

The Relationship between Income and Fertility for the Case of Nannies – A utility maximization approach

ERASMUS UNIVERSITY ROTTERDAM
Erasmus School of Economics
Department of Economics

Supervisor: Dr. Sacha Kapoor

Name: Hrisyana Doytchinova

Exam number: 358282

E-mail address: hrisyanadoytchinova@yahoo.com

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Introduction

In 1798 Malthus wrote one of the most cited papers in economic history where he argues that as men become richer they would use all of the resources causing for everyone not to have enough and thus the population to decrease. In Malthus's mind the relationship between income and fertility was positive, however what we actually observe nowadays is the opposite – a negative relationship. (Malthus, 1798)

Some years later, another scholar - Gary S. Becker, wrote a paper on the determinants of fertility. He first discussed the negative relationship between income and fertility. (Becker, 1960)

Nowadays, we generally observe that the higher an income a household has, the less number of children it will have relatively to a household with less income. Thus, establishing a negative relationship between fertility and income and meaning that as the world develops and becomes richer, the rate of growth of the population will slow down.

Returning back to the core of this paper is the discussion whether the relationship between income and fertility is really a negative one. This suggestion has been frequently proven by theoretical papers. Nevertheless, the empirical researches tend to accuse this theory of being a statistical fluke and argue that the relationship is actually positive. In their argument they provide a reason for this as one of a missing variable.

There are more theories trying to explain what influences people to have the number of children they do. Two particularly important ones unlike the income fertility relationship look at what causes the parents to desire fewer children or more than average.

One such theory is the Quantity Quality theory, first introduced in Becker's paper from 1960. There he argues that parents who do not value human capital

enough and prefer leisure time would have more children as opposed to parents who value more human capital. This theory would be used in accordance to the models later on discussed to show that having the income fertility negative relationship remains. The reason for this is the following: parents who prefer quality over quantity are typically parents who value human capital a lot and they would spend more for their children. As they would spend more they would also have less children. Since they value human capital more this also means that their income would be higher in comparison to parents who do not value human capital.

Another theory which will only be mentioned here is the Investment theory which assumes that parents view their children either as an investment so they prefer to have less children the more they invest in them.

Changes when nannies are involved as nowadays many parents (especially in the developed world) use their services. Most of all, this is a particularly interesting case as one of the main assumptions of the majority of the models concerning fertility and income is that the main cost for parents when having children is time. Thus since a nanny would be allowed in the model, this should clear up more time and therefore parents with higher income would hire a nanny just because this makes more time available for them. Therefore, in a way children become a normal good.

In this paper the framework of Jones, Schoonbrodt & Tertilt (2008) is used as a 'benchmark' to guide the discussion for fertility model allowing for nannies. This model is based on the utility maximization and is optimized using the Lagrange multiplier. With the help of simple extensions to their nanny model, this paper looks into the case where there are nannies and preference heterogeneity. Afterwards, to this case the quality of children is added as well. Then to obtain a more holistic view of the situation, the choice model for nannies is also looked at. Finally the paper finishes with a discussion whether parents find difficulties in the search for a quality nanny.

Literature Review

The literature on the relationship between income and fertility was at first divided between two opinions. One is that this relationship is positive as Malthus once concluded and the other that it is negative. Modern empirical researches argue that this negative relationship encountered is a statistical mistake due to a missing variables problem namely that of the participation of women in the market. Furthermore, they believe that the relationship between income and fertility is actually the relationship of female wages and fertility. The business cycles to affect women and fertility, but there are also other factors.

First a paper which considers the relationship to be actually between female wages and fertility and dependent on the business cycle will be discussed. This discussion will be followed by a more modern paper which is an extension of the first where it is established that the relationship between female wages and fertility is dependent not only on the business cycle but also on other factors. Afterwards, this will be contrasted with the view that there is negative relationship between fertility and wages.

The first paper by Butz and Ward (1979), uses an empirical model based on data from the US from the period 1948-74. They analyse the microeconomic determinants of fertility. Furthermore, they distinguish in their results between models where the wife is working and not, and also models where only the gender specific earnings are taken into account. In this way they hope to reveal whether the fact that the wife is working matters to the relationship or not and if there are any gender specific differences. Their empirical results show that the 1950's "baby boom" can be explained as a response to rising male income. The following baby bust of the 1960's they explain by primarily the increases in female wages and income. Their prediction for the future is that fertility movements can be expected to move countercyclically. (Butz & Ward, 1979)

This statement they base on the statement that the employment of young women and the variation in their wages are more than high enough to induce continuing countercyclical fertility movements. In addition, they also expect the female employment ratio to continue its secular increase and female wages to rise as long as the economy expands. (Butz & Ward, 1979)

A research which is actually an extension of the paper by Burtz and Ward is the one by Ahn and Mira (2009). The authors use a panel of OECD aggregate fertility and labor market data since 1970 to verify the conclusions reached by Burtz and Ward. Furthermore, they extend the research by commenting on paid child care and unemployment. (Ahn & Mira, 2009)

