Downside-risk hedging by Quantile-specification

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Abstract

This paper addresses the hedging problem of an excess exposure to domestic markets, and uses a foreign market as a hedging instrument. Conventionally hedging concerns minimum variance portfolios, though this paper mainly focuses on hedging downside –risk. Linear Quantile Regressions and Normal- and Student T-copula quantiles are specified to find quantile specific (downside) optimal hedge ratios. By simulation these methods are used to estimate optimal hedge-ratios during several years and performance is measured. This paper concludes OLS-based minimum variance hedges outperform downside-risk hedges on both variance reduction and portfolio's value lost at extreme downside deviations.

Contents

1. Introduction

An investor is exposed to the risk he has taken on in earlier investment. This exposure can be reduced by taking, usually opposing, positions in alternative investments. These alternative investments then serve as hedging instruments. Initial exposure can be a result of particular business conducted by the individual, but it can also be the result of holding certain assets. One could think of a classic investing mistake like investing only in a domestic market, this leaves the investor fully exposed to this market. Investing in additional markets might be a good way to diversify, and thereby hedge away some of the domestic risk one was earlier exposed to. This paper considers an initial exposure to the AEX index and the S&P500 index will serve as a hedging instrument, so that we hedge excessive exposure to a certain market by taking a position in another large market. Data on indexes mentioned above is available over the interval of 12/10/1992 until 30/4/2014, this are 1125 weekly observations. The use of weekly observations will reduce the influence of extreme observations. Also, due to the time difference between Europe and the U.S. it is unclear how trading days relate, for weekly observations this is less of a problem.

Classically optimal hedging positions are determined by means of an ordinary least squares regression, providing the in-sample minimum variance position. This classical approach will serve as a benchmark. Lien has written many articles that describe methodology of determining hedge ratios. Lien (2005) concludes, in one of his later papers, OLS methodology tends to always outperform methods that make use of error-correction and cointegration relationships, as well as bivariate GARCH methods, whenever the goal is variance reduction.

Clearly minimum variance is desirable whenever negative deviations occur, while on the other hand positive deviations bring along an increase in portfolio-value. Hence, selectively treating positive and negative deviations could be of great use for taking financial positions. A method that provides the possibility of incorporating aversion to losses rather than gains is called the Linear Quantile Regression (LQR). Similarly copula functions can be used to specify quantiles of the joint process of the hedging instrument and exposure, from which the quantile specific optimal hedge ratio can be determined. This hedge ratio represents the optimal position at the occurrence of a certain quantile, so the optimal hedge ratio found in the lower quantile will be optimal whenever large losses are incurred. Both quantile specifying methods, linear- and copula-quantile regressions, are used and described in Alexander (2008). Sadly no clear conclusions are drawn concerning performance of these methods, and only one year's data is used to estimate hedge ratios. This paper aims to use a more realistically sized dataset and provide an insight into the applicability of quantile specifying methods to hedge downside risk. Bouyé and Salmon (2002) introduced a more general and complete approach to copula-quantile regression modeling, but it focuses mainly on (tail-) dependence structure determination. This paper focuses on hedging purposes and aims to provide a more practical view on these models' applicability for these purposes.

The hedging techniques and different approaches mentioned above will be tested in a simulation-like manner. Initial model specification will be done on a sample up until 2006, while the part of data that remains will serve as an out-of-sample period, and therefore unknown. Positions taken in accordance to one of the methods mentioned above will be judged on whether it serves its goal, but will also be compared to other methods with identical goals. By doing this it will hopefully become clear what methods to address for given goals. For clarity, the goals considered in this paper are variance reduction and (mainly) downside risk reduction.

Although the methodology of quantile-regressions and copula-quantiles seems very useful at first sight, results are disappointing. No method has shown the ability to really hedge downside-risk. Also the choice of quantile seems to have barely any effect on the positions taken. Conventional OLS hedges provide relatively good downside protection, even though these hedges are actually minimum variance hedges. Techniques that specify quantiles in order to take downside-risk hedging positions have shown no effectiveness. Also minimum variance hedges found by means of copulas are outperformed by OLS.

The section expands on the dataset and its most obvious statistical properties will be discussed, followed by a detailed description of the methodology. Sections 5 and 6 will provide results and conclusions respectively.

2. Data

Weekly observation of both the AEX- and the S&P500-index are considered respectively as exposure and hedging instrument. Data is collected over the period between 10/12/1992 and 5/5/2014, resulting in 1125 weekly observations. The use of weekly observations reduces influence of the different trading hours, also the less outliers are to be expected. Data has been adjusted for stock splits and dividends. The dataset will typically be divided into a 'sample' that will serve as estimation material for the initial hedging position, and an 'out-of-sample' period which will be used to measure effectiveness of the hedging positions considered. Financial conditions observed in the last few years have shown plenty down-side deviations, and will therefore be a good period to test hedging methodology that claims to account for this. Therefore the 'out of sample' period will start as off 12/25/2006.

When one looks at the returns of the AEX and the S&P500 typically negative skewness and excess kurtosis are found. Heteroskedasticity and serial correlation are shown present in these series by respectively an ARCH-test and a Breusch-Godfrey-test. The table (1) below shows p-values for the tests mentioned above performed within the sample.

