Dye	3	20	10.03	201
Toothbrushes, Nail polish, Shampoo	3	20	9.83	197
Cream Zdrave, Nail polish, Toothpaste	3	20	9.80	196
Wet wipes, Face cream day, Universal face cream	3	20	9.75	195
Nail care, Nail polish, Universal face cream	3	20	9.73	195
Hair mask, Toothbrushes, Toothpaste	3	20	9.71	194
Natural oils, Hand cream, Toothpaste	3	20	9.58	192
Hair mask, Hand cream, Toothpaste	3	20	9.45	189
Cream Zdrave, Nail polish, Universal face cream	3	20	9.26	185
Natural oils, Hand cream, Universal face cream	3	20	8.80	176
Bar soap, Make-up lips, Toothpaste	3	20	8.64	173
Wet wipes, Cream Zdrave, Shampoo	3	20	8.15	163
Bar soap, Make-up lips, Hand cream	3	20	7.95	159
Bar soap, Nail polish, Hair Dye	3	20	7.94	159
Wet wipes, Cream Zdrave, Toothpaste	3	20	7.52	150
Glycerine soap, Hair Dye, Toothpaste	3	20	7.51	150
Cream Zdrave, Hand cream, Universal face cream	3	20	7.50	150
Toothbrushes, Wet wipes, Nail polish	3	20	6.98	140
Bar soap, Universal face cream, Hair Dye	3	20	6.46	129

Store 4 – Shopping basket bundled items – bundle size 2

Table 8

Bundle of items	Bundle size	Number of sales	Average Value Per Sale	Overall value of Bundle
Hair Dye, Shampoo	2	47	7.02	330
Hair Dye, Toothpaste	2	47	6.34	298
Nail polish, Hair Dye	2	45	7.46	336
Shampoo, Toothpaste	2	44	6.60	290
Jewellery, Toothpaste	2	42	13.50	567
Nail polish, Universal face cream	2	42	6.59	277
Universal face cream, Hair Dye	2	42	6.01	252

Universal face cream, Toothpaste	2	42	5.45	229
Hand cream, Hair Dye	2	42	5.25	220
Hand cream, Toothpaste	2	42	4.55	191
Universal face cream, Shampoo	2	41	6.22	255
Jewellery, Hair Dye	2	40	14.96	598
Make-up eyes, Nail polish	2	40	9.00	360
Face cream day, Toothpaste	2	40	8.51	340
Make-up eyes, Toothpaste	2	40	8.29	331
Jewellery, Nail polish	2	39	14.56	568
Face cream day, Nail polish	2	39	9.58	374
Face cream day, Shampoo	2	39	9.14	357
Nail polish, Shampoo	2	39	7.68	299
Hand cream, Shampoo	2	39	5.52	215
Toothbrushes, Toothpaste	2	39	4.81	188
Hand cream, Universal face cream	2	39	4.50	175
Jewellery, Shampoo	2	38	15.22	578
Face cream day, Hair Dye	2	38	9.13	347
Face cream day, Hand cream	2	38	7.61	289
Nail polish, Toothpaste	2	38	6.86	261
Make-up lips, Jewellery	2	37	16.73	619
Make-up lips, Toothpaste	2	37	8.74	323
Face cream day, Universal face cream	2	37	8.37	310
Make-up eyes, Universal face cream	2	37	8.14	301
Make-up eyes, Hand cream	2	37	7.01	260
Hand cream, Nail polish	2	37	5.79	214
Hand cream, Jewellery	2	36	12.10	436
Make-up eyes, Hair Dye	2	36	9.07	326
Jewellery, Universal face cream	2	35	13.26	464
Cream Zdrave, Shampoo	2	35	6.66	233
Cream Zdrave, Toothpaste	2	35	5.87	205
Toothbrushes, Hair Dye	2	35	5.35	187
Make-up lips, Make-up eyes	2	34	12.04	409
Make-up lips, Nail polish	2	34	10.34	352
Make-up lips, Hair Dye	2	34	9.35	318
Make-up lips, Hand cream	2	34	7.97	271
Wet wipes, Hair Dye	2	34	4.94	168
Wet wipes, Toothpaste	2	34	4.55	155
Face cream day, Jewellery	2	33	16.60	548
Wet wipes, Jewellery	2	33	12.15	401
Make-up eyes, Shampoo	2	33	8.65	285
Make-up lips, Universal face cream	2	33	8.65	285
Wet wipes, Nail polish	2	33	5.18	171
Wet wipes, Shampoo	2	33	5.02	166
Toothbrushes, Hand cream	2	33	3.81	126
Make-up eyes, Jewellery	2	32	15.96	511
Make-up eyes, Face cream day	2	32	11.01	352
Make-up lips, Face cream day	2	31	11.42	354
Anti-wrinkle face care, Hair Dye	2	31	11.36	352
Make-up lips, Shampoo	2	31	9.31	289
Cream Zdrave, Universal face cream	2	31	5.58	173

