Abstract

This paper concentrates on the relation between information search and weighting of rare events in decision from experience. Previous research found a consistent description-experience gap in weighting of rare events, as people overweight rare events when facing description-based decisions and underweight rare events when facing experience-based decisions. Using Erev et al. (2010) database, I identify two search strategies drawn from Bayesian decision theory: Pascal strategy and binomial strategy. In choices made after Pascal strategy, where the decision to stop searching is conditioned on the encountering of *r* rare events, subjects over-weight rare events more than in choices made after binomial strategy, where the decision to stop searching is unaffected by the number of rare events encountered. Splitting choices in Pascal/binomial is a way of interpreting the search process. Another way is dividing subjects into avid and frugal searchers (Hertwig & Pleskac 2010). Making a synthesis of those two ways of interpreting the search process could shed light on the link between information search and the description-experience gap.

Index

Introduction……3

1. Literature review……4

2. Research question 1: relation between search strategy and decision…………………………………………11

3. Method……14

4. Research question 2: Pascal/binomial determinants……………………………………………………………17

5. Dataset……19

6. Results – Estimation database……………………………………………………………………………………22

7. Results – Competition database ……………………………………………………………………………………32

8. Conclusion………37

Appendix……39

References………44

Introduction

Decision making research has traditionally focused on decision from description (henceforth DFD). In this paradigm, people are given full information about the probability distribution of the lotteries among which they are asked to choose. Choices between explicit lotteries provide researchers with the means to understand processes that should have some bearing upon important real-world decisions under uncertainty. Recently, decision from experience (henceforth DFE), a paradigm in which people have to acquire information about the lotteries by sampling from a static probability distribution, has drawn many researchers' attention. DFE represents a major recent breakthrough in behavioral decision research and a shift of attention away from the study of decision from description, where choices are made from explicitly stated payoffs and associated probabilities.

There are two main reasons why researchers find decision from experience interesting. First, the DFE paradigm resembles the way people make a large number of choices in the real world. Second, the way subjects acquire information may be decisive for their decision making process. It has been found that choice patterns in DFE consistently differs from DFD, hence the description-experience gap. The most researched and probably most relevant feature of the DFD-DFE gap is the underweighting of rare events in DFE (black swan effect in de Palma et al., forthcoming) compared to the overweighting of rare events in DFD (predicted by prospect theory and extensively supported by the literature).

This paper concentrates on one aspect of DFE, namely the relation between information search and decision. I assume that subjects search for information using one of the two strategies outlined in Bayesian decision theory: Pascal or binomial strategy (Winkler 1972). I find that the strategy subjects use to search for information impacts the weighting of rare events. In particular, Pascal choices lead to over-weight rare events more than binomial choices. Secondly I investigate whether the use of Pascal/binomial strategy can be predicted by personality traits of the subjects and by ecological features of the prospects. It turns out that personal traits of subjects and ecological properties of the prospects are correlated with the use of one or the other strategy (Pascal/binomial).

The paper is structured as follows. Next section is dedicated to a literature review of decision under risk and uncertainty from early developments of theory of probability with the introduction of the concept of expected value to decision from experience. Section 2 is dedicated to the discussion of the first research question (search strategy and decision), section 3 to the method used to tackle it. Section 4 discusses the second research question (prediction of search strategy). In section 5 the database used for analysis is described. Section 6 contains the results of the study. Section 7 concludes, summarizing the study and considering prospects for future research.

1. Literature review

Decision making from expected value to prospect theory[1]**.** In the 17th century, two French mathematicians, Blaise Pascal and Pierre de Fermat developed independently the idea to multiply probabilities by outcomes to find the mathematical expectation of a gambling problem they were working on. The expectation operator or "Expected Value" has been the basis for a theory of decision making under risk since its invention. In the 18th century two Swiss mathematicians, Gabriel Cramer and Daniel Bernoulli, introduced the concept of utility, originally meant to solve the St. Petersburg paradox. Introducing utility, the two mathematicians substituted the objective lottery values with the subjective utilities, thus allowing each subject to value lotteries in light of his own financial circumstances[2]. In the words of Cramer: "the mathematicians estimate money in proportion to its quantity, and men of good sense in proportion to the usage that they may make of it" (Cramer 1728). Two centuries later, Von Neumann and Morgenstern (1947) outlined four axioms which have to hold for an individual preferences to be represented by expected utility. The individual whose choice follow expected utility theory was defined as rational, implying the normative value of the theory.

Soon after, experiments shed on lights on systematic violations of expected utility theory. For instance, Allais (1953) showed that, contrary to the independence axiom, people do not disregard a common element shared by a pair of lotteries when making a choice between them. The

[1] For a deeper discussion on the history of probability and decision making from Pascal onwards, see Bernstein & Bernstein (1996) "Against the gods: The remarkable story of risk". If interested on the history of probability and decision making before Pascal, I suggest the following read: Franklin (2002) "The science of conjecture: Evidence and probability before Pascal".

[2] With respect to expected value expression: $\sum p_i x_i$, the expected utility permits the individual to value the objective payoff according to the subjective desirability the individual himself attaches to it : $\sum p_i u(x_i)$

phenomenon was named common consequence effect or Allais paradox. To take into account systematic violations of expect utility theory, several modifications to it were proposed. The most famous was undoubtedly prospect theory (Kanheman & Tversky 1979). Prospect theory retained the fundamental mathematical element of multiplying probabilities with utilities, using a utility function predicting risk aversion in the gain domain and risk seeking in the loss domain and introducing a non-linear probability weighting function, predicting overweighting of small probabilities[3] and underweighting of large probabilities. Two more points of novelty were represented by the introduction of loss aversion, the fact that losses loom larger than gains, and of the notion of reference point, which serves to separate the two regions of losses and gains and is settled to zero when a new prospect or set of prospects is evaluated in such a way that each change in wealth brings some utility or disutility even if it has little impact on the individual's financial situation. The most distinctive implication of prospect theory is the fourfold pattern of risk attitudes. The shapes of the value and the probability weighting functions imply risk-averse preferences when gains have high probability and losses have small probability, whereas they imply risk seeking preferences when gains have low probability and losses have high probability (Tversky & Kahneman 1992). In contrast to expected utility theory, prospect theory does not make any claim on a normative ground. It is solely a descriptive theory of decision making.

DFE: three paradigms. In the last ten years a wave of research on decision from experience showed a consistent DFD-DFE gap. In DFE experiments, subjects are asked to choose between two buttons of a "money machine". The two buttons are associated with unknown probability distributions. Each click results in a random draw from the designed distribution. Three DFE paradigms are typically researched (Hertwig & Erev 2009). The first is the free sampling paradigm, in which people first explore available distributions sampling as many outcomes as they want and only then decide from which distribution to make a final draw influencing their payoff. The search phase is thus separated from the one-shot decision phase. In the full-feedback paradigm, each draw contributes to people's

[3] Kahneman and Tversky indeed thought of small probabilities as being neglected or exaggerated as proven by a passage in 1979 paper: "Because people are limited in their ability to comprehend and evaluate extreme probabilities, highly unlikely events are either ignored or over-weighted, and the difference between high probability and certainty is either neglected or exaggerated". However, subsequent research has shown that overweighting of small probabilities is more common (Tversky & Kahneman 1992).

payoff and they receive draw-by-draw feedback on the obtained and the forgone payoffs. The partial-feedback paradigm is identical to the full-feedback one, except that people only learn about the obtained payoffs. In this paradigm people face a trade-off between exploration (gathering more information on a distribution) and exploitation (maximizing their payoff). The focus of the study is the free sampling paradigm. When I mention DFE, I refer to the free sampling paradigm unless stated otherwise.

DFE-DFD gap and reasons for it. While DFD is well predicted by prospect theory, decision from experience differs from it mainly because subjects tend to underweight small probabilities which leads to more risk-seeking behavior in presence of bad rare events and hence to a reversed fourfold pattern (e.g. Hertwig & Erev 2009). Moreover Barron & Erev (2003), who focused on the partial feedback paradigm, find a reversed reflection effect and thus claim that a different decision-making condition not only acts on the probability weighting function, but also on the shape of people's utility function, a point which will be advocated by several other researchers in the following years (Ungemach et al. 2009, Ludvig & Spetch 2011). Another feature of decision from experience not shared with description, namely the search for information, may influence the decision makers deviating choices from the description case (e.g. Hills & Hertwig 2010).

Undoubtedly the most debated element of the gap is the underweight of small probabilities in DFE compared to similar decisions made from description (de Palma et al., forthcoming). It can have significant implications in finance, insurance pricing and risk management. See the discussion in Taleb (2004) on selling small probabilities in finance, the chapter "Principles of insurance underwriting" in Buffett (2001) or the relation between underweighting of small probabilities and industrial accidents in Barkan et al. (1998).

While Barron and Erev (2003) drew attention on DFE by analyzing the partial feedback paradigm, Hertwig et al. (2004) defined the free sampling paradigm and substantially stimulated the debate on the topic. Hertwig et al. built six binary lotteries and divided the participants' pool in two groups. The first group made classic description-based decisions, the second group were told to sample from two payoff distributions for each lottery until they were ready to express their preference for one over the other. Results showed the choices clearly depended on the condition in which decision was taken. Facing the same lottery, decision makers put consistently less weight on the rare event when making

DFE than when making DFD. The experimenters proposed two explanations for their results. Firstly, the DFE paradigm carries a sampling error as far as designed probabilities differ from experienced probabilities obtained by drawing from the distributions. Because subjects themselves decide when to stop sampling and make a decision, the relative frequencies observed only rarely match with the actual probabilities of outcomes. The sampling error is amplified by the limited search effort made by subjects during the experiment, because small samples cause the rare event to be encountered less frequently than expected. That is because when *p* is small and *n* is small, the binomial distribution is positively skewed and it is likely to encounter the rare event less than the mathematical expectations (*np*). The theoretical argument is confirmed by experimental data. Indeed 78% of the subjects encountered the rare event less frequently than expected (*np*), whereas only 22% sampled it as frequently as expected or more frequently than expected.

Secondly, decision makers do not weight each trial they sample in an equal way. They give higher weights to more recent trials. Due to their rarity, rare events do not appear as often as common events in recent trials, which leads to underweighting of rare events. This phenomenon was named recency effect. Hertwig et al. (2004) find that the second halves of their samples predicted the choices better than the first halves.