Table 1

	AEX	S&P500
ARCH *(no heterosked.)*	0,62	0,05
BREUSCH-G. *(no serial corr.)*	0	0

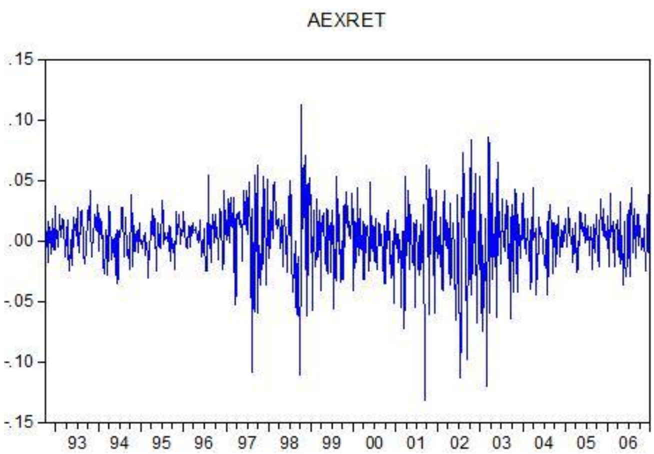

Figure 1

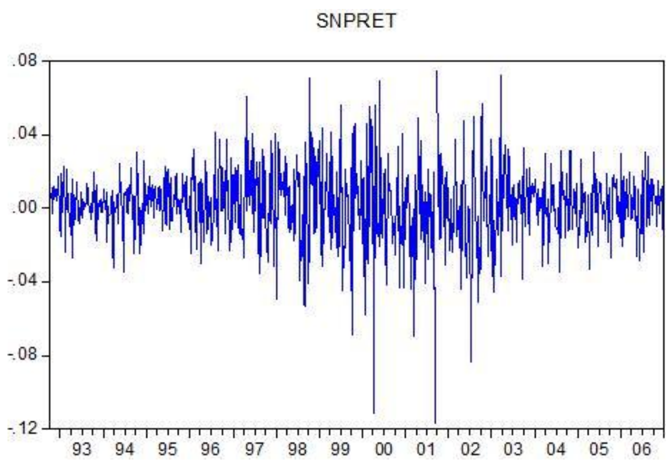

Figure 2

Although homoskedasticity is not deliberately rejected by the ARCH-test, figures (1 and 2) above do seem to be somewhat more volatile around '98. This is right around the time of the 'internet bubble' clearly US markets were effected more severely. Unrest as of 9/11/2001 seems to have a similar impact. Although testing failed to point out heteroskedasticity, we can see that between '96 and '04 increased volatility and unrest are present.

Scatter plotting all observations could provide insights into the dependence structure present in the data if dependence structures are very strong. For the dataset considered in this paper no clear relation appears, the graph is to be found in the appendix as graph 1. To determine the dependence structure for our dataset more extensive testing and analysis are needed, which is done more extensively in the results section.

3. Methods

Risk that comes along with a certain investment can be reduced in some cases by applying hedging techniques to find a less risky position. For finding such positions it is of importance to understand the dependence structure among financial assets. To do so, one can use copula functions. Though, before copulas are to be used, marginal distributions must be caught in a model such that standardization provides homoskedastic series. There are methods to test whether such models properly explain, or significantly fit, the data. After specification of the underlying process different paths can be chosen, one can use copula functions and their quantile specifications, to focus on the hedge on downside-risk. One can also use simpler methods like ordinary least squares, which does not require standardization. The sections below expand on this short summary and describe the extraction of hedge ratios as well as the measurements of performance.

3.1 Standardization

Financial data typically shows heteroskedastic characteristics and serial correlation. Also distributions of financial returns are often skewed (negatively) and show excess kurtosis. Hence, it is important to use models that catch these characteristics and allow for error distributions that are not normal. Autoregressive models are commonly used to model serial correlation and heteroskedasticity, this paper uses so called GJR-GARCH models. These models allow for asymmetric relations, and describe a time-variant error distribution variance. But maybe even more important is the distribution that is assigned to the errors. This paper considers the skew-T (ST) distribution as given by Hansen (1994), to catch the remaining skewness and kurtosis in the error distribution. The model is given by the formulas (1, 2, and 3) below.

Equations 1, 2, 3

$$Y_t = \delta + \sigma_t * Z_t$$

$$\sigma_t^2 = \mu + \alpha * \varepsilon_{t-1}^2 + \beta * \sigma_{t-1}^2 + \gamma * \mathrm{I}_{(\varepsilon_{t-1}<0)} * \varepsilon_{t-1}^2$$

$$Z \sim ST(\tau, \lambda)$$

In the equations above Y_t are the financial returns that are being modeled, described by a constant and random Z_t scaled by time-variant variance parameter. This variance parameter is defined as given in equation 2, respectively by a constant, a delayed error, a delayed variance estimate and finally another delayed error that only takes value if this delayed error is negative. More delayed variables can be added as long as additional parameter estimates are significant. Additional autoregressive variables could also be considered, this would mean adding a Y_{t-i} to equation 1. For these autoregressive variables the optimal model can be found by looking for a minimum Schwarz Info Criterium (SIC) value. This criterion indicates an optimal equilibrium between explanatory power and amount of variables. The variance estimates of the formula (2) above can be used to standardize $Y_t - \mu$, resulting in a series that is distributed ST(τ, λ). Since this specification generally concerns a time-constant distribution, heteroskedasticity should not be present in the standardized residuals. Also the fit of the distribution that has been assigned to the standardized residuals must be tested. If one of these two most obvious demands is not pleased by the model, it has not been specified incorrectly. To test the fit of the distribution there are so called Goodness of Fit measures, these measures are based on the difference between quantiles of the distribution assigned and empirical quantiles. This difference must off course not be too large. In the section goodness of fit (1) these measures will be described for the case of marginal distributions.

3.2 Copula functions

This paper focuses on the application of copula-based techniques in the practice of hedging. When an individual is confronted with a certain exposure and is keen to reduce the risk that comes along, a hedging decision is to be made. Besides finding a nicely fitted model for the exposure as well as the hedging instrument, it is crucial to find a good model for the dependence among these two variables. Such relationships can be described by copula functions, the equations (4, 5, and 6) below describe the general case.

Equations 4, 5, 6

$$X \sim G \ and \ Y \sim F$$

$$s.t. \quad \mathrm{P}(\mathrm{X} < x) = \mathrm{G}(\mathrm{x}) \text{ and } \mathrm{P}(\mathrm{Y} < y) = \mathrm{F}(\mathrm{y})$$

$$\exists \ C\big(G(x), F(y)\big) = c \in (0,1)$$

Hence, C describes the joint distribution of the processes X and Y, the copula function binds the separate processes to one process. There are many different copula functions that all have their own specific characteristics. These characteristics refer to whether the separate processes show larger dependence at the center or at the tails of the observations. And if tail-dependence is present there is a distinction between upper-, lower-, and equal tail-dependence. To catch the most common dependence structures this paper considers the normal, student, Gumbel, and the Clayton copula, respectively modelling no tail dependence, symmetric tail dependence, upper tail dependence, and lower tail dependence. The models that allow single tail dependence can easily be rotated, so that the dependence structure changes from upper to lower or the other way around. The reason to incorporate both the Gumbel and the Clayton copulas is that although both copulas can model upper and lower tail dependence the dependence-structures differ. The upper tail of the Clayton copula shows less dependence than the rotated Gumbel, this is visible in the figures (3 and 4) below.