They first look at female employment rates along with fertility rates for the corresponding years and conclude that high unemployment in the 'low' participation countries is likely to have contributed to a faster decrease in fertility. Next on they also check the trends of the fertility and the business cycles and conclude that for the period observed fertility was countercyclical in 15 out of 21 countries. (Ahn & Mira, 2009)

They disprove the explanation by Burtz and Ward about the cause of an end to a baby boom and the beginning of a baby bust. This explanation states that the relationship between income and fertility according to them is dependent on women's wages and varies with the business cycles of the country. To understand on what basis Ahn and Mira base their disproof, first it has to be noticed that 'good times' are the most expensive times to have children (due to high wages) according to Burtz and Ward. Therefore, a great number of women in the labor force will yield a negative association between business cycles and fertility. This also implies that the negative correlation between fertility and participation in the cross-section of OECD countries during the 70's is consistent with the hypothesis of dominant substitution effects. Nevertheless, the problem arises once one asks why then the correlation becomes positive in the 90's when the average fraction of working women in the cross section was even larger.

Therefore, making the explanation by Burtz and Ward for the US time series not consistent with the cross-sectional facts. (Ahn & Mira, 2009)

One part of the paper which is of particular interest to this paper is where they look at child care. The authors study how purchased child care will affect female wages and fertility. They find that at higher wages the direct effect of higher female wages on fertility becomes less negative (or more positive) when market child care is considered, facilitating the reversal of the correlation between fertility and female participation. (Ahn & Mira, 2009)

The majority of papers trying to investigate the negative relationship between income and fertility are theoretical papers. This paper in particular is a development to the paper by Jones, Schoonbrodt & Tertilt (2008). They show several different basic models and give a suggestion for a continuation of the model for a nanny but this time taking in consideration the quality of the nanny by the parents. Originally the idea of how to construct the utility and the budget constraints comes from Gary Becker's famous paper from 1960 "An Economic Analysis of Fertility" where he presents a utility maximization model for the parents, in which the increase of income causes a decline in the number of children. Furthermore, he analyses the desire of parents to have children as if it is a supply and a demand problem. He also concludes that children could be classified as less liquid assets which he explains by the simple fact that children cannot be bought and sold. Therefore, this in return causes the "purchase" of children to be postponed. (Becker, 1960)

As already mentioned Becker does notice a negative relationship between income and fertility. However he also reminds his readers that there are other variables which change along with income and fertility. He provides several suggestions, amongst whom: decline in child mortality, increase in contraceptive knowledge and rise in the cost of children. He explains that each one of these has to be checked in detail, as the precise estimation of the magnitude of influence from each on fertility seems difficult.

Becker also makes another important remark and that is the theory of Quantity and Quality of Children. By observing the increase in expenditure for children, he also observes an increase in their quality. He proclaims legislation changes such as legislation prohibiting child labour and requiring children to have education to have most contributed for higher expenditures on children. The paper concludes with the thought that it is impossible to determine whether there is or there is not a negative correlation between income and fertility.

Contrary to this, somewhat almost 50 years later Jones, Schoonbrodt and Tertilt use his determinants of fertility in order to create a utility model with budget constraints which tries to estimate the relationship between income and fertility. Through the Lagrange multiplier method, they obtain optimal solutions to the utility maximisations and show that the negative relationship between income and fertility remains. In the sections where it is not reached immediately, it is shown to exist under specific assumptions about the model in view.

In summary, it seems that over the past 50 years research from both opinions is contracting as both sides notice that fertility is not dependent on wages but on other factors. Furthermore, wages themselves are also dependent on a number of factors which also have an effect on the fertility. Therefore, all of these should be considered when one wants to look into the relationship.

Methodology

This paper looks at the determinants of the number of children parents have. The negative relationship between income and fertility will be shown and will be proved to remain constant for two scenarios: when a nanny is included and when both a nanny is included but also the fact that parents care about their child's/children's quality. These suppositions are derived from the fact that it is assumed parents with high incomes enjoy working very much and thus if a nanny is introduced that would liberate more time for them, and so the relationship would remain. Furthermore, it is assumed that wealthier parents have higher wages because they value human capital more. From that it is also assumed that they value the quality of their children. Therefore not only is there a negative relationship between wages and fertility but also for the parents desiring higher quality children would mean that they would spend more on providing for their children and due to this they would have less children.

The negative relationship between income and fertility has been frequently proven in past papers with cross section data. Such examples include Becker's paper from 1960, Jones & Tertilt (2008), Borg (1989), Docquier (2004) and Westoff (1954).

Since the paper will also look into the case when parents hire a nanny, the annual nanny survey from the INA website from 2013 will be discussed. This survey was completed by 640 completed and 177 partial responses. Responses from this survey were split between nannies and nannies/household managers, which are also household managers. A large share of the nannies had more than 5 years of experience, namely 71%. An important finding is associated with the characteristics of higher paid nannies namely more tenure, more experience and advanced degrees. Surprisingly however, couples where at least one parent does not work pay higher. This could be explained by the fact that these are families where one of the parents has high enough salary for other not to seek work and to still have money for a nanny.

