Erasmus University Rotterdam

Bachelor Thesis

An Essay on the Three-State Matching Model of Labour Flow

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This thesis is submitted in fulfilment of the requirements for the Bachelor of Science in Economics and Business Economics

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July 2015

Erasmus University Rotterdam

Abstract

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by Gilbert SANJAYA

This thesis employs a calibrated Diamond-Mortensen-Pissarides (DMP) model, also known as matching model, by considering out-of-the-labour-force individuals into the analysis. While the conventional DMP model considers all individuals within the workforce, this thesis offers different perspective by taking into account that some individuals voluntarily decide to be out of the labour market. Much scrutiny is put on the searching behaviour of individuals and firms that shape the equilibrium (un)employment rate in our model. We consider three separate scenarios for our general equilibrium model; when agents are assumed to be homogenous, heterogenous, and heterogenous and naïve, respectively. Our finding informs that the distinction between involuntary and voluntary unemployment provides an interesting consideration for policy analyses.

Keywords: Matching model, DMP model, search behaviour, involuntary unemployment, voluntary unemployment, naivety, business cycles.

JEL Classification Code: E24, J21, J22, J23, J64.

Acknowledgements

First and foremost, I would like to thank Dr. Dana Sisak, for only because of her guidance and supervision that the completion this thesis is made possible. Without her constructive criticisms, provoking ideas, and nuanced guidance on the state-of-the-art of economics modelling, this thesis' quality would have never been elevated up to this level.

Special thanks to Dr. Björn Brügemann from Tinbergen Institute, whose lecture on labour economics served as the main motivation of this thesis. My sincere thanks also goes to Dr. Brigitte Hoogendorn, the coordinator of two honours classes that I have attended for the past two years, for her encouragements have sparked a deep research interest within myself. Lastly, a special thanks to my friend Jean-Paul van Brakel, who has assisted me in covering my deficiencies in programming.

I would also like to express my gratidude and sincere thanks to my family, who had been influential in providing mental supports throughout my study. Especially, I express my biggest thanks to my parents, Mr Steve Sanjaya and Mrs Nurnaningsih Sumarsono, who had provided tremendous supports throughout my study. I will be forever indebted to them. Also to my brother, Richard Sanjaya, and my sister, Michelle Sanjaya, whose prayers and insights have kept my motivations and ambitions intact. I am confident that my family's supports have been the critical reason as to why I am able to succesfully wrap my undergraduate study up.

Lastly, I am grateful to my God Jesus Christ, for only because of His blessing hand that my entire undergraduate study had been a blessing and can be successfully concluded with the submission of this thesis.

Rotterdam, 8^{th} July 2015

Gilbert Sanjaya

Contents

Symbols

θ	labour mmrket tightness
$q(\theta)$	labour market tightness per vacancies
$p(\theta)$	labour market tightness per unemployed workers
w	bargained wage
β	bargaining power of workers
$J(w)$	net present asset value of working
U	net present asset value of being voluntarily unemployed
$E(w)$	net present asset value of finding a match
V	net present asset value of opening up a new vacancy
ψ	individual's search cost
κ	firm's search or vacancy cost
z	utility derived by individuals from non-working activities
v	available vacancy in the economy
u or L	the size of the household or all the unemployed workers in the economy
u_i	involuntary unemployment rate
u_v	voluntary unemployment rate
U	total unemployment rate $(u_i + u_v)$
L^s	labour supply or individuals searching for job
L^d	labour demand or total vacancy opened by firms
L^*	equilibrium employment level
L_i^s	amount of time spent on searching by an individual
l	amount of time spent on leisure by an inidividual
L_i^d	labour demand of an individual firm
L^0	initial matches or individuals that are currently working

Chapter 1

Introduction

"One should hardly have to tell academicians that information is a valuable resource: knowledge is power. And yet it occupies a slum dwelling in the town of economics. Mostly it is ignored: the best technology is assumed to be known; the relationship of commodities to consumer preferences is a datum."

Joseph Stigler - *The Economics of Information* (1961).

The central element of conventional labour economics analyses is that individuals work for an equilibrium clearing wage, where 'upward-sloping' labour supply and 'downward-sloping' labour demand curves intersect. Although this model has been the mainstream handbook for elementary economists, it fails to describe some important facts. Most importantly, contemporary scholars have highlighted the negligence of neoclassical model towards the scrutiny of unemployment. In traditional neoclassical model, individuals whose preferences are not satiated by the equilibrium clearing wage level will choose to be out of the labour market and will be omitted from the model.

Furthermore, within this framework, there is no distinction between these out-of-the-labour-force individuals and individuals who fail to find a job in the labour force - they are simply referred to as being unemployed. Not distinguishing between these types of unemployment is a major fallacy, especially from utilitarian perspective. Indeed, individuals that decide to not enter the labour force realize that they would be better off by not working, since their out-of-the-labour-force action reflects a 'utility-maximization' behaviour, all things considered. Contemporarily, we refer to this as a voluntary unemployment case. On the other hand, involuntary unemployment is referred to as the case in which an individual, who is better off working at the equilibrium clearing wage, fail to find a job.

Taking into account these definitions, a simple microeconomics comparative static is thus insufficient to capture the essence of unemployment. It was not until Stigler (1961) seminal proposition that economists realized the fundamental nature behind *involuntary* unemployment. As stated on the quotation above, then-economists did not consider numerous barriers, notably caused by asymmetric information, that pose a hindrance for an individual to find a matching job. Developing on this intuition, he convened the search theory. Within a microeconomics framework, search theory explains an individual's strategy in facing potential opportunities with some random probabilities. The crucial assumption here is that delaying choice bears a high opportunity costs. Hence, search decision has to be done in a quick and successive fashion. Applying this to the context of labour market, the notion that job opportunities are attained with some probabilities serves as an explanation behind the frictions in labour market that hinders full employment to be attained.

Building on the fundamental of search theory, contemporary macroeconomists have been working on the extension of search model that encompasses aggregate economies by considering the interaction between searching individuals and firms. The interaction thus formed a general equilibrium model of unemployment due to mismatch between individual and firm. This type of unemployment is commonly known as frictional unemployment, whereas this method of analyzing unemployment is known as search and matching theory, or simply matching theory. Arguably, Mortensen (1978) was the first to connect the link between individual and firm searching behaviour. Diamond (1982) delivered some major contributions to the development of this model, most prominently by proposing a preliminary solution to the wage determination problem in the search and matching context. The developments of matching theory are comprehensively treated and summarized in a book by Pissarides (2000). His book is now used as a standard postgraduate textbook in analyzing unemployment in a macroeconomics scale. For their contributions, these three economists went to win the Nobel Prize in 2009 and the matching model is now colloquially known as Diamond-Mortensen-Pissadires (DMP) model.

With the emergence of the matching model, the static-neoclassical approach to unemployment, which considers little usage of the dynamic allocation process, is deemed obsolete. This is especially problematic in analyzing macroeconomics data where frictional and structural unemployment, temporal and permanent reallocation, and sectoral shifts in employment take place (Entorf, 1998). Implicatively, frictionless neoclassical model cannot accurately capture the nature behind the workers transition over time and hence would be a poor predictor of the business cycle fluctuations (Pissarides, 1974; Yashiv, 2007). Perhaps, the most notable coup de grâce chastise on the neoclassical model is the fact that it fails to explain as to why full employment is never attainable and how vacancy and searching individual coexist at the same time (Yashiv, 2007). Search and matching framework are able to cover these tumultuous questions.

However, the search and matching model did not inescapably avoid criticisms. Although the matching theory serves an excellent explanan behind frictional unemployment, it does not mention the nature behind *voluntary* unemployment, since it considers all agents in the economy as a part of the labour force (Tüzemen, 2012). In other word, the model is deficient in a sense that it does not capture the fact that some individuals choose to be unemployed. Moreover, Shimer (2005) argued that the standard matching model is insufficient in capturing the fluctuations of unemployment rate in the various stage of business cycles. Although Hall (2005) and Pissarides (2009) infer that the model's predictability is enchanced when wage is sufficiently rigid, there are normative suggestions to increase the applicability of the matching model across a wider range of circumstances.

Since the topic of unemployment is particularly a subject of scrutiny in recent crisis, the societal relevance of such topic is not trivial. Hence, this thesis offers possible explanations to aforementioned criticisms. Building on a simple microeconomics framework, we show that incorporating voluntary unemployment in the matching framework is possible. This is done so by accounting that voluntarily unemployed individual derives some benefits by refraining to enter labour force. Thus, this forms a three-way relationship between searching individuals, working individuals, and out-of-labour-force individuals. By doing so, we are able to decompose the nature of both type of unemployment - voluntary and involuntary - and hence enabling us to isolate and determine the frictional (involuntary) unemployment apart from the voluntary unemployment. This juxtaposition would be the heart of this thesis.

In addition, our model also covers the heterogeneity across individuals. With heterogeneities, it is conceivable as to why some individuals decide to enter the labour force and some do not. Utilizing DMP framework, we aim to analyze labour supply-demand relationships while simultaneously disintegrates the aggregate unemployment structure into voluntary and involuntary unemployment. We do so by scrutinizing on the agents' behaviour in the micro-level structure before aggregating into the macro level. In short, our fundamental aim is then to study the nature of unemployment from grassroots level in order to get a better picture of the implication of such behaviour in macro level. Indeed, quoting Hommes et al (2012):

"What is the relationship between heterogeneous individual learning at the micro level and the emerging aggregate macro behaviour which it co-creates?"

We shall show that the distinction between the two types of unemployment is crucial, as the involuntary unemployment entails a larger efficiency loss compared to the voluntary unemployment. The main idea is that individuals who are voluntarily unemployed still benefit from the out-of-the-labour-force activities, such as leisure and unemployment benefits, whereas involuntary unemployed individuals sacrifice such benefits by searching. If policy makers or government plan to cut loose unemployment, we suggest that cutting the involuntary unemployment should be the foremost objective. The intuition behind this suggestion would be elaborated throughout this thesis.

Moreover, to give a different perspective on the possible determinant of aggregate unemployment level and to possibly explain the underestimated fluctuations by DMP model as suggested by Shimer (2005), we extend our comparative statics by relaxing the notion of rational expectations. We infer that the model with naivety could better explain the fluctuations over the business cycles. Thus, we consider a matching model with backward-looking agents that based their decisions on the incidence in the past. Although mainstream matching model are meant to be utilized in a macroeconomics environment, our strategy to analyze matching theory by emphasizing on agents' decision allows for an integration of behavioural complexity. Precisely, we employ a dynamic iteration model, specifically Cobweb Model, as introduced by Nerlove (1958) and, most recently, developed by Hommes (2013). Thus, this extension provides an additional insight to the matching model by putting much scepticism on the notion of rationality.

In short, we utilize the matching framework in our general equilibrium model to provide an overview on the composition of unemployment in the equilibrium. Moreover, by scrutinizing on the micro-level behaviour, our approach enables us to integrate complex analysis of the interaction between economic agents to picture the dynamic of (un)employment rate. To conclude, the main purpose and scientific relevance of this thesis are to complement the existing theory of unemployment by providing a more nuanced overview of the complexion of unemployment.

The structure of this thesis will be as follows. Chapter 2 discusses some seminal literatures on the development of matching theory as well as the ontology behind our model. Chapter 3 deals with the fundamental of our model, which is based on the framework of DMP model. Chapter 4 describes the equilibria and comparative statics of our model. Chapter 5 briefly discusses an econometrics technique in assessing unemployment using the intuition of our model. At the closure of this thesis, the conclusion and some policy recommendations will be stated in the concluding chapter.

