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MSc. Thesis Econometrics
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Economic Capital Forecasting of Interest Rate Risk in a Multi-Year Framework using Non-parametric Least Squares Monte Carlo for a With-Profit Insurance Product

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Chapter 1

Introduction

The fear of losing assets has caused humans to protect their belongings for a very long time; as far back as the Chinese and Babylonian traders in 2000 B.C., mankind has used the concept of insurance in order to protect their goods. Sailing merchants would receive a loan to fund the ship of travelling sailors. However, a sailor could pay the merchant a premium that would cancel this loan, in case the shipment would never arrive. In a sense, this is a very simple example of an insurance product (Vaughan, 1997).

The insurers of today's world are a corner stone in running the economies and protect individuals. The concept of insuring is as simple as it is elegant: events which would have a large (financial) consequence but with a small probability for an individual, are collected by a large institute and covered. Individuals can protect their assets from a potential loss by paying a (recurrent) premium beforehand, erasing this large potential impact for them. An insurer would typically gather a big amount of these premiums, in order to cover the amount of money needed when this event would happen at one of their policyholders. Insurers therefore play a crucial role in the economy, protecting individual players from large risks they cannot cover through paying relatively small premiums.

Insurers generally have two variables that can easily be adjusted by them, both having a major impact on their business model and strategy. First of all the premiums, especially when reoccurring, generate a cash flow to cover large claims. The determination of the height of this premiums, is based on a relative simple concept. For example: when an negative event would cost \$10000 having a $0,1\%$ probability a year, the insurer should have asked a $P_{premium} \geq \$10$ a year, to cover these claims. Too low premiums could result in not being able to cover these claims when occurring. However, asking too high premiums would result in clients walking to other firms, given the very competitive landscape in the insurers world. An equilibrium between these two contrary interests should be seeked by an insurer.

Another variable that could influence an insurer's performance, is the amount of capital that it has reserved for future claims. An insurer typically makes a small margin on the premiums they receive, due to the competitive industry. However, the way an insurer invests that premium is potentially more profitable. An insurer typically has a significant amount of long term investments, yielding an higher profit compared to investments with a short maturity. Too many long maturing investments in the portfolio will lead to liquidity problems, in case more claims are received than expected.

An insurer goes into default whenever it cannot meet its obligations (in a very simple world); in other words, whenever the liabilities side of an insurer (its obligations) has a higher value than the assets side of this insurer. Moreover, given that the value of the assets and liabilities change over time, an accurate projection of both sides is essential for a good health of the organisation. It is therefore of great importance that the insurer can predict its value of the obligations in the future (on the liability side) and the value of the products in their portfolio and incoming cash flows (on the asset side) accurately. An accurate valuation could, as explained before, result in a competitive premium for the policyholders while reserving an optimal amount of money for future claims.

Not only from a economic point of view, but also from a regulatory point of view it is of interest for an insurer to predict their obligations and liabilities. As a consequence of the new Solvency 2 regulations, an insurer has to calculate its Solvency Capital Requirement (SCR). This term can be explained as a required surplus above the regular liabilities, which is the amount of capital insurance companies must hold to reduce the risk of insolvency. Solvency 2 has been developed and adjusted over the last thirteen years. The first steps have already been taken, and a full implementation of the new regulatory framework will be required by the 1st of January 2016. Not complying to these regulations could eventually lead to the withdrawal of the insurers licence by regulators.

There are several methods to calculate this SCR. EIOPA, the European regulator, has proposed a standard formula to calculate the SCR. Experts argue however that this standard formula uses unacceptable assumptions and therefore conclude that the standard SCR does not estimate the risks accurately (Kousaris, 2011). This, combined with the fact that under some restrictions an insurer is allowed to calculate its SCR using their own models, results in insurers looking for alternative ways to accurately predict SCR's. For more background information on Solvency 2 regulations, refer to Appendix A.

There are numerous factors that could influence the position or value of an insurer's portfolio. Factors like asset location, price changes on the stock exchanges, and the correlation between different risks can influence the value of the liabilities and assets an insurer possesses. However, one factor has a structural influence on the insurer's solvency position: the interest rate. The effect of a change in interest rate works both on the assets- and liabilities side of an insurer. Insurers have a significant amount of money invested in long-term maturing (government) bonds. The value of these obtained bonds are influenced by interest rate movements of today's market; both the value of the bonds already possessed are affected as the yield of new bonds. Moreover, the change of interest also has an impact on the liabilities side of an insurer, since the value of (future) cash flows is directly influenced by a change in discount rates. As interest rates go up, discount rates rise as well. A higher discount rate results in a loss of value of future premiums. Typically, the potential effect interest change has on future liabilities is bigger than the asset value fluctuations. Insurers are typically exposed to a significant interest (market) risk and therefore highly interested in the consequences of the value of their assets and liabilities when interest rates change.

1.1 Challenges when valuing an insurer's assets and liabilities

In the previous section we have introduced the basic concept of insuring and showed the relevance of accurate valuations for an insurer. This section will elaborate on this argument when introducing some of the challenges an insurer would encounter when valuing their asset cash flows and liabilities (A&L).

The need of an accurate valuation is apparent, however the subject of how to obtain this accurate valuation has not been touched upon. As already mentioned, interest rate development is crucial for an accurate valuation of the A&L. Predicting how the interest rate will develop without any major uncertainty is a tricky and difficult job. The amount of underlying parameters influencing the interest rate is gigantic, so forecasting an accurate estimation is not feasible. Also for the optionality in some common (profit-sharing) products, a deterministic model using only one forecasted curve is not sufficient. Since an insurer's value of the A&L is significantly influenced by the interest rate curve, a wrongly predicted curve could have major consequences.

To overcome this problem, insurers would typically predict multiple possible interest rate developments. These predictions are translated into scenarios, which are sample paths of interest rate developments of a specified stochastic process. Typically, thousands of scenarios are used, from which the SCR for each path could be determined. These scenarios, also called Real-World (RW) scenarios, show potential interest rate curve developments projected in a multi-year time frame. To calculate the SCR, a pre-determined percentile of sample paths is selected, resulting in the most negative outcomes for the insurer.

In addition to the Real-World scenarios used to project the interest rate curve in the future, another stochastic set is used to estimate the real value of the asset or liability. This set consists of Risk-Neutral (RN) scenarios and is in general market-consistent and risk-free. Typically, a RN scenario set would contain thousands of those interest rate scenarios. Insurance products with guarantees, like profit-sharing products, are very difficult to value closed-form due to their non-linear behaviour. Valuing profit-sharing products with only one deterministic curve could lead to inaccurate valuations, with disastrous consequences (O'Brien, 2006). There is a substantial amount of profit-sharing products in insurer's portfolios; in the U.K. only, the value of the assets backing up these products is estimated at more than £400 bn. (Harrison, 2007). Using a whole set of interest rate scenarios enables us to accurately value a complex product with embedded optionality, at a certain moment in time.

It is very important to understand the different ways RW- and RN scenarios are used in projecting the correct value of A&L in the future. Although the combination of RW and RN scenarios will probably result in a very accurate value of the A&L, it requires a lot of computational power. This is also known as the full stochastic approach. If one would use this full stochastic approach, only for t=1 one would have 5000 RW-world scenarios and for all these scenarios 5000 RN scenarios should be used to validate the product in each of the 5000 scenarios. A typical balance sheet calculation is not very simple, let alone this number of calculations revolving around it. One might imagine that this approach projected over a number of years and for multiple products, is not feasible.

Recently, more and more attention is drawn towards reliable multi-year projections. Horig (2013) gives two general reasons for this increase of interest. First of all, accurate forecasts gives management an idea of (financial) consequences of strategic choices made. Insurers are especially interested how capital requirements and capital coverages develop in specified cases. Having insight in these future states can influence certain (financial) decisions, like portfolio management and risk appetite. Secondly, from a regulatory point of view it is obliged to calculate the development of the solvabilitity through time. The previous argument is settled within regulations, in a more abstract way under the so-called "Own Risk and Solvency Assessment" (ORSA). In the Solvency II directive Article 45-2 (European Parlament, 2009), ORSA is described as:

"*... The Own-Risk and Solvency Assessment shall be an integral part of the business strategy and shall be taken into account on an ongoing basis in the strategic decisions of the undertaking.*"

Hence, the main reason for executing an ORSA, is to get an idea of the implications of a certain strategic choice and potential economic stress scenarios. A technical interpretation of this statement is given by the European Insurance and Occupational Pensions Authority (EIOPA). In their final consultation report "on the Proposal for Guidelines on Forward Looking Assessment of Own Risks (based on the ORSA principles)"(Eiopa (2013), in 5.43) they denote that:

"*The undertaking needs to project its capital needs at least over its business planning period, taking into account medium and long term risk, as appropriate. This projection is to be made taking into consideration any likely changes to the risk profile and business strategy over the projection period and the sensitivity of the assumptions used.*"

Hence, EIOPA agrees that there should be a mechanism which gives an indication of the consequences of certain decisions. They continue in point 5.44, arguing that if an insurer would generate a new business plan, their solvency should be tested using a range of possible scenarios in order to provide a proper basis for decision-making. Although an exact time-frame is not given, three to five years is often regarded as appropriate. In practice however, simple deterministic forecasts for economical capital are used by insurance companies at the moment.

In this thesis an Accelerated Convergence Technique (ACM) in the form of Least Squares Monte Carlo, will be used to circumvent the usage of full real-world scenarios on risk-neutral scenarios. Moreover, the use of such an accelerated convergence technique is also supported by an analytical framework and intuitively explained. A sample product is constructed to illustrate the techniques, for a one year forecast horizon. In the end, the horizon is enlarged to a multi-year setting.

1.2 Objective

The objective of this thesis is:

To implement the Least Squares Monte Carlo approach to predict the value of the assets and liabilities cash flows of an insurer's profit sharing sample product, more accurately than existing approaches, concerning interest rate risk for multiple years.

In this objective we define the word "*accurately*" rather broadly: possible biases flowing out of imposed restrictive assumptions could lead to inaccurate forecasts. Hence, bypassing these restrictions is, in our opinion, also a way of making your predictions more accurate. In order to achieve this objective, a number of underlying questions should be answered. The research questions are:

1. What is a suitable performance measure and benchmark to asses the outcome of the LSMC technique?
2. How can we incorporate real-world scenarios and risk-neutral scenarios for the calculation of the profit sharing interest rate SCR accurately?
3. How does the ACM perform compared to the benchmark?
4. How can we extend the predictions into a multi-year framework?

1.3 Outline of this report

In this section a short overview of the research structure and outline of the report will be given. Our research consists of two phases; a simulation phase and a fitting phase. In the simulation phase we define appropriate interest rate risk-drivers using market data, set up a product with non-linear cash-flows and show two different ways of valuing this product. This phase leads to certain values of the product NAV, given a set of risk-drivers. The next phase is the fitting phase. Upon the resulting valuations, a fit will be constructed for both approaches. Eventually, we will end up with a fitted NAV, from which a SCR can be derived. This structure of the report is depicted in Figure 1.1.

Based on this structure, the outline of the report is as follows. In the preceding chapter a short introduction of the problem is given together with the objective of this thesis and the relevance of the subject. In Chapter 2 we will introduce the methodology for respectively the cash-flow calculations (Section 2.1), the real-world interest rate scenario generation (Section 2.2), the risk-neutral interest rate scenario generation (Section 2.3). In Section 2.4 the models are combined into a full nested approach to value the product. The LSMC approach is discussed in Section 2.5, a non-parametric fitting scheme is presented in Section 2.6 and we finish the chapter with a multi-year extension (Section 2.7). We will continue in Chapter 3 to give a brief introduction on the data used. Given the LSMC and the full nested approaches, results

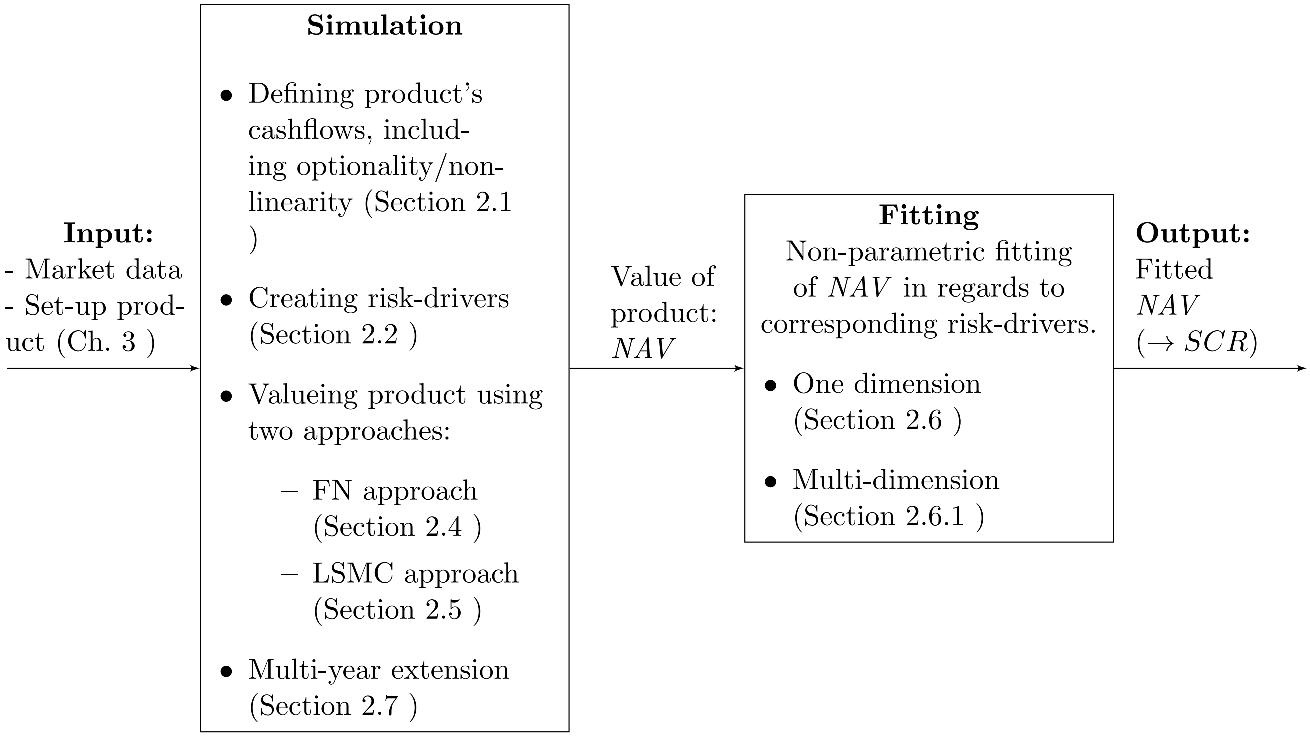


Figure 1.1: The general structure of this report, with corresponding sectioning.

are presented in Chapter 4. For this chapter, the same structure as used in Chapter 2 is kept to present the results. The report finalizes with concluding remarks denoted in Chapter 5 and a global discussion of the results presented in Chapter 6.

Chapter 2

Methodology

In this chapter the models used for the analysis, are discussed. In Section 2.1, the general cash flow calculations involved in an insurance company are stated. We try to answer the question "How are insurance companies affected by a change in the yield curve?". In the following section, we propose a method to generate real-world interest rate scenarios, based on the Diebold-Li model (Diebold & Li, 2006). The introduction of a model that is able to simulate risk-neutral valuation interest rate scenarios, is stated in Section 2.3. This is done using a Hull-White one-factor model (Hull & White, 1990). By using these models, in combination with the earlier mentioned insurance product calculations, we are able to value a profit-sharing insurance product. A profit-sharing product allows policy holders to partly benefit from the surplus an insurer makes on this policies. Using the Full-Nested approach, explained in Section 2.4, the value is calculated. However, since this method is not feasible in practice, an alternative Accelerated Convergence Method (ACM) is discussed, in the form of the Least Squares Monte Carlo (LSMC) approach. The LSMC approach is discussed in detail in Section 2.5, including the corresponding (non-parametric) fitting details stated in Section 2.6. We conclude this chapter with a proposed multi-year extension.

2.1 General Assets and Liabilities calculations

In this section, a general idea of the balance sheet calculations of an insurance company are briefly explained. We base our balance sheet on a profit-sharing product and assume a straightforward asset portfolio. The methodology in this section remains simple, to keep our focus on the LSMC curve fitting and forecasting. Extensions could be implemented in the future.

An insurer's assets, A_t, is defined as all the possessions an insurance company has at time t. Typically an insurer would have a mix of different types of government and corporate bonds, equity and cash. The incoming premiums are collected and converted into investments in one of those categories. Since the majority of an insurer portfolio contains government bonds, and in order to keep the model simple, we assume that the insurer is all-in on government bonds.

On the other side of the balance sheet the liabilities are situated, denoted by L_t. For a life insurer, typically these would primarily consist of liquidized claims, in case of death, lapse of a policyholder or at a specified maturity. The available capital, or own funds, can be seen as a buffer in times of difficult circumstances. This term is denoted as AC_t (or V_t), and is defined as:

$$AC_t = A_t - L_t. \tag{2.1}$$

Recently, A&L problems have been subjected to a lot of attention throughout insurer's landscape. The new European regulatory framework for insurers, Solvency II, has been developed since 2002 and recently the first real implementation steps are taken. Under these new regulations, insurers are obliged to calculate their SCR given their portfolio. Insurers can choose to use a standard, rather conservative, formula developed by EIOPA, or develop their own method to do these calculations. When choosing for the lather option, extensive validation and documentation is required, in order to get regulatory approval. For a more extensive description of Solvency II, please consider Appendix A.

Under Solvency II, from a regulatory perspective, an insurer should be able to withstand scenarios occurring once every 200 years. The corresponding probability is based on the predicted value of the insurer in one year NAV_1, and can be formulated as:

$$P(NAV_1 \geq 0 | NAV_0 = SCR) \geq 99.5\%, \tag{2.2}$$

in which SCR denotes the minimum value of NAV_0 for which this constraint is satisfied. Therefore, one needs to calculate the AC at $t = 1$; the exact relation between the introduced terms will we explained later.

An important variable, closely alleviated with the interest rate, is the discount curve (or zero-coupon curve). The discount curve has an impact on both the assets and liabilities of an insurance company. When forecasting the value of future liabilities, the time-value of money can have a serious impact on an insurer's obligations.

In this paper a profit-sharing product is selected. A profit sharing product is a popular life insurance product, where clients are also profiting from the return the company makes on its assets. A guaranteed rate is issued, allowing policyholders to save capital. Whenever the Forward Rate exceeds this guaranteed rate, the policy holder partly benefits from this surplus. If the return is below the certain guaranteed level, $r_G(t)$, the guaranteed rate is issued. The simplified pricing of this profit sharing product and determination of the guaranteed rate is done by comparing a one years Forward Rate, denoted by $f(t, 1)$ with the guaranteed rate. The one years forward rate is defined by:

$$f(t, t+1) = \frac{d(t)}{d(t+1)} - 1, \tag{2.3}$$

in which $d(t)$ denotes the discount factor for a certain time t.

As already explained, clients will receive either $r_G(t)$ or a rate where they profit from a return greater that the guaranteed rate. This relation can be written as:

$$r_{APS}(t) = \begin{cases} r_G(t) + G \cdot (f(t, t+1)(t) - r_G(t)), & \text{for } f(t, t+1) \geq r_G(t) \\ r_G(t), & \text{else} \end{cases} \tag{2.4}$$

in which G denotes the fraction of the return above the guaranteed rate that will be shared and the $r_{APS}(t)$ the resulting rate after profit sharing.

