Detecting speculative bubbles in GARCH processes: *

A time-series based test for predicting explosive behavior

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Abstract

This paper provides further evidence for the usefulness of the speculative bubble test proposed by Franses (2014). The test is proven to be effective in predicting bubbles in high-frequency data. Three real world examples as well as Monte Carlo simulations show that the test gives signals when a speculative bubble occurs. Furthermore, this paper has adapted the recursive residuals method for GARCH processes. For stable processes, this method is even more accurate, but it cannot be used for detecting explosive behavior.

Keywords: Speculative Bubbles, Non-Gaussian Processes, Recursive Residuals, High-frequency Data

JEL Classifications: C22, G10

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Contents

1 Introduction

People have experienced speculative bubbles and bubble bursts already in the Dutch Golden Age during the famous Tulip Mania period. Afterwards we have identified many more, amongst others in real estate and stock markets. The huge impact of sudden price drops after a period of large price increases has led to many researchers trying to capture the nature of such speculative bubbles. Recently, time series based econometric techniques are being increasingly used for determining bubble-like patterns and predicting the crash of explosive processes.

Despite the large amount of interest in speculative bubbles, high-frequency speculative bubbles have been very little exposed in existing literature. The emersion of high frequency trading makes the subject particularly relevant. This paper builds on a recently proposed bubble diagnostic by Franses (2014), modified to allow for GARCH like patterns in the data. Abreu and Brunnermeier (2003) and Guenster et al. (2009) argue that when a speculative bubble occurs, the price of the asset grows faster than fundamental values, in combination with a sudden acceleration of real price growth. Franses (2014) finds that speculative bubbles in time-series indeed display an unbalance between growth $(1 - L)$ and acceleration $(1 - L)^2$, where L represent the lag operator. Recursive residuals, an econometric technique for assessing model stability, can be used to detect such speculative bubbles and predict whether a collapse is near.

Literature on speculative bubbles can roughly be divided into three groups: theoretical studies, empirical studies and econometric testing. Theoretical papers are Blanchard and Watson (1983), Tirole (1982), Tirole (1985), Evans (1989) and Schiller (2000) amongst others. Empirical studies include Shiller (1980), West (1988), Campbell and Shiller (1988), Diba and Grossman (1988) and Evans (1991). Finally, some major papers about the econometric testing (of bubbles) are Bhargava (1986), Kim (2000), Busetti and Taylor (2004), Phillips et al. (2011) and Homm and Breitung (2012).

High-frequency financial data can often not be characterized by ARMA processes, due to conditional heteroskedasticity. Therefore, the aforementioned test is adjusted in such a way that the test can be applied to higher-frequency financial data. Afterwards, real world high-frequency financial data with bubble-like patterns illustrates the usefulness of the bubble diagnostic in practice. Finally, Monte Carlo simulations are performed to assess the power of the proposed test. This paper uses a dataset consisting of numerous intraday stock returns on IPO days, where one observes bubble-like behavior. The data are obtained from the *Wharton Research Data Services*.

Section 2 briefly summarizes existing bubble literature. Section 3 outlines the model and econometric methods used in this paper. Section 4 illustrates the bubble diagnostic with examples from artificially generated data sets and demonstrates the practical relevance of the test by applying the test on high-frequency speculative bubbles on IPO days. Section 5 presents Monte Carlo simulations confirming the power of the test. Finally, section 6 summarizes the aforementioned findings and concludes.

2 Model specification

In this section, I will illustrate the idea of the proposed test by showing some of the properties of explosive processes and derive the test afterwards. The data in this section are artificially generated. Three different cases are considered in particular: stationary time series ($\alpha_1 < 1$), random walks ($\alpha_1 = 1$) and non-stationary or explosive time series ($\alpha_1 > 1$). As we will see with actual stock prices, the random walk representation best describes financial time series.

Figure 1 shows that for stationary time series and random walks, growth and acceleration are in balance. Explosive processes however show an imbalance between those two variables. Due to the high-frequency data, this is not as clear as in Franses (2014), but still we see that with explosive processes, the data points are far less concentrated on one spot. This becomes even more clear from Figure 2, where the imbalance of acceleration and growth of the non-stationary process is very much like in Franses (2014), with the data points moving away from the initial cloud. Figure 2 shows the results from a generated time series with conditional heteroskedasticity, which can be seen by looking at the volatility clustering, but also by looking at the few data points relatively far from the cloud of data points in the stable processes. We see some points in the upper right and lower left corners, whereas this is far less the case in Figure 1. Since growth and acceleration are only in balance when the

Figure 1: Properties of Stationary time series, Random walks and Non-stationary time series

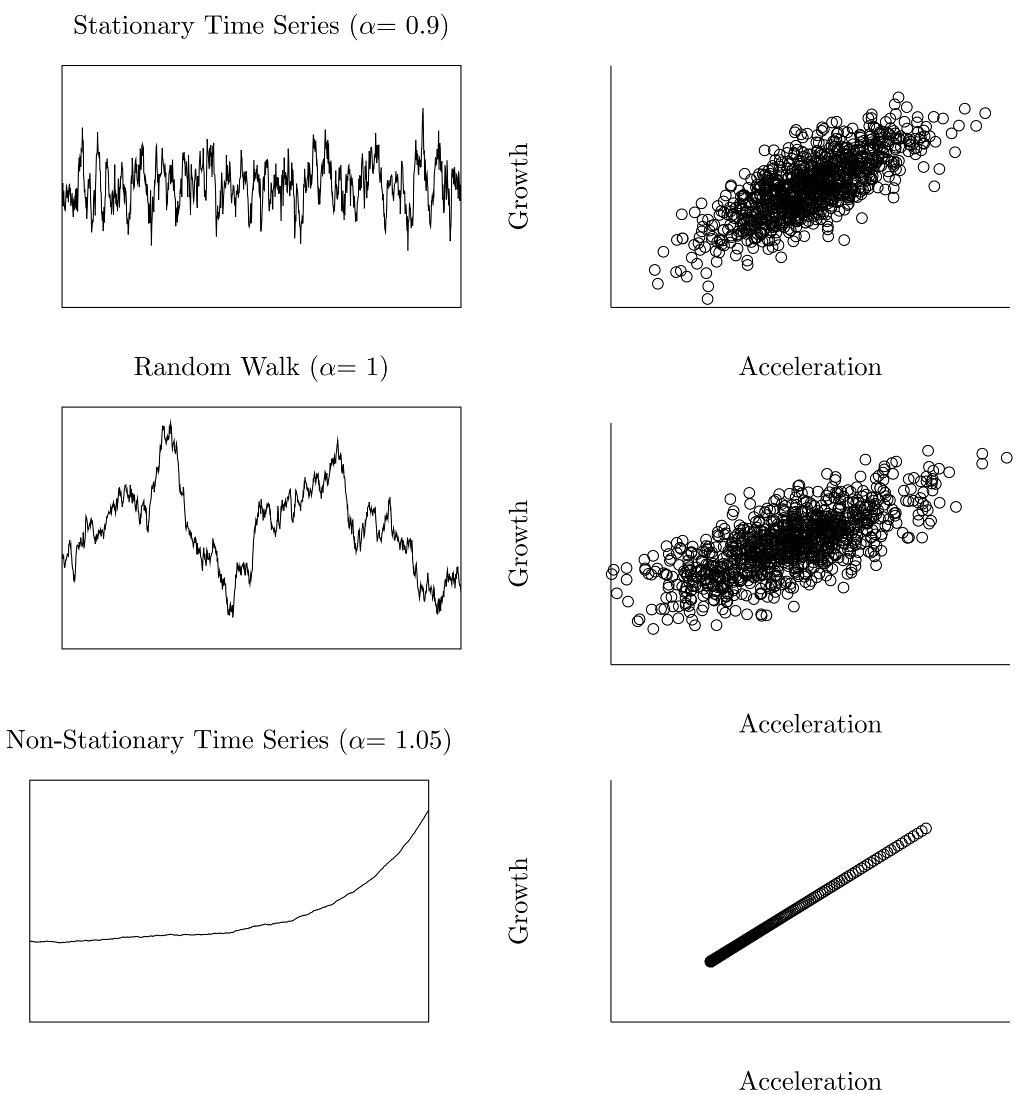
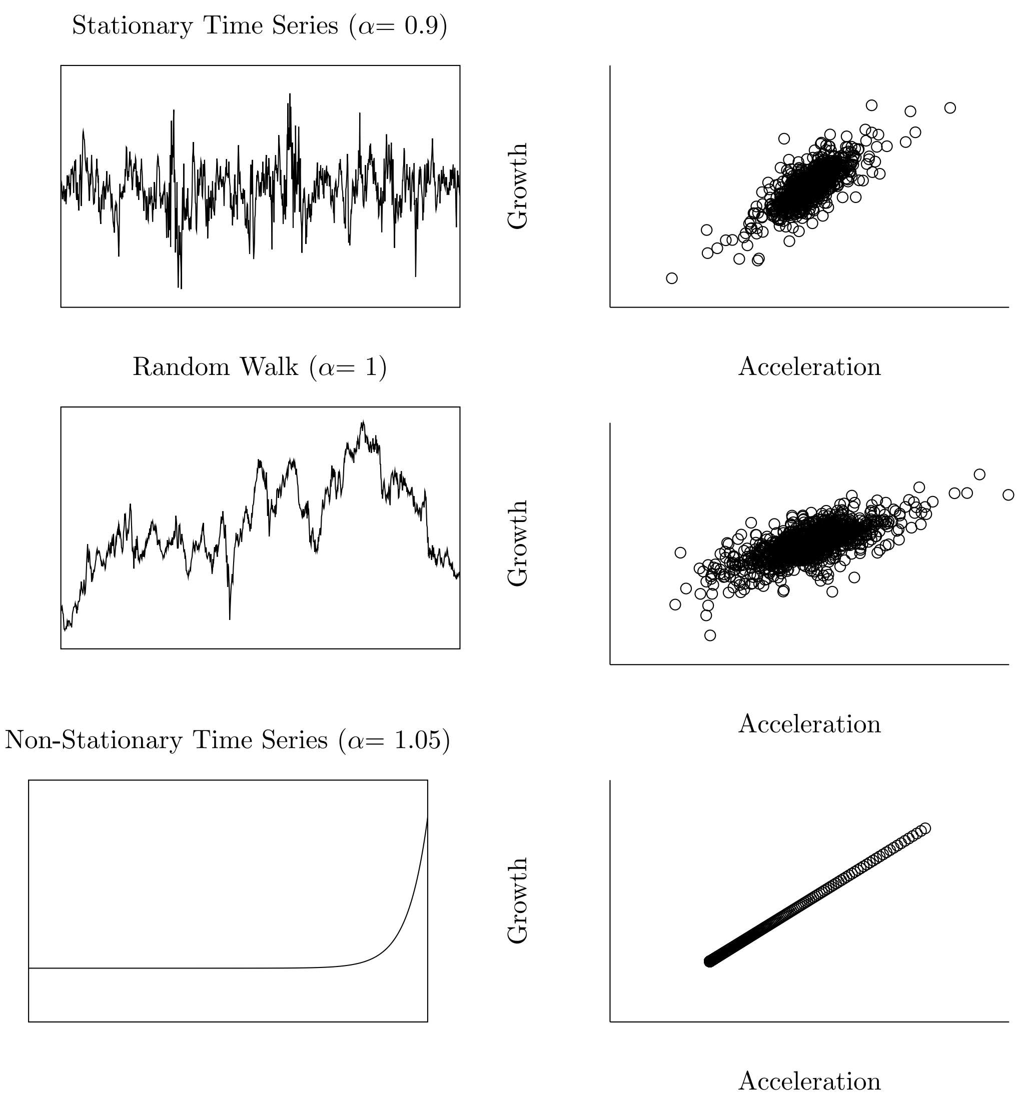