Many studies investigated whether the sampling error is the sole cause of the gap. The evidence are ambiguous. I will report four studies which are representative of the sampling error literature. In the first, Rakow et al. (2008) show that sampling variability accounts for most of the DFD–DFE gap in the free sampling paradigm. In their study, the difference between biased and unbiased samples (samples with sampling error and without sampling error) had larger consequences than the difference between description and experience per se. Participants responded consistently to the probabilistic information encountered. The way participants acquired those information (description-experience) was of no consequence, which leads the authors to think of the description-experience gap as a purely statistical phenomenon. Their view was shared by Fox & Hadar (2006) who considered that it is premature to claim that a new theory of risky choice was needed after the results drawn by DFE, since they showed that prospect theory parameters of Tversky & Kahneman (1992) explains the DFE data of Hertwig et al. (2004) when experienced probabilities are taken into account rather than designed probabilities. Fox & Hadar (2006) called for future research to explore search patterns and termination of search rules of subjects, which had promising outlook. The third study (Hau et al. 2008) showed that increasing financial incentives (which stimulates more search) and forcing subjects to draw large samples from each deck reduces description-experience gap, but does not eliminate it. In particular, while the distribution including the rare event was selected 59% of the times in DFD and only 34% of the times in free sampling DFE, when the experimenter increased financial incentives and when subjects were forced to draw larger samples, the distribution including the rare event was selected respectively 46% and 44% of the times. Hence the gap is reduced but not fully eliminated. In the fourth study, Ungemach et al. (2009) controlled for the sampling error as they devised the lotteries in such a way that experienced probabilities equaled designed probabilities for every lottery and every subject. They found DFE-DFD gap to be smaller in absence of the sampling error, but still present. The recency effect, which earlier studies (Barron & Erev 2003, Hertwig et al. 2004) claimed had significant effect on the gap, was found to be insignificant by both Rakow et al. and Ungemach et al.

Since results indicated that the sampling error and the recency effect did not close the gap, researchers proposed other complementary explanations for it. One was the judgment error, i.e. misjudgment of probability when gathering information from experience leading to deviations with respect to the description condition. This hypothesis was not supported by Hau et al. and Ungemach et al. In the two studies, in order to check for a correct estimation of probabilities, subjects were asked to report the estimated payoff distributions of the lotteries after making their decisions. Reported probabilities approximately resembled experienced probabilities, which rules out the judgment error as a source for the DFD-DFE gap[4].

Hadar and Fox (2009) advocate that information asymmetry is the main cause of DFD-DFE gap. Information asymmetry is twofold: under sampling of the rare event because of subjects limited effort (sampling error) and underweighting certainty of the safe option because of context effects or because outcome certainty is not explicitly stated by the experimenter. The authors maintain that, when experienced probabilities are similar to designed probabilities and outcome certainty is explicit for the safe option, decision-experience gap disappears.

[4] Rakow et al. find a slight underweighting of high probabilities, together with accurate estimation of low probabilities. Ungemach et al. find accurate estimation of probabilities ranging from 0 to 1.

In a series of studies conducted in 2008, Erev, Glozman and Hertwig find that mere context and presentation effects have an impact on the weighting of the rare event. In one study they label the two sampling buttons with all possible alternatives coming from each probability distribution. For instance, if the lottery is (4, 0.8) or (3, 1), they label the first button “4 or 0” and the second button “3”. Merely stating the possibility of drawing the rare event causes decision makers to over-weight the frequency of the rare event. In another study, the authors present a capped-payoff version of the St. Petersburg paradox in which they describe the winning amount for each outcome up to the 20th draw. The list included payoffs ranging from 2 Israeli shekels when k=1 (p=1/2) to 1,048,576 shekels when k=20 (p <1/10^{6}). Knowing that such a huge payoff is possible, although very unlikely, participants are willing to pay an higher price to play the game with respect to the condition where no description of prizes was presented. Their attention is lured to the jackpot.

To conclude the review of the decision experience gap, it is interesting to turn to research done by Ludvig & Spetch (2011), who researched both the free sampling and the partial feedback paradigms. The experiment they designed to analyze the gap between conditions is a standard one (binary risky choice versus safe option), but they introduced the novelty of studying 50-50 probabilities instead of choices involving a rare event. Since a gap in their experiment cannot be explained by probability weighting, then, maintaining the Bernoullian framework of multiplying outcomes for probabilities, it had to be explained by a change in utility function passing from decision to experience. Their findings suggest that a new set of utility curves specifically fit for DFE may be needed and that the description-experience gap cannot be explained only in light of probability weighting. Instead, subjective utility curves based on experienced outcomes may overweight extreme values, growing faster than linear in the gain region (being convex instead of concave). Thus, extreme values carry proportionally more weight in decisions based on experience, leading to risk seeking for gains and risk aversion for losses or a reversed reflection effect.

Ludvig et al. (2013) do a follow-up study with slight modifications in their experiment setting and again excluding rare events to focus on decision involving 50-50 probabilities. They eliminate the comparison between different conditions by leaving out DFD problems and zero outcomes, which, not absolute gains nor losses, may be treated as a special kind of outcomes (Shampanier et al. 2007).

This time, the reversed reflection effect was significant only when extreme outcomes were present. That leads Ludvig et al. to argue that an extreme choice heuristic (over-weight extreme outcomes) may be crucial in DFE, as subjects who are not provided with an explicit description of probabilities are particularly influenced by the highest and the lowest outcome, especially when they are extreme compared to other lotteries tackled during the experiment.

Search patterns and DFE. Studies in decision making have traditionally neglected the role of information search because of the format of the studies themselves. In DFD information about probabilities and outcomes is provided with no margin error, so that subjects do not need to explore the lotteries, but only evaluate information provided by the researcher (Lejarraga 2010).

On the contrary, the process of DFE requires subjects to search for information and base their decisions on how they perceived the data they were provided with. Consequently, researchers turned their attention to the way people search for information, the strategies that they may adopt in the process and their correlation with observable decisions.

Hertwig and Pleskac (2010) note that limited search amplifies the sampling error and extensive search reduces it, as experienced probabilities converge to designed probabilities over an infinite number of trials. They hypothesize that this amplification effect makes subjects' decision easier and their preferences stronger. To test the hypothesis the authors divide subjects into two groups, frugal searchers and avid searchers, using the median number of trials at subject level as cutoff point. Those who sampled more than the median are avid searchers, the rest are frugal searchers. The former indeed have both stronger preferences, i.e. their choices are more distant from 50-50, and rate their choices as easier than the latter when explicitly asked by the experimenters.

In the paper where they discussed reasons for description-experience gap, Ungemach et al. (2009) suggest looking into search patterns to future research. They noted that during sampling various subjects switched frequently between the two buttons and had the intuition that the process of continuously comparing the two options may trigger a cognitive mechanism which is fundamentally different from DFD, hence the cause of the gap. Their suggestion was accepted by Hills and Hertwig (2010) who titled their paper "Sampling foreshadows decisions", following their findings indicating that different search strategies influence decision making. Switching buttons frequently when sampling is positively correlated to round-wise strategy (choosing *A* when it is better than *B* in $(n+1)/2$

draws), while first sampling from a button and switching only once is positively correlated to summary strategy (choosing *A* when mean(*A*) >mean (*B*)). Frequent switchers underweighted rare events more as they were likely paying attention mostly to ranking one outcome against the other regardless of the absolute amount, while infrequent switchers tended to underweight rare events less as they were likely using a natural mean strategy. Although their research implies that choice on search strategy is made a priori, Hills and Hertwig conclude their paper by acknowledging that they do not have conclusive evidence to back up this assumption.

Lejarraga et al. (2012) challenge the assumption on the ground of the evidence they gathered. In fact, their findings support that people adapt their search effort to the events that they draw. In particular, Lejarraga et al. focus on the relation between the domain in which subjects are making decisions (gain – loss), the amount of variance experienced in the sampling phase and termination of search. Their analysis builds on the premises that there is evidence that, on a physiological level, losses appear to trigger increased autonomic arousal relative to equivalent gains (Gonzalez, Dana, Koshino & Just, 2005). Lejarraga et al. find that subjects explored more lotteries in the loss domain which represent increased attraction or vigilance towards those outcomes. That can be seen as a product of loss aversion, insofar as decision makers want to avoid the high cost of making sub-optimal choices in the loss domain. The authors also find that experiencing more variance was positively correlated with more search. They find that this effect should not be imputed to avid searchers experiencing variance as a consequence of their already planned heavy sampling, because subjects' sampling was quite variable and positively correlated to experienced variance.

Contrary to speculation of Lejarraga et al. the results of a recent paper by Mehlhorn et al. (2013) suggest that the relationship between search and variability is largely driven by the increased chance to observe variability in larger samples, rather than by increased search following observed variability. Thus, Mehlhorn et al. share the idea of Hills and Hertwig (2010), which thought of search strategy as largely being conceived independently of the sampling phase.

2. Research question 1: relation between search strategy and decision

Relatively little is known about how search is related to properties of lotteries observed by subjects, and how these properties affect subsequent choice. Existent research can be divided in the two streams mentioned above.

The first one claims that the strategy subjects use to gather information impacts decision (Hills and Hertwig 2010) and that variance in experienced events is not a significant factor in the decision to extend search (Mehlhorn et al. 2013).

The second stream claims that ecological properties of the lotteries, in particular variance and choice domain, shape search, which in turn conditions decision (Lejarraga et al. 2012).

My research builds on the premise that the way subjects search is both determined by some a priori strategy and by the unfolding of the events. It can be considered an extension of the three papers above: differently from Hills and Hertwig (2010) I do not examine search between prospects but within prospects (with attention to the stopping rule), and differently from Lejarraga I assume that sampled outcomes do not enter the decision to terminate search if the subject is following a certain strategy. In the next sub-section I will outline the search strategies defined in Bayesian decision theory (Winkler 1972), that I will use to state my research questions.

Definition of search strategies. Let search occur according to one of the following two strategies. In the first the number of rare events the subject encounters before terminating search is fixed at r (sampling until the r^{th} rare event). The number of trials n needed to draw r rare events is a random variable and the distribution of n given p and r is a Pascal distribution of the form:

$$P(n|r,p) = \binom{n-1}{r-1} p^r (1-p)^{n-r}$$

In the second strategy the sample size n is fixed, in which case r is a random variable and the distribution of r given p and n is a binomial distribution:

$$P(r|n,p) = \binom{n}{r} p^r (1-p)^{n-r}$$

Notice that the two sampling distributions differ only in the first terms, the combinatorial terms. The reason for this is that the last trial must be a success in Pascal sampling because of the way Pascal sampling is defined, while there is not such restriction in binomial sampling. It turns out that in the application of Bayes' theorem, the combinatorial terms are irrelevant since they do not involve p, the variable of interest. The only important part of the sampling distribution for Bayesian purposes is the part involving p. Therefore, the likelihood function for the Bernoulli process can be taken to be equal to $p^r (1-p)^{n-r}$, so that it is not even necessary to know whether the sampling is done with n fixed or with r fixed. To determine the posterior distribution, the only information needed about the sample consists of the values of the sample statistics, r and n. Hence r and n are sufficient statistics. The procedure used to tell the statistician when to stop sampling is called a stopping rule, and if the stopping rule has no effect on the posterior distribution, then it is said to be non-informative. The two stopping rules discussed above (sample until you have n trials and sample until you have r successes) are both non-informative (see p. 157 of Winkler (1972) for a formal proof).

The fact that these stopping rules are non-informative in Bayesian decision theory mean that they should have no effect on probability judgment. My hypothesis instead is that using different search strategies leads to different evaluation of probabilistic information. To clarify the point I outline the way the experiment was conducted in the next paragraph and give examples of choices made after searching with Pascal strategy and choices made after searching with binomial strategy.