Now that we know the surplus of the profit sharing policy, lets define the cash-flows concerned with it. The accounting reserve, denoted by $CF_{ACR}(t)$, values the capital that is reserved in the future. This accounting reserve is defined as:

$$CF_{ACR}(t) = CF_{ACR}(t-1) \cdot (1 - R_L - R_M) \cdot (1 + r_{APS}(t)), \tag{2.5}$$

in which the variable R_L denotes the lapse rate. Lapse risks are the obligations that flow out of a percentage of the clients choosing to stop the policy (the outflow). Furthermore, the variable R_M denotes the mortality rate; capital that is reserved for policyholder that pass away. For most life insurance policies, a certain amount of money is paid out whenever the client dies or after a certain maturity. Again, we only model interest rate risk and assume R_L and R_M fixed rates. We assume $CF_{ACR}(0)$ is known and equals a positive number. Furthermore both R_L and R_M are assumed to be fixed annual percentiles of the total clientèle.

In order to estimate the outgoing cash flows every year, the variable $CF_L(t)$ is introduced. It is defined as:

$$CF_L(t) = CF_{ACR}(t-1) \cdot (R_L + R_M). \tag{2.6}$$

The final estimate of your liabilities (also known as Best Estimate Liabilities) can be regarded as the sum of all the $CF_L(t)$, adjusted for the time value of money with a time varying discount rate, in other words:

$$L(T = 0) = \sum_{t=1}^{\tau} d(t) \cdot CF_L(t). \tag{2.7}$$

The corresponding duration is defined as:

$$D_L(T = 0) = \sum_{t=1}^{\tau} d(t) \cdot CF_L(t) \cdot t. \tag{2.8}$$

On the assets side, we assume that the portfolio only contains government bonds. In this way we are able to isolate behaviour of future cash flows adequately. The government bonds are assumed to have a maturity τ, a certain issued coupon $C_{GB}(t)$ and a notional N_{GB}. A typical resulting cash flow of such a bond can be observed in Figure 2.1.

The mathematical interpretation is denoted in Equation 2.9. When t is smaller than the maturity of the bond τ, it would yield the coupon. On its maturity it yields its notional.

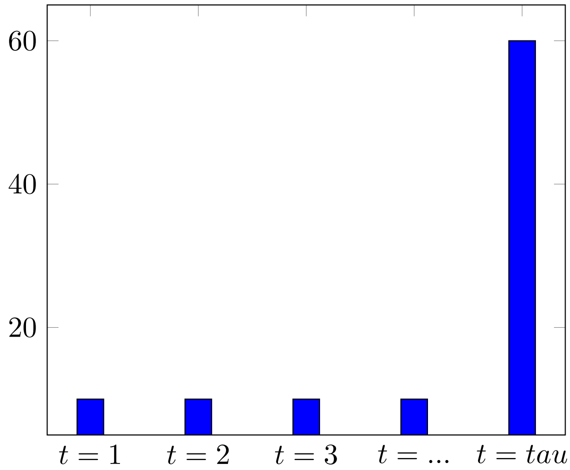


Figure 2.1: Cash flows of a normal government bonds until maturity. The bonds yields an annual coupon to the bondholder, until it has reached its maturity on $t = \tau$, on which moment the notional is given out.

$$CF_{GB}(t) = \begin{cases} N_{GB} \cdot C_{GB}(t), & \text{if } 0 < t < \tau \\ N_{GB} \cdot (1 + C_{GB}(t)), & \text{if } t = \tau \\ 0, & \text{else.} \end{cases} \tag{2.9}$$

Again, to quantify the magnitude of the assets the cash flows are discounted,like:

$$A(T = 0) = \sum_{t=1}^{\tau} d(t) \cdot CF_{GB}(t). \tag{2.10}$$

Consequently the duration rolls out:

$$D_A(T = 0) = \sum_{t=1}^{\tau} d(t) \cdot CF_{GB}(t) \cdot t. \tag{2.11}$$

So far, all calculations lead to the current value of our assets and liabilities at $T = 0$. However, since we want to predict the behaviour of those values in the future, we'll have to cope with $T = 1$. This implies that some of our variables are calculated slightly different.

First of all, the new premiums, NP, are defined as $NP(T = 1) = L(T = 0) \cdot g(T = 1)$. In this equation the growth of the product is assumed to be a known parameter. Knowing the NP, you can easily obtain:

$$IN_{GB}(T) = NP(T) - CF_L(1|T-1) + CF_{GB}(1|T-1), \tag{2.12}$$

where $IN_{GB}(T)$ denotes the new inflow into the portfolio. Equation 2.12 is essential, since it links the asset side of the portfolio with the liabilities side of the portfolio. The forecasted interest curve, denoted by $r(t)$, can be determined using the discount rate at a certain t:

$$r(t) = \left(\frac{1}{d(t)}\right)^{1/t} - 1. \tag{2.13}$$

In Section 2.2 adjustments on the interest rates are implemented into these interest rates.

Obviously, the new inflow of premiums have an effect on the magnitude of previous calculated variables. The accounted capital reserve and the guaranteed rate are adjusted as:

$$CF_{ACR}(t = 1|T) = NP(T) \cdot (1 - R_L - R_M) \cdot (1 + r_G(T)), \tag{2.14}$$

$$r_G(T) = r_G(T-1) \cdot CF_{ACR}(1|T-1) + \frac{r(D_L) \cdot CF_{ACR}(1|T)}{CF_{ACR}(1|T-1) + CF_{ACR}(1|T)}. \tag{2.15}$$

It can be observed that the values of $r_G(T)$ and $CF_{ACR}(t = 1|T)$ should be obtained iteratively. Interpretatively, adjustments on the guaranteed rate are made, based on new premiums on the expected movement on the interest rate. Also the cash flows on the liabilities side are influenced by the new premiums:

$$CF_L(t|T) = \begin{cases} CF_L(t+1|T-1) + NP(T) \cdot (R_L + R_M), & \text{if } t = 1 \\ CF_L(t+1|T-1) + CF_{ACR}(t-1|T) \cdot (R_L + R_M), & \text{if } t > 1 \\ 0, & \text{else.} \end{cases} \tag{2.16}$$

Moreover, we can calculate that:

$$CF_{GB}(t|T) = \begin{cases} NI(T) \cdot r(T) + CF_{GB}(t+1|T-1)(R_L + R_M), & \text{if } t < \tau \\ NI(T) \cdot (1 + r(T)), & \text{if } t = \tau \\ 0, & \text{else.} \end{cases} \tag{2.17}$$

From there on, the $L(T)$ and $A(T)$ can be calculated similarly using respectively Equations 2.7 and Equation 2.10. This summarizes the product's cash flows calculations for one year. The chosen product only serves as a example to measure the performance of the different approaches. One might imagine that in reality both the asset- and liabilities side of an insurer are much more complex and a great variety of products exist.

2.2 Real-world interest rate scenarios

In this section, the approach used to generate real-world interest rate scenarios is discussed. The calibration of the model is discussed, and how the model is used for sampling y simulating selected risk-drivers.

2.2.1 The model

To generate the forecasts for the interest rates, we simulate different yield curves based on historical observations. To keep the model agile, the Diebold-Li model (Diebold & Li, 2006) is used. In order to use the Diebold-Li model, first the Nelson-Siegel model (Nelson & Siegel, 1987) should be considered, which equals:

$$y = \beta_0 + (\beta_1 + \beta_2)\frac{\tau}{m}(1 - e^{\frac{-m}{\tau}}) - \beta_2 e^{\frac{-m}{\tau}}. \tag{2.18}$$

This model was the start point for Diebold & Li (2006). They have rewritten the above expression into a model, which is able capture the dynamics of the yield curves. This model, also known as the Dynamic Nelson Siegel model is suitable when one is interested in forecasting yield curves (Christensen et al., 2011). The model is defined as:

$$y_{t,t+\tau} = \beta_{1t} + \beta_{2t}\left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_{3t}\left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right), \tag{2.19}$$

in which τ denotes the maturity of the bond, λ a given constant and $\{\beta_{1,t}, \beta_{2,t}, \beta_{3,t}\}$ factors through the different maturities. These factors have also an interpretation: β_{1t} denotes the long term *level* of the yield curve, β_{2t} denotes the corresponding *slope* and β_{3t} the *curvature* of the yield curve. A visual representation of these factors can be found in Figure 2.2. The magnitude of λ is fixed and Diebold et al. (2006) have shown that $\lambda = 0.0609$ in their selected time frame, for monthly observations. We apply this value in our calculations, neglecting the fact that it might be incorrect given our different range.

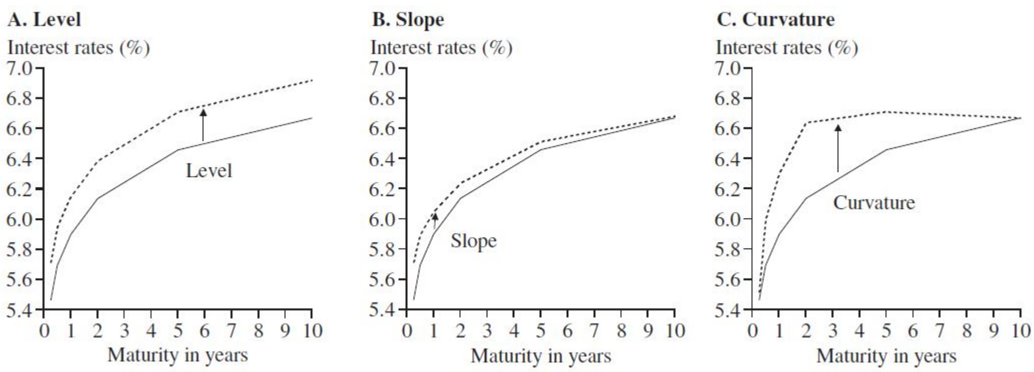


Figure 2.2: The effect of changes in the different Diebold-Li factors on a yield curve. The level factor is denoted by β_1, the slope factor by β_2 and the curvature factor by β_3. Obtained from Wu (2003).

2.2.2 Forecasts

Furthermore, Diebold & Li (2006) extend the model towards a forecasting methods, by implying that the corresponding estimators have predictive parameters. They propose both an autoregressive model (AR) and vector-autoregressive (VAR) process to obtain forecasts. We consider the risk factors simultaneously and select scenarios in which the SCR crosses an certain threshold, thus realistic combinations of the factors should be generated. Furthermore, given the chaotic economic circumstances of the last decade, we cannot ignore possible recent cross-correlation between the different factors. In addition, the VAR model imposes more flexibility than the AR model. We therefore use a VAR model in which $\{\hat{\beta}_{1,t}, \hat{\beta}_{2,t}, \hat{\beta}_{3,t}\}$ follow an VAR(1) process, defined as:

$$\hat{\beta}_{n,t+h|t} = \hat{\mu}_n + \hat{\Gamma}_n \hat{\beta}_{n,t}, \tag{2.20}$$

where $n = \{1, 2, 3\}$, h denotes the forecast horizon, $\hat{\mu}_n$ are model constants and $\hat{\Gamma}$ is a vector which contains estimators for the (cross) correlations. For the estimation of the parameters two-step OLS will be used. The significance of the estimated model parameters will be tested.

Once we have estimated our model, we will be able to generate m different real-world interest rate scenarios, by drawing from a normal distribution of the error term $\hat{\nu}_{n,t}$, which can be added to the estimated Equation 2.20. Since we can easily determine the discount curves from zero-coupon bond prices, the interest rate from Equation 2.13 is affected by a different yield curve. In order to create a yield curve based on the maturities at a certain date, some simple handlings are performed. Between the maturities, a basic spline interpolation is used, to form a continuous curve. We assume that interest rates exceeding the last known maturity, equal the last observed interest rate (in our case observations with a maturity of 30 years). Hence, a flat extrapolation of the yield curve is performed, a general accepted way to extrapolate the curve. This whole process will ultimately lead to a changes in the value of the assets and liabilities of the insurance company.

2.3 Risk-neutral interest rate scenarios

Numerous authors have claimed that, due to the complexity of the profit sharing product, a closed form valuation of a profit sharing portfolio is not feasible (f.i. Bauer, Bergmann, & Kiesel (2010) or Corsaro et al. (2008)). Therefore, a numerical approach based on a large number of (interest) scenarios is a general accepted way to value these products. This section focuses on sampling scheme of risk-neutral scenarios. A model to generate these scenarios is proposed, based on the paper of Hull & White (1990). The relation between the real-world scenarios and the risk-neutral scenarios is discussed in Section 2.4.

2.3.1 The model

In order to generate risk-neutral scenarios, also known as "inner" scenarios in a SCR calculation, a Hull White 1-factor model is used. The Hull-White 1-factor model (Hull & White, 1990) is regarded as an extension to the Vasicek model (Vasicek, 1977), introducing an approach to incorporate the initial term structure as well. The model assumes no-arbitrage (as will be used in Section 2.4) and models the instantaneous short rate $r(t)$, which equals the $f(t, t)$. The behaviour of this short rate can be captured as a Stochastic Differential Equation(SDE):

$$dr(t) = (\theta(t) + \alpha r(t))dt + \sigma dW^{\mathbb{Q}}(t), \tag{2.21}$$

where $\theta(t)$ denotes a deterministic function of the time to fit the initial term structure, $dW^{\mathbb{Q}}(t)$ a Brownian motion in a $\mathbb{Q}$-measure (see Section 2.4), α a constant denoting the mean-reverting speed and σ a instantaneous volatility constant. Not making α and σ time-dependent, prevents over-parametrization and instability of the parameters Carverhill (1995) and this adjustment will be adopted in our process as well.

The general relationship between bond prices and forward rates is defined by:

$$f^M(0,T) = -\frac{\delta}{\delta T}\log P^M(0,T), \tag{2.22}$$

in which $P^M(0,T)$ denotes the market discount factor for a maturity T and $f^M(0,T)$ the market instantaneous forward rate at t=0. The SDE of Equation 2.21 can, under some technical conditions, be integrated into:

$$r(t) = r(0)e^{-\alpha t} + \int_0^t \theta(u)e^{-\alpha(t-u)}du + \int_0^u e^{-\alpha(t-u)}dW(u). \tag{2.23}$$

Hull & White (1990) show that this process matches the initial term structure if we define $\theta(t)$ as:

$$\theta(t) = \frac{\delta f(0,t)}{\delta t} + \alpha f(0,t) + \frac{\sigma^2}{2\alpha}(1 - e^{-2at}). \tag{2.24}$$

A rather more intuitive notation of the process can be seen in the book of Pelsser (2000), where he rewrites the process into:

$$r(t) = y(t) + \varphi(t), \tag{2.25}$$

in which:

$$\varphi(t) = f(0,t) + \frac{\sigma^2}{2\alpha}(1 - e^{-\alpha t})^2, \tag{2.26}$$

and

$$dy = -\alpha y dt + \sigma dW^{\mathbb{Q}}, \tag{2.27}$$

where $y(0) = 0$.

It can be shown that the short term model is normally distributed with:

$$r(t) \sim N\Big(\big[f(u,t) + \frac{\sigma^2}{2\alpha^2}(1 - e^{-\alpha(t-u)})^2\big], \big[\frac{\sigma^2}{2\alpha}(1 - e^{-2\alpha(t-u)})\big] \Big). \tag{2.28}$$

The assumption that the Brownian motion is normally distributed (explained in Section 2.4), implies that negative interest rates can be sampled, given that the probability that a negative short rate is sampled equals:

$$\begin{aligned}\Pr(r(t) < 0) &= \Pr\left(\sqrt{Var(r(t))} \cdot Z + E[r(t)] \leq 0\right) \\ &= \Phi\left(-\frac{\varphi(t)}{\sqrt{\frac{\sigma^2}{2\alpha}(1 - e^{-2-\alpha t})}}\right).\end{aligned} \tag{2.29}$$

This probability is larger than zero, hence negative rates do occur. This is generally regarded as one of the main drawbacks of the one factor model.

Under the so-called "affine assumptions" we are able to construct the bond prices in the HW-model, again using Pelsser (2000):

$$P(t,T) = A(t,T) \cdot e^{-B(t,T) \cdot r(t)}, \tag{2.30}$$

where

$$A(t,T) = \frac{P^*(0,T)}{P^*(0,t)} \exp(B(t,T) \cdot f^*(0,t)) - \frac{\sigma^2}{4\alpha}(1 - e^{-2\alpha t} B^2(t,T)), \tag{2.31}$$

and

$$B(t,T) = \frac{1 - e^{-\alpha(T-t)}}{\alpha}. \tag{2.32}$$

This is a popular representation of the process, widely used within risk-neutral scenario generation.

2.3.2 The simulation

Now that we have got a model, sampling is possible. One of the interesting outcomes for our purposes are priced zero-coupon bonds, given that discount rates can be derived from there on. The simulation follows a number of steps, denoted in this section. These steps are based on the original Hull-White paper (Hull & White, 1990) and Pelsser (2000).

The parameters σ and α are linked to market data of either swaptions, caps or floors and can be calibrated using these instruments. A detailed calibration process can be read in Appendix C. We, however, fix the mean reversing constant and implied volatility parameter, which will be explained in Section 2.3.3.

In the previous section, we have explained the continuous Hull-White model. However, for our simulation purposes, discretization of the model is required, so Equation 2.21 can be rewritten into:

$$\delta r = (\theta(t) - ar(t))\delta t + \sigma Z \sqrt{\delta t}, \tag{2.33}$$

in which $r(t)$ is the short rate, Z is a random variable from the standard normal distribution δt is a small time step and $\theta(t)$ is defined in Equation 2.24.

The initial zero-curve can be expressed in the initial forward curve using:

$$
\begin{aligned}
f(0,t) &= R + t\frac{dR}{dt} \\
&\approx_{\lim_{\delta t \to 0}} R(t+\delta t) + t\frac{R(t+\delta t - R(t))}{\delta t},
\end{aligned}
\tag{2.34}
$$

in which R(t) denotes the initial zero curve. Hence, from there on, Equation 2.24 can be rewritten into:

$$
\theta(t)\delta t = F(0,t+\delta t) - F(0,t) + aF(0,t)\delta t + \frac{\sigma^2}{2\alpha}(1-e^{-2\alpha t})\delta t. \tag{2.35}
$$

This means that the short rate can be modelled as:

$$
r((n+1)\delta t) = r(n\delta t) + \theta(n\delta t)\delta t - ar(n\delta)\delta t + \sigma Z_{n+1}\sqrt{\delta t}, \tag{2.36}
$$

with r(0) = R(0) and n the time step in the future. The short rate can be constructed using small time-steps δt, thus the discrete model is defined.

Now using Equation 2.33 - 2.36, we are able to obtain the deliverable $P(t,T)$. Obviously, to change this value into the simulated interest rate, one should calculate:

$$
R(t,T) = \frac{-\ln P(t,T)}{T}. \tag{2.37}
$$

2.3.3 Estimation of the model

Before the simulation phase begins, the model should be estimated using market data. However, two important parameters have not been determined yet; the mean-reversion constant α and the implied volatility σ. A choice should be made whether the parameters should be calibrated or fixed.

There are several options to determine the values of the constants when one would choose a calibration of the risk-neutral scenario set. We could either use (Euro) caps, swaptions or floors. In the book of Brigo & Mercurio (2006), a simple model is showed calibrated Euro caps data. This model turns out to be mean-diverging (due to a second mean-reverting factor incorporated in cap prices), hence not a realistic outcome. A very clear paper regarding the different calibration methods of a Hull-White one factor model is written by Gurrieri et al. (2009). In the end, the use of payer swaptions for calibration are preferred.

