# Strict Rules and Asymmetric Discretion Introducing Uncertainty in HRM Strategies

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## 1. Introduction

Many decisions in organisations are governed by pre-set rules. Especially when a firm needs to decide about promotions, job-assignment or hiring employees, rules seem to dictate to managers what to do. Assistant professors are often only promoted if they succeed in publishing a certain number of papers in well-established journals and good reviews by students (Erasmus School of Economics, 2010). Moreover, factory workers have to meet a certain production level in their probationary period in order to be retained by the company in many cases (Saint-Cyr, 2011). Also other areas face these probationary periods, from academics to government (New York University, 2000; Liff, 2007). Another example would be bonus schemes for executives, where bonuses are determined on the basis of a scheme that has been set at an earlier stage (Royal Dutch Shell, 2012; DSM, 2013). These rules seem puzzling sometimes, since the manager or supervisor in charge is often very capable of making such decisions. Bewley (1999) interviews more than 300 businessmen and is surprised that so many constrain their decision-making by bureaucratic rules. This practice is not only limited to firms. At what level of secondary education students in the Netherlands will start, is mostly determined by the grade for one centralized test and not, as one might expect, by the teacher that had the opportunity to assess the skills of the students for more than a year. This test, the CITO test, is taken at the end of eight years of primary education and the score determines what level of secondary education the particular student can go to. To be admitted by a level, a certain threshold score needs to be passed in that very test.

These kinds of rules have received substantial attention in economic literature. One of the explanations given for the preference of rules over discretion is that discretion causes rent seeking behaviour. Employees that are affected by human resource decisions will be led to spend valuable resources and time trying to influence managers and executives holding discretionary power over the decision (Milgrom, 1988). By limiting the discretionary power and by substituting it with rules, firms can prevent such costly behaviour. The other explanation that has been provided in the literature is that committing to rules and human resource strategies can provide an incentive and motivation for employees to higher levels of effort performance (Dev06; Dev06; Dev06). Koch & Nafziger (2012) indicate that the large literature on careers and incentives emphasize two roles of promotion and job assignment decisions. The first relates to learning about the abilities of employees and signalling this information and about promotions as a source of incentive. This paper will focus on learning the abilities of the employee and promotions as a source of incentive. It will abstract from the signalling function of job assignment decisions. In this paper the firm faces the trade off between efficient job allocation and the use of this job allocation for incentives, which has been discussed a lot in economic literature.

Such incentives work especially well for incentivising the acquisition of firmspecific human capital. In this paper I will consider a situation where an employee has the possibility of investing in firm-specific human capital. Naturally, such an investment is costly. The manager decides afterwards to keep or replace the employee. By committing to rules that make it more likely for the employee to be retained if he has invested, the manager can promote investment. I find that it is indeed optimal for the manager to commit to job allocation rules in a contract. Making it easier for investing employees to pass the bar and more difficult for non-investing employees to do so pays off by increasing the likelihood that the employee indeed invests. Ex post, these rules are not optimal. However, if the firm will commit to these ex ante optimal rules, it will maximize its payoff in the long run.

So far this analysis follows existing literature. However, this analysis implicitly assumes that these rules can be based on levels that are measured with certainty. This model incorporates uncertainty in the measurement of productivity or other information promotional rules are based on and investigates the implications of this uncertainty. Especially if the true ability can be learned with certainty afterwards, the firm faces a dilemma; basing the decision whether to keep an employee based on the true ability will cause the investments incentives to be ineffective, leading a lower share of employees to invest. On the other hand, if the firm bases its firing decision on the (imperfect) productivity thresholds, it may fire skilled employees and keep unskilled ones, which can be very costly as well. This is the problem this paper will focus on.

It seems that a combination of both options is optimal; the firm uses the imperfect productivity thresholds to promote investment by the employee, but gives itself room for discretion regarding the decision, albeit this discretion has to be asymmetric to prevent the threshold incentive from breaking down. By doing this the manager can partly fix the costly measurement error. But more notably, asymmetric discretion will provide even stronger incentives to the agent to invest in human capital. Hence, this paper provides an explanation for the use of asymmetrical discretionary power by managers when they feel that the outcome by a promotional rule does not fit the specific situation.

Taking the example of the assistant professor, the promotion decision can often be perfectly made by the dean who has had the possibility of observing the skills and potential of the candidate for a long period. A talented academic may expect to get the promotion easily. Setting a rule that requires a certain number of publications before such a promotion is considered, independently of the skills of the concerning candidate, incentivises skilled assistant professors to exert substantial effort as well. Universities signal that these rules are strict, which can be understood as a commitment to these rules. However, the rules often provide as well in some discretionary power for the dean; an academic who does not meet the standard due to influences outside his or her control but who is very talented and expected to do well, can be given an exemption from the rule. However, an assistant professor without any capabilities who passes the rule due to luck will never be subject to such discretion that allows the dean to fire him anyways. This is an example of how asymmetric discretion can partly help to solve the imperfection in the rule. This discretion needs to be asymmetric though, otherwise setting the rule will be useless because academics expect to be solely judged on their performance, eliminating any incentive stemming from the preset rule.

## 2. Related literature

In business the importance of HRM strategies and their application is well understood, especially in relation to the acquisition of firm-specific human capital (Lado & Wilson, 1994). Hence, many economists have investigated and discussed the incentives coming from these HRM strategies. Gibbons & Waldman (1999) give a survey of this literature. They provide an overview of models relating to the various fields HRM strategies can affect, such as job assignment, human capital acquisition, incentive contracting, efficiency wages and tournaments. They argue that these models are so called "building-blocks" as they can "be combined and enriched to address various aspect of careers" (Gibbons & Waldman, 1999, p.2374).

One of these aspects is the difference between ex ante and ex post optimal promoting rules. This difference has to do with time inconsistency when promotions are used both for job allocation and as an incentive. Ex ante it is optimal to promise to take future effort and human capital investment by an employee into account when deciding about possible promotions to incentivise more effort and investment. However, after the effort and investment has been performed, the expost optimal strategy is to focus purely on job assignment. Waldman (2003) discusses this inconsistency and argues that firms therefore commit to internal promotion in order to reward firm-specific investment and prior performance. He discusses several options to do so, ranging from prohibiting promoting outside employees to setting higher standards for outside employees when the costs of the first option are too high. This paper adds to the last argument of using different standards for employees, based on their behaviour. In this paper employees that invested in firm-specific human capital are rewarded by a standard that is relative easy to pass. However, this instrument works two ways. Certain behaviour can also be discouraged by setting higher standards for inside employees. This paper provides in this argument since it allows the manager to set relative higher standards for employees that did not invest. Bayo-Moriones & Ortín-Ángel (2006) find empirical evidence in a sample of 653 Spanish firms that commitment to internal promotion schemes indeed is used to reward effort and skills acquisition. This commitment tends to be higher when firms are better able to measure the employee's skills. This is of significant importance for this paper, since I assume that managers can observe whether employees have invested in firm-specific human capital. Furthermore, I assume that it is possible to measure output, although with a measurement error, and ability.

Other authors also offer possible ways of committing to HRM strategies that incentivise and award good behaviour by employees. One of these stands is that firms can commit credibly to these HRM strategies by taking the decisions out of the hands of the managers who have an interest in making the best possible assignment and give it to someone whose interest are in following the stated policy (Milgrom & Roberts, 1988). This could serve as an explanation for the existence of personnel departments that decide on promotion decisions according to a set of rules. Milgrom and Roberts observe that many managers and employees complain about personnel departments that are, in their view, bureaucratic, unresponsive and rule-bound. The authors show that this might actually be efficient for the firm. I show similar results in this paper, but also find that some responsiveness and discretion allowing is optimal for a firm when measurement whether employees meet the rules cannot be done perfectly. Another reason for rule bound HRM strategies is that dismissing an employee is often a painful process. Managers might be inclined to keep the employee and forgo the trouble of dismissing him (Dominguez-Martinez & Swank, 2011). In this paper I assume all agents, both manager and employee, act solely in their own interest. This, of course, is an abstraction from reality, but provides an opportunity to study the interaction between rules and discretion without interferences of such unpredictable behavior.

Also Prendergast (1993) and Carmichael (1983) show how promotion can be used as an incentive for optimal human capital acquisition. Prendergast observes that a worker only acquires new skills in the face of a higher wage when he does so. The employer then has an incentive to claim the worker did not invest in these skills in order to keep wage costs low. This is in fact a moral hazard problem that can be solved by commitment to a wage scale for different tasks. The worker, in turn, is rewarded for skills acquisition by promotion to another job. Kahn and Huberman (1988) offer the upor-out scheme that is in place in some industries as a solution to the commitment problem. However, the possibility of being fired in this case reduces the value of any firm-specific human capital investment and, hence, employees are less likely to actually invest (Gibbons, 1998).

DeVaro (2006) studies the incentives coming from these differences in wages. In a cross-sectional study he considers a sample of skilled workers in establishments located in four metropolitan areas in the United States. He finds that a difference in wages between those who are not promoted and those who are, incentives work effort and skills acquisition. Workers are motivated by larger spreads. In this paper, the incentive does not stem from a difference in wages, but a difference in standards. Investments or effort is rewarded by relative lower standards that need to be achieved during a probationary period or for a promotion. These non-monetary rewards can be as valuable to employees. Another interesting finding is that worker effort decreases when the stochastic component of production increases (DeVaro, 2006). This is of importance for this paper since I consider a model where production is indeed measured with some random factor.

Lastly, a part of the literature focuses on the information that job assignment and promotion decisions reveal. The manager may distort job assignments to influence information in three ways (Koch & Nafziger, 2012). The firm might want to influence the information about employees' that the company itself (Meyer, 1991) or outsiders receive (Waldman, 1984; Bernhardt, 1995). Job assignments and promotions can also reveal information to the employee himself. Since a positive decision signals a high ability, the employee is likely to put more effort or invest in costly human capital that only pays off with a high ability. Hence, firms might be inclined to put standards in probationary periods or for promotions lower, since the too low ability will be compensated by higher effort. For papers on this topic, see Ishida (2006), Nazfiger (2010) and Dominguez-Martinez & Swank (2011). I will not consider any of these informational effects of HRM strategies, because this will unnecessarily complicate the analysis without giving more information about the relevant factors under study.

The studies of Bayo-Moriones & Ortín-Ángel (2006) and DeVaro (2006) are examples of empirical evidence that firms indeed commit to promotion rules in order to incentivise firm-specific human capital investment. In this paper I develop the argument one step further by showing that a combination of rules and asymmetric discretion are optimal to achieve the best outcome when a firm faces the trade off between efficient job allocation and incentives. To the best of my knowledge no empirical papers investigate asymmetric discretion in promotion and job allocation decisions; much work still needs to be done in this field of research.

## 3. The Model

Consider a manager, who represents the firm, running a unit with one employee. Output of the unit depends solely on the ability of the employee, *a*. As explained before, I do not incorporate effort choice in the model, since it would unnecessarily complicate the analysis and would abstract from the important implications. This is because effort choice is affected as well by the HRM strategy of the firm and the discretion in the form of confidence management. The employee, however, can increase his productivity by investing in firm specific human capital. For reasons of simplicity, I will call this training of the employee. However, human specific human capital is a concept that applies to a broader range of investments than training alone. In case the employee does not invest in training his output is equal to *a* and when he does invest in training his output equals *ba* (with b > 1).

One can think of this model as a two-period model. In the first period the employee finds himself in a probation period. He makes an investment decision, i.e. he can either invest or not invest,  $I \in \{0,1\}$ . The employee does not know his own ability when he has to make his investment decision. He only knows that *a* is a random variable that is distributed between 0 and 1. To simplify matters, a discrete uniform distribution is used, where a can take the values 0, 1/3, 2/3 and 1, which all occur with a probability of 25 per cent. Hence,  $a \in \{0, \frac{1}{3}, \frac{2}{3}, 1\}$ . Investment is costly for the employee. The investments cost, *c*, is uniformly distributed on  $[0, \overline{c}]$ . During this period, both the manager and the employee observe the signal about the employees ability,  $y^1$ . The signal measures the output or ability imperfectly, giving rise to a measurement error,  $\varepsilon$ . This measurement error has a discrete uniform distribution as well and can be either 0, negative 1/3 or positive 1/3, i.e.  $\varepsilon \in \{-\frac{1}{3}, 0, \frac{1}{3}\}$ . The signal both players observe after period one is  $y = a + \varepsilon$ . After the first period, the manager also observes the employee's investment decision and, because of years of experience in the industry, the true ability of the employee. This makes *I* a verifiable signal, whereas *a* is observable but not verifiable. Also measured output, v, is verifiable.

In the second period the manager decides whether or not to keep the employee and sends him his decision,  $M \in \{1,0\}$ . He can base this decision on all information he has, i.e. *a*, *b*, *z*, *y* and *I*. However, if he wants to use a HRM strategy and commit to it, he has to stipulate rules based on verifiable signals, i.e. the investment decision, *I*, and the measured output, *y*. The manager can set so called minimum productivity levels the employee has to meet in order to be retained by the company. I will call these minimum productivity levels thresholds from here onwards. It is possible to set different thresholds for both outcomes of *I*. Committing to thresholds based on the ability of the employee is not possible however, since this information is not verifiable. Only the manager can observe the employee's ability.