Chapter 2

Literature Review

The conventional approach in analyzing labour market is the neoclassical framework of demand-supply interaction. Starting from 1960's, many economists, notably Stigler (1962), Holt and David (1966), and Phelps et al (1970), started to infer that neoclassical model is not sophisticated enough to accurately depict the reality, especially with regards to the determinant of unemployment. Ever since Stigler (1961) stressed the importance of information in economics, economists' view on the behaviour of economics agents has drastically changed. In his paper, he elaborated on the 'search' behaviour of buyer and seller in finding a suitable price due to incomplete information. Applying this concept to the context of labour market, he convened the search theory of unemployment, which has been highly influential in advancing economists' understanding on the nature of unemployment. This theory infers that heterogeneity among workers, in addition to incomplete information, as the main cause of the friction in labour market and dysfunctionality of neoclassical model (Stigler, 1962).

Building upon the fundamental of search theory, economists have begun to put more emphasize on the interaction between *two* searching parties: individuals and firms, thus constructing a flow approach in analyzing unemployment. Diamond, Mortensen, and Pissarides are the most pronounced intellectuals that are deemed to have delivered substantial contributions to this model. Mortensen (1978) was arguably among the first who convened this intuition, where he put forth the idea behind 'two-sided search' and the dynamic of the movement in-and-out of labour market. Diamond and Maskin (1979) later conceptualized matching function. Matching function is a mathematical concept that defines the explicit count of the matching job between firm and worker. Matching function can be defined as the number of matching job as a function of the number of employed worker and vacancy available. The first economist to test the applicability of the matching model empirically was Christopher Pissarides. Pissarides (1979) studied the macroeconomic behaviour of equilibrium unemployment by utilizing matching function on U.S. data.

One of the major refinements in the matching model was the insight as to how wage is determined. While wage is exogenously given in neoclassical model, Diamond (1982) argued that endogenous bargaining process affects the efficiency in searching, which in turn negate the notion of 'instant allocation of resources' and frictionless labour market. This proposition was seminal and is still widely used as a fundamental wage-determination model in modern matching model. His insight also motivates further research that utilizes the integration of endogenous wage component to DMP framework. A seminal paper by Pissarides (1985) is one example. In this paper, he analyzed the effect of cyclical fluctuations to the short-run level of unemployment and wage bargaining dynamics.

Arguably, the last major improvement in the matching model was the inclusion of endogenous job creation and destruction in matching framework by Mortensen and Pissarides (1994). The endogenous job creation implies that potential employers also consider the state of nature before opening up any vacancy. Thus, Mortensen and Pissarides (1994) considered a case where a job is subjected to idiosyncratic job-specific risk, in which the extent to which jobs are created or destructed is dependent on this notation. In short, with their model, the state of economy, such as boom or recession, can also be captured and reflected to equilibrium unemployment rate to some extent. This model is particularly useful as a proxy in catering cyclical changes in unemployment found in U.S. data.

Based on Mortensen and Pissarides (1994), Pissarides (2000) wrote, arguably his *magnum opus*, a well-known textbook that comprehensively assesses unemployment based on the matching model. In this book, Pissarides compactly defines the matching function, endogenous wage determination, and equilibrium unemployment rate based on endogenous job creation/destruction and vacancy-unemployed match. Mortensen and Pissarides (1994) paper and Pissarides' (2000) book has since been the mainstream standard handbook theory of unemployment.

We can summarize in three reasons as to why the matching model is deemed to have advanced scholars' understanding regarding the labour market dynamic. First, it rejects the notion that the wage is exogenously given and is the main determinant of employment rate in the equilibrium. It proposes the idea where equilibrium employment rate is determined by the interaction between searching workers and vacancies. Second, it integrates the idea of Nash bargaining to determine the wage level in the equilibrium. This put forth the complexity of the interaction between job creation and endogenous wage determination as a fundamental determinant of the equilibrium unemployment rate. Lastly, the advantage of analyzing labour market equilibrium through the interaction between vacancies and unemployed is that it provides a more sophisticated overview of the implication of various labor market policies to the equilibrium employment rate. Indeed, wages alone cannot explain the cyclical fluctuations over the business cycles and the scrutiny on the dynamic of the searching behaviour might be a pivotal key to understand the dynamic of the labour market.

The model presented here is similar to the endogenous labour force participation (LFP) model by Tüzemen (2012). This model utilizes a three-state labour market structure, instead of a standard two-state model exhibited in DMP model. Whereas DMP model considers all agents to be in the labour force, the 'third' state introduced in endogenous LFP model is the disequilibrium choice of voluntarily unemployed individual. To analyze the interaction between individuals and firms in the labour market, we utilize Agent-based Computational Economics (ACE) as a tool to model the equilibrium (un)employment rate based on this interaction.

In this thesis, we notably put forth a circumstance where the notion of rationality is relaxed. We cite Hommes (1994) and Hommes (2013) as the fundamental literatures basis of our ACE study with naivety. Hommes (1994) emphasized on the implication of adaptive expectations in shaping the pricing dynamic in conventional supply-demand model. In this thesis, we apply this intuition into our matching framework. Hommes' (2013) book, which investigates complex economics system with boundedly rational interacting agents, is the basic foundation of our model with naïve individuals. Using this theory, we hope to shed a light on Shimer (2005) criticism on standard matching model that seems to underestimate business cycle fluctuations under productivity shocks.

Chapter 3

The Model

3.1 Matching Function

We first define the concept of matching function as suggested by the DMP model. The matching function is defined as the number of job match as a function of unemployed individuals and number of vacancies available.

$$M = m(u, v) \tag{3.1}$$

where M is twice differentiable and concave. Variables u and v are the number of unemployed and vacancies as a fraction of the labour force L, respectively. [1]

One plausible assumption is that matching function exhibits constant return to scale (Pissarides, 2000). CRS assumption ensures that matching function grows proportionally in u and v. Now, we can derive the number of matching count per vacancy:

$$q(\theta) = \frac{M}{v} = m(\frac{u}{v}, \frac{v}{v}) = m(\theta^{-1}, 1) \tag{3.2}$$

where $\theta(= \frac{v}{u})$ is the labour market tightness. High θ implies that there is a small number of job-seekers relative to the available vacancy, making it more difficult to fill the vacancy and labor market is said to be 'tight'.

[1]Matching function can take several forms. For example, Mortensen (1982) proposed a linear matching function $M(u, v) = a_1 v + a_2 u$, where a_1 and a_2 are the frequency of contacts. Alternatively, one can consider a quadratic form of $M(u, v) = (a_1 + a_2)uv$. Another realistic assumption is that matching function takes a Cobb-Douglas form: $M(u, v) = u^{\alpha} v^{1-\alpha}$. This implies that whenever u or v equals 0, then M is equal to 0; matching cannot happen in case of either no unemployed worker or vacancy exist in the economy. Nevertheless, the exact specification of the matching function will not alter our result by any mean.

We formally define $q(\theta)$ as the probability of matching and filling a vacancy. It is easy to notice that $q(\theta)\theta$ is therefore the probability of finding a matching job for an unemployed worker. To simplify the notation, we denote:

$$q(\theta)\theta = p(\theta) \tag{3.3}$$

where $p(\theta) = \frac{M}{u} = m(1, \theta)$[2].

3.2 The Economy

Households We suppose that our economy consists of a large number of households, in which each household is composed of continuum of individuals. Households will allocate each of its individual member to search for a job or not. Searching is equivalent to being in the labour force. Individual who searches will get a job with probability $p(\theta)$ as defined before. This implies that in equilibrium, some individuals matched with the firm, some fail to match with the firm, and some decide not to enter labour force. The household derives some utility from consumption, in which consumption can only be substantiated through the working individuals. To understand the result of our model, it is pivotal to unequivocally define these following operationalization of unemployment in our framework

DEFINITION 1 ***Frictional Unemployment*** *A set of individuals that are involuntarily unemployed due to the failure in finding a job match in the labour market.*

Time Period However, one could also argue that frictional unemployment is involuntary when mismatch happens because an individual can reject an offer in hand. Nevertheless, in our model, we scrutinize the searching behaviour only in one period such that a worker would accept any offer that is above her reservation preferences. With this, we rule out the voluntary component in mismatch. Hence, individuals who decide to enter the labour force but fail to find a match must experience utility loss. Contrastingly, voluntarily unemployed individual will derive benefits outside the labour market. We refer to all the benefits pertinent to this as leisure.

DEFINITION 2 ***Voluntary Unemployment*** *A set of individuals that are out of labour force and derive some benefits from leisure.*

[2] Matching process is defined within a fixed time interval and the probability distribution of a worker finding a job match or a firm filling a vacancy is therefore defined in a Poisson process:

$$q(\theta) \sim Poisson(\mu),\ p(\theta) \sim Poisson(\mu)$$

where μ is the absolute mean match count.

At the begininng of the period, all inidividuals within households are unemployed and are included within u. We denote $u \equiv L$ for notational simplicity. Let $L^s \subseteq L$ denotes the rate of individuals across households who are allocated to search. $L - L^s = u_v$ would then be the proportion of households who decide to be voluntarily unemployed. Among L^s, only a portion of $L^* \in L^s$ will find matching jobs. The remaining portion u_i of households fails to find a match. Variable u_i is thus equivalent to the involuntary unemployment rate. Clearly, the total unemployment rate in the economy is $U = L - L^*$, where U can also be written as a linear combination of u_I and u_v, that is, $U = u_i + u_v$.

<u>Definition 3</u> ***Households' Component*** *A plane* $\mathbb{R}^3$ *that consists of subspaces* $\begin{bmatrix} L^s & u_v & u_i \end{bmatrix}$, *where* $L^* \in L^s \subseteq L \supseteq u_v$*. Moreover,* u_i *and* L^* *of* $\mathbb{R}^2$ *are the bases of* L^s*.*

Variable	Description
L	Households size or all individuals in the economy
L^s	Individuals searching for job
L^*	Searching individuals that secure matching
U	Total unemployment rate
u_i	Involuntary unemployment rate, or searching individuals that fail to match
u_v	Voluntary unemployment rate, or individuals who decide to leisure

Table 3.1: Households' Component Summarized

Firms On the demand side, consider a perfectly competitive market with many firms, where aggregate firms demand L^d unit of labour. Not all firms search, only a portion of $L^d \subseteq v$ does so, where only a fraction of $L^* \in L^d$ matches. For simplification purpose, we suppose that labour is the only input of production and each individual member in the household is a perfect substitute input for the firm. Firm will search for a worker and match with probability $q(\theta)$. Analogously, $L^d - L^*$ depicts the unfilled vacancy rate.

Equilibrium The resulting equilubrium from this interaction is a level of θ for which a portion of L^* of households matches with firms. Formally, the definition of an equilibrium is as follows:

<u>Definition 4</u> ***Matching Equilibrium****: An allocation of* (L^s, u_i, u_v) *of households' problem and* (L^d, v) *of firms' taking* $(\beta, z, y, \psi, \gamma, \theta)$ *as given such that labour market clears.*

Above definition infers that households optimally choose a portion of individuals to search, which directly implies that the portion of individuals that is allocated to leisure is also chosen. Similarly, firms choose how many worker that they demand out of all available vacancies in the economy. Clearly, if *all* firms would search, then labour demand is essentially equal to the number of vacancies available ($L^d = v$).

By definition, there exists a level of market tightness as a function of matching labour that define the equilibrium in our matching model. This implies that we take variable θ as given, similar to the variable wage in the neoclassical model. Conceivably, since there exists a large number of households, the decision of one household will not change the labour market tightness. Hence, each household will take the labour market tightness as given. This is also applicable to each individual firm. Since there are many firms, the externality imposed by changing v is negligible. The main intution behind this model is that the equilibrium level of employment boils down to the level of θ, or the ratio of vacancy to unemployment chosen by the respective party.

