Optimal Pricing with Network Externalities

The Influence of Network Externalities on the Optimal Pricing

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26-8-2014

In this paper the influence of network externalities on pricing will be reviewed. In case of two firms in two periods, network externalities can influence the bandwagon if there is no price discrimination and can influence the price in both ways. If there are strong network externalities the price will increase and decrease if the network externalities are weak.

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1. Introduction

There are many products for which the utility that a user derives from consumption of the good increases with the number of other agents consuming the good (Katz & Shapiro, 1985). This is a quote from two economists, which they wrote in an article about network externalities. The mobile phone is an example of a product with network externalities. The more people use a mobile phone, the more useful it is to also have a mobile phone. Another example is a product like Facebook, Facebook is more attractive when more people use it. There are lots of other examples like those products.

It is an interesting appearance how products can be worth more through the use of many people. Facebook is good examples of that. During the IPO of Facebook the shares were sold for $38, which meant that Facebook was worth $104 billion (Posthumus, 2012).

Katz and Shapiro have another article about network externalities, where they use the network externalities with the adoption of technologies. In that paper they study the dynamic of industry evolution in a market with technological change, where two incompatible technologies subject to network externalities (Katz & Shapiro, 1986).

When a product has network externalities, and different firms have different ways to make those products, it is beneficial for the firms in question to use the same technology together, because the benefits of the network externalities will be less when the firms make it in different ways. They will make a standard for the product, which will lead to a better use of the network externalities. Because when there is one standard product that most customers will use, it is more attractive for customers who did not use any of those products yet to use it also. Just like Facebook, more people will join because of the people around them who already joined, it will become more and more attractive to join. In that way the firms get more demand which will lead to greater benefits in the market and for the firm itself. Firms who can cooperate to make a standard product can get higher demand and push other firms out of the market because their product will be less attractive.

Network externalities in combination with new technologies can be beneficial. It can be beneficial for different parties, also for the producer of the technology, because when more people use their technology it will become more attractive. Because that could become the only way to produce the product. In this paper the side of the producer will be reviewed. The leading question of this paper is about the influence of network externalities on the price that the producer will ask to maximize his profits.

In the literature network externalities is a subject what is treated in different ways. There are of course papers written about products with network externalities. Katz and Shapiro have written an article about product introduction with network externalities. The article is about the presumption that in markets of network externalities excess inertia exhibits with a bias towards the existing product. In contrast Katz and Shapiro provide conditions under which equilibrium there is a tendency to rush into new technologies (Katz & Shapiro, 1992). Excess inertia is a socially excessive reluctance to switch to a superior new standard when important network externalities are present in the current one (Farrell & Saloner, 1986). Katz and Shapiro did write another article about network externalities and products, which was about expectation from the consumers about which seller would be dominant. If consumers expect a seller to be dominant, he will be dominant and consumers are willing to pay more for his product (Katz & Shapiro, 1985).

Another article from Katz and Shapiro about network externalities is about the adoption of a new technology in the presence of network externalities. This article is about whether technologies are sponsored or not, when an inferior technology is sponsored it has a strategic advantage. But when two competing technologies are sponsored, the technology that will be superior has a strategic advantage. (Katz & Shapiro, 1986)

Another article, from Janssen and Mendys-Kamphorst, about network externalities is about the evolution of market shares with repeated purchases. In that article they investigate how market shares change when new, superior technology with network externalities is introduced in a market which is initially dominated by an old technology. They look to the influence of sponsoring the old and new technology, or only the new one, and the quality difference of both technologies. (Janssen & Mendys-Kamphorst, 2007)

Nicholas Economides wrote an article about network externalities and the willingness to share the technology. An exclusive holder of a technology can have incentives to share his technology because of strong network externalities. The quantity leader has incentives to invite entry and license his technology without charge, and with very strong network externalities the leader can pay a subsidy to the invited followers. (Economides, 1995)

From the side of firms, who introduce network externalities, it is important to understand which user characteristics confer the greatest network benefits on other potential adopters. Catherine Tucker did a research to those characteristics, with the introduction of a massage by video technology in an investment bank. The adoption of higher placed workers has a larger impact than the adoption of ordinary workers. So with the launch of a product with network externalities it is important to look to which players you want to target first. (Tucker, 2008)

The article of Farrell and Saloner is an article that is used as basis for this paper. Their model is used to make an own model with a few adaptions. Their research is when different types of firms want to change to new technologies. First they look at a model with sequential decisions and complete information. Later on they look at the influence of incomplete information and communication, in a model with more periods, on the change to a new technology. (Farrell & Saloner, 1985)

This paper builds on the model of Farrell and Saloner, the model is changed at a few points. In the article of Farrell and Saloner there is no price for the new technology, in this paper the price is added. In this paper another subject than Farrell and Saloner studied is examined. However we will use their model in this paper. In the next chapter the model and the way to work with the model will be explained.

There are similarities between the articles above and this paper. The influence of network externalities is the most important one. Network externalities can have influence on different points and different levels, the level of firms, also firms who are leading in a market, or consumers. The use of network externalities as influence on adopting new technologies is the same as in this paper. Also the use of different periods and players are similar to some articles.

But there are also differences between those articles and this paper. None of the papers look at the optimal price for the new technology from the view of a producer, the paper of Economides talks about incentives to share a technology of an exclusive holder. That is different with the research of this paper, because it is a firm that stands in the market and has benefits from the network externalities. This research is about a producer that stands outside the market and does not benefit directly from the network externalities. Only the paper of Farrell and Saloner from 1985 looks at a third party who is outside the market, but they do not look at the price that can be set.

The added value of this paper is about the way to look at the influence of network externalities from a different perspective. We will look at the influence of network externalities on the price that can be asked from a producer, who does not participate in the market. Both sides are partly covered by Farrell and Saloner or Economides, but both sides are not combined yet. The conclusion of what kind of influence network externalities have on the price is not yet treated in one of the articles above.

The research is done in the following order. First the model will be explained, the way it is made, the players who participate, the choices they can make and the assumptions of the model are treated. Also the way to make the model mathematical to get results will be treated in the second chapter. The possible scenarios will be explained, two of the four scenarios will be treated afterwards in the chapters after the model.

The first scenario that will be examined is the case where the price cannot be changed, there is no price discrimination possible. There will be looked what kind of influence network externalities have on the decision of the firms, in the different periods.

The second scenario is when the possibility to adjust the price is a surprise for the firms and also for the producer, this will be explained in the next chapter. With those results the conclusion of the influence of network externalities on the pricing of the new technology will be made.