If the manager decides to fire the employee he can hire a new employee. For simplicity it is assumed that the manager has neither time nor willingness to invest effort in learning the true ability of the new employee. To express the expected ability of this outside option,  $E(a^{new}|M = 0) = z$  is used, where E is the expected value operator. I also assume that a new employee will not invest in training. This assumption can be

<sup>&</sup>lt;sup>1</sup> y can as well be a signal for the agent's output in period one, based on his ability. This distinction in interpretation is irrelevant for the analysis.

defended by the notion that this new employee will only work one period for the firm. Hence, a training that takes one period to complete will be unlikely to pay off.

The output generated by the employee that eventually works for the unit in period two represents the payoff to the manager. Hence, the payoff equals  $v_I = b^I a$ , with b > 1 and  $I \in \{0,1\}$ . The payoff to the employee is equal to his own output as well, given that he is retained. However, the employee has to pay for the possible investments costs. Hence, his payoff equals  $u_I = b^I a - c^I$ , with b > 1,  $c \in [0, \bar{c}]$  and  $I \in \{0,1\}$ .

So all together, this model consists of the following stages. First, nature draws *a* and *b*. Second, the manager can set thresholds for retaining the employee, where he can vary the thresholds based on *I*. Afterwards, the worker observes *c* and makes his investment decision based on this. In the fourth stage both the manager and the employee observe the signal for output, *y*. The manager also observers *a* and determines his job assignment decision, possibly guided by the threshold rules. In the last stage the second-period payoffs are realized. This model will be solved using a Perfect Bayesian equilibrium.

The analysis is divided into three sections. First I consider a HRM policy where the manager cannot commit himself to certain thresholds. Secondly, I consider the cases where this is possible and the manager sets two different output thresholds for the employee to meet, both based on the imperfect output measurement. Lastly, I consider the effect of introducing the possibility for discretion regarding the HRM policy.

#### 3.1. No commitment

First I assume that the manager cannot commit to a specific HRM strategy based on measure *y*. In this case the model is easily solved. The manager observes the investment decision and ability of the employee and decides whether to retain or fire him. For this he will use a threshold strategy, taking into account the expected value of a new employee. If the ability of the employee is higher than threshold  $t_I^*$ , where  $I \in \{1,0\}$ , the manager keeps the employee. Otherwise, the employee is replaced. Given I = 0, the manager's output will be  $v_0 = a$ , given the observed *a*. Clearly, a new employee yields an expected payoff of *z*. Hence, the manager will retain the employee if  $v_0 \ge v_{new}$ , or to put differently, if  $a \ge z$ . Hence, the threshold will be  $t_0^* = z$ . The same holds if the employee does decide to invest, but the manager's output will be  $v_1 = ba$  in this case. Hence,  $t_1^* = \frac{z}{b}$ . Since a new employee cannot invest in productivity increasing training, the incumbent faces a lower threshold if he does invest in training, simply because his output given a certain ability will be higher.

Based on these thresholds, the employee will determine his investment decision. If he does decide to invest, his expected payoff is

$$u_1 = \Pr(M=1) \cdot E\left(ba \middle| a \ge \frac{z}{b}\right) + \Pr(M=0) \cdot 0 - c \tag{1}$$

where  $Pr(M = 1) = Pr(a \ge t_1^*)$  and  $Pr(M = 0) = Pr(a < t_1^*)$ . Consequently, if the agent does not invest, his payoff will be

$$u_0 = \Pr(M = 1) \cdot E(a|a \ge z) + \Pr(M = 0) \cdot 0$$
 (2)

where  $Pr(M = 1) = Pr(a \ge t_0^*)$  and  $Pr(M = 0) = Pr(a < t_0^*)$ . Let  $c^*$  be the cost of investment when the agent is indifferent between investing in training and not

investing. The agent chooses to invest in training if  $c < c^*$ . Remember that a has a discrete continuous distribution and, hence, this investment decision can only be solved for different combinations of z and b, which determine the threshold levels. For example,  $\Pr(a \ge t_0^*)$  will be 1/4 when z = 1, 1/2 when  $2/3 \le z < 1$ , 3/4 when  $1/3 \le z < 2/3$  and 1 when z < 1/3. To give an example of the calculations, assume that z = 1/2 and b = 2. Substituting we get

$$u_{1} = \Pr(a \ge 1/4) \cdot E(ba|a \ge 1/4) + \Pr(a < 1/4) \cdot 0 - c$$
  
=  $3/4 \cdot (2/3b) - c$   
=  $1/2b - c$ 

The same can be done for I = 0, giving a payoff equal to

$$u_{0} = \Pr(a \ge 1/2) \cdot E(a|a \ge 1/2) + \Pr(Ma < 1/2) \cdot 0$$
  
=  $1/2 \cdot 5/6$   
=  $5/12$ 

Solving for  $u_0 = u_1$  gives us

$$c^* = \frac{1}{2} b - \frac{5}{12}$$

The  $c^*$  for all combinations of z and b are given in Table 1 below. For clarity reasons I include the corresponding thresholds. Note that it is optimal to set  $t_0^* = z$  and  $t_1^* = \frac{z}{b}$ , but the exact number on the interval does not matter since these intervals lie between the possible abilities. If  $z = \frac{1}{2}$ , setting a  $t_0^* = \frac{1}{2}$  has the same effect as setting  $t_0^* = \frac{2}{3}$ , given any  $t_1^*$ .

Z	b	$t_0^*$	$t_1^*$	С*
$\frac{2}{3} < z \le 1$	$1 < b < \frac{3}{2}z$	$\frac{2}{3} < t_0^* \le 1$	$\frac{2}{3} < t_1^* < 1$	$\frac{1}{4}b - \frac{1}{4}$
$\frac{2}{3} < z \le 1$	$\frac{3}{2} z \le b < 3z$	$\frac{2}{3} < t_0^* \le 1$	$1/3 < t_1^* \le 2/3$	$\frac{5}{12}b - \frac{1}{4}$
$\frac{2}{3} < z \le 1$	$3z \leq b$	$\frac{2}{3} < t_0^* \le 1$	$0 < t_1^* \le \frac{1}{3}$	$\frac{1}{2}b - \frac{1}{4}$
$1/3 < z \leq 2/3$	1 < b < 3z	$1/3 < t_0^* \le 2/3$	$1/3 < t_1^* \le 2/3$	$\frac{5}{12}b - \frac{5}{12}$
$1/3 < z \leq 2/3$	$b \ge 3z$	$1/3 < t_0^* \le 2/3$	$0 < t_1^* \le \frac{1}{3}$	$\frac{1}{2}b - \frac{5}{12}$
$0 < z \leq \frac{1}{3}$	b > 1	$0 < t_0^* \le \frac{1}{3}$	$0 < t_1^* \le \frac{1}{3}$	$\frac{1}{2}b - \frac{1}{2}$
z = 0	b > 1	$t_{0}^{*} = 0$	$t_1^*=0$	$\frac{1}{2}b - \frac{1}{2}$

Table 1

Given this *c*\*, it is possible to determine the payoff of the manager. This is equal to

$$v = \Pr(l = 0) \cdot (\Pr(a < t_0^*) \cdot z + \Pr(a \ge t_0^*) \cdot E[a|a \ge t_0^*]) + \Pr(l = 1) \cdot (\Pr(a < t_1^*) \cdot z + \Pr(a \ge t_1^*) \cdot E[ba|a \ge t_1^*])$$
(3)

where  $\Pr(I = 1) = \frac{c^*}{\bar{c}}$  and  $\Pr(I = 0) = \frac{(1 - c^*)}{\bar{c}} = 1 - \frac{c^*}{\bar{c}}$ . Note that when

 $\bar{c} \leq c^*$ ,  $\Pr(l = 1) = 1$  and  $\Pr(l = 0) = 0$ . When the maximum investment cost is sufficiently small, i.e. it is lower that the indifference point, every agent will invest in training. For the purposes of this paper, it is especially interesting when some of the employees will invest in training and some of them will not. If this is not the case, the choice of  $t_0^*$  is not particularly important. Hence, I assume that  $\bar{c} \ge c^*$ holds.

Again, I use the case where  $z = \frac{1}{2}$  and b = 2 as an example to illustrate the calculations. In this case, the manager's payoff is

$$v = \left(1 - \frac{(1/2b - 5/12)}{c}\right) \cdot \left(\Pr(a < 1/2) \cdot z + \Pr(a \ge 1/2) \cdot E[a|a \ge 1/2]\right) + \left(\frac{(1/2b - 5/12)}{c}\right) \cdot \left(\Pr(a < 1/4) \cdot z + \Pr(a \ge 1/4) \cdot E[ba|a \ge 1/4]\right)$$
$$= \left(1 - \frac{(1/2b - 5/12)}{c}\right) \cdot \left(1/2z + 1/2 \cdot (5/6)\right) + \left(\frac{(1/2b - 5/12)}{c}\right) \cdot \left(1/4z + 3/4 \cdot (2/3b)\right)$$
$$= \frac{1}{2}z + \frac{5}{12} + \frac{1}{c} \cdot (1/2b - 5/12) \cdot (1/2b - 5/12 - 1/4z)$$

The expected payoff for other sets of *z* and *b* follow the same intuition. They can be found in Table 2.

Table 2
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Tuble 2		
Z	b	V
$\frac{2}{3} < z \le 1$	$1 < b < \frac{3}{2}z$	$3/4z + 1/4 + 1/\overline{c} \cdot (1/4b - 1/4) \cdot (1/4b - 1/4)$
$\frac{2}{3} < z \le 1$	$\frac{3}{2}z \le b < 3z$	$\frac{3}{4}z + \frac{1}{4} + \frac{1}{\bar{c}} \cdot \left(\frac{5}{12}b - \frac{1}{4}\right)$
		$\cdot \left( \frac{5}{12} b - \frac{1}{4} - \frac{1}{4} z \right)$
$\frac{2}{3} < z \le 1$	$3z \leq b$	$\frac{3}{4}z + \frac{1}{4} + \frac{1}{\bar{c}} \cdot (\frac{1}{2}b - \frac{1}{4})$
		$\cdot (\frac{1}{2}b - \frac{1}{4} - \frac{1}{2}z)$
$\frac{1}{3} < z$	1 < b < 3z	$\frac{1}{2}z + \frac{5}{12} + \frac{1}{\bar{c}} \cdot \left(\frac{5}{12}b - \frac{5}{12}\right) \cdot \left(\frac{5}{12}b - \frac{5}{12}\right)$
$\leq \frac{2}{3}$		
$\frac{1}{3} < z$	$b \ge 3z$	$\frac{1}{2}z + \frac{5}{12} + \frac{1}{\bar{c}} \cdot \left(\frac{1}{2}b - \frac{5}{12}\right)$
$\leq 2/3$		$\cdot \left(\frac{1}{2}b - \frac{5}{12} - \frac{1}{4}z\right)$
$0 < z \leq \frac{1}{3}$	b > 1	$\frac{1}{4}z + \frac{1}{2} + \frac{1}{\bar{c}} \cdot (\frac{1}{2}b - \frac{1}{2}) \cdot (\frac{1}{2}b - \frac{1}{2})$

z = 0	b > 1	$\frac{1}{2} + \frac{1}{\bar{c}} \cdot (\frac{1}{2}b - \frac{1}{2}) \cdot (\frac{1}{2}b - \frac{1}{2})$
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Clearly, the expected payoffs are increasing in both *b* and *z*. These are logical results, since a higher *b* implies that the investment is worth more and the employee is more likely to invest. A higher expected value for the outside option increases the minimum in the equation.

So, when a manager cannot commit to a HRM strategy, he will simply observe the investment decision by and the ability of the employee after period 1. After comparing this to his outside option, i.e. a new employee, he will determine whether to keep or fire this employee. His expected payoff given the outside option, *z*, and the return on the investment, *b*, are given in Table 2.

#### 3.2. Commitment

Next I consider the possibility for the manager to commit to a specific HRM strategy, where he stipulates the two thresholds before he observes the employee's ability and investment decision. Since ability is not a verifiable measure, these thresholds need to be based on the measured output. As stated already, I assume this measurement to be imperfect, i.e. a measurement error  $\varepsilon$  exists. Hence, y is not a perfect measure for ability *a*, nor for output. Stipulating and committing to these thresholds can, however, increase the likelihood of the employee investing in training. Decreasing  $t_1^*$ relative to Z/h increases the chance that an agent will be retained when he invests. Hence, the likelihood that an agent invests will increase. The same effect can be created by increasing  $t_0^*$  relative to z, which is the optimal level for this threshold under a situation without commitment. Making it less likely that an employee that did not invest makes the threshold will make investment more attractive to the employee. These effects could make it optimal for the manager to commit to thresholds different to those found in section 1, despite the fact that he has to base those thresholds on the imperfect signal y. To see what happens, I first determine the likelihood of a certain ability a, given the measured output y. From Bayesian updating, it follows that

$$Pr(a = 0|y = 0) = \frac{2}{3}$$

$$Pr(a = \frac{1}{3}|y = 0) = \frac{1}{3}$$

$$Pr(a = 0|y = \frac{1}{3}) = \frac{1}{3}$$

$$Pr(a = \frac{1}{3}|y = \frac{1}{3}) = \frac{1}{3}$$

$$Pr(a = \frac{2}{3}|y = \frac{1}{3}) = \frac{1}{3}$$

$$Pr(a = \frac{1}{3}|y = \frac{2}{3}) = \frac{1}{3}$$

$$Pr(a = \frac{2}{3}|y = \frac{2}{3}) = \frac{1}{3}$$

$$Pr(a = \frac{1}{y} = \frac{2}{3}) = \frac{1}{3}$$

$$Pr(a = \frac{1}{y} = \frac{2}{3}) = \frac{1}{3}$$

$$Pr(a = \frac{2}{3}|y = 1) = \frac{1}{3}$$

$$Pr(a = \frac{1}{y} = 1) = \frac{2}{3}$$

Note that a negative output or an output larger than one makes no sense given the possible abilities. Hence, a negative measured output will be regarded as a zero output with a = 0 and an output larger than one will be regarded as an output of one and a = 1.