Figure 2: Properties of Stationary time series, Random walks and Non-stationary time series with GARCH error terms



time series is stable, we can derive a test for detecting unstable processes by using this fact (Franses, 2014).

When growth and acceleration are in balance, this implies that the series

$$(1-L)y_t \tag{1}$$

and

$$(1-L)^2 y_t \tag{2}$$

are in balance as well. The regression line connecting growth and acceleration can be seen as the regression line connecting

$$\epsilon_t - \epsilon_{t-1} \tag{3}$$

and

$$\epsilon_t - 2\epsilon_{t-1} + \epsilon_{t-2} \tag{4}$$

with regression coefficient equal to

$$\frac{\sigma^2_{\epsilon,t} + 2\sigma^2_{\epsilon,t-1}}{\sigma^2_{\epsilon,t} + 4\sigma^2_{\epsilon,t-1} + \sigma^2_{\epsilon,t-2}} \tag{5}$$

In case of homoscedasticity, the regression coefficient is thus equal to 0.5. In case of conditional heteroskedasticity, the regression coefficient does not have to be exactly 0.5. However, if we take the limit of the steps $n \to \infty$, the consecutive variances will approximately be the same and therefore we again have a regression coefficient of 0.5. Thus, it follows that

$$(1-L)y_t - 0.5(1-L)^2 y_t = 0.5(1-L^2)y_t \tag{6}$$

is stable when growth and acceleration are in balance. We can therefore construct a test for the presence of a speculative bubble by regressing $(1-L^2)y_t$ on a constant and using recursive residuals to assess model stability.

2.1 Recursive residuals

We recursively estimate the regression coefficient. The estimated coefficient is used to predict the next value of the dependent variable. The one-step ahead forecast error resulting from this prediction is the recursive residual, defined as:

$$w_t = \frac{y_t - x_t' b_{t-1}}{(1 + x_t'(X_{t-1}'X_{t-1})^{-1}x_t)^{1/2}} \tag{7}$$

Those recursive residuals are independently and normally distributed with zero mean and variance σ^2, the variance of the regression.

However, since high-frequency financial time series can better be modeled by a GARCH process, we use the model proposed by Bollerslev (1986) instead of an AR model, namely:

$$p_t = \alpha_0 + \alpha_1 p_{t-1} + \epsilon_{p,t}, \quad \epsilon_{p,t} \sim NID(0, \sigma^2_{p,t}) \tag{8}$$

$$\sigma^2_{p,t} = \gamma_0 + \gamma_1 \epsilon^2_{p,t-1} + \gamma_2 \sigma^2_{p,t-1} \tag{9}$$

Now we can also adapt the recursive residuals method, by recursively estimating the GARCH parameters and instead of using the constant regression variance, we can use the one-step ahead predicted GARCH variance. In the following section, both versions of the test are considered.

3 Results

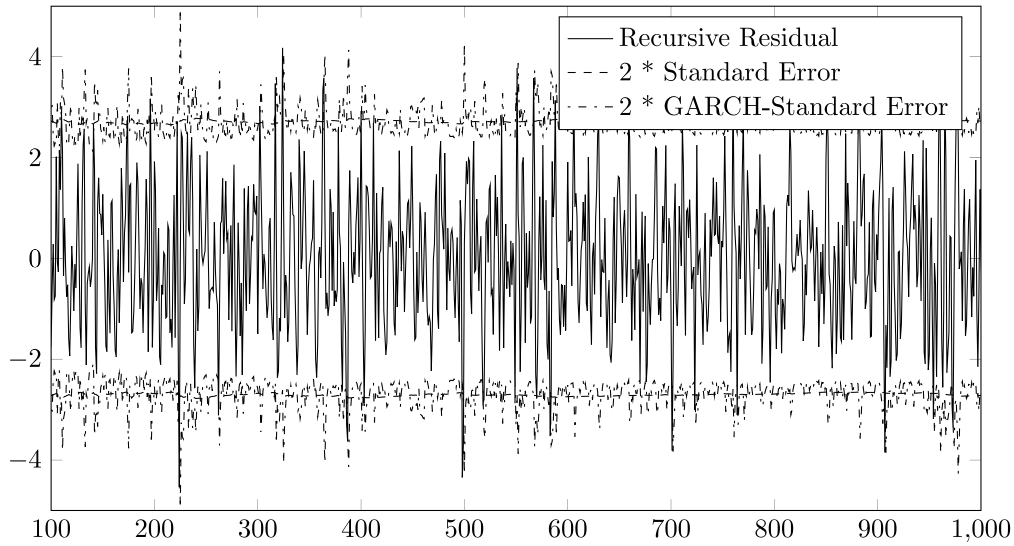
This section presents the results of this paper. First of all, there is a brief part in which the test will be visualized. Afterwards we examine three real world examples of stocks with bubble-like patterns within a single day. Finally, the results of Monte Carlo simulations are presented.

3.1 Artificial data

This section serves as a short illustration of the test applied on artificial data. The data used are the same as in Figure 1 and Figure 2. Now the recursive residuals following from the regression of $(1 - L^2)y_t$ on a constant are visualized together with the corresponding standard errors and one-step ahead predicted GARCH standard errors. The first three figures are based on homoskedatic data, whereas the latter three figures contain conditional heteroskedasticity.

Figure 3, Figure 4 and Figure 5 are very much in line with the results from Franses (2014). We see that the stable processes (Figure 3 and Figure 4), show stable behavior and stay within the bounds derived from the regression standard errors. Note that the GARCH standard errors are in both cases very near the constant standard errors, which is in line

Figure 3: Recursive estimation of $(1 - L^2)y_t$ on a constant for a stationary time-series (α= 0.9)



with the expectations, because constant standard errors have been used when generating the process. Figure 5 is very different from the stable processes. Here we clearly see that the recursive residuals move upwards and thereby leaving the bounds. However, if we consider the GARCH standard errors, the recursive residuals remain within the bounds. Apparently the explosive behavior goes together with an increase in estimated standard error by means of GARCH estimation. Since the recursive residuals exceed the constant standard error bounds and remain within the GARCH standard error bound, the first version would predict the bubble, whereas the latter would not.