In the last chapter the conclusion of the paper will be made and there will be given some opportunities for further research.

2. The model

The model that will be used to research will be introduced and explained in this chapter. First the model will be explained and the players who play a role will be looked at. After the introduction of the model and players, the symbols that are used for the mathematics and the assumptions for the math will be explained. There also will be a few scenarios explained that will be treated in this paper.

The model is a simplified model, there are three players in this model, the producer and two firms. There will be looked at the pricing of a new technology through a firm that does not compete in the market, the producer. The producer only sells the technology to the firms in the market. There will be a few different scenarios how the transactions and price settings will be. Those will be discussed later on.

In this model there will be looked at a producer, two firms, two technologies, and two periods. The firms can choose to switch to a new technology(Y) or stay at the old technology(X), they can choose to buy it in the first or second period. The benefits for firm *j* can be defined by $B_j(m,k)$. Where *k* stands for the kind of technology that is used and m is defined by the number of firms that use that kind of technology. $B_j(2,X)$ are the benefits when both firms stay at the old technology, this is normalized to zero in the status quo. Thus $B_j(2,X)=0$. In this model there are network externalities, which implies that a technology gives more benefit when more firms have that technology. The producer does not have any new costs to sell one or two technologies to the firms.

There is incomplete information in this model, so the benefits function will be $B^i(m,k)$, where *i* denotes a firm's type, the subscript *j* is not needed anymore, because there is a difference captured with the *i*. The higher *i*, the stronger the preference to change to the new technology. We assume that all types are equally probable, they are uniformly distributed on [0,1].

There are two periods, the firm can choose to switch in period 1, period 2 or not at all. The first round move depends only on the firm's *i* and the price that is set by the producer. The possible change in the second period depends on their own *i* and the price, but also on the movement in the first period. If one firm switched in the first period, it is more attractive for the other firm to switch in the second period, because of the network externalities. If a player switched in the first period, he has no further decisions to make.

There are a few assumptions that we need to make in this model, the source of these assumptions is the paper from Farrell and Saloner (Farrell & Saloner, 1985).

Assumption 1. $B^i(2,k)>B^i(1,k)$, k=X and Y.

This means that networks are beneficial, when both players switch or stay the same, it is always more beneficial when both firms have the same technology.

Assumption 2. $B^i(2,Y)$ and $B^i(1,Y)$ are continuous and strictly increasing in *i*.

This assumption is about the types of the firms, the higher *i* the more eager the firms are to switch to Y.

Assumption 3. $B^1(1,Y)>0$ and $B^0(2,Y)<B^0(1,X)$.

Unilateral switching is at least beneficial for one firm, and there are also firms that would rather remain at the old technology than join the other firm with the new technology. This implies also that there are firms for whom the decision depends on the choice of the other firm.

Assumption 4. $B^i(2,Y)-B^i(1,X)$ is monotone in i.

If a firm with a type *i'* prefers a combined switch to then new technology, than so do all firms with *i>i'*, they all would follow the lead if that was possible.

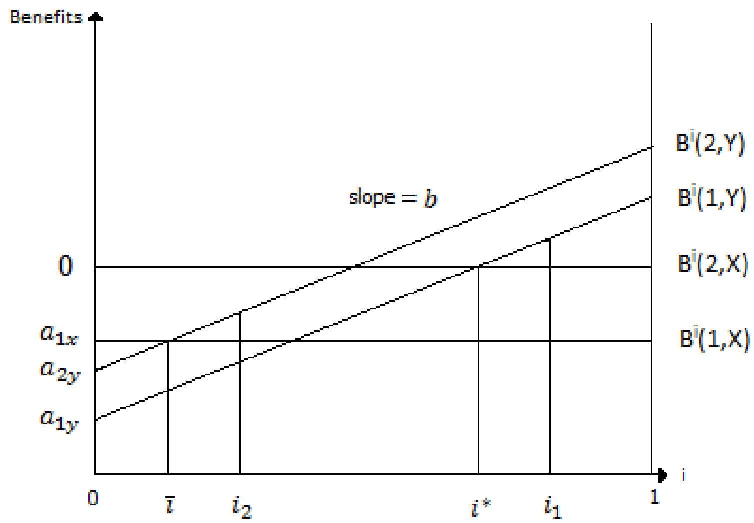

Figure 1 – The model in a graph

Figure 1 is made with the assumptions 1 till 4. There is a difference with the figure from Farrell and Saloner, this figure has straight lines, while the lines in the paper of Farrell and Saloner are curved. For simplicity we have assumed that the benefits are linear functions of *i*. The following functions are made: $B^i(2,Y)=a_{2y}+bi$. With a_{2y} as a negative parameter, because it starts below $B^i(2,X)$, which is zero. The function of $B^i(1,Y)= a_{1y}+b*i$, also a_{1y} is negative. $B^i(2,Y)$ is always above $B^i(1,Y)$, as shown in the figure, this is because of assumption 1, this means also that $a_{2y}>a_{1y}$. The b stands for the slope of the line, the higher the *i* of the firm*,* the higher the firm benefits from the new technology. $B^i(1,X)=a_{1x}$ is the last function, this function is always negative, because of assumption 1, in this case it is just a negative constant line.

The first one is about the intersection when $B^i(2,X) = B^i(1,Y)$, the firm with type *i** is indifferent between move alone to the new technology, and stay at the old one with the other firm.[1] So every type of *i>i** will move in directly to the new technology, in case there is no price for the technology. The second intersection, $\bar{\imath}$, is where $B^i(1,X)$ and $B^i(2,Y)$ intersect. A firm with type $\bar{\imath}$ is indifferent between stay alone at the old technology or join the other firm with the new technology.[2] Every firm with type $i<\bar{\imath}$ will never change technology, they rather want to stay alone at the old technology than join the other firm with the new technology. There are two more intersections, those are not interesting for the decisions that the firms have to make in the case of this paper. They are useful to know the preferences of the firms. To see what they prefer alone, the old or new technology ($B^i(1,X)$ and $B^i(1,Y)$), and what they prefer together, the new or old technology ($B^i(2,X)$ and $B^i(2,Y)$). The preference for what the firms want together can be interesting if the firms can communicate together and choose their technology, but in this paper this is not the subject of research.