Timing and Mechanism of Matching Throughout our analysis, we suppose that there is no initial match, which means that every individuals are initially unemployed and no individual can find a job without searching and subsequently matching with a firm. This assumption is particularly handy since we are interested in inspecting frictional unemployment. Hence, our focus is pinned down solely to individuals that are seeking for jobs, and not to those who already have one. The point where the number of matching count per vacancy and number of matching count per unemployed worker are equalized will define the number of individuals employed in the equilibrium. Finally, we summarize the timing of our matching model as follows

1. At the begininng of the period, all individuals within households are unemployed and are included within the pool of unemployed workers $u(= L)$, whereas firms' available vacancies amounts to v

2. From here, labour market tightness θ is constructed and competition will force both parties to take θ as given

3. Households allocate a proportion L^s of individuals to search for a job, whereas firms demand L^d amount of labour. Individuals that are not allocated to search are included within u_v and derive some benefit from leisure

4. Matching process occurs with probability as defined by the labour market tightness. Post-matching process, a fraction of u_i individuals fail to secure a match

3.3 Wage Determination

As suggested by Diamond (1982), to get a more realistic depiction of the nature of unemployment, we will not treat wage as an exogenous variable. Rather, it is endogenously determined through the bargaining process between firm and worker[3]. Herein, we define Bellman Equation and utilize dynamic programming to solve the Nash Equilibrium outcome of bargaining. The bargained outcome is vital in deciding whether individuals and firms would search or not.

For simplicity, assume a single bi-directional relationship between a single worker and a firm. Since we are focusing on the frictional unemployment, we scrutinize only to the case where an unemployed individual finds its initial match. Furthermore, both parties are risk neutral and the planning horizon is infinite. From an individual's perspective, she will decide to work if it yields her a positive surplus. This surplus is equal to the present-discounted value of working (denoted $E(w)$) minus the present-discounted value of non-working activities (denoted U). Similarly, firm will only hire this worker if it gives it a positive surplus. The surplus is the difference between the present-discounted value of profit from hiring this individual (denoted $J(w)$) and present-discounted value of opening up a vacancy (V). Adopting an intermediate Cobb-Douglas form, the bargaining equation is thus straightforward:

$$w = \underset{w}{\operatorname{argmax}} (E(w) - U)^{\beta} (J(w) - V)^{1-\beta} \tag{3.4}$$

where β captures the bargaining power of workers.

Basically, we can substitute the asset value of each term in (3.4) to solve for the bargaining problem. We have put the comprehensive derivations in Appendix A. In short, the resulting bargaining equation is

$$w = \beta(y + \gamma\theta) + (1 - \beta)z \tag{3.5}$$

where y is the surplus from matching, γ is the vacancy or search costs of the firm, and z is utility derived from out-of-the-labour-force activities by the individual. It is noteworthy to mention that herein, we assume 'free-entry condition', that the rent from opening additional vacancies, V, is 0 (Pissarides, 1985; Mortensen and Pissarides, 1994; Pissarides, 2000). Conceivably, if $V > 0$, then opening up vacancy entails non-negative benefits. With free entry condition, more firm will enter the market to open the vacancy until the benefit from leaving the job vacant is eroded [4].

[3] Although Diamond (1982) was the one who first proposed endogenous wage model in matching framework, mainstream economists contemporarily utilize Shapiro-Stiglitz efficiency wages model to determine the bargaining outcome. The wage formula that is shown here is derived from Pissarides (2000). See: Shapiro and Stiglitz (1984) for the efficiency wages model.

[4] The implication is that there will be a limit for which the number of v is opened by firms. If $V > 0$, more firms will open up new vacancies and labour market tightness goes up. Since probability of finding a match for the firm ($q(\theta)$) is decreasing in θ, it must be that the profit from opening additional vacancy diminishes with more firm opening up new vacancies. Free entry assumption ensures that this rent is exhausted.

However, contrast to Pissarides (2000), we will consider a simple one-period setting in our analysis. This would lead to a slightly different outcome of the bargaining

$$w = \beta y + (1 - \beta) z \tag{3.6}$$

Notice that the term $\gamma\theta$ is considered as sunk in one-period setting[5].

The intuition of this equation can easily be seen by considering two extreme cases. When $\beta = 0$, the wage paid to the individual is equal to the reservation wage as workers have no bargaining power. Contrastingly, when $\beta = 1$, the worker has all the bargaining power and will reap all the profit from the firm.

From above bargaining equation, one could inquire that workers essentialy play an ultimatum game since they have an alternative option of z if she chooses not to search. To see why, we rewrite

$$w = z + \beta(y - z) \tag{3.7}$$

We can see that the wage paid to the worker needs to be at least equal to her utility from leisure plus some surplus from matching. However, the firm has no such alternative option and is forced to choose between getting zero profit or sharing the surplus amounting to $z + \beta(y - z)$ with the worker. Hence, we can argue that, implicitly, $\lim_{\beta \to 1} \beta \approx 1$ and since each firm's surplus is equal to $y - w = y - z - \beta(y - z)$, $\beta \approx 1$ implies that all firms will earn close-to-zero profit in the equilibrium and would be indifferent between searching and not.

Empirically, however, this deduction is not realistic since firms are observed to possess substantially more power than workers. Throughout the history, prominent scholars, economist, and contract theorist have scrutinized the matter of the inequality of bargaining power and found that employers are significantly more powerful in determining the bargaining outcome (Marx, 1893; Webb & Webb, 1897; Pigou, 1920). Webb and Webb (1897) argued that workers are more pressured to sell their labour than the employers are to buy. This is because that "bargaining power depends on which party has the resources to hold out the longest in the negotiations" (Kaufman, 2009). Nevertheless, with the notion of efficiency wages, minimum wages and performance pay, it is unlikely that Marx's deduction, whereby inequality of bargaining power would drive wages into the subsistence level, holds true. Therefore, although we take variable β exogenous, abovesaid enquiries raise an assumption that the β in our model takes some intermediate value, instead of an extreme one, and hence the surplus are always shared in the

[5] Intuitively, the term $\gamma\theta$ denotes the expected vacancy cost that is saved if a vacancy is filled. If γ or θ increases, it necessitates firms to pay higher wage to share the surplus with the worker, in which the amount is dependent on the bargaining power of workers.

equlibrium. Thus, the asymmetric bargaining components are still within a reasonable boundary.

Bargaining Timing Contrast to mainstream wage determination model in on-the-job-search framework, our bargaining phase occurs before the matching process takes place. This bargaining method is considered as a bargaining wage approach under a perfect information scheme, whereby the outcome of the bargain is more less certain before employee and employers met. This implies that in our environment, firms and workers can readily estimate the surplus value of matching and firms are aware that individuals would search only if the wages fall in between $[z, y]$. Thus, given variable β, the equilibrium wage must fall under $[z, y]$ to induce individuals to search. In reality, one can think of a job-posting wage determination approach, whereby the expertise of workers are expected and the salary are known in advance before matching takes place (Hall & Krueger, 2012). Thus, in our model, we stress that wage merely act as a proxy in catering an individual's search decision. By taking into account that individuals know the expected value of matching, we can focus on the parameters that motivate individuals to search.

Chapter 4

Result

4.1 Equilibrium with Homogenous Households' Component and Firms

The simplest case of our model concerns the determination of unemployment where both household and firm are homogenous entities.

Household's Problem We first determine the optimization problem of the household. Consider an instantaneous expected welfare function of the household, $EU(.) : \mathbb{R}^{3+} \to \mathbb{R}$, as a function of consumption, leisure, and searching activity:

$$EU(U(c), u_v, L^s) = U(c) + zu_v - \psi L^s \tag{4.1}$$

Recall that z is the utility derived by leisuring individuals, whereas ψ is the cost of searching. Let $z > 0, \psi > 0,$ and $U(c) > 0$ be our assumptions. For simplicity, assume that utility from consumption is linear: $U(c) = c$. Since the utility function is linear, leisure and search decision are perfect substitutes, which is logical.

Here, we introduce three constraints:

1. Budget constraint. Consumption can only be substantiated through working and thus satisfies $c = wL^*$.
2. Allocation constraint. The proportion of leisuring household is equal to the proportion of household who does not search: $u_v = L - L^s$. Whereas out of L^s, only a portion of $L^* \in L^s$ will find a match. Thus, we can write $L^s = L^* + u_i$. The proportion of household that is voluntarily unemployed is then $u_v = L - L^* - u_I$.

3. Search constraint. Labor supply can only be realized through matching. We can directly infer from here that, out of L^s, only a proportion of $p(\theta)u$ would find a match, whereas the remaining $u_i = (1 - p(\theta))u$ fails to find a match and hence be frictionally unemployed. Search constraint is thus $L^* = p(\theta)L^s$. Rearranging yields $L^s = \frac{L^*}{p(\theta)}$ as the search constraint.

Substituting these constraints will transform abovementioned multivariate optimization into a static-constrained optimization[1].

$$EU(L^*) = wL^* + z(L - L^* - u_i) - \psi\frac{L^*}{p(\theta)} \tag{4.2}$$

From this model we derive this following Lemma:

Lemma 1: *In optimum condition, households with homogenous component will allocate all individuals to the labour market when* $p(\theta) \geq \frac{\psi}{\beta(y+\gamma\theta-z)}$

Proof: Since the probability of finding a match is increasing in the labour market tightness, it is intuitive that an individual's labour force participation decision is strictly increasing in θ. Conceivably, there exists a range of 'tightness' in which some individual would search and supply positive labour. Before maximizing household's problem with respect to the labour supply, we substitute the Nash solution into wage.

$$EU(L^*) = (\beta y + (1-\beta)z)L^* + z(L - L^* - u_i) - \psi\frac{L^*}{p(\theta)} \tag{4.3}$$

Next, the first-order condition is: :

$$\beta(y - z) + z - \frac{\psi}{p(\theta)} = 0 \tag{4.4}$$

Individuals will supply positive labour supply in a region where equation (4.4) is strictly increasing in L^*.

$$L^* = \max\left\{\beta(y - z) + z - \frac{\psi}{p(\theta)}, 0\right\} \tag{4.5}$$

Aggregating across households and expressing in term of probability of finding a match, we can express (4.5) as

[1] Alternatively, one can construct this following lagrangian with multiple constraints: .

$$\mathcal{L}(c, u_v, L^s, \lambda, \jmath, \ell) = c + zu_v + \psi L^s - \lambda(c - wL^*) - \jmath(l - L - L^* - u_i) - \ell(u - \frac{L^*}{p(\theta)})$$

$$L^*(\theta) = \begin{cases} 0 & \text{if } p(\theta) < \frac{\psi}{\beta(y-z)+z} \\ [0, p(\theta)u] & \text{if } p(\theta) = \frac{\psi}{\beta(y-z)+z} \\ p(\theta)u & \text{if } p(\theta) > \frac{\psi}{\beta(y-z)+z} \end{cases} \tag{4.6}$$

If the condition $p(\theta) < \frac{\psi}{\beta(y-z)}$ holds, no one will be allocated to the labour force and everyone would be voluntarily unemployed. Conversely, if the argument (4.5) is strictly increasing in L^*, that is, if $p(\theta) > \frac{\psi}{\beta(y-z)}$, all the individuals will be allocated to search. Of course, when the terms are equal, households will be indifferent between allocating its member to the workforce or leisure. In conclusion, households' choice boils down to the level of θ, instead of wages, and the searching decision will be positive if and only if θ is such that $p(\theta) > \frac{\psi}{\beta(y-z)}$. This concludes the proof to our lemma. ■

We mentioned that households optimally allocate $L^s \subseteq L$ individuals to search. However, in this setting, households' component consists of homogenous individuals and since the households' utility function is linear, the household is unable to make a trade-off. Therefore, it must be the case that either *all* individuals are volutarily unemployed (if $\theta < \theta^s$) or *all* individuals are allocated to search (if $\theta > \theta^s$). Labour supply choice is thus written as a function of the labour market tightness ($L^*(\theta)$).

