Erasmus University Rotterdam

Bachelor thesis

Econometrie & Operationele Research

“Mixed integer and heuristics model for the inventory routing problem in fuel delivery.”

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Juli 6, 2015

Abstract

This paper follows the same procedure as Vidović et al. (2014). We present solution approaches for the multi-product multi-period Inventory Routing Problem (IRP) in fuel delivery. A homogeneous fleet of vehicles with trailers is used for the distribution of full compartments of fuel from a single depot to a set of fuel stations. The daily fuel consumptions follow a discrete probability function. To solve the IRP, a Mixed Integer Programming (MIP) model is proposed. To observe the impact of the fleet size costs on the obtained solutions we propose heuristics with and without fleet size costs. The heuristics model is based on constructive heuristics and is further improved with two search types: a local intra-period and a large intra-period neighborhood search. By testing on some of the assumptions made in Vidović et al. (2014), we question some of the results they presented in their paper.

Contents

1 Introduction

In the world as we know it today there are vehicles everywhere, in fact we can't imagine a world without them anymore. All these vehicles run on a certain type of fuel, like diesel for example. If we run out of fuel we just buy some new one at a nearby fuel station. Those fuel stations also do not have unlimited fuel, they have a maximum fuel capacity. For a logistic manager it would be preferable to deliver the fuel at his stations just before he runs out of it. This is quite complicated because most of the vehicles, delivering fuel, visit more than one station per day. They drive a certain route visiting a couple of stations. It is of course preferred for all vehicles to leave the depot fully loaded and arrive empty at the end of the day but that is a difficult problem concerning the supply and demand of fuel. Furthermore, it would be a waste of money if the route the truck driver needs to drive would be longer than needed. In fact, the route distance needs to be minimized. So in the supply chain there must be an interrelationship between the inventory allocation and the vehicle routing. In logistics this problem is known as the integrated Inventory Routing Problem (IRP) and its objective is finding a balance between the inventory and routing costs such that the total costs are minimized. This integrated IRP can be found in many industries, supermarkets for example or other industrial industries like Liquefied Natural Gas. Vidović et al. (2014) tries to tackle a practical problem of the transportation of different fuel types from a single depot location to a set of petrol stations during a predefined planning horizon. The IRP in this paper is known as a so called multi-product multi-period deterministic IRP. To solving this problem a Mixed Integer Programming (MIP) model is proposed and a heuristic is used for a case with and without fleet size cost. The purpose of the distinction of two cases is observing the impact of the fleet size cost on the already obtained solutions. After the heuristic there are two improvement techniques used, a large inter-period neighborhood search and a local intra-period search. After solving the heuristic and improvement techniques the results are compared to the MIP model on a set of small-size test examples. Furthermore, some conclusions are drawn from the obtained results. With this conclusions Vidović et al. (2014) solve a set of large-size problems using the same heuristic and improvement techniques. In our paper we remake the MIP models and the heuristic approach. We evaluate and question some of the results presented by Vidović et al. (2014) and implement some extensions. We have organized the paper in the following way. A literature review is presented in Section 2. In Section 3 the problem and heuristics are formulated and the MIP models are introduced. Also the searches of improvement are introduced here. In Section 4 the results are evaluated and compared to the results of Vidović et al. (2014). Some results are based on the questions that arise after reading the paper of Vidović et al. (2014). Finally, Section 5 presents some concluding remarks and directions for further research.

2 Literature review

Bell et al. (1983) were among the first papers who introduced the IRP. The IRP in its original form and the metered version discussed in Herer and Levy (1997) are both closely related to the Vehicle Routing Problem (VRP). Toth and Vigo (2001) covers three main variants of VRP and extends it with time windows, backhauls and pickup and delivery. The VRP is all about 'vehicle scheduling' and solving this in different ways, like the methods in Christofides (1976) and Gendreau et al. (1994). IRP however, also includes the inventory concerns and can therefore be interpreted as an extension of the VRP (see Ball (1998)).

There are two kinds of IRP's, a strategic IRP and a tactical IRP (see Webb and Larson (1995)). In our paper we implement both strategic and tactical and define it as IRP and IRPF. The IRP is implemented in all kinds of branches, like supermarkets. Gaur and Fisher (2004) describes and implements a system to solve a periodic IRP at a leading supermarket chain in the Netherlands. But also in other industrial areas like the Liquefied Natural gas (LNG) industry IRP is used (see Grønhaug et al. (2010)) or the less familiar satellite industry (see Bard et al. (1998)). In our paper we assume that there is never a stock-out, by declaring a minimum fuel capacity. However, Federgruen and Zipkin (1984) took the cost per unit shortage at location **i** into consideration. Huang and Lin (2010) also implemented stock-out costs but with uncertain demand. The conventional ant colony optimization (ACO) algorithm is used to optimize the tradeoff between transportation costs and stock-out costs. In Herer and Levy (1997) they also took the stock-out costs into consideration and combined this with the use of a concept of temporal distances, which was already introduced by Herer (1996). In Herer and Levy (1997) they also used the heuristic developed by Clarke and Wright (1964), who attempt to solve the clustering of customers to trucks and finding the best route together. Our assignment heuristic is also based on Clarke and Wright (1964). Last but not least, Herer and Levy (1997) also considered vehicle outsourcing, meaning that every route incurs at a fixed cost when hiring a vehicle. We make use of this idea in the IRPF model, but we use a heuristic in such way that there is never outsourcing needed. When deliveries to customers are made by more than one vehicle it is named a "split delivery". Dror and Trudeau (1989) was the first to introduce the split delivery. This could be needed when there is a very large demand at some day. In this paper the VRP constraint that every customer is served by only one vehicle is relaxed. Yu et al. (2008) developed a model which also incorporated split delivery. Bertazzi et al. (2013) applied an order-up-to-level policy and proposed a hybrid roll-out algorithm to solve it. In our models we only work with one depot but also a multi-depot petrol station heuristic is possible, which can be seen in Cornillier et al. (2012). They look at a multi-depot petrol station replenishment problem with time windows and assume a heterogeneous fleet. It is also possible to propose an algorithm based on the allocation of first route second strategy. In Moin et al. (2011) they proposed a hybrid genetic algorithm for solving this. At last, in our paper we make a simplification of the reality by defining a discrete probability density function for the daily fuel consumption but in Kleywegt et al. (2004) they formulate a Markov decision process model with stochastic demand.

3 Methodology

3.1 Problem Formulation

For all **t** days in the planning horizon **T** we define a certain set of **I** petrol stations each offering **J** fuel types. For each of these fuel types, customers have a certain daily demand $\mathbf{q_{ij}}$. The intensity of the demand varies for each station **i** and type **j** but we assume the demand to be constant for all days through the entire planning horizon **T**. Every petrol station is equipped with **J** underground reservoir tanks, one for each type. It is not allowed for inventory levels for any type of fuel to fall below their defined daily consumption $\mathbf{q_{ij}}$ nor above the maximum capacity $\mathbf{Q_{ij}}$. It is not allowed for inventory levels for any type of fuel to fall below $\mathbf{q_{ij}}$ to prevent negative inventory. It can be seen as an extra approximation. At the beginning of the planning horizon the tanks have a certain $\mathbf{S}_{ij}^0$ stock level left. Since most of the time the daily demand over the planning horizon is more than the stock level, fuel needs to be delivered to the stations. Delivery is done by one type of vehicle, which has two compartments ($\mathbf{K} = 2$). This truck can tow a trailer with two additional compartments. Therefore, in one route, at most four compartments ($\mathbf{K} = 4$) can be delivered. The capacity is assumed to be equal for all compartments and only full compartments are delivered to the stations. So a vehicle towing a trailer can visit either one, two, three or four stations per route. For the sake of simplicity, stations can be served only once per day.