Let us now look at cases in which we have generated GARCH processes. Unlike in the previous examples, this time the bounds of constant standard errors and GARCH standard errors for stable processes are much less alike. We observe that the GARCH standard errors do take into account when there is a period of more volatility, whereas the constant standard error bounds are far less able to adapt. Figure 6 and Figure 7 suggest that the GARCH standard errors are more useful when processes are stable, because we will not be given signals at times of high volatility. If we look at Figure 8, however, we see that the test based on GARCH standard errors is unable to correctly predict a speculative bubble. We

Figure 4: Recursive estimation of $(1-L^2)y_t$ on a constant for a random walk (α= 1)

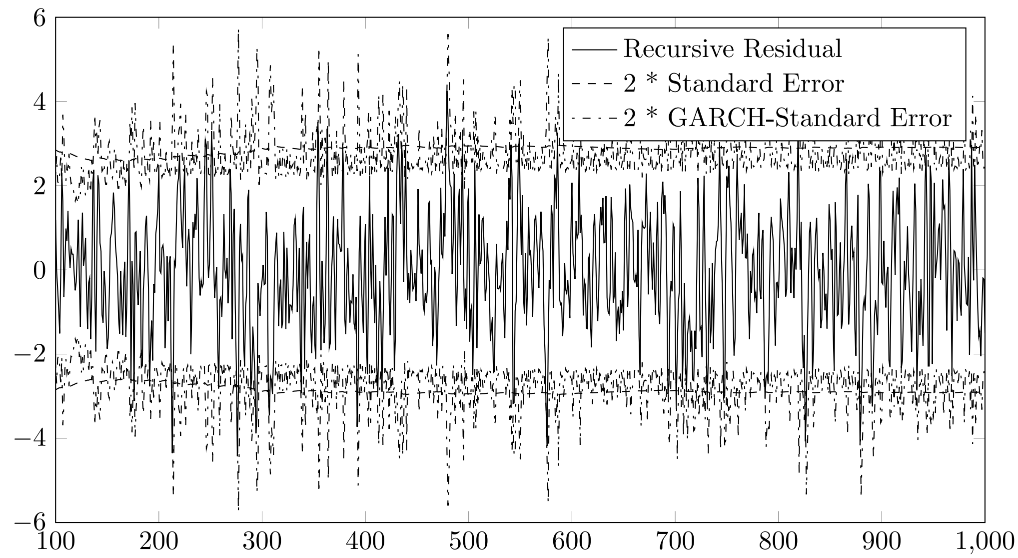


Figure 5: Recursive estimation of $(1-L^2)y_t$ on a constant for a non-stationary time-series (α= 1.05)

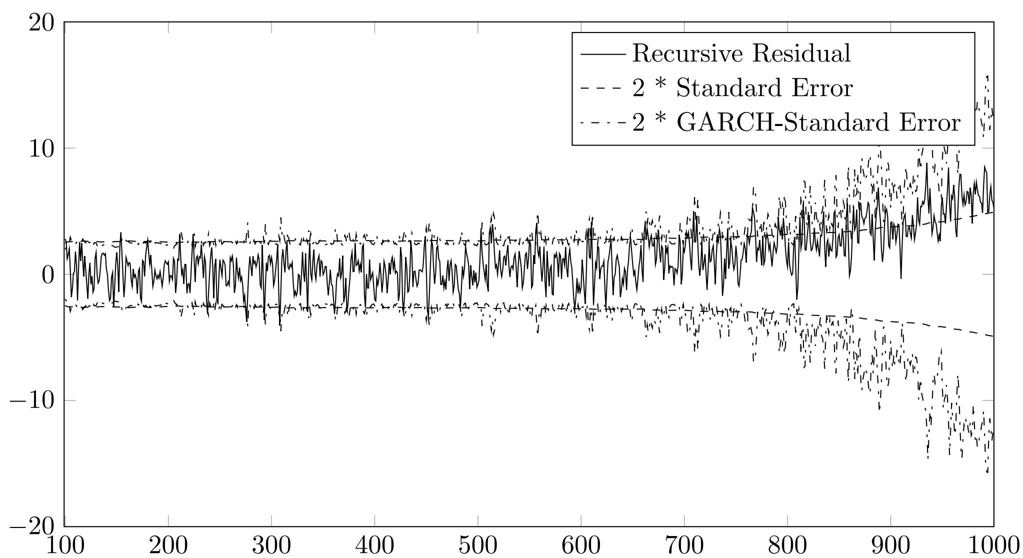
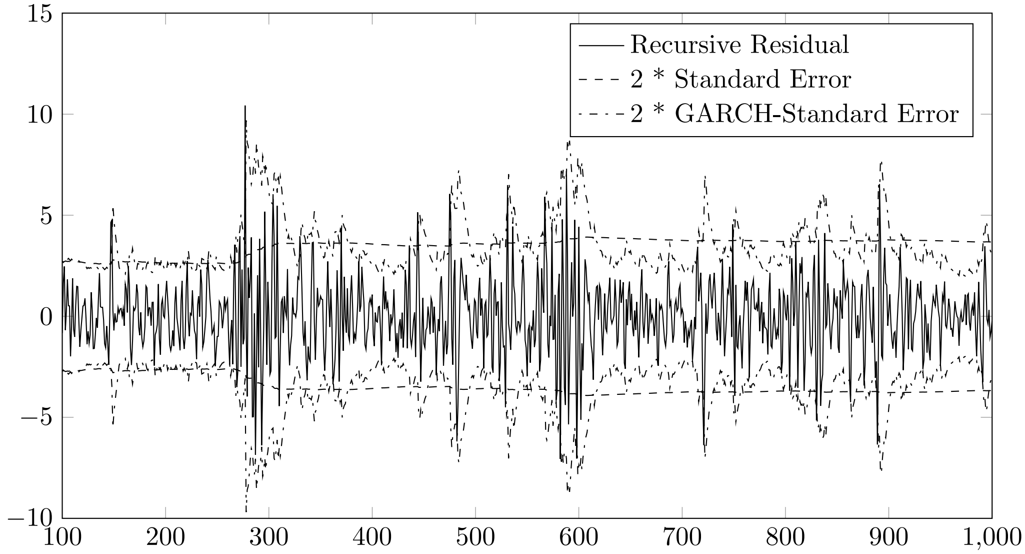


Figure 6: Recursive estimation of $(1-L^2)y_t$ on a constant for a stationary time-series (α= 0.9) with GARCH error terms



see almost the same pattern as in Figure 5, namely that the recursive residuals go above the constant standard error bounds, but stay within the bounds based on GARCH standard errors. Thus, for stable processes, GARCH estimation might lead to more accurate results, but unlike with constant standard errors, it is unable to predict a speculative bubble.

3.2 Real data

In order to investigate the usefulness of the proposed test, some real-life examples have been examined. I consider three stocks, listed on either the New York Stock Exchange (NYSE) or the National Association of Securities Dealers Automated Quotations (NASDAQ), which have showed a bubble-like pattern on a single trading day. In all of the three cases we look at the day on which the IPO of the stock took place. The three stocks are LinkedIn Corporation (Ticker:LNKD) and Youku Inc. (Ticker:YOKU) listed on the NYSE and Baidu Inc. (Ticker:BIDU) listed on the NASDAQ. LinkedIn is an American business-oriented social networking service, Youku a Chinese video hosting service and Baidu a Chinese web services company. Their IPOs were on May 18 in 2011, December 8 in 2010 and the August 5 in 2005 respectively. It is important to note that the choice of stocks is only based on the

Figure 7: Recursive estimation of $(1 - L^2)y_t$ on a constant for a random walk (α= 1) with GARCH error terms

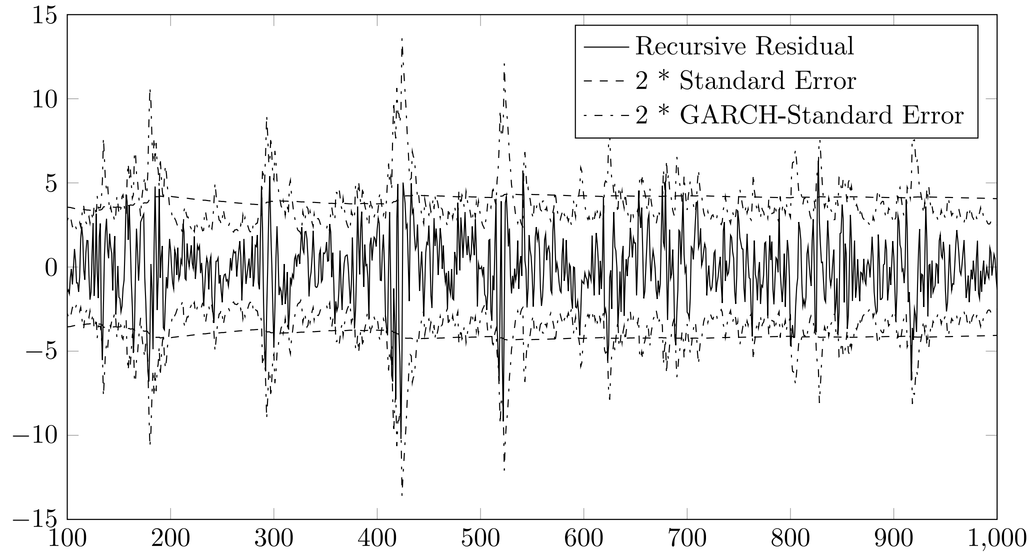
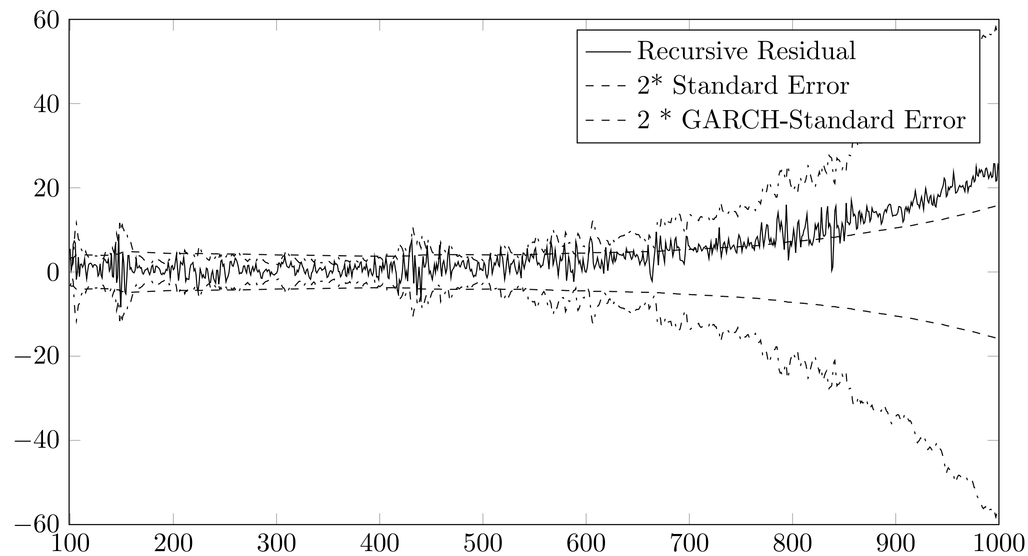