In the typical DFE framework, shared by the experiment that generated the data I am working on, subjects can choose between two alternatives, a safe and a risky prospect. The safe prospect gives a medium outcome (hereafter *M*) 100% of the times. The risky prospect gives a high outcome (*H*) with probability p_h and a low outcome (*L*) with probability p_l. Given this framework, when a subjects samples from the safe prospect he always draws *M*. On the other hand when a subject samples from the risky prospect he can draw *H* or *L*, one of the two being the rare event. Because there is no variability in the safe prospect, no single event from the safe prospect can have an influence on respondents' decision to terminate search and on his final decision. Instead there is variability in the risky prospect, which is represented by the rare (and significant) event. That is why I entirely focus on the exploration of the risky prospect and do not take into account the exploration of the safe prospect. I think that terminating one's search just after drawing a rare event leads the subject to put greater weight on it with respect to the situation when the rare event does not lead to the decision to terminate search.

I classify choices made stopping search after *r* rare events as choices made searching with Pascal strategy. All the other choices for which search was not stopped after encountering a rare event are choices made searching with binomial strategy. I also make two slight modifications to the search rules that seem logical thinking about the patterns of search in a typical DFE experiment.

First, when a subject samples one more time after encountering the rare event for the first time and then immediately stop searching. I included this particular termination rule into the set of Pascal strategies because after sampling a rare event for the first time a subject may be uncertain whether that event is truly a rare event. Then he checks that with one more draw from the risky prospect before terminating search.

Second, when a subject stops searching after encountering the rare events more than 5 times. I assume that if a subject did not stop searching having met the rare event for 5 times his search rule is not influenced by the outcomes he encounters. So, the fact that he stops searching after the r^{th} rare event, if r>5, is pure coincidence. I would then include such a stopping rule in the binomial strategy.

Research question. Do subjects over-weight rare events more when making choices after searching with Pascal strategy with respect to binomial strategy?

Hypothesis. I expect Pascal choices, for which search is concluded by the draw of a rare event (save the two exceptions) to lead to over-weighting of rare events more compared to binomial choices. The rationale of the hypothesis is that, for Pascal choices, the draw of a rare event is central in the cognitive process that leads to the decision of terminating exploration. On the other hand, a binomial choice is made after *n* draws containing both the common and the rare event from the risky prospect with the last draw being a common event. That can signal a search strategy which works as follows: “I will sample a predetermined amount of times from the risky prospect regardless of the outcomes drawn. Then I will evaluate its desirability”.

3. Method

In order to test the hypothesis that Pascal strategy choices lead to over-weight rare events more than Binomial strategy choices, I build a probit model with overweight/underweight of rare events as dependent variable and Pascal/Binomial as independent variable.

I also add some control variables possibly correlated with the dependent variable based on previous literature. The control variables are the following: one making up for the utility differential arising between the lotteries due to sampling error, one related to extreme outcomes, one controlling for median number searches at subject level, one related to the order of the question within the experiment and two controlling for the domain of choice (only gain, mixed, only loss). I wrote about my independent variable (Pascal/binomial strategy) extensively in the section "Research question 1: relation between search strategy and decision". In this section I will describe the dependent variable and the control variables.

The dependent variable. Probability weighting is unobservable, only decisions are. I infer probability weighting of rare events from decisions using a qualitative rule that allows me to create the dummy variable underweighting/over-weighting. The rule is: every time the rare event is a bad one and the subject chooses the safe prospect, or every time the rare event is a good one and the subject chooses the risky prospect, he is over-weighting the rare event; vice versa every time the rare event is a bad one and the subject chooses the risky prospect, or every time the rare event is a good one and the subject chooses the safe prospect, he is underweighting the rare event. This simple qualitative rule does not account for the utility differential between the safe and the risky prospect, which is handled through a continuous variable explained in the next paragraph.

Treatment of utility and utility differential variable. Different people can have different utility functions and different probability functions. Given that I am interested in probability weighting, I have to keep the utility function fixed. It follows that I have to assume a standard utility function for each subject. I have two alternative functions that suit the purpose, so I will run the model twice, assuming prospect theory function with Kahneman-Tversky parameters[5] (Tversky & Kahneman 1992) the first time and linear utility function the second time. Prospect theory is the most successful descriptive theory of decision making in the last 30 years and its utility function is considered suitable

---

[5] $v(x) = x^{\alpha}$ *if* $x \_> 0$; $-\lambda (-x)^{\beta}$ *if* $x < 0$ the parameters are $\alpha = 0.88$, $\beta = 0.88$, $\lambda = 2.25$

to represent people's preferences accurately in a variety of situations. Linear utility is a basic model of people's preferences which assumes constant sensitivity and equal impact of gains and losses. Linear utility is representative of preferences when dealing with small lotteries (see discussion in Wakker 2010, sections 1.5 – 1.6).

The scope of the utility differential variable in the model is to compensate for the utility differential between the risky and safe prospect, so that the dependent variable of the model is not influenced by this factor. Some problems were designed to offer two lotteries with equal expected utility, while some others offer a lottery which gives slightly higher expected utility than the other. Moreover experienced probabilities can exacerbate the expected utility differential (the amplification effect noted by Hertwig & Pleskac 2010). In particular, in small samples the expected utility difference between lotteries increases.

Let me clarify how the variable was built in detail. In 243 out of the 356 observations of the estimation database there is an incentive in terms of expected utility to select the option over-weighting rare events, following the qualitative definition of over-weighting given before. Since the dummy dependent variable over/underweighting takes the value 0 when subjects underweight rare events and 1 when subjects over-weight rare events, the coefficient of a continuous utility differential variable in favor of over-weighting ('LU_DIFF_OVERW' in case of linear utility and 'PTU_DIFF_OVERW' in case of prospect theory utility) will be positive if the utility differential will matter in subject's choice. Variable formula is the following: LU_DIFF_OVERW = $+|EV(R)-EV(S)|$ if difference is in favor overweighting and LU_DIFF_OVERW = $-|EV(R)-EV(S)|$ if difference is in favor of underweighting.

Other control variables. a) **Extreme outcomes.** When facing multiple numerical problems in the same context, people are especially sensitive to extreme outcomes, both high and low ones (Ludvig & Spetch 2011, Ludvig, Madan & Spetch 2013). A dummy 'extreme outcomes' variable controls for increased sensitivity to very high or very low outcomes. I defined extreme outcomes as those rare events higher than 20 or lower than -20. The cutoff is arbitrary, but justified by the small fraction of data that fit the definition and the saliency of such events compared to other events which may serve as context to subjects (Erev, Glozman & Hertwig 2008).

b) **Sample size at subject level**. I take the definition of frugal searchers and avid searchers from Hertwig & Pleskac (2010). Due to the possible effect of this personal characteristic on underweight/over-weighting, I create a dummy variable to control for it. To build the variable I split subjects' sample in two: those whose mean number of searches (both from the risky and the safe option) is higher than the median number of searches at subject level are classified as avid searchers; the rest are classified as frugal searchers.

c) **Order of the problem**. Search depends on subjects' attention and motivation. Hau et al. (2008) gives evidence that increasing financial incentives induces subjects to draw larger samples. As motivation is altered by money, attention can similarly be altered by length of the task. I believe that the order of the problem within the set of problems may be relevant because subjects may have either limited attention span and willingness to stay in the lab such that they might answer later questions quickly and carelessly due to the desire to leave as soon as possible, or there could be learning effects such that a subject needs the first few problems to understand how to give appropriate answers throughout the set of problems.

d) **Domain of choice**. Subjects may behave differently depending on the position of lottery outcomes with respect to a reference point, mostly assumed to be 0. There is evidence that search is influenced by the domain of choice (Lejarraga et al. 2012). Moreover several papers claim that prospect theory reflection effect is reversed in DFE, which means that in DFE people are risk seeking for gains and risk averse for losses (Barron & Erev 2003, Erev et al. 2010, Ludvig & Spetch 2011). Since there are three kinds of lotteries, gain, mixed and loss, the model includes two dummy variables to control for the domain of choice.

4. Research question 2: Pascal/binomial determinants

Pascal and binomial strategies are defined at choice level, i.e. search and decision are linked for every single choice, not at subject level. Therefore subjects may alternate the two strategies or stick to one in particular. It is curious to see to what extent, if any, subjects' use of a strategy can be predicted by personal characteristic or by ecological properties of the prospects.

To check for it I build a probit model with Pascal/binomial as dependent variable and all the control variables listed for research question 1 (median number of searches at subject level, extreme outcomes, order of the problem and two domain variables) as independent variables. I will also check whether the strategy used by a subject was consistent throughout the problems or whether a subject used different strategies for different problems.

Based on reasoning, I expect avid searchers make an extra effort compared to frugal searchers mostly to obtain some frequency information other than just the information concerning where the rare event is. Therefore, I believe that being an avid searcher is positively correlated to using a binomial strategy. If that is not true, meaning avid and frugal searchers indifferently use Pascal and binomial strategies, it would imply that outcomes influence search if a subject is using Pascal strategy and do not influence search if a subject is using binomial strategy. On the other hand, if being an avid searcher is positively correlated to the use of binomial strategy, it would mean that some of the effect imputed to the strategy should be attributed to the effort in searching. In this case the probability that a frugal searcher stopped searching right after the first rare event not because he is influenced by it but because he devotes little effort to search would increase. The point here is that there are some binomial choices that can mistakenly be classified as Pascal choices while the opposite is not true. For the sake of the example consider Pascal choices only those made immediately after encountering the first rare event. Assume all choices were made after binomial strategy search with stopping after n trials. The probability that a choice be classified as a Pascal choice is: $(1-p)^{n-1} * p$ where p is the probability of the rare event. As n increases, this probability approaches 0, whereas this probability is non-negligible when n is small. Since all search processes included in the database for this study include at least one rare event (more in the next section), this effect is to be considered. In other words, even if we assume that all choices are made after binomial strategy, some of them will appear as made after Pascal strategy.

Now assume that all choices were made after Pascal strategy. All choices would then be classified after Pascal strategy given the definition of the strategy itself. Because the set of Pascal strategies always end with a rare event (except for one case still classified as Pascal, where r=1 and the subject makes one last draw before terminating search), no choice would be mistakenly classified as binomial. In other words, when we assume that all choices are made after Pascal strategy, all choices appear as being made after Pascal strategy. The same does not hold when we assume all choices are made after binomial strategy. That is why the interaction between Pascal/binomial and avid/frugal may be relevant.

5. Dataset

Estimation dataset. I first analyzed the estimation dataset from the Choice-prediction competition organized by Erev et al. (2010). The dataset was assembled by Erev et al. and made public with the aim to give material to researchers so that they could come up with models predicting decision making in various conditions. The goodness of fit of the models would have been measured by a later study (the competition one) and the best models for each condition were declared winners of the competition. The estimation dataset contains DFD and both partial feedback and free sampling paradigms of DFE. I reduced the dataset to observations suited to the purpose of my study (free sampling).

