Erasmus University Rotterdam

Bachelor Thesis

Logistics

Analyzing Pricing and Production Decisions with Capacity Constraints and Setup Costs

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Abstract

In this Bachelor Thesis we consider a problem for maximizing profit over the horizon, where we make use of setup costs and capacity constraints as explained in Deng and Yano (2006). For this problem we need to determine prices and demands and need to choose the best production patterns. We characterize properties of the optimal solution, consider cases with constant, increasing and seasonal demand and different capacity levels. For generating this, we use the *Lagrange Multiplier Method* and the *Shortest Path Algorithm*. An interesting aspect is that the optimal price increases when the length of the Regeneration Interval changes for increasing capacity. Another interesting thing is when the capacity increases the average unit price stays the same under certain settings, which means that the capacity is too big for the given demand. Finally, we discuss what kind of extensions are possible for our problem.

Contents

1 Introduction **2**

2 Problem Description **2**

3 Literature Review **3**

4 Methodology **4**
4.1 Problems 4
4.1.1 General Problem 5
4.1.2 Reduced Problem 6
4.1.3 Special Case 7
4.2 Regeneration Intervals 7
4.3 Methods 9
4.3.1 Lagrange Multiplier Method 9
4.3.2 Shortest-Path Algorithm 11

5 Results **12**
5.1 Price Pattern for Homogeneous Demand 12
5.2 Average Unit Price 13
5.3 Price Pattern for non-Homogeneous Demand 15

6 Conclusion & Discussion **15**

7 Further Research **16**

1 Introduction

The interest in the integration of pricing and production/inventory has increased over the last few years. A lot of research about this topic is done, but none of them made use of capacity constraints or they assume that the production costs are linear or piecewise linear and convex. All the models that are used now, assume that the supplier decides which products will be purchased. This is one of the reasons why capacity constraints are not taken into consideration. Another point is that pricing decisions, capacity constraints, and production scale economies, as said in Deng and Yano (2006), significantly complicate the problem.

Our research was motivated by the decision problems for typical customers who want to buy durable goods with seasonal demands. When we speak about seasonal demands, it means that there is a fluctuation in the demand during the year, which is every year the same. Due to this phenomenon, it is hard to avoid overcapacity. Industries who face this problem are the automobile industry, furniture and jewelry shops and so on. This are products that do not need to be purchased that frequently, because of the long existence of the products. Furthermore, durable goods also face setup costs, (Deng and Yano, 2006), which are in some periods too large to ignore.

In this bachelor thesis we assume that the capacity and demand curve are known and that the inventory holding costs are incurred on the end of the period. The production costs consist of setup costs for each production run and linear variable costs. To simplify the problem we only look at single product cases. In this bachelor thesis we are going to explain the algorithm for determining the optimal prices and want to gain an understanding of the relationship between capacity and optimal prices, as explained in Deng and Yano (2006). Results show that when capacity constraints and setup costs get inserted, it may be optimal to charge higher prices and to produce less frequently. We are going to examine this phenomenon.

Another aspect we are going to evaluate is what happens with the optimal price when we change the capacity level and how does the average unit price change as a function of the capacity. Further, we are going to compare these average unit prices for homogeneous, increasing and seasonal demand and explain why the average unit prices differ.

Organization of the report. The outline of the report is as follows. In Section 2 the problem statement is given and we will explain what we could achieve with this information. The Literature Review will be shown in Section 3. Here, we explain the background information, the theories and algorithms we need to use for solving this problem. Further, the Methodology will be described in Section 4. Here, we introduce the parameters and variables we use for our problem and we elaborate on the theories and algorithms as mentioned in Section 3. Other aspects we are going to explain here are the *Lagrange Multiplier Method* and the *Shortest-Path Algorithm*. Section 5 shows our results of the methods we apply and we present some numerical examples that illustrate the effects of capacity on prices. Next, we make some concluding remarks in Section 6 and in Section 7 we discuss model extensions and future research directions.

2 Problem Description

The research problem is setting prices and choosing production quantities for a single product over a finite horizon for a capacity constrained manufacturer facing price-sensitive demands. The general intuition is that optimal prices may increase when capacity increases. In this thesis we will examine if this is generally the case or that there will be a change in the pattern of the price when the capacity increases.

We do not consider promotion or sale prices in our general case. In each period the costs consist of setup costs, which only holds if production takes place, variable production costs per unit and per-unit inventory holding costs on end-of-period inventory. These costs could vary in time.

In general, the firm decides the price of the products and this will define the demand and selects the production quantity. This is in contradiction with our case, we have a given demand function and with this demand we determine the optimal price. We assume that the product is durable or semi-durable and that the demand curves are independent across periods. Another requirement in our case is that back-orders are not allowed. Our goal is to maximize the profit over the horizon. For achieving our goal we make use of Regeneration Intervals(RIs). A RI is defined as a set of consecutive periods, where the start and end inventory is equal to zero and with strictly positive inventory in all periods. We make use of RIs, because it is an useful way to consider all the possible options for maximizing the profit.

For our problem we have a known demand curve, which may vary from period to period and is differentiable and strictly decreasing with respect to the price. The demand curves for the different periods are independent. In our problem we do not make use of competitive effects, which are multiple firms who have similar products.

In the following Section we will give some background information about our problem and explain which theories we use.