3.1.1 IRP MIP model

The routing costs (**RC**) depends on the distance all vehicles need to travel, whereas the inventory costs (**IC**) depend on the sum of the average stock levels on each day of the planning horizon. The complete model of the minimization of the total costs (**IC+RC**) is described in Vidović et al. (2014). Since it is too hard to minimize the **IC** and **RC** at the same time, we relax the inventory routing problem (IRP) in such way that we first minimize the inventory costs. This relaxation is referred to as the relaxed MIP model. Given the solution of this minimization we use the same heuristic as described in Vidović et al. (2014). Before we make use of this heuristic we first need to create the original fuel delivery plan. This is done using a route construction method, which we will describe later on. On this delivery plan we run the heuristic and try to lower the total costs by creating more optimal delivery routes. We therefore first introduce the relaxed MIP model and propose the heuristic afterwards. For the complete model of minimizing the routing cost (**RC**) and the inventory cost (**IC**) at the same time we refer to Vidović et al. (2014).

3.1.2 Mathematical formulation of the relaxed IRP MIP model

Indices:

i	fuel stations	($\mathbf{i} \in \{1,2,...,\mathbf{I}\}$)
j	fuel types	($\mathbf{j} \in \{1,2,...,\mathbf{J}\}$)
t,z	time period or day in the planning horizon **t,z**	($\mathbf{j} \in \{1,2,...,\mathbf{J}\}$)
k	compartment number	($\mathbf{k} \in \{1,2,...,\mathbf{K}\}$)

Decision variable:

$$\mathbf{X}_{ijtk} = \begin{cases} 1, & \text{if fuel station } \mathbf{i} \text{ is supplied with fuel type textbfj in time period } \mathbf{t} \text{ with } \mathbf{k} \text{ compartments} \\ 0, & \text{otherwise} \end{cases}$$

$\mathbf{S}_{ij}^0$	stock level of fuel type **j** at station **i** at the beginning of the planning horizon
$\mathbf{q}_{ij}$	consumption of the fuel type **j** at station **i**
$\mathbf{c}_{inv}$	inventory carrying costs per day
$\mathbf{Q}_{ij}$	capacity of the underground reservoir for the fuel type **j** at station **i**

Objective function:

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T}((S_{ij}^0 - t \cdot q_{ij} + \frac{q_{ij}}{2}) + \sum_{z=1}^{t}\sum_{k=1}^{K} x_{ijzk} \cdot d_k) \cdot c_{inv} \tag{1}$$

Subject to:

$$S_{ij}^0 + \sum_{z=1}^{t}\sum_{k=1}^{K} x_{ijzk} \cdot d_k - \sum_{t=1}^{z-1} q_{ij} \leqslant Q_{ij} \qquad \forall i \in I, \forall j \in J, \forall z \in T \tag{2}$$

$$S_{ij}^0 + \sum_{z=1}^{t}\sum_{k=1}^{K} x_{ijzk} \cdot d_k - \sum_{t=1}^{z} q_{ij} \geqslant q_{ij} \qquad \forall i \in I, \forall j \in J, \forall z \in T \tag{3}$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K} x_{ijzk} \cdot d_k \leqslant K \qquad \forall i \in I, \forall z \in T \tag{4}$$

$$x_{ijzk} \in \{0,1\} \qquad \forall i \in I, \forall j \in J, \forall t \in T, \forall k \in K \tag{5}$$

The objective function (1) attempts to minimize the total inventory costs of the stored fuel in all of the underground reservoir tanks during the whole planning horizon. The storage fluctuates due to the amount of daily demand and whether there will be a fuel delivery or not. Constraint (2) makes sure that the maximum quantity of the stored fuel in the reservoir tanks will not exceed the reservoir capacity. Constraint (3) is the exact opposite, it makes sure there is always sufficient fuel to meet the daily demand. Since we assumed the fuel delivery to a station only done once per day, constraint (4) is needed to prohibit multiple direct deliveries. At last, Constraint (5) defines the decision variable to be binary.

3.1.3 IRPF MIP model

The fuel delivery route will be constructed by assigning utilities to all possible delivery routes. This will result into a set of routes for all days that needs to be driven by the truck drivers. However, each delivery company has only a limited number of vehicles and truck drivers. This is not taken into account in the IRP MIP model. To include this the previous model is extended by including the fleet size and the corresponding fleet size costs (**FC**). We assume that it takes the truck driver the whole day to finish the route. So each route represents one vehicle that is used during the whole day. Therefore, the fleet size of day **t** in the observed planning horizon **T** equals the maximum number of delivery routes. The required fleet size on the planning horizon **T** equals the maximum of the daily required fleet sizes. In reality it is often the case that there are not enough vehicles available to satisfy the required fleet size. To include this fact into our model we define a number of dummy vehicles such that the required fleet size is met. For the sake of simplicity we assume that the number available truck drivers are equal to the available fleet size.

3.1.4 Additional mathematical formulations for the IRPF MIP model

The IRPF model minimizes the inventory, routing and fleet size cost simultaneously. The formulation is added to the already proposed IRP model. We first introduce the additional required notations.

$\mathbf{F}_a$	fleet size that is available on each day **t** on the planning horizon **T**
$\mathbf{F}_t$	required number of vehicles per each time period **t** on the planning horizon **T**
$\mathbf{F}$	required fleet size to satisfy the fleet size on the planning horizon **T**
$\mathbf{c}_v$	fixed cost of the available fleet size per truck per day
$\mathbf{c}_m$	fixed cost of the dummy vehicles fleet

Objective function: **IC** + **RC** + **FC**, where

$$\mathbf{FC} = \sum_{t=1}^{T} F_a \cdot c_v + (F - F_a) \cdot c_m \tag{6}$$

$$\sum_{p=1}^{I}(y_{pt} + \sum_{q=p+1}^{I}(y_{pqt} + \sum_{w=q+1}^{I}(y_{pqwt} + \sum_{e=q+1}^{I} y_{pqwet}))) = F_t \qquad \forall t \in T \tag{7}$$

$$F \geq F_t \qquad \forall t \in T \tag{8}$$

$$F \geq F_a \tag{9}$$

The IRPF model is solved through the same iterative procedure as described in Vidović et al. (2014). In the first iteration we set $\mathbf{F}_a$ equal to zero, since $\mathbf{F}_a$ must be defined to be less than the fleet size that is sufficient to realize all the deliveries. In this case only dummy vehicles are used for the deliveries. Since this value is not adequate enough for satisfying all of the deliveries without using the dummy vehicles, we increase $\mathbf{F}_a$ by one vehicle and solve the model again. We repeat this procedure until no dummy vehicles are used.

3.2 Heuristic approach

By solving the relaxed MIP model we minimize the total inventory costs for all days of the planning horizon. Minimizing the inventory costs forces delivery of the smallest possible quantities, in our case one full compartment, at the latest possible day. We use a heuristic approach to route the vehicles also as good as possible. If we would construct the vehicle routes directly on the obtained delivery plan, the travel distances would be far from optimal because the minimization of the inventory cost does not take the distance between stations into account.

Since we also need to minimize the routing cost we make use of a heuristic. In this heuristic we attempt to change the delivery plan by moving the deliveries of quantities to one or a few days earlier. This is possible since the relaxed MIP model forced delivery at the latest possible time period. However, each delivery that will be moved earlier in the planning horizon will give additional inventory cost and has influence on the routing of vehicles. This influence is not only on the day where the delivery is moved from but also on the day where the delivery is moved to If the movement is possible and it leads to a lower total cost, we expect not only to see a decline in the routing cost but also an increase in the inventory cost. However the increase in the inventory cost has to be lower comparing to the decline in the routing cost. Since it could be very time-consuming if we just calculated the possible benefits on every potential transfer Vidović et al. (2014) only observed the **E** most eligible transfers. An eligibility calculation is used for this, which runs for all stations over the planning horizon.

3.2.1 Eligibility calculation

The eligibility calculation is the same as described in Vidović et al. (2014) and is based on the values of the three criteria $V^{\alpha}_{it\delta}$, which is a measurement of the effects if we transfer a single compartment from station **i** on day **t** to day **t**-δ, where $\alpha \in$\{1,2,3\} describes the type of criteria. We define $\mathbf{K}_{it}$ as the number of compartments to be delivered to petrol station **i**, on day **t**.