The estimation experiment was run with 40 university students (from Technion, Israel) participating in the free sampling paradigm, randomly assigned to two different sub-groups. Each sub-group contained 20 participants who were presented with a representative sample of 30 problems from the estimation set. The participants were told that the experiment included several games, and in each game they were asked to choose once between two decks of cards represented by two buttons on the screen. It was explained that before making this choice they would have been able to sample the two decks. Each game started with the sampling stage, and the participants were asked to press the choice stage key when they felt they had sampled enough, but not before sampling at least once from each deck. The outcomes of the sampling were determined by the relevant problem. One deck corresponded to the safe alternative: all the virtual cards in this deck provided the medium payoff. The second deck corresponded to the payoff distribution of the risky option; e.g. sampling the risky deck in problem 21 resulted with the payoff +2 Israeli Shekels in 10% of the cases, and payoff -5.7 Shekels in the other cases. During the choice stage participants were asked to select once between the two virtual decks of cards. Their choice yielded a covert random draw of one card from the selected deck and was considered at the end of the experiment to determine the final payoff. The 60 problems composing the estimation dataset are presented in Appendix 1. The order of problems was randomized for each subject. A typical screen and the instructions are presented in Appendix 2.

Competition dataset. I also have access to the competition dataset from Erev et al. I will use it to evaluate the robustness of results found using the estimation dataset. This procedure of out-of-sample testing can corroborate or disconfirm the evidence gathered in this study.

The competition session was identical to the estimation session in each condition with two exceptions: different problems were randomly selected, and different subjects participated. The 60 problems used in the competition session are presented in Appendix 3. The 40 participants to the free sampling condition were drawn from the same population used in the estimation session (university students from Technion) without replacement. That is, the participants in the competition study did not participate in the estimation study, and the choice problems were new problems randomly drawn from the same distribution.

Adjustment to datasets. In order to test my research questions, I excluded choices coming from a search process in which the rare event was absent. These choices are trivial because they compare two certain outcomes of different value. For instance: (8, 100%) or (10,100%).

I also excluded lotteries in which a dominant strategy is clearly identifiable because those kind of observations have no relevance to my research questions and presumably were added to the set of problems only to check subjects' reliability.

Example of a dominant choice: risky option is (-3, 100%) or (-3, 80%; -12, 20%)

safe option is (-3, 100%)

Lastly, I excluded observations in which a "probability reversal" occurs in the sampling phase because the expected value of the two options differs markedly, making one of the two prospects unattractive and changing the goal of the experiment.

Example of a probability reversal: risky option is (18.8, 80%; 7.6, 20%)

safe option is (15.5, 100%)

Experienced probability of H is 0.33 instead of 0.8. As a consequence the common event becomes the rare event and vice versa, making the risky option desirable only for a person with strong risk seeking preferences.

Descriptive statistics estimation. The free sampling estimation dataset contains responses to 30 problems for each of 40 subjects participating in the experiment, for a total of 1200 observations. In 809 of them the rare event is absent from experienced outcomes, and 35 of them consist either of decisions where dominant choices are clearly identifiable or of probability reversal problems. The final sample contains 356 observations.

35% of the 356 were Pascal observations divided as follows. 21% of the respondents terminated sampling immediately after encountering the rare event or after doing a single more trial after encountering of the rare event (r=1), 7% of the respondents terminated sampling immediately after encountering the rare event for the second time (r=2) and 6% of the respondents terminated sampling immediately after encountering the rare event for the third, the fourth or the fifth time (r>2).

Subjects over-weighted rare events 62% of the times and underweighted them 38% of the times. Notice that these choices were all made after having encountered the rare event at least once. If trivial choices are included and assuming subjects could spot dominant strategies, rare events were over-weighted around 19% of the times, in line with the description-experience gap.

Approximately 1/3 of the problems involve only losses, 1/3 only gains and the rest mixed lotteries.

Descriptive statistics competition. The free sampling competition dataset includes subjects who sampled the rare outcome at least once in at least 10 out of 30 choice problems (in total 28 subjects), for a total of 838 observations. In 349 observations the rare event is absent from experienced outcomes, and 70 of them consist either of decisions where dominant choices are clearly identifiable or of probability reversal problems. The final sample contains 419 observations.

40% of the 419 were Pascal observations divided as follows. 27% of the respondents terminated sampling immediately after encountering the rare event or after doing a single more trial after encountering of the rare event (r=1), 6% of the respondents terminated sampling immediately after encountering the rare event for the second time (r=2) and 6% of the respondents terminated sampling immediately after encountering the rare event for the third, the fourth or the fifth time (*r*>2).

Subjects over-weighted rare events 72% of the times and underweighted them 28% of the times. Notice that these choices were all made after having encountered the rare event at least once. If trivial choices are included and assuming subjects could spot dominant strategies, rare events were over-weighted around 39% of the times, in line with the DFE-DFD gap.

Approximately 1/3 of the problems involve only losses, 1/3 only gains and the rest mixed lotteries.

6. Results – Estimation database

Results corroborate the main hypothesis of the study, namely that subjects over-weight rare events more in Pascal choices than in binomial choices. As you can see from the tables below, the coefficient Pascal/Binomial is significant at 1% level in the version of the model including linear utility as well as in the one including prospect theory utility function. Although the direction of the effect is clearly identifiable, the magnitude of the effect caused by using one strategy or another cannot be interpreted directly from coefficients as the marginal effect of a coefficient on the probability of over-weighting/underweighting rare events depends on the level of all the other coefficients.

Table 1

Dependent Variable: OVER_UNDERW

Method: ML - Binary Probit (Quadratic hill climbing)

Included observations: 356

Convergence achieved after 3 iterations

Covariance matrix computed using second derivatives

Variable	Coefficient	Std. Error	z-Statistic	Prob.
PAS_BIN	0.540478	0.159065	3.397840	0.0007
EXTREME	-0.868554	0.320202	-2.712522	0.0067
ORDER	0.001235	0.007427	0.166312	0.8679
NEG_DUMMY	0.058594	0.157728	0.371489	0.7103
MIXED_DUMMY	-0.157120	0.175928	-0.893093	0.3718

AVID_FRUGAL	0.233515	0.143251	1.630110	0.1031
LU_DIFF_OVERW	0.231516	0.049715	4.656883	0.0000

Mean dependent var	0.623596	S.D. dependent var	0.485165
S.E. of regression	0.447857	Akaike info criterion	1.184109
Sum squared resid	70.00093	Schwarz criterion	1.260301
Log likelihood	-203.7713	Hannan-Quinn criter.	1.214417
Deviance	407.5426	Restr. deviance	471.5409
Avg. log likelihood	-0.572391		

Obs with Dep=0	134	Total obs	356
Obs with Dep=1	222		

Table 2

Dependent Variable: OVER_UNDERW

Method: ML - Binary Probit (Quadratic hill climbing)

Included observations: 356

Convergence achieved after 3 iterations

Covariance matrix computed using second derivatives

Variable	Coefficient	Std. Error	z-Statistic	Prob.
PAS_BIN	0.542993	0.158552	3.424695	0.0006
EXTREME	-0.649195	0.321605	-2.018608	0.0435
ORDER	-0.000596	0.007410	-0.080433	0.9359
NEG_DUMMY	0.109658	0.158238	0.692992	0.4883
MIXED_DUMMY	-0.076261	0.173261	-0.440149	0.6598
AVID_FRUGAL	0.181036	0.142711	1.268546	0.2046
PTU_DIFF_OVERW	0.140211	0.033307	4.209653	0.0000

Mean dependent var	0.623596	S.D. dependent var	0.485165
S.E. of regression	0.451634	Akaike info criterion	1.196479
Sum squared resid	71.18680	Schwarz criterion	1.272671
Log likelihood	-205.9732	Hannan-Quinn criter.	1.226787
Deviance	411.9464	Restr. deviance	471.5409
Avg. log likelihood	-0.578576		

Obs with Dep=0	134	Total obs	356
Obs with Dep=1	222		

Among the control variables, only extreme events influence the weighting of rare events. The sign of the effect, though, is the opposite of what predicted before. When subjects face a rare event of very high significance, they underweight it with respect to the baseline case consisting of non-extreme event. This is a puzzling result. It goes against Ludvig & Spetch (2011) and Ludvig et al. (2013). I will return on the topic after discussing the other control variables.

None of the variables 'order', 'neg_dummy' and 'mixed_dummy' has an effect on the dependent variable. They can be excluded from the model with little risk of losing valuable information in the process.

I find some directional effect for the variable 'avid_frugal', but not enough to draw solid conclusions. Looking at the results, it is possible that frugal searchers suffer from the amplification effect and are thus induced to underweight rare events simply because they have encountered less of them. More study on this variable would help clarify its relation with weighting of rare events.

Finally, I find evidence that differences in utility matter. A subject is more prone to choose a risky prospect containing the rare event when the risk he is taking is justified by some gain of expected utility. By the same token, when the safe prospect yields a higher expected utility, its attractiveness to the subjects increases. The two versions of the model incorporating different utility functions are similar. The coefficient of the main variable of interest, Pascal/Binomial, is practically equal. There is a small difference in the effect of the utility differential, which is likely due to difference in the measurement of utility. Overall the two models seem interchangeable.

Given the many variables involved in the regression, there may be some multicollinearity problems distorting coefficients. Multicollinearity is a statistical phenomenon in which two or more predictor variables in a multiple regression model are highly correlated, meaning that one can be linearly predicted from the others with a non-trivial degree of accuracy. In this situation the coefficient estimates of the multiple regression may not only carry their net effect but also the effect of other highly correlated variables. I suspect the coefficient of the variable 'extreme' may be distorted due to a multicollinearity issue, which would explain the sign of the coefficient being the opposite of what the literature predicts. So I take out the variable 'extreme' from the regression to see what happens to the coefficients of other variables. It turns out that all the signs of the other coefficients remain unaltered[6].

Table 3

Dependent Variable: OVER_UNDERW
Method: ML - Binary Probit (Quadratic hill climbing)
Included observations: 356
Convergence achieved after 3 iterations
Covariance matrix computed using second derivatives

Variable	Coefficient	Std. Error	z-Statistic	Prob.
PAS_BIN	0.476773	0.156215	3.052019	0.0023
ORDER	0.001765	0.007364	0.239760	0.8105
NEG_DUMMY	0.044212	0.156241	0.282971	0.7772
MIXED_DUMMY	-0.112528	0.174546	-0.644691	0.5191
AVID_FRUGAL	0.176835	0.141038	1.253809	0.2099
LU_DIFF_OVERW	0.246336	0.049042	5.022959	0.0000

Mean dependent var	0.623596	S.D. dependent var	0.485165
S.E. of regression	0.451430	Akaike info criterion	1.200415
Sum squared resid	71.32601	Schwarz criterion	1.265722
Log likelihood	-207.6738	Hannan-Quinn criter.	1.226393
Deviance	415.3476	Restr. deviance	471.5409
Avg. log likelihood	-0.583353		

Obs with Dep=0	134	Total obs	356
Obs with Dep=1	222		

As you can see below, when removing the variable 'avid_frugal' (Table 4) or when removing the variable 'lu_diff_overw' (Table 5) which may be the source of multicollinearity with 'extreme', no sign

[6] I use linear utility differential in this regression and the following ones for convenience. If prospect theory utility differential is used, results are qualitatively equal.

of any coefficient changes, no insignificant coefficient turns significant or the reverse. This suggests that the variable 'extreme' is not highly correlated with any other variable. A likelihood ratio test on the model presented in Table 1 also shows that the consequences of imposing the restriction to take out 'extreme' are non-negligible, suggesting the larger model is not mis-specified.