Furthermore, most of the nannies were for new-borns and also most of the nannies work full time (70%). Experience varies greatly from case to case. Around 85% of nannies have some sort of college education. Nevertheless, education has been found irrelevant for payment for nannies.

Most nannies care for one or two children (77%) and the age of the children varies with a peak at 3-5 years, mostly no nanny care for older than 12 year old children.

Most nannies worked for current employer for less than 3 years (76%) around 40% less than a year.

Despite highest paying couples being the ones where at least one partner does not work, most nannies work for couples where both parents are working (79%).

In order to show that the negative relationship is present for the case with nannies and for the case for nannies, given that the parents care about the quality of the nannies, the model used in the paper by Jones, Schoonbrodt & Tertilt (2008) will be developed to a case where there is a nanny and when the parents care about the quality of the nanny. This is a basic Lagrange Multiplier exercise as there is a utility function and several constraints (depending on the specific case). By considering the utility of the parents and taking into concern the different constraints the Lagrange Multiplier method finds optimal solutions.

The utility that parents receive is assumed to depend on their level of consumption, the number of children that they have, the level of leisure they consume and the quality of their child. As mentioned previously above, the constraints depend on the case, in the most general case there will be a time constraint, a budget constraint, a wage constraint and a quality constraint. This case is further discussed in the Model Development Section in the Subsection Basic Model.

Now the variables which are used will be more explained and related to practice. First off, the model assumes that the parents utility depends on the following four variables: consumption, number of children, leisure time and quality of children. The consumption level which the paper has in mind is the consumption per month per household. The number of children is just the number of children. Leisure time is the time the parents have on their own, so not working or taking care of their children. The quality of a child is something more ambiguous in the sense that it is harder to define it in real life, however in the model we assume that it is about the education of the child/children and the quality of the nanny.

Models

The model which will be developed at the end of this section relates to the Model with nannies and preference heterogeneity from the paper “Fertility Theories: Can they explain the negative fertility-income relationship?”. To be able to explain how the desired model works, first the Basic Model from that paper must be explained. From then on, the next model which will be discussed is an alteration of the Basic Model for nannies and the third one which is nannies and preference heterogeneity. Then on a model from the perspective of a nanny will be simulated and the paper will finish with a discussion on whether highly qualified nannies are scarce.

The Basic Model

The Basic Model from the paper “Fertility Theories: Can they explain the negative fertility-income relationship?” (Jones, Schoonbrodt, & Tertilt, 2008) assumes that the utility of people depends on their consumption, the number of children they have, the quality of their children and leisure. People maximise utility subject to three constraints: time, budget and child quality constraints. Or otherwise:

$$\begin{aligned} \max_{c,n,q,l} \quad & u_c(c) + u_n(n) + u_q(q) + u_l(l) \\ \text{s. t.} \quad & l_w + b_1 n + \ell \leq 1 \\ & c + (b_0 + s)n \leq y + w l_w \\ & q = f(s) \end{aligned}$$

In the above-described model c stands for consumption, n for the number of children, q for the quality of children and ℓ for leisure. Furthermore, the utility of the parents is separable and assumed to have a constant elasticity of substitution (CES). Thus the utility function of each of the four variables is:

$$u_x(x) = \frac{\alpha_x(x^{1-\sigma_x} - 1)}{1 - \sigma_x}$$

The first constraint: $l_w + b_1n + \ell \leq 1$ represents the time constraint. We can see that the time is either devoted to working l_w , taking care of the children (b_1n where b_1 is units of time devoted to caring about children) and leisure time which the parents spend on purely themselves ℓ .

The second constraint $c + (b_0 + s)n \leq y + wl_w$ is the budget constraint. The total wealth consists of labour income wl_w and y represents any extra earnings on top of the wage. This has to be bigger or equal to the sum of consumption along with the units of goods spend on children and education child inputs times the number of children or otherwise $c + (b_0 + s)n \leq y + wl_w$.

Lastly, there is also a child quality constraint which is represented by $q=f(s)$. The nature of the function will be specified according to the model.

Model 2: Childcare as a market good through the help of nannies

Assumptions and model

In the seventh section of the paper “Fertility Theories: Can they explain the negative fertility-income relationship?” (Jones, Schoonbrodt, & Tertilt, 2008) the Basic Model is adapted so that now it allows for parents to buy childcare. This assumption changes the view on children because now childcare could be considered a good. In order to be able to show that, first a simple model with a nanny added would be explained. This model from the paper will be later on in the next two sections developed.

