

ERASMUS UNIVERSITY ROTTERDAM

MASTER THESIS

MSC IN ECONOMETRICS AND MANAGEMENT SCIENCE
WITH SPECIALIZATION IN QUANTITATIVE FINANCE

Systemic capital requirements for Nordic banks

Author:

Jokubas MARKEVICIUS

student number: 379331

Academic supervisor:

Dr. Wing WAH THAM

Co-reader:

Dr. H.J.W.G. KOLE



February 4, 2015

Abstract

Maintaining financial stability in the economy requires sufficient capitalization of the banking sector. Due to the interconnectedness and correlated banks' exposures the probability of banks' defaults can be highly amplified in times of economic distress. Therefore, these two factors should be taken into account when setting capital requirements for banks. This paper uses the interbank network construction techniques and the structural credit risk modelling approach within the constrained optimization problem to find optimal systemic capital requirements for the largest banks in the Nordic-Baltic region. Based on the papers by Elsinger et al. (2006) and Webber and Willison (2011), it provides the mapping from the estimated risk levels to the capital surcharges required for keeping systemic risks within tolerable limits. The resulting quantification of the systemic capital requirements can either serve as the supporting indicator for the central bankers for setting banks' capital buffers or the indicator of the stance of the individual bank's riskiness after taking its systemic importance into account.

Contents

1	Introduction	3
2	Common approaches to modelling systemic risk	7
3	Methodology	7
3.1	Estimating the interbank network	9
3.1.1	Maximum Entropy method	10
3.1.2	Minimum Density method	11
3.2	Credit risk modelling	14
3.2.1	Merton structural credit risk model	15
3.2.2	The First Passage model	17
3.2.3	Implementing the credit risk models	18
3.3	Interbank network clearing	19
3.3.1	Eisenberg and Noe algorithm	19
3.3.2	Adding correlation and interbank network clearing to credit risk models	21
3.4	Optimizing over the level of capitalization	22
4	Data	24
5	Results	26
5.1	Interbank network	26
5.2	Aggregate loss distribution	29
5.3	Comparative static exercises: changing the structure of the banking system . . .	32
5.4	Systemic capital requirements	33
6	Conclusions	38
7	Literature	41
8	Appendix	43

1 Introduction

The Great Financial Crisis revealed gaps in the regulation of the banking system as the systemic risk factors were left unaccounted in setting capital requirements. The risks of the banking sector have historically been assessed by considering balance sheets of individual banks regardless their heterogeneity and importance to the whole financial system. In other words, traditional risk management approach has been focused on individual banks exposures rather than risks of the systemic nature. In particular, regulators have ignored two major systemic risk factors: correlation in banks' exposures to non-banks assets and inter-linkages between the banks themselves. Both of them tend to kick in during the stress periods and amplify the magnitude of the losses. In the first case, similarities between banks' asset exposures generate a tendency for banks' solvency positions to deteriorate together when the financial system is affected by a negative macro-financial shock. In the second case, an individual bank's inability to cover its liabilities may lead to its default and trigger contagious losses to other banks which are exposed to the defaulting bank via the interbank lending. Hence, due to systemic risk factors, focusing narrowly on the health on individual banks may leave the banking system under-capitalized and vulnerable.

To ensure financial stability in the economy it is necessary to set banks' capital requirements such that the whole financial system is sufficiently capitalized after accounting for systemic risk factors. Setting higher capital requirements for systemically important banks reduces their credit risk and the possible negative impact to the real economy. In other words, higher capital buffers enables banks to withstand strong macro-financial shocks. On the other hand, setting the requirements too high may induce banks to significantly reduce their lending or increase loan interest rates, thus, creating an undesirable effect to the economy. This can be the case if capital is more expensive than debt and the Modigliani Miller theorem (1956) does not hold (Webber and Willison (2011)). According to J. Vickers (2012), the latter theorem fails then applied to banks because generally their riskiness is lowered by implied guarantees such as government deposit guarantee schemes, potential government bail-outs (in case of solvency problems) or monetary policy actions directed at improving balance sheets of troubled banks. These risk mitigating factors distort markets enabling banks to benefit from cheaper borrowing as opposed to issuing more equity. In fact, Bank of Sweden (Sveriges Riksbank, 2011) estimated that during 2002-2010 55% of the large Swedish banking groups' profits were earned due to indirect government guarantees. Such guarantees reduced average funding interest rates

by about 0.9 percentage point throughout the aforementioned period. Hence, to optimize the trade-off between risk taking and efficiency, a level of capital for the banking system needs to be achieved such that the probability of default is minimized across the banking sector at the minimum cost to the society.

The primary focus of this paper is on maintaining financial stability in the Nordic-Baltic region which is dominated by relatively few large Scandinavian banks as well as their branches and subsidiaries. Therefore, financial stability in such region can effectively be ensured by achieving sufficient level of capitalization of these banks. This level should be just sufficient as holding too much capital may not be beneficial for the society (in terms of reduced and/or more costly lending). The goal of this paper, hence, can be defined as finding systemic optimal level of capitalization for each of the largest banks in the Nordic banking system based on both the balance sheet and market data. The need to include systemic risk factors into the Scandinavian banks credit risk modelling here is of particular importance. First, these banks tend to have relatively similar exposures to non-banks assets. Second, the Nordic banking system is very concentrated and interconnected. According to the Bank of Sweden (2014), high concentration and large interbank network are the main risk sources that can lead to substantial costs to the real economy through the amplification of the losses in the banking sector should a strong macro-financial shock occurs. Furthermore, Nordic banks tend to have relatively small share of capital on their balance sheets which makes them even more sensitive to the systemic risk factors.

To find optimal systemic capital requirements this paper employs a system-wide risk management approach which accounts for correlation of banks' assets as well as the inter-linkages between banks. The research is largely based on the paper by Webber and Willison (2011) in which authors develop a framework for estimating systemic capital requirements for the largest banks in the UK. As in the latter paper, in this research the correlation effect is modelled within the structural credit risk modelling framework while the effect of the interbank lending is induced by using a network clearing algorithm. Together these two effects increase the probability of a bank's default and generate a need for higher and more accurate capital requirements. Then, such systemic optimal capital requirements are calibrated using the optimization procedure, also proposed by Webber and Willison (2011). This optimization is done in two steps. First, the optimal level of total capitalization of the banking system is calibrated subject to the tolerable risk level. Second, the estimated system-wide capitalization

level is further optimized by changing the relative shares of such capitalization held by individual banks. As a result, this paper estimates the level of capitalization required for each bank in the sample such that the risk of the banking sector as a whole is minimized by the subjective level of risk tolerance based on the variance-at-risk of the simulated loss distribution.

Modelling banks credit risk lies at the heart of the optimization. This paper uses two credit risk models: Merton (1974) and the First Passage (1976) where the latter is a more sophisticated and closer-to-real-world extension of the former. Both of these models are popular tools among the risk managers for estimating credit risk of the exchange-traded companies. This paper considers structural credit risk models for particularly three reasons. First, they allow to estimate banks' credit risk from the publicly available market and balance sheet data. Second, they allow to separately estimate the impact of correlation between banks exposures to the banks credit risk. Third, using structural risk models allows solving the constrained optimization problem which in this paper is constructed to calibrate the optimal level of total system-wide capital and the optimal relative shares of such capital held by individual banks. The latter reason is the most significant one as the optimization process is crucial in finding optimal systemic capital requirements.

To account for the interbank market risk factor in modelling banks credit risk this paper uses the estimated interbank networks rather than the observed exposures. Although the actual bilateral data of the interbank lending is preferable as it provides the real world linkages between banks, this type of data is usually unreported for the sake of confidentiality or even unobserved. Hence, to overcome this issue and fill in the bilateral interbank exposure matrix this paper uses two different quantitative techniques: Maximum entropy and Minimum density. The first method provides dense and fully diversified networks while the second method assumes network scarcity and concentration. Although such estimated networks may not be totally precise, they are able to represent the fact that banks with largest interbank exposures are most vulnerable to the default by their counter-party in the financial system. Moreover, using two rather distant network estimation methods adds more robustness to the results. While the Maximum density method tends to underestimate the impact of contagion, the Minimum density method is claimed to overestimate it. In effect, the confidence interval of the true interbank contagion effect is narrowed.

The contribution of this thesis is three-fold. First, it incorporates the interbank network es-

timization techniques into the systemic capital requirements modelling framework proposed by Webber and Willison (2011). Hence, this new framework can be used for more general cases when the bilateral data on interbank lending is not available. Second, in addition to the Merton model used in Webber and Willison (2011), this paper also considers an alternative credit risk model - the First Passage model - which is believed to be more realistic. Third, this thesis uses the data of the largest Scandinavian banks, given the focus on the financial stability in the Nordic-Baltic region.

To estimate systemic capital requirements this paper uses the data of the 6 largest exchange-traded Scandinavian banks. The resulting deviations of the observed banks capitalization from the estimated optimal requirements are expected to be used as a systemic risk indicator. For example, those banks with a substantial or increasing negative deviation are believed to becoming more probable to default, hence, more attention for risk-management of such banks is needed. In particular, central banks in countries where a branch of such bank operates may consider adding a systemic capital add-on or the counter-cyclical capital buffer to reduce the riskiness of such bank. This add-on/buffer may be based on the fractions (depending on the size of the branch) of the quantified capital surcharges estimated in this paper. To explore the dynamics of the systemic optimal capital requirements this paper considers 10 years of time series data from 2004.

The estimated systemic capital surcharges suggest that after taking the correlation and interbank network effects into account all banks were highly under-capitalized during the main stress periods of 2007-2009 and 2012. Moreover, under the market stress scenario the majority of the banks are now required additional capital surcharges in order to reduce their solvency risk. The results supports the worries of the central bank of Sweden that Nordic banks have too little equity and are exposed to the risks of systemic nature. Indeed, high portion of the model-suggested capital add-ons comes from the correlation among banks assets and the high degree of interconnectedness.

The structure of this paper is as follows: in section 2 I compare the common approaches to modelling systemic risk; in section 3 I overview the methodology as well as the model implementation algorithms necessary to achieve the desired results; section 4 gives the overview of the input data; section 5 talks about the results and compares them; finally, section 6 provides with the conclusion.

2 Common approaches to modelling systemic risk

Although there have been several different attempts to estimate the level of systemic risk in the banking sector following the Great Financial Crisis in 2007-2009, they are typically rather limited in terms of explaining how the systemic risk should be reflected in banks' capital requirements.

The majority of the introduced approaches to measure bank's systemic risk have been based on the Value-at-Risk (VaR) type methodologies. In particular, the use of Conditional Value-at-Risk (CoVaR) have become the most propagated among the financial stability institutions in various countries. Brunnermeier *et al* (2009), proposed to use CoVar technique to capture banks' systemic risks by quantifying the extent to which tail risks faced by banks move together. In this set-up, a bank's contribution to the systemic risk can be perceived as the difference between bank's CoVaR and the unconditional $\alpha\%$ VaR for the system. Authors propose to hold capital to protect others in the financial system, however, the mapping of the estimated level of systemic risk to the banks capital requirements is not clear. Acharya *et al* (2010) employ similar approach, only instead of holding more capital, they propose that banks should pay a so called insurance premium which is based on a bank's $\alpha\%$ VaR conditional on the banking system making losses equal to its $\alpha\%$ VaR. Gauthier *at all* (2010) use CoVaR and component VaR in the structural credit risk modelling set-up to allocate a fixed amount of capital in the banking system among the individual banks. In doing so, authors weigh the fixed level of capital by the share of bank's CoVaR contributions in the sum of such contributions. However, this approach does not identify the optimal level of capital in the system and thus can be seen as inferior to the approach used by Lewis and Webber and in this paper. Finally, Tarashev *et al* (2010) measure a bank's contribution to systemic risk using Shapley value concept from cooperative game theory (Shapley (1953)). This approach is similar to this paper in its objective as it seeks to allocate systemic risks fairly across banks, however, it is quite distant in terms of the rationale.

3 Methodology

This paper combines quantitative techniques proposed in different sources of the literature to estimate the bilateral interbank exposure network using the balance sheet data, assess the credit risk of the exchange-traded banks using the market data and optimize the level of capital such

that the systematic risk of the banking sector is minimized subject to the minimal impact on the efficiency. Hence, the complete estimation procedure can be structured in 3 main parts.

In part one, I estimate the unknown interbank lending relationships using 2 quantitative techniques that are rather extreme cases in terms of the resulting (estimated) interbank links. While the Maximum entropy method constructs the network which is as disperse as possible given the constraints (i.e. observed total interbank assets and liabilities), the Minimum density approach operates under the rationale that adding or maintaining interbank links is costly, hence, the network is assumed to be sparse and concentrated. Therefore, the former method is claimed to have a downward bias on the contagious effect while the latter, in theory, overestimates contagion. In order to ensure the robustness of the estimated capital requirements both such methods are considered. In such way, the bounds of the interbank network effect on contagion can be set, limiting the uncertainty. Although estimating interbank network rather than using observed data may not provide a completely accurate picture of the real network, it should give the main insights of where the interbank linkages are concentrated and what the magnitude of the contagious impact of banks' inter-linkages might be. The results in this step will prove useful then estimating the simulated contagious cascades of defaults in further parts of this paper.

In part two I employ the well-known Merton (1974) structural credit risk model and a modification of this model (i.e. the First Passage model) to estimate the asset value of the banks in the sample using the market data on such banks' equity. In doing so I closely follow the papers by Elsinger *et al.* (2006) and Webber and Willison (2011). These papers introduce two sources of systemic risk in modelling the banks' credit risk with structural credit risk models. First, since banks often have same (type of) exposures, in the event of a drop in the prices of such exposures, i.e. during the crisis, banks may experience a simultaneous drop in their asset value. To take this effect into account in the Merton credit risk model, authors of the aforementioned papers introduce the correlation among banks' asset values. Second, to account for the possibility of contagious default cascades as a result of a failure of one bank, the researchers add interbank network exposures into the model. For inducing the contagious effect Webber and Willison (as well as this paper) employ a widely-used interbank clearing algorithm developed by Eisenberg and Noe (2001). Together, these two effects further increase the probability of banks' defaults and the requirement to hold more capital.

In part three, I optimize the level of capitalization for each bank in the sample such that the systemic risk of the whole banking sector is minimized with as little capital as possible (maintaining efficiency). Here I again follow the paper by Webber and Willison (2011) and use their proposed algorithm to obtain the optimal capitalization level of each bank with respect to minimizing the aggregate simulated loss of all banks in the sample.