With these probabilities, the investment decision *I* can be analysed. This has to be done for every set of possible thresholds. Note that although the thresholds can take any value between 0 and 1, it makes no sense to put them between any of the four possible output outcomes: 0, 1/3, 2/3, 1. Any threshold between 1/3 and 2/3 will have the same effect as stating the threshold at 2/3. Moreover, stipulating a HRM strategy where  $t_0^* > t_1^*$  makes no sense as well, since no manager will deliberately discourage his employee to make a positive investment decision. Therefore, we are left with the following set of thresholds:

$$t_{0}^{*} = 1$$
 with  $t_{1}^{*} = 1$ ,  
 $t_{1}^{*} = \frac{2}{3}$ ,  
 $t_{1}^{*} = \frac{1}{3}$  or  $t_{1}^{*} = 0$   
 $t_{0}^{*} = \frac{2}{3}$  with  $t_{1}^{*} = \frac{2}{3}$ ,  
 $t_{1}^{*} = \frac{1}{3}$  or  $t_{1}^{*} = 0$   
 $t_{0}^{*} = \frac{1}{3}$  with  $t_{1}^{*} = \frac{1}{3}$  or  $t_{1}^{*} = 0$   
 $t_{0}^{*} = 0$  with  $t_{1}^{*} = 0$ 

Because every set of thresholds will lead to a different investment decision, all possible cases need to be considered. For example, given  $t_0^* = 1$  and  $t_1^* = 1$  the employee's payoff when he invests can be determined.

$$u_{1} = \Pr(y \ge t_{1}^{*}) \cdot E[ba|y \ge t_{1}^{*}] + \Pr(y < t_{1}^{*}) \cdot 0 - c$$
(3)  
=  $\Pr(y = 1) \cdot E[ba|y = 1] + \Pr(y < 1) \cdot 0 - c$   
=  $\frac{2}{9}b - c$ 

The same can be done when I = 0. This leads to the following equation

$$u_{0} = \Pr(y \ge t_{0}^{*}) \cdot E[a|y \ge t_{0}^{*}] + \Pr(y < t_{0}^{*}) \cdot 0$$

$$= \Pr(y = 1) \cdot E[a|y = 1] + \Pr(y < 1) \cdot 0$$

$$= \frac{2}{9}$$
(4)

Equating both payoffs gives

$$c^* = \frac{2}{9}b - \frac{2}{9}$$

The agent chooses to invest in training if  $c < c^*$  and chooses not to invest otherwise. The calculations for every  $c^*$  and its corresponding set of thresholds follow according to the example above. The results are shown below in Table 3.

Table	Table 3				
$t_0^*$	$t_1^*$	С*			
1	1	$\frac{2}{9}b - \frac{2}{9}$			
1	$^{2}/_{3}$	$\frac{7}{18}b - \frac{2}{9}$			
1	1/3	$\frac{17}{36}b - \frac{2}{9}$			
1	0	$\frac{1}{2}b - \frac{2}{9}$			
$^{2}/_{3}$	$^{2}/_{3}$	$\frac{7}{18}b - \frac{7}{18}$			
$^{2}/_{3}$	$^{1}/_{3}$	$\frac{17}{36}b - \frac{7}{18}$			
$^{2}/_{3}$	0	$\frac{1}{2}b - \frac{7}{18}$			
1/3	$^{1}/_{3}$	$\frac{17}{36}b - \frac{17}{36}b$			
$\frac{1}{3}$	0	$\frac{1}{2}b - \frac{17}{36}$			
0	0	$\frac{1}{2}b - \frac{1}{2}$			

Several things can be observed from Table 1. First,  $c^*$  increases when  $t_1^*$  decreases given a certain  $t_0^*$ . To put differently, the agent is more likely to invest in training when the threshold for employees that invest is lower, given a certain threshold for non-investing employees. Moreover, the benefits of a lower  $t_1^*$  increase in b, the benefits of the training. Both effects occur at a decreasing rate. Lastly, the agent is less likely to invest if the threshold when I = 0 is lower, given a certain  $t_1^*$ . The overall effect is that the higher the spread between  $t_1^*$  and  $t_0^*$ , the more likely the agent is to invest in training.

With these indifference costs, it is possible to calculate the expected payoff for the manager for every set of thresholds. When these thresholds are known, it will be easy to see what HRM strategy maximises the expected payoff to the manager. The expected payoff is again equal to equation (3)

$$v = \Pr(I = 0) \cdot (\Pr(y < t_0^*) \cdot z + \Pr(y \ge t_0^*) \cdot E[a|y \ge t_0^*]) + \Pr(I = 1) \cdot (\Pr(y < t_1^*) \cdot z + \Pr(y \ge t_1^*) \cdot E[ba|y \ge t_1^*])$$

where  $\Pr(I = 1) = \frac{c^*}{\bar{c}}$  and  $\Pr(I = 0) = \frac{(1 - c^*)}{\bar{c}} = 1 - \frac{c^*}{\bar{c}}$ . Again it is assumed that

 $\bar{c} \ge c^*$  holds. For  $t_0^* = 1$  and  $t_1^* = 1$ , the manager's expected payoff is

$$v = \Pr(I = 0) \cdot (\Pr(y < 1) \cdot z + \Pr(y \ge 1) \cdot E[a|y \ge 1]) + \Pr(I = 1) \cdot (\Pr(y < 1) \cdot z + \Pr(y \ge 1) \cdot E[ba|y \ge 1]$$
  
=  $\Pr(I = 0) \cdot (\frac{3}{4}z + \frac{2}{9}) + \Pr(I = 1) \cdot (\frac{3}{4}z + \frac{2}{9}b)$   
=  $\frac{3}{4}z + \frac{2}{9} + \frac{c^*}{c} \cdot (\frac{2}{9}b - \frac{2}{9})$ 

Substituting *c*\* gives

$$v = \frac{3}{4}z + \frac{2}{9} + \frac{1}{\bar{c}} \cdot \frac{2}{9}b - \frac{2}{9} \cdot \frac{2}{9}b - \frac{2}{9}$$

The calculations for the other set of thresholds follow in the same way. The outcomes for all calculations are given in Table 4 below.

Tabl	e 4	
$t_0^*$	$t_1^*$	V
1	1	$\frac{3}{4}z + \frac{2}{9} + \frac{1}{\bar{c}} \cdot \frac{2}{9}b - \frac{2}{9} \cdot \frac{2}{9}b - \frac{2}{9}$
1	$^{2}/_{3}$	$\frac{3}{4}z + \frac{2}{9} + \frac{1}{\bar{c}} \cdot (\frac{7}{18}b - \frac{2}{9}) \cdot (\frac{7}{18}b - \frac{2}{9} - \frac{1}{4}z)$
1	$1/_{3}$	$\frac{3}{4}z + \frac{2}{9} + \frac{1}{\bar{c}} \cdot (\frac{17}{36}b - \frac{2}{9}) \cdot (\frac{17}{36}b - \frac{2}{9} - \frac{1}{2}z)$
1	0	$\frac{3}{4}z + \frac{2}{9} + \frac{1}{\bar{c}} \cdot (\frac{1}{2}b - \frac{2}{9}) \cdot (\frac{1}{2}b - \frac{2}{9} - \frac{3}{4}z)$
$^{2}/_{3}$	$^{2}/_{3}$	$\frac{1}{2}z + \frac{7}{18} + \frac{1}{\bar{c}} \cdot (\frac{7}{18}b - \frac{7}{18}) \cdot (\frac{7}{18}b - \frac{7}{18})$
$^{2}/_{3}$	$1/_{3}$	$\frac{1}{2}z + \frac{7}{18} + \frac{1}{\bar{c}} \cdot (\frac{17}{36}b - \frac{7}{18}) \cdot (\frac{17}{36}b - \frac{7}{18} - \frac{1}{4}z)$
$^{2}/_{3}$	0	$\frac{1}{2}z + \frac{7}{18} + \frac{1}{\bar{c}} \cdot (\frac{1}{2}b - \frac{7}{18}) \cdot (\frac{1}{2}b - \frac{7}{18} - \frac{1}{2}z)$
1/3	$1/_{3}$	$\frac{1}{4}z + \frac{17}{36} + \frac{1}{\bar{c}} \cdot (\frac{17}{36}b - \frac{17}{36}) \cdot (\frac{17}{36}b - \frac{17}{36})$
$\frac{1}{3}$	0	$\frac{1}{4}z + \frac{17}{36} + \frac{1}{\bar{c}} \cdot (\frac{1}{2}b - \frac{17}{36}) \cdot (\frac{1}{2}b - \frac{17}{36} - \frac{1}{4}z)$
0	0	$\frac{1}{2} + \frac{1}{\bar{c}} \cdot (\frac{1}{2}b - \frac{1}{2}) \cdot (\frac{1}{2}b - \frac{1}{2})$

The payoffs in Table 4 give insight in some crucial and important tendencies. To analyse the different effects, one can best split up the payoffs in three parts; the expected payoff when no employee invests, the chance that an employee invests and the expected value of this investment to the manager. This can best be seen be rewriting equation (5) into

 $v = (\Pr(y < t_0^*) \cdot z + \Pr(y \ge t_0^*) \cdot E[a|y \ge t_0^*]) +$  $\Pr(I = 1) \cdot (\Pr(y \ge t_1^*) \cdot E[ba|y \ge t_1^*] - \Pr(y \ge t_0^*) \cdot E[a|y \ge t_0^*] + (\Pr(y < t_1^*) - \Pr(y < t_0^*)) \cdot z)$ 

 $\Pr(y < t_0^*) \cdot z + \Pr(y \ge t_0^*) \cdot E[a|y \ge t_0^*]$  represents the expected payoff when no employee invests,  $\Pr(I = 1) = \frac{c^*}{c}$  does the same for the probability that the employee does invests and the net expected value of this investment to the manager is equal to  $\Pr(y \ge t_1^*) \cdot E[ba|y \ge t_1^*] - \Pr(y \ge t_0^*) \cdot E[a|y \ge t_0^*] + (\Pr(y < t_1^*) - \Pr(y < t_0^*)) \cdot z$ All three parts have their own influence on the expected payoff to the manager, depending on the outside option, *z*, the value of the investment, *b*, the maximum cost of the investment,  $\bar{c}$ , and thresholds  $t_0^*$  and  $t_1^*$ .

The expected value to the manager when the employee never invests depends on both the outside option and his choice of thresholds. The first thing that should be noted is that the higher the outside option, *z*, the higher the value of this first part is. The expected value when the employee never invests is increasing in *z*. Furthermore, one can see that this expected value consists of a trade off between two choices. The first term in the equation is the value of hiring a new employee times the probability that a new employee has to be hired. Obviously, this value goes down when  $t_0^*$  decreases; the lower the threshold for non-investing employees, the lower the probability that the employee has to leave and a new one is hired. The magnitude of this effect is determined by the expected value of this new employee, *z*. Decreasing  $t_0^*$ , however, leads to an increase in the value of the second term. This term represents the expected value of keeping the employee when he did not invest. Lowering the threshold makes it more likely the employee is allowed to stay, leading to a higher expected value. However, since employees with a lower ability a are allowed to stay when the threshold is decreased, the expected value increases at a decreasing rate. The expected payoff to the manager of a non-investing employee depends on both these terms. Whether lowering  $t_0^*$  increases or decreases the expected value is determined by the trade off between both terms and, thus, by the expected value of the new employee z.

The probability that the agent invests in training is equal to the product between the indifference  $\cot c^*$  and the inverse of  $\bar{c}$ . Hence, the impact of different thresholds follows along the same lines as the one about the indifference  $\cot c^*$ , which has already been explained. The only difference is that this probability is inversely related to the maximum costs of the training. The higher these costs, the more likely the training will be too expensive for the employee to invest in. Hence, the probability of investing goes down when  $\bar{c}$  increases. Following the same reasoning, as the maximum possible cost of investment for the employee decreases, the more likely the choice for investment is.

Lastly, the expected value of investment for the manager follows largely the same patterns as the chance of investment. Decreasing  $t_1^*$  will increase the probability that an employee is retained. Hence, his extra expected value over the employee that does not invest, increases, as can be seen from the increase in the factor that *b* is multiplied with. As was also the case with the value for the non-investing employee, this factor increases at a decreasing rate, since lowering the  $t_1^*$  will make lower ability employees stay in the company. Investing has also opportunity costs however. To put simply, the opportunity costs of investment are the foregone benefits of having a non-investing employee or a new employee. It can be seen that this opportunity cost increases when  $t_0^*$  decreases, since a decrease in  $t_0^*$  increases the value of a non-investing employee. These opportunity costs, however, depend also on the difference in height between  $t_0^*$  and  $t_1^*$ . Lowering a threshold implies that there is a lower probability a new employee is hired, lowering the expected value of this outside option. If the two thresholds are equal, the chances of hiring a new employee are equal as well for both I = 1 and I = 0. Hence, investment will not lead to an opportunity cost in terms of having a lower probability that a new employee is hired when the employee invests. When the threshold for I = 1starts to drop in comparison to  $t_0^*$ , the opportunity cost increases due to a lower expected value of the outside option in the investment state relative to the noninvestment state. All in all, lowering  $t_1^*$  leads to an increase in the gross investment value, but increases opportunity costs as well. This can be counterfeited by lowering  $t_0^*$ . but this increases the opportunity costs by a higher foregone expected value of a noninvesting employee. The overall effect of the two thresholds on the value of investment depends on both *b* and *z*.