To be able to allow for the nanny to be included in the basic model, the utility of the parents will be made much simpler. In this case we would only consider that they derive utility from only two things – consumption and the number of children. Furthermore, the only constraint that will be considered is the budget constraint. In order to include the option for parents that a nanny could be hired as well, variable gamma is added. Gamma can take values between $[0,1]$. Moreover, gamma represents the proportion of the day that the

parents spend with their children. Thus when gamma is 0 this means that the parents spend no time at all with their children. Conversely when gamma is 1 – all their time is spent on the their children. Thus the model looks like this:

$$\begin{aligned} \max_{c,n} \quad & \alpha_c u(c) + \alpha_n u(n) \\ \text{s. t.} \quad & c + w_n(1 - \gamma)b_1 n \leq w(1 - \gamma b_1 n) \end{aligned}$$

In this case the utility function of a parent is $\alpha_c u(c) + \alpha_n u(n)$ and the constraint is $c + w_n(1 - \gamma)b_1 n \leq w(1 - \gamma b_1 n)$. The introduction of gamma is vital for the Nanny Model as one of the main assumptions is that the only reason why parents would like to have less children is due to the lack of time to take care of them. Thus by including the option where we could control this time, we can observe how the model changes when the time changes.

Comparison between wage of nannies to that of parents

One could also notice that in this model of great importance is the wage of the nanny w_n compared to the wage of the parent w . Thus we could notice that if $w_n > w$, it will be not optimal to hire a nanny. This could be explained by the fact that nannies are too expensive and the parents simply do not have enough money. Consequently, if the case is the opposite $w_n < w$, then the parents would hire a nanny.

Optimization of Model

If we want to find the optimal levels of consumption, number of children and childcare for the children from the parents γ , we have to use the Lagrange multiplier method. Therefore:

$$\begin{aligned} \mathcal{L} &= \alpha_c u(c) + \alpha_n u(n) - \lambda(w(1 - \gamma b_1 n) - c - w_n(1 - \gamma)b_1 n) \\ \frac{d\mathcal{L}}{dc} &= \alpha_c u'(c) - \lambda(-1) = 0 \\ \frac{d\mathcal{L}}{dn} &= \alpha_n u'(n) - \lambda(w(-\gamma b_1) - w_n(1 - \gamma)b_1) = 0 \end{aligned}$$

$$\frac{d\mathcal{L}}{d\gamma} = -\lambda(w(-b_1n) - w_n(-1)b_1n) = 0$$

$$\frac{d\mathcal{L}}{d\lambda} = -(w(1 - \gamma b_1n) - c - w_n(1 - \gamma)b_1n) = 0$$

And so:

$$\lambda = -\alpha_c u'(c)$$

$$\lambda = \frac{(w(-\gamma b_1n) - w_n(1 - \gamma)b_1n)}{\alpha_n u'(n)} \quad (1)$$

$$-\lambda(w(-b_1n) - w_n(-1)b_1n) = 0,$$

where either $\lambda = 0$ or

$$w(-b_1n) = w_n(-1)b_1n \quad (2)$$

$$w(1 - \gamma b_1n) = c + w_n(1 - \gamma)b_1n \quad (3)$$

or simply put as the total amount of wages earned from the budget constraint being equal to the expenses which in this case are the consumption c and the amount of money spend on childcare to a nanny $w_n(1 - \gamma)b_1n$.

Optimum level for the wage of parents and Implications

First off, from equation (2) we get that:

$$w(b_1n) = w_n b_1n$$

therefore: $w^* = w_n^*$

Thus we have obtained that at the optimal level, the wage for the parents should be the same as the wage for the nannies. In this case the parents will be indifferent between hiring a nanny and not hiring one. This is typical because as expected at the optimal level everyone is the same thus the wages of the nannies and of the parents have equalized and they do not influence gamma.

Optimum level of consumption and Implications

Using this result in equation (3) we obtain:

$$w(1 - \gamma b_1 n) = c + w(1 - \gamma) b_1 n$$

From which follows:

$$c^* = w(1 - b_1 n)$$

This means that the optimal level of consumption is a function of the wage of the parent w and the number of children and units of goods spent on children. This means that when the wage, the number of children or number of units spent per child increase in magnitude, the consumption increases as well.

Another important factor that we could notice is that gamma disappears because of the same wage for nannies and parents. Thus at the optimal level it is not a variable which causes an influence on the optimal level of consumption and the number of children.

Consumption Change with number of Children

Furthermore, if we differentiate with respect to the number of children the last equation we can see that:

$$\frac{dc}{dn} = -wb_1$$

Thus the consumption decreases with the number of children and is a function of the wage and the units of goods spend on children.

Consumption Change with Wage

Once this function is differentiated with respect to the wage, we get:

$$\frac{dc}{dw} = 1 - b_1 n$$

Since 1 is only a constant we could also consider this derivative as a decreasing function of the number of children n times the units of goods spend on children b_1 . This derivative then shows that the consumption decreases with increasing the wage.