Figure 1 depicts the broad set-up of the general model (also see figure 34 in appendix). The shaded areas represent bilateral interbank assets and liabilities (i.e. bank 2 holds assets within bank 1 and bank 3). Surrounded by the hatched line are the market values of the banks' balance sheets estimated in steps 1 and 2 (as explained previously in this chapter). From these estimates and including the systemic risk effects (asset correlation and interbank exposure) we can construct a simulated distribution of asset shortfalls below promised debt liabilities. Such distribution can be shifted to the desired level by changing the size of each bank's capital. In other words, the optimal capital levels for each bank can be calibrated (the blue line) so that the tail of the aggregate loss distribution is equal to the chosen target level (for more details see chapter 3.4).

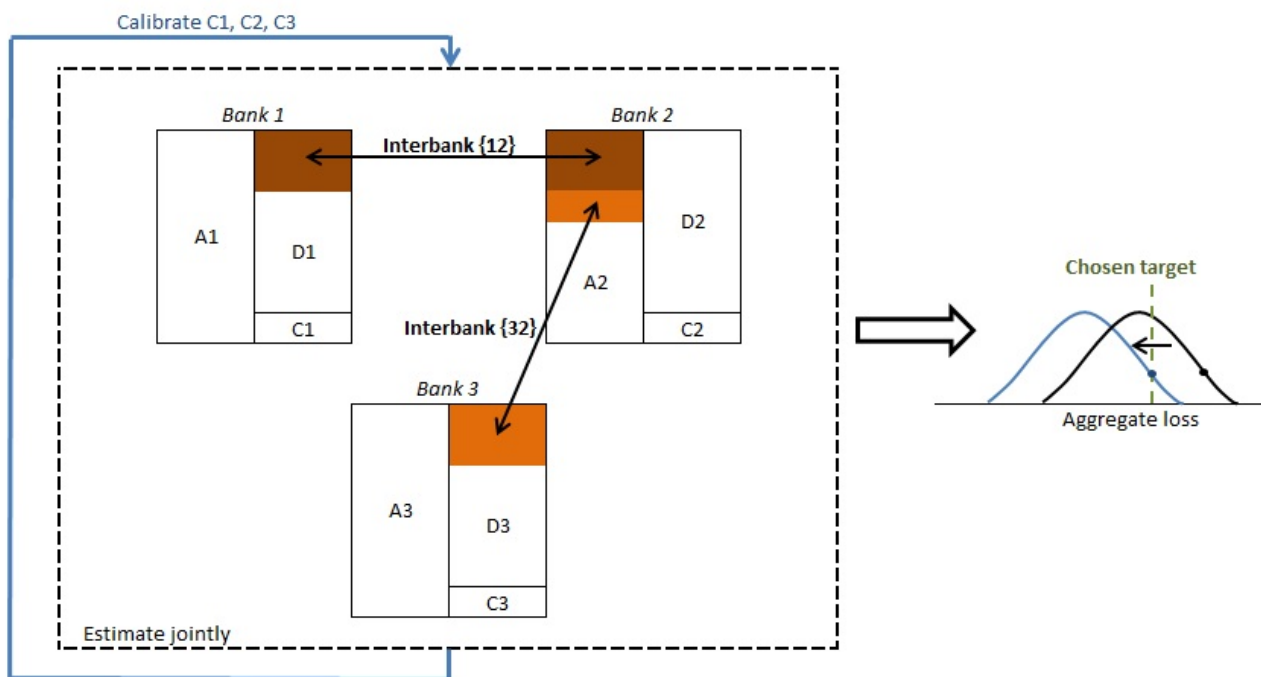


Figure 1: The broad set-up of a general model. A, D and C refer to banks' assets, debt and capital respectively.

3.1 Estimating the interbank network

Interbank linkages are the important characteristic of the financial system. The degree and variability of the interconnectedness in the banking sector provides information on how the

failure of one agent can contagiously affect other agents in the system. Ideally, this information is observed by some supervisory institution in a country. However, often times the data on bilateral banks' exposures is either not collected or not reported due to confidentiality, limiting the possibility of the effective research.

To overcome the issue of data unavailability on interbank exposures, researchers have proposed different quantitative techniques. The leading method in the literature is called Maximum Entropy (Upper 2011, Elsinger et al. 2013). Under this approach the blanks of the interbank liability matrix are filled as evenly as possible knowing only the total interbank assets and liabilities of each bank. The largest link in ME by construction is between a bank with the biggest amount of interbank assets and a bank with the highest value of interbank liabilities and vice versa. Also, it is assumed that banks try to distribute their lending evenly across all other banks in the market. Hence, this technique serves as a good starting point when little is known about the interbank market (one rather general assumption) and it has the advantage of being relatively easy to implement. However, Maximum Entropy method has been criticised (Mistrulli 2011, Upper 2011 and Markose et al.) for introducing a downward bias when the estimated interbank networks are used for stress testing, thus, underestimating the true extent of contagion. Cocco et al. (2009) claimed that interbank networks are typically sparse because interbank activity is based on relationships while Craig and von Peter (2014) argued that smaller banks use a limited set of money centre banks. Taking these findings into account, Kartik Anand *et al.* (2014) proposed an attractive alternative approach called the Minimum density. This method creates sparse and more concentrated interbank lending matrices as compared to the ME. In order to reduce the model risk, this paper considers both of these approaches. The combination of the estimated systemic capital requirements under both methods allows drawing more robust conclusions, i.e. the real network is expected to lie somewhere in between the networks estimated with the two different approaches.

3.1.1 Maximum Entropy method

The Maximum Entropy technique is rather easy to comprehend. Suppose the interbank lending at one point in time is represented by the $N \times N$ matrix X , where N is the number of banks in the system. The element x_{ij} in the matrix X represents the amount bank i lends to bank j . The sum of the columns in row i represents how much bank i is lending to other banks in the system, i.e. its total interbank assets, which can be expressed as $a_i = \sum_{j=1}^N x_{ij}$. Similarly,

the sum of the rows in column j represents the total interbank liabilities of bank j , expressed as $l_j = \sum_{i=1}^N x_{ij}$. If nothing is known about the distribution of bilateral exposures, the matrix X has $N^2 - 2N$ unknowns that need to be estimated. Under the assumption that banks try to maximize the dispersion of their interbank lending, the bilateral exposures could be given by a simple solution $x_{ij}^* = a_i * l_j$. However, to rule out the fact that banks do not borrow from or lend to themselves it is necessary to assume that: $x_{ij}^* = 0, \forall i = j$.

The problem is then to estimate bilateral exposures such that the matrix \hat{X} (obtained by the maximisation) becomes as close as possible to matrix X^* . This is generally obtained by minimising the cross-entropy between the two matrices (Mistrully 2007):

$$\begin{aligned} \min_{\hat{x}_{ij}} \sum_{i=1}^N \sum_{j=1}^N \ln \left(\frac{\hat{x}_{ij}}{x_{ij}^*} \right) \\ s.t. \end{aligned} \tag{1}$$

$$a_i = \sum_{j=1}^N x_{ij} \quad l_j = \sum_{i=1}^N x_{ij} \quad \hat{x}_{ij} \geq 0 \quad \forall j \neq i \quad \hat{x}_{ij} = 0 \quad \forall i = j$$

In this paper this is done by using iterative proportional fitting technique, also known as RAS (Blien and Graef 1997, Elsinger et al. 2013) algorithm.

3.1.2 Minimum Density method

The Minimum density (MD) approach was proposed by Anand, Craig and von Peter (2014) as a more realistic alternative to the Maximum entropy technique. In contrast to the latter, Minimum density method is based on the rationale that interbank linkages are costly to add and maintain, hence, the interbank network is sparse. These costs may be associated with information processing, risk management or creditworthiness checks. Moreover, according to Bench and Atalay (2010) and Iori et al. (2008), the relationships among banks are also disassortative: less-connected banks are more likely to trade with well-connected banks than with other less-connected banks. This reflects the economic rationale that smaller banks, rather than transacting with each other, typically use a small set of money centre banks as intermediaries (Craig and von Peter 2014). Consequently, Minimum Density approach identifies the most probable interbank links, taking into account the cost minimization, and loads them with the largest possible exposure consistent with the balance sheet data on total interbank assets and liabilities.

As with the previously discussed Maximum entropy method, the blanks of the unknown interbank lending matrix X need to be filled knowing only the marginals A_i and L_i (total interbank assets and liabilities respectively). If I set c to represent a fixed cost of establishing a link, then, the Minimum Density approach can be formulated as the constrained optimization problem for the matrix \hat{X} :

$$\min_{\hat{X}} c \sum_{i=1}^N \sum_{j=1}^N I_{[\hat{X}_{ij}>0]} \text{ st.} \quad (2)$$

$$\sum_{j=1}^N \hat{X}_{ij} = A_i \quad \forall i = 1, 2, \dots, N; \quad \sum_{i=1}^N \hat{X}_{ij} = L_j \quad \forall j = 1, 2, \dots, N; \quad \hat{X}_{ij} \geq 0 \quad \forall i, j,$$

where integer function I equals one only if bank i lends to bank j and matrix \hat{X} represents an unknown bilateral interbank exposure network. Vectors A and L represent total interbank assets and liabilities respectively and are thought as the constraints. Then, in the objective function (2), which is used to assign links, the constrains are softened by assigning penalties for deviations from the marginals,

$$AD_i = \left(A_i - \sum_{j=1}^N \hat{X}_{ij} \right) \quad (3)$$

$$LD_i = \left(L_i - \sum_{j=1}^N \hat{X}_{ji} \right),$$

where LD_i measures bank's i current deficit; how much of its bilateral lending falls short of the total amount it needs to raise, L_i . When these penalties are introduced in (2), the objective function that needs to be maximized becomes:

$$V(\hat{X}) = -c \sum_{i=1}^N \sum_{j=1}^N 1_{[\hat{X}_{ij}>0]} - \sum_{i=1}^N [\alpha_i AD_i^2 + \sigma_i LD_i^2] \quad (4)$$

Now, sparse \hat{X} networks that minimize the deviations from the marginals are more efficient and attain higher values in the objective function $V(\hat{X})$.

In addition to being sparse, interbank networks are typically disassortative: small banks seek to match their lending and borrowing needs through relationship with larger banks that are well placed to satisfy those needs. The relevant measure of size here is a bank's current surplus AD_i and deficit LD_i to be met in the interbank market. Let $Q \equiv \{Q_{ij}\}$ be the set of probabilities for relationships between i and j . The probability that i lends to j increases if either i is large

lender to a small borrower j , or i is a small lender to a large borrower j :

$$q_{ij} = \max\left\{\frac{AD_i}{LD_j}, \frac{LD_j}{AD_i}\right\} \quad (5)$$

$$Q_{ij} = \frac{q_{ij}}{\sum_{i=1}^N \sum_{j=1}^N q_{ij}},$$

where N is a number of banks in the market and q is an auxiliary criterion that gives a higher probability Q to a link being chosen if either the asset deficit of bank i is high relative to the liability deficit of bank j or the liability deficit of bank j is high relative to the asset deficit of bank i .

According to the objective function (4), networks with a lower density have higher values. At the same time, the priors Q specify the characteristic interbank feature that small banks typically have links with large banks. To generate networks that are both sparse and disassortative, Anand, Craig and von Peter propose a trade-off mechanism. Let $P(\hat{X})$ be the probability distribution over all possible network configurations which is derived by maximizing the sum of two terms. The first is the expected value of networks: $\sum_{\hat{X}} V(\hat{X})P(\hat{X})$ - networks that have few links and a high value should be more likely. At the same time, to ensure that the most likely network solutions are disassortative, the probability distribution P should be close to our prior Q . Formally, this is achieved by maximizing the relative entropy $\theta R(P\|Q) = \theta \sum_{\hat{X}} P(\hat{X}) \log(P(\hat{X})/Q(\hat{X}))$ between the distribution P and prior Q , and where θ is a scaling parameter that weighs the information in the prior against the information incorporated within the objective function. Thus, the distribution, P , over possible network configuration can be derived as the solution to:

$$\max_P \sum_{\hat{X}} P(\hat{X})V(\hat{X}) + \theta R(P\|Q), \quad (6)$$

which can be solved using the first order condition as:

$$P(\hat{X}) \propto Q(\hat{X}) \exp^{\theta V(\hat{X})} \quad (7)$$

In words, this expression means that a candidate \hat{X} has a higher likelihood of being chosen than the prior Q specifies if the departure from Q raises the value of the objective (4) which defines the maximization problem.

Anand, Craig and von Peter proposed a computationally feasible heuristic algorithm to handle

large sets of data, based on simulated annealing technique. However, as this paper deals with a relatively small sample of banks, the algorithm is modified to reduce the randomness of the resulting estimated interbank matrices. The Minimum Density estimation algorithm can be defined as follows:

- At each iteration a link (i, j) is selected with probability Q_{ij} , where Q_{ij} is defined as in (5). Since there are only 6 banks in the sample and it is desirable for the results to be stable, the randomness effect is eliminated by raising the auxiliary criterion q_{ij} (see equation 5) to the power of n (where n is a large number). In this way, at each iteration the most probable link is selected.
- Next, the exposure \hat{X}_{ij} is loaded with the maximum value that this pair of banks can transact given their current asset and liability positions, i.e. $\hat{X}_{ij} = \min(AD_i, LD_j)$
- If adding this link increases the value function, $V(\hat{X} + \hat{X}_{ij}) > V(\hat{X})$, the allocation is retained.
- If, however, the addition of \hat{X} diminishes the value function:
 - a) The link is retained as long as the network including \hat{X} is more likely than without the link, i.e., with probability $P(\hat{X} + \hat{X}_{ij})/P(\hat{X}) \simeq \exp(V(\hat{X} + \hat{X}_{ij}) - V(\hat{X}))$.
 - b) The link is otherwise rejected.
- Finally, once positions have been updated, proceed to the next iteration until the total interbank market volume has been allocated.

This paper sets the cost of establishing the interbank relationship $c = 0$ as it can be assumed that these type of cost effects for large banks are not very significant. Moreover, the scaling parameter θ is set to be equal to 1 as this paper assumes the information in prior and in the objective function to be of the same importance.