In case of a set price, for instance i_1, which means that $P = B^{i_1}(1,Y) - B^{i_1}(2,X) \ = a_{1y} + b * i_1 - 0$, a firm will buy the new technology when the benefits of the new technology minus the benefits of the old technology is equal to zero or higher than zero. His intersection will be i_1, because when $i \geq i_1$ the benefits of the new technology minus the price are higher than the benefits of the old technology. When a firm has $i \leq i_1$ it is not beneficial to change technology. This is the case when the firms only look at this period and do not know about possible price changes. The firms can react

[1] As the firm is indifferent between staying alone the new technology or together the old technologies, they will switch to the new technology.
[2] As the firm is indifferent between together the new technology or staying alone at the old technologies, they will switch to the new technology.

differently on different situations. It depends on the information they have, but also the information that the producer has can lead to different outcomes. Therefor multiple scenarios will be reviewed.

The first scenario is a scenario without price discrimination, the producer cannot change his prices. It is not allowed to change the price in the second period, so the price that will be set in the first period is also the price in the second period. The firms do not know about their influence on other parties. There are three kinds of actions that the firms can consider in this scenario. Switch at time 1, switch at time 2, if and only if the opponent switched at time 1, and do not switch at time 2 even if the opponent switched at time 1. There is a fourth action possible, switch at time 2 if the opponent did not switch at time 1, but this option is dominated by the first, switch at time 1. If a firm want to switch, regardless the choice of the opponent, he prefers to switch at time 1 (Farrell & Saloner, 1985).

In the second scenario the players do not know anything about price changes in the first period, it is a surprise to all that the producer can change his price. Also the producer did not know in the first period that he is able to change his prices for the second period. The firms only look at their own preference in the first period, if it is beneficial to change to the new technology. The firms have in this scenario the same options as in scenario one, but their choice can be influenced by the price that is set in the second period. Also the fourth action would be possible, if both firms do not change. In that case the producer will lower his price in the second period, and both firms will make a new decision caused by the new price. This case can occur when the regulations will be adjusted between the two periods. This can happen when the regulations do not allow price discrimination between periods at first, but in the second period the regulations will be adjusted so the producer can ask a different price in the second period.

There are two other possible scenarios, but those are out of the scope of this thesis for complexity[3].

In the paper of Farrell and Saloner is the *"bandwagon strategy"* explained. It is a more "political" middle group that wait with their decision, they are "for" the measure if the bandwagon begins to roll and "against" otherwise (Farrell & Saloner, 1985). This strategy is also important for this research, because with the decision for the price it is important to get as much players who will buy the new technology. This group is a target for the producer of the technology, the producer needs first to find a player who want his technology, after that also the players who have the bandwagon strategy will follow, which ensures a bigger profit for the producer. In Figure 1 you can see which types of firms use the bandwagon strategy, namely the firms with an $i^*>i>\bar{\imath}$, those types change if the price is zero. With a positive price the bandwagon will differ, we will see this in the first scenario. If the price is set with i_1 and one firm will switch, the other firm has to choose between $B^i(1,X)$ and $B^i(2,Y) – P$. The price stays the same and the i_2, the firms who are indifferent in the second period to switch and not to, will be smaller than the i_1. The firms with an $i_1>i>i_2$ will be the types who use the bandwagon strategy.

In this paper the two scenarios will be reviewed and compared. The first scenario is without price discrimination and the second scenario includes price discrimination as a surprise for everyone. In the first scenario the bandwagon strategy will be reviewed, to see which firms will follow if one firm already changed. The price cannot change, but the price will be used to look at the second scenario,

[3] In the third possible scenario only the producer knows that he can change his price in the second period, for the firms that have to choose if they want to change to the new technology it is a surprise.
In the fourth possible scenario it is common knowledge that the producer can change his price, so all the players can participate on price changes. They can participate with their own expectations of the price changes.

because that is the optimal price set by the producer. If he finds out in the second period that he can change the price he will search for an optimal price again. There are three situations that can occur in the first period: both firms will change, one firm will change or no firm will change. The first one is not interesting, because there is no firm anymore to sell the new technology to. The third situation is also not the main part of the research, because that price will decrease if possible, because otherwise no firm will buy it the second period. This is the case because the option to change to the new technology in the second period, if the other firm does not change it the first period, is dominated by changing in the first period. The second option is the most interesting option, because the price can increase or decrease. There are two movements if one firm changes to the new technology, namely the distribution of i will be between 0 and i_1 instead of 0 and 1, with $i_1<1$. Which implies that the firm who is left in the distribution valuates the new technology less than the firm who already changed. This can lead to a price decrease. But the other movement is about the technologies, the new technology will be valuated more because of the network externalities, and the old technology will be valuated less because the firm is alone. This can lead to a price increase. In the next chapters there will be looked at which effect dominates.

3. No price discrimination

Model

The first scenario that will be reviewed is the scenario without price discrimination. The producer can set the price of the new technology in the first period and cannot change it. So the firms will look in the first period if it is beneficial to change to the new technology. There are three possibilities after the first period, both firms, one firm or no firm changed to the new technology. There is no second period if both firms change. Also if none of the firms change there is no second period, because if one of the firms would change in a situation with the same price in both periods, they would change in the first period. So there would only be a second period needed when one of the firms will change. As can be seen in the graph the producer has to make a profit-maximizing decision between the probability of change of the firms and the price. The choice of P will determine the i_1, which determines which types of firms will change their technology. The price that is set in the first period also determines the price in the second period and with that the i_2. This choice for the producer will be reviewed in this chapter.

An important assumption of make in this model is the assumption of myopia of the buying firms. This assumption indicates that the firms assume that the other firm does not switch. Also the firms expect that they have no influence on the future, which indicates that the price that they expect no price changes, which is important for the research of the decisions of the firms. The firms have a lack of long-range perspective in thinking, they do not look further than the current period. So in the first period the firms do not take a possible change of the other firm into account.