Consumption Change with the units of goods spend on children

Lastly, the change of the consumption at optimum with the units of goods spend on children is negative and proportional to the wage and the number of children:

$$\frac{dc}{db_1} = -wn$$

Knowing this, we can now substitute in (1):

$$\frac{(w(-\gamma b_1) - w(1 - \gamma)b_1)}{\alpha_n u'(n)} = -\alpha_c u'(c)$$

Optimal Wage level in terms of changes in utility and b_1

Which becomes:

$$w^* = \frac{\alpha_c u'(c) \times \alpha_n u'(n)}{b_1}$$

This means that the optimum level of wage is proportional to the change in utility of consumption with respect to consumption and the change of utility with respect to the number of children ($\alpha_c u'(c) \times \alpha_n u'(n)$) but inversely proportional to the number of goods in units spent on children: b_1 .

Once we take a closer look at the differentiation of the utility of the number of children it actually appears that there might indeed exist a negative relationship, as if we substitute that, the equation now becomes:

$$w^* = \frac{\alpha_c u'(c) \times \alpha_n^2}{b_1 \times n^{\sigma_n}}$$

Nevertheless, this depends on what the value of σ_n is. If it is the case that $\sigma_n < 1$ this means that there is high elasticity of substitution but also that increasing the number of children n will increase the wage as well. If it is $\sigma_n > 1$ this means that there is low elasticity of substitution and that increasing n brings for a decrease in the wage w^* .

Conclusion

So far, we have seen that if not at optimal level, the wage of the parent compared to the wage of the nanny is the main indicator of what gamma might be. Hence, by knowing gamma we know whether the parents are more or less likely to use a nanny.

Therefore, if the parents have a higher wage, then they could hire a nanny and then they would have more children. The contrary is also holds true: when the parents have a lower wage than the market wage for nannies, they would rather take care of their children and thus have less children. Thus, this shows that income is proportional to the number of children – if it increases so will fertility and if it decreases- fertility will decrease as well. In the next model we will explain how making the correct assumption the negative relationship between income and fertility can be restored.

A problem is the fact that the negative relationship between the number of children and the wage of the parents is very unstable. In reality, this model does not tell us much and shows us exactly what we expected but it a vital point for understanding the next section. In fact the conclusion of it has already been discussed in the paper by Jones, Schoonbrodt, & Tertilt, however in this paper we would explore beyond that and thus we would further look at the case in which both the quality of nannies and preference heterogeneity is considered; from the point of view of a nanny and lastly a discussion whether parents face troubles finding a nanny.

Model 3: Nannies and Preference Heterogeneity

Assumptions and Model

In order to re-establish the negative relationship between fertility and income in the case for nannies, a preference assumption concerning the parents' life style choice is introduced. In this model parents have two options: either choose for the "market-consumption life style" or "family-leisure life style". (Jones, Schoonbrodt, & Tertilt, 2008) The former assumes that the parents invest more in human capital l_s and thus have higher wages, whereas the second one that they enjoy their leisure time relatively more and thus their human capital investment is less profitable. This has the following model structure:

$$\begin{aligned} \max_{c,n,\ell} \quad & \alpha_c \ln c + \alpha_n \ln n + \alpha_\ell \ln \ell \\ \text{s. t.} \quad & l_s + l_w + \ell + \gamma b_1 n \leq 1 \\ & w = a l_s \\ & c + w_n (1 - \gamma) b_1 n \leq w l_w \end{aligned}$$

Here the parents' utility is based on their consumption c , number of children n and leisure time ℓ . The original function in the paper is with $\log x$, however for this example it suffices to have the natural logarithm. Furthermore, they maximise their utility subject to three constraints: the first one is a time constraint which divides the time of the parents between time spent on human capital l_s , time spend working l_w , leisure time ℓ and time spent looking after their children $\gamma b_1 n$. This all needs to be less than or equal to one.

The second constrain is a function definition of the wage which is defined by the ability a of the parent and the time he spent on studying l_s .

Finally, the last constraint is the budget constraint, where the amount earned by the parents $w l_w$ has to be bigger or equal to their the sum of their consumption and the money spent on nannies $w_n (1 - \gamma) b_1 n$.

Optimization of Model

Once again the Lagrange multiplier method will be used:

$$\mathcal{L} = \alpha_c \ln c + \alpha_n \ln n + \alpha_\ell \ln \ell - \lambda(l_s + l_w + \ell + \gamma b_1 n - 1) - \beta(w - al_s) - \theta(c + w_n(1 - \gamma)b_1 n - wl_w)$$

$$\frac{d\mathcal{L}}{dc} = \frac{\alpha_c}{c} - \theta(1) = 0$$

Optimal Level of Consumption

From which follows:

$$c^* = \frac{\alpha_c}{\theta} \text{ or } \theta^* = \frac{\alpha_c}{c} \quad (1)$$

Furthermore, differentiating with respect to the number of children n we obtain:

$$\frac{d\mathcal{L}}{dn} = \frac{\alpha_n}{n} - \lambda(\gamma b_1) - \theta(w_n(1 - \gamma)b_1) = 0$$