(I) The value of the first criterion $V^1_{it\delta}$ defines if it is even possible to move the delivery from station **i** from day **t** to day **t**-δ. If transfer is possible, $\mathbf{K}_{it} > 0 \bigcap K_{it-\delta} < 4$ set $V^1_{it\delta} = 1$; otherwise set $V^1_{it\delta} = 0$.
(II) The value of the second criterion $V^2_{it\delta}$ is based on the change in the number of "to be served" fuel stations incurred by the transfer of the compartment from day **t** to **t**-δ

Case 1: $\mathbf{K}_{it} = 1 \bigcap 0 < \mathbf{K}_{it-\delta} < 4$, then $V^2_{it\delta} = 2$
This is the best option, since station **i** will be deleted from the route on day **t** and its delivery quantity will be added to the already existing route on day **t**-δ
Case 2: $\mathbf{K}_{it} > 1 \bigcap 0 < \mathbf{K}_{it-\delta} < 4$, then $V^2_{it\delta} = 1$
The number of stations that needs to be served on day **t** and **t**-δ remain the same.
Case 3: $\mathbf{K}_{it} = 1 \bigcap \mathbf{K}_{it-\delta} = 0$, then $V^2_{it\delta} = 1$.
Station **i** will be deleted from the route on day **t** but this station will then be added to the route on day **t**-δ

Otherwise set $V^2_{it\delta} = 0$.

(III) The value of the third criterion $V^3_{it\delta}$ is based on the routing distance. The idea is to find a station on day **t** which is the most isolated for the other stations on the delivery route. It must be so isolated that it is closer to a station in day **t**-δ. This has a high potential for generating savings in the routing costs. The psuedo code for the calculation of the value of the third criterion is given in table 1.

For every station $\mathbf{i}$
if $\mathbf{K}_{it} > 0$
Find the largest distance $\mathbf{r}_{ip}$ where $\mathbf{p} \in \mathbf{Ii} \backslash \mathbf{i}$ such that $\mathbf{K}_{pt} > 0$
Find the shortest distance $\mathbf{r}_{iq}$ where $\mathbf{q} \in \mathbf{Ii} \backslash \mathbf{i}$ such that $0 < \mathbf{K}_{pt} < 4$
$V^3_{it\delta} = \mathbf{r}_{ip}$ - $\mathbf{r}_{iq}$

Table 1: Pseudo code for the calculation of the third criterion $V^3_{it\delta}$

To determine the most eligible stations we first sort the first criterion in non-increasing order. Since the value of the first criterion varies only between a zero and a one, many transfer possibilities will be of equal value. To this end, we put the the possible transfers with the first criterion equal to zero in a matrix and the transfers equal to one in another matrix. Next, for both matrices we sort the stations on the second criterion again in non-increasing order. Because the second criterion again has different values (two, one and zero), we put them again in new matrices, now each with equal values for criterion one and two. At last, all matrices are sorted in non-increasing order by the third criterion. To get the most eligible transfers we make one large matrix combining all these matrices. The first rows of the matrix has now the highest criterion values and the last rows the lowest criterion values. So, the $\mathbf{E}$ most eligible transfers are in the first rows of the matrix and are selected for calculating the actual transfer benefits. To calculate the benefit of the transfer we need to construct the routes.

3.2.2 Route construction using assignment heuristics

Now it is time to construct the delivery routes for the trucks. We base the construction on the value of assignment utilities, which is inspired by savings concept described in Clarke and Wright (1964). This is the same algorithm as used in Herer and Levy (1997). The procedure for route construction has to be done for every day $\mathbf{t}$ in the planning horizon $\mathbf{T}$.

Let us first define the vehicle utilization. The vehicle utilization depends on the number of compartments. Therefore we define $\varepsilon = \{0.25, 0.5.\ 0.75, 1\}$ corresponding to one, two, three or all four compartments. So, a vehicle with all four compartments filled give a higher utilization than a vehicle with only one full compartment. The distance from the depot to station $\mathbf{i}$ is represented as $\mathbf{r}_i$ (**i = p, q, w or e, i** $\in$ **I**). The distance $\mathbf{r}_{pq}$ represents the distance between station $\mathbf{p}$ and $\mathbf{q}$, which is calculated using the Euclidean distance. The distance between three or four stations ($\mathbf{r}_{pqw}$ and $\mathbf{r}_{pqwe}$ is a little bit more complicated. To calculate the minimum lengths for three and four stop routes we use enumeration. We just enumerate every route possibility. For example to calculate the minimum lengths for a three stop route, the route is given as follows:

depot - station p - station q - station w - depot

If we now change the place of station **p** and **q** we get the following:

depot - station q - station p - station w - depot

This gives most of the time a different route length, since the distance from station q to the depot is most of the time different to the distance from station p to the depot. If we now try every possible combination given the stations p, q and w we enumerate every route possibility. Finally we select the route with minimum length. The same idea holds for a four stop route.

Since we have a maximum of four compartments there are only four different routes possible: direct delivery, two station delivery, three station delivery and four station delivery. For every station we calculate, if possible, all the route types. This results in the following utilities:

$$u_p = K_{pt} \cdot r_p - \varepsilon \cdot r_p \tag{10}$$

$$u_{pq} = K_{pt} \cdot r_p + K_{qt} \cdot r_q - \varepsilon \cdot r_{pq} \tag{11}$$

$$u_{pqw} = K_{pt} \cdot r_p + K_{qt} \cdot r_q + K_{wt} \cdot r_w - \varepsilon \cdot r_{pqw} \tag{12}$$

$$u_{pqwe} = K_{pt} \cdot r_p + K_{qt} \cdot r_q + K_{wt} \cdot r_w + K_{et} \cdot r_e - \varepsilon \cdot r_{pqwe} \tag{13}$$

Every fuel station has now been given more than one utility. Our objective is to construct a delivery route with the highest possible utility for every station. We do this by making a utility matrix with the calculated utilities sorted in non-increasing order. Afterwards we select the delivery with the highest utility. All stations in this route will be set in the set of routes but first we need to delete all these stations in the other utilities. We do this until there is no more station left in the utility matrix. The pseudo code is given in table 2.

```
Input: $\mathbf{K}_{it}$, $u_p$, $u_{pq}$, $u_{pqw}$, $u_{pqwe}$,
Set_of_routes_all_days = []
for day in planning horizon:
    Set_of_routes = []
    Utility = [$u_p$; $u_{pq}$;$u_{pqw}$; $u_{pqwe}$]
    Sort Utility in descending order
    while Utility is not empty
        set the Set_of_routes equal to the first row of Utility
            for station in Set_of_routes
                for route in Utility
                    if station in route
                        delete route from Utility
insert Set_of_routes to Set_of_routes_all_days
```

Table 2: Pseudo code of the route construction

Before we try to decrease the total cost we need to calculate the routing cost immediately after the relaxed MIP. We call this the original total cost. For the **E** most eligible transfers we rerun the route construction code. Before we do this we have to check if by switching the delivery from day **t** to day **t**-δ the inventory restrictions are still met. If there is no violation then we compare the obtained total cost to the original cost. If the obtained total cost is lower we replace the original total cost in this total cost and the delivery schedule is updated. The whole heuristic algorithm is shown in figure 1.