On the other hand, the created model should stay feasible. A shortcoming of the above methods is the computational power required. Using a correct calibration of the model's mean-reversion and volatility of the Full Nested approach (based on payer swaptions for example), implies also using it for the Least Squares Monte Carlo model. While keeping the calibration approaches similar, direct comparison between the methods are allowed. The amount of outer scenarios for the LSMC approach however, is usually a multitude compared to FN approach. This implies that calibration should be performed for every outer scenario. This is a costly exercise computational wise. Furthermore, a correct calibration of the risk-neutral set is a rather complex exercise. There are methods to simply fix the parameters for every real-world scenario, and although these methods are not always recommended, this approach is used in our model. The values of α and σ are determined, using the paper of Gurrieri et al. (2009). Although no explicit advise is given, some values are mentioned. The fixation of the parameters can be considered as a shortcoming of the model, which is further discussed in Chapter 6. A calibration scheme based on payer swaptions, is suggested in Appendix C.

2.3.4 Validation of the scenario sets

In the previous sections we have described how to form a risk-neutral scenario set for a given real-world scenario. In our model, we have chosen to fix the values of α and σ. However, we have not touched the validation of the outcome. In this section we will test an important assumption of the scenario set, namely the lack of arbitrage in the model (which is necessary for the proposed method of Section 2.4). This assumption will be tested using Martingale's Theorem. An application of Martingale's theorem (Also known as the "1=1" rule), basically tests whether the average discounted present values of the cash flows are equal to its current value. (Wyatt, 2007). A Doob martingale is a stochastic process $M(t)$, which satisfies the following "fair-game" assumption, (assuming some technical assumptions are satisfied):

$$E[M(t+s)|\mathscr{F}_t] = M(t), \tag{2.38}$$

in which $\mathscr{F}_t$ is a σ-algebra, discussed in Section 2.4. In an economical framework, the above assumption implies the absence of arbitrage in the scenario set. In the case of our risk-neutral scenario set, Martingale's Theorem states, given the last item in the above list:

$$E_Q\left[M(t)\right] = 0, \tag{2.39}$$

in the case that:

$$E[M(0)] = 0. \tag{2.40}$$

So at $t = 0$ the discount factor is by construction zero, implying that the expected value of the discount curve over other maturities is zero as well. In order to handle the increasing volatility in time of the scenarios, a time-dependent confidence level is proposed, based on (ActuarialModeling, 2013). The confidence bands can be defined as:

$$CI(t) = \pm\, t_{n-1,1-\alpha/2} \cdot \frac{\mathrm{SD}(M(t))}{\sqrt{n}}, \tag{2.41}$$

where $t_{n-1,1-\alpha/2}$ denotes a quantile of a Student-distribution, α is the confidence level, $\mathrm{SD}(M(t))$ Standard Deviation of the scenario set and n the number of observations.

2.4 Full Nested approach

Now that we have both real-world scenarios and risk-neutral scenarios, we could combine these scenarios into a forecast, computing the required available capital in one year (SCR). In this section, the so-called "Full Nested" approach (FN) is discussed, combining these introduced scenario sets.

In Section 2.2 a general approach is discussed to simulate real-world interest rate scenarios, using a Diebold-Li model. These simulations are sometimes called "outer scenarios". Furthermore, in Section 2.3 we have imposed a method for generating risk-neutral valuation scenarios. These simulations are sometimes called "inner scenarios". We will now show why the combination of outer- and inner scenarios will generate reliable estimates of the capital required given a set of risk-drivers.

Let us assume a complete filtered probability set, denoted by $(\Omega, \mathscr{F}, \mathbb{P}, \mathbf{F} = (\mathscr{F}_t)_{t\in[0,T]})$, in which T denotes the maturity of the liabilities and Ω represents the space with all possible states of the financial market. The $\mathscr{F}_t$ denotes a σ-algebra; a filtered probability space consisting of the history of the market up until time t. The $\mathbb{P}$ denotes an objective market-measure, reflecting the real-world uncertainty of the market. Some technical conditions are satisfied in $\mathbf{F}$.

Moreover, let us assume a collection of asset price state processes $(Y_t)_{t\in[0,T]}$, a numéraire $N(t)$ and a risk-neutral probability measure $\mathbb{Q}_N$, which is "equivalent" to the $\mathbb{P}$-measure. Given the model is arbitrage-free, the risk-neutral pricing formula states that:

$$\frac{V(t)}{N(t)} = E_t^{\mathbb{Q}_N}\left[\frac{V(T)}{N(T)}\middle| \mathscr{F}_t\right], \tag{2.42}$$

in which $V(t)$ denotes the value of a certain defined asset and $t < T$ and $\frac{V(t)}{N(t)}$ a martingale under $\mathbb{Q}_N$.

In our case, the numéraire is defined as the money market account $B(t)$ given by $B(t) = e^{\int_0^t r(u)du}$. This implies that Equation 2.42 can be rewritten into:

$$\begin{aligned} V(t) &= E^{\mathbb{Q}_B}\left[\frac{B(t)}{B(T)}V(T)\middle| \mathscr{F}_t\right] \\ &= E^{\mathbb{Q}_B}\left[e^{-\int_t^T r(u)du}V(T)\middle| \mathscr{F}_t\right], \end{aligned} \tag{2.43}$$

in which $\mathbb{Q}_B$ is the equivalent martingale measure. Note that the numéraire $B(t)$ is equal to the discount rate at time t. Equation 2.43 is an important formula, since it states a relationship between value of a product at a given time and the (discounted) the cash-flows in the future.

From Equation 2.43, we are able to introduce the Full Nested (FN) method. We start by simulating an outer scenario with discounting factors for different maturities. This will lead to a forecasted curve and a state at $T = 1$. A prediction of how the interest rate will move in the coming years is a result. In order to value our profit sharing product at that instance, we'll have to generate interest rate scenarios for a large amount of years in the future, until $T = T^*$. For every interest rate scenario, the calculation discussed in Section 2.1 is performed. This is done for a large number of outer scenarios. In this way, for every outer scenario, a corresponding value or AC_t is calculated.

Using Equation 2.43 on our model, this implies that the net asset value at time 1, NAV_1, can be written as:

$$NAV_1^i = E^{\mathbb{Q}}\left[\sum_{T=2}^{T=T^*} AC_T \cdot d(T) \middle| X^i\right], \tag{2.44}$$

in which $d(t)$ denotes a discount scenario based on the interest rate and X^i a (interest) state vector with certain risk-drivers, (in our case the Diebold-Li factors) under the i-th interest rate. Since we have simulated our risk-drivers based on historical data, we know that $X^i = f_t(Y_{s\in[0,t]})$. A similar line of thought can be found in Bauer, Bergmann, & Reuss (2010).

Given our complex relation between the development of the market and both the assets and liabilities of the insurer, an analytical solution of Equation 2.44 is not feasible. However, an application of Girvanov's (Girsanov, 1960) theorem is used to come to a solution.

Suppose that a process X_t is given by:

$$dX_t = \mu(t, X_t)dt + \sigma(t, X_t)dW_t^{\mathbb{P}}, \tag{2.45}$$

and that we have constructed a Brownian motion under $\mathbb{Q}$:

$$dW_t^{\mathbb{Q}} = \lambda_t dt + dW_t^{\mathbb{P}}, \tag{2.46}$$

than, using Girsanov's theorem, under some technical conditions, we are allowed to change the measure in Equation 2.45 (changing the drift) into:

$$dX_t = (\mu(t, X_t) - \sigma(t, X_t)\lambda_t)dt + \sigma(t, X_t)dW_t^{\mathbb{Q}}. \tag{2.47}$$

This implies that, if we generate M risk-neutral scenarios, using $\Delta dW_t^{\mathbb{Q}} \sim N(0, \Delta t)$, we are allowed to calculate the X_T^i:

$$E^{\mathbb{Q}}[f(X_t)] \approx \frac{1}{M}\sum_{i=1}^{M} f(X_t^i). \tag{2.48}$$

Given that we use M inner scenarios and using the similarities of Equation 2.47 and the Hull-White model defined in Equation 2.21, we can use the above expressions to rewrite 2.44 into:

$$N\hat{A}V_1^i(M) = \frac{1}{M^i}\sum_{j=1}^{M}\sum_{T=2}^{T=T^*}\left[AC_T \cdot d(T)^{(i,j)}\right] - N\hat{A}V_0. \tag{2.49}$$

In this equation $d(T)^{(i,m)}$ denotes the discount rate at time T in sample path j given the risk-drivers i. The $N\hat{A}V_0$ denotes the value of the product at $T = 0$. When forecasting, a risk-

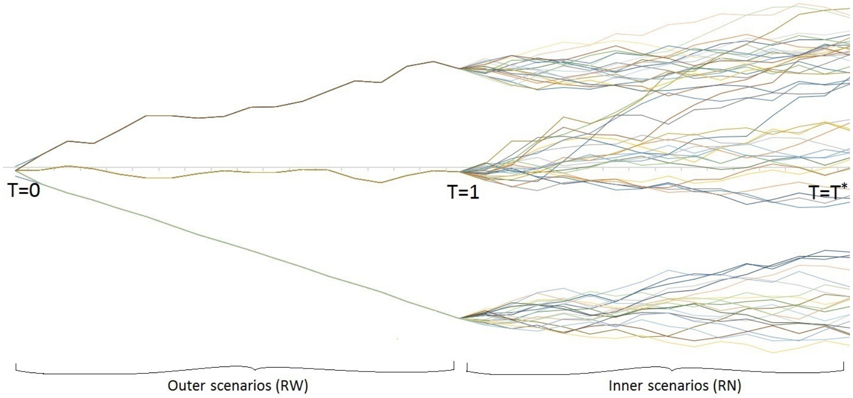


Figure 2.3: A schematic overview of the relationship between outer- and inner scenarios in the Full-Nested (FN) approach. On each forecasted real-world scenario for $T = 1$ ("outer scenario"), with underlying simulated risk-factors, a tree of risk-neutral scenarios ("inner scenario") is implemented until T^* to value its effect.

neutral tree is used to value the product at the moment, resulting in a constant value throughout the real-world scenarios. The corresponding sample standard deviation $\hat{\sigma}_1^i(M)$ is defined as:

$$\hat{\sigma}_1^i(M) = \sqrt{\frac{1}{M^i - 1}\sum_{j=1}^{M}\left(\sum_{T=2}^{T=T^*}\left[AC_T \cdot d(T)^{(i,j)}\right] - N\hat{A}V_1^i(M)\right)^2}. \tag{2.50}$$

In Figure 2.3 the Full Nested approach (FN approach) is illustrated. In this figure, T^* denotes a time in the future (typically 30 years) for which risk-neutral interest scenarios are generated. Furthermore, for illustrative purposes, only three outer scenarios are depicted in this figure. However, when the analysis is really performed, typically more than 5000 RW scenarios (and 5000 RN scenarios) are required to obtain robust estimates. The nesting of the inner scenarios in every outer scenario, is why this approach is referred to as "Full" Nested.

Our definition of "FN approach" deserves a short further explanation. Due to the small uncertainty that remains in the valuations (explained later) and due to the continuous nature of many risk-drivers, a truly "full nested" valuation is not feasible here. However, if we make sure that the amount of valuations is large (more than 5000), the resulting uncertainty between data in fits will be neglect-able. Hence, we will regard the resulting fit as the "FN approach".

The results of this approach can be regarded as being exact (Vedani & Fabien, 2013). However, in practice this approach turns out to be non-feasible. The computational power requested for this set-up, a full real-world scenario set on a full risk-neutral scenario set, for an insurance portfolio is very high. This means that the time needed to calculate the risk under certain circumstances, is too high. Zhao (Zhao, 2013) estimated the time needed for the calculation in the case of 1000 outer scenarios on 1000 inner scenarios and 1000 liability points. Using 100 modern CPU's, this calculation would take 58 days. This implies that consequences of the managerial decisions for these insurance products, are difficult to measure.

2.5 Least Squares Monte Carlo approach

When considering the previous section, the urgency for an alternative approach is clear. There exist several accelerated convergence techniques in this context, such as curve fitting and replicating portfolios. However, in this thesis we focus on the use of a popular approach, called the Least Squares Monte Carlo (LSMC). We will start by explaining the intuition of this approach in this section, followed by the curve fitting methodology.

2.5.1 The model

The idea of Least Squares Monte Carlo originates from Longstaff & Schwartz (2001). In their paper *"Valuing American Options by Simulation: A Simple Least-Squares Approach"*, they introduce a new way to value options of the American kind. The investigated American options are callable; they can be surrendered at any time by the owner of the option. Since the value of these options do have a accepted closed-form solution introduced by Black & Scholes (1973), the accuracy of the method can be investigated.

In the paper of Longstaff & Schwartz (2001) they try to minimize the uncertainty of the prediction of the value of the option, by implying a structured regression across different risk-neutral paths. They start from the ultimate maturity of the option and work backwards in time steps. At every step the immediate value of the exercised option is compared with the expected cash flow of the option when continuing. In the case the immediate value is higher than the expected value, the option is called. The ultimate deliverable is an optimal time to call the option.

The general idea of LSMC is as follows. Although the name of the approach is adopted from Longstaff & Schwartz (2001), the way the approach is implemented in an insurance environment, is slightly different. Instead of using a large number of inner scenarios for every outer scenario (the FN approach), we now use only one inner scenario for every outer scenario. Although this method will result in inaccurate valuations when taking observations separately, a whole set of these valuations can reveal a certain pattern. When all valuations are collected, we are able to fit a curve through these valuations. The idea is that this fit results in an comparable valuation for the selected risk-drivers (in our case fluctuation of the interest rate). From this fit, a certain lower boundary can be selected to identify the 1-in-200 year scenario.

Another popular ACM available for valuing complex products, is the "replicating portfolio" technique. This approach is somewhat more practical; one would try to replicate the cash-flows of the complex product, using instruments which have known closed-form valuations. When suitable instruments are found that walk parallel with the portfolio's cash-flows, these instruments are valued; the resulting value is assumed to be equal to the value of the underlying product. One of the main shortcomings of this technique, is finding suitable instruments for exotic products, especially when incorporating non-market risk (Hörig & Leitschkis, 2012). Finding proper instruments for more complex portfolios with embedded optionality and non-linearity, can be very challenging. In these cases, the use of a risk-neutral scenario set is more or less inevitable, making LSMC an attractive alternative.

LSMC approach is suitable for an illustrative explanation as well. In Figure 2.4 one can see the limited number of inner scenarios per outer scenarios that are used under LSMC. Every different interest rate outer scenario i, based on risk-driver X_i, leads to a new situation. Now, to value the real-world scenario, instead of using a large number of inner scenarios, only one inner scenario is sampled to simulate interest rates. This implies that the computational power is much less compared to the FN approach.

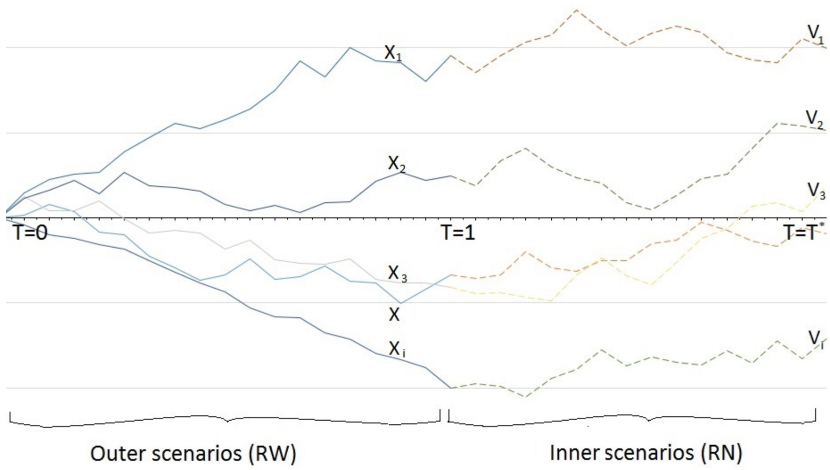


Figure 2.4: A schematic overview of the relationship between outer- and inner scenarios in the Least Squares Monte Carlo (LSMC) approach. On each forecasted real-world scenario for $T = 1$ ("outer scenario"), with underlying simulated risk-factors, a only one risk-neutral scenario ("inner scenario") is implemented, until T^*, to value its effect. A fit upon these inaccurate valuations will lead to trustworthy results.

2.6 Non-parametric fitting

An important aspect of both the FN process and the LSMC process, is finding an accurate fit. Although we have only three explanatory variables, the technique introduced in this section, could be implemented in circumstances with more risk-drivers (f.i. incorporating spread risk, lapse risk, longevity risk etc.). The non-parametric fitting scheme enables us to consider these risk-drivers simultaneously, without assuming constant correlation among them. This is an advantage compared to generic parametric fittings, where risk-drivers are often considered independently.

Estimator

We have introduced three interest rate variables, X_1, X_2 and X_3, representing the three factors of the Diebold-Li model $(\beta_1, \beta_2, \beta_3)$ (see Section 2.2). Our expectation is that these factors explain the volatility of the Net Asset value (NAV), defined as the dependent parameter Y. In this multi-dimension context, a non-parametric method is introduced for fitting purposes. In non-parametric regressions the form of the fit is solely based on the observations, and not on a predefined function. Moreover, since our dependant variable roles out of a non-linear calculation, no assumptions regarding the distribution can be made. Especially in situations with a large number of risk-drivers as input, restrictions concerning the distribution is not preferable. Non-parametric approaches do not require a defined distribution and is therefore more robust compared to its parametric equivalent.

One of the properties of the proposed VAR model, is that correlation may exist between our sampled risk-drivers. Considering this characteristic, a suitable estimator proposed by Scott (1992), is the Nadaraya-Watson estimator (Nadaraya (1964) and Watson (1964)). In their analysis they derive a regression estimator in one dimension $\hat{m_h}$ as:

$$\hat{m}_h(X) = \frac{\sum_{i=1}^{n} K_h(X - X_i)Y_i}{\sum_{i=1}^{n} K_h(X - X_i)}, \tag{2.51}$$

in which n denotes the total number of observations, K denotes the Kernel function and h denotes the bandwidth of the fit. Nadaraya (1964) has showed that the estimator is asymptotic normally distributed. Furthermore, consistency of this estimator was proven by Yamato (1973). Precise conditions for the asymptotic normality of the estimator were provided by Ralescu & Sun (1993). A more extensive overview of characteristics of the estimator and corresponding kernels is given in the paper of Cheng & Sun (2006).

Kernel

A logic choice for a kernel function would be the Gaussian function, since our underlying distribution is normal distributed. However, this Gaussian function also puts a relatively high weight to observations in the tails of the kernel. In our fits, especially the LSMC fit, the NAV-outliers should not be put a too high weight on, due to their sampled risk-neutral assignation. Therefore the Epanechnikov kernel is introduced (Epanechnikov, 1969). A big advantage of the Epanaechnikov kernel is its limited weight in the far ends; it will only assign a weight to observations in one bandwidth length, where the standard Guassian kernel includes observations that have a distance of two times the bandwidth.