3 Literature Review

Loads of literature can be found for this topic, but all the articles look at the topic differently. Yano and Gilbert (2004) have a comprehensive review about coordinating production and pricing decision making. They do not consider capacity constraints, which is important for our research.

The concept of RIs is also introduced in Thomas (1970). His first discrete-time model with concave costs shows that the optimal solution consists of RIs and that prices can be determined by solving a non-linear optimization problem. This thesis considers a problem of a monopolist. We generalize his model and include capacity constraints. With the algorithm as shown in Thomas (1970) we get optimal prices and production quantities.

Florian and Klein (1971) already take capacity constraints in consideration. They make use of a discrete-time concave-cost production problem with capacity constraints, but without pricing decisions. They prove that at most one production quantity is strictly positive and less than the capacity for the optimal production schedule within an RI.

Another article that takes capacity in consideration is Gaimon (1988). They assume that capacity may be acquired to either increase or replace the firms existing productive capacity. Second they assume that the acquisition of new capacity causes a reduction in the firm's production costs.

Zhao and Wang (2002) consider the same problem about maximizing their profit when coordinating joint pricing-production decisions in a supply chain. Their goal is to show the existence of a manufacturer's price schedule that induces distributor to adopt decisions in the decentralized setting to achieve the performance of a centralized supply chain, (Zhao and Wang, 2002). They focus on developing an incentive scheme for the manufacturer to achieve channel coordination. Their research is different from our search in a way that they do not consider capacity constraints.

In Lee and Kim (2002) they propose a hybrid approach combining the analytic and simulation model for solving integrated production-distribution problems in supply chain management. They consider multiple constraints including capacity constraints.

In Jans and Degraeve (2008) are different lot sizing problems described. They focus on the same aspects: the set ups, the characteristics of the production process, the inventory, demand side and rolling horizon. As in our problem we also take a close look to the set ups and the demand side. There main aim is to discuss models that have been developed for optimizing production planning and inventory management. In Jans and Degraeve (2008) are several similarities compared to the formulation of the problem.

In another article they model a joint manufacturing/pricing decision problem, accounting for that portion of demand realized in each period that is induced by the interaction of pricing decisions in the current period and in previous periods, (Ahn H. and Kaminsky, 2007). They formulate programming models and develop solution techniques to solve this problem.

The methods we are going to use to solve our problem are the *Lagrange Multiplier Method* and the *Shortest Path Algorithm*. For solving the relaxation of our problem we make use of the *Lagrange Multiplier Method*.

In Fraser (1992) the achievement in Lagrange's method of multipliers is documented and they consider its precise character as an advance over earlier methods and results. For this they consider the *Isoperimetric Problem*.

We use the theory of the *Lagrange Multiplier Method* for determining the profits and all production patterns within a RI. Next, we use the *Shortest-Path Algorithm* for connecting the RIs and to find the maximum profit for the whole period, as explained in Knuth (1977) and Dijkstra (1959). In Dijkstra (1959) the general idea of this method is explained. There are different types of the *Shortest-Path Method*. There is the single-source shortest path problem, single-destination shortest path problem and the all-pairs shortest path problem. For our problem we make use of the single-source shortest path problem, because we have to find the shortest paths between two nodes. To develop our problem we use Dijkstra's Algorithm, (Knuth, 1977). Here, the general case of this problem is expounded.
We use all this research for solving our problem, which is further elaborated in Section 4.

4 Methodology

Our main goal is to realize the optimal production schedule and prices. There are several steps to achieve this. In Section 4.1 we explain the problem and give the general and reduced form of the problem. Furthermore, we give the special case under certain conditions we used for determining our results. After having a clear view of the problem, we will show how we determine this problem for a RI. Next, we introduce in Section 4.3 the methods we use to solve our problem.

4.1 Problems

Deng and Yano (2006) have several forms to represent the problem of setting prices and choosing production quantities over a finite horizon with capacity constraints and setup costs. In this part we will first introduce the general problem followed by the reduced problem and give our special case with the conditions we used.

4.1.1 General Problem

The general form of the problem is NP (non-deterministic Polynomial time) - hard even without the introduction of pricing decisions, which means that other forms of the problem can be transformed in polynomial time to the general problem. The price does not influence the optimal production decisions. Factors that do influence the structure of the optimal solution are:

- whether capacity is time varying or constant
- whether there is speculative motive for holding inventory
- whether the setup costs are non-increasing over the horizon

With speculative motive for holding inventory in period t we mean that the production costs per unit in period t are higher than when you produce your product in period $t-1$ and hold the unit: $v_t > v_{t-1} + h_{t-1}$. Thus, this is an important factor to check. Furthermore, for determining the optimal production schedule and prices we make use of the following variables and parameters:

T:	number of periods
t:	period index, $t = 1, ..., T$
K_t:	setup cost in period t
v_t:	unit production cost in period t
h_t:	inventory holding cost per unit remaining at the end of period t
C_t:	capacity in period t (in units)
D_t:	demand in period t (decision variable)
$P_t(D_t)$:	price in period t, which is the inverse of the demand function in period t (implicit decision)
x_t:	production quantity in period t (decision variable)
δ_t:	setup indicator in period t (1 if $x_t > 0$; 0 otherwise)
I_t:	inventory remaining at the end of period t