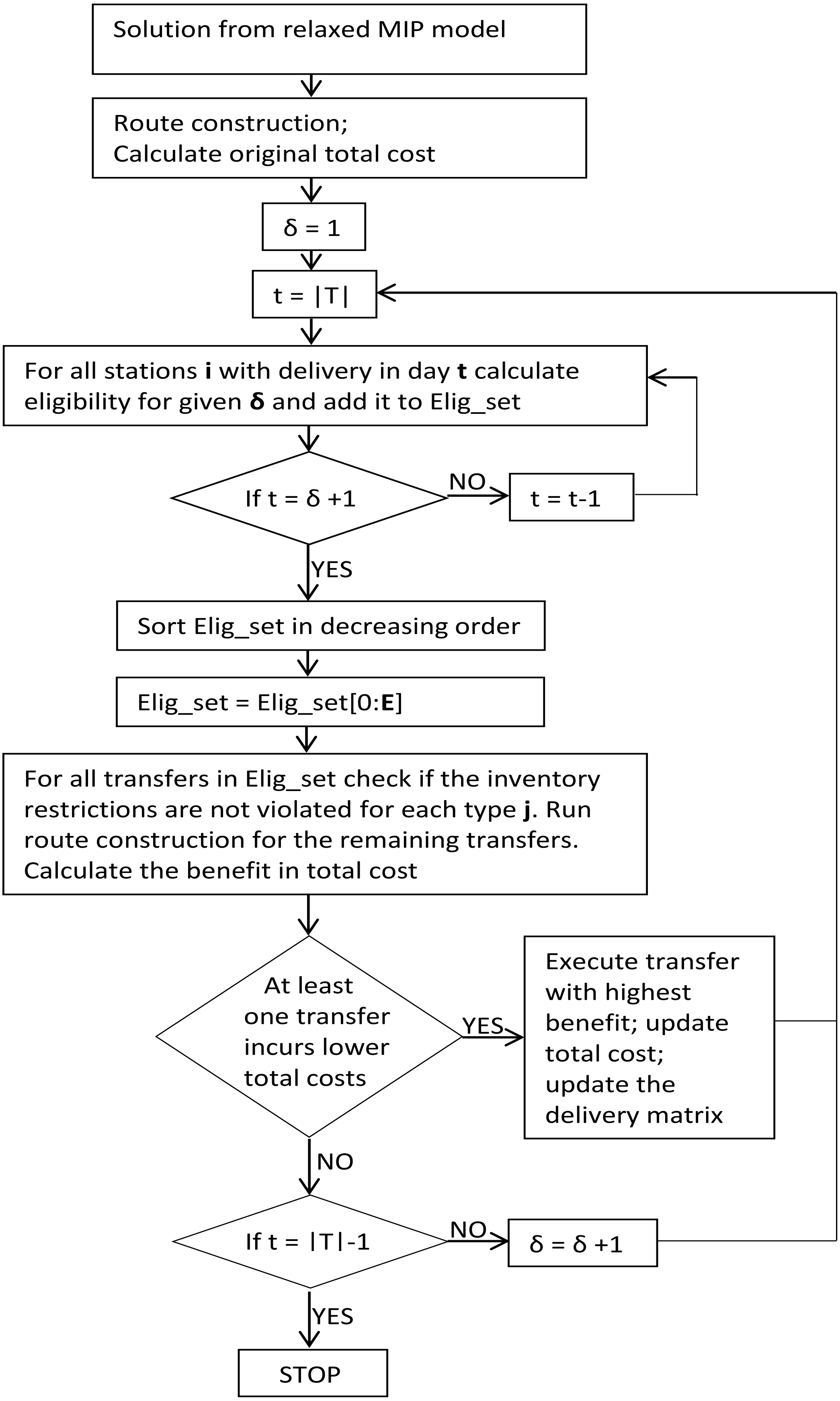

Figure 1: Heuristic algorithm

3.3 Local and large neighborhood searches

3.3.1 Local neighborhood searches

In Vidović et al. (2014) the local variable neighborhood search (LVS) and the large variable neighborhood search (LNS) are part of the heuristic. In this paper these searches are left out of the heuristic, since it is an improvement of the heuristic. It could give a wrong indication of the performance of the heuristic. If in Vidović et al. (2014) the heuristic does not succeed in finding a more optimal solution and the combination of LVS and LNS do, it will look like the heuristic overall did perform well.

The local search is divided in three neighborhood structures each trying to improve the routing cost (and therefore the total cost) of deliveries on every day in the planning horizon. To check whether the solution can be improved by the local search on day $\mathbf{t}$ there must be at least 2 routes on day $\mathbf{t}$. The first local search (1) interchanges a single station between two routes on the same day. The second local search (2) removes a single station from one route and inserts it into another route on the same day. The third local search (3) removes two arcs, a connection between either two stations or a station and a depot, and tries to find the best possible reconnection. There will only be one arc removed per route. For all local search structures it must hold that the solution is feasible. So we verify for each day in the planning horizon if the inventory levels are still above the daily fuel consumption $\mathbf{q}_{ij}$ and below the maximum reservoir capacity $\mathbf{Q}_{ij}$.Next, one station cannot have more than $\mathbf{K}$ compartments on a single day and on a single route a maximum of $\mathbf{K}$ compartments can be delivered.

There are two ways to implement the local variable neighborhood search (LVS), the first implementation is the same as Vidović et al. (2014) and is given in table 3. (in the remainder of this paper, referred to as LVS). LVS tries to find a more optimal solution with local search structure (1). If there is no improvement, structure (2) is used and if that also does not lead to an improvement structure (3) is used.

Another way is a procedure we came up ourselves and will be referred to as local variable extension neighborhood search (LVE). This procedure differ from LVS since we try to improve the solution with all three local searches and do not stop the improvement of local search (3) if we found an improvement of local search (1). The pseudo-code for the three structures are given in tables 4, 5 and 6.

```
Input: Heuristic solution
while improvement
  improvement = False
  Procedure: Stations interchange of a single station between two routes on the same day
  for each pair of routes in all routes of day t
    for all feasible swaps of a pair of stations from two routes
      if the interchange gives shorter routes and therefore lower routing costs
        improvement = True
        update Heuristic solution
        break while loop
  Procedure removal/insertion of one route
    for route_from in all routes of day t
      for all stations in route_from
        for route_to in all route
          remove station from route_from and insert into route_to
          if solution is feasible and gives shorter routes (lower routing costs)
            improvement = True
            update Heuristic solution
            break while loop
  Procedure: removal of two arcs
  for route_1 in all routes of day t
    for route_2 in all routes of day t
      for arc_1 in route_1
        for arc_2 in route_2
          delete arc_1 and delete arc_2
          make all feasible reconnections possible
          if there exists a reconnection with shorter routes (lower routing cost)
            improvement = True
            update Heuristic solution
            break while loop
```

Table 3: Pseudo code LVS, which is the same as Vidović et al. (2014)

```
Input: Heuristic Solution
for day t in planning horizon T:
  if number of routes > 2
    while improvement = false
      for each pair of routes (i,j) on day t
        for each station x on route i
          for each station y on route j
            if swap (x,y) is feasible
              calculate the new routing cost of routes (i,j)
              if interchange incurs lower routing cost
                Heuristic solution = heuristic solution with stations interchange
                improvement = true
                break the algorithm
```

Table 4: Pseudo code local search structure 1: Stations interchange

```
Input: Heuristic Solution
for day t in planning horizon T:
  if number of routes > 2
    while improvement = false
      for each pair of routes (route_from,route_to) on day t
        for each station x on route_from
          for each station y on route_to
            remove station x from route_from and insert in route_to
              if solution is feasible
                calculate the new routing cost of routes (i,j)
                if removal/insertion incurs lower routing cost
                  Heuristic solution = heuristic solution with removal/insertion
                  improvement = true
                  break the algorithm
```

Table 5: Pseudo code local search structure 2: Removal/insertion

```
Input: Heuristic Solution
for day t in planning horizon T:
  if number of routes > 2
    while improvement = false
      for each pair of routes (i,j) on day t
        for each station x on route i
          for each station y on route j
            for each arc_1 in route i
              for each arc_2 in route j
                delete arc_1 and arc_2
                make all possible reconnections
                if there exists reconnection
                  calculate the new routing cost of routes (i,j)
                  if removal of two arcs incurs lower routing cost
                    Heuristic solution = heuristic solution with removal of two arcs
                    improvement = true
                    break the algorithm
```

Table 6: Pseudo code local search structure 3: Removal of two arcs

3.3.2 Large neighborhood search

The large neighborhood search (LNS) is the final step in obtaining the best possible solution of the problem. For this search **all** delivery quantities of a single stations are removed from one day and are moved to another day in the planning horizon. If a shorter route exist and therefore a lower total cost is obtained, we do the same feasibility verification check as done in the local search procedure. If the solution is feasible, the move is realized. This procedure continues with searching until no improvement can be found. The LNS is the same as given in Vidović et al. (2014) and is shown in table 7.

```
Input: Heuristic Solution
while improvement:
  improvement = False
  Procedure: removal/insertion of stations
  for day_from in planning horizon T
    for day_to in planning horizon T
      for station in day_from
        remove station from day_from
        insert station in day_to
        if solution is feasible with lower total cost
          improvement = True
          update Heuristic solution
          Go to line 1
```

