HEAVY and Realized (E)GARCH models

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This paper investigates the out-of-sample performance of several models that predict unobserved conditional variance. The models that are considered are the HEAVY, RealGARCH(1,1) and the RealEGARCH(1,1) model. These models are also extended, using the squared daily return as extra regressor and adding an indicator function for negative returns multiplied with the realized measure. With these models, forecasts are made and compared with two benchmark models, being the GARCH(1,1) model and the HAR-3 model. The loss function that is used to compare these models is the QLIKE loss function, with the squared daily returns, realized variance and realized kernel as a proxy. The data that are considered, are the indices of the FTSE100, DAX30, CAC40, AEX, SSMI, IBEX35 and the EUROSTOXX50 from January 2000 to March 2014. It turns out that the models using realized measures do not beat the benchmark models out-of-sample for most of the models. The only models that beat the HAR(3)-RV benchmark model regularly are the forecast combinations based on the in-sample discounted MSPE, for the 21-day ahead forecasts. Another major conclusion is that the extension of the models with the squared daily return and the non-linear models do not perform better than the standard models out-of-sample.

Keywords: high-frequency volatility, forecasting, HEAVY models, Realized GARCH models, Realized EGARCH models.

Contents

1 Introduction

One of the most important concepts in finance is the use of volatility. In all kind of subjects this measure is considered, for example for the use of derivative pricing and value-at-risk measures. On the other hand, it is also an important concept for portfolio choice.

The best known model for conditional volatility is the Generalized AutoRegressive Conditional Heteroskedacity model, better known as the GARCH model, which was first proposed by Bollerslev (1986). He considers the volatility being time different and uses the past (unobserved) volatility and squared shocks of the returns as explaining variables for today's volatility.

In the recent years, the topic of volatility has been subject of many academic papers. There have been several suggestions considering high-frequency (intra-day) data to estimate daily volatility. Examples of such predictors are the realized volatility (for example, Andersen and Bollerslev, 1998), realized kernel (Hansen and Lunde, 2006) and daily range (Gallant, Hsu and Tauchen, 1999). However, many more high-frequency volatility estimators exist.

More recent papers have suggested to use the high-frequency data as predictors in GARCH-like models. Three of those models are the high-frequency based volatility (HEAVY) models (Shephard and Shephard, 2010) and the realized GARCH models and realized EGARCH models (Hansen, Huang and Shek, 2012). This combination has been proven to be fruitful in out-of-sample volatility estimation.

In these papers, the focus lies especially at the market of the United States of America. In this research, the focus lies on several (West-)European markets. Especially, the indices that are considered, are the AEX, CAC40, DAX, EuroSTOXX, FTSE100, IBEX35 and SSMI indices.

Another aspect that has not been researched in detail in the existing literature is the use of more recent proposed high-frequency volatility measures. These more recent proposed realized measures have in common that they include effects of jumps in the prices and are therefore more robust to these jumps. Other measures only look at negative returns that have been proven to have more influence on the variance by for example Andersen et al. (2001). In the papers of Shephard and Shephard (2010) and Hansen, Huang and Shek (2012), the only realized measures that are considered, are the realized variance, realized kernel and daily range. In this research, the (5-minute) realized bi-power variation as proposed by Barndoff-Nielson and Sheppard (2004), the (5-minute) semi-variance proposed by Barndoff-Nielsen, Kinnebrock and Sheppard (2008) and the median truncated realized variance proposed by Andersen, Dobrev and Schaumberg (2008) are investigated too. The used realized measures from Shephard and Shephard (2010) and Hansen, Huang and Shek (2012) will also be considered. Furthermore, for the realized variance, realized semi-variance and the realized bi-power variation, 1-minute sub-sampling will also be considered. Since the daily range is sometimes below 0, this paper uses the squared daily range. This is justifiable by the fact that the daily range is a return, and the squared returns are a measure for volatility too.

The realized measures are different from one another in their robustness. First of all, the daily range, realized measure and realized semivariance are not robust to noise at all. The difference between the three measures is the intra-day data that they use. The realized measure uses all the intra-day data that is available, the realized semivariance is calculated using only the negative returns on the specific day, while the daily range is calculated by subtracting the lowest log price of the day from the highest log price of the day. The realized kernel is robust to some noise of the data. It uses a kernel weighting function to weight the different intra-day returns in order to calculate the realized measure. The median truncated variance and the realized bipower variation are realized measures that are robust to jumps in the prices. The difference between those realized measures is the weighting function that is applied.

A third part that deviates from the existing literature, is by considering the extensions proposed by Shephard and Shephard (2010) in their section 5. These models consider statistical leverage effects by adding an indicator function for when daily returns drop below 0. The motivation for this extension is that it is proven in for example Andersen et al. (2001), that positive returns have less effect on future volatility than negative returns. Another proposed extension is to add the squared return to the HEAVY-r part of the equations in Shephard and Shephard (2010). There are two main reasons to add this extra regressor to the model. First of all, it is a natural way of incorporating the standard GARCH model into the HEAVY model. The second reason is that this extension adds more momentum to the HEAVY model, according to Shephard and Shephard (2010). The same extensions can also be used in the realized GARCH model of Hansen, Huang and Shek (2012).

This paper shows that for the estimation of the HEAVY, RealGARCH(1,1) and RealEGARCH(1,1) models and their extensions, most of the parameters are stable over time, with some slight differences over time. An exception to this is the estimation of the RealEGARCH(1,1) model with the squared daily return as extra explaining variable. These parameters turn out to become very unstable, and therefore this model will be disregarded in this paper.

The relation between the realized bi-power variation and the realized variance is examined in this paper too. This is of special interest, since the realized variance is biased due to sudden jumps in the prices. The realized bi-power variation accounts for these jumps, so in the limit, the average of the realized bi-power variation will go to the unconditional variance. It turns out that both the jumps and the realized bi-power variation have equal influence on the conditional variance.

The rest of the research is based on the out-of-sample forecasts at a horizon of 1, 5, 10 and 21 days. There are two benchmark models, the GARCH(1,1) model of Bollerslev(1986) and the HAR(3)-RV model of Corsi (2009), that are compared to the different models that are estimated. The models are compared using the QLIKE loss function and are investigated all individually and combined using the mean and median of the forecasts. Furthermore, the forecasts are combined using weights based on the in-sample discounted MSPE.

This paper shows that the benchmark models in general perform better than the HEAVY, RealGARCH(1,1) and RealEGARCH(1,1) model, with only minor exceptions. This is in contradiction to the results for the US markets that were investigated in the papers of Shephard and Shephard (2010) and Hansen, Huang and Shek (2012). They found that the out-of-sample performance of these models outperformed the GARCH model. The reason for this difference could be the fact that they use likelihood ratio tests to compare the models to the benchmark models. Furthermore, the RealGARCH and RealEGARCH models are estimated using different lags. It is also shown that the extensions to the models do not improve the out-of-sample performance of the standard models, except the non-linear extension for the HEAVY model.

In order to test whether the differences between the QLIKE results are significant, model confidence sets, introduced by Hansen, Lunde and Nason (2011), are build. This paper shows that for almost all time horizons and indices, the set only consists out of the best performing model. The only exceptions to this are the FTSE100, SSMI and EUROSTOXX50 at a 1-day forecasting horizon.

Combining the models does improve the out-of-sample performance. Depending on the way that the forecasts are combined, it even turns out that for the 21-day ahead forecasting the model combinations based on in-sample discounted MSPE beats the HAR(3)-RV model in most of the cases.

The rest of this paper is constructed as follows. In 'Section 2' the different models and methods used are explained in detail. 'Section 3' reports the data that is used. Also, it shows some summary statistics and dynamics of the data that is used. The results are shown in 'Section 4'. This section is divided into two parts, one discussing the estimates of the models and the other discussing the out-of-sample forecasting results. Finally, 'Section 5' concludes and discusses possible extensions to this research.

2 Methods

This section introduces the different models that are used in this paper. First of all, some notation is introduced. The time series of daily return data are denoted as:

$$r_1, r_2, ..., r_T.$$

The daily returns are the standard log daily returns of the index. Besides the daily returns, the realized measures are denoted in general as:

$$RM_1, RM_2, ..., RM_T.$$

In general, the structure of the models is as given in Equation (1).

$$r_t = \sqrt{h_t} z_t,$$

$$h_t = f(h_{t-1}, RM_{t-1}, z_t, r_t | \mathcal{F}_{t-1}),$$

$$RM_t = g(h_t, h_{t-1}, RM_{t-1}, z_t | \mathcal{F}_{t-1}), \tag{1}$$

where z_t is assumed to be standard normally distributed and independent of all other measures. Furthermore, z_t and z_s are independent when $t \neq s$. The choice of the functions $f(\cdot)$ and $g(\cdot)$ depends on the model used. The history of r_t and RM_t is denoted as $\mathcal{F}_t$.

2.1 HEAVY models

The first model under consideration is the HEAVY model. This model is developed by Shephard and Shephard (2010). The model is given in Equation (2).

$$RM_t = \eta_t \mu_t, \text{ where } \mathbb{E}[\eta_t | \mathcal{F}_{t-1}] = 1,$$

$$\begin{pmatrix} h_t \\ \mu_t \end{pmatrix} = \begin{pmatrix} \omega \\ \omega_R \end{pmatrix} + \begin{pmatrix} \alpha \\ \alpha_R \end{pmatrix} RM_t + \begin{pmatrix} \beta & 0 \\ 0 & \beta_R \end{pmatrix} \begin{pmatrix} h_{t-1} \\ \mu_{t-1} \end{pmatrix}. \tag{2}$$

Equation (2) shows that the HEAVY model consists out of two parts. The first part explains the development of the unobserved conditional variance and the second part explains the development of the realized measures. Those two parts are not depending on one another in a direct way. Both parts of the model are described by their first lag and the lag of the actual realized measure.

The use of the HEAVY model is motivated by Shephard and Shephard (2010) by the fact that it is simple to estimate and it is build on the literature regarding the GARCH models of Bollerslev (1986). The model structure is simple and therefore the understanding of the general features is easy. Furthermore, Shephard and Shephard (2010) have proven that the use of the realized measures give additional gains in out-of-sample forecasting.

In order to estimate the parameters, we use dimensional reduction for ω_R. We use the unconditional mean of the realized measure and the parameters α_R and β_R. This gives $\omega_R = \mu_R(1 - \alpha_R - \beta_R)$, with μ_R the unconditional mean of the realized measure. The reason that this is not done for ω, is that the ratio of the unconditional means between r_t^2 and RM_t is high, which results in unstable results.

2.2 RealGARCH(1,1) models

Another model that is used in this paper, is the RealGARCH(1,1) model. This model was proposed by Hansen et al. (2012). This model is given in Equation (3).

$$\begin{aligned} h_t &= \omega + \alpha RM_{t-1} + \beta h_{t-1}, \\ RM_t &= \xi + \phi h_t + \tau_1 z_t + \tau_2(z_t^2 - 1) + u_t, \end{aligned} \tag{3}$$

where u_t is assumed to be standard normally distributed and independent of z_t. Furthermore, u_t and u_s are independent when $t \neq s$. The reason to use z_t and $z_t^2 - 1$ is to include leverage effects of the returns into the equation. The use of $z_t^2 - 1$ is used in stead of z_t^2 in order to keep the expectation equal to zero, so ξ is not biased.

At first sight, the RealGARCH(1,1) model does not look different compared to the HEAVY model. Especially, the part explaining the unobserved conditional variance seems to be the same. However, the main difference is that today's conditional variance is also an regressor for today's realized measure. This means that the values of α and β are also based on the part explaining the realized measure. This second part also differs from the HEAVY model. The realized measure is explained by today's conditional variance and the random variable z_t. This z_t is not observed directly, so it is calculated using today's volatility and return, by using Equation (1). z_t and $z_t^2 - 1$ are included to add the leverage effect to the model, which is not done in the HEAVY model.

The RealGARCH model is a logical model that is easily derived from the standard GARCH model. One could say that the regular GARCH model is nested in these more general RealGARCH models, with the squared daily return as 'realized measure'. Its advantage over the RealEGARCH model that will be explained below is the simpleness. It is easier to interpret the results from RealGARCH than from the RealEGARCH model.

2.3 RealEGARCH(1,1) models

The third model of the standard models that is used, is the RealEGARCH(1,1) model. This model is referred to in the same paper as the RealGARCH(1,1) model (Hansen et al., 2012). The model is similar to the RealGARCH(1,1) model, only now the log transformations are considered. This results into the model of Equation (4).

$$h_t = \exp(\omega + \alpha \log(RM_t) + \beta \log(h_t) + \tau_1 z_t + \tau_2 (z_t^2 - 1))$$

$$\log(RM_t) = \xi + \phi log(h_t) + u_t, \tag{4}$$

where u_t is again assumed to be standard normally distributed, independent of z_t and independent of u_s when $t \neq s$.

This model is almost similar to the RealGARCH(1,1) model, now taking a look at explaining $\log(h_t)$, using $\log(RM_t)$. The only difference is that the leverage effect is now taken into the conditional variance part in stead of the realized measure part.

The volatility of the indices can never become lower than 0. The RealEGARCH model naturally provides this restriction, since in practice the logarithmic transformation of the conditional variance is estimated. Therefore the model does not depend on the estimates of the parameters for the regressors in order to have positive conditional variances.

2.4 Models with r_t^2

Besides the standard models as given above, some extensions to the models are examined too. The first extension uses the squared return as an additional regressor. The reason for using this regressor comes from the standard GARCH(1,1) model, where this squared residual is used to estimate the volatility. Thereby, the GARCH models are general accepted models that provide good results. It therefore follows naturally to add the squared daily returns as additional regressor to the models. This results in the models of Equations (5) and (6) for the HEAVY, RealizedGARCH(1,1) model, respectively.

$$h_t = \omega + \alpha RM_t + \beta h_t + \gamma r_t^2,$$

$$\mu_t = \omega_R + \alpha_R RM_t + \beta_R \mu_t. \tag{5}$$

$$h_t = \omega + \alpha RM_t + \beta h_t + \gamma r_t^2,$$

$$RM_t = \xi + \phi h_t + \tau_1 z_t + \tau_2 (z_t^2 - 1) + u_t. \tag{6}$$

The parameter γ explains what the additional information is of the squared daily return. These models are also useful in comparing the influence of the realized measures and the squared daily return to the conditional variance.

When the value of α is higher than γ, this implies that the realized measure is a better regressor for explaining the conditional variance and vice versa.

The RealizedEGARCH(1,1) is not extended with the squared daily return, since it's value can be 0 at some times. This results in the fact that some observations should be disregarded, which would mean a loss of information. Another problem with estimating the RealizedEGARCH(1,1) model with $\log(r_t^2)$, is the instability of the parameters. Therefore, this model will be not further investigated.

2.5 Non-linear models

Another set of models to extend the standard models, is to use non-linear models. There exist lots of non-linear models to use. However, in this paper only the use of an indicator function is considered. This indicator function is used, since it is generally accepted that conditional volatility reacts stronger to negative returns than to positive returns (see for example Andersen et al., 2001). To the standard models of Equation (2) up to including (4) the part $\gamma \mathbb{I}_{\{r_t<0\}} RM_t$ is added, with $\mathbb{I}_{\{\cdot\}}$ being the indicator function giving 1 if $\{\cdot\}$ is true and 0 otherwise. This results into the models of Equation (7), (8) and (9) for the HEAVY, RealizedGARCH(1,1) and RealizedEGARCH(1,1), respectively.

$$h_t = \omega + \alpha RM_t + \beta h_t + \gamma \mathbb{I}_{\{r_t\}} RM_t,$$

$$\mu_t = \omega_R + \alpha_R RM_t + \beta_R \mu_t. \tag{7}$$

$$h_t = \omega + \alpha RM_t + \beta h_t + \gamma \mathbb{I}_{\{r_t\}} RM_t,$$

$$RM_t = \xi + \phi h_t + \tau_1 z_t + \tau_2 (z_t^2 - 1) + u_t. \tag{8}$$

$$h_t = \exp(\omega + \alpha \log(RM_t) + \beta \log(h_t) + \tau_1 z_t + \tau_2 (z_t^2 - 1)$$

$$+ \gamma \mathbb{I}_{\{r_t\}} \log(RM_t)),$$

$$\log(RM_t) = \xi + \phi log(h_t) + u_t. \tag{9}$$

The parameter α can be interpreted as the factor of the realized measure when the returns are positive. When the returns are negative, the parameter γ explains how much the influence of the realized measure increases. Furthermore, the model also gives a distinction in the volatility when the market is in crisis or expansion. In periods of extension one sees often that the volatility is low, while in periods of crises the volatility is high. This non-linear model accounts for this distinction too.

2.6 Combination of realized bi-power variation and realized variance

In general, it would be irrelevant to estimate combinations of realized measures into one model. All the realized measures are based on the same data set, so it is assumable that their correlations are very high, and therefore there is no significant improvement in the models.

One combination, however, is relevant to research. Not from a forecasting point-of-view, but from a theoretical view. This is the combination between the realized bi-power variation and realized variance. From the theory, the realized bi-power variation adjusts to jumps in the volatility, while the realized variance ignores those jumps. This leads to $\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} BV5_t = \bar{\sigma}^2$ and $\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} RV_t = \bar{\sigma}^2 + \bar{J}$, with $\bar{\sigma}^2$ the asymptotic unconditional variance, $\bar{J}$ the average variance caused by jumps in the prices, $BV5_t$ the 5-minute bi-power variation and $RV5_t$ the 5-minute realized variance.

This means that if we would estimate the models with the realized bi-power variation as one realized measure and the difference between the realized variance and realized bi-power variation as another measure, we can see how much influence the jumps have on the models. I.e., Equation (10) shows how this would result in the HEAVY-model.

$$h_t = \omega + \alpha_1 BV5_t + \alpha_2 \max(0, RV5_t - BV5_t) + \beta h_t, \tag{10}$$

where $BV5_t$ is the realized bi-power variation and $RV5_t$ is the 5-minute realized variance, both without subsampling. The maximization of the difference and 0 is taken, since there should not be negative variances. The implication of the maximization is that negative jumps are not taken into account, since these are set to 0 by the maximization function.

However, most simple alternatives would make it harder to examine the difference between the effect of the jumps in prices. Examples of these alternatives include adding the minimum of the difference and 0, the squared difference and the absolute difference. In the first case, this would eventually imply estimating $h_t = \omega + \alpha_1 BV5_t + \alpha_2 RV5_t$, which does not give explanation on the effect of the jumps on the variance. The latter two would not add in the explanation of positive and negative jumps, since negative and positive jumps are considered as positive jumps.

The maximum of $(RV5_t - BV5_t)$ and 0 is a significant amount of times equal to 0, due to the fact that the difference is negative at some periods. This would mean that the log of this value is equal to $-\infty$. If this combination would be applied for the RealizedEGARCH(1,1) model, this would mean that a lot of observations have to be disregarded. Therefore, these combinations are not done for the RealizedEGARCH(1,1) model.

2.7 Out-of-sample forecasts

In order to check whether the models are good at predicting the conditional variance, there has to be made a comparison out-of-sample. In order to do this, there will be made use of a loss function that uses some of the realized measures as proxies. The proxies will be the squared daily return, the realized variance based on 5-minute intervals, without subsampling, and the realized kernel. There will be used several measures as proxy to test the robustness of the forecasts. The loss function that will be used to compare the models to is the quasi likelihood (QLIKE), which is given in Equation (11)

$$QLIKE_j = \frac{1}{P}\sum_{t=T}^{T+P-j} log(\hat{h}_{t+j|T}) + \frac{p_{t+j}}{\hat{h}_{t+j|T}}, \tag{11}$$

where P is the number of observations, p_t is the proxy measure of the variance, j the forecasting horizon and $\hat{h}_{t+j|T}$ the forecast from the model. The forecasting horizon will be taken at 1 day, 5 days (1 trading week), 10 days (2 trading weeks; also the horizon that financial institutions have to use when reporting their Value-at-Risk and other downside risk measures under Basel regulations) and 21 days (1 trading month). A longer horizon is not considered, since the overall consensus, as given by e.g. Christoffersen and Diebold (2000), is that volatility is predictable, but only up to a limited number of periods ahead.

The choice for the QLIKE loss function is made, since it is proven to be a loss function that is robust to noise in the volatility proxy (Patton, 2011). The advantage of this loss function is that the conditional variance of the standardized forecasting error is approximately 2. This means that the loss function is less affected by the most extreme observations. This is an advantage above the MSPE loss function, which gives variable conditional variances, depending on the forecasting errors (Patton, 2011), which can become very large when there are a lot of 'extreme' observations.

For the estimation of the h days ahead forecast, there will be made use of the integrated volatility. This means that all the volatilities of the past h

days are being summed up together. In order to do this, the approximation of Shephard and Shephard (2010) will be used. They use the assumption that $var(r_{t+1} + r_{t+2} + \cdots + r_{t+h}) = \sum_{j=1}^{h} var(r_{t+j})$. The individual variances can be easily computed from the models. In order to have QLIKE results in the same order, the total variance will be divided by the horizon length, i.e. $\frac{1}{h}\sum_{j=1}^{h} var(r_{t+h})$.

In order to forecast the extended models over more than 1 day ahead, a model for r_t has to be chosen. There are a lot of different models that would apply for the forecasting of r_t. However, since the emphasis in this paper lies on the forecasting of volatility and not on r_t, the assumption is made that the returns are constant with a shock, as is shown in Equation (12).

$$r_t = \mu + \varepsilon_t, \tag{12}$$

with ε_t normally distributed, independent to all previous mentioned variables, and independent to ε_s when $t \neq s$. The constant is estimated as the mean over the window that the parameters of the volatility are estimated.

Besides the comparison between the above mentioned models, there will also be some benchmark models. These models will be the standard GARCH(1,1) model by Bollerslev (1986) and the HAR(3)-RV model proposed by Corsi (2009). This last model is a regression model using the realized volatility and is described in Equation (13).

$$RV5_t^{(d)} = c + \beta_d RV5_t^{(d)} + \beta_w RV5_t^{(w)} + \beta_m RV5_t^{(m)} + \varepsilon_t, \tag{13}$$

where the superscripts d, w, and m stand for daily, weekly and monthly realized volatility, respectively. The weekly realized volatility is calculated as $RV5_t^{(w)} = \frac{1}{5}\sum_{i=0}^{4} RV5_{i,t-i}^{(d)}$. The monthly realized volatility is calculated in a similar way, but with 21 days.

The HAR(3)-RV model uses the realized variance to construct three factors. This implies that the conditional variance of today is not only based on the realized variance of the day before, but also by the variance of the past week and past month. This means that the estimate of the conditional variance is affected less by extreme observations. The model also accounts for some long-run memory in the model, since the volatility of 21 days ago is still explaining, albeit only a little, today's conditional variance.

2.7.1 Combination of forecasts

Besides using the single forecasts, the forecasts themselves will be combined with each other. This is done in three different ways.

The first two ways of combining the forecasts is by taking the mean or by taking the median of the forecasts. It has been proven in the literature that these non-parametric weighting combinations are hard to beat in practice, see for example Bunn (1985), Clemen and Winkler (1986), and Timmermann (2006).

The third forecast combination method is based on the in-sample Mean Squared Prediction Error (MSPE) and is proposed by Timmermann (2006). Even though the P in MSPE assumes that prediction errors are taken, this is not the case in this paper. The errors that are taken are the estimation errors. In this way, the term in-sample is justified. The weight is taken by calculating a deviant of the in-sample MSPE, exponentially discounted by the time. The weights are taken in such an order, that they are time-varying. Since the estimation periods are taken over the past 5 years, the same is done with the weights. This results in the calculation of the in-sample discounted

MSPE as in Equation (14) at time t.

$$DMSPE_t = \sum_{t=1}^{T} \lambda^t e_t^2, \tag{14}$$

where e_t is the forecasting error $\hat{h}_{t|T} - p_t$ and λ is the discount factor. One can easily see that if $\lambda = 1$, the discounted MSPE is equal to the regular MSPE. However, it is more likely to use a factor of λ between 1.05 and 1.10 (Timmermann, 2006). Since the in-sample time-series has about 1500 observations (depending on the index and time-frame), a λ of 1.05 will be used in this paper.

The time-varying weights are calculated using the inverse of the discounted MSPE. This is done, since the MSPE is a penalty function and a high discounted MSPE indicates poor forecasting power. It can be done since the MSPE is always bigger than 0, while for example the QLIKE function is not. Therefore, the weights are calculated using Equation (15).

$$\theta_t = \frac{DMSPE_t^{-1}}{\sum_{i=1}^{N} DMSPE_t^{-1}}, \tag{15}$$

where N is the number of models that are taken into account. In this way the weights all add up to 1. Since the discounted MSPE cannot take a value less than 0, we thereby have that the weights are all between 0 and 1.

The reason to base weights on the discounted MSPE and not on a loss function similar to the QLIKE function, comes from the fact that the discounted MSPE is always bigger than 0, while the QLIKE might not be bigger than 0. This makes it easier to use the inverse of the discounted MSPE, while with the QLIKE several adjustments for the negative values have to be made.

There are 15 different combinations that are used. First of all, for the 8 different models the forecasts of all the realized measures are combined. Next, we will combine all the forecasts from the HEAVY (standard, with r_t^2

and non-linear), the RealizedGARCH(1,1) and the RealizedEGARCH(1,1) models. Also, all the standard models, non-linear models and models with r_t^2 are combined. Finally, all available forecasts are combined with each other.

2.7.2 Comparison of forecasts

There exist many methods to compare the forecasts to one another and test if their significantly different. Most of these tests have to compare every set of two forecast time series separately. Since in this paper, many models with different realized measures are taken, this would result in a lot of results, from which only a small part are important. Therefore, the choice is made to use the model confidence set by Hansen, Lunde and Nason (2011).

The model confidence set is the equivalent of a confidence interval for parameters. It takes all the different results and test which results fall into a $(1-\alpha)$ confidence set, where α is a user pre-defined significance level. It selects iteratively which model, where the word model can be used in the broadest sense, performs the worse and throws this out of the confidence set, until the p-value is below α. In order to implement the model confidence set procedure, bootstrapping is used. Next, the algorithm is explained together with the choices made in this paper. For a detailed explanation, one is referred to the paper of Hansen, Lunde and Nason (2011) [1].

The algorithm for selecting the models that perform the best consists out of three steps. First of all, a choice should be made for the number of bootstrap (B) results and a block length (l). The value of B should not be taken too small, to avoid depending on extreme observations. The value of l

[1]For background information, made assumptions and consequences of using this method, one is referred to the paper itself. One who wants to apply the model confidence set can go directly to the web appendix, where the bootstrap procedure is explained as algorithm.

does not matter too much, however, when a large l is chosen, it results into adding more models to the model confidence set. This paper uses $B = 1000$ and $l = 5$. With the choice of these values, a sample size of n (the number of forecasts) discrete values is selected. First of all, ν_{b_1} is selected randomly from a discrete uniform distribution between 1 and the number of observations. Next, $(\lambda_{b_1}, \ldots, \lambda_{b_l}) = (\nu_{b_1}, \nu_{b_1} + 1, \ldots, \nu_{b_1} + l - 1)$. This is repeated until a sample size of n is created and repeated for all samples $b = 1, \ldots, B$.

The second step constructs the results of the loss function for every single model and point forecast. For each different model, the average of these loss function values ($\bar{L}_i$, $i = 1, \ldots, m$) are taken. The corresponding bootstrap variables are given by $L_{i,b,t} = L_{i,\lambda_{b,t}}$ for $b = 1, \ldots, B$, $i = 1, \ldots, m$ and $t = 1, \ldots, n$, with the sample averages $\bar{L}_{b,i}$. Finally in this step, the values $\nu_{b,i} = \bar{L}_{b,i} - \bar{L}_i$ are constructed.

The final step iteratively removes the worst performing model. At first, all models are considered. For all these models, the average point differences between the models are calculated. For all models $i = 1, ..., m$, the averages per model are calculated and used to calculate T-statistics. The model with the maximum T-statistic (T_{max}) is nominated to be eliminated from the model confidence set. The p-value is than calculated as $p = \frac{1}{B} \sum_{b=1}^{B} \mathbb{I}_{\{T_{max} > T_{b,max}\}}$, with $T_{b,max}$ the maximum T-statistic of the b-th bootstrap statistics. If this p-value is higher than α, the set is rejected and the model with the maximum T-statistic is removed from the set. The third step is repeated until the p-value is below α. The models that are left over are the models of the model confidence set at a $1 - \alpha$ significance level.

2.8 Implementation Issues

In order to estimate the different models, two implementation issues have to be resolved. Since the different models are estimated dynamically, a choice has to be made whether an expanding or a moving window is used. The choice has been made to use an moving window of 5 years. Since conditional volatility is estimated, it is more logical to dismiss results from older periods, since these do not influence today's volatility anymore. Due to the excessive time it takes to estimate the models, the window is not moved on a daily basis, but on a monthly basis. This results into 72 time windows per model that are estimated.

The other implementation issue to resolve is the way to estimate the different models. Since the conditional volatility is unobserved, standard estimation techniques cannot be used. However, in accordance with Shephard and Shephard (2010) and Hansen, Huang and Shek (2012), Quasi-Likelihood estimation is used. This estimation technique is similar to the estimation technique used for the standard GARCH models and therefore is not discussed in detail. For a detailed reading on the estimation of the parameters, one is referred to the papers of Shephard and Shephard (2010) or Hansen, Huang and Shek (2012).

3 Data

This paper uses the database 'Oxford-Man Institute's realized library' version 0.2, which has been produced by Heber et al. (2009) [2].

The data that is used from the database are the indices of the FTSE100, DAX30, CAC40, AEX, SSMI, IBEX35 and the EUROSTOXX50. These data

[2] Available at http://realized.oxford-man.ox.ac.uk

are available from January 3 2000 up to including the current day. In this paper the data available from January 2000 up to including March 2014 is used.

The database contains a lot of different realized measures. The measures that will be used to research the models are the realized variance (at 5 and 10-minute intervals) (for example, Andersen and Bollerslev, 1998), the realized kernel (Hansen and Lunde, 2006), the (5-minute) realized bi-power variation (Barndoff-Nielsen et al., 2004), the median truncated realized variance (Andersen et al., 2008), the 5-minute realized semivariance (Barndoff-Nielsen et al., 2008) and the daily range (Gallant et al., 1999). The realized variance, bi-power variation and semivariance will be taken with and without subsampling. Instead of considering the daily range (which sometimes is below 0), the squared daily range is used.

The estimation will be done initially from the period of January 2000 up to including December 2005. With these estimates, the forecasts will be made for the next month. For the rest of the period up to including March 2014, a moving window will be applied to update the estimates. This is done, since it is likely that the parameters will change over time. Furthermore, since volatility is time-dependent, the data from the period of 2000 is not likely to be relevant for the estimates in, for example 2011.

3.1 Summary statistics

Table 1 gives summary statistics for the realized measures and the (squared) log daily returns for each index. The table shows that for all measures, the mean values of the realized measures are almost the same for each index, with exceptions for the realized semivariances and the squared daily range. The means of the realized semivariances are all approximately half of the other realized measures. This can be explained by the fact that for the realized semivariance only a look is taken at the downside risk. Therefore, to be an unbiased estimator for the variance, the measure has to be scaled by 2. The means of the squared daily range are significantly larger than of the other realized measures. This is caused by the fact that according to Parkinson (1980) the squared daily range has to be scaled by $\frac{1}{4\log(2)} \approx \frac{1}{2.772}$. If this scaling is performed on the mean, this results into a mean of 0.94, which lies in the same range as the other realized measures.

When the standard deviations for the realized measures are considered, we see the same features as for the means. The standard deviations of the realized semivariances are all about half of the other realized measures. The standard deviations of the squared daily range is again significantly higher than of the other realized measures. When the scaling is applied again, this results in a standard deviation of 1.74, which again lies in the same range as of the other realized measures. A difference with the means, is that the standard deviations of the squared daily returns are quite higher than of the realized variance. This implies that the realized variance is a better estimator for the volatility than the squared daily returns.

When a look is taken at the daily returns, we see that the mean value is a little below 0. However, since the standard deviation of the returns is about 100 times larger than of the mean, we can assume that the mean return is equal to 0. This justifies the model of Equation (1), where the mean value is disregarded.

Table 1

Summary Statistics

This table reports summary statistics for the different realized measures, the daily returns and the squared daily returns. All values have to be multiplied by 10^{-4}, except for the standard deviation of the returns (which should be multiplied by 10^{-2}). Panel A reports the means of the measures and returns, while Panel B reports the standard deviations of the measures and returns. In the panels, RV5 is the 5-minute realized variance, RK the realized kernel, RV5ss the 5-minute realized variance with subsampling, RV10 the 10-minute realized variance, RV10ss the 10-minute realized variance with subsampling, BV5 the 5-minute bi-variate power variation, BV5ss the 5-minute bi-variate power variation with subsampling, MEDRV the median truncated realized variance, RS5 the 5-minute realized semivariance, RS5ss the 5-minute realized semivariance with subsampling and DR the squared daily range.