For the general problem: the firm decides the price in each period, which then defines the demand, and selects the production quantity. The objective function to maximize the profit over the horizon for the general case will look as follows:

$$\max_{\vec{D},\vec{x}} \sum_{t=1}^{T} [P_t(D_t)D_t - \delta_t K_t - v_t x_t - h_t I_t] \tag{1}$$

$$\text{s.t. } I_t = I_{t-1} + x_t - D_t,\ t = 1, ..., T \tag{2}$$

$$0 \leq x_t \leq \delta_t C_t,\ t = 1, ..., T \tag{3}$$

$$I_t, P_t(D_t), D_t \geq 0,\ t = 1, ..., T \tag{4}$$

Where $P_t(D_t)D_t$ is the revenue function in the objective function and that this function is concave in D_t and achieves in D_t a maximum, which is strictly positive but finite. Due to this assumption we eliminated 2 unrealistic options. The first possibility that $D_t = 0$ is optimal cannot be reached. Secondly, the infinite quantity cannot be sold for a negative price.

For finding optimal production schedules and prices of the general problem and prices are different ways:

- Non-constant capacity, speculative motive for holding inventory, and arbitrary setup costs

- Non-constant capacity, no speculative motive for holding inventory and arbitrary setup costs
- Constant capacity, no speculative motive for holding inventory, and arbitrary setup costs
- Constant capacity, no speculative motive for holding inventory, and non-increasing setup costs

We will focus on constant capacity, no speculative motive for holding inventory, and non-increasing setup costs.

4.1.2 Reduced Problem

However, the general form as mentioned in Section 4.1.1 can be reformulated by taking advantage of the characteristics of the optimal production policy. The reason why we consider a reformulated problem is that the challenge for solving a standard capacitated lot-sizing problem in a RI, even without pricing decisions, is not trivial, and the pricing decisions further complicate the problem. Therefore, the new formulation, as introduced in Deng and Yano (2006), suggests that solving the joint production and pricing problem for the reduced problem has the same fundamental complexity as the general problem. In the reduced form we have t_i, which denotes the period in which the ith setup within the RI occurs. Next, t_f denotes the fractional production if it exists. We have a fractional production when the demand does not meet the capacity. We will explain this more comprehensive in the special case part. However, the production pattern for the fractional production is determined differently:

$$x_{t_f} = \sum_{j=1}^{t_f-1} D_j - \sum_{t_i \in S: i \neq f} C_{t_i} \tag{5}$$

Here, S stands for the production pattern. When we implement (5) in the general problem we get the following objective function and restrictions:

$$\max_{\vec{D}} \sum_{t=1}^{t_f-1} (P_t(D_t) - v_{t_f} + \sum_{j=t}^{t_f-1} h_j) D_t + \sum_{t=t_f}^{n} (P_t(D_t) - v_{t_f} - \sum_{j=t_f}^{t-1} h_j) D_t + \kappa \tag{6}$$

$$s.t. \sum_{j=1}^{n} D_j \leq \sum_{t_i \in S: t_i \leq t} C_{t_i} \text{ for } t = 1, ..., t_f - 1 \tag{7}$$

$$\sum_{j=1}^{t} D_j \leq \sum_{t_i \in S: i \neq f, t_i \leq t} C_{t_i} + (\sum_{j=1}^{n} D_j - \sum_{t_i \in S: i \neq f} C_{t_i}) \text{ for } t = t_f, ..., n \tag{8}$$

$$0 \leq \sum_{j=1}^{n} D_j - \sum_{t_i \in S: i \neq f} C_{t_i} \tag{9}$$

$$\sum_{j=1}^{n} D_j - \sum_{t_i \in S: i \neq f} C_{t_i} \leq C_{t_f} \tag{10}$$

$$P_t(D_t) \geq 0 \text{ for } t = 1, ..., n \tag{11}$$

$$D_t \geq 0 \text{ for } t = 1, ..., n \tag{12}$$

where the constant κ is defined as:

$$\kappa = \sum_{i \in S - \{t_f\}} (v_{t_f} - v_i) C_i - \sum_{j=1}^{m} K_{t_j} - \sum_{j=1}^{f-1} C_{t_j} [\sum_{i=t_j}^{t_f - 1} h_i] + \sum_{j=f+1}^{m} C_{i_j} [\sum_{i_{t_f}}^{t_j - 1} h_i] \tag{13}$$

In this problem, constraint (7) and (8) ensure that the demand is satisfied for all periods and do not exceed the capacity for that period. Constraint (9) and (10) ensure that the production quantity in the fractional production period is positive and within the limits of the capacity level.

When we look at constraint (11) we see that it is not profitable to have $P_t = 0$ and D_t positive. In this case, the revenue will be zero but there are production costs and other costs, which gives a loss. The conclusion we can draw here is that this constraint is not binding. The same holds for constraint (7) and (8) except for $t = n$. These constraints are not binding at optimality because they ensure non-negativity of inventory at the end of intermediate periods in the RI. With this taking in consideration we made some adjustments in the problem and got the following relaxed problem:

$$\max_{\vec{D}} \sum_{t=1}^{t_f - 1} (P_t(D_t) - v_{t_f} + \sum_{j=t}^{t_f - 1} h_j) D_t + \sum_{t=t_f}^{n} (P_t(D_t) - v_{t_f} - \sum_{j=t_f}^{t-1} h_j) D_t + \kappa \tag{14}$$

$$\text{s.t.} \sum_{j=1}^{n} D_j \leq \sum_{t_i \in S} C_{t_i} \tag{15}$$

$$D_t \geq 0 \text{ for } t = 1, ..., n \tag{16}$$

4.1.3 Special Case

For our special case we can define $P_t(D_t)$ in a different way. In our numerical examples we use an inverse demand function as noted in Equation (17). The reason for this is that we consider constant costs. In addition, each unit of production uses one unit of capacity.