Redundant Variables Test

Specification: OVER_UNDERW PAS_BIN EXTREME ORDER NEG_DUMMY MIXED_DUMMY AVID_FRUGAL LU_DIFF_OVERW

Redundant Variables: EXTREME

	Value	df	Probability
Likelihood ratio	7.804977	1	0.0052

LR test summary:

	Value	df
Restricted LogL	-207.6738	350
Unrestricted LogL	-203.7713	349

Consequently, the fact that subjects underweighted extreme events in the experiment merits some considerations. First, it could be that my definition of extreme event is not suited to the database, possibly because there is no event which is immensely significant for subjects' payoff, i.e. I mistakenly treated large events as extreme events. Second, it could be that subjects underweighted extreme rare events as the decision procedure looked like the one outlined by Hills and Hertwig (2010): choosing *A* when it is better than *B* in (n+1)/2 draws. Subjects who use this decision rule would be particularly insensitive to extreme events as they are merely comparing two outcomes to express a simple preference without any degree of strength attached to this preference.

Table 4

Dependent Variable: OVER_UNDERW

Method: ML - Binary Probit (Quadratic hill climbing)

Included observations: 356

Convergence achieved after 3 iterations

Covariance matrix computed using second derivatives

Variable	Coefficient	Std. Error	z-Statistic	Prob.
PAS_BIN	0.567157	0.157681	3.596859	0.0003
ORDER	0.007024	0.006491	1.082051	0.2792
EXTREME	-0.793453	0.314314	-2.524394	0.0116
NEG_DUMMY	0.129129	0.150967	0.855343	0.3924
MIXED_DUMMY	-0.049969	0.162540	-0.307425	0.7585
LU_DIFF_OVERW	0.231016	0.049497	4.667246	0.0000

Mean dependent var	0.623596	S.D. dependent var	0.485165
S.E. of regression	0.449028	Akaike info criterion	1.185958
Sum squared resid	70.56928	Schwarz criterion	1.251266
Log likelihood	-205.1005	Hannan-Quinn criter.	1.211936
Deviance	410.2010	Restr. deviance	471.5409
Avg. log likelihood	-0.576125		

Obs with Dep=0	134	Total obs	356
Obs with Dep=1	222		

Table 5

Dependent Variable: OVER_UNDERW

Method: ML - Binary Probit (Quadratic hill climbing)

Included observations: 356

Convergence achieved after 3 iterations

Covariance matrix computed using second derivatives

Variable	Coefficient	Std. Error	z-Statistic	Prob.
PAS_BIN	0.662606	0.152602	4.342047	0.0000
EXTREME	-1.017512	0.306379	-3.321094	0.0009
ORDER	0.001629	0.007218	0.225645	0.8215
NEG_DUMMY	0.057445	0.154370	0.372123	0.7098
MIXED_DUMMY	0.039147	0.166759	0.234755	0.8144
AVID_FRUGAL	0.228040	0.137895	1.653724	0.0982

Mean dependent var	0.623596	S.D. dependent var	0.485165

S.E. of regression	0.461718	Akaike info criterion	1.243374
Sum squared resid	74.61409	Schwarz criterion	1.308681
Log likelihood	-215.3205	Hannan-Quinn criter.	1.269352
Deviance	430.6410	Restr. deviance	471.5409
Avg. log likelihood	-0.604833		
Obs with Dep=0	134	Total obs	356
Obs with Dep=1	222		

Pascal/binomial and avid/frugal. This part reports results related to second research question of the study, namely the relation between the variable Pascal/binomial and all the other control variables. Before regressing the variable Pascal/binomial on the other control variables, I regressed 'over_underw' on all the control variables, linear utility differential included. If the effect of Pascal/binomial is captured by another variable, then I would suspect that variable to be correlated with Pascal/binomial.

Table 6

Dependent Variable: OVER_UNDERW

Method: ML - Binary Probit (Quadratic hill climbing)

Included observations: 356

Convergence achieved after 3 iterations

Covariance matrix computed using second derivatives

Variable	Coefficient	Std. Error	z-Statistic	Prob.
EXTREME	-0.696581	0.303786	-2.293000	0.0218
ORDER	0.006279	0.007173	0.875435	0.3813
NEG_DUMMY	0.078823	0.155508	0.506872	0.6122
MIXED_DUMMY	-0.134234	0.174156	-0.770773	0.4408
AVID_FRUGAL	0.286774	0.141234	2.030490	0.0423
LU_DIFF_OVERW	0.260887	0.048787	5.347442	0.0000

Mean dependent var	0.623596	S.D. dependent var	0.485165
S.E. of regression	0.453384	Akaike info criterion	1.211743
Sum squared resid	71.94493	Schwarz criterion	1.277051

Log likelihood	-209.6902	Hannan-Quinn criter.	1.237721
Deviance	419.3805	Restr. deviance	471.5409
Avg. log likelihood	-0.589018		
Obs with Dep=0	134	Total obs	356
Obs with Dep=1	222		

From Table 6 you can see that the coefficient of 'avid_frugal' turns significant at 5% level. It suggests a correlation between Pascal/binomial and avid/frugal. The hypothesis of a negative correlation is supported by the following regression.

Table 7

Dependent Variable: PAS_BIN

Method: ML - Binary Probit (Quadratic hill climbing)

Included observations: 356

Convergence achieved after 3 iterations

Covariance matrix computed using second derivatives

Variable	Coefficient	Std. Error	z-Statistic	Prob.
EXTREME	0.108112	0.279231	0.387179	0.6986
ORDER	0.006940	0.006817	1.018011	0.3087
NEG_DUMMY	-0.332689	0.151253	-2.199549	0.0278
MIXED_DUMMY	-0.161997	0.161570	-1.002643	0.3160
AVID_FRUGAL	-0.331156	0.135529	-2.443422	0.0145

Mean dependent var	0.345506	S.D. dependent var	0.476202
S.E. of regression	0.480879	Akaike info criterion	1.324946
Sum squared resid	81.16681	Schwarz criterion	1.379369
Log likelihood	-230.8403	Hannan-Quinn criter.	1.346594
Deviance	461.6807	Restr. deviance	458.9694
Avg. log likelihood	-0.648428		
Obs with Dep=0	233	Total obs	356
Obs with Dep=1	123		

Indeed it seems being an avid searcher is correlated to using binomial strategy and being a frugal searcher is correlated to using Pascal strategy. That leads me to think that the dichotomy Pascal/binomial and avid/frugal overlap to the extent that some subjects may be influenced by events drawn (Lejarraga et al. 2012), whereas others allocate to search a predetermined amount of effort (Hills and Hertwig 2010, Mehlhorn et al. 2013). Also the coefficient of 'neg_dummy' is negative and significant, indicating subjects tend to use binomial strategy in the loss domain more than in gain domain. This negative correlation endorses Lejarraga's statement that losses trigger more search for information.

Adding the linear utility differential to the picture (Table 8), I find that the larger the difference in utility between the risky and the safe prospect, the more choices occur after search follows Pascal strategy.

Table 8

Dependent Variable: PAS_BIN

Method: ML - Binary Probit (Quadratic hill climbing)

Included observations: 356

Convergence achieved after 3 iterations

Covariance matrix computed using second derivatives

Variable	Coefficient	Std. Error	z-Statistic	Prob.
EXTREME	0.245697	0.289925	0.847448	0.3967
ORDER	0.005746	0.006943	0.827589	0.4079
NEG_DUMMY	-0.369466	0.153301	-2.410062	0.0159
MIXED_DUMMY	-0.332330	0.170018	-1.954676	0.0506
AVID_FRUGAL	-0.353584	0.137592	-2.569810	0.0102
LU_DIFF_OVERW	0.163237	0.045633	3.577191	0.0003

Mean dependent var	0.345506	S.D. dependent var	0.476202
S.E. of regression	0.472700	Akaike info criterion	1.293392
Sum squared resid	78.20573	Schwarz criterion	1.358700
Log likelihood	-224.2238	Hannan-Quinn criter.	1.319370
Deviance	448.4475	Restr. deviance	458.9694

Avg. log likelihood	-0.629842		
Obs with Dep=0	233	Total obs	356
Obs with Dep=1	123		

Before turning to the competition database, let me summarize the main findings of this section. First, the use of Pascal/binomial strategy has an impact on the weighting of rare events: Pascal choices lead subjects to over-weight rare events more. Second, when an extreme rare event is present, subjects underweight it. Third, the utility differential between the two prospects is a factor subjects incorporate in their decision making procedure. Fourth, the use of Pascal and binomial strategy is correlated with the personal trait avid/frugal, with the domain of choice of prospects and with the utility differential: binomial strategy is more frequent among avid searchers, among decisions including only loss prospects and among decision between prospects divided by a small utility differential.

Summary of the results, estimation database

	under/over	pas/bin	extreme	order	neg_d	mixed_d	avid_frugal	lu_diff	ptu_diff
table 1	dep. Variable	***	***	not sign	not sign	not sign	not sign	***	not present
table 2	dep. Variable	***	**	not sign	not sign	not sign	not sign	not present	***
table 3	dep. Variable	***	not present	not sign	not sign	not sign	not sign	***	not present
table 4	dep. Variable	***	**	not sign	not sign	not sign	not present	***	not present
table 5	dep. Variable	***	***	not sign	not sign	not sign	*	not present	not present
table 6	dep. Variable	not present	**	not sign	not sign	not sign	**	***	not present
table 7	not present	dep. Variable	not sign	not sign	**	not sign	**	not present	not present
table 8	not present	dep. Variable	not sign	not sign	**	not sign	**	***	not present

*=significant at 10% level **=significant at 5% level ***=significant at 1% level

7. Results – Competition database

The analysis on the competition database serves as a robustness check to previous findings. In this case using linear utility differential variable or prospect theory utility differential variable brings some unexpected differences. Regressing over/underweighting on all the other variables, the coefficient of Pascal/binomial is significant at 5% level only in the model containing linear utility. In the one containing prospect theory utility, the directional effect is still there, but the coefficient turns insignificant. It is hard to explain why there is such a difference between the two models diverging from one another only for utility measurement.