Wet wipes, Hand cream	2	31	3.25	101
Toothbrushes, Shampoo	2	30	5.47	164
Toothbrushes, Universal face cream	2	30	4.44	133
Toothbrushes, Jewellery	2	29	12.70	368
Cream Zdrave, Make-up lips	2	29	9.59	278
Cream Zdrave, Face cream day	2	29	8.87	257
Toothbrushes, Face cream day	2	29	7.79	226
Wet wipes, Face cream day	2	29	6.99	203
Wet wipes, Make-up eyes	2	29	6.76	196
Cream Zdrave, Hair Dye	2	29	6.54	190
Cream Zdrave, Hand cream	2	29	4.96	144
Bar soap, Hair Dye	2	29	3.86	112
Cream Zdrave, Jewellery	2	28	13.91	390
Bar soap, Jewellery	2	28	12.00	336
Anti-wrinkle face care, Shampoo	2	28	11.40	319
Toothbrushes, Make-up eyes	2	28	7.44	208
Cream Zdrave, Nail polish	2	28	6.86	192
Toothbrushes, Nail polish	2	28	5.96	167
Bar soap, Shampoo	2	28	4.12	115
Wet wipes, Universal face cream	2	28	4.00	112
Bar soap, Toothpaste	2	28	3.35	94
Anti-wrinkle face care, Nail polish	2	27	11.85	320
Anti-wrinkle face care, Toothpaste	2	27	11.26	304
Anti-wrinkle face care, Hand cream	2	27	9.66	261
Anti-wrinkle face care, Wet wipes	2	27	9.39	254
Face cream night, Toothpaste	2	27	8.43	228
Bar soap, Make-up lips	2	27	6.34	171
Anti-wrinkle face care, Jewellery	2	26	18.74	487
Anti-wrinkle face care, Make-up eyes	2	26	13.07	340
Anti-wrinkle face care, Universal face cream	2	26	10.39	270
Face cream night, Hair Dye	2	26	9.15	238
Hair mask, Nail polish	2	26	8.58	223
Cream Zdrave, Make-up eyes	2	26	8.25	214
Hair mask, Shampoo	2	26	8.09	210
Hair mask, Hair Dye	2	26	7.69	200
Wet wipes, Make-up lips	2	26	7.40	192
Toothbrushes, Wet wipes	2	26	3.33	87
Face cream night, Face cream day	2	25	11.59	290
Natural oils, Universal face cream	2	25	7.80	195
Face cream night, Hand cream	2	25	7.65	191
Hair mask, Toothpaste	2	25	7.56	189
Glycerine soap, Toothpaste	2	25	3.94	98
Bar soap, Hand cream	2	25	2.21	55
Glycerine soap, Jewellery	2	24	11.83	284
Natural oils, Toothpaste	2	24	8.34	200
Face cream night, Universal face cream	2	24	8.31	199
Toothbrushes, Make-up lips	2	24	8.31	199
Hair mask, Universal face cream	2	24	7.54	181
Hair mask, Hand cream	2	24	6.53	157

Toothbrushes, Cream Zdrave	2	24	5.02	120
Bar soap, Nail polish	2	24	4.27	103
Bar soap, Universal face cream	2	24	3.09	74
Anti-wrinkle face care, Make-up lips	2	23	14.20	327
Hair mask, Face cream day	2	23	10.38	239
Face cream night, Shampoo	2	23	9.42	217
Nail care, Nail polish	2	23	7.05	162
Glycerine soap, Make-up lips	2	23	6.82	157
Natural oils, Hand cream	2	23	6.40	147
Hair conditioner, Shampoo	2	23	6.34	146
Bar soap, Make-up eyes	2	23	6.26	144
Wet wipes, Cream Zdrave	2	23	4.59	106
Glycerine soap, Hand cream	2	23	2.66	61
Anti-wrinkle face care, Face cream day	2	22	13.39	295
Make-up skin, Hair Dye	2	22	12.11	266
Natural oils, Shampoo	2	22	8.59	189
Natural oils, Hair Dye	2	22	8.34	184
Hair conditioner, Nail polish	2	22	6.72	148
Glycerine soap, Make-up eyes	2	22	6.54	144
Bar soap, Face cream day	2	22	6.06	133
Nail care, Universal face cream	2	22	5.55	122
Hair styling, Hand cream	2	22	4.76	105
Glycerine soap, Hair Dye	2	22	4.39	97
Bar soap, Cream Zdrave	2	22	3.75	82
Make-up skin, Jewellery	2	21	18.54	389
Hair mask, Jewellery	2	21	15.47	325
Nail care, Jewellery	2	21	14.56	306
Natural oils, Face cream day	2	21	10.91	229
Make-up skin, Universal face cream	2	21	10.43	219
Make-up skin, Hand cream	2	21	9.94	209
Natural oils, Nail polish	2	21	9.10	191
Make-up blush, Hand cream	2	21	7.46	157
Nail care, Shampoo	2	21	6.84	144
Hair conditioner, Hair Dye	2	21	6.16	129
Hair mask, Wet wipes	2	21	6.12	129
Baby care, Hair Dye	2	21	5.27	111
Baby care, Toothpaste	2	21	4.58	96
Bar soap, Toothbrushes	2	21	2.19	46
Bar soap, Wet wipes	2	21	1.87	39
Make-up skin, Shampoo	2	20	12.39	248
Make-up skin, Nail polish	2	20	12.09	242
Make-up blush, Make-up eyes	2	20	11.06	221
Anti-wrinkle face care, Cream Zdrave	2	20	10.82	216
Make-up skin, Toothpaste	2	20	10.56	211
Natural oils, Make-up eyes	2	20	10.29	206
Hair mask, Make-up eyes	2	20	9.78	196
Anti-wrinkle face care, Toothbrushes	2	20	9.74	195
Hair mask, Cream Zdrave	2	20	8.17	163
Face cream night, Toothbrushes	2	20	7.81	156
Baby care, Face cream day	2	20	7.53	151