$$P_t(D_t) = A_t - B_t D_t \tag{17}$$

Here, A_t is the expected demand in period t and B_t is the slope for the demand function. We can implement this price function in (14) and this will give the new objective function in (18).

$$\max_{\vec{D}} \sum_{t=1}^{t_f - 1} (A_t - B_t D_t - v_{t_f} + \sum_{j=t}^{t_f - 1} h_j) D_t + \sum_{t=t_f}^{n} (A_t - B_t D_t - v_{t_f} - \sum_{j=t_f}^{t-1} h_j) D_t + \kappa \tag{18}$$

4.2 Regeneration Intervals

In the previous Sections we introduced the general and reduced problem. There exist many efficient algorithms for determining this problem with a concave objective function and a single constraint. We made use of Algorithm 1 to solve the general problem and to make use of the problem when considering fractional productions. Here we determine RIs in the first part of the Algorithm.

The first step we take is determining the total number of periods(n). Every period can exists of multiple RIs. We can explain this in more detail with Figure 1.

Result: Maximize profit
initialization;
for *Start period* **do**
    for *End period* **do**
        for *Number of periods producing* **do**
            Determine corresponding D_t and P_t;
            Check for producing period;
            Determine inventory level and profit;
        end
    end
end
Initialization;
for *Number of Periods* **do**
    Determine corresponding D_t and P_t given no restrictions;
    Calculate inventory level and profit;
end
Shortest path Algorithm;
Get optimal solution for capacity;

Algorithm 1: Algorithm for maximizing profit

In this example we chose a period of length 4. In total there are 10 different options for a RI as shown. When you look at row 7 in Figure 1, this RI starts in period 2 and ends in period 4. When the total number of periods increases, the number of possibilities for RIs increases too.

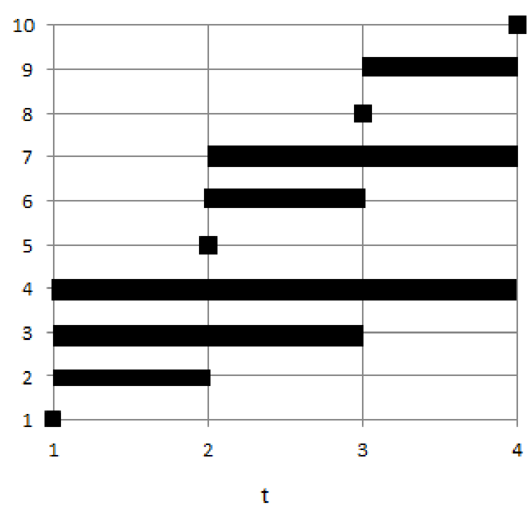


Figure 1: Possible RIs for $n = 4$

Further, there are several possibilities for every RI and we will compare these options with each other in the following loop in Algorithm 1. Here we also calculate the demand and price for every period, we check the producing level and determine the inventory level for calculating the profit.

Next, we compare the profit for the same RI length and choose the maximum and store this option. We continue with these steps until we got all the possible RIs.

Next, we check the profit for the fractional production. For this option we ignore restriction (15). This implies that for the first period the demand can vary between 0 and the capacity level and that the total capacity does not need to be a multiple of the given capacity. Taking this in consideration we can consider the permission of the fractional production. After determining the highest profit for all the possible RIs for both, the fractional production and the the general production, we get the optimal solution for a given capacity level.

We consider all these steps of Algorithm 1 to find the optimal price in a RI and to get eventually the optimal solution for the whole generating period. If none of the options for a given RI are viable, the RI can be eliminated from consideration. There are only 2 aspects of the production pattern that play a role: t_f, the fractional product period and the sum of the capacities in the periods with setups.

In the next Section we discuss which methods we use to construct the price, demand and production pattern for all RIs.

4.3 Methods

For solving the general and reduced problem as mentioned in Section 4.1, we use certain methods. The first method is the *Lagrange Multiplier Method*. With the use of this method we determine the price, demand and associated profit for every t in a RI. When we constructed all RIs we used a *Shortest Path Algorithm* to find the optimal solution. In this algorithm we determine the combinations of RIs which give the highest profit.