From which:

$$\frac{\alpha_n}{n} = \lambda(\gamma b_1) + \theta(w_n(1 - \gamma)b_1) \quad (2)$$

Finally, differentiating with respect to leisure ℓ :

$$\frac{d\mathcal{L}}{d\ell} = \frac{\alpha_\ell}{\ell} - \lambda(1) = 0$$

Which translates into:

$$\lambda^* = \frac{\alpha_\ell}{\ell} \quad (3)$$

Optimal Level of Number of Children

Thus substituting (1) and (3) into (2) we obtain the following solution:

$$\frac{\alpha_n}{n} = \left(\frac{\alpha_\ell}{\ell}\right) \times (\gamma b_1) + \left(\frac{\alpha_c}{c}\right) \times (w_n(1 - \gamma)b_1)$$

Thus the optimal number of children is:

$$n^* = \frac{\alpha_n}{b_1 \left(\frac{\alpha_\ell \times \gamma}{\ell} + \left(\frac{\alpha_c}{c} \right) \times (w_n(1 - \gamma)) \right)}$$

Thus the relationship between the wage of the nanny and fertility is negative at the optimum as if the wage increases this means (if all things constant) that the number of children will decrease. Furthermore, looking back at the budget constraint we can notice that an increase in the wage for a nanny means that the wage must increase as well if all things held constant. Thus, we could make the relation that given that nannies with higher wages would be hired by people with higher wages. These people with higher wages then would tend to have less children relatively than the people with lower wages which can afford a lower wage demanding nanny.

Case for $\gamma = 0$

Let us now have a look at the solutions for the two extremes for γ when $\gamma = 0$ (it is always optimal to hire a nanny) and $\gamma = 1$ (it is never optimal to hire a nanny). If we take the first case, when it is always optimal to hire a nanny and thus $\gamma = 0$, the optimal level of the number of children we obtain the following result:

$$n^* = \frac{\alpha_n}{b_1 \left(\frac{\alpha_c \times w_n}{c} \right)}$$

In this case the relationship between income and fertility is still negative. What we could also notice is that even though the number of children increases with both leisure and consumption, the preference for leisure and most of all the preference for consumption when increased cause a decrease in the number of children. Also, an increase in the time spent on raising children also causes a decrease on the number of children.

Case for $\gamma = 1$

Now taking the second case: when $\gamma = 1$, we get:

$$n^* = \frac{\alpha_n}{b_1 \left(\frac{\alpha_\ell}{\ell} \right)}$$

This result simply implies that when the parents spend the whole time available with their children, their wage is irrelevant to their choice of number of children.

Model 4: Quality of Children, Nannies and Preference Heterogeneity

Assumptions and Model

Until now we have assumed that the quality of the nannies is fixed. However what happens in practice is that actually parents with higher salaries (or more able parents in our case) care about the quality of their nannies as they prefer to have fewer children but of high quality and therefore spend more on their children. This section will try to prove precisely that.

This model will assume the same as the model from the previous section, however there is one alteration – now the quality of children is factor to the utility and is a function of educational child inputs s and nanny quality q . This arises from the Quality-Quantity Theory initially introduced by Becker where it is argued that richer parents prefer quality of their children over quantity. (Becker, 1960) It is important to note that this relation is relative, while richer parents might still have a lot of children, they care about the quality of their children rather than the quantity, thus relative to less financially able parents they would have less children.

The utility maximisation looks in this case like this:

$$\begin{aligned} \max_{c,n,\ell,Q} \quad & \alpha_c \ln c + \alpha_n \ln n + \alpha_\ell \ln \ell + \alpha_Q \ln Q \\ \text{s. t.} \quad & l_s + l_w + \ell + \gamma b_1 n \leq 1 \\ & Q = f(s, q) \\ & w = a l_s \\ & c + w_n(1 - \gamma) b_1 n \leq w l_w \end{aligned}$$

In this case we can see that parents maximise utility with vision of their consumption, number of children and leisure but also the quality of children. This utility is all subject to five constraints: a time constraint, a child quality constraint, a wage constraint and a budget constraint. The child quality constraint is assumed to be an increasing function as with increasing either the

child educational inputs or the quality of the nanny the child quality will increase as well. Apart from the child quality constraint, all the other constraints have previously been discussed, thus they will not be repeated.

Optimization of Model

If we perform the Lagrange multiplier method we obtain the following solutions:

$$\begin{aligned} \mathcal{L} = & \alpha_c \ln c + \alpha_n \ln n + \alpha_\ell \ln \ell + \alpha_Q \ln Q - \beta(l_s + l_w + \ell + \gamma b_1 n - 1) \\ & - \theta(Q - f(s, q)) - \lambda(w - al_s) - \mu(c + w_n(1 - \gamma)b_1 n - wl_w) \end{aligned}$$

Optimal Solutions

From where we obtain the following optimal solutions:

$$\begin{aligned} c^* &= \frac{\alpha_c}{\mu} \\ \ell^* &= \frac{\alpha_\ell}{\beta} \\ Q^* &= \frac{\alpha_Q}{\theta} \\ n^* &= \frac{\alpha_n}{b_1 \left(\frac{\alpha_\ell \times \gamma}{\ell} + \left(\frac{\alpha_c}{c} \right) \times (w_n(1 - \gamma)) \right)} \end{aligned}$$

Conclusion

This looks familiar, as apart from the optimal solution obtained for the quality of children, the optimal number of children is the same as the already obtained answer from before. To the already reached conclusions in the previous section we could add that at the optimum level of number of children the quality of children does not affect it. From this we can conclude that parents which invest more in child educational inputs and nanny quality, and thus care more about child quality have higher salaries and less children.