Table 9

Dependent Variable: OVER_UNDERW

Method: ML - Binary Probit (Quadratic hill climbing)

Included observations: 418

Convergence achieved after 4 iterations

Covariance matrix computed using second derivatives

Variable	Coefficient	Std. Error	z-Statistic	Prob.
PAS_BIN	0.364268	0.152430	2.389734	0.0169
EXTREME	-0.438841	0.279968	-1.567472	0.1170
ORDER	0.021910	0.007155	3.062136	0.0022
NEG_DUMMY	0.047544	0.157647	0.301584	0.7630
MIXED_DUMMY	-0.354215	0.178343	-1.986143	0.0470
AVID_FRUGAL	0.075745	0.136433	0.555184	0.5788
LU_DIFF_OVERW	0.579716	0.071273	8.133695	0.0000
Mean dependent var	0.727273	S.D. dependent var		0.445895
S.E. of regression	0.396598	Akaike info criterion		0.971179
Sum squared resid	64.64615	Schwarz criterion		1.038759
Log likelihood	-195.9765	Hannan-Quinn criter.		0.997895
Deviance	391.9530	Restr. deviance		489.8564
Avg. log likelihood	-0.468843			
Obs with Dep=0	114	Total obs		418
Obs with Dep=1	304			

Table 10

Dependent Variable: OVER_UNDERW

Method: ML - Binary Probit (Quadratic hill climbing)

Included observations: 418

Convergence achieved after 4 iterations

Covariance matrix computed using second derivatives

Variable	Coefficient	Std. Error	z-Statistic	Prob.
PAS_BIN	0.249847	0.147787	1.690593	0.0909
EXTREME	0.380855	0.252554	1.508011	0.1316
ORDER	0.017092	0.006881	2.483889	0.0130
NEG_DUMMY	0.201824	0.153626	1.313735	0.1889
MIXED_DUMMY	-0.052743	0.169252	-0.311622	0.7553
AVID_FRUGAL	-0.096617	0.133444	-0.724024	0.4691
PTU_DIFF_OVERW	0.364140	0.047347	7.690952	0.0000

Mean dependent var	0.727273	S.D. dependent var	0.445895
S.E. of regression	0.403340	Akaike info criterion	1.021406
Sum squared resid	66.86272	Schwarz criterion	1.088985
Log likelihood	-206.4738	Hannan-Quinn criter.	1.048121
Deviance	412.9476	Restr. deviance	489.8564
Avg. log likelihood	-0.493956		
Obs with Dep=0	114	Total obs	418
Obs with Dep=1	304		

In both models the utility differential variable is significant at 1% level, carrying the ordinary effect of over-weighting rare events when there are incentives to it. Also the variable 'order' is positive and significant in both models, representing a tendency to overweight rare events more towards the end of the experimental session. It can be the consequence of higher reliance on an heuristic like: "chase good rare events, avoid bad rare events" that can gain importance during the experiment as subjects get tired and less willing to make some computational effort. You can also see the tendency to underweight rare events in mixed prospects with respect to positive prospects.

Moving to second research question, I regress Pascal/Binomial on all the control variables (order, extreme events, negative prospects, mixed prospects, avid/frugal and utility differential) as I did for the estimation dataset. Table 11 contains linear utility, Table 12 contains prospect theory utility.

Table 11

Dependent Variable: PAS_BIN

Method: ML - Binary Probit (Quadratic hill climbing)

Included observations: 418

Convergence achieved after 3 iterations

Covariance matrix computed using second derivatives

Variable	Coefficient	Std. Error	z-Statistic	Prob.
EXTREME	-0.554311	0.243046	-2.280682	0.0226
ORDER	0.008149	0.005579	1.460655	0.1441
NEG_DUMMY	-0.296675	0.136815	-2.168448	0.0301
MIXED_DUMMY	-0.326913	0.153270	-2.132922	0.0329
AVID_FRUGAL	-0.302045	0.117662	-2.567046	0.0103
LU_DIFF_OVERW	0.099508	0.043746	2.274702	0.0229

Mean dependent var	0.394737	S.D. dependent var	0.489380
S.E. of regression	0.483707	Akaike info criterion	1.334208
Sum squared resid	96.39660	Schwarz criterion	1.392134
Log likelihood	-272.8495	Hannan-Quinn criter.	1.357107
Deviance	545.6991	Restr. deviance	560.8054
Avg. log likelihood	-0.652750		
Obs with Dep=0	253	Total obs	418
Obs with Dep=1	165		

Table 12

Dependent Variable: PAS_BIN

Method: ML - Binary Probit (Quadratic hill climbing)

Included observations: 418

Convergence achieved after 3 iterations

Covariance matrix computed using second derivatives

Variable	Coefficient	Std. Error	z-Statistic	Prob.
EXTREME	-0.379972	0.244608	-1.553396	0.1203
ORDER	0.006441	0.005698	1.130331	0.2583
NEG_DUMMY	-0.270488	0.134788	-2.006756	0.0448
MIXED_DUMMY	-0.279697	0.148931	-1.878023	0.0604
AVID_FRUGAL	-0.331203	0.116351	-2.846591	0.0044
PTU_DIFF_OVERW	0.075081	0.033002	2.275036	0.0229

Mean dependent var	0.394737	S.D. dependent var	0.489380
S.E. of regression	0.483788	Akaike info criterion	1.334189
Sum squared resid	96.42881	Schwarz criterion	1.392114
Log likelihood	-272.8455	Hannan-Quinn criter.	1.357088
Deviance	545.6910	Restr. deviance	560.8054
Avg. log likelihood	-0.652740		

Obs with Dep=0	253	Total obs	418
Obs with Dep=1	165		

The model containing prospect theory utility (Table 12) closely resembles the model examined in the estimation section (Table 8). In these models the variables having significant effect on Pascal/binomial are the negative dummy, avid/frugal and the utility differential (linear utility in Table 8, PT utility in Table 12). In Table 11, the three variables having an effect in Table 12 maintain this effect. In addition to those, 'extreme' is negative and significant, indicating that when a rare event is extreme subjects tend to use Binomial search, and 'order' is positive and significant, indicating that subjects opt for Pascal strategy later in the search process.

Summary of the results, competition database

	under/over	pas/bin	extreme	order	neg_d	mixed_d	avid_frugal	lu_diff	ptu_diff
table 9	dep. Variable	**	not sign	***	not sign	**	not sign	***	not present
table 10	dep. Variable	*	not sign	**	not sign	not sign	not sign	not present	***
table 11	not present	dep. Variable	**	not sign	**	**	**	**	not present
table 12	not present	dep. Variable	not sign	not sign	**	*	***	not present	**

*=significant at 10% level **=significant at 5% level ***=significant at 1% level

7. Conclusion

Search strategy affects probability weighting in decision from experience. Pascal strategy, where subjects fix to *r* the number of rare events they will draw, leads to over-weighting of rare events more than binomial strategy, where subjects fix to *n* the number of draws they will make from the risky prospect. While, according to Bayesian decision theory, using one or the other strategy should be non-informative and thus should have no effect on decision whatsoever, choices following different strategy conduce to different sensitivity to rare events. The effect of search strategies on decision found in the estimation study is to large extent supported by results of the competition study.

Search strategy can be predicted by the average number of draws from the risky prospect by a dummy variable dividing the sample of subjects between avid searchers and frugal searchers, in accordance with the definition given by Hertwig & Pleskac (2010). Avid searchers are inclined to use binomial strategy and frugal searchers are inclined to use Pascal strategy. Search strategy is also influenced by the domain of choice and the utility differential between prospects. In the loss domain and when the utility differential is small, binomial search is more frequent. In the gain domain and when the utility differential is large, Pascal search is more frequent.

Researchers have been struggling with the decision-experience gap in the last decade. Explanations of the gap involving sampling error, recency effects and probability judgment did not close gap. Research focusing on search patterns look promising, although far from being exhaustive. In my opinion studying the connection between Bayesian strategies (Pascal/binomial) and subjects' effort (avid/frugal) may help in gaining a deeper understanding of the description-experience gap.
With regard to that, the following point is worth mentioning. While the division between avid and frugal searchers at subject level is justified on the ground that number of draws within subject does not vary much (Hertwig & Pleskac 2010), it would be interesting to look at the management of Bayesian strategies within subjects: do people alternate Pascal and binomial strategies or do they use one of the two strategies to tackle most of the problems? If the latter is true, then it would be possible to divide subjects into Pascal searchers and binomial searchers who may approach both

search and decision phases differently from each other. In this study this question cannot be answered because the sample of choices for each subject is small, thus a Chi-square test to check for the greater use of one strategy over another does not bring any significant result except for a few subjects.

Appendix 1: estimation set problems.

Table 1: the aggregate proportion of choices of the estimation session.

	Risk Gamble			Safe	Proportion of choices in Risk			average number
Problem	High	P(High)	Low	Medium	Description	E-Sampling	E-repeated	of samples
1	-0.3	0.96	-2.1	-0.3	0.2	0.25	0.33	10.4
2	-0.9	0.95	-4.2	-1	0.2	0.55	0.50	9.7
3	-6.3	0.3	-15.2	-12.2	0.6	0.5	0.24	13.9
4	-10	0.2	-29.2	-25.6	0.85	0.3	0.32	10.7
5	-1.7	0.9	-3.9	-1.9	0.3	0.8	0.45	9.9
6	-6.3	0.99	-15.7	-6.4	0.35	0.75	0.68	9.9
7	-5.6	0.7	-20.2	-11.7	0.5	0.6	0.37	11.1
8	-0.7	0.1	-6.5	-6	0.75	0.2	0.27	13.9
9	-5.7	0.95	-16.3	-6.1	0.3	0.6	0.43	11.0
10	-1.5	0.92	-6.4	-1.8	0.15	0.9	0.44	11.8
11	-1.2	0.02	-12.3	-12.1	0.9	0.15	0.26	11.9
12	-5.4	0.94	-16.8	-6.4	0.1	0.65	0.55	11.2
13	-2	0.05	-10.4	-9.4	0.5	0.2	0.11	10.4
14	-8.8	0.6	-19.5	-15.5	0.7	0.8	0.66	12.1
15	-8.9	0.08	-26.3	-25.4	0.6	0.3	0.19	11.6
16	-7.1	0.07	-19.6	-18.7	0.55	0.25	0.34	11.0
17	-9.7	0.1	-24.7	-23.8	0.9	0.55	0.37	15.1
18	-4	0.2	-9.3	-8.1	0.65	0.4	0.34	11.2
19	-6.5	0.9	-17.5	-8.4	0.55	0.8	0.49	14.9
20	-4.3	0.6	-16.1	-4.5	0.05	0.2	0.08	10.9
21	2	0.1	-5.7	-4.6	0.65	0.2	0.11	8.8
22	9.6	0.91	-6.4	8.7	0.05	0.7	0.41	9.2
23	7.3	0.8	-3.6	5.6	0.15	0.7	0.39	10.7
24	9.2	0.05	-9.5	-7.5	0.5	0.05	0.08	14.6
25	7.4	0.02	-6.6	-6.4	0.9	0.1	0.19	8.9
26	6.4	0.05	-5.3	-4.9	0.65	0.15	0.20	13.4
27	1.6	0.93	-8.3	1.2	0.15	0.7	0.50	8.9
28	5.9	0.8	-0.8	4.6	0.35	0.65	0.58	10.6
29	7.9	0.92	-2.3	7	0.4	0.65	0.51	10.6
30	3	0.91	-7.7	1.4	0.4	0.7	0.41	10.0
31	6.7	0.95	-1.8	6.4	0.1	0.7	0.52	11.0
32	6.7	0.93	-5	5.6	0.25	0.55	0.49	11.0
33	7.3	0.96	-8.5	6.8	0.15	0.75	0.65	11.1
34	1.3	0.05	-4.3	-4.1	0.75	0.1	0.3	11.4
35	3	0.93	-7.2	2.2	0.25	0.55	0.44	12.8
36	5	0.08	-9.1	-7.9	0.4	0.2	0.09	14.6
37	2.1	0.8	-8.4	1.3	0.1	0.35	0.28	10.9
38	6.7	0.07	-6.2	-5.1	0.65	0.2	0.29	10.9
39	7.4	0.3	-8.2	-6.9	0.85	0.7	0.58	12.7
40	6	0.98	-1.3	5.9	0.1	0.7	0.61	13.5
41	18.8	0.8	7.6	15.5	0.35	0.6	0.52	9.0
42	17.9	0.92	7.2	17.1	0.15	0.8	0.48	10.8
43	22.9	0.06	9.6	9.2	0.75	0.9	0.88	9.9