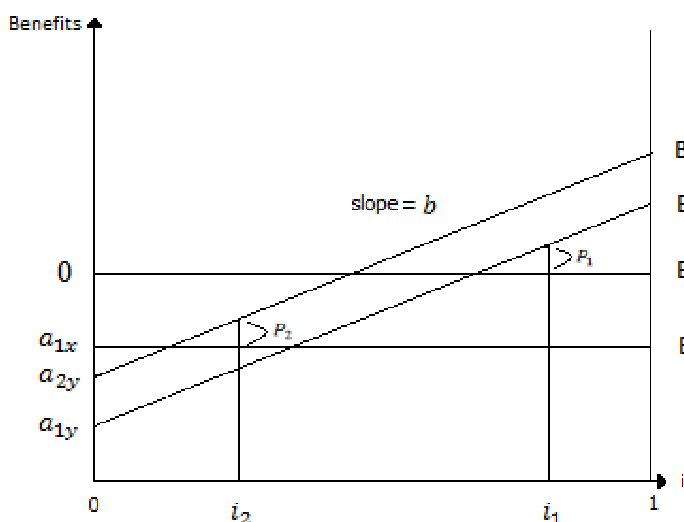

Figure 2 – Graphical approach of the price

Without price discrimination

First there will be an assumption of the price in case of the change of one firm, $P_1 = P_2$.

$$P_1 = a_{1y} + b * i_1$$
$$P_2 = a_{2y} + b * i_2 - a_{1x}$$
$$i_2 = \frac{a_{1y} + a_{1x} - a_{2y}}{b} + i_1$$

Now i_2 is defined in terms of i_1, we will look at the different situations and the probabilities that they occur. In these situations the revenues of the producer will be reviewed. The probability of a firm to switch is $(1-i_1)$, because the firms with an $i \geq i_1$ will switch, so the probability that this firm does not switch is i_1. Which leads to the following probabilities of: both firms switch, one firm switches and no firm switches.[4]

$$\Pr(2Y) = (1-i_1)^2$$
$$\Pr(1Y) = 2(i_1 - i_1^2)$$
$$\Pr(0Y) = i_1^2$$

With those probabilities and P_1, we can make the following expected profit function for the first period.

$$\pi_1 = (1-i_1)^2 * 2 * (a_{1y} + b * i_1) + 2(i_1 - i_1^2) * (a_{1y} + b * i_1) + i_1^2 * 0$$
$$\pi_1 = (2 - 2i_1) * (a_{1y} + b * i_1)$$

The probability, in case of one switched firm, that the other firm will switch in the second period is $\frac{i_1 - i_2}{i_1}$. If a firm has $i > i_1$, he would have been switched in the first period. And if a firm has $i < i_2$, he will not switch in the second period. The expected profits for period two will be:

$$\pi_2 = \frac{(i_1 - i_2)}{i_1} * (a_{2y} + b * i_2 - a_{1x})$$
$$\pi_2 = \frac{\left(i_1 - \left(\frac{a_{1y} + a_{1x} - a_{2y}}{b} + i_1\right)\right)}{i_1} * \left(a_{2y} + b * \left(\frac{a_{1y} + a_{1x} - a_{2y}}{b} + i_1\right) - a_{1x}\right)$$
$$\pi_2 = \frac{\left(-\frac{a_{1y} + a_{1x} - a_{2y}}{b}\right)}{i_1} * (a_{1y} + b * i_1)$$

The probability to get this profit is $2(i_1 - i_1^2)$, so with this information the total expected profit function can be made. The optimal i_1 can be calculated by differentiating the following function:

$$\pi_{total} = \pi = (2 - 2i_1) * (a_{1y} + b * i_1) + 2(i_1 - i_1^2) * \frac{\left(-\frac{a_{1y} + a_{1x} - a_{2y}}{b}\right)}{i_1} * (a_{1y} + b * i_1)$$
$$\pi = (2 - 2i_1) * (a_{1y} + b * i_1) + (2i_1 - 2) * \left(\frac{a_{1y} + a_{1x} - a_{2y}}{b}\right) * (a_{1y} + b * i_1)$$
$$\frac{\partial \pi}{\partial i_1} = -2 * (a_{1y} + b * i_1) + (2 - 2i_1) * b + 2 * \left(\frac{(a_{1y} + a_{1x} - a_{2y})}{b}\right) * (a_{1y} + b * i_1) + (2i_1 - 2) * \left(\frac{(a_{1y} + a_{1x} - a_{2y})}{b}\right) * b = 0$$
$$-2b * i_1 + (a_{1y} + a_{1x} - a_{2y}) * i_1 + (a_{1y} + a_{1x} - a_{2y}) * i_1 = a_{1y} - b - \left(\frac{a_{1y}}{b}\right) * (a_{1y} + a_{1x} - a_{2y}) + (a_{1y} + a_{1x} - a_{2y})$$
$$i_1 = \frac{a_{1y} - b + \left(1 - \frac{a_{1y}}{b}\right) * (a_{1y} + a_{1x} - a_{2y})}{2 * (a_{1y} + a_{1x} - a_{2y} - b)}$$

With the i_1 we can find the optimal i_2 and P. After finding the i_2 and P we can interpret the outcome and results.

$$i_2 = \frac{a_{1y} + a_{1x} - a_{2y}}{b} + i_1$$

[4] $\Pr(2Y) = (1-i_1) * (1-i_1) = (1-i_1)^2$
$\Pr(1Y) = (1-i_1) * i_1 + i_1 * (1-i_1) = 2(i_1 - i_1^2)$
$\Pr(0Y) = i_1 * i_1 = i_1^2$

$$i_2 = \frac{a_{1y} + a_{1x} - a_{2y}}{b} + \frac{a_{1y} - b + \left(1 - \frac{a_{1y}}{b}\right) * \left(a_{1y} + a_{1x} - a_{2y}\right)}{2 * \left(a_{1y} + a_{1x} - a_{2y} - b\right)}$$

$$P_1 = a_{1y} + b * \frac{a_{1y} - b + \left(1 - \frac{a_{1y}}{b}\right) * \left(a_{1y} + a_{1x} - a_{2y}\right)}{2 * \left(a_{1y} + a_{1x} - a_{2y} - b\right)}$$

$$P_1 = a_{1y} + \frac{b * \left(a_{1y} - b\right) + \left(b - a_{1y}\right) * \left(a_{1y} + a_{1x} - a_{2y}\right)}{2 * \left(a_{1y} + a_{1x} - a_{2y} - b\right)}$$

$$P_1 = a_{1y} + \frac{b - a_{1y}}{2} = \frac{1}{2} * \left(a_{1y} + b\right)$$

$$P_2 = a_{2y} + b * \left(\frac{a_{1y} + a_{1x} - a_{2y}}{b} + \frac{a_{1y} - b + \left(1 - \frac{a_{1y}}{b}\right) * \left(a_{1y} + a_{1x} - a_{2y}\right)}{2 * \left(a_{1y} + a_{1x} - a_{2y} - b\right)}\right) - a_{1x}$$

$$P_2 = a_{1y} + b * \frac{a_{1y} - b + \left(1 - \frac{a_{1y}}{b}\right) * \left(a_{1y} + a_{1x} - a_{2y}\right)}{2 * \left(a_{1y} + a_{1x} - a_{2y} - b\right)}$$

$$P_2 = a_{1y} + \frac{b - a_{1y}}{2} = \frac{1}{2} * \left(a_{1y} + b\right) = P_1$$