Table 7: Pseudo code for the LNS

4 Results

Before we get to the results, we first have to define a couple of things. All of the following characteristics are equal to those described in Vidović et al. (2014):

- Number of fuel stations **I** equals 10 in the small scale problem and 50 in the large scale.
- 3 different fuel types **J**=3
- All vehicles tow a trailer, so there are four compartments **K = 4**. Each compartment has 8 tons of capacity.
- The fuel stock level $\mathbf{S}_{ij}^{0}$ at the beginning of the planning horizon is randomly, based on the uniform distribution, generated between 2 and 10 ton.
- The daily fuel consumption $\mathbf{q}_{ij}$ are generated using a discrete probability density function. With a probability of $\mathbf{p}_1 = 0.4$ it takes the value of 1 ton, with $\mathbf{p}_2 = 0.5$ 2 tons and 3 tons with a probability of $\mathbf{p}_3 = 0.1$.
- Reservoir capacities $\mathbf{Q}_{ij}$ are equal to the size of 20 tons or 30 tons, which are randomly, based on the uniform distribution, assigned to fuel stations for all types.
- The distance coordinates of the stations are randomly, based on the uniform distribution, generated. Both **x** and **y** coordinates can take any value in the interval [-50 50]km. We assume the depot to be located at (0,0)km.
- The day cost of carrying inventory $\mathbf{c}_{inv} = 1.09$ €/t.
- The cost per traveled kilometer is $\mathbf{c}_r = 2$ €/km.
- The fixed cost of fleet size is $\mathbf{c}_v =$ €200 per vehicle per day for the available vehicles.
- The fixed cost of fleet size is $\mathbf{c}_m =$ €1000 per vehicle per day for the dummy vehicles.

At last we define the planning horizon length **T** equal to 5. In Vidović et al. (2014) they make use of different planning horizon lengths, but we try to be as consistent as possible.

The first step in our procedure is to solve the relaxed IRP MIP model. The obtained solution has to give a minimum inventory cost. Furthermore, since we did not take the routing cost into account, it seems logical that the routing cost can be reduced. As mentioned before we use a couple parameters as input with a given value, these values need to be declared in advance. The values for one instance is given in table 8.

I	$\mathbf{q}_{i1}$	$\mathbf{q}_{i2}$	$\mathbf{q}_{i3}$	$\mathbf{S}^0_{i1}$	$\mathbf{S}^0_{i2}$	$\mathbf{S}^0_{i3}$	$\mathbf{Q}_{i1}$	$\mathbf{Q}_{i2}$	$\mathbf{Q}_{i2}$	$\mathbf{x}_i$	$\mathbf{y}_i$
1	1	2	1	4	7	2	30	20	20	-47	-32
2	1	2	1	10	4	4	30	30	30	25	-26
3	2	2	2	5	7	9	30	30	20	0	39
4	1	2	2	3	8	2	20	30	30	-2	-48
5	3	2	2	10	3	10	20	20	30	41	-1
6	1	1	2	10	3	8	30	20	30	11	-34
7	1	1	1	5	4	6	30	20	30	12	48
8	1	2	2	3	4	7	20	30	20	36	21
9	1	1	2	4	5	4	20	20	30	31	0
10	1	2	2	5	6	6	30	30	20	8	-3

Table 8: Parameters used as input for one instance of problem ($\mathbf{q}_{ij}$ [t], $\mathbf{S}^0_{ij}$ [t], $\mathbf{Q}_{ij}$ [t], $(\mathbf{x},\mathbf{y})$ [km])

4.1 Standard Case

In Vidović et al. (2014) is described that to speed up the process of finding a better solution, only the **E** most Eligible candidates are used for further benefit calculation. Since they set **E**=3 in the case of 10 different stations, we do the same in our standard case. We compare the obtained costs with the solution when running the heuristic. Afterwards, we also calculate the heuristic solution combined with the local search presented in Vidović et al. (2014) (LSV) or the local search extension (LSE). Finally, we calculate the heuristic solution combined with one of the two different local search procedures combined with a large neighborhood search. For obtaining a good average result, we did the heuristic for 50 different input combinations. The results are given in table 9.

Notice: the original case is the relaxed IRP MIP model without making use of the heuristic. H LSV is the heuristic solution combined with the LSV and H LSE LNS is the heuristic solution combined with the LSE and the LNS.

Standard case	Total	Costs			Routing	Cost		Inventory	Cost			
E = 3	avg	min	max	stdev	avg	min	max	stdev	avg	min	max	stdev
original	3764	2671	5709	590	2778	1728	4700	575	987	889	1070	39.4
Heuristic	2932	2071	3703	306	1849	1012	2661	294	1082	950	1245	62.7
H LSV	2877	1999	3537	304	1795	940	2495	290	1082	950	1245	62.7
H LSE	2829	1907	3553	302	1747	848	2511	287	1082	950	1245	62.7
H LSV LNS	2753	2046	3292	259	1592	917	2132	239	1161	979	1331	81.1
H LSE LNS	2761	1907	3269	272	1644	847.8	2201	259	1117	957	1280	74.0

Table 9: results based on the characteristics of Vidović et al. (2014)

The LSV gives a higher average cost than the LSE. To calculate whether these costs differ significantly we make us of the so called Welch-Satterthwaite t-test as given in Moser and Stevens (1992). This t-test is known as the so called unequal variance t-test. All the information we need is already given in table 9. The t-statistic is calculated as:

$$t = \frac{\mu_1 - \mu_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \tag{14}$$

And the calculation of the corresponding degrees of freedom (v) is given by:

$$v = \frac{(\frac{1}{n_1} + \frac{u}{n_2})^2}{\frac{1}{n_1^2(n_1-1)} + \frac{u^2}{n_2^2(n_2-1)}}, where \qquad u = \frac{s_2^2}{s_1^2} \tag{15}$$

If we calculate (t,v) for the LSV compared to the LSE we conclude that these values do not differ significantly on a 95% confidence interval, since (t,v) equals (0.792 ,97.5). So the LSE does not perform significantly better. But if we look in figure 2 we see that the LSE is never worse than the LSV. With this in mind and knowing that the corresponding calculation times are very close to each other, namely 2.3 seconds for LSE and 1.9 seconds for LSV, we have reason enough to leave the LSV behind and continue the paper with only LSE. One remark however, is that the heuristic combined with the LSV and LNS gives a slightly lower cost. The gap is not as large as with the comparison just described and far from significant ((t,v) = (-0.151, 97.3)). An explanation could be that the LSV and the LSE give different

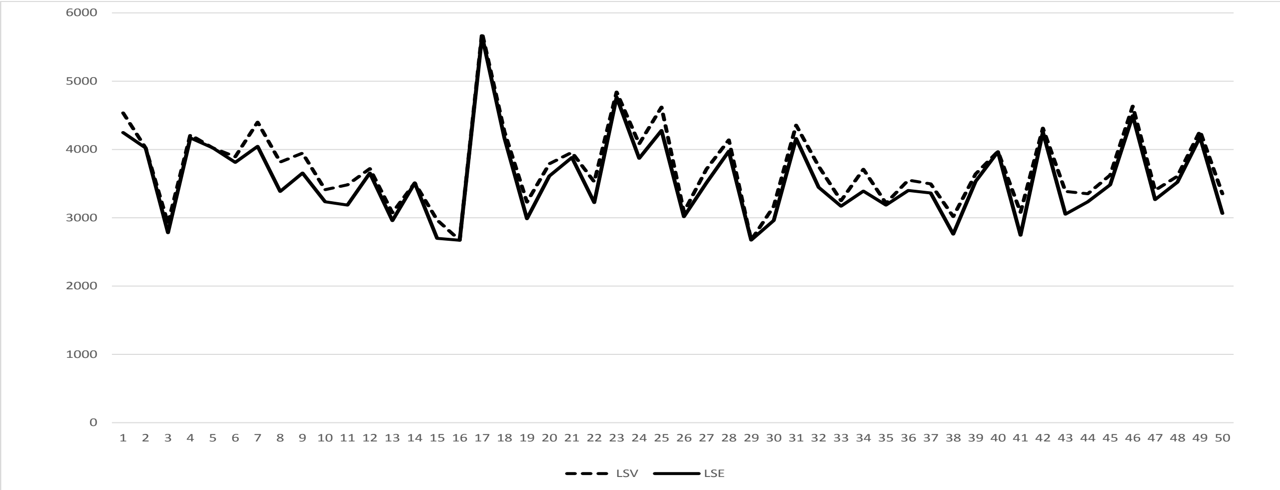

Figure 2: The solutions of the heuristic in combination with the LSV or LSE for 50 different instances

solutions and therefore different set of routes for all days to deliver compartments. Therefore it could be the case that a removal and insertion is feasible in one large search and in the other it is not. This could give a slightly lower total cost in the end. So, it has more to do with how well the LNS performs instead of the performance of LSV versus LSE.