Panel A

	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	STOXX50
RV5	0.96	1.95	1.55	1.34	0.95	1.56	1.81
RK	0.95	1.93	1.55	1.36	0.90	1.54	1.77
RV5ss	0.89	1.82	1.52	1.31	0.91	1.51	1.62
RV10	1.00	1.93	1.56	1.37	0.94	1.60	1.82
RV10ss	0.90	1.73	1.51	1.33	0.90	1.53	1.58
BV5	0.85	1.68	1.44	1.25	0.87	1.44	1.50
BV5ss	0.83	1.66	1.43	1.25	0.86	1.40	1.46
MEDRV	0.65	1.49	1.19	1.04	0.85	1.22	1.22
RS5	0.49	1.01	0.80	0.69	0.48	0.80	0.94
RS5ss	0.46	0.94	0.78	0.68	0.46	0.78	0.84
DR	2.62	4.83	4.04	3.72	2.64	4.40	4.60
r_t	-3.81	-3.60	-4.48	-4.78	-2.11	-4.79	-3.20
r_t^2	0.98	1.88	1.60	1.49	1.03	1.73	1.94

Panel B

	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	STOXX50
RV5	1.79	3.29	2.52	2.21	1.65	2.13	3.48
RK	1.65	3.40	2.48	2.33	1.50	2.13	3.36
RV5ss	1.58	3.05	2.43	2.18	1.56	2.09	2.86
RV10	1.82	3.30	2.66	2.35	1.66	2.49	3.59
RV10ss	1.59	2.89	2.43	2.30	1.61	2.22	2.60
BV5	1.68	2.81	2.31	2.05	1.55	2.03	2.52
BV5ss	1.50	2.85	2.27	2.12	1.51	1.95	2.42
MEDRV	1.23	2.59	1.81	1.62	1.21	1.44	2.01
RS5	0.95	1.86	1.37	1.20	0.86	1.14	2.27
RS5ss	0.82	1.68	1.29	1.17	0.81	1.12	1.76
DR	4.83	8.64	6.78	7.39	5.37	6.96	8.08
r_t	0.99	1.37	1.27	1.22	1.02	1.32	1.39
r_t^2	2.42	4.92	3.98	4.36	3.02	4.71	4.92

Table 2 reports the correlation matrix for the different realized measures for the FTSE100. The upper diagonal shows the Pearson's autocorrelations, while the lower diagonal shows the Spearman's rank correlations. As we can see in Table 2, the Pearson's correlations between the different measures are very high, with 0.79 being the lowest value. This is caused by the fact that the underlying data that determines the measure are the same for every measure, only calculated into a different way.

For the Spearman's rank correlations, Table 2 shows that the correlations are even higher, with a minimum of 0.83. This shows that when there is accounted for extreme values, it shows that the realized measures are even more correlated with one another.

For the correlation matrices of the other indices, the same results hold as for the FTSE100. This means that it not useful to combine realized measures in estimating the above mentioned models, since they would not add a lot of extra information to the model.

Figure 1 shows some summary results for the 5-minute realized variance of the FTSE100. In subfigure a), one can see the development of the 5-minute realized variance over the entire period that is considered. It shows that between 2000 and 2003 and 2008 to 2012 the index is very volatile, with high peaks around 2003, late 2008 and late 2011. The latter two peaks mark different time periods in the most recent financial crisis, caused by the global bank crisis. The first peak marks a period during the collapse of the internet bubble. In these periods, stock markets globally were very volatile. For the other indices, the same features can be seen. One striking difference with other realized measures for the FTSE100 is that the peak around 2003 is the highest for the realized variance. For the realized kernel, the peak is the highest during the start of the most recent financial crisis, starting at the end of 2008. This fact is peculiar and the only explanation that could be given to this is the way that the realized measure is calculated gives these differences.

Subfigure b) and c) of Figure 1 tell something about the distribution of the realized variance. In this paper, it is sometimes implicitly assumed that the realized measures follow a (mixed) normal distribution or lognormal distribution. Subfigure b) and c) show that the realized variance most definitely does not follow a normal distribution. This can already be concluded from the fact that the values of the realized measures can never take a negative value. Subfigure b) likes to show that the realized variance follows a lognormal distribution. The red line (the density function of a lognormal distribution) follows the bars of the histogram quite good. When the natural logarithmic transformation is taken, the results seem to follow a normal distribution that is not completely symmetric. This follows also from the

Table 2

Correlation Matrix Realized Measures FTSE100

This table reports the correlations between the different realized measures of the FTSE100 index. The upper diagonal shows the Pearson autocorrelations, while the lower diagonal shows the Spearman's rank correlations. In this table, RV5 is the 5-minute realized variance, RK the realized kernel, RV5ss the 5-minute realized variance with subsampling, RV10 the 10-minute realized variance, RV10ss the 10-minute realized variance with subsampling, BV5 the 5-minute bi-variate power variation, BV5ss the 5-minute bi-variate power variation with subsampling, MEDRV the median truncated realized variance, RS5 the 5-minute realized semivariance, RS5ss the 5-minute realized semivariance with subsampling and DR the squared daily range.

	RV5	RK	RV5ss	RV10	RV10ss	BV5	BV5ss	MEDRV	RS5	RS5ss	DR
RV5	1.00	0.95	0.97	0.94	0.93	0.96	0.96	0.91	0.94	0.93	0.85
RK	0.99	1.00	0.97	0.98	0.97	0.90	0.95	0.89	0.90	0.95	0.87
RV5ss	0.99	0.99	1.00	0.95	0.98	0.96	0.99	0.94	0.91	0.96	0.87
RV10	0.98	0.98	0.98	1.00	0.95	0.88	0.92	0.86	0.89	0.92	0.86
RV10ss	0.98	0.98	0.99	0.98	1.00	0.91	0.98	0.91	0.87	0.95	0.88
BV5	0.98	0.98	0.99	0.97	0.98	1.00	0.97	0.91	0.89	0.91	0.82
BV5ss	0.98	0.98	1.00	0.97	0.99	0.99	1.00	0.94	0.89	0.95	0.86
MEDRV	0.96	0.97	0.98	0.94	0.97	0.97	0.98	1.00	0.83	0.89	0.79
RS5	0.95	0.94	0.95	0.94	0.94	0.95	0.95	0.93	1.00	0.94	0.81
RS5ss	0.95	0.95	0.96	0.94	0.96	0.95	0.96	0.95	0.99	1.00	0.85
DR	0.88	0.88	0.89	0.89	0.90	0.87	0.88	0.85	0.83	0.84	1.00

skewness and the kurtosis of the realized variance, which are 0.42 and 3.05 respectively. The skewness of 0.42 shows that the distribution of $\log(RV5_t)$ is not symmetric and can therefore not be normally distributed. The kurtosis of 3.05 lies very close to 3, meaning that the tails are as fat as of a normal distribution. The Jarque-Bèra test statistic is 105.4, meaning that the realized variance is most definitely not lognormally distributed. For the other realized measures, the skewness of their log transformations lies between 0.21 (DR) and 0.66 ($MEDRV$), while the kurtosis lies between 2.81(DR) and 3.49 ($MEDRV$). Therefore, for none of the realized measures it holds that they are lognormally distributed.

Finally, the autocorrelations of the realized measures, shown in subfigure d) of Figure 1, are pretty high and only slowly declining. For the normal realized variance, the value starts at 0.56 and only declines slowly over the 100 lags towards 0.11. For the log transformation, the autocorrelation at 1 lag is about 0.82 and declines to 0.43. The models that are used in this papers implicitly already assumed this persistence. The graph of Figure 1 d) now shows that this assumption is legit.

4 Results

4.1 Estimates

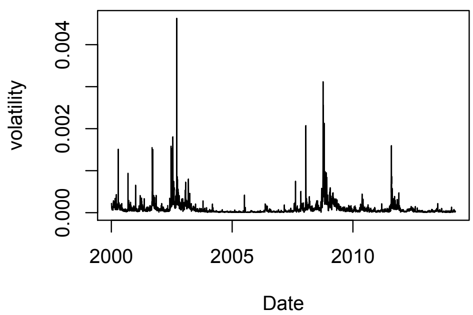
First of all, the estimates of the models are examined. Not all the models are shown, since this does not contribute to the understanding of the article. Moreover, the emphasis of this paper lies in the forecasting of the models. Therefore, the estimates are only discussed briefly and only for the realized variance of the FTSE100.

Figure 1

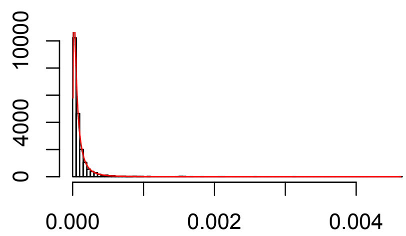
Summary Graphs

This figure reports graphs for the 5-minute realized variance of the FTSE100 and its log transformation. Subfigure a) shows the realized variance over time from January 2000 - March 2014. Subfigure b) shows a histogram of the 5-minute realized variance. The red line is a density function of the lognormal distribution, with the mean and variance of the log transform of the realized variance as parameters. Subfigure c) shows a histogram of the natural logarithm of the realized variance. The red line is the density function of a normal distribution with the mean and variance of the log realized variance as parameters. Finally, Subfigure d) shows the autocorrelation function of the first 100 lags of the realized variance (blue) and its log transformation (red).

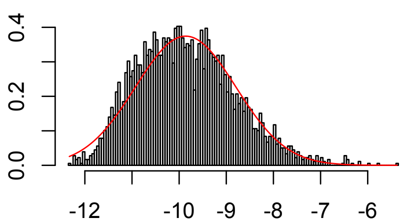
(a) $RV5_t$ plot



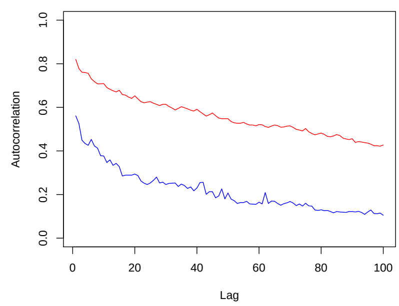
(b) $RV5_t$ histogram



(c) $\log(RV5_t)$ histogram



(d) Autocorrelations



4.1.1 Standard models

At first, the estimates of the standard models are examined. Figure 2 reports the estimates of the three standard models that are explained in Equations (2) - (4).

Figure 2 shows that for the HEAVY model, ω is approximately constant over time and the sum of α and β is constant over time. This means that the unconditional variance is approximately constant over time. The fact that the sum of the α and β is approximately 1 for the entire period implies that the persistence of the variance is very high. This is one of the assumptions and features that GARCH models are based on. Figure 2 shows that this feature also holds for the realized variance of the FTSE100 in the HEAVY model. Figure 2 also shows that α and β are tending more towards each other over time. This means that in the beginning the conditional unobserved variance plays a bigger role in explaining the conditional variance a day later than the realized measure. However, at the end the contribution to the conditional variance is approximately the same.

For the μ_t part, we see that the parameters of α_R and β_R are very close together and cross each other two times over time. This means that for the realized measure, the past unobserved value and the realized value are explaining approximately the same part of the prediction of the next value for the realized measure. The sum of the two parameters is again constant over time and a little below 1. This means that the sum of the parameters is robust over time, and also for the μ_t part the persistence is high.

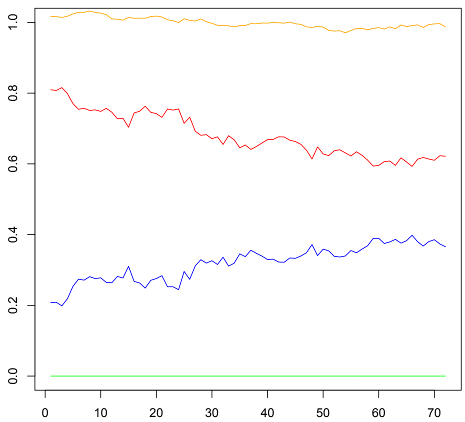
Figure 2 shows that for the RealGARCH(1,1) model, there is a strange jump around time window 28, which complies with adding August 2008 to the sample. This was around the beginning of the latest financial crisis, so it seems reasonable that the parameters would change around this period, since

Figure 2

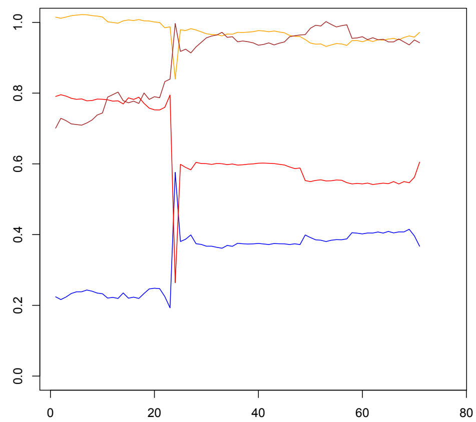
Estimates of the standard models

This figure reports the estimates of the parameters of the standard models. The lines correspond to the following parameters: green is ω, blue is α, red is β, yellow is γ, orange is $\alpha + \beta + 0.5\gamma$, purple is ξ, brown is ϕ, black is τ_1 and grey is τ_2.

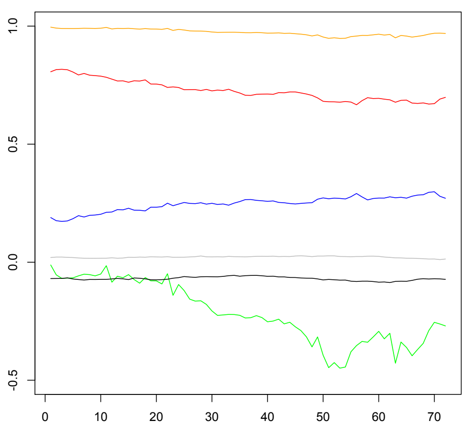
(a) HEAVY model (h_t)



(b) RealGARCH(1,1) model (large parameters)



(c) RealGARCH(1,1) model (h_t)



(d) HEAVY model (μ_t)

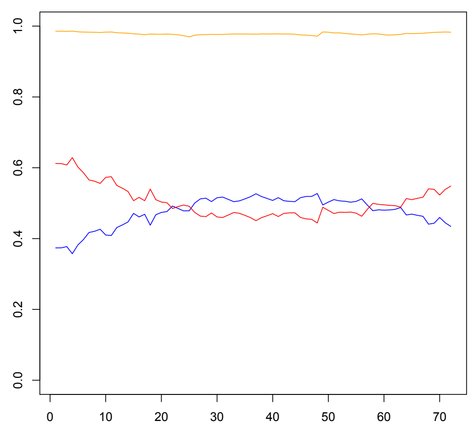
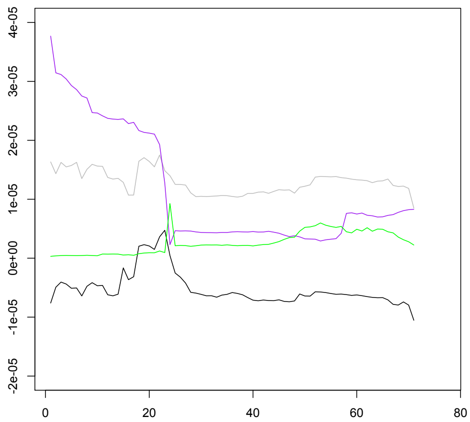
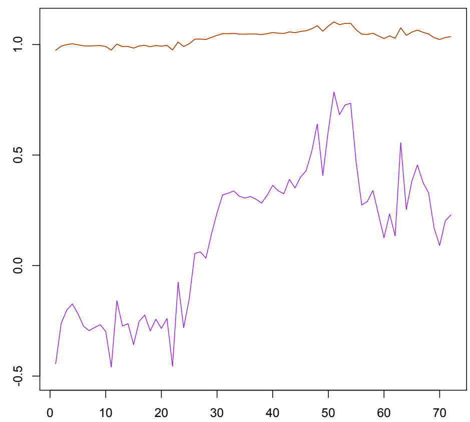


Figure 2 continued

(e) RealGARCH(1,1) model (small parameters)



(f) RealGARCH(1,1) model (RM_t)



the volatility got higher from this moment on. This indicates that there is a break in the constancy of the parameters. However, the sum of α and β is only altered in a small way, which makes it seems like this sum is robust over time and the persistence of the RealGARCH(1,1) model is high.

For display reasons, the parameters of the RealGARCH(1,1) model are divided into a figure for the large parameters and the small parameters. The small parameters are the parameters that are in the order of 10^{-5}. These parameters include ω, ξ, τ_1 and τ_2. The large parameters are all in the order of 0.1 to 1. These parameters include α, β, $\alpha + \beta$ and ϕ.

For the large parameters, figure 2 shows that before and after the jump around August 2008, the parameters α and β are approximately constant. This indicates that these parameters are robust over time and depend only little on the window that is chosen to estimate the parameter. For ϕ, almost the same holds, only around the sudden jump for the other parameters, ϕ

goes gradually to a new level. For the rest of the time it only differs a little, meaning that it is likely that ϕ is robust too. This means that throughout the complete sample, both h_t is a good (unobserved) measure to explain the realized variance and vice versa.

For the small parameters, Figure 2 shows that most of the parameters are approximately constant. The only parameter that deviates from this trend is ξ. This parameter starts relatively big and declines over time. This, together with the fact that ϕ is almost constant over time, indicates that the unconditional mean of the realized variance declines over time and after the sudden jump, it levels out to be approximately the same for the rest of the period.

Finally, Figure 2 shows that for the RealEGARCH(1,1) model, the parameters τ_1 and τ_2 seem to be constant over time, albeit that they are very close to 0, in comparison with the other parameters. This means that the leverage effect of the model is only small, but constant. Another observation is that the τ_1 parameter is constantly below 0 and the τ_2 parameters is constantly above 0. This means that z_t has a negative impact on the conditional variance and $z_t^2 - 1$ a positive effect on the conditional variance. Furthermore, we see that for the conditional variance, the effects for α, β and $\alpha + \beta$ are similar to the trend for these parameters in the HEAVY model, and therefore the persistence is also harboured into the RealEGARCH model. For ω, we see that the value starts of just below 0 and that over time this value gradually declines to around -0.4. This means that the other parameters overestimate the conditional variance and a factor smaller than 1 has to be applied to correct for this overestimation.

Figure 2 shows at last that for the estimating part of RM_t, ϕ is a parameter that constantly lies around 1. This suggests that $\log(RM_t)$ and $\log(h_t)$

have the same relation and trends, and only differ by a constant. This constant ξ, however, is very unstable, since it has a lot of peaks, as can be seen in Figure 2. It starts around -0.5 and than goes gradually, though be it unsteady, to 0.5, before it declines again to 0.25. This means that it is probably hard to predict RM_t and therefore hard to predict h_t a couple of days ahead.

4.1.2 Models with r_t^2

Next, the estimates of the models with the squared daily return as an additional regressor are considered. It should be recalled that this expansion is done only for the HEAVY and RealGARCH(1,1) model, since the estimates of the RealEGARCH(1,1) model are very unstable. Figure 3 reports the estimates for these models. The figure does not show the estimates for the μ_t part of the HEAVY model, since those are the same as in Figure 2.

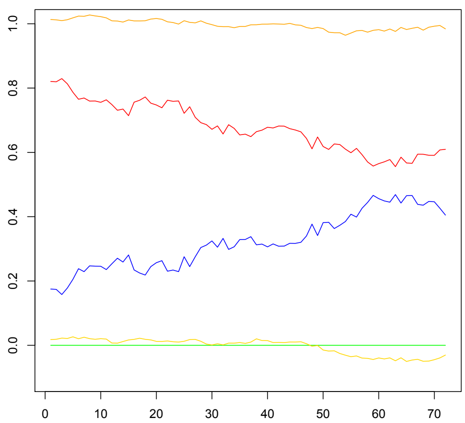
For the HEAVY model, Figure 3 shows that for ω, α and β, we have the same pattern as for the standard model. Therefore, these parameters will not be discussed again. An interesting parameter to look at, is the γ parameter, since it tells what the extra value of r_t^2 is. We see that the value of γ lies around 0 for the complete part, meaning that the regressor adds, relatively to the realized variance, little value in explaining the unconditional variance. This means that the realized variance is a better proxy for the conditional variance than the squared daily return. We do again see that the sum of α, β and γ constantly lies around 1, meaning that the persistence is still harboured into the model, even after adding an additional regressor. One significant detail is that the value of the sum of these parameters is sometimes larger than 1. This is something that is not allowed into the regular GARCH models. However, since the mean of the realized variance and of the squared daily return are not the same, this means that for checking if the

Figure 3

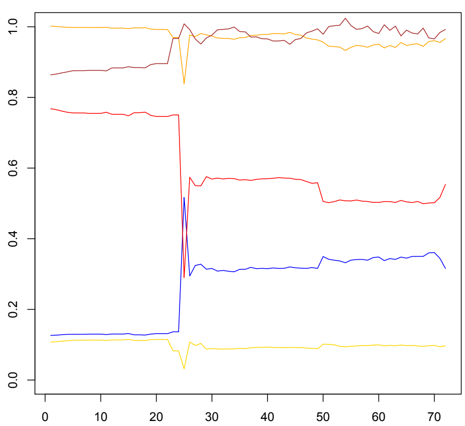
Estimates of the standard models

This figure reports the estimates of the parameters of the HEAVY and Real-GARCH(1,1) models with r_t^2 as additional regressor. The lines correspond to the following parameters: green is ω, blue is α, red is β, yellow is γ, orange is $\alpha+\beta+\gamma$, purple is ξ, brown is ϕ, black is τ_1 and grey is τ_2.

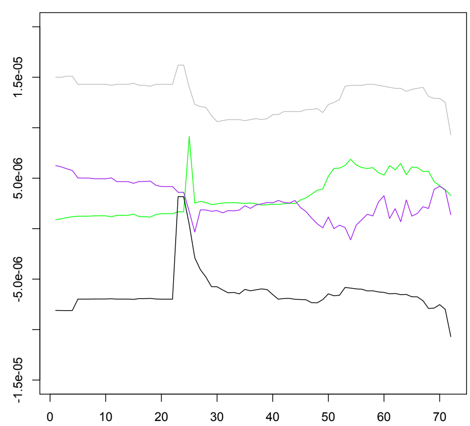
(a) HEAVY model (h_t)



(b) RealGARCH(1,1) model (large parameters)



(c) RealGARCH(1,1) model (h_t)



model is stationary, there has to be made an adjustment for the difference in means. After this adjustment is made, the model is still stationary.

For the large parameters of the RealGARCH(1,1) model, Figure 3 shows that all the parameters show the same trend as in the standard models. The parameter of γ lies around 0.1 for almost the complete time, except around the time window that includes August 2008. The same jump as for the other parameters is show in γ, which goes to just above 0. Figure 3 shows that γ has a higher value in the RealGARCH(1,1) model than in the HEAVY model. This means that the influence of the squared daily return is of bigger importance in the RealGARCH(1,1) model than in the HEAVY model. However, since α is bigger than γ, the realized variance explains the conditional variance better than the squared daily return.

For the small parameters of the RealGARCH(1,1) model, Figure 3 shows, in contradiction to the large parameters, some difference with the estimates of the standard RealGARCH(1,1) model. We see that ξ is more constant than in the standard RealGARCH(1,1) model.

4.1.3 Non-linear models

The third set of models that are estimated are the non-linear models. The estimates are presented in Figure 4. In Figure 4, the μ_t part of the HEAVY model is not taken into account, since this is the same as in Figure 2.

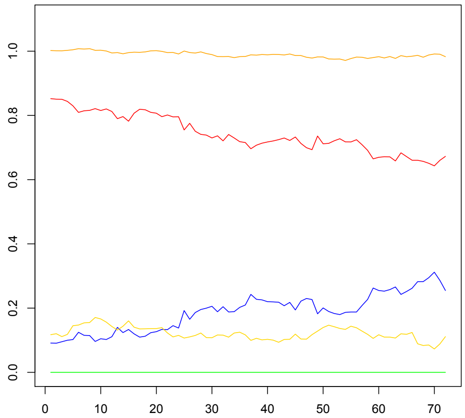
Figure 4 shows that for the HEAVY model, the parameter of α is a lot smaller than in the standard models. This is due to the fact that part of the explanatory power of the realized variance lies now in γ. Figure 4 shows that γ lies around 0.1 for all the time windows, which is approximately the difference between the α of Figure 2 and Figure 4. This means that when the returns are negative, the realized variance contributes big to the value of

Figure 4

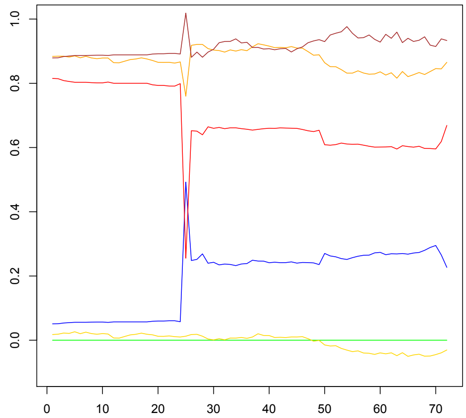
Estimates of the standard models

This figure reports the estimates of the parameters of the three models with $\mathbb{I}_{\{r_t<0\}}RM_t$ as additional regressor. The lines correspond to the following parameters: green is ω, blue is α, red is β, yellow is γ, orange is $\alpha+\beta+\gamma/2$, purple is ξ, brown is ϕ, black is τ_1 and grey is τ_2.

(a) HEAVY model (h_t)



(b) RealGARCH(1,1) model (large parameters)



(c) RealGARCH(1,1) model (h_t)

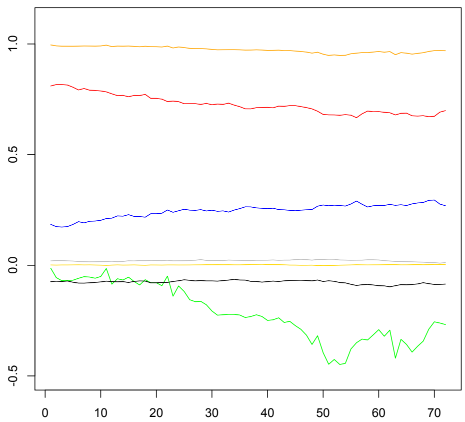
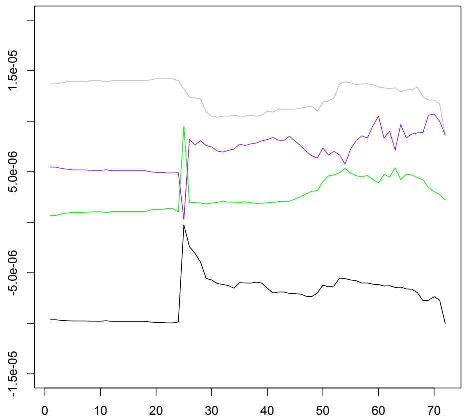
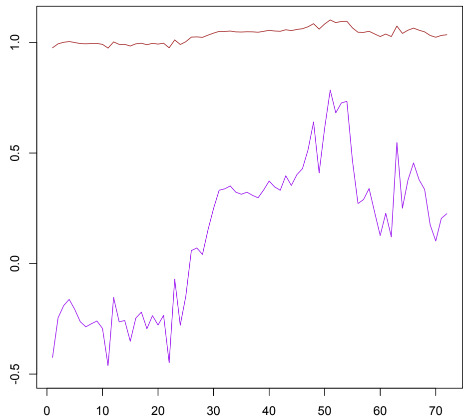


Figure 4 continued

(d) RealGARCH(1,1) model (small parameters)



(e) RealGARCH(1,1) model (RM_t)



the conditional variance. This suggests that the assumption that volatility is higher when returns are negative are underlined by the non-linear HEAVY model. For β, Figure 4 shows that it follows the same pattern as in the standard model. Also the sum of the parameters α, β and $\frac{1}{2}\gamma$ lies around 1. Only the half of γ is taken, since $\mathbb{I}_{\{r_t<0\}}$ is 0 (approximately) half of the time. This indicates that the persistence in the non-linear HEAVY model is still present.

Figure 4 shows that for the large parameters of the RealGARCH(1,1) model, α has declined approximately 0.2 for the entire period in comparison to α in the standard RealGARCH(1,1) model. One would expect that this would result in a large γ. However, this is not the case. Figure 4 shows that γ does not deviate much from 0. This is peculiar and no explanation for this can be given. What can be said about γ is that the effect of the realized measure when $r_t < 0$ is not much more than when $r_t > 0$. This means that the assumption that the volatility is higher when returns are negative is not underlined by the RealGARCH(1,1) model. The rest of the parameters are developing in approximately the same way as in the standard RealGARCH(1,1) model.

Figure 4 shows, for the small parameters of the RealGARCH(1,1) model, that they are all approximately the same as for the RealGARCH(1,1) model with the daily squared return as additional regressor. Again, we see that in comparison with the parameters of the standard RealGARCH(1,1) model, the parameters change all smoother over time and that ξ is more constant over time.

Figure 4 shows that for the conditional variance part of the RealEGARCH(1,1) model, that γ is almost 0 for every time frame. The parameter for α has not changed a lot in comparison to the standard RealEGARCH(1,1) model. This means that the effect of the negative realized measure is not very big. The suggestion that the volatility is higher when the returns are negative is therefore not underlined by the RealEGARCH(1,1) model. Since the effect of splitting up the model for positive and negative returns is not very big, the other parameters have not changed a lot too.

Finally, Figure 4 shows that for the realized measure part of the RealEGARCH(1,1) model, the parameters of ϕ and ξ have not noticeably changed in comparison to the standard RealEGARCH(1,1) model. This is due to the fact that the value of γ lies very close to 0, while the other parameters did not change much too. This means that the unconditional variance has not changed a lot, which results in approximately the same parameters ξ and ϕ as in the standard RealEGARCH model.

4.1.4 Combination of realized bi-power and realized variance

As explained in the section ‘Methods’, estimating models with combinations of realized measures is not fruitful. As the ‘Data’ section shows, the correlations are in general quite high between the realized measures. However, the estimates for the combination of the realized bi-power and realized variance are interesting from a theoretical point-of-view. Figure 5 reports the values of α_1 and α_2 of Equation (10) over time for the HEAVY and RealGARCH(1,1) model for the FTSE100.

Figure 5

Estimates of combination of $RV5_t$ and $BV5_t$

This figures show the parameters α_1 and α_2 of Equation (10) for the HEAVY and RealGARCH(1,1) models. The results are for the FTSE100, taken over the 72 time windows. The blue line represents α_1 and the red line represents α_2. Subfigure a) shows this for the HEAVY model and subfigure b) for the RealGARCH(1,1) model.

(a) HEAVY model **(b)** RealGARCH(1,1) model

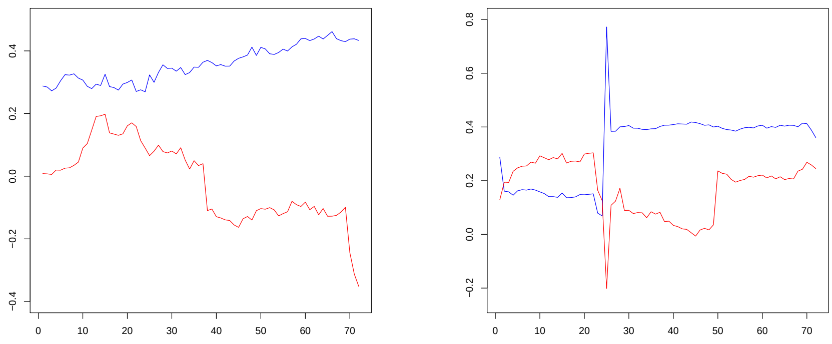


Figure 5 shows that for both the HEAVY as the RealGARCH(1,1) model, the jumps have a great contribution to the volatility. For the RealGARCH(1,1) model, the jumps even have a bigger influence on the conditional variance than the realized bi-power variation in the first 20 time frames. For both models it holds though, that the contribution of the different regressors differs quite a lot and are most likely not robust over time. Also, Figure 5 shows that there are sudden jumps in the parameters for the difference regressor.

A significant detail is that for both models the parameters differ a lot between the models. As for the RealGARCH(1,1) model, the jump parameter is almost always above 0, the parameter of the HEAVY model jumps below 0 after about time window 35, which is around March 2009. This shows that the influence of the model is also a big part of the contribution on how well the realized measures predict the conditional variance.