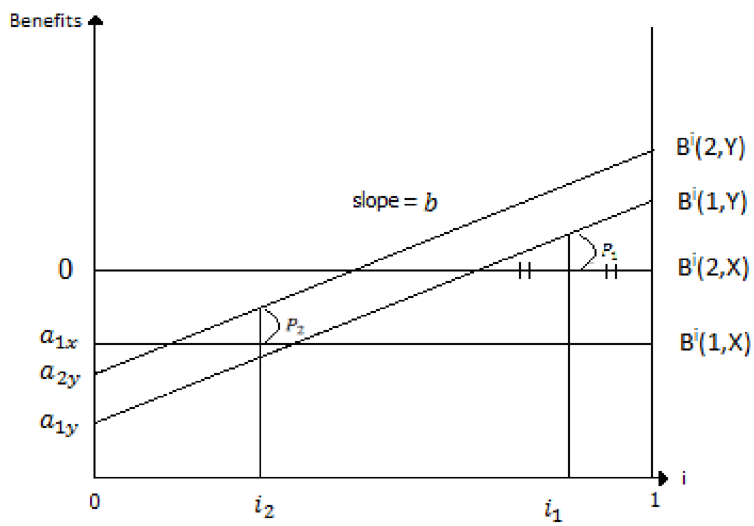

Figure 3 - Graphical approach of P_1 and P_2

As shown in Figure 3 the price lies in the middle of the part from line $B^i(1,Y)$ above the zero line. P_1 is the half of the difference between a_{1y} and b, the difference is the height of the line at $i = 1$. From P_1 we can derive i_2, because P_2 is the same as P_1, and the i_2 is important to know the bandwagon.

In the formula of P_1 we can interpret the influence of the different variables on the price. The influence of variable b is simple to interpret; it has a positive influence on the price. The higher b the higher the price, this is quite logical, because if the slope of the line of the benefits will be steeper the benefits on any point on the line will be higher. Also a_{1y} has a positive influence on the price, when a_{1y} increases, the price will be higher. If the difference between the start point of the new technology and the old one, which is zero, is smaller, than the new technology benefits more than before, which will lead to a higher price.

As shown above the variable a_{2y} and a_{1x} has no influence on the price, which is explainable, because in the first period, where the price is set, the producer does not take the second period price into account, because the price stays the same. The a_{2y} only has influence on i_2, even as the variable a_{1x}. Their presence change the decisions of the firms in the bandwagon, which is from i_2 till i_1, they have influence on the probability that the producer sells the technology to the second firm. a_{2y} has a negative influence on i_2, which means that the i_2 will be lower if a_{2y} increases. So it has a positive effect on the decision of the second firm to change, more types of i will change to the new technology. The opposite is true for a_{2x}, it has a positive influence on i_2, which means that the higher a_{2x}, the less types of i will change to the new technology.

In the formula of $i_2 = \frac{a_{1y}+a_{1x}-a_{2y}}{b} + i_1$, the difference in the value of the new technology for the second firm is directly visible. The difference is $a_{2y} - a_{1x} - a_{1y}$, $(a_{2y} - a_{1x})$ is the part of benefits that the firm will earn in the second period, a_{1y} is the part of benefits that the firm missed in the first period. The total difference divided by the slope (b) shows the type of firms who will change to the new technology in the second period if the other firms switched.

With the same prices in period one and two, if one firm will switch in the first period, the other firm will switch also if he is in the area of the bandwagon, which is from i_2 till i_1. The increased value of the new technology, which is $a_{2y} - a_{1y}$, and the decreased value of the old technology, which is a_{1x}, will lead to a higher probability of the change of the residual firm. There are two ways to look at the formula of i_2, the value change of the periods and the value change of the technologies. Both give the same difference for the second firm. The way of looking at the technology is the most logical way in this research. The difference in the value of the technologies will determine the bandwagon. If the positive network externality of the new technology is big, which means that the difference between the use of the new technology together or alone ($a_{2y} - a_{1y}$), than the bandwagon will be bigger. The probability that the second firm will change will be bigger. Also if the positive network externality of the old technology is big, which means the use of the old technology together or alone ($0 - a_{1x}$), the bandwagon will be bigger. The firm that has the choice to switch in the second period, if the other firm switched in the first period, did lose the benefits of the positive network externality of the old technology.

If there are more firms in a model, with more periods, and also network externalities, than if one firm will set the “train” in motion, there is a possibility that more firms will follow because the new technology will be more attractive and the old one less attractive. Every period the new technology can get more attractive, so it is very important to get the “train” rolling. If there are more firms, and the firms are uniformly distributed, the producer can set higher prices, because the probability that one of the firms has an $i > i_1$ is higher. In the second period the technology will be worth more and the other firms can jump the bandwagon in one of the remaining periods.

4. Price change as surprise to all

In this chapter there will be looked at a different second period than before. At the end of the first period the regulations about price discrimination are adjusted, it is a force from the outside of the model. None of the players could take this adjustment into account. In this scenario the producer will set the same price in period one as before, he will optimize his profits in the first period, with the assumption that the price will stay the same in the second period. Also the firms do not know anything about a price change, they will react the same as before. So the first period will stay the same as before, only the second period will change. There are three possible outcomes in the first period, both firms changed, one firm changed or no firm changed. The price $(P_2(i_1, n))$ for the second period is influenced by the amount of changed firms n, and het chosen i_1. In this chapter there will be looked at the price change of the last two possible outcomes, because the first outcome does not change the price. In the first case of $n = 2$, there is no firm left to sell the technology to, so it is not interesting. The last case of $n = 0$, is less interesting because the price will always drop, otherwise the technology will never be sold. The case of $n = 1$ is the most interesting, and also the topic of this research.

Second period with $n = 1$

First the case of $n = 1$ will be treated, one firm did already buy the new technology, now the regulations are adjusted and the producer can change his price. The question treated is: what will happen to the price? A reason to believe that the price will decrease is because the firm who assigned the most value to the new technology changed already. In the first period all firms were normally distributed between 0 and 1. But now the distribution is between 0 and i_1, with $i_1 < 1$. But there are more movements after the first period, namely the value of the technology will be higher, through the network externalities. The value did change from B(1,Y) to B(2,Y). Also the value of the old technology did change, because there were also positive network externalities, the value did change from B(2,X) to B(1,X). The negative change of the old technology and the positive change of the new technology are reasons to believe that the price will increase.