Table 9 and Table 10 – Store 4 Recommendations.

Selected Item	Recommendation	Sales of Selected Items	Linked Sales	% of linked sales	Average value of recommendation	Overall value of linked sales
Shower cream	Jewellery	22	17	77.27 %	8.68	191.14
Lip care	Jewellery	20	15	75.00 %	8.38	167.65
Bar soap	Jewellery	39	28	71.79 %	8.24	321.72
Nail care	Jewellery	31	21	67.74 %	8.01	248.47
Anti-wrinkle set	Jewellery	17	13	76.47 %	7.95	135.29
Wet wipes	Jewellery	45	33	73.33 %	7.84	352.91
Glycerine soap	Jewellery	34	24	70.59 %	7.66	260.64
Hair conditioner	Jewellery	26	17	65.38 %	7.56	196.7
Shower gel	Jewellery	18	12	66.67 %	7.56	136.1
Make-up lips	Jewellery	51	37	72.55 %	7.55	385.3
Make-up skin	Jewellery	28	21	75.00 %	7.49	209.8
Hair Dye	Jewellery	62	40	64.52 %	7.48	463.79
Hair styling	Jewellery	26	17	65.38 %	7.28	189.49
Toothbrushes	Jewellery	43	29	67.44 %	7.26	312.2
Make-up blush	Jewellery	24	15	62.50 %	7.20	172.99
Anti-acne	Jewellery	16	12	75.00 %	7.17	114.82
Cleansing face	Jewellery	22	16	72.73 %	7.13	156.99
Shampoo	Jewellery	62	38	61.29 %	7.06	438.19
Anti-wrinkle face care	Jewellery	40	26	65.00 %	7.06	282.66
Nail polish	Jewellery	59	39	66.10 %	7.04	415.62

Selected Item	Recommendation	Sales of Selected Items	Linked Sales	% of linked sales	Average value of recommendation	Overall value of linked sales
Aftershave lotion	Shampoo	12	12	100.00 %	3.83	46.01
Anti-wrinkle set	Universal face cream	17	16	94.12 %	2.41	41.04
Anti-acne	Toothpaste	16	15	93.75 %	2.56	40.99
Anti-age face care	Shampoo	11	10	90.91 %	3.35	36.93
Anti-age face care	Toothpaste	11	10	90.91 %	2.77	30.53
Massaging oil	Toothpaste	11	10	90.91 %	2.73	30.1
Massaging oil	Universal face cream	11	10	90.91 %	2.18	24
Anti-age face care	Toothbrushes	11	10	90.91 %	1.71	18.83
Toothbrushes	Toothpaste	43	39	90.70 %	2.59	111.53
Face cream night	Toothpaste	30	27	90.00 %	2.42	72.85
Hair conditioner	Shampoo	26	23	88.46 %	3.05	79.5
Anti-wrinkle set	Hand cream	17	15	88.24 %	1.79	30.55
Anti-acne	Make-up lips	16	14	87.50 %	5.30	84.85
Anti-age set	Face cream day	16	14	87.50 %	4.97	79.66
Anti-acne	Universal face cream	16	14	87.50 %	2.26	36.19
Make-up blush	Hand cream	24	21	87.50 %	1.55	37.34
Face cream night	Hair Dye	30	26	86.67 %	2.93	87.98
Cleansing face	Make-up eyes	22	19	86.36 %	4.95	109.11
Cleansing face	Toothpaste	22	19	86.36 %	2.62	57.64
Hair Dye Prof.	Shampoo	20	17	85.00 %	2.79	55.99