The used Epanaechnikov kernel is defined as:

$$K(u) = \frac{3}{4} \cdot (1 - \left(\frac{u}{h}\right)^2) \mathbb{1}_{|\frac{u}{h} \leq 1|}, \tag{2.52}$$

in which K denotes the kernel weight function, h denotes the bandwidth of the fit and u is a bandwidth-dependant distance parameter. This distance u can be rewritten in the Euclidian space as:

$$u = \sqrt{\frac{X_1}{h_1}^2 + \frac{X_2}{h_2}^2 + \frac{X_3}{h_3}^2}. \tag{2.53}$$

So, more intuitively, u tries to minimize the distance between the observations and the fit, with respect to the corresponding bandwidth. To estimate the confidence interval, a simple bootstrap method is used. In this approach, a large number of re-samples is performed, each with their own fit. Ultimately, the average fit is selected, with their confidence boundaries plotted around it. The confidence level α is set to 5%.

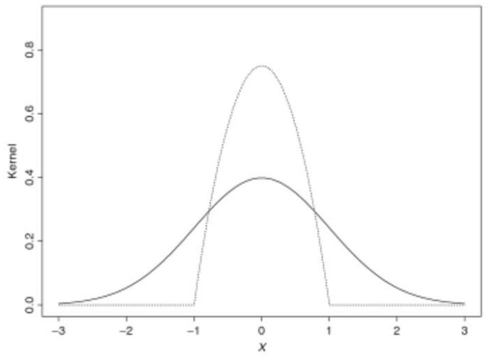


Figure 2.5: This figure shows a Gaussian kernel, the solid line and a Epanechnikov kernel, denoted with the dashed line (Obtained from Tsay (2005))

2.6.1 Multi-dimension

In a lot of the literature the effect of a risk-driver is regarded independently of other considered risk-drivers, especially in LSMC literature. However, it is unlikely that the underlying correlation between the effects of the risk-drivers is always constant. Hence, a method that would enable us to consider multiple risk-drivers simultaneously would yield more accurate results. An extension of our non-parametric model to a multiple dimension analysis (so considering multiple risk-drivers), can be implemented rather straightforward ((Scott, 1992)). Again a so-called binning algorithm is engaged, ensuring a fast and robust computation. Bins represent the amount of segments the range of a risk-driver is divided in. When incorporating all three risk-drivers, the non-parametric multivariate kernel density estimator becomes:

$$\hat{f}_{h_1,h_2,h_3}(x_1, x_2, x_3) = \frac{1}{n}\sum_{i=1}^{n} \frac{1}{h_1}\frac{1}{h_2}\frac{1}{h_3} \cdot \prod_{j=1}^{3} \frac{3}{4}\left\{1 - \left(\frac{x_j - X_{i,j}}{h_j}\right)^2\right\} \mathbb{1}_{|\frac{x_j - X_{i,j}}{h_j} \leq 1|}. \tag{2.54}$$

Our corresponding multivariate Nadaraya-Watson estimator becomes:

$$\hat{m}_{\mathbf{h}(\mathbf{X})} = \frac{\sum_{i=1}^{n} K_{\mathbf{h_x}}(\mathbf{X} - \mathbf{X_i}) \cdot Y_i}{\sum_{i=1}^{n} K_{\mathbf{h_x}}(\mathbf{X} - \mathbf{X_i})}, \tag{2.55}$$

where now instead of single numbers, vectors of observations are considered, compared to Equation 2.51. This approach can be extended to even more dimensions rather straightforward. Hence, the non-parametric fit can be seen as a fitted hyperplane.

The selection of appropriate bandwidths h is a very delicate matter. The bandwidth can be thought of as the width of the bars in a histogram, implicitly used for the estimation. Selecting a too small bandwidth will lead to volatile and noisy fits. Trends and effects are hard to spot, due to the random noise of the fit. However, a too large bandwidth will lead to an over-smoothed fit; the information of certain values of the risk-drivers are taken too far. Again trends and effects are hard to spot, they are flattened out. The "local" property of the Nadaraya-Watson estimator is violated and a bias is the result. There are several (automated) methods for selecting an appropriate bandwidth (f.i. Hyndman et al. (2004)) or rules-of-thumbs (f.i. Scott (1992) and Silverman (1986)). However, due to the limited amount of considered risk-drivers and the special origin of the LSMC data, the bandwidths are selected manually. This ensurers that the bandwidths selected have the right characteristics. The optimal bandwidths are, besides a function of the dimensionality and the sample size, a function of the standard deviation of the risk-drivers. The different rules-of-thumbs are used as a starting point. Further research could lead to appropriate automated algorithms, discussed in Chapter 6.

Another problem that could occur, revolves around the curse of dimensionality. When increasing the amount of dimensions of the non-parametric fit (hence, including more risk-drivers in our case), more data is necessary to ensure a consistent fit. Given the fact that the values are simulated, this implies increasing the amount of observations. An increase of simulated real-world scenarios would therefore be needed, not to lose accuracy. However, this implies an increase in computation time. Future research could revolve around this shortcoming with regards to LSMC, as discussed in Chapter 6.

One might notice that the Least Squares character is diminished, given that no explicit root-squares is taken of the explanatory observations (Equation 2.55). However, Scott (1992) shows that our local constant estimator of the Nadaraya-Watson can be derived from the least-squares criterion, given some technical conditions. Therefore, we are allowed to use the least squares terminology, although we do not use least squares directly in our estimation.

It is worth mentioning that the LSMC fits are expected to be more robust towards problems around dimensionality compared to FN fits. For the LSMC approach, a big number of observations based on different real-world for fitting purposes are needed anyway; also in lower dimensions. Furthermore observations are created rather fast compared to the FN method, making the method presumably more applicable for the LSMC. However, the exact problem and consequences of curse of dimensionality in this context will not be considered in this research.

2.6.2 LSMC performance

When both the FN fit and the LSMC fit are constructed, the performance of the LSMC approach can be evaluated. Since we consider the FN approach as the benchmark value for the set of risk-drivers, the mismatch of LSMC compared to FN denotes the performance of the LSMC fit.

The comparison is performed in two ways. First of all, a visual comparison is executed. Within the fits (both the curves and the plains) confidence intervals are plotted using sampling bootstrapping techniques. The trend, steepness and noise of the fits are considered. This is done for the one and two dimension fits. Both approaches are plotted on top of each other, visualizing the mismatches. This will give us a global idea how the fit performs and where possible problems occur.

For a numerical comparison between the FN fit and the LSMC fit, the bins of the fits are regarded. As already mentioned in the previous section, the bins represent the Y value of a fit/regression. Hence, by selecting a lower quantile of Y values based on its corresponding NAV, we are able to see which combination of risk-drivers leads to the most negative results. The corresponding explanatory values are listed and patterns among the risk-driver are evaluated. This approach will be used when considering two and three risk-drivers, but can be used in situations with more risk-drivers.

However, the selected bins do not necessarily correspond to the amount of observations represented by this bin. Hence, the lowest few bins cannot directly be translated into an absolute percentage of the observations. To solve this representation problem, we consider the lowest percentage of *observations*, from the FN approach. Hence, by linking the bins to this threshold, we are able to see how many observations the bin-set represents.

However, this approach has some drawbacks as well. Under the opposed threshold, no restrictions on the risk-drivers are made. Furthermore, if the LSMC approach would be used stand alone, this approach cannot be used, given the big amount of unreliable valuations. A possible solution for this problem, can focus on the uncertainty in the bins. Every bin would get assigned a weight, based on the number of observations it reflects. This might be linked to the uncertainty around the selected bin. This procedure assumes that the inner scenarios are assigned randomly and bootstrapped with large amount of iterations. However, for our purposes this proposed methodology is out of scope and hence we use the earlier mentioned FN threshold.

2.7 Multi-year framework

In the previous sections we have proposed a method that is able to fit valuations of a sample insurance product in a LSMC non-parametric fashion for one year. In this section we will extend this model into a multi-year context.

As already mentioned in the introduction, the ignorance towards multi-year setting has translated into a scarce amount of publications concerning this topic. While there are some scientific articles published for SCR calculations using LSMC, multi-year extensions are very rare.

One of the only papers that is focused on LSMC in multi-year setting, was written by Horig (2013). By extending previous work on SCR for one year (Leitschkis & Horig, 2012), propose a method that uses LSMC in a multi-year context. The different net assets values are ranked throughout time and divided in three categories, based on the Solvency II coverage ratio. A rough indication is given how the solvability moves throughout time. They are able to couple expected positions of year one to multi-year positions. However, considerations regarding the uncertainty of their results, are missing. In addition, no clear explanation of the techniques followed can be found.

Another article worth mentioning was written by Morrison et al. (2013). They demonstrate how a simple LSMC can be extended to a multi-year environment. Although their results look promising, some remarks concerning the methodology could be made. For instance, the handling of the risk-drivers happens individually. Furthermore, they ignore a growing uncertainty within their calculations. And again, exact methodologies and assumptions of their approach are missing. Very recently, more and more commercial proposals are released revolving around this topic. However, the multi-year analyses are often limited and in addition the exact methodologies are often missing in these paper. Therefore these articles are not considered here.

2.7.1 Extension of method

One of the biggest changes for the multi-year setting, revolves around the approach of prediction horizon h. In the one year forecast, we considered the horizon as a fixed parameter, set up at the beginning of the calculations. Using this parameter, all the calculations are performed. In the multi-year setting however, the forecast horizon is treated differently. We include the forecast period as an added risk-driver, which we let vary throughout the outer scenarios, just like the other risk-drivers. In this manner we are able to see how a bigger forecast horizon influences the net asset value throughout time. The effect on the solvability is observed and interpreted. In our case, this implies adding a fourth risk factor besides the existing Diebold-Li factors. However, since the forecast horizon has only indirect effect on the NAV, we forecast our risk-drivers conditional on a certain horizon. The asset value is now considered as $NAV(\beta_1, \beta_2, \beta_3|h)$. The performance of the LSMC fit is again compared using visual techniques, similar as discussed in Section 2.6.2.

The considered product is not changed, hence the same assumptions are made. We choose to not to use solvability results from the previous years, to enhance future solvability forecasts. The forecasts of the risk-drivers is again simulated using a VAR(1) model.

Chapter 3

Data

In this chapter, the data used in the research is discussed. The sampling of the yield curves is based on historical data described in the first section and the set-up of the sample insurance product is described in the second section.

3.1 Yield curves

We have retrieved the data from the website of the Federal Reserve Bank of St. Louis (`http://research.stlouisfed.org/`, retrieved December 2013). The data is not seasonally adjusted. The selected curves are Zero-Coupon Bonds and have a Treasury Constant Maturity Rate. Observations with a monthly frequency are satisfactory for this application. The original dataset has a timespan from October 1983 to October 2013. The selected curves have a maturity of 3 months, 6 months, 1 year, 2 years, 3 years, 5 years, 7 years, 10 years, 20 years and 30 years. The dataset is depicted in Figure 3.1.

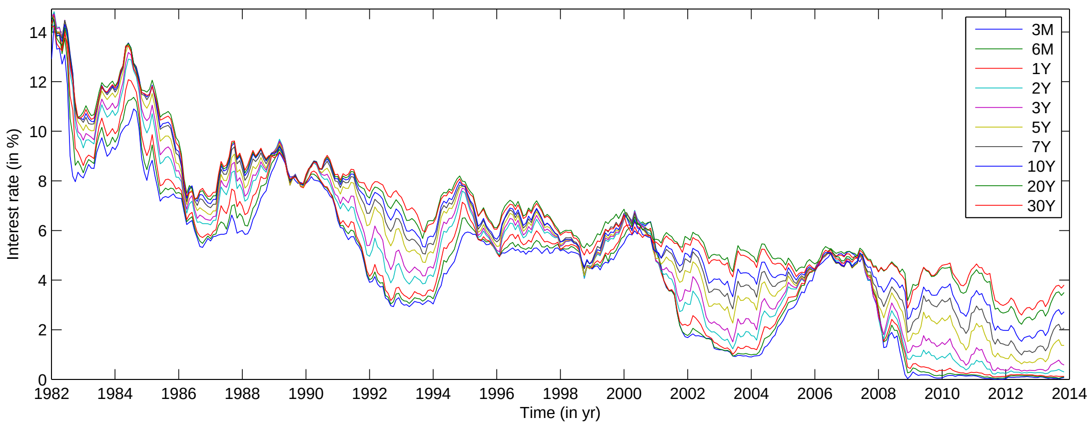


Figure 3.1: The yields for different maturities as retrieved from the FED

The short missing timespan for the thirty year maturity is filled using the volatility of bond of the twenty year maturity. The whole set is used to calibrate the vector autoregressive model.

3.2 Set-up sample product

We will show the performance of the LSMC using a sample product described in Section 2.1. The assigned values of the parameters can be observed in Table 3.1. The values are based on rough assumptions/expert judgements; it is not relevant to what extend the assigned values are very realistic, given our purposes. The big number of variables implies that resulting values have mainly an explanatory function.

Table 3.1: Parameters of the sample with-profit product

Parameter		Value
N_{GB}	Gov. bond notional	950000
τ_{GB}	Gov. bond maturity	10 years
τ_{Lia}	Liability maturity	15 years
$CF_{ACR}(0)$	Accounting reserve	650000
$r_G(0)$	Guaranteed rate	1.8%
R_L	Lapse rate	1.5%
R_M	Mortality rate	1%
G	Profit sharing rate	70%
$g_{T=1}$	Growth of product	3%
α_{HW}	Mean-reversion constant	0.005
σ_{HW}	Implied volatility	0.01

This table shows how the sample product is set-up. Definitions of the parameters can be found in Chapter 2.

Chapter 4

Results

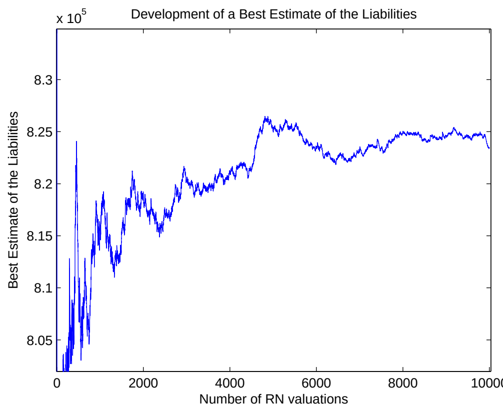
In this chapter, an overview will be given of the results obtained, based on the proposed methods. The majority of the analyses conducted are described in Chapter 2. In Section 4.1 results coming form the elementary cash flow calculations are listed. Estimators used for simulation of the outer scenarios coming from the Diebold-Li model, can be observed in Section 4.2. Furthermore, results from the risk-neutral scenarios, rolling out of the Hull-White model, are denoted in Section 4.3. Next, this chapter includes resulting LSMC fits and its performance, denoted in Section 4.4 and Section 4.4.1. We conclude this chapter with the multi-year extension, results denoted in Section 4.5. All of the analyses were performed in MATLAB (2013).

4.1 Cash flow calculations

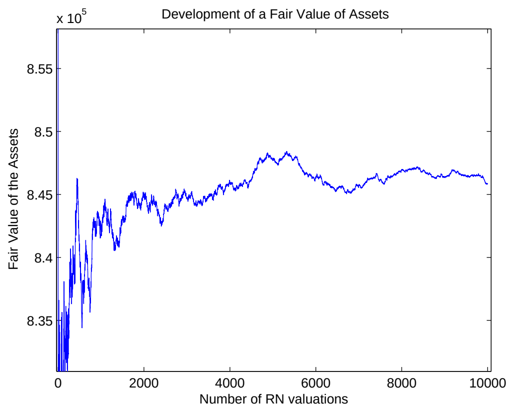
As already mentioned, we will first look at some results from the underlying cash flow equations for the sample insurance with-profit product. These calculations for a profit sharing product show how the proposed techniques perform in practice.

An interesting relation to take a look at, is how the number of risk-neutral valuations of the assets and liabilities of the product interacts with its iterated average value. This will give a rough indication how many inner scenarios are necessary to produce reliable outcomes for the FN approach. This process is depicted in Figure 4.1 for randomly drawn set of risk-drivers. For both the calculation of the Best Estimate of the Liabilities (depicted in Figure 4.1a) and of the Fair Value of the Assets (depicted in Figure 4.1b) a large number of risk-neutral scenarios is needed. We look how the average values develop throughout a total of 10000 risk-neutral scenarios. Similar analyses show that 5000 inner scenarios will give a fairly reliable valuation, taking in regard the magnitude of the y-axes. Therefore, in our research 6000 RN scenarios will be used to value outer scenarios in the FN approach, unless stated different in the individual analyses.

The other cash-flow calculations are performed in the background as well, however none of the outcomes are relevant for the following sections.



(a) Best Estimate of the Liabilities (BEL).



(b) Fair Value of the Assets (FV).

Figure 4.1: Development of the value of the assets (FV) and liabilities (BEL) of our insurance product for a given set of risk-drivers, put against the amount of (risk-neutral) iterations.

4.2 Real-world scenarios

In this section, the results obtained from the simulation of realistic outer yield curves can be observed. Results coming from the calibration, estimation and simulation of the model are listed here.

Using the equation defined in Section 2.2.1, we are able to see how β_1 (depicting the level shift of the yield curve), β_2 (known as the slope of the yield curve) and β_3 (known as the curvature of the yield curve) are moving through time. In Figure 4.2 these historical movements are depicted. Several interesting characteristics can be observed. What immediately strikes the eye is the correlation between the three factors. In general, the slope and curvature move simultaneously up, whenever the level goes down. In times of recession the factor representing the level of the yield curve, tends to move downwards. Investors are generally looking for risk-free investments and consider the long-end of the yield curve safe, pressing the rate (hence level) downwards. Furthermore, the volatility of the curvature has a bigger magnitude compared to the two other factors.

As already mentioned in the methodology section, we have chosen to use a VAR(1) model to estimate the movement of the factor loadings. The estimators of this analysis are denoted in Table 4.1. The different levels of significance are denoted with asterisks. All factors use information from other factors to some extend, given the big number of significant parameters. Furthermore, for all three factors the most important source of information is the lagged value of themselves. This is line with our expectations.

Moreover, these resulting values are in line with observations found in literature (f.i. Diebold et al. (2006)). As expected, the factors are most explained by their own lagged series, given their large values compared to cross-correlations. The slope- and level factors are also able to explain each other to some extend. The fact that all the estimators turned out to be significant, has primarily to do with the range used. Decreasing the sample size leads to a smaller amount of significant estimated parameters.

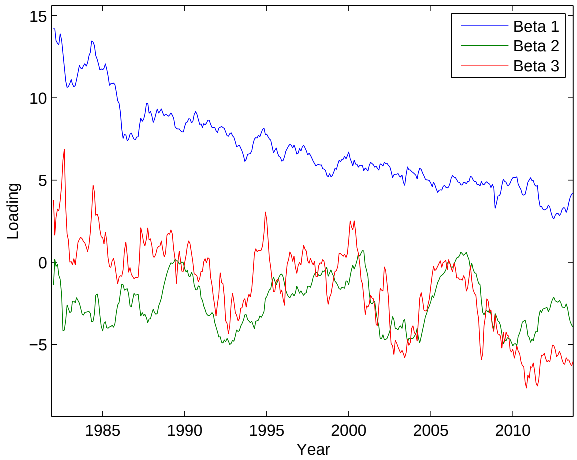


Figure 4.2: The development of the loadings of the three Diebold-Li factors, put against corresponding year. The blue line denotes the first level factor β_1, the second slope factor β_2 is depicted by the green line and red line denotes the third curvature factor β_3.