For the second period the probability that the second firm will change to the new technology is $\frac{i_1-i_2}{i_1}$. The price of the technology is $P_2 = a_{2y} + b * i_2 - a_{1x}$, which is the combination between the value of the new technology $(a_{2y} + b * i_2)$ and the decreased value of the old technology (a_{1x}). The expected profit of the second period can be calculated with this information. And from that the optimal i_2 can be calculated.

$$\pi_2 = \frac{i_1 - i_2}{i_1} * (a_{2y} + b * i_2 - a_{1x})$$

$$\frac{\partial \pi_2}{\partial i_2} = -\frac{1}{i_2} * (a_{2y} + b * i_2 - a_{1x}) + b * \frac{i_1 - i_2}{i_1} = 0$$

$$2b * i_2 = a_{1x} - a_{2y} + b * i_1$$

$$i_2 = \frac{1}{2} i_1 + \frac{a_{1x} - a_{2y}}{2b}$$

With this result the optimal P_2 can be calculated. The i_1 and P_1 can be taken from the previous chapter. After that the P_2 is calculated it can be compared with P_1.

$$P_1 = \frac{1}{2} * (a_{1y} + b)$$

$$P_2 = a_{2y} + b * i_2 - a_{1x}$$

$$P_2 = a_{2y} + b * \left(\frac{1}{2} i_1 + \frac{a_{1x} - a_{2y}}{2b}\right) - a_{1x}$$
$$P_2 = \frac{1}{2} * \left(a_{2y} - a_{1x}\right) + \frac{1}{2} b * i_1$$
$$P_2 = \frac{1}{2} * \left(a_{2y} - a_{1x}\right) + \frac{1}{2} b * \frac{a_{1y} - b + \left(1 - \frac{a_{1y}}{b}\right) * \left(a_{1y} + a_{1x} - a_{2y}\right)}{2 * \left(a_{1y} + a_{1x} - a_{2y} - b\right)}$$
$$P_2 = \frac{1}{2} * \left(a_{2y} - a_{1x}\right) + \frac{1}{2} * \frac{b - a_{1y}}{2} = \frac{1}{2} * \left(a_{2y} - a_{1x}\right) + \frac{1}{4} * \left(b - a_{1y}\right)$$

To see which price is higher they will be compared. The results will be discussed after the comparison.

$$P_2 \gtreqless P_1$$
$$\frac{1}{2} * \left(a_{2y} - a_{1x}\right) + \frac{1}{4} * \left(b - a_{1y}\right) \gtreqless \frac{1}{2} * \left(a_{1y} + b\right)$$
$$\frac{1}{4} b + \frac{1}{2} * \left(a_{2y} - a_{1x} - \frac{1}{2} a_{1y}\right) \gtreqless \frac{1}{4} b + \frac{1}{2} * \left(a_{1y} + \frac{1}{2} b\right)$$
$$a_{2y} - a_{1x} - \frac{1}{2} a_{1y} \gtreqless a_{1y} + \frac{1}{2} b$$
$$a_{2y} - a_{1x} - a_{1y} \gtreqless \frac{1}{2} * \left(a_{1y} + b\right)$$
$$a_{2y} - a_{1x} - 1\frac{1}{2} a_{1y} \gtreqless \frac{1}{2} b$$

No conclusion can be made from this equation directly. P_2 can be higher than P_1, but also lower. There are two ways to look at the equation, the first is $a_{2y} - a_{1x} - a_{1y} \gtreqless \frac{1}{2}\left(a_{1y} + b\right)$. The left side of the equation is the positive network externality, and the right side is the same as P_1. This way is the easiest way to compare the prices in the different periods. The second way is $a_{2y} - a_{1x} - 1\frac{1}{2} a_{1y} \gtreqless \frac{1}{2} b$, with this equation it is the easiest to see which variable has a positive or negative effect on the price change. So both ways will be used to get results.

First there will be looked at the effect of the variables. It is easy to deduce the effect of the variables. The first variable, a_{2y}, has a positive effect on P_2. a_{2y} can only affect P_2 if one firm changed in the first period, this is because of the assumption of myopia. If the variable will be higher the P_2 will be higher, which means that the probability that $P_2 > P_1$ will be bigger. This is reasonable, because if the start point of $B^i(2,Y)$ changes, positively, and all other variable stay the same, the technology will be worth more and the price can increase. The second variable, a_{1x}, has a negative effect on P_2, if the variable will be higher the probability that $P_2 > P_1$ will be smaller. Also this is straightforward, a_{1x} is the other option for the second firm, the firm has to choose between the new and old technology. The new technology is worth less if the other option, the old technology, is worth more. In that case the firm is willing to pay less for the new technology. The third variable, a_{1y}, is also negative. It has the same effect as a_{1x}, only the effect of a_{1y} is stronger, namely 1.5 instead of 1. This is also logical, if a_{1y} is higher, the new technology is worth more in the first period, which lead to a higher P_1, but also the network externalities are lower, which lead to a lower P_2. That explains why the effect of a_{1y} is stronger than a_{1x}, because it has an effect in both periods. The last variable to explain is b, it has a positive effect on both prices, but a greater positive effect on P_1. So the effect on the price difference is negative, if b is higher the probability that $P_2 > P_1$ will be smaller. This is reasonable, because the effect of b on P_1 is bigger. Which is logical, because the influence of b depends on the i, and the i is higher at P_1, because all firms are still in the market. In the second period the firm the highest i already changed to the new technology, so the remaining firm values the technology less than the firm that already changed.

The other way to look at the price difference was $a_{2y} - a_{1x} - a_{1y} \gtrless \frac{1}{2}(a_{1y} + b)$, with the network externalities on the left side and the price in the first period on the right side. The left side shows the benefits that the remaining firm will have from changing in the second period. The left side shows the benefits that he can gain in the second period ($a_{2y} - a_{1x}$), minus the benefit that he missed in the first period ($a_{1y} - 0$). The following conclusion can be made: when the network externalities are bigger, a price increase is more likely. Which is a quite logical conclusion, because if the network externalities are bigger, the remaining firm values the new technology in the second period more, and is willing to pay more for the technology.