Firm's Problem Next, we solve firms' maximization problem. We suppose that the profit function of a firm is also linear:

$$\Pi(L^*) = (y - w)L^* - \gamma v \tag{4.7}$$

The search constraint for the firm is the number of vacancy that matches with workers that search for a job, such that it will define equilibrium labour demand equation as $L^* = q(\theta)L^d$. Rearranging yields $L^d = \frac{L^*}{q(\theta)}$ as the search constraint. Profit function would then be:

$$\Pi(L^*) = (y - w)L^* - \gamma \frac{L^*}{q(\theta)} \tag{4.8}$$

From here, our second lemma is derived:

Lemma 2: *All firms will search and labour demand is positive when* $q(\theta) \geq \frac{\gamma}{(1-\beta)(y-z)+\beta\gamma\theta}$

Proof: First we substitute Nash solution into (4.8)

$$\Pi(L^*) = (y - \beta y - (1-\beta)z)L^* - \gamma \frac{L^*}{q(\theta)} \tag{4.9}$$

Maximizing (4.9) with respect to labour demand yields:

$$y - z - \beta(y-z) - \frac{\gamma}{q(\theta)} = 0 \tag{4.10}$$

Aggregating across multiple firms, the optimal labour demand choice is thus:

$$L^*(\theta) = \begin{cases} 0 & \text{if } q(\theta) < \frac{\gamma}{(1-\beta)(y-z)} \\ [0, q(\theta)v] & \text{if } q(\theta) = \frac{\gamma}{(1-\beta)(y-z)} \\ q(\theta)v & \text{if } q(\theta) > \frac{\gamma}{(1-\beta)(y-z)} \end{cases} \tag{4.11}$$

Since firms' profit function are also linear, we similarly develop a corner solution whereby *all* firms will either search to fill the vacancies or not at all. ■

With the decision of both parties specified, we are ready to determine the equilibrium outcome in our matching model. On the supply side, we know that the function $p(\theta)$ is increasing in θ. The impact of the labour market tightness is thus positive to the labour supply. At sufficiently large value of θ, there exists a range in which $p(\theta) > \frac{\psi}{\beta(y-z)}$ and labour supply is positive. Let θ^s be the reservation tightness that induces household to be indifferent between allocating its members to the labour force and not to allocate at all. Of course, by definition, θ^s is the level of tightness when $p(\theta) = \frac{\psi}{\beta(y-z)}$

Similarly, on the demand side we denote θ^d as the reservation tightness that induces homogenous firms to be indifferent between searching and not. The argument from the demand side is exactly the opposite to that of labour supply since the probability to find a match for the firm, $q(\theta)$ is decreasing in θ. Thus, if θ is sufficiently high, there might be a condition where *all* firms would not demand any labour.

The equilibrium employment rate is defined when the matching rate of both parties are equalized:

$$p(\theta)u = q(\theta)v \tag{4.12}$$

With $\frac{L^d}{L^s} = \theta$, we have

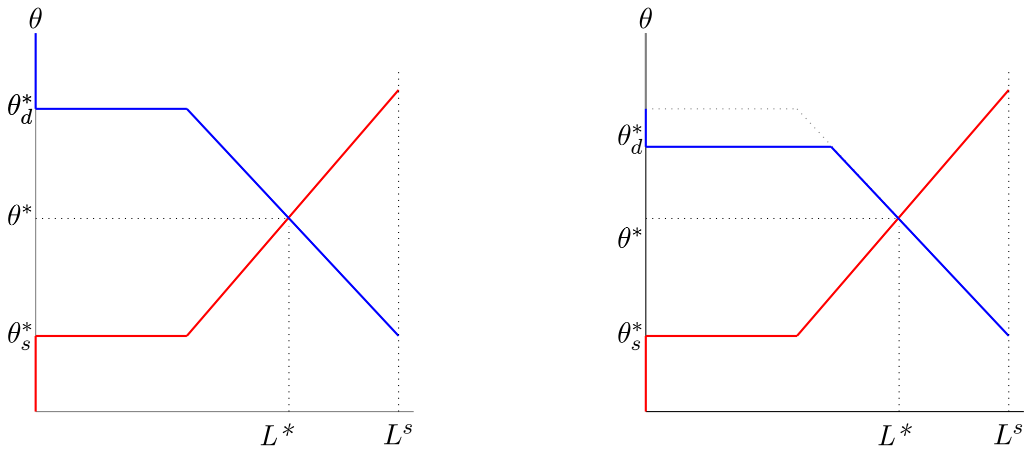
$$p(\theta) = q(\theta)\theta \tag{4.13}$$

This is essentially the condition depicted in (3.3). After substituting the search constraint, the equilibrium labour market tightness is thus

$$\theta^* = \frac{L^d}{L^s} \tag{4.14}$$

This is essentially the condition depicted in (3.3). After substituting the search constraint, the equilibrium labour market tightness is thus

$$\theta^* = \frac{L^d}{L^s} \tag{4.15}$$



(A) Matching Equilibrium. (B) Equilibrium after an increase in γ.

FIGURE 4.1: Depiction of matching equilibrium in its steady state and when it is fed by an exogenous shock

The implication of the corner solution is that the equilibrium level of employment is fixed. From Figure 4.1, we depict a condition where an increase in γ does not change the equilibrium employment rate. Since equilibrium condition boils down solely to θ, variable γ that does not affect the probability of matching does not alter our equilibrium. The only thing that change is firms' reservation tightness, θ^d, which has gone down because searching becomes less attractive.

Moreover, we see that equilibrium can only exist if $\theta^d > \theta^s$, that is, when firms' marginal tightness threshold to search is higher than households. Hence, the only possibility where an exogenous shock would alter our equilibrium is when the shock induces disequilibrium condition of $\theta^d < \theta^s$ to hold. For example, with a positive shock in γ, an alteration from equilibrium to disequilibum condition is possible only if the shock is large enough to induce $\theta^d < \theta^s$, or otherwise our equilibrium (un)employment rate will not change. Since our equilibrium entails a corner solution, the existence of an equilibrium implies that no one derives a benefit from leisure since everyone is practically allocated to search. Conversely, a case where an equilibrium does not exist suggest that all household members' are voluntarily unemployed. Notice that the distance $L^s - L^*$ measures the involuntary unemployment rate, or the rate in which the 'searching group' of individuals fails to find match[2]. We conclude this section by proposing this following argument

Proposition 1 *Homogenous households will allocate $L^s = L$ individuals to search if the condition in Lemma 1 is satisfied and homogenous firms will demand $L^d = v$ labours if the condition in Lemma 2 is satisfied. Equilibrium can only exist if $\theta^d > \theta^s$ and exogenous shocks will not change the equilibrium employment rate unless it leads to a disequilibrium case whereby exogenous shocks trigger the condition $\theta^d < \theta^s$ to hold.*

4.2 Equilibrium with Heterogenous Households' Component and Firms

A model with homogenous households' member is unrealistic in many senses. On theoretical ground, the model is too rigid to capture the circumstance whereby some individuals are allocated to search and some to the voluntary unemployment pool within a household. On empirical ground, it is inconceivable for individuals within a household to have homogenous preferences. The resulting comparative static in case of homogenous household's member is also inconceivable, since in most instances the equlibrium (un)employment rate is unaltered in a presence of an exogenous shock. In this section, we introduce some heterogeneities across individuals within households and firms to depict a more realistic case whereby households allocates only some proportion of individuals to search and not all firms intend to fill their vacancies.

Assumption 1 *z_i is heterogenous across individuals within households, ceteris paribus*

Household's Problem Let us suppose that individuals have different valuation of z. Furthermore, let $z_i \in [\underline{z}, \bar{z}]$ be uniformly distributed across individuals in the household. Let n be the number of individuals in the household. The value function of each household is now

$$EU(U(c), u_v, L^s) = U(c) + \sum_{i=1}^{n} z_i \, u_v - \psi L^s \tag{4.16}$$

Solving above's maximization will not change the conclusion depicted in section 4.1 since $\sum_{i=1}^{n} z_i = z$ in our model. Since ultimately we want to depict the case where heterogeneity in z_i would lead to an interior solution of the household's labour supply decision, the household's linear utility function would not be a plausible explanation. Neverthelss, we can employ a case whereby each individual, whose utility function is also linear, has a liberty to choose her optimal search decision. From here, the decision of each respective individual will be aggregated to represent the household choice of

[2]Recall that $L^s = L$, since all individuals are allocated to search

labour supply. With this specification, it is possible to have some individuals working and searching at the same time within a household.

We write an instantaneous expected utility function $Eu_i(.) : \mathbb{R}^2 \to \mathbb{R}$ of an individual i as follows

$$Eu_i(\mathrm{L}, u_v) = \mathrm{L}w - L_s^i\psi + z_i l \tag{4.17}$$

where $\sum_{i=1}^n Eu_i(.) = EU(.)$. Variable L denotes the realization of labour supply if an individual i matches. An inidividual chooses the amount of time allocated to search, L_i^s, and to leisure, l. Here we introduce 2 constraints

1. Search Constraint. Individual secures matching with some probabilities. Search constraint is defined by $\mathrm{L} = p(\theta)L_i^s$, where L_i^s is the labour supply of an individual.
2. Time Constraint. Let 1 be the available time. If l denotes the amount of time spent on leisure, $L_i^s = 1 - l$ would thus be the time spent on searching or, equivalently, supplying labour.

Lemma 3 *Suppose that an instantaneous linear utility function w_i is strictly increasing in z_i, where $z_1 \neq z_2 \neq ... \neq z_n$. An individual's choice boils down to a corner solution where $L_i^s(\theta|z_i) = 1$ if $p(\theta)w - \psi < z_i$ or $L_i^s(\theta|z_i) = 0$ if $p(\theta)w - \psi > z_i$, whereas the aggregate individuals' choice within a household depicts an interior labour supply decision $L^s \subset L$ of a household. The aggregate labour supply curve would thus be upward sloping in θ.*

Proof We substitute above constraints to find

$$Eu_i(L_i^s) = L_i^s(p(\theta)w - \psi) + z_i(1 - L_i^s) \tag{4.18}$$

Taking first-order condition with respect to labour supply gives us

$$p(\theta)w - \psi - z_i = 0 \tag{4.19}$$

Or alternatively, we can write

$$L_i^s(\theta|z_i) = \begin{cases} 0 & \text{if } p(\theta)w - \psi < z_i \\ [0,1] & \text{if } p(\theta)w - \psi = z_i \\ 1 & \text{if } p(\theta)w - \psi > z_i \end{cases} \tag{4.20}$$

From here, the interpretation is straightforward. If the expected benefits from searching are less than leisure, an individual will decide not to search at all. Conversely, when the expected benefits of searching outweigh the benefits from leisure, an individual will allocate all available time on searching. This corner solution is achieved due to the nature of a linear utility function. Note that we do not substitute the Nash solution since it would not change our conclusion. To see why, let us write the labour supply decision in following from

$$L_i^s(\theta|z_i) = \max\left\{p(\theta)(\beta y + (1-\beta)z_i) - \psi - z_i, 0\right\} \tag{4.21}$$

Hence, although variable z_i affects wage, its influence is compressed by the probability of matching and bargaining parameter. All in all, high z_i would make it more likely for condition $p(\theta)(\beta y + (1-\beta)z_i) - \psi < z_i$ to hold.

Because each individual has different valuation of z_i and noting that $z_1 \neq z_2 \neq ... \neq z_n$, in equlibrium we will see some individuals searching for jobs and some individuals being voluntarily unemployed *within and between households*. In aggregate level, we have $L^s \subset L$ individuals allocated by the households in the labour force and since participation chance increases in θ, the aggregate labour supply curve will be upward sloping in θ. ■

Firm's Problem Since firms are homogenous, their labour demand decisions will not change and our conclusion in Section 4.1 still holds. Recall that we suppose that the wage determination occurs before matching takes place. If firms form homogenous expectation, say $\hat{z}$, the wages paid to all workers will be the same

$$\bar{w} = \beta y + (1-\beta)\hat{z}_i \tag{4.22}$$

With homogenous expectations, all firms will either search or not at all and only a handsful of individuals whose problem satisfy $p(\theta)\bar{w} - \psi > \hat{z}_i$ would search. The only thing in which the heterogeneity in z_i would affect firms' labour demand decision is when we assume that firms are also heterogenous with respect to the expected level of z_i that it will encounter in the labour market. Such assumption is meaningful. Indeed, if firms know that workers are heterogenous, would we expect all firms, who are competing againts each others, form exactly the same expectations regarding the type of worker that they will meet in the labour force? Perhaps not. Let us suppose such non-trivial assumption

Sub-Assumption 1.1 *Each firm forms heterogenous expectations with respect to z_i that it will encounter in the labour market*

With this sub-assumption, setting z_i heterogenous would indirectly impact the labour demand decision across firms since variable z_i enters the wage equation.