Furthermore, if we go back to the original purpose of hiring a nanny, this implies that since now the option of a nanny is possible parents who initially value more leisure time and have less children and thus earn more, will of course hire a nanny because that means that they will have more leisure time. The quantity quality fits this because these parents earn a lot and thus like to invest in their children.

Model 5: Choice Model for Nannies

Assumptions and Model

In order to obtain a more holistic view on the issue it would be also of interest to have a look at the choice model nannies face, as they are the ones who decide to take the job initially. This would give us further insight to not only understand what nannies parents would chose for their child(ren) but also will these nannies actually take the job and reveal any possible misunderstanding between the two sides.

First of all let us have a look at the model but this time from the perspective of a nanny:

$$\begin{aligned} \max_{c_n, n_n, \ell_n, Q} \quad & \alpha_{c_n} \ln c_n + \alpha_{n_n} \ln n_n + \alpha_{\ell_n} \ln \ell_n + \alpha_Q \ln Q \\ \text{s. t.} \quad & \ell_n + (1 - \ell_n)n_n \leq 1 \\ & Q = f(s, q) \\ & w_n = q(1 - \ell_n) \\ & c_n \geq y_n + w_n \end{aligned}$$

One could notice that the utility function is almost identical to the one of the parents however all the variables are marked for the case of a nanny. A nanny is considered to have the same utility as a parent because she is the one watching over the children and in the same atmosphere. However, what might be different from the parents' utility is the likeness a nanny places on each value and thus the values for the different alphas. Also, the constraints are different from the model from the parents perspective and are made so to imitate the wants and constraints of the nanny.

The first one is the time constraint, where ℓ_n represents the leisure time of a nanny and it is a fraction of the whole time available for a nanny. Thus the expression $(1 - \ell_n)$ represents the time a nanny spends taking care of a child.

This is then multiplied by the number of children a nanny would prefer to have and added to the leisure time available for a nanny must equal one.

The second constraint is the same as the constraint from the previous section as this constraint determines the quality of a nanny and that is dependent upon the educational expenses of the parents for the child/children and the quality of the nanny.

The third constraint represents the wage of a nanny which is equal to the quality of the nanny times the time spends working.

Finally the last constraint is the nannies budget constraint. This constraint looks much simpler as in comparison to the parent's one since nannies are assumed to not have children in this example. Therefore, the total consumption of a nanny should be less or equal to the wage which a nanny receives per month and any extra income that she might have.

Optimization of Model and the Optimal levels for consumption, leisure time, quality of children and number of children

After having all this in mind, now the maximisation problem becomes:

$$\mathcal{L} = \alpha_{c_n} \ln c_n + \alpha_{n_n} \ln n_n + \alpha_{\ell_n} \ln \ell_n + \alpha_Q \ln Q - \beta_n(\ell_n + (1 - \ell_n)n_n - 1) - \theta_n(Q - f(s, q)) - \lambda_n(w_n - q(1 - \ell_n)) - \mu_n(c_n - y_n - w_n)$$

Once again we are faced with a solution which requires the Lagrangian multiplier and so we obtain the following solutions:

$$c_n^* = \frac{\alpha_{c_n}}{\mu_n} \quad (1)$$

$$\ell_n^* = \frac{\alpha_{\ell_n}}{\beta_n(1 - n_n) + \lambda_n q} \quad (2)$$

$$Q^* = \frac{\alpha_Q}{\theta_n} \quad (3)$$

$$n_n^* = \frac{\alpha_{n_n}}{\beta_n(1 - \ell_n)} \quad (4)$$

Optimal level of consumption for a nanny

$$c_n^* = \frac{\alpha_{c_n}}{\mu_n} \quad (1)$$

Analysing the results nothing surprising has come our way as expected. First of all we notice that the optimal consumption level is dependent on two factors – the likeness a nanny places on consuming α_{c_n} and inversely proportional to the value for the budget constraint.

Optimal level of leisure time for a nanny

$$\ell_n^* = \frac{\alpha_{\ell_n}}{\beta_n(1 - n_n) + \lambda_n q} \quad (2)$$

A more interesting result is (2) where we find the optimal level of leisure for a nanny. It is as usual directly proportional to the likeness a nanny places for leisure time and inversely proportional to several factors. The most important of these factors is the number of children, however once again that result should be of no surprise as normally the more children one has the less of leisure time. Another fact is that the optimal leisure time is also inversely proportional to the quality of a nanny, which perhaps indicates that the higher of a quality a nanny has the more she is going to work.