4.2 Forecasts

4.2.1 Standard models

At first, the out-of-sample forecast performance of the standard models, as described in Equations (2) - (4), are discussed. Table 3 reports the values that are obtained using the QLIKE loss function as described in Equation (11). The proxies that are used are the squared daily return, the 5-minute realized variance and the realized kernel.

Table 3 and Table 9 in the appendix show that almost all of the used models, with all the different realized measures, do not perform better out-of-sample than the benchmark models. The regular GARCH(1,1) model is not beaten by any of the models, while the HAR(3)-RV model is only beaten by a few models with only a few realized measures. However, we will discuss for the realized variance as proxy and each model separate the results in the

Table 3

QLIKE results for standard models

This table reports the results for the QLIKE loss function (Equation (11)) of all the standard models that were described in Equations (2) - (4). The abbreviations for the realized measures are the same as used in Table 1. The model reports results for the HEAVY (H), RealGARCH(1,1) (RG), RealEGRACH(1,1) (REG), GARCH(1,1) (G) and HAR(3)-RV (HAR) models. The proxy that is used in this table is the realized variance. For the other proxies, the numerical results are presented in the appendix. The forecasting periods is from January 2006 - March 2014. All values have to be multiplied with 10^4.

		h=1					h=5					h=10					h=21				
		H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR
	RV5	-1.738	-1.743	-1.741			-1.723	-1.732	-1.733			-1.722	-1.717	-1.723			-1.715	-1.680	-1.697		
	RK	-1.728	-1.743	-1.739			-1.719	-1.729	-1.732			-1.718	-1.703	-1.722			-1.710	-1.492	-1.696		
	RV5ss	-1.731	-1.747	-1.743			-1.716	-1.738	-1.736			-1.714	-1.723	-1.727			-1.704	-1.617	-1.701		
	RV10	-1.723	-1.743	-1.741			-1.712	-1.728	-1.733			-1.712	-1.678	-1.723			-1.706	-1.352	-1.695		
FTSE100	RV10ss	-1.729	-1.745	-1.742			-1.711	-1.734	-1.735			-1.710	-1.720	-1.725			-1.701	-1.681	-1.699		
	BV5	-1.723	-1.744	-1.739			-1.710	-1.729	-1.730			-1.711	-1.709	-1.720			-1.708	-1.646	-1.692		
	BV5ss	-1.727	-1.745	-1.741			-1.709	-1.733	-1.733			-1.711	-1.713	-1.723			-1.708	-1.530	-1.696		
	MEDRV	-1.708	-1.746	-1.736			-1.696	-1.736	-1.728			-1.703	-1.684	-1.718			-1.699	-1.461	-1.690		
	RS5	-1.635	-1.747	-1.734			-1.622	-1.737	-1.724			-1.651	-1.726	-1.711			-1.674	-1.695	-1.677		
	RS5ss	-1.624	-1.744	-1.735			-1.637	-1.735	-1.726			-1.658	-1.723	-1.716			-1.671	-1.660	-1.686		
	DR	-1.684	-1.741	-1.746			-1.677	-1.725	-1.743			-1.676	-1.655	-1.737			-1.670	-1.079	-1.723		
	-				-1.771	-1.751				-1.783	-1.746				-1.785	-1.738				-1.778	-1.721
	RV5	-1.625	-1.633	-1.627			-1.608	-1.621	-1.614			-1.602	-1.595	-1.591			-1.591	-1.248	-1.530		
	RK	-1.621	-1.634	-1.631			-1.609	-1.626	-1.620			-1.600	-1.603	-1.604			-1.586	-1.238	-1.569		
	RV5ss	-1.625	-1.633	-1.630			-1.611	-1.625	-1.618			-1.604	-1.604	-1.603			-1.592	-1.245	-1.567		
	RV10	-1.617	-1.633	-1.626			-1.605	-1.621	-1.615			-1.597	-1.591	-1.598			-1.583	-1.257	-1.562		
DAX35	RV10ss	-1.614	-1.634	-1.631			-1.599	-1.626	-1.620			-1.597	-1.605	-1.604			-1.587	-1.256	-1.569		
	BV5	-1.607	-1.635	-1.629			-1.595	-1.627	-1.617			-1.590	-1.609	-1.600			-1.582	-1.415	-1.561		
	BV5ss	-1.618	-1.633	-1.630			-1.601	-1.627	-1.618			-1.594	-1.609	-1.601			-1.581	-1.273	-1.564		
	MEDRV	-1.610	-1.634	-1.630			-1.594	-1.627	-1.618			-1.590	-1.613	-1.601			-1.583	-1.360	-1.563		
	RS5	-1.494	-1.632	-1.619			-1.476	-1.623	-1.605			-1.487	-1.608	-1.585			-1.508	-1.510	-1.538		
	RS5ss	-1.452	-1.632	-1.623			-1.457	-1.625	-1.609			-1.482	-1.610	-1.590			-1.505	-1.521	-1.545		
	DR	-1.591	-1.633	-1.641			-1.582	-1.622	-1.635			-1.579	-1.606	-1.628			-1.570	-1.575	-1.611		
	-				-1.659	-1.639				-1.684	-1.634				-1.687	-1.626				-1.682	-1.610

Table 3 continued

		h=1					h=5					h=10					h=21				
		H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR
CAC40	RV5	-1.663	-1.668	-1.659			-1.637	-1.645	-1.640			-1.628	-1.611	-1.622			-1.613	-1.262	-1.580		
	RK	-1.653	-1.668	-1.659			-1.634	-1.644	-1.641			-1.620	-1.609	-1.624			-1.614	-1.249	-1.585		
	RV5ss	-1.662	-1.667	-1.658			-1.634	-1.645	-1.640			-1.621	-1.614	-1.623			-1.613	-1.265	-1.583		
	RV10	-1.658	-1.668	-1.656			-1.632	-1.646	-1.638			-1.616	-1.616	-1.620			-1.607	-1.316	-1.579		
	RV10ss	-1.658	-1.667	-1.657			-1.630	-1.643	-1.639			-1.622	-1.609	-1.622			-1.614	-1.242	-1.583		
	BV5	-1.657	-1.667	-1.657			-1.640	-1.644	-1.638			-1.625	-1.614	-1.620			-1.610	-1.293	-1.575		
	BV5ss	-1.656	-1.666	-1.658			-1.627	-1.644	-1.639			-1.616	-1.614	-1.621			-1.611	-1.278	-1.580		
	MEDRV	-1.642	-1.665	-1.656			-1.612	-1.645	-1.637			-1.605	-1.621	-1.618			-1.599	-1.309	-1.572		
	RS5	-1.570	-1.668	-1.655			-1.536	-1.648	-1.635			-1.539	-1.624	-1.614			-1.550	-1.328	-1.565		
	RS5ss	-1.570	-1.666	-1.655			-1.543	-1.647	-1.635			-1.540	-1.623	-1.614			-1.551	-1.340	-1.565		
	DR	-1.608	-1.672	-1.657			-1.591	-1.664	-1.645			-1.579	-1.606	-1.634			-1.571	-1.537	-1.610		
	-				-1.682	-1.673				-1.697	-1.664				-1.699	-1.656				-1.692	-1.638
AEX	RV5	-1.713	-1.724	-1.708			-1.691	-1.695	-1.689			-1.685	-1.625	-1.669			-1.679	-1.174	-1.621		
	RK	-1.712	-1.724	-1.709			-1.690	-1.697	-1.690			-1.678	-1.645	-1.671			-1.673	-1.244	-1.626		
	RV5ss	-1.716	-1.726	-1.710			-1.692	-1.695	-1.692			-1.680	-1.647	-1.673			-1.669	-1.147	-1.629		
	RV10	-1.708	-1.725	-1.707			-1.687	-1.696	-1.687			-1.678	-1.623	-1.666			-1.674	-1.114	-1.615		
	RV10ss	-1.713	-1.725	-1.709			-1.690	-1.701	-1.690			-1.680	-1.673	-1.671			-1.676	-1.460	-1.626		
	BV5	-1.711	-1.724	-1.707			-1.690	-1.699	-1.687			-1.676	-1.664	-1.666			-1.667	-1.293	-1.614		
	BV5ss	-1.710	-1.721	-1.709			-1.683	-1.705	-1.690			-1.674	-1.689	-1.671			-1.671	-1.642	-1.624		
	MEDRV	-1.709	-1.724	-1.710			-1.687	-1.717	-1.692			-1.681	-1.707	-1.674			-1.670	-1.600	-1.629		
	RS5	-1.616	-1.724	-1.703			-1.599	-1.705	-1.682			-1.610	-1.675	-1.658			-1.630	-1.298	-1.598		
	RS5ss	-1.621	-1.688	-1.706			-1.588	-1.676	-1.686			-1.604	-1.659	-1.663			-1.626	-1.615	-1.608		
	DR	-1.656	-1.672	-1.706			-1.637	-1.671	-1.692			-1.629	-1.667	-1.678			-1.625	-1.658	-1.646		
	-				-1.743	-1.727				-1.755	-1.718				-1.756	-1.710				-1.748	-1.693
SSMI	RV5	-1.711	-1.715	-1.716			-1.706	-1.703	-1.710			-1.701	-1.683	-1.700			-1.694	-1.568	-1.681		
	RK	-1.704	-1.717	-1.716			-1.700	-1.707	-1.711			-1.693	-1.684	-1.702			-1.681	-1.353	-1.683		
	RV5ss	-1.709	-1.715	-1.715			-1.703	-1.704	-1.708			-1.698	-1.679	-1.698			-1.691	-1.376	-1.678		
	RV10	-1.700	-1.716	-1.715			-1.699	-1.703	-1.708			-1.694	-1.677	-1.697			-1.686	-1.293	-1.676		
	RV10ss	-1.703	-1.714	-1.713			-1.696	-1.699	-1.707			-1.690	-1.671	-1.697			-1.680	-1.288	-1.677		
	BV5	-1.702	-1.715	-1.715			-1.699	-1.704	-1.708			-1.696	-1.683	-1.699			-1.690	-1.356	-1.678		
	BV5ss	-1.703	-1.714	-1.714			-1.696	-1.701	-1.708			-1.691	-1.675	-1.698			-1.681	-1.333	-1.678		
	MEDRV	-1.719	-1.716	-1.713			-1.712	-1.707	-1.706			-1.707	-1.692	-1.696			-1.697	-1.653	-1.676		
	RS5	-1.615	-1.718	-1.714			-1.618	-1.711	-1.705			-1.631	-1.699	-1.694			-1.649	-1.634	-1.670		
	RS5ss	-1.601	-1.717	-1.711			-1.623	-1.709	-1.703			-1.631	-1.694	-1.691			-1.642	-1.642	-1.666		
	DR	-1.666	-1.721	-1.717			-1.662	-1.715	-1.715			-1.659	-1.709	-1.713			-1.652	-1.697	-1.707		
	-				-1.738	-1.723				-1.762	-1.721				-1.765	-1.714				-1.759	-1.701

Table 3 continued

		h=1					h=5					h=10					h=21				
		H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR
IBEX35	RV5	-1.598	-1.594	-1.609			-1.582	-1.578	-1.596			-1.576	-1.556	-1.581			-1.567	-1.502	-1.545		
	RK	-1.584	-1.611	-1.610			-1.573	-1.593	-1.597			-1.565	-1.566	-1.582			-1.556	-1.498	-1.547		
	RV5ss	-1.599	-1.610	-1.608			-1.580	-1.590	-1.596			-1.573	-1.561	-1.582			-1.565	-1.422	-1.547		
	RV10	-1.597	-1.613	-1.607			-1.581	-1.597	-1.594			-1.575	-1.576	-1.578			-1.563	-1.517	-1.540		
	RV10ss	-1.591	-1.620	-1.609			-1.576	-1.614	-1.596			-1.571	-1.607	-1.582			-1.564	-1.593	-1.548		
	BV5	-1.586	-1.619	-1.608			-1.573	-1.613	-1.595			-1.566	-1.606	-1.580			-1.559	-1.593	-1.543		
	BV5ss	-1.589	-1.607	-1.608			-1.569	-1.587	-1.595			-1.562	-1.560	-1.580			-1.554	-1.495	-1.544		
	MEDRV	-1.582	-1.600	-1.595			-1.565	-1.582	-1.582			-1.560	-1.558	-1.566			-1.552	-1.499	-1.530		
	RS5	-1.443	-1.621	-1.605			-1.458	-1.614	-1.590			-1.468	-1.607	-1.569			-1.480	-1.594	-1.520		
	RS5ss	-1.480	-1.621	-1.607			-1.457	-1.615	-1.592			-1.463	-1.608	-1.572			-1.478	-1.594	-1.525		
	DR	-1.554	-1.614	-1.616			-1.546	-1.608	-1.609			-1.543	-1.602	-1.602			-1.538	-1.589	-1.589		
	-				-1.630	-1.613				-1.650	-1.607				-1.653	-1.602				-1.646	-1.589
Eurostoxx50	RV5	-1.566	-1.575	-1.558			-1.532	-1.549	-1.544			-1.532	-1.504	-1.526			-1.527	-1.209	-1.481		
	RK	-1.550	-1.577	-1.564			-1.526	-1.557	-1.553			-1.527	-1.524	-1.539			-1.527	-1.416	-1.502		
	RV5ss	-1.577	-1.578	-1.567			-1.553	-1.560	-1.556			-1.542	-1.531	-1.542			-1.531	-1.277	-1.505		
	RV10	-1.564	-1.579	-1.560			-1.540	-1.552	-1.548			-1.539	-1.504	-1.531			-1.534	-1.050	-1.491		
	RV10ss	-1.545	-1.580	-1.569			-1.517	-1.560	-1.559			-1.521	-1.531	-1.545			-1.522	-1.418	-1.509		
	BV5	-1.523	-1.579	-1.567			-1.502	-1.563	-1.553			-1.511	-1.540	-1.536			-1.513	-1.481	-1.492		
	BV5ss	-1.556	-1.577	-1.566			-1.530	-1.566	-1.555			-1.526	-1.549	-1.541			-1.524	-1.510	-1.501		
	MEDRV	-1.528	-1.575	-1.571			-1.509	-1.569	-1.560			-1.507	-1.551	-1.545			-1.507	-1.512	-1.504		
	RS5	-1.385	-1.579	-1.555			-1.368	-1.560	-1.540			-1.382	-1.527	-1.519			-1.411	-1.357	-1.471		
	RS5ss	-1.309	-1.578	-1.553			-1.349	-1.559	-1.540			-1.367	-1.525	-1.520			-1.405	-1.308	-1.475		
	DR	-1.543	-1.586	-1.583			-1.528	-1.574	-1.581			-1.525	-1.558	-1.577			-1.518	-1.499	-1.565		
	-				-1.622	-1.579				-1.648	-1.581				-1.654	-1.576				-1.649	-1.565

remainder of this section.

For the HEAVY models, we see that in general the best performing models, are the models that use the 'classic' realized measures as realized measure, being the 5-minute realized variance (with and without subsampling) and the realized kernel. For the forecasting at a horizon of 21 days, we even see that the realized variance is the measure that forecasts the best for the HEAVY model, with exception of the Eurostoxx50, where the 10-minute realized variance forecasts the best.

For the RealGARCH(1,1) model, it is difficult to say something general about which realized measure performs best. The best performing realized measures, looking at the out-of-sample results, differs per index. However, almost never the best performing realized measures are the realized variance (with and without subsampling, 5- and 10-minute), and the realized kernel. The other realized measures have at least at one of the horizons and indices that they are the best performing. For the IBEX35, for example, the realized semivariance with subsampling performs best at all time horizons, while for the SSMI this is the squared daily range.

For the RealEGARCH(1,1) model, Table 3 reports that when using the realized variance as proxy, the best realized measure to use is the squared daily range. The only index that this is not the best realized measure to forecast out-of-sample, is the AEX at a 1-day forecasting horizon. For the AEX, the best realized measure to use is the median truncated realized variance or 5-minute realized variance with subsampling. A remark for the RealEGARCH(1,1) model is that it beats the HAR(3)-RV model with some indices. For example, for the Eurostoxx50, the RealEGARCH(1,1) model with the squared daily range performs the same as the HAR(3)-RV model at a 5-day and 21-day horizon and even beats the model at a 1-day and 10-day horizon.

Between the models, the results are mixed, but the best model is for almost all indices and horizons a trade-off between the RealGARCH(1,1) and RealEGARCH(1,1) model. Only for the 21-day ahead forecasts of the AEX index, the HEAVY model outperforms the other two models. For the AEX, SSMI and IBEX35, the RealGARCH(1,1) model outperforms the other models, with exception of the 21-day ahead forecasts of the AEX (HEAVY) and the SSMI (RealEGARCH(1,1)). For the FTSE100, DAX30 and EUROSTOXX50, the RealEGARCH(1,1) model outperforms the other models, with exception of the 1-day ahead forecasts of the FTSE100 (RealGARCH(1,1)). For the CAC40, the best model is for the 1-day and 5-day ahead forecasts the RealGARCH(1,1) model and for the 10-day and 21-day ahead forecast the RealEGARCH(1,1) model. In general, the results show that for the shorter horizons the RealGARCH(1,1) model is the best, while for the longer horizons the RealEGARCH(1,1) model is the best.

Table 9 in the appendix shows that when the realized kernel or the squared daily return is used as a proxy, the results do not differ a lot from the results with the realized variance as a proxy. We see that for all indices at all time horizons, the same models turn out to be the best. The only difference is that at some indices and time horizons the best performing realized measure differs. However, the general trend stays the same, and therefore the QLIKE results are robust to the chosen proxy.

4.2.2 Models with r_t^2

For the HEAVY and RealGARCH(1,1) model, the out-of-sample performance is also examined with the squared daily return as an extra regressor. The values of the QLIKE loss function are given in Table 4. Since the results of the standard models show that there are not many differences between the proxies, only the results that use the realized variance as a proxy are discussed in detail. The results for the other proxies can be found in the 'Appendix', but are not discussed.

Table 4 shows that for the HEAVY models with the realized variance as proxy, in general the 'classic' realized measures deliver the best results, which is also the case for the standard HEAVY model. The only exception to this pattern, is for the IBEX35 and for the 5-day, 10-day and 21-day ahead forecasts of the EUROSTOXX50. For these indices and time horizons, the squared daily range is the best realized measure to use for the HEAVY model. However, none of these models is better than the benchmark models.

For the RealGARCH(1,1) model, Table 4 shows that the best realized measure to use is in general the squared daily range. The only exceptions are the 10-day and 21-day ahead forecasts of the FTSE100 and the 21-day ahead forecasts of the AEX and SSMI. For the FTSE100 exceptions, the best realized measure to use is the realized semivariance with subsampling. For the AEX exception, the best realized measure is the 10-minute realized variance without subsampling and for the SSMI, the best realized measure is the 5-minute bipower variation. This is different from the case with the standard RealGARCH(1,1) model, where it is shown that it depends per index and time horizon which realized measure performs best. None of the realized measures defeats any of the benchmark models.

Comparing the RealGARCH(1,1) model and HEAVY model with r_t^2 as an extra regressor, we see that both models do not defeat the benchmark models with r_t^2 as proxy. Also, the standard models are not defeated when r_t^2 is considered as proxy. This means that the extension of r_t^2 is not suitable for predicting the conditional variance, when one believes that a proxy for

Table 4

QLIKE results for models with r_t^2

This table reports the results for the QLIKE loss function (Equation (11)) of all the models with the squared daily return that were described in Equations (5) and (6). The abbreviations for the realized measures are the same as used in Table 1. The model reports results for the HEAVY (H), RealGARCH(1,1) (RG), RealEGRACH(1,1) (REG), GARCH(1,1) (G) and HAR(3)-RV (HAR) models. The table shows the results for the realized variance as proxy. The forecasting period is from January 2006 - March 2014. All values have to be multiplied with 10^4.

		h=1					h=5					h=10					h=21				
		H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR
	RV5	-1.657	-1.716				-1.641	-1.658				-1.646	-1.559				-1.643	-1.169			
	RK	-1.700	-1.726				-1.696	-1.673				-1.704	-1.576				-1.705	-1.039			
	RV5ss	-1.695	-1.729				-1.682	-1.686				-1.685	-1.615				-1.679	-1.406			
	RV10	-1.682	-1.720				-1.668	-1.664				-1.672	-1.565				-1.669	-1.089			
	RV10ss	-1.686	-1.730				-1.685	-1.682				-1.697	-1.596				-1.699	-1.201			
FTSE100	BV5	-1.646	-1.714				-1.638	-1.658				-1.639	-1.566				-1.636	-1.286			
	BV5ss	-1.682	-1.727				-1.675	-1.682				-1.693	-1.608				-1.700	-1.410			
	MEDRV	-1.653	-1.711				-1.649	-1.672				-1.663	-1.614				-1.665	-1.467			
	RS5	-1.555	-1.724				-1.557	-1.692				-1.598	-1.644				-1.627	-1.422			
	RS5ss	-1.531	-1.734				-1.564	-1.703				-1.611	-1.653				-1.647	-1.454			
	DR	-1.621	-1.746				-1.614	-1.724				-1.612	-1.599				-1.605	-0.736			
	-				-1.771	-1.751				-1.783	-1.746				-1.785	-1.738				-1.778	-1.721
	RV5	-1.575	-1.621				-1.576	-1.583				-1.578	-1.513				-1.572	-1.274			
	RK	-1.601	-1.622				-1.593	-1.590				-1.590	-1.529				-1.580	-1.347			
	RV5ss	-1.549	-1.625				-1.535	-1.596				-1.530	-1.542				-1.520	-1.367			
	RV10	-1.581	-1.618				-1.577	-1.581				-1.576	-1.514				-1.568	-1.344			
	RV10ss	-1.565	-1.624				-1.556	-1.595				-1.557	-1.541				-1.549	-1.382			
DAX35	BV5	-1.519	-1.625				-1.534	-1.594				-1.534	-1.542				-1.531	-1.421			
	BV5ss	-1.575	-1.627				-1.568	-1.600				-1.568	-1.552				-1.562	-1.428			
	MEDRV	-1.518	-1.629				-1.513	-1.597				-1.514	-1.541				-1.510	-1.266			
	RS5	-1.348	-1.623				-1.370	-1.600				-1.403	-1.567				-1.445	-1.492			
	RS5ss	-1.367	-1.627				-1.393	-1.608				-1.423	-1.580				-1.455	-1.515			
	DR	-1.568	-1.630				-1.558	-1.622				-1.553	-1.611				-1.544	-1.566			
	-				-1.659	-1.639				-1.684	-1.634				-1.687	-1.626				-1.682	-1.610

Table 4 continued

		h=1					h=5					h=10					h=21				
		H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR
	RV5	-1.635	-1.656				-1.613	-1.607				-1.607	-1.538				-1.599	-1.369			
	RK	-1.588	-1.656				-1.570	-1.603				-1.557	-1.530				-1.549	-1.366			
	RV5ss	-1.633	-1.657				-1.605	-1.609				-1.595	-1.545				-1.596	-1.402			
	RV10	-1.611	-1.652				-1.588	-1.597				-1.575	-1.520				-1.571	-1.334			
	RV10ss	-1.610	-1.656				-1.582	-1.608				-1.572	-1.542				-1.569	-1.399			
CAC40	BV5	-1.634	-1.654				-1.616	-1.603				-1.602	-1.537				-1.594	-1.396			
	BV5ss	-1.590	-1.655				-1.563	-1.608				-1.553	-1.547				-1.546	-1.416			
	MEDRV	-1.592	-1.657				-1.568	-1.619				-1.571	-1.570				-1.573	-1.461			
	RS5	-1.492	-1.656				-1.467	-1.614				-1.482	-1.559				-1.508	-1.429			
	RS5ss	-1.468	-1.657				-1.442	-1.618				-1.445	-1.567				-1.464	-1.461			
	DR	-1.628	-1.658				-1.603	-1.650				-1.592	-1.642				-1.589	-1.622			
	-				-1.682	-1.673				-1.697	-1.664				-1.699	-1.656				-1.692	-1.638
	RV5	-1.675	-1.715				-1.656	-1.208				-1.652	-1.233				-1.646	-1.245			
	RK	-1.657	-1.715				-1.641	-1.213				-1.631	-1.238				-1.632	-1.249			
	RV5ss	-1.689	-1.716				-1.671	-1.205				-1.668	-1.232				-1.667	-1.245			
	RV10	-1.627	-1.713				-1.606	-1.222				-1.604	-1.245				-1.603	-1.255			
	RV10ss	-1.679	-1.714				-1.661	-1.217				-1.653	-1.242				-1.661	-1.253			
AEX	BV5	-1.667	-1.713				-1.655	-1.205				-1.644	-1.233				-1.639	-1.247			
	BV5ss	-1.651	-1.712				-1.628	-1.209				-1.623	-1.236				-1.627	-1.249			
	MEDRV	-1.665	-1.714				-1.656	-1.186				-1.658	-1.217				-1.656	-1.233			
	RS5	-1.539	-1.719				-1.531	-1.219				-1.545	-1.239				-1.564	-1.247			
	RS5ss	-1.494	-1.719				-1.503	-1.213				-1.543	-1.235				-1.586	-1.245			
	DR	-1.641	-1.720				-1.622	-1.261				-1.615	-1.251				-1.614	-1.242			
	-				-1.743	-1.727				-1.755	-1.718				-1.756	-1.710				-1.748	-1.693
	RV5	-1.665	-1.711				-1.664	-1.683				-1.660	-1.643				-1.656	-1.565			
	RK	-1.676	-1.713				-1.686	-1.694				-1.685	-1.662				-1.681	-1.594			
	RV5ss	-1.638	-1.710				-1.633	-1.685				-1.630	-1.646				-1.622	-1.574			
	RV10	-1.669	-1.709				-1.679	-1.682				-1.680	-1.640				-1.679	-1.562			
	RV10ss	-1.680	-1.707				-1.680	-1.678				-1.681	-1.635				-1.678	-1.562			
SSMI	BV5	-1.616	-1.705				-1.621	-1.698				-1.620	-1.688				-1.616	-1.663			
	BV5ss	-1.670	-1.708				-1.675	-1.679				-1.677	-1.639				-1.676	-1.570			
	MEDRV	-1.647	-1.711				-1.642	-1.692				-1.639	-1.665				-1.630	-1.616			
	RS5	-1.512	-1.714				-1.556	-1.692				-1.584	-1.659				-1.618	-1.600			
	RS5ss	-1.522	-1.712				-1.557	-1.693				-1.589	-1.663				-1.622	-1.609			
	DR	-1.601	-1.711				-1.599	-1.703				-1.596	-1.694				-1.589	-1.090			
	-				-1.738	-1.723				-1.762	-1.721				-1.765	-1.714				-1.759	-1.701

Table 4 continued

		h=1					h=5					h=10					h=21				
		H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR
	RV5	-1.564	-1.596				-1.552	-1.542				-1.549	-1.462				-1.543	-1.284			
	RK	-1.544	-1.596				-1.536	-1.541				-1.530	-1.464				-1.523	-1.299			
	RV5ss	-1.544	-1.596				-1.527	-1.541				-1.522	-1.464				-1.517	-1.300			
	RV10	-1.564	-0.655				-1.555	-1.036				-1.553	-0.981				-1.545	-0.829			
	RV10ss	-1.514	-1.594				-1.505	-1.527				-1.503	-1.435				-1.496	-1.213			
IBEX35	BV5	-1.548	-1.252				-1.538	-1.306				-1.537	-1.236				-1.533	-1.082			
	BV5ss	-1.539	-1.593				-1.521	-1.537				-1.517	-1.460				-1.511	-1.302			
	MEDRV	-1.523	-1.588				-1.511	-1.547				-1.506	-1.487				-1.501	-1.354			
	RS5	-1.343	-1.597				-1.365	-1.549				-1.384	-1.483				-1.412	-1.348			
	RS5ss	-1.412	-1.598				-1.393	-1.553				-1.398	-1.491				-1.410	-1.371			
	DR	-1.568	-1.579				-1.558	-1.565				-1.555	-1.535				-1.551	-1.496			
	-				-1.630	-1.613				-1.650	-1.607				-1.653	-1.602				-1.646	-1.589
	RV5	-1.536	-1.559				-1.507	-1.499				-1.511	-1.400				-1.511	-1.001			
	RK	-1.448	-1.563				-1.434	-1.505				-1.448	-1.414				-1.453	-1.087			
	RV5ss	-1.534	-1.567				-1.512	-1.519				-1.507	-1.451				-1.502	-1.273			
	RV10	-1.487	-1.550				-1.472	-1.480				-1.473	-1.373				-1.467	-0.965			
	RV10ss	-1.402	-1.566				-1.444	-1.517				-1.478	-1.448				-1.490	-1.289			
Eurostoxx50	BV5	-1.443	-1.568				-1.450	-1.523				-1.475	-1.460				-1.486	-1.323			
	BV5ss	-1.470	-1.566				-1.437	-1.524				-1.445	-1.467				-1.449	-1.356			
	MEDRV	-1.441	-1.568				-1.439	-1.536				-1.448	-1.485				-1.458	-1.383			
	RS5	-1.292	-1.557				-1.305	-1.515				-1.325	-1.454				-1.361	-1.202			
	RS5ss	-1.092	-1.572				-1.232	-1.533				-1.265	-1.461				-1.318	-1.049			
	DR	-1.532	-1.575				-1.520	-1.562				-1.530	-1.547				-1.530	-1.511			
	-				-1.622	-1.579				-1.648	-1.581				-1.654	-1.576				-1.649	-1.565

the conditional variance is the realized variance.

4.2.3 Non-linear models

At last, the out-of-sample performance of the non-linear models is discussed. The values of the QLIKE loss function are given in Table 5. As with the previous models, Table 5 only shows the results with the realized variance as proxy. The results for the squared daily return and realized kernel as a proxy, can be found in the 'Appendix', but are not discussed.

Table 5 shows for the HEAVY models with the realized variance as a proxy, that it differs per index and time horizon what the best realized measure is. This is in contradiction with the results of the standard models and the model with the squared daily returns as an extra regressor, where it is mostly the 'classic' realized measures that perform best. For example, for the IBEX35 and the EUROSTOXX50, the best realized measure is the daily range, while for the SSMI, the best realized measure is the median truncated realized variance. The performance of the non-linear model, however, is not better than of the standard HEAVY model. In comparison with the model with the squared daily return as extra regressor, the non-linear model performs better, albeit not with large differences (between 0 and 100).