3.2 Credit risk modelling

Modelling banks' credit risk lies at the core of the estimation of the systemic capital requirements. This paper employs structural risk modelling framework for the estimation of the credit risk of each bank in the sample. In this setting, the value of the exchange-traded company is modelled within the option pricing theoretical framework developed by Black-Scholes (1973). The rationale here is that the pay-off of holding equity is identical to the pay-off of holding

a call option (see figure 2). Equity holders have the right but not the obligation to sell their shares. Hence, the market price of the equity is thought as the price of the call option with the asset value representing the price of the underlying and the default threshold mimicking the strike price. The use of structural risk models, then, allows solving the constrained optimization problem, proposed by Lewis and Webber (2011), to obtain the optimal level of capital in the banking system. This paper considers two types of credit risk models. The first was developed by Merton (1974) and became a popular choice for credit risk managers since then. The second - the First Passage model - is the extension of the former and is often claimed to be more realistic in terms of less strict assumptions than in the case with the Merton model.

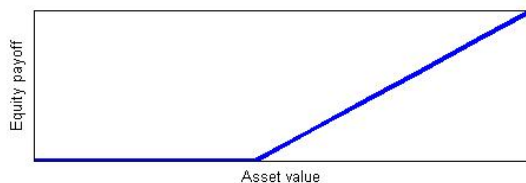


Figure 2: Structural credit risk model rationale

3.2.1 Merton structural credit risk model

The standard method to model company's credit risk from the market information is based on the paper by Merton (1974). Author assumes that the capital structure of a bank is comprised of two basic elements: zero-coupon debt and equity. If the bank's assets were insufficient to pay the face value of its debt when it fell due the bank would default. Figure 3 illustrates the latter argument by suggesting different possible paths that a bank's asset value can potentially follow from time t (today) to time T (future reference day). If a bank's asset value fell below its debt liabilities at time T , equity holders would receive nothing and bond holders would recover whatever the bank's assets were after paying bankruptcy costs. Therefore, we can model the market value of the bank as the price of the call option for the equity holders, with the market value of bank's assets regarded as the underlying asset and the value of the zero-coupon debt as the strike price.

Assume that the market value of the bank's asset follows a Brownian Motion:

$$\frac{dV_{ti}}{V_{ti}} = \mu_i dt + \sigma_{vi} dW_{ti}^P, \quad (8)$$

where μ_i and σ_{vi} is the mean and standard deviation of the logarithmic incremental returns of the asset value of bank i , and W_{ti}^P is the bank i specific Brownian Motion at time t under the

real world probability measure P . In addition, the shocks may be correlated across banks:

$$dW_i^P dW_j^P = \rho_{ij} dt \neq 0, \quad (9)$$

where ρ_{ij} is the correlation coefficient between two Brownian Motions at time t . In fact, the correlation between random shocks creates the desired systemic risk factor effect. Then, using the Black-Scholes formula for a call option we have:

$$E_0 = V_0 N(d_1) - D \exp(-rt) N(d_2), \quad (10)$$

where E_0 and V_0 represent the value of bank's equity and assets at time $t = 0$, D stands for the default threshold, r - for the market interest rate, T - for a time horizon in years and N - for the standard normal distribution. Then, from Black-Scholes formula we know that:

$$d_1 = \frac{\ln(V_0/D) + (r + \frac{\sigma_v^2}{2})T}{\sigma_v \sqrt{T}} \quad (11)$$

$$d_2 = d_1 - \sigma_v \sqrt{T} \quad (12)$$

Using the Ito lemma we can show that:

$$\sigma_E E_0 = \frac{\delta E}{\delta V} \sigma_v V_0 = N(d_1) \sigma_v V_0, \quad (13)$$

which can be rearranged to look like:

$$\sigma_v = \left(\frac{V_0}{E_0} \frac{\delta E}{\delta V} \right)^{-1} \sigma_E \quad (14)$$

V_0 and σ_v are unobserved and need to be estimated. Merton (1974) proposed to calibrate these unknowns by solving equations (10) and (14) simultaneously. However, this method is claimed to be rather sensitive to the starting values of the unknowns. Instead, this paper follows the technique implemented in the paper by Webber and Willison (2011). The authors first invert equation (10) to obtain the series of asset values; then use the Maximum Likelihood procedure, proposed by Duan (1994), to estimate the standard deviation of the change in the logarithmic process of this series, i.e. σ_v . The expected likelihood function for Merton model which needs

to be maximized can be written as:

$$L^E(\mu_v, \sigma_v; E_1, E_h, \dots, E_{nh}) = \ln \left(\prod_{k=1}^n \frac{1}{\sqrt{2\pi dt} \sigma_v} \exp \left(- \frac{\left(\ln \left(\frac{\hat{V}_{\sigma_k}}{\hat{V}_{\sigma(k-1)}} \right) - (\mu_v - 0.5\sigma_v^2) dt \right)^2}{2\sigma_v^2 dt} \right) \right), \quad (15)$$

where it is assumed that the asset returns follow the log-normal distribution, hence, the mean of such returns can be estimated as:

$$\hat{\mu}_{v_i} = \frac{E[\ln V_{i,t+dt}] - E[\ln V_{i,t}]}{dt} + 0.5\sigma_{v_i}^2, \quad (16)$$

and σ_v is calibrated so that $\max_{\sigma_i} (L(\sigma_i))$ is obtained (see section 3.2.3).

3.2.2 The First Passage model

Due to its oversimplifying set of assumptions Merton model is often assumed to be quite unrealistic; by construction it tends to overestimate companies' credit risk. Indeed, under the Merton approach it is assumed that the company can only default at time T . In other words, despite the path of the company's asset value up to time T , the losses or gains to the equity holders can only be evaluated at the maturity. However, in reality a firm can default at any time and the investors should be aware of that and hence incorporate this risk into the price of the equity. Therefore, for the same market value of the equity, the market asset value of the company at time $t = 0$ should be higher.

To capture the effect of a possibility of a bank reaching default barrier at any time up to T this paper considers the First Passage model. This model estimates the value of the firm by using the pricing of the down-and-out option technique. In this case, the value of a bank's equity at time T is $E_T = (V_T - D)^+ 1_{\{\min_{0 \leq t \leq T} V_t > K\}}$, hence the formula for estimating equity:

$$E_t = C_B(V_t, \sigma_v, r, D, T) + D e^{-rT} \left(\frac{K}{V_t} \right)^{2r/\sigma_v^2 - 1} N(h_-) - V_t \left(\frac{K}{V_t} \right)^{2r/\sigma_v^2 + 1} N(h_+), \quad (17)$$

where C_B is the price of the European call option, which in this setting can be expressed as E_0 in equation (10). In addition, K is the default threshold up to time T which in this case is set

to be equal to D ; and

$$h(\pm) = \frac{\ln\left(\frac{K^2}{DV_t}\right) + (r \pm 0.5\sigma_v^2)T}{\sigma_v\sqrt{T}}, \text{ when } D \geq K \quad (18)$$

$$h(\pm) = \frac{\ln\left(\frac{K}{V_t}\right) + (r \pm 0.5\sigma_v^2)T}{\sigma_v\sqrt{T}}, \text{ when } D < K$$

Figure 4 illustrates the First Passage model's assumption of a firm's default up to time T . Now, in order to calibrate the unknown variables V_t and σ_v , this paper uses the Maximum Likelihood estimation technique proposed by Duan (1994). According to Duan, one needs to maximize the expected likelihood function, which in the case for the FPM is defined as:

$$L^E(\mu_v, \sigma_v; E_1, E_{dt}, \dots, E_{dtn}) = \ln \left(\prod_{k=1}^n \frac{1}{\sqrt{2\pi dt} \sigma_v} \exp \left(-\frac{\left(\ln \left(\frac{\hat{V}_{\sigma k}}{\hat{V}_{\sigma(k-1)}} \right) - (\mu_v - 0.5\sigma_v^2) dt \right)^2}{2\sigma_v^2 dt} \right) \right) - \sum_{k=1}^n \ln \left(N \left(\frac{\ln \left(\frac{\hat{V}_{dtk}(\sigma)}{F} \right) + (r + 0.5\sigma_v^2)(T - dtk)}{\sigma_v \sqrt{T - dtk}} \right) \right) \quad (19)$$

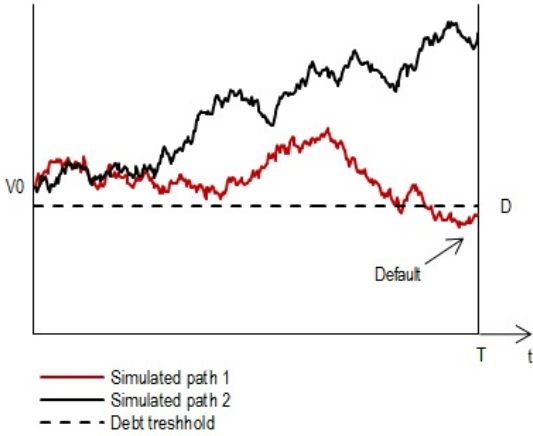


Figure 3: Merton model: it is assumed that a firm defaults if its market value V crosses the default threshold (e.g. total value of its liabilities D) at time T . Simulated path 1 here is a defaulting path while simulated path 2 is a non-defaulting path.

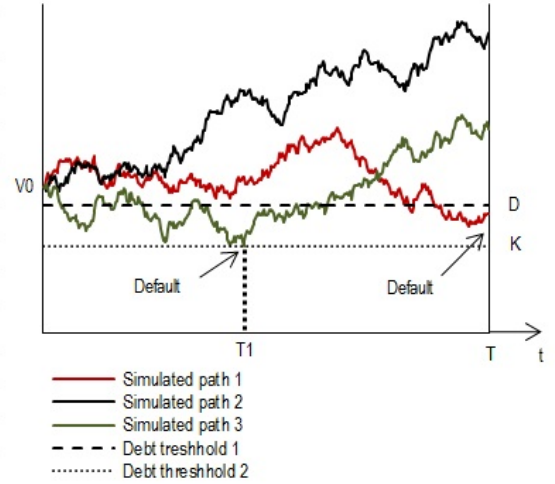


Figure 4: The First Passage model: it is assumed that a firm defaults if its market value V crosses the default threshold (e.g. total value of its liabilities D) at time T . In addition, it is assumed that a firm defaults if at any time up to T its market value falls below threshold K (e.g. at time $T1$). Simulated paths 1 and 3 here are the defaulting paths while simulated path 2 is a non-defaulting path.

3.2.3 Implementing the credit risk models

In calibrating the asset values of the banks in the sample, this paper relies upon the iteration procedure used in the paper by Webber and Willison (2011). For each bank $i_{i=1,2,\dots,n}$:

- Choose exogenously an initial value of the unobserved volatility of asset returns σ_i .
- Using time series of observed bank equity prices and the risk-free spot interest rate of maturity $\tau = \tau_i$ over a given time interval $t \in [-\tau, 0]$, invert the credit risk model (either Merton or FPM) to back-out a corresponding time series of asset values $V_{i,t}$ over interval $t \in [-\tau, 0]$. In this paper for inverting the credit risk models I use the *fsolve* function in Matlab.
- Calculate an estimate for the drift rate for bank i , $\mu_i = \hat{\mu}_i$, from equation 16.
- Compute the value of the log-likelihood function in equation (15) for Merton model and (19) for FPM from $\sigma_i, V_{i,t}(\sigma_i)$ and $\hat{\mu}_i(V_{i,t}(\sigma_i))$.
- Numerically solve for σ_i such that $\max_{\sigma_i} (L(\sigma_i))$ is obtained (this paper uses the *fminsearch* function in Matlab). This yields the maximum likelihood estimate of the diffusion parameters μ_i, σ_i for each bank viewed separately.
- If estimated σ_i is the same (with a particular tolerance level) as in the previous iteration, stop the algorithm. If, however, σ_i differs from its value at the previous iteration, start from step 2 using a newly estimated σ_i .

3.3 Interbank network clearing

For the model to be capable of capturing the risks stemming from the interlinkages between banks, an appropriate algorithm with the property of inducing cascade defaults is needed. In other words, if one or more banks in the system defaults on its interbank liabilities, other banks will lose a proportion of their interbank assets which might imply their default as well. This paper uses the network clearing algorithm proposed by Eisenberg and Noe (2001).

3.3.1 Eisenberg and Noe algorithm

Eisenberg and Noe (2001) developed a clearing mechanism that solves the interbank payment vectors of all banks in the system simultaneously. As of today, this mechanism is the leading method for estimating the cascades of default in the financial system.

To implement the Eisenberg and Noe network clearing algorithm, I come back to the interbank exposure matrix X presented in section 3.3. Once again, suppose that total interbank

liabilities by bank i are expressed as $L_i = \sum_{j=1}^N X_{ij}$. Next, I define the relative liabilities matrix $\Pi \in R^{n \times n}$ by :

$$\pi_{ij} = \begin{cases} X_{ij}/L_i & \text{if } L_i > 0 \\ 0 & \text{otherwise} \end{cases} \quad (20)$$

Then, following the Eisenberg and Noe (2001), I denote $e_i \geq 0$ the net assets of bank i from sources outside the banking system. In this paper I assume that e_i represent the asset surplus over the liabilities at time T . The corresponding vector of net assets is denoted by e . Now, I can define the financial system as (X, e) , where X can be thought as a liabilities matrix. Eisenberg and Noe consider a clearing vector which specifies payments between the banks in the system which are consistent with the three rules:

1. Limited liabilities: Each node never pays more than its available cash flow.
2. Priority of debt claims over equity: Paying off the liabilities X_{ij} has priority, even if the external assets e_i have to be used for that.
3. Proportionality: If default occurs the defaulting bank pays all claimant banks in proportion to the size of their nominal claims on the assets of the defaulting bank.

Given these rules, a clearing vector for the financial system (X, e) can be defined as a vector $L^* \in [0; L]$, such that $L^* = \phi(L^*)$, where ϕ is the function defined by:

$$\phi(X)_i = \begin{cases} L_i, & \text{if } L_i \leq e_i + \sum_{j=1}^n X_j \pi_{ij} \\ L_i = e_i + \sum_{j=1}^n X_j \pi_{ij} & \text{else} \end{cases} \quad (21)$$

The clearing vector L^* can be interpreted as the cash which bank i has available to pay ot to other banks. The value of the assets available to bank i will be the sum $e_i + \sum_{j=1}^n X_j \pi_{ij}$, and if this is at least L_i , then bank i is able to meet its obligations. If this inequality does not hold, then bank i is in default and must call in its assets.