Although one might question the added value of a VAR model when comparing its results to a simple one-lag autoregressive (AR(1)) model, we will stick to this model. An important reason is for this ignorance, is related to our appliance. In this study, we simulate thousands of yield curves that could possibly occur in the (near) future. Each of these thousand simulations gets assigned an equal probability. Hence, our interest is in the scenario set as a whole, and therefore a very accurate base forecast yield curve has not our primarily interest.

From the parameters obtained in Table 4.1 and using Equation 2.20 we are able to derive an one-year forecast from the estimation. The first Diebold-Li factor β_1 is expected to increase to 4.366 in one year, the second factor β_2 decreases to -4.379 and β_3 increases to -5.601.

Table 4.1: The parameters of the estimated VAR(1) model

	(1)	(2)	(3)
	β_1	β_2	β_3
$\beta_{1,t-1}$	0.994230***	-0.046306***	0.060441***
	(0.00885)	*(0.01159)*	*(0.02214)*
$\beta_{2,t-1}$	0.016453*	0.930439***	0.049496*
	(0.01078)	*(0.01411)*	*(0.02696)*
$\beta_{3,t-1}$	-0.11409	0.060331***	0.919719***
	(0.00903)	*(0.01183)*	*(0.02259)*
μ	0.037353	0.230502***	-0.440176***
	(0.06301)	*(0.08252)*	*(0.15761)*

In this table, the values of the estimators of the VAR(1) model are denoted. The standard errors, between the brackets, are calculated using an inverse Hessian. The significance is denoted by asterisks; * indicates significance under 10% confidence level, ** indicates significance under 5% confidence level and *** indicates the parameter being significant under 1% confidence level

4.3 Risk-neutral scenarios

In this section, the results from sampling of risk-neutral scenarios are presented. A scenario set for a random outer scenario is depicted in Figure 4.3. Some analyses are performed on this scenario set.

When taking a closer look at Figure 4.3, the volatility immediately strikes the eye, with negative interest rates exceeding -15%. This is a consequence (and one of the main drawbacks) of using a Hull-White one-factor model. However, for valuation purposes these extreme scenarios are not expected to be harmful, especially when using a set of thousands of curves. Risk-neutral scenarios do not necessarily have to be realistic, as long as they are arbitrage free and used in big numbers.

To test whether the scenarios are indeed risk-neutral, the Martingale test, explained in Section 2.3.4, is performed upon this scenario set. The result of the test is presented in Figure 4.4. The line in the middle represents the $E[M(0)] = 0$ martingale, as discussed in Section 2.3.4, while the dotted lines represent the corresponding confidence bands. Note that the confidence band is diverging for longer maturities. This is logic, since the volatility of the inner scenarios also increases for longer maturities, denoted in Figure 4.3. Using the corresponding confidence intervals, we are able to see that the resulting martingale falls into the 95% intervals. Analyses performed on other scenario sets for different combinations of risk-factors, lead to comparable results. This gives an indication that the calculated inner scenario set is indeed arbitrage free.

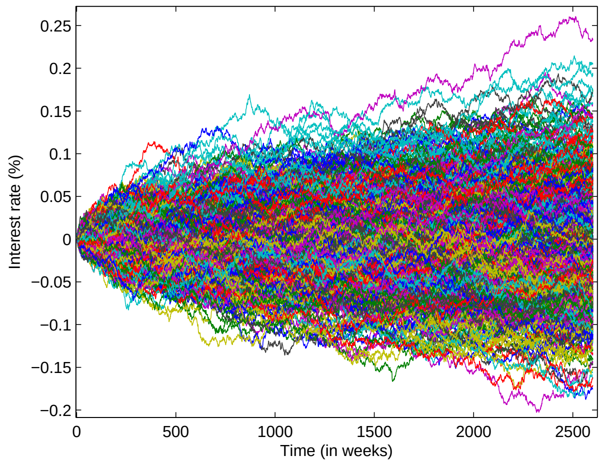


Figure 4.3: A risk-neutral scenario set, given a set of risk-drivers, for illustrative purposes. The coloured lines represent different risk-neutral interest rate developments for the coming 50 years, which are simulated based on the one factor Hull White model.

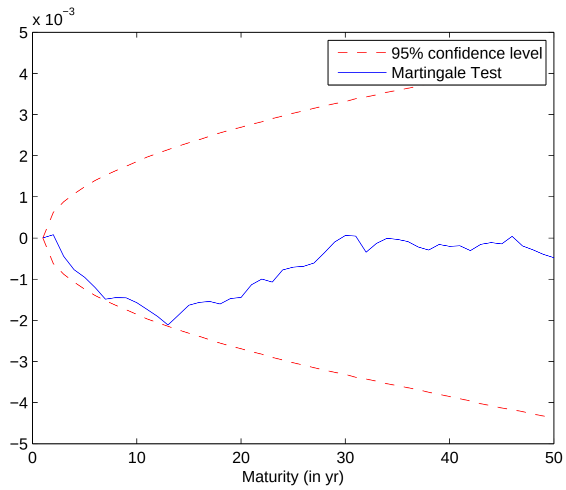


Figure 4.4: Martingale test including 95 % confidence bands for a random risk-neutral scenario set. The solid line denotes the $E[M(0)] = 0$ martingale, as discussed in Section 2.3.4, while the dotted lines represent the corresponding confidence bands.

4.4 Least Squares Monte Carlo

Now that we have the results of the underlying models, the results coming from the LSMC calculation can be observed in this section. Note that these analyses are a combination of all previous techniques and lead to the ultimate resulting figures. We present the different fits of the non-parametric approaches and compare the performance of the LSMC- with the FN approach.

First of all, we will discuss the relevance of the proposed LSMC technique. One of the main reasons for using LSMC involves its expected reduced calculation time. The limited number of inner scenarios per outer scenarios in the LSMC approach should result in a decrease of calculation time compared to an equal number of the outer-scenarios within the FN approach. This is confirmed when looking at the results in Table 4.2. When using 300 real-world outer scenarios, the Full Nested approach will take on average 1079 seconds to obtain results. With the same configuration the approach will take a little more than 12 seconds. When extending the complexity by increasing the number of outer scenarios to 3000, the FN approach takes 14209 seconds to calculate, while the LSMC approach takes 232 seconds to calculate on average. Hence, the LSMC approach indeed takes less time to calculate a fit compared to the FN approach. We should realize that this difference is obtained for a relative simple with-profit sharing product; complexer products lead to even bigger differences. This is one of the main reasons why insurance companies are looking for alternative (faster) ways to get an indication of risks exposed.

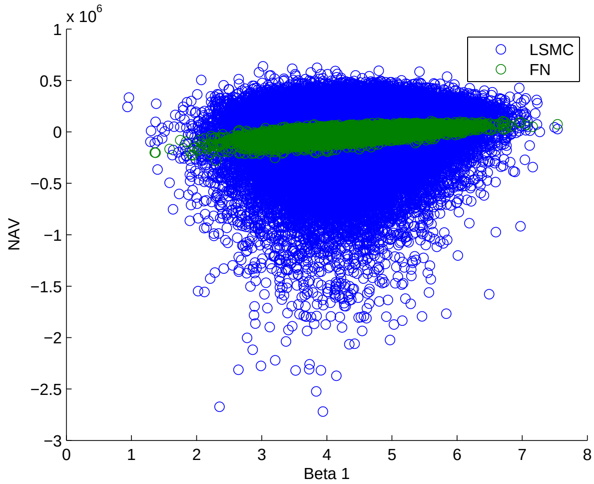
Table 4.2: Comparison of the calculation time

	(1)	(2)	(3)
	Number of RW-scenarios		
	RW = 300	RW = 3000	RW = 30000
Full Nested approach	1079 s	14209 s	-
LSMC approach	12 s	232 s	2412 s

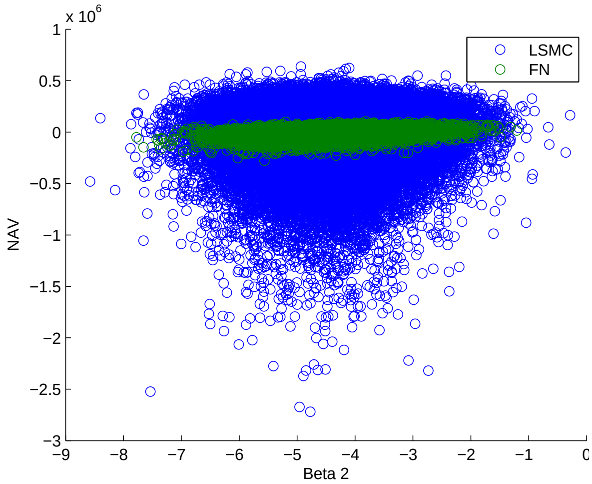
This table denotes the difference in calculation time of the NAV's for both the FN approach and the LSMC approach in seconds. For every calculation 6000 risk-neutral inner scenarios are used in the FN approach and 1 inner scenarios in the LSMC approach. Calculation time corrected for CPU workload.

However, one might question the difference in the quality of the resulting fit and its accuracy when using the different approaches. Results that could answer this inquiry are presented in the remainder of this chapter. If not explicitly mentioned otherwise, we use 3000 outer scenarios in the FN approach compared to 30000 outer scenarios in the LSMC approach. This should be enough; further research could indicate optimal scenario allocation budgets in this context.

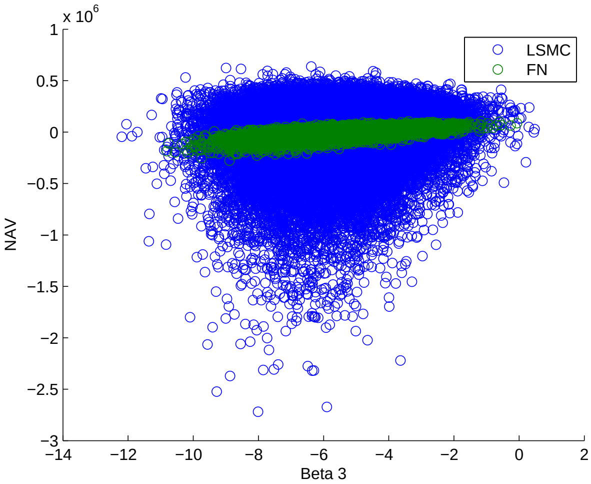
To get an intuition of the influence of the different risk-drivers upon the SCR, the individual effects of the risk-drivers are considered. We will start by plotting the *observations* of both methods against the calculated value of the product, the earlier mentioned NAV. Although these figures do not necessarily show interaction between certain risk-drivers, the figures do give an idea of the explanatory capacities of the risk-drivers. In Figure 4.5a the influence of the



(a) Beta 1



(b) Beta 2



(c) Beta 3

Figure 4.5: Observations of the three Diebold-Li factors plotted against the Net Asset Value for both the FN- and the LSMC approach. The blue markers denote LSMC observations, using only one risk-neutral path for every risk-driver. The green markers denote FN valuations, where a set of 6000 risk-neutral paths are used to value the product, given the risk-drivers.

β_1, the level of the yield curve, on the NAV is depicted. The figure reveals several interesting characteristics that deserve a further explanation. First of all, there seems to be a non-linear correlation between the first Diebold-Li term and the net asset value. The level of a yield curve seems to explain the NAV to some extend. Furthermore, a somewhat triangular shape of the LSMC observations for all three risk-drivers can be observed. Whenever an risk-driver gets assigned a HW inner scenario with a very negative interest rate, this results in an extreme low valuation. In contrary, extreme high interest rates have a decreased mirrored upwards effect on the valuations; the discounting of the liabilities is balanced by the discounted future incoming cash-flows. This explains the triangular shape. In the FN approach, extreme low interest rates are averaged out by other inner scenarios and no triangular shape can be observed. In the remainder of this chapter we will further research these characteristics.

Moreover, the observations for the other two Diebold-Li factors are depicted in Figure 4.7 and Figure 4.8. At a first glance, there does seem to be a correlation for β_2 and for β_3 as well. It looks like an increase in the second Diebold-Li factor has a positive effect on the NAV, while an increase in the third Diebold-Li factor implies an increase in the net asset value as well. In the next section we will further exploit these assumptions. Furthermore we will test the performance of the LSMC approach, when correcting for the mutual dependencies between the risk-drivers.

An overall note when looking at the three figures could be made, concerning the magnitude of the volatilities for both methods. Due to the random assignment of inner scenarios for every LSMC outer scenario, the valuations vary to a large extend. Individual valuations of the LSMC approach lack accuracy, hence a combined set should be considered to obtain accurate valuations. The triangular shape mentioned earlier can be seen for all three factors. The spread of the FN method is smaller and it is expected to decrease even further, when considering the combined effect of the factors on the NAV.

4.4.1 Non-parametric fitting

In this section the results of the non-parametric fitting based on the mentioned observations, are listed. Referring back to Figure 1.1, the fitting "block" is now entered. We will begin by showing the fits when using one factor, followed by a two factor analysis. These approaches leave room for visual interpretation. We will finish this section with a non-visual multi-factor analysis, in our case considering all three factors.

One factor

We will start by regarding the three risk-drivers independently. Sampling of the different Diebold-Li factors is based on a vector autoregressive model, so there does exist correlation among the risk factors.

Firstly, the non-paramatric Nadaray-Watson (NW) fit of β_1 on the net value of the assets, depicted in Figure 4.6, will be discussed. The red curve denotes the FN fit, the blue curve the LSMC fit and the dotted lines represent the 95% confidence interval of the LSMC fit.

The fits are simulated using sixty bins and the Epanechnikov kernel has a bandwidth of 0.4.

This is slightly bigger than mentioned rules-of-thumbs would suggest (Scott's rule suggests a bandwidth of around 0.3), but in the same order of magnitude. To be able to see a trend, a smoothed fit has our primarily interest. Too much noise can cause unclear conclusions as of the effect of risk-drivers in this way is rather indistinct. In addition, a bigger bandwidth also implies a robuster fit concerning generated outliers. In the appendices, one can find some sensitivity analyses for the bandwidth selection. For the LSMC approach valuations individually are not to be trusted, sets of observations should be considered. Hence increasing the bandwidth (or, in our case, fixing the bandwidth to the FN approach given the bigger amount of observations), will decrease the noise of this fit (and marginal increase of the bias of the fit).

The figure reveals the expected relation between β_1 and the NAV. A shift up of the yield curve results in a higher value of the assets. This is line with practices from the last decade; a low interest rate environment has pressing consequences on an insurer. The fact that the confidence intervals lay close to the LSMC fit, indicate that β_1 has a big influence on the asset value of the insurer. This is confirmed by the steepness of the curves: starting from around -150000 up to 60000. Apparently, this particular set-up leads to a rather negative outlook for the insurer. Possibly the issued guaranteed rate is not reached. A more thorough inquiry of this specific set-up, is out-of-scope.

More interesting is to analyse how the FN fit compares to the LSMC fit. The LSMC fit tends to follow the FN fit in a proper fashion. Throughout the whole range the FN fit is situated within the uncertainty of the LSMC fit. The maximum deviation between the two fits is 6%, with 30000 outer scenarios for the LSMC approach and 3000 for the FN method. Due to the sampling process, at the far ends of the fits the number of observations is limited, the deviation increases. In the middle of the fits, where many observations are available, the fits lay practically on top of each other. The uncertainty increases at the far ends, leading to a bigger spread in the confidence intervals. A reason why the two fits do not overlap each other perfectly, lays within the accuracy of the boundary values of the observations. We have engaged the fitting scheme a small number of times, showing the influence of outliers on the fits. The confidence boundaries of the FN are left out, since they lay very close to the FN fit. These boundaries give limited extra information regarding the fitting capabilities of the LSMC curve. Note that, due to the extra generated outer scenarios for the LSMC approach, this fit is longer at both ends. However, the number of observations at those ends is very limited; hence, the reliability of the fit is hard to verify and presumably not accurate.

When looking how the β_2 interacts with the NAV a certain correlation is revealed as well, as depicted in Figure 4.7. The bandwidth for both fits is again set to 0.4. The fit is less steep; starting at -90000 to -20000. It is important to realize the negative correlation between the movement of β_1 and β_2, as can be observed in Figure 4.2. Hence, given our sampling scheme, the drawn values of β_1 are negatively correlated with the drawn values of β_2. Assuming that the level of the yield curve has a strong effect on the value of the product, the influence of β_2 on the net asset value is smaller. Recall that the second Diebold-Li factor represents the shorter end of the yield curve, often called the "slope" factor. Low values of β_2 result in a low value of the NAV whereas the higher values lead to an increase for the NAV. However, the influence of β_1

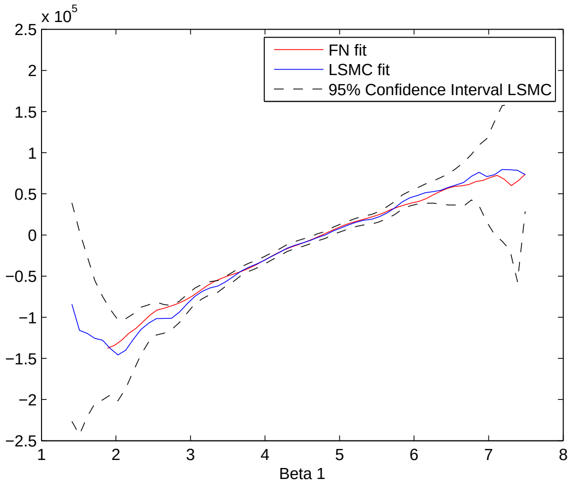


Figure 4.6: Nadaraya-Watson fits with an Epanechnikov kernel of the LSMC- and the FN approach of Beta 1 against NAV. The red line denotes the FN fit, the blue line the LSMC fit and the dotted black lines the corresponding LSMC 95% confidence bands. *(Bandwidth: 0.4; Bins: 60)*

on the long end of the yield curves diminishes this effect and a increasing fit with a smaller slope is the result. Furthermore, observe the slightly larger confidence intervals around the fits. This can be interpreted as a smaller explanatory performance by this factor, where other factors have influence on the exact location of the underlying observations. Similar characteristics could be found when regarding confidence intervals of the FN fits.

More interesting for us is to see how the LSMC fit behaves when compared to the benchmark. When comparing both fits, the results are again comparable. The LSMC fits tends to follow the FN fit throughout the whole curve. Both approaches suffer from slightly bigger uncertainty compared to β_1. The FN fit lays within the uncertainty throughout the whole range. The fits suffer from the fact that observations with a low β_2 depend on the magnitude of the other risk-drivers. Increasing the amount of outer scenarios reduces this uncertainty.

The relation between the third Diebold-Li factor and the net asset value is depicted in Figure 4.8. The bandwidth is enlarged to 0.5, given the different volatility of this risk-driver. There is a rather strong effect of the “curvature” factor on the value of the assets. This is somewhat unexpected and is probably caused by the correlation between the first and third Diebold-Li factor, again referring to Figure 4.2. The origin of the upward slope is related to this correlation. Again the confidence intervals around the fit is slightly bigger than the first risk-driver. This confirms the assumption that the explanatory capabilities of this factor is rather limited.

When comparing the two fits, again the LSMC approach seems to follow the FN nicely. The LSMC fit is unreliable at the far ends, due to extreme valuations from outliers. The FN fit lays within the uncertainty of the LSMC fit.