For the RealGARCH(1,1) models, Table 5 shows, when the realized variance is used as a proxy, there are differences between the index and time horizon which realized measure performs best. For example, for the SSMI and the EUROSTOXX50, Table 5 shows that the best realized measure is the squared daily range. However, for the FTSE100, the best realized measures are the realized bi-power variation with subsampling and 10-minute realized variance. In general, the non-linear RealGARCH(1,1) model does not outperform the standard RealGARCH(1,1) model. In comparison with

Table 5

QLIKE results for the non-linear models

This table reports the results for the QLIKE loss function (Equation (11)) of all the non-linear models that were described in Equations (7) - (9). The abbreviations for the realized measures are the same as used in Table 1. The model reports results for the HEAVY (H), RealGARCH(1,1) (RG), RealEGRACH(1,1) (REG), GARCH(1,1) (G) and HAR(3)-RV (HAR) models. The table shows the results for the realized variance as proxy. The forecasting period is from January 2006 - March 2014. All values have to be multiplied with 10^4.

		h=1					h=5					h=10					h=21				
		H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR
FTSE100	RV5	-1.714	-1.664	-1.742			-1.701	-1.590	-1.733			-1.709	-1.484	-1.722			-1.707	-1.239	-1.691		
	RK	-1.700	-1.690	-1.740			-1.696	-1.626	-1.732			-1.704	-1.531	-1.721			-1.705	-1.303	-1.690		
	RV5ss	-1.714	-1.705	-1.743			-1.702	-1.658	-1.735			-1.709	-1.590	-1.726			-1.708	-1.435	-1.698		
	RV10	-1.694	-1.691	-1.741			-1.692	-1.631	-1.733			-1.702	-1.541	-1.722			-1.703	-1.316	-1.690		
	RV10ss	-1.686	-1.710	-1.743			-1.685	-1.653	-1.735			-1.697	-1.566	-1.725			-1.699	-1.355	-1.696		
	BV5	-1.694	-1.614	-1.739			-1.685	-1.546	-1.730			-1.697	-1.459	-1.719			-1.701	-1.292	-1.687		
	BV5ss	-1.682	-1.740	-1.741			-1.675	-1.724	-1.733			-1.693	-1.696	-1.723			-1.700	-1.615	-1.693		
	MEDRV	-1.670	-1.743	-1.736			-1.668	-1.724	-1.727			-1.687	-1.692	-1.717			-1.694	-1.554	-1.685		
	RS5	-1.535	-1.587	-1.735			-1.555	-1.539	-1.724			-1.606	-1.481	-1.709			-1.646	-1.376	-1.669		
	RS5ss	-1.530	-1.721	-1.736			-1.564	-1.692	-1.726			-1.611	-1.649	-1.713			-1.647	-1.532	-1.677		
	DR	-1.689	-1.689	-1.746			-1.685	-1.657	-1.742			-1.687	-1.615	-1.737			-1.683	-1.528	-1.724		
	-				-1.771	-1.751				-1.783	-1.746				-1.785	-1.738				-1.778	-1.721
DAX35	RV5	-1.575	-1.597	-1.627			-1.576	-1.532	-1.614			-1.578	-1.432	-1.597			-1.572	-1.202	-1.557		
	RK	-1.591	-1.614	-1.631			-1.589	-1.557	-1.619			-1.587	-1.462	-1.603			-1.578	-1.239	-1.568		
	RV5ss	-1.596	-1.607	-1.630			-1.586	-1.551	-1.618			-1.583	-1.466	-1.602			-1.574	-1.275	-1.565		
	RV10	-1.581	-1.611	-1.625			-1.577	-1.543	-1.614			-1.576	-1.418	-1.599			-1.568	-1.083	-1.566		
	RV10ss	-1.569	-1.606	-1.630			-1.570	-1.554	-1.619			-1.573	-1.472	-1.604			-1.568	-1.287	-1.569		
	BV5	-1.549	-1.566	-1.630			-1.560	-1.500	-1.617			-1.563	-1.408	-1.599			-1.560	-1.230	-1.557		
	BV5ss	-1.573	-1.569	-1.630			-1.567	-1.513	-1.617			-1.567	-1.434	-1.600			-1.561	-1.272	-1.561		
	MEDRV	-1.560	-1.573	-1.631			-1.558	-1.508	-1.617			-1.561	-1.419	-1.599			-1.558	-1.236	-1.556		
	RS5	-1.348	-1.572	-1.623			-1.369	-1.534	-1.604			-1.402	-1.485	-1.577			-1.444	-1.379	-1.508		
	RS5ss	-1.330	-1.622	-1.625			-1.384	-1.592	-1.608			-1.424	-1.549	-1.585			-1.464	-1.448	-1.524		
	DR	-1.603	-1.398	-1.642			-1.592	-1.498	-1.635			-1.589	-1.506	-1.628			-1.580	-1.496	-1.612		
	-				-1.659	-1.639				-1.684	-1.634				-1.687	-1.626				-1.682	-1.610

Table 5 continued

		h=1					h=5					h=10					h=21				
		H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR
	RV5	-1.635	-1.608	-1.656			-1.613	-1.527	-1.640			-1.607	-1.423	-1.626			-1.599	-1.211	-1.596		
	RK	-1.635	-1.614	-1.655			-1.615	-1.531	-1.641			-1.600	-1.424	-1.628			-1.600	-1.210	-1.600		
	RV5ss	-1.633	-1.622	-1.656			-1.605	-1.548	-1.641			-1.595	-1.450	-1.627			-1.596	-1.247	-1.597		
	RV10	-1.636	-1.616	-1.649			-1.611	-1.539	-1.636			-1.600	-1.432	-1.625			-1.599	-1.191	-1.602		
	RV10ss	-1.630	-1.630	-1.654			-1.605	-1.561	-1.639			-1.597	-1.471	-1.626			-1.595	-1.285	-1.599		
CAC40	BV5	-1.634	-1.613	-1.655			-1.616	-1.533	-1.639			-1.602	-1.428	-1.625			-1.594	-1.221	-1.592		
	BV5ss	-1.627	-1.624	-1.655			-1.598	-1.555	-1.639			-1.592	-1.462	-1.626			-1.593	-1.268	-1.594		
	MEDRV	-1.592	-1.642	-1.654			-1.568	-1.585	-1.638			-1.571	-1.506	-1.623			-1.573	-1.316	-1.588		
	RS5	-1.474	-1.648	-1.654			-1.455	-1.610	-1.634			-1.473	-1.564	-1.615			-1.502	-1.464	-1.572		
	RS5ss	-1.483	-1.651	-1.654			-1.466	-1.616	-1.635			-1.475	-1.572	-1.616			-1.501	-1.472	-1.574		
	DR	-1.628	-1.602	-1.640			-1.603	-1.600	-1.632			-1.592	-1.597	-1.628			-1.589	-1.585	-1.617		
	-				-1.682	-1.673				-1.697	-1.664				-1.699	-1.656				-1.692	-1.638
	RV5	-1.690	-1.698	-1.709			-1.672	-1.637	-1.689			-1.670	-1.545	-1.668			-1.671	-1.279	-1.618		
	RK	-1.691	-1.708	-1.710			-1.672	-1.655	-1.691			-1.663	-1.572	-1.672			-1.665	-1.265	-1.626		
	RV5ss	-1.688	-1.708	-1.711			-1.670	-1.657	-1.692			-1.668	-1.573	-1.673			-1.667	-1.192	-1.627		
	RV10	-1.682	-1.703	-1.706			-1.662	-1.643	-1.687			-1.659	-1.546	-1.667			-1.663	-1.241	-1.622		
	RV10ss	-1.679	-1.708	-1.710			-1.661	-1.654	-1.690			-1.653	-1.563	-1.669			-1.661	-1.103	-1.620		
AEX	BV5	-1.690	-1.695	-1.708			-1.674	-1.634	-1.688			-1.666	-1.541	-1.668			-1.666	-1.258	-1.618		
	BV5ss	-1.680	-1.707	-1.710			-1.659	-1.658	-1.690			-1.656	-1.579	-1.670			-1.661	-1.231	-1.621		
	MEDRV	-1.665	-1.707	-1.710			-1.656	-1.663	-1.692			-1.658	-1.593	-1.673			-1.656	-1.220	-1.626		
	RS5	-1.533	-1.708	-1.705			-1.537	-1.674	-1.682			-1.561	-1.623	-1.655			-1.596	-1.485	-1.588		
	RS5ss	-1.494	-1.716	-1.708			-1.503	-1.687	-1.686			-1.543	-1.644	-1.660			-1.586	-1.521	-1.596		
	DR	-1.674	-1.443	-1.710			-1.650	-1.548	-1.695			-1.647	-1.573	-1.680			-1.648	-1.578	-1.644		
	-				-1.743	-1.727				-1.755	-1.718				-1.756	-1.710				-1.748	-1.693
	RV5	-1.686	-1.638	-1.718			-1.689	-1.561	-1.709			-1.689	-1.448	-1.697			-1.687	-1.188	-1.671		
	RK	-1.676	-1.677	-1.718			-1.686	-1.623	-1.710			-1.685	-1.536	-1.698			-1.681	-1.159	-1.671		
	RV5ss	-1.685	-1.669	-1.716			-1.689	-1.607	-1.707			-1.688	-1.510	-1.694			-1.684	-1.286	-1.664		
	RV10	-1.669	-1.657	-1.718			-1.679	-1.585	-1.708			-1.680	-1.479	-1.693			-1.679	-1.247	-1.660		
	RV10ss	-1.672	-1.660	-1.715			-1.676	-1.593	-1.706			-1.677	-1.494	-1.693			-1.676	-1.287	-1.663		
SSMI	BV5	-1.673	-1.645	-1.716			-1.681	-1.577	-1.707			-1.683	-1.476	-1.694			-1.682	-1.239	-1.662		
	BV5ss	-1.670	-1.665	-1.715			-1.675	-1.603	-1.706			-1.677	-1.511	-1.693			-1.676	-1.307	-1.662		
	MEDRV	-1.697	-1.654	-1.714			-1.697	-1.585	-1.705			-1.695	-1.477	-1.689			-1.689	-1.135	-1.656		
	RS5	-1.510	-1.700	-1.717			-1.552	-1.670	-1.705			-1.582	-1.628	-1.687			-1.617	-1.530	-1.643		
	RS5ss	-1.513	-1.704	-1.715			-1.552	-1.681	-1.702			-1.586	-1.645	-1.682			-1.620	-1.567	-1.634		
	DR	-1.674	-1.707	-1.721			-1.671	-1.696	-1.717			-1.669	-1.687	-1.713			-1.662	-1.675	-1.704		
	-				-1.738	-1.723				-1.762	-1.721				-1.765	-1.714				-1.759	-1.701

Table 5 continued

		h=1					h=5					h=10					h=21				
		H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR
IBEX35	RV5	-1.564	-1.574	-1.610			-1.552	-1.490	-1.596			-1.549	-1.387	-1.579			-1.543	-1.207	-1.535		
	RK	-1.558	-1.579	-1.610			-1.555	-1.501	-1.597			-1.551	-1.405	-1.581			-1.546	-1.246	-1.541		
	RV5ss	-1.571	-1.578	-1.609			-1.555	-1.497	-1.596			-1.551	-1.398	-1.581			-1.545	-1.232	-1.544		
	RV10	-1.564	-1.552	-1.609			-1.555	-1.448	-1.594			-1.553	-1.325	-1.576			-1.545	-1.122	-1.531		
	RV10ss	-1.559	-1.571	-1.610			-1.550	-1.472	-1.597			-1.547	-1.357	-1.583			-1.543	-1.178	-1.548		
	BV5	-1.548	-1.572	-1.609			-1.538	-1.490	-1.595			-1.537	-1.393	-1.577			-1.533	-1.232	-1.532		
	BV5ss	-1.557	-1.580	-1.608			-1.542	-1.508	-1.595			-1.539	-1.422	-1.578			-1.536	-1.279	-1.535		
	MEDRV	-1.541	-1.578	-1.596			-1.530	-1.528	-1.582			-1.527	-1.455	-1.565			-1.521	-1.176	-1.524		
	RS5	-1.343	-1.600	-1.610			-1.365	-1.563	-1.590			-1.384	-1.512	-1.559			-1.412	-1.407	-1.475		
	RS5ss	-1.387	-1.602	-1.609			-1.379	-1.567	-1.591			-1.398	-1.520	-1.563			-1.424	-1.426	-1.491		
	DR	-1.568	-1.406	-1.617			-1.558	-1.527	-1.609			-1.555	-1.531	-1.603			-1.551	-1.525	-1.588		
	-				-1.630	-1.613				-1.650	-1.607				-1.653	-1.602				-1.646	-1.589
Eurostoxx50	RV5	-1.505	-1.382	-1.548			-1.481	-1.266	-1.539			-1.501	-1.130	-1.528			-1.505	-0.899	-1.501		
	RK	-1.487	-1.481	-1.558			-1.475	-1.392	-1.551			-1.492	-1.280	-1.542			-1.500	-1.079	-1.519		
	RV5ss	-1.534	-1.486	-1.562			-1.512	-1.399	-1.555			-1.507	-1.292	-1.546			-1.502	-1.107	-1.521		
	RV10	-1.466	-1.475	-1.548			-1.481	-1.381	-1.542			-1.501	-1.259	-1.533			-1.506	-1.027	-1.512		
	RV10ss	-1.399	-1.530	-1.563			-1.443	-1.463	-1.556			-1.477	-1.376	-1.549			-1.489	-1.219	-1.527		
	BV5	-1.409	-0.940	-1.562			-1.438	-1.064	-1.552			-1.470	-1.025	-1.540			-1.482	-0.923	-1.509		
	BV5ss	-1.499	-1.505	-1.561			-1.474	-1.433	-1.554			-1.484	-1.344	-1.545			-1.490	-1.190	-1.520		
	MEDRV	-1.441	-1.542	-1.568			-1.439	-1.486	-1.560			-1.448	-1.410	-1.549			-1.458	-1.281	-1.520		
	RS5	-1.183	-1.529	-1.555			-1.255	-1.487	-1.539			-1.294	-1.429	-1.516			-1.346	-1.306	-1.466		
	RS5ss	-1.089	-1.515	-1.562			-1.227	-1.466	-1.548			-1.269	-1.401	-1.529			-1.328	-1.274	-1.483		
	DR	-1.532	-1.572	-1.582			-1.520	-1.553	-1.582			-1.530	-1.537	-1.580			-1.530	-1.515	-1.572		
	-				-1.622	-1.579				-1.648	-1.581				-1.654	-1.576				-1.649	-1.565

the RealGARCH(1,1) model with the squared daily return, it performs a bit worse, with not very big difference (between 0 and 150).

With the realized variance as a proxy, Table 5 shows for the RealEGARCH(1,1) that the results are more or less the same as with the standard models. An exception to this is for the CAC40 index, with 1-day and 5-day ahead forecasts. Where in the standard model the squared daily range is the best realized measure, it is for the non-linear RealEGARCH(1,1) model the 5-minute realized variance. For all the other indices and time horizons, the daily range is the best realized measure for the non-linear RealEGARCH(1,1). This model also outperforms the HAR(3)-RV model at some time horizons and indices. For example, for the EUROSTOXX50, the non-linear RealEGARCH(1,1) model is better than the HAR(3)-RV model for all time horizons.

In general, the non-linear models do not outperform the standard models out-of-sample, except for the HEAVY model. Between the three non-linear models, for almost all indices and time horizons, the RealEGARCH(1,1) model performs better than the other two models. The only exceptions are the 1-day ahead forecast of the AEX and the SSMI, where the RealGARCH(1,1) model is the best of the three models. Another exception is the 21-day ahead forecasts of the AEX, where the HEAVY model is the best model of the three non-linear models.

4.2.4 Stability of the QLIKE results

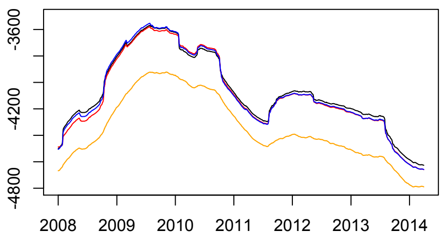
Besides investigating the QLIKE loss function results for the complete sample, the stability of the QLIKE results are considered too. However, this is only done for the realized variance of the FTSE100 of the standard models, with the realized kernel as a proxy. The QLIKE loss function results are taken with a moving window of 2 years and updated every day until the end of the period. The dates on the x-axis mark the ending of the period the QLIKE is calculated. This results in Figure 6.

Figure 6

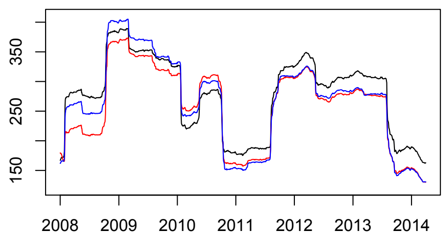
QLIKE loss function over time

This figure shows the results of the QLIKE loss function values (subfigure a)) for the standard models of the HEAVY (black), RealGARCH(1,1) (red) and RealE-GARCH(1,1) (blue) models and the difference to the lowest value possible (subfigure b)). The QLIKE results are taken for two years and updated every day until the end of the period. The forecasting period that is taken is from January 2006 - March 2014. The dates on the x-axis mark the ending date of the period that the QLIKE is calculated. The orange line shows the minimum value that the loss function can take.

(a) HEAVY model (h_t)



(b) HEAVY model (h_t)



The results in Figure 6 show that the resulting values of the loss function vary over time. However, when the values are compared with one another, subfigure a) of Figure 6 shows that the pattern is more or less the same over time.

However, subfigure b) shows that for the model, the differences between the results of the QLIKE values of the models and the lowest value possible differs over time and has some sudden jumps. These jumps are marked at the beginning of 2008, at the end of 2008, at the beginning, halfway and at the end of 2010, and halfway 2011, 2012 and 2013. These jumps can be explained by the fact that the volatility is not homogeneous over the period 2006-2014. As subfigure a) of Figure 1 has already shown, there are also shocks in the realized variance over this period. Especially the shocks of 2008, beginning of 2010 and halfway 2011 can be clearly seen in the graph of the realized variance over time. These periods mark times in the financial crisis that started halfway 2008.

Even though it is clear that the performance of the results of the QLIKE vary over time, this will not be further examined for the other horizons, proxies, indices and models, since it is expected that the observations will not change too much from the observations of Figure 6.

4.2.5 Model confidence set

For all the models presented before (standard, with squared daily returns and non-linear), a model confidence set is made. The results are that for almost all the indices and time horizons, the set only consists out of the best performing model, at a significance level of 95%. The only exceptions are at a forecasting horizon of 1 day for the FTSE100, SSMI and the EUROSTOXX50. The models that are in these model confidence sets are given in Table 6, together with the corresponding p-value.

Table 6 shows that in order to be included into the model confidence set, the QLIKE results have to be more or less the same, with only a maximum difference of about 10 between the QLIKE values. The fact that for only those three indices a model confidence set can be build, means that for the

Table 6

Model Confidence Set results

This table reports the different models that are included in the model confidence set of the FTSE100, SSMI and EUROSTOXX50 with a forecasting horizon of 1 day. The realized measures that are mentioned in the columns of the table are the combinations of the model that is included into the model confidence set at a significance level of 95%. In the last column the corresponding p-values are given for the model confidence set. The forecasts are taken from the period January 2006 - March 2014.

Index	standard			squared return		non-linear			p-value
	H	RG	REG	H	RG	H	RG	REG	
FTSE 100		RV5ss, BV5, BV5ss, MEDRV, RS5, RS5ss	DR		DR		MEDRV	DR	0.030
SSMI	MED-RV	RS5, RS5ss, DR	DR					RV5, RK, RV10, DR	0.034
EURO STOXX 50	RV5ss	all except RV5	DR		DR			DR	0.040

other indices the QLIKE results are significantly different from one another.

As already stated in Section 2.7.2., a higher value of l changes the results slightly, by the fact that more models are included into the model confidence set. In order to test the robustness of the choice of B, also several other values for B (5000 and 10000) are chosen and the lengths of the different model confidence sets are examined. It turns out that there are no changes with the amount of models that are included into the model confidence set. Therefore, we can say that this method of comparing the forecasting results is robust to the choice of B, when B is at least chosen sufficiently large.

4.2.6 Combination of Forecasts

Besides considering every model with every realized measure apart, the combinations of the forecasts are considered too. The forecasts are combined using their mean, their median and their in-sample discounted MSPE. Since the combinations using the median always performs worse than using the mean, the median combinations are not discussed. The results for these combinations can be found in the 'Appendix'.

First of all, the mean combinations are discussed. Only the results for the 1-day ahead and the 21-day ahead forecasts are discussed. For the 5-day ahead and 10-day ahead, the results are given in the 'Appendix', though they are not discussed. Table 7 reports the values of the QLIKE loss function.

Table 7 shows that when the squared daily return is taken as a proxy, none of the mean combination models beat the GARCH(1,1) model. At a 1-day ahead forecast horizon, the best combination model is the model that combines the standard RealGARCH(1,1) models. The only exception is for the DAX30, where the combination of all the standard models is the best model. Table 7 shows also that for some of the indices the mean models out-

Table 7

QLIKE results for combination models (mean)

This table reports the QLIKE results for the combinations models, based on the mean, for 1-day and 21-day ahead forecasts. The results are reported for the combination of all standard HEAVY (H), all HEAVY models with the squared daily return (H+r_t^2), all non-linear HEAVY models (H+$\mathbb{I}_{\{r_t<0\}}$ and all HEAVY models (All H). The same applies for the RealGARCH(1,1) (RG) and RealEGARCH(1,1) models. Finally, the results for the GARCH(1,1) (G) and HAR(3)-RV (HAR) models are also presented. Panel A uses the squared daily return as a proxy, Panel B the realized variance and Panel C the realized kernel as a proxy.

Panel A

	h=1							h=21						
	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50
H	-1.744	-1.633	-1.662	-1.712	-1.710	-1.578	-1.579	-1.718	-1.605	-1.622	-1.668	-1.691	-1.549	-1.556
H+r_t^2	-1.732	-1.608	-1.649	-1.697	-1.694	-1.550	-1.540	-1.713	-1.588	-1.608	-1.657	-1.684	-1.521	-1.525
H+$\mathbb{I}_{\{r_t<0\}}$	-1.724	-1.603	-1.641	-1.694	-1.688	-1.539	-1.525	-1.713	-1.584	-1.603	-1.656	-1.683	-1.516	-1.520
All H	-1.737	-1.619	-1.654	-1.704	-1.701	-1.561	-1.555	-1.717	-1.596	-1.615	-1.663	-1.688	-1.534	-1.539
RG	-1.752	-1.636	-1.665	-1.718	-1.712	-1.589	-1.584	-1.022	-1.247	-1.270	-1.072	-1.314	-1.474	-1.024
RG+r_t^2	-1.735	-1.627	-1.657	-1.707	-1.710	-1.547	-1.571	-0.711	-1.291	-1.471	-1.447	-1.061	-0.690	-0.981
RG+$\mathbb{I}_{\{r_t<0\}}$	-1.719	-1.607	-1.634	-1.693	-1.683	-1.543	-1.528	-1.467	-1.337	-1.411	-1.165	-1.265	-1.124	-1.289
All RG	-1.742	-1.629	-1.657	-1.711	-1.706	-1.571	-1.571	-0.731	-1.053	-1.205	-0.984	-0.715	-0.701	-0.776
REG	-1.745	-1.627	-1.648	-1.688	-1.704	-1.569	-1.568	-1.698	-1.569	-1.583	-1.601	-1.672	-1.496	-1.510
REG+$\mathbb{I}_{\{r_t<0\}}$	-1.745	-1.627	-1.644	-1.689	-1.706	-1.571	-1.564	-1.694	-1.565	-1.598	-1.599	-1.657	-1.483	-1.522
All REG	-1.745	-1.627	-1.646	-1.689	-1.705	-1.570	-1.566	-1.696	-1.567	-1.592	-1.600	-1.665	-1.490	-1.517
All standard	-1.749	-1.637	-1.664	-1.714	-1.711	-1.585	-1.582	-1.040	-1.273	-1.301	-1.087	-1.340	-1.484	-1.057
All r_t^2	-1.736	-1.624	-1.658	-1.705	-1.705	-1.554	-1.564	-0.740	-1.383	-1.578	-1.531	-1.103	-0.864	-1.069
All $\mathbb{I}_{\{r_t<0\}}$	-1.737	-1.622	-1.648	-1.698	-1.699	-1.558	-1.555	-1.658	-1.497	-1.580	-1.275	-1.421	-1.297	-1.498
All models	-1.743	-1.630	-1.658	-1.708	-1.706	-1.571	-1.571	-0.740	-1.111	-1.277	-1.030	-0.756	-0.759	-0.835
G	-1.809	-1.720	-1.734	-1.786	-1.789	-1.673	-1.687	-1.782	-1.689	-1.698	-1.749	-1.762	-1.641	-1.656
HAR	-1.755	-1.636	-1.667	-1.711	-1.714	-1.574	-1.583	-1.723	-1.610	-1.638	-1.680	-1.697	-1.558	-1.567

Panel B

	h=1							h=21						
	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50
H	-1.744	-1.638	-1.671	-1.724	-1.720	-1.609	-1.581	-1.717	-1.605	-1.626	-1.683	-1.696	-1.580	-1.554
H+r_t^2	-1.731	-1.618	-1.659	-1.712	-1.706	-1.589	-1.541	-1.712	-1.588	-1.612	-1.675	-1.691	-1.562	-1.522
H+$\mathbb{I}_{\{r_t<0\}}$	-1.723	-1.610	-1.654	-1.708	-1.698	-1.585	-1.528	-1.712	-1.584	-1.608	-1.675	-1.690	-1.558	-1.516
All H	-1.736	-1.626	-1.665	-1.718	-1.711	-1.598	-1.558	-1.716	-1.596	-1.620	-1.680	-1.695	-1.571	-1.537
RG	-1.748	-1.640	-1.671	-1.732	-1.721	-1.622	-1.585	-1.014	-1.249	-1.269	-1.092	-1.317	-1.511	-1.034
RG+r_t^2	-1.732	-1.631	-1.663	-1.724	-1.718	-1.596	-1.573	-0.707	-1.296	-1.469	-1.452	-1.059	-0.771	-0.990
RG+$\mathbb{I}_{\{r_t<0\}}$	-1.718	-1.611	-1.642	-1.713	-1.694	-1.595	-1.532	-1.461	-1.336	-1.410	-1.189	-1.270	-1.197	-1.300
All RG	-1.740	-1.633	-1.663	-1.728	-1.716	-1.613	-1.573	-0.728	-1.055	-1.205	-0.989	-0.709	-0.744	-0.788
REG	-1.742	-1.631	-1.658	-1.710	-1.716	-1.609	-1.567	-1.699	-1.570	-1.584	-1.626	-1.683	-1.548	-1.508
REG+$\mathbb{I}_{\{r_t<0\}}$	-1.742	-1.631	-1.655	-1.711	-1.718	-1.610	-1.563	-1.695	-1.566	-1.598	-1.624	-1.669	-1.539	-1.521
All REG	-1.742	-1.631	-1.657	-1.710	-1.717	-1.610	-1.565	-1.698	-1.569	-1.591	-1.625	-1.677	-1.544	-1.515
All standard	-1.747	-1.640	-1.672	-1.729	-1.721	-1.619	-1.583	-1.033	-1.274	-1.305	-1.105	-1.344	-1.518	-1.064
All r_t^2	-1.735	-1.631	-1.665	-1.721	-1.715	-1.597	-1.565	-0.737	-1.384	-1.581	-1.537	-1.103	-0.910	-1.072
All $\mathbb{I}_{\{r_t<0\}}$	-1.736	-1.626	-1.657	-1.716	-1.710	-1.603	-1.556	-1.656	-1.496	-1.583	-1.289	-1.429	-1.340	-1.497
All Models	-1.742	-1.634	-1.667	-1.724	-1.717	-1.611	-1.572	-0.738	-1.111	-1.282	-1.036	-0.753	-0.788	-0.842
G	-1.771	-1.659	-1.682	-1.743	-1.738	-1.630	-1.622	-1.778	-1.682	-1.692	-1.748	-1.759	-1.646	-1.649
HAR	-1.751	-1.639	-1.673	-1.727	-1.723	-1.613	-1.579	-1.721	-1.610	-1.638	-1.693	-1.701	-1.589	-1.565

Panel C

	h=1							h=21						
	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50
H	-1.753	-1.633	-1.672	-1.723	-1.727	-1.611	-1.593	-1.723	-1.601	-1.628	-1.681	-1.702	-1.582	-1.568
H+r_t^2	-1.741	-1.612	-1.660	-1.710	-1.715	-1.591	-1.557	-1.718	-1.583	-1.615	-1.673	-1.698	-1.563	-1.542
H+$\mathbb{I}_{\{r_t<0\}}$	-1.735	-1.604	-1.655	-1.705	-1.708	-1.587	-1.545	-1.719	-1.578	-1.611	-1.672	-1.697	-1.559	-1.536
All H	-1.747	-1.621	-1.666	-1.716	-1.720	-1.600	-1.573	-1.723	-1.591	-1.622	-1.678	-1.701	-1.572	-1.554
RG	-1.758	-1.634	-1.672	-1.730	-1.729	-1.623	-1.599	-1.023	-1.243	-1.272	-1.089	-1.324	-1.512	-1.050
RG+r_t^2	-1.744	-1.626	-1.664	-1.722	-1.727	-1.598	-1.589	-0.717	-1.290	-1.473	-1.452	-1.067	-0.773	-1.020
RG+$\mathbb{I}_{\{r_t<0\}}$	-1.728	-1.604	-1.643	-1.711	-1.704	-1.596	-1.552	-1.471	-1.325	-1.416	-1.184	-1.285	-1.198	-1.336
All RG	-1.750	-1.627	-1.664	-1.726	-1.724	-1.614	-1.589	-0.735	-1.050	-1.208	-0.988	-0.715	-0.745	-0.808
REG	-1.754	-1.624	-1.660	-1.708	-1.725	-1.611	-1.585	-1.711	-1.564	-1.587	-1.622	-1.692	-1.550	-1.531
REG+$\mathbb{I}_{\{r_t<0\}}$	-1.754	-1.625	-1.656	-1.709	-1.727	-1.612	-1.581	-1.708	-1.560	-1.601	-1.620	-1.679	-1.541	-1.543
All REG	-1.754	-1.625	-1.658	-1.708	-1.726	-1.612	-1.583	-1.710	-1.562	-1.595	-1.621	-1.686	-1.546	-1.538
All standard	-1.758	-1.635	-1.673	-1.727	-1.729	-1.620	-1.597	-1.040	-1.270	-1.307	-1.102	-1.349	-1.519	-1.077
All r_t^2	-1.745	-1.625	-1.666	-1.719	-1.724	-1.599	-1.581	-0.742	-1.380	-1.584	-1.536	-1.110	-0.911	-1.093
All $\mathbb{I}_{\{r_t<0\}}$	-1.747	-1.620	-1.659	-1.713	-1.719	-1.604	-1.573	-1.666	-1.490	-1.586	-1.286	-1.437	-1.340	-1.520
All Models	-1.752	-1.629	-1.668	-1.722	-1.725	-1.612	-1.587	-0.742	-1.107	-1.283	-1.036	-0.757	-0.789	-0.856
G	-1.778	-1.655	-1.684	-1.742	-1.744	-1.631	-1.634	-1.784	-1.679	-1.693	-1.746	-1.763	-1.648	-1.658
HAR	-1.759	-1.634	-1.674	-1.725	-1.731	-1.614	-1.594	-1.727	-1.606	-1.640	-1.690	-1.706	-1.590	-1.580

perform the HAR(3)-RV model. For example, for the AEX, the combination of all standard HEAVY models, of all standard RealGARCH(1,1) models, all RealGARCH(1,1) models and all standard models, the HAR(3)-RV model is outperformed out-of-sample. In comparison with the standard models of Equations (2) - (4), only for the FTSE100 and AEX the mean combination outperforms those models.

For the 21-day ahead forecasts, Table 7 shows that with the squared daily return as proxy, the combination of the standard HEAVY models is the best of the combination models. However, these combinations do not outperform any of the benchmark models or the standard models from Equation (2) - (4). Only for the AEX, the mean combination outperforms the standard models.

Table 7 shows that when the realized variance or the realized kernel is used as a proxy, the results are more or less the same, with a few differences. For example, the best model is the same as the best model with the squared daily return as a proxy. However, when the realized variance or kernel is used as a proxy, the combinations of all standard HEAVY models do not beat the HAR(3)-RV benchmark model anymore.

Besides the mean combinations, the combinations using the in-sample discounted MSPE are considered too. The discounted MSPE is calculated according to Equation (14). The results for the QLIKE loss function using this combination are given in Table 8. Again, only the 1-day and 21-day ahead forecasts are considered.

Table 8 shows that when the squared daily return is used as a proxy, the best combination model is the combination of all standard models. Only for the FTSE100 and the DAX30, the combination of the standard Real-GARCH(1,1) models is better than the other models. Table 8 also shows

Table 8

QLIKE results for combination models (MSPE)

This table reports the QLIKE results for the combinations models, based on the in-sample discounted MSPE as given in Equations (14) and (15), for 1-day and 21-day ahead forecasts. The results are reported for the combination of all standard HEAVY (H), all HEAVY models with the squared daily return (H+r_t^2), all non-linear HEAVY models (H+$\mathbb{I}_{\{r_t<0\}}$ and all HEAVY models (All H). The same applies for the RealGARCH(1,1) (RG) and RealEGARCH(1,1) models. Finally, the results for the GARCH(1,1) (G) and HAR(3)-RV (HAR) models are also presented. Panel A uses the squared daily return as a proxy, Panel B the realized variance and Panel C the realized kernel as a proxy.