To determine a clearing vector I employ the following iterative algorithm:

- initialize $L^* = L$, and calculate the net value of bank i , $V_i = e_i + \sum_{j=1}^n X_j \pi_{ij} - L_i^*$. If $\forall i, V(i) \geq 0$, it mean no bank defaults and the clearing payment vector is $L^* = L$, algorithm terminates; otherwise proceed to the next step.
- Find banks with net value $V < 0$ and denote them by U . These banks can only pay part of their liabilities to other banks. Estimate the ratio $\theta = ((e_i + \sum_{j=1}^n X_j \pi_{ij})/L_i^*)$. Under

the assumption that only banks in U default, replace X_{ij} by $\theta \times X_{ij}$, so that the limited liability criterion is met, and thus get new X_{ij}, Π_{ij}, L_i^* and V_i . Return to step 1 and continue the iteration until U is empty - no more cascade defaults occur.

3.3.2 Adding correlation and interbank network clearing to credit risk models

Having defined the estimation procedure of interbank lending matrix as well as the clearing vector, I now move forward to introduce the systemic risk effects in the Merton and the First Passage models described in section 3.2. In doing so I follow the papers by Elsinger, Hehar and Summer (2006) as well as Webber and Willison (2011).

In section 3.2.3 I showed how the iterative algorithm is used to estimate the asset values and the diffusion parameters (μ_i, σ_i) for each bank in the sample viewed separately. Now I move further and introduce the correlation between banks' asset returns into the credit risk model. I estimate the realised variance-covariance matrix Σ between banks' asset returns and corresponding correlation structure from the estimates of $A_{i,t}$ over period $t \in [-\tau, 0]$ recovered in section 3.2.3. Next, I re-compute asset values for each bank $i_{i=1,2,\dots,n}$ using new variances (diagonal elements) of the full variance-covariance matrix Σ . Then, as in Webber and Willison (2011), I simulate forward the correlated asset value distributions for the system of banks $i_{i=1,2,\dots,n}$ for a period $t \in [0, T]$ using the Cholesky decomposition of the correlation matrix. Figure 5 depicts this simulation procedure. Next, for each bank in the sample, I calculate the distribution of asset shortfalls below the promised debt liabilities based on their fundamental solvency positions before accounting for the default cascades. Finally, I clear the network of interbank exposures using the Eisenberg and Noe (2001) algorithm described previously in the paper and mark down the assets of any contagiously failing banks from the value reached under the diffusion process at time T . I complete this step by identifying the z th percentile of the resulting loss distribution for the system as a whole aggregating across banks.

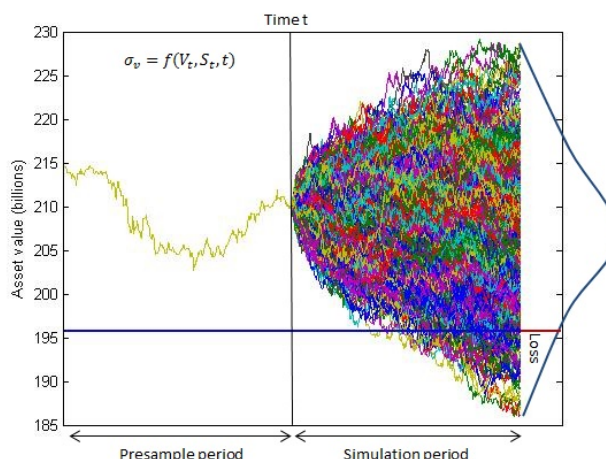


Figure 5: Introducing correlation to credit risk model. The pre-sample periods refers to a period in which the series of the banks' asset values V_i are estimated using the maximum likelihood procedure. The logarithmic changes of these asset values are then used to estimate the realized variances σ_v for each bank in the sample as well as realized μ_v and the correlation matrix. Next, using these diffusion parameters and the correlation structure I simulate forward the correlated asset values for each bank from time t to time T . At time T I obtain the distribution of the simulated asset values for each bank which are needed in the capital optimization step

3.4 Optimizing over the level of capitalization

The final step in this framework of modelling systemic capital requirements for banks is finding the optimal level of total capital in the banking system as well as the optimal proportions of such capital held by individual banks. In doing so this paper relies on the calibration procedure proposed by Webber and Willison (2011).

The simplest way to reduce the credit risk in the banking sector would be to impose high capital requirements. However, holding high levels of capital is costly for banks and can also be costly for the society. Not only large levels of capital can result in a forgone revenue for banks, smaller ROE's (return-on-capital) for investors but also too much capital reduces social welfare as the lending to the economy becomes constrained. Therefore, having a lot of equity to insure against the systemic risk can bring more damage to society than positive impact related to reducing the credit risk. According to Webber and Willison (2011), a social planner faces a trade off between systemic risk and the efficiency of the banking sector at the same time. The efficiency of the banking system is a decreasing function of the total capital held across banks, thus, policy-makers confront a non linear constrained optimization problem in which they need to minimize the level of capital held by each bank in the system (i.e $\min_{i=1,2,..,n}(\sum_i C_i)$), subject to the chosen systemic risk objective. Webber and Willison (2011) propose to define this systemic risk objective in terms of a target for the location of the z th percentile of the distribution

of system losses relative to promised debt liabilities:

$$\min_{\{C_i\}_{i=1,2,\dots,n}} \left(\sum_i C_i \right) \quad s.t. \quad VaR_z^{system}(\{C_i\}_{i=1,2,\dots,n}) = 0 \quad (22)$$

In other words, the policy-maker tries to minimise inefficiency (total capital) subject to the banking system remaining solvent with a chosen target probability and the parameter z reflects the trade-off between the systemic risk and efficiency objectives.

The estimation of C_i for all banks in the sample as in equation (22) is rather difficult and computationally cumbersome owing to many non-linearities in the estimation techniques discussed earlier in this paper. However, Webber and Willison (2011) describes a rather efficient (although still quite time consuming) calibration algorithm based on iteration. The algorithm goes as follows:

1. Optimize over the total level of system capital $\sum_i C_i$, holding fixed the relative shares held by each bank. In this paper it is assumed that $z = 95$ percent. This total level of capital is observed at time $t = 0$, i.e. before the simulation of the correlated asset return distributions is performed. The optimization is done by increasing (decreasing) $\sum_i C_i$ at time $t = 0$ if $VaR_z^{system}(\{C_i\}_{i=1,2,\dots,n}) > (<)0$. This effectively anchors the approximate level of capital that the system requires overall to be sufficiently robust. In this paper this is done by minimizing the objective function using the *fminsearch* tool in MATLAB.
2. Adjust the share of aggregate capital held by each bank, $(C_i)_{i=1,2,\dots,n}$ to $(\tilde{C}_i)_{i=1,2,\dots,n}$, such that the chosen measure of systemic risk is reduced, $VaR_z^{system}(\{\tilde{C}_i\}_{i=1,2,\dots,n})$. If this is possible, the allocation $(\tilde{C}_i)_{i=1,2,\dots,n}$ must be superior to $(C_i)_{i=1,2,\dots,n}$ because systemic risk is lowered for the same level of efficiency since $\sum_i \tilde{C}_i = \sum_i C_i$. In this paper, I perform this step using a *fmincon* function in Matlab.
3. Reduce the level of system capital by a small amount ϵ , allocated pro-rata across banks, and perform the optimization in the previous step again. Since reducing the total capital level by a very small amount at each iteration would require enormous computational time, I follow the Webber and Willison and reduce the total system capital in increments of 1 percent of its initial level. According to the authors, smaller perturbations had been found to have little impact on the results.
4. Repeat steps (1)-(2) until it is no longer possible to further reduce system capital and simultaneously achieve the policymakers' chosen tolerable level of systemic risk. This

yields the minimum level of aggregate capital that can be allocated across banks and simultaneously meet the chosen systemic risk constraint.

4 Data

The Nordic-Baltic region is dominated by 6 Scandinavian-based large exchange-traded banks, their branches and subsidiaries. These six banks differ significantly from other competitors in terms of the size of their market share and their balance sheets. Every one of them can be considered as too-big-to-fail and thus are of huge importance to the financial stability of the region. Four of these banks are based in Sweden, one in Norway and one in Denmark. Figure 6 provides a brief overview of these banks. With the book asset value of EUR 641 billion Nordea was the largest bank in Scandinavia in 2014 01, followed by Danske with EUR 430 billion. Together, these two banks accounted for half of the aggregate six banks' assets. SEB, Handelsbanken and DNB had balance sheets of similar size at around EUR 290 billion while Swedbank's assets were slightly lower at EUR 216. On the other hand, in 2014 01 Swedbank was the most capitalized bank in the sample group with its equity amounting to more than 10 % of the asset value. Danske and Nordea, on contrary, were the most leveraged with capital-to-assets ratio at 3.9 % and 6.2 % respectively. Such ratio of SEB, Handelsbanken and DNB stood at between 7 % and 8 %. In 2014 01 the most interbank assets relative to total balance sheet assets had Danske (8 %), followed by Swedbank (7 %). Such ratios of SEB and Handelsbanken were at around 6 % while Nordea and DNB were least exposed to the interbank market with the interbank assets-to-total assets ratio at 3 % and 2,1 % respectively. The level of interbank assets held by each bank, however, varied significantly during 2004 - 2014. Hence, the role of interconnectedness as the systemic risk factor varied as well. It was larger before the Global Financial Crisis and has diminished since then. With the abundance of liquidity in the international money markets followed by the expansionary monetary policy actions by the major central banks the interbank market has been shrinking. However, for some banks interbank exposures remain significant part in their balance sheets, thus, the risk of interbank contagion is still present.

Bank	Country	Total assets 2014-01 (billions euro)	% total assets of aggregate system asset value 2014-01	% total assets of aggregate asset value max	% total assets of aggregate asset value min	% equity of asset value 2014-01	% equity of asset value max	% equity of asset value min	% interbank assets of total assets 2014-01	% interbank assets of total assets max	% interbank assets of total assets min
SEB	Sweden	287,7	13,3	16,2	11,8	7,3	8,0	0,8	6,4	13,9	5,8
SWEDBANK	Sweden	216,3	10,0	11,4	9,3	10,7	10,9	0,5	7,0	14,0	6,4
NORDEA	Sweden	640,8	29,7	32,6	23,7	6,2	9,3	2,0	3,0	10,3	2,9
HANDELSBANKEN	Sweden	291,2	13,5	14,5	11,5	7,8	8,8	2,6	6,2	10,8	5,0
DANSKE	Denmark	429,9	19,9	27,7	19,2	3,9	6,7	0,7	8,0	21,0	4,4
DNB	Norway	291,1	13,5	14,9	8,5	7,2	10,3	1,3	2,1	6,7	1,4

Figure 6: Summary of the banks in the sample. Min and Max correspond to the minimum and maximum values from 2004 to 2014 respectively.

Each of the aforementioned Scandinavian banks has either their subsidiaries or branches which dominate in other Nordic-Baltic countries. Hence, the failure of one of such bank not only would directly affect the domestic country, but would also spread to other countries in the region through channels such as increased funding cost for subsidiaries/ branches, frozen interbank market due to the loss in confidence and subdued confidence among the depositors and investors both for the subsidiaries/branches of the failing parent bank and for other banks in the region. Ensuring financial stability in the Nordic-Baltic region thus can be defined as limiting credit risk of these 6 banks.

The estimation of structural credit risk models requires both the balance sheet data and the market data. Both of these types of data are available at the *BLOOMBERG* terminal. Regarding the market data, I use daily observations of the market capitalization of the 6 banks in the sample (see figure 7) as well as the 12 months Euribor as a proxy for the market interest rate. With regards to the balance sheet data, I use the quarterly observed data on total short term and long term liabilities. I define the default threshold to be equal to total liabilities for each bank minus its total interbank liabilities. In addition, I set both the pre-sample period τ and the estimation period T to be equal to 1 year. Therefore, the loss distribution at date $t=0$ includes the simulated asset values below the default threshold at date $t=T$. For simulating the asset values I use 1000 paths each time. To ensure convergence, it is important that the random errors which are used for simulations would remain the same during all optimization steps. Therefore, these errors are generated and stored before the beginning of the algorithm.

For constructing the interbank network of bilateral exposures, I use quarterly observations on banks total interbank assets and total liabilities to other financial institutions (interbank liabilities). Since this quarterly data is very volatile due to accountancy reasons, I try to smooth it by using yearly averages (i.e. averages of the observed values in 4 previous periods, see figure 8). Averaging such type of data helps avoid spikes in the estimation of system loss VaR, how-

ever, leaving the model still representative of the real world. Furthermore, as the assumption of the 6 banks in the sample being bilaterally interconnected only between themselves can be too strict, I use 50 per cent of banks total interbank assets and liabilities to construct the interbank networks. Indeed, Nordic banks tend to use other global banks or money funds to obtain their short term funding which usually falls under the definition of liabilities to other financial institution which is assumed to be a proxy for interbank liabilities. Similarly, banks in the sample are expected to have interbank assets with financial institution based in other regions. Therefore, an assumption that 50 % of all interbank assets and liabilities of the banks in the sample are distributed among these institutions is a rather reasonable one.

For the estimation of system-wide VaR of the aggregate loss distribution at date T I use the subjective risk tolerance level of 5 per cent. Finally, to obtain the dynamic and illustrative results, I consider a 10 years period from 2004-07-01 to 2014-07-01. For the results to be still illustrative but less time-consuming, the loss distribution and the systemic optimal capital requirements are estimated at every half a year.

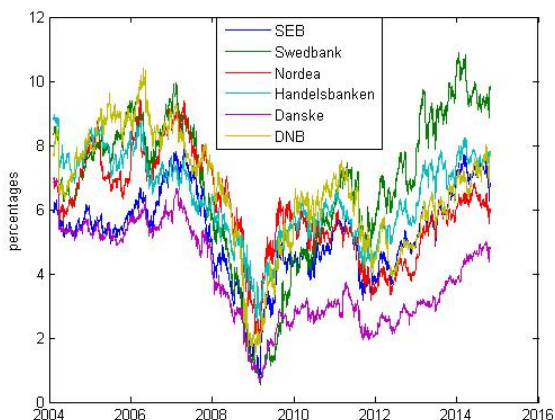


Figure 7: Market capitalization as the share of book value of total assets

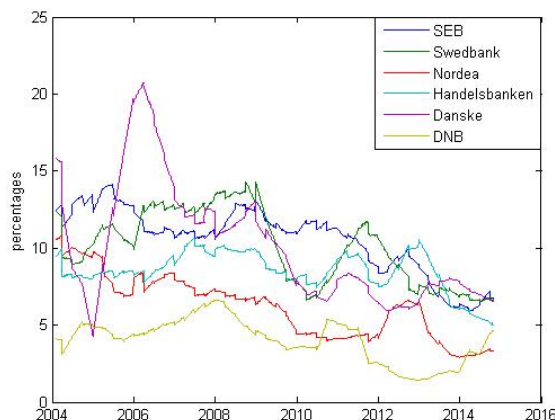


Figure 8: Interbank assets as the share of book value of total assets

5 Results

5.1 Interbank network

I estimate the interbank matrix of bilateral exposures using two methods described in section 3.1.1. The estimations are done at every half a year from 2004 to 2014, a sample period discussed in the data section. Figures 9 and 10 provide estimated matrices of the interbank network using Maximum Entropy and Minimum Density methods respectively for 2014-01-01.