4.2 Is E = 3 enough/too much?

The next question that arises is whether the use of the 3 most eligible ($\mathbf{E} = 3$) in Vidović et al. (2014) is correct, since it is not based on any scientific result. To verify if $\mathbf{E} = 3$ is not enough, enough or too much, we run the same 50 input combinations putting $\mathbf{E}$ equal to 1, 3 and higher until we see no more improvement or a change. The results of the heuristic with different values of eligibility are shown in table 10 and the results of the heuristic combined with the LSE and LNS is given in table 11.

Our primary focus lies on the solutions of the heuristic, which are given in table 10. We immediately see that there is a reasonable decline in total cost if we set $\mathbf{E} = 3$ instead of 1, in fact the difference is significant ((t,v) = (2.115,83.6)), so $\mathbf{E} = 1$ is rejected. If we continue we see that after $\mathbf{E} = 6$ the average total cost increases. This is a result, which we did not expect to see. A reason behind this could be that at some point the seventh most eligible transfer give a lower total cost in the heuristic, but because of this transfer change another, more beneficial solution, would have given an even lower total cost. Due to the fact that the seventh most eligible transfer is already changed, the other transfer change is not longer possible. This could happen when δ equals 1 and the seventh most eligible transfer happens to a station i on day 3. After the change the fuel delivery to station i will now be done on day 2. If we continue our heuristic we set δ equal to 2, there could be no more benefit calculation on day 2, but if $\mathbf{E}$ was not equal to 7 there was a transfer possible from station i, which would lead to an even lower total cost.

Since the best possible solution is when the $\mathbf{E} = 6$ and since there is just a small average calculation time difference between the solutions with $\mathbf{E} = 3$ and $\mathbf{E} = 6$, we decide to continue to work with a $\mathbf{E}$ equal to 6.

Heuristic	Total	Cost			Routing	Cost			Inventory	Cost		
	avg	min	max	stdev	avg	min	max	stdev	avg	min	max	stdev
E = 1	3100	2039	4848	471	2040	980	3778	472	1060	953	1186	53
E = 3	2932	2071	3703	306	1849	1012	2661	294	1082	951	1245	63
E = 4	2894	2071	3551	306	1809	1012	2457	287	1085	966	1245	60
E = 5	2889	2070	3551	315	1805	1012	2457	293	1084	966	1219	55
E = 6	**2883**	**2071**	**3551**	**314**	**1799**	**1012**	**2457**	**291**	**1084**	**966**	**1236**	**56**
E = 7	2889	2071	3438	309	1805	1012	2396	285	1084	966	1236	57
E = inf	2886	2071	3551	322	1804	1012	2457	304	1082	966	1227	51

Table 10: Results of the heuristic for 50 different input combinations and different values of **E**

Heuristic + LSE + LNS	Total	Cost			Routing	Cost			Inventory	Cost			avg. calc.
	avg	min	max	stdev	avg	min	max	stdev	avg	min	max	stdev	time (s)
E = 1	2828	2008	3628	362	1715	949	2608	369	1113	953	1340	93	2.7
E = 3	2761	1907	3269	272	1644	848	2201	259	1117	957	1280	74	2.3
E = 4	2765	196	3466	272	1608	848	2354	259	1136	966	1306	86	2.3
E = 5	2748	1907	3345	298	1637	848	2251	267	1112	966	1276	72	3.4
E = 6	**2740**	**1907**	**3345**	**291**	**1633**	**848**	**2251**	**261**	**1107**	**966**	**1276**	**73**	**3.6**
E = 7	2776	1976	3344	286	1633	848	2127	259	1129	966	1280	80	4.0
E = inf	2776	1907	3394	305	1676	848	2347	287	1100	966	1288	67	7.8

Table 11: Results of the heuristic combined with LSE and LSV with 50 different input combinations and different values of **E**.

4.3 Evaluation of different heuristic procedures

Our problem consist of a couple of pieces. We have our relaxed MIP model, the heuristic, the local searches (both LSV and LSE) and the large neighborhood search. As a result, we can make different kind of heuristic combinations and compare there solutions. All the different combinations possible are given in table 12. The worst possible solution is the 'original solution' from the relaxed IRP MIP model. If we immediately construct the routes for this delivery plan, the routing cost will be far from optimal. As explained before, this is because it does only take the inventory costs into account. The best possible solution is the heuristic with **E** = 6 including the LSE and LNS. We define this solution as the 'lower bound'. The heuristic algorithm without further improvement methods is defined as 'H'. The 'standard case' is based on Vidović et al. (2014), where **E** = 3, but with the LSE instead of the LSV.

To get an idea of the performance of the different procedures, we calculated the percentual difference between the heuristic combination and the lower bound. This is given in table 12 as *HS-LB(%)*.

Model	Total cost			RC		IC		avg. Calc.
	avg	stdev	HS-LB (%)	avg	stdev	avg	stdev	time (s)
Upper bound	3764	590	37.17%	2778	575	987	39	0.4
Heuristic (H)	2883	314	5.20%	1799	291	1084	56	2.0
LSV	3728	594	35.80%	2741	578	987	39	6.6
LSE	3566	607	29.88%	2580	592	987	39	6.9
LNS	2941	354	7.32%	1738	336	1203	89	1.7
H + LS	2778	310	1.32%	1694	285	1084	56	3.2
H + LNS	2770	276	1.17%	1613	256	1156	82	2.7
LSE + LNS	3060	379	11.76%	1929	392	1131	127	7.3
Standard case	2761	272	0.87%	1644	259	1117	74	2.3
Lower bound	2741	291	0.00%	1633	261	1107	73	3.2

Table 12: Heuristic solutions that were obtained by different improvement procedures for 50 instances

As we can see in table 12 the upper bound and the lower bound differ quite allot. The local searches without the heuristic performs not so much better than the upper bound and are quite time consuming compared to the rest of the combinations. The heuristic on its own performs pretty well since it differs 5.2% from the best possible solution. Also, the LNS procedure comes quite close (7.32%) to the lower bound without any previous calculation of the heuristic but if we combine the LSE with the LNS it does not come as close as the LNS procedure on its own. There is not any real explanation for this and the result differs from Vidović et al. (2014). But what we do see that the LNS performs very well compared to the heuristic on its own and the calculation time is not significantly higher, the calculation is even lower. Another fact is that the LNS in Vidović et al. (2014) is even lower compared to the heuristic. So we do question the quality of the heuristic procedure. Therefore, we plotted the upper bound (UB), lower bound (LB) and large neighborhood search (LNS) for all 50 instances. The results are shown in figure 3.