4.3.1 Lagrange Multiplier Method

For addressing the relaxed problem in Equation (14 - 16) for constant capacity with no speculative motive for holding inventory, and non-increasing setup costs we use the *Lagrange Multiplier Method*. For implementing this reduced problem we use the method as described in Fraser (1992). Equation (19 & 20) represents the same problem as described in (14) and (15) only defined with $f(D)$, $g(D)$ and b, where $f(D)$ represents the objective function in (18), $g(D) = \sum_{j=1}^{n} D_j$ and $b = \sum_{t_i \in S} C_{t_i}$. In Equation (21) we get the Lagrange function $\mathcal{L}(D, \lambda)$ for our problem and in (22) are the functions implemented, where λ represents the Langrange Multiplier.

$$\max_{\vec{D}} f(D) \tag{19}$$

$$\text{s.t. } g(D) = b \tag{20}$$

$$\mathcal{L}(D, \lambda) = f(D) + \lambda(g(D) - b) \tag{21}$$

$$\begin{aligned} \mathcal{L}(D, \lambda) = \sum_{t=1}^{t_f - 1} (A_t - B_t D_t - v_{t_f} + \sum_{j=t}^{t_f - 1} h_j) D_t + \sum_{t=t_f}^{n} (A_t - B_t D_t - v_{t_f} - \\ \sum_{j=t_f}^{t-1} h_j) D_t + \kappa + \lambda(\sum_{j=1}^{n} D_j - \sum_{t_i \in S} C_{t_i}) \end{aligned} \tag{22}$$

The next step, is to take the derivative of the Lagrange function. However, for determining λ we found out that there exists a certain pattern under our conditions, which are a constant λ within a

RI and the other parameters are given. As mentioned earlier, t_f denotes the fractional production period. There are only n possible choices when $t_f = 1$. But for the sum of capacity, we must consider all possible combinations of production periods. Under the condition that the fractional production only takes place in the first period, will give the the following problem for the Lagrange multiplier function:

$$\mathcal{L}(D,\lambda) = \sum_{t=1}^{n}(A_t - B_t D_t - v_{t_f} - \sum_{j=1}^{t-1} h_j) D_t + \kappa + \lambda(\sum_{j=1}^{n} D_j - \sum_{t_i \in S} C_{t_i}) \tag{23}$$

With this Equation we will determine the derivative with respect to D_t and λ. This gives the following equations:

$$\frac{\partial \mathcal{L}(D_t,\lambda)}{\partial D_t} = A_t - 2B_t D_t - v_t - (t-1)h_t + \lambda = 0 \tag{24}$$

$$\frac{\partial \mathcal{L}(D,\lambda)}{\partial \lambda} = \sum_{j=1}^{n} D_j - \sum_{t_i \in S} C_{t_i} = 0 \tag{25}$$

With these equations and the given condition that λ is constant within a RI we can generate a general function for λ:

$$\lambda = \frac{\sum_{t_i \in S} C_{t_i} - \sum_{t=1}^{n}(\frac{A_t}{2 \cdot B_t} - \frac{v_t}{2 \cdot B_t} - \frac{(t-1) \cdot h_t}{2 \cdot B_t})}{0.5 \cdot n} \tag{26}$$

Where, n is the number of periods within a RI. Given λ we can determine the corresponding demand and price for all periods in the RI. Our formula for calculating the demand is as follows:

$$D_t = \frac{\sum_{t_i \in S} C_{t_i}}{n B_t} - \sum_{t=1}^{n}(\frac{A_t}{0.5n} - \frac{v_t}{0.5n} - \frac{(t-1) \cdot h_t}{0.5n}) + \frac{A_t}{2B_t} - \frac{v_t}{2B_t} - \frac{(t-1) \cdot h_t}{2B_t} \tag{27}$$

With this demand function we calculate the price with Equation (17). This price function only holds under the following conditions: homogeneous linear demand, constant capacity, constant variable costs, constant holding costs and constant setup costs. The optimal price within a RI increases, because all the parameters are constant. The only part that causes a different price are the holding costs, which increases every period with h_t. Thus, the optimal price increases within a RI and than declines when it goes to the consecutive RI. The value of the holding costs is very important for the variation of the optimal price.

However, for calculating the fractional production we consider the optimization problem without restrictions. This implies that λ is equal to zero. In this case, the demand function will look as follows:

$$D_t = \sum_{t=1}^{n} \frac{A_t - v_t - (t-1) \cdot h_t}{2 \cdot B_t} \tag{28}$$

For we can use our solutions for the fractional production in the *Shortest Path Algorithm*, we need to check if the option is feasible. When this is checked, with the given price and demand given from Equations (17,27) we can calculate the sales and the profit for both scenarios, with and without fractional production. In Equation (1) the formula for the profit for the general form is given. We compare the profit for both scenarios starting in period 1 and choose the one with the highest profit, we can check this n times. When the profit of the fractional production is higher, we take this option instead of the normal one as RI. When all the RIs are determined, we go to the following step for optimizing our problem, which will be explained in the next Section.

4.3.2 Shortest-Path Algorithm

In Section 4.3.1 we generated all possible RIs, with optimal demand for every period, given capacity and number of periods. The next step is to implement the information of the *Lagrange Multiplier Method* in the *Shortest Path Algorithm.* Here, we ascertain all possible combinations for connecting the RIs with each other. For the example in Figure 1 in Section 4.2 there are 8 different options for connecting the RIs with each other:

$$[1, 5, 8, 10], [1, 5, 9], [1, 6, 10], [1, 7], [2, 8, 10], [2, 9], [3, 10], [4]$$

We generated the *Shortest Path Algorithm* as follows. First we check for a RI of period 1, which end is in period 1, the maximum profit. Thereafter, we look for all the RIs and check what is the end en start period. For example, when the start & end period is equal to 2, which means that the RI only exists of one period, we count this profit with the profit which only produces in period 1. When all the possible combinations for a period, which starts at 1 and ends at 2 are calculated, we compare the total profit for the different options and take the maximum. We put the combination of RIs, which corresponds to the maximum profit, at the place for period 2 with the corresponding profit. We continue this process until we have the whole period n. We can visualize our steps with Equation (29 - 31). $p(i, j)$ represents the profit of RI $[i, j]$, where i stands for the start period of the RI and j for the end period. $\pi(t)$ means the profit until period t. In this example we considered n equal to 3, but for greater values of n the same process continues only the different options for $\pi(t)$ will increase.