Lemma 4 *Suppose there exists heterogeneities in z_i, where $z_1 \neq z_2 \neq ... \neq z_n$, and each firm forms different expectations regarding the level of z_i. A firm will search for a worker if condition $q(\theta) > \frac{\gamma}{(1-\beta)(y_i - E(z_i)) + \beta\gamma\theta}$ strictly holds, where $E(z_i)$ denotes the* *expectations concerning the heterogenous parameter. The aggregate labour demand curve would be downward sloping in θ.*

PROOF We first write a firm's demand equation after substituting the search constraint and Nash solution, which essentially mirrors equation (4.9), as follows

$$\Pi(L_i^d) = (y_i - \beta y_i - (1-\beta)E(z_i))L_i^d - \gamma \frac{L_i^d}{q(\theta)} \tag{4.23}$$

where $L_i^d \in \{0, 1\}$ depicts a firm's labour demand decision, where a value of '1' implies to search and '0' to not search.

The difference between (4.23) and (4.9) is that parameter z_i is heterogenous in (4.23). The expectations term $E(.)$ indicates that a firm will 'speculate' the level of z_i that it will encounter in the labour market before deciding to search for in the labour market.

The optimization problem of a firm would then follow exactly the same intuition as in section 4.1.

$$L_i^d(\theta|z_i) = \begin{cases} 0 & \text{if } q(\theta) < \frac{\gamma}{(1-\beta)(y-E(z_i))} \\ [0,1] & \text{if } q(\theta) = \frac{\gamma}{(1-\beta)(y-E(z_i))} \\ 1 & \text{if } q(\theta) > \frac{\gamma}{(1-\beta)(y-E(z_i))} \end{cases} \tag{4.24}$$

Now, each firm would form a different expectation on the expected value of the surplus $E(z_i)$ before deciding to search. Since the expectation term $E(z_i)$ varies across firms, it implicates that the value of θ where $\theta = \theta^d$ also varies. Thus, in equilibrium, some firms would decide to search while some others would not. Since labour demand decision is decreasing in θ, our labour demand curve would then be downward sloping with respect to θ. ■

Notice the similarity between this derivation and the resulting individual's labour supply choice that we previously described. For labour supply decision with heterogenous individuals, a portion of L^s workers will be searching while the rest leisures. Assuming heterogenous expectations, for labour demand decision, aggregate firms would demand $L^d \subset v$ unit of labour. In another word, labour demand is less than the total vacancy rate since some firms decide not to search.

However, the heterogeneity in z_i makes less sense if we do not impose a strong assumption of heterogeneity in workers' skills. Specifically, it has to hold that an individuals' skills are increasing in z_i, that is, high z_i denotes a relatively high-skilled worker with high y_i. Because the Nash wage equation suggests that high z_i necessitates firms to pay higher wage, this assumption would rule out the possibility of a 'complete randomness' in job search. Without this assumption, the profit obtained by firms in equilibrium would purely based on luck - a firm that finds a worker with the lowest z_i gets the highest surplus. Since (4.23) ensures that a firm's profit is strictly increasing in y_i, imposing the assumption that higher z_i leads to higher y_i compensates the higher wage that a firm has to pay in case it were to meet an individual with high z_i.

What happen if we waive the assumption of $z_1 > z_2 > ... > z_n$ and $y_1 > y_2 > ... > y_n$? Now, besides the labour market tightness, the matching probability is also conditional to the fact that the surplus $(y_i - E(z_i))$ would be such that $q(\theta) > \frac{\gamma}{(1-\beta)(y_i - E(z_i))}$. Moreover, since z_i and y_i are randomly distributed, it would be difficult for a firm to form a consistent expectation and the aggregate labour demand equation is likely to be discontinuous at some point.

Assumption 2 *γ is heterogenous across firms, ceteris paribus*

Firm's Problem Let us assume that heterogeneities are now firm-specific. For example, let us assume that the search costs across m firms are heterogenous, that is, $\gamma_1 \neq \gamma_2 \neq ... \neq \gamma_m$.

Lemma 5 *Suppose that γ_i is heterogenous across firms, where $\gamma_1 \neq \gamma_2 \neq ... \neq \gamma_m$. Firms labour demand decision is now*

$$L_i^d(\theta|\gamma_i) = \begin{cases} 0 & \textit{if } q(\theta) < \frac{\gamma_i}{(1-\beta)(y-z)} \\ [0,1] & \textit{if } q(\theta) = \frac{\gamma_i}{(1-\beta)(y-z)} \\ 1 & \textit{if } q(\theta) > \frac{\gamma_i}{(1-\beta)(y-z)} \end{cases} \tag{4.25}$$

whereas the aggregate firms' choice is $L^s \subset v$. The aggregate labour demand curve would thus be downward sloping in θ.

<u>Proof</u> For the firms, we optimize

$$\Pi(L_i^d) = (y - \beta y - (1-\beta)z)L_i^d - \gamma_i \frac{L_i^d}{q(\theta)} \tag{4.26}$$

After taking the first-order condition, notice that nothing would change in a firm's optimization problem since we would ultimately arrive to the same conclusion as stated by Lemma 4.

$$L_i^d(\theta|\gamma_i) = \begin{cases} 0 & \text{if } q(\theta) < \frac{\gamma_i}{(1-\beta)(y-z)} \\ [0,1] & \text{if } q(\theta) = \frac{\gamma_i}{(1-\beta)(y-z)} \\ 1 & \text{if } q(\theta) > \frac{\gamma_i}{(1-\beta)(y-z)} \end{cases} \tag{4.27}$$

Recall that '1' means that the firm would search and '0' means that it ould not. Since γ is heterogenous, the demand curve would also be downward sloping, just as our conclusion under Lemma 4. ■

Household's Choice The heterogeneity in γ would not affect individuals' decision because γ does not affect individuals' decision by any mean.

Assumption 3 *ψ and/or y is heterogenous, ceteris paribus*

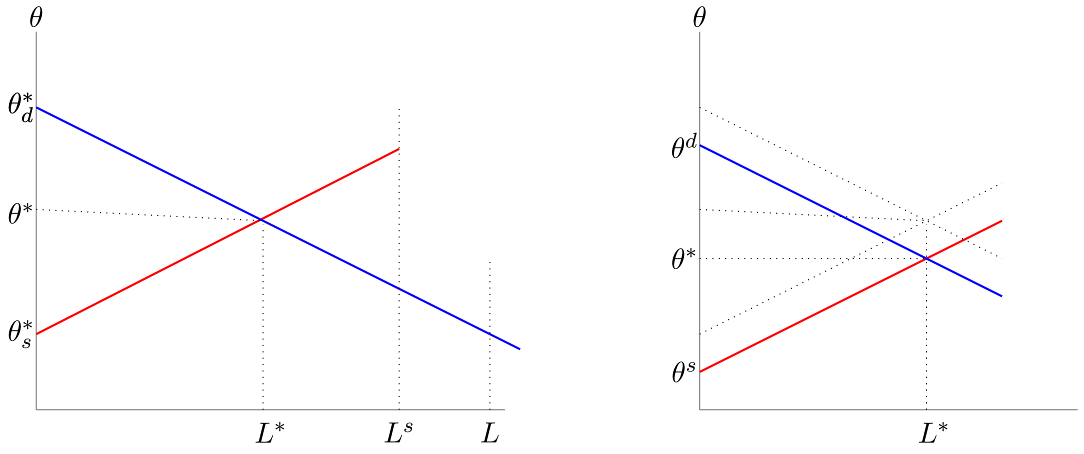
Alternatively, one can also set the search costs ψ heterogenous across inidividuals. In any case, this would not change our resulting conclusion of an interior solution of the aggregate labour supply decision of the households. With this specification, we would then have an upward sloping labour supply curve. Since variable ψ does not enter the labour demand equation of individual firm, keeping all other variables constant would mean that we still preserve the homogenous corner solution decision across firms.

Similarly, setting the surplus y_i heterogenous across firms would lead to an interior solution of the aggregate labour demand and labour demand curve would then be upward sloping. If we hold all other variables constant, the labour supply decision across households is of an interior solution since variable y_i enters the labour supply decision of individuals through the wage equation.

Equilibrium and Comparative Static Suppose that the heterogeneities components affect both labour supply and demand, that is, the resulting labour supply is upward sloping and labour demand is downward sloping. The resulting equilibrium of this interaction is thus similar to the neoclassical supply-demand model, albeit we utilize the labour market tightness as the determinant of the equlibrium employment rate. With this model, we are able to isolate between individuals that are inside and outside the labour market. The 'insider' can be distinguished between working individuals (L^*) and involuntarily unemployed individuals (u_i). Figure 4.2. illustrates the comparative static of our matching equilibrium.

Figure 4.2(A) visualizes the steady state equilibrium of our model under the assumption that both parties are heterogenous. The distance $L^s - L^* = u_i$ measures the proportion of individual that fails to find a match (involuntary unemployment). The distance $L - L^s = u_v$ measures the voluntary unemployment rate. On Figure 4.3(B), we show that when the elasticity of substitution between labour demand and supply with respect to the bargaining power of workers are the same, an increase in β has no effect on the equlibrium employment level. Due to heterogeneities, an exogenous shock could now change the resulting equilibrium (un)employment rate because heterogeneous individuals and firms adjust their expectations diferently due to a change in exogenous variables.

Notice that the interpretation of neccessary equilibrium condition is slightly different after we introduce heterogeneities in our model. Suppose that we have $\theta^d < \theta^s$. On supply side, *marginal household* will only search when the labour market is sufficiently tight, whereas *marginal firm* will search if the labour market is sufficiently loose. In



(A) Matching Equilibrium. (B) Equilibrium after an increase in β.

FIGURE 4.2: After introducing some heterogeneities, the comparative statics is now more intuitive with interior solutions of the households' problem

short, when marginal household demands tighter labour market compared to marginal firm, this particular labour market will not exist since no one will find a match.

PROPOSITION 2 *Heterogeneities across individuals within households and firms cause respective agents to react differently towards exogenous shocks that might lead to a change in the equlibrium (un)unemployment rate.*

Summarizing our finding in this setting, Table 4.1 depicts the most important conclusion of this section. It exhibits the impact of each change in each exogenous variable to both labour demand and supply, and to equilibrium employment level. For example, consider

Variable	**Description**	**Labour Supply**	**Labour Demand**	**Net Effect on Employment**
β	Bargaining Power of Workers	Shift out	Shift In	Ambiguous
y	Surplus from Matching	Shift Out	Shift Out	Increase
z	Benefits from Leisure	Shift In	Shift In	Decrease
ψ	Individual Search Costs	Shift In	Unchanged	Decrease
γ	Vacancy Costs	Unchanged	Shift In	Decrease

TABLE 4.1: Comparative statics of our matching model with heterogeneities in case of positive shocks

an increase in worker bargaining power, *ceteris paribus*. This will shift the supply curve to the right while also simultaneously shift the demand curve to the left. It is then up to the elasticity of labour supply and demand with respect to the bargaining power that determines the net change in unemployment.

4.3 Equilibrium with Naive Households

So far, our equilibrium accounts that households behave according to the standard economics assumptions with rational expectations. Here, we try to inspect our equilibrium from a different prespective by assuming that households form naive expectations. In this section, we keep the heterogeneities across agents intact. Assuming rationality, in each period, household will allocate the optimal amount of L^s and u_v given the state of θ and other exogenous variables. However, if we assume that individuals are backward looking and naïve, the equilibrium behavior is much more complex and the convergence towards equilibrium employment rate is conditional. In this section, emphasize will be put on the implication of the naïve expectation to the behaviour of labour supply of households and therefore we will not explain the quantitative and mathematical aspects in detail.