Second period with $n = 0$

Now the case of $n = 0$ will be treated, the producer did not sell anything in the first period. The producer will try to sell the technology to one or both firms, so he will lower the price. Both firms have to take into account that if the other firm's changes in the second period that they cannot jump on the bandwagon, because this is the last period. It is important to know the assumption that both firms do not know about the possible price change. Otherwise the first period will not be the same as in the previous chapter.

In case of $0Y$ the firms have to make a decision between changing to the new technology or continuing using the old one. They will look at the probability that the other firm will change. The firms have to calculate what their payoff is with what probability.

$$\Pr(other\ firm\ Y) = \frac{i_1 - i'_2}{i_1}$$
$$B(Y) = a_{2y} + b * i'_2 - P_2$$
$$B(X) = a_{1x}$$
$$\Pr(other\ firm\ X) = \frac{i'_2}{i_1}$$
$$B(Y) = a_{1y} + b * i'_2 - P_2$$
$$B(X) = 0$$
$$P_2 = a_{1y} + b * i_2$$

Above here the probability that the other firm chooses X or Y are shown, also the benefits (B) for the other firm to choose. This scheme is the same for both firms; they have to make the same decision. There is a difference between i'_2 and i_2, P_2 is set by the producer, and from the P_2 the i_2 can be derived. The i'_2 is the point where the firms are indifferent between switching to the new technology or staying at the old one. First the point of indifference needs to be calculated.

$$\frac{i_1 - i'_2}{i_1} * (a_{2y} + b * i'_2) + \frac{i'_2}{i_1} * (a_{1y} + b * i'_2) - (a_{1y} + b * i_2) = \frac{i_1 - i'_2}{i_1} * a_{1x} + \frac{i'_2}{i_1} * 0$$
$$(i_1 - i'_2) * (a_{2y} + b * i'_2) + i'_2 * (a_{1y} + b * i'_2) - i_1 * (a_{1y} + b * i_2) = (i_1 - i'_2) * a_{1x}$$
$$i'_2 * \left(a_{2y} - a_{1y} - a_{1y} - b * i_1\right) = i_1 * \left(a_{2y} - a_{1y} - a_{1x} - b * i_2\right)$$
$$i'_2 = i_1 * \frac{a_{2y} - a_{1y} - a_{1x} - b * i_2}{a_{2y} - a_{1y} - a_{1y} - b * i_1}$$
$$i'_2 = i_1 * \left(1 + \frac{b * (i_1 - i_2)}{a_{2y} - a_{1y} - a_{1y} - b * i_1}\right)$$

Now that the point where the firms are indifferent is calculated, the profit function for the second period can be made. With the profit function the producer can calculate the point where he can maximize his profit. First the probabilities for the outcomes[5]:

$$\Pr(2Y) = \frac{(i_1 - i_2')^2}{{i_1}^2}$$

$$\Pr(1Y) = 2 * \frac{i'_2 * (i_1 - i'_2)}{{i_1}^2}$$

$$\Pr(0Y) = \frac{{i'_2}^2}{{i_1}^2}$$

$$\pi_2 = \frac{(i_1 - i_2')^2}{{i_1}^2} * 2 * \left(a_{1y} + b * i_2\right) + 2 * \frac{i'_2 * (i_1 - i'_2)}{{i_1}^2} * \left(a_{1y} + b * i_2\right)$$

$$\pi_2 = \frac{2 * \left(i_1^2 - 2i_1 i'_2 + i'^2_2\right) + 2i_1 i'_2 - 2i'^2_2}{{i_1}^2} * \left(a_{1y} + b * i_2\right)$$

$$\pi_2 = \frac{2(i_1 - i'_2)}{i_1} * \left(a_{1y} + b * i_2\right)$$

$$\pi_2 = \frac{2\left(i_1 - i_1 * \left(1 + \frac{b * (i_1 - i_2)}{\left(a_{2y} - a_{1y} - a_{1x} - b * i_1\right)}\right)\right)}{i_1} * \left(a_{1y} + b * i_2\right)$$

$$\pi_2 = -2 * \frac{b * (i_1 - i_2)}{\left(a_{2y} - a_{1y} - a_{1x} - b * i_1\right)} * \left(a_{1y} + b * i_2\right)$$

The profit function is now known, the producer can derive this function to i_2. The i_2 that will be found shows the point where the producer can maximize his profits. The producer has to fill that i_2 in the price function.

$$\frac{\partial \pi_2}{\partial i_2} = \frac{2b}{\left(a_{2y} - a_{1y} - a_{1x} - b * i_1\right)} * \left(a_{1y} + b * i_2\right) + b * \left(-2 * \frac{b * (i_1 - i_2)}{\left(a_{2y} - a_{1y} - a_{1x} - b * i_1\right)}\right) = 0$$

$$2b * \left(a_{1y} + b * i_2\right) - 2b * \left(b * (i_1 - i_2)\right) = 0$$

$$2b * i_2 = b * i_1 - a_{1y}$$

$$i_2 = \frac{1}{2} i_1 - \frac{a_{1y}}{2b}$$

With the found i_2 the producer can calculate his optimal P_2.

$$P_2 = a_{1y} + b * i_2$$

$$P_2 = a_{1y} + b * \left(\frac{1}{2} i_1 - \frac{a_{1y}}{2b}\right)$$

$$P_2 = \frac{1}{2} a_{1y} + \frac{1}{2} b * \left(\frac{a_{1y} - b + \left(1 - \frac{a_{1y}}{b}\right) * \left(a_{1y} + a_{1x} - a_{2y}\right)}{2 * \left(a_{1y} + a_{1x} - a_{2y} - b\right)}\right)$$ [6]

$$P_2 = \frac{1}{2} a_{1y} + \frac{1}{2} * \frac{b - a_{1y}}{2} = \frac{1}{4} * (a_{1y} + b)$$

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[5] $\Pr(2Y) = \frac{i_1 - i'_2}{i_1} * \frac{i_1 - i'_2}{i_1} = \frac{(i_1 - i_2')^2}{{i_1}^2}$

$$\Pr(1Y) = \frac{i_1 - i'_2}{i_1} * \frac{i'_2}{i_1} + \frac{i'_2}{i_1} * \frac{i_1 - i'_2}{i_1} = 2 * \frac{i'_2 * (i_1 - i'_2)}{{i_1}^2}$$

$$\Pr(0Y) = \frac{i'_2}{i_1} * \frac{i'_2}{i_1} = \frac{{i'_2}^2}{{i_1}^2}$$

[6] See page 7 for the steps and i_1

This price can be compared with the price from the previous case, and more important, with the price in the first period. So the movement of the price can be seen if no firm will change to the new technology in the first period.