Panel A

	h=1							h=21						
	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50
H	-1.735	-1.608	-1.647	-1.695	-1.696	-1.547	-1.547	-1.730	-1.602	-1.627	-1.674	-1.695	-1.539	-1.549
H+r_t^2	-1.704	-1.564	-1.621	-1.673	-1.660	-1.505	-1.492	-1.701	-1.560	-1.591	-1.639	-1.660	-1.492	-1.500
H+$\mathbb{I}_{\{r_t<0\}}$	-1.689	-1.543	-1.599	-1.648	-1.649	-1.474	-1.471	-1.684	-1.541	-1.568	-1.608	-1.645	-1.467	-1.478
All H	-1.719	-1.611	-1.656	-1.710	-1.695	-1.580	-1.538	-1.714	-1.598	-1.630	-1.688	-1.698	-1.564	-1.537
RG	-1.749	-1.632	-1.661	-1.716	-1.706	-1.589	-1.580	-1.742	-1.624	-1.644	-1.704	-1.706	-1.584	-1.577
RG+r_t^2	-1.728	-1.622	-1.650	-1.699	-1.703	-1.534	-1.564	-1.719	-1.612	-1.630	-1.681	-1.703	-1.527	-1.561
RG+$\mathbb{I}_{\{r_t<0\}}$	-1.710	-1.598	-1.624	-1.687	-1.665	-1.531	-1.508	-1.694	-1.591	-1.602	-1.669	-1.666	-1.523	-1.504
All RG	-1.736	-1.629	-1.659	-1.723	-1.716	-1.612	-1.569	-1.734	-1.617	-1.638	-1.707	-1.716	-1.603	-1.564
REG	-1.745	-1.627	-1.648	-1.688	-1.703	-1.570	-1.568	-1.739	-1.617	-1.636	-1.678	-1.702	-1.565	-1.570
REG+$\mathbb{I}_{\{r_t<0\}}$	-1.745	-1.627	-1.643	-1.689	-1.706	-1.572	-1.564	-1.740	-1.617	-1.631	-1.679	-1.705	-1.567	-1.565
All REG	-1.742	-1.631	-1.656	-1.710	-1.717	-1.610	-1.565	-1.742	-1.618	-1.638	-1.695	-1.717	-1.602	-1.564
All standard	-1.748	-1.641	-1.672	-1.729	-1.721	-1.619	-1.585	-1.747	-1.629	-1.653	-1.715	-1.721	-1.611	-1.581
All r_t^2	-1.679	-1.595	-1.632	-1.697	-1.513	-1.502	-1.506	-1.672	-1.581	-1.603	-1.674	-1.515	-1.478	-1.501
All $\mathbb{I}_{\{r_t<0\}}$	-1.737	-1.628	-1.659	-1.718	-1.711	-1.606	-1.558	-1.735	-1.615	-1.636	-1.699	-1.711	-1.597	-1.555
All models	-1.733	-1.625	-1.657	-1.716	-1.709	-1.601	-1.553	-1.731	-1.608	-1.634	-1.696	-1.707	-1.589	-1.548
G	-1.809	-1.720	-1.734	-1.786	-1.789	-1.673	-1.687	-1.782	-1.689	-1.698	-1.749	-1.762	-1.641	-1.656
HAR	-1.755	-1.636	-1.667	-1.711	-1.714	-1.574	-1.583	-1.723	-1.610	-1.638	-1.680	-1.697	-1.558	-1.567

Panel B

	h=1							h=21						
	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50
H	-1.739	-1.626	-1.665	-1.720	-1.713	-1.599	-1.561	-1.736	-1.613	-1.643	-1.701	-1.713	-1.590	-1.558
H+r_t^2	-1.718	-1.600	-1.649	-1.701	-1.696	-1.577	-1.525	-1.716	-1.585	-1.620	-1.678	-1.695	-1.565	-1.524
H+$\mathbb{I}_{\{r_t<0\}}$	-1.710	-1.588	-1.641	-1.693	-1.685	-1.568	-1.506	-1.707	-1.575	-1.609	-1.667	-1.687	-1.556	-1.510
All H	-1.721	-1.608	-1.654	-1.709	-1.694	-1.581	-1.533	-1.717	-1.595	-1.628	-1.686	-1.695	-1.568	-1.533
RG	-1.746	-1.636	-1.667	-1.729	-1.716	-1.622	-1.580	-1.744	-1.624	-1.647	-1.715	-1.716	-1.614	-1.574
RG+r_t^2	-1.726	-1.626	-1.657	-1.717	-1.713	-1.592	-1.566	-1.722	-1.612	-1.635	-1.699	-1.714	-1.579	-1.559
RG+$\mathbb{I}_{\{r_t<0\}}$	-1.710	-1.604	-1.630	-1.707	-1.674	-1.586	-1.509	-1.700	-1.591	-1.605	-1.688	-1.674	-1.574	-1.501
All RG	-1.737	-1.631	-1.661	-1.724	-1.715	-1.610	-1.570	-1.734	-1.619	-1.639	-1.708	-1.715	-1.601	-1.564
REG	-1.742	-1.631	-1.658	-1.709	-1.715	-1.610	-1.567	-1.741	-1.618	-1.640	-1.695	-1.715	-1.602	-1.567
REG+$\mathbb{I}_{\{r_t<0\}}$	-1.742	-1.631	-1.653	-1.710	-1.718	-1.611	-1.563	-1.742	-1.618	-1.635	-1.696	-1.718	-1.603	-1.561
All REG	-1.739	-1.628	-1.655	-1.708	-1.715	-1.607	-1.562	-1.739	-1.615	-1.637	-1.694	-1.715	-1.598	-1.561
All standard	-1.743	-1.633	-1.667	-1.725	-1.712	-1.613	-1.574	-1.740	-1.620	-1.646	-1.709	-1.713	-1.604	-1.569
All r_t^2	-1.732	-1.614	-1.646	-1.706	-1.696	-1.592	-1.535	-1.729	-1.600	-1.621	-1.687	-1.696	-1.580	-1.532
All $\mathbb{I}_{\{r_t<0\}}$	-1.722	-1.612	-1.646	-1.706	-1.684	-1.579	-1.535	-1.718	-1.599	-1.620	-1.686	-1.686	-1.565	-1.531
All Models	-1.737	-1.626	-1.660	-1.718	-1.710	-1.605	-1.561	-1.734	-1.612	-1.637	-1.699	-1.710	-1.594	-1.557
G	-1.771	-1.659	-1.682	-1.743	-1.738	-1.630	-1.622	-1.778	-1.682	-1.692	-1.748	-1.759	-1.646	-1.649
HAR	-1.751	-1.639	-1.673	-1.727	-1.723	-1.613	-1.579	-1.721	-1.610	-1.638	-1.693	-1.701	-1.589	-1.565

Panel C

	h=1							h=21						
	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50
H	-1.750	-1.620	-1.666	-1.718	-1.722	-1.601	-1.576	-1.747	-1.607	-1.645	-1.699	-1.721	-1.591	-1.575
H+r_t^2	-1.731	-1.593	-1.650	-1.699	-1.705	-1.579	-1.542	-1.728	-1.579	-1.623	-1.676	-1.704	-1.567	-1.544
H+$\mathbb{I}_{\{r_t<0\}}$	-1.724	-1.580	-1.642	-1.691	-1.696	-1.570	-1.524	-1.720	-1.569	-1.611	-1.665	-1.696	-1.557	-1.531
All H	-1.719	-1.609	-1.622	-1.709	-1.701	-1.576	-1.472	-1.717	-1.589	-1.593	-1.680	-1.699	-1.555	-1.469
RG	-1.757	-1.630	-1.669	-1.727	-1.724	-1.623	-1.594	-1.754	-1.619	-1.650	-1.712	-1.723	-1.615	-1.590
RG+r_t^2	-1.739	-1.620	-1.658	-1.715	-1.722	-1.593	-1.582	-1.733	-1.607	-1.637	-1.697	-1.722	-1.581	-1.578
RG+$\mathbb{I}_{\{r_t<0\}}$	-1.720	-1.596	-1.632	-1.705	-1.685	-1.588	-1.529	-1.709	-1.584	-1.608	-1.686	-1.684	-1.576	-1.523
All RG	-1.737	-1.631	-1.660	-1.725	-1.715	-1.609	-1.570	-1.734	-1.618	-1.639	-1.709	-1.715	-1.599	-1.564
REG	-1.754	-1.625	-1.659	-1.707	-1.724	-1.612	-1.584	-1.753	-1.612	-1.643	-1.693	-1.723	-1.603	-1.586
REG+$\mathbb{I}_{\{r_t<0\}}$	-1.754	-1.625	-1.655	-1.708	-1.726	-1.613	-1.580	-1.753	-1.612	-1.638	-1.694	-1.725	-1.604	-1.581
All REG	-1.739	-1.628	-1.655	-1.708	-1.715	-1.607	-1.561	-1.739	-1.615	-1.637	-1.694	-1.715	-1.599	-1.560
All standard	-1.742	-1.635	-1.669	-1.727	-1.719	-1.613	-1.578	-1.740	-1.623	-1.649	-1.712	-1.717	-1.601	-1.574
All r_t^2	-1.729	-1.612	-1.646	-1.707	-1.693	-1.592	-1.534	-1.723	-1.598	-1.620	-1.687	-1.693	-1.582	-1.529
All $\mathbb{I}_{\{r_t<0\}}$	-1.721	-1.613	-1.646	-1.706	-1.681	-1.581	-1.540	-1.714	-1.599	-1.621	-1.686	-1.682	-1.569	-1.536
All Models	-1.737	-1.626	-1.660	-1.718	-1.710	-1.605	-1.560	-1.734	-1.612	-1.637	-1.700	-1.710	-1.594	-1.556
G	-1.778	-1.655	-1.684	-1.742	-1.744	-1.631	-1.634	-1.784	-1.679	-1.693	-1.746	-1.763	-1.648	-1.658
HAR	-1.759	-1.634	-1.674	-1.725	-1.731	-1.614	-1.594	-1.727	-1.606	-1.640	-1.690	-1.706	-1.590	-1.580

that for the CAC40, AEX, SSMI, IBEX35 and EUROSTOXX50, there are multiple combination models that outperform the HAR(3)-RV model. However, the GARCH(1,1) model is not outperformed by any of the combination models. In comparison to the mean combination models, the results are more or less the same. For some of the indices the mean combination model is better, while for other indices the combination models using the discounted MSPE performs better.

For the 21-day ahead forecasts, Table 8 shows that with the squared daily return as a proxy, the best combination model to use is for all indices the combination of all standard models. Even though, none of the models outperforms the GARCH(1,1) model, most of the combination models using the discounted MSPE outperform the HAR(3)-RV model. In comparison to the other models discussed in this paper, the best combination model using the discounted MSPE outperforms all of these models.

Table 8 shows that when the realized kernel and the realized variance are used as a proxy, the same results hold for the 21-day ahead forecasts. However, for the 1-day ahead forecasts there are some differences in results. Table 8 shows that for the realized variance and realized kernel as a proxy, there are not many combination models anymore that beat the HAR(3)-RV benchmark model. However, the combination model that is the best under the squared daily return as a proxy, is still the best when the realized variance or kernel is used as a proxy.

Since the model confidence sets for the non-combined forecasts show that two results are already significantly different from one another when the difference is more than 10, the model confidence sets are not constructed for the forecast combinations. In stead, it is assumed that all the results from Tables 7 and 8 are significantly different from each other. To be completely sure about this, the model confidence sets should be build, but that is left for further research.

5 Conclusion and Discussion

In this paper, different models that use realized measures as an explaining variable have been considered and their performance out-of-sample to estimate and predict the conditional variance. This has been done using the data of the FTSE100, DAX30, CAC40, AEX, SSMI, IBEX35 and EUROSTOXX50 data from January 2000 up to including March 2014.

First of all, the estimates of these models have been discussed for the realized variance as the explaining variable in the HEAVY, RealGARCH(1,1) and RealEGARCH(1,1) model for the FTSE100. In the standard model, model with the squared daily return as extra regressor and the non-linear models, it has been shown that most of the parameters are robust over time. An exception to this is the RealEGARCH(1,1) model with the squared daily return as an extra regressor, which has very unstable parameters. Therefore this model has not been discussed. Furthermore, there were some parameters that did change a lot over time for the other RealEGARCH(1,1) models.

The RealGARCH(1,1) and HEAVY model have also been estimated using both the realized bi-power variation and the realized variance. These estimates showed what the effect is of the jumps in the variance, which is taken into account with the realized bi-power variation, but not with the realized variance. It turns out that in the HEAVY model, the effect of the jumps explains approximately the same as the realized bi-power variation, but as time goes by this effect declines. For the RealGARCH(1,1) model, this effect stays the same throughout the entire period and explains almost an equal part as the realized bi-power variation.

Regarding the out-of-sample forecasts, the results showed that the models using a realized measure as an explaining variable do not beat the benchmark GARCH(1,1) model. For most of the standard models, non-linear models and models with the squared daily return as an extra regressor, the HAR(3)-RV model cannot be beaten too, however, there are a few exceptions for the RealEGARCH(1,1) model.

The results between the different models have been put together using a model confidence set. However, it turns out that this set is only filled with the best performing model. The only exceptions to this are the FTSE100, SSMI and EUROSTOXX50 at a 1-day forecasting horizon. For these three models, it turns out that they have respectively 10, 9 and 14 models in the model confidence set, at a significance level of 95%.

The models have also been used to combine the forecasts using the mean and median of the forecasts and weighting according to the models in-sample performance. This in-sample performance has been measured using the discounted MSPE. For the combination models using the mean, the models did slightly improve in some cases, however, the best models still did not beat the benchmark models. For the combinations using the in-sample discounted MSPE as a weighting, the models did not beat the HAR(3)-RV model with the 1-day ahead forecasts. However, for the 21-day ahead forecasts, many of the combination models did beat the HAR(3)-RV model. The results of the median forecast combinations were always worse than the results of the mean forecast combinations, and were therefore not discussed.

All in all, the best way to forecast the conditional variance stays the GARCH(1,1) model by Bollerslev (1986). However, when one wants to use realized measures in order to forecast the conditional variance, the best way is to do this for long-term forecasting. As this paper has shown, the best model for this is to use a combination model between all the three standard models and realized measures, weighted according to their in-sample discounted MSPE.

Even though the best effort is done to make this research as complete as possible, there are always questions for further research in this topic. One of the possible extensions that could be made to this research is by investigating more general RealGARCH(p,q) and RealEGARCH(p,q) models. In this research, only models with p=q=1 are considered. Hansen, Huang and Shek (2012) have shown that models with a second lag for either the realized measure as the lagged conditional variance give good results out-of-sample too. Similarly, there could be more lags included into the HEAVY model.

Another extension of the models could be to use a model for r_t too. Even though the emphasis in this paper does not lay on predicting r_t, it is used in a number of models too (for instance via z_t). In this paper, the simple choice of using a random walk model has been used. In stead of the random walk model, there have been a lot of suggestions that turn out well to forecast r_t. This might lead to even better results out-of-sample. However, the risk of using too many parameters rises, which could lead to high estimation errors and therefore poor forecasts.

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Appendix

Table 9

QLIKE results for the standard models

This table reports the results for the QLIKE loss function (Equation (11)) of all the standard models that were described in Equations (2) - (4). The abbreviations for the realized measures are the same as used in Table 1. The model reports results for the HEAVY (H), RealGARCH(1,1) (RG), RealEGRACH(1,1) (REG), GARCH(1,1) (G) and HAR(3)-RV (HAR) models. Panel A uses the squared daily return as a proxy and Panel B the realized kernel. The forecasting period lasts from January 2006 - March 2014. All values have to be multiplied with 10^4.

Panel A

		h=1					h=5					h=10					h=21				
		H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR
FTSE100	RV5	-1.736	-1.745	-1.743			-1.727	-1.737	-1.736			-1.727	-1.723	-1.725			-1.716	-1.684	-1.694		
	RK	-1.738	-1.748	-1.744			-1.723	-1.734	-1.737			-1.722	-1.706	-1.725			-1.710	-1.497	-1.695		
	RV5ss	-1.738	-1.750	-1.746			-1.723	-1.741	-1.740			-1.718	-1.724	-1.729			-1.704	-1.622	-1.699		
	RV10	-1.735	-1.746	-1.743			-1.719	-1.732	-1.736			-1.719	-1.680	-1.724			-1.708	-1.363	-1.693		
	RV10ss	-1.732	-1.748	-1.746			-1.719	-1.739	-1.739			-1.717	-1.724	-1.727			-1.702	-1.682	-1.697		
	BV5	-1.718	-1.747	-1.742			-1.711	-1.735	-1.735			-1.718	-1.713	-1.723			-1.709	-1.639	-1.690		
	BV5ss	-1.726	-1.749	-1.745			-1.718	-1.738	-1.738			-1.717	-1.716	-1.726			-1.709	-1.532	-1.695		
	MEDRV	-1.728	-1.749	-1.741			-1.706	-1.738	-1.734			-1.710	-1.682	-1.722			-1.701	-1.464	-1.688		
	RS5	-1.648	-1.750	-1.738			-1.638	-1.742	-1.729			-1.664	-1.728	-1.715			-1.679	-1.693	-1.676		
	RS5ss	-1.586	-1.750	-1.741			-1.645	-1.742	-1.733			-1.665	-1.727	-1.720			-1.670	-1.661	-1.685		
	DR	-1.680	-1.748	-1.747			-1.680	-1.732	-1.744			-1.679	-1.660	-1.738			-1.671	-1.084	-1.722		
	-				-1.809	-1.755				-1.795	-1.749				-1.793	-1.741				-1.782	-1.723
DAX35	RV5	-1.619	-1.629	-1.624			-1.606	-1.620	-1.612			-1.602	-1.594	-1.586			-1.592	-1.248	-1.519		
	RK	-1.615	-1.629	-1.626			-1.602	-1.625	-1.617			-1.597	-1.603	-1.603			-1.585	-1.235	-1.568		
	RV5ss	-1.605	-1.629	-1.625			-1.607	-1.623	-1.616			-1.600	-1.603	-1.601			-1.591	-1.243	-1.566		
	RV10	-1.617	-1.630	-1.622			-1.606	-1.620	-1.612			-1.596	-1.590	-1.596			-1.583	-1.257	-1.560		
	RV10ss	-1.607	-1.629	-1.626			-1.596	-1.624	-1.617			-1.596	-1.604	-1.602			-1.587	-1.254	-1.568		
	BV5	-1.582	-1.631	-1.626			-1.588	-1.625	-1.616			-1.588	-1.609	-1.599			-1.582	-1.415	-1.561		
	BV5ss	-1.613	-1.627	-1.625			-1.601	-1.625	-1.615			-1.592	-1.609	-1.600			-1.581	-1.272	-1.563		
	MEDRV	-1.605	-1.628	-1.626			-1.590	-1.625	-1.616			-1.588	-1.613	-1.600			-1.583	-1.362	-1.563		
	RS5	-1.492	-1.630	-1.616			-1.474	-1.622	-1.603			-1.487	-1.608	-1.584			-1.507	-1.509	-1.536		
	RS5ss	-1.476	-1.630	-1.618			-1.455	-1.624	-1.607			-1.483	-1.610	-1.589			-1.504	-1.519	-1.543		
	DR	-1.587	-1.630	-1.639			-1.582	-1.619	-1.633			-1.578	-1.603	-1.626			-1.571	-1.571	-1.610		
	-				-1.720	-1.636				-1.702	-1.633				-1.698	-1.626				-1.689	-1.610

Panel A continued

		h=1					h=5					h=10					h=21				
		H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR
	RV5	-1.655	-1.662	-1.649			-1.626	-1.638	-1.631			-1.622	-1.607	-1.616			-1.611	-1.262	-1.579		
	RK	-1.648	-1.662	-1.648			-1.616	-1.637	-1.631			-1.607	-1.605	-1.617			-1.607	-1.248	-1.584		
	RV5ss	-1.650	-1.660	-1.647			-1.621	-1.638	-1.630			-1.607	-1.611	-1.616			-1.607	-1.266	-1.582		
	RV10	-1.652	-1.662	-1.646			-1.617	-1.639	-1.628			-1.602	-1.611	-1.613			-1.602	-1.317	-1.578		
	RV10ss	-1.643	-1.661	-1.647			-1.615	-1.636	-1.630			-1.613	-1.606	-1.616			-1.610	-1.243	-1.583		
CAC40	BV5	-1.639	-1.660	-1.647			-1.628	-1.638	-1.629			-1.616	-1.612	-1.614			-1.606	-1.295	-1.575		
	BV5ss	-1.638	-1.658	-1.646			-1.612	-1.637	-1.629			-1.601	-1.611	-1.615			-1.605	-1.280	-1.580		
	MEDRV	-1.638	-1.655	-1.644			-1.597	-1.638	-1.627			-1.593	-1.619	-1.612			-1.596	-1.311	-1.572		
	RS5	-1.558	-1.662	-1.644			-1.508	-1.641	-1.624			-1.525	-1.621	-1.607			-1.547	-1.331	-1.564		
	RS5ss	-1.552	-1.660	-1.643			-1.522	-1.641	-1.624			-1.524	-1.621	-1.607			-1.546	-1.344	-1.565		
	DR	-1.598	-1.672	-1.651			-1.584	-1.663	-1.640			-1.570	-1.606	-1.632			-1.566	-1.542	-1.612		
	-				-1.734	-1.667				-1.714	-1.657				-1.710	-1.651				-1.698	-1.638
	RV5	-1.702	-1.708	-1.686			-1.661	-1.668	-1.662			-1.660	-1.590	-1.644			-1.663	-1.149	-1.597		
	RK	-1.701	-1.709	-1.687			-1.658	-1.670	-1.662			-1.650	-1.613	-1.645			-1.655	-1.221	-1.601		
	RV5ss	-1.703	-1.710	-1.688			-1.671	-1.668	-1.664			-1.655	-1.617	-1.648			-1.653	-1.110	-1.604		
	RV10	-1.702	-1.712	-1.686			-1.659	-1.671	-1.660			-1.650	-1.590	-1.640			-1.656	-1.081	-1.590		
	RV10ss	-1.697	-1.710	-1.686			-1.662	-1.676	-1.661			-1.655	-1.648	-1.644			-1.659	-1.431	-1.601		
AEX	BV5	-1.690	-1.707	-1.685			-1.663	-1.673	-1.658			-1.650	-1.641	-1.640			-1.649	-1.262	-1.589		
	BV5ss	-1.697	-1.709	-1.688			-1.654	-1.681	-1.662			-1.644	-1.669	-1.645			-1.653	-1.622	-1.599		
	MEDRV	-1.703	-1.714	-1.690			-1.662	-1.703	-1.666			-1.658	-1.693	-1.649			-1.654	-1.583	-1.605		
	RS5	-1.595	-1.708	-1.681			-1.548	-1.682	-1.654			-1.569	-1.654	-1.631			-1.602	-1.271	-1.572		
	RS5ss	-1.593	-1.668	-1.685			-1.531	-1.655	-1.658			-1.559	-1.637	-1.636			-1.599	-1.589	-1.582		
	DR	-1.644	-1.659	-1.686			-1.620	-1.658	-1.670			-1.610	-1.655	-1.657			-1.614	-1.646	-1.626		
	-				-1.786	-1.711				-1.765	-1.699				-1.761	-1.694				-1.749	-1.680
	RV5	-1.703	-1.704	-1.705			-1.696	-1.692	-1.697			-1.693	-1.671	-1.687			-1.689	-1.557	-1.670		
	RK	-1.701	-1.708	-1.705			-1.691	-1.697	-1.698			-1.684	-1.673	-1.689			-1.675	-1.346	-1.671		
	RV5ss	-1.697	-1.706	-1.703			-1.692	-1.693	-1.695			-1.687	-1.668	-1.685			-1.683	-1.368	-1.667		
	RV10	-1.696	-1.707	-1.703			-1.689	-1.692	-1.694			-1.685	-1.664	-1.683			-1.680	-1.286	-1.664		
	RV10ss	-1.685	-1.705	-1.701			-1.685	-1.688	-1.693			-1.680	-1.658	-1.683			-1.673	-1.281	-1.665		
SSMI	BV5	-1.684	-1.704	-1.703			-1.687	-1.693	-1.696			-1.686	-1.671	-1.685			-1.684	-1.350	-1.666		
	BV5ss	-1.694	-1.704	-1.702			-1.687	-1.691	-1.695			-1.680	-1.664	-1.685			-1.674	-1.327	-1.667		
	MEDRV	-1.712	-1.704	-1.700			-1.702	-1.696	-1.693			-1.697	-1.681	-1.682			-1.691	-1.644	-1.664		
	RS5	-1.606	-1.710	-1.701			-1.604	-1.701	-1.691			-1.618	-1.690	-1.679			-1.642	-1.625	-1.658		
	RS5ss	-1.602	-1.709	-1.699			-1.614	-1.699	-1.689			-1.623	-1.685	-1.676			-1.638	-1.634	-1.653		
	DR	-1.657	-1.716	-1.704			-1.655	-1.711	-1.703			-1.653	-1.705	-1.701			-1.646	-1.693	-1.698		
	-				-1.789	-1.714				-1.774	-1.713				-1.772	-1.708				-1.762	-1.697

Panel A continued

		h=1					h=5					h=10					h=21				
		H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR
	RV5	-1.564	-1.565	-1.568			-1.546	-1.547	-1.553			-1.539	-1.518	-1.534			-1.531	-1.455	-1.492		
	RK	-1.552	-1.573	-1.571			-1.536	-1.550	-1.556			-1.525	-1.515	-1.536			-1.520	-1.434	-1.495		
	RV5ss	-1.555	-1.570	-1.569			-1.543	-1.546	-1.554			-1.532	-1.508	-1.535			-1.529	-1.361	-1.496		
	RV10	-1.561	-1.575	-1.566			-1.546	-1.554	-1.550			-1.540	-1.524	-1.528			-1.531	-1.457	-1.486		
	RV10ss	-1.549	-1.589	-1.570			-1.539	-1.582	-1.554			-1.533	-1.575	-1.536			-1.527	-1.560	-1.497		
IBEX35	BV5	-1.539	-1.587	-1.566			-1.530	-1.581	-1.551			-1.523	-1.574	-1.532			-1.520	-1.559	-1.489		
	BV5ss	-1.546	-1.565	-1.566			-1.531	-1.543	-1.552			-1.519	-1.508	-1.533			-1.517	-1.432	-1.490		
	MEDRV	-1.546	-1.556	-1.549			-1.521	-1.534	-1.533			-1.513	-1.502	-1.513			-1.509	-1.431	-1.472		
	RS5	-1.336	-1.589	-1.565			-1.392	-1.583	-1.545			-1.405	-1.576	-1.519			-1.421	-1.561	-1.461		
	RS5ss	-1.413	-1.590	-1.568			-1.386	-1.584	-1.548			-1.394	-1.576	-1.522			-1.417	-1.562	-1.466		
	DR	-1.539	-1.583	-1.581			-1.530	-1.576	-1.573			-1.523	-1.570	-1.565			-1.520	-1.556	-1.553		
	-				-1.673	-1.574				-1.655	-1.572				-1.653	-1.567				-1.641	-1.558
	RV5	-1.562	-1.574	-1.560			-1.530	-1.551	-1.548			-1.533	-1.510	-1.530			-1.529	-1.201	-1.482		
	RK	-1.550	-1.577	-1.565			-1.521	-1.558	-1.557			-1.530	-1.528	-1.543			-1.530	-1.418	-1.503		
	RV5ss	-1.574	-1.577	-1.567			-1.560	-1.562	-1.560			-1.546	-1.536	-1.546			-1.535	-1.281	-1.507		
	RV10	-1.565	-1.579	-1.561			-1.540	-1.554	-1.551			-1.541	-1.510	-1.535			-1.537	-1.044	-1.492		
	RV10ss	-1.548	-1.580	-1.570			-1.520	-1.562	-1.562			-1.525	-1.534	-1.549			-1.526	-1.415	-1.511		
Eurostoxx50	BV5	-1.462	-1.577	-1.566			-1.489	-1.563	-1.556			-1.512	-1.541	-1.539			-1.514	-1.484	-1.494		
	BV5ss	-1.551	-1.576	-1.565			-1.534	-1.567	-1.558			-1.530	-1.551	-1.544			-1.527	-1.512	-1.503		
	MEDRV	-1.540	-1.574	-1.571			-1.515	-1.569	-1.563			-1.513	-1.552	-1.548			-1.510	-1.512	-1.506		
	RS5	-1.402	-1.579	-1.557			-1.363	-1.562	-1.545			-1.382	-1.533	-1.524			-1.411	-1.357	-1.474		
	RS5ss	-1.396	-1.577	-1.554			-1.354	-1.560	-1.545			-1.376	-1.529	-1.526			-1.408	-1.298	-1.479		
	DR	-1.532	-1.580	-1.585			-1.527	-1.568	-1.584			-1.526	-1.552	-1.581			-1.520	-1.493	-1.567		
	-				-1.687	-1.583				-1.668	-1.584				-1.665	-1.580				-1.656	-1.567

Panel B

		h=1					h=5					h=10					h=21				
		H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR
	RV5	-1.748	-1.754	-1.753			-1.732	-1.743	-1.745			-1.731	-1.729	-1.735			-1.722	-1.693	-1.708		
	RK	-1.740	-1.754	-1.752			-1.730	-1.740	-1.745			-1.727	-1.714	-1.735			-1.717	-1.503	-1.709		
	RV5ss	-1.744	-1.758	-1.755			-1.725	-1.749	-1.748			-1.722	-1.733	-1.739			-1.710	-1.627	-1.713		
	RV10	-1.733	-1.754	-1.753			-1.721	-1.738	-1.745			-1.720	-1.690	-1.735			-1.713	-1.366	-1.707		
FTSE100	RV10ss	-1.739	-1.756	-1.755			-1.721	-1.746	-1.747			-1.718	-1.732	-1.738			-1.707	-1.694	-1.711		
	BV5	-1.734	-1.755	-1.752			-1.721	-1.740	-1.743			-1.721	-1.721	-1.733			-1.715	-1.659	-1.704		
	BV5ss	-1.739	-1.756	-1.753			-1.720	-1.744	-1.745			-1.720	-1.724	-1.736			-1.715	-1.541	-1.709		
	MEDRV	-1.722	-1.757	-1.748			-1.707	-1.745	-1.740			-1.712	-1.692	-1.731			-1.706	-1.469	-1.702		
	RS5	-1.653	-1.757	-1.746			-1.638	-1.747	-1.736			-1.663	-1.736	-1.724			-1.682	-1.705	-1.691		
	RS5ss	-1.640	-1.755	-1.747			-1.651	-1.745	-1.739			-1.669	-1.733	-1.729			-1.679	-1.669	-1.699		
	DR	-1.689	-1.748	-1.757			-1.682	-1.732	-1.753			-1.681	-1.664	-1.747			-1.674	-1.088	-1.732		
	-				-1.778	-1.759				-1.789	-1.753				-1.791	-1.744				-1.784	-1.727
	RV5	-1.620	-1.628	-1.620			-1.602	-1.615	-1.608			-1.597	-1.588	-1.585			-1.587	-1.242	-1.523		
	RK	-1.616	-1.628	-1.625			-1.604	-1.621	-1.614			-1.595	-1.598	-1.598			-1.582	-1.232	-1.563		
	RV5ss	-1.619	-1.628	-1.624			-1.605	-1.620	-1.612			-1.599	-1.598	-1.597			-1.587	-1.239	-1.561		
	RV10	-1.611	-1.627	-1.620			-1.600	-1.615	-1.608			-1.592	-1.584	-1.592			-1.579	-1.250	-1.555		
DAX35	RV10ss	-1.608	-1.628	-1.624			-1.593	-1.620	-1.614			-1.591	-1.599	-1.598			-1.582	-1.250	-1.563		
	BV5	-1.601	-1.629	-1.623			-1.589	-1.621	-1.611			-1.585	-1.603	-1.594			-1.577	-1.410	-1.555		
	BV5ss	-1.612	-1.627	-1.623			-1.596	-1.621	-1.612			-1.589	-1.603	-1.595			-1.576	-1.268	-1.557		
	MEDRV	-1.604	-1.628	-1.624			-1.588	-1.622	-1.612			-1.584	-1.607	-1.595			-1.577	-1.354	-1.556		
	RS5	-1.482	-1.627	-1.613			-1.465	-1.617	-1.598			-1.478	-1.602	-1.578			-1.500	-1.504	-1.531		
	RS5ss	-1.445	-1.626	-1.616			-1.447	-1.619	-1.602			-1.474	-1.605	-1.584			-1.498	-1.515	-1.538		
	DR	-1.588	-1.629	-1.636			-1.580	-1.618	-1.630			-1.576	-1.602	-1.623			-1.568	-1.571	-1.607		
	-				-1.655	-1.634				-1.681	-1.629				-1.684	-1.622				-1.679	-1.606