Optimal level of quality for a child in the view of a nanny

$$Q^* = \frac{\alpha_Q}{\theta_n}$$

The result in (3) is the identical result as from the previous section for parents as already explained because of the fact that the utility function is kept

the same apart from putting all the variables of interest from the perspective of a nanny and also the child quality constraint being kept the same.

Optimal level of number of children a nanny would choose

$$n_n^* = \frac{\alpha_{n_n}}{\beta_n(1 - \ell_n)}$$

Finally, the optimal level of the number of children a nanny would desire to have is dependent on the likeness a nanny holds for having a certain amount of children and is inversely proportional to leisure time, as already established.

Discussion: Scarcity Problem of Highly Qualified Nannies

In the previous section it was established what the criteria are for a nanny to take a job and what levels of her main assumed utility bringing variables in life give her satisfaction. A question that follows is whether assuming that parents care about the quality of a nanny, do they then face scarcity in the search of one? From the model from the previous section one could assume that parents might face scarcity, as the quality of a nanny is directly proportional to the wage of one. Thus, leaving only a few parents being able to afford one.

In basic “supply and demand language” scarcity means first off that the supply of a certain product or service, in this case qualified nannies, is limited. In comparison, the demand is higher than the supply and thus some people do not receive the products or services, which they desire.

To be able to understand this better, one should first think when would this occur. Or more precisely, what makes the qualified nannies less than the number of parents who desire a high qualified nanny. From common knowledge the total number of nannies (qualified and not qualified) is less than the total number of parents (desiring a qualified nanny, not desiring one and being indifferent). This is logical as there are not many people who wish to be nannies in comparison to people with children. Furthermore, in our case to be a qualified nanny means to have more years of experience (more than 10 years for example). Therefore, because the number of nannies is so little with more than 10 years of experience it is definitely less than the number of parents who would like to hire one. However, this is a supply and demand problem and if the market is correctly working should be able to fix itself.

Nevertheless, not always the market is able to do that. Here comes another question, what would prevent the market from reaching an equilibrium quantity of well qualified nannies and parents desiring to hire one? Simply because the wage for a well qualified nanny is high. If the wage for a high quality nanny was

not that high, in theory a majority of the parents would prefer to give their children in the hands of such a professional.

However, as already seen because qualified nannies in most of the cases are considered nannies with more years of experience which have went to college (although as seen from the survey a high majority of all nannies have went to college) and thus invested more in themselves. However, another factor which might be of greater importance is the fact that not many people want to be nannies and thus the ones which have a lot of years of experience are very few. Therefore, since they are so little for such a big group of “consumers” they can demand high wages which not all parents can afford to pay for.

To actually test this one would need to give a good and precise definition of a well qualified nanny and also make a difference between the services a nanny provides – what age of children does she take care of, is she only a nanny or does she have also other responsibilities in the house, take into consideration the geographical area in which a nanny is operating and so on. This issue would have to be looked in future research as it is beyond the scope of this paper.

Conclusion

This paper looked into the negative relationship between income and fertility. Models with preference heterogeneity and nannies were considered and one such with both of these assumptions. Later on the quality of the child/children was added as well. All these studies helped for the understanding of the decisions behind the process with which parents make choices about the number of children they would like to have. It was seen that wealthier parents which are assumed to hold more money due to the fact that they have value more human capital have less children but also care about the quality of a nanny that they chose. Thus by parents having the option to hire a nanny those parents who are wealthier and value human capital more would go for a nanny. This could be explained by the fact that for parents the most important cost to having children is time costs, thus once the possibility of a nanny becomes possible they would choose that.

To see this from a different angle a similar model (but with different budget constraints) was created for nannies. There it was found that the nannies would demand more money the more they work and that the number of children a nanny takes care of is inversely proportional to the leisure time.

Also raised was the question whether parents face scarcity in trying to find a highly qualified nanny and this issue was discussed. In general however what is noticed from the survey conducted for 2013 from INA and from the results obtained is that the parents who mostly seek the help of a nanny are parents of new-borns or small children until 5, where both parents work. The education of a nanny still does not have an influence on her salary but her experience does. Furthermore, an important determinant for the salary might be the suggestion of one family to another about a nanny.

Limitations and Recommendations

In case future research is to be conducted on this subject, one should distinguish between the age of the children and also what a nanny has to do (does she have further activities in the house apart from taking care of the children?). Also, the geographical area should be considered and perhaps a good idea for an empirical research would be to compare nanny working hours and income from a rich neighbourhood to a poor one to be able to tell whether wealthier families do indeed consider taking a nanny. Moreover, as different researches have already seen it this model should also include the economic situation of the geographical region or country. This would help to actually build a dynamic model which would capture the changes in fertility and income with the changes in the business cycle.

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