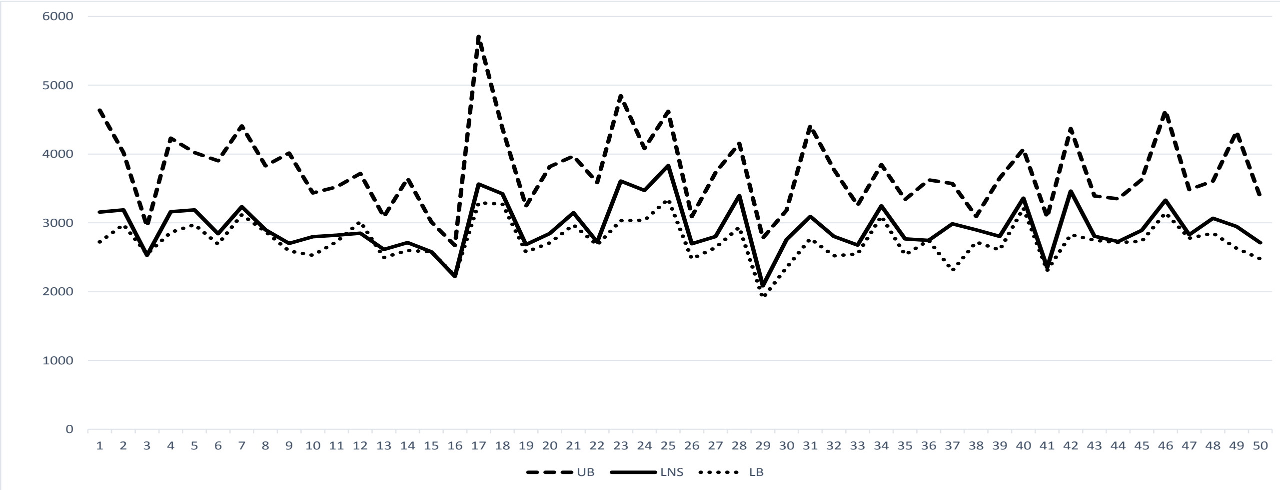

Figure 3: UB, LB and LNS solutions of 50 different instances

What we do see is that the LNS most of the time comes very close. It has just a couple of outliers. This allows us to question the quality of the heuristic even more.

4.4 Large scale problem

In Vidović et al. (2014) they run the heuristic on two kinds of problems, a small-scale and a large-scale problem. The small-scale problem has 10 different fuel stations and the large-scale problem has 50. In the limited time for this research a large scale problem was not possible to do. But we do evaluate the decision of Vidović et al. (2014) to eliminate the delivery of four stations per single route ($\mathbf{y}_{pqwe}$). Before the elimination was possible they evaluated the calculation time and the quality of the IRP and IRPF models with different maximum allowed numbers of stations per single routes. They define a quadruple, triple and double assignment of stations per route. Which means that in case of a triple assignment at most three stations can be visited per route. The maximum number of compartments delivered per route however stay the same because the maximum capacity of the vehicles stay the same. The results of the different assignments are given in table 13. The same procedure is used as in Vidović et al. (2014), namely the heuristic combined with the searches is used. The only difference is that we used LSE instead of LSV. The results match with the results and conclusion of Vidović et al. (2014). The triple assignment gives solutions that are very much negligible different, whereas the computational time is significantly lower than for the quadruple assignment. Therefore, the model can be restrained to the triple assignment, which enables the testing on a large-scale problem. The solutions of the IRPF MIP model on a 5-day horizon are compared to the solutions of the IRPF model in table 3 of Vidović et al. (2014) quite higher. We will discuss the quality of these solutions later on.

	IRP			IRPF		
	Quad. assign.	triple assign.	doub. assign.	Quad. assign.	triple assign.	doub. assign.
avgIC	1107.36	1109.98	1161.25	1119.92	1107.01	1196.66
avgRC	1633.29	1627.44	1557.10	2274.77	2367.62	1370.45
avgFC	-	-	-	2540.00	2460.00	2980.00
avg total	**2740.65**	**2737.42**	**2718.35**	**5934.69**	**5934.63**	**5547.11**
avg. Calc. time (s)	3.65	1.73	0.83	12.14	5.55	1.73

Table 13: A comparison of the results for the quadruple, triple and double assignment of the MIP IRP and IRPF models for 50 instances with 10 petrol stations and a 5-day planning horizon.

However, in our opinion the quality of the heuristic is based on the heuristic without the LSE/LSV and the LSN. So to really compare the results for the quadruple, triple and double assignment of the MIP IRP and IRPF models, the LSE and LSN are left out. The results are shown in table 14 and are different. For the IRP MIP model the quadruple and triple assignment still are close, but less closer than before but most significant difference is at the IRPF MIP model. For the IRPF model the triple and double assignment are now negligible close to each other. This was not a problem if the quadruple assignment where close to them. However, the quadruple assignment of the IRPF model is not really close to the other assignments. This questions the correct rejection of the quadruple assignment for the large-scale problem.

	IRP			IRPF		
	Quad. assign.	triple assign.	doub. assign.	Quad. assign.	triple assign.	doub. assign.
avgIC	1084.17	1079.29	1101.78	1067.60	1067.43	1109.46
avgRC	1693.71	1709.74	1710.12	2569.41	2455.88	2233.53
avgFC	-	-	-	2580.00	2540.00	2720.00
avg total	**2777.88**	**2789.02**	**2811.90**	**6217.01**	**6063.30**	**6062.99**
avg. Calc. time (s)	3.24	1.36	0.73	10.78	3.95	1.41

Table 14: A comparison without **LSE and LSN** of the results for the quadruple, triple and double assignment of the MIP IRP and IRPF models for 50 instances with 10 petrol stations and a 5-day planning horizon.

4.5 Demand extension

In reality the daily fuel consumption $\mathbf{q}_{ij}$ fluctuates far more than the discrete probability function as described in the computational results of Vidović et al. (2014). To make it more realistic we focus on the fact that it is possible that on a day $\mathbf{t}$ on the planning horizon a large fuel consumption arises. A reason could be that it is the start of a holiday season for the whole country and the station is close to the country border. Another possibility could be a very large event, like the world championship football, taking place close to the fuel station. To include this we redefine the daily fuel consumption $\mathbf{q}_{ij}$ as a discrete probability density function, taking the values of 1 ton with a probability of $\mathbf{p}_1 \approx 0.3967$, 2 tons with a probability of $\mathbf{p}_2 \approx 0.4967$, 3 tons with a probability of $\mathbf{p}_3 \approx 0.967$ and 6 tons with a probability of $\mathbf{p}_4 = 0.01$.

4.6 Most optimal cases

Now that we have evaluated the different kind of local searches and use of different eligibility transfers, we finally can calculate our most optimal cases. We distinguish the solutions with the demand extension and without the demand extension. We set $\mathbf{E}$ equal to 6 and we use the LSE to get our best results. The results are shown in table 15.

Demand	MIP	TC				RC				IC				FC	Avg. calc.
		avg	min	max	stdev	avg	min	max	stdev	avg	min	max	stdev	avg	time (s)
No	IRP	2741	1907	3345	291	1633	848	2251	261	1107	966	1276	73	-	3.65
	IRPF	5935	4397	11779	1212	2275	891	8258	1222	1120	966	1305	81	2540	12.14
Yes	IRP	2877	2109	4456	540	1772	1257	3096	438	1105	791	1482	152	-	1.42
	IRPF	6304	4158	9277	1300	2568	707	5348	1246	1076	791	1423	159	2660	8.61

Table 15: most optimal heurtic results for 50 instances with 50 fuel stations and a 5-day planning horizon

To evaluate the quality of the solution with respect to the routing and inventory segment as already described in Campbell et al. (2001) and in Vidović et al. (2014) some metrics are used. We used the same metrics as described in Vidović et al. (2014) but we made some extra metrics to further evaluate the quality of the solution. $\mathbf{aR}_{MAX}$- the average of the maximum number of routes from the set of all of the days in the planning horizon, which is in case of the IRPF model equal to the fleet size. **aR**- the average total vehicle stops. **aD**- the average total travel distance in km. **adD**- the average distance from the depot to the station or vice versa. **adD** is an important addition to the metrics of Vidović et al. (2014) to evaluate the solution quality because if this value is very high the location of the depot is wrong or could be better. **asD**- the average distance from a station to the next station. This is also an addition to the already existing metrics. $\mathbf{aQ}_{PER}\mathbf{D}$- the average quantity of fuel per traveled distance (t/km). $\mathbf{aI}_{BEF}JI$- the average of the inventory level per station per fuel type. In this paper the $\mathbf{aI}_{BEF}$ is given for all the types, for a better insight in the solutions. The solutions are given in table 16 and in case of the IRP model match quite good with the solutions in table 5 of Vidović et al. (2014). A suggestion why the solutions of the IRPF model are much higher compared to the solutions in Vidović et al. (2014) might be that the average total travel distance **aD** is way higher. The fleet size however, do match pretty well. The average distance between the depot and the first/last station on the route seems to be quite large compared to the average distance between two stations on the route. This suggest that it could be profitable to look at a multi depot problem or at another location of the depot.