As can be seen, the effect that the thresholds have on the expected payoff to the manager is determined by z, b and  $\bar{c}$  in many ways. The easiest way to observe the effect of the various variables is to take a look at certain cases. First, let us see what happens when the expected value of a new employee is equal to the average of all available abilities, i.e. z = 1/2. Furthermore, the maximum cost of investment,  $\bar{c}$ , is assumed to be equal to 3. With this value, the constraint  $\bar{c} \ge c^*$  is certain to be satisfied for this range of b. Later on, I will investigate what happens when  $\bar{c}$  takes on higher values. For these values, the optimal HRM strategies for different returns to investment are given in Graph 1. In order to make this graph orderly and accessible, I numbered the available set of thresholds from 1 to 10, in the order they are displayed in the tables. For example, number 6 corresponds with a HRM strategy that sets  $t_0^* = 2/3$  and  $t_0^* = 1/3$ . The graphs are not on scale; they only serve to make patterns visible. Furthermore, I choose to only

investigate the strategies for values of *b* up to 4. Employees that invest are 4 times more productive compared to non-investing employees at this maximum value. Higher returns to investment seem unlikely. Moreover, considering even higher values of *b* does not add anything to the findings. It even distorts the analysis, because  $\bar{c} \ge c^*$  will not hold at very high values of *b*. Every employee, irrespective of the investment cost he faces, will invest in training. For those reason I decided to limit the analysis to  $b \in (1,4]$ . The position of the indifference points are given by numbers that round off, since this benefits the main analysis. For exact numbers, also for other indifference points in the rest of the paper, I refer to the Appendix.

Graph 1

$$\begin{array}{c}z''=''1/2''\\ \bar{c}''='3''\\b'' + & & \\1'' & 1.12'' & 2.24'' & 2.64'' & 4''\\ & 5'' & 6'' & 3'' & 4''\end{array}$$

As can be seen from this graph, the optimal strategy when investment incurs almost no productivity gains is to put both thresholds above the outside option of a  $1/_2$ . This makes sense, because allowing employees with a lower output to stay, makes it likely that employees with a too low ability are retained, especially when taking into account the possible measurement error. However, as b rises, it becomes quickly efficient to put  $t_1^*$  on 1/3, keeping  $t_0^*$  on 2/3. For 1.12 < b < 3/2 this implies that it is more efficient to put  $t_1^*$  below the outside option  $\frac{z}{b}$  and keep  $t_0^*$  above z. The job assignment decision is distorted to induce the employee to invest. For  $3/2 \le b \le 2.24$ , the outside option drops below the threshold of 1/3. The value of the investment is not high enough in this range to justify a corresponding decrease in the threshold. This changes from a *b* of 2.24 onwards. From this value it is optimal to make the gap between the thresholds wider in order to reward investment even more. Interestingly, it is not  $t_1^*$ that has to be lowered, as one might expect since the expected value of the alternative option for firing an investing employee decreases with b. Instead, it is payoff maximizing to increase  $t_0^*$  to 1, far above the expected value of the outside option of 1/2. The manager commits to a contract that makes it relatively very difficult for a non-investing employee to meet the standard when he does not invest.

To see why it may be optimal to increase  $t_0^*$  instead of lowering  $t_1^*$ , remember that an employee with a measured output of y = 0, has a 2/3 probability of having an actual ability of 0. These employees are worthless to the firm, even if they have invested. Only a very high increase in productivity after investment would justify decreasing the threshold to 0, since at the margin only 1/3 of the employees now allowed to stay benefit the firm. It appears that every *b* above 2,64 satisfies this criterion. This productivity increase justifies a maximum spread in thresholds to reward investment, even if this implies that there is a high change that ex post an employee is retained that should have been fired or one that is fired but should have been kept. These are the optimal HRM strategies for values of *b* up till 4, given the assumptions that z = 1/2 and  $\bar{c} = 3$ . It is interesting to see what happens to the optimal strategies when these assumptions are altered. Let's first assume that the maximum cost of investment is higher, suppose 5. The optimal HRM strategies for various values of *b*, are displayed in Graph 2.

Graph 2



A higher  $\bar{c}$  makes it less likely that investment is profitable for employees and, correspondingly, lowers the probability that the employee invests. For low values of *b*, this does not change anything to the optimal HRM strategy. Both thresholds are initially set at  $\frac{2}{3}$  and  $t_1^*$  is optimally lowered to  $\frac{1}{3}$  when *b* increases above 1.12, as was the case under the assumption of a lower  $\bar{c}$ . However, when b is even higher, it is not optimal any more to have a higher  $t_0^*$ . Instead it is optimal to keep the threshold for noninvesting employees and set  $t_1^* = 0$ . So for a higher  $\bar{c}$  it is too costly to set a higher  $t_0^*$ , since the chance that an employee will invest is lower and, hence, the proportion of employees that would have an ability high enough ex post but that have to be fired because they did not invest is higher. The incentive for investment is strengthened by lowering the threshold for investing employees instead. Note that this is done at a higher b compared to the previous situation, since the optimal switching point to the new set of thresholds is at 2.53 instead of 2.24. Looking at the payoff functions for the manager, a higher  $\bar{c}$  reduces the relative importance of the investment, leading to a change in optimal strategies and to managers being more conservative in giving incentives for investment. This holds also for giving the maximum incentive possible, i.e.  $t_1^* = 0$  and  $t_0^* = 1$ . This spread is now only justified by a much larger return on investment, since 3.11 > 2.64.

So a higher maximum investment cost will make the manager be more conservative in giving incentives for investments. Moreover, he will let these incentives first come from a lower threshold for investing employees, before increasing the threshold for non-investing employees.

Not only the maximum investment cost will change the optimal HRM strategy for the manager. The expected ability of a new employee also has its influence. Therefore, I first consider the situation of relative high ability outside workers in Graph 3 and 4. For this case I use  $z = \frac{2}{3}$ . Next, I will consider a situation where the expected ability of new employees is relatively low, i.e.  $z = \frac{1}{3}$ . The optimal strategies for this situation can be seen in Graph 5, 6 and 7.





Given  $\bar{c} = 3$ , a relative high z of  $\frac{2}{3}$  changes the optimal strategy for every return on investment quite a lot. For low values of *b* it is optimal to set  $t_1^* = \frac{2}{3}$  and  $t_0^* = 1$ . So,  $t_1^*$  is set slightly above the value of the outside option. It is not worth it to stipulate a lower threshold for investing employees, because of the discrete distribution of ability and output. The first value that a lower threshold can take is 1/3, which is apparently not justified by the value and chance of a possible investment. An incentive for investment is given though, by setting the threshold for non-investing employees higher, i.e.  $t_0^* = 1$ . Note that this not only incentives employees to invest, it also makes perfectly sense given the measurement error. Setting the threshold at 1 ensures the manager it will only keep non-investing employees with an ability of at least 2/3. A lower threshold would give rise to a probability of having an employee with an ability of 1/3, which is lower than the expected ability of a new employee. The consequence of this value for  $t_0^*$ , however, is that there is a chance the manager has to fire an employee with an ability of  $^{2}/_{3}$ . This does not matter for the manager since he replaces an employee who does not have any valuable firm-specific human capital and because the new employee has an expected ability equal to that of the fired employee. Hence, setting  $t_0^* = 1$  does not cause any costs for the manager.

Since  $t_0^*$  is already at its maximum, a manager that want to further incentivise his employee to invest in training must lower  $t_1^*$ . As is visible in Graph 3, this is indeed optimal at higher returns. From b = 1.45 onwards it is optimal to set a lower threshold, i.e.  $t_1^* = \frac{1}{3}$ . Note that this is lower than the value of the outside option for b < 2, so the threshold for investing is relatively low. For a higher *b* this is not the case; the outside option is lower than this threshold on this range. The value of investment does not justify a lower threshold. As can be seen, only when the investing employees are more than 3.49 as productive as non-investing employees is a relatively easy standard justified. Hence, at levels of *b* higher than this 3.49 the spread between the thresholds is maximal.

Increasing  $\bar{c}$  does not change anything to the optimal HRM strategies in this case. Even the optimal switching points stay the same. The maximum height of the investment does not matter for determining the optimal strategy. At this value of z, i.e. 2/3 the expected value when no employee invests is equal for the threshold sets 2, 3 and 4. Hence, only the last part of the pay off strategies matters for the optimal strategy. Since all equations have the same  $\bar{c}$ , this value does not matter either for determining the optimal strategy and the switching point between the strategies.

Graph 4



I now have considered the cases with a relative high expected ability of a new employee. Next I will move to the opposite assumption; a relative low z of 1/3. For this value of z, I consider three cases with maximum investment values of 3, 4 and 5. I have chosen to include  $\bar{c} = 4$  as well, because it helps showing the effect of a higher  $\bar{c}$  properly.

The first case, when  $\bar{c}$  is relatively low, i.e.  $\bar{c} = 3$ , is depicted in Graph 5. As can be seen, the optimal HRM strategy at low returns to investment in training for the manager is to set the threshold for investing employees slightly higher than the outside option,  $t_1^* = \frac{1}{3}$ . Putting a relatively low threshold for an employee that invests is not justified by the benefits, for the same reason as it was when  $z = \frac{1}{2}$  and  $\bar{c} = 3$ ; an even lower threshold increases the chance that employees with an ability of 0 are allowed to stay. These employees are worthless to the firm, even when they invested. However, giving incentives for investment is optimal though, even at low levels of *b*. This incentive stems from setting a standard for non-investing employees higher than the outside option. Hence,  $t_0^*$  is optimally set at  $\frac{2}{3}$ . When *b* is high enough, higher than 1.74 in this case, it is optimal to increase this incentive by setting the lowest threshold possible for agents that have invested in training. At last it is optimal to set  $t_0^*$  at its maximum when b > 2.89.

Graph 5



These optimal set of thresholds stay mostly the same when the maximum cost of investment,  $\bar{c}$ , increases. At  $\bar{c} = 4$ , the strategies for low values of *b* and the switching

point between strategy 6 and 7 at 1.74 stay the same. The lower probability of the agent investing in training is only reflected by a higher optimal switching point from strategy 7 to 4. When  $\bar{c}$  is even higher, this optimal switching point takes on an even higher value as well; for this value of  $\bar{c}$  it lies even out of the chosen range of  $b \in (1,4]$ .



Graph 6

This all shows us that if the manager has the possibility to commit to a specific HRM strategy, it is optimal to do so. The manager will give incentives for investment by setting a relative low  $t_1^*$ , a relative high  $t_0^*$ , or both. If such incentives would not be optimal, we would have found that the optimal thresholds were similar to those in section 3.2, where no commitment is possible. Which one of the incentives are optimal to give depend on the value of investment, the expected ability of a new employee and the maximum cost of investment. With a relative high z, the incentive is given at first by a higher  $t_0^*$ . When this incentive is not sufficient any more because of very high returns to investment,  $t_1^*$  is increased. When z decreases to exactly a half, the optimal strategy is to start setting a slightly lower  $t_1^*$  and a slightly higher  $t_0^*$ . For higher values of b the incentive is increased by increasing  $t_0^*$  first before lowering  $t_1^*$ , given low values of  $\bar{c}$ . The opposite is the case when  $\bar{c}$  is high; first  $t_1^*$  is lowered, before increasing  $t_0^*$ . Lastly, when z is relatively low, the incentive at low values of b stems from a higher threshold for non-investing employees. Then,  $t_1^*$  is lowered and when that is not sufficient any more,  $t_0^*$  is set at its maximum.

### 3.3. Discretion

#### 3.3.1 General model with discretion

As seen above, committing to a HRM strategy with thresholds can increase the manager's payoff by inducing the employee to invest in training. Ex post, respecting the contract might not be optimal in individual cases, but the manager has to in order to

prevent the incentive given by thresholds from breaking down. However, using imperfect measures for these thresholds come at an even greater cost for the manager; he might learn after period one that the agent meets the threshold according to the imperfect measure, but that he does not according to his true ability. Respecting the HRM strategy is even more costly in this case. Another example is that an agent does not meet the threshold due to the measurement error, whereas his true ability is high. The manger faces a dilemma; he can commit to the strategy to induce investment. However, this can be very costly because he has to base thresholds on a verifiable, but imperfect, measure. Not respecting these thresholds, however, whenever the measurement turns out to not reflect ability truthfully, will cause the HRM strategy to break down. In this case the situation will move back to model 1, where the manager cannot commit to any HRM strategy.

We have already seen that commitment is still optimal. But there might be a better solution however. Suppose the agent invested, so I = 1. According to the measured signal, he does not meet the threshold and needs to be fired. However, the manager learns that the agent's true ability is high, even so high that the true ability is higher than that of the outside option. This very situation is displayed below in Graph 8.

Graph 8



In this case the manager can build a clause allowing discretion in the contract. Ex post, the manager has an incentive to retain the agent, albeit  $y < t_1^*$ . Doing so, would not influence the incentives coming from threshold  $t_1^*$ , since the agent would not have made it anyways based on the measured signal y. Agents that would have made the threshold, are still retained. It only gives the agent that invested and has a higher ability than the outside option, the chance to stay in the company, despite he did not make the threshold due to a measurement error. To put simply, it can only benefit agents that have invested and, hence, makes the incentive to invest even larger, not smaller. Note that this is the only case the manager can apply this kind of discretion when I = 1. Not committing to the contract and firing employees that made the threshold, but who have a lower true ability than the outside option or even the threshold, is not possible. Ex post this might be optimal, but it will cause the whole HRM strategy with investment incentives to break down. Hence, the discretion the manager can use is asymmetrical.