As expected, the results obtained by two different approaches are very different. The network estimated under the Maximum Entropy approach is dense (the matrix has a full rank) while the one estimated using a Minimum density technique is sparse and concentrated. In other words, under the former method, banks diversify their interbank lending at the maximum possible level while the latter method assumes that the number of interbank relationships that each bank has is limited.

Figure 9 shows the interbank network estimated under the Maximum Entropy method. As expected, the largest in value links are between Nordea and Danske. This comes from the fact that these banks have largest volumes of total interbank assets and liabilities. Indeed, as the algorithm suggests, the values of the interbank links estimated under the Maximum Entropy method are proportional to the total values of banks' interbank assets and liabilities. Figure 10, on the other hand, shows a completely different picture. The provided interbank matrix is sparse - only few interbank links are suggested. The largest banks - Nordea and Danske - have more than one bank which they are liable to via the interbank market while the rest of the banks have only one interbank counter-party on a liability side. Moreover, smaller banks, such as DNB and Handelsbanken are assumed to be exposed to only one other bank, and the rest of the banks have two counter-parties in terms of interbank assets.

	SEB	SWEDBANK	NORDEA	HANDELSBANKEN	DANSKE	DNB
SEB	€ -	€ 354	€ 4.315	€ 877	€ 3.690	€ 168
SWEDBANK	€ 767	€ -	€ 295	€ 597	€ 2.515	€ 196
NORDEA	€ 1.652	€ 520	€ -	€ 1.286	€ 5.413	€ 2.359
HANDELSBANKEN	€ 775	€ 244	€ 2.968	€ -	€ 2.538	€ 116
DANSKE	€ 1.973	€ 621	€ 7.562	€ 1.536	€ -	€ 2.818
DNB	€ 785	€ 247	€ 36	€ 62	€ 258	€ -

Figure 9: Bilateral interbank liabilities matrix estimated with the Maximum Entropy method for 2014-01-01. Interbank assets are provided horizontally and interbank liabilities - vertically. Millions of EUR

	SEB	SWEDBANK	NORDEA	HANDELSBANKEN	DANSKE	DNB
SEB	€ -	€ 1.986	€ 8.858	€ -	€ -	€ -
SWEDBANK	€ 5.952	€ -	€ 1.965	€ -	€ -	€ -
NORDEA	€ -	€ -	€ -	€ 498	€ 6.322	€ -
HANDELSBANKEN	€ -	€ -	€ 7.631	€ -	€ -	€ -
DANSKE	€ -	€ -	€ 234	€ -	€ -	€ 8.987
DNB	€ -	€ -	€ -	€ -	€ 7.219	€ -

Figure 10: Bilateral interbank liabilities matrix estimated with the Minimum Density method for 2014-01-01. Interbank assets are provided horizontally and interbank liabilities - vertically. Millions of EUR

Such estimated bilateral interbank lending relationships are illustrated visually in figures 11 and 12, which show the net interbank lending linkages (i.e. bilateral assets minus bilateral liabilities). It is evident that under the minimum density method the estimated connections between banks are much stronger compared to the maximum entropy approach. Again, this illustrates the fact that the Minimum Density method is likely to overestimate the contagious effect while the maximum entropy - underestimate it. In fact, since we are dealing with a relatively small sample of large and systemically important banks, the Maximum entropy method seems to be somewhat more representative. The banks in the sample operate in one relatively small region which covers 7 countries, thus, given their large size, it is highly likely that they are all interconnected between themselves with no missing links. Nevertheless, the Minimum Density method serves well by adding robustness in the estimation of the capital requirements.

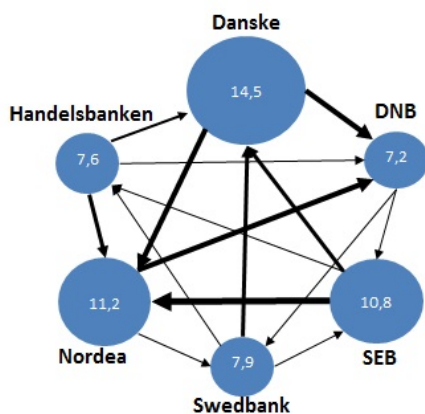


Figure 11: Bilateral net lending relationships, estimated using the Maximum Entropy method for 2014-01-01. The sizes of the bubbles are proportional to individual banks' total interbank assets, represented by the numeric values inside the bubbles (in billions of Euro). The arrows represent whether the estimated net bilateral lending is positive or negative, i.e. an arrow drawn from bank A to bank B means that bank A has more assets in bank B than liabilities (positive net lending). The thickness of the lines is proportional to the size of the net bilateral interbank lending.

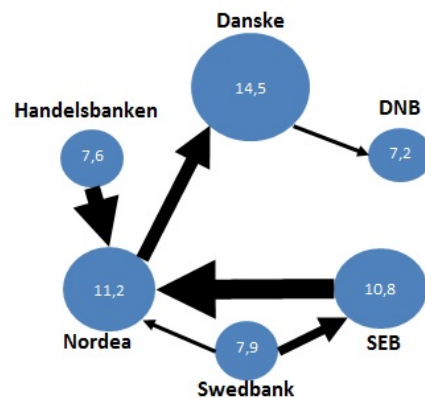


Figure 12: Bilateral net lending relationships, estimated using the Minimum Density method for 2014-01-01. The sizes of the bubbles are proportional to individual banks' total interbank assets, represented by the numeric values inside the bubbles (in billions of Euro). The arrows represent whether the estimated net bilateral lending is positive or negative, i.e. an arrow drawn from bank A to bank B means that bank A has more assets in bank B than liabilities (positive net lending). The thickness of the lines is proportional to the size of the net bilateral interbank lending.

5.2 Aggregate loss distribution

Before looking at the calibrated systemic capital requirements for banks it is worth to explore the intermediate results. As in the case with the interbank network matrix, the aggregate loss distributions are estimated at points in time with half a year intervals in between. They include the simulated losses for all banks in the sample where the loss is defined as the bank's asset value which falls below the default threshold (a bank's total liabilities) at time T . In addition, the losses (asset values below the threshold) are expressed as the percentage of the aggregate sum of these default thresholds (total system-wide liabilities) of individual banks. Figure 13 presents the dynamics of the 95th percentile of the aggregate loss distributions estimated using different models and parameters. As expected, the tails of the loss distributions have two major peaks. The first one is associated with the Great financial crisis (2007-2009) while the second occurs during the European sovereign debt crisis (2012). The magnitude of losses depends on the model as well as the parameters used. The First Passage model with no systemic risks factors appears to provide the lowest system losses; under the Merton credit risk approach system losses increase substantially. Adding risk factors (i.e. correlation between banks' assets and the interbank network effect) widens the loss distribution further. Finally, fixing the diffusion parameters to the level estimated during the crisis in 2008 increases the losses substantially for other time points in the sample. For further discussion this paper considers the Merton model with no systemic effects as the benchmark model.

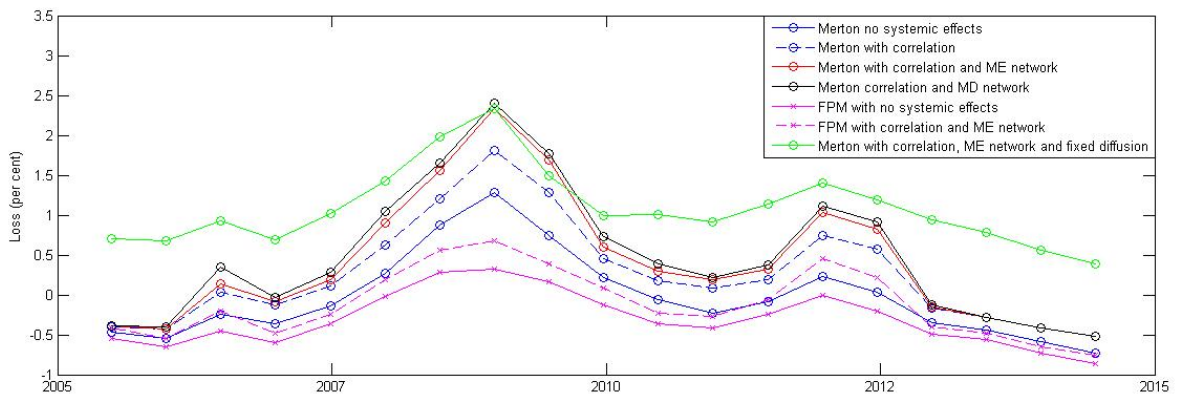


Figure 13: The 95th percentile of the aggregate simulated loss distributions from 2005 to the interim of 2014. The loss at time t is defined as the difference between the simulated asset value and the default threshold at time T . The losses at the 95th percentile are expressed as the percentage of the aggregate system-wide liabilities.

The inclusion of the correlation between the banks' assets widens the distribution of simulated losses (see figure 15). This is due to the fact that the correlations among banks' assets are mostly positive throughout the sample period (see appendix I). Moreover, as correlations appear to increase in times of financial market stress, simulated losses are more strongly magnified

during the crisis periods in 2008 and 2012. Figure 14 shows the difference between the 95th percentile of the simulated losses estimated with Merton model with and without including correlation. It times of market turbulence this difference reaches up to 0.55 percentage points of system-wide liabilities.

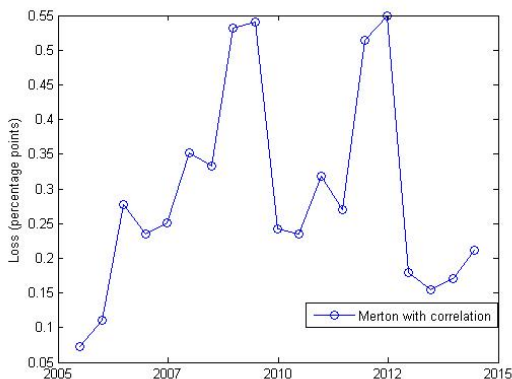


Figure 14: Correlation effect: the difference of in the 95th percentile of the aggregate losses (expressed as the percentage of total system liabilities) estimated with Merton model with and without the correlation effect

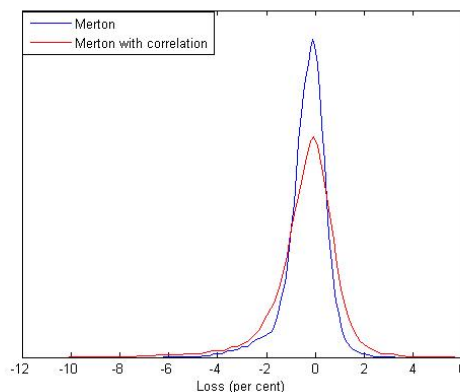


Figure 15: Correlation effect 2: the distributions of the aggregate simulated losses estimated in 2009, expressed as the percentage of the system-wide liabilities

The inclusion of network magnifies the losses of the banking system substantially. Although it has little effect during the periods of small market volatility, such network effect kicks in when the turbulence in the financial markets increases. Figure 16 shows the difference between the 95th percentile of the system losses estimated by Merton model without the interbank network effect and the one with such effect. This difference is the highest during the peak of the financial crisis in 2008-2009 and again in 2012. As is shown in figure 17, the inclusion of the contagion network effect alters the tail of the loss distribution by making it fatter. In other words, extreme losses become more likely. Moreover, different network estimation models show different magnitudes of losses. The losses under the Minimum Density model appear to be slightly higher, indicating that more concentrated networks have stronger negative contagion effect. However, the difference does not appear to be substantial. The average gap between the 95th percentile of the system losses estimated under both network models is 0.07 percentage points. As mentioned in section 3, Maximum entropy tends to underestimate the contagion effect while Minimum density overestimates it. Hence, the true effect is rather likely to lie somewhere in between. In other words, the results by the two different network estimation methods can be interpreted as the some sort of boundaries of where the true effect might be.

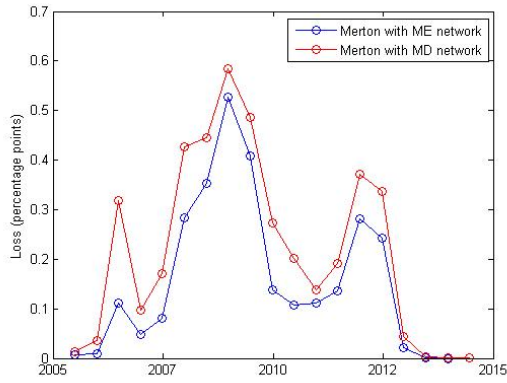


Figure 16: Network effect: the difference of the 95th percentile of the system losses (expressed as the percentage of total system liabilities) estimated with Merton model with and without the network effect

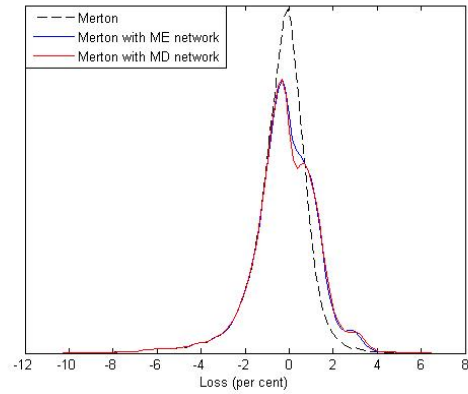


Figure 17: Network effect: the distributions of the aggregate simulated losses estimated in 2009, expressed as the percentage of the system-wide liabilities

As expected, the First Passage model seems to provide lower losses compared to the Merton model (see figure 18). Again, this comes from the rationale of the option pricing theory. While under the Merton approach bank's equity price is modelled as a price of the plain vanilla European call option, under the First Passage approach the price of the equity is assumed to be represented by the price of the down-and-out call option. The second option is more risky for the holder, therefore, should in theory be less expensive. As the price of the option as well as the default threshold, i.e. the price of the equity and the value of liabilities, are given it must be that the price of the underlying is higher in the case of the down-and-out option. Therefore, the asset values estimated under the First Passage approach are lower compared to Merton, leading to smaller simulated losses. Moreover, as the estimated asset values increase, the corresponding logarithmic asset returns decrease, reducing the volatility of such returns. Figure 19 shows that the system loss distribution under the First Passage modelling approach is more concentrated and less exposed to the positive values.