Figure 8: Recursive estimation of $(1 - L^2)y_t$ on a constant for a non-stationary time-series (α= 1.05) with GARCH error terms



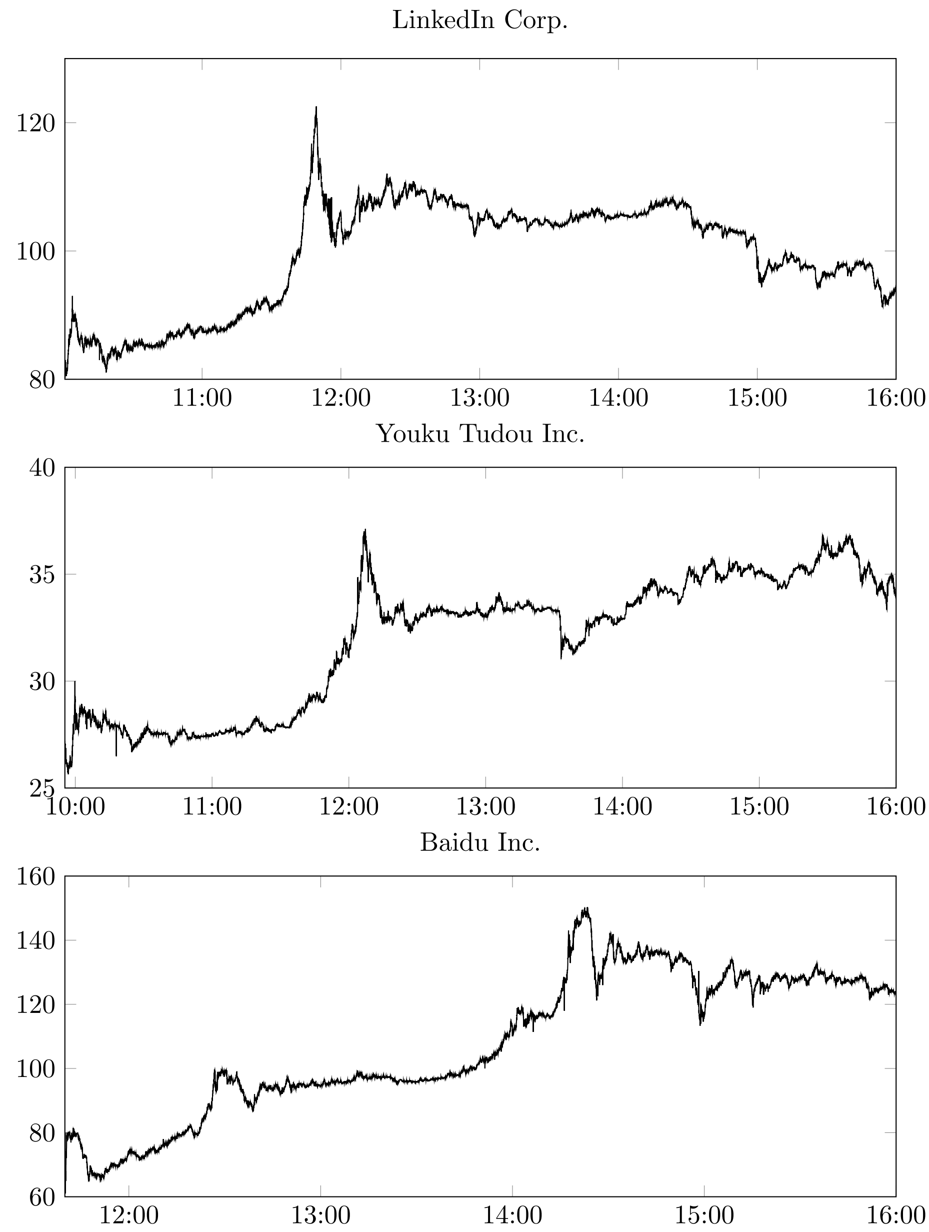
fact that they have shown explosive behavior on a single day and have no further meaning. The data are sampled at a 1-second frequency and are obtained from the *Wharton Research Data Services*.

As we can see from Figure 9, all of the three stocks show the typical bubble-like pattern. We see that the bubbles of LinkedIn, Youku and Baidu burst around 11:50, 12:10 and 14:20 respectively. LinkedIns IPO price was $45, it opened at around $80 and the stock closed at a price of $94,25.At its peak, the stock price was even above $120. The pattern we see with the share price of Youku is very similar: the stock closed at $33.44, 160 percent above the offering price of $12,80. The peak of the bubble that we examine was at nearly $36. Baidu opened at $66, more than double its IPO price of $27. After having reached the peak of more than $140, Baidu closed at a price of $122,54.

Let us now examine the results of the proposed test. Figure 10 and Figure 11 show in black the share price of LinkedIn. The grey areas denote the periods in which we receive a signal that there might be a bubble. Note that due to the high-frequency data, one cannot exactly see in the figures at which seconds there was a signal. However, in order to illustrate the usefulness of the test, it is sufficient to show roughly the periods in which signals are observed. The difference between Figure 10 and Figure 11 is that in Figure 10 the test is based on constant standard errors whereas in Figure 11 the test is based on the one-step ahead predicted GARCH standard errors. The obtained results are rather different, as can be seen in the figures.

Figure 10 shows that the signals are highly concentrated around the peak of the bubble. Note that at a 5% confidence interval, we see a considerable amount of signals already from 11:30. Those signals become less visible when decreasing the confidence interval to 1% and even less so when decreasing it to 0.1%. The signals just before the bubble burst and right afterwards are in all three cases clearly visible. This suggests that the bubble could have been predicted by means of the test. Interesting to mention is that the periods where there is no bubble are far less highlighted in grey, meaning that in relatively stable periods, the test does not provide us with false signals. At a 5% confidence interval, we see though some periods of sudden price changes where we observe signaling, but at a 0.1% level, the size of those false signals is almost neglectible.

Figure 9: Share price development



As can be seen in Figure 11, using predicted GARCH standard errors for obtaining bubble signals results in a completely different picture. Instead of receiving signals when a bubble collapse is near, this method provides signals in almost every period, irrespective of a sudden change or upcoming collapse. We can therefore conclude that the size of the test is very low. Not predicting a bubble when the test should is in line with the results obtained from the artificial data. When there are large expected changes in the share price, the GARCH estimation also predicts large standard errors. Consequently the large recursive residual does not exceed the confidence bounds due to the large standard errors. It is therefore not useful to consider this version of the bubble test.

Youku and Baidu show a very similar picture and are therefore shown in the Appendix. The main difference is that LinkedIn and Youku show a much clearer bubble, resulting in more concentrated signals. Baidu has more than one bubble-like pattern, which also becomes clear from the grey areas in more than one period. We can see however that most of the signals are given during the period of the main bubble around 14:20.

3.3 Power of the test

In this section the power of the proposed test by using Monte Carlo simulations is assessed. I use a similar data-generating process as Phillips et al. (2011), with the fundamental price following a random walk with drift and an independent bubble process, which is a linear explosive process. Unlike Phillips et al. (2011), the bubble process does collapse and instead of constant standard errors, a GARCH process is used. The sample size is 5000, the start of the bubble at $t = 3000$ and the collapse at $t = 4000$, resulting in the following data generating process:

$$p_t = p_t^f + b_t,\ p_t^f = \alpha_0 + p_{t-1}^f + \epsilon_{p,t},\ \ \epsilon_{p,t} \sim NID(0, \sigma_{p,t}^2) \tag{10}$$

$$\sigma_{p,t}^2 = \gamma_0 + \gamma_1 \epsilon_{p,t-1}^2 + \gamma_2 \sigma_{p,t-1}^2 \tag{11}$$

$$b_t = (1+g) b_{t-1}\ \ if\ \ t \in [3000, 4000],\ \ 0\ \ otherwise \tag{12}$$

I used 5000 seconds of the LinkedIn share price around the speculative bubble to estimate the parameters for the data-generating process by means of a state space approach. The