$$\pi(1) = p(1, 1) \tag{29}$$

$$\pi(2) = \begin{cases} \pi(1) + p(2, 2) \\ p(1, 2) \end{cases} \tag{30}$$

$$\pi(3) = \begin{cases} \pi(2) + p(3, 3) \\ \pi(1) + (2, 3) \\ p(1, 3) \end{cases} \tag{31}$$

The main motivation to use the *Shortest Path Algorithm* is for the reduce of the computation time and number of iterations for the whole problem. For this case, not all the possible options need to be calculated.

Next, the total profit is the profit which belongs to $\pi(n)$. With this $\pi(n)$ we get the corresponding price, demand and production pattern for every time period. We could continue all these steps for other capacity levels.

In conclusion, the whole process starts with initializing all parameters followed by determining the price and demand for every time period within a RI with use of the *Lagrange Multiplier Method.* At the end, we determine the combination of RIs with the highest profit, using the *Shortest Path Algorithm*, and solve the problem. In the next Section we give some examples for these models and problems.

5 Results

In this Section, we demonstrate the theory as explained in Section 4 and present some numerical examples. We look at what happens with the optimal price when we change the capacity. First, we show what happens with the price pattern for homogeneous demand when we change the capacity. In Section 5.2 we present the average unit price and next we give the optimal solution given seasonal demand for a given capacity in Section 5.3.

In the examples followed by Deng and Yano (2006) we consider 6 time periods and each unit of production uses one unit of capacity. Under these conditions we will look for homogeneous demand and constant costs. The parameters are as follows:

$$
\begin{aligned}
B &= 1 \\
A_t &= 10 \\
K_t &= 10 \\
v_t &= 1 \\
h_t &= 0.1
\end{aligned}
$$

5.1 Price Pattern for Homogeneous Demand

In this case, capacity is constant over the horizon, but we consider different capacity levels to illustrate the effect of changes in capacity for the optimal solution. In our examples, we use the inverse demand function as noted in (17) for determining the price for the case of homogeneous demand and constant costs.

In Figure 2 the optimal price against the time periods is given for capacity varying from 7 to 14. The results almost correspond to the results in Deng and Yano (2006). The only difference is that for a capacity of 10 the production pattern is different. In Deng and Yano (2006) they have 3 RIs. This cannot be a fractional production pattern, because in my case this option is not feasible. The total demand for the 3 periods is greater than 10, so we must produce in more than one period. However, when we produce the remaining in period 1, which is not a multiple of 10, ensures that the demand is greater than the production. This is the reason why the fractional production is not feasible. For the general production pattern is this ([1,2] , [3,4], [5,6]) not the optimal solution. Other options, including my ([1,2,3] , [4,5,6]), have a higher profit. This is the reason for the difference for a capacity level of 10.

It is also interesting in Figure 2 that for every capacity level there is a certain pattern. Each curve has a different shape depending on the optimal production pattern.

For a capacity level of 7, you could see that the optimal solution exists of 3 RIs namely: [1,2], [3,4] and [5,6]. Within a RI the optimal price raises every time period. This can be explained by the fact that the price linearly depends on the demand (17), as already mentioned in Section 4.1.3.

In the case of Figure 2 we have constant demands and production costs. Furthermore, λ is the same for every period in a RI because the general problem (6 - 8) has a single constraint, only the increase in the inventory holding costs provides a decrease in the demand when t increases. The price in period t is linear depending on the demand. This explains why the price raises for every period in a RI. In the first period of the next RI is the price dropped compared to the last time period of the previous RI. Considering our restrictions, the value of the first period in every RI is the same for all RIs. Because of this phenomenon every price curve gets a sawtooth shape.

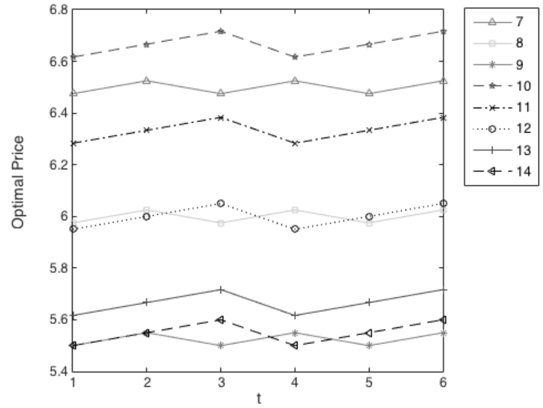


Figure 2: Optimal Prices vs. Time for Capacity Levels 7 to 14

Moreover, for the capacity levels 7 to 9 the length of the RIs are equal to two and for 10 to 14 they are three, as shown in Figure 2. An increase in the capacity level for the same length of the RIs ensures that the optimal price drops. Further, when the number of periods within a RI changes, which in our case happens when the capacity increases from 9 to 10, the level of the optimal price increases again. Next, when the capacity increases and the RI length stays the same the optimal price drops. Here, we can find a certain pattern. For the same length of RIs, the price decreases when the capacity increases. However, when the number of periods within a RI changes, when the capacity increases, the optimal price increases again.