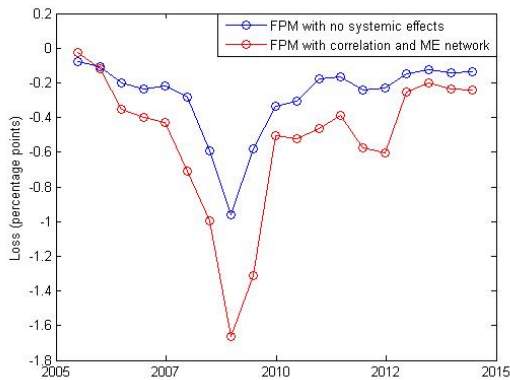


Figure 18: Credit risk model effect: the difference between the 95th percentile of the system loss distribution estimated by the Merton and First Passage models with and without network effect

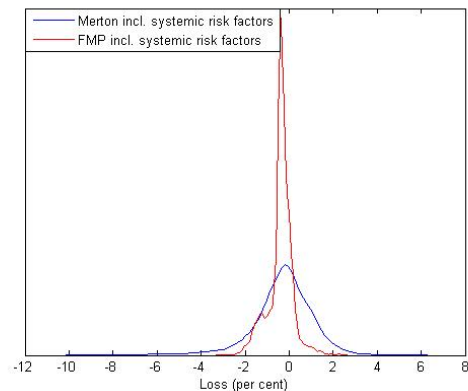


Figure 19: Credit risk model effect 2: the distributions of the aggregate simulated losses estimated in 2009, expressed as the percentage of the system-wide liabilities

5.3 Comparative static exercises: changing the structure of the banking system

Changes in the structure of the banking system may alter the effect that a given bank has on the solvency of the system overall when it fails. This paper briefly considers 3 such changes: diffusion parameters become fixed as in the stress period, one bank doubles in size and the cost of contagious defaults increases.

The issue with the estimated loss distributions so far was that they tend to be pro-cyclical. In other words, the estimated loss distributions at time T are based on the balance sheet information and the diffusion parameters which are in turn estimated from the period $[-\tau, 0]$. In order to reduce this pro-cyclicality, I estimate the 95th percentile of the simulated loss distribution using the stress period diffusion parameters. Figure 20 shows the difference between the 95th percentile of the simulated loss distributions estimated with Merton model with systemic risk factors, with and without fixed diffusion parameters. The difference appears quite substantial as it reaches up to 1 percentage point of the system-wide liabilities in times of little market turbulence.

As in the case with setting diffusion parameters to the stress levels, doubling the bank's balance sheet or increasing the costs of contagion defaults increases simulated system losses. Figure 21 shows that in each previously described stress testing scenarios the estimated loss distribution shifts rightwards compared with the benchmark. Detailed assessment of the different stress testing scenarios could prove useful for policy makers, however, it is beyond scope of this paper.

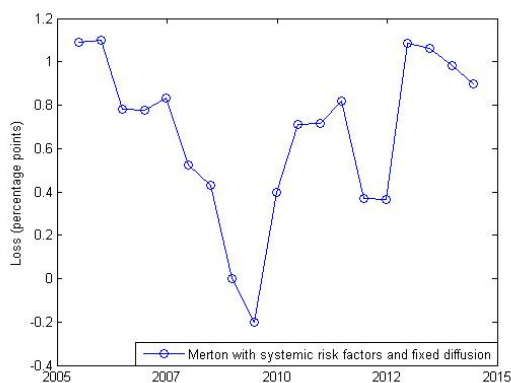


Figure 20: Fixing diffusion parameters: the difference between the 95th percentile of the system loss distribution estimated with the Merton model incl. systemic risk factors and such model with the diffusion parameters fixed to the 2008 level.

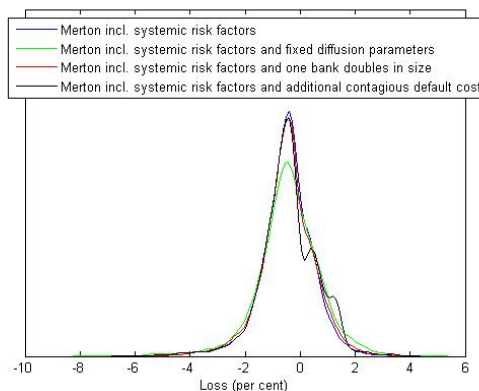


Figure 21: Different stress-testing scenarios: impact to the system loss distribution

5.4 Systemic capital requirements

The optimization procedure as described in the methodology part of this paper provides a way to map the estimated loss distributions into the systemic capital surcharges for each bank. In other words, the optimal and risk adjusted levels of capitalization are calibrated in such a way that the 95 percentile of the simulated loss distribution is set to be equal to zero. Figure ?? shows the capital surcharges as a percentage of the asset value for each bank in the sample between 2005 and 2014 interim estimated with Merton model with no systemic effect. In effect, these surcharges show how much each bank has to contribute in terms of issuing new equity so that the 95 percentile of the system losses is equal to zero. This post-optimization adjustment of system loss distribution is illustrated in figure 23. After the increase in aggregate system capitalization level the distribution shifts leftwards so that its 95th percentile is zero. Negative values in the capital surcharges mean that either the 95th percentile of the loss distribution pre-optimization showed negative losses or the losses by other banks are more important to the system - they need to increase their capital levels far more.

The dynamics of the resulting capital surcharges are in line with the dynamics of the estimated loss distribution. For example, when estimated under the Merton credit risk approach with no systemic risk factors, the level of such surcharges climbs up to 6 percent for some banks around the 2008 but falls back sharply from 2012 onwards. However, the amount of capital surcharges varies considerably among individual banks. For instance, the Handelsbanken would have been short of capital basically only during the main stress period from 2007 to 2009 with the required capital level reaching 3 % of bank's asset value at the peak. In other periods Handelsbanken would have been seen as in effect not causing significant risk to the Nordic financial system. Danske bank, on the other hand, would have been short of capital basically from 2007 up to 2012, indicating that capital shortage of this bank is much more significant to the system.

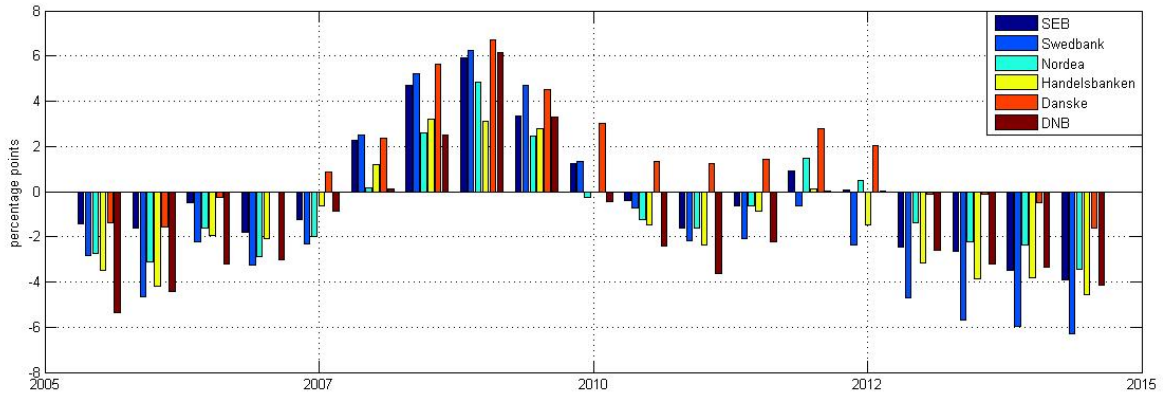


Figure 22: Systemic capital surcharges with Merton model and no systemic risk factors. Bars represent change in the ratio of capital to assets for each bank in the network following the optimisation.

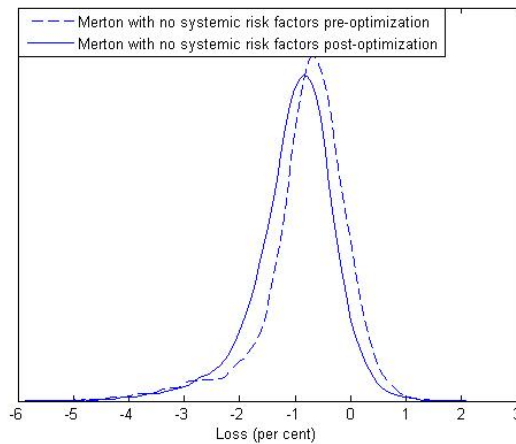


Figure 23: System loss distributions pre and post-optimization (2009) estimated with Merton model with no systemic risk factors. Losses expressed as a fraction of system wide liabilities.

Adding the correlation between banks' assets effect increases capital surcharges significantly. Figure 24 depicts the proposed capital surcharges estimated with the Merton approach after taking correlation into account. For some banks the desired increase in capitalization would have been up to 9 % of their asset value during the great financial crisis in 2008. Figure 25 shows the capital add-on due to the introduction of correlation (i.e. the difference between the capital-to-assets ratio between the Merton model with and without correlation). As expected, this difference is largest during the stress periods when the correlations between banks' asset values are highest. In particular, the effect is large for *Nordea* bank with the proposed capital add-on due to the correlation reaching above 4 % of its asset value in the interim of 2009.

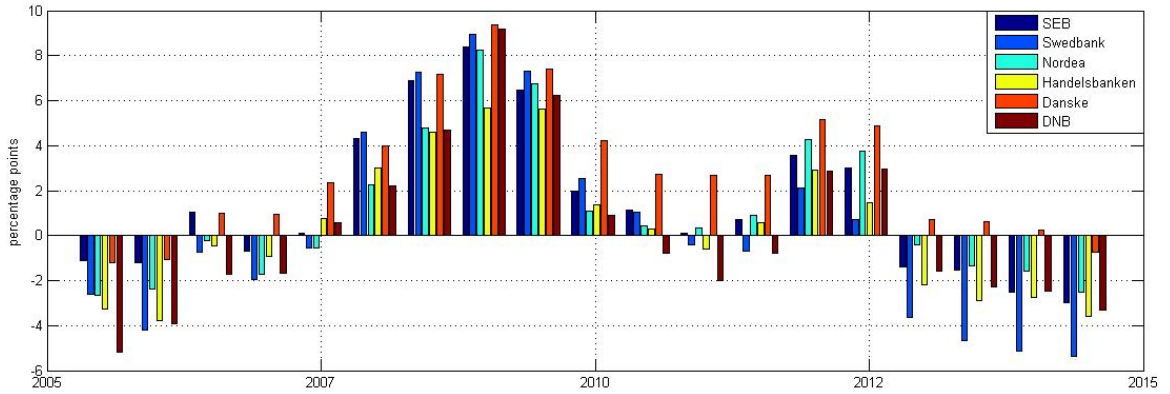


Figure 24: Systemic capital surcharges with Merton model incl. correlation. Bars represent change in the ratio of capital to assets for each bank in the network following the optimisation.

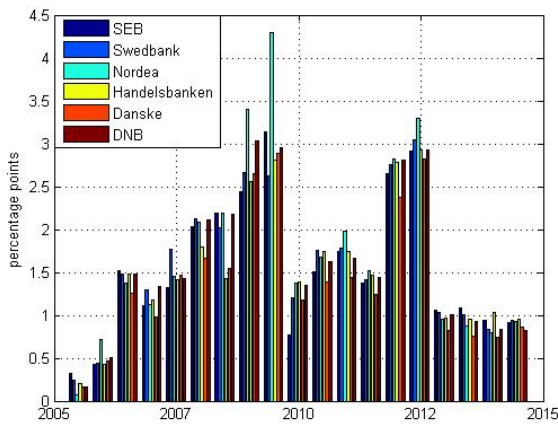


Figure 25: Correlation effect on capital surcharges. Bars represent change in the ratio of capital to assets after adding the correlation effect to the simple Merton model.

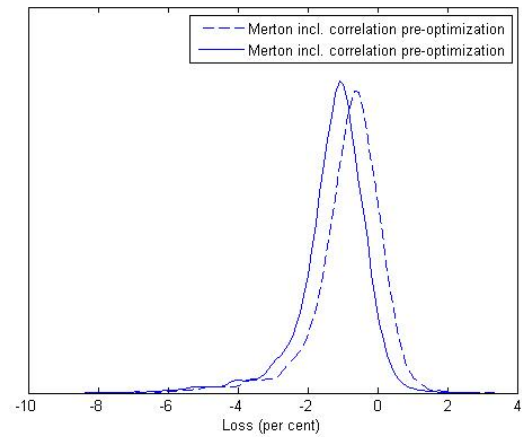


Figure 26: System loss distributions pre and post-optimization (2009) estimated with Merton model incl. correlation. Losses expressed as a fraction of system wide liabilities.

Going further into the estimation of systemic capital surcharges, the second systemic risk factor - the interbank network effect - is added (to the Merton model). As in the case with the first risk factor (i.e. correlation), the estimated capital surcharges increase substantially. Figure 27 shows the increase in the capital-to-asset ratio for the banks in the sample estimated with Merton model with correlation and the Maximum Entropy network structure. The risk of the contagious defaults due to the interbank linkages raises the estimated capital surcharges for some banks up to 12 % of their asset value in 2009. Figures 28 and 29 show the network effect estimated with the Maximum Entropy and Minimum Density methods. As discussed in previous sections, the ME method tends to underestimate the contagious losses, therefore, the proposed capital surcharges estimated with this network construction model is somewhat lower as compared to the MD approach. However, the differences do not appear to be substantial. The simulated losses caused by the interbank linkages map into estimated capital surcharges reaching up to around 3% of asset values for some banks in 2007-2009.