Given the fact that β_1 and β_2 have a correlation with the resulting value (neglecting their mutual dependencies), those two risk-drivers will be used for the two-factor analyses. However,

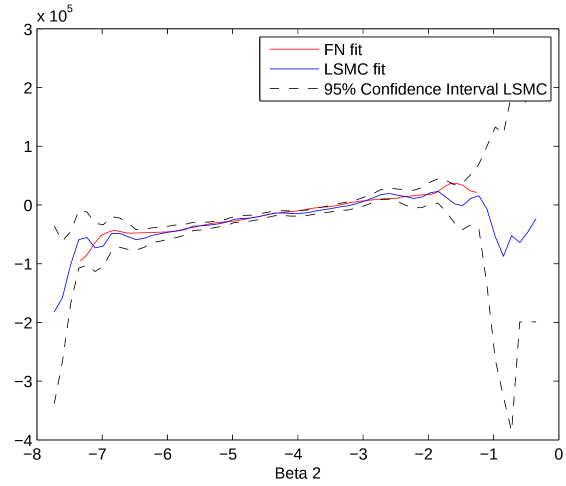


Figure 4.7: Nadaraya-Watson fits with an Epanechnikov kernel of the LSMC- and the FN approach of Beta 2 against NAV. The red line denotes the FN fit, the blue line the LSMC fit and the dotted black lines the corresponding LSMC 95% confidence bands. *(Bandwidth: 0.4; Bins: 60)*

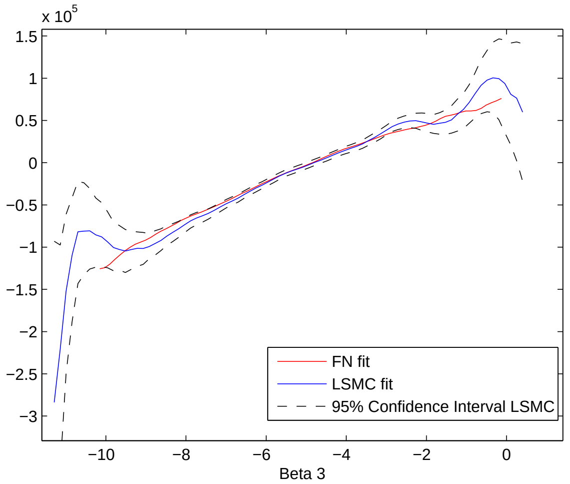


Figure 4.8: Nadaraya-Watson fits with an Epanechnikov kernel of the LSMC- and the FN approach of Beta 3 against NAV. The red line denotes the FN fit, the blue line the LSMC fit and the dotted black lines the corresponding LSMC 95% confidence bands. *(Bandwidth: 0.5; Bins: 60)*

we could have used other risk-drivers as well.

For comparison purposes, LSMC bins that are created out of FN range are not considered in the further analyses. Hence, the observations are still used, also for other bins, but some rows of bins at the far ends are not depicted. The added value of these bins could be questioned anyway, since individual LSMC observations as in the far ends of the fit, are not reliable individually. When using the stand-alone non-parametric LSMC approach, this is a problem that could be encountered: from which point on do we consider the LSMC fit reliable? Techniques that could be used in this context deal with the robustness of fits, and the law of large numbers. These topics are however, out-of-scope for our purposes.

Two factors

Next, two factor fits will be presented for both approaches. We will use the parameters β_1 and β_2 to perform a fitting on. The two fits are compared visually and with the use of techniques introduced in Section 2.6.2.

In Figure 4.9 the Nadaraya-Watson non-parametric fit with an Epanaechnikov kernel is denoted, for the risk-drivers β_1 and β_2 regarding the NAV for the FN fit. The bandwidth is increased compared to the previous analyses; 0.6 for β_1 and 0.6 for β_2. Hence, the added dimension implies that observations are considered in a bigger range. The fits are bootstrapped 3000 times. The black vertical lines represent the bootstrapped 95% confidence intervals at the different bins. This fitting is carried out a number of times, to ensure reliable fits.

The 3D visualization reveals that a combination of the two factor indeed results in the most negative asset value. Low values of β_1 in combination with low values of β_2 lead to the lowest solvability for the insurance company. Furthermore, the effect of a change in β_1 has slightly more effect than a change of β_2, although the influence is comparable. This is depicted by the small variance of bins for values of β_1 (Figure 4.9b) compared to the rather bigger spread of the bins for values of β_2 (Figure 4.9c). Furthermore, the figure denoted that the small number of observations leads to bigger uncertainties, as can be observed at the edges of the plain. Increasing the number of observations will decrease the uncertainty, confirmed by the small uncertainties in the middle of the plain.

For the LSMC approach, the same methodology is applied. The bandwidth of the fit is increased to .6 for both factors. In Figure 4.10 the resulting fit, where only one inner scenarios for every outer scenario is used, can be observed. The resulting fit is comparable to the FN fit. Again, a combination of β_1 and β_2 leads to the lowest net value of the assets. And again, the most uncertainty is caught in the far ends of the fit, due to the limited observations here. When comparing the uncertainty intervals with the FN two factor fit, LSMC shows more uncertainty at the borders. This is caused by the low accuracy of individual valuations, which have big biasing influence in the tails. Increasing the number of observations here, leads to smaller uncertainties at these locations. Again the β_1 has a bigger influence on the NAV than the β_2, in line with findings for the FN- and one dimension fit (Section 4.4.1). To put the fit in perspective, Figure 4.10d includes individual observations to the fit. Hence, the fitted plain of

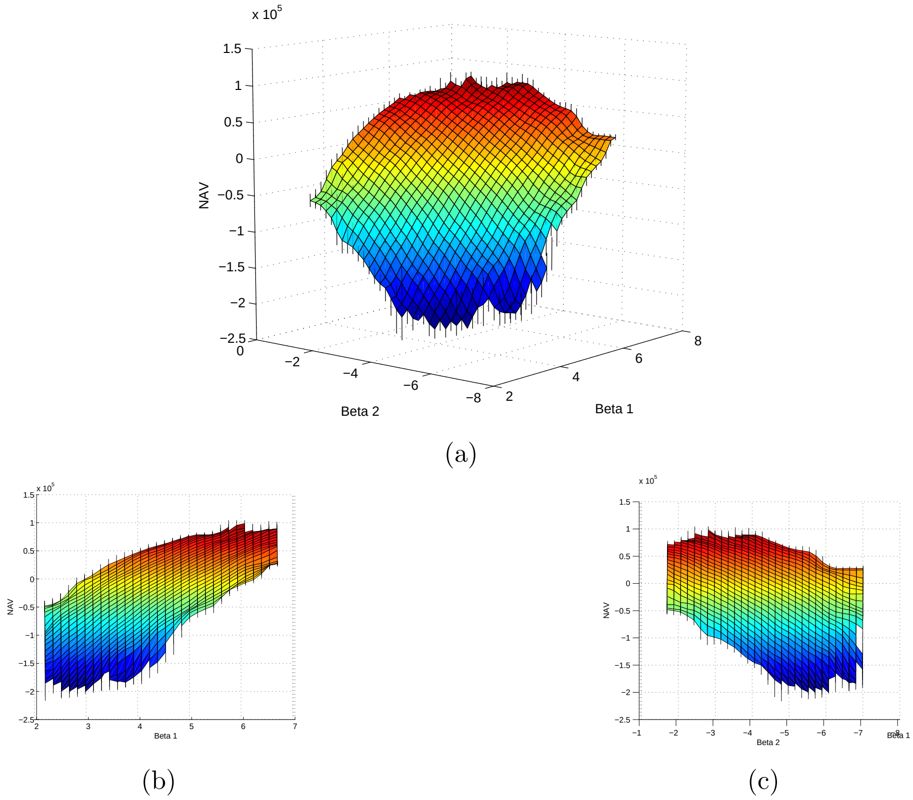


Figure 4.9: The value *NAV* fitted on Beta 1 and Beta 2 for the FN approach, using 6000 risk-neutral scenarios. The vertical lines denote the 95 % confidence intervals at the different locations. *(Bandwidth*$_{\beta_1}$*: 0.6; Bandwidth*$_{\beta_2}$*: 0.6; Bins: 795)*

the risk-drivers takes place on a relative small range of the NAV.

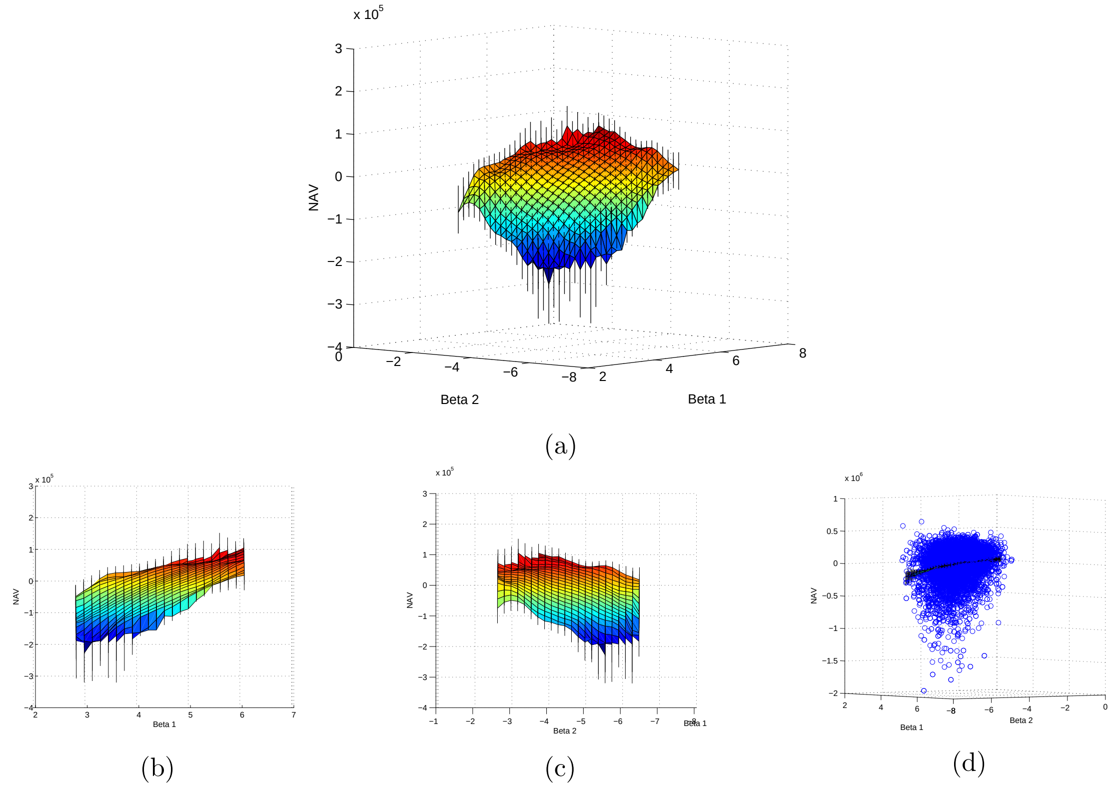


Figure 4.10: The value NAV fitted on Beta 1 and Beta 2 for the LSMC approach, using one risk-neutral scenario for every risk-driver set. The vertical lines denote the 95 % confidence intervals at the different locations. *(Bandwidth*$_{\beta_1}$*: 0.6; Bandwidth*$_{\beta_2}$*: 0.6; Bins: 438)*

The next step in the two factor analysis, is comparing both fits to each other. Here, we can see how the multi-factor LSMC fit performs. We will start by comparing them visually. The two fits are plotted in Figure 4.11, in which the blue plain represents the LSMC fit and the red plain represents the FN fit. As expected, the two fits do match each other quiet well in general. In the middle the two plains follow each other nicely; the same curvature can be seen throughout different places. At the far ends of the distributions, where the number of observations is limited, the fit is the most volatile. Also, given our sampling scheme, increased variance can be seen in places where both factors have the same sign/magnitude.

In practice, one would specifically be interested in situations where the net asset value is at its lowest. As already mentioned, a combination of a low β_1 and β_2 results in a low NAV for both approaches. From a regulatory point of view, insurers are especially interested in these low bins of the fits. In other words: "under what circumstances do I need to reserve more capital to keep my insurance company solvent and how much capital would than be needed?" Selecting the lowest part of both fits, will give us an idea of the performance. The 0.5% bins representing scenarios indicating these negative outcomes are listed in Table 4.3. As explained in Section 2.6.2, we base this percentile on the FN observations. The threshold equals -170780, so only bins under that value are regarded. Because we only consider LSMC bins that fall within the FN range, the grid of both approach differs. This results in an unequal amount of bins under the threshold. For the FN approach, a total of 795 bins were created while for the LSMC approach 438 bins were considered. Given that for the FN approach 23 bins fall below the 0.5% treshhold and for the 20, this implies that for the FN approach 3.6% of the bins are selected while for LSMC approach 4.6%. Hence, although the number of selected bins is unequal, it is representative for a comparable area of the fit.

The resulting values of the bins are in line with our expectations, based on the previous figures. The lowest values of the FN fits are -200352, -199983 and -199728 while the LSMC produces -227916 and -194850. These values are close to each other, but not comparable without considering the risk-drivers. When looking at the corresponding values of the risk-drivers, comparable results are obtained as well. The average values of β_1 differs 3.5% while β_2 differs less than 1%. The bigger differences can be explained by looking a the start points of β_2 for both fits; the FN fit's edge is compiled starting with less extreme values. Hence, the lower NAV for the bins can be explained when regarding the risk factors. Notice that for both approaches the value of the bins of β_1 is leading throughout the scenarios. This indicates that for both fits β_1 has a bigger influence on the NAV than β_2. This is in line with earlier findings. However, it is again a combination of the two factor that explain low outcomes. When comparing the values of NAV to the one factor fits, lower values have rolled out in two factor analyses. The average values illustrated in the plots are in line with the findings of the one factor fits. Here, the power of multi-factor comparison is already shown to some extend. The multi-factor analyses shows how combinations of risk-drivers lead to very negative results. In conclusion, very comparable results.

Off course, these results should be considered with care. While the volatility of the observations, especially at the borders, also produces LSMC fits that do differ from the FN fit, increasing the amount of observations/simulations dismisses this effect to a large extend. Furthermore, the magnitude of the bandwidth can change the value of the assets. However, when comparing the two fits relatively to each other, this effect is neglect-able.

Although the fits do match in this case, they remain relatively sensitive to fluctuations in the assigned bandwidths. It is therefore advised when handling new risk-drivers, to first select an appropriate bandwidth. In our case, we have used the bandwidths found for the one factor NW fits as a starting point. From there one the bandwidths were increased gradually, since observations are used in a smaller space due to the extra dimension. Algorithms exist that can automatically select bandwidths, however these were not specifically tested in our application.

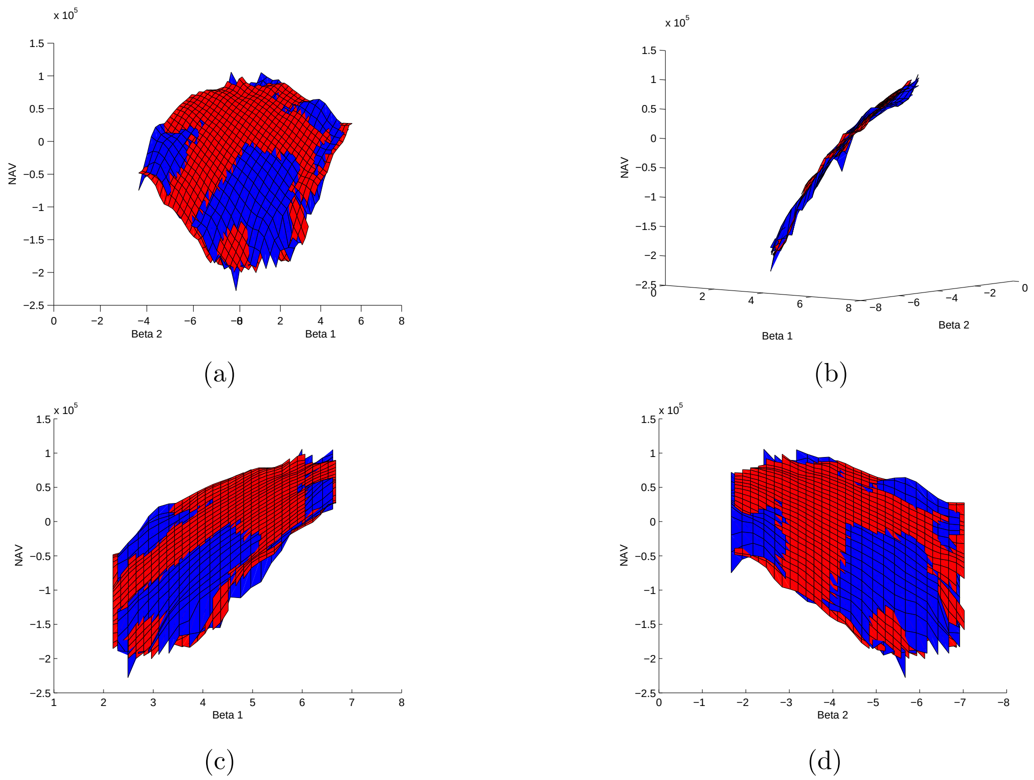


Figure 4.11: Comparison of the fitted values of *NAV* fitted on Beta 1 and Beta 2 for both approaches. The blue plain represents the LSMC approach, while the red plain represents the FN approach. Confidence intervals are left out, for illustrative purposes.

Table 4.3: Selected lower risk cases for two DL factors

FN approach				LSMC approach			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	$NAV \cdot 10^5$	β_1	β_2		$NAV \cdot 10^5$	β_1	β_2
Bin 1	-2.00352	2.96	-6.12	Bin 1	-2.27916	2.49	-5.67
Bin 2	-1.99983	2.65	-5.75	Bin 2	-1.94850	2.49	-5.17
Bin 3	-1.99728	2.50	-5.57	Bin 3	-1.94282	3.11	-6.42
Bin 4	-1.96034	2.81	-5.93	Bin 4	-1.93420	2.70	-5.42
Bin 5	-1.93922	2.50	-5.39	Bin 5	-1.92889	2.70	-5.92
Bin 6	-1.91209	2.81	-5.75	Bin 6	-1.92397	3.32	-6.67
Bin 7	-1.89671	2.65	-5.57	Bin 7	-1.92247	2.90	-6.17
Bin 8	-1.89023	2.34	-5.21	Bin 8	-1.90517	2.70	-5.67
Bin 9	-1.89021	2.96	-5.93	Bin 9	-1.89553	2.49	-5.42
Bin 10	-1.87437	2.34	-5.02	Bin 10	-1.88719	2.28	-5.42
Bin 11	-1.86550	3.12	-6.12	Bin 11	-1.88375	2.90	-5.67
Bin 12	-1.85465	2.19	-4.84	Bin 12	-1.87890	2.90	-5.92
Bin 13	-1.83771	3.74	-6.85	Bin 13	-1.85573	2.28	-4.92
Bin 14	-1.82788	3.58	-6.66	Bin 14	-1.83076	3.73	-6.92
Bin 15	-1.82497	2.50	-5.21	Bin 15	-1.78094	2.70	-5.17
Bin 16	-1.81258	3.58	-6.85	Bin 16	-1.75409	2.90	-5.42
Bin 17	-1.80828	2.96	-5.75	Bin 17	-1.74832	2.28	-5.17
Bin 18	-1.80124	2.65	-5.39	Bin 18	-1.72266	2.49	-4.92
Bin 19	-1.79710	3.12	-5.93	Bin 19	-1.71934	3.32	-6.42
Bin 20	-1.79343	2.81	-5.57				
Bin 21	-1.79171	3.43	-6.66				
Bin 22	-1.77866	3.74	-6.66				
Bin 23	-1.76980	2.34	-4.84				
Bin 24	-1.76793	2.19	-4.66				
Bin 25	-1.75913	3.27	-6.12				
Bin 26	-1.74918	3.43	-6.48				
Bin 27	-1.74796	3.89	-6.85				
Bin 28	-1.73666	2.50	-5.02				
Bin 29	-1.71231	2.34	-4.66				
Average	*-1.84139*	*2.89*	*-5.77*	*Average*	*-1.87115*	*2.79*	*-5.73*

In this table, the bins representing scenarios triggering the lowest 0.5 percent bins are listed for both approaches. In column (1) the resulting values of the scenarios are listed and in columns (2) and (3) the corresponding risk-drivers.