Demand	MIP									J1		J2		J3	
		aRmax	aR	aS	aD	aDperS	adD	asD	aQperD	aIbef	aIaft	aIbef	aIaft	aIbef	aIaft
No	IRP	2.94	8.30	16.18	842.65	52.39	37.99	17.41	0.26	0.27	1.56	0.34	1.31	0.23	1.62
	IRPF	2.52	7.66	16.66	2266.20	137.54	39.26	20.02	0.10	0.37	1.37	0.45	1.43	0.32	1.44
Yes	IRP	2.84	8.56	16.64	919.02	55.01	39.04	18.93	0.25	0.65	1.82	0.70	1.90	0.51	1.72
	IRPF	2.66	8.44	17.98	2511.54	140.28	39.95	22.25	0.09	0.68	1.46	0.75	1.76	0.51	1.45

Table 16: comparison of the metrics for the MIP models

5 Conclusions

This paper follows the same procedure as Vidović et al. (2014). We present the solutions of two MIP models, namely the IRP and IRPF, for the multi-product multi-period IRP in fuel delivery. We also present the heuristics approach for the IRP and IRPF. The MIP models are formulated as the assignment of fuel stations to individual routes. This formulation also takes the inventory costs into consideration. We begin the heuristic by relaxing the routing costs of the MIP model to obtain an initial solution with minimum inventory costs. This is done since it is too hard to minimize the inventory and routing costs at the same time. As a result of minimum inventory costs, the delivery of fuel compartments is done at the latest possible time period and with the lowest quantity. In obtaining a more optimal routing cost, the heuristic tries to transfer deliveries over one or more time periods earlier. This incurs an increase in the inventory costs but a (larger) decrease in routing costs. To optimize the solution even further, local and large neighborhood searches are used.

An interesting conclusion can be drawn from the comparison of the local searches in the standard case. Our own local search seems to outperform the local search of Vidović et al. (2014). Next, the use of a maximum of 3 most eligible transfers is questioned. We conclude that it is not based on any scientific result and find that the use of a maximum of 6 most eligible transfers incurs even lower total costs. Since our heuristic procedure consists of a variety of parts, we evaluate different heuristic procedure combinations. The main conclusion is that the LNS procedure performs quite similar to the main heuristic procedure. Since the LNS in Vidović et al. (2014) even outperforms the main heuristic procedure and the computation time is not much larger, sometimes even lower, we question the quality of the heuristic procedure. In Vidović et al. (2014) they also calculate a large-scale problem with 50 different fuel stations. Because of a large calculation time for a problem of this size, they first compare the assignment of a different maximum number of stations per route. The conclusion of removing the

quadruple assignment is questioned in this paper. If the heuristic is combined with the local and large neighborhood search, we come to the same conclusion as Vidović et al. (2014). However, if we leave out the local and large neighborhood search, since they try to improve the heuristic and are not part of it, we can not conclude anything anymore. This allows us to question the paper by Vidović et al. (2014) even more. Finally we make the discrete probability density function a little more realistic, by introducing a large demand. With all the previous results we calculate the most optimal cases, using $\mathbf{E} = 6$ and the LSE. To evaluate the quality of the solution with respect to the routing and inventory segment, the same metrics of Vidović et al. (2014) are introduced. We even introduce some of our own metrics.
Further research can be concerned with transforming the homogeneous fleet into a heterogeneous fleet, since the problem of Vidović et al. (2014) is concerned with vehicles with two compartment who can tow a trailer of another two compartments. A distinction needs to be made between those two. To make the daily consumption of fuel function more realistic a stochastic model needs to be used instead of a discrete. At last, it could be profitable to compare the results of a single depot problem to a multi depot problem, since the distance between a station and a depot seems to be quite large.

6 Bibliography

M.O. Ball. Allocation/routing: models and algorithms. *Vehicle routing:methods and studies*, pages 194–221, 1998.

Jonathan F Bard, Liu Huang, Patrick Jaillet, and Moshe Dror. A decomposition approach to the inventory routing problem with satellite facilities. *Transportation science*, 32(2):189–203, 1998.

Walter J Bell, Louis M Dalberto, Marshall L Fisher, Arnold J Greenfield, Ramchandran Jaikumar, Pradeep Kedia, Robert G Mack, and Paul J Prutzman. Improving the distribution of industrial gases with an on-line computerized routing and scheduling optimizer. *Interfaces*, 13(6):4–23, 1983.

Luca Bertazzi, Adamo Bosco, Francesca Guerriero, and Demetrio Lagana. A stochastic inventory routing problem with stock-out. *Transportation Research Part C: Emerging Technologies*, 27:89–107, 2013.

Ann M. Campbell, Lloyd W. Clarke, and Martin W. P. Savelsbergh. The vehicle routing problem. pages 309–330, 2001.

Nicos Christofides. The vehicle routing problem. *Revue française d'automatique, d'informatique et de recherche opérationnelle. Recherche opérationnelle*, 10(1):55–70, 1976.

G u Clarke and John W Wright. Scheduling of vehicles from a central depot to a number of delivery points. *Operations research*, 12(4):568–581, 1964.

Fabien Cornillier, Fayez Boctor, and Jacques Renaud. Heuristics for the multi-depot petrol station replenishment problem with time windows. *European Journal of Operational Research*, 220(2):361–369, 2012.

Moshe Dror and Pierre Trudeau. Savings by split delivery routing. *Transportation Science*, pages 141–145, 1989.

Awi Federgruen and Paul Zipkin. A combined vehicle routing and inventory allocation problem. *Operations Research*, 32(5):1019–1037, 1984.

Vishal Gaur and Marshall L Fisher. A periodic inventory routing problem at a supermarket chain. *Operations Research*, 52(6):813–822, 2004.

Michel Gendreau, Alain Hertz, and Gilbert Laporte. A tabu search heuristic for the vehicle routing problem. *Management science*, 40(10):1276–1290, 1994.

Roar Grønhaug, Marielle Christiansen, Guy Desaulniers, and Jacques Desrosiers. A branch-and-price method for a liquefied natural gas inventory routing problem. *Transportation Science*, 44(3):400–415, 2010.

Yale T Herer and Roberto Levy. The metered inventory routing problem, an integrative heuristic algorithm. *International Journal of Production Economics*, 51(1):69–81, 1997.

Y.T. Herer. A temporal approach to the inventory routing problem. *Working Paper*, 1996.

Shan-Huen Huang and Pei-Chun Lin. A modified ant colony optimization algorithm for multi-item inventory routing problems with demand uncertainty. *Transportation Research Part E: Logistics and Transportation Review*, 46(5):598–611, 2010.

Anton J Kleywegt, Vijay S Nori, and Martin WP Savelsbergh. Dynamic programming approximations for a stochastic inventory routing problem. *Transportation Science*, 38(1):42–70, 2004.

Noor Hasnah Moin, Said Salhi, and NAB Aziz. An efficient hybrid genetic algorithm for the multi-product multi-period inventory routing problem. *International Journal of Production Economics*, 133 (1):334–343, 2011.

Barry K Moser and Gary R Stevens. Homogeneity of variance in the two-sample means test. *The American Statistician*, 46(1):19–21, 1992.

Paolo Toth and Daniele Vigo. *The vehicle routing problem.* Society for Industrial and Applied Mathematics, 2001.

Milorad Vidović, Dražen Popović, and Branislava Ratković. Mixed integer and heuristics model for the inventory routing problem in fuel delivery. *International Journal of Production Economics*, 147: 593–604, 2014.

Ian R Webb and Richard C Larson. Period and phase of customer replenishment: A new approach to the strategic inventory/routing problem. *European Journal of Operational Research*, 85(1):132–148, 1995.

Yugang Yu, Haoxun Chen, and Feng Chu. A new model and hybrid approach for large scale inventory routing problems. *European Journal of Operational Research*, 189(3):1022–1040, 2008.