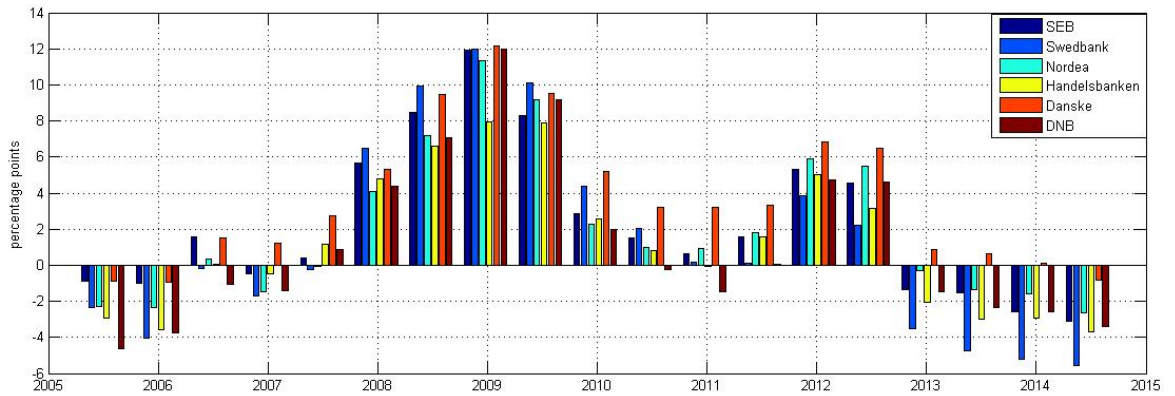


Figure 27: Systemic capital surcharges with Merton model incl. correlation and interbank network. Bars represent change in the ratio of capital to assets for each bank in the network following the optimisation.

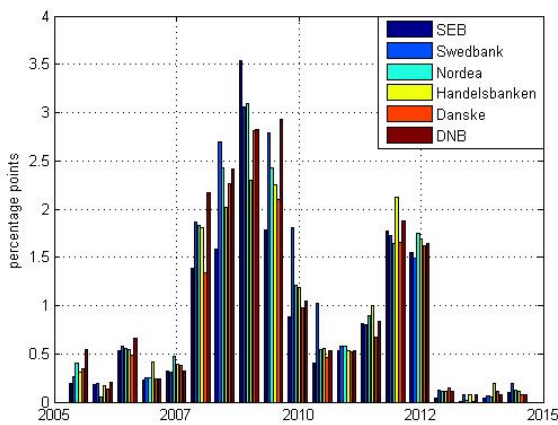


Figure 28: Network effect: difference between capital surcharges estimated with Merton model plus ME network and Merton model with no network

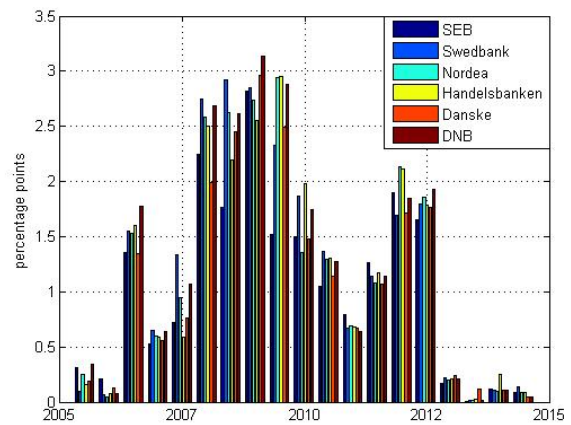


Figure 29: Network effect: difference between capital surcharges estimated with Merton model plus MD network and Merton model with no network

Next, this paper considers calibration of systemic capital surcharges based on the First Passage model approach. The estimated capital add-ons under such approach (including both systemic risk factors) are shown in figure 30. In line with the evolution of the 95th percentile of the system loss distribution the proposed capital surcharges appear to be substantially smaller than under the Merton approach. The largest proposed add-on reaches 4.5 % of asset value in 2008. In addition, figure 31 shows the difference between such surcharges estimated with the First Passage and Merton models. This difference reaches up to 8-9 percentage points for some banks during the great financial crisis, supporting the fact that one's perception of credit risk is highly dependent upon perceived assumptions of the behaviour of the economic agent (i.e. equity holder).

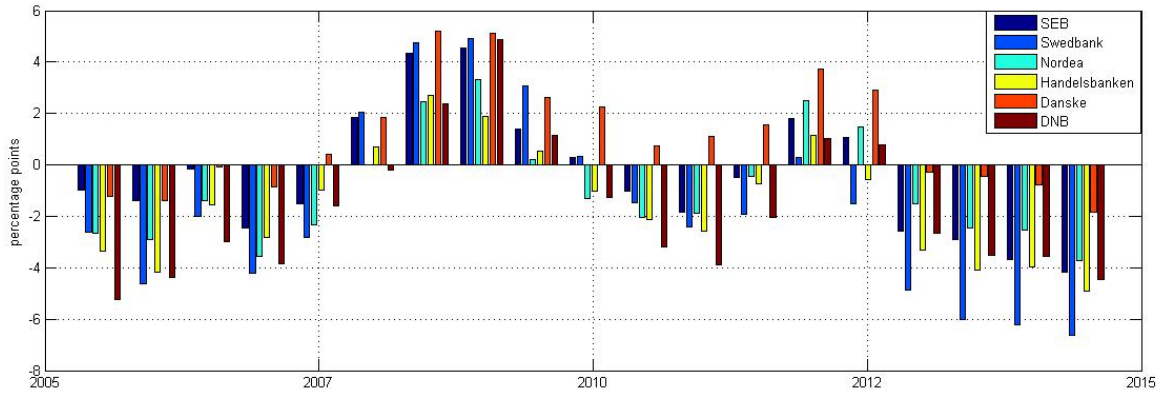


Figure 30: Systemic capital surcharges with the First Passage model incl. correlation and interbank network. Bars represent change in the ratio of capital to assets for each bank in the network following the optimisation.

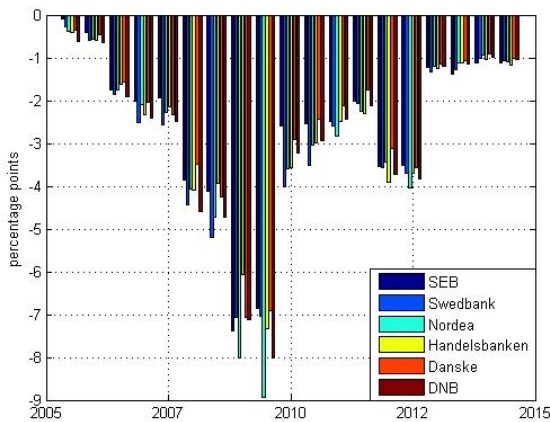


Figure 31: Credit risk model effect on capital surcharges. Bars represent the difference between the ratios of capital to assets estimated with FPM and Merton approaches

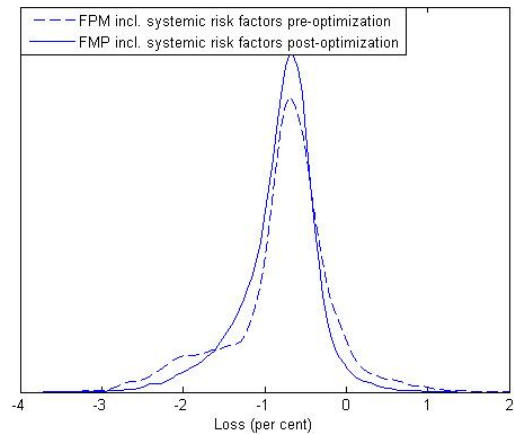


Figure 32: System loss distributions pre and post-optimization (2009) estimated with First Passage model with systemic risk factors. Losses expressed as a fraction of system wide liabilities.

As already discussed, the issue with using structural credit risk models to estimate systemic capital requirements for banks is that proposed capital surcharges tend to be pro-cyclical. At time t it is not known what macro-financial shocks will occur at time $t + \tau$, therefore, the estimation of banks' credit risk can only be based on previously observed historical data. As a result, the proposed capital add-ons may sometimes be insufficient. To deal with this issue this paper considers fixing the market stress scenario for all the time points in the sample. Clearly, for maintaining financial stability in the economy it is crucial that banks were able to withstand even extremely turbulent periods in financial markets. Therefore, the assumption that banks should always have enough capital to withhold such periods is a rather reasonable one. The most turbulent period from 2004 to 2014 was the Great financial crisis in 2007-2009. Hence, by fixing the diffusion parameters, such as banks asset return volatility, mean and the correlation structure, to the level estimated in 2008 for all the time points, the stress-robust capital surcharges can be calibrated. It is a matter of choice for a policy maker which model

to use for banks' credit risk estimation. We have seen that model choice indeed matters. This paper considers estimating fixed-diffusion stressed capital surcharges with the Merton model with correlation and MD network, as it provides the worst case scenario. Figure 33 shows the capital surcharges for each bank estimated with Merton model including both systemic risk factors and with diffusion parameters fixed at the 2008 level. The results indicate that all banks in the sample have been under-capitalized to withstand significant volatility in the market price of their equity throughout the period of 2005 - 2014. In other words, the 95th percentile of the simulated system loss distribution for the Nordic banks would have been higher than zero during the whole sample period if the market was as volatile as in 2008. Furthermore, the degree of the estimated capital surcharges varies considerably among banks, indicating the heterogeneity in their importance for the financial system. In particular, *Danske* appears to have been requiring additional capital the most in the recent period (i.e. from 2012). Similar situation holds for *Nordea* and *SEB*. On the other hand, *Swedbank* appears to have significantly improved its solvency position since 2008 and under this modelling framework would not require additional capital.

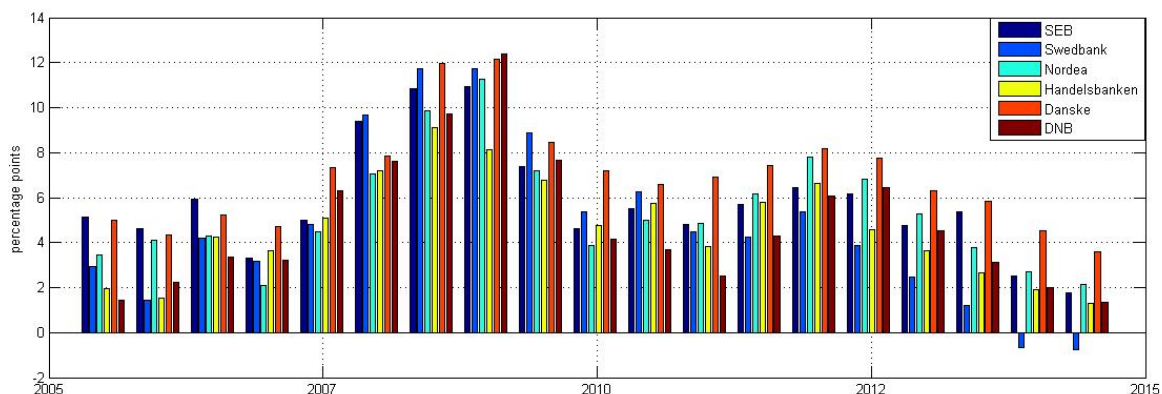


Figure 33: Systemic capital surcharges with Merton model incl. correlation and MD interbank network; the diffusion parameters are fixed to the level estimated in 2008. Bars represent change in the ratio of capital to assets for each bank in the network following the optimisation.

6 Conclusions

Focussing narrowly on the health of individual banks may leave the banking system vulnerable to systemic macro-financial shocks. Therefore, the risks of the systemic nature should be assessed and mapped into banks' capital requirements so that the whole banking sector is sufficiently capitalized. Two of such risks are of particular attention, i.e. the correlation of the banks exposures which tends to deteriorate banks' solvency positions in tandem and the inter-linkages among banks which could create the contagion effect during the stress periods.

On the other hand, increasing capital requirements by too much might result in the inefficiency in terms of the forgone revenues for banks, lower ROE's for investors and increased loan costs for the society. As a result, optimal systemic capital requirements for each bank need to be found so that the system-wide risk is constrained within tolerable limits with as limited increase in banks' capital level as possible.

This paper uses the structural credit risk modelling approach mapped in the constrained optimization problem to find optimal systemic capital requirements for the 6 largest banks in the Nordic-Baltic region. In doing so it uses two bilateral interbank network estimation methods, two structural credit risk models, the network clearing algorithm and the optimization procedure proposed in Webber and Willison (2011).

Results show that adding systemic risk factors, such as the effect of correlation among banks' assets and the interbank contagion effect, increases the calibrated capital surcharges for banks substantially. Moreover, the level of such surcharges varies considerably from one bank to another depending on the size of their balance sheet, the degree of their asset correlation and the interconnectedness with other banks in the system. The capital add-ons estimated due to the interbank network effect are network-model dependent. Although the true such effect is unknown due to the data unavailability, the difference between capital add-ons estimated with the Maximum entropy and the Minimum density methods are not substantial, adding robustness to the calibrated results. Furthermore, the choice of using the appropriate credit risk model matters as the results obtained by using Merton credit risk model and the First Passage model differs substantially (with the First Passage model suggesting considerably smaller capital requirements). Since the choice of using one model over another is based on the set of assumptions that one is comfortable relying upon, this paper does not draw firm conclusions with respect to the model choice and leaves it up to the policy-maker.

The capital estimation framework discussed in this paper suffers from the issue of pro-cyclicality. To deal with this problem, the stress scenario in the financial markets is considered. Under the conservative scenario, where the systemic capital requirements are estimated with the Merton model including the systemic risk factors and fixing the diffusion parameters to the stressed level estimated in 2008, almost all banks were found under-capitalized throughout the entire period of 2005 - 2014. These results confirm the worry of the central bank of Sweden that the Nordic banks are too little capitalized and, therefore, are vulnerable to the risks of systemic

nature.

7 Literature

Brunnermeier, M, Crockett, A, Goddhart, C, Persaud, A D and Shin, H (2009), 'The fundamental principles of financial regulations', Geneva Reports on the World Economy.

Craig, B and von Peter, G (2014), 'Interbank tiering and money center banks. Journal of Financial Intermediation, In Press.

Eisenberg, L and Noe, T H (2001), 'Systemic risk in financial systems', Management Science, Vol. 47, No. 2, pages 236-49.

Elsinger, H, Lehar, A and Summer, M (2006), 'Using market information for banking system risk assessment', International Journal of Central Banking, Vol. 2(1), pages 156-65

Elsinger, H, Lehar, A and Summer, M (2013), 'Network models and systemic risk assessment, In J.-P. Fouque and J. Langsam (Eds.), Handbook on Systemic Risk, pages 287-305, Cambridge University Press.

Gauthier, C, Lehar, A and Souissi, M (2010), 'Macroprudential regulation and systemic capital requirements', Bank of Canada Working Paper No. 2010-4

Anand, K, Craig, B and von Peter, G (2014), 'Filling in the Blanks: Network Structure and Interbank Contagion', Bank of International Settlements, working paper no. 455.

Merton, R (1974), 'On the pricing of corporate debt: the risk structure of interest rates', Journal of Finance, Vol. 29, pages 449-70.

Mistrulli, P (2011), 'Assessing financial contagion in the interbank market: Maximum entropy versus observed interbank lending patterns', Journal of Banking and Finance 35(5), pages 1114-1127.