Three factors

The next logical step is to extend the above method to a multivariate model with three dimensions. In our study this implies including the third Diebold-Li factor, but this approach could also be used with other risk-drivers. The proposed method can be extended to even more risk-drivers, although this might bring some related problems.

In order to keep the results interpretable and comparable, we will focus on the tabellized results. Implementing all three risk-drivers, implies that an easy visual comparison is not longer possible. However, taking into account the ultimate application of this analysis, we can select certain outcomes. Under Solvency II, insurers are obliged to keep enough capital reserved to survive a shock that will occur once every 200 years. With this in mind, selecting the combination of risk-drivers resulting in the lowest net values of the assets, will give us insight in the interaction between the risk-drivers considering the effect on the value.

However, we are specifically interested in the performance of the FN-approach compared to the LSMC-approach. When the most negative circumstances are selected for both approaches, we can compare whether the same scenarios are selected and the whether corresponding magnitude of the underlying values are equal. We fit 12 bins in every dimension, and again only consider bins lower than -170780, corresponding to the 1-in-200 VaR. To be able to compare the fits, LSMC bins with no FN observations are again not considered. Again by focusing on the resulting bins, a comparison can be made. The bandwidth of the fits are slightly increased to .8, .8 and .9, since an extra dimension implies that data is used for a smaller range of risk-drivers. By increasing the bandwidth, observations are carried further. This makes the fits, especially at the tails of the LSMC approach, more stable but possibly more biased.

The results are denoted in Table 4.4. In the left part of the table, bins of the FN fit can be found while the right part contains bins of the LSMC approach. Let us first consider the values of the net asset value. The LSMC bins range from -233020 to -176550 with an average value of -200220. The FN bins vary between -212740 and -177770 with an average value of -195530. It is remarkable that the values of the β_3 of the LSMC are in general higher than expected. This has to do with the way the extreme bins are fitted in this dimension. The most extreme line of bins had an high β_3 value, but had to be ignored since no FN observation of that magnitude was available. Hence, the second row as the outside bins for LSMC, having an effect on the comparison. Extra fittings/observations could diminish this effect. Note that the values of the three factor fits are lower than the two factor fits. This can be explained by considering two characteristics. First of all, since we are now using the data in rather small range, the fits (especially the LSMC approach) is very sensitive to inaccurate outliers. This results in rather negative values of the bins. Secondly, taking into account the third factor as well, leads to lower values for the extreme bins. The effect of an extra dimension can be observed.

In general, the values of the β_1 do not vary much in both approaches. This is in line with our earlier findings, where we saw that β_1 has the biggest influence on the NAV. Especially in the FN approach, the first 23 bins are fitted on the lowest four bin-values of β_1. The influence of β_2 is slightly lower, but still only the lowest rows of bins produce selected scenarios. For the LSMC approach, the some effect of outliers can be observed (for example Bin 7). However, this is an exception; moreover, more observations would average out this effect. Also for the third factor only low bin values are selected.

The 0.5% Value-at-Risk would in our case be equal to the highest value in the list; so for the FN approach -177770 and for the LSMC approach -17655. This critical scenarios have a β_1 of respectively 3.27 and 2.65, a β_2 of -6.85 and -4.98 and a β_3 of -6.88 and -7.91. The difference of NAV can be explained by looking at the β_1; given that this value is slightly higher for the LSMC approach could have a positive effect of its NAV. These VaR scenarios expose one of the main weaknesses of VaR method; if multiple risk-drivers have significant influence, the triggering scenario is selected rather randomly. It does not give us information of a possible trend in risk-drivers.

When we compare the average value of the VaR, the FN approach denotes -195530 and the LSMC denotes -200220. This value, equivalent to the 99.5% Expected Shortfall, gives an idea of how severe the damage would be in case the VaR would occur (zooming in on the tails of the distribution). Expected Shortfall is another well-known risk measure, however not used in Solvency 2. The results for both approaches are, again, comparable. The higher value can be explained by the outliers at the borders of the hyperplane. When not considering them, the values differ 0.84%. Again, this can be explained by looking at the different values of the risk-drivers.

The values are about 5% lower compared to the two factors fits of Table 4.3. This is mainly due to the amount of extreme low bins. Within the VaR, we do not give any bin a weight based on the amount of observations it represents. So, while more extreme bins are created with specific negative factor combinations, they are treated as important as lower bins when calculating the average bin value. Hence, the drawback of weightless bins becomes apparent when considering more dimensions.

In the end, the fits for three factors show somewhat comparable values to the two factor fits, when considering the different values of the risk-drivers. The proposed bin selection scheme can be extended to even more simultaneous risk factors consideration. However, the curse of dimensionality should be avoided; already with three factors, outliers at the edges lead to non-robust fits. This could be prevented by bigger simulation budgets or more sophisticated mathematical approaches. Information of how outer scenarios were simulated could be used in the non-parametric fitting process. However, this would disable direct market data implementation. In addition, research focussed on weigh allocation should give a more balanced overview.

Table 4.4: Selected lower risk cases for three DL factors

FN approach					**LSMC approach**				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	$NAV \cdot 10^5$	β_1	β_2	β_3		$NAV \cdot 10^5$	β_1	β_2	β_3
Bin 1	-2.1274	2.34	-5.75	-8.64	Bin 1	-2.3302	2.65	-6.23	-8.90
Bin 2	-2.1064	2.81	-6.30	-6.88	Bin 2	-2.2161	2.65	-5.61	-4.95
Bin 3	-2.1023	2.34	-5.21	-8.64	Bin 3	-2.1472	2.65	-6.23	-4.95
Bin 4	-2.1023	2.81	-5.75	-8.64	Bin 4	-2.1345	2.65	-4.35	-6.92
Bin 5	-2.0863	2.34	-4.66	-9.51	Bin 5	-2.1345	2.65	-4.98	-3.96
Bin 6	-2.0863	2.34	-4.66	-8.64	Bin 6	-2.1233	3.16	-6.23	-8.90
Bin 7	-2.0175	3.27	-6.30	-9.51	Bin 7	-2.0940	4.71	-1.85	-8.90
Bin 8	-2.0135	2.81	-5.75	-9.51	Bin 8	-1.9359	2.65	-6.23	-6.92
Bin 9	-2.0135	3.27	-5.75	-9.51	Bin 9	-1.9295	2.65	-5.61	-7.91
Bin 10	-2.0089	3.74	-6.85	-9.51	Bin 10	-1.8382	3.68	-5.61	-4.95
Bin 11	-1.9611	2.34	-5.21	-8.64	Bin 11	-1.8238	3.16	-6.23	-8.90
Bin 12	-1.9526	3.27	-6.30	-8.64	Bin 12	-1.7918	2.65	-6.23	-7.91
Bin 13	-1.9357	3.74	-6.85	-8.64	Bin 13	-1.7667	2.65	-5.61	-8.90
Bin 14	-1.9177	2.81	-5.75	-6.88	Bin 14	-1.7655	2.65	-4.98	-7.91
Bin 15	-1.8795	3.74	-6.30	-9.51					
Bin 16	-1.8768	3.74	-6.30	-8.64					
Bin 17	-1.8749	2.34	-4.66	-8.64					
Bin 18	-1.8740	2.81	-5.21	-8.64					
Bin 19	-1.8483	3.74	-6.85	-7.76					
Bin 20	-1.8081	3.27	-5.75	-8.64					
Bin 21	-1.8047	2.81	-4.66	-9.51					
Bin 22	-1.7958	3.27	-6.30	-7.76					
Bin 23	-1.7777	3.27	-6.85	-6.88					
Average	*-1.9553*	*3.01*	*-5.82*	*-8.60*	*Average*	*-2.0022*	*2.94*	*-5.43*	*-7.21*

In this table, the bins representing scenarios triggering the lowest 0.5 percent net asset value bins are listed for both approaches. In column (2) the resulting values of the scenarios are listed and in columns (3), (4) and (5) the corresponding risk-drivers can be observed regarding the for the FN approach. Similarly, in column (7) the resulting values of the scenarios are listed and in columns (8), (9) and (10) the corresponding risk-drivers can be observed regarding the for the LSMC approach

4.5 Multi-year forecast

Now that we gathered results for a one year horizon, the forecast horizon will be increased to five years. Both approaches are analysed with a smaller number of simulations: 400 outer scenarios per year for the FN approach (each using 6000 inner scenarios), and 7000 per year for the LSMC approach, each using one inner scenario. The parameters of the sample product are left unchanged. This section denotes the results of this multi-year analysis. The difference in calculation time between the FN- and the LSMC approach is again striking. The FN approach takes 1121 seconds cpu time, whereas the LSMC approach consumes 65 seconds. This is line with the findings denoted in Table 4.2.

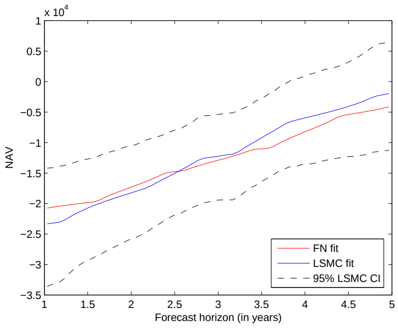
As already mentioned in Section 2.7, the horizon is regarded as an additional conditional risk-driver. Using an one-factor Nadaraya-Watson fit, the uncorrected effect can be observed in Figure 4.12. Within this figure, Figure 4.12b denotes the FN approach while Figure 4.12c fit denotes the LSMC approach. The bandwidth is set to 1.4 for the FN fit and 1.4 for the LSMC fit, creating a smooth fit through time. When comparing the FN fit with the LSMC fit, we see a comparable trend through time, depicted in Figure 4.12a. Despite the limited amount of outer scenarios for the LSMC approach, it still follows the FN fit quiet well. Both fits do lie well in each others uncertainties.

When zooming in on the dynamics of the fits, two things can be observed. First of all, the value of the product seems to increase throughout time. This can be explained by looking at the two most important factors: both the β_1 and β_3 are increasing, leading to a higher NAV. Secondly, the uncertainty of the forecast barely changes through time. This is an interesting result, moreover since uncertainty is the primarily source of solvability related calculations.

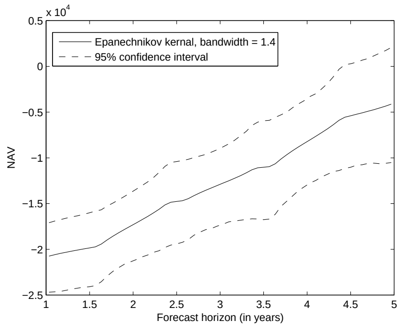
To explain this small increase in uncertainty, we will have to go back to our other risk-drivers. The Diebold-Li factors are the underlying risk-drivers, causing the change in net asset value. When the horizon of the Diebold-Li factors is extended, the same VAR(1) model is used. Results of these forecasts are denoted in Table 4.5. The results in the table show us that the value of the first factor increases to a value of 4.434. The β_2 evens out to value of -3.767 and the β_3 equals -4.602 for a five year forecast. This gradual approximation to the series' mean is in line with expectations of a VAR(1) process.

Denote also that for β_1 and β_2, the variance gradually increases to respectively 1.234 and 1.529. When comparing these values to the variance for a one year forecast, the increase is relatively small. When regarding the β_3, the increase of the standard deviation goes up to 2.001. Apparently the variance around the one year forecast barely increases when extending it to a multi-year horizon for all three risk-drivers.

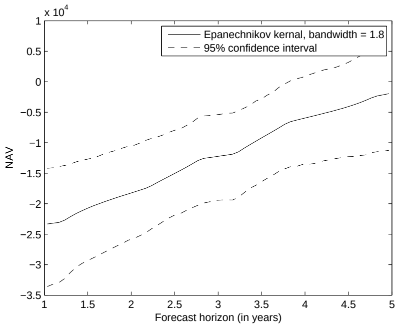
This gradual change in the uncertainty, results in the absence of diverging confidence bands. The confidence bands run almost parallel to the fit, for both approaches. An insurer could also extend its SCR using a simple deterministic model (using only one expected scenario). We therefore can conclude that, with these risk-drivers and for this product, given the small increase of uncertainty, the stochasticity in the multi-year ORSA forecast does not necessarily add value. We have only performed an one-dimension analyses on the horizon, so the effect between forecast



(a) Comparison of both fits



(b) FN approach



(c) LSMC approach

Figure 4.12: The different fits for a multi-year forecast. The forecast horizon is considered as new risk-factor, the underlying DL factors are not depicted. *(Bandwidth: 1.4; Bins: 60)*

horizon and other risk-drivers is assumed constant.

A strong remark should be made concerning the selected risk-drivers. In our case, the forecast horizon does not have a direct effect on the uncertainty around the NAV (and hence the SCR). When one would use risk-drivers with diverging uncertainties, this would most probably lead to another outcome. This implies that stochasticity and therefore LSMC approaches could be required for an accurate risk measure. In that case, the proposed method in which the horizon is treated as an risk-driver gives a fair indication of an insurer's solvability position in the future. Even using a VAR(p) model with $p > 1$ could have a serious effect on the results.

Table 4.5: The estimated DL estimators

	(1)	(2)	(3)	(4)	(5)	(6)
	Forecast horizon in months					
	h=0	h=12	h=24	h=36	h=48	h=60
β_1	4.192	4.366	4.388	4.391	4.401	4.423
$St.Dev_{\beta_1}$		(0.769)	(0.993)	(1.112)	(1.183)	(1.234)
β_2	-3.741	-4.379	-4.297	-4.097	-3.914	-3.767
$St.Dev_{\beta_2}$		(0.929)	(1.225)	(1.397)	(1.486)	(1.529)
β_3	-6.932	-5.601	-5.135	-4.899	-4.729	-4.602
$St.Dev_{\beta_3}$		(1.535)	(1.782)	(1.894)	(1.958)	(2.001)

In this table, the estimators following from the Diebold-Li VAR(1) model including the corresponding standard deviations (between brackets) are listed. The forecast horizon is extended from 12 months to 60 months, as expected in ORSA regulations.

Chapter 5

Conclusion

In this report we have implemented non-parametric fits on data obtained by a Least Squares Monte Carlo (LSMC) process. To our knowledge, no research is conducted in which the LSMC approach is combined with non-parametric kernel fitting techniques. Furthermore, we have proposed a LMSC framework to value insurance products with embedded optionality. Closed-form formulas for those products are not available and replication techniques are not always feasible. The proposed non-parametric way of fitting has some theoretical advantages compared over regular parametric LSMC fits and the results look very promising. Especially when incorporating risk-drivers in which the effect on value depends on other risk-drivers as well, multivariate analysis is necessary to give a fair representation of risks exposed. Non-parametric estimation imposes little restrictions on the observations and has, in theory, an infinite dimensional parameter space. Hence, these aspects make forecasts more accurate and empirical compared to regular parametric fits.

First of all, when considering some of the advantages, non-parametric fitting makes very few assumptions upon the fitted data. Furthermore, the non-parametric way of fitting does not impose a certain distribution on observations. Extensions in which market data is directly implemented in the model could enrich the model. In addition, this type of fitting is applicable on multiple risk-drivers simultaneously, revealing specific risks of risk-driver combinations. However, the proposed LSMC fitting scheme has shown some drawbacks as well. Since the non-parametric technique only uses information gathered by observed data, extrapolation is not possible. Although extrapolation is normally not required for risk metrics, this characteristic indirectly leads to more volatility at the ends of the fit. Furthermore, adding extra dimensions leads to the curse of dimensionality. One can opt to increase the number of samples, specify the fitting-space or increase bandwidths, using information in a bigger range. Caution is required when considering the latter option, since observations used too far in the fit lead to biases.

To show how the non-parametric LSMC performs in practice, a sample insurance product was created. The insurance product has a profit-sharing characteristic, in which holders are rewarded with at least a guaranteed rate, regardless of the current state of the economy. If the interest rate was above this guaranteed rate, the surplus was shared with the policyholder. The interest rates were generated using a three-factor vector autoregressive VAR(1) model, as proposed in the paper of Diebold & Li (2006). To calculate a value of the product given a scenario, a risk-neutral scenario set was generated based on an one-factor Hull & White (1990) model. Two different approaches were compared with each other; a Full Nested (FN) approach consisting of a complete risk-neutral scenario set for every risk-driver combination (the benchmark) and the LSMC approach where only one risk neutral scenario was assigned to a combination of risk-drivers randomly. The results were considered in the Solvency II regulatory perspective.

The LSMC characteristic of valuing products with embedded optionality, has proven to be remarkably faster compared to approaches consisting of a whole risk-neutral tree. The non-parametric LSMC has proven to be a reliable method to fit observations on, compared to the non-parametric FN fit, as shown in Section 4.4.1. The results have denoted that the LSMC approach outcomes are comparable to the FN approach outcomes, which are considered as the benchmark. Although the LSMC fit shows uncertainty on points on which the amount of observations is limited, on average the results overlap the FN fits. Hence, for the calculation of the Solvency Capital Requirements (SCR) for one year, the proposed LSMC scheme can lead to faster valuations for difficult insurance products without losing a lot of accuracy. When considering the effect of multiple risk-factors simultaneously, the LSMC fit still showed reliable results.

Under current regulations, a stochastic multi-year forecast is not explicitly required. Nevertheless, the proposed model is extended into a multi-year framework to see some of its specific implications. Instead of simply extending the forecast horizon, we treat the horizon as an extra risk-driver. This allows us to explore the effect of increasing horizons on the position of the insurer, while considering effects of other risk-drivers simultaneously. In our set-up the effect on the value of the assets is relatively small. However, it is suspected that this outcome is strongly affected by the limited increase of volatility of the specific risk-drivers. Further research, incorporating significant risk-drivers with strongly increasing uncertainty in a multi-year context, should confirm this suspicion.

Further research concerning bandwidth selection and bin treatment could lead to an implemented standalone non-parametric LSMC model that calculates reliable outcomes, that the FN approach shows accurate valuations and assuming that our simple product is illustrative. This implemented LSMC model will have a positive effect on the calculation time needed, compared to FN methods and will be able to value complex non-linear products, where other approaches are not effective. This could ultimately lead to risk management information systems, playing a more prominent role in decision making of organisations.

Chapter 6

Discussion

In this chapter, some of the challenges we encountered during the research are summarized. Uncertainties are listed and directions for further research are given.

There are some remarks that could be made regarding the risk-drivers selected, simulating the real-world scenarios. We have chosen to use three Diebold-Li factors as our risk-drivers, limiting ourselves to interest rate risk. However, in the Solvency II framework (and also in reality), there are numerous risks that should be taken into account as well. The framework created allows us to change the type of risk-driver rather easy. For life- and non-life insurers other types of market- and credit risks are interesting to take into consideration. In our context spread risk could have an enormous effect on the net value of the assets. All of these risks could easily be implemented in the proposed framework. We have taken lapse risk and mortality/longevity risks as a fixed rates, but in reality extensive models could be implemented here as well. More generally, our proposed model could function as a basis in which simplifications could be replaced by sophisticated models rather easy. This would give us more insight in the effect that risks has on aspects of the SCR.