We express the equation for aggregate labour supply as

$$L_t^s = L_t^0 + p(\theta)\theta_{t-1} + \epsilon \tag{4.28}$$

Here, L^s and L^0 are the labour supply and initial match, respectively. Initial match describes the proportion of individuals who are currently working. The probability of matching tunes the steepness of the curve, whereas ϵ denotes error terms due to exogenous variables. Observe that individuals base their labour decision on the observability of the past value of the labour market tightness.

The evidences that workers based their decision to search for jobs on their expectations on the availability of vacancies are initially proposed by Strand and Dernburg (1964) and are considered within the framework of the dynamic labour flow analysis by Holt and David (1966). Contemporarily, U.S. Bureau of Labor Statistics (2009) highlighted that the most pronounced reason for discouraged workers to enter the labour force in case of recessions is that workers expect that there is no job available in the labour market. With the assumption of naïve expectations, we hypothesize that individuals lagging expectations on the availability of vacancies alter the instant adjustment to the new equilibrium after exogenous shocks occured. For example, after a recession occured, individuals still expect that the vacancies from the past are still available while in reality it does not. Over the period of shock, the expectations are continuously dwindling and some individuals are moving in and out of the labour force. Thus, the adjustment to the lower equilibrium rate is not instant, which in turn justifies the empirical finding of free-falling employment rate over the bust period of business cycles.

Next, we write the aggregate labour demand equation as

$$L_t^d = L_t^0 - q(\theta)\theta_t + \epsilon \tag{4.29}$$

To ensure the existence of an equilibrium, let map L^{-1} be twice differentiable and reservation tightness satisfies $\theta^d > \theta^s$. Moreover, we suppose that both ε and ϵ satisfy i.i.d. condition, that is, $N \sim [0, \sigma^2_{\varepsilon,\epsilon}]$. Finally, let L^* be the equilibrium or steady state or fixed point of the employment level.

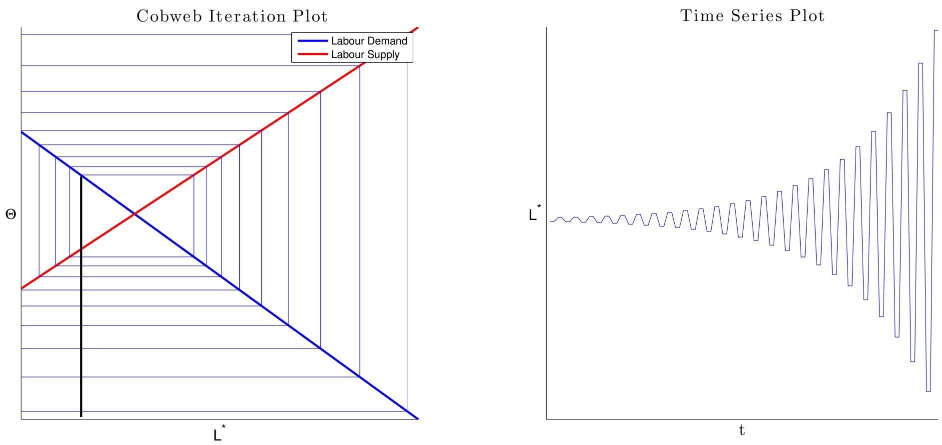


FIGURE 4.3: An example of Diverging Case

As indicated previously, we consider the comparative static in our preceeding section to familiarize us with the application of dynamic mapping in labour market setting. Assume that we are initially at the equilbrium level of L^*. In the subsequent period, there is a positive technological shock whereby firms' search costs decrease. Our model infer that labour demand curve would shift to the right and employment rate increases. However, with naïve individuals, the adjustment to the new equilibrium is not instant. Systematically, the adjustment process in case of negative shock to γ is as follows:

1. Firms open additional vacancies and labour market tightness goes up.

2. Some voluntarily unemployed individuals will enter the labour market resulting in more individuals searching for a job.

3. More individuals searching for a job implies that labour market tightness has to go down in the next period. This decreases the confidence of some marginal individuals and they will choose to be out of the labour market

4. Fewer individuals searching for a job implies that labour market tightness adjusts to a higher level again in subsequent period

5. Repeat to step 2. These adjustment cycles repeat themselves indefinitely.

The convergence to the steady state level of employment is conditional upon the slope of supply and demand curve. If we assume that $p(\theta) > |q(\theta)|$, that is, when the absolute value of the slope of supply curve is steeper than demand curve, then fluctuations decrease in magnitude per iteration, which lead to a converging case in the long run. Contrastingly, when $p(\theta) < |q(\theta)|$, the system will not converge to the new fixed point but instead diverging. Clearly, $p(\theta) = |q(\theta)|$ causes a stable 2-cycle over time.

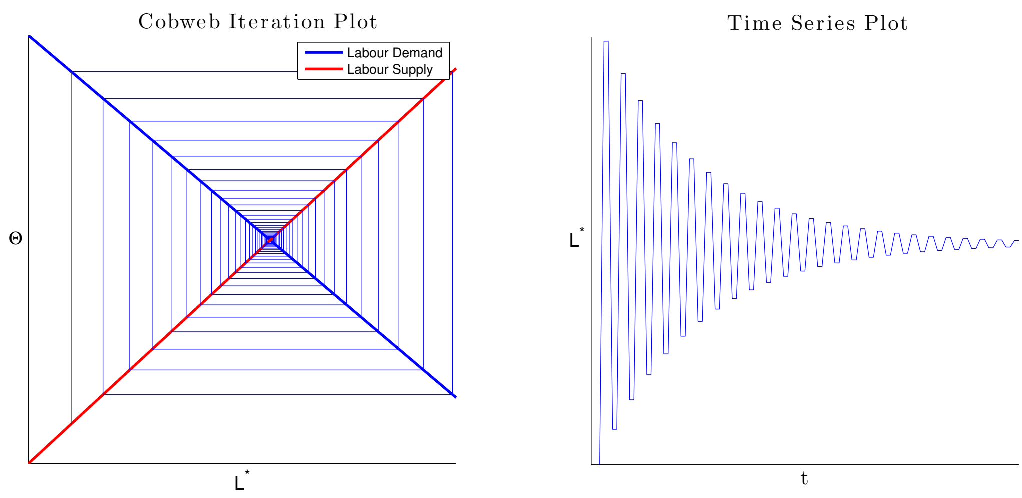


FIGURE 4.4: An example of Converging Case

Alternatively, we can inspect the stability of map θ: $\mathbb{R}^3 \to \mathbb{R}^n$ from its first-order condition with respect to the endogenous parameter L. The notion of **Conditional Convergence** inquires that the iteration of L converges to the steady state L^* only if this following first-order condition is satisfied

$$-1 < \frac{\theta'(L^s)}{\theta'(L^d)} < 1 \tag{4.30}$$

or otherwise the iteration diverges.

In our previous depiction of equilibria with rational agents, fluctuations in employment level happen due to a rational and efficient response towards shocks over different time periods. If we assume that all variables stay constant within the period of observation, the (un)employment count will be consistent over time. Conversely, with naïve agent, the fluctuations happens because individuals are backward looking. Consequently, even if we hold all variables constant throughout a period after a shock has occured, the fluctuations in unemployment due to a primordial shock can still be observed in subsequent periods. Hence, the effect of a shock lasts much longer in this model. From here, we try to answer some empirical questions by utilizing this concept as a foundation.

Fluctuations in Employment Rate Shimer (2005) argued that the search and model is not sufficient in explaining the fluctuations over the business cycles in a presence of significant productivity shock. He quoted that *"in the United States, the standard deviation of the vacancy-unemployment ratio is almost 20 times as large as the standard deviation of average labor productivity, while the search model predicts that the two variables should have nearly the same volatility"*.

In our model, a productivity shock is equivalent to a change in y. When the shock is positive, it increases the equilibrium employment rate and when the shock is negative, it decreases the equilibrium employment rate. We propose that Shimer (2005) finding can be attributed to the naivety of workers that cause a more pronounced fluctuations of the labour market tightness and (un)employment rate. This, in turn, causes the standard deviation of labour market tightness to escalate.

Proposition 3 *Naive expectations with respect to shocks intensify the dynamic of (un)employment over the business cycles and cause an inflation of the standard error of the labour market tightness.*

A more sophisticated explanation is provided by Hommes (1994) in his analysis of dynamic behaviour of supply and demand of commodities over time. He inferred that a strong non-linear tendency causes a chaotic behaviour in the dynamical system of pricing behaviour of commodities. Hommes (1994) proposed that a non-linear sigmoid shape of the labour supply curve and adaptive expectations[3] might lead to a chaos.

[3]Here, Hommes (1994) distinguished between adaptive and naive expectations. Our abovementioned cobweb model is a model with naive expectations. The adaptive expectations term can be defined as follow

$$\hat{\theta}_t = (1-x)\hat{\theta}_{t-1} + x\theta_{t-1} \tag{4.31}$$

where x is the *expectations weight factor*. $\hat{\theta}$ defines the expected labour market tightness whereas θ is the actual labour market tightness. Thus, equation above describes the individual expectations of labour market tightness on time t is dependent on the weighted expectations of old-expected labour market tightness and old-actual labour market tightness. In case $x = 1$, we would have a traditional cobweb model with fully naive expectations as we have defined before. Hommes (1994) proved that chaotic behaviour might occur if x is such that $0 < x < 1$ and supply function is of sigmoid shape.

Using the same intuition, we consider a case where labour supply curve is defined in a sigmoid shape. The intuition behind an S-shaped labour supply curve is still within the realm of diminishing return. When the level of labour market tightness is low, increasing the level of tightness slightly will not induce significant portion of marginal individuals to enter the workforce. Similarly, when the level of labour market tightness is high, the effectiveness of it in attracting marginal worker diminishes. With abovementioned specification, the time-series behaviour might neither be converging nor diverging. Rather, the series exhibits a tendency towards random walk which lead to a random fluctuations over the business cycles that inflate the standard deviation of vacancy-unemployed ratio. The mathematical treatment of this chaotic behaviour is tedious and will not be discussed in this thesis. In short, the graph below shows examples of chaotic movement in employment rate over time.

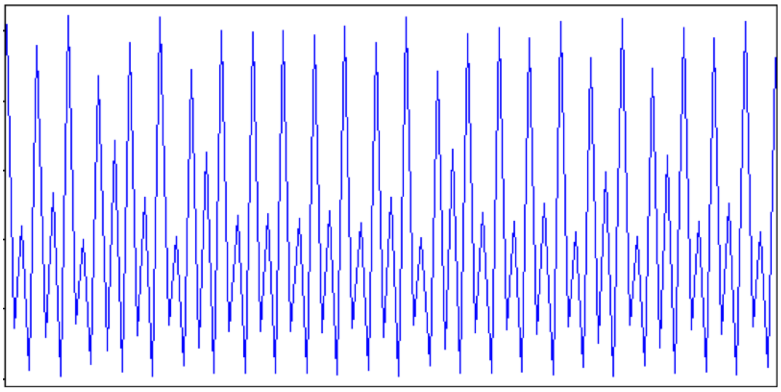


FIGURE 4.5: An Example of a Chaotic Time Series Plot

Chapter 5

Econometrics Estimation

In this section, we offer a possible methodology to analyze the labour market flow from our model's perspective. Here, our econometrics specification predominantly aims to estimate the aggregate labour supply. Recall that the aggregate labour supply in our model refers to the individuals who search in the labour market. We infer that the data for employment rate and ratio of vacancies to unemployed can easily be accessed. Thus, by estimating the aggregate labour supply, we can essentially calculate our parameters of interest: the involuntary and voluntary unemployment.

To depict a more realistic setting, assume that each household represents a muncipality. Each muncipality has its respective individuals and firms that coexist and search for a match. Hence, each muncipality will have its own level of labour market tightness. Let us assume a free movement of labour across different muncipalities. The total (un)employment rate across muncipalities will define the aggregate (un)employment rate in our system. One relevant question that we propose is: *what is the relationship between aggregate labour market tightness and aggregate labour supply given the behaviour of the labour supply of each household to a specific level of tightness?* To answer this question, we must then draw an inference across muncipalities and investigate the idiosyncratic components that link each of the muncipality to their respective level of labour market tightness. From here, we proceed to determine the aggregate level of labour supply at one point of time.