Now suppose that the agent has not invested. In this case ex post discretion is possible as well. Consider the situation displayed in Graph 9 below.



Graph 9



In this situation the agent passes the threshold, but the manager learns after period one that the true ability of the agent is lower than the outside option. In this case, the manager has the possibility to deviate from his threshold rule, for two reasons. First, the manager has an incentive to deviate ex post; the expected value of a new employee is higher than the value of keeping the employee. The other reason is that deviation will not cause his HRM strategy to break down. Recall that the incentives coming from this strategy originate from the fact that it is more likely to be retained for the firm in case the agent invested relative to the situation where he did not invest. So, putting an extra constraint on the hiring decision when I = 0 does not cause this incentive to break down. On the contrary, it makes not investing potentially even more costly for the employee.

Note that for I = 0 this is the only potential case where discretion is possible. Deciding to keep an employee that did not make  $t_0^*$ , but whose true ability lies above the outside option (or even the threshold) does not satisfy the requirements that make discretion possible. It is true that the manager has an incentive to keep the employee ex post, but doing so would make the threshold useless. Hence, discretion is asymmetrical for I = 0 as well. Asymmetric discretion can provide not only a way to solve the extra costs stemming from the measurement error, it can even increase the incentive for investment.

To assess the effect of these two possibilities for discretion on the manager's payoff and to observe if allowing asymmetric discretion would be profit maximizing, we have to start with the agent's payoff. A rational agent will take into account the effect of the possibility of asymmetric discretion on his payoffs and will adjust his investment decision accordingly. His payoff when he decides to invest will now be

$$u_{1} = \Pr(y \ge t_{1}^{*}) \cdot E[ba|y \ge t_{1}^{*}] + \Pr(y < t_{1}^{*}) \cdot (\Pr(a \ge \frac{z}{b}|y < t_{1}^{*})) \cdot E[ba|a \ge \frac{z}{b}, y < t_{1}^{*}] + \Pr(a < \frac{z}{b}|y < t_{1}^{*}) \cdot 0) - c$$
(5)

and

$$u_{0} = \Pr(y \ge t_{0}^{*}) \cdot (\Pr(a \ge z | y \ge t_{0}^{*}) \cdot E[a | a \ge z, y \ge t_{0}^{*}] + \Pr(a < z | y \ge t_{0}^{*}) \cdot 0) + \Pr(y < t_{0}^{*}) \cdot 0$$
(6)

#### when I = 0.

Solving for  $u_1 = u_0$  will give  $c^*$ . However, it should be noted that some of the probabilities depend on the level of z and b relative to the true ability of the employee. Because a has a discrete distribution it is not possible to give a general solution. The model needs to be solved for various sets of z and b to compare with the outcomes of section 2 and to see the effect of discretion. I will come back to specific cases later in this section.

Given these set of investment decisions, it is again possible to compute the payoff to the manager. If the manager applies his possibility of discretion, his expected payoff will be

$$v = \Pr(I = 0) \cdot$$

$$\left(\Pr(y < t_0^*) \cdot z + \Pr(y \ge t_0^*) \cdot \left( \begin{array}{c} \Pr(a \ge z | y \ge t_0^*) \cdot E[a | a \ge z, y \ge t_0^*] \\ + \Pr(a < z | y \ge t_0^*) \cdot z \end{array} \right) \right) + \Pr(I = 1)$$

$$\cdot \left( \frac{\Pr(y < t_1^*) \cdot \left(\Pr\left(a \ge z/_b | y < t_1^*\right) \cdot E\left[ba | a \ge z/_b, y < t_1^*\right] + \Pr\left(a < z/_b | y < t_1^*\right) \cdot z\right)}{+ \Pr(y \ge t_1^*) \cdot E[ba | y \ge t_1^*]} \right)$$

$$(7)$$

where  $\Pr(I = 0) = 1 - \frac{c^*}{c}$  and  $\Pr(I = 1) = \frac{c^*}{c}$ . Again, it is assumed that  $\bar{c} \ge c^*$ . Note that the payoff again depends on specific levels of *z* and *b* relative to *a*. To see what the effect of discretion is on the optimal HRM strategy for the manager, we have to consider specific cases. This is done in the next sections. I will consider two cases, one with a relative low *z* and one with a relative high *z*.

## 3.3.2 Discretion with $0 < z \le \frac{1}{3}$

To assess the potential of discretion as part of the HRM strategy, specific cases have to be considered. As can be seen in section 2, the optimal thresholds to set by the manager when z is relatively low is a relative higher threshold for non-investing employees at 2/3. When a higher b justifies a larger incentive to invest,  $t_1^*$  is lowered. When that is not sufficient any more,  $t_0^*$  is set at its maximum.

This section analyses if it is optimal for the manager to use his potential for discretion and if this changes the optimal strategy found in section 2. Using  $0 < z \le 1/3$ , and given that b > 1, implies that the outside option for I = 0 lies on the interval (0, 1/3) and on (0, 1/3) for I = 1. With these sets of outside options, the investment decision of the employee can be determined for every set of threshold. The specific value of the outside option is not important for the investment decision within these intervals, since the distribution of the agent's ability is discrete. For the calculations given as examples in this section z = 1/3 and z/b = 1/6 are used, but any number in the given intervals can be chosen to determine the probability that the ability of the employee is higher than the expected outside option. To see this, remember that  $\Pr(a \ge 1/3) = \Pr(a \ge 1/6) = \Pr(a \ge 1/10)$  etcetera.

Again, consider the situation where  $t_0^* = 1$  and  $t_1^* = 1$ . Substituting in equations (5) and (6) gives

$$u_{1} = \Pr(y \ge 1) \cdot E[ba|y \ge 1] + \Pr(y < 1) \cdot (\Pr(a \ge 1/6|y < 1)) \cdot E[ba|a \ge 1/6, y < 1] + \Pr(a < 1/6|y < 1) \cdot 0) - c$$
$$= 1/2 b - c$$

and

$$u_{0} = \Pr(y \ge 1) \cdot (\Pr(a \ge \frac{1}{3} | y \ge 1) \cdot E[a|a \ge \frac{1}{3}, y \ge 1] + \Pr(a < \frac{1}{3} | y \ge t_{0}^{*}) \cdot 0) + \Pr(y < 1) \cdot 0 = \frac{2}{9}$$

Substituting for  $u_0 = u_1$  gives  $c^* = \frac{1}{2}b - \frac{2}{9}$ . These two payoffs give some interesting insights in the implications of discretion. Basically, The manager bases his decision whether to keep the employee only on *a*, which he learns after the first period, when the employee has invested. This is because everyone who passes  $t_1^*$  has an ability that is bigger than  $\frac{1}{3}$  for sure and the manager can use discretion if the employee does not meet the threshold but does have an ability of  $\frac{1}{3}$  or higher. This will not be the case for every low thresholds. On the other hand, the possibility of discretion has no effect on the payoff for the non-investing employee, also because no employee that met the threshold of 1 will have an ability lower than the outside option of  $\frac{1}{3}$ . This is again not the case for every threshold. For every set of thresholds,  $c^*$  is given in Table 5.

Table	e 5	
$t_0^*$	$t_1^*$	С*
1	1	$\frac{1}{2}b - \frac{2}{9}$
1	$^{2}/_{3}$	$\frac{1}{2}b - \frac{2}{9}$
1	$1/_{3}$	$\frac{1}{2}b - \frac{2}{9}$
1	0	$\frac{1}{2}b - \frac{2}{9}$
$^{2}/_{3}$	$^{2}/_{3}$	$\frac{1}{2}b - \frac{7}{18}$
$^{2}/_{3}$	$1/_{3}$	$\frac{1}{2}b - \frac{7}{18}$
$^{2}/_{3}$	0	$\frac{1}{2}b - \frac{7}{18}$
$\frac{1}{3}$	$^{1}/_{3}$	$\frac{1}{2}b - \frac{17}{36}$
$\frac{1}{3}$	0	$\frac{1}{2}b - \frac{17}{36}$
0	0	$\frac{1}{2}b - \frac{1}{2}$

The value of  $t_1^*$  seems to have no influence on the investment decision. For thresholds higher than zero, an agent that does not meet the threshold but that has an ability that is higher than the outside option will be retained anyways. For  $t_1^* = 0$ , discretion is not possible and, hence, the  $u_1$  does not change for this threshold. This means that the payoffs to the agent when he invests are higher for every threshold

except for zero. The value of  $t_0^*$  has the same effect as when no discretion is used. Discretion in the case that the employee did not invest is not possible when  $t_0^* \ge 2/3$  since all abilities of the employees that meet the threshold will be higher than the outside option. For thresholds of 1/3 and lower, the employee can be fired, although he met the threshold. This will not change his payoff however, because this will only be done when his ability is equal to zero and, correspondingly, his payoff would have been equal to zero anyways. All in all, the possibility of discretion does not change the payoffs to the non-investing employees. Given that these payoffs do not change and the payoffs to the investing employee rises, investing becomes more attractive and the chance that the employee invest rises.

The next step is to calculate payoff v to the manager for every set of thresholds given that he uses the possibility of discretion. As we will see, the effect of this discretion will be larger on his payoff compared to that to the employee. This is because the outside option of the manager is not zero, as contrary to the agent. Using again  $t_0^* = 1$  and  $t_1^* = 1$ to exemplify the calculations made, equation (7) gives

$$v = \Pr(I = 0) \cdot \left( \Pr(y < 1) \cdot z + \Pr(y \ge 1) \cdot \left( \frac{\Pr(a \ge \frac{1}{3} | y \ge 1) \cdot E[a|a \ge \frac{1}{3}, y \ge 1]}{+ \Pr(a < \frac{1}{3} | y \ge 1) \cdot z} \right) \right) + \Pr(I = 1) \cdot \left( \frac{\Pr(y < 1) \cdot \left( \Pr(a \ge \frac{1}{6} | y < 1) \cdot E[ba|a \ge \frac{1}{6}, y < 1] + \Pr(a < \frac{1}{6} | y < 1) \cdot z}{+ \Pr(y \ge 1) \cdot E[ba|y \ge 1]} \right) \right) + \Pr(I = 1) \cdot \left( \frac{1}{2} b + \frac{1}{4} z \right) = \frac{3}{4} z + \frac{2}{9} + \frac{c^*}{c} \cdot \left( \frac{1}{2} b - \frac{2}{9} - \frac{1}{2} z \right) \right)$$

Substituting *c*<sup>\*</sup> gives

$$v = \frac{3}{4}z + \frac{2}{9} + \frac{1}{\bar{c}} \cdot \left(\frac{1}{2}b - \frac{2}{9}\right) \cdot \left(\frac{1}{2}b - \frac{2}{9} - \frac{1}{2}z\right)$$

The whole set of payoffs can be found in Table 6 below.

Table	e 6	
$t_0^*$	$t_1^*$	V
1	1	$\frac{3}{4}z + \frac{2}{9} + \frac{1}{\bar{c}} \cdot (\frac{1}{2}b - \frac{2}{9}) \cdot (\frac{1}{2}b - \frac{2}{9} - \frac{1}{2}z)$
1	$^{2}/_{3}$	$\frac{3}{4}z + \frac{2}{9} + \frac{1}{\bar{c}} \cdot (\frac{1}{2}b - \frac{2}{9}) \cdot (\frac{1}{2}b - \frac{2}{9} - \frac{1}{2}z)$
1	1/3	$\frac{3}{4}z + \frac{2}{9} + \frac{1}{\bar{c}} \cdot (\frac{1}{2}b - \frac{2}{9}) \cdot (\frac{1}{2}b - \frac{2}{9} - \frac{7}{12}z)$
1	0	$\frac{3}{4}z + \frac{2}{9} + \frac{1}{\bar{c}} \cdot (\frac{1}{2}b - \frac{2}{9}) \cdot (\frac{1}{2}b - \frac{2}{9} - \frac{3}{4}z)$
$^{2}/_{3}$	$^{2}/_{3}$	$\frac{1}{2}z + \frac{7}{18} + \frac{1}{\bar{c}} \cdot (\frac{1}{2}b - \frac{7}{18}) \cdot (\frac{1}{2}b - \frac{7}{18} - \frac{1}{4}z)$
$^{2}/_{3}$	$1/_{3}$	$\frac{1}{2}z + \frac{7}{18} + \frac{1}{\bar{c}} \cdot (\frac{1}{2}b - \frac{7}{18}) \cdot (\frac{1}{2}b - \frac{7}{18} - \frac{1}{3}z)$
$^{2}/_{3}$	0	$\frac{1}{2}z + \frac{7}{18} + \frac{1}{\bar{c}} \cdot (\frac{1}{2}b - \frac{7}{18}) \cdot (\frac{1}{2}b - \frac{7}{18} - \frac{1}{2}z)$
1/3	$1/_{3}$	$\frac{1}{3}z + \frac{17}{36} + \frac{1}{\bar{c}} \cdot (\frac{1}{2}b - \frac{17}{36}) \cdot (\frac{1}{2}b - \frac{17}{36} - \frac{1}{6}z)$
$\frac{1}{3}$	0	$\frac{1}{3}z + \frac{17}{36} + \frac{1}{\bar{c}} \cdot (\frac{1}{2}b - \frac{17}{36}) \cdot (\frac{1}{2}b - \frac{17}{36} - \frac{1}{3}z)$

$0 \qquad 0 \qquad \frac{1}{4}z + \frac{1}{2} + \frac{1}{\bar{c}} \cdot (\frac{1}{2}b - \frac{1}{2}) \cdot (\frac{1}{2}b - \frac{1}{2} - \frac{1}{4}z)$	
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The most striking fact in this table is the effect of  $t_1^*$  on the payoffs. As one could already see before, lowering the threshold for investing employees given any  $t_0^*$  does not increase the chance of investing. Nor does it increase the value of the investment, as can be seen in Table 6. The reason for this is that the expected value of a new employee is relatively low, especially in comparison to an employee that invested. Hence, lowering the threshold for investing employees does not benefit the employee who invested nor does it increase the payoff to the manager. An employee that previously did not make the higher threshold, but who was allowed to stay anyways because his ability was higher than the expected value of the new employee, will now be allowed to stay because of the lower threshold. The ground on which the employee is allowed is different, but de facto no changes for the incumbent employee are made. For that reason the employee is not more likely to invest and the value of investment to the manager does not increase.