Panel B continued

		h=1					h=5					h=10					h=21				
		H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR
	RV5	-1.664	-1.669	-1.660			-1.638	-1.647	-1.642			-1.629	-1.614	-1.624			-1.616	-1.265	-1.584		
	RK	-1.655	-1.669	-1.660			-1.635	-1.645	-1.643			-1.621	-1.612	-1.626			-1.616	-1.252	-1.588		
	RV5ss	-1.663	-1.669	-1.660			-1.635	-1.646	-1.642			-1.622	-1.616	-1.625			-1.615	-1.268	-1.586		
	RV10	-1.659	-1.670	-1.658			-1.632	-1.647	-1.640			-1.617	-1.618	-1.622			-1.609	-1.320	-1.583		
CAC40	RV10ss	-1.659	-1.669	-1.659			-1.631	-1.645	-1.641			-1.623	-1.612	-1.625			-1.616	-1.245	-1.587		
	BV5	-1.659	-1.668	-1.659			-1.642	-1.645	-1.640			-1.626	-1.617	-1.622			-1.612	-1.296	-1.579		
	BV5ss	-1.657	-1.668	-1.659			-1.628	-1.645	-1.641			-1.617	-1.616	-1.624			-1.613	-1.281	-1.584		
	MEDRV	-1.643	-1.666	-1.657			-1.613	-1.646	-1.639			-1.607	-1.623	-1.621			-1.602	-1.311	-1.576		
	RS5	-1.572	-1.669	-1.657			-1.537	-1.649	-1.637			-1.541	-1.626	-1.616			-1.553	-1.331	-1.569		
	RS5ss	-1.570	-1.667	-1.657			-1.545	-1.648	-1.637			-1.542	-1.625	-1.617			-1.554	-1.343	-1.569		
	DR	-1.609	-1.673	-1.659			-1.591	-1.665	-1.648			-1.579	-1.608	-1.638			-1.572	-1.540	-1.614		
	-				-1.684	-1.674				-1.699	-1.665				-1.701	-1.657				-1.693	-1.640
	RV5	-1.711	-1.722	-1.706			-1.690	-1.693	-1.687			-1.683	-1.622	-1.666			-1.677	-1.168	-1.617		
	RK	-1.710	-1.723	-1.707			-1.689	-1.695	-1.688			-1.677	-1.643	-1.669			-1.671	-1.238	-1.622		
	RV5ss	-1.715	-1.724	-1.708			-1.691	-1.693	-1.690			-1.679	-1.645	-1.671			-1.667	-1.143	-1.625		
	RV10	-1.707	-1.724	-1.705			-1.685	-1.694	-1.684			-1.676	-1.621	-1.663			-1.672	-1.108	-1.611		
AEX	RV10ss	-1.711	-1.723	-1.707			-1.689	-1.699	-1.688			-1.678	-1.670	-1.668			-1.673	-1.455	-1.622		
	BV5	-1.710	-1.722	-1.706			-1.689	-1.697	-1.685			-1.675	-1.662	-1.663			-1.665	-1.288	-1.610		
	BV5ss	-1.708	-1.720	-1.707			-1.681	-1.703	-1.688			-1.672	-1.687	-1.668			-1.669	-1.639	-1.620		
	MEDRV	-1.707	-1.723	-1.708			-1.686	-1.716	-1.690			-1.679	-1.705	-1.671			-1.667	-1.598	-1.625		
	RS5	-1.613	-1.722	-1.701			-1.596	-1.703	-1.680			-1.607	-1.673	-1.655			-1.626	-1.293	-1.594		
	RS5ss	-1.617	-1.683	-1.704			-1.585	-1.671	-1.684			-1.601	-1.654	-1.661			-1.622	-1.608	-1.604		
	DR	-1.655	-1.669	-1.704			-1.637	-1.668	-1.689			-1.629	-1.664	-1.675			-1.625	-1.655	-1.643		
	-				-1.742	-1.725				-1.754	-1.716				-1.755	-1.708				-1.746	-1.690
	RV5	-1.720	-1.723	-1.725			-1.713	-1.712	-1.718			-1.709	-1.693	-1.709			-1.700	-1.578	-1.690		
	RK	-1.712	-1.725	-1.725			-1.707	-1.715	-1.719			-1.700	-1.694	-1.711			-1.687	-1.361	-1.692		
	RV5ss	-1.717	-1.723	-1.723			-1.711	-1.713	-1.717			-1.706	-1.689	-1.707			-1.697	-1.385	-1.688		
	RV10	-1.709	-1.725	-1.724			-1.707	-1.713	-1.717			-1.701	-1.688	-1.706			-1.693	-1.303	-1.685		
SSMI	RV10ss	-1.711	-1.723	-1.722			-1.704	-1.708	-1.716			-1.698	-1.681	-1.706			-1.687	-1.298	-1.686		
	BV5	-1.711	-1.723	-1.724			-1.707	-1.712	-1.717			-1.704	-1.692	-1.708			-1.697	-1.364	-1.687		
	BV5ss	-1.711	-1.723	-1.723			-1.704	-1.710	-1.716			-1.698	-1.685	-1.707			-1.687	-1.342	-1.687		
	MEDRV	-1.727	-1.725	-1.722			-1.720	-1.716	-1.715			-1.714	-1.701	-1.705			-1.704	-1.664	-1.685		
	RS5	-1.630	-1.726	-1.723			-1.633	-1.720	-1.714			-1.644	-1.708	-1.703			-1.659	-1.643	-1.680		
	RS5ss	-1.618	-1.726	-1.720			-1.636	-1.718	-1.712			-1.642	-1.703	-1.700			-1.651	-1.652	-1.676		
	DR	-1.671	-1.726	-1.726			-1.667	-1.720	-1.723			-1.664	-1.714	-1.720			-1.656	-1.702	-1.713		
	-				-1.744	-1.731				-1.766	-1.727				-1.769	-1.720				-1.763	-1.706

Panel B continued

		h=1					h=5					h=10					h=21				
		H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR
	RV5	-1.600	-1.595	-1.610			-1.583	-1.579	-1.598			-1.577	-1.557	-1.583			-1.568	-1.502	-1.547		
	RK	-1.586	-1.613	-1.612			-1.575	-1.594	-1.599			-1.566	-1.568	-1.584			-1.557	-1.499	-1.549		
	RV5ss	-1.601	-1.611	-1.610			-1.582	-1.591	-1.598			-1.575	-1.562	-1.584			-1.566	-1.423	-1.549		
	RV10	-1.599	-1.614	-1.609			-1.583	-1.598	-1.596			-1.577	-1.577	-1.580			-1.564	-1.519	-1.542		
IBEX35	RV10ss	-1.593	-1.621	-1.611			-1.578	-1.615	-1.598			-1.572	-1.608	-1.584			-1.565	-1.595	-1.550		
	BV5	-1.588	-1.621	-1.610			-1.575	-1.614	-1.596			-1.568	-1.608	-1.582			-1.560	-1.594	-1.545		
	BV5ss	-1.591	-1.609	-1.609			-1.571	-1.588	-1.597			-1.563	-1.561	-1.582			-1.555	-1.497	-1.545		
	MEDRV	-1.583	-1.602	-1.597			-1.567	-1.583	-1.583			-1.561	-1.559	-1.568			-1.553	-1.500	-1.531		
	RS5	-1.449	-1.622	-1.607			-1.462	-1.616	-1.592			-1.470	-1.609	-1.572			-1.482	-1.595	-1.522		
	RS5ss	-1.483	-1.622	-1.609			-1.461	-1.616	-1.594			-1.465	-1.609	-1.574			-1.480	-1.596	-1.527		
	DR	-1.554	-1.615	-1.618			-1.547	-1.609	-1.611			-1.544	-1.603	-1.604			-1.539	-1.590	-1.591		
	-				-1.631	-1.614				-1.652	-1.608				-1.655	-1.603				-1.648	-1.590
	RV5	-1.579	-1.589	-1.577			-1.549	-1.566	-1.564			-1.548	-1.525	-1.548			-1.543	-1.233	-1.507		
	RK	-1.565	-1.592	-1.582			-1.543	-1.573	-1.572			-1.544	-1.544	-1.560			-1.545	-1.441	-1.525		
	RV5ss	-1.591	-1.592	-1.584			-1.568	-1.576	-1.575			-1.558	-1.551	-1.563			-1.548	-1.297	-1.529		
	RV10	-1.577	-1.593	-1.578			-1.556	-1.569	-1.567			-1.555	-1.525	-1.553			-1.551	-1.074	-1.516		
Eurostoxx50	RV10ss	-1.562	-1.594	-1.586			-1.536	-1.576	-1.577			-1.540	-1.549	-1.565			-1.541	-1.441	-1.533		
	BV5	-1.540	-1.594	-1.584			-1.521	-1.579	-1.573			-1.530	-1.557	-1.557			-1.531	-1.502	-1.517		
	BV5ss	-1.573	-1.591	-1.583			-1.548	-1.581	-1.574			-1.545	-1.566	-1.561			-1.543	-1.530	-1.525		
	MEDRV	-1.547	-1.589	-1.587			-1.529	-1.583	-1.578			-1.528	-1.567	-1.564			-1.527	-1.531	-1.528		
	RS5	-1.414	-1.594	-1.574			-1.399	-1.576	-1.561			-1.412	-1.546	-1.542			-1.439	-1.381	-1.498		
	RS5ss	-1.341	-1.593	-1.572			-1.381	-1.575	-1.560			-1.400	-1.543	-1.543			-1.435	-1.327	-1.502		
	DR	-1.549	-1.598	-1.599			-1.535	-1.588	-1.597			-1.533	-1.573	-1.594			-1.527	-1.518	-1.582		
	-				-1.634	-1.594				-1.659	-1.596				-1.663	-1.591				-1.658	-1.580

Table 10

QLIKE results for models with r_t^2

This table reports the results for the QLIKE loss function (Equation (11)) of all the models with the squared daily returns that were described in Equations (5) and (6). The abbreviations for the realized measures are the same as used in Table 1. The model reports results for the HEAVY (H), RealGARCH(1,1) (RG), RealEGRACH(1,1) (REG), GARCH(1,1) (G) and HAR(3)-RV (HAR) models. Panel A uses the squared daily returns as a proxy and Panel B the realized kernel. The forecasting period lasts from January 2006 - March 2014. All values have to be multiplied with 10^4.

Panel A

		h=1					h=5					h=10					h=21				
		H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR
	RV5	-1.657	-1.716				-1.649	-1.660				-1.655	-1.560				-1.644	-1.174			
	RK	-1.709	-1.730				-1.702	-1.678				-1.711	-1.578				-1.706	-1.041			
	RV5ss	-1.700	-1.731				-1.689	-1.690				-1.689	-1.616				-1.679	-1.399			
	RV10	-1.683	-1.723				-1.673	-1.671				-1.679	-1.571				-1.672	-1.089			
	RV10ss	-1.697	-1.732				-1.695	-1.686				-1.707	-1.598				-1.701	-1.195			
FTSE100	BV5	-1.643	-1.712				-1.644	-1.659				-1.648	-1.566				-1.637	-1.276			
	BV5ss	-1.686	-1.728				-1.687	-1.685				-1.702	-1.610				-1.702	-1.403			
	MEDRV	-1.677	-1.717				-1.659	-1.679				-1.668	-1.618				-1.666	-1.465			
	RS5	-1.561	-1.728				-1.574	-1.700				-1.613	-1.650				-1.634	-1.426			
	RS5ss	-1.532	-1.740				-1.577	-1.710				-1.620	-1.657				-1.648	-1.448			
	DR	-1.620	-1.749				-1.619	-1.727				-1.615	-1.600				-1.606	-0.737			
	-				-1.809	-1.755				-1.795	-1.749				-1.793	-1.741				-1.782	-1.723
	RV5	-1.561	-1.615				-1.571	-1.583				-1.578	-1.516				-1.572	-1.274			
	RK	-1.593	-1.617				-1.587	-1.587				-1.587	-1.528				-1.579	-1.338			
	RV5ss	-1.526	-1.619				-1.529	-1.593				-1.527	-1.541				-1.521	-1.358			
	RV10	-1.580	-1.614				-1.575	-1.579				-1.574	-1.512				-1.569	-1.339			
	RV10ss	-1.562	-1.620				-1.556	-1.592				-1.556	-1.539				-1.548	-1.374			
DAX35	BV5	-1.491	-1.620				-1.522	-1.593				-1.530	-1.542				-1.530	-1.422			
	BV5ss	-1.561	-1.622				-1.567	-1.597				-1.566	-1.550				-1.562	-1.422			
	MEDRV	-1.505	-1.623				-1.507	-1.595				-1.514	-1.542				-1.511	-1.263			
	RS5	-1.338	-1.620				-1.364	-1.598				-1.401	-1.566				-1.444	-1.490			
	RS5ss	-1.358	-1.625				-1.393	-1.607				-1.422	-1.579				-1.453	-1.511			
	DR	-1.566	-1.627				-1.557	-1.620				-1.552	-1.608				-1.545	-1.562			
	-				-1.720	-1.636				-1.702	-1.633				-1.698	-1.626				-1.689	-1.610

Panel A continued

		h=1					h=5					h=10					h=21				
		H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR
	RV5	-1.621	-1.648				-1.597	-1.598				-1.598	-1.531				-1.595	-1.366			
	RK	-1.582	-1.649				-1.557	-1.595				-1.549	-1.525				-1.544	-1.367			
	RV5ss	-1.611	-1.649				-1.583	-1.601				-1.577	-1.541				-1.590	-1.403			
	RV10	-1.609	-1.644				-1.573	-1.588				-1.559	-1.512				-1.565	-1.330			
	RV10ss	-1.587	-1.650				-1.558	-1.601				-1.557	-1.539				-1.566	-1.400			
CAC40	BV5	-1.610	-1.647				-1.599	-1.596				-1.588	-1.533				-1.588	-1.399			
	BV5ss	-1.576	-1.647				-1.555	-1.601				-1.542	-1.543				-1.540	-1.418			
	MEDRV	-1.580	-1.646				-1.544	-1.610				-1.558	-1.567				-1.569	-1.465			
	RS5	-1.474	-1.650				-1.433	-1.607				-1.462	-1.555				-1.502	-1.429			
	RS5ss	-1.430	-1.651				-1.408	-1.612				-1.423	-1.566				-1.461	-1.465			
	DR	-1.615	-1.660				-1.591	-1.652				-1.580	-1.643				-1.584	-1.625			
	-				-1.734	-1.667				-1.714	-1.657				-1.710	-1.651				-1.698	-1.638
	RV5	-1.661	-1.696				-1.628	-1.207				-1.629	-1.232				-1.630	-1.244			
	RK	-1.640	-1.696				-1.601	-1.212				-1.596	-1.237				-1.610	-1.248			
	RV5ss	-1.676	-1.697				-1.638	-1.204				-1.636	-1.231				-1.647	-1.244			
	RV10	-1.624	-1.697				-1.573	-1.221				-1.575	-1.244				-1.586	-1.254			
	RV10ss	-1.658	-1.694				-1.621	-1.217				-1.617	-1.241				-1.640	-1.252			
AEX	BV5	-1.643	-1.693				-1.632	-1.204				-1.619	-1.233				-1.621	-1.246			
	BV5ss	-1.634	-1.693				-1.590	-1.208				-1.587	-1.235				-1.607	-1.248			
	MEDRV	-1.661	-1.699				-1.623	-1.185				-1.629	-1.217				-1.635	-1.232			
	RS5	-1.526	-1.702				-1.486	-1.219				-1.514	-1.238				-1.540	-1.246			
	RS5ss	-1.469	-1.702				-1.430	-1.212				-1.487	-1.235				-1.553	-1.244			
	DR	-1.627	-1.708				-1.603	-1.260				-1.594	-1.250				-1.602	-1.241			
	-				-1.786	-1.711				-1.765	-1.699				-1.761	-1.694				-1.749	-1.680
	RV5	-1.653	-1.703				-1.651	-1.674				-1.649	-1.633				-1.649	-1.561			
	RK	-1.674	-1.704				-1.675	-1.683				-1.673	-1.651				-1.673	-1.586			
	RV5ss	-1.633	-1.701				-1.627	-1.675				-1.621	-1.636				-1.615	-1.568			
	RV10	-1.667	-1.701				-1.667	-1.671				-1.669	-1.628				-1.672	-1.555			
	RV10ss	-1.665	-1.699				-1.667	-1.667				-1.669	-1.624				-1.671	-1.556			
SSMI	BV5	-1.598	-1.695				-1.606	-1.686				-1.608	-1.675				-1.609	-1.652			
	BV5ss	-1.661	-1.698				-1.663	-1.669				-1.663	-1.629				-1.668	-1.566			
	MEDRV	-1.644	-1.700				-1.634	-1.682				-1.630	-1.656				-1.624	-1.611			
	RS5	-1.507	-1.708				-1.538	-1.684				-1.568	-1.652				-1.609	-1.599			
	RS5ss	-1.512	-1.705				-1.545	-1.683				-1.577	-1.655				-1.614	-1.605			
	DR	-1.594	-1.711				-1.591	-1.701				-1.589	-1.691				-1.583	-1.092			
	-				-1.789	-1.714				-1.774	-1.713				-1.772	-1.708				-1.762	-1.697

Panel A continued

		h=1					h=5					h=10					h=21				
		H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR
	RV5	-1.511	-1.543				-1.506	-1.473				-1.500	-1.370				-1.498	-1.152			
	RK	-1.510	-1.544				-1.498	-1.473				-1.489	-1.372				-1.484	-1.173			
	RV5ss	-1.486	-1.543				-1.481	-1.473				-1.473	-1.373				-1.475	-1.174			
	RV10	-1.510	-0.502				-1.509	-0.953				-1.506	-0.876				-1.502	-0.699			
	RV10ss	-1.466	-1.541				-1.468	-1.457				-1.469	-1.339				-1.462	-1.081			
IBEX35	BV5	-1.487	-1.187				-1.484	-1.225				-1.482	-1.136				-1.485	-0.956			
	BV5ss	-1.487	-1.539				-1.480	-1.467				-1.468	-1.367				-1.467	-1.174			
	MEDRV	-1.474	-1.536				-1.459	-1.481				-1.452	-1.400				-1.454	-1.237			
	RS5	-1.261	-1.548				-1.286	-1.489				-1.302	-1.402				-1.337	-1.237			
	RS5ss	-1.340	-1.551				-1.330	-1.492				-1.337	-1.412				-1.351	-1.265			
	DR	-1.547	-1.544				-1.537	-1.527				-1.530	-1.495				-1.527	-1.451			
	-				-1.673	-1.574				-1.655	-1.572				-1.653	-1.567				-1.641	-1.558
	RV5	-1.532	-1.557				-1.506	-1.499				-1.513	-1.404				-1.513	-0.992			
	RK	-1.440	-1.561				-1.422	-1.505				-1.448	-1.417				-1.455	-1.067			
	RV5ss	-1.532	-1.565				-1.519	-1.519				-1.511	-1.454				-1.506	-1.273			
	RV10	-1.498	-1.547				-1.478	-1.479				-1.478	-1.373				-1.471	-0.935			
	RV10ss	-1.409	-1.566				-1.440	-1.517				-1.482	-1.450				-1.493	-1.286			
Eurostoxx50	BV5	-1.314	-1.565				-1.438	-1.521				-1.477	-1.461				-1.487	-1.323			
	BV5ss	-1.433	-1.564				-1.433	-1.524				-1.446	-1.469				-1.453	-1.356			
	MEDRV	-1.457	-1.567				-1.443	-1.535				-1.453	-1.486				-1.462	-1.383			
	RS5	-1.323	-1.555				-1.309	-1.516				-1.331	-1.457				-1.362	-1.184			
	RS5ss	-1.232	-1.570				-1.232	-1.534				-1.272	-1.463				-1.321	-1.040			
	DR	-1.519	-1.572				-1.518	-1.559				-1.532	-1.542				-1.533	-1.503			
	-				-1.687	-1.583				-1.668	-1.584				-1.665	-1.580				-1.656	-1.567

Panel B

		h=1					h=5					h=10					h=21				
		H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR
	RV5	-1.667	-1.729				-1.651	-1.673				-1.655	-1.577				-1.649	-1.190			
	RK	-1.715	-1.739				-1.709	-1.688				-1.715	-1.595				-1.714	-1.061			
	RV5ss	-1.707	-1.742				-1.693	-1.701				-1.695	-1.632				-1.687	-1.430			
	RV10	-1.694	-1.733				-1.679	-1.680				-1.681	-1.587				-1.676	-1.112			
	RV10ss	-1.700	-1.743				-1.698	-1.698				-1.708	-1.616				-1.708	-1.223			
FTSE100	BV5	-1.659	-1.726				-1.649	-1.673				-1.649	-1.584				-1.643	-1.310			
	BV5ss	-1.697	-1.740				-1.688	-1.697				-1.704	-1.626				-1.709	-1.434			
	MEDRV	-1.669	-1.724				-1.663	-1.685				-1.674	-1.628				-1.673	-1.482			
	RS5	-1.577	-1.735				-1.574	-1.705				-1.611	-1.660				-1.636	-1.438			
	RS5ss	-1.553	-1.745				-1.583	-1.716				-1.626	-1.668				-1.657	-1.474			
	DR	-1.626	-1.752				-1.619	-1.731				-1.618	-1.607				-1.610	-0.741			
	-				-1.778	-1.759				-1.789	-1.753				-1.791	-1.744				-1.784	-1.727
	RV5	-1.568	-1.615				-1.569	-1.576				-1.572	-1.505				-1.566	-1.264			
	RK	-1.594	-1.616				-1.587	-1.583				-1.584	-1.522				-1.574	-1.337			
	RV5ss	-1.543	-1.619				-1.529	-1.589				-1.525	-1.535				-1.516	-1.358			
	RV10	-1.573	-1.612				-1.570	-1.574				-1.570	-1.505				-1.563	-1.333			
	RV10ss	-1.558	-1.618				-1.550	-1.589				-1.551	-1.534				-1.543	-1.373			
DAX35	BV5	-1.512	-1.619				-1.527	-1.588				-1.528	-1.534				-1.525	-1.412			
	BV5ss	-1.567	-1.621				-1.561	-1.593				-1.562	-1.545				-1.556	-1.420			
	MEDRV	-1.511	-1.623				-1.507	-1.591				-1.509	-1.534				-1.504	-1.258			
	RS5	-1.332	-1.616				-1.356	-1.594				-1.391	-1.561				-1.435	-1.485			
	RS5ss	-1.357	-1.621				-1.381	-1.602				-1.413	-1.574				-1.446	-1.508			
	DR	-1.565	-1.626				-1.555	-1.617				-1.550	-1.606				-1.541	-1.562			
	-				-1.655	-1.634				-1.681	-1.629				-1.684	-1.622				-1.679	-1.606

Panel B continued

		h=1					h=5					h=10					h=21				
		H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR
	RV5	-1.637	-1.657				-1.614	-1.609				-1.609	-1.541				-1.602	-1.374			
	RK	-1.590	-1.658				-1.572	-1.606				-1.559	-1.533				-1.551	-1.371			
	RV5ss	-1.635	-1.658				-1.606	-1.611				-1.597	-1.548				-1.599	-1.407			
	RV10	-1.613	-1.653				-1.589	-1.599				-1.576	-1.523				-1.573	-1.340			
	RV10ss	-1.611	-1.658				-1.583	-1.610				-1.573	-1.546				-1.572	-1.404			
CAC40	BV5	-1.636	-1.656				-1.618	-1.606				-1.604	-1.540				-1.597	-1.402			
	BV5ss	-1.592	-1.656				-1.565	-1.610				-1.554	-1.550				-1.548	-1.421			
	MEDRV	-1.595	-1.658				-1.569	-1.621				-1.574	-1.573				-1.576	-1.467			
	RS5	-1.494	-1.657				-1.470	-1.616				-1.485	-1.561				-1.512	-1.433			
	RS5ss	-1.468	-1.658				-1.443	-1.619				-1.447	-1.570				-1.468	-1.466			
	DR	-1.629	-1.659				-1.604	-1.651				-1.593	-1.643				-1.591	-1.624			
	-				-1.684	-1.674				-1.699	-1.665				-1.701	-1.657				-1.693	-1.640
	RV5	-1.673	-1.713				-1.655	-1.208				-1.650	-1.233				-1.644	-1.245			
	RK	-1.655	-1.713				-1.639	-1.212				-1.629	-1.237				-1.629	-1.249			
	RV5ss	-1.686	-1.714				-1.669	-1.205				-1.666	-1.232				-1.665	-1.245			
	RV10	-1.625	-1.711				-1.604	-1.222				-1.602	-1.244				-1.601	-1.255			
	RV10ss	-1.677	-1.711				-1.659	-1.217				-1.652	-1.241				-1.658	-1.253			
AEX	BV5	-1.665	-1.711				-1.653	-1.205				-1.642	-1.233				-1.636	-1.247			
	BV5ss	-1.649	-1.710				-1.626	-1.208				-1.622	-1.236				-1.625	-1.248			
	MEDRV	-1.661	-1.712				-1.654	-1.185				-1.656	-1.217				-1.653	-1.233			
	RS5	-1.536	-1.716				-1.528	-1.219				-1.542	-1.238				-1.560	-1.247			
	RS5ss	-1.488	-1.717				-1.499	-1.213				-1.539	-1.235				-1.581	-1.245			
	DR	-1.640	-1.718				-1.621	-1.260				-1.615	-1.251				-1.613	-1.242			
	-				-1.742	-1.725				-1.754	-1.716				-1.755	-1.708				-1.746	-1.690
	RV5	-1.673	-1.720				-1.671	-1.694				-1.668	-1.655				-1.663	-1.579			
	RK	-1.688	-1.722				-1.695	-1.704				-1.694	-1.673				-1.689	-1.607			
	RV5ss	-1.646	-1.719				-1.642	-1.695				-1.638	-1.657				-1.629	-1.587			
	RV10	-1.681	-1.719				-1.690	-1.692				-1.689	-1.652				-1.688	-1.577			
	RV10ss	-1.691	-1.716				-1.691	-1.688				-1.690	-1.647				-1.687	-1.576			
SSMI	BV5	-1.626	-1.714				-1.629	-1.707				-1.628	-1.697				-1.623	-1.674			
	BV5ss	-1.680	-1.718				-1.686	-1.690				-1.686	-1.651				-1.685	-1.584			
	MEDRV	-1.655	-1.720				-1.650	-1.702				-1.647	-1.676				-1.637	-1.628			
	RS5	-1.533	-1.723				-1.573	-1.702				-1.599	-1.670				-1.630	-1.612			
	RS5ss	-1.543	-1.721				-1.574	-1.703				-1.604	-1.674				-1.634	-1.621			
	DR	-1.606	-1.717				-1.603	-1.709				-1.601	-1.701				-1.593	-1.095			
	-				-1.744	-1.731				-1.766	-1.727				-1.769	-1.720				-1.763	-1.706

Panel B continued

		h=1					h=5					h=10					h=21				
		H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR
	RV5	-1.565	-1.597				-1.554	-1.544				-1.551	-1.465				-1.545	-1.287			
	RK	-1.545	-1.597				-1.538	-1.543				-1.532	-1.466				-1.524	-1.303			
	RV5ss	-1.546	-1.597				-1.529	-1.543				-1.523	-1.466				-1.519	-1.303			
	RV10	-1.566	-0.646				-1.558	-1.036				-1.555	-0.982				-1.547	-0.831			
	RV10ss	-1.515	-1.596				-1.507	-1.529				-1.504	-1.437				-1.497	-1.216			
IBEX35	BV5	-1.551	-1.264				-1.541	-1.310				-1.539	-1.240				-1.535	-1.085			
	BV5ss	-1.542	-1.594				-1.523	-1.539				-1.519	-1.462				-1.513	-1.306			
	MEDRV	-1.524	-1.590				-1.512	-1.549				-1.507	-1.489				-1.503	-1.357			
	RS5	-1.347	-1.598				-1.369	-1.551				-1.388	-1.485				-1.414	-1.351			
	RS5ss	-1.415	-1.599				-1.396	-1.554				-1.400	-1.492				-1.412	-1.373			
	DR	-1.569	-1.580				-1.560	-1.565				-1.556	-1.536				-1.552	-1.496			
	-				-1.631	-1.614				-1.652	-1.608				-1.655	-1.603				-1.648	-1.590
	RV5	-1.554	-1.576				-1.526	-1.521				-1.531	-1.431				-1.531	-1.042			
	RK	-1.466	-1.579				-1.453	-1.527				-1.467	-1.444				-1.471	-1.130			
	RV5ss	-1.553	-1.583				-1.532	-1.540				-1.528	-1.479				-1.524	-1.313			
	RV10	-1.500	-1.568				-1.488	-1.504				-1.490	-1.405				-1.485	-1.005			
	RV10ss	-1.426	-1.583				-1.469	-1.538				-1.501	-1.477				-1.513	-1.328			
Eurostoxx50	BV5	-1.467	-1.584				-1.475	-1.544				-1.499	-1.487				-1.509	-1.359			
	BV5ss	-1.487	-1.581				-1.456	-1.545				-1.464	-1.494				-1.469	-1.392			
	MEDRV	-1.467	-1.583				-1.465	-1.555				-1.474	-1.510				-1.484	-1.418			
	RS5	-1.327	-1.574				-1.337	-1.537				-1.357	-1.481				-1.392	-1.235			
	RS5ss	-1.133	-1.588				-1.270	-1.552				-1.303	-1.485				-1.353	-1.078			
	DR	-1.541	-1.589				-1.531	-1.578				-1.541	-1.564				-1.542	-1.530			
	-				-1.634	-1.594				-1.659	-1.596				-1.663	-1.591				-1.658	-1.580

Table 11

QLIKE results for non-linear models

This table reports the results for the QLIKE loss function (Equation (11)) of all the non-linear models that were described in Equations (7) - (9). The abbreviations for the realized measures are the same as used in Table 1. The model reports results for the HEAVY (H), RealGARCH(1,1) (RG), RealEGRACH(1,1) (REG), GARCH(1,1) (G) and HAR(3)-RV (HAR) models. Panel A uses the squared daily returns as a proxy and Panel B the realized kernel. The forecasting period lasts from January 2006 - March 2014. All values have to be multiplied with 10^4.