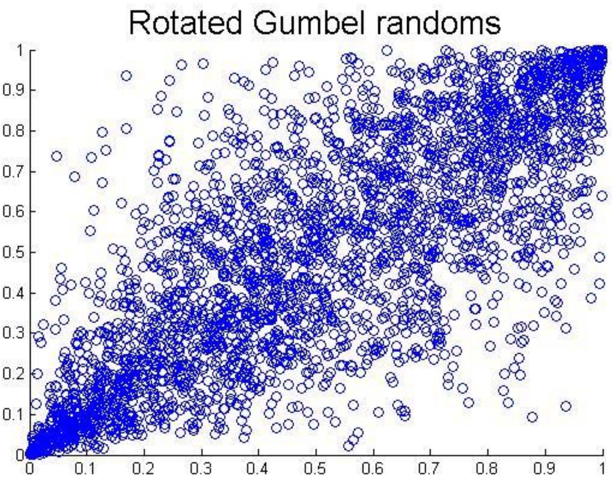

Figure 3

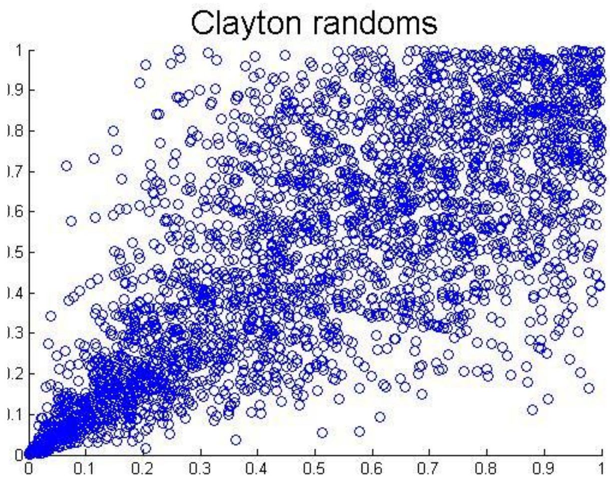

Figure 4

By estimating all of these copula functions and analyzing the likelihoods the dependence structure that best fits the data can be found. Once a copula has been found, similarly to when marginal distributions are found, the goodness of fit can be tested. This can be done in a similar manner, which is to be discussed in section goodness of fit (2).

3.3 Goodness of Fit (1)

This paper considers two measures that indicate whether or not a certain dataset significantly differs from the distribution one has specified. Specifically, these measures will be used to check whether there is no significant difference between the so called Emperical Distribution (ED), given in formula (7) below, and the skew-T distribution that follows from the model mentioned in the section (Standardization) above.

Equation 7

$$ED(Y_i) = {}^{i}/_{n} \quad for\ i = 1, \dots, n \ \ and\ Y_i\ is\ the\ i^{th} smallest\ value\ of\ Y$$

This empirical approach of specifying the quantiles of the distribution follows from the fact that random p-values from a certain distribution are uniformly distributed. The measures mentioned above are the Cramer von Mises and the Kolmogorov-Smirnov test statistics, respectively given in the formulas (respectively 8 and 9) below.

Equation 8, 9

$$CvM = \sum_{i=1}^{T} (U_i - {}^{i}/_{T})^2$$

$$KS = \max_t |U_i - {}^{i}/_{t}|$$

Where U_i is the i^{th} smallest p-value that is found for the standardized series of Y_t - δ (equation 1), which was assumed to be distributed ST(τ, λ) (formula 3). Hence, these measures are based upon the distance between empirical and assumed distributions' p-value estimates.

Since we have fully specified our model the asymptotic distribution of these test statistics does not apply, and simulation is needed in order to find the relevant critical values. Simulation of the Cramer von Mises- and Kolmogorov-Smirnof-test statistics will be done in the following way.

- Generate a random sequence of Y_t- δ according to the specification of the GJR-GARCH model specified in formulas 1, 2, and 3. This random sequence must be very long in order to minimize any starting bias that is the result of the starting values of the autoregressive model.

- Re-estimate the model specified in the formulas (1, 2, and 3) in the section above, and extract standardized residuals and the estimated distribution.

- Use these estimates to find U-values and determine the test statistics, and store these. Sometimes the random data will result in distribution estimates that have very extreme parameters, typically very large degrees of freedom. In this case simulation should restart, since the choice of model would have altered it is not realistic to stick to the same model.

- Repeat a 1000 times and take the (1- alfa)*1000th element of the sorted sequence of test statistics, this should be larger than the test statistics found for the original dataset.

3.4 Goodness of Fit (2)

The goodness of Fit tests can be performed in the same manner as for the marginal distributions. Instead of an empirical distribution as in formula (7) above, an empirical copula is constructed similarly as stated in formula (10) below.

Equation 10

$$ED_{copula}(u) = {}^{1}\!/_{T} * \sum_{t=1}^{T} \prod_{i=1}^{n} I_{(U_{it} < u)}$$

The same test statistics, the Cramer von Mises and the Kolmogorov-Smirnov statistics can be calculated as stated below in the formulas *(11 and 12).*

Equations 11, 12

$$CvM = \sum_{i=1}^{T} (C(U_i; \hat{\theta}) - ED_{copula}(U_i))^2$$

$$KS = \max_{t} |C(U_i; \hat{\theta}) - ED_{copula}(U_i)|$$

These measures are very similar to the univariate case discussed earlier, they depend on distance between assumed and empirical joint process specifications. Critical values for the copula functions' test statistics are constructed by means of simulation, just like discussed above for the marginal distributions. The only difference between copula and standard goodness of fit testing is that U values mentioned in step three should be used as described in the formulas of this section instead of the formulas from the section above.

3.5 Additional dependence measures

Dependence structures can be determined by estimating copula function that have certain dependence characteristics, by comparing the likelihood of different copulas the best fitting structure can be selected. There are also other measures of dependence that can be observed be observed from the dataset, by constructing the empirical distributions. One of these measures is the population rank correlation that uses the fact that U-values, or p-values of a distribution, are uniformly distributed. The formula (13) below states this measure, and indicates whether dependence is positive or negative and also the strength of this dependence.

Equation 13

$$\hat{\rho} = \frac{12}{T} \sum_{t=1}^{T} U_{1t} U_{2t} - 3$$

Another measure that is to be withdrawn from the dataset in a similar manner is a measure of quantile dependence. This measure can give insights into symmetry of dependence. Upper quantiles might show differences from lower quantiles, this information can point out which characteristics the optimal copula ought to have. The formula of quantile dependence is given as the formulas (14) below. This measure practically counts the amount of times that both assets coincide in the lower quantile. This means that the higher this measure's value, the more observations coincide below (above) a lower (upper) quantile.

Equation 14

$$\widetilde{\lambda_q} = \begin{cases} \dfrac{1}{Tq} \displaystyle\sum_{t=1}^{T} I\{U_{1t} < q, U_{2t} < q\} \\ \dfrac{1}{T(1-q)} \displaystyle\sum_{t=1}^{T} I\{U_{1t} > q, U_{2t} > q\} \end{cases}$$

3.6 Hedging

Once a model has been constructed to explain financial returns and their variance regimes, a hedger can extract an 'optimal' position from his or her model. A conventional hedging method is to make use of ordinary least squares to acquire the in-sample minimum variance position. Literature like Lien (2005) claims this simple method is in fact the best method to use, foremost when trying to find minimum variance positions. Copula functions can also be used to estimate a minimum variance position but will probably be outperformed on this criterion. To get a complete view into the applicability of the techniques mentioned in the sections before performance on the conventional minimum variance criteria will be compared. But the main focus of this paper is to find methods that provide hedging positions that specifically reduce the occurrence of large losses, or down-side risk. To do so this paper considers two methods, the linear quantile-regression, and the copula-based quantile-regression. In the sections below these methods will be described along with the basic methods these more sophisticated techniques expand on.