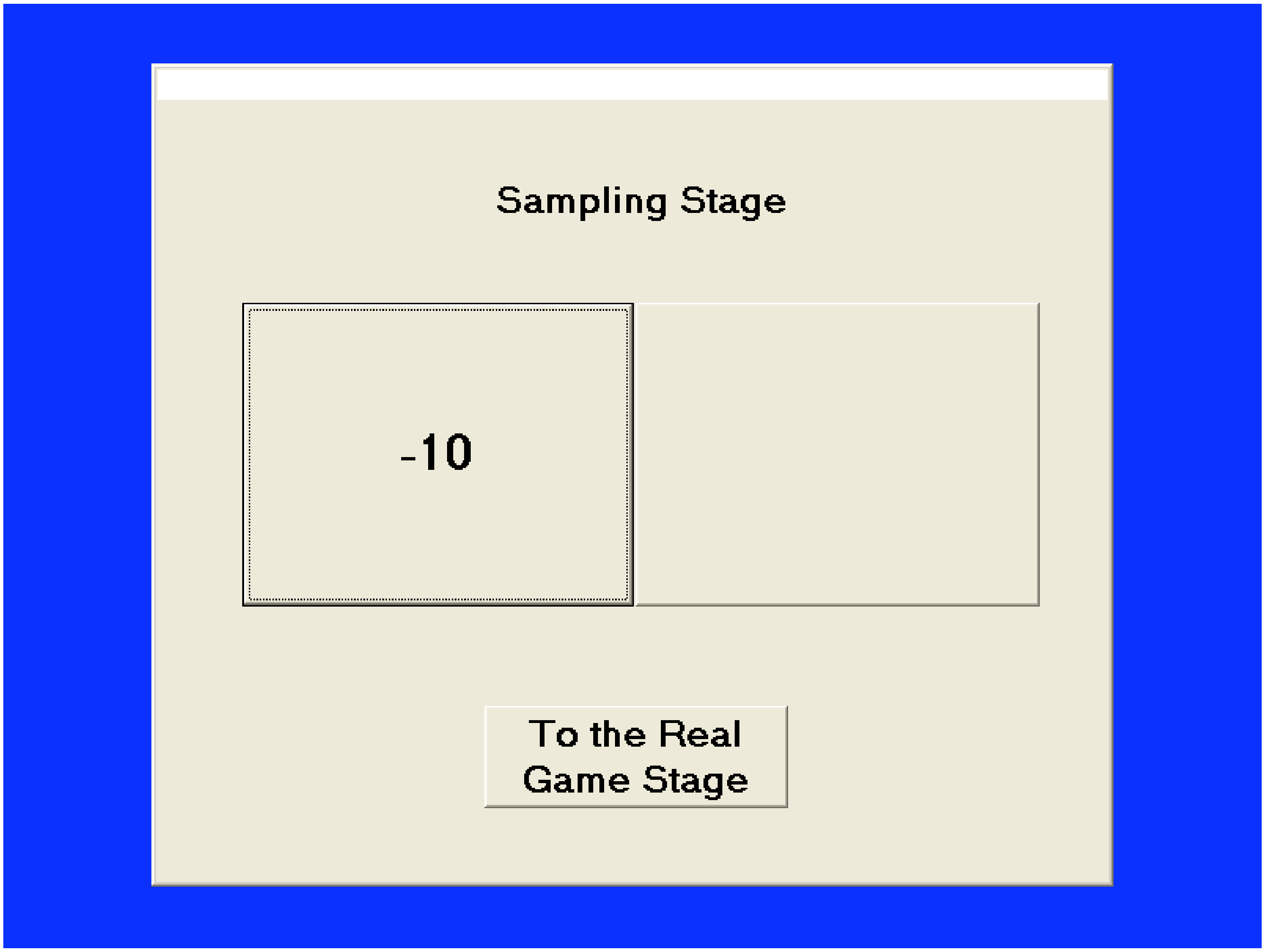
44	10	0.96	1.7	9.9	0.2	0.7	0.56	10.1
45	2.8	0.8	1	2.2	0.55	0.7	0.48	19.4
46	17.1	0.1	6.9	8	0.45	0.2	0.32	9.2
47	24.3	0.04	9.7	10.6	0.65	0.2	0.25	11.8
48	18.2	0.98	6.9	18.1	0.1	0.75	0.59	9.0
49	13.4	0.5	3.8	9.9	0.05	0.45	0.13	8.9
50	5.8	0.04	2.7	2.8	0.7	0.2	0.35	10.0
51	13.1	0.94	3.8	12.8	0.15	0.65	0.52	9.0
52	3.5	0.09	0.1	0.5	0.35	0.25	0.26	11.9
53	25.7	0.1	8.1	11.5	0.4	0.25	0.11	9.0
54	16.5	0.01	6.9	7	0.85	0.25	0.18	13.4
55	11.4	0.97	1.9	11	0.15	0.7	0.66	9.6
56	26.5	0.94	8.3	25.2	0.2	0.5	0.53	14.3
57	11.5	0.6	3.7	7.9	0.35	0.45	0.45	10.0
58	20.8	0.99	8.9	20.7	0.25	0.65	0.63	12.9
59	10.1	0.3	4.2	6	0.45	0.45	0.32	10.1
60	8	0.92	0.8	7.7	0.2	0.55	0.44	10.2

Appendix 2: instructions and typical screens

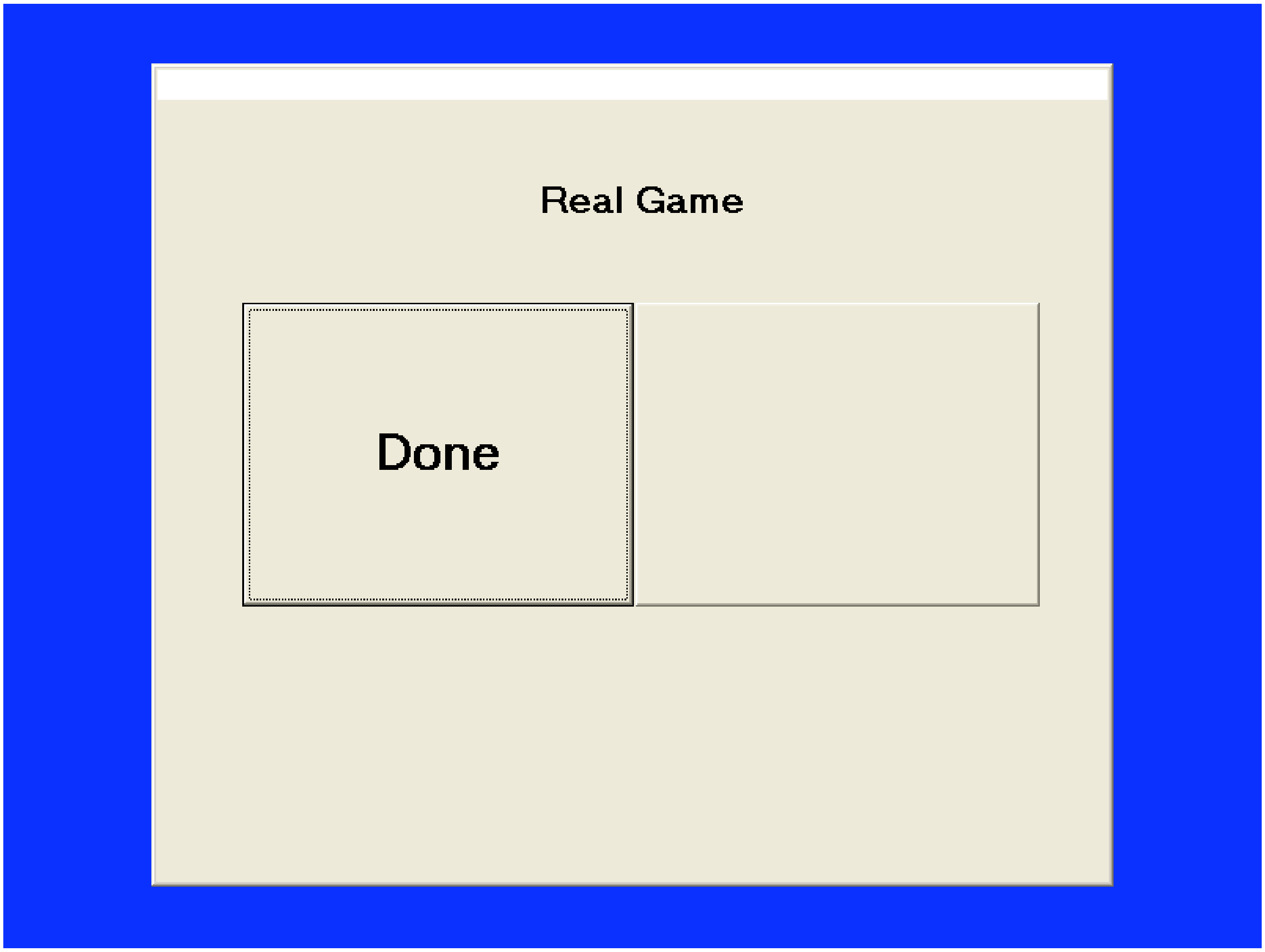
This experiment includes several games. Each game includes two stages: The sampling stage and the choice stage. At the choice stage (the second stage) you will be asked to select once between two virtual decks cards (two buttons). Your choice will lead to a random draw of one card from this deck, and the number written on the card will be the "game's outcome." During the sampling stage (the first stage) you will be able to sample the two decks. When you feel that you have sampled enough press the "choice stage" key to move to the choice stage. At the end of the experiment one of the games will be randomly drawn (all the games are equally likely to be drawn). Your payoff for the experiment will be the outcome (in Sheqels) of this game.

Good luck!

Experimental screen (a) after sampling the deck associated with the safer option in Problem 4 during the sampling stage:



Experimental screen (b) – After choosing the deck associated with the safer option in Problem 4 during the real game stage:



Appendix 3: competition set problems

Table 4.2: the aggregate proportion of choices of the competition session.