Modigliani, F and Miller, M (1958), 'The cost of capital, corporation finance and the theory of investment', American Economic Review, Vol. 58, pages 261-97.

Shapley, L S (1953), 'A value for n-person games', reprinted in Roth, A E (ed.), The Shapley value, Cambridge University Press.

Sveriges Riksbank (2011), 'Appropriate capital ratio in major Swedish banks'

Tarashev, N, Borio, C and Tsatsaronis, K (2010), 'Attributing systemic risk to individual institutions', Bank for International Settlements Working Paper No. 308.

Upper, C (2007), 'Using counterfactual simulations to assess the danger of contagion in interbank markets', Bank for International Settlements Working Paper No. 234.

Vickers, J (2012), 'Some economics of banking reform', University of Oxford Department of Economics, Discussion paper series.

Webber, L, Willison, M (2011), 'Systemic capital requirements', Bank of England, Working Paper No. 436

8 Appendix

A) The set-up of general model

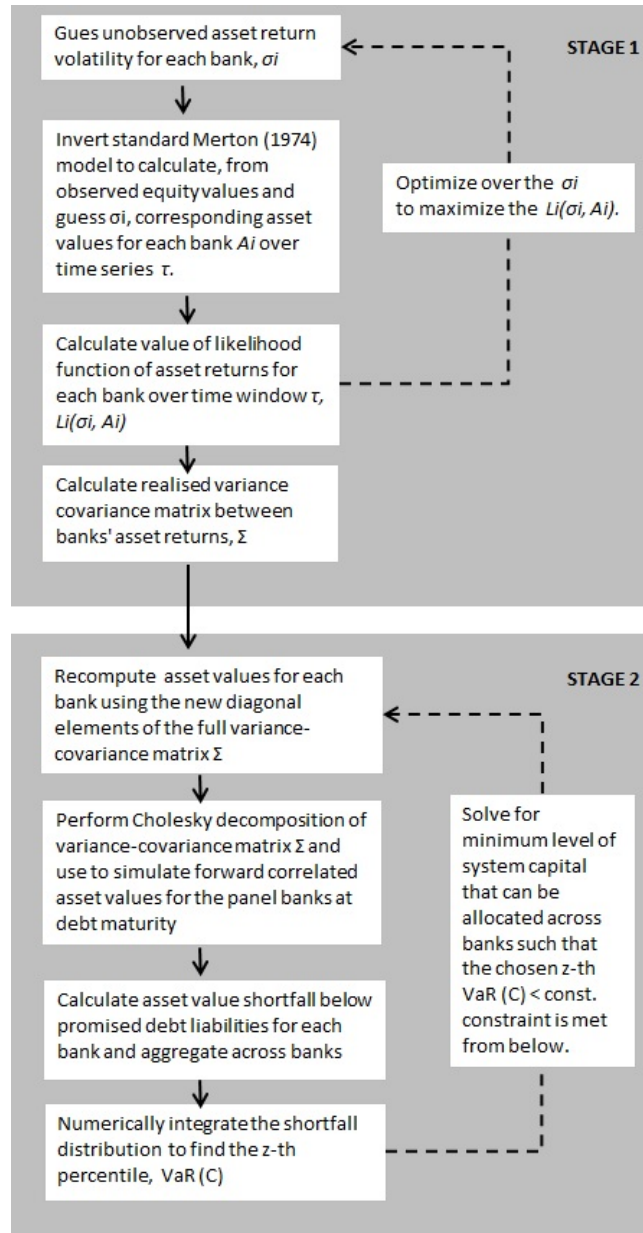


Figure 34: Step-wise illustration of the procedure (algorithms) of the estimation of optimal systemic capital requirements for banks

B) Diffusion parameters

2005 01 01					2005 07 01					2006 01 01					2006 07 01					2007 01 01									
1,00	0,60	0,66	0,65	0,58	0,23	1,00	-0,09	-0,03	0,08	0,41	0,37	1,00	0,36	0,04	0,33	0,44	0,33	1,00	0,71	0,49	0,77	0,50	0,39	1,00	0,61	0,49	0,74	0,33	0,43
0,60	1,00	0,89	0,94	0,90	-0,09	-0,09	1,00	0,42	0,49	0,18	0,11	0,36	1,00	0,57	0,62	0,31	0,25	0,71	1,00	0,37	0,70	0,38	0,34	0,61	1,00	0,35	0,61	0,39	0,40
0,66	0,89	1,00	0,92	0,87	-0,12	-0,03	0,42	1,00	0,36	0,36	-0,06	0,04	0,57	1,00	0,50	0,42	0,06	0,49	0,37	1,00	0,52	0,21	0,13	0,49	0,35	1,00	0,51	0,13	0,14
0,65	0,94	0,92	1,00	0,94	-0,19	0,08	0,49	0,36	1,00	0,09	0,14	0,33	0,62	0,50	1,00	0,21	0,25	0,77	0,70	0,52	1,00	0,45	0,35	0,74	0,61	0,51	1,00	0,26	0,38
0,58	0,90	0,87	0,94	1,00	-0,30	0,41	0,18	0,36	0,09	1,00	0,16	0,44	0,31	0,42	0,21	1,00	0,25	0,50	0,38	0,21	0,45	1,00	0,50	0,33	0,39	0,13	0,26	1,00	0,50
0,23	-0,09	-0,12	-0,19	-0,30	1,00	0,37	0,11	-0,06	0,14	0,16	1,00	0,33	0,25	0,06	0,25	0,25	1,00	0,39	0,34	0,13	0,35	0,50	1,00	0,43	0,40	0,14	0,38	0,50	1,00

Figure 35: Correlation matrices between the logarithmic daily changes of banks' asset values estimated with Merton model

	SEB	Swedbank	Nordea	Handelsbanken	Danske	DNB	
2005 01 01	0,00091	0,00008	0,00090		0,00001	0,00083	0,00052
2005 01 07	0,00090	0,00025	0,00087		0,00015	0,00078	0,00050
2006 01 01	0,00075	0,00052	0,00111		0,00063	0,00053	0,00077
2006 01 07	0,00085	0,00064	0,00128		0,00124	0,00025	0,00087
2007 01 01	0,00037	0,00089	0,00110		0,00057	0,00047	0,00078
2007 01 07	0,00051	0,00086	0,00058		0,00034	0,00083	0,00071
2008 01 01	0,00058	0,00084	0,00069		0,00069	0,00012	0,00075
2008 01 07	0,00054	0,00052	0,00058		0,00021	0,00104	0,00067
2009 01 01	0,00044	0,00010	0,00076		0,00039	0,00036	0,00058
2009 01 07	0,00120	0,00012	0,00165		0,00091	0,00036	0,00116
2010 01 01	0,00078	0,00211	0,00102		0,00107	0,00028	0,00115
2010 01 07	0,00048	0,00188	0,00073		0,00090	0,00012	0,00082
2011 01 01	0,00092	0,00094	0,00105		0,00058	0,00038	0,00065
2011 01 07	0,00025	0,00039	0,00030		0,00041	-0,00016	0,00073
2012 01 01	0,00047	0,00078	0,00096		0,00083	0,00038	0,00095
2012 01 07	0,00051	0,00090	0,00072		0,00060	0,00027	0,00139
2013 01 01	0,00058	0,00092	0,00023		0,00049	0,00020	0,00080
2013 01 07	0,00082	0,00076	-0,00021		0,00021	0,00006	0,00028
2014 01 01	0,00018	0,00017	-0,00007		0,00046	-0,00019	0,00009
2014 01 07	0,00026	0,00041	0,00026		0,00039	0,00023	0,00013

Figure 36: Mean values μ of logarithmic changes of banks asset values estimated with Merton model and used for simulations

	SEB	Swedbank	Nordea	Handelsbanken	Danske	DNB	
2005 01 01	0,02582	0,02000	0,02527		0,02475	0,01789	0,01516
2005 01 07	0,02332	0,01727	0,02406		0,01724	0,01401	0,01775
2006 01 01	0,01985	0,01658	0,02454		0,01971	0,01346	0,01968
2006 01 07	0,01986	0,02241	0,03796		0,02515	0,01426	0,02595
2007 01 01	0,02214	0,02427	0,03886		0,02538	0,01520	0,02653
2007 01 07	0,02379	0,02584	0,02267		0,02170	0,01748	0,02222
2008 01 01	0,02799	0,03016	0,02356		0,02307	0,02492	0,02194
2008 01 07	0,02860	0,02725	0,02497		0,02536	0,03220	0,02478
2009 01 01	0,03474	0,03088	0,03265		0,03345	0,03220	0,03591
2009 01 07	0,05458	0,03834	0,04747		0,03837	0,02566	0,04625
2010 01 01	0,04037	0,05377	0,03929		0,02952	0,02043	0,03897
2010 01 07	0,02295	0,04327	0,02528		0,02367	0,01381	0,03076
2011 01 01	0,02007	0,02199	0,02196		0,02111	0,01536	0,02819
2011 01 07	0,01949	0,02004	0,02000		0,01793	0,01734	0,02626
2012 01 01	0,02243	0,03137	0,02532		0,02289	0,01981	0,03158
2012 01 07	0,02371	0,03174	0,02651		0,02282	0,01540	0,03265
2013 01 01	0,01765	0,02278	0,01623		0,01594	0,01088	0,02033
2013 01 07	0,01560	0,02933	0,01430		0,01810	0,01112	0,01750
2014 01 01	0,01779	0,02838	0,01689		0,02100	0,01023	0,01925
2014 01 07	0,01764	0,02488	0,01680		0,01938	0,01143	0,01944

Figure 37: Volatility values σ_v of logarithmic changes of banks asset values estimated with Merton model and used for simulations

	SEB	Swedbank	Nordea	Handelsbanken	Danske	DNB
2005 01 01	0,03055	0,02764	0,02376	0,02392	0,00937	0,01367
2005 01 07	0,02353	0,01727	0,02412	0,01724	0,01402	0,01775
2006 01 01	0,02000	0,01658	0,02464	0,01971	0,01346	0,01969
2006 01 07	0,02007	0,02243	0,03975	0,02534	0,01426	0,02601
2007 01 01	0,02231	0,02436	0,04085	0,02551	0,01520	0,02661
2007 01 07	0,02388	0,02593	0,02269	0,02172	0,01751	0,02224
2008 01 01	0,02942	0,03196	0,02363	0,02319	0,02570	0,02198
2008 01 07	0,04004	0,03948	0,02578	0,02648	0,03603	0,02534
2009 01 01	0,03647	0,03500	0,04592	0,04568	0,03114	0,04184
2009 01 07	0,02665	0,02911	0,04414	0,04290	0,02503	0,03942
2010 01 01	0,03454	0,03619	0,04437	0,03784	0,02122	0,04266
2010 01 07	0,03473	0,03812	0,04217	0,03782	0,02070	0,04282
2011 01 01	0,03415	0,03616	0,03658	0,02157	0,02175	0,03136
2011 01 07	0,01998	0,02030	0,02056	0,01799	0,02357	0,02706
2012 01 01	0,03307	0,04793	0,03485	0,03418	0,02439	0,04657
2012 01 07	0,03282	0,04709	0,03275	0,03588	0,01985	0,04488
2013 01 01	0,01905	0,02321	0,01725	0,01599	0,01247	0,02332
2013 01 07	0,01592	0,02977	0,01438	0,01812	0,01147	0,01768
2014 01 01	0,01786	0,02856	0,01698	0,02103	0,01030	0,01934
2014 01 07	0,01766	0,02489	0,01682	0,01939	0,01146	0,01948

Figure 38: Volatility values σ_v of logarithmic changes of banks asset values estimated with the First Passage model and used for simulations

C) Estimated optimal capital-to-asset ratios

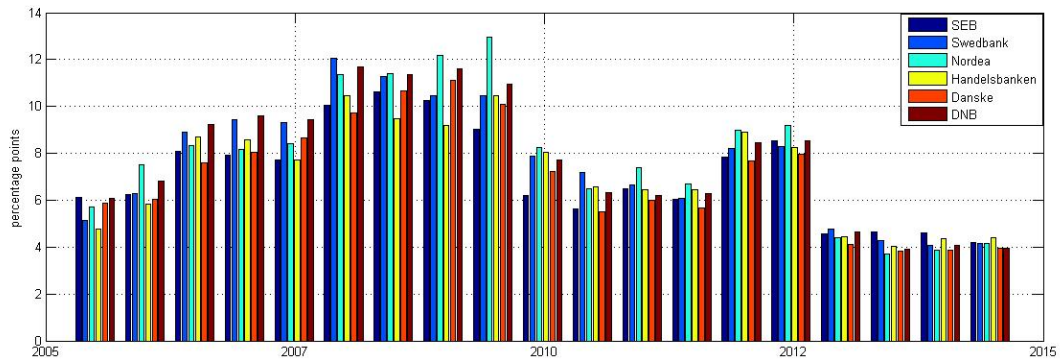


Figure 39: Optimal banks' capital-to-total assets ratio estimated with Merton model including systemic risk factors (asset correlation and interbank exposures).

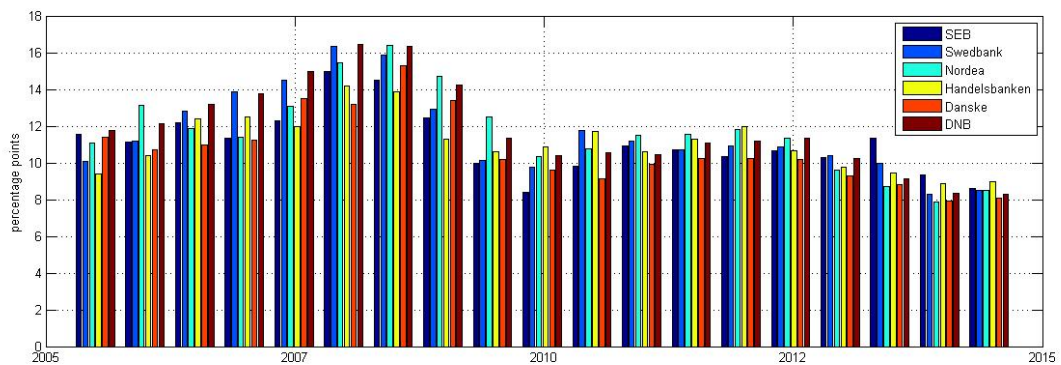


Figure 40: Optimal banks' capital-to-total assets ratio estimated with Merton model including systemic risk factors (asset correlation and interbank exposures), holding the diffusion parameters fixed at the 2008 crisis levels (stress scenario).