First the comparison with P_2 in the previous case: $\left(\frac{1}{2} * \left(a_{2y} - a_{1x}\right) + \frac{1}{4} * \left(b - a_{1y}\right)\right)$. It is obvious that the price will be lower. Both prices have $\frac{1}{4} * b$, but in the case of 0Y the only other parameter is negative, while in the case of 1Y two of the three parameters are positive. It is also verbal explainable, because as in the previous case there are no firms left with $i \geq i_1$, but the technology is not valued more as in the previous case. So the price in the case of 0Y must be under the price of 1Y. But also the price must even be lower than P_1, as discussed before, otherwise the producer cannot earn anything anymore.

The price from the first period is two times the price from the second period in this case. $P_1 = \frac{1}{2} * (a_{1y} + b)$ and $P_2 = \frac{1}{4} * (a_{1y} + b)$, which could be explained on the basis of the first period. In the first period the price is on the middle of the part above the zero line, now the part above the zero line is smaller, because the distribution is between zero and i_1. In the second period the producer maximizes his expected profit again, and will be again in the middle of the part above the zero line, which is the half of P_1.

It is important to know that this is the case when there are no costs for the producer to sell the new technology, or to reproduce the technology for both players, otherwise the decision from the producer will be influenced by the costs. This could change the outcome of the fifty percent price drop.

5. Conclusion

A short recap of what is studied in this paper; first the model of Farrell and Saloner is used and adjusted to make a mathematical approach. There were made a few possible scenarios where network externalities had different influence on the pricing or decisions of the firms. The first scenario was without price discrimination, the influence of network externalities we visual in the probability that the remaining firm would change to the new technology if the other firm already changed in the first period. The network externalities influenced the bandwagon where firms choose to follow the other firm. In the second scenario the possible price change was a surprise to all parties, both, the firms and producer, could not anticipate on the price change. In this scenario the price in the first period stays the same as in the first scenario. The price in the second scenario will change, so the producer can optimize his price in the second period again. There were two possibilities for the second period where the prices could change, if one firm or no firm changed in the first period. The producer did not have to set a new price if both firms changed in the first period, because there would be no firm to sell it to.

Results and conclusion

In the first scenario the price that was set was at the half of the part of the graph above the normalized zero line. The optimal pricing of the producer had to weigh the probability of selling the new technology with the revenues, the price. The higher he prices the new technology, the less probability he had that one of the firms, or both firms would change. The start point of the new technology (a_{1y}), had a positive influence on the price, the higher the start point, which means closer to zero, the higher the price would be. Also the slope of the line (b), had a positive influence on the price, the steeper the slope would be, the higher the price would be. The network externalities did not influence the price in the second period, because the price must be the same as in the first period, because price discrimination was not allowed.

The network externalities influenced the bandwagon, if one firm decides to change in the first period, the probability that the other firm would change in the second period becomes bigger. The increased value of the technology $(a_{2y} - a_{1y})$ increased the probability of the change of the remaining firm, also the decreased value of the old technology (a_{1x}) increased the probability of the change.

In the second scenario the i_1 and P_1 could be taken from the first scenario. The price could change in the second period, the two possible scenarios where the price could change were if one firm switched or if no firm switched. When one firm changed to the new technology the price could change both ways, it depends on the size of the network externalities. There are three movements when one firm changed to the new technology, one movement that would decrease the price is that the firm who values the new technology the most already changed. The other firm has a lower i than the changed firm. But there are also two movements which lead to a price increase, the first is the higher value of the new technology, because of the firm that joined the new technology it becomes more attractive. The last movement is the lower value of the old technology, because the changed firm leaves the old technology it becomes less attractive to stay at the old one. The formula for which price would be higher was $a_{2y} - a_{1x} - 1\frac{1}{2}a_{1y} \gtrless \frac{1}{2}b$. The movements are shown in the formula, the first movement is visible in the $\frac{1}{2}b$, because the b has more influence on the price of the first period than the second. The second movement is visible in $a_{2y} - 1\frac{1}{2}a_{1y}$, which shows the difference between the old and new technology. The third movement is visible in $-a_{1x}$, which shows the decrease of the old technology. There cannot be drawn a direct conclusion, but there can be looked at the variables and their influence. The conclusion that can be drawn is the bigger the network externality is, the greater the probability is that the price in the second period is bigger than in the first. So strong network externalities will increase the price in the second period.

The price will drop when no firm switched in the first period, which is a logical conclusion, because if no one would change to the new technology in the first period at the set price no one would buy it in the second period against that price. But how much the price would decrease was questionable. The result for that question was fifty percent. $P_2 = \frac{1}{4} * (a_{1y} + b)$ is the half of $P_1 = \frac{1}{2} * (a_{1y} + b)$. This could be expected because it was again in the part of the graph above the zero line, in the middle. One important thing to know about this result is that there are no costs for the producer to sell another technology to the second firm, otherwise the price could be different.

Recommendations

There are a few recommendations for further research about the influence of network externalities on the price of a new technology. First, the third and fourth possible scenario could be examined, the results could be different from the results of the second scenario. With those scenarios it is very important to look very good at the expectations of the firms about the price change. They will anticipate on possible price changes and their influence on the other firm. If they change the probability that the other firm would change will be bigger. But if both firms do not change the price will be lower, so it is the most profitable for both firms to wait and change in the second period. But if one firm does not change in the second period it would be less profitable, because after the second period the other firm cannot change anymore. Those thoughts have to be included in the model.

Other thoughts for further research are about different slopes between one firm in the new technology or two firms in the new technology. This could be written as b_{1y} and b_{2y}. This could change the pricing in the first and second period. And if $b_{2y} > b_{1y}$ it increases the willingness for the already changed firm that the other firm will change. This in combination with the article of Economides[7], where he does research to subsidies from the quantity leader to other firms to join his technology, can lead to an interesting research. What if the changed firm will pay a part of the price for the remaining firm? Maybe the producer can increase his price more if the changed firm will pay a part of the price.

Another subject for further research is when there are more players and more periods, what influence will the network externalities have on the bandwagon, or what influence will it have on the price? And if the price rises because of the increasing value of the technology, will there be a period that more firms will change because they know that the price will increase.

Also the influence of a subsidy can be reviewed. What if the producer gives a firm who value the new technology less with a subsidy in the first period, so the firm who values it more has to pay a much higher price in the second period? But important in this case is if it is even possible, because there is incomplete information, and the producer does not know which firm values it the least.

[7] See page 3

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