In the following Section we will show what happens with the price for different levels of capacity when we use the average unit price.

5.2 Average Unit Price

In Section 5.1 we examined what happens with the price level in every period when the capacity changes. Here, we will look what happens with the average price for the whole period when the capacity changes. For calculating the average unit price we used the following formula:

$$p_a = \frac{\sum_{t=1}^{T} p_t D_t}{\sum_{t=1}^{T} D_t} \tag{32}$$

where p_a is the demand-weighted average price. In Figure 3 is the average unit price for constant demand shown, $A = [10]$. For this option we look for a capacity from 1 to 30. An interesting thing to see is that for certain periods the average unit price is constant. We could conclude that for the capacity of 14 until 20 they used fractional production, because the average unit price is here exactly alike. The average unit price is here equal to €5,55 The same holds for a capacity of 27 and higher with a price of €5,62. We can underpin our arguments by the fact that the average unit price is the same, which could not be the case when we make use of restriction (15). For low capacity levels, small changes in capacity lead to a decline in the average unit price, which indicates that the demand increases. So more customers are willing to buy the product.

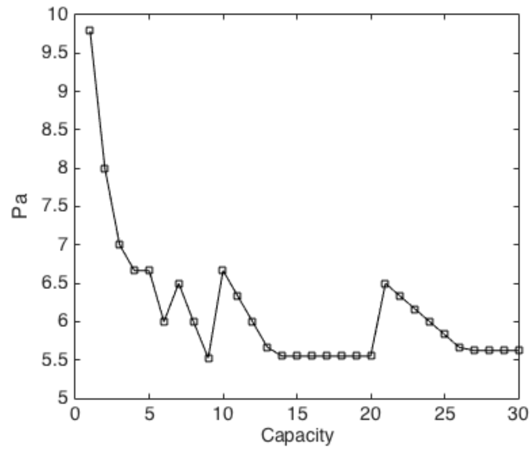
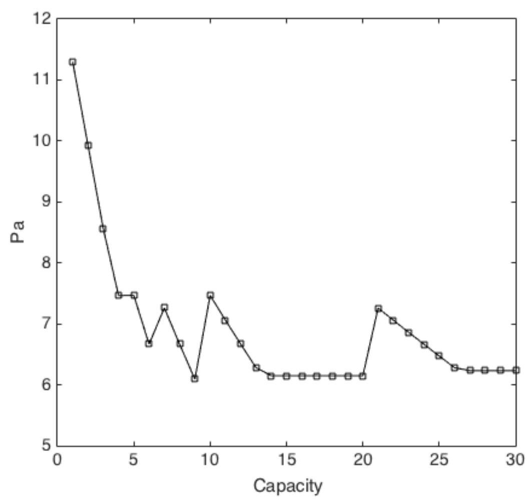
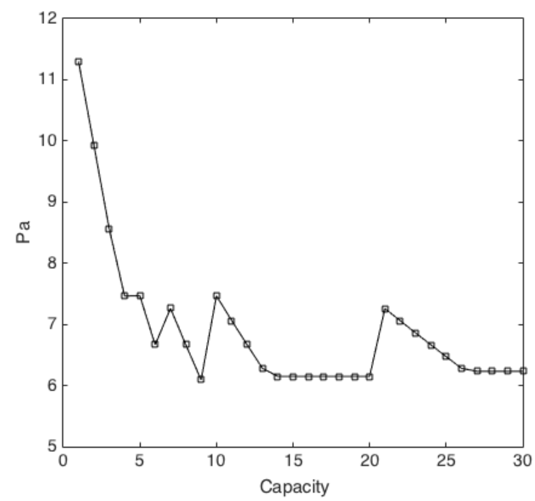


Figure 3: Average Unit Price with Constant Demand as a Function of Capacity

We also did some calculation for increasing demand and seasonal demand functions. We made use of the following increasing demand: $A = [7.5, 8.5, 9.5, 10.5, 11.5, 12.5]$ and for the seasonal demand we used $A = [10, 14, 6, 10, 14, 6]$. In Figure 4a & 4b the average unit prices are shown as a function of the capacity.



(a) Average Unit Price with Increasing Demand as a Function of Capacity



(b) Average Unit Price with Seasonal Demand as a Function of Capacity

Figure 4

An interesting aspect to notice is that for Figure 4a the average unit price is the same for a capacity from 16 - 20 namely €5,71 and also for a capacity level of 27 and greater, which equals €5,79. For the average price with seasonal demand is the capacity of 14 until 20 equal to €6,15 and for a capacity of 27 and greater €6,23. An aspect that stands out is that for all the three cases: homogeneous, increasing and seasonal demand, the average unit price converges at the same capacity level, namely 27, to a constant price. We can conclude from this phenomenon that a capacity level greater than 27 does not influence the average unit price. However, these three constant prices differ. For a constant demand is the price equal to €5,62, increasing demand €5,79 and for seasonal demand is it equal to €6,23. We conclude from this difference that the demand scenario has an influence on the height of the average unit price. But in general, the shape of the three Figures (3, 4a, 4b) has approximately the same form only shifted. In the next Section we will continue with the seasonal demand and explain in more detail the corresponding price and demand for given capacity levels.