Some remarks could be made more specifically on the simulation of the yield curves. We have used an vector autoregressive process to forecast the Diebold-Li factors for the simulation of outer scenarios. Although this method gives reliable combinations of factors, the absolute effect of the risk-driver on the net asset value cannot be observed without the correlation. One could argue that other models could outperform the proposed model.

The 1-factor Hull-White model used in our research, is a relative limited model to generate valuation interest rates. By not calibrating the mean-reversing parameter and the implied volatility parameter, we have not used the model to its full potential. Calibration would generate more reliable values, given a certain real-world scenario. However, the 1f-HW model has some structural limitations as well. It is known that the model produces a significant amount of negative interest rate scenarios, curves that in reality almost never occur. In order to decrease the amount of those scenarios, a more sophisticated model could be used (f.i. a 2-factor Hull-White model (G2++)). Under all circumstances, an increase of calculation time should be put against the win in accuracy/robustness of the model.

One of the problems we have encountered in the non-parametric fitting, is the selection of bandwidths. For our risk-drivers, automated algorithms (like the different rules-of-thumbs) did not give satisfactory outcomes. Therefore, given the limited amount of risk-drivers we have incorporated, the bandwidths were selected manually. However, when extending this into a multi-dimension model, automated bandwidth selection is more feasible. Furthermore, the manual action makes an implementation not possible without "experts judgement". How the bandwidths should be determined, both theoretically and in practice, is unclear.

As already mentioned, we have chosen to use non-parametric fitting once the observations of the LSMC and the FN approach were generated. Although the non-parametric approach has advantages compared to parametric fits, one of the main drawbacks revolves around fitting locations with a small amount of observations. We have avoided this situation by simply generating sufficient data points, but in practice only a limited scenario budget is available. Literature exists on dynamic bandwidths, however to our knowledge not subjected to this topic. Also the sensitivity of LSMC subjected to the curse of dimensionality, could be an interesting topic for further research. Although the proposed method should be able to cope more risk-drivers, its approach has not been tested using more than three risk-drivers. It could be interesting to research how our approach would withstand this, especially when dealing with multiple influential risk-drivers.

All of the mentioned improvements could lead to a reliable stand-alone non-parametric LSMC valuation model, not only for interest rate risk but all kind of other risks as well. In the end, insurers would have better insights in their exposed portfolio's risks. With risk management information systems giving accurate forecasts, their role within financial institutions could reshape the way managerial decisions are made.

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Appendices

Appendix A

Solvency II

In this appendix, a small description concerning Solvency 2 is given. We focus on the general structure of the regulations and characteristics that are relevant for our purposes. For an exact description of the framework, please consider reading the original directive (European Parlament, 2009). In the European Union a new solvency regime will be required for all insurers. Solvency II, as the new regime is called, requires adjustments in the way insurers calculate and report the risks they take. After a process of decades, a full implementation will required on the 1st of January 2016. For the regulators the new regime gives insights in how risks are calculated and allows comparison between insurers situated in the European Union. For insurers on the other hand, the new regulations could give incentives for reduced capital requirements in addition to a better insight in exposed risks. Solvency II is constructed from a risk's perspective; to ensure the future payments to policyholders. Both life and non-life insurance companies are regulated by means of Solvency II.(European Parlament, 2009). The basis of Solvency II was modelled upon the Basel II structure which was used within the banking world; a similar 3 pillar set-up was created to create a level playing field for market participants (Gründl & Gal (2013)).

Pillar 1

Pillar 1 focusses on the Quantitative requirements imposed by the regulator. It consists of a standard from which the valuation of the liabilities to policyholders can be calculated. Furthermore, it gives an idea of the capital required to reserve.

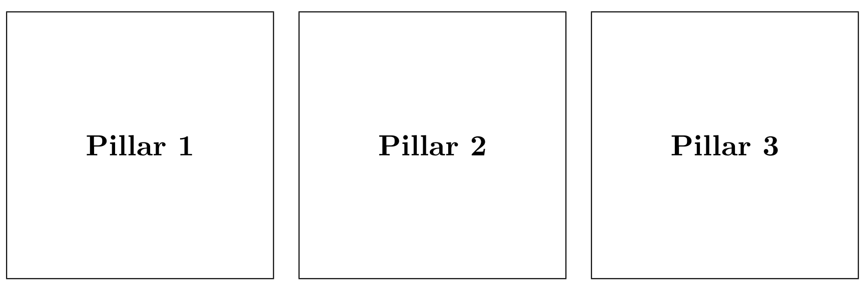


Figure A.1: A representation of the structure of the Solvency II framework.

It is important to understand the difference between the Minimum Capital Requirements (MCR) and the Solvency Capital Requirements (SCR). The MCR gives a value for the capital form which short-term liabilities given the current portfolio of the insurer, the contracts and customers at this time and the state of the economy at this time. However, the MCR gives no guarantees when on the factors just mentioned change in a disadvantageous manner. The MCR capital has to be covered using only basic own funds. The MCR can be seen as the lowest amount of capital an insurer should hold in all cases; otherwise the supervisor will revoke the firms license.

The SCR on the other hand, gives a value of the "target amount of capital. It indicates the amount of capital a firm should hold in order to survive very harsh economic conditions in a 1 year horizon. There are two ways and insurer can calculate the SCR. An insurer could chose to calculate this SCR based on the QIS5 standard formula, or by means of an own developed model. When an insurer would chose for the latter, this implies that the used model is also proposed and accepted by the supervisor.

The European Insurance and Occupational Pensions Authority (EIOPA) is an official independent advisory institute, which delivers to bodies like the European Parliament, the Council of the European Union and the European commission. One of EIPOAs main deliverables is the Quantitative Impact Study; in 2011 the 5th and probably the last (QIS5) was performed. The goal of QIS5 is to give a global impression of the potential impact that Solvency II will have on insurers en to test the feasibility of certain modules within Solvency II.

Pillar 2

Pillar 2 concentrates on the more qualitative features of the risk module, as well as the approach to supervisory review. Pillar 2 requirements make sure that the insurer meets the proposed risk management standards and that the insurer is adequately capitalised. Another important feature of Solvency II is Pillar's 2 so called: Own Risk and Solvency Assessment (ORSA). This ORSA has to be performed by the insurer, regardless of the choice of method for calculating SCR under Pillar 1 (standard formula or own calculation). An short description of ORSA with its implications for our research, can be found in Section 2.7.

Pillar 3

Pillar 3 primarily concerns the method of reporting the risks and other disclosure requirements. The main goal of Pillar 3 is to create market transparency. Insurers should collect relevant information, that will be distributed to regulators, shareholders and analysts.

Value-at-Risk

In Solvency II, Value-at-Risk (VaR) is used to as benchmark method. This is well known risk-measure, also used in other regulations. Basically, given a large number of possible scenarios, the VaR considers the α-percentile lowest resulting values. The value at risk equals the resulting value of the insurer for this percentile. Often, a financial institution should be able to withstand

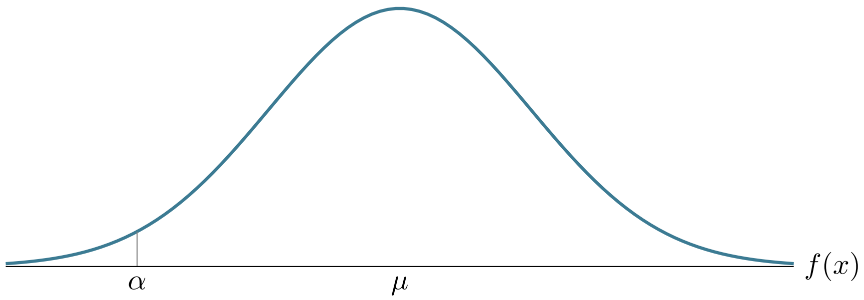


Figure A.2: The Value-at-Risk methodology illustrated. Given a return distribution $f(x)$, only the lowest results are considered. The Value-at-Risk is equivalent to the return corresponding to the α-percentile. When a loss distribution is considered, the methodology is mirrored.

the circumstances happening under α and keep at least enough capital corresponding to this VaR. So in Figure A.2 values are assumed to be normally distributed by $f(x)$ and mean μ. The corresponding VaR would be equal to resulting SCR, hence in this figure the value of the normal distribution at α. One of the main drawback of VaR is its limited information; it does not consider the magnitude of the loss, given the scenario under α is happening. The amount of capital required to absorb shocks within assets and liabilities for Solvency 2, is set up to withstand circumstances occurring only once every 200 years. In other words, with a 99.5% confidence level, the capital should be able to survive a 1-years forecast. This forecast includes all kinds of risks an insurer company is exposed to.

Methods for executing the ORSA

As already mentioned, as a part of Pillar 2, an ORSA should be executed. Generally speaking, two alternive methods exist for this assessment: a deterministic approach and a stochastic approach.

In the deterministic method an insurer's portfolio is projected using predefined deterministic paths. After a (in general) simple simulation, the effects on this portfolio in the selected time frame are considered. Other names for this method are Covariance Matrix Method or Deterministic Balance Sheet Projection. In essence, one would try to point out all the risk-drivers (the indicators driving the insurance company's risks) and use them to project SCR and Economic Capital within the selected time frame. Possible variations of this approach would try to capture a Best Estimate in given (macro) economic circumstances.

A possible deterministic approach is to shock every single risk indicator on an event happening once every 200 years (a 99.5% shock). To be able to calculate the resulting loss in the first place, two assumptions are made.

1. *Assumption 1*: The economic risks follow a multivariate normal distribution

2. *Assumption 2*: The losses are a linear function of our risks

Using these assumptions, one would could come up with both a correlation and covariance matrix which could lead to the required capital. This approach is incorporated into the standard QIS5

SCR formula, proposed by the regulator as a possible method for calculating the Solvency Capital Reserve.

The main advantages focus on the relative simplicity of this approach. The method can be easily implemented and due to it's clarity, communication towards non-experts is also a strong point. The method is proven to be robust and has been implemented in the standard regulatory formula. Furthermore, the sector is familiar with this approach and it is widely used.

However, also some disadvantages are attached to this projection method. The weaknesses focus on the assumptions made regarding the correlation matrix. In the paper by (Kousaris, 2011) these drawbacks are clearly presented. The "linearity" assumed in the covariance matrix approach, can be interpreted on two pillars.

First of all, the correlation between economic risks are summarized as a linear Pearson correlation coefficient. The multivariate character imply limitations regarding the distributions used. The risks, hidden in the fat tails of the return distribution, when not corrected of tail dependency could imply an underestimation of number of extreme cases.

Secondly, it is assumed that the liabilities react linear to changes in risk-drivers. In reality however, non-linearity can be observed in this dependency. Kousaris, illustrates this by means of a plain vanilla put option. He shows that the value of a put option increases more than the linear model suggests, when both the equity and interest rate fall. This non-linearity in the liabilities side has been researched as well by (Cardi & Rusnak, 2007). They find that especially for life insurers this assumption implies as structural underestimation of the capital requirement. In the end, Kousaris conclude that the standard formula in the Solvency II framework contain *"glaring and substantial flaws that lead to a high level of inaccuracy"*. Measures to compensate for this inaccuracy imposed by some firms, like a "big bang" or "medium bang", are rather redundant.

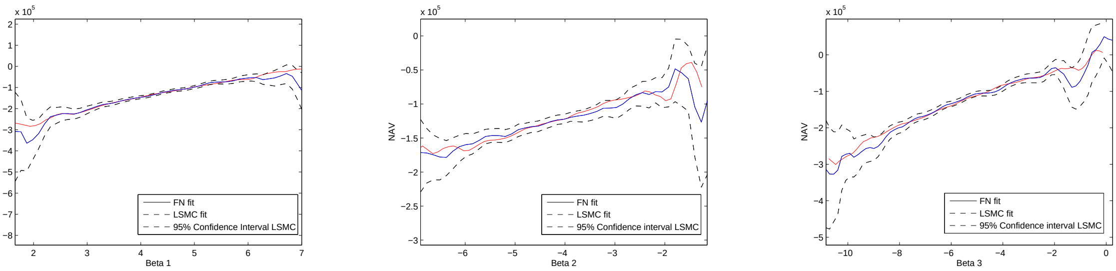
Hence, the demand for more sophisticated methods is clear. A more accurate method revolves around the use of stochastic real-world senario's and valuations. These techniques are used to calculate SCR for one year, but can be extended to multi year scenarios. An example of such an technique can be found in Chapter 2.7.

Appendix B

Sensitivity Analyses

In this appendix some sensitivity analyses will be performed, substantiating statements made in the report. Furthermore, this will lead to a better understanding in effect of some parameters upon the resulting outcomes.

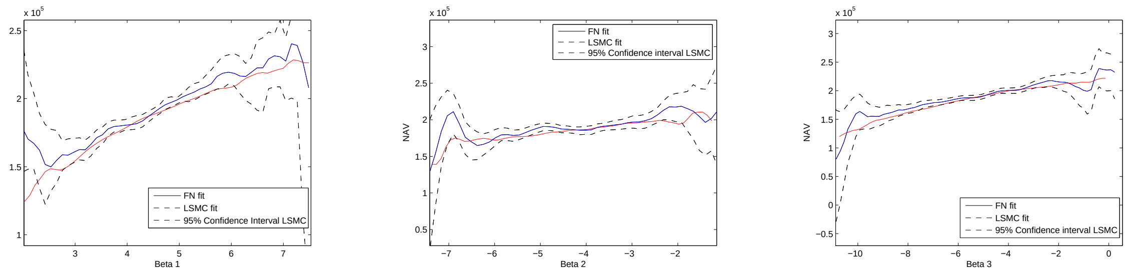
Effect of increasing the guaranteed rate



(a) Beta 1 (b) Beta 2 (c) Beta 3

Figure B.1: Observations of the three Diebold-Li factors plotted against the Net Asset Value for both the FN- and the LSMC approach (respectively the red and the blue fit). Compared to the original set-up, the guaranteed rate is increased from 1.8% to 2.8%, ceteris paribus. As expected, this leads to a more negative value of the profit-sharing product, compared to the original set-up.

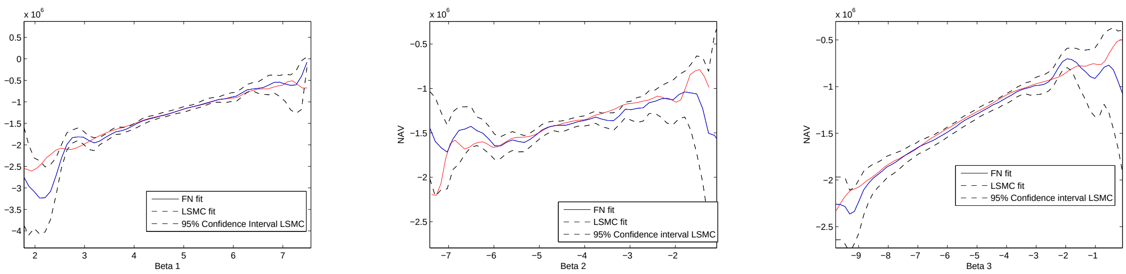
Effect of decreasing the profit sharing rate



(a) Beta 1 (b) Beta 2 (c) Beta 3

Figure B.2: Observations of the three Diebold-Li factors plotted against the Net Asset Value for both the FN- and the LSMC approach (respectively the red and the blue fit). Compared to the original set-up, the profit sharing rate is decreased from 70% to 30%, ceteris paribus. As expected, this leads to a more positive forecast of the value of the profit-sharing product, compared to the original set-up. A reducting of the profit-sharing rate, results in a decreased influence of β_2 and β_3.

Effect of increasing the maturity of the liabilities



(a) Beta 1 (b) Beta 2 (c) Beta 3

Figure B.3: Observations of the three Diebold-Li factors plotted against the Net Asset Value for both the FN- and the LSMC approach (respectively the red and the blue fit). Compared to the original set-up, the liability maturity is increased from 15 years to 25 years, ceteris paribus. As expected, this leads to a less positive forecast of the value of the profit-sharing product, compared to the original set-up. Furthermore, the influence of the β_1 increases while the influence of β_2 decreases, as expected.

Appendix C

Calibration of Hull-White model

In Section 2.3.3 we have showed an argumentation to use fixed parameters for σ and α. The quality of risk-neutral set would however benefit from calibrated values of α and σ. In this appendix, one can find a proposed calibration scheme for the parameters using payer swaptions.

A swaption is a dealing between two parties, trading a fixed interest rate for a floating interest rate (or vice versa). The holder of the payer swaption recieves a floating interest rate based on forward rates traded for a fixed (strike) rate K for a swap time T and corresponding cash-flows at T_{pay} . The value of such a payer swaption can be modelled as put options on zero-coupon bonds using the Hull-White:

$$V_{pay}(T, T_{pay}, K) = \sum_{t=1}^{n-1} K \cdot \tau_t \cdot P_k^*(T, T+t) + P_k^*(T, T+n)(1 + K \cdot \tau_n) \tag{C.1}$$

in which τ_t denotes the year fraction between the observations, P_k^* denotes the price of an put option maturing at T and written on a zero-coupon (pure discount) bond at $(T+t)$ and k the strike rate. Furthermore, we know that:

$$K = \frac{P(0,T) - P(0,T+n)}{\sum_{t=1}^{n} \tau_t \cdot P(0, T+t)} \tag{C.2}$$

The strike rate can be calculated using:

$$k(t) = A(T, T+t) \cdot e^{-B(T,T+t)r^X}, \tag{C.3}$$

in which r^X equals a spot rate for which:

$$\sum_{t=1}^{n-1} K \cdot \tau_t \cdot A(T, T+t) \cdot e^{-B(T,T+t)r^X} + (1 + K \cdot \tau_n) \cdot A(T, T+t) \cdot e^{-B(T,T+t)r^X} = 1. \tag{C.4}$$

Intuitively, this imposes that all the discounted cash flows of the bond at time T are equal to one.

$$P^*_{k(t)} = k(t) \cdot P(0,T) \cdot \Phi(-h + \sigma_p) - P(0, T+t) \cdot \Phi(-h), \tag{C.5}$$

in which σ_p and h are constants for a given t, equal to:

$$\sigma_p = \sigma \sqrt{\frac{1 - e^{-2\alpha T}}{2\alpha}} \cdot B(T, T+t), \tag{C.6}$$

and

$$h = \frac{1}{\sigma_p} \cdot \ln\left(\frac{P(T, T+t)}{k(t) \cdot P(0,T)}\right) + \frac{\sigma_p}{2}. \tag{C.7}$$

Given the analytical price of the swaption in Equation C.5, we can compare the analytical outcomes to the modelled outcomes. The α and σ we are after, are equal to the minumum of the MSE:

$$\{\alpha, \sigma\} = \min_{\alpha,\sigma} \sum_T \sum_n \left(V_{pay}(T, T_{pay}, K) - V^*_{pay}(T, T_{pay}, K)\right)^2 \tag{C.8}$$

where $V^*_{pay}(T, T_{pay}, K)$ denotes the observed payment swaptions with maturities T and that pay cash-flows at T_{pay} until $T + n$. Hence, using these constants, we are finally able to obtain our simulated interest rates. These can be used to value the profit sharing product.