Figure 10: Share price of LinkedIn Corp. and signals given by the proposed test

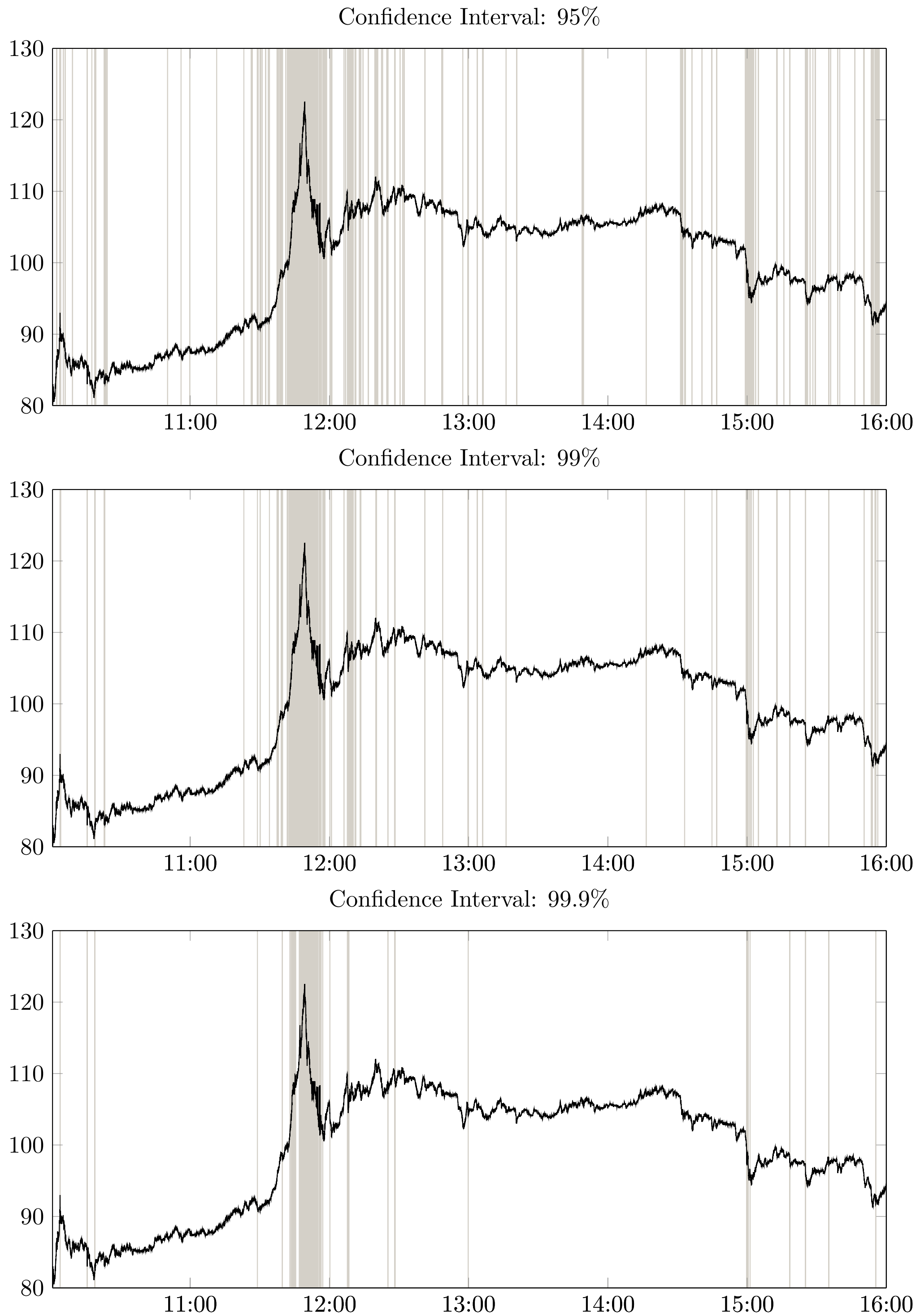
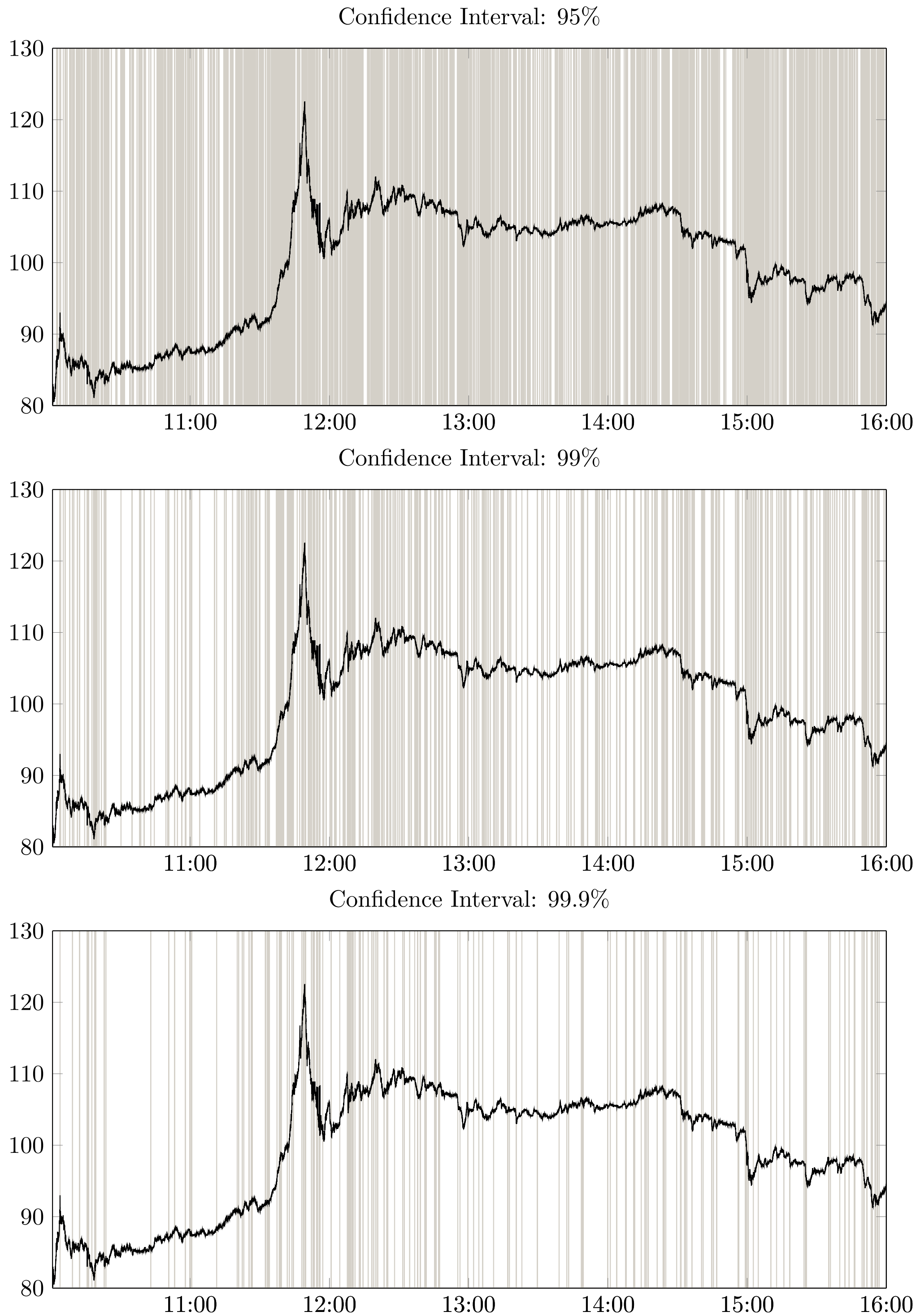


Figure 11: Share price of LinkedIn Corp. and signals given by the proposed test based on predicted GARCH standard errors

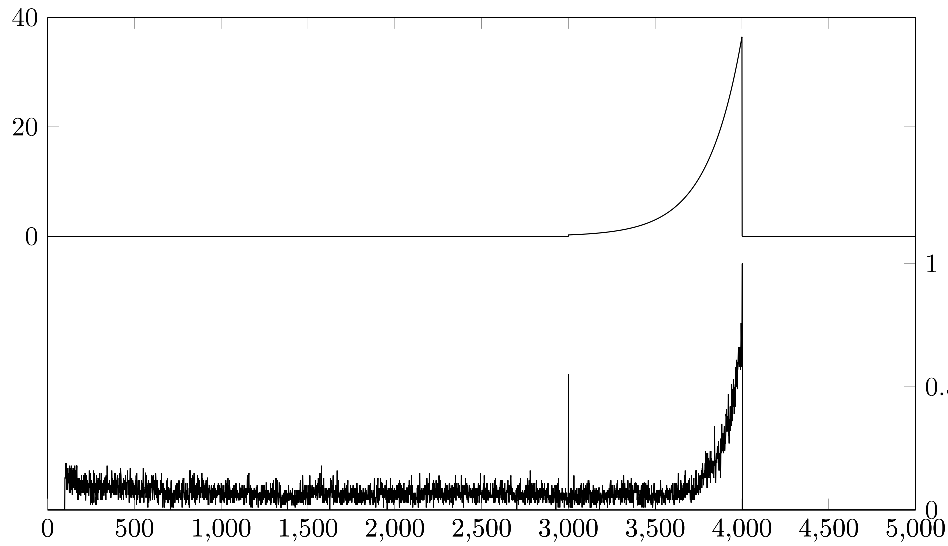


estimated parameter values are $\alpha_0 = 0$, $\gamma_0 = 0.0001$, $\gamma_1 = 0.06$ and $\gamma_2 = 0.93$. The starting value of the bubble is set at $b_{3000} = 0.25$ and the growth of the bubble at $g = 0.005$. Note that the parameter values are much lower than those used by Phillips et al. (2011), due to the higher data-frequency.

To assess the effectiveness of the proposed test, the implied power of the test is shown in Figure 12 and Figure 13. The results are based on 100 Monte Carlo simulations. The power of the test can be defined as the average number of times that there is a signal given on time t. The upper part of the figures show the rational bubble component, for a convenient overview of where the bubble collapse takes place. The lower part of the figures denotes the power of the test, ranging from 0 to 1, where 0 means that on a certain time t, there was never a signal given and 1 means that at time t, all the simulations provided a signal. In Figure 12 I made use of the test with constant standard errors and in Figure 13 GARCH standard errors were used. Both figures show test results based on a 5% significance level. The same figures based on 1% and 0.1% significance levels can be found in the Appendix. The picture of those figures is almost the same, the only difference is that the power is lower because of the higher confidence intervals.

Figure 12 makes very clear the usefulness of the bubble test. First, until the start of the bubble, we see the power of the bubble moving around 5%. This was to be expected since the process is stable in this period and should not show explosive or any other unstable behavior. At $t = 3000$, we observe a power of approximately 0.5, which can be explained by the sudden shift in observed price due to the initial value of the rational bubble component. Until $t = 3500$, the power remains around 5%, since the bubble component is not yet very large, but afterwards the power starts to rise. Ultimately, the power of the test reaches levels of around 0.7 just before the collapse of the bubble. Note that this may not sound very large, however, if you take into account that the data are sampled at second-frequency, this is a lot. Just before the bubble collapse, the test would provide signals hinting towards a collapse in more than 50% of the cases. After the collapse of the bubble we see that the power of the test is 0, from $t = 4001$ onwards. This can be explained by the larger standard errors and therefore smaller rejection regions, due to the previous uncertainty in the bubble period. This could cause problems in cases of periodically collapsing bubbles. In order to

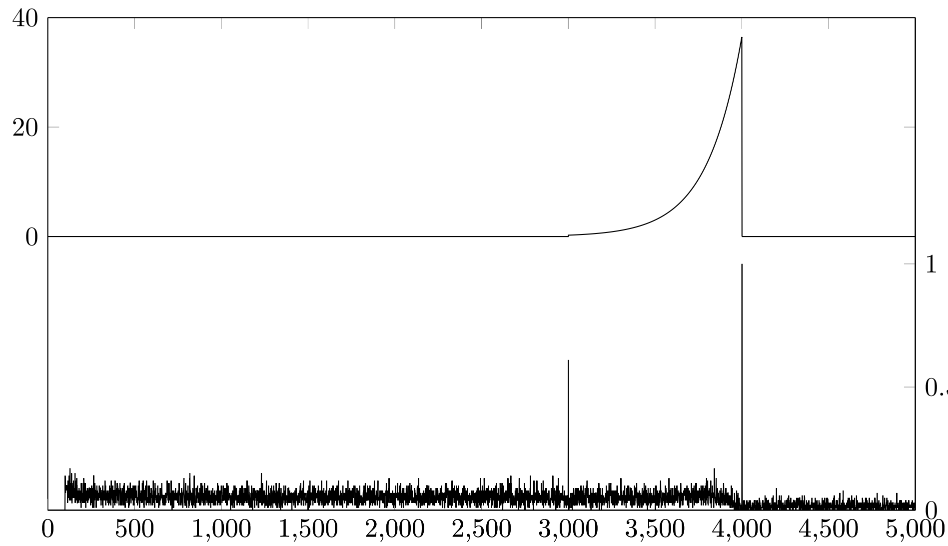
Figure 12: Power of the test based on 100 Monte Carlo simulations (Confidence interval 95%)



prevent this problem, one should not include data before and during the previous bubble when predicting a new one.

Figure 13 shows, as expected, completely different results. The first period until the start of the bubble is similar to the previous figure. The power of the test moves around 5%, which is correct since the confidence interval was set at 95%. Again we see the high power at $t = 3000$ because of the starting bubble value. However the power of the test remains the same and even decreases during the bubble period. As we have seen in Subsection 3.1, this can well be explained by the fact that the GARCH standard errors increase even more than the recursive residuals, resulting in too small rejection regions. Therefore we again see confirmed that using GARCH standard errors makes the test useless. In the period after the bubble, the power is close to, but above 0. This is again too low, but due to the conditional heteroskedasticity, standard errors move to normal level relatively quickly and do not remain at very high levels caused by the speculative bubble.

Figure 13: Power of the test with GARCH standard errors based on 100 Monte Carlo simulations



4 Conclusion

This paper provides further evidence for the usefulness of the speculative bubble test proposed by Franses (2014). Secondly stock price data shows that the test can be used in practice, even with very high data frequencies. The power of the test is relatively high, above 0.5 in the period before the speculative bubble collapses. Performing the test is very easy and can give meaningful insights about a possible bubble.

This paper also proposes a new method of the use of recursive residuals for assessing model stability. Instead of using constant standard errors from the regression, one can estimate the time series as a GARCH process and use the one-step ahead forecasted GARCH standard error. This method happens to be very accurate for stable processes, but is not effective for predicting speculative bubbles. Using bounds based on GARCH standard errors drastically decreases the power of the test. Besides, using recursive residuals with GARCH standard errors increases computational complexity a lot.

4.1 Limitations and topics for further research

Due to computational complexity, it was not possible to perform more than 100 Monte Carlo simulations. Although this size is relatively small, I do not expect results to deviate much when increasing the number of simulations. In order to have a stronger proof for the power of the test, I would propose repeating the experiment with 1000 simulations or more. Besides, it would be interesting to vary the parameters used in the Monte Carlo simulations in order to see how much the bubble has to grow in order to receive signals from the test.

This paper encourages conducting further research on high-frequency speculative bubbles in general. I propose a comparison of the power of different econometric tests for higher frequency GARCH processes and stimulate the research on more real-life applications.

This paper has not attempted to use the signals in such a way that they can efficiently forecast the time of collapse. One can think of using for example a linear combinations of the last x signals as a proxy for the chance of a bubble collapsing. Further analysis of the signal patterns is needed in order to make an even more efficient and accurate test.

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5 Appendix

Figure 14: Share price of Youku Inc. and signals given by the proposed test

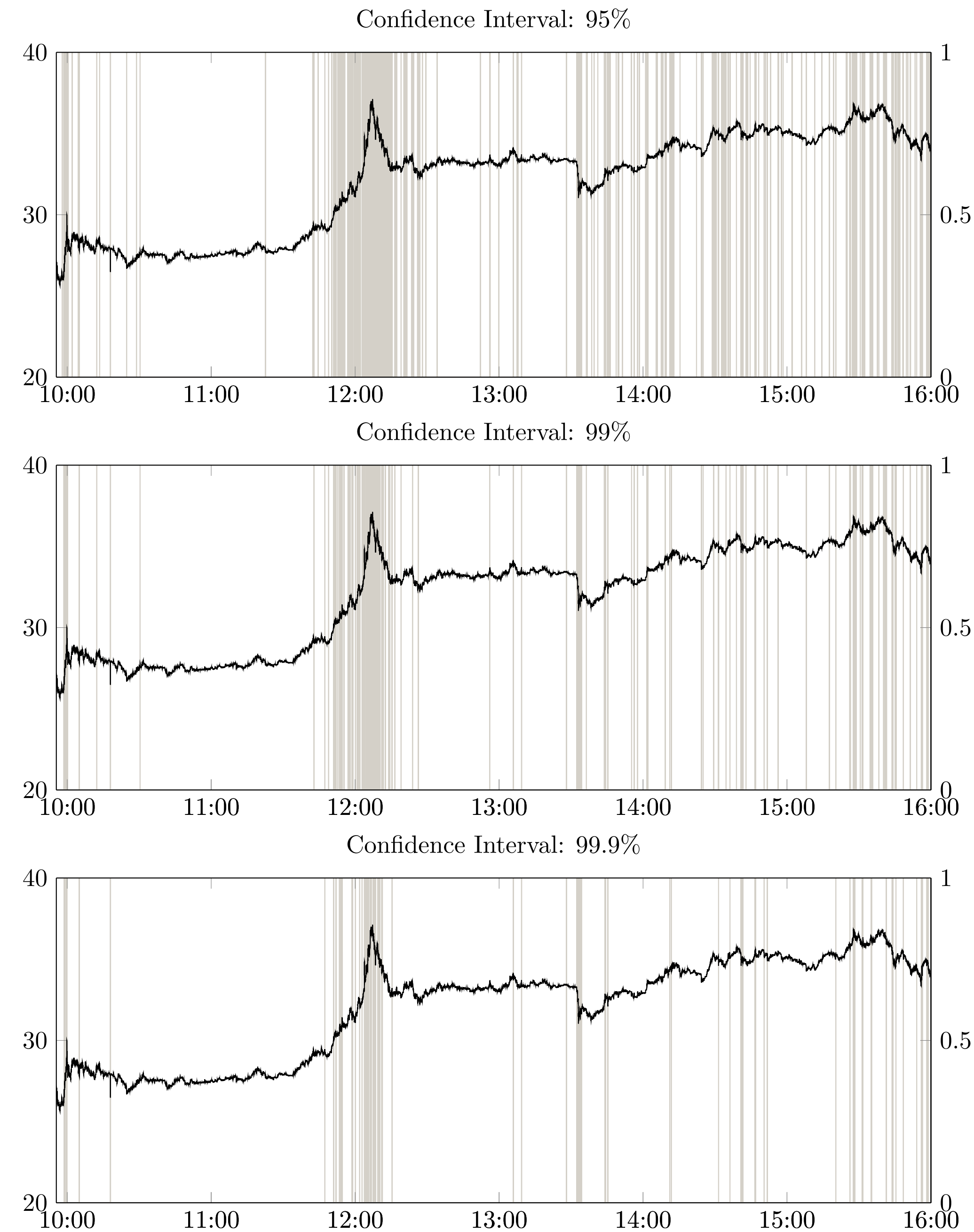

Figure 15: Share price of Youku Inc. and signals given by the proposed test based on predicted GARCH standard errors

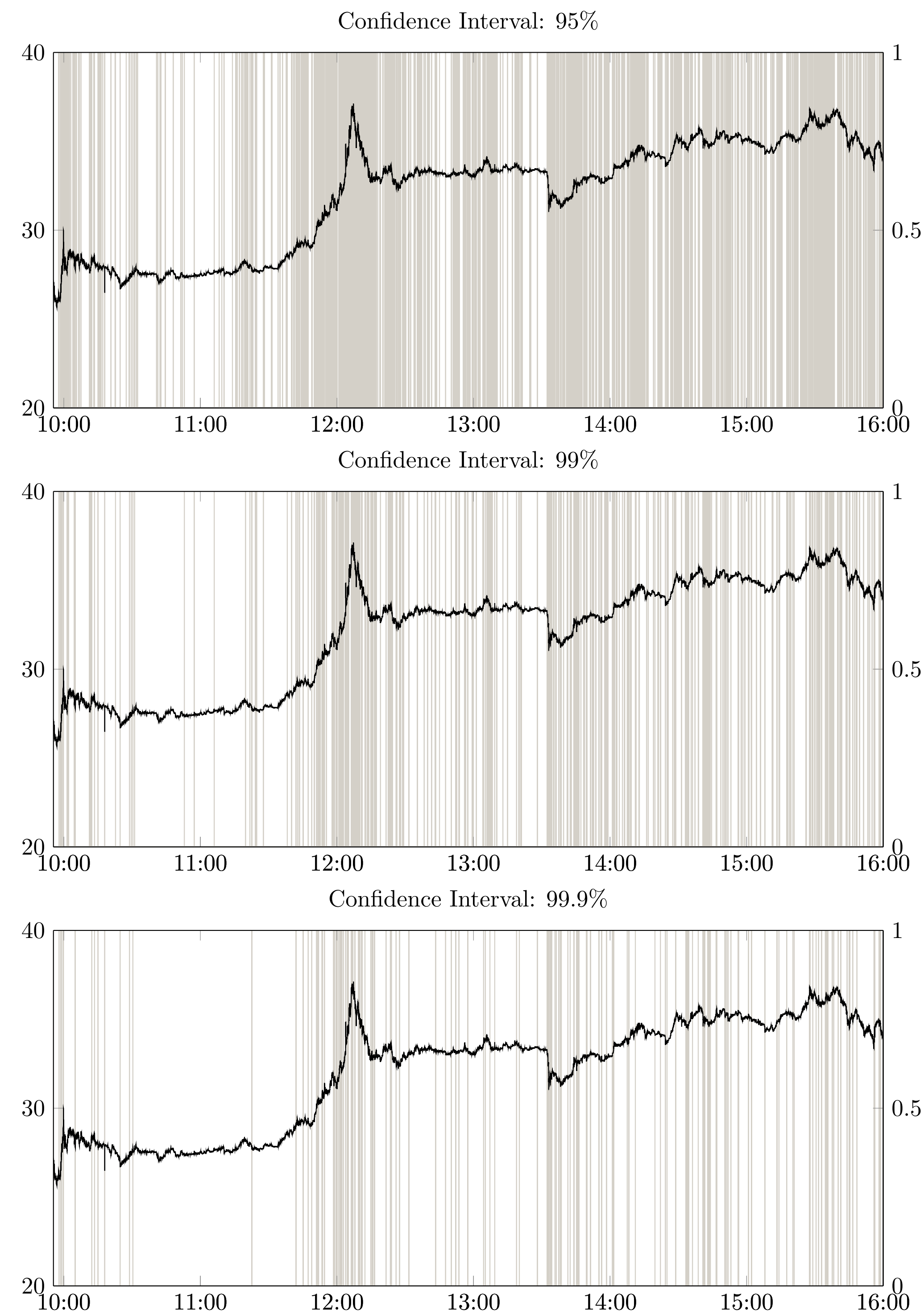

Figure 16: Share price of Baidu Inc. and signals given by the proposed test

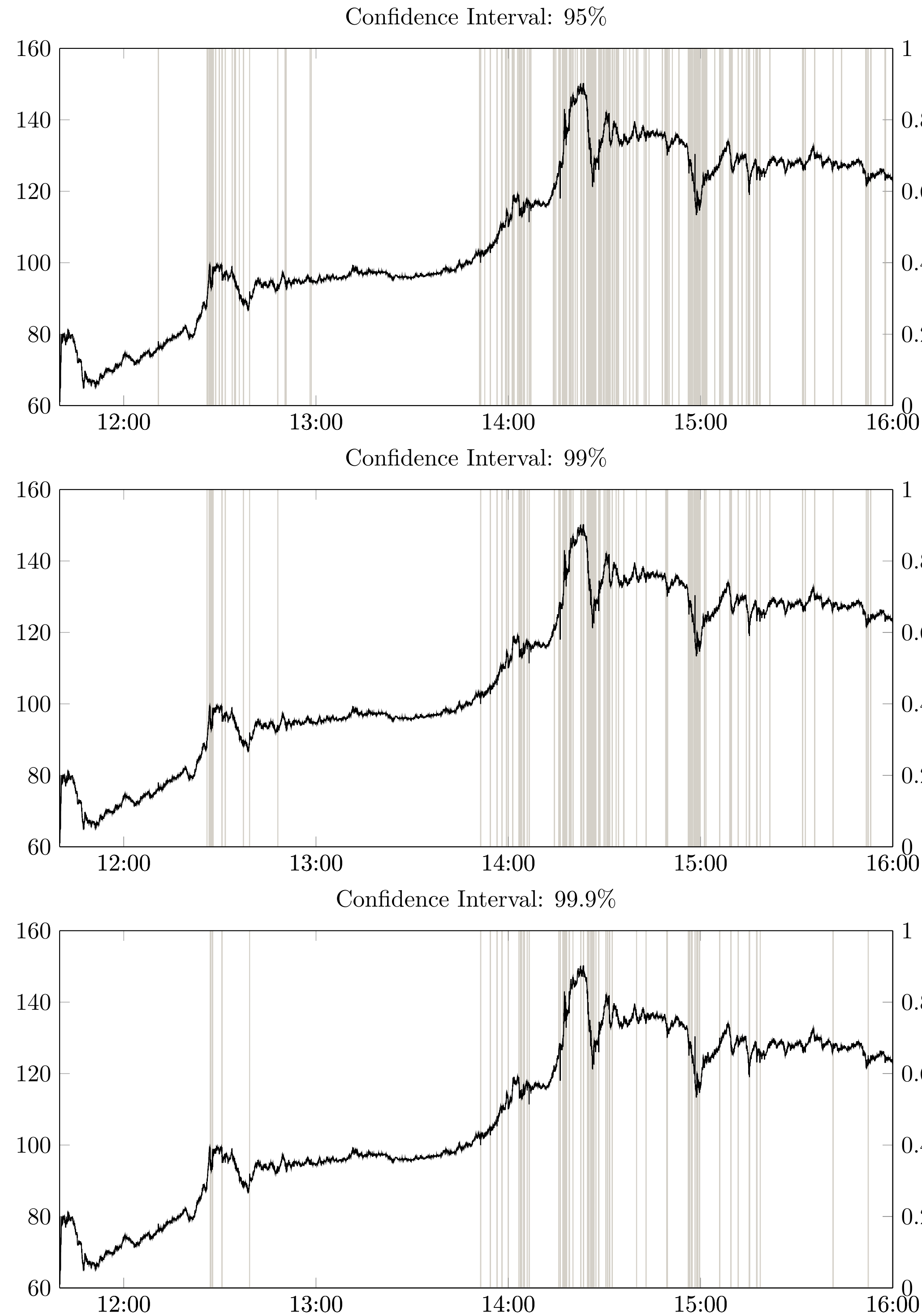

Figure 17: Share price of Baidu Inc. and signals given by the proposed test based on predicted GARCH standard errors

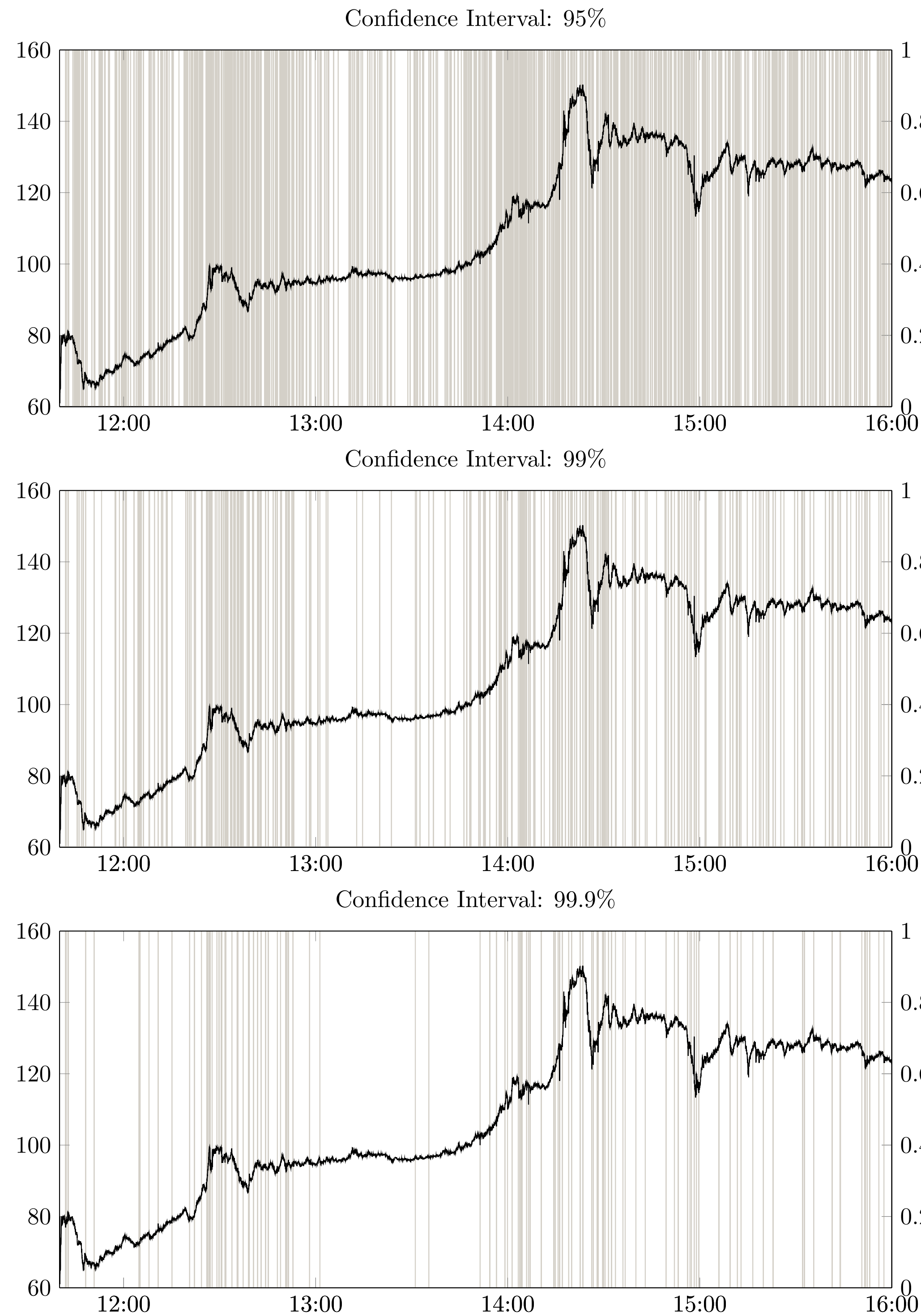

Figure 18: Power of the test based on 100 Monte Carlo simulations (Confidence interval 99%)

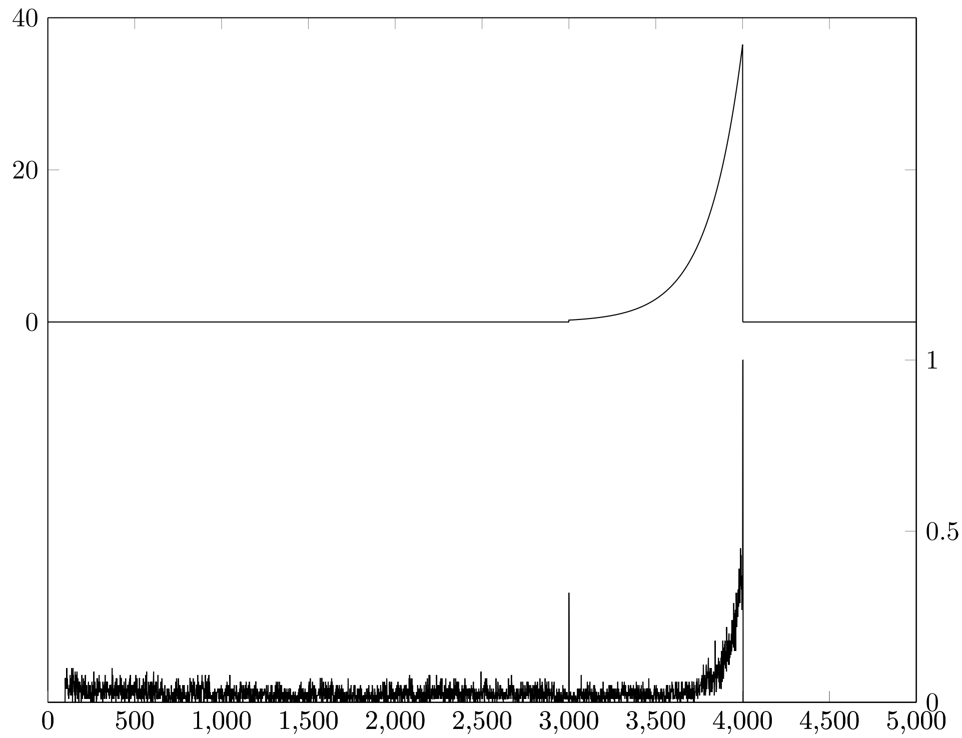

Figure 19: Power of the test with GARCH standard errors based on 100 Monte Carlo simulations (Confidence interval 99%)

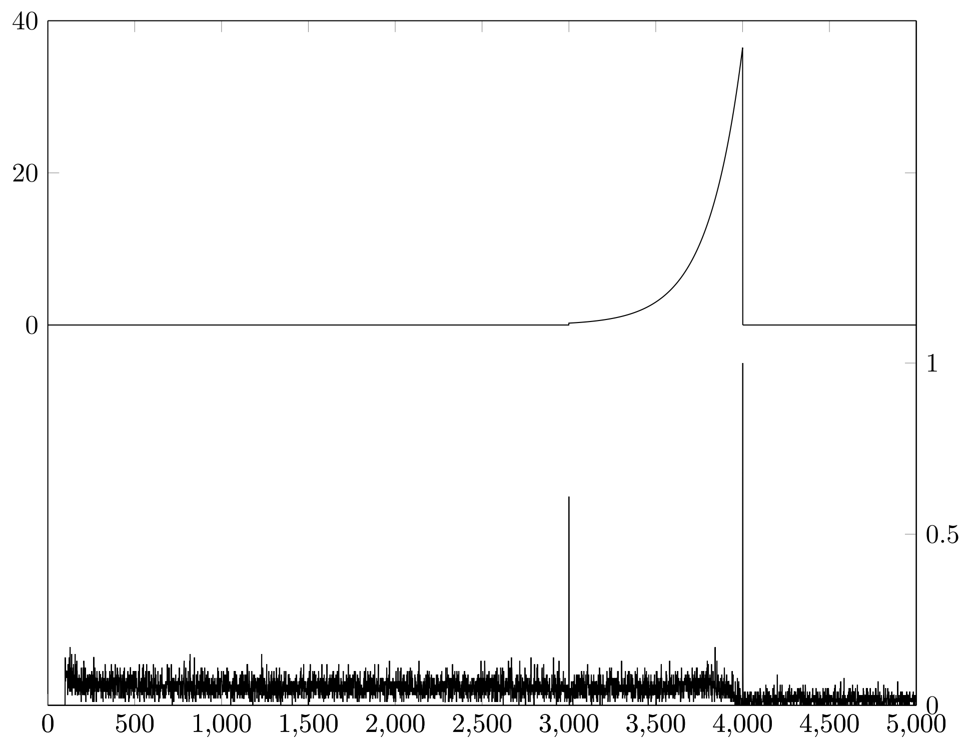

Figure 20: Power of the test based on 100 Monte Carlo simulations (Confidence interval 99.9%)

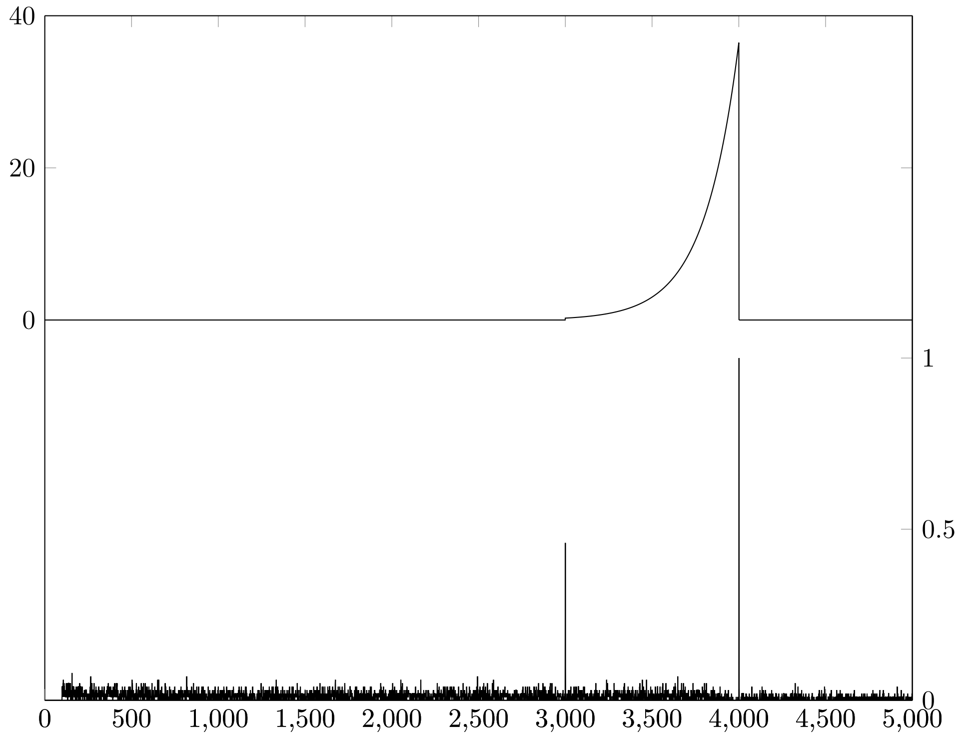

Figure 21: Power of the test with GARCH standard errors based on 100 Monte Carlo simulations (Confidence interval 99.9%)