Panel A

		h=1					h=5					h=10					h=21				
		H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR
FTSE100	RV5	-1.710	-1.658	-1.744			-1.707	-1.587	-1.736			-1.716	-1.482	-1.724			-1.709	-1.235	-1.689		
	RK	-1.709	-1.692	-1.745			-1.702	-1.628	-1.736			-1.711	-1.532	-1.724			-1.706	-1.303	-1.689		
	RV5ss	-1.717	-1.705	-1.746			-1.711	-1.659	-1.739			-1.715	-1.591	-1.728			-1.708	-1.431	-1.696		
	RV10	-1.710	-1.691	-1.744			-1.700	-1.633	-1.737			-1.709	-1.541	-1.723			-1.705	-1.308	-1.688		
	RV10ss	-1.697	-1.712	-1.746			-1.695	-1.656	-1.739			-1.707	-1.568	-1.727			-1.701	-1.352	-1.694		
	BV5	-1.689	-1.604	-1.742			-1.690	-1.541	-1.735			-1.706	-1.463	-1.722			-1.703	-1.306	-1.686		
	BV5ss	-1.686	-1.737	-1.745			-1.687	-1.722	-1.738			-1.702	-1.696	-1.726			-1.702	-1.618	-1.692		
	MEDRV	-1.691	-1.745	-1.742			-1.680	-1.725	-1.733			-1.695	-1.690	-1.720			-1.696	-1.551	-1.683		
	RS5	-1.550	-1.594	-1.739			-1.572	-1.545	-1.730			-1.621	-1.489	-1.713			-1.651	-1.390	-1.669		
	RS5ss	-1.532	-1.724	-1.741			-1.577	-1.696	-1.732			-1.620	-1.652	-1.718			-1.648	-1.537	-1.677		
	DR	-1.687	-1.696	-1.747			-1.690	-1.664	-1.744			-1.691	-1.622	-1.738			-1.684	-1.534	-1.723		
	-				-1.809	-1.755				-1.795	-1.749				-1.793	-1.741				-1.782	-1.723
DAX35	RV5	-1.561	-1.593	-1.623			-1.571	-1.531	-1.612			-1.578	-1.434	-1.595			-1.572	-1.207	-1.556		
	RK	-1.578	-1.609	-1.626			-1.582	-1.555	-1.617			-1.584	-1.464	-1.602			-1.577	-1.243	-1.566		
	RV5ss	-1.574	-1.602	-1.625			-1.581	-1.550	-1.616			-1.579	-1.467	-1.600			-1.574	-1.279	-1.563		
	RV10	-1.580	-1.608	-1.621			-1.575	-1.542	-1.611			-1.574	-1.420	-1.596			-1.569	-1.094	-1.564		
	RV10ss	-1.559	-1.602	-1.626			-1.567	-1.552	-1.617			-1.572	-1.474	-1.602			-1.568	-1.291	-1.567		
	BV5	-1.522	-1.560	-1.626			-1.554	-1.497	-1.615			-1.561	-1.408	-1.597			-1.559	-1.232	-1.557		
	BV5ss	-1.560	-1.568	-1.625			-1.566	-1.514	-1.615			-1.565	-1.438	-1.598			-1.562	-1.278	-1.559		
	MEDRV	-1.546	-1.566	-1.626			-1.551	-1.505	-1.615			-1.558	-1.419	-1.598			-1.558	-1.240	-1.556		
	RS5	-1.338	-1.567	-1.620			-1.364	-1.530	-1.603			-1.400	-1.484	-1.576			-1.444	-1.383	-1.507		
	RS5ss	-1.318	-1.620	-1.621			-1.379	-1.592	-1.606			-1.421	-1.550	-1.583			-1.463	-1.450	-1.522		
	DR	-1.604	-1.404	-1.640			-1.593	-1.490	-1.633			-1.589	-1.499	-1.626			-1.581	-1.489	-1.610		
	-				-1.720	-1.636				-1.702	-1.633				-1.698	-1.626				-1.689	-1.610

Panel A continued

		h=1					h=5					h=10					h=21				
		H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR
	RV5	-1.620	-1.598	-1.645			-1.597	-1.516	-1.631			-1.598	-1.414	-1.621			-1.595	-1.209	-1.596		
	RK	-1.626	-1.605	-1.644			-1.593	-1.522	-1.631			-1.583	-1.418	-1.622			-1.593	-1.212	-1.600		
	RV5ss	-1.611	-1.613	-1.645			-1.583	-1.539	-1.631			-1.577	-1.444	-1.622			-1.590	-1.250	-1.597		
	RV10	-1.630	-1.607	-1.638			-1.592	-1.529	-1.626			-1.583	-1.424	-1.619			-1.593	-1.193	-1.602		
	RV10ss	-1.611	-1.621	-1.642			-1.584	-1.553	-1.629			-1.584	-1.466	-1.621			-1.591	-1.288	-1.600		
CAC40	BV5	-1.610	-1.603	-1.644			-1.599	-1.521	-1.630			-1.588	-1.419	-1.620			-1.588	-1.217	-1.594		
	BV5ss	-1.605	-1.614	-1.644			-1.577	-1.545	-1.630			-1.576	-1.456	-1.620			-1.587	-1.270	-1.595		
	MEDRV	-1.579	-1.630	-1.642			-1.544	-1.575	-1.628			-1.558	-1.501	-1.618			-1.569	-1.320	-1.589		
	RS5	-1.452	-1.640	-1.641			-1.418	-1.600	-1.623			-1.453	-1.556	-1.609			-1.496	-1.459	-1.572		
	RS5ss	-1.451	-1.643	-1.641			-1.436	-1.608	-1.624			-1.454	-1.567	-1.610			-1.497	-1.472	-1.574		
	DR	-1.615	-1.605	-1.632			-1.591	-1.603	-1.626			-1.580	-1.600	-1.624			-1.584	-1.591	-1.619		
	-				-1.734	-1.667				-1.714	-1.657				-1.710	-1.651				-1.698	-1.638
	RV5	-1.679	-1.676	-1.686			-1.635	-1.603	-1.662			-1.641	-1.504	-1.643			-1.651	-1.226	-1.594		
	RK	-1.674	-1.687	-1.688			-1.635	-1.622	-1.663			-1.630	-1.533	-1.646			-1.644	-1.208	-1.601		
	RV5ss	-1.675	-1.688	-1.689			-1.638	-1.623	-1.664			-1.635	-1.534	-1.647			-1.647	-1.123	-1.602		
	RV10	-1.679	-1.686	-1.685			-1.624	-1.610	-1.660			-1.623	-1.504	-1.642			-1.642	-1.201	-1.598		
	RV10ss	-1.658	-1.687	-1.687			-1.621	-1.619	-1.661			-1.616	-1.522	-1.642			-1.640	-1.046	-1.594		
AEX	BV5	-1.665	-1.672	-1.685			-1.641	-1.600	-1.660			-1.634	-1.503	-1.642			-1.645	-1.206	-1.593		
	BV5ss	-1.667	-1.687	-1.688			-1.622	-1.625	-1.662			-1.620	-1.543	-1.644			-1.639	-1.165	-1.595		
	MEDRV	-1.661	-1.689	-1.690			-1.623	-1.632	-1.666			-1.629	-1.561	-1.648			-1.635	-1.167	-1.602		
	RS5	-1.507	-1.687	-1.683			-1.473	-1.646	-1.655			-1.510	-1.593	-1.628			-1.564	-1.450	-1.561		
	RS5ss	-1.469	-1.696	-1.687			-1.430	-1.658	-1.658			-1.486	-1.613	-1.633			-1.553	-1.487	-1.569		
	DR	-1.662	-1.403	-1.690			-1.626	-1.522	-1.674			-1.623	-1.546	-1.660			-1.635	-1.556	-1.625		
	-				-1.786	-1.711				-1.765	-1.699				-1.761	-1.694				-1.749	-1.680
	RV5	-1.676	-1.627	-1.706			-1.678	-1.547	-1.697			-1.679	-1.433	-1.684			-1.680	-1.181	-1.659		
	RK	-1.674	-1.665	-1.707			-1.675	-1.609	-1.698			-1.673	-1.519	-1.685			-1.673	-1.129	-1.659		
	RV5ss	-1.675	-1.655	-1.705			-1.676	-1.589	-1.695			-1.675	-1.489	-1.680			-1.676	-1.268	-1.652		
	RV10	-1.667	-1.646	-1.706			-1.667	-1.571	-1.694			-1.669	-1.463	-1.679			-1.672	-1.239	-1.647		
	RV10ss	-1.656	-1.645	-1.704			-1.662	-1.574	-1.693			-1.665	-1.473	-1.678			-1.668	-1.273	-1.650		
SSMI	BV5	-1.656	-1.632	-1.705			-1.668	-1.560	-1.695			-1.671	-1.458	-1.680			-1.675	-1.230	-1.650		
	BV5ss	-1.661	-1.649	-1.703			-1.663	-1.584	-1.694			-1.663	-1.490	-1.679			-1.668	-1.290	-1.650		
	MEDRV	-1.688	-1.638	-1.702			-1.686	-1.567	-1.691			-1.684	-1.454	-1.675			-1.682	-1.123	-1.643		
	RS5	-1.505	-1.690	-1.706			-1.536	-1.658	-1.691			-1.566	-1.616	-1.671			-1.608	-1.528	-1.629		
	RS5ss	-1.502	-1.692	-1.703			-1.541	-1.665	-1.688			-1.573	-1.629	-1.667			-1.612	-1.555	-1.619		
	DR	-1.668	-1.710	-1.708			-1.663	-1.699	-1.705			-1.661	-1.692	-1.701			-1.656	-1.680	-1.695		
	-				-1.789	-1.714				-1.774	-1.713				-1.772	-1.708				-1.762	-1.697

Panel A continued

		h=1					h=5					h=10					h=21				
		H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR
IBEX35	RV5	-1.511	-1.520	-1.571			-1.506	-1.422	-1.553			-1.500	-1.295	-1.530			-1.498	-1.084	-1.478		
	RK	-1.514	-1.527	-1.572			-1.512	-1.434	-1.555			-1.505	-1.317	-1.533			-1.503	-1.132	-1.487		
	RV5ss	-1.515	-1.525	-1.570			-1.510	-1.430	-1.554			-1.502	-1.309	-1.534			-1.501	-1.115	-1.490		
	RV10	-1.510	-1.494	-1.569			-1.509	-1.371	-1.550			-1.506	-1.219	-1.525			-1.502	-0.986	-1.473		
	RV10ss	-1.500	-1.517	-1.570			-1.504	-1.400	-1.555			-1.500	-1.261	-1.536			-1.499	-1.054	-1.496		
	BV5	-1.487	-1.514	-1.568			-1.484	-1.419	-1.551			-1.482	-1.302	-1.527			-1.485	-1.112	-1.474		
	BV5ss	-1.501	-1.526	-1.568			-1.496	-1.442	-1.551			-1.488	-1.336	-1.529			-1.490	-1.168	-1.479		
	MEDRV	-1.489	-1.524	-1.550			-1.475	-1.459	-1.532			-1.470	-1.365	-1.509			-1.469	-1.088	-1.462		
	RS5	-1.261	-1.555	-1.573			-1.285	-1.509	-1.545			-1.301	-1.444	-1.504			-1.336	-1.319	-1.402		
	RS5ss	-1.297	-1.557	-1.572			-1.296	-1.513	-1.546			-1.317	-1.452	-1.510			-1.352	-1.341	-1.423		
	DR	-1.547	-1.360	-1.583			-1.537	-1.483	-1.574			-1.530	-1.489	-1.565			-1.527	-1.488	-1.550		
	-				-1.673	-1.574				-1.655	-1.572				-1.653	-1.567				-1.641	-1.558
Eurostoxx50	RV5	-1.488	-1.374	-1.550			-1.478	-1.256	-1.544			-1.503	-1.118	-1.532			-1.508	-0.877	-1.501		
	RK	-1.479	-1.481	-1.559			-1.468	-1.391	-1.555			-1.496	-1.278	-1.546			-1.503	-1.068	-1.519		
	RV5ss	-1.532	-1.484	-1.563			-1.519	-1.396	-1.559			-1.511	-1.288	-1.550			-1.506	-1.095	-1.522		
	RV10	-1.453	-1.474	-1.550			-1.479	-1.380	-1.546			-1.505	-1.257	-1.537			-1.510	-1.018	-1.512		
	RV10ss	-1.408	-1.531	-1.564			-1.440	-1.463	-1.560			-1.481	-1.375	-1.552			-1.493	-1.212	-1.528		
	BV5	-1.304	-0.836	-1.561			-1.427	-1.027	-1.555			-1.472	-0.994	-1.543			-1.484	-0.894	-1.510		
	BV5ss	-1.468	-1.503	-1.561			-1.474	-1.431	-1.558			-1.488	-1.341	-1.549			-1.493	-1.180	-1.521		
	MEDRV	-1.457	-1.541	-1.568			-1.443	-1.484	-1.563			-1.452	-1.408	-1.552			-1.462	-1.273	-1.521		
	RS5	-1.188	-1.530	-1.557			-1.249	-1.488	-1.544			-1.298	-1.431	-1.521			-1.348	-1.303	-1.468		
	RS5ss	-1.244	-1.514	-1.564			-1.237	-1.465	-1.553			-1.280	-1.399	-1.534			-1.332	-1.266	-1.486		
	DR	-1.519	-1.567	-1.585			-1.518	-1.547	-1.585			-1.532	-1.531	-1.583			-1.533	-1.508	-1.573		
	-				-1.687	-1.583				-1.668	-1.584				-1.665	-1.580				-1.656	-1.567

Panel B

		h=1					h=5					h=10					h=21				
		H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR
	RV5	-1.726	-1.676	-1.754			-1.712	-1.602	-1.745			-1.719	-1.496	-1.734			-1.715	-1.250	-1.704		
	RK	-1.715	-1.702	-1.753			-1.709	-1.638	-1.744			-1.715	-1.543	-1.734			-1.714	-1.315	-1.704		
	RV5ss	-1.728	-1.717	-1.755			-1.714	-1.670	-1.748			-1.720	-1.603	-1.738			-1.716	-1.447	-1.711		
	RV10	-1.707	-1.703	-1.753			-1.704	-1.643	-1.745			-1.712	-1.553	-1.734			-1.711	-1.327	-1.703		
	RV10ss	-1.700	-1.722	-1.755			-1.698	-1.666	-1.747			-1.708	-1.579	-1.737			-1.708	-1.367	-1.709		
FTSE100	BV5	-1.708	-1.625	-1.752			-1.698	-1.558	-1.743			-1.708	-1.470	-1.732			-1.710	-1.305	-1.701		
	BV5ss	-1.697	-1.746	-1.753			-1.688	-1.730	-1.745			-1.704	-1.703	-1.735			-1.709	-1.622	-1.707		
	MEDRV	-1.687	-1.751	-1.748			-1.682	-1.732	-1.740			-1.699	-1.700	-1.730			-1.703	-1.562	-1.698		
	RS5	-1.560	-1.593	-1.747			-1.575	-1.543	-1.736			-1.621	-1.485	-1.723			-1.655	-1.378	-1.683		
	RS5ss	-1.553	-1.732	-1.748			-1.583	-1.702	-1.739			-1.625	-1.659	-1.727			-1.657	-1.542	-1.691		
	DR	-1.695	-1.695	-1.756			-1.691	-1.664	-1.752			-1.693	-1.623	-1.747			-1.688	-1.535	-1.734		
	-				-1.778	-1.759				-1.789	-1.753				-1.791	-1.744				-1.784	-1.727
	RV5	-1.568	-1.590	-1.620			-1.569	-1.523	-1.608			-1.572	-1.421	-1.590			-1.566	-1.186	-1.551		
	RK	-1.584	-1.607	-1.625			-1.583	-1.549	-1.613			-1.581	-1.452	-1.597			-1.572	-1.224	-1.561		
	RV5ss	-1.589	-1.599	-1.624			-1.579	-1.543	-1.612			-1.577	-1.456	-1.596			-1.569	-1.261	-1.559		
	RV10	-1.572	-1.604	-1.618			-1.570	-1.534	-1.608			-1.570	-1.406	-1.592			-1.563	-1.064	-1.560		
	RV10ss	-1.561	-1.599	-1.624			-1.563	-1.545	-1.613			-1.566	-1.462	-1.598			-1.562	-1.272	-1.563		
DAX35	BV5	-1.542	-1.557	-1.623			-1.553	-1.489	-1.611			-1.556	-1.396	-1.592			-1.553	-1.214	-1.551		
	BV5ss	-1.565	-1.561	-1.624			-1.560	-1.504	-1.611			-1.561	-1.423	-1.594			-1.555	-1.258	-1.554		
	MEDRV	-1.552	-1.564	-1.625			-1.550	-1.498	-1.611			-1.554	-1.407	-1.593			-1.551	-1.221	-1.549		
	RS5	-1.332	-1.564	-1.617			-1.356	-1.525	-1.598			-1.391	-1.475	-1.570			-1.435	-1.368	-1.500		
	RS5ss	-1.318	-1.615	-1.619			-1.370	-1.585	-1.602			-1.413	-1.541	-1.578			-1.455	-1.437	-1.516		
	DR	-1.600	-1.393	-1.637			-1.588	-1.492	-1.630			-1.585	-1.500	-1.623			-1.576	-1.492	-1.608		
	-				-1.655	-1.634				-1.681	-1.629				-1.684	-1.622				-1.679	-1.606

Panel B continued

		h=1					h=5					h=10					h=21				
		H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR
	RV5	-1.637	-1.609	-1.657			-1.614	-1.530	-1.642			-1.609	-1.427	-1.628			-1.602	-1.219	-1.599		
	RK	-1.638	-1.615	-1.657			-1.616	-1.534	-1.642			-1.601	-1.428	-1.630			-1.602	-1.217	-1.603		
	RV5ss	-1.635	-1.624	-1.657			-1.606	-1.551	-1.642			-1.597	-1.454	-1.629			-1.599	-1.255	-1.600		
	RV10	-1.638	-1.618	-1.651			-1.612	-1.542	-1.638			-1.601	-1.437	-1.627			-1.602	-1.201	-1.605		
	RV10ss	-1.631	-1.631	-1.655			-1.607	-1.564	-1.641			-1.599	-1.475	-1.628			-1.598	-1.293	-1.602		
CAC40	BV5	-1.636	-1.614	-1.656			-1.618	-1.535	-1.641			-1.603	-1.432	-1.627			-1.597	-1.228	-1.596		
	BV5ss	-1.630	-1.626	-1.657			-1.600	-1.557	-1.641			-1.594	-1.466	-1.628			-1.596	-1.276	-1.598		
	MEDRV	-1.595	-1.644	-1.655			-1.569	-1.588	-1.640			-1.574	-1.511	-1.625			-1.576	-1.325	-1.591		
	RS5	-1.477	-1.649	-1.655			-1.457	-1.612	-1.636			-1.476	-1.567	-1.618			-1.506	-1.469	-1.575		
	RS5ss	-1.484	-1.653	-1.655			-1.467	-1.619	-1.637			-1.477	-1.575	-1.619			-1.505	-1.477	-1.577		
	DR	-1.629	-1.603	-1.642			-1.604	-1.601	-1.635			-1.593	-1.597	-1.631			-1.591	-1.586	-1.620		
	-				-1.684	-1.674				-1.699	-1.665				-1.701	-1.657				-1.693	-1.640
	RV5	-1.687	-1.695	-1.707			-1.670	-1.635	-1.687			-1.669	-1.541	-1.665			-1.668	-1.272	-1.615		
	RK	-1.689	-1.706	-1.708			-1.671	-1.653	-1.689			-1.662	-1.568	-1.669			-1.663	-1.258	-1.622		
	RV5ss	-1.685	-1.706	-1.709			-1.669	-1.654	-1.690			-1.666	-1.569	-1.670			-1.665	-1.185	-1.623		
	RV10	-1.679	-1.701	-1.704			-1.661	-1.641	-1.685			-1.657	-1.542	-1.665			-1.661	-1.236	-1.618		
	RV10ss	-1.677	-1.706	-1.708			-1.659	-1.651	-1.688			-1.651	-1.560	-1.667			-1.658	-1.095	-1.616		
AEX	BV5	-1.688	-1.693	-1.706			-1.672	-1.632	-1.686			-1.663	-1.537	-1.665			-1.663	-1.251	-1.614		
	BV5ss	-1.678	-1.705	-1.708			-1.657	-1.655	-1.688			-1.654	-1.576	-1.667			-1.658	-1.225	-1.617		
	MEDRV	-1.661	-1.705	-1.709			-1.654	-1.661	-1.690			-1.656	-1.590	-1.670			-1.653	-1.215	-1.622		
	RS5	-1.529	-1.706	-1.703			-1.534	-1.671	-1.680			-1.558	-1.620	-1.652			-1.592	-1.480	-1.584		
	RS5ss	-1.488	-1.714	-1.706			-1.499	-1.685	-1.684			-1.539	-1.641	-1.658			-1.581	-1.517	-1.592		
	DR	-1.673	-1.437	-1.708			-1.650	-1.539	-1.693			-1.646	-1.564	-1.677			-1.647	-1.571	-1.640		
	-				-1.742	-1.725				-1.754	-1.716				-1.755	-1.708				-1.746	-1.690
	RV5	-1.697	-1.652	-1.726			-1.699	-1.578	-1.718			-1.698	-1.469	-1.706			-1.694	-1.216	-1.680		
	RK	-1.688	-1.689	-1.726			-1.695	-1.637	-1.718			-1.694	-1.553	-1.707			-1.689	-1.181	-1.680		
	RV5ss	-1.696	-1.681	-1.725			-1.698	-1.622	-1.716			-1.697	-1.529	-1.703			-1.692	-1.312	-1.674		
	RV10	-1.681	-1.670	-1.727			-1.690	-1.601	-1.717			-1.689	-1.498	-1.702			-1.688	-1.273	-1.670		
	RV10ss	-1.683	-1.673	-1.724			-1.686	-1.609	-1.715			-1.687	-1.513	-1.702			-1.685	-1.311	-1.673		
SSMI	BV5	-1.684	-1.658	-1.725			-1.691	-1.593	-1.716			-1.692	-1.496	-1.703			-1.690	-1.266	-1.672		
	BV5ss	-1.680	-1.677	-1.724			-1.686	-1.618	-1.715			-1.686	-1.530	-1.702			-1.685	-1.331	-1.673		
	MEDRV	-1.707	-1.667	-1.724			-1.706	-1.601	-1.714			-1.704	-1.497	-1.699			-1.697	-1.160	-1.666		
	RS5	-1.531	-1.710	-1.726			-1.570	-1.681	-1.714			-1.597	-1.640	-1.696			-1.629	-1.544	-1.655		
	RS5ss	-1.534	-1.714	-1.724			-1.569	-1.691	-1.711			-1.600	-1.657	-1.692			-1.632	-1.581	-1.646		
	DR	-1.680	-1.713	-1.728			-1.676	-1.702	-1.724			-1.674	-1.693	-1.720			-1.667	-1.681	-1.711		
	-				-1.744	-1.731				-1.766	-1.727				-1.769	-1.720				-1.763	-1.706

Panel B continued

		h=1					h=5					h=10					h=21				
		H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR	H	RG	REG	G	HAR
	RV5	-1.565	-1.575	-1.612			-1.554	-1.492	-1.598			-1.551	-1.388	-1.581			-1.545	-1.209	-1.537		
	RK	-1.559	-1.580	-1.612			-1.557	-1.502	-1.599			-1.553	-1.407	-1.583			-1.547	-1.249	-1.543		
	RV5ss	-1.574	-1.579	-1.611			-1.558	-1.499	-1.598			-1.553	-1.400	-1.583			-1.547	-1.235	-1.546		
	RV10	-1.566	-1.553	-1.611			-1.558	-1.449	-1.596			-1.555	-1.326	-1.578			-1.547	-1.123	-1.533		
	RV10ss	-1.561	-1.573	-1.612			-1.552	-1.473	-1.599			-1.549	-1.358	-1.585			-1.544	-1.180	-1.550		
IBEX35	BV5	-1.551	-1.573	-1.611			-1.541	-1.491	-1.597			-1.539	-1.395	-1.579			-1.535	-1.234	-1.534		
	BV5ss	-1.559	-1.582	-1.610			-1.544	-1.510	-1.596			-1.541	-1.424	-1.580			-1.538	-1.281	-1.537		
	MEDRV	-1.542	-1.580	-1.597			-1.532	-1.530	-1.583			-1.528	-1.457	-1.566			-1.523	-1.177	-1.525		
	RS5	-1.347	-1.602	-1.612			-1.369	-1.565	-1.592			-1.388	-1.514	-1.561			-1.414	-1.409	-1.477		
	RS5ss	-1.391	-1.603	-1.612			-1.383	-1.569	-1.593			-1.401	-1.521	-1.565			-1.427	-1.428	-1.493		
	DR	-1.569	-1.411	-1.619			-1.560	-1.528	-1.611			-1.556	-1.532	-1.604			-1.552	-1.526	-1.589		
	-				-1.631	-1.614				-1.652	-1.608				-1.655	-1.603				-1.648	-1.590
	RV5	-1.524	-1.408	-1.568			-1.503	-1.299	-1.560			-1.521	-1.172	-1.550			-1.526	-0.954	-1.524		
	RK	-1.507	-1.504	-1.576			-1.498	-1.421	-1.570			-1.514	-1.318	-1.562			-1.522	-1.129	-1.540		
	RV5ss	-1.553	-1.508	-1.580			-1.532	-1.427	-1.574			-1.528	-1.327	-1.566			-1.524	-1.154	-1.543		
	RV10	-1.485	-1.498	-1.568			-1.503	-1.411	-1.562			-1.522	-1.298	-1.554			-1.527	-1.082	-1.534		
	RV10ss	-1.423	-1.550	-1.581			-1.468	-1.488	-1.575			-1.500	-1.409	-1.569			-1.512	-1.262	-1.548		
Eurostoxx50	BV5	-1.434	-0.957	-1.580			-1.464	-1.092	-1.571			-1.494	-1.059	-1.561			-1.506	-0.966	-1.532		
	BV5ss	-1.519	-1.525	-1.579			-1.497	-1.459	-1.573			-1.508	-1.377	-1.565			-1.513	-1.232	-1.541		
	MEDRV	-1.466	-1.560	-1.584			-1.465	-1.509	-1.578			-1.474	-1.440	-1.568			-1.484	-1.320	-1.541		
	RS5	-1.225	-1.549	-1.575			-1.293	-1.510	-1.559			-1.331	-1.456	-1.539			-1.381	-1.340	-1.493		
	RS5ss	-1.134	-1.536	-1.580			-1.268	-1.490	-1.568			-1.310	-1.429	-1.551			-1.365	-1.309	-1.508		
	DR	-1.541	-1.587	-1.598			-1.531	-1.569	-1.598			-1.541	-1.555	-1.596			-1.542	-1.534	-1.587		
	-				-1.634	-1.594				-1.659	-1.596				-1.663	-1.591				-1.658	-1.580

Table 12

QLIKE results for combination models (median)

The table reports the QLIKE results for the combinations models, based on the median, for 1-day, 5-day, 10-day and 21-day ahead forecasts. The results are reported for the combination of all standard HEAVY (H), all HEAVY models with the squared daily return (H+r_t^2), all non-linear HEAVY models (H+$\mathbb{I}_{\{r_t<0\}}$ and all HEAVY models (All H). The same applies for the RealGARCH(1,1) (RG) and RealEGARCH(1,1) models. Finally, the results for the GARCH(1,1) (G) and HAR(3)-RV (HAR) models are also presented. Panels A and B use the squared daily return as a proxy, Panels C and D the realized variance and Panels E and F the realized kernel as a proxy. Panels A, C and E report results for the 1-day and 21-day ahead forecasts. Panels B, D and F report results for the 5-day and 10-day ahead forecasts.

Panel A

	h=1							h=21						
	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50
H	-1.739	-1.619	-1.656	-1.705	-1.704	-1.557	-1.558	-1.718	-1.594	-1.614	-1.665	-1.691	-1.527	-1.532
H+r_t^2	-1.708	-1.570	-1.625	-1.677	-1.674	-1.511	-1.503	-1.708	-1.570	-1.594	-1.648	-1.677	-1.497	-1.500
H+$\mathbb{I}_{\{r_t<0\}}$	-1.702	-1.557	-1.612	-1.668	-1.667	-1.498	-1.466	-1.707	-1.567	-1.588	-1.645	-1.676	-1.491	-1.491
All H	-1.719	-1.587	-1.636	-1.684	-1.683	-1.526	-1.519	-1.712	-1.577	-1.599	-1.653	-1.682	-1.505	-1.508
RG	-1.750	-1.631	-1.662	-1.711	-1.707	-1.583	-1.579	-1.022	-1.244	-1.262	-1.052	-1.297	-1.445	-1.021
RG+r_t^2	-1.730	-1.621	-1.650	-1.698	-1.703	-1.542	-1.565	-0.705	-1.243	-1.404	-1.445	-1.030	-0.602	-0.963
RG+$\mathbb{I}_{\{r_t<0\}}$	-1.703	-1.600	-1.617	-1.687	-1.655	-1.528	-1.498	-1.403	-1.235	-1.270	-1.094	-1.095	-0.973	-1.170
All RG	-1.737	-1.625	-1.651	-1.703	-1.704	-1.553	-1.570	-0.722	-1.041	-1.153	-1.044	-0.693	-0.608	-0.749
REG	-1.744	-1.626	-1.648	-1.687	-1.703	-1.569	-1.567	-1.695	-1.563	-1.580	-1.598	-1.667	-1.492	-1.502
REG+$\mathbb{I}_{\{r_t<0\}}$	-1.744	-1.626	-1.644	-1.688	-1.705	-1.570	-1.562	-1.691	-1.561	-1.596	-1.596	-1.651	-1.480	-1.518
All REG	-1.744	-1.626	-1.646	-1.688	-1.704	-1.570	-1.565	-1.693	-1.562	-1.586	-1.597	-1.660	-1.486	-1.508
All standard	-1.747	-1.630	-1.659	-1.707	-1.706	-1.571	-1.575	-1.032	-1.252	-1.277	-1.065	-1.328	-1.445	-1.036
All r_t^2	-1.729	-1.618	-1.650	-1.696	-1.700	-1.541	-1.559	-0.732	-1.341	-1.520	-1.524	-1.084	-0.799	-1.021
All+$\mathbb{I}_{\{r_t<0\}}$	-1.734	-1.614	-1.636	-1.687	-1.690	-1.547	-1.544	-1.653	-1.483	-1.570	-1.258	-1.411	-1.279	-1.481
All Models	-1.741	-1.624	-1.652	-1.699	-1.702	-1.561	-1.565	-0.740	-1.096	-1.257	-1.053	-0.753	-0.738	-0.819
G	-1.809	-1.720	-1.734	-1.786	-1.789	-1.673	-1.687	-1.782	-1.689	-1.698	-1.749	-1.762	-1.641	-1.656
HAR	-1.755	-1.636	-1.667	-1.711	-1.714	-1.574	-1.583	-1.723	-1.610	-1.638	-1.680	-1.697	-1.558	-1.567

Panel B

	h=5							h=10						
	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50
H	-1.727	-1.606	-1.623	-1.668	-1.698	-1.540	-1.533	-1.728	-1.602	-1.614	-1.658	-1.695	-1.532	-1.533
H+r_t^2	-1.703	-1.574	-1.594	-1.635	-1.675	-1.504	-1.489	-1.713	-1.575	-1.589	-1.633	-1.675	-1.498	-1.496
H+$\mathbb{I}_{\{r_t<0\}}$	-1.698	-1.566	-1.583	-1.627	-1.669	-1.496	-1.456	-1.710	-1.570	-1.579	-1.627	-1.672	-1.491	-1.482
All H	-1.709	-1.583	-1.600	-1.643	-1.681	-1.513	-1.498	-1.717	-1.582	-1.594	-1.640	-1.681	-1.507	-1.505
RG	-1.738	-1.625	-1.639	-1.676	-1.695	-1.568	-1.563	-1.656	-1.605	-1.569	-1.610	-1.672	-1.546	-1.538
RG+r_t^2	-1.686	-1.594	-1.603	-1.449	-1.678	-0.918	-1.521	-1.554	-1.546	-1.544	-1.452	-1.644	-0.810	-1.452
RG+$\mathbb{I}_{\{r_t<0\}}$	-1.657	-1.485	-1.548	-1.599	-1.593	-1.375	-1.425	-1.588	-1.406	-1.460	-1.515	-1.501	-1.260	-1.335
All RG	-1.709	-1.547	-1.610	-1.656	-1.682	-0.928	-1.533	-1.572	-1.513	-1.515	-1.594	-1.648	-0.856	-1.470
REG	-1.737	-1.616	-1.630	-1.663	-1.696	-1.554	-1.558	-1.726	-1.600	-1.616	-1.645	-1.685	-1.534	-1.543
REG+$\mathbb{I}_{\{r_t<0\}}$	-1.737	-1.615	-1.630	-1.663	-1.695	-1.553	-1.557	-1.725	-1.599	-1.620	-1.644	-1.680	-1.530	-1.547
All REG	-1.737	-1.616	-1.630	-1.663	-1.695	-1.553	-1.558	-1.726	-1.599	-1.618	-1.644	-1.683	-1.532	-1.545
All standard	-1.737	-1.621	-1.638	-1.676	-1.698	-1.556	-1.563	-1.660	-1.605	-1.575	-1.618	-1.686	-1.537	-1.545
All r_t^2	-1.702	-1.597	-1.610	-1.546	-1.683	-0.940	-1.524	-1.607	-1.566	-1.574	-1.534	-1.665	-0.879	-1.475
All+$\mathbb{I}_{\{r_t<0\}}$	-1.723	-1.535	-1.610	-1.621	-1.680	-1.460	-1.526	-1.714	-1.517	-1.593	-1.601	-1.668	-1.434	-1.508
All Models	-1.729	-1.549	-1.626	-1.647	-1.692	-0.960	-1.547	-1.623	-1.532	-1.562	-1.592	-1.678	-0.927	-1.518
G	-1.795	-1.702	-1.714	-1.765	-1.774	-1.655	-1.668	-1.793	-1.698	-1.710	-1.761	-1.772	-1.653	-1.665
HAR	-1.749	-1.633	-1.657	-1.699	-1.713	-1.572	-1.584	-1.741	-1.626	-1.651	-1.694	-1.708	-1.567	-1.580

Panel C

	h=1							h=21						
	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50
H	-1.738	-1.625	-1.666	-1.720	-1.714	-1.598	-1.560	-1.717	-1.594	-1.619	-1.683	-1.697	-1.567	-1.529
H+r_t^2	-1.706	-1.584	-1.640	-1.690	-1.682	-1.565	-1.507	-1.707	-1.570	-1.599	-1.668	-1.685	-1.543	-1.496
H+$\mathbb{I}_{\{r_t<0\}}$	-1.697	-1.572	-1.630	-1.684	-1.676	-1.557	-1.473	-1.706	-1.567	-1.594	-1.666	-1.684	-1.538	-1.488
All H	-1.717	-1.599	-1.649	-1.700	-1.694	-1.575	-1.522	-1.711	-1.577	-1.604	-1.673	-1.689	-1.549	-1.504
RG	-1.746	-1.635	-1.668	-1.726	-1.716	-1.620	-1.579	-1.014	-1.246	-1.261	-1.076	-1.300	-1.491	-1.031
RG+r_t^2	-1.728	-1.626	-1.657	-1.716	-1.711	-1.595	-1.567	-0.701	-1.249	-1.403	-1.451	-1.025	-0.708	-0.970
RG+$\mathbb{I}_{\{r_t<0\}}$	-1.703	-1.605	-1.626	-1.708	-1.668	-1.580	-1.500	-1.399	-1.232	-1.271	-1.124	-1.108	-1.066	-1.179
All RG	-1.735	-1.630	-1.659	-1.720	-1.713	-1.602	-1.571	-0.719	-1.044	-1.154	-1.057	-0.684	-0.668	-0.763
REG	-1.741	-1.630	-1.658	-1.709	-1.715	-1.609	-1.567	-1.697	-1.565	-1.581	-1.623	-1.678	-1.545	-1.500
REG+$\mathbb{I}_{\{r_t<0\}}$	-1.741	-1.630	-1.655	-1.710	-1.717	-1.610	-1.561	-1.692	-1.562	-1.596	-1.621	-1.663	-1.537	-1.516
All REG	-1.741	-1.630	-1.657	-1.710	-1.716	-1.609	-1.564	-1.695	-1.563	-1.587	-1.622	-1.672	-1.541	-1.507
All standard	-1.744	-1.634	-1.667	-1.723	-1.716	-1.611	-1.574	-1.025	-1.253	-1.279	-1.088	-1.332	-1.489	-1.045
All r_t^2	-1.727	-1.624	-1.657	-1.714	-1.709	-1.592	-1.561	-0.729	-1.345	-1.521	-1.529	-1.084	-0.859	-1.027
All+$\mathbb{I}_{\{r_t<0\}}$	-1.732	-1.618	-1.648	-1.708	-1.701	-1.594	-1.543	-1.652	-1.484	-1.572	-1.273	-1.421	-1.324	-1.479
All models	-1.739	-1.628	-1.660	-1.716	-1.713	-1.604	-1.565	-0.738	-1.097	-1.260	-1.066	-0.749	-0.769	-0.827
G	-1.771	-1.659	-1.682	-1.743	-1.738	-1.630	-1.622	-1.778	-1.682	-1.692	-1.748	-1.759	-1.646	-1.649
HAR	-1.751	-1.639	-1.673	-1.727	-1.723	-1.613	-1.579	-1.721	-1.610	-1.638	-1.693	-1.701	-1.589	-1.565