	Risk Gamble			Safe	Proportion of choices in Risk			average number
Problem	High	P(High)	Low	Medium	Description	E-Sampling	E-repeated	of samples
1	-8.7	0.06	-22.8	-21.4	0.7	0.45	0.25	16.35

2	-2.2	0.09	-9.6	-8.7	0.6	0.15	0.27	15.65
3	-2	0.1	-11.2	-9.5	0.45	0.1	0.25	15.6
4	-1.4	0.02	-9.1	-9	0.85	0.2	0.33	15.9
5	-0.9	0.07	-4.8	-4.7	0.8	0.35	0.37	15.55
6	-4.7	0.91	-18.1	-6.8	0.5	0.75	0.63	14.75
7	-9.7	0.06	-24.8	-24.2	0.95	0.5	0.3	20.95
8	-5.7	0.96	-20.6	-6.4	0.35	0.65	0.66	15.85
9	-5.6	0.1	-19.4	-18.1	0.75	0.2	0.31	15.5
10	-2.5	0.6	-5.5	-3.6	0.45	0.5	0.34	17.15
11	-5.8	0.97	-16.4	-6.6	0.4	0.65	0.61	17.35
12	-7.2	0.05	-16.1	-15.6	0.75	0.4	0.25	16.85
13	-1.8	0.93	-6.7	-2	0.25	0.55	0.44	11.85
14	-6.4	0.2	-22.4	-18	0.7	0.15	0.21	12.05
15	-3.3	0.97	-10.5	-3.2	0.1	0.1	0.16	18.2
16	-9.5	0.1	-24.5	-23.5	0.9	0.7	0.39	15.7
17	-2.2	0.92	-11.5	-3.4	0.25	0.65	0.47	14.7
18	-1.4	0.93	-4.7	-1.7	0.3	0.55	0.41	16.5
19	-8.6	0.1	-26.5	-26.3	0.9	0.6	0.49	16.25
20	-6.9	0.06	-20.5	-20.3	1	0.6	0.25	15.95
21	1.8	0.6	-4.1	1.7	0.05	0.1	0.08	10.8
22	9	0.97	-6.7	9.1	0	0.15	0.14	14.85
23	5.5	0.06	-3.4	-2.6	0.4	0.2	0.28	18.05
24	1	0.93	-7.1	0.6	0.25	0.65	0.46	14.05
25	3	0.2	-1.3	-0.1	0.35	0.25	0.21	14.5
26	8.9	0.1	-1.4	-0.9	0.7	0.25	0.23	17.65
27	9.4	0.95	-6.3	8.5	0.2	0.55	0.67	13.25
28	3.3	0.91	-3.5	2.7	0.25	0.65	0.58	12.95
29	5	0.4	-6.9	-3.8	0.75	0.7	0.39	15.1
30	2.1	0.06	-9.4	-8.4	0.5	0.3	0.33	18.1
31	0.9	0.2	-5	-5.3	1	0.95	0.88	14.8
32	9.9	0.05	-8.7	-7.6	0.65	0.3	0.21	19.7
33	7.7	0.02	-3.1	-3	0.9	0.35	0.28	15.95
34	2.5	0.96	-2	2.3	0.2	0.5	0.52	15.85
35	9.2	0.91	-0.7	8.2	0.15	0.6	0.56	14.7
36	2.9	0.98	-9.4	2.9	0	0.35	0.34	18.15
37	2.9	0.05	-6.5	-5.7	0.6	0.35	0.3	15.3
38	7.8	0.99	-9.3	7.6	0.2	0.75	0.62	15.25
39	6.5	0.8	-4.8	6.2	0	0.35	0.32	11
40	5	0.9	-3.8	4.1	0.1	0.5	0.46	13.4
41	20.1	0.95	6.5	19.6	0.15	0.65	0.5	13.7
42	5.2	0.5	1.4	5.1	0.05	0.05	0.08	12
43	12	0.5	2.4	9	0	0.25	0.17	14.35
44	20.7	0.9	9.1	19.8	0.15	0.55	0.44	11.85
45	8.4	0.07	1.2	1.6	0.9	0.25	0.2	14.8
46	22.6	0.4	7.2	12.4	0.75	0.3	0.41	15.3
47	23.4	0.93	7.6	22.1	0.35	0.65	0.72	13.2
48	17.2	0.09	5	5.9	0.85	0.5	0.24	14
49	18.9	0.9	6.7	17.7	0.15	0.45	0.57	11.6
50	12.8	0.04	4.7	4.9	0.65	0.3	0.26	15.45
51	19.1	0.03	4.8	5.2	0.7	0.25	0.22	18.75
52	12.3	0.91	1.3	12.1	0.1	0.35	0.41	10.5
53	6.8	0.9	3	6.7	0.2	0.4	0.41	11.6

54	22.6	0.3	9.2	11	0.85	0.85	0.6	10.55
55	6.4	0.09	0.5	1.5	0.35	0.4	0.28	10.55
56	15.3	0.06	5.9	7.1	0.4	0.25	0.17	17.75
57	5.3	0.9	1.5	4.7	0.3	0.65	0.66	15.6
58	21.9	0.5	8.1	12.6	0.85	0.8	0.47	11.35
59	27.5	0.7	9.2	21.9	0.35	0.25	0.42	15.4
60	4.4	0.2	0.7	1.1	0.75	0.7	0.38	12.6

References

- Barkan, R., Zohar, D., & Erev, I. (1998). Accidents and decision making under uncertainty: A comparison of four models. *Organizational behavior and human decision processes*, *74*(2), 118-144.

- Barron, G., & Erev, I. (2003). Small feedback-based decisions and their limited correspondence to description-based decisions. *Journal of Behavioral Decision Making*, *16*(3), 215-233.

- Bernstein, P. L., & Bernstein Peter, L. (1996). *Against the gods: The remarkable story of risk*. New York: Wiley.

- Buffett, W. (2001). Letter to shareholders of Berkshire Hathaway Inc. http://www.berkshirehathaway.com/2001ar/2001letter.html

- Camilleri, A. R., & Newell, B. R. (2009). The role of representation in experience-based choice. *Judgment and Decision Making*, *4*(7), 518-529.

- Cramer (1728). Letter from Cramer to Nicolas Bernoulli. London, 21st May 1728. http://cerebro.xu.edu/math/Sources/NBernoulli/correspondence_petersburg_game.pdf

- de Palma, A., Abdellaoui, M., Attanasi, G., Ben-Akiva, M., Fehr-Duda, H., Erev, I., ... & Weber, M. Beware of black swans and do not ignore white ones?.

- Erev, I., Ert, E., Roth, A. E., Haruvy, E., Herzog, S. M., Hau, R., ... & Lebiere, C. (2010). A choice prediction competition: Choices from experience and from description. *Journal of Behavioral Decision Making*, *23*(1), 15-47.

- Erev, I., Glozman, I., & Hertwig, R. (2008). What impacts the impact of rare events. *Journal of Risk and Uncertainty*, *36*(2), 153-177.

- Fox, C. R., & Hadar, L. (2006). Decisions from experience= sampling error+ prospect theory: Reconsidering Hertwig, Barron, Weber & Erev (2004). *Judgment and Decision Making*, *1*(2), 159-161.

- Hadar, L., & Fox, C. R. (2009). Information asymmetry in decision from description versus decision from experience. *Judgment and Decision Making*,*4*(4), 317-325.

- Hau, R., Pleskac, T. J., & Hertwig, R. (2010). Decisions from experience and statistical probabilities: Why they trigger different choices than a priori probabilities. *Journal of Behavioral Decision Making*, *23*(1), 48-68.

- Gonzalez, C., Dana, J., Koshino, H., & Just, M. (2005). The framing effect and risky decisions: Examining cognitive functions with fMRI. *Journal of economic psychology*, *26*(1), 1-20.

- Hau, R., Pleskac, T. J., Kiefer, J., & Hertwig, R. (2008). The description–experience gap in risky choice: The role of sample size and experienced probabilities. *Journal of Behavioral Decision Making*, *21*(5), 493-518.

- Hertwig, R. (2012). The psychology and rationality of decisions from experience. *Synthese*, *187*(1), 269-292.

- Hertwig, R., & Erev, I. (2009). The description–experience gap in risky choice. *Trends in cognitive sciences*, *13*(12), 517-523.

- Hertwig, R., Barron, G., Weber, E. U., & Erev, I. (2004). Decisions from experience and the effect of rare events in risky choice. *Psychological Science*,*15*(8), 534-539.

- Hertwig, R., & Pleskac, T. J. (2010). Decisions from experience: Why small samples?. *Cognition*, *115*(2), 225-237.

- Hills, T. T., & Hertwig, R. (2010). Information Search in Decisions From Experience Do Our Patterns of Sampling Foreshadow Our Decisions?.*Psychological Science*, *21*(12), 1787-1792.

- Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica: Journal of the Econometric Society*, 263-291.

- Lejarraga, T. (2010). When experience is better than description: Time delays and complexity. *Journal of Behavioral Decision Making*, *23*(1), 100-116.

- Lejarraga, T., Hertwig, R., & Gonzalez, C. (2012). How choice ecology influences search in decisions from experience. *Cognition*, *124*(3), 334-342.

- Ludvig, E. A., & Spetch, M. L. (2011). Of black swans and tossed coins: is the description-experience gap in risky choice limited to rare events?. *PloS one*,*6*(6), e20262.

- Ludvig, E. A., Madan, C. R., & Spetch, M. L. (2013). Extreme outcomes sway risky decisions from experience. *Journal of Behavioral Decision Making*.

- Mehlhorn, K., Ben-Asher, N., Dutt, V., & Gonzalez, C. (2013). Observed Variability and Values Matter: Toward a Better Understanding of Information Search and Decisions from Experience. *Journal of Behavioral Decision Making*.

- Rakow, T., Demes, K. A., & Newell, B. R. (2008). Biased samples not mode of presentation: Re-examining the apparent underweighting of rare events in experience-based choice. *Organizational Behavior and Human Decision Processes*, *106*(2), 168-179.

- Shampanier, K., Mazar, N., & Ariely, D. (2007). Zero as a special price: The true value of free products. *Marketing Science*, *26*(6), 742-757.

- Taleb, N. N. (2004). No, Small Probabilities Are Not" Attractive to Sell": A Comment. *Journal of Behavioral Finance*, *5*(1), 2-7.

- Tversky, A., & Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and uncertainty*, *5*(4), 297-323.

- Ungemach, C., Chater, N., & Stewart, N. (2009). Are probabilities overweighted or underweighted when rare outcomes are experienced (rarely)?. *Psychological Science*, *20*(4), 473-479.

- Verbeek, M. (2008). *A guide to modern econometrics*. John Wiley & Sons.

- Von Neumann, J., & Morgenstern, O. (1947). Theory of games and economic behavior (2d rev.)

- Wakker, P. P. (2010). *Prospect theory: For risk and ambiguity* (Vol. 44). Cambridge: Cambridge University Press.

- Winkler, R. L. (1972). *An introduction to Bayesian inference and decision*. New York: Holt, Rinehart and Winston.