5.3 Price Pattern for non-Homogeneous Demand

In the example above we looked at the optimal prices against the capacity levels with homogeneous demand. Now we consider non-homogeneous demands. The prices in different periods may move in different directions and the demand weighted average price may move in unexpected directions, because the demand in every period differs. In the following example we use seasonal demand $A = \{10, 14, 6, 10, 14, 6\}$. All other parameters stay the same. In Table 1 the production pattern, demand and corresponding price are given for a capacity of 5 & 6. The profit is €88,74 and €95,21 for a capacity of 5 & 6 respectively. The optimal solutions for a capacity of 5 are different from Deng and Yano (2006). The optimal solution Deng and Yano (2006) used are a feasible solution but not the optimal solution, because it has a lower profit. The optimal solution for seasonal demand is shown in Table 1.

Table 1: Optimal Solutions for the Seasonal Demand with C= 5 and 6

t	1	2	3	4	5	6
A	10	14	6	10	14	6
$C = 5$						
x_t*	5	5	0	5	5	0
D_t*	3,383	5,333	1,283	3,383	5,333	1,283
P_t	6,617	8,667	4,717	6,617	8,667	4,717
$C = 6$						
x_t*	6	6	0	6	6	0
D_t*	4,050	6,000	1,950	4,050	6,000	1,950
P_t	5,950	8,000	4,050	5,950	8,000	4,050

In Table 1 we observe that when there is a positive or negative change in A the same pattern happens for the optimal demand in that period for the demand D_t and the optimal price P_t. Another thing that stands out is when the capacity raises, the price declines for the same length of RIs, which is here equal to 3. The same happened for the homogeneous demand in Section 5.1, where for the same length of the RIs the optimal price drops when the capacity increases. In this case, the demand-weighted average unit price (p_a) for $C = 5$ is equal to 7,47 and for $C = 6$ is $p_a = 6,67$. For these capacities, the average unit price increases when the capacity decreases.

6 Conclusion & Discussion

In this Bachelor Thesis, we examine the problem of joint production and pricing decisions with given capacity constraints for a given time period with setup costs. We clarify how to solve this problem for maximizing the profit over the horizon. For completing our steps we made use of Regeneration Intervals(RIs). The models we used for determining the RIs are the *Lagrange Multiplier Method* and the *Shortest Path Method*. We used the *Lagrange Multiplier Method* for computing all the RIs. Furthermore, first we found all the RIs for a particular capacity level and then we got the optimal production pattern out of the *Shortest Path Algorithm*.

In Section 5 we found several interesting aspects. The price pattern for homogeneous demand follows a certain pattern. The price in period t depends linearly on the demand, which explains why the price raises for every period in a RI. A change in the length of RIs for different capacity levels influences the optimal price as shown in Figure 2. The pattern we can deduce here is that the optimal price drops, for RIs with the same length, when the capacity increases. The optimal price raises again when there is a change in the length of RIs for increasing capacity.

The following aspect we examined is the average unit price. When the capacity increases the average price converges to a certain value. We compared the prices for constant, increasing and seasonal demand and conclude that the average price for seasonal demand is the highest when the capacity does not influence the price level, which is for a capacity greater than 27. In Section 5.3 we deepen into the scenario for seasonal demand. We observed that when A increases or decreases in the next period the same pattern happens for the optimal demand(D_t) in that period. Another thing that stands out is when the capacity raises, the price declines for the same length of RIs for every t, which is the same conclusion as we made for homogeneous demand in Section 5.1.

Further, we could check for many different scenario's certain aspects. These scenario's are explained in Section 7.

7 Further Research

For determining the maximum profit over the horizon are different ways. In this Bachelor Thesis we considered constant capacity, no speculative motive for holding inventory, and non-increasing setup costs. For further research we also could check the maximum profit for the other three options as mentioned in Section 4.1.1. One of these options is a change in the setup costs. In our research we made use of non-increasing setup costs. But in this case we could consider arbitrary setup costs. When the setup costs are arbitrary, the demand given from the demand function stays the same, it will only influence the profit. This could have a greater influence for the RIs which will be chosen when maximizing the profit.

Further, another aspect we could determine is non-constant capacity. In this case, every RI could consist of multiple capacity levels, which would give different conditions and much more combinations for RIs. For these options other assumptions must be made, which means that we need to change parts of the algorithms.

Another extension that can be made is calculating the maximum profit with constant price instead of constant demand. A question to ask here, is how much benefit can be obtained when the prices are flexible instead of constant. Another question that follows is how much influence has homogeneous, increasing and seasonal demand when we consider constant prices.

Further we can compare our optimal solution with myopic solutions. The myopic policy compares the prices and resulting demands with those for independent single-period problems with a setup in each period. An interesting question is whether, when taking seasonal changes in mind, does it benefit to set prices in low seasons, which means to have overcapacity, for satisfying the demand in high seasons. For some cases it is not optimal to use the myopic policy. Cases for which this hold are when the gross margin does not cover the setup cost.

The last point we could consider is instead of a single product looking at multiple product scenario's. Here, a decision maker can explore different allocations of capacity to products in the various time periods for better understanding of the interrelated effects of capacity allocations and prices on profits.

Further research is needed to consider other realistic factor that could happen during a production process. Examples of these factors could be inter-temporal substitutions by customers, uncertain demand and dynamic pricing.

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