To the contrary, lowering  $t_1^*$  to values of 1/3 or 0, even decreases the value of investment. At higher values of investment the manager is able to retain the employee only when he has an ability higher than the outside option. A lower threshold forces him to also keep the employees with a true ability of 0, since his discretionary power has to be asymmetric. At the higher threshold he could fire the low ability worker and get an expected value of *z* instead. Hence, lowering  $t_1^*$  given any  $t_0^*$ , will always decrease the expected payoff to the manager when *z* is this low and discretion is possible. The only exception to this rule are threshold for investing employees of 1 and 2/3, which will give the same payoff for any  $t_0^*$ . The reason for this is that given these thresholds the employee would have been retained anyways, independently of his ability. The consequence of these impacts of  $t_1^*$  is that for any given  $t_0^*$  it is optimal for the manager to set  $t_1^*$  as high as possible. This is always true, irrespectively of  $\bar{c}$  and *b*. Only for  $t_0^* = 1$  the manager will be indifferent between a  $t_1^*$  of 1 or 2/3.

The effect of  $t_0^*$  is not as clear-cut. Decreasing this threshold will lead to a lower probability that the employee will not be fired when he decides not to invest. This has several consequences. First of all, the chance that a new employee will be hired decreases. This has a negative effect on the payoff when every employee would not invest, the first part in the equation. On the other hand, employees who were previously fired will now be retained. Albeit their ability is relatively low because high ability employees were retained also under high thresholds, the discretionary power makes it possible to still fire employees that have a true ability below *z*. Hence, the added value always offsets the lost expected value due to a lower chance of having a new employee. Consequently, the value of the payoff to the manager when the employee would never invest strictly decreases in  $t_0^*$ .

At the same time, increasing  $t_0^*$  incentivises the employee to invest. Despite the fact that  $t_1^*$  will increase in par with  $t_0^*$ , doing so will increase the expected value of investing. This is because the discretionary power the manager has is asymmetric, leaving him the option to keep an employee who has invested and has a high ability, even at the higher threshold. This is not the case for the employee who does not invest; the discretionary power works in the opposite direction in his case. Hence, increasing  $t_0^*$  also has two positive effects on the expected payoff to the manager. First, it increases the chance that the employee will invest. Second, the actual value of investment to the manager increases.

All in all, the overall effect will depend on the return to investment and spread investment costs can take. When *b* is very high,  $t_0^*$  will be set high to induce investment. When *b* is low or  $\bar{c}$  is high, incentivising investment is not worthwhile and, hence,  $t_0^*$  will be set low. This is also visible from Graph 10 and 11, that display the optimal set of thresholds for any *b* given  $\bar{c} = 3$  and  $\bar{c} = 5$ .

Graph 10



The optimal sets of thresholds fit the predictions. When  $\bar{c} = 3$ , it is optimal to set low thresholds when *b* is low; incentivising investment is not worth the costs.  $t_0^*$  and  $t_1^*$ are both set at 1/3. At higher levels of *b*, it becomes optimal to set a higher  $t_0^*$ , i.e. 2/3. Since a lower  $t_1^*$  only influences the payoff in a negative way, this threshold is set at the level of 2/3 as well. The manager is indifferent between the two sets of thresholds when  $b \approx 2.14$ . Again, for the exact values of the indifference points, see the appendix. If *b* takes a value higher than 2.94, it even pays off to the manager to increase  $t_0^*$  to 1. Increasing  $t_1^*$  as well is not necessary, since a profit maximising manager will be indifferent between threshold set 1 and 2.

Graph 11 shows that the effect of  $\bar{c}$  also follows the predictions. A larger spread of possible investments costs makes it more unlikely that the employee will invest in training. This reduces the value of incentivising investment, shifting the indifferent points to the right. HRM strategy 5 is now only optimal at a *b* higher than 2.78, the indifference point with strategy 1 or 2 has even shifted out of the considered range.

Graph 11



These two graphs show that applying asymmetric discretion changes the optimal HRM strategies. The question is whether it is also profit maximising to apply asymmetric discretion in the first place, or to commit to a contract without any possible discretion. Comparing Table 4 with Table 6 reveals that the first term of the payoffs did not change for any set of thresholds, except for set 10. This is because discretionary power allows

the manager to fire a non-investing employee with ability 0, even when the threshold is 0. Since set 10 is never an optimal strategy when  $z = \frac{1}{3}$ , these changes are irrelevant.

Next, for all sets the probability of investing has increased compared to the no discretion case, since discretion equips the manager to retain high ability employees, even when the measurement error caused them to not pass the threshold. Next to that, the manager can fire a non-investing employee with a low ability who passed the threshold due to the measurement error. This causes the chance an investing employee to increase relative to a non-investing employee. To be more precise, the probability rises to the same level when  $t_1^* = 0$ , given any  $t_0^*$  under the case of no discretion. The employee, facing discretion, acts like  $t_1^*$  is set at 0 under no discretion, increasing the probability of investing to the maximum given any  $t_0^*$ . For the same reason, the value of the investment increases. However, increasing the chance an employee is allowed to stay at the company increases the opportunity costs of investment as well, since the chance of hiring a new employee has decreased. This negative effect is low in comparison with the increase in value and chance of investment. Note that the thresholds sets where  $t_1^* = 0$  are equal to the case when no discretion is used. These sets, HRM strategy 4 and 6, are optimal at high levels of b without discretion. This shows that asymmetric discretion is optimal to incorporate in the contract for two reasons. First, the payoffs of these strategies 4 and 6 are equal under both situations. Second, remember that given that particular  $t_0^*$ , it is profit maximising to increase  $t_1^*$  to the same level as  $t_0^*$ . Hence, under discretion a higher expected payoff is always possible for high levels of *b* compared to the case of no discretion. For lower levels of *b* under the case of no discretion,  $t_0^* = \frac{2}{3}$  and  $t_1^* = \frac{1}{3}$  constitute the optimal strategy to follow for the manager. Substituting  $z = \frac{2}{3}$  in the payoff functions of this HRM strategy under both discretion and no discretion, shows that the payoff under discretion is higher for every value of b. Moreover, the manager can always increase his expected payoff further when discretion is possible by setting  $t_1^* = \frac{2}{3}$ . All in all, allowing asymmetric discretion, when a new employee has a relative low expected ability of 1/3, always increases the expected payoff to the manager, irrespectively of b and  $\bar{c}$ . These results also hold for a z lower than  $1/_{3}$ .

## 3.3.3 Discretion with $z = \frac{2}{3}$

In the previous section the optimal strategy with discretion when the outside option is equal to or smaller than 1/3. Next, I will consider the case with a relative high z. This analysis is slightly more complicated than the case of a low z. When  $z \le 1/3$ ,  $\Pr(a \ge z) = \Pr(a \ge \frac{z}{b}) = \frac{3}{4}$ , irrespective of b. This is not the case for high values of z. Hence, the optimal HRM strategies are determined for both 1 < b < 2 and  $b \ge 2$ . This is necessary, since  $\Pr(a \ge z) = \Pr(a \ge \frac{z}{b}) = \frac{1}{2}$  for 1 < b < 2 and  $\Pr(a \ge z) = \frac{1}{2}$ ,  $\Pr(a \ge \frac{z}{b}) = \frac{1}{4}$  for  $b \ge 2$ . The probabilities that the true ability of an investing employee exceeds the outside option is different on both intervals of b. Furthermore, note that any z on the interval  $[\frac{2}{3}, 1]$  meets the following analysis, but the value of b

that makes  $Pr(a \ge z/b)$  turn from 1/2 to 1/4 is different. I choose to use z = 2/3, in order to be able to compare with section 2.

First, the optimal strategies for  $b \in (1,2)$  are derived. Using equations (5) and (6), the indifference points between investment and no investment for the employee can be determined. See section 3.3.2 for a reminder of the calculations. The only difference is that *z* takes on a different value. The indifference points are given in Table 7.

Table	e 7	
<i>t</i> <sub>0</sub> *	$t_1^*$	С*
1	1	$\frac{5}{12}b - \frac{2}{9}$
1	$^{2}/_{3}$	$\frac{4}{9}b - \frac{2}{9}$
1	$^{1}/_{3}$	$\frac{17}{36}b - \frac{2}{9}$
1	0	$\frac{1}{2}b - \frac{2}{9}$
$^{2}/_{3}$	$^{2}/_{3}$	$\frac{4}{9}b - \frac{13}{36}$
$^{2}/_{3}$	$^{1}/_{3}$	$\frac{17}{36}b - \frac{13}{36}$
$^{2}/_{3}$	0	$\frac{1}{2}b - \frac{13}{36}$
1/3	$^{1}/_{3}$	$\frac{17}{36}b - \frac{5}{12}$
<sup>1</sup> / <sub>3</sub>	0	$\frac{1}{2}b - \frac{5}{12}$
0	0	$\frac{1}{2}b - \frac{5}{12}$

Contrary to the case when  $z \leq 1/3$ , does  $t_1^*$  influence the indifference point between investment and no investment. Discretionary power will cause most employees to be retained when the expected ability of a new employee is low. Lowering  $t_1^*$  does not make a difference, since the employee is retained anyways when he invests. This is not the case with a higher expected ability for the new employee. The chance that  $a \geq \frac{z}{b}$  is lower. Hence, lowering the threshold for an employee who invests does provide an extra incentive to indeed invest.

For  $t_1^* = 1$  and  $t_1^* = \frac{2}{3}$ , the incentive is actually higher compared to the situation where no discretion is possible. This reason for this is that at these high thresholds the discretionary power allows the manager to retain more employees that did not make the threshold due to the measurement error. At lower thresholds none of the investing employees do not make it due to the measurement error. Hence, the discretionary power of the manager does not increase the incentive to invest for these cases.

Similar to all studied cases, a lower  $t_0^*$  provides a decreased incentive to invest in costly training. However, the asymmetric discretion makes the incentive decrease less when  $t_0^*$  is lowered, compared to the case of no discretion and the case of discretion but with a relative low *z*. A high *z* causes many non-investing employees that initially make the threshold to be fired anyways.

The next step is again to use these indifference points to calculate the payoffs to the manager and to determine what set of thresholds maximises his payoff. The calculations following equation (6) are given in Table 8.

Table	e 8	
$t_0^*$	$t_1^*$	V
1	1	$\frac{3}{4}z + \frac{2}{9} + \frac{1}{\bar{c}} \cdot (\frac{5}{12}b - \frac{2}{9}) \cdot (\frac{5}{12}b - \frac{2}{9} - \frac{1}{4}z)$
1	$^{2}/_{3}$	$\frac{3}{4}z + \frac{2}{9} + \frac{1}{\bar{c}} \cdot \frac{4}{9}b - \frac{2}{9} \cdot \frac{4}{9}b - \frac{2}{9} - \frac{1}{3}z$
1	$^{1}/_{3}$	$^{3}/_{4}z + ^{2}/_{9} + ^{1}/_{\bar{c}} \cdot (^{17}/_{36}b - ^{2}/_{9}) \cdot (^{17}/_{36}b - ^{2}/_{9} - ^{1}/_{2}z)$
1	0	$\frac{3}{4}z + \frac{2}{9} + \frac{1}{\bar{c}} \cdot (\frac{1}{2}b - \frac{2}{9}) \cdot (\frac{1}{2}b - \frac{2}{9} - \frac{3}{4}z)$
$^{2}/_{3}$	$^{2}/_{3}$	$\frac{7}{12}z + \frac{13}{36} + \frac{1}{\bar{c}} \cdot \frac{4}{9}b - \frac{13}{36} \cdot \frac{4}{9}b - \frac{13}{36} - \frac{1}{6}z$
$^{2}/_{3}$	$^{1}/_{3}$	$\frac{7}{12}z + \frac{13}{36} + \frac{1}{\bar{c}} \cdot (\frac{17}{36}b - \frac{13}{36}) \cdot (\frac{17}{36}b - \frac{13}{36} - \frac{1}{3}z)$
$^{2}/_{3}$	0	$\frac{7}{12}z + \frac{13}{36} + \frac{1}{\bar{c}} \cdot (\frac{1}{2}b - \frac{13}{36}) \cdot (\frac{1}{2}b - \frac{13}{36} - \frac{7}{12}z)$
1/3	$^{1}/_{3}$	$\frac{1}{2}z + \frac{5}{12} + \frac{1}{\bar{c}} \cdot (\frac{17}{36}b - \frac{5}{12}) \cdot (\frac{17}{36}b - \frac{5}{12} - \frac{1}{4}z)$
1/3	0	$\frac{1}{2}z + \frac{5}{12} + \frac{1}{\bar{c}} \cdot (\frac{1}{2}b - \frac{5}{12}) \cdot (\frac{1}{2}b - \frac{5}{12} - \frac{1}{2}z)$
0	0	$\frac{1}{2}z + \frac{5}{12} + \frac{1}{\bar{c}} \cdot (\frac{1}{2}b - \frac{5}{12}) \cdot (\frac{1}{2}b - \frac{5}{12} - \frac{1}{2}z)$

Naturally, the analysis of the probabilities that the employee invests follows the argumentation about the indifference points. Furthermore, lowering  $t_0^*$  again increases the expected value of a non-investing employee. The increases in value are lower compared to the non-commitment case however, because an employee that does not invest and passes the threshold can still be fired under certain conditions. For the same reason, it is more likely that a new employee is hired. Hence, decreasing  $t_0^*$  lowers the chance that a new employee is indeed hired, but less than when no discretion is possible.