For simplicity, we assume linear dependency of the relationship between variables depicted in our previously-mentioned comparative statics. Let there be n muncipalities. We write the labour supply equation for muncipality i as

$$L_i^s = L_0 + \kappa_1 \theta_i + \kappa_2 \beta_i + \kappa_3 y_i + \kappa_4 z_i + \kappa_5 \psi_i + \kappa_6 \gamma_i \tag{5.1}$$

Here, κ denotes the coefficient that measures the effect of each variable to $Ł^s$. Clearly, as we defined in Section 4.1, labour supply increases in θ, β, and y and decreases in z and ψ. Aggregate labour supply equation can be represented in this following matrix form

$$\begin{pmatrix} L_1^s \\ L_2^s \\ \vdots \\ L_i^s \end{pmatrix} = \begin{pmatrix} 1 & \theta_1 & \beta_1 & y_1 & z_1 & \psi_1 & \gamma_1 \\ 1 & \theta_2 & \beta_2 & y_2 & z_2 & \psi_2 & \gamma_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \theta_i & \beta_n & y_n & z_n & \psi_n & \gamma_n \end{pmatrix} \begin{pmatrix} L_0 \\ \kappa_1 \\ \vdots \\ \kappa_6 \end{pmatrix}$$

Since our equilibrium condition boils down to θ, we will map individuals' labour supply reaction to a change in variable θ. We instrumentized all others variables, which we have assumed to be exogenous throughout this paper. In short, let labour supply of muncipality i be defined as

$$L_i^s(t) = L_0(t) + \kappa\theta_i(t) + \Phi C_i(t) \tag{5.2}$$

where

$$C_i(t) = a_0(t) + a_2\beta_i(t) + a_3 y_i(t) + a_4 z_i(t) + a_5\psi_i(t) + a_6\gamma_i(t) \tag{5.3}$$

In matrix form, we would then have a simplified form of labour supply equation

$$\begin{pmatrix} L_1^s \\ L_2^s \\ \vdots \\ L_n^s \end{pmatrix} = \begin{pmatrix} 1 & \theta_1 & C_1 \\ 1 & \theta_2 & C_1 \\ \vdots & \vdots & \vdots \\ 1 & \theta_n & C_n \end{pmatrix} \begin{pmatrix} L_0 \\ \kappa \\ \Phi \end{pmatrix}$$

If $n > 3$, the exact solution to the level of aggregate labour supply does not exist. Although technically there can exist an exact solution when n=3, it is almost impossible to calculate the level of unemployment based on each muncipality's data without any measurement error. Realistically, the aggregate level of employment can only be determined by approximation to its least square. The quantitative treatment on this can be seen on Appendix B.

Empirically, ordinary least square regression is the widely used method to estimate aggregate (un)employment level. Assuming that we have solved the 'best-approximation' fit, our labour supply equation would then be

$$Ł^s = Ł_0 + \kappa\Theta + \Omega\mathbf{C} \tag{5.4}$$

where $Ł^s$, $Ł_0$, Θ and $\mathbf{C}$ are $n \times 1$ vectors that denote aggregate labour supply, initial matches or currently-working population, labour market tightness and exogenous component respectively, whereas κ and Ω are some eigenvalues that measure the common factors shared by muncipalities. Aggregate labour supply can be estimated by summing the row in vector Ł^s.

Note that vector Ł^s is *not* the aggregate employment rate, but rather it denotes the aggregate labour supply. If vector Ł^* denotes the equilibrium employment rate, one can easily account for the number of mismatch, or involuntary unemployment rate, by substracting Ł^* from Ł^s. Similarly, voluntary unemployment size can be calculated by substracting Ł^s from the vector that denotes the total working age population, say Ł. By doing so, we essentially disentangle the two different types of unemployment with this approach.

Chapter 6

Conclusion

In this thesis, we analyze the dynamic of labour market flow using a calibrated DMP framework. We notably utilize the three-state matching model, where out-of-the-labour-force individuals are also considered and juxtaposed with the mismatched individuals. We operationalize involuntary unemployment as individuals who fail to find a match, whereas voluntary unemployment is defined as inidividuals who decide not to search. To dissect the involuntary component in mismatch, we suppose that our model is confined within one period, which in turn would mean that each individual and firm would accept any offer in hand that is above their marginal preferences. Although this could lead to an over-exaggeration of efficiency loss in mismatch, this assumption gives us an inidication that involuntary unemployment is relatively more detrimental compared to the voluntary unemployment. In summary, the purpose of this thesis is to provide simple extensions to existing literatures and to provide a different theoretical perspective in analyzing unemployment.

Firstly, we analyze the equilibrium when all agents in the economy are assumed to be homogenous. We assume that the utility functions of both household and firm are linear. We develop a corner solution whereby households would either allocate all inidividuals to search or not. Similarly, all firms will either search or not. This model is able to account for an equilibrium where employment rate boils down to the labour market tightness, or the interaction between vacancies and unemployed individuals looking for jobs. Nevertheless, due to the homogeneity and corner solutions, this model is not sophisticated enough to capture the existence of voluntary and involuntary unemployment altogether. Moreover, exogenous shocks do not seemingly change the equilibrium employment rate, which is empirically not compelling.

Next, we introduce heterogeneities across individuals and firms. With this specification, individuals within a household can freely choose to allocate their time to search and to leisure based on their preferences. Since we again assume linearity in the objective function, an individual choose to either allocate all her time to search or not. The resulting equilibrium would be that each individual would have different reservation tightness in the equilibrium. Aggregating this fact across households and taking into account that some individuals would search and some would not, labour supply decision is increasing in labour market tightness. This would mean that an increase in the labour market tightness would induce some extra individuals to search. Similarly, imposing heterogeneities across firms would mean that some firms will decide not to search when the labour market tightness goes up. The interaction forms the equilibrium employment rate in this setting. Although individuals' decision is still of an extreme choice, households' develop an interior solution whereby not all individuals supply labour. Using this intuition, we can substract the realization of labour supply match in equilibrium from the labour supply choice of household to find the frictional, or involuntary, unemployment rate. Individuals that decide to not search are considered within the voluntary unemployment pool.

Introducing heterogeneities results in a very different comparative statics of our matching model. Now, heterogenous agents would react differently to a change in exogenous variables. The resulting equilibrium employment rate would then be altered in a presence of exogenous shock. Our model is thus able to predict the change in unemployment rate due to various government policies that would be otherwise uncaptured in a simple neoclassical framework. For example, consider a policy of an increase in the retirement age. Our model asserts that the benefits from leisure z would go down and some additional individuals, notably older workers, would search. At the same time, the wage required to induce marginal individual to work also goes down and hence more firms would search. Clearly, this results in a higher employment rate in the equilibrium. A research by Staubli and Zweimüller (2011), which inspected the employment behaviour after a policy of an increase in the minimum retirement age in Austria between 2000 and 2006 confirms our hypothesis.

Lastly, we consider a variaton in the comparative static of our model with heterogenous agents. Instead of conventional ratioanlity, we suppose that individuals are backward looking and naive in choosing their labour supply decision. The naivety causes a heightened fluctuations in the labour market tightness and (un)employment rate over time in presence of exogenous shocks. In our example, we consider a lagging adjustment to the new equilibrium after a shock occured. One can infer the consequences if the system is fed by multiple shocks over a period of business cycles; that the fluctuations and measurement error of the labour market tightness will be even more pronounced. With this deduction, we propose that the naivety in individuals' expectations as a possible explanation to Shimer's (2005) proposal pertaining to the insufficiency of the matching model in capturing the fluctuations of (un)employment rate over the business cycles.

6.1 Policy Recommendations and Implications

Descriptive Power Our model can arguably depict a more nuanced view on the implication of government policies to the dynamic of the labour market. According to our model, there are some ways in which goverment can alter the unemployment rate in the economy by influencing the exogenous variables. We infer that the easiest variable to be influenced by a government is the bargaining power of workers. A government can, for instance, change the regulation with regards to the minimum wage, strikes, or lockouts. For now let us focus on the regulation with regards to the minimum wage. According to the neoclassical model, minimum wages policy would shift the labour demand curve to the left and thereby reducing the employment rate. However, research by Card and Krueger (1995) disputed the external validity of this theory, as they have found an empirical paradox whereby employment increases after an imposition of minimum wages policy.

What does our model tell about this? Since our wage equation is determined endogenously, let us rewrite the Nash equation with minimum wage regulation as follows

$$\bar{w} > \beta y + (1 - \beta) z \tag{6.1}$$

where $\bar{w}$ is the minimum wages level. To pin down our analysis to the searching behaviour of individuals that are directly affected by this policy, suppose that we scrutinize the searching behaviour of low-skiled workers. To find the solution to this Nash bargaining problem, the endogenous barganing equation suggests that the bargaining power of low-skilled workers has to increase

$$\bar{w} = \hat{\beta} y + (1 - \hat{\beta}) z \tag{6.2}$$

where $\hat{\beta} > \beta$ is the new bargaining power of workers. Our model does not automatically assert that the increase in the bargaining power of workers would lead to a lower employment rate. If the elasticity of labour supply with respect to the bargaining power of workers is higher than labour demand, more individuals will search compared to firms that bail out from the labour market. The resulting equilibrium would be that more individuals would find a match in the equlibrium. Since now we have less vacancies available compared to the unemployed worker searching for jobs, the equlibrium labour market tightness θ^* has to go down. Figure 6.1. depicts this circumstance.

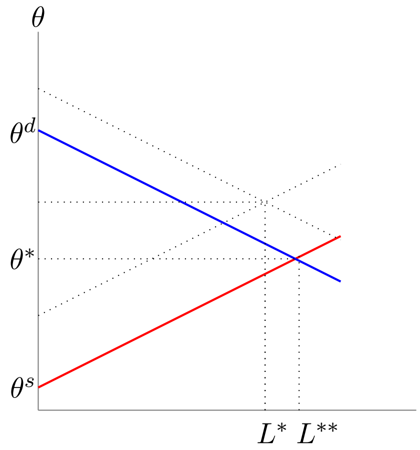


FIGURE 6.1: Minimum Wage Paradox

The empirical evidences pertaining to the impact of minimum wages to the employment rate however, remain mixed. Nevertheless, our model could provide a theoretical explanation behind those phenomena. For example, a finding by Dube, Lester, and Reich (2010), which concluded that imposition of minimum wages in low-skilled sectors does not change the employment rate might signify that the elasticity of labour demand and supply with respect to the bargaining power are approximately equal.

Hence, if a government plans to decrease the short-run unemployment rate, one policy intervention that can be done is to either (i) *decrease* the minimum wages when firms are more reactive towards the change in β or to (ii) *increase* the minimum wages if otherwise. These policies would implicate to a higher employment rate in the equilibrium. Of course, a preliminary estimation of the elasticity of labour supply and demand with respect to the bargaining power needs to be put forth. Moreover, if we take individuals' naivety into account, the rate of adjustment to the potential equilibrium also needs to be considered. Indeed, this result is merely a recommendation from positive economics' perspective. A normative approach to the government intervention will be elaborated on the subsequent section.

Government's Policies Implications From welfare perspective, we suggest that policies concerning (un)employment rate should be carefully and thoroughly analyzed before being implemented. Consider an example of the minimum wage paradox above. We saw that the equilibrium employment rate increases from L^* to L^{**}. Here, we do not visually show the exact rate of increase in the labour supply in our graph. Let L_0^s be the old labour supply and L_1^s be the new one. With our model, we can estimate the increase in labour supply by estimating $\Delta L^s = L_1^s - L_0^s$. Next, we want to estimate its impact to unemployment. Previously, our involuntary unemployment rate is $u_i = L_0^s - L^*$ and voluntary unemployment rate is $u_v = L - L_0^s$. The new involuntary unemployment rate is thus $u_i^* = L_1^s - L^{**}$, whereas the new voluntary unemployment rate is $u_v^* = L - L_1^s$.