Panel D

	h=5							h=10						
	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50
H	-1.722	-1.609	-1.639	-1.697	-1.709	-1.581	-1.532	-1.722	-1.604	-1.626	-1.687	-1.705	-1.575	-1.530
H+r_t^2	-1.695	-1.578	-1.613	-1.671	-1.686	-1.552	-1.489	-1.705	-1.577	-1.603	-1.666	-1.687	-1.549	-1.493
H+$\mathbb{I}_{\{r_t<0\}}$	-1.690	-1.572	-1.604	-1.666	-1.683	-1.545	-1.458	-1.702	-1.572	-1.595	-1.662	-1.684	-1.543	-1.478
All H	-1.702	-1.587	-1.618	-1.678	-1.693	-1.560	-1.498	-1.710	-1.585	-1.608	-1.672	-1.692	-1.555	-1.502
RG	-1.733	-1.626	-1.645	-1.701	-1.705	-1.606	-1.562	-1.654	-1.606	-1.570	-1.640	-1.683	-1.591	-1.534
RG+r_t^2	-1.682	-1.597	-1.610	-1.456	-1.688	-0.980	-1.521	-1.551	-1.547	-1.548	-1.458	-1.653	-0.892	-1.452
RG+$\mathbb{I}_{\{r_t<0\}}$	-1.656	-1.489	-1.557	-1.631	-1.609	-1.439	-1.427	-1.587	-1.406	-1.467	-1.551	-1.520	-1.346	-1.338
All RG	-1.705	-1.550	-1.618	-1.679	-1.693	-0.980	-1.533	-1.572	-1.516	-1.518	-1.620	-1.658	-0.920	-1.469
REG	-1.733	-1.618	-1.640	-1.690	-1.709	-1.596	-1.555	-1.724	-1.602	-1.622	-1.670	-1.699	-1.581	-1.540
REG+$\mathbb{I}_{\{r_t<0\}}$	-1.733	-1.618	-1.639	-1.691	-1.708	-1.596	-1.554	-1.723	-1.600	-1.626	-1.670	-1.694	-1.579	-1.543
All REG	-1.733	-1.618	-1.640	-1.690	-1.708	-1.596	-1.554	-1.723	-1.601	-1.624	-1.670	-1.697	-1.580	-1.541
All standard	-1.732	-1.623	-1.647	-1.701	-1.709	-1.597	-1.560	-1.659	-1.606	-1.580	-1.646	-1.698	-1.583	-1.541
All r_t^2	-1.697	-1.600	-1.620	-1.555	-1.694	-0.994	-1.524	-1.602	-1.567	-1.581	-1.542	-1.676	-0.943	-1.474
All+$\mathbb{I}_{\{r_t<0\}}$	-1.718	-1.538	-1.622	-1.653	-1.693	-1.508	-1.522	-1.710	-1.520	-1.602	-1.630	-1.683	-1.488	-1.505
All models	-1.724	-1.553	-1.636	-1.672	-1.703	-1.000	-1.544	-1.621	-1.535	-1.568	-1.620	-1.691	-0.971	-1.516
G	-1.783	-1.684	-1.697	-1.755	-1.762	-1.650	-1.648	-1.785	-1.687	-1.699	-1.756	-1.765	-1.653	-1.654
HAR	-1.746	-1.634	-1.664	-1.718	-1.721	-1.607	-1.581	-1.738	-1.626	-1.656	-1.710	-1.714	-1.602	-1.576

Panel E

	h=1							h=21						
	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50
H	-1.749	-1.619	-1.667	-1.718	-1.722	-1.600	-1.576	-1.724	-1.589	-1.622	-1.681	-1.704	-1.568	-1.548
H+r_t^2	-1.719	-1.577	-1.642	-1.687	-1.694	-1.567	-1.527	-1.715	-1.564	-1.602	-1.666	-1.693	-1.545	-1.519
H+$\mathbb{I}_{\{r_t<0\}}$	-1.712	-1.564	-1.632	-1.681	-1.688	-1.559	-1.495	-1.714	-1.561	-1.597	-1.664	-1.692	-1.540	-1.511
All H	-1.730	-1.592	-1.651	-1.698	-1.704	-1.577	-1.542	-1.719	-1.571	-1.607	-1.670	-1.697	-1.550	-1.526
RG	-1.757	-1.629	-1.670	-1.724	-1.725	-1.621	-1.593	-1.023	-1.240	-1.263	-1.072	-1.308	-1.492	-1.047
RG+r_t^2	-1.741	-1.620	-1.658	-1.714	-1.720	-1.597	-1.583	-0.712	-1.242	-1.408	-1.450	-1.035	-0.711	-1.002
RG+$\mathbb{I}_{\{r_t<0\}}$	-1.715	-1.597	-1.628	-1.706	-1.680	-1.581	-1.521	-1.411	-1.218	-1.278	-1.118	-1.128	-1.066	-1.222
All RG	-1.746	-1.624	-1.660	-1.718	-1.722	-1.603	-1.587	-0.726	-1.038	-1.156	-1.055	-0.691	-0.669	-0.786
REG	-1.753	-1.623	-1.660	-1.707	-1.724	-1.611	-1.584	-1.709	-1.558	-1.584	-1.619	-1.688	-1.547	-1.524
REG+$\mathbb{I}_{\{r_t<0\}}$	-1.753	-1.624	-1.656	-1.708	-1.726	-1.612	-1.579	-1.706	-1.555	-1.599	-1.617	-1.673	-1.538	-1.539
All REG	-1.753	-1.624	-1.658	-1.708	-1.725	-1.611	-1.582	-1.708	-1.557	-1.590	-1.618	-1.682	-1.543	-1.530
All standard	-1.755	-1.628	-1.668	-1.721	-1.725	-1.612	-1.590	-1.034	-1.248	-1.281	-1.085	-1.339	-1.490	-1.061
All r_t^2	-1.739	-1.618	-1.659	-1.712	-1.719	-1.594	-1.578	-0.735	-1.339	-1.525	-1.528	-1.092	-0.861	-1.053
All+$\mathbb{I}_{\{r_t<0\}}$	-1.744	-1.611	-1.649	-1.706	-1.711	-1.596	-1.563	-1.664	-1.477	-1.575	-1.270	-1.430	-1.325	-1.505
All models	-1.751	-1.622	-1.662	-1.714	-1.722	-1.606	-1.582	-0.743	-1.092	-1.262	-1.064	-0.753	-0.770	-0.844
G	-1.778	-1.655	-1.684	-1.742	-1.744	-1.631	-1.634	-1.784	-1.679	-1.693	-1.746	-1.763	-1.648	-1.658
HAR	-1.759	-1.634	-1.674	-1.725	-1.731	-1.614	-1.594	-1.727	-1.606	-1.640	-1.690	-1.706	-1.590	-1.580

Panel F

	h=5							h=10						
	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50
H	-1.733	-1.603	-1.640	-1.696	-1.717	-1.583	-1.550	-1.732	-1.598	-1.628	-1.685	-1.712	-1.576	-1.549
H+r_t^2	-1.707	-1.571	-1.614	-1.669	-1.696	-1.554	-1.511	-1.716	-1.570	-1.605	-1.664	-1.696	-1.551	-1.516
H+$\mathbb{I}_{\{r_t<0\}}$	-1.703	-1.564	-1.605	-1.664	-1.693	-1.548	-1.483	-1.713	-1.565	-1.597	-1.660	-1.694	-1.545	-1.502
All H	-1.714	-1.580	-1.620	-1.676	-1.703	-1.562	-1.519	-1.720	-1.578	-1.610	-1.670	-1.701	-1.557	-1.523
RG	-1.744	-1.621	-1.647	-1.699	-1.714	-1.607	-1.578	-1.665	-1.600	-1.573	-1.638	-1.693	-1.592	-1.553
RG+r_t^2	-1.697	-1.591	-1.612	-1.455	-1.698	-0.982	-1.542	-1.568	-1.540	-1.551	-1.458	-1.665	-0.894	-1.479
RG+$\mathbb{I}_{\{r_t<0\}}$	-1.667	-1.480	-1.560	-1.628	-1.624	-1.441	-1.453	-1.599	-1.396	-1.471	-1.547	-1.538	-1.347	-1.370
All RG	-1.716	-1.544	-1.620	-1.676	-1.702	-0.982	-1.552	-1.583	-1.509	-1.520	-1.618	-1.670	-0.922	-1.495
REG	-1.745	-1.612	-1.642	-1.688	-1.717	-1.598	-1.574	-1.736	-1.595	-1.624	-1.668	-1.708	-1.583	-1.560
REG+$\mathbb{I}_{\{r_t<0\}}$	-1.745	-1.611	-1.641	-1.689	-1.717	-1.598	-1.573	-1.735	-1.594	-1.628	-1.667	-1.703	-1.581	-1.563
All REG	-1.745	-1.612	-1.642	-1.688	-1.717	-1.598	-1.573	-1.736	-1.595	-1.626	-1.667	-1.706	-1.582	-1.562
All standard	-1.744	-1.617	-1.648	-1.699	-1.718	-1.599	-1.577	-1.670	-1.600	-1.582	-1.644	-1.706	-1.584	-1.560
All r_t^2	-1.711	-1.593	-1.621	-1.554	-1.704	-0.996	-1.544	-1.615	-1.560	-1.584	-1.541	-1.686	-0.945	-1.499
All+$\mathbb{I}_{\{r_t<0\}}$	-1.730	-1.531	-1.624	-1.650	-1.703	-1.509	-1.544	-1.722	-1.513	-1.605	-1.627	-1.693	-1.490	-1.528
All models	-1.736	-1.547	-1.638	-1.669	-1.712	-1.002	-1.563	-1.632	-1.528	-1.570	-1.618	-1.700	-0.973	-1.537
G	-1.789	-1.681	-1.699	-1.754	-1.766	-1.652	-1.659	-1.791	-1.684	-1.701	-1.755	-1.769	-1.655	-1.663
HAR	-1.753	-1.629	-1.665	-1.716	-1.727	-1.608	-1.596	-1.744	-1.622	-1.657	-1.708	-1.720	-1.603	-1.591

Table 13

QLIKE results for combination models (mean)

The table reports the QLIKE results for the combinations models, based on the mean, for 5-day and 10-day ahead forecasts. The results are reported for the combination of all standard HEAVY (H), all HEAVY models with the squared daily return (H+r_t^2), all non-linear HEAVY models (H+$\mathbb{I}_{\{r_t<0\}}$ and all HEAVY models (All H). The same applies for the RealGARCH(1,1) (RG) and RealEGARCH(1,1) models. Finally, the results for the GARCH(1,1) (G) and HAR(3)-RV (HAR) models are also presented. Panel A uses the squared daily return as a proxy, Panel B the realized variance and Panel C the realized kernel as a proxy.

Panel A

	h=5							h=10						
	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50
H	-1.736	-1.622	-1.640	-1.679	-1.704	-1.564	-1.564	-1.732	-1.615	-1.630	-1.670	-1.699	-1.555	-1.562
H+r_t^2	-1.722	-1.600	-1.617	-1.657	-1.691	-1.534	-1.522	-1.723	-1.596	-1.608	-1.650	-1.688	-1.524	-1.527
H+$\mathbb{I}_{\{r_t<0\}}$	-1.716	-1.595	-1.608	-1.651	-1.686	-1.528	-1.511	-1.721	-1.592	-1.601	-1.646	-1.684	-1.519	-1.520
All H	-1.728	-1.609	-1.626	-1.666	-1.696	-1.546	-1.540	-1.728	-1.605	-1.618	-1.659	-1.693	-1.537	-1.542
RG	-1.740	-1.629	-1.646	-1.701	-1.702	-1.577	-1.569	-1.658	-1.611	-1.578	-1.646	-1.685	-1.563	-1.546
RG+r_t^2	-1.696	-1.605	-1.620	-1.452	-1.691	-0.934	-1.533	-1.568	-1.566	-1.576	-1.454	-1.665	-0.848	-1.472
RG+$\mathbb{I}_{\{r_t<0\}}$	-1.681	-1.507	-1.585	-1.622	-1.644	-1.422	-1.472	-1.626	-1.452	-1.528	-1.563	-1.593	-1.349	-1.406
All RG	-1.718	-1.550	-1.626	-1.553	-1.688	-0.960	-1.540	-1.593	-1.521	-1.545	-1.519	-1.662	-0.913	-1.492
REG	-1.738	-1.617	-1.631	-1.664	-1.697	-1.554	-1.560	-1.727	-1.602	-1.617	-1.647	-1.687	-1.535	-1.546
REG+$\mathbb{I}_{\{r_t<0\}}$	-1.738	-1.617	-1.630	-1.665	-1.696	-1.554	-1.559	-1.726	-1.601	-1.621	-1.647	-1.683	-1.531	-1.549
All REG	-1.738	-1.617	-1.631	-1.665	-1.696	-1.554	-1.560	-1.727	-1.602	-1.619	-1.647	-1.685	-1.533	-1.548
All standard	-1.741	-1.629	-1.646	-1.693	-1.705	-1.574	-1.573	-1.668	-1.618	-1.589	-1.642	-1.696	-1.562	-1.563
All r_t^2	-1.721	-1.612	-1.630	-1.531	-1.696	-0.973	-1.542	-1.642	-1.596	-1.610	-1.535	-1.687	-0.936	-1.516
All $\mathbb{I}_{\{r_t<0\}}$	-1.728	-1.547	-1.624	-1.640	-1.687	-1.475	-1.541	-1.718	-1.531	-1.609	-1.620	-1.675	-1.451	-1.526
All Models	-1.733	-1.557	-1.636	-1.618	-1.698	-0.974	-1.556	-1.628	-1.543	-1.576	-1.579	-1.688	-0.944	-1.535
G	-1.795	-1.702	-1.714	-1.765	-1.774	-1.655	-1.668	-1.793	-1.698	-1.710	-1.761	-1.772	-1.653	-1.665
HAR	-1.749	-1.633	-1.657	-1.699	-1.713	-1.572	-1.584	-1.741	-1.626	-1.651	-1.694	-1.708	-1.567	-1.580

Panel B

	h=5							h=10						
	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50
H	-1.731	-1.624	-1.650	-1.703	-1.713	-1.596	-1.564	-1.729	-1.617	-1.639	-1.693	-1.708	-1.589	-1.560
H+r_t^2	-1.717	-1.604	-1.632	-1.688	-1.702	-1.575	-1.522	-1.717	-1.599	-1.620	-1.679	-1.698	-1.569	-1.524
H+$\mathbb{I}_{\{r_t<0\}}$	-1.710	-1.597	-1.625	-1.683	-1.697	-1.571	-1.511	-1.715	-1.593	-1.614	-1.676	-1.695	-1.565	-1.516
All H	-1.723	-1.612	-1.640	-1.695	-1.707	-1.584	-1.539	-1.723	-1.606	-1.629	-1.686	-1.703	-1.578	-1.540
RG	-1.735	-1.631	-1.651	-1.720	-1.712	-1.611	-1.569	-1.655	-1.612	-1.579	-1.667	-1.696	-1.600	-1.544
RG+r_t^2	-1.691	-1.607	-1.626	-1.459	-1.701	-0.990	-1.534	-1.566	-1.567	-1.578	-1.460	-1.675	-0.918	-1.473
RG+$\mathbb{I}_{\{r_t<0\}}$	-1.680	-1.510	-1.593	-1.649	-1.657	-1.481	-1.477	-1.625	-1.453	-1.533	-1.593	-1.606	-1.422	-1.412
All RG	-1.715	-1.553	-1.632	-1.561	-1.699	-1.002	-1.541	-1.592	-1.523	-1.547	-1.528	-1.673	-0.962	-1.493
REG	-1.734	-1.619	-1.641	-1.691	-1.710	-1.596	-1.557	-1.725	-1.604	-1.623	-1.672	-1.701	-1.582	-1.543
REG+$\mathbb{I}_{\{r_t<0\}}$	-1.734	-1.619	-1.639	-1.692	-1.709	-1.596	-1.555	-1.724	-1.603	-1.626	-1.672	-1.697	-1.580	-1.546
All REG	-1.734	-1.619	-1.640	-1.691	-1.709	-1.596	-1.556	-1.724	-1.603	-1.625	-1.672	-1.699	-1.581	-1.544
All standard	-1.738	-1.631	-1.655	-1.715	-1.716	-1.609	-1.572	-1.666	-1.619	-1.595	-1.666	-1.707	-1.600	-1.561
All r_t^2	-1.716	-1.615	-1.639	-1.541	-1.706	-1.016	-1.543	-1.637	-1.598	-1.619	-1.543	-1.697	-0.984	-1.516
All $\mathbb{I}_{\{r_t<0\}}$	-1.724	-1.550	-1.635	-1.666	-1.700	-1.520	-1.540	-1.714	-1.532	-1.617	-1.646	-1.687	-1.502	-1.524
All models	-1.729	-1.560	-1.646	-1.631	-1.709	-1.011	-1.555	-1.626	-1.546	-1.583	-1.594	-1.699	-0.984	-1.534
G	-1.783	-1.684	-1.697	-1.755	-1.762	-1.650	-1.648	-1.785	-1.687	-1.699	-1.756	-1.765	-1.653	-1.654
HAR	-1.746	-1.634	-1.664	-1.718	-1.721	-1.607	-1.581	-1.738	-1.626	-1.656	-1.710	-1.714	-1.602	-1.576

Panel C

	h=5							h=10						
	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50
H	-1.740	-1.619	-1.651	-1.702	-1.720	-1.598	-1.577	-1.736	-1.612	-1.640	-1.692	-1.714	-1.591	-1.574
H+r_t^2	-1.726	-1.598	-1.633	-1.686	-1.710	-1.577	-1.541	-1.726	-1.593	-1.621	-1.678	-1.706	-1.571	-1.543
H+$\mathbb{I}_{\{r_t<0\}}$	-1.721	-1.591	-1.626	-1.681	-1.706	-1.573	-1.531	-1.724	-1.587	-1.616	-1.675	-1.703	-1.567	-1.536
All H	-1.732	-1.607	-1.641	-1.693	-1.715	-1.586	-1.556	-1.731	-1.601	-1.630	-1.684	-1.710	-1.579	-1.557
RG	-1.745	-1.625	-1.653	-1.718	-1.720	-1.612	-1.584	-1.666	-1.607	-1.582	-1.665	-1.704	-1.601	-1.561
RG+r_t^2	-1.705	-1.601	-1.628	-1.458	-1.710	-0.991	-1.554	-1.580	-1.560	-1.581	-1.459	-1.685	-0.919	-1.499
RG+$\mathbb{I}_{\{r_t<0\}}$	-1.689	-1.502	-1.595	-1.646	-1.669	-1.482	-1.500	-1.635	-1.444	-1.536	-1.589	-1.619	-1.423	-1.440
All RG	-1.725	-1.547	-1.634	-1.560	-1.708	-1.004	-1.559	-1.604	-1.516	-1.550	-1.527	-1.683	-0.963	-1.516
REG	-1.746	-1.613	-1.642	-1.689	-1.718	-1.598	-1.575	-1.737	-1.598	-1.626	-1.669	-1.710	-1.584	-1.563
REG+$\mathbb{I}_{\{r_t<0\}}$	-1.746	-1.613	-1.641	-1.690	-1.718	-1.598	-1.574	-1.736	-1.597	-1.629	-1.669	-1.706	-1.582	-1.566
All REG	-1.746	-1.613	-1.642	-1.689	-1.718	-1.598	-1.575	-1.736	-1.597	-1.627	-1.669	-1.708	-1.583	-1.564
All standard	-1.747	-1.626	-1.656	-1.713	-1.723	-1.610	-1.587	-1.676	-1.614	-1.596	-1.664	-1.714	-1.601	-1.577
All r_t^2	-1.727	-1.609	-1.641	-1.540	-1.715	-1.018	-1.561	-1.648	-1.592	-1.620	-1.543	-1.705	-0.985	-1.536
All $\mathbb{I}_{\{r_t<0\}}$	-1.735	-1.543	-1.636	-1.664	-1.709	-1.521	-1.558	-1.725	-1.526	-1.619	-1.644	-1.697	-1.503	-1.545
All models	-1.739	-1.555	-1.647	-1.629	-1.718	-1.012	-1.572	-1.636	-1.540	-1.584	-1.593	-1.708	-0.985	-1.553
G	-1.789	-1.681	-1.699	-1.754	-1.766	-1.652	-1.659	-1.791	-1.684	-1.701	-1.755	-1.769	-1.655	-1.663
HAR	-1.753	-1.629	-1.665	-1.716	-1.727	-1.608	-1.596	-1.744	-1.622	-1.657	-1.708	-1.720	-1.603	-1.591

Table 14

QLIKE results for combination models (MSPE)

The table reports the QLIKE results for the combinations models, based on the in-sample discounted MSPE, for 5-day and 10-day ahead forecasts. The results are reported for the combination of all standard HEAVY (H), all HEAVY models with the squared daily return (H+r_t^2), all non-linear HEAVY models (H+$\mathbb{I}_{\{r_t<0\}}$ and all HEAVY models (All H). The same applies for the RealGARCH(1,1) (RG) and RealEGARCH(1,1) models. Finally, the results for the GARCH(1,1) (G) and HAR(3)-RV (HAR) models are also presented. Panel A uses the squared daily return as a proxy, Panel B the realized variance and Panel C the realized kernel as a proxy.

Panel A

	h=5							h=10						
	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50
H	-1.737	-1.611	-1.633	-1.677	-1.696	-1.543	-1.551	-1.736	-1.609	-1.631	-1.674	-1.693	-1.539	-1.551
H+r_t^2	-1.706	-1.568	-1.599	-1.640	-1.663	-1.501	-1.502	-1.706	-1.566	-1.594	-1.638	-1.658	-1.495	-1.502
H+$\mathbb{I}_{\{r_t<0\}}$	-1.689	-1.550	-1.574	-1.610	-1.648	-1.474	-1.476	-1.691	-1.548	-1.569	-1.602	-1.644	-1.467	-1.478
All H	-1.716	-1.610	-1.643	-1.699	-1.698	-1.577	-1.539	-1.716	-1.606	-1.637	-1.693	-1.697	-1.571	-1.539
RG	-1.749	-1.633	-1.652	-1.710	-1.707	-1.588	-1.580	-1.747	-1.630	-1.649	-1.707	-1.704	-1.585	-1.579
RG+r_t^2	-1.728	-1.623	-1.639	-1.684	-1.703	-1.532	-1.564	-1.726	-1.620	-1.635	-1.682	-1.701	-1.528	-1.563
RG+$\mathbb{I}_{\{r_t<0\}}$	-1.706	-1.599	-1.613	-1.675	-1.663	-1.529	-1.508	-1.702	-1.597	-1.609	-1.673	-1.662	-1.524	-1.507
All RG	-1.735	-1.629	-1.651	-1.716	-1.717	-1.610	-1.569	-1.735	-1.624	-1.646	-1.712	-1.716	-1.607	-1.566
REG	-1.746	-1.627	-1.640	-1.677	-1.703	-1.567	-1.572	-1.745	-1.623	-1.639	-1.678	-1.701	-1.564	-1.572
REG+$\mathbb{I}_{\{r_t<0\}}$	-1.746	-1.626	-1.635	-1.679	-1.705	-1.569	-1.567	-1.745	-1.623	-1.633	-1.679	-1.703	-1.567	-1.568
All REG	-1.742	-1.628	-1.648	-1.703	-1.717	-1.607	-1.565	-1.743	-1.624	-1.643	-1.699	-1.716	-1.604	-1.565
All standard	-1.748	-1.639	-1.664	-1.723	-1.722	-1.616	-1.584	-1.748	-1.635	-1.660	-1.719	-1.721	-1.614	-1.583
All r_t^2	-1.677	-1.594	-1.620	-1.688	-1.517	-1.495	-1.508	-1.676	-1.589	-1.613	-1.681	-1.515	-1.488	-1.504
All $\mathbb{I}_{\{r_t<0\}}$	-1.737	-1.626	-1.649	-1.709	-1.712	-1.602	-1.558	-1.737	-1.622	-1.643	-1.704	-1.711	-1.600	-1.557
All Models	-1.732	-1.621	-1.646	-1.706	-1.709	-1.596	-1.553	-1.733	-1.616	-1.641	-1.702	-1.707	-1.593	-1.551
G	-1.795	-1.702	-1.714	-1.765	-1.774	-1.655	-1.668	-1.793	-1.698	-1.710	-1.761	-1.772	-1.653	-1.665
HAR	-1.749	-1.633	-1.657	-1.699	-1.713	-1.572	-1.584	-1.741	-1.626	-1.651	-1.694	-1.708	-1.567	-1.580

Panel B

	h=5							h=10						
	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50
H	-1.737	-1.624	-1.654	-1.711	-1.714	-1.595	-1.560	-1.738	-1.620	-1.649	-1.706	-1.713	-1.592	-1.559
H+r_t^2	-1.714	-1.598	-1.635	-1.690	-1.697	-1.572	-1.527	-1.716	-1.594	-1.627	-1.683	-1.695	-1.568	-1.526
H+$\mathbb{I}_{\{r_t<0\}}$	-1.705	-1.587	-1.623	-1.680	-1.689	-1.562	-1.510	-1.708	-1.584	-1.615	-1.671	-1.687	-1.558	-1.510
All H	-1.717	-1.607	-1.641	-1.698	-1.695	-1.575	-1.535	-1.719	-1.603	-1.634	-1.691	-1.695	-1.572	-1.534
RG	-1.746	-1.635	-1.659	-1.723	-1.717	-1.619	-1.579	-1.745	-1.631	-1.655	-1.719	-1.716	-1.617	-1.577
RG+r_t^2	-1.725	-1.625	-1.648	-1.709	-1.715	-1.586	-1.565	-1.725	-1.620	-1.643	-1.704	-1.714	-1.583	-1.562
RG+$\mathbb{I}_{\{r_t<0\}}$	-1.707	-1.602	-1.621	-1.699	-1.674	-1.581	-1.508	-1.705	-1.598	-1.615	-1.695	-1.673	-1.578	-1.505
All RG	-1.735	-1.631	-1.652	-1.717	-1.716	-1.607	-1.569	-1.735	-1.627	-1.647	-1.713	-1.715	-1.604	-1.567
REG	-1.742	-1.628	-1.650	-1.702	-1.716	-1.606	-1.568	-1.743	-1.624	-1.646	-1.699	-1.715	-1.604	-1.568
REG+$\mathbb{I}_{\{r_t<0\}}$	-1.743	-1.628	-1.645	-1.703	-1.718	-1.607	-1.563	-1.743	-1.624	-1.641	-1.700	-1.717	-1.605	-1.562
All REG	-1.739	-1.626	-1.647	-1.701	-1.716	-1.603	-1.562	-1.740	-1.622	-1.643	-1.698	-1.715	-1.601	-1.562
All standard	-1.742	-1.631	-1.658	-1.718	-1.713	-1.610	-1.573	-1.742	-1.627	-1.653	-1.714	-1.712	-1.607	-1.571
All r_t^2	-1.731	-1.612	-1.635	-1.697	-1.697	-1.587	-1.536	-1.731	-1.608	-1.629	-1.693	-1.695	-1.583	-1.534
All $\mathbb{I}_{\{r_t<0\}}$	-1.720	-1.611	-1.635	-1.696	-1.687	-1.575	-1.536	-1.720	-1.606	-1.628	-1.691	-1.685	-1.570	-1.533
All Models	-1.735	-1.624	-1.650	-1.709	-1.711	-1.600	-1.561	-1.735	-1.620	-1.644	-1.704	-1.710	-1.597	-1.559
G	-1.783	-1.684	-1.697	-1.755	-1.762	-1.650	-1.648	-1.785	-1.687	-1.699	-1.756	-1.765	-1.653	-1.654
HAR	-1.746	-1.634	-1.664	-1.718	-1.721	-1.607	-1.581	-1.738	-1.626	-1.656	-1.710	-1.714	-1.602	-1.576

Panel C

	h=5							h=10						
	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50	FTSE100	DAX30	CAC40	AEX	SSMI	IBEX35	EUROSTOXX50
H	-1.748	-1.618	-1.655	-1.709	-1.722	-1.597	-1.576	-1.749	-1.614	-1.651	-1.704	-1.721	-1.594	-1.576
H+r_t^2	-1.726	-1.592	-1.636	-1.688	-1.706	-1.574	-1.545	-1.728	-1.588	-1.629	-1.681	-1.704	-1.570	-1.545
H+$\mathbb{I}_{\{r_t<0\}}$	-1.718	-1.580	-1.624	-1.678	-1.699	-1.564	-1.528	-1.721	-1.577	-1.616	-1.670	-1.696	-1.560	-1.530
All H	-1.714	-1.604	-1.609	-1.696	-1.701	-1.574	-1.476	-1.717	-1.599	-1.599	-1.685	-1.699	-1.567	-1.470
RG	-1.756	-1.630	-1.660	-1.721	-1.725	-1.620	-1.593	-1.756	-1.626	-1.656	-1.717	-1.723	-1.618	-1.592
RG+r_t^2	-1.737	-1.619	-1.649	-1.707	-1.723	-1.588	-1.582	-1.736	-1.615	-1.644	-1.702	-1.722	-1.585	-1.580
RG+$\mathbb{I}_{\{r_t<0\}}$	-1.717	-1.595	-1.623	-1.697	-1.685	-1.583	-1.528	-1.714	-1.591	-1.617	-1.693	-1.683	-1.580	-1.526
All RG	-1.736	-1.630	-1.651	-1.718	-1.716	-1.606	-1.569	-1.736	-1.626	-1.647	-1.714	-1.715	-1.602	-1.567
REG	-1.754	-1.623	-1.652	-1.700	-1.724	-1.608	-1.585	-1.754	-1.619	-1.648	-1.697	-1.723	-1.606	-1.586
REG+$\mathbb{I}_{\{r_t<0\}}$	-1.754	-1.623	-1.647	-1.702	-1.726	-1.609	-1.581	-1.754	-1.619	-1.643	-1.698	-1.725	-1.607	-1.582
All REG	-1.739	-1.626	-1.647	-1.701	-1.716	-1.603	-1.561	-1.740	-1.622	-1.643	-1.697	-1.715	-1.601	-1.561
All standard	-1.741	-1.634	-1.660	-1.720	-1.719	-1.610	-1.578	-1.741	-1.630	-1.656	-1.716	-1.717	-1.607	-1.576
All r_t^2	-1.726	-1.610	-1.635	-1.698	-1.695	-1.588	-1.534	-1.726	-1.606	-1.629	-1.693	-1.693	-1.585	-1.532
All $\mathbb{I}_{\{r_t<0\}}$	-1.718	-1.611	-1.635	-1.697	-1.683	-1.576	-1.541	-1.718	-1.607	-1.629	-1.692	-1.681	-1.573	-1.538
All Models	-1.735	-1.624	-1.650	-1.709	-1.711	-1.600	-1.560	-1.735	-1.620	-1.644	-1.705	-1.710	-1.597	-1.558
G	-1.789	-1.681	-1.699	-1.754	-1.766	-1.652	-1.659	-1.791	-1.684	-1.701	-1.755	-1.769	-1.655	-1.663
HAR	-1.753	-1.629	-1.665	-1.716	-1.727	-1.608	-1.596	-1.744	-1.622	-1.657	-1.708	-1.720	-1.603	-1.591