3.6.1 Simple Approaches

The first hedging methods to be discussed is the most well known and simple approach. By means of ordinary least squares (OLS) returns on the unhedged position are regressed on returns on the hedging instrument, as given in the formula (15) below.

Equation 15

$$Y_{exposure} = \beta X_{hedging\ instrument} + \varepsilon$$

The coefficient β of this equation is the hedge ratio that represents the in-sample minimum variance position. Minimum variance is clearly not the position this paper focuses on, but it is quite similar to the next method to be discussed. The OLS method will be used to serve as a benchmark for the linear quantile regression (LQR) that is to be described below.

The linear quantile regression is a method that provides the possibility of incorporating aversion to losses rather than gains. Negative returns are given a larger penalty than positive returns by means of the loss (L) function given in the formula (16) below.

Equation 16

$$L = [Y - (\beta X)] * [q - I\{Y < \beta X\}]$$

X are in this case returns on the hedging instrument, while Y are the returns on the current position. I is an indicator function that give value 1 whenever a negative error occurs, and q is the quantile that is considered. The second part of the product is responsible for the difference between positive and negative errors and the loss that is assigned to it. Since the sign of both parts are either simultaneously positive or negative, absolute errors are considered and the slope of the loss function is q-1 for negative errors while it is q for positive.

3.6.2 Copula-based approaches

Problems might arise because of the linearity of dependence that is assumed by the linear quantile regression. By using copula functions the assumption of linear dependence can be dropped. One can namely express a quantile as a function of copula parameters and underlying distributions. Bouyé and Salmon (2002) describe this technique in their paper as the copula quantile regression. To illustrate this technique assume that marginal distributions F(X) and G(Y) of X and Y have been specified. From these distributions it is possible to construct a copula function that provides the missing link to the joint multivariate process of X and Y. Once such a copula function is found can be used to specify the quantiles, to illustrate this the formula (17) below describes the q quantile of the normal copula. (Alexander II.7.2 (2008))

Equation 17

$$Q(X,q,\text{б}) = \mathrm{G}^{-1}\left[\text{б}\Phi^{-1}(\mathrm{F(X)}) + \sqrt{1-\text{б}^2}\,\Phi^{-1}(\mathrm{q})\right]$$

In this equation б is the correlation coefficient of the normal copula function, and Φ the normal CDF. This formulation of quantiles changes for the use of different copulas. This specification of the conditional quantiles of Y can be used to perform a so called copula quantile regression, of which the solution is found by the following minimization problem.

Equation 18

$$\min_{\text{б}} \sum_{t=1}^{T} (q - I\{Y_t < Q(X,q,\text{б})\}) * (Y_t - Q(X,q,\text{б}))$$

The solution found to this problem, the estimate of б, at the specified quantile allows us to determine the hedge ratio (h^*) as:

Equation 19

$$h^* = \text{б} * \frac{\mathrm{S_y}}{\mathrm{S_x}}$$

In which S_y and S_x are the standard deviations of the exposure and hedging instrument, these are extracted from the GARCH models that are used to standardize their returns. By doing so, the current regime will be better described than by the sample variance. Note that this approach assumes correctly specified underlying distributions F and G. Simultaneously re-estimating parameters for F and G as well as for б demands sophisticated optimization routines that go beyond the scope of this research.

3.6.3 Practical circumstances

Since markets continuously change the optimal position proposed by a hedger's models also varies over time. Sadly it is not possible to adjust financial positions day-to-day because of transaction costs. This makes hedging a somewhat tactical game. One could choose to rebalance whenever the current position (taken in the past) deviates too much from the currently optimal position. Another option would be to rebalance at predetermined points in time, which is off course quite practical since no full time market analysis is required. For simplicity this paper considers this last method, rebalancing at predetermined points in time. This will be done on a 24 weeks interval, or about every half year. At every rebalancing moment parameters of the marginal distributions and copula functions are re-estimated on the 'grown' set of available data.

3.6.4 Comparison

Whether or not the methods above indeed protect the hedger against downside risk will be visible in the portfolio value. As of 2006 there are some strong drops in asset prices and therefore it should be visible to what extent large losses in the exposure, the AEX index, have an effect on the hedged positions. Normally in hedging literature variance reduction is mentioned as a measure of performance, but this measure is not very useful for investigating downside risk hedging. For the ordinary (minimum variance) hedge ratios extracted directly from the copula itself variance reduction will be considered. Since the OLS solution is the in-sample minimum variance solution it would be extraordinary to achieve higher variance reduction with models under as many assumptions as copula-based models.

4. Results

In this section model selection will be discussed, after which the models chosen will be tested for goodness of fit. Finally hedging performance of both downside risk hedges and minimum variance hedges will be discussed and compared for the different methods that are discussed in earlier sections.

4.1 Model Selection

As described in the section Data excess kurtosis and skewness are present in the returns-series of the AEX- and S&P500-index. Clearly normality of marginals is no reasonable assumption, a student's T distribution would seem like a better idea because of the ability of this distribution to catch excess kurtosis. Still skewness would have to be ignored in order to argue in favor of the assumption of a student's T distribution. A more realistic approach would be to use a skewed T-distribution as specified by Hansen (1994). Indeed standardized residuals acquired by means of a GARCH-model under assumption of a skewed-T error distribution seem very useful.

The GJR-GARCH-model specifications and parameter estimates are given in table 1 of Appendix B. These estimates serve as an illustration of the model used, and have been estimated over the initial sample, i.e. before 2007. Since parameters are estimated every time the portfolio is rebalanced actual values are not that relevant. One GJR-term has been removed due to insignificance, and the parameters are used as in equation 2.

The GJR-GARCH estimates in table (1) of Appendix B can be used to construct, time-variant, variance estimates for the zero mean series of the AEX and the S&P. These variance estimates are used to standardize the zero mean series. These standardized residuals of the AEX and S&P are distributed according to the skewed-T distribution with estimated degrees of freedom (skewness parameters) are respectively 18.9 (-0.26) and 17.5 (-0.18). To test whether the assumptions concerning the error distribution for the model above are reasonable, goodness of fit measures will be considered for these distributions. The table (2) below states the Cramer von Mises and Kolmogorov-Smirnov scores for the distributions found for the AEX and S&P500, accompanied by the simulated critical value. The table shows that the distributions estimated significantly fit the data, since all values are smaller than critical values found.

Table 2

	CvM-statistic	Critical value (10%)	KS-statistic	Critical value (10%)
AEX	0,0455	0,1188	0,0246	0,0308
S&P500	0,0388	0,1191	0,0252	0,0314

The standardized residuals that follow from this model are not heteroskedastic and only for the S&P500 serial correlation occurs. The strong implication of no heteroskedasticity confirms that a constant error distribution is likely. The ARCH-LM test and Breusch-Godfrey provided the p values stated in the table 2 of Appendix B.

By means of estimated probability integral transformations a sequence of values is converted to p-values according to a certain distribution. The standardized series discussed above are assumed to be skew-T distributed, so by means of the skew-T CDF p-values can be acquired. Such series serve as input for the construction of copula functions. Since there are many different copula functions that model very different dependence structures, it is common practice to estimate a range of copulas and look for the one with the highest likelihood. The table (3) below shows the copula functions considered and their specific characteristic, and finally the negative log-likelihood. The student-t copula performs best in our sample, followed by the normal-, and rotated Gumbel copulas. These copulas all allows a different kind of tail dependence, as can also be seen in the table. A scatter-plot of the U-values that are used to model the copula functions above could provide insights into tail-dependence. This plot can be found as graph 2 of Appendix A and shows slight tail dependence, but no conclusions can be drawn. Additional dependence measures might provide more of a confirmation.

Table 3

Copula	**Characteristic**	**(–) log-likelihood**
Normal	No tail-dependence	**-166,7**
Student-t	Equal tail-dependence	**-170,4**
Clayton	Lower tail-dependence	-145,1
Rotated Gumbel	Lower tail-dependence	**-166,5**
Rotated Clayton	Upper tail-dependence	-109,7
Gumbel	Upper tail-dependence	-143,1

4.2 Additional dependence measures

The rank-correlation that can be retrieved from the data displayed in the scatter-plot above is 0.88. The scatter-plot indeed shows that negative returns coincide with negative returns, as for positive returns. Another, more meaningful, measure that can be used to describe dependence structure is the 'near tail' dependence measure. It is clear that the largest part of this dependence indeed shows a symmetric dependence, just like in the student's t and the normal copulas. The extreme quantiles seem to show significant differences, though this is to be explained by a lack of observations for these quantiles. A plot of the 'near tail' dependence measure for the dataset can be found as graph 4 in Appendix A. No real conclusions can be drawn from these additional measures, therefore the three models that show the best likelihood scores will be used for further investigation.

4.3 Goodness of fit

Besides testing the goodness of fit for the error distributions that are assumed during the standardization phase, these same tests can be performed for copula functions. The table (4) below shows test statistics and critical values for the Cramer von Mises and the Kolmogorov-Smirnov tests at a significance level of 10%. It can be concluded that not only the marginal distributions that have been assigned to the data, but also the copula functions indeed have a good fit, since no test statistic exceeds its critical value.

Table 4

Copula	CvM-statistic	Critical value (10%)	KS-statistic	Critical value (10%)
Student-T	0,0393	0,0906	0,0273	0,0368
Rotated Gumbel	0,1112	0,293	0,0392	0,0501

4.4 Downside hedges

After selecting the right standardization models and copula functions that significantly fit the dataset, these can be used to test practical applications. This paper is concerned with the applicability of the models constructed in the sections above for the purpose of hedging. Specifically this paper tries to identify methods that offer the opportunity to hedge downside risk. This means that the hedged position is expected to incur less extreme losses when the unhedged position experiences extreme losses. These extreme losses can be seen as, and actually are, the lower quantiles of the returns-distribution on the financial assets.

First the OLS- and LQR-approaches will be considered, since these are simplest. A hedging decision of taking a position in the S&P500 to reduce risk that AEX exposure brings along is simulated over the period as of 2006 until 2014. The figure (5) below shows the results achieved by applying OLS and LQR for different quantiles along with the initial exposure, the AEX-index. Results are disappointing, there is no clear difference between the different techniques. Indeed the size of the large loss around the 80th week is strongly reduced by the linear quantile portfolios, but the conventional OLS approach yields approximately the same results.

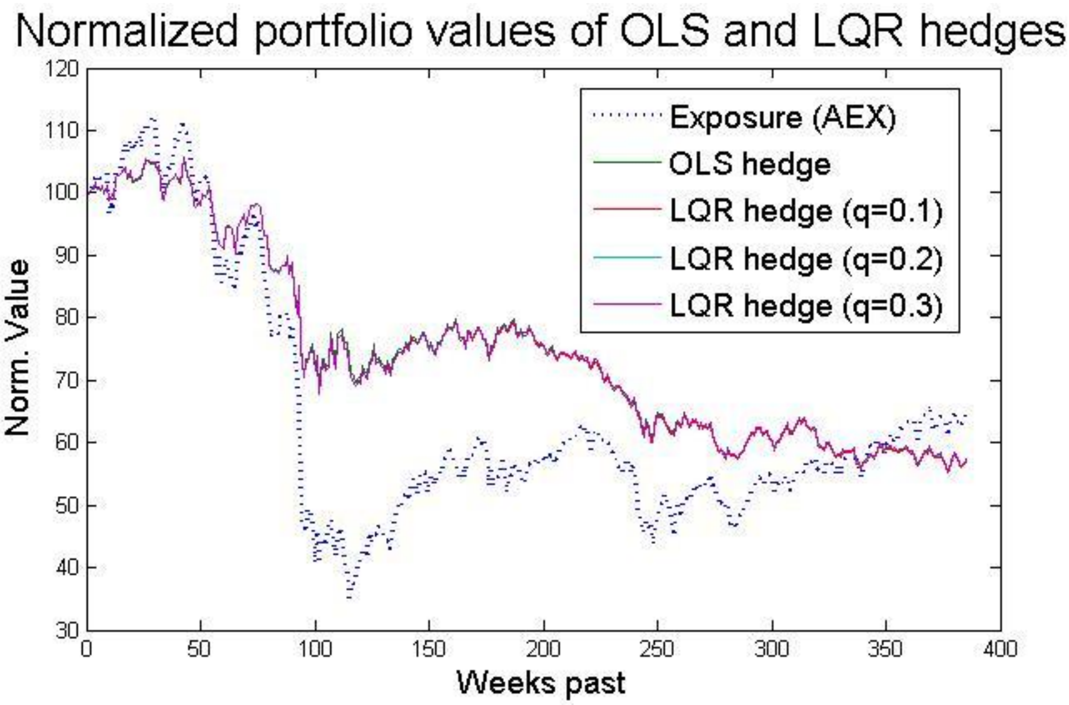

Figure 5

The use of quantiles has not shown to be useful for the linear case, since it has not distinguished itself from conventional OLS. The use of more sophisticated copula functions and copula-quantiles might show more distinct performances. As stated earlier we will consider the three copulas, the student-t, the rotated Gumbel, and the normal copula. These three copulas scored the highest likelihood and significantly fit the dataset. However, there is no closed form of Gumbel quantiles and those will therefore not be considered. Besides the fact that results for OLS and LQR are so similar, the choice of quantile seems to hardly have any effect on the hedging positions taken. For the hedges based on the T-Copula this is also the case, as can be seen in the figure (6) below. On the other hand, for the normal-Copula based hedges there is a clear difference in positions taken for different quantiles, as can be seen in figure (7) below.

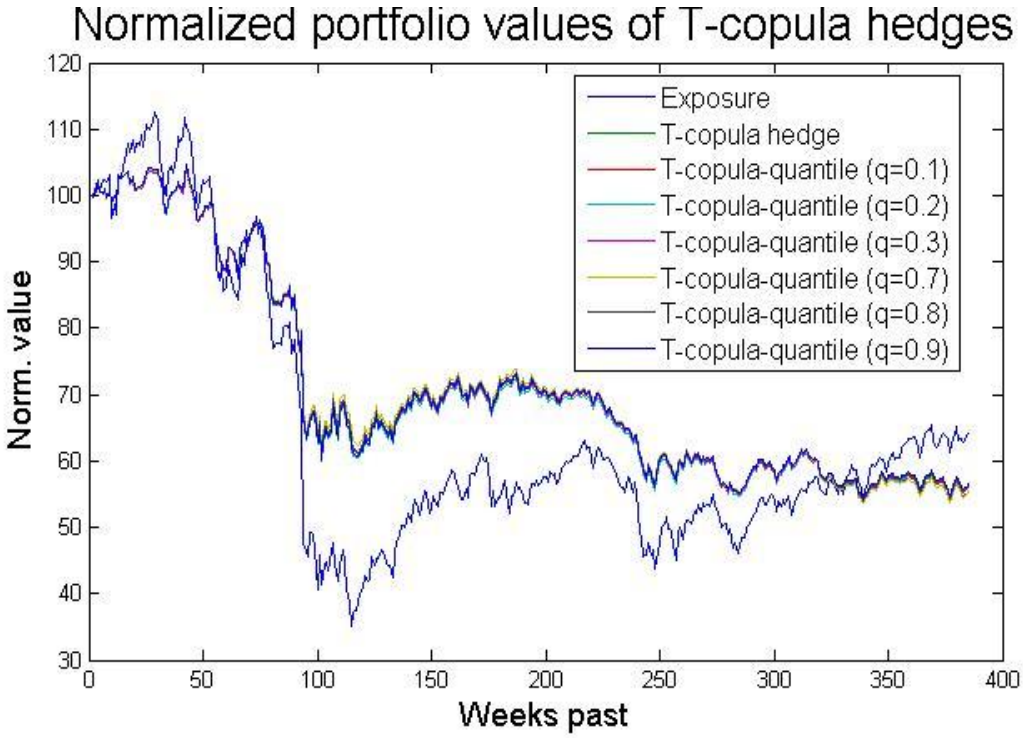

Figure 6

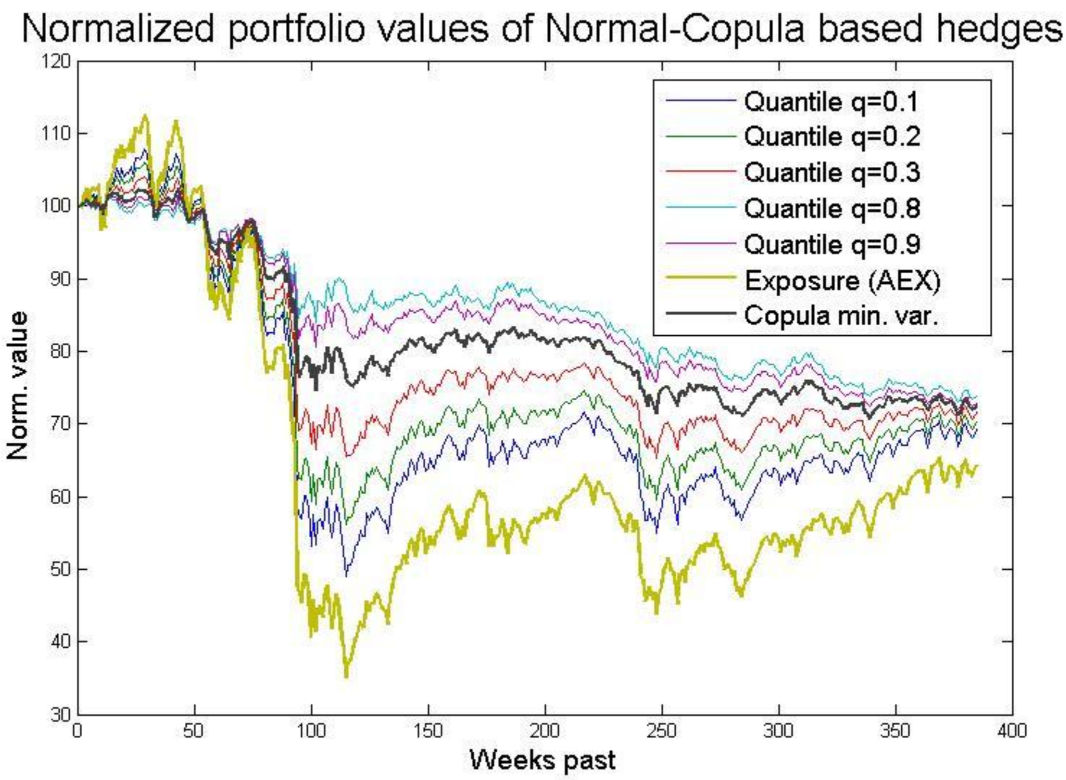

Figure 7

There is a problem though, concerning the results found for the normal-copula based hedges. The portfolios constructed to minimize downside-risk suffer the worst from the large downside deviation around the 80th week. The higher quantiles seem to offer very good protection against a crash, while theoretically the lower quantiles should. Estimated optimal hedge-ratio grows as the quantile is set higher for this dataset, so the higher the quantile the larger the short position in S&P500 stock. The crisis that is visible in the graph had worldwide effects, resulting in huge gains for the ones that had a short position in any market. The best outcomes of the Normal-Copula quantile-regression are first off all a lucky result of misspecification, and secondly outperformed by a simple method like OLS. Graph 3 of Appendix A illustrates these points.

The specification of copula-quantiles in order to find downside-risk hedge-ratios has not shown to be effective in case of a crisis. The simpler alternative of linear quantile specification results in practically the same hedges as conventional linear regression, minimum variance, hedge-ratios. Because of these two findings it is unclear whether quantile-specification is the way to hedge downside-risk. As an illustration, the figure (8) below shows the different techniques' hedging results for the 0.2 quantile.

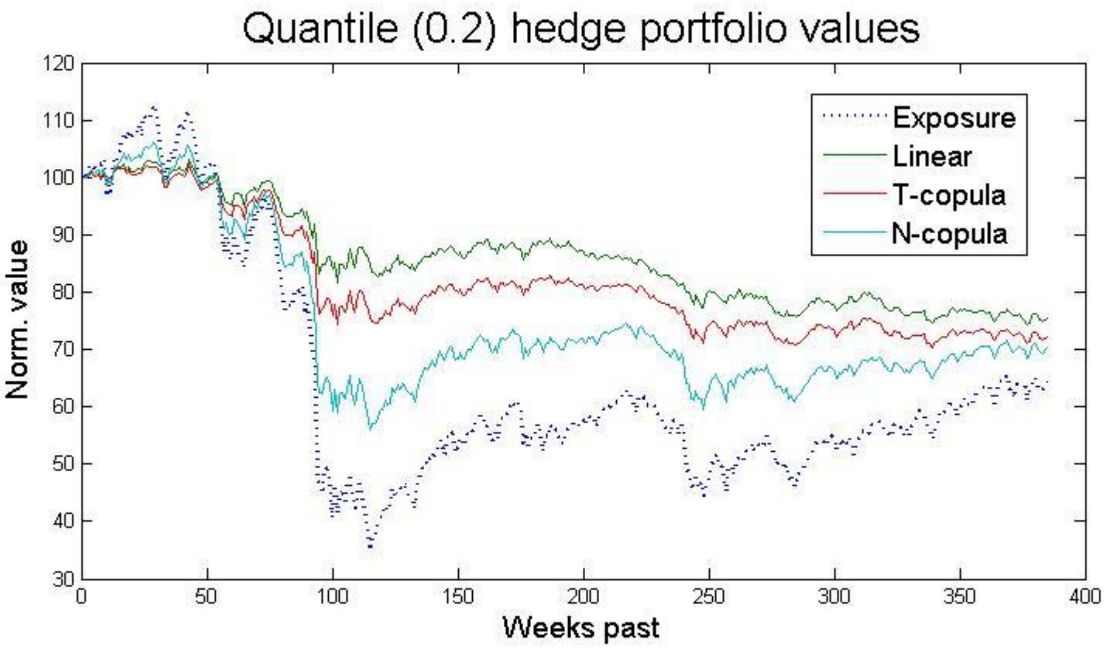

Figure 8

4.5 Minimum variance hedges

Copula functions can also be used to determine minimum variance hedge-ratios, the danger is though that the amount of parameters increases uncertainty. Hedge-ratios computed by means of OLS are in-sample minimum variance positions computed as simple as possible.

The table (5) below illustrates that indeed OLS outperforms copula-based minimum variance hedges when considering variance reduction as a measure of performance. The difference is not large but still substantially larger than the difference between the different copula-based methods.

Table 5

Method	**Variance reduction achieved (%)**
OLS	91,8
LQR	91,5
Normal-Copula	88,8
T-Copula	88,9
Rotated Gumbel-Copula	88,8

5. Conclusion

This paper has considered the practice of hedging by means of two simple methods, ordinary least squares and linear quantile-regression, and three somewhat more complicated copula-based models. Copula functions model dependence structure amongst two or more variables, which sounds very useful for hedging. Sadly the use of copula function for hedging has not shown very useful, since OLS beat the copula-based hedges on both variance reduction and downside-risk reduction.

Downside-risk hedge-ratios can be found, according to theory, by copula-quantile specification as well as by a simpler linear quantile-regression. The copula-based alternative was outperformed by the OLS-hedge, which was not even a downside-risk hedge. Portfolio value of the copula-based hedges dropped almost as much as the initial exposure at the occurrence of the crisis. The linear quantile-regression did save most of the portfolio value when the crisis occurred, but results were practically the same as for the OLS. Copula-quantile specification requires many parameters to be estimated and therefore involves a lot of uncertainty, while OLS involves very few estimates to be made. Hence, downside-risk hedging by means of the copula-based methods this paper considers is a poor strategy.

Besides the fact that downside-risk hedges did not protect against downside-risk, the influence of the quantile considered was in most cases insignificant. In one case, for the Normal-copula, there was a difference visible but the quantile chosen influenced the portfolios' values in the wrong direction. For the normal-copula downside-risk hedges suffered the worst from downside-deviations in the form of a crisis, and therefore seem not very useful for this purpose. This leaves us to conclude that quantile specification does not really work. Since the effects of the quantile considered are very small or sometimes even opposed to expectations it is unclear if it is even possible to hedge downside-risk in this manner.

This paper considered downside-risk hedging methods that were tested over a period in which a crisis, i.e. real downside-risk, occurred. Performance of copula-based models was very bad, and the use of linear quantile-regression was approximately equal to conventional OLS. Besides the fact that performance was poor, the methods showed very strange, or barely any, effects of changes in the quantile considered. This paper also briefly considered minimum variance hedges, which once again confirmed that copula-based models did not do so well. It seems as if simplicity is the way to go for a hedging problem as considered here.

6. Literature

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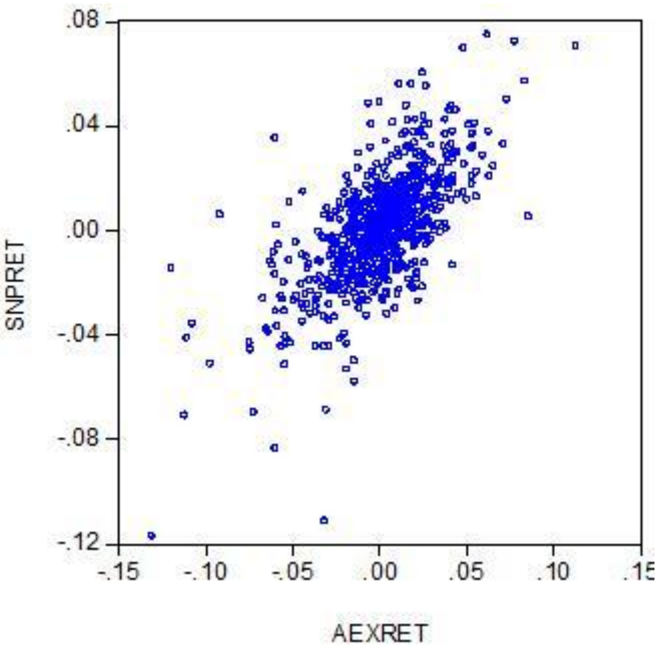
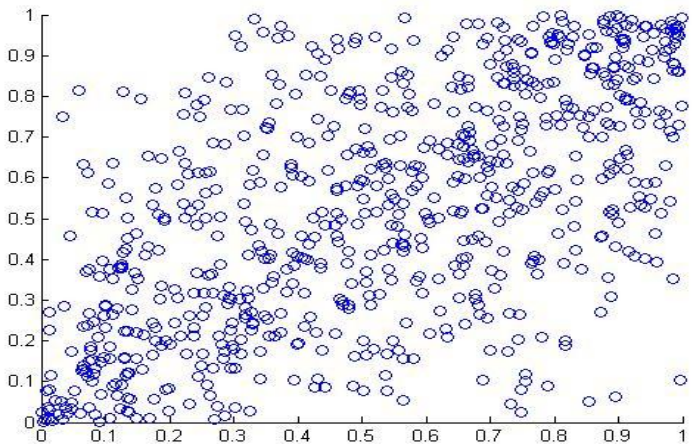
APPENDICES

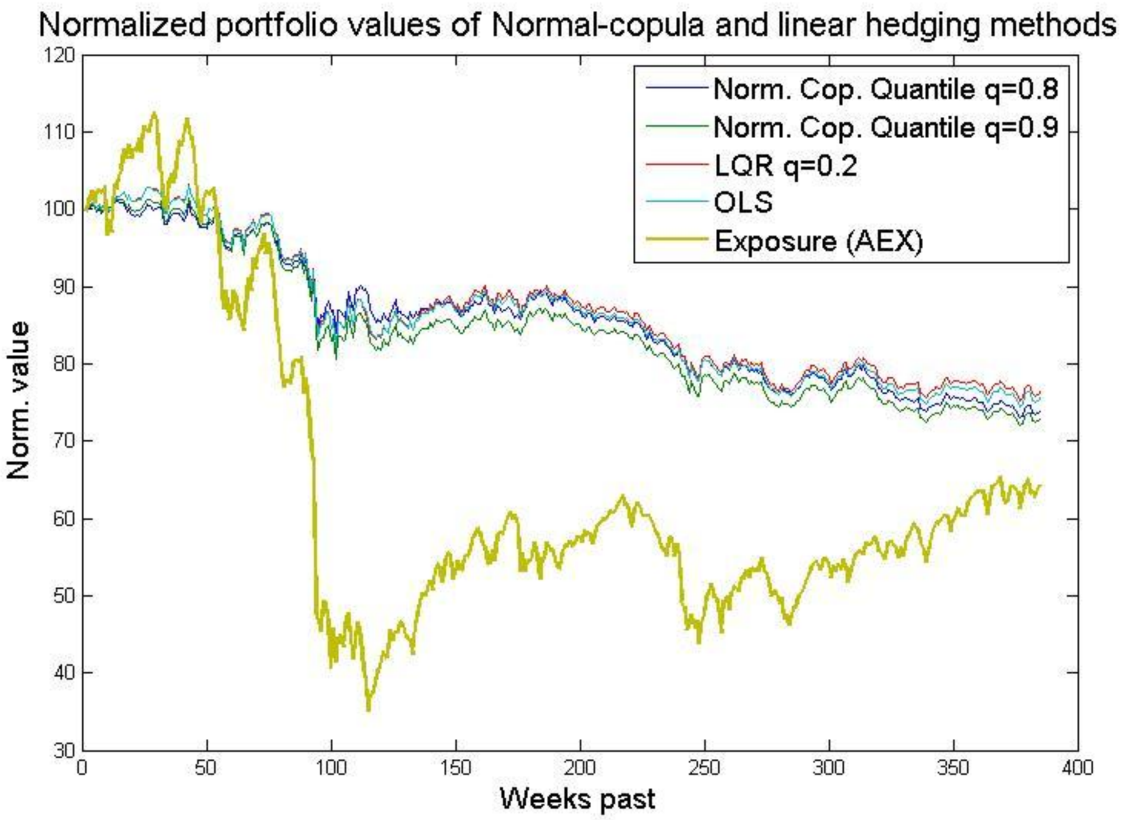
APPENDIX A



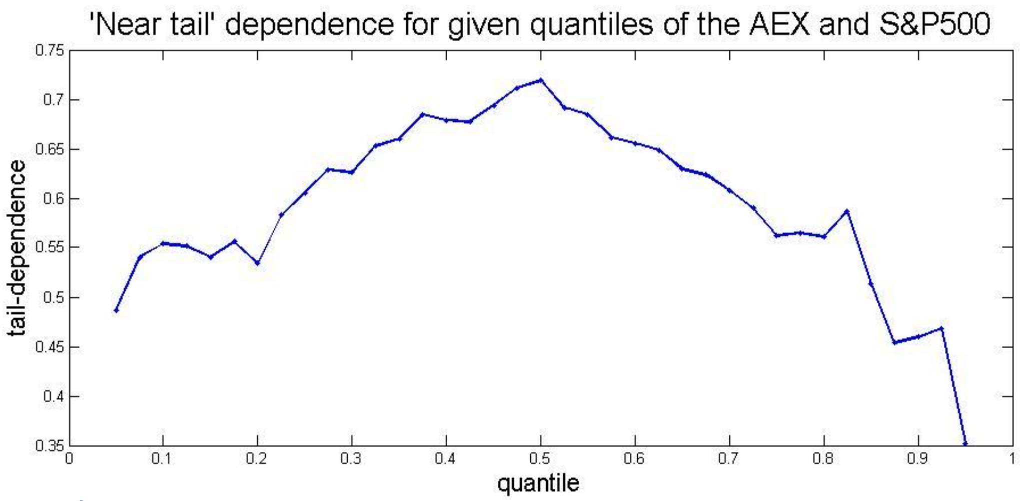
Graph 1



Graph 2



Graph 3



Graph 4

APPENDIX B

Table 1

parameters	AEX	Std. err.	S&P	Std. err.
Mu	2,00 (*10^-5)	7,46 (*10^-6)	1,16 (*10^-5)	7,03 (*10^-6)
α_1	2,52 (*10^-7)	1,63 (*10^-7)	4,93 (*10^-7)	6,17 (*10^-5)
α_2	0,08	0,03	0,05	0,03
γ	0,13	0,04	0,17	0,08
β	0,83	0,04	0,84	0,07
τ (D.o.F.)	18,9	12,16	17,49	9,11
λ (Skewness)	-0,26	0,05	-0,18	0,05

Table 2

	AEX	**S&P500**
ARCH *(no heterosked.)*	0.90	0.79
BREUSCH-G. *(no serial corr.)*	0.17	0.01