Thus, one can calculate the net effect of such policies to both types of unemployment by taking the difference between the two

$$\Delta u_i = u_i^* - u_i = L_1^s - L^{**} - L_0^s + L^* = \Delta L^s - \Delta L^{eq} \tag{6.3}$$

Hence, when the increase in labour supply outweighs the increase in equilibrium employment rate, the involuntary unemployment rate will go up. The net effect on the voluntary unemployment rate is obvious. Since we assume the case whereby labour supply increases, it must come in expense of voluntarily unemployed individuals

$$\Delta u_v = u_v^* - u_v = L - L_1^s - L + L_0^s = -\Delta L^s \tag{6.4}$$

Concluding, the trade-off here is thus clear-cut. When minimum wage induces too many individuals to enter the labour market, the involuntary unemployment rate will go up although the employment rate has gone up. Therefore, we suggest that if minimum wages policy is forecasted to increase employment rate, the best scenario for it to be implemented is when the forecasted net increase in employment rate outweighs the net increase in participation decision.

Generalizing, if the government aims to reduce unemployment rate, it has to primarily try to compress the involuntary unemployment rate, since the impact of involuntary unemployment rate is more detrimental to the welfare. If the government implements a policy that increase employment rate - say - by increasing minimum retirement age, employment would increase and some individuals are made better off by the outside option and matching. Nevertheless, the government should also consider that an increase in labour supply would mean that *involuntary* unemployment rate could also increase. Some individuals that are no longer voluntarily unemployed and are failed to find a match are now shifted to the involuntary unemployment pool and lost some of her voluntary benefits. Moreover, since out-of-the-labour-force activities become less attractive, the benefits derived from voluntary unemployment decreases for some voluntarily unemployed individuals. Therefore, we suggest for governments to conduct a cost-benefit analysis before implementing any policy, even to a seemingly straightforward policy that would increase employment rate, to avoid any detrimental effect to the welfare. By any mean, government should aim on enhancing the degree of information's acquisition and clarity to reduce inefficiencies in the labour market due to a mismatch.

Moreover, our model gives some indication to policy makers and governments as to what would happen to the equilibrium employment rate as a result of agents' behaviour. For instance, consider a case of a horizontal merger that increases the market power of the merging firm. If this merger is large enough such that the firm could influence the labour market tightness, there is a threat whereby a firm will limit its capacity and hire less workers, thereby leading to an increase in unemployment rate. Indeed, the government regulations can do little when the firm has a market power to influence the tightness level and employment rate. This signals for consideration to disallow such merger.

6.2 Limitations and Recommendations for Further Research

Lastly, we would like to highlight some limitations of our model and possible recommendations that might be useful for further developments of the matching model. In this thesis, we suppose linear objective functions of agents to greatly simplify our analysis. This might lead to an exaggeration of efficiency loss due to a mismatch, since searching individuals practically allocate all their time to search and derive no benefits from leisure. Conceivably, one can utilize an intermediate form of objective functions, such as Cobb-Douglas form, to depict an interior solution of inidividuals in heterogenous model. By doing so, we can scrutinize the difference in the search intensity between heterogenous worker in our setting.

Moreover, we solely scrutinize the behaviour of searching individuals and firms. To depict a more accurate overview of the aggregate employment rate and welfare condition in the economy, one could also take into account the size of the existing workers in the analysis. This enhancement would make it possible for our model to provide a more deliberate policy recommendations and to be generalized over a wider range of circumstances.

Furthermore, there are some factors in our model that have been held constant for simplification purpose. Our model imposes no disctinction between high and low skilled workers, does not consider the difference in the degree of information acquisition among inidividuals and firms, and assumes one period setting which rules out the possibility for workers or firms to reject an offer in hand. All these stylized assumptions can be considered and waived in future research to enhance the applicability and reduce the artificiality of our model.

Appendix A

Wage Determination Derivation

Dynamic Environment We first substitute the decision choice of both firm and worker into the equation (3.4) to find the equilibrium wage *ex-post* the bargaining phase. We first solve for the individual. We write the present value of working as

$$rE(w) = w + \sigma(U - E(w)) \tag{A.1}$$

where r is the interest rate. Here, at rate σ, the job is exogenously destroyed and relationship ends.

On the other hand, it is plausible to think that the present value of being unemployed is equal to the benefit of being unemployed z plus the probability of getting employed and getting employment benefit.

$$rU = z + p(\theta)(E(w) - U) \tag{A.2}$$

Using the same intuition, we can derive the expected present value of matching for the firm.

$$rJ(w) = y - w - \sigma(J(w) - V) \tag{A.3}$$

Here we assume labour as the only production's input to simplify our analysis. y is thus the joint output obtained when a worker and a firm produce together. V is the asset value of vacancy, which we will elaborate subsequently.

For a non-filled vacancy, a firm has to bear an opportunity cost of γ. The value of leaving the job vacant is thus:

$$rV = -\gamma + q(\theta)(J(w) - V) \tag{A.4}$$

where at a rate $q(\theta)$ a firm manages to fill the vacancy and obtains a surplus of $(J(w)-V)$.

It is worthwhile to notice that firms will not stop creating jobs as long as each additional worker grants them positive marginal profits. Herein, we assume a competitive case of endogenous job creation, that is, in the natural equilibrium there is no entry deterrence and more firms are willing to create vacancy as long as the surplus from it is positive. By 'free-entry' assumption, such rent is eroded and it implies that we have $V = 0$ in the equilibrium. Therefore, we would then have:

$$J(w) = \frac{\gamma}{q(\theta)} \tag{A.5}$$

We are now ready to determine the bargaining outcome. With $V = 0$, recall that we have

$$w = \underset{w}{\operatorname{argmax}}(E(w) - U)^{\beta}(J(w))^{1-\beta} \tag{A.6}$$

for the Nash bilateral bargaining problem.

Substituting (A.1), (A.2) and (A.3) gives:

$$w = \underset{w}{\operatorname{argmax}} \left(\frac{w-z}{r+\sigma+p(\theta)}\right)^{\beta}\left(\frac{y-w}{r+\sigma}\right)^{1-\beta} \tag{A.7}$$

Solving above maximization yields us this following expression:

$$\beta\frac{y-w}{r+\sigma} = (1-\beta)\frac{w-z}{r+\sigma+p(\theta)} \tag{A.8}$$

Finally, rearranging the terms and substituting (A.5) give us this following wage equation[1]:

$$w = \beta(y+\gamma\theta) + (1-\beta)z \tag{A.9}$$

Static Environment In a static environment, the bargaining is similar to a one-shot-one-period bargaining game. Since there is no future, an individual regards the benefit from matching as

$$E(w) = w \tag{A.10}$$

[1] Note that equation (A.5) can be expressed as:

$$\frac{y-w}{r+\sigma} = \frac{\gamma\theta}{p(\theta)} \rightarrow y-w = (r+\sigma)\frac{\gamma\theta}{p(\theta)}$$

Realize that there is no future consideration and hence we calculate the benefit without any consideration for time value of money and hence interest rate. Moreover, an individual will not take the probability of job destruction since this will occur later in the future.

Using the same intuition, we derive the benefit from not searching, as follows

$$U = z \tag{A.11}$$

Clearly, if one decide to search, then there is no going back - one would simply choose to not enter the labour force in this single period.

For the firm, the free-entry assumption still holds and hence the asset value of vacancy V is still equal to zero. The benefit of matching is simply

$$J(w) = y - w \tag{A.12}$$

Finally, we substitute above constraints to the wage bargaining equation

$$w = \underset{w}{\operatorname{argmax}} (w - z)^{\beta} y^{1-\beta} \tag{A.13}$$

Solving

$$\beta y = (1 - \beta)(w - z) \tag{A.14}$$

Finally, with some abuse of notations, we derive the Nash bargaining solution in a static environment

$$w = \beta y + (1 - \beta) z \tag{A.15}$$

Appendix B

Least Square Derivation for the Aggregate Labour Supply

Let us rewrite the term as $Ł = A\mathbf{x}$, where

$$Ł = \begin{pmatrix} L_1^s \\ L_2^s \\ \vdots \\ L_n^s \end{pmatrix}, \ A = \begin{pmatrix} 1 & \theta_1 & C_1 \\ 1 & \theta_2 & C_1 \\ \vdots & \vdots & \vdots \\ 1 & \theta_n & C_n \end{pmatrix}, \ \mathbf{x} = \begin{pmatrix} L_0 \\ \kappa \\ \Phi \end{pmatrix}$$

Notice than A is an $n \times 3$ matrix. Suppose that n=3. By definition, we can utilize $\mathbf{x} = A^{-1}L$ to find a solution for vector $\mathbf{x}$. The inverse of A can be found using a standard Gauss Elimination method

$$A^{-1} = \left(\begin{array}{ccc|ccc} 1 & \theta_1 & C_1 & 1 & 0 & 0 \\ 1 & \theta_2 & C_2 & 0 & 1 & 0 \\ 1 & \theta_3 & C_3 & 0 & 0 & 1 \end{array}\right)$$

This, however, requires $\det |A| \neq 0$ for matrix A to be invertible.

If $n > 3$, then matrix A is non-squared and thus non-invertible. With this, we can only estimate paramaters in $\mathbf{x}$ using least square approximation. We can derive the least square approximation using rank theorem. Firstly, we need to ensure that each coloumn in A is independent. By definition, if each coloumn in A is independent, then rank(A^TA) is equal to the number of parameter in $\mathbf{x}$ (=3). Since rank(A^TA) = rank(A) = 3, it follows that matrix A^TA is invertible. Using this as a fundamental, we derive

$$Ł = A\mathbf{x} \tag{B.1}$$

$$(A^T A)^{-1} A^T \text{Ł} = (A^T A)^{-1} A^T \mathbf{x} \tag{B.2}$$

$$\mathbf{x} = (A^T A)^{-1} A^T \text{Ł} \tag{B.3}$$

Equation (B.3) is a commonly used formula for the least square approximation. Applying this formula to our problem, we write

$$\begin{pmatrix} L_0 \\ \kappa \\ \Phi \end{pmatrix} = \left[\begin{pmatrix} 1 & 1 & \dots & 1 \\ \theta_1 & \theta_2 & \dots & \theta_n \\ C_1 & C_2 & \dots & C_n \end{pmatrix} \begin{pmatrix} 1 & \theta_1 & C_1 \\ 1 & \theta_2 & C_1 \\ \vdots & \vdots & \vdots \\ 1 & \theta_n & C_n \end{pmatrix} \right]^{-1} \begin{pmatrix} 1 & 1 & \dots & 1 \\ \theta_1 & \theta_2 & \dots & \theta_n \\ C_1 & C_2 & \dots & C_n \end{pmatrix} \begin{pmatrix} L_1^s \\ L_2^s \\ \vdots \\ L_n^s \end{pmatrix}$$

$$\begin{pmatrix} L_0 \\ \kappa \\ \Phi \end{pmatrix} = \begin{pmatrix} 1 & \sum_{i=1}^{n} \theta_i & \sum_{i=1}^{n} C_i \\ \sum_{i=1}^{n} \theta_i & \sum_{i=1}^{n} \theta_i^2 & \sum_{i=1}^{n} \theta_i C_i \\ \sum_{i=1}^{n} C_i & \sum_{i=1}^{n} \theta_i C_i & \sum_{i=1}^{n} C_i^2 \end{pmatrix}^{-1} \begin{pmatrix} \sum_{i=1}^{n} L_i^s \\ \sum_{i=1}^{n} \theta_i L_i^s \\ \sum_{i=1}^{n} C_i L_i^s \end{pmatrix}$$

Without specifying the value of θ_i and C_i, the solution to above inverse is tedious and hence will not be shown. If one were to conduct an empirical research, the exact specification of abovementioned variables can be observed and least square approximation can be determined.

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