The effects of exclusive contracts on competition in Cournot markets

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Abstract

This thesis deals with the viability and effects of exclusive contracts when Cournot competition is introduced, as opposed to the Bertrand competition that is assumed in the existing literature. An exclusive contract involves a buyer agreeing to buy a certain part or all of his input from a seller. If he breaches this contract, he has to pay damages. It is found that if a monopolist or multiple sellers in a Cournot monopsony close a (partially) exclusive contract with the buyer, it depends on the method of calculation of expectation damages whether entry by homogenous competitors will be blocked or not. In a Bertrand upstream monopoly with multiple Cournot retailers and a simultaneous contracting phase, the options to deter entry from a more efficient competitor are limited. Whether entry is deterred at best depends on the perception of each buyer about his competitors. However, entry deterrence is still possible under certain circumstances. Finally, when two upstream Bertrand firms sell to Cournot retailers and contracting happens sequentially, they can use exclusive contracts to co-exist and both make a profit, despite behaving competitively.
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1 Introduction

The question if incumbent firms can use so-called exclusive contracts to inefficiently deter entry has been a topic of debate among economists, lawyers and policy makers for quite some time. An unequivocal answer, or even a majority opinion, has not been found yet (Simpson & Wickelgren 2007). The issue continues to be relevant in a time in which many industries are subject to a supply oligopoly, especially in manufacturing. An exclusive contract can impose limitations on either the manufacturer or the buyer, or both of them. In this thesis, an exclusive contract is defined as vertical agreement where a buyer agrees to buy all or a certain part of his purchases from a certain seller. An current example where this kind of contract plays a central role is the billion euro fine that the European Commission imposed on processor manufacturer Intel for (allegedly) abusing its dominant position in the market for mainstream CPUs by paying clients to restrict their usage of the CPUs of Intel’s competitor AMD. The European General Court upheld the decision, which is now under review by the Court of Justice.\(^1\) Another example, which provided inspiration for this thesis, is the Dutch hotel and catering industry. The vast majority of cafés and restaurants has an exclusive agreement with one brewing company.

Exclusive contracts are not bad per se. The most prevalent argument was made by Scherer (1980), who argued that "for manufacturers, exclusive dealing arrangements are often appealing, because they ensure that their products will be merchandised with maximum energy and enthusiasm." Also, if a buyer signs an exclusive contract, the supplier gets some certainty about future revenues. The seller may therefore give him easier access to credit facilities, or invest in smoother logistics, because he knows he can earn his investment back.\(^2\) However, if an exclusive contract impedes or even prevents competition, it may lead to an inefficient outcome. An anticompetitive motivation for exclusive contracts could be to make entry of competing firms more difficult or impossible altogether (e.g. Preston 1965). It is therefore difficult to decide if this kind of contract should be legal or not.

This issue has been analyzed in several papers, with various conclusions. It has been argued that an incumbent monopolist could never inefficiently limit competition with an exclusive contract, because he could never compensate the buyer for the deadweight loss incurred by his monopoly (e.g. Posner 1976 and Bork 1978). Fumagalli and Motta (2006) find that if buyers are not final consumers but retailers fierce competition among them prevents the incumbent from using exclusivity to deter entry. Simpson and Wickelgren (2007) came to the opposite conclusion, arguing that fierce competition would in fact enable the seller to use exclusive contracts.

A number of important differences between these papers originate from the exact setting of the model and the selection of which factors are taken into account. Some noteworthy distinctions

\(^1\) Intel Corp. v European Commission, Judgment of the General Court of 12 June 2014, ECLI:EU:T:2014:547. The use of exclusive or quasi-exclusive contracts has been found to be incompatible with art. 102 TFEU, unless it is justified by exceptional circumstances, by the Court of Justice in the precedents Hoffmann LaRoche (ECLI:EU:C:1978:108) and Tomra (ECLI:EU:C:2012:221).

\(^2\) Hold-up problems can occur, but that debate is not in the scope of this thesis. See e.g. Hart & Moore (1990). The fact that this problem will not be discussed here does not mean that it is not important to take into account. A hold-up problem can influence the outcome of negotiations about an exclusive contract.
are whether the buyer is the final consumer or a competing retailer, the presence of economies of scale and fixed entry costs, and whether one-part or two-part tariffs are used. A common factor in the models of the aforementioned papers is the starting point that there is a monopolist incumbent who faces Bertrand competition, meaning the price would drop to (or under) the incumbent’s marginal costs if any competitor entered the market. This offers a good basis for analysis of this problem. However, not all markets fit this setting. Certain kinds of exclusive contracts may also be used in markets with multiple incumbents, who do not necessarily set their price equal to marginal costs. Take, for example, the Dutch market for drinks in cafés, that has a supply oligopoly rather than monopoly. The aim of this thesis is to examine the use of exclusive contracts to deter entry and limit competition in markets where there is already some degree of competition, and compare the results to earlier papers that assume a monopolist and Bertrand competition in case of entry.

The implications of introducing quantity competition for the feasibility of exclusive contracts obviously depend on whether the competition on quantity takes place in the upstream market or in the downstream market. This thesis looks at both possibilities. Firstly, it is shown under which circumstances exclusive contracts can be used to limit competition when there is only upstream competition. Secondly, the same is done for the scenario where there is downstream competition and upstream competition is on price. The latter circumstance resembles existing literature on exclusive contracts with downstream competition.

More specifically, the first part of this thesis is the analysis of a model with an indefinite amount of incumbent sellers who compete as Cournot firms, and one final buyer. The most important qualities of this part compared to the existing literature is that it allows for multiple incumbents and analyses how they behave under Cournot rather than Bertrand competition. It examines the dynamic of multiple incumbents tendering a contract at the same time, leading to several partially exclusive contracts.

The second part is the analysis of a model with one or two incumbent seller(s) in a Bertrand upstream market, who sells to an indefinite amount of buyers who are retailers in a Cournot downstream market. The main additions of this part to the existing literature are that it examines Cournot competition among retailers, and that it allows for an arbitrary amount of retailers. The largest part deals with simultaneous contracting, but I also analyse a sequential game. The latter is not as exhaustive as could be, but still is an interesting exploration.
Table 1: Timeline

<table>
<thead>
<tr>
<th>Period 1.1</th>
<th>Period 1.2</th>
<th>Period 2</th>
</tr>
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<tbody>
<tr>
<td>Incumbents offer contracts;</td>
<td>Entrants decide, all firms</td>
<td>Buyer buys input (and decide</td>
</tr>
<tr>
<td>buyer decides</td>
<td>produce simultaneously</td>
<td>on breach)</td>
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Part I

Partially exclusive contracts

2 The buyer as final consumer

2.1 The general model

Consider a market with one active buyer who is the final consumer of the traded goods. His demand is given by \( P = a - Q \). In period 1, he buys from \( n \) sellers, who are homogeneous Cournot competitors with constant marginal costs \( c < a \) and no fixed costs. Each individual firm \( i \in \{1, \ldots, n\} \) will then sell \( q_i = \frac{a-c}{n+1} \) for the equilibrium price \( \frac{a+cn}{n+1} \), earning profit \( \pi_i = \frac{(a-c)^2}{(n+1)^2} \).

After period 1, each firm has to decide on its strategy for the next period. It is assumed from here on that all incumbents remain active in period 2. Now suppose that one extra firm (the entrant), which is identical to the incumbents except for not having produced in period 1, enters the market in period 2. If the incumbents anticipate this, the new Cournot equilibrium price will be \( \frac{a+cn}{n+2} \) and each firm will sell \( q_i = \frac{a-c}{n+2} \), earning \( \pi'_i = \frac{(a-c)^2}{(n+2)^2} \). Define \( \pi_i \) as the profit of an incumbent in period \( t \) and \( CS_t \) as the consumer surplus in \( t \). Compared to period 1, simple calculation gives that each incumbent firm earns \( \pi_1 - \pi'_1 = \frac{(a-c)^2(2n+3)}{(n+1)^2(n+2)^2} \) less and consumer surplus is \( CS_2 - CS_1 = \frac{(a-c)^2(2n^2+4n+1)}{2(n+1)^2(n+2)^2} \) higher. Each incumbent would be willing to pay up to \( \pi_1 - \pi'_1 \) to prevent the entrant from entering the market, meaning that all incumbents together are prepared to pay \( n \) times this amount.

Since there is one buyer and there are multiple sellers, we will consider a kind of contract that the buyer can sign with the incumbents except for not having produced in period 1, enters the market in period 2. If the incumbents anticipate this, the new Cournot equilibrium price will be \( \frac{a+cn}{n+2} \) and each firm will sell \( q_i = \frac{a-c}{n+2} \), earning \( \pi'_i = \frac{(a-c)^2}{(n+2)^2} \). Define \( \pi_i \) as the profit of an incumbent in period \( t \) and \( CS_t \) as the consumer surplus in \( t \). Compared to period 1, simple calculation gives that each incumbent firm earns \( \pi_1 - \pi'_1 = \frac{(a-c)^2(2n+3)}{(n+1)^2(n+2)^2} \) less and consumer surplus is \( CS_2 - CS_1 = \frac{(a-c)^2(2n^2+4n+1)}{2(n+1)^2(n+2)^2} \) higher. Each incumbent would be willing to pay up to \( \pi_1 - \pi'_1 \) to prevent the entrant from entering the market, meaning that all incumbents together are prepared to pay \( n \) times this amount.

Since there is one buyer and there are multiple sellers, we will consider a kind of contract that the buyer can sign with multiple sellers, rather than one exclusive contract. The effect of this contract would be that he agrees to obtain part of his purchases from a certain firm. This kind of contract will be referred to as a partially exclusive contract. Take, for example, a buyer’s commitment to allocate a certain part of his promotional space to the product of a seller, as mentioned by Klein and Murphy (2008). Assuming the amount of promotional space is limited, this would limit the buyer’s possibilities to use the products of other sellers. Usually the seller will have to compensate the buyer for restricting its opportunity set, for example with a lumpsum payment.

Suppose that each incumbent can offer the buyer a partially exclusive agreement in which the buyer commits himself to buy \( \frac{1}{n} \) of his total demand in period 2 from that seller for the old equilibrium price \( p_1 \) in exchange for a signing fee, irrespective of the exact shape and background.
of the contract.\textsuperscript{3} The reason for offering a contract that mandates the buyer to obtain a relative part of his demand from the seller is that it makes competition harder, because a higher demand (as a result of competing offers) will obligate the buyer to buy more from that seller too. If all sellers sign such a contract with the buyer, he is obligated to buy his entire demand from them, thus making entry impossible without breach of contract. The possibility for the buyer to breach (some of the) contracts and pay expectation damages will be examined, but first the basic model without breach will be discussed.

The timeline of the model is presented in table 1. If the buyer signs such a ‘partially exclusive agreement’ with each incumbent, he will be committed to buy his entire demand from the incumbents, \textit{de facto} excluding the entrant.\textsuperscript{4} The buyer will be better off signing all of these contracts than signing none of them if the total compensation he receives from the firms is higher than the would-be increase of his consumer surplus if the entrant had been able to enter market. Define $\Pi_t$ as the combined profits of all $n$ incumbents in period $t$, so that $\Pi_t = \sum_n \pi^i_t$.

\textbf{Lemma 1} Partially exclusive agreements can be profitable for the buyer and the incumbents if it holds that $\Pi_1 - \Pi_2 \geq CS_2 - CS_1$. Then there exists a signing fee $F \in \left[\left((CS_2 - CS_1)/n\right), (\pi^1_1 - \pi^2_2)\right]$ such that each incumbent \{i, ..., n\} can induce the buyer to sign, if the buyer cannot communicate with the entrant before signing.

Calculating the standard Cournot equilibrium price and quantity for both situations and substituting then renders $\frac{(a-c)^2(2n^2 + 3n)}{(n+1)^2(n+2)^2} \geq \frac{(a-c)^2(2n^2 + 4n + 1)}{2(n+1)^2(n+2)^2}$. This holds for all $n \geq 1$, meaning exclusion is possible, despite the deadweight loss this induces. To understand this, it helps to distinguish two effects of entry: a lower market price and changes in production quantities. The incumbents sell less products and receive a lower price, while on the other side, the consumer surplus increases because of the lower price and the increased total production. The effect of the price decrease on its own evens out between the producers and the buyer and has no direct effect on total welfare. The relevant direct variable here is quantity. Because the entrant not only creates new supply but also takes over part of the incumbent’s production, he imposes a negative externality on the incumbents. The size of this externality is larger than the size of the positive externality for the consumer, who benefits from the newly created supply. As long as the incumbent profits that "disappear" in the pockets of the entrant together with the "new" profits he creates cannot be (partly) transferred to the buyer, the incumbents can induce him to block entry. The buyer is better off than in the case of entry without any payments, but worse off than if he had not signed the contracts and received a payment from the entrant.

The threat of entry gives the buyer market power, but the question is on whom he can exercise it. The preceding result relies heavily on the assumption that the entrant is unable to communicate with the buyer before the buyer decides to sign the contract. He can offer him more than the incumbents, and the buyer would never sign if he receives that offer. Note that

\textsuperscript{3}The possibility of tendering a contract that entitles the seller to more than $1/n$ is beyond the scope of this thesis and will not be discussed.

\textsuperscript{4}Assuming breaching the contract is not allowed; the possibility of efficient breach will be examined later.
the entrant will only make an offer if there is a credible threat of entry deterrence, meaning an offer of the incumbents must still be on the table.

The buyer does not necessarily know the entrant is considering entry. The incumbents may choose to pay the buyer for partial exclusivity as a precaution, when there is no concrete threat of entry yet. The assumption about the sequence of communication can also be relaxed in the presence of entry costs such that the entrant cannot offer more compensation than the incumbents. This could be the case in markets with small margings or high investment requirements, for example. If we allow for this possibility, we can stretch lemma 1 so that we can formulate a more general proposition.

**Lemma 2** There exists a level of entry costs for which the incumbents are able to deter entry, even if the buyer can communicate with the entrant before he has to decide whether he signs the partially exclusive agreements.

**Proposition 1** Suppose contracts must be observed. Then there is a range of parameters such that any number of incumbents can induce the buyer to sign an partially exclusive agreement that includes a signing fee $F$. All incumbents offer this contract and the buyer has no reason not to sign all of them, so entry is deterred.

### 2.2 Extension to multiple entrants

The concept of using partially exclusive contracts to deter entry can be applied more generally if we allow for multiple potential entrants. In a case with $k$ potential entrants, the decrease in total incumbent profits after entry is equal to

$$\Pi_1 - \Pi_2 = \frac{kn(2n + 2 + k)(a - c)^2}{(n + 1)^2(n + 1 + k)^2},$$

(1)

and the increase in consumer surplus equals

$$CS_2 - CS_1 = \frac{k(2n^2 + 2nk + 2n + k)(a - c)^2}{2(n + 1)^2(n + 1 + k)^2}.$$  

(2)

Using lemma 1 and equation 1 and 2, we get

$$F \in \left[ \frac{k(2n^2 + 2nk + 2n + k)(a - c)^2}{2n(n + 1)^2(n + 1 + k)^2}, \frac{k(2n + 2 + k)(a - c)^2}{(n + 1)^2(n + 1 + k)^2} \right].$$

(3)

Such an $F$ exists if and only if $2n^2 + 2n \geq k$. This and lemma 2 prove the following proposition.

**Proposition 2** Suppose contracts must be observed. Then there is a range of parameters such that $n$ incumbents in a homogenous Cournot market can deter entry of $k$ identical firms if and only if $2n^2 + 2n \geq k$. They each pay the buyer a signing fee $F$ as shown in equation 3. The buyer will sign this contract with each incumbent.
3 Efficient breach

3.1 General conditions

An important point in the discussion about the anticompetitive potential of exclusive contracts is the possibility that the buyer can choose to breach the contract he has signed and pay expectation damages instead of complying (Fumagalli & Motta 2006; Simpson & Wickelgren 2007). In general, whether the counterparty can sue for compliance rather than damages depends on the law the contract is subject to.\(^5\) As a rule of thumb, if a party fails to meet a contract, he has to compensate his counterparty so that the counterparty is put in the same position where he would have been if the contract had been complied with. Suppose, in our case, that the buyer signs the previously mentioned partially exclusive contract with each incumbent. The incumbents and the entrants have knowledge of each contract and of the quantities and price that can be expected in each relevant scenario. Regarding the latter, we have to distinguish the incumbents’ strategies if they do not expect entry and their strategies when they do. We will hypothesize that they do not expect entry because breach will be deterred, since the possibility of this scenario is the subject of our analysis. If we find such a scenario to exist under the condition that the incumbents expect entry to be deterred, its existence on its own proves the correctness of its underlying assumption.

Define \(q_i\) as the production of each incumbent. Suppose the entrants also have full knowledge of the values of \(n, k\), and \(q_i\), and also know that the buyer can breach his contracts. If they expect this to happen, they will decide to produce, and behave as followers in a Stackelberg game. The incumbents can be seen as the leaders in this game, although they will not be able to choose their normal optimal strategy as leaders, because they assume they will continue to operate in the old Cournot setting. Note that this is beneficial for them, because they are committed to a higher level of production than would be the case in a normal Stackelberg game.

The buyer will breach the contracts and buy from both incumbents and entrants if the amount he has to pay in compensations is less than the increase in his consumer surplus. There can be some debate about reimbursing the signing fee, the result of which will likely depend on the exact terms of the contract and the legal provisions in the relevant jurisdiction. Without specific stipulations, it can be assumed that the the buyer can keep the signing fee as long as he is put in the same monetary position as in the case of compliance.

**Lemma 3** Define \(CS_2 - CS_1 = \sigma\) and damages payable to the incumbents as \(\mu\). Breach of contract will occur if \(\sigma \geq \mu\).

Whether breach will happen or not depends to a large degree on the calculation of the expectation damages. Before discussing that point, a number of general assumptions will first

\(^5\)In the case that is considered here, the buyer cannot comply with the contracts anymore after he has bought from the entrant. Aside from the fact that he breaches at least one contract by not buying his total demand from the incumbents, note that the production phase has ended at that time and the duped incumbents cannot deliver more products than they already produced. The alternative way of complying, reverting the purchases made with the entrants, is impossible too. The purchase agreements between the buyer and the entrants are still valid and will generally not affected by agreements that the entrant was no party of.
be established. Define \( p_2 \) as the resulting equilibrium price in period 2, \( Q_N \) as the total amount produced by the incumbents (\( = \sum_n q_n \)), \( q_k \) as each entrant’s production and \( Q_K \) as the total amount produced by the entrants (\( = \sum_k q_k \)). We know that \( Q_N = n \frac{(a-c)}{(n+1)} \), which renders the optimal production decision for each individual entrant using his Stackelberg reaction function

\[ q_k = \arg \max q_k \pi^k = \frac{a-c}{(k+1)(n+1)}. \]

The equilibrium price can then be calculated as \( p_2 = a - Q_N - Q_K = \frac{(a-c)(k+n+kn)}{(k+1)(n+1)} \). The entrants will each earn

\[ \pi^k = (p_2 - c)q_k = \frac{(a-c)^2 (k+1)^2 (n+1)^2}{2(k+1)^2 (n+1)^2}. \]

It can generally be assumed that the buyer buys \( Q_N \) for the net price \( p_1 \) from the incumbents, regardless whether the incumbents stick to their old price \( p_1 \) and demand that part of the agreement be met, or compete and drop their price to \( p_2 \). In the latter case they would claim the differential \( p_1 - p_2 \) in additional damages, which would lead to the same result.\(^6\) This means the extra competition does not make trading with the incumbents any more attractive for the buyer. The only increase in consumer surplus comes from the extra purchases from the entrants for the lower price \( p_2 \). The increase in consumer surplus is given by

\[ \sigma = (p_1 - p_2)Q_K/2 = \frac{k^2 (a-c)^2}{2(k+1)^2 (n+1)^2}. \]

not taking into account costs incurred by having to pay back the signing fee to the incumbents.

The assessment of payable damages \( \mu \) is more debatable. Simpson and Wickelgren (2007) argue that expectation damages should be calculated based on the, as they call it, "but-for" quantity, which is the quantity that the buyer would have bought if he observed the contract. However, the case presented in this section is different from theirs, since the contract examined here includes stipulations regarding quantity and a fixed price, and. As will be shown, a literal interpretation of the contract makes exclusion possible.

### 3.2 Assessing damages: the individual or literal approach

If each contract with an incumbent is considered individually, it could be argued that the incumbent should be compensated for all missed profits resulting from the fact that he was not allowed to sell \( 1/n \) of the buyer’s total demand. This is a literal interpretation of each contract. The fact that the incumbents’ profits are as high as in the situation of no breach is ignored, because it is irrelevant. The only thing that matters is the buyer’s commitment to purchase a certain part of his buyings from his counterparty, regardless of the circumstances or alternatives. Damages would be equal to the difference in profits between observing and not observing. If every incumbent is succesful in making such a claim, the extra purchases \( Q_K \) would cost considerably more than their actual price \( p_2 \). The cost premium, consisting of damages payable to the incumbents, would in fact be so high that it prevents entry.

This is shown by the following. Suppose the incumbents sold \( q_n \) for the agreed price \( p_1 \). As explained earlier, this comes down to the same as selling for the competitive price \( p_2 \) and then

\(^6\)Note that the incumbents have already transferred more than this amount to the buyer as \( F \) to compensate him for his lower-than-would-be consumer surplus.
claiming the differential as damages. The total claims by the incumbents amount to the extra profit they would have made selling the additional $Q_K$ for their price $p_1$. The compensation for all $n$ firms together will be equal to $\mu = Q_K(p_1 - c) = \frac{k(a-c)^2}{(k+1)(n+1)^2}$. Since $\mu \geq \sigma$ for all $n, k \in \mathbb{N}$, the buyer will not breach his contracts.

It makes no difference if the entrants offer to pay the buyer if he lets them sell to him, if he has already signed the partially exclusive agreements. The combined payments of the entrants would at most be equal to their benefit from entering, $\sum \pi_k = k \frac{(a-c)^2}{(k+1)^2(n+1)^2}$. Simple algebra gives that $\sigma + k\pi_k \leq \mu$ for all $n, k \in \mathbb{N}$, so this does not change the result.

**Proposition 3** If each contract between the buyer and incumbent $\{i, \ldots, n\}$ is considered individually and expectation damages would be determined as the difference between the obtained $\pi_i^1$ and the profits $i$ would have made if he had sold $1/n$ of total $Q$ against $p_1$, the result would be that entry is deterred. In this case it is irrelevant whether the buyer has to reimburse the signing fees he received. This is also the case if the entrants could pay the buyer to breach.

### 3.3 Assessing damages: the "en bloc" approach

If the contracts are not considered individually but collectively, it can be argued that by making one all-embracing,"en bloc" breach decision, the buyer caused no damages to the incumbents at all, as long as he buys $Q_N$ against $p_1$ from them. He would after all never have bought more if he had observed every contract. The "but-for" quantity is equal to the quantity sold to the incumbents. In this approach, the total damage is less than the sum of its parts: it is non-existent. Assuming only that $\sigma \geq 0$, the preceding means that entry will never be deterred and competition will not be limited.

**Proposition 4** If the "but-for" quantity is calculated as the quantity an incumbent would have sold if none of the agreements had been breached, entry will not be deterred.

This requires some legal consideration. In many systems, the buyer would in principle be forced to put the seller in the same position as he would have been in if the buyer had bought $1/n$ of his demand against $p_1$ from him. However, proposition 3 shows that this is undesirable. Therefore, a judge (or lawmaker) should consider discarding any claim of this kind by the sellers. The legal basis for doing so could be that obliging oneself to buying $1/n$ of his variable demand from $i$ is anticompetitive and therefore against anticompetition law, if applicable.\footnote{In the European Union, article 102 TFEU allows for such decisions. In the United States similar precedents exist (starting with the Supreme Court’s decision in 1922 in the case Standard Fashion Co. v. Magrane-Houston Co.; see e.g. Marvel 1982 for more discussion).}

### 3.4 Alternative contracts and their implications

#### 3.4.1 Enforcable quantities, no fixed price

Another possible contract is an agreement to buy $Q/n$ from $i$ against the market price. If all incumbents close this contract with the buyer, the result without breach would be the same as of
a contract with a price agreement \( P = p_1 \), so proposition ?? still holds. The market price in case of breach and entry will be \( p_2 \). In this case, the increased competition by letting \( k \) entrants into the market does have an effect of the existing trade between the buyer and the incumbents. The buyer will pay \( p_2 \) for its entire demand, meaning that \( \sigma \) will increase by \( (p_1 - p_2)Q_N \) compared to the value found in equation 4. The increase in consumer surplus is now described by

\[
\sigma' = (p_1 - p_2)Q_K/2 + (p_1 - p_2)Q_N.
\]  

(5)

In this case, it is not so clear that the incumbents cannot call upon the quantity clause. They can argue that each of them would have been happy to sell \( q_n + Q_K/n \) against \( p_2 \). While this claim is doubtful and difficult to assess on its legal merits, as discussed above, we can examine what the consequences would be if it was awarded. Using the individual approach of expectation damage assessment, we can define the collective damages as \( \mu' = (p_2 - c)Q_K \). Calculating \( \sigma' \geq \mu' \) renders \( \frac{k(k+2n+2kn)(a-c)^2}{2(k+1)\tau(n+1)^2} \geq \frac{k(a-c)^2}{(k+1)\tau(n+1)^2} \), which holds for all \( n,k \in \mathbb{N} \), so breach would occur.

**Proposition 5** If the contract consists of a commitment to buy \( 1/n \) from \( i \) without a price agreement, entry is not deterred, even if damages are calculated based on a literal interpretation of each contract.

### 3.4.2 Signing fee as provisional clause

Consider a different case, where the contract awards the buyer with a bonus (in the range of \( F \)) if he buys \( 1/n \) of his total demand from \( i \). The buyer can choose to earn this bonus or discard the possibility and make an autonomous decision. By making \( F \) a provisional payment (a bonus) rather than an unconditional part of the contract, there can be no breach and no damages. Any price agreement as part of the contract will be meaningless because the incumbents will not sell anything if they do not conform to the competitive market price. Since conforming to the market price would be a voluntary action, still no damages could be claimed. Entry will only be deterred if the combined quantity bonuses are higher than the increase in consumer surplus after entry. Since the price will drop to the competitive price \( p_2 \), the increase in consumer surplus is the same as found in equation 5. The maximal value of \( F \) is then still insufficient to prevent breach. Take the maximal value of \( F \) for each incumbent, \( F_{\text{max}} = \frac{k(2n+2k)(a-c)^2}{(n+1)^2(n+1+k)^2} \). Using equation 5, calculation gives that \( \sigma' \geq nF_{\text{max}} \) holds for all \( n,k \in \mathbb{N} \). Hence, entry is not deterred here either.

**Proposition 6** If the contract consists of a conditional reward equal to \( F \) for buying \( 1/n \) from \( i \), entry is not deterred.

### 3.5 Conclusion

It has been shown that the implications of a partially exclusive contract depend on the interpretation of the expectation damages and on the specifics of the contract. In cases with a credible price agreement, the deadweight loss from the too high price for the incumbents’ part of the
production is left out of the equation, which makes entry deterence possible. In the case of a contract with an enforcable quantity ratio and fixed price, exclusion is possible when expectation damages are calculated based on a literal interpretation of each contract. However, when the interpretation of the damage is put in a broader perspective, it can be concluded that it is nonexistent and entry will not be deterred. When entry also leads to more competitive prices for the incumbents, entry will not be deterred. The full deadweight loss of the difference between the original Cournot market and the quasi-Stackelberg market cannot be compensated by the incumbents. A contract that contains only a quantity ratio and no fixed price will not deter entry, and the existence of such a contract does not necessarily have to ring alarm bells. These contracts should be viewed favourably if their existence does not lead to any apparent anticompetitive effects. However, partially exclusive contracts with clauses regarding both a fixed price and a quantity ratio should be treated negatively. If the buyer or the entrants have reason to suspect that these contracts are enforcable by a judge, they will deter entry and their mere existence has anticompetitive effects.

A note should be made with regard to the relevance of the model. It may be unlikely that multiple sellers all close the same partially exclusive agreement with a buyer. It is hard to find data on the extent to which these contracts occur in the business world. One could imagine a few examples, like a manufacturer buying store space for his products. In any case, the results we find are applicable to the case where there is just one seller (n = 1). In that case there would be just one, pure exclusive agreement and the buyer would be obligated to buy from a monopolist seller. Our results show under which circumstances this would lead to entry deterrence. Like in the case of multiple sellers, the question is how damages are calculated. If the monopolist can sue the buyer for not buying exclusively from him, entry by any number of competitors will be deterred. Under these circumstances an exclusive agreement between a buyer and a Cournot seller inefficiently limits competition.

Part II

Exclusive contracts with retailers

4 Simultaneous contracting

4.1 Model and definitions

In the last sections, we considered a Cournot market with an arbitrary number of sellers and a single final buyer. Sellers offered a partially exclusive agreement. We will now look at a case with completely exclusive contracts and downstream competition. Consider a market chain with Bertrand competition in the upstream market and Cournot competition in the downstream market. Suppose that there is one upstream monopolist α and that there are n homogenous
downstream firms, who are retailers rather than final consumers of the traded good. The downstream firms, who will also be referred to as the buyers\(^8\), face the inverse demand function \( p = a - Q \). Their only cost is the wholesale price \( w \). The upstream incumbent firm’s only cost is the variable cost \( c \) and his profit function is \( \pi_o = (w_o - c)q_o \), where \( q_o \) denotes his total sales.

In period 1, the monopolist offers the buyers the same exclusive contract, that stipulates that the buyer will only buy from the monopolist. The contracting period is a simultaneous game without discrimination. Buyers have no knowledge of any entrant and have no incentive not to sign with the monopolist. In period 2, a possible entrant emerges. Each buyer must then decide whether they buy from the incumbent or from the entrant in a simultaneous game. In the former case, the buyer remains captive. In the latter case, the buyer breaches his contract, and he will be a free buyer. In period 3, the buyers decide what quantity they buy, knowing the number of captive and free buyers.

The only cost the buyers incur is the wholesale price of the products they sell, \( w \). Suppose the entrant has a marginally lower cost price than the monopolist, \( c' \), so that \( c - c' \to 0 \).\(^9\) For simplicity, entry costs are ignored in the model. To make a proof of principial, we assume that the entrant’s goal is maximising his market share without incurring an operational loss. To capture as many buyers as possible he sets his wholesale price to be as low as possible, i.e. equal to his marginal costs, so \( w_e = c' \).

In period 2 each buyer has two options; remaining captive and breaching. A buyer has to choose between them and a combination is not allowed. Let \( h \) denote the set of strategies that start with honouring the contract, and \( b \) the set of strategies that follow from breaching and buying from the entrant. The subscripts \( h \) and \( b \) will also be used to separate variables in different scenario’s: \( h \) is used to indicate the scenario in which an arbitrary but fixed buyer honours the agreement and \( b \) to indicate that in which the same buyer decides to breach. Define the number of captive buyers as \( v \), so the amount of free buyers is equal to \( n - v \). Let \( i \) be an arbitrary captive buyer and \( j \) an arbitrary free buyer. Define the optimal quantity for any captive buyer as \( q_i \) and the optimal quantity for any free buyer as \( q_j \), and total quantity \( Q = \sum q_i \).

Similarly to the previous part, a free buyer has to compensate the incumbent for his loss of revenues. He has to pay expectation damages to put the incumbent in the position where the latter would have been if the buyer had honoured the agreement. The amount of these damages is denoted as \( \tau \), and consists of the missed sales to the breaching buyer and the damage he causes to the monopolist because of his competition with captive buyers. The payoff of an arbitrary buyer when he chooses \( h \) is \( \pi_{ih} = (p_h - w_o)q_{ih} \) and his payoff when he chooses \( b \) is \( \pi_{jb} = (p_b - w_e)q_{jb} - \tau \).

\(^8\)Not to be confused with the final consumers, who will not be mentioned.

\(^9\)The difference between \( c \) and \( c' \) is defined to be so marginally small that it makes as good as no difference in Cournot competition, but will make a difference in a Bertrand setting. Therefore, an incumbent is not able to make any sales to a free buyer, but otherwise, the difference can be ignored.
Table 2: Timeline

<table>
<thead>
<tr>
<th>Period 1</th>
<th>Period 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incumbent offers contract; buyers sign</td>
<td>Entrant emerges, buyers choose to remain captive or buy from entrant</td>
</tr>
</tbody>
</table>

4.2 General analysis

Each buyer will sign the exclusive contract with the monopolist in period 1, because he is not worse off by doing so. He may receive a signing fee $F$, which can be marginally low.\textsuperscript{10}

Since we assume that each buyer’s decision in period 1 is a given and strategies in period 3 are standard equilibrium Cournot strategies, the focus of our analysis is on period 2. Each buyer will choose the strategy that maximizes his payoff, so his optimal strategy set is $\arg\max_{b,h}\{\pi_{ih}, \pi_{jb}\}$.

A buyer will breach if $\pi_{jb} > \pi_{ih}$, which can be written as

\[
(p_h - w_e)q_{jb} + \tau > (p_h - w_c)q_{ih}.
\]

(6)

It is obvious that a buyer could never be worse off by breaching if we leave $\tau$ out of the equation, since $w_e < w_c$ by assumption, the buyer is free to choose $q$ and buyers’ strategies are strategic substitutes (meaning the other buyers will only decrease their quantity if they expect more competitive behaviour). He can always choose a $q$ such that the result is exactly the same as it would have been if he had remained captive. Therefore, the only thing that can prevent him from breaching is the expectation damages he has to pay.

Lemma 4 Because $w_e < w_c$ and lower marginal costs lead to more competitive behaviour, and quantities are strategic substitutes, we know that $Q_b^{-j} < Q_h^{-i}$. This implies that an arbitrary buyer who breaches can always achieve a benchmark outcome where he sells the would-be quantity of a captive and receives a higher price. Therefore there exists a set of breaching strategies that lead to a higher payoff than the best available honouring strategy, in the absence of recoverable damages.

We assume that the amount of damages that a breacher has to pay to the monopolist is equal to the marginal damage that a breacher causes. That consists of 1) the monopolist’s missed profit on the would-be sales to the breacher, plus 2) the missed profit because the monopolist sells less to the captive buyers due to the increased downstream competition. The first part is a direct loss of revenue and will be denoted as $\tau_r$. The direct loss of revenue is equal to $\tau_r = (w_h - c)q_{ih}$. The second part is an indirect loss by competition and will be denoted as $\tau_c$. The indirect loss by competition is equal to $\tau_c = v(q_{ih} - q_{jb})(w_h - c) + v(w_h - w_b)q_{ib}$. That is: the difference between the actual and would-be quantity times the would-be wholesale price minus production costs (the quantity effect), plus the difference between the actual and the would-be wholesale price times the actual quantity (the price effect). Note that these two effects do not necessarily both occur.

\textsuperscript{10}Or negative, combined with a low wholesale price. This will be discussed later.
Since $r_e = (w_c - c)q_{ih}$ and $w_e = c'$, we can write (6) as

$$(p_b - c')q_{jb} - \tau_e > (p_h - c)q_{ih}. \tag{7}$$

If the direct loss $r_e$ were the only recoverable damages, a buyer would breach if $(p_b - c')q_{jb} > (p_h - c)q_{ih}$. Using lemma 4 this proves that breaching is always profitable, since we assumed that $c' < c$. Even if the entrant has no efficiency advantage, breaching is at least a weakly dominant strategy in any case. That means that breach can only be prevented if a breacher has to compensate the monopolist for the increased downstream competition as well. This implies that an exclusive contract with fixed quantities can never prevent breach, as that would rule out any external influence on the purchasing behaviour of captive buyers.

Note that a free buyer could buy and sell the same quantity as he would have being captive, thereby not causing any real increase in competition. The pointe of the competition damage is that the firms that remain captive expect the breacher to act more competitively. It is only the captive buyers' anticipation on extra competition in period 3 that makes him lose revenue. Expecting more competition, they will act less competitively, i.e. buying less if the price remains the same. This makes the term "expectation damages" extra fitting.

**Proposition 7** Breach can only be prevented if breaching causes a recoverable loss of profit for the incumbent due to the other buyers' anticipation of increased competition. This implies that a contract with a fixed quantity cannot prevent breach.

Throughout this analysis it is assumed that the resulting loss of profit is recoverable by the incumbent, because there exists at least a degree of causality between the breach and the loss. If the buyer has to pay this damage, he has no reason to not behave more competitively. Therefore we can assume that the breachers treat $\tau$ as exogenous in period 3. Following from lemma 4 and proposition 7, $\tau$ being exogenous is a necessary condition for breach to be prevented. If $\tau$ were endogenous in period 3, there would always be at least one breaching strategy that strictly dominates remaining captive: becoming a free buyer, otherwise copycatting a captive buyer's strategy, thereby avoiding incurring $\tau_e$ and profiting from the gap between $c'$ and $c$. This implies that breach is not prevented if a breacher does not have to pay for the damage of anticipated competition.

Knowing that the quantity must remain variable, there are two relevant types of contract to consider. The first is the contract that contains a fixed wholesale price, and the second is the contract that contains a variable wholesale price. As mentioned, quantities are variable in both cases. The following argument (partly) relies on calculations presented in appendix A.1.

### 4.3 Fixed wholesale price

With a fixed wholesale price, there are two subtypes of contracts that we need to consider.
4.3.1 Predatory pricing

The first is a contract with a wholesale price below the normal competitive price and a negative signing fee ($F < 0$). The buyer pays a subscription fee and pays low prices in return, so to say. The wholesale price may even be below the incumbent’s cost price ($w_a < c'$). A buyer who has signed this contract would have no incentive to breach if the entrant offers a wholesale price equal to $c'$. Note, however, that breaching damages do not play a role here, since the monopolist makes no profit on his sales. The entrant could offer a similar contract with a negative signing fee that makes breaching attractive. A buyer would pay the difference between $w_a$ and $w_e$ times the quantity he expects to buy as signing fee. This means the entrant can shape his contract such that a buyer who breaches his contract with the incumbent and signs with the entrant is at least equally well off, *ceteris paribus*. Using lemma 4 we know that this is strictly dominant, so all firms will breach and buy from the entrant.

**Lemma 5** A contract including a fixed wholesale price below marginal costs and a negative signing fee does not prevent entry by a more efficient firm.

4.3.2 Contract with profit margin

We will now consider a contract with a wholesale price above the incumbent’s production costs ($w_a > c$). Suppose the entrant offers a wholesale price equal to his variable production costs, $c'$. The question is if the monopolist can offer a contract with a wholesale price such that buyers will never breach, i.e. remaining captive is each buyer’s dominant strategy. All calculations used for this subsection can be found in appendix A.2. Since we assumed that $c - c' \to 0$, we will from here on substitute $c'$ with $c$. This means that we slightly underestimate the profit of a free buyer, so from here on, $\pi_{jb} > \pi_{ih}$ suffices for breaching as the strictly dominant strategy (rather than $\pi_{jb} > \pi_{ih}$). We can write (7) as

$$\pi_j = (p - c)q_j - \tau_c \geq (p_h - c)q_{ih}. \quad (8)$$

The profit of a free buyer $j$ following his optimal strategy is given by

$$\pi_j = (p - c)q_j - \tau = \left(\frac{a + vw - c(v+1)}{n+1}\right)^2 - \tau, \quad (9)$$

where $v$ is the amount of buyers that remain captive. The gross profit of a free buyer, excluding payable damages, decreases with the amount of free buyers. The more firms remain captive, the larger the competitive advantage of breaching becomes. On the other hand, the competitive damages inflicted on the monopolist are higher too. We write $\tau = (w_a - c)q_{ih} + v(w_a - c)(q_{ib} - q_{ih})$. Since the wholesale price effect is fixed, we see a quantity effect ($q_{ib} - q_{ih}$) but no price effect. If all buyers follow their optimal strategy, this yields $\tau = \frac{w_a - c}{n+1}(a - w + (w_a - c)(2v - n - 3))$ in the equilibrium. The damages a free buyer is due to the monopolist also rise with the amount.
of buyers who remain captive. The overall effect of the amount of free buyers on the profit of a free buyer is ambiguous.

The profit of a captive buyer in the equilibrium is equal to

$$\pi_i = (p - w_\alpha)q_i = \left(\frac{a - (n - v + 1)w + (n - v)c}{n + 1}\right)^2. \tag{10}$$

This is the last factor in analysing the profitability of breaching. The profit a buyer could obtain by remaining captive is his opportunity cost of breaching. The captive buyer’s profit obviously increases in $v$, as captive rivals act less competitively than free rivals.

To assess whether a buyer will breach, we take an arbitrary firm and compare his payoff as a breacher to his payoff as a captive buyer, keeping the breaching decision of the other buyers equal. Their strategy set in period 3 depends on his decision, but their strategy set in period 2 is exogenous. Define $\hat{v}$ as the total amount of captive buyers when the firm decides not to breach (which will be denoted with the subscript $h$). If he decides to become a free buyer (subscript $b$), there will be $\hat{v} - 1$ captive buyers.

Using (8) the condition $\pi_{ih}^{\hat{v}} > \pi_{ib}^{v+1}$ gives us one set of outcomes. Breach is prevented only if $w_\alpha > c + \frac{(n-1)(a-c)}{2v-n-2}$ and $\hat{v} > \frac{n+2}{2}$. This means that setting $w_\alpha$ above that level is necessary but not sufficient to prevent breach. Note that setting $w_\alpha \leq \frac{an-a-c}{n-2}$ implies an upper bound for $\hat{v}$ alongside the existing lower limit. The contract with the best chance of preventing breach therefore has $w_\alpha > \frac{an-a-c}{n-2}$ (allowing for $\hat{v} = n$). Note that $w_\alpha > \frac{an-a-c}{n-2}$ implies a higher wholesale price than the normal optimal monopoly wholesale price for $v = n$, which is $\frac{a+c}{2}$. A captive buyer will therefore pay a higher wholesale price than a free buyer who buys from the monopolist (and pays the normal monopoly price). The monopolist will therefore have to pay a signing fee to let a buyer sign in the first place.

With a contract with $w_\alpha > \frac{an-a-c}{n-2}$, remaining captive is a buyer’s best strategy if and only if the condition

$$\hat{v} > \frac{n + 2}{2}$$

is met. For $\hat{v} \leq \frac{n+2}{2}$ and for any contract with $w_\alpha \leq c + \frac{(n-1)(a-c)}{2v-n-2}$, breaching is dominant for each buyer. This means that the decision in period 2 is made under uncertainty. From the perspective of one buyer, the strategies of the other buyers are mixed, making it unclear what his optimal strategy is. This makes the outcome of period 2 unpredictable.

The more competitors a buyer expects to remain captive, the lower the threshold for $w$ is. As mentioned before, the damages caused by the anticipation of increased downstream competition play a crucial role in preventing breach. When more buyers remain captive, the incumbent has a larger part of the market, so he is harmed more by increased competition. When less buyers remain captive, more harm is caused to the former monopolist, but the marginal damage decreases. Each individual breacher only pays for the marginal damage he causes, so breaching becomes easier. For $v \leq \frac{n+2}{2}$, the damage is so low that it cannot prevent breach. To compensate
for the decreased damage to quantities, the wholesale price $w$ has to rise, so the result of the multiplication is sufficiently high to prevent others from breaching.

Summarizing, breach and entry can be prevented because of the exorbitant damages a breacher has to pay to the monopolist. Whether this is successful or not depends on each buyer’s expectation of the actions of his competitors.

**Proposition 8** Suppose a monopolist in a Bertrand upstream market offers all retailers in a Cournot downstream market an exclusive contract with a fixed price and variable quantity in a simultaneous game. This contract can deter entry from an entrant with marginally lower production costs if and only if it contains a price above the normal monopoly wholesale price. If it contains a sufficiently high signing fee and the buyers do not expect the entrant, all buyers will sign it. If the entrant offers a wholesale price equal to his marginal production costs, a buyer with incomplete information has no dominant strategy if his only cost of breaching is the marginal damage he causes to the incumbent. He will remain captive if and only if he expects at least $\frac{n+2}{2}$ other firms to remain captive.

### 4.3.3 Other fixed costs

Up until now, we assumed the cost of breaching was equal to the marginal damage to the incumbent. However, switching costs (material and non-material) can also play an important role. It is also possible to include a (fixed) breaching fee in the contract. These costs depend strongly on the circumstances of the market and each specific buyer. We will briefly examine the role of these costs. Say that the sum of all costs of breaching (e.g. damages, fees, switching costs and opportunity costs [of network externalities]) is captured in $\Phi$, so that $\pi_{jb} = (p_b - c')q_{jb} - \Phi$. A buyer will breach when $(p_b - c')q_{jb} - \Phi > (p_h - w_a)q_{ih}$. In order to prevent breach, the costs must be sufficiently high. We find that $\Phi > \frac{(c-w)(w-a+(a-c)n+(w-c)(n^2))}{(n+1)^2}$. Because $\frac{\partial \Phi}{\partial w} = \frac{(n-1)(w-c)^2}{(n+1)^2}$ is always positive, the threshold value of $\Phi$ that will prevent breach ($\Phi^*$) is increasing in $w$. Hence the threshold value of $\Phi$ that will make remaining captive the dominant strategy for all buyers is found for $v = n$. This yields

$$\Phi^* = \frac{(w-c)((w-c)n^2 + (a-w)(n-1))}{(n+1)^2}.$$  

If we substract $\tau$, the variable damages, we get the critical level of the sum of all other costs than damages (e.g. switching costs) that will lead to entry being deterred, $\varphi^*$. This yields that entry is deterred if and only if

$$\varphi > \frac{(w-c)((2n^2 - 2n - 1)w - 2a - c)}{(n+1)^2}.$$
4.4 Exclusive contracts with variable wholesale price

If the contract contains no fixed wholesale price, the timeline is the same as before, except we add a period 2.1 in which the incumbent sets \( w_\alpha \) (rather than in period 1). All calculations for this subsection can be found in appendix A.3.

The incumbent determines his wholesale price by maximizing his profit function to \( w_\alpha \), knowing the amounts of captive and free buyers. He uses their reaction functions to maximize his profit.\(^\text{11}\) His optimal wholesale price is \( w_\alpha = \frac{a + c + 2c(n - v)}{2(n + v + 1)} \). We find that the resulting quantity of a captive buyer is constant at \( q_i = \frac{a - c}{2(n + v + 1)} \) (\( = q_{ih} = q_{ib} \)). This is the same profit maximizing quantity as would result from a monopoly without entry, which is logical, because otherwise there would have to be some kind of effect of \( v \) on \( q_{ih,b} \). The difference with monopoly is that \( w \) is variable and needs to be adjusted properly to maintain the optimal quantity. The profit maximizing \( w_\alpha \) logically increases in \( v \) (\( \frac{\partial w_\alpha}{\partial v} = \frac{a - c}{2(n + v + 1)^2} \)) and vice versa decreases when competition increases. In terms of \( q_i \), his marginal revenue and marginal costs for every captive buyer remain the same no matter how many there are, so his optimal \( q \) does not change.

As for the damages we see a price effect but no quantity effect. \( \tau = (w_h - c)q_i + (\bar{v} - 1)(w_h - w_b)q_i \)

A buyer will only remain captive if both \( n > 3 \) and \( \bar{v} > n - \frac{\sqrt{2n + 3} - 3}{2} \). Again each individual buyer’s optimal strategy depends on the strategies of the other buyers and there is no predictable outcome of period 2. Define \( v^* \) as the threshold of \( v \), i.e. the minimum number of captives for condition (28) to hold. \( v^* \sim n - \frac{\sqrt{2n + 3} - 3}{2} \), so we write

\[
v^* \sim n - \frac{\sqrt{2n + 3} - 3}{2}.
\]

The derivative of \( v^* \) to \( n \) is as follows:

\[
\frac{\partial v^*}{\partial n} = 1 - \frac{1}{2\sqrt{2n + 3}}.
\]

The higher the amount of buyers becomes, the larger a fraction of the buyers has to remain captive in order to make remaining captive the best strategy for an individual firm. The required fraction of cooperators \( \frac{v^*}{n} \) gives an indication of how likely it is that the condition will be met. The higher the ratio, the more cooperation is required, and the lower the probability that remaining captive is the best strategy for an individual firm. The threshold ratio and therefore the expected payoff of breaching compared to remaining captive is increasing in \( n \) when \( n \geq 13 \) (with the corresponding \( v^* = 9 \)) and approaches 1 for a high \( n \). Generally we can say that when the downstream market is more crowded, breach becomes more likely. Still, entry can be deterred.

**Proposition 9** Suppose a monopolist in a Bertrand upstream market offers all retailers in a Cournot downstream market an exclusive contract with a variable price and quantity in a simul-

\(^{11}\) \( \tau \) is exogenous for the incumbent as well. If the incumbent would intentionally increase \( \tau \), he would be causing the damage to himself. Therefore, that part would not be recoverable from the breachers.
taneous game. When the entrant offers a wholesale price equal to his marginal production costs, entry may or may not be deterred. A buyer has no dominant strategy if his only cost of breaching is the marginal damage he causes to the incumbent. If a buyer expects at least $v^*$ competitors to remain captive with certainty, he will remain captive. If he expects at least $n - v^*$ competitors to breach with certainty, he will breach. Breach becomes likelier with a higher $n$.

4.5 Conclusion

We found that in a market chain with a monopolist incumbent in the upstream Bertrand market and one or more retailers in the downstream Cournot market, breach can only be prevented only if the indirect loss of the incumbent due to the anticipation of competition can be claimed from the breaching buyers. The direct loss of sales to a breacher alone is not enough to prevent breach and entry from a marginally more efficient competitor. A contract with a fixed quantity does not prevent breach in any case, and neither will a contract that uses predatory pricing.

A contract with a fixed price and variable quantity can only prevent entry if the price is above the monopoly wholesale price. To induce the buyers to sign, they must not expect entry and receive a sufficiently high signing fee. Even then, remaining captive is not their dominant strategy in all cases. The more downstream firms remain captive, the less attractive it becomes to breach and vice versa. Because all buyers make their decision simultaneously, the outcome cannot be predicted. Entry may be deterred or may not be.

When the contract leaves both price and quantity variable, the outcome is ambiguous too. Again, each buyer’s optimal strategy depends on the decisions his competitors make. When more firms breach, breaching becomes a better strategy for all other firms. Breach becomes increasingly likelier with a higher number of buyers, making entry deterrence more difficult. The result can be either way, depending on what firms expect their competitors to do. When they expect inertia, entry remains blocked.

5 Brief discussion of an alternative: sequential contracting

5.1 The model

It is interesting to consider a rather different setting, where we look at the competitive dynamic between two symmetric incumbents, in the absence of a threat of entry. This section is less extensive than the previous ones, but shows some interesting results. The goal is not to present a comprehensive analysis but to provide quick insight into an interesting possibility.

Let us assume the entrant from the preceding argument eventually entered the market, and has now become completely identical to the former monopolist. The upstream market is now a symmetrical Bertrand duopoly, and the downstream market is still Cournot. Contracting happens sequentially (i.e. one buyer after the other), which may fit many real markets better than simultaneous contracting. The timeline is as follows. In period 1 an arbitrary buyer is
Table 3: Timeline

<table>
<thead>
<tr>
<th>Period 1.1</th>
<th>Period 1.2</th>
<th>Period 2</th>
<th>Period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arbitrary buyer is offered a contract</td>
<td>Buyer decides between being captive and being free</td>
<td>Buyers buy input</td>
<td>Buyers trade</td>
</tr>
</tbody>
</table>

offered a contract by one or both upstream firm(s). Their offers are simultaneous. The buyer decides whether he signs, and if yes, with which firm. He makes the choice that will lead to the market equilibrium with the highest profit for him in the next period. This equilibrium is predictable because he has full information about the number of signers with either upstream firm. Period 2 is a production phase where all buyers decide what quantity they trade, with full knowledge of the market. Then they trade in period 3, after which the next buyer is offered a contract in period 4, etc. The timeline is an infinite cycle with 3n stages in each cycle.

The contract contains no fixed price in our model. Suppose that the upstream firms do not sell to free buyers for a lower price than to their captive buyers (for example because the exclusive contracts prohibit it - as we will see, it is profitable for the upstream firms to limit themselves in this way).

5.2 Analysis

The main difference between this sequential model and a simultaneous game where all buyers are offered the same contract at the same time (like in all preceding cases) is that now it is not possible to capture the entire market in one blow by offering a slightly more attractive contract. In this model, the use of exclusive contracts can actually allow the producers to make a profit. All buyers will ultimately sign a contract, because there is no cheaper alternative to buy from, and signing at least allows them to capture some of the incumbent’s profit. If the contract contains no fixed wholesale price, the primary strategic variable in the competition between the upstream firms will be the number of signers. In the equilibrium all buyers are offered and sign the same contract, because of re-negotiations and/or time limits on contracts. In this case all buyers will receive a signing fee equal to the marginal profit an incumbent makes when he captures that buyer, as shown in appendix B. If we assume an equilibrium where each upstream firm captures half of the buyers, the marginal profit of signing is relatively low. The intuition behind this is that if one firm does not sign with the buyer, the other one will. The signing firm will charge a

12 This spares us the interesting but complicated possibility of offering a contract that maximises the industry profits as if there were no competition, and transfers the entire profits of the fictional monopolist to the buyers as signing fee. This contract would beat any other normal offer, and therefore the monopoly situation could actually be achieved by foreclosing the competitor. The foremost problem is that both firms can offer it, and if they do, they both end up with half of the market. They will compete in the production phase anyway, and therefore they will never be able to earn back the high signing fees. The second issue is that the competing upstream firm could anticipate the offer and find a reaction such that it is more attractive for some firms to sign his reaction contract, which would contain a lower wholesale price, thereby giving the signers a competitive advantage. This would be an interesting topic of future research.
price that is pretty close to the one the non-signing firm would have asked, so the non-signing firm does not suffer much from any increased competition. It is good for both upstream firms if the downstream firms have no cheap alternative. They are able to split the market, and pay only the marginal profit on the marginal signer as signing fee. Since the marginal profits they make on each non-marginal signer are higher they make a profit, even though they are Bertrand competitors.

5.3 Breach

A limitation of this result is that the possibility of breach was not taken into account. An offer of one of the upstream firms to the buyers that involves half of the buyers breaching would be an all-in attack on the competitor, targeting all buyers simultaneously. It would make little sense to limit oneself to the last signer. The latter case would likely also make little difference for our results. A simultaneous contracting game with competition between upstream sellers evokes some complications that make it too extensive to include in this thesis, as explained in the last footnote. In a nutshell, a firm could try to offer a contract which maximises industry profits. This spawns some complicated consequences, because his competitor may have a reaction that ruins his plan and profits. It would a very interesting topic of future research.

Despite the possibility that breach would happen in this model, the preliminary result is still interesting and relevant. There may be numerous reasons why breach does not happen. For example, competition legislation that limits a company’s market share could rule out a large scale attack, thereby facilitating the less competitive outcome of the sequential game.

5.4 Conclusion

Supposing there is no breach, the model shows that even when upstream firms act competitively, they can use exclusive contracts to make the market less competitive. It indicates that the use of exclusive contracts with variable wholesale prices can happen and limit competition under certain, not entirely fictional circumstances. This possibility can shed some light on how the use of exclusive contracts should be judged by antitrust authorities. Especially in markets with high entry barriers (ergo a low threat of entry) or high switching costs, exclusive contracts can reduce competition.

Proposition 10 Suppose duopolists in a Bertrand upstream market offer all retailers in a Cournot downstream market an exclusive contract in a sequential game. The contract cannot be breached. If there is no threat of entry and free buyers cannot get a lower price than captive buyers, the upstream firms pay the marginal profit of capturing an extra buyer as signing fee and all buyers sign with one of the two firms. In this case, the use of exclusive contracts limits competition and allows the duopolists to make a profit in the equilibrium, even though they actively compete for contracts.
6 Final conclusion and discussion

We can conclude that exclusive agreements can very well lead to the deterrence of entry by new competitors, even if they are (marginally) more efficient. We looked at a number of different cases. Both when the sellers compete in a Cournot market and when they compete in a Bertrand market but sell to Cournot retailers, exclusive agreements can be used to limit competition. Much can depend on the way expectation damages are calculated. In a Bertrand upstream market with a Cournot downstream market, multiple firms can be active and make a profit, despite their competitive behaviour. This gives reason to look into markets that resemble those characteristics, for example the Dutch beer market for pubs. Consumers may be paying too much.

There are several other circumstances that are largely outside of the scope of this thesis that can make exclusive contracts more or less effective. There may be additional costs of switching after having signed to consider. Switching costs may lead to a lower or even non-existent signing fee or threat of entry because it is hard for the buyer to switch. Network effects may increase the profitability of exclusive agreements. The seller expands his network by capturing an extra buyer, increasing its value and making it more attractive for all buyers, thus lowering the threshold for buyers to enter. Product diversification, on the other hand, could make being a free buyer more attractive. For example, if two friends want to visit a pub, and one likes one brand of beer and the other likes another brand, they may end up choosing a pub that offers both.

We reach a conclusion. Whether exclusive agreements have anticompetitive effects that harm market entrants and/or final consumers depends on a lot of circumstances. This thesis gave some insight into some of them. The most important conclusion is that anticompetitive effects can indeed occur. The dynamic is very different from that under pure Bertrand competition. This is the case even when firms only meet indirect Cournot competition (through their retailers). The results are less radical than in papers that examine Bertrand competition because Cournot allows for some differences in price and efficiencies. In my opinion, it shows more resemblance to reality in that aspect. Because of that, I hope that this thesis can give some meaningful insight into the actual implications of exclusive contracts.

7 Appendices

A Exclusive contracts in a simultaneous game

A.1 General calculations

Define $q_i$ as the quantity traded by each buyer who buys from the monopolist (i.e. remains captive) and define $q_j$ as the quantity traded by each free buyer who buys from the entrant (i.e. breaches the contract). There are $n$ buyers, of whom $v$ remain captive, so there are $n - v$ free
buyers. Let \( i \) denote an arbitrary buyer who decides to remain captive and \( j \) an arbitrary buyer who becomes a free buyer. A buyer in \( i \) pays the wholesale price set by the monopolist, \( w \). A buyer in \( j \) pays the wholesale price set by the entrant, which is assumed to be \( c' = \lim_{c \to c'} c' \).

The reaction functions that describe the optimal quantities are

\[
\begin{align*}
q_i &= \frac{a - \sum_{n-v} q_i - w}{n+1} \\
q_j &= \frac{a - \sum_{n-v} q_j - \lim_{c \to c'} c'}{n+1}
\end{align*}
\]

Substitution and writing \( \sum_v q_i = vq_i, \sum_{n-v} q_j = (n-v)q_j \) and \( \lim_{c \to c'} c' \approx c \) gives

\[ q_j = \frac{a - vq_i - c}{n - v + 1}. \tag{13} \]

Substituting (13) in the reaction function for \( q_i \) and solving for \( q_i \) renders

\[ q_i = \frac{a - (n - v + 1)w + (n - v)c}{n + 1}, \tag{14} \]

so

\[ q_j = \frac{a - c + (w - c)v}{n + 1} \tag{15} \]

and

\[ p = \frac{a + (n - v)c + vw}{n + 1}. \tag{16} \]

### A.2 Fixed wholesale price

Suppose an arbitrary buyer \( i \) decides to remain captive and buy from the monopolist. In that case, we say that there are \( \bar{v} \) captive buyers, so \( v = \bar{v} \), where \( \bar{v} \) can be any integer in \([1, n]\). We write

\[
q_{ih} = \frac{a - (n - \bar{v} + 1)w + (n - \bar{v})c}{n + 1},
\]

\[
q_{jh} = \frac{a - c - \bar{v}c + \bar{v}w}{n + 1}
\]

and

\[ p_x = \frac{a + (n - \bar{v})c + \bar{v}w}{n + 1}. \]

This gives

\[ \pi_{ih} = (p_n - w)q_{ih} = \left( \frac{a - (n - \bar{v} + 1)w + (n - \bar{v})c}{n + 1} \right)^2. \]

Suppose an arbitrary buyer \( j \) decides to become a free buyer and breach the contract. In this case, there is one less captive buyer than in the case where \( j \) buys from the monopolist. Since all buyers make their decision independently, their decisions will be the same as in the last case.
(where $v = \bar{v}$), so $v = \bar{v} - 1$. Substituting $v$ in (14), (15) and (16) gives

$$q_{ib} = \frac{a - (n - \bar{v} + 2)w + (n - \bar{v} + 1)c}{n + 1},$$

$$q_{jb} = \frac{a - w + \bar{v}(w - c)}{n + 1}$$

and

$$p_b = \frac{a + (\bar{v} - 1)w + (n - \bar{v} + 1)c}{n + 1}.$$

If a buyer breaches the contract, he has to pay expectation damages to the monopolist. Let $\tau$ denote the damages a buyer who breaches the contract has to pay. This gives

$$\pi_{jb} = (p_b - c)q_{ib} - \tau = \left(\frac{a + (\bar{v} - 1)w - c\bar{v}}{n + 1}\right)^2 - \tau.$$

Expectation damages are assumed to be equal to the marginal damage that the breach causes, i.e. 1) the monopolist’s missed profit on the would-be sales to $j$, and 2) the missed profit because the monopolist sells less to the captive buyers because of the more competitive behaviour of $j$. This gives

$$\tau = (w - c)q_{ih} + (\bar{v} - 1)(w - c)(q_{ih} - q_{ib}).$$

assuming $w \geq c$, because otherwise there are no damages. Since $q_{ix} - q_{iy} = \frac{w - c}{n + 1}$, we get

$$\tau = \frac{(w - c)q_{ih} + (\bar{v} - 1)(w - c)^2}{n + 1} = \frac{w - c}{n + 1} (a + c(n - 2v + 1) + w(2v - n - 2))$$

(17)

An arbitrary buyer will decide to uphold the contract and remain captive if and only if $\pi_{ih} > \pi_{jb}$, which yields $(p_h - w)q_{ih} > (p_b - c)q_{ib} - \tau$.\(^{13}\) Using (17), this condition can be written as

$$(p_h - c)q_{ih} > (p_b - c)q_{ib} - \frac{(\bar{v} - 1)(w - c)^2}{n + 1}.$$ 

This gives

$$\frac{(a - c + (w - c)\bar{v})(a - w + (c - w)(n - \bar{v}))}{(n + 1)^2} > \left(\frac{a + (\bar{v} - 1)w - c\bar{v}}{n + 1}\right)^2 - \frac{(\bar{v} - 1)(w - c)^2}{n + 1}. \quad (20)$$

For algebraic simplicity, we split $w$ into two variables; the monopolist cost price $c$ and his profit

\(^{13}\)Note that $\pi_j^{\bar{v} - 1} = \pi_j^{\bar{v} - 1} - \tau$ is not enough, because we wrote $\lim_{v' \to c'} c' \simeq c$ earlier on. If $\pi_j^{\bar{v} - 1}(\bar{v}, \bar{v}, \bar{c}, \bar{c}) = \pi_j^{\bar{v} - 1}(\bar{v}, \bar{v}, \bar{c}, \bar{c}) - \tau$ in this line of calculation, $\pi_j^{\bar{v} - 1}$ is actually (marginally) smaller than $\pi_j^{\bar{v} - 1} - \tau$, so the buyer will breach.
margin $\gamma$, so $w = c + \gamma$. Substituting this in the equation and simplifying yields

$$\gamma(2\bar{v} - 2 - n) > (n - 1)(a - c). \quad (21)$$

This gives one outcome:

$$\gamma > \frac{(n-1)(a-c)}{2\bar{v}-n-2} \quad \text{for} \quad \bar{v} > \frac{n+2}{2}.$$ 

$\gamma < \frac{(n-1)(a-c)}{2\bar{v}-n-2} \quad \text{for} \quad \bar{v} < \frac{n+2}{2}$ is no outcome because it yields $\gamma < 0$, which does not fulfill the condition $w \geq c$ presented in lemma 5.

This means there is no $\gamma$ for which $\arg \max_{h,b}\{\pi_ih, \pi_jb\}$ is no function of $v$.

If all fixed costs are captured in $\Phi$, a buyer will remain captive only if $\frac{(a-c+w-c)\varepsilon(a-w+c-w)(n-v)}{(n+1)^2} > \left(\frac{a+c+(w-c)v}{n+1}\right)^2 - \Phi$. This gives

$$\Phi > \frac{(c-w)(w-a + (a-c)n + (w-c)(n\bar{v} - \bar{v}))}{(n+1)^2}.$$ 

Define $\Phi^* = \frac{(c-w)(w-a + (a-c)n + (w-c)(n\bar{v} - \bar{v}))}{(n+1)^2}$ as the threshold value of $\Phi$ that will make remaining captive the strictly dominant strategy. Since $\frac{\partial \Phi^*}{\partial v} = \frac{(n-1)(w-c)^2}{(n+1)^2} > 0$, the threshold rises with $v$, so $\Phi^*$ is found for $v = n$. This yields

$$\Phi^* = \frac{(w-c)((w-c)n^2 + (a-w)(n-1))}{(n+1)^2}.$$ 

If we substract $\tau$ using (17), we get the critical level of the sum of all other costs than damages (e.g. switching costs), $\varphi^*$. This yields

$$\varphi^* = \frac{(w-c)((2n^2 - 2n - 1)w - 2a - c)}{(n+1)^2}.$$ 

**A.3 Variable wholesale price**

This section continues from the end of A.1.

To find the optimal $w = \arg \max_w \pi(w)$, we analyse the profit function

$$\pi = (w-c)\varepsilon_i = (w-c)\varepsilon_i \frac{a - (n-v+1)w + (n-v)c}{n+1} \quad \text{[using (14)].} \quad (22)$$

$\frac{\partial \pi}{\partial w} = 0$ yields

$$w = \frac{a + c + 2c(n-v)}{2(n-v+1)}. \quad (23)$$

Combining (14) and (23) we find

$$q_i = \frac{a - c}{2(n+1)}. \quad (24)$$
and with (13)
\[ q_j = \frac{(a - c)(2n - v + 2)}{2(n + 1)(n - v + 1)}. \] (25)

Then we calculate \( p = a - vq_i - (n - v)q_j \), which gives
\[ p = \frac{2(a + cn + cn^2) - v(a + c + 2cn)}{2(n + 1)(n - v + 1)}. \] (26)

Suppose an arbitrary buyer \( i \) decides to remain captive and buy from the monopolist. In that case, we say that there are \( v \) captive buyers, so \( v = \bar{v} \), where \( \bar{v} \) can be any integer in \([1, n]\). We write
\[ q_{ix} = \frac{(a - c)(2n - \bar{v} + 2)}{2(n - \bar{v} + 1)(k + \bar{v} + 1)}, \]
\[ p_h = \frac{2(a + an + cn + cn^2) - \bar{v}(a + c + 2cn)}{2(n + 1)(n - \bar{v} + 1)} \]
and
\[ w_h = \frac{a + c + 2c(n - \bar{v})}{2(n - \bar{v} + 1)}. \]
This gives
\[ \pi_{ih} = (p_h - w_h)q_i = