Value at Risk and Dependency over time for Exchange Rates with Time-Varying Copulas

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Abstract

We use constant and time-varying copulas, both Normal and Symmetrized Joe-Clayton copulas, to describe the dependency of exchange rates. We test wether copulas describe the exchange rates better than more conventional models as the AR(p)-GARCH(P,Q) and test whether there is a difference between the stability of the copula parameters between emerging markets and advanced markets versus the Euro. We find evidence that the copula models estimate a better VaR than the conventional model, but no evidence that there is a difference in stability of the parameters for the different type of markets.

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1 Introduction

It has long been assumed that exchange rate returns follow a normal distribution, however various studies such as Westerfield (1977) and McFarland et al. (1982) have shown us that this assumption does not hold. Violations of normality often include but are not limited to an asymmetric distribution and 'fat tails'. A commonly used measure of dependence between asset returns is Pearson's correlation coefficient, denoted as ρ. This measure only completely captures the dependency relationship between two random variables when the joint distribution is elliptical. Elliptical distributions are those that, in their bivariate form have elliptical contours. These include the bivariate normal and bivariate Student's t-distribution.

A commonly used measure of dependence between asset returns is Pearsons correlation coefficient, denoted as ρ. This measure only completely captures the dependency relationship between two random variables when the joint distribution is elliptical. Elliptical distributions are those that, in their bivariate form have elliptical contours. These include the bivariate normal and bivariate Student's t-distribution.

The paper by Patton (2006) shows us that deviations from multivariate normality occur, in the sense that the joint distribution of the Deutsche mark/U.S. dollar and the Yen/U.S. Dollar exchange rate exhibits asymmetric dependence. He models this joint distribution using a time-varying conditional copula. Erb et al. (1994), Longin and Solnik (2001) and Ang and Chen (2002) show that correlation between two exchange rates is higher when they both depreciate, for example during a crisis, than when they appreciate. The paper by Patton (2006) shows that deviations from multivariate normality occur for exchange rates, and specifically that it can exhibit assymetric dependence. He uses a conditional copula to model this assymetry. Theoretical evidence provided in the paper by Patton is substantial, but there is a missing link to financial risk measures. From the view of an investor (or multinational trying to hedge their income), its interesting to make the connection to widely used risk measures such as the value at risk (VaR). In Wang et al. (2010) exchange rates are also modeled by copulas, but they focus solely on important currencies for China.

From an investor's point of view, it is interesting to make the connection to Value-at-Risk (VaR) and Expected Shortfall (ES). These riskmeasures, required for banks since BaselII (2006), are widely used. Therefore, we examine whether VaR and ES estimates created with copulas for portfolios is better than estimates with a more standard AR-GJR-GARCH model.

Our main hypothesis is that copulas forecast exchange rates and their dependencies better than solely the conventional AR(p)-GARCH(P,Q) models that are the input for the copulas. We estimate Value at Risk and Expected Shortfall for the exchange rate pairs Euro - Pound and Real - Peso to test this. We evaluate the 99%, 95% and 90% Value at Risk with the Christoffersen test and find that all copulas outperform the conventional AR(p)-GARCH(P,Q) models.

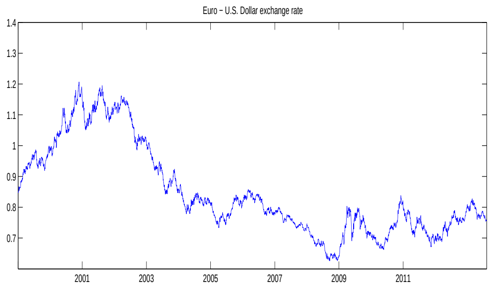
We also test if the copula parameters of ...

We proceed as follows. In Section 2, we discuss the data set. We continue with the methodology in Section 3. The results for our hypothesis are stated in Section 4. We conclude in Section 5.

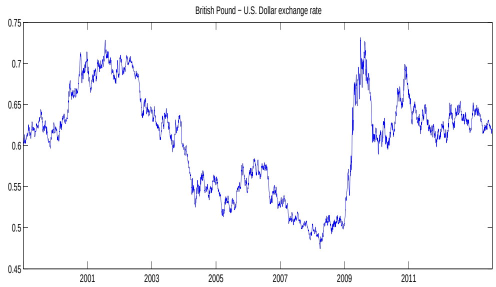
2 Data

We examine daily data on 21 exchange rates[1], for the period January 1999 to December 2012. This is divided into eleven advanced markets and ten emerging markets IMF (2013). The countries with their status are found in Appendix A.1, Table 13. We use four exchange rates to construct risk measures, namely Euro - U.S. Dollar, British Pound - U.S. Dollar, Brasilian Real - U.S. Dollar, Mexican Peso - U.S. Dollar. We construct an advanced pair, the Euro and the Pound, and an emerging pair, the Peso and the Real. Figure 1 shows the movements of these exchange rates for our sample period.

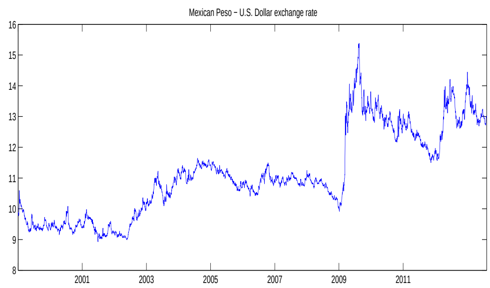
Figure 1: Course of the four exchange rates against the U.S. Dollar



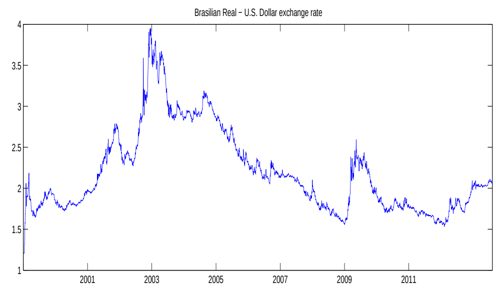
(a) Euro - U.S. Dollar



(b) Pound - U.S. Dollar



(c) Peso - U.S. Dollar



(d) Real - U.S. Dollar

NOTE: This figure shows the course of the Euro, Pound Peso and Real against the U.S. Dollar for the period January 1999 to December 2012.

Remarkable is the overall appriciation of the U.S. Dollar around 2009 against all exchange rates except the Euro. This was due to the fact that the Federal Reserve started the large scale Quantative Easing program by the Fed (Fed (2009)), which led to a flee to the U.S. Dollar.

We test for a unit root with the Augmented Dickey-Fuller (ADF) test. Table 14 in Appendix A.3 shows the results. We do no reject the null hypothesis, the presence of a unit root, for the levels of the series, so we use the first log differences of these time series. We do reject the null hypothesis, the presence of a unit root, for the first log differences of all exchange rates. The summery statistics for the four currencies used for risk measures are given in Table 1, while the summery statistics

[1]collected from WM/Reuters

for all exchange rates can be found in Appendix A.3, Table 15.

Table 1: Descriptive statistics for the returns of the exchange rates

	Euro	Pound	Peso	Real
Mean	-0,003	0,000	0,007	0,014
Maximum	3,844	3,918	7,552	10,800
Minimum	-4,617	-4,473	-4,766	-11,778
Std. Dev.	0,642	0,587	0,646	1,095
Skewness	-0,184	0,038	0,679	0,499
Kurtosis	5,429	7,316	15,646	18,840
Jarque-Bera	<0,001	<0,001	<0,001	<0,001
Observations		3652		

NOTE: This table contains the descriptive statistics for the returns of the exchange rates against the U.S. Dollar, calculated as ret $= 100 \cdot (\log(x_t) - \log(x_{t-1}))$. The sample period is 1/1/1999 until 31/12/2012.

The rejection of the Jarque-Bera test shows that a normal distrubition is not appropriate. Bollerslev (1986) suggests the use of Student's t distribution for financial data. The Euro is the only currency that appreciated against the U.S. - Dollar in our time frame. The variance of the Euro, Pound and Peso are more ore less the same, while the variance of the Real is a lot higher. This may be due the fallout of the economic crisis in Brasil in 1999 and the recovery that took place afterwards. There is excess kurtosis in all currencies, especially in the Peso and the Real.

3 Methodology

In this section we show our used methodology. First we introduce the marginal models for the currencies, later the conditional copula models to describe the dependency structure. We introduce the goodness of fit test for copulas (first introduced by Patton (2006)). After that we describe the riskmeasures and the tests to evaluate if they're correct. In the last section, the Mann-Whitney U test is explained.

3.1 Marginal models

Mandelbrot (1963) shows us that financial assets exhibit volatility clustering, and Engle (1982) and Bollerslev (1986) introduced models that were capable of capturing that volatility clustering. Since the introduction of these models, they are widely researched (Bollerslev et al. (1992)) and used in the financial world (Diebold et al. (1998)). Erb et al. (1994), Longin and Solnik (2001) and Ang and Chen (2002) show that the dependence in the upper and lower tail can be substantially different. Multiple extensions of the GARCH-models can model this behavior.

We use two of these extensions for our marginal models; the more simple Glosten-Jagannathan-Runkle-GARCH model (GJR-GARCH) (introduced by Glosten et al. (1993)) and the more complex exponential GARCH-model (EGARCH)(introduced by Nelson (1991)) to model the conditional variance. Both allow to model for the stylized facts of financial returns of volatility clustering and take into account that conditional variance tends to be higher after a decrease in return than after an equal increase. The GJR-GARCH model has the restriction that all parameters must be larger than 0, and the sum of all parameters, except the constant, is smaller than one. When $\sum \gamma_i + \sum \alpha_j + 0.5 \sum \xi_j \to 1$, it might be better to choose an EGARCH model. We use the BIC and AIC information criteria to decide the model selection. This model has fewer restrictions on the parameters (for example: negative parameters are allowed). For reasons given in Section 2, we choose a t_v-distribution for the errors.

3.1.1 ARMA(p,q)-GJR-GARCH(P,Q) model

The ARMA(p,q)-GJR-GARCH(P,Q) model with t-distributed errors is given by:

$$X_t = \mu_x + \sum_{i=1}^{p} \phi_{i,x} X_{t-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} + \varepsilon_t \tag{1}$$

with $\varepsilon_t = \sigma_t z_t$. $z_t \sim t_v$ with v degrees of freedom and

$$\sigma_t^2 = \kappa + \sum_{i=1}^{P} \gamma_i \sigma_{t-i}^2 + \sum_{j=1}^{Q} \alpha_j \varepsilon_{t-j}^2 + \sum_{j=1}^{Q} \xi_j I[\varepsilon_{t-j} < 0] \varepsilon_{t-j}^2 \tag{2}$$

The indicator function $I[\varepsilon_{t-j} < 0]$ equals 1 if $\varepsilon_{t-j} < 0$ and 0 otherwise. For stationarity and positivity, the GJR model has the following constraints:

$$\begin{aligned} \kappa &> 0 \\ \gamma_i, \alpha_j &\geq 0 \\ \alpha_j + \xi_j &\geq 0 \\ \sum_{i=1}^{P} \gamma_i + \sum_{j=1}^{Q} \alpha_j + \frac{1}{2} \sum_{j=1}^{Q} \xi_j &< 1 \end{aligned}$$

3.1.2 ARMA(p,q)-EGARCH(P,Q) model

The ARMA(p,q)-EGARCH(P,Q) model with t-distributed errors is given by:

$$X_t = \mu_x + \sum_{i=1}^{p} \phi_{i,x} X_{t-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} + \varepsilon_t \tag{3}$$

where $\varepsilon_t = \sigma_t z_t$. $z_t \sim t_v$ with v degrees of freedom and

$$\log \sigma_t^2 = \kappa + \sum_{i=1}^{P} \gamma_i \log \sigma_{t-i}^2 + \sum_{j=1}^{Q} \alpha_j \left[\frac{|\varepsilon_{t-j}|}{\sigma_{t-j}} - E\left\{ \frac{|\varepsilon_{t-j}|}{\sigma_{t-j}} \right\} \right] + \sum_{j=1}^{Q} \xi_j \left(\frac{\varepsilon_{t-j}}{\sigma_{t-j}} \right) \tag{4}$$

where

$$E\left\{ \frac{|\varepsilon_{t-j}|}{\sigma_{t-j}} \right\} = E\{z_{t-j}|\} = \sqrt{\frac{v-2}{\pi}} \frac{\Gamma(\frac{v-1}{2})}{\Gamma(\frac{v}{2})} \tag{5}$$

3.2 Copula models

Sklar (1959) introduces copulas to describe the dependency between random variables. The general bivariate copula is :

$$F(x, y) = C(F_x(x), F_y(y)), \qquad \text{or} \tag{6}$$

$$f_{xy}(x, y) = f_x(x) \cdot f_y(y) \cdot c(F_X(x), F_Y(y)) \tag{7}$$

This means that a joint distribution for two variables can be exactly decomposed into the two marginal distributions and a copula function. This is not limited to a linear measure, like the correlation coefficient. To describe the dependency correct, the copula

Now declare the variables X and Y as the variables to describe, and W as the conditioning variable. Let the joint distribution of (X, Y, W) be F_{XYW}, and the probability of (X, Y) given W be $F_{XY|W}$. The conditional distributions are $F_{X|W}$ and $F_{Y|W}$ for $X|W$ and $Y|W$ respectively. The conditional bivariate copula distribution of $(X, Y)|W$ can be derived from the unconditonial joint distribution for (X, Y, W) by taking the following derivative:

$$F_{XY|W}(x, y, |w) = f_w(w)^{-1} \cdot \frac{\delta F_{XYW}(x, yw)}{\delta w}, \qquad \text{for } w \in \mathbb{W} \tag{8}$$

where f_w is the unconditional density of W and $\mathbb{W}$ is the support of W. Then Patton (2006) states that the conditional copula of $(X, Y)|W = w$, where $X|W = w \sim F_{X|W}(\cdot|w)$ and $Y|W = w \sim F_{Y|W}(\cdot|w)$ is the conditional joint distribution function of $U \equiv F_{X|W}(X|w)$ and $V \equiv F_{Y|W}(Y|w)$ given $W = w$.

According to Alexander (2008) $F_x(x)$ and $F_y(y)$ are uniform distributed. The substitutions $u = F_x(x)$ and $v = F_y(y)$ are probability integral transforms (PIT).

A great benefit of using copulas is that in can apply to any combination of marginal distributions. For example, we can combine marginal t-distribution with a normal distribution. The only complication introduced when extending Sklar's theorem to conditional distributions is that the conditioning variable(s), W, must be the same for both marginal distributions and the copula. This is important in the construction of conditional density models using copula theory. Failure to use the same conditioning variable for $F_{X|W}$, $F_{Y|W}$, and C will, in general, lead to a failure of the function $F_{XY|W}$ to satisfy the conditions for it to be a joint conditional distribution function.

We follow Patton (2006) and use the conditional bivariate normal copula and the conditional bivariate Symmetrized Joe-Clayton (SJC) copula. We use the same models with no time-variation as a benchmark. Figure 2 shows how different copulas have a different dependency structure.

3.2.1 Normal copula

The normal copula model with no time-variation is described in equation 9.

$$C(u, v|\rho_t) = \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi\sqrt{1-\rho_t^2}} \exp\left[\frac{-(r^2 - 2\rho_t rs + s^2)}{2(1-\rho_t^2)}\right] drds \tag{9}$$

where Φ^{-1} is the inverse of the standard normal $c.d.f$ and $\rho_t = \rho$ is the correlation. We use equation 10 to condition the parameters on time.

$$\rho_t = \tilde{\Lambda}\left(\omega_\rho + \beta_\rho \cdot \rho_{t-1} + \alpha \cdot \frac{1}{10}\sum_{j=1}^{10} \Phi^{-1}(u_{t-j}) \cdot \Phi^{-1}(v_{t-j})\right) \tag{10}$$

$\tilde{\Lambda}(x) = (1 - e^{-x})(1 + e^{-x})^{-1}$ makes sure that the correlation stays within$(-1, 1)$.

3.2.2 Symmetrized Joe-Clayton copula

The Symmetrized Joe-Clayton (SJC) copula with no time-variation is introduced by Joe (1997) and is a variant of the Joe-Clayton copula. The equation for the SJC copula is:

$$C_{SJC}(u, v \mid \tau_t^U, \tau_t^L) = \frac{1}{2}(C_{JC}(u, v \mid \tau_t^U, \tau_t^L) + C_{JC}(1-u, 1-v \mid \tau_t^U, \tau_t^L) + u + v - 1) \tag{11}$$

where C_{JC} is the Joe-Clayton copula, given by:

$$C_{JC}(u, v|\tau^U, \tau^L) = 1 - \left(\{[1-(1-u)^\kappa]^{-\gamma} + [1-(1-v)^\kappa]^{-\gamma} - 1\}^{-1/\gamma}\right)^{1/\kappa} \tag{12}$$

where

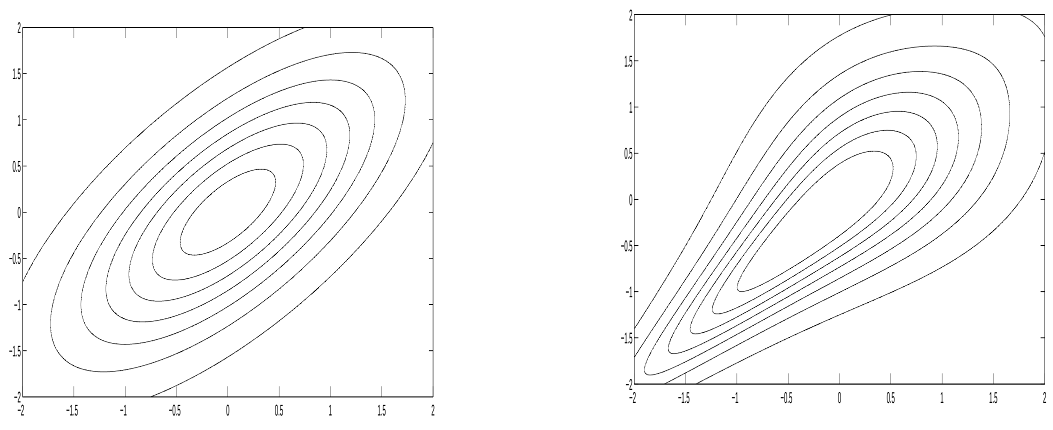
$$\begin{aligned} \kappa &= \log_2(2-\tau^U)^{-1} \\ \gamma &= -\log_2(\tau^L)^{-1} \\ \tau^U &\in (0,1) \\ \tau^L &\in (0,1) \end{aligned}$$

$\tau_t^U = \tau^U$ and $\tau_t^L = \tau^L$ are the upper and lower tail dependence coefficients. We use equation 13 for $x = \{U, L\}$ to condition on time.

$$\tau_t^x = \Lambda\left(\omega_x + \beta_x \tau_{t-1}^x + \alpha_x \frac{1}{10}\sum_{j=1}^{10} \mid u_{t-j} - v_{t-j} \mid\right) \tag{13}$$

The logistic transformation $\Lambda(x) = (1+e^{-x})^{-1}$ makes sure that the τ^U and τ^L are within (0,1).

Figure 2: Contour plots of copulas



(a) Bivariate normal copula with $\rho = 0.7$

(b) Bivariate SJC-copula with $\tau^U = 0.2$ and $\tau^L = 0.7$

NOTE: This figure shows the contour plots of the bivariate normal copula and the bivariate Symmetrized Joe-Clayton (SJC) copula. The difference in dependency structure for the upper and lower tail is cleary visible for the SJC-copula.

3.3 Pseudo Maximum Likelihood Estimation

The variables u and v that are needed for the copula models are the probability integral transforms of the data. However, as we do not know the true distribution of the marginal models we have to use a different approach. Since the copula corresponding to a random pair (X,Y) is invariant under monotonic increasing transformations (see Genest and Favre (2007)), we can obtain the same copula parameters without using the CDF for the probability integral transforms. According to Oakes (1982) the ranks of data pairs retain the greatest amount of information of the underlying dependence structure. Thus, we first transform the standardized residuals to ranks and then use these as data for the copula models. The ranks are made by equation 14, where R_i is the rank of X_i among $X_1, \ldots, X_n$ and S_i is the rank of Y_i among $Y_1, \ldots, Y_n$. We rescale the axes by $1/(n+1)$ to create a set in the unit square $[0,1]^2$

$$R^* = \frac{R_i}{n+1}, \quad S^* = \frac{S_i}{n+1} \tag{14}$$

Using maximum likelihood, it is possible to estimate the marginal model and copula parameters at the same time. However, as this is computationally more demanding we opt to estimate the copula parameters with pseudo maximum likelihood, see Genest and Favre (2007). This means that we estimate the marginal models and copula parameters seperately. So we use the ranks as described above to apply maximum likelihood in two seperate stages.

3.4 Goodness of Fit

We use the Likelihood Ration test proposed by Patton (2006) in his appendix to test if the real data fits the estimated density well. For the marginal densities we use five[2] regions, while we use seven[3] regionsfor the copula densities (see Appendix B.1, Figure 9). We test the fit of the regions individually, and all regions combined.

3.4.1 Individual test

The null hypothesis that the model is adequately specified in each of the $K+1$ regions individually is: $H_0 : Hit_t^j \sim inid^4\ Bernoulli(p_{jt})$, and $H_1 : Hit_t^j \sim Bernoulli(\pi_{jt})$. With

$$\pi_{jt} = \pi_j(Z_{jt}, \beta, p_{jt}) = \Lambda\left(Z_{jt} \cdot \beta_j - \ln\left[\frac{1-p_{jt}}{p_{jt}}\right]\right) \tag{15}$$

where $\Lambda(x) \equiv (1+e^{-x})^{-1}$ is the logistic transformation. Z_{jt} is a (1×4) vector that includes a constant and the number of hits in the past one, five and ten days. β_j is a 4×1 vector of parameters to be estimated. If $\beta_j = 0$, then $\pi_{jt} = \pi_j(Z_{jt}, 0, p_{jt}) = p_{jt}$. Thus, H_0 is equal to $\beta_j = 0$ and $H_1 = \beta_j \neq 0$. We use maximum likelihood to estimate β_j, see the equation below.

$$\mathcal{L}(\pi_j(Z_j, \beta_j, p_j | Hit^j) = \sum_{t=1}^{T} \left(Hit_t^j \cdot \ln(\pi_j) + (1 - Hit_t^j) \cdot \ln(1-\pi_j)\right) \tag{16}$$

The test statistic LR_j is equal to $-2 \cdot (\mathcal{L}(p_j|Hit^j) - \mathcal{L}(\pi_j(Z_j, \hat{\beta}_j, p_j|Hit^j)) \sim \chi^2_{k_j}$.

[2]For the marginal densities we look at the lower and upper 10% tail, the upper and lower 10th to 25th quantile and the remaining middle 50%. This is equivalent to a positive or negative extreme event, positive or negative moderate event and a median event.

[3]For the copula densities we look at the joint lower and upper 10% tail, the joint upper and lower 10th to 25th quantile, the joint 25th to 75th quantile, the lower (upper) 25% tail of U with the upper (lower) 25% tail of V.

[4]independent but not identically distributed

3.4.2 Joint test

We define $\Pi_t \equiv [\pi_{0t}, \pi_{1t}, ..., \pi_{Kt}]'$ and $P_t \equiv [p_{0t}, p_{1t}, ..., p_{Kt}]'$. The null hypothesis is $H_0 : M_t \sim \ inid\ Multinomial(P_t)$ versus $H_1 : M_t \sim \ Multinomial(\Pi_t)$. Again, Π_t is modeled using P_t and Z_t. Then,

$$\pi_1(Z_t\boldsymbol{\beta}, P_t) = \Lambda\left(\lambda_1(Z_{1t}, \beta_1) - \ln\left[\frac{1-p_{1t}}{p_{1t}}\right]\right) \tag{17}$$

$$\pi_j(Z_t, \boldsymbol{\beta}, P_t) = \left(1 - \sum_{i=1}^{j-1} \pi_{it}\right) \cdot \Lambda\left(\lambda_j(Z_{jt}, \beta_j) - \ln\left[\frac{1-\sum_{i=1}^{j} p_{it}}{p_{jt}}\right]\right), \tag{18}$$

for j = 2,...,K

$$\pi_{0t} = 1 - \sum_{j=1}^{K} \pi_j(Z_t, \boldsymbol{\beta}, P_t) \tag{19}$$

where $Z_t \equiv [Z_1, ..., Z_K]'$ and $\beta \equiv [\beta_1, ..., \beta_K]'$. The competing hypotheses may be expressed as $\boldsymbol{\beta}$=0 versus $\boldsymbol{\beta} \neq 0$. The likelihood function to be maximized to obtain $\boldsymbol{\beta}$ is

$$\mathcal{L}(\Pi(Z, \boldsymbol{\beta}, P)|Hit) = \sum_{t=1}^{T} \sum_{j=0}^{K} \ln \pi_{jt} \cdot \mathbf{1}\{M_t = j\} \tag{20}$$

We also use a likelihood ratio test this hypothesis, which is specified as $LR_{ALL} = -2 \cdot (\mathcal{L}(P|Hit) - \mathcal{L}(\Pi(Z, \hat{\boldsymbol{\beta}}, P)) \sim \chi^2_{K_\beta}$.

3.5 Riskmeasures

We construct two equally weighted portfolios, one for both currency pairs. We calculate the one-day Value-at-Risk (VaR) (Linsmeier and Pearson (2000)) and Expected Shortfall (ES) (Rockafellar and Uryasev (2000)) with MonteCarlo simulation. We use the normal- and SJC-copula, both conditional and unconditional, and an AR(1)-GJR-GARCH(1,1) model as benchmark. The parameter estimation is based on a 500 day moving window.

3.5.1 Value-at-Risk

For a given confidence interval $\alpha \in (0, 1)$, the VaR of the portfolio is the biggest loss you expect with $(1 - \alpha)$ probability.

$$\text{VaR}_\alpha(L) = \inf\{l \in \mathbb{R} : P(L > l) \leq 1 - \alpha\} = \inf\{l \in \mathbb{R} : F_L(l) \geq \alpha\} \tag{21}$$

This risk measure gives a great insight in that largest possible loss for the forecasted window. However, it does not tell how big that loss can be in $(1 - \alpha)$% of the cases, which can be harmfull because of fat tails that exists for financial assets.

3.5.2 Expected Shortfall

The method expected shortfall (ES) covers this limitation of the Value-at-Risk. It calculates the average of the losses in the worst α% of the cases. This gives a better insight in the downside risk. It is calculated as follows:

$$\text{ES}_\alpha = \frac{1}{\alpha} \int_0^\alpha \text{VaR}_\gamma(X) \text{d}\gamma \tag{22}$$

where VaR_γ is the γ% Value-at-Risk and α is the size of the lower quantile.

A graphical representation of the difference between the VaR and ES is given in Figure 11 in Appendix D.

3.6 Christoffersen test

We use the test proposed by Christoffersen (1998) to test the estimated Value at Risk. It is a likelihoodhood ratio test, decomposed in two parts; the unconditional coverage test and the indepency test. If we ignore the first observation (because LR_{ind} is a first-order Markov chain) we can calculate the test statistic as follows:

$$LR_{cc} = LR_{uc} + LR_{ind} \quad \sim \chi^2(2) \tag{23}$$

The elaboration of these tests are found in appendix C.

3.7 Mann-Whitney U-test

The Mann-Whitney U-test (described in Wackerly et al. (2002)) is a nonparametric test for equality of populations medians of two independent samples X and Y.

The Mann-Whitney U-test statistic, U, is the number of times an x precedes a y in an ordered arrangement of the elements in the two independent samples X and Y. For example, if $x = [5\ 6\ 7\ 11]$ and $y = [8\ 9\ 12\ 15]$, then in the ordered observations $[5\ 6\ 7\ 8\ 9\ 11\ 12\ 15]$ $u_{y=8} = 3$, $u_{y=9} = 3$, $u_{y=12} = 4$ and $u_{y=15} = 4$, so $U = 3+3+4+4 = 14$. Very large or very small values of U imply a shift in sample mean between the populations of x and y.

The test is equivalent to the Wilcoxon rank sum test. It is calculated as follows:

$$U = n_x n_y + \frac{n_x(n_x+1)}{2} - W \tag{24}$$

where n_x is the length of x, n_Y is the length of y and W is the Wilcoxon rank sum statistic (as described in Wackerly et al. (2002)), calculated as follows

$$W = |\sum_{i=1}^{N_r}[\text{sgn}(x_i - y_i) \cdot R_i]| \tag{25}$$

where N_r is the reduced sample size and R_i is the rank of $|x_i - y_i|$.

4 Results

Here we show our results. To be consise, we show here only the results for the four exchange rates that are used for the risk measures. All other information can be found in the appendix.

4.1 Marginal models

We choose an GJR-GARCH or EGARCH model, based on the AIC and BIC information criteria. These values are found in Table 17 in Appendix E.1. We choose the appropriate (p, q, P, Q) lags for the models, to eliminate any autocorrelation in the standardized residuals and squared standardized residuals. We use the ARCH-LM test to test for any heteroskedacity left. Table 2 shows the selected models for the four main currencies.

Table 2: Selected models for the Euro, Pound, Peso and Real

	Euro	Pound	Peso	Real
Model	EGARCH	EGARCH	GJR	EGARCH
AR lags	{1}	{1}	{1}	{1}
GARCH lags	{1}	{1}	{1}	{1}
ARCH lags	{1,2}	{1}	{1}	{1}

NOTE: This table shows the selected models for the four main currencies.

We see that we choose a EGARCH model for the Euro, Pound and Real, while the GJR-model provides a better fit for the Peso. The ability of the EGARCH-model to have the sum of the parameters larger than one, seems to be important for the Pound and Real. The GJR model for the euro hits the non-negative bounderies, and subsequently the Euro is better described by the EGARCH-model. The parameters estimates and standard errors for the marginal models are found in Table 3. In Table 4 we see that the residuals are homoskedastic, while Table 23 in Appendix E.1 shows that there is no autocorrelation left in the standardized residuals and standardized squared residuals.

We see in Table 3 that the AR-component is significant for the Real, but the first lag is necessary to remove the autocorrelation for all models. Furthermore .....

Table 4: Results for the ARCH-LM test for the marginal distributions

	Euro	Pound	Peso	Real
ARCH-LM	0.33	0.70	0.55	0.57

NOTE: This table shows the p-values for the ARCH-LM test proposed by Engle (1982). A p-value larger than 0.05 means that we cannot reject the null of homoskedacity

In Tables 5 and 6 we show that we only need univariate models for the marginal distributions. No lags of the other variables is significant. We follow Patton (2006) and test whether the first lag of the first exchange rate is important for the conditional mean of the second exchange rate by regressing the residuals of the second exchange rate on the first lag of the first exchange rate and test if that regression coefficient is equal to 0. This works both ways. To test the conditional variance models, we regress the standardized squared residuals of one exchange rate on the lagged squared residuals of the other exchange rate and test whether these regression coefficients are equal to zero.
Interesting to see is that for the Euro, Pound and Real pass the

Table 7 shows that the results for LM test for serial independence of the probability integral transforms, U and V, the results for the Kolmogorov-Smirnov (KS) test of the density specifiation and the joint hit test described in Section 3.4, with the following five regions: the upper and lower 10%-tail, the upper and lower 10th to 25th quantile and the middle 50% (25th to 75th quantile).

Table 3: Results for the marginal distributions

	Euro	Pound	Peso	Real
Constant	-0,006*	-0,006*	-0,016*	-0,010*
	(0,009)	(0,008)	(0,010)	(0,008)
AR(1)	-0,002*	0,013*	0,044	0,025*
	(0,014)	(0,017)	(0,017)	(0,017)
GARCH constant	-0,006	0,002	-0,015	0,007
	(0,002)	(0,001)	(0,004)	(0,001)
GARCH(1)	0,994	0,955	0,970	0,889
	(0,002)	(0,007)	(0,005)	(0,012)
ARCH(1)	-0,141	0,051	0,277	0,132
	(0,039)	(0,008)	(0,021)	(0,014)
ARCH(2)	0,215			
	(0,039)			
Leverage (1)	0,033*	-0,026	0,083	-0,091
	(0,026)	(0,009)	(0,013)	(0,017)
Leverage (2)	-0,028*			
	(0,025)			
DoF	9,945	12,121	6,897	8,093
	(1,572)	(2,276)	(0,609)	(0,950)

* indicates that we cannot reject $\beta = 0$ at the 0.05 level.
NOTE: This table shows the maximum likelihood estimates, with asymptotic standard errors in the parentheses of the parameters for the marginal distributions for the four exchange rates.

Table 7: Tests of the marginal distribution models

	Euro	Pound	Real	Peso
First moment LM test	0.37	0.19	0.13	0.84
Second moment LM test	0.12	0.33	0.80	0.26
Third moment LM test	0.07	0.29	0.97	0.52
Fourth moment LM test	0.40	0.06	0.37	0.32
K-S test	0.29	0.28	0.84	0.003
Joint hit test	0.52	0.28	0.88	$<$0.001

NOTE: In this table we show the p-values from LM tests of serial independence of the first four moments of the variables U_t and V_t, described in the text from the models described in the text. We regress $(u_t - \bar{u})^k$ and $(v_t - \bar{v})^k$ on 10 lags of both variables, for $k = 1, 2, 3, 4$. The test statistic is $(T-20) \cdot R^2$ for each regression and is distributed under the null as χ^2_{20}. We also report the p-value for the Kolmogorov-Smirnov (KS) test. This shows the adequacy of the marginal models. We also report the p-value from the joint hit test described in Section 3.4 that describes if the density model fits well in the five regions. Table 16 in Appendix B.2 shows the results for the individual regions.

Table 5: Testing the influence of the " other" variable in the mean and variance model for the Euro and Pound

	p-Value
X_{t-1} in mean model for Y_t	0.08
Y_{t-1} in mean model for X_t	0.60
ε^2_{t-1} in conditional variance model for Y_t	0.21
η^2_{t-1} in conditional variance model for X_t	0.76

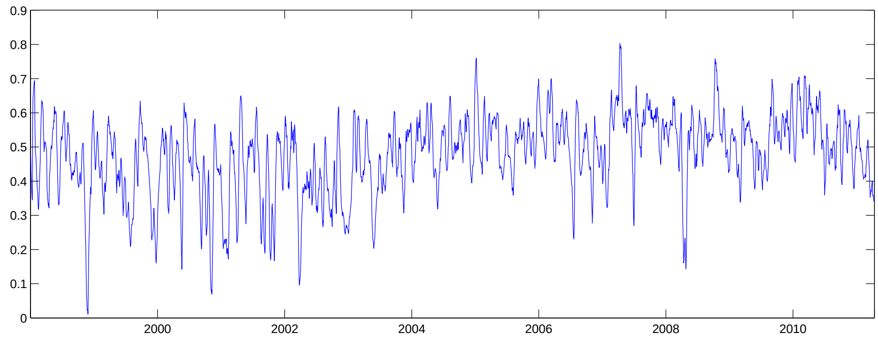
NOTE: Variable X with corresponding variance ε^2 is the Euro, and variable Y with corresponding variance η^2 is the Pound. We report p-values that the variables have coefficients equal to 0; a p-value greater than 0.05 means we cannot reject the null at the 0.05 level.

Table 6: Testing the influence of the " other" variable in the mean and variance model for the Peso and Real

	p-Value
X_{t-1} in mean model for Y_t	0.41
Y_{t-1} in mean model for X_t	0.21
ε^2_{t-1} in conditional variance model for Y_t	0.11
η^2_{t-1} in conditional variance model for X_t	0.94

NOTE: Variable X with corresponding variance ε^2 is the Peso, and variable Y with corresponding variance η^2 is the Real. We report p-values that the variables have coefficients equal to 0; a p-value greater than 0.05 means we cannot reject the null at the 0.05 level.

Figure 3: ρ over time for the Real-Peso copula

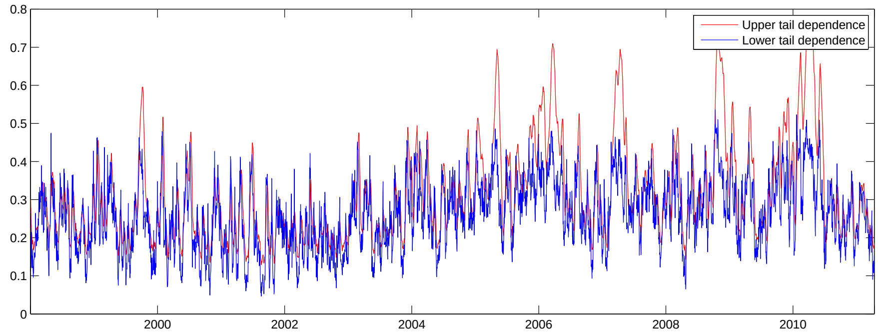


NOTE: This figure shows the value of ρ for the Real and Peso over time.

4.2 Copula models

We present here the results for our estimated copula models. Table 8 shows the maximum likelihood estimates for copula parameters.

Figure 4: τ^U and τ^L over time for the Real-Peso copula



NOTE: This figure shows the values of τ^U and τ^L for the Real and Peso over time.

Table 8: Results for the copula models

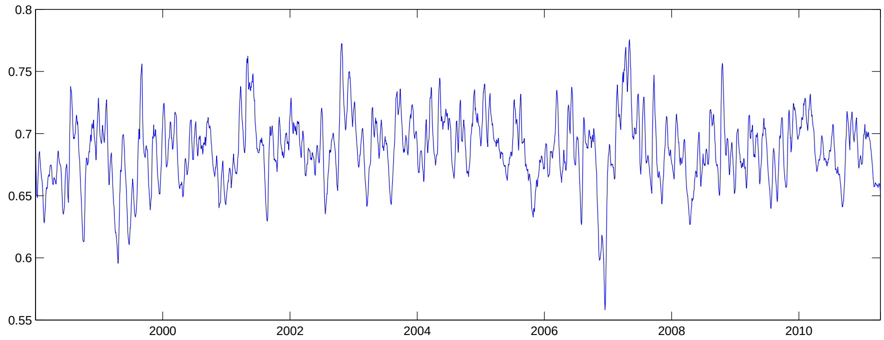
	Euro - Pound	Peso - Real
Constant normal copula		
$\bar{\rho}$	0.688	0.470
	(0.007)	(0.012)
Copula likelihood	1170.90	456.74
Constant SJC copula		
$\bar{\tau}^U$	0.408	0.329
	(0.018)	(0.018)
$\bar{\tau}^L$	0.545	0.220
	(0.012)	(0.022)
Copula likelihood	1155.56	464.56
Time-varying normal copula		
Constant	-0.577	0.023*
	(0.003)	(0.029)
α	0.065	0.201
	(0.012)	(0.040)
β	3.240	1.965
	(0.010)	(0.094)
Copula likelihood	1184.08	500.30
Time-varying SJC copula		
Constant^U	-1.923	1.061*
	(0.023)	(1.118)
α^U	-0.488	-6.434
	(0.122)	(2.341)
β^U	4.001	-1.149*
	(0.029)	(2.009)
Constant^L	-1.978	-1.601
	(0.006)	(0.120)
α^L	-0.172	-2.611
	(0.071)	(0.607)
β^L	4.018	3.961
	(0.017)	(0.122)
Copula likelihood	1205.29	514.33

* indicates that we cannot reject $\beta = 0$ at the 0.05 level.
NOTE: This table shows the maximum likelihood estimates of

Figures 5, 6, 3 and 4 give a better insight in the values of Table 8. We see that the time-varying normal copula for the Euro and Pound moves mostly between 0.65 and 0.75, with the non-time varying ρ is 0.688. The normal time-varying copula for the Real and Peso shows a lot more variation than the time-varying normal copula for the Euro and Pound, but both show no break in the parameters.

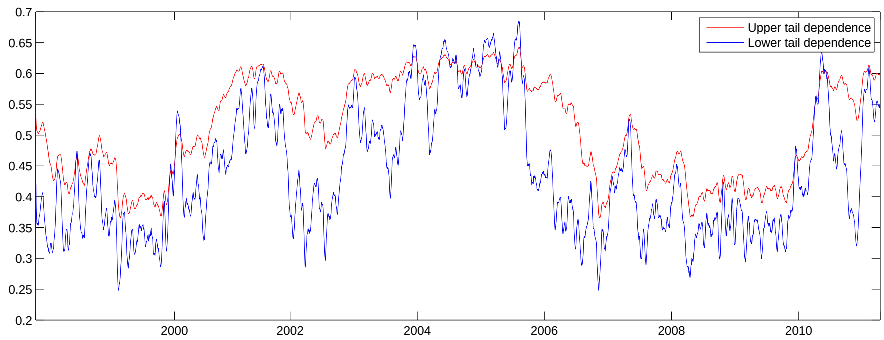
We see that there is a lot more variation in the upper and lower tail dependency for the Euro and the Pound over time than the time-varying normal coupa parameter, but the parameters move together. The upper and lower taildependence of the time-varying SJC copula for the Real and Peso seem to move perfectly together, until 2005, 2006. Than the uppertail dependence tend to be higher than the lower taildepence.

Figure 5: ρ over time for the Euro-Pound copula



NOTE: This figure shows the value of ρ for the Euro and Pound over time.

Figure 6: τ^U and τ^L over time for the Euro-Pound copula



NOTE: This figure shows the values of τ^U and τ^L for the Euro and Pound over time.

4.3 Hit tests

We perform the hittest described in Section 3.4 to test the fit of the copula models. If we use $p = 0.05$, we find that the SJC-copula, both constant and time-varying, fit both pairs.

An interesting result is however, that the time-varying SJC-copula does not pass the test at $p = 0.10$, while the SJC-copula with non-timevarying parameters still passes that test. This can be due to the fact that the hittest for the time-varying SJC-copula almost takes the number of datapoints times as much time, than the hittest for the constant SJC-copula. This may result in a lower precision of the calculation of the hits, and subsequently the test. The time-varying normal copula performs also well, and better than the constant time-varying copula.

Table 9: Hit test results for the Euro-Pound copula models

	Constant		Time-varying	
	Normal Copula	SJC Copula	Normal Copula	SJC Copula
Regio 1	0,393	0,223	0,298	0,391
Regio 2	0,404	0,513	0,479	0,387
Regio 3	0,020	0,076	0,340	0,338
Regio 4	0,161	0,130	0,073	0,125
Regio 5	0,063	0,193	0,209	0,126
Regio 6	0,024	0,183	0,066	<0,001
Regio 7	0,242	0,628	0,002	0,584
Joint	0,013	0,271	0,134	0,074

NOTE: We report the p-values from the hit tests that the models are correctly specified in that (all) regions. A p-value larger than 0.05 means that the copula model is well specified in that region.

All copulas describing the Peso and Real pass our hittest, although all fail for the first region. This is the lower

$$0.1, 0.1$$

region, and possibly the most interesting region when looking at high VaR-predictions (the 99% VaR) for example.

Table 10: Hit test results for the Peso-Real copula models

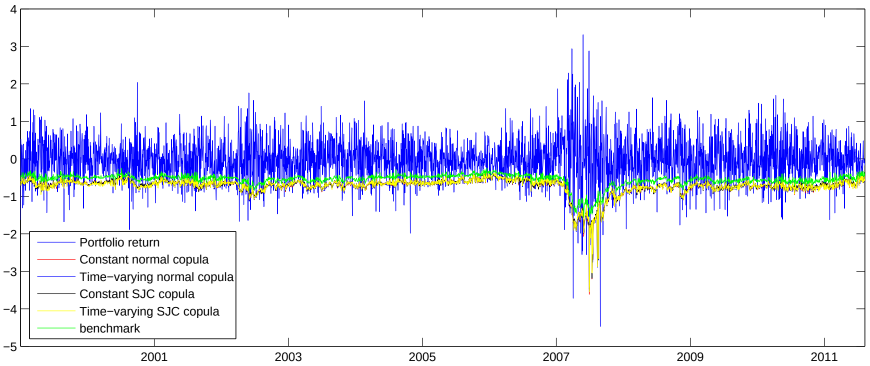
	Constant		Time-varying	
	Normal Copula	SJC Copula	Normal Copula	SJC Copula
Region 1	0,026	0,023	0,027	0,042
Region 2	0,746	0,170	0,936	0,811
Region 3	0,256	0,377	0,347	0,391
Region 4	0,771	0,780	0,540	0,451
Region 5	0,169	0,388	0,230	0,385
Region 6	0,072	0,615	0,198	0,273
Region 7	0,880	0,677	0,936	0,913
Joint	0,396	0,667	0,655	0,738

NOTE: We report the p-values from the hit tests that the models are correctly specified in that (all) regions. A p-value larger than 0.05 means that the copula model is well specified in that region.

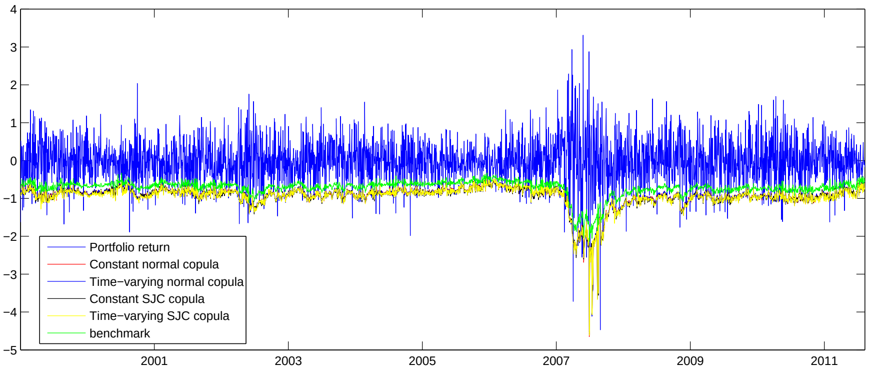
4.4 Risk measures

We estimate the Value-at-Risk for an equally weighted portfolio for both currency pairs. Table 11 and Figure 7 show the results for the Euro and Pound portfolio. Interesting to see is that the benchmark fails the Christoffersen test for all q, while all copula models pass the test.

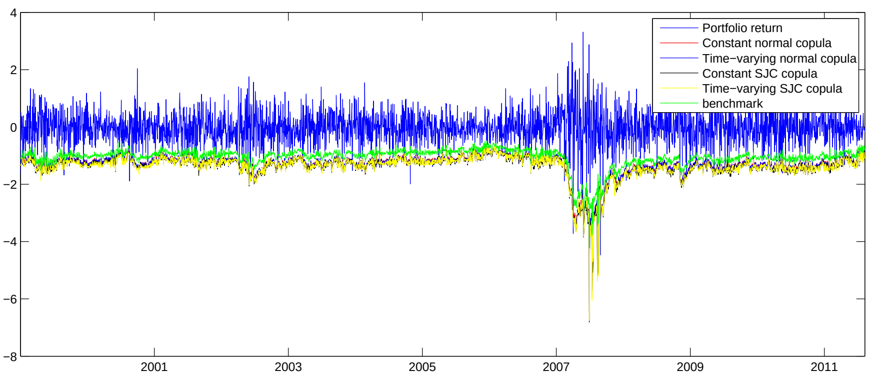
Figure 7: Value-at-Risk estimates for an equally weigthed portfolio with the euro for 2001-2012



(a) Forecasted VaR 90%



(b) Forecasted VaR 95%



(c) Forecasted VaR 99%

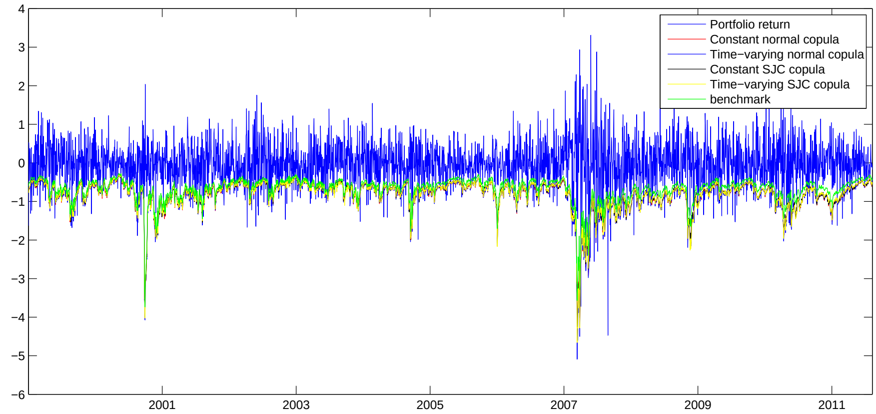
Table 11: Christoffersen test results for the Euro Pound estimated Value-at-Risk

q		Unconditional	Independent	Conditional Coverage
99%	Benchmark	<0,001	0,644	<0,001
	Constant normal copula	0,051	0,619	0,132
	Time-varying normal copula	0,035	0,647	0,097
	Constant SJC copula	0,791	0,359	0,634
	Time-varying SJC copula	0,539	0,407	0,587
95%	Benchmark	<0,001	0,673	<0,001
	Constant normal copula	0,658	0,075	0,187
	Time-varying normal copula	0,779	0,204	0,429
	Constant SJC copula	0,899	0,123	0,302
	Time-varying SJC copula	0,971	0,037	0,114
90%	Benchmark	<0,001	0,021	<0,001
	Constant normal copula	0,587	0,326	0,533
	Time-varying normal copula	0,716	0,638	0,838
	Constant SJC copula	0,854	0,317	0,596
	Time-varying SJC copula	0,716	0,498	0,744

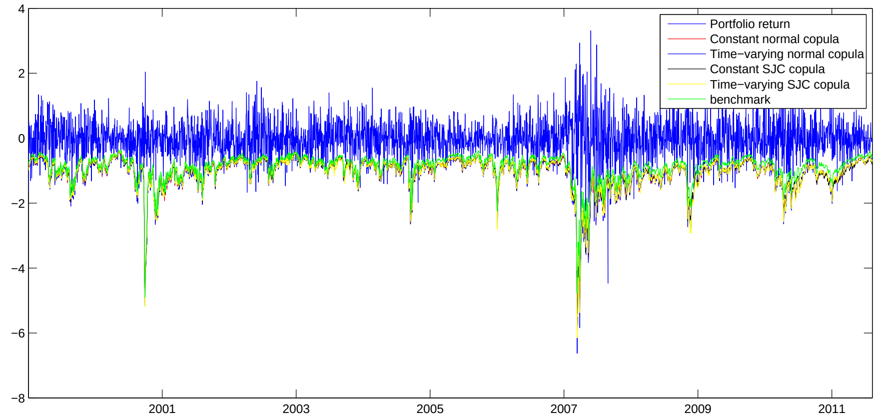
NOTE: We report the p-values from the Christoffersen tests for the estimated 99% Value-at-Risk. A p-value larger than 0.05 means that the Value-at-Risk is well specified. A (-) means that there were not enough hits to compute the test.

Table 12 and Figure 8 show the results for the Value-at-Risk estimates for the Real and Peso. We see that for the 99% Value-at-Risk the costant and time-varying normal copula does not generate two VaR-violations, and the Independency test does not return an value. We see that the copula models perform well for the 99% and 95% VaR, but not for the 90% VaR. This may be due to the relativly low correlation and high differences we saw in Section 4.2.

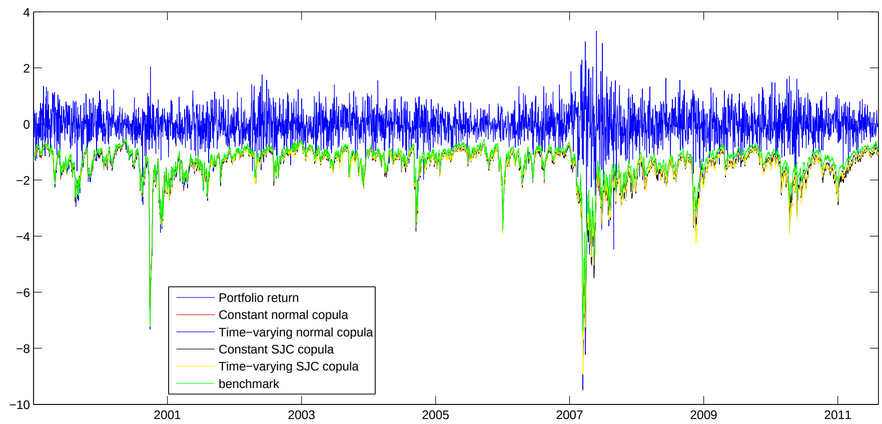
Figure 8: Value-at-Risk estimates for an equally weigthed portfolio with the Peso and Real for 2001-2012



(a) Forecasted VaR 90%



(b) Forecasted VaR 95%



(c) Forecasted VaR 99%

Table 12: Christoffersen test results for the Real and Peso estimated Value-at-Risk

q		Unconditional	Independent	Conditional Coverage
99%	Benchmark	0,001	0,065	0,001
	Constant normal copula	0,649	-	-
	Time-varying normal copula	0,785	-	-
	Constant SJC copula	0,649	0,271	0,492
	Time-varying SJC copula	0,927	0,313	0,599
95%	Benchmark	<0,001	0,426	<0,001
	Constant normal copula	0,048	0,341	0,091
	Time-varying normal copula	0,048	0,180	0,058
	Constant SJC copula	0,122	0,255	0,158
	Time-varying SJC copula	0,227	0,327	0,299
90%	Benchmark	<0,001	0,128	<0,001
	Constant normal copula	<0,001	0,748	<0,001
	Time-varying normal copula	<0,001	0,792	<0,001
	Constant SJC copula	0,007	0,315	0,017
	Time-varying SJC copula	0,004	0,677	0,015

NOTE: We report the p-values from the Christoffersen tests for the estimated 99% Value-at-Risk. A p-value larger than 0.05 means that the Value-at-Risk is well specified. A (-) means that there were not enough hits to compute the test.

4.5 Dependency differences

We test the copula parameters from Tables 25 and 26 with Wilcoxon Mann-Whitney U-test. Unfortunatly we find for the normal non-timevarying copula and the SJC non-timevarying copula no significant differences, with all p-values of 0.5. The problem may be the subsample, and that the correlation between the Euro and certain (more closely placed) markets is higher, disregarding whether the economey is an emerging or an advanced one.

5 Conclusion

The paper by Patton (2006) shows us that the exchange rates of the DM-USD en the Yen-USD exhibit asymmetric dependence, that is, dependence during joint positive events is smaller than during joint negative events. He does this by applying four copula models, namely the normal and SJC copula, both with and without time varying parameters. In our paper, we apply these copulas to two other exchange rate pairs, the Euro-Pound and the Mexican Peso-Real.

Our main hypothesis is that copulas forecast exchange rates and their dependencies better than solely the conventional AR(p)-GARCH(P,Q) models that are the input for the copulas. We estimate Value at Risk and Expected Shortfall for the exchange rate pairs Euro - Pound and Mexican Peso - Real to test this. We evaluate the 99%, 95% and 90% Value at Risk with the Christoffersen test and find that all copulas outperform the conventional AR(p)-GARCH(P,Q) models. We find that all copula models perform well according to the Christoffersen test, while the VaR estimated with solely the AR(p)-GARCH models do not pas the test. So the copula models outperform the more conventional models.

We find no significant difference between the parameters for emerging markets and advanced markets,

For further research, one can take a closer a look at the subsample for the different markets for the dependencys.

A Data description

A.1 List of countries

Table 13: Advanced and emerging markets

Advanced markets	Emerging markets
Australia[5]	**Brasil**[6]
Canada[7]	Chili [8]
Great Brittain[9]	Colombia [10]
Isreal[11]	Czech Republic[12]
Japan[13]	Hungary[14]
New Zealand[15]	**Mexico**[16]
Norway[17]	Peru[18]
South-Korea[19]	South Africa[20]
Sweden[21]	Poland[22]
Switserland[23]	Thailand[24]
European Union[25]	

NOTE: This table shows what countries are considered advanced markets and emerging markets by IMF (2013). The exchange rates of the bold stated countries agianst the U.S. Dollar are used for an indepth analysis, the complete sample is used to determine the dependency of advanced and emerging markets.

[25]Free floating since 1983, Stevens (2013)
[25]Free floating since 1970, of Canada (2010)
[25]Free floating since 1973, Gwartney et al. (2009)
[25]Not yet floating, Bufman and Leidermand (2001)
[25]Free floating since 1973, Ito and Krueger (1999)
[25]Free floating since 1985, of New Zealand (2013)
[25]Free floating since 1992, Kleivset (2012)
[25]Free floating since 1992, Larsson (2004)
[25]Free floating since 1997, Chung et al. (2000)
[25]Free floating since 1973, Peytrignet (2000)
[25]Free floating since 1979, Ickes (2002)
[25]Floating since 1999, Williamson (2010)
[25]Floating since 1999, Frenkel and Rapetti (2010)
[25]Free floating since 1999, Vargas (2005)
[25]Managed floating since 1997, Begg (1998)
[25]Floating since 2008, Zoican (2009)
[25]Free floating since 1994, Li et al. (2000)
[25]Free floating since -, van der Merwe (1996)

A.2 ADF-test results

Table 14: Results for the Augmented Dickey-Fuller test

	price	return
Australia	0,248	<0,001
Canada	0,141	<0,001
Israel	0,473	<0,001
Japan	0,364	<0,001
New Zealand	0,298	<0,001
Norway	0,338	<0,001
South Korea	0,476	<0,001
Sweden	0,419	<0,001
Switzerland	0,295	<0,001
United Kingdom	0,610	<0,001
Euro	0,485	<0,001
Argentina	0,996	<0,001
Brasil	0,661	<0,001
Chili	0,587	<0,001
Colombia	0,648	<0,001
Czech Republic	0,255	<0,001
Hungary	0,568	<0,001
Mexico	0,813	<0,001
Polad	0,477	<0,001
Slovakia	0,243	<0,001
South-Africa	0,689	<0,001

NOTE: This table shows the p-values for the Augmented Dickey-Fuller test for the prices and returns of the exchange rates. A p-value smaller than 0.05 means a rejection of the null that a unit root is present.

A.3 Descriptive statistics

Table 15: Descriptive statistics for the returns of the exchange rates

	Australia	Canada	Israel	Japan	New Zealand	Norway	South Korea	Sweden	Switzerland	United Kingdom	Euro
Mean	-0,014	-0,012	-0,003	-0,007	-0,012	-0,009	-0,003	-0,006	-0,011	0,001	-0,003
Maximum	8,828	4,338	4,575	3,077	6,646	4,795	10,351	3,541	8,475	3,919	3,844
Minimum	-6,701	-5,046	-2,873	-4,610	-5,878	-6,458	-13,265	-5,547	-5,451	-4,474	-4,617
Std. Dev.	0,852	0,594	0,487	0,653	0,873	0,764	0,692	0,773	0,695	0,587	0,641
Skewness	0,872	-0,063	0,337	-0,337	0,426	-0,076	-0,705	-0,204	0,320	0,038	-0,183
Kurtosis	15,453	8,660	8,417	6,431	8,156	7,493	61,644	6,400	11,758	7,316	5,429
Jarque-Bera	<0,001	<0,001	<0,001	<0,001	<0,001	<0,001	<0,001	<0,001	<0,001	<0,001	<0,001
Observations						3652					

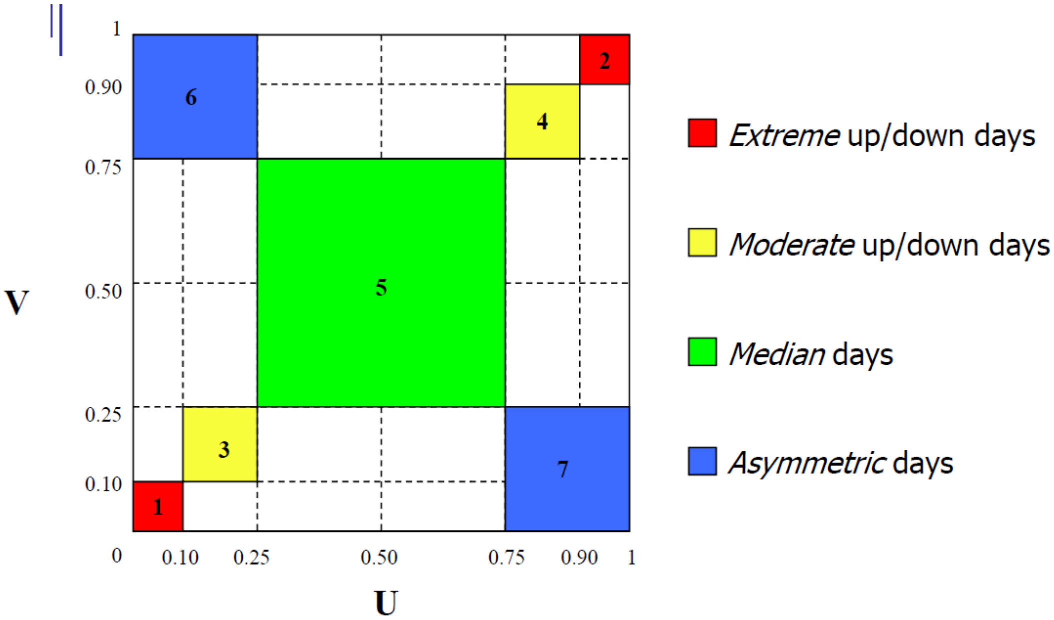
	Argentina	Brasil	Chili	Colombia	Czech Republic	Hungary	Mexico	Poland	Slovakia	South-Africa
Mean	0,044	0,014	0,000	0,004	-0,012	0,001	0,007	-0,013	0,010	-0,003
Maximum	33,647	10,801	5,462	4,871	4,029	6,305	7,552	3,789	9,808	5,696
Minimum	-13,353	-11,778	-3,799	-4,675	-5,219	-5,200	-4,766	-4,617	-8,523	-6,697
Std. Dev.	0,966	1,095	0,647	0,674	0,788	0,930	0,646	0,691	1,059	0,890
Skewness	15,341	0,499	0,634	0,370	-0,161	0,071	0,679	-0,120	0,326	0,158
Kurtosis	502,593	18,840	9,475	11,455	6,050	6,420	15,646	4,950	8,700	7,560
Jarque-Bera	<0,001	<0,001	<0,001	<0,001	<0,001	<0,001	<0,001	<0,001	<0,001	<0,001
Observations						3652				

NOTE: This table contains the descriptive statistics for the returns of the exchange rates against the U.S. Dollar, calculated as ret $= 100 \cdot (\log(x_t) - \log(x_{t-1}))$. The sample period is 1/1/1999 until 31/12/2012.

B Hit test

B.1 Visual representations of the regions

Figure 9: Regions for the hittest



B.2 Individual hit tests

Table 16: Results for the hit test in the individual regions

	Euro	Pound	Peso	Real
Regio 1	0,004	0,012	<0,000	<0,001
Regio 2	0,077	0,228	0,078	0,313
Regio 3	<0,001	<0,001	<0,001	<0,001
Regio 4	0,401	0,313	0,703	0,610
Regio 5	<0,001	0,141	<0,001	<0,001
Joint	<0,001	0,003	<0,001	<0,001

NOTE:

C Christoffersen test

The Christoffersen test is divided in two stages;

C.0.1 Unconditional coverage test

Observe a sample path $\{y_t\}_{(t=1)}^T$, of the time series y_t. Define an indicator function

$$I_t = \begin{cases} 1, & \text{if } y_t \in [L_{t|t-1}(p), U_{t|t-1}(p)] \\ 0, & \text{if } y_t \notin [L_{t|t-1}(p), U_{t|t-1}(p)] \end{cases} \tag{26}$$

The likelihood under the null hypothesis is:

$$L(p; I_1, I_2, ..., I_t) = (1-p)^{n_0} p^{n_1} \tag{27}$$

And under the alternative:

$$L(\pi; I_1, I_2, ..., I_t) = (1-\pi)^{n_0} \pi^{n_1} \tag{28}$$

In the likelihood the n_0 and n_1 are the total amount of zeros and ones in the indicator, respectively.

$$LR_{uc} = -2\log\left(\frac{L(p; I_1, I_2, ..., I_t)}{L(\hat{\pi}; I_1, I_2, ..., I_t)}\right) \sim \chi^2(s-1) = \chi^2(1), \tag{29}$$

C.0.2 Independency test

For the alternative hypothesis Christoffersen uses a binary first-order Markov chain, with transition probability matrix

$$\Pi_1 = \begin{bmatrix} 1-\pi_{01} & \pi_{01} \\ 1-\pi_{11} & \pi_{11} \end{bmatrix}, \text{where } \pi_{ij} = Pr(I_t = j \mid I_{t-1} = i). \tag{30}$$

The likelihood function for this process is:

$$L(\Pi_1; I_1, I_2, ..., I_T) = (1-\pi_{01})^{n_{00}} \pi_{01}^{n_{01}} (1-\pi_{11})^{n_{10}} \pi_{11}^{n_{11}}, \tag{31}$$

where n_{ij} is the total amount of observations with value i follwed by value j. The maximum likelihood estimation for Π_1 is:

$$\hat{\Pi}_1 = \begin{bmatrix} n_{00}/(n_{00}+n_{01}) & n_{01}/(n_{00}+n_{01}) \\ n_{10}/(n_{10}+n_{11}) & n_{11}/(n_{10}+n_{11}) \end{bmatrix} \tag{32}$$

For the null hypothesis of independence Christoffersen uses again a binary first order Markov chain,

$$\Pi_2 = \begin{bmatrix} 1-\pi_2 & \pi_2 \\ 1-\pi_2 & \pi_2 \end{bmatrix} \tag{33}$$

Here the different Markov states are independent of each other. The likelihood under the null becomes:

$$L(\Pi_2; I_1, I_2, ..., I_T) = (1-\pi_2)^{n_{00}+n_{10}} \pi_2^{n_{01}+n_{11}} \tag{34}$$

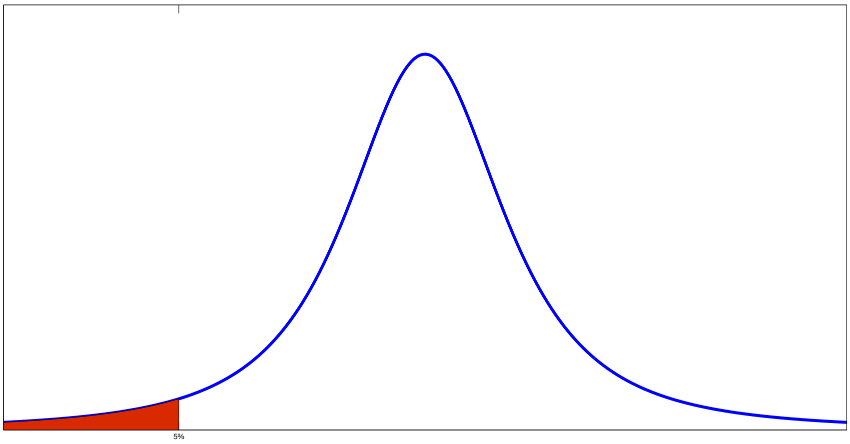
The maximum likelihood estimate is: $\hat{\pi}_2 = (n_{01}+n_{11})/(n_{00}+n_{10}+n_{01}+n_{11})$. The LR test becomes:

$$LR_{ind} = -2log\left(\frac{L(\hat{\Pi}_2; I_1, I_2, ..., I_T)}{L(\hat{\Pi}_1; I_1, I_2, ..., I_T)}\right) \sim \chi^2((s-1)^2) = \chi^2(1) \tag{35}$$

D Risk measures

D.1 Visual representation

Figure 10: Contour plots of copulas



NOTE: This figure shows the visual representation of the Value-at-Risk and Expected Shortfall for $q = 5\%$. The VaR is the value at the 5% mark, while the ES is the average of the red area.

E Results all currencies

E.1 Marginal models

Table 17: Selected models for all currencies

	GJR		EGARCH	
	AIC	BIC	AIC	BIC
Euro	6702	6757	6689*	6745*
Australia	7883*	7933*	7887	7937
Canada	5439*	5483*	5443	5486
Israel	3871	3914	3866*	3910*
Japan	6773	6816*	6769*	6825
New Zealand	8578*	8621*	8597	8641
Norway	7654*	7698*	7665	7708
South Korea	4553*	4602*	4563	4607
Sweden	7754*	7810*	7758	7814
Switserland	7234	7289	7214*	7270*
United Kingdom	5763*	5806*	5777	5820
Brasil	8669	8713	8659*	8702*
Chili	6073*	6122*	6080	6130
Colombia	5800	5862	5358*	5414*
Czech	7963*	8007*	7974	8017
Hungary	8851*	8895*	8859	8902
Mexico	5625*	5668*	5625	5668
Peru	2085	2135	2080*	2129*
Poland	8279*	8323*	8289	8332
Slovakia	7382*	7426*	7392	7436
South Africa	9727	9770	9719*	9762*

NOTE: This table shows the model selection for each of the currencies. The appropriate lags for the relevant regressors are chosen, to remove autocorrelation in both the residuals as the squared residuals.

Table 18: Selected models for all currencies

	AR	Variance	GARCH	ARCH
Euro	1	EGARCH	1	[1 , 2]
Australian Dollar	1	GJR	[1, 2]	1
Canadian Dollar	1	GJR	1	1
Israeli Shekel	1	EGARCH	1	1
Japanese Yen	1	GJR	1	1
New Zealand Dollar	1	GJR	1	1
Norwegian Krone	1	GJR	1	1
South Korean Won	[2, 4]	GJR	1	1
Swedish Krona	1	GJR	1	[1, 2]
Swiss Franc	1	EGARCH	1	2
United Kingdom Pound	1	EGARCH	1	1
Brasilian Real	1	EGARCH	1	1
Chilean Peso	[1, 4]	GJR	1	1
Colombian Peso	[1, 4, 9]	EGARCH	[1, 2]	1
Czech Koruna	1	GJR	1	1
Hungarian Forint	1	GJR	1	1
Mexican Peso	1	GJR	1	1
Peruvian Sol	1	EGARCH	1	1
Polish Zloty	1	GJR	1	1
Slovak Koruna	1	GJR	1	1
South Africa Rand	1	EGARCH	1	1

NOTE: This table shows the model selection for each of the currencies. The appropriate lags for the relevant regressors are chosen, to remove autocorrelation in both the residuals as the squared residuals.

Table 19: Results for the marginal models of all currencies

	Australia	Canada	Israel	Japan	New Zealand	Norway	South Korea	Sweden	Switzerland	UK	Euro
Constant	-0,029	-0,010	-0,007	0,003	-0,031	-0,012	-0,020	-0,008	-0,009	-0,007	-0,006
	(0,011)	(0,007)	(0,006)	(0,009)	(0,012)	(0,011)	(0,006)	(0,011)	(0,010)	(0,008)	(0,009)
AR(i)	0,015	-0,008	0,027	-0,038	0,019	0,024	0,015	0,008	-0,021	0,015	-0,002
	(0,017)	(0,017)	(0,016)	(0,016)	(0,017)	(0,017)	(0,016)	(0,017)	(0,016)	(0,017)	(0,014)
AR(j)							0,028				
							0,016				
Garch Constant	0,005	0,001	0,002	0,005	0,008	0,003	0,004	0,004	0,004	-0,009	-0,006
	(0,002)	(0,000)	(0,001)	(0,002)	(0,002)	(0,001)	(0,001)	(0,001)	(0,002)	(0,003)	(0,002)
GARCH(i)	0,922	0,953	0,913	0,962	0,959	0,961	0,866	0,958	0,964	0,993	0,994
	(0,385)	(0,006)	(0,010)	(0,007)	(0,007)	(0,006)	(0,012)	(0,007)	(0,007)	(0,002)	(0,002)
GARCH(j)	0,025										
	(0,367)										
ARCH(i)	0,063	0,051	0,088	0,014	0,044	0,043	0,154	0,034		0,088	-0,141
	(0,022)	(0,007)	(0,012)	(0,007)	(0,007)	(0,007)	(0,019)	(0,025)		(0,012)	(0,039)
ARCH(j)								0,013	0,018		0,215
								(0,026)	(0,022)		(0,039)
Leverage(i)	-0,040	-0,014	-0,007	0,023	-0,032	-0,020	-0,039	-0,034	0,012	0,016	0,034
	(0,017)	(0,010)	(0,015)	(0,009)	(0,009)	(0,009)	(0,021)	(0,031)	(0,026)	(0,007)	(0,026)
Leverage(j)								0,012	0,007		-0,028
								(0,031)	(0,026)		(0,025)
DoF	8,093	14,176	5,494	6,124	6,914	8,931	4,568	12,109	7,831	11,589	9,959
	(0,931)	(3,077)	(0,541)	(0,583)	(0,768)	(1,344)	(0,370)	(2,441)	(0,870)	(2,054)	(1,576)

NOTE:

Table 20: Results for the marginal models of all currencies

	Brasil	Chili	Colombia	Czech Republic	Hungary	Mexico	Peru	Poland	Slovakia	South-Africa
Constant	0,003	-0,016	-0,008	-0,017	-0,022	-0,005	-0,010	-0,031	-0,017	0,003
	(0,001)	(0,011)	(0,008)	(0,006)	(0,011)	(0,012)	(0,008)	(0,011)	(0,011)	(0,013)
AR(i)	-0,016	0,041	0,066	0,072	0,025	0,033	0,025	0,056	0,021	0,025
	(0,015)	(0,018)	(0,017)	(0,015)	(0,017)	(0,017)	(0,017)	(0,017)	(0,017)	(0,017)
AR(j)			0,050	0,022						
			(0,016)	(0,013)						
AR(k)				0,029						
				(0,012)						
Garch Constant	0,000	0,015	0,006	0,042	0,003	0,004	0,007	0,008	0,006	0,003
	(0,000)	(0,003)	(0,001)	(0,009)	(0,001)	(0,001)	(0,001)	(0,002)	(0,002)	(0,001)
GARCH(i)	0,847	0,840	0,893	0,284	0,959	0,954	0,889	0,915	0,965	0,931
	(0,008)	(0,012)	(0,011)	(0,080)	(0,006)	(0,006)	(0,012)	(0,010)	(0,007)	(0,007)
ARCH(i)	0,154	0,215	0,111	0,364	0,043	0,061	0,132	0,092	0,038	0,084
	(0,014)	(0,021)	(0,014)	(0,070)	(0,007)	(0,008)	(0,014)	(0,011)	(0,007)	(0,009)
ARCH(j)				0,436						
				(0,090)						
Leverage(i)	-0,003	-0,123	-0,029	-0,169	-0,015	-0,040	-0,091	-0,035	-0,030	-0,030
	(0,019)	(0,022)	(0,017)	(0,068)	(0,009)	(0,010)	(0,017)	(0,014)	(0,008)	(0,012)
DoF	4,045	6,954	6,571	2,795	9,744	8,847	8,101	8,253	8,954	7,696
	(0,161)	(0,650)	(0,579)	(0,193)	(1,593)	(1,253)	(0,951)	(1,066)	(1,241)	(1,035)

NOTE:

Table 21: Results the ARCH-LM test for all currencies

Currency	p-value
Euro	0,986
Australia	0,532
Canada	0,525
Israel	0,171
Japan	0,436
New Zealand	0,561
Norway	0,480
South Korea	0,708
Sweden	0,146
Switzerland	0,963
United Kingdom	0,699
Brasil	0,869
Chili	0,782
Colombia	0,227
Czech Republic	0,166
Hungary	0,556
Mexico	0,287
Peru	0,574
Poland	0,980
Slovakia	0,448
South-Africa	0,263

NOTE: We report the p-values of the ARCH-LM test for all currencies. A p-value larger than 0.05 means that we reject the null hypothesis that there are ARCH-effects left in the residuals.

Table 22: Results the KS test for all currencies

Currency	p-value
Euro	0,302
Australia	0,487
Canada	0,617
Israel	0,775
Japan	0,755
New Zealand	0,668
Norway	0,870
South Korea	0,045
Sweden	0,879
Switserland	0,199
United Kingdom	0,167
Brasil	0,580
Chili	0,686
Colombia	0,346
Czech Republic	0,471
Hungary	0,383
Mexico	0,221
Peru	0,002
Poland	0,460
Slovakia	0,823
South Africa	0,335

NOTE: We report the p-values of the Kolmogorov-Smirnov test for all currencies. A p-value larger than 0.05 means that we reject the null hypothesis that the residuals are normally distributed, a condition needed for the proper use of copula models. Almost none of the models satisfy this criteria, so we use a work-around. This involves taking the rank, as described in Section 3.3.

Table **??** shows the results for the Ljung-Box tests on the standardized residuals and the squared standardized residuals. We follow Tsay (2005) and choos $\log(T) = 9$ as the number of lags to test. A p-value less than 0.05 indicates autocorrelation in the standardized (squared) residuals.

Table 23: Results for the Ljung-Box tests

		Standardized residuals				Standardized squared residuals			
	Lag	AC	PAC	Q-stat	P-value	AC	PAC	Q-stat	P-value
	1	0,022	0,022	1,701	0,192	0,019	0,019	1,252	0,263

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Table 23 – continued from previous page

		Standardized residuals				Standardized squared residuals			
	Lag	AC	PAC	Q-stat	P-value	AC	PAC	Q-stat	P-value
	2	0,011	0,011	2,171	0,338	-0,007	-0,007	1,423	0,491
	3	-0,009	-0,009	2,450	0,484	-0,012	-0,012	1,950	0,583
	4	0,014	0,015	3,192	0,526	0,016	0,016	2,854	0,583
Euro	5	0,007	0,007	3,383	0,641	-0,005	-0,006	2,938	0,710
	6	-0,009	-0,010	3,707	0,716	0,008	0,008	3,152	0,789
	7	0,019	0,020	5,099	0,648	-0,023	-0,023	5,104	0,647
	8	0,016	0,015	5,989	0,648	-0,001	0,000	5,107	0,746
	1	0,012	0,012	0,493	0,483	-0,010	-0,010	0,385	0,535
	2	-0,010	-0,010	0,845	0,656	0,007	0,007	0,589	0,745
	3	-0,017	-0,017	1,952	0,582	-0,003	-0,002	0,612	0,894
Australia	4	0,024	0,025	4,137	0,388	0,014	0,014	1,325	0,857
	5	0,016	0,015	5,036	0,411	0,002	0,002	1,335	0,931
	6	-0,026	-0,026	7,458	0,281	0,003	0,002	1,360	0,968
	7	0,012	0,014	8,018	0,331	-0,003	-0,003	1,399	0,986
	8	0,024	0,023	10,100	0,258	0,012	0,011	1,905	0,984
	1	0,000	0,000	0,001	0,976	-0,010	-0,010	0,401	0,527
	2	-0,015	-0,015	0,788	0,674	-0,030	-0,030	3,588	0,166
	3	-0,006	-0,006	0,930	0,818	-0,009	-0,010	3,897	0,273
Canada	4	-0,001	-0,001	0,931	0,920	0,015	0,014	4,681	0,322
	5	-0,009	-0,009	1,212	0,944	-0,005	-0,005	4,759	0,446
	6	-0,020	-0,020	2,664	0,850	-0,025	-0,024	7,011	0,320
	7	0,007	0,006	2,828	0,900	-0,001	-0,001	7,014	0,427
	8	0,024	0,023	4,931	0,765	0,032	0,031	10,836	0,211
	1	0,009	0,009	0,284	0,594	0,037	0,037	4,875	0,027
	2	-0,001	-0,001	0,290	0,865	-0,004	-0,005	4,931	0,085
	3	-0,003	-0,003	0,325	0,955	-0,003	-0,003	4,970	0,174
Isreal	4	0,027	0,027	2,899	0,575	0,018	0,019	6,209	0,184
	5	0,021	0,020	4,484	0,482	-0,007	-0,008	6,367	0,272
	6	-0,016	-0,016	5,437	0,489	0,016	0,016	7,277	0,296
	7	0,011	0,011	5,843	0,558	0,020	0,019	8,727	0,273
	8	0,024	0,023	7,946	0,439	-0,005	-0,006	8,811	0,358
	1	0,011	0,011	0,449	0,503	0,013	0,013	0,612	0,434
	2	0,002	0,002	0,467	0,792	0,036	0,036	5,342	0,069
	3	-0,015	-0,015	1,272	0,736	-0,023	-0,024	7,261	0,064
Japan	4	-0,001	0,000	1,274	0,866	-0,004	-0,005	7,316	0,120
	5	0,010	0,010	1,654	0,895	-0,002	0,000	7,331	0,197
	6	-0,026	-0,026	4,088	0,665	0,005	0,005	7,415	0,284
	7	0,013	0,014	4,726	0,693	-0,016	-0,016	8,313	0,306
	8	0,012	0,012	5,247	0,731	-0,013	-0,013	8,888	0,352
	1	0,020	0,020	1,394	0,238	-0,010	-0,010	0,338	0,561
	2	-0,017	-0,017	2,445	0,294	0,022	0,022	2,183	0,336
	3	-0,018	-0,018	3,692	0,297	0,004	0,004	2,231	0,526
New Zealand	4	0,002	0,002	3,705	0,447	0,035	0,035	6,700	0,153
	5	0,010	0,010	4,095	0,536	-0,004	-0,004	6,770	0,238

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Table 23 – continued from previous page

		Standardized residuals				Standardized squared residuals			
	Lag	AC	PAC	Q-stat	P-value	AC	PAC	Q-stat	P-value
	6	-0,019	-0,020	5,486	0,483	-0,009	-0,010	7,043	0,317
	7	0,011	0,012	5,896	0,552	-0,012	-0,013	7,614	0,368
	8	0,019	0,018	7,184	0,517	-0,011	-0,012	8,096	0,424
Norway	1	0,003	0,003	0,039	0,844	-0,012	-0,012	0,508	0,476
	2	-0,004	-0,004	0,102	0,950	-0,019	-0,019	1,832	0,400
	3	-0,027	-0,027	2,678	0,444	0,010	0,009	2,175	0,537
	4	0,006	0,007	2,827	0,587	0,021	0,021	3,770	0,438
	5	-0,001	-0,001	2,833	0,726	0,011	0,012	4,187	0,523
	6	-0,009	-0,010	3,142	0,791	0,013	0,014	4,775	0,573
	7	0,015	0,016	3,986	0,781	-0,006	-0,006	4,926	0,669
	8	0,009	0,009	4,297	0,829	0,026	0,026	7,473	0,487
Korea	1	0,029	0,029	2,976	0,085	0,006	0,006	0,143	0,706
	2	0,020	0,019	4,380	0,112	-0,017	-0,017	1,180	0,554
	3	0,023	0,022	6,291	0,098	-0,012	-0,012	1,704	0,636
	4	0,010	0,008	6,630	0,157	-0,018	-0,018	2,908	0,573
	5	0,025	0,024	8,930	0,112	-0,023	-0,024	4,905	0,428
	6	0,017	0,014	9,935	0,127	-0,005	-0,005	4,982	0,546
	7	0,019	0,017	11,321	0,125	-0,021	-0,022	6,618	0,470
	8	-0,005	-0,008	11,418	0,179	-0,016	-0,017	7,511	0,483
Sweden	1	0,010	0,010	0,394	0,530	-0,024	-0,024	2,109	0,146
	2	-0,001	-0,001	0,397	0,820	-0,013	-0,014	2,729	0,256
	3	-0,023	-0,023	2,262	0,520	0,015	0,015	3,578	0,311
	4	0,004	0,004	2,319	0,677	0,017	0,018	4,666	0,323
	5	-0,011	-0,011	2,755	0,738	-0,009	-0,007	4,942	0,423
	6	0,008	0,008	3,016	0,807	0,008	0,008	5,161	0,523
	7	0,031	0,031	6,445	0,489	0,000	0,000	5,161	0,640
	8	0,020	0,019	7,896	0,444	0,039	0,039	10,635	0,223
Switzerland	1	0,015	0,015	0,856	0,355	0,024	0,024	2,050	0,152
	2	0,008	0,007	1,069	0,586	0,045	0,044	9,432	0,009
	3	-0,013	-0,014	1,732	0,630	-0,003	-0,005	9,457	0,024
	4	-0,002	-0,002	1,754	0,781	-0,003	-0,005	9,490	0,050
	5	0,011	0,011	2,173	0,825	-0,002	-0,001	9,502	0,091
	6	0,005	0,005	2,267	0,894	-0,009	-0,009	9,819	0,132
	7	0,034	0,034	6,472	0,486	0,003	0,003	9,851	0,197
	8	0,013	0,012	7,080	0,528	0,005	0,005	9,933	0,270
United Kingdom	1	0,005	0,005	0,096	0,757	-0,014	-0,014	0,718	0,397
	2	-0,010	-0,010	0,462	0,794	-0,002	-0,002	0,731	0,694
	3	-0,009	-0,009	0,740	0,864	0,001	0,001	0,733	0,865
	4	0,023	0,023	2,694	0,610	-0,011	-0,011	1,198	0,878
	5	-0,006	-0,006	2,815	0,728	0,015	0,014	1,996	0,850
	6	0,010	0,010	3,147	0,790	-0,005	-0,005	2,083	0,912
	7	0,017	0,017	4,158	0,761	0,007	0,007	2,275	0,943
	8	0,014	0,013	4,850	0,773	0,019	0,019	3,583	0,893

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Table 23 – continued from previous page

		Standardized residuals				Standardized squared residuals			
	Lag	AC	PAC	Q-stat	P-value	AC	PAC	Q-stat	P-value
Brasil	1	0,022	0,022	1,703	0,192	-0,005	-0,005	0,102	0,749
	2	-0,008	-0,008	1,930	0,381	0,007	0,007	0,261	0,878
	3	0,011	0,011	2,348	0,503	-0,003	-0,002	0,284	0,963
	4	0,028	0,027	5,185	0,269	-0,010	-0,010	0,645	0,958
	5	0,013	0,012	5,795	0,327	-0,007	-0,007	0,801	0,977
	6	0,007	0,007	5,971	0,426	0,002	0,002	0,810	0,992
	7	0,021	0,021	7,640	0,365	-0,003	-0,003	0,851	0,997
	8	-0,001	-0,003	7,643	0,469	0,000	0,000	0,851	0,999
Chili	1	0,027	0,027	2,601	0,107	0,003	0,003	0,028	0,868
	2	0,001	0,001	2,608	0,271	-0,009	-0,009	0,300	0,861
	3	0,012	0,012	3,160	0,368	-0,006	-0,006	0,437	0,932
	4	0,011	0,011	3,625	0,459	-0,010	-0,010	0,831	0,934
	5	0,004	0,003	3,674	0,597	0,001	0,001	0,832	0,975
	6	-0,009	-0,009	3,959	0,682	0,000	0,000	0,832	0,991
	7	-0,026	-0,026	6,378	0,496	-0,008	-0,008	1,057	0,994
	8	0,046	0,048	14,260	0,075	-0,004	-0,004	1,124	0,997
Colombia	1	0,030	0,030	3,248	0,072	0,021	0,021	1,647	0,199
	2	-0,001	-0,002	3,251	0,197	0,008	0,007	1,866	0,393
	3	0,006	0,006	3,378	0,337	-0,018	-0,019	3,087	0,378
	4	0,015	0,015	4,219	0,377	0,006	0,007	3,221	0,521
	5	-0,008	-0,009	4,481	0,482	-0,008	-0,008	3,446	0,632
	6	0,018	0,018	5,616	0,468	-0,004	-0,004	3,504	0,743
	7	0,010	0,009	5,969	0,543	-0,014	-0,014	4,248	0,751
	8	-0,002	-0,002	5,978	0,650	-0,005	-0,005	4,349	0,824
Czech	1	0,016	0,016	0,922	0,337	-0,020	-0,020	1,470	0,225
	2	0,030	0,030	4,180	0,124	-0,016	-0,017	2,439	0,295
	3	-0,017	-0,018	5,198	0,158	0,018	0,017	3,569	0,312
	4	0,010	0,010	5,598	0,231	0,003	0,004	3,605	0,462
	5	0,013	0,014	6,259	0,282	-0,002	-0,002	3,625	0,605
	6	0,016	0,015	7,192	0,303	0,024	0,023	5,674	0,461
	7	0,019	0,018	8,490	0,291	0,007	0,008	5,867	0,555
	8	0,017	0,016	9,610	0,294	-0,026	-0,025	8,338	0,401
Hungary	1	0,011	0,011	0,405	0,525	-0,023	-0,023	1,901	0,168
	2	0,005	0,005	0,510	0,775	0,012	0,011	2,385	0,303
	3	-0,013	-0,013	1,162	0,762	0,000	0,001	2,385	0,496
	4	0,003	0,003	1,194	0,879	0,012	0,012	2,880	0,578
	5	0,007	0,007	1,381	0,926	0,032	0,032	6,515	0,259
	6	-0,003	-0,003	1,414	0,965	-0,008	-0,007	6,752	0,344
	7	0,013	0,013	2,072	0,956	-0,003	-0,005	6,796	0,450
	8	0,001	0,000	2,073	0,979	0,021	0,021	8,474	0,389
	1	0,022	0,022	1,723	0,189	0,026	0,026	2,501	0,114
	2	-0,007	-0,007	1,900	0,387	0,053	0,052	12,609	0,002
	3	-0,013	-0,013	2,515	0,473	0,022	0,019	14,343	0,002

Continued on next page

Table 23 – continued from previous page

		Standardized residuals				Standardized squared residuals			
	Lag	AC	PAC	Q-stat	P-value	AC	PAC	Q-stat	P-value
Mexico	4	-0,015	-0,014	3,327	0,505	0,004	0,001	14,410	0,006
	5	-0,022	-0,022	5,096	0,404	0,021	0,018	15,955	0,007
	6	-0,014	-0,014	5,848	0,440	-0,006	-0,007	16,067	0,013
	7	0,006	0,006	5,974	0,543	0,011	0,009	16,491	0,021
	8	0,014	0,013	6,720	0,567	-0,004	-0,004	16,540	0,035
Peru	1	0,016	0,016	0,973	0,324	-0,001	-0,001	0,004	0,948
	2	0,019	0,019	2,276	0,320	0,019	0,019	1,364	0,506
	3	0,026	0,026	4,783	0,188	0,007	0,007	1,554	0,670
	4	0,003	0,002	4,819	0,306	0,028	0,028	4,507	0,342
	5	0,024	0,023	6,975	0,223	0,009	0,009	4,794	0,442
	6	-0,005	-0,007	7,082	0,313	-0,010	-0,011	5,165	0,523
	7	0,020	0,020	8,606	0,282	-0,002	-0,003	5,182	0,638
	8	0,006	0,004	8,731	0,365	-0,004	-0,004	5,234	0,732
Poland	1	0,023	0,023	1,867	0,172	0,018	0,018	1,129	0,288
	2	0,023	0,023	3,843	0,146	-0,018	-0,018	2,279	0,320
	3	-0,016	-0,017	4,722	0,193	0,008	0,009	2,542	0,468
	4	0,001	0,002	4,729	0,316	-0,008	-0,009	2,804	0,591
	5	0,002	0,003	4,750	0,447	0,023	0,023	4,675	0,457
	6	-0,024	-0,025	6,940	0,326	0,002	0,001	4,696	0,583
	7	0,022	0,023	8,723	0,273	-0,005	-0,004	4,796	0,685
	8	0,006	0,006	8,866	0,354	-0,009	-0,010	5,115	0,745
Slovakia	1	0,008	0,008	0,216	0,642	0,000	0,000	0,001	0,982
	2	0,023	0,022	2,072	0,355	0,011	0,011	0,441	0,802
	3	-0,028	-0,028	4,929	0,177	-0,015	-0,015	1,265	0,738
	4	0,009	0,009	5,242	0,263	0,009	0,008	1,533	0,821
	5	0,016	0,017	6,149	0,292	-0,001	0,000	1,535	0,909
	6	-0,003	-0,005	6,183	0,403	0,012	0,011	2,047	0,915
	7	0,012	0,012	6,693	0,462	-0,015	-0,015	2,908	0,893
	8	0,017	0,018	7,715	0,462	-0,014	-0,014	3,587	0,892
South Africa	1	0,020	0,020	1,456	0,228	0,020	0,020	1,476	0,224
	2	-0,015	-0,015	2,226	0,329	0,020	0,019	2,869	0,238
	3	-0,016	-0,016	3,201	0,362	-0,015	-0,015	3,647	0,302
	4	0,006	0,007	3,353	0,501	0,033	0,033	7,679	0,104
	5	0,010	0,009	3,729	0,589	0,010	0,009	8,014	0,155
	6	-0,023	-0,024	5,692	0,459	-0,006	-0,008	8,154	0,227
	7	0,020	0,021	7,126	0,416	0,017	0,018	9,184	0,240
	8	0,025	0,024	9,466	0,304	0,004	0,003	9,241	0,322

E.2 Copula models

Table 24: Testing the influence of the " other" variable in the mean and variance model for the Euro and Pound

	Mean model		Variance model	
	X_{t-1} for Y_t	Y_{t-1} for X_t	ε^2_{t-1} for Y_t	η^2_{t-1} for X_t
Australia	0,955	0,202	0,454	0,082
Canada	0,434	0,966	0,080	0,077
Israel	0,068	0,590	0,539	0,330
Japan	0,741	0,576	0,502	0,051
New Zealand	0,052	0,096	0,582	0,189
Norway	0,193	0,961	0,152	0,264
South Korea	0,177	0,738	0,209	0,272
Sweden	0,599	0,747	0,068	0,612
Switzerland	0,439	0,514	0,028	0,485
United Kingdom	0,084	0,669	0,200	0,711
Argentina	0,098	0,774	0,991	0,575
Brasil	0,065	0,543	0,824	0,872
Chili	0,685	0,416	0,713	0,300
Colombia	0,853	0,885	0,239	0,699
Czech Republic	0,338	0,063	0,134	0,797
Hungary	0,097	0,097	0,088	0,109
Mexico	0,835	0,376	0,114	0,407
Polad	0,967	0,233	0,079	0,436
Slovakia	0,712	0,365	0,203	0,442
South-Africa	0,976	0,628	0,296	0,831

NOTE: Variable X with corresponding variance ε^2 is the Euro, and variable Y with corresponding variance η^2 is the Pound. We report p-values that the variables have coefficients equal to 0; a p-value greater than 0.05 means we cannot reject the null at the 0.05 level.

Table 25: Results for the copula models of the Euro vs ...

	Australia	Canada	Israel	Japan	New Zealand	Norway	South Korea	Sweden	Switzerland	UK
					Constant normal copula					
$\bar{\rho}$	0,560	0,395	0,308	0,286	0,523	0,832	0,181	0,850	0,874	0,688
Copula likelihood	687	310	182	156	583	2148	61	2338	2632	1169
					Constant SJC copula					
$\bar{\tau}^U$	0,324	0,195	0,144	0,103	0,294	0,635	0,026	0,644	0,722	0,408
$\bar{\tau}^L$	0,404	0,245	0,145	0,165	0,350	0,680	0,056	0,709	0,770	0,544
Copula likelihood	742	375	293	239	620	2151	90	2349	2827	1182
					Time-varying normal copula					
Constant	-0,018	0,015	0,004	0,030	-0,004	7,050	0,001	-2,672	-0,105	-0,581
α	0,156	0,148	0,127	0,158	0,156	-0,215	0,031	0,051	0,612	0,064
β	2,175	1,970	1,988	1,868	2,100	-5,377	2,005	6,061	2,701	3,246
Copula likelihood	704	342	202	187	583	2142	66	2314	2836	1153
					Time-varying SJC copula					
Constant	1,067	2,308	-0,226	1,641	-1,982	0,455	0,148	-0,822	0,562	-1,926
α^U	-6,607	-11,992	-7,633	-15,380	-0,526	-4,606	-11,550	-3,089	0,631	-0,506
βU	-1,663	-4,824	2,218	0,637	4,141	1,095	-14,784	2,781	0,570	4,003
Constant	-1,896	-1,795	1,340	1,970	0,536	-0,252	0,647	0,065	0,325	-1,977
α^L	-0,422	-1,378	-13,582	-12,956	-6,174	-1,776	-10,142	-3,408	-3,137	-0,174
β^L	3,937	4,019	0,627	-1,896	0,175	1,818	-9,040	1,764	1,910	4,017
Copula likelihood	738	373	282	269	612	2171	71	2369	3231	1205

NOTE: Variable X with corresponding variance ε^2 is the Euro, and variable Y with corresponding variance η^2 is the Pound. We report p-values that the variables have coefficients equal to 0; a p-value greater than 0.05 means we cannot reject the null at the 0.05 level.

Table 26: Results for the copula models of the Euro vs ...

	Argentina	Brasil	Chili	Colombia	Czech Republic	Hungary	Mexico	Polad	Slovakia	South-Africa
					Constant normal copula					
$\bar{\rho}$	0,046	0,215	0,209	0,181	0,864	0,829	0,151	0,694	0,920	0,425
Copula likelihood	4	86	82	61	2500	2122	42	1200	3429	364
					Constant SJC copula					
$\bar{\tau}^U$	0,000	0,106	0,060	0,030	0,695	0,639	0,051	0,513	0,815	0,234
$\bar{\tau}^L$	0,000	0,046	0,072	0,064	0,715	0,689	0,030	0,510	0,816	0,244
Copula likelihood	4	184	177	93	2521	2144	191	1316	3491	434
					Time-varying normal copula					
Constant	0,159	0,011	0,008	0,001	-2,914	-1,143	0,002	0,043	-4,742	0,004
α	0,152	0,136	0,123	0,028	0,086	0,175	0,093	0,402	0,250	0,178
β	-1,552	1,911	1,921	2,009	6,336	4,078	1,982	2,087	8,410	2,002
Copula likelihood	3	108	92	72	2508	2165	62	1242	3625	380
					Time-varying SJC copula					
Constant^U	-45,378	2,303	1,737	0,188	-0,586	2,037	1,903	-0,188	-0,279	3,733
α^U	-0,010	-16,802	-16,117	-10,740	-5,841	3,234	-17,295	-5,423	1,138	-17,804
β^U	-0,014	-2,329	-2,063	-18,082	2,960	-3,482	-1,544	2,106	1,992	-3,550
Constant^L	-59,653	3,512	1,903	-0,202	-2,082	-0,532	1,753	-1,963	1,437	-1,597
α^L	-0,007	-22,912	-15,925	-9,440	-0,393	-5,015	-17,559	-0,383	-4,593	-2,362
β^L	0,000	-2,940	-1,903	1,572	4,265	2,876	-0,312	4,054	0,388	3,886
Copula likelihood	0	160	129	80	2577	2283	121	1411	3781	461

NOTE: Variable X with corresponding variance ε^2 is the Euro, and variable Y with corresponding variance η^2 is the Pound. We report p-values that the variables have coefficients equal to 0; a p-value greater than 0.05 means we cannot reject the null at the 0.05 level.

F Mann-Whitney U test

Figure 11: Contour plots of copulas

Critical values of the Mann-Whitney U Test

n_1 \ n_2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	—	—	—		—	—	—	—	—	—	—	—	—		—	—	—	—	0	0
																			—	**—**
2	—	—	—	—	0	0	0	1	1	1	1	2	2	2	3	3	3	4	4	4
					—	**—**	**—**	**0**	**0**	**0**	**0**	**1**	**1**	**1**	**1**	**1**	**2**	**2**	**2**	**2**
3	—	—	0	0	1	2	2	3	3	4	5	5	6	7	7	8	9	9	10	11
	—	**—**	**—**	**—**	**0**	**1**	**1**	**2**	**2**	**3**	**3**	**4**	**4**	**5**	**5**	**6**	**6**	**7**	**7**	**8**
4	—	—	0	1	2	3	4	5	6	7	8	9	10	11	12	14	15	16	17	18
	—	**—**	**—**	**0**	**1**	**2**	**3**	**4**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**11**	**12**	**13**	**13**
5	—	0	1	2	4	5	6	8	9	11	12	13	15	16	18	19	20	22	23	25
	—	**—**	**0**	**1**	**2**	**3**	**5**	**6**	**7**	**8**	**9**	**11**	**12**	**13**	**14**	**15**	**17**	**18**	**19**	**20**
6	—	0	2	3	5	7	8	10	12	14	16	17	19	21	23	25	26	28	30	32
	—	**—**	**1**	**2**	**3**	**5**	**6**	**8**	**10**	**11**	**13**	**14**	**16**	**17**	**19**	**21**	**22**	**24**	**25**	**27**
7	—	0	2	4	6	8	11	13	15	17	19	21	24	26	28	30	33	35	37	39
	—	**—**	**1**	**3**	**5**	**6**	**8**	**10**	**12**	**14**	**16**	**18**	**20**	**22**	**24**	**26**	**28**	**30**	**32**	**34**
8	—	1	3	5	8	10	13	15	18	20	23	26	28	31	33	36	39	41	44	47
	—	**0**	**2**	**4**	**6**	**8**	**10**	**13**	**15**	**17**	**19**	**22**	**24**	**26**	**29**	**31**	**34**	**36**	**38**	**41**
9	—	1	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54
	—	**0**	**2**	**4**	**7**	**10**	**12**	**15**	**17**	**20**	**23**	**26**	**28**	**31**	**34**	**37**	**39**	**42**	**45**	**48**
10	—	1	4	7	11	14	17	20	24	27	31	34	37	41	44	48	51	55	58	62
	—	**0**	**3**	**5**	**8**	**11**	**14**	**17**	**20**	**23**	**26**	**29**	**33**	**36**	**39**	**42**	**45**	**48**	**52**	**55**
11	—	1	5	8	12	16	19	23	27	31	34	38	42	46	50	54	57	61	65	69
	—	**0**	**3**	**6**	**9**	**13**	**16**	**19**	**23**	**26**	**30**	**33**	**37**	**40**	**44**	**47**	**51**	**55**	**58**	**62**
12	—	2	5	9	13	17	21	26	30	34	38	42	47	51	55	60	64	68	72	77
	—	**1**	**4**	**7**	**11**	**14**	**18**	**22**	**26**	**29**	**33**	**37**	**41**	**45**	**49**	**53**	**57**	**61**	**65**	**69**
13	—	2	6	10	15	19	24	28	33	37	42	47	51	56	61	65	70	75	80	84
	—	**1**	**4**	**8**	**12**	**16**	**20**	**24**	**28**	**33**	**37**	**41**	**45**	**50**	**54**	**59**	**63**	**67**	**72**	**76**
14	—	2	7	11	16	21	26	31	36	41	46	51	56	61	66	71	77	82	87	92
	—	**1**	**5**	**9**	**13**	**17**	**22**	**26**	**31**	**36**	**40**	**45**	**50**	**55**	**59**	**64**	**67**	**74**	**78**	**83**
15	—	3	7	12	18	23	28	33	39	44	50	55	61	66	72	77	83	88	94	100

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