The asymmetric discretion also influences the effect of the thresholds on the value of investment; for high thresholds an employee who invests is more likely to be retained. Hence, the value of investments for these thresholds is higher with discretionary power. At the same time, the opportunity costs of the investing employee, i.e. a new employee, are higher since this new employee is less likely to be hired. This can be seen in the payoffs of  $t_1^* = 1$  and  $t_1^* = \frac{2}{3}$ . Besides these differences is the influence of the thresholds on the value of investment is equal to the case of no discretion.

Asymmetric discretion has a much larger influence on the payoff functions when z is high. Therefore, it is also less straightforward to observe whether the optimal strategy changes and whether discretion increases the expected payoff. Before we proceed to this question, I first construct the payoffs when  $b \ge 2$ .

 $c^*$  is constructed in exactly the same way, except that  $\Pr(a \ge \frac{z}{b}) = \frac{1}{4}$ . This gives us the indifference costs that are shown in Table 9.

Tabl	e 9	
$t_0^*$	$t_1^*$	С*
1	1	$\frac{1}{2}b - \frac{2}{9}$
1	$^{2}/_{3}$	$\frac{1}{2}b - \frac{2}{9}$
1	$1/_{3}$	$\frac{1}{2}b - \frac{2}{9}$
1	0	$\frac{1}{2}b - \frac{2}{9}$
$^{2}/_{3}$	$^{2}/_{3}$	$\frac{1}{2}b - \frac{13}{36}$
$^{2}/_{3}$	$1/_{3}$	$\frac{1}{2}b - \frac{13}{36}$
$^{2}/_{3}$	0	$\frac{1}{2}b - \frac{13}{36}$
1/3	$^{1}/_{3}$	$\frac{1}{2}b - \frac{5}{12}$
$\frac{1}{3}$	0	$\frac{1}{2}b - \frac{5}{12}$
0	0	$\frac{1}{2}b - \frac{5}{12}$

Basically is this table a combination of two tables previously under consideration; The influence of  $t_0^*$  is exactly the same as when 1 < b < 2, which is quite logical since the value *b* takes on does not influence the hiring decision when I = 0, even when discretion is applied. The influence of  $t_1^*$  on  $c^*$ , however, is the same as when  $z \le 1/3$  and b > 1. Also this makes sense, since both combinations of *z* and *b* make that the outside option for an investing employee lies on the interval (0, 1/3]. Since this outside value determines whether discretion is applied,  $t_1^*$  leads to the same effects in both cases.

One might expect that also the payoffs to the manager will be a combination of section 3.3.2 and the beginning of this section when *b* takes on low values. Table 10, constructed again according to equation (6), confirms this.

Table 10		
$t_0^*$	$t_1^*$	V
1	1	$\frac{3}{4}z + \frac{2}{9} + \frac{1}{\bar{c}} \cdot (\frac{1}{2}b - \frac{2}{9}) \cdot (\frac{1}{2}b - \frac{2}{9} - \frac{1}{2}z)$
1	$^{2}/_{3}$	$\frac{3}{4}z + \frac{2}{9} + \frac{1}{\bar{c}} \cdot (\frac{1}{2}b - \frac{2}{9}) \cdot (\frac{1}{2}b - \frac{2}{9} - \frac{1}{2}z)$
1	$1/_{3}$	$\frac{3}{4}z + \frac{2}{9} + \frac{1}{\bar{c}} \cdot (\frac{1}{2}b - \frac{2}{9}) \cdot (\frac{1}{2}b - \frac{2}{9} - \frac{7}{12}z)$
1	0	$\frac{3}{4}z + \frac{2}{9} + \frac{1}{\bar{c}} \cdot (\frac{1}{2}b - \frac{2}{9}) \cdot (\frac{1}{2}b - \frac{2}{9} - \frac{3}{4}z)$
$^{2}/_{3}$	$^{2}/_{3}$	$\frac{7}{12}z + \frac{13}{36} + \frac{1}{\bar{c}} \cdot (\frac{1}{2}b - \frac{13}{36}) \cdot (\frac{1}{2}b - \frac{13}{36} - \frac{1}{3}z)$
$^{2}/_{3}$	$1/_{3}$	$\frac{7}{12}z + \frac{13}{36} + \frac{1}{\bar{c}} \cdot (\frac{1}{2}b - \frac{13}{36}) \cdot (\frac{1}{2}b - \frac{13}{36} - \frac{5}{12}z)$
$^{2}/_{3}$	0	$\frac{7}{12}z + \frac{13}{36} + \frac{1}{\bar{c}} \cdot \frac{1}{2}b - \frac{13}{36} \cdot \frac{1}{2}b - \frac{13}{36} - \frac{7}{12}z$
$\frac{1}{3}$	$^{1}/_{3}$	$\frac{1}{2}z + \frac{5}{12} + \frac{1}{\bar{c}} \cdot (\frac{1}{2}b - \frac{5}{12}) \cdot (\frac{1}{2}b - \frac{5}{12} - \frac{1}{3}z)$

1/3	0	$\frac{1}{2}z + \frac{5}{12} + \frac{1}{\bar{c}} \cdot (\frac{1}{2}b - \frac{5}{12}) \cdot (\frac{1}{2}b - \frac{5}{12} - \frac{1}{2}z)$
0	0	$\frac{1}{2}z + \frac{5}{12} + \frac{1}{\bar{c}} \cdot (\frac{1}{2}b - \frac{5}{12}) \cdot (\frac{1}{2}b - \frac{5}{12} - \frac{1}{2}z)$

The first part of the equation, the expected value when the employee never invests, is equal to the ones when  $z = \frac{2}{3}$  and 1 < b < 2. Again, this is for the reason that the hiring decision when I = 0 does not depend on *b*. The probability that an employee invests consists of the combination described above, where  $c^*$  is discussed. The value of investments basically follows these lines as well. The expected value of an investing employee that is hired is equal to the case when  $z \leq 1/3$  and asymmetric discretion can be applied. Hence, for the same reasons as explained in section 3.3.2 does  $t_1^*$  not influence the value of investment positively. A lower  $t_1^*$  does decrease the chance a new employee is hired, which is reflected by a lower value of investment. This is reflected by the negative part of the investment value, which consist of the foregone expected values when a employee invests. The first parts is the foregone value of an employee when he did not invest, which follows the part where z = 2/3 and 1 < b < 2, since the value of an employee that does not invest depends on a hiring decision independent of *b*. The chance that a new employee is hired is determined, however, by an interaction between the hiring decision when I = 0 and the decision when I = 1. Therefore, the opportunity costs in terms of the foregone value of a new employee is not comparable to one of the two sections. It follows the same tendency, however, that this opportunity cost increases when  $t_1^*$  decreases relative to  $t_0^*$ . Moreover, no difference exists in setting  $t_1^* = 1$  and  $t_1^* = \frac{2}{3}$ , which is similar when the new employee has a relative low expected value.

The fact that a lower  $t_1^*$  does always decrease the payoff to the manager, implies again that this threshold is always set always equal to  $t_0^*$ . Except when  $t_0^* = 1$ , when the threshold can set either at 2/3 or 1. What level of  $t_0^*$  is chosen depends on the return of investment and the range the cost of investment can take, as it did in section 3.3.2. Combining both the payoffs for 1 < b < 2 and  $b \ge 2$  gives us the following optimal HRM strategies when  $\bar{c} = 3$ , shown in Graph 12, and  $\bar{c} = 5$ , shown in Graph 13.

Graph 12



Graph 13



Without discretion  $t_0^*$  is optimally set at 1 to induce investment and  $t_1^*$  at 2/3. When *b* is higher, the manager will set a lower  $t_1^*$  to induce investment even further. With discretion this pattern is different. For  $b \in (1,2)$  it is optimal to set both thresholds at 2/3. This is also the case for a part on the range  $b \ge 2$ . However, at one point the value of investment is high enough to justify an increase of the incentive to invest in training by setting  $t_0^*$  at 1. At this point the manager is indifferent between leaving  $t_1^*$  at 2/3 or setting it at 1 as well. The incentive stems from the fact that it is less likely for a non-investing employee to be retained, whereas the chance does not chance for an investing employee.

When  $\bar{c}$  is higher, which causes the probability of investment to be lower, the switching point from HRM strategy 5 to 1 or 2 lies at a higher value of *b*. This is similar to what we have seen in previous cases.

For  $b \ge 2$  no positive incentive can be given by setting a relative lower  $t_1^*$ . This is not the case when 1 < b < 2, which can be seen when the maximum cost of investment is really low, say  $\bar{c} = 2$ . Note that at this value, the assumption that  $\bar{c} > c^*$  is still satisfied. When the maximum cost of investment is this low it is optimal to incentivise investment already at lower values of b. This can be seen in Graph 14. From b = 1.72onwards, the manager optimally sets  $t_0^* = 1$  and leaves  $t_1^*$  at 2/3, since this difference in thresholds does incentivise investment. Strategy 2 is strictly optimal to strategy 1 in this range. When b is higher than 2,  $t_1^*$  has no positive influence any more and, hence, the manager is indifferent between  $t_1^* = 2/3$  and  $t_1^* = 1$ .

Graph 14



As said earlier, whether it is optimal to allow asymmetric discretion is less easy to see when *z* is relatively high, because it changes more to the payoffs. Almost none of the payoffs of strategies that were optimal without discretion are equal to those when discretion can be used. There is one exception however; the payoff of strategy 4 is exactly the same when  $b \ge 2$ . Since we have seen that strategy 4 is optimal at high levels of *b* without discretion and that the manager can improve his expected payoff by setting

a higher  $t_1^*$  when discretion is allowed, we can conclude that allowing asymmetric discretion indeed increases the expected payoff to the manager for very high values of *b*. Also, optimal strategy 3 when no discretion is allowed has a lower expected payoff than strategy 1 or 2 with discretion, because

$$\frac{3}{4^{z} + 2} + \frac{1}{\bar{c}} \cdot (\frac{1}{2^{b} - 2}) \cdot (\frac{1}{2^{b} - 2}) \cdot (\frac{1}{2^{b} - 2}) = \frac{3}{4^{z} + 2} + \frac{1}{\bar{c}} \cdot (\frac{17}{36^{b} - 2}) \cdot (\frac{17}{36^{b} - 2}) \cdot (\frac{17}{36^{b} - 2}) \cdot (\frac{17}{36^{b} - 2}) = \frac{1}{2^{z}}$$

One can see that the payoff with discretion is always higher. For 1 < b < 2 one can only compare the payoffs by substituting z = 2/3 in the equations. Doing this shows that for the range 1 < b < 2, HRM strategy 5 with discretion indeed provides a higher expected payoff than HRM strategy 2 or 3 when no discretion is used. In total, allowing asymmetric discretion always provides a higher expected payoff when z = 2/3.

## 4. Discussion

Section 3 provides an analysis of the optimal HRM strategies for a manager in three cases: no commitment, commitment and commitment with asymmetric discretion. Here I will summarise and discuss the found results.

When no commitment is possible, a manager will simply assess the ability of the employee after period one. He will compare this to the outside option and use this information to decide whether to keep or fire the employee. Basically, a non-investing employee will be retained if his ability is higher or equal to z, an investing employee has to have an ability of at least  $\frac{z}{b}$ . The standard an investing employee faces is lower, simply for the reason that it is assumed that a new employee does not invest in training. An investing employee with the same ability as a new employee is therefore worth more. Note, however, that the abilities have a discrete distribution. Therefore, if both z and  $\frac{z}{b}$  fall between the same abilities, both an investing and non-investing employee face de facto the same threshold.

The possibility to commit to thresholds based on the verifiable measure y before the employee makes the investment decision changes the story quite a bit. Given this possibility, the manager can incentivise valuable investment in firm-specific human capital. Under no commitment he was only concerned about efficient job allocation. Now he faces a trade off between incentives and efficient job allocation, a dilemma well described in economic literature. The higher the difference between the threshold for a non-investing employee and an investing employee is, the larger the incentive for investment. However, setting lower or higher thresholds than the outside option distorts efficient job allocation. This trade off is observable in the results. At low values of *b* the thresholds are set close to the outside option. As *b* is higher, incentivising training is worth more. At these higher values of investment the manager will set thresholds with a higher spread between the two. The higher *b* is, the larger spread is justified. Whether the incentive is given by setting a lower  $t_1^*$  or a higher  $t_0^*$  depends mostly on the expected ability of a new employee and this ability divided by b, since deviating the threshold from the outside options is costly. When z is relatively high and has a value of 2/3, it is optimal at low values of b to just set  $t_1^* = 2/3$  and let the incentive for investment come from a high threshold for non-investing employees, i.e.  $t_0^* = 1$ . Only when investment is worth more, the incentive is increased by setting a lower  $t_1^*$ . A relatively low z at 1/3 shows a slightly different pattern. Again, first the incentive stems from a higher  $t_0^*$  at  $\frac{2}{3}$ , whereas  $t_1^*$  is set at  $\frac{1}{3}$ . Giving a stronger incentive by increasing  $t_0^*$  is very costly however, since this implies a large deviation from the outside option. Therefore,  $t_1^*$  is decreased first before  $t_0^*$  is increased to its maximum, to incentivise investment at high values of *b*. Finally, I considered the case when z = 1/2. When investment has practically no value, no incentive and both thresholds are set at  $^{2}/_{3}$ . When *b* is higher  $t_1^*$  is set at 1/3. This is an incentive for investment, but at higher levels of *b* it simply reflects the outside option when an investing employee is replaced. For even higher levels of b, it depends on the maximum cost of investment what strategy is optimal. When  $\bar{c} = 3$  it is optimal to increase  $t_0^*$  before setting  $t_1^*$  at its lowest possible value. When  $\bar{c} = 5$ , i.e. the maximum cost investment can take is higher and employees are less likely to invest,  $t_1^*$  is decreased before  $t_0^*$  is increased. Besides this influence of the maximum cost of investment on the strategies, I have found that a higher  $\bar{c}$  reduces the relative importance of the investment, leading to managers being more conservative in giving incentives for investment. The switching point to a more incentivising strategy shifts up as  $\bar{c}$  takes on a higher value.

Most importantly, I proved that it is optimal to use commitment when possible. This result corresponds with other papers described in the related literature section. However, I assumed that the signal committed thresholds need to be based on, is not a perfect measurement for ability. This causes commitment to be extra costly. The efficient job allocation is not only distorted by the incentives, but also by the imperfectness of the signal alone. Even with this measurement error, it is worth committing to a HRM strategy incentivising investment.

However, I also found that it is optimal to build in room for discretion in the contract when this uncertainty in the signal exists. This discretion can partly correct for the mistakes made due to this measurement error. The discretion needs to be asymmetric however, to prevent the incentive system from breaking down. Moreover, the asymmetry has another advantage: it benefits employees who invests and handicaps non-investing employees. Hence, the incentive for investment is increased by the asymmetric discretion alone, even without any spread between the thresholds.

The results show that at low values of z the incentive coming from  $t_1^*$  is gone, basically because the discretion offers the possibility to retain able employees that do not pass the standard due to measurement error. This is extremely valuable for the manager, since it allows him to give an incentive without bearing the cost of the measurement error. If this incentive does not prove to be efficient given the value of investment,  $t_0^*$  is increased, making it more difficult for an employee to pass the threshold when he does not invest. Since a spread between  $t_1^*$  and  $t_0^*$  can only have a negative influence when z is relatively low,  $t_1^*$  is increased accordingly. Basically, the manager gives himself the room to apply the discretion.

At high levels of z, a lower  $t_1^*$  relative to  $t_0^*$  can induce investment. Hence, it is not necessarily optimal to set  $t_1^* = t_0^*$ , as the case when  $\bar{c} = 2$  shows. As b increases, the outside option  $\frac{z}{b}$  for investing employees decreases to low levels. At this point, when  $b \ge 2$  in this paper, the analysis follows the same lines as when z is low.

All in all, I conclude that when one leaves the assumption that HRM strategies can be based on a perfect measure, it is still optimal to commit to such a HRM strategy in order to provide incentives for firm-specific human capital investments. A pareto improvement is achievable when the manager allows for asymmetric discretion in the contract. This discretion partly corrects the measurement error and even provides an incentive for investment by itself.

## **5.** Conclusion

The time inconsistency of HRM strategies regarding job allocations and promotions as a way to reward firm-specific human capital investment is elaborately discussed in economic literature. Many authors have shown that commitment to schemes that promote investment are optimal for the firm when these investments are valuable. However, these HRM strategies need to be based on verifiable signals. It is often implicitly assumed that these signals are a perfect measure for ability, effort, etc. This paper shows that when this assumption is dropped and uncertainty in the signal is introduced, it is still possible that it is worth to commit to incentivising schemes. It also shows that too rigid rules are not optimal when the signal is imperfect. The manager that makes the hiring decision should be allowed to exercise discretion to some degree, although this discretion should be asymmetrical. It should only benefit employees that invest in human capital and hinder non-investing employees.

This provides an explanation for the observation that rules are not always as strictly applied as theory suggests. To get back at the promotion decision of the assistant professor discussed in the introduction, it happens that promising academics are still promoted when they did not meet the publication requirement, especially when the reason for this are so called "external factors". This is optimal from the point of view of the university, since the amount of publication so far is not a perfect measure for performance in the future. Especially when the dean can show the talented academic has exerted sufficient effort to meet the threshold and not passing the standard was due to other factors, promoting the dean will not cause the incentive system to break down. It will only provide even stronger incentives, since other assistant professors will now that if they are talented enough, not meeting the publication requirement does not impede promotion, as long as they tried hard enough. It takes away some uncertainty that may cause employees to invest less.

The results provide also important lessons for other situations. Firms should establish strict rules if they want to promote investment, but should take into account that allowing the manager to use asymmetric discretion is optimal, even ex ante, when rules are based on imperfect measures. Given almost no signals are perfect in reality, the possibility of asymmetric discretion concerning job allocation decisions should be considered carefully in many organisations. In this paper the job allocation decision under study is a probation period, but the results can be as easily applied to other promotion or job allocation decisions. Also schools that base the allocation of students to different levels at one test that never perfectly measures their ability, should allow some discretionary power to the teacher when he or she believes that a particular student is smart enough for a certain level, although he did not passed the test.

This paper also has its limitations. In the model many assumptions are used that are clearly abstractions from reality. Ability for example is assumed to have a discrete

distribution, which will not be the case in real life. Moreover, this is a two period model, whereas job allocation decisions in organisations will have much more periods in reality. It is assumed that an employee does not have an outside option to work for. This is obviously not the case in real life and a participation constraint should be considered when these HRM strategies are applied. Furthermore, I make a strong assumption about the payoff functions. Although these assumptions and abstractions make it possible to focus on the interactions under study, they can be important for the results. Further research should focus on the question whether the results are robust to other distributions of the variables and payoff functions for example. Moreover, introducing a multi-period game can be very interesting in this light.

Lastly, I assume that the measurement error takes on a maximum value of one ability level from the true ability. Although this paper provides a good start for analysing the implications of this measurement error, is this assumption essential for the found results. I expect that as the measurement error is smaller, the potential value of asymmetric discretion to decrease. Conversely, I expect that a higher measurement error increases the potential value. However, when the imperfection of the signal becomes to large relative to the benefits of setting thresholds, no firm will use the signal any more, even when asymmetric discretion is possible. Further research should focus on the effects of measurement errors of different magnitudes.

Hence, the results of this paper should be interpreted with care. Nevertheless, I believe that it provides important insights in the effect of imperfect measurements regarding HRM strategies and job allocation. One should use these insights in the working of asymmetric discretion in promotion decisions as one of the "building blocks" described by Gibbons & Waldman (1999).

## Bibliography

Bayo-Mariones, A., & Ortín-Ángel, P. (2006). Internal Promotion Versus External Recruitment in Industrial Plants in Spain. *Industrial and Labor Relations Review*, 59 (3), 451-470.

Bernhardt, D. (1995). Strategic Promotion and Compensation . *The Review of Economic Studies*, 62 (2), 315-339.

Bewley, T. (1999). *Why wages don't fall during a recession.* Cambridge, MA: Harvard University Press.

Carmichael, L. (1983). Firm-Specific Human Capital and Promotion Ladders. *The Bell Journal of Economics*, 14 (1), 251-258.

DeVaro, J. (2006). Internal promotion competitions in firms. *RAND Journal of Economics*, *37* (3), 521-542.

Dominguez-Martinez, S., & Swank, O. (2011, February 11). On the Role of Pre-Determined Rules for HRM Policies. *Tinbergen Institute Discussion Paper 11-034/1*. DSM. (2013). *Managing Board's renumeration*. Retrieved at July 1, 2014, from DSM: http://www.dsm.com/corporate/about/corporate-governance/managing-board-corporate-governance/managing-board-remuneration.html

Erasmus School of Economics. (2010, January). *Erasmus School of Economics, Career and development.* Retrieved at July 1, 2014, from Career: http://www.eur.nl/ese/english/information\_for/ese\_staff/personnel\_and\_organisation /career\_and\_development/career/

Gibbons, R. (1998, August). Incentives in Organizations. NBER Working Paper 6695.

Gibbons, R., & Waldman, M. (1999). Careers in Organizations: Theory and Evidence. In O. Ashenfelter, & D. Card, *Handbook of Labor Economics* (Vol. 3, pp. 2373–2437). Elsevier Science B.V.

Kahn, C., & Huberman, G. (1988). Two-Sided Uncertainty and "Up-or-Out" Contracts. *Journal of Labor Economics*, 6 (4), 423-444.

Koch, A., & Nafziger, J. (2012). Job Assignments under Moral Hazard: The Peter Principle Revisited. *Journal of Economics & Management Strategy*, *21* (4), 1029-1059.

Lado, A., & Wilson, M. (1994). Human Resource Systems and Sustained Competitive Advantage: A Competency-Based Perspective. *The Academy of Management Review*, *19* (4), 699-727.

Liff, S. (2007). Managing Government Employees: How to Motivate Your People, Deal with Difficult Issues, and Achieve Tangible Results. New York: AMACON Division of American Management Association.

Meyer, M. (1991). Learning from Coarse Information: Biased Contests and Career Profiles . *The Review of Economic Studies*, *58* (1), 15-41.

Milgrom, P. R. (1988). Employment Contracts, Influence Activities, and Efficient Organization Design. *Journal of Political Economy*, 96 (1), pp. 42-60.

Milgrom, P., & Roberts, J. (1988). An Economic Approach to Influence Activities in Organizations. *American Journal of Sociology*, *94*, S154-S179.

New York University. (2000, April 1). *Probationary Period*. Retrieved at July 1, 2014, from New York University: http://www.nyu.edu/about/policies-guidelines-compliance/policies-and-guidelines/probationary-period.html

Prendergast, C. (1993). The Role of Promotion in Inducing Specific Human Capital Acquisition. *The Quarterly Journal of Economics*, *108* (2), 523-534.

Royal Dutch Shell. (2012). *Shell Annual Report and Form 20-F 2012*. Retrieved at July 1, 2014, from Shell: http://reports.shell.com/annual-report/2012/servicepages/downloads/files/executive\_directors\_shell\_ar12.pdf

Saint-Cyr, Y. (2011, August). *Probationary period* – *a complex human resource topic*. Retrieved at July 1, 2014, from HR Infodesk: http://www.hrinfodesk.com/preview.asp?article=36087

Waldman, M. (1984). Job Assignments, Signalling, and Efficiency . *The RAND Journal of Economics*, *15* (2), 255-267.

# Appendix

Indifference points between HRM strategies

Graph 1	
Between strategy 5 and 6	$b = \frac{1}{124} \left( 107 + \sqrt{1033} \right) \approx 1.12$
Between strategy 6 and 3	$b = \frac{38}{17} \approx 2.24$
Between strategy 3 and 4	$b = \frac{1}{35} (53 + \sqrt{1549}) \approx 2.64$
Graph 2	_
Between strategy 5 and 6	$b = \frac{1}{124} \left( 107 + \sqrt{1033} \right) \approx 1.12$
Between strategy 6 and 7	$b = \frac{1}{140} \left( 227 + \sqrt{16249} \right) \approx 2.53$
Between strategy 7 and 4	$b = \frac{28}{9} \approx 3.11$
Graph 3 and 4	
Between strategy 2 and 3	$b = \frac{4}{31} \left(7 + 3\sqrt{2}\right) \approx 1.45$
Between strategy 3 and 4	$b = \frac{4}{35} \left( 17 + 2\sqrt{46} \right) \approx 3.49$
Graph 5	_
Between strategy 6 and 7	$b = \frac{1}{70} \left( 85 + \sqrt{1345} \right) \approx 1.74$
Between strategy 7 and 4	$b = \frac{26}{9} \approx 2.89$
Graph 6	
Between strategy 6 and 7	$b = \frac{1}{70} \left( 85 + \sqrt{1345} \right) \approx 1.74$
Between strategy 7 and 4	$b = \frac{32}{9} \approx 3.56$
Graph 7	
Between strategy 6 and 7	$b = \frac{1}{70} \left( 85 + \sqrt{1345} \right) \approx 1.74$
Graph 10	
Between strategy 8 and 5	$b = \frac{193}{90} \approx 2.14$
Between strategy 5 and 1,2	$b = \frac{25}{9} \approx 2.78$

<i>Graph 11</i> Between strategy 8 and 5	$b = \frac{53}{18} \approx 2.94$
<i>Graph 12</i> Between strategy 5 and 1,2	$b = \frac{221}{108} \approx 2.05$
<i>Graph 13</i> Between strategy 5 and 1,2	$b = \frac{293}{108} \approx 2.71$
<i>Graph 14</i> Between strategy 5 and 2 Between strategy 2 and 1,2	$b = \frac{55}{32} \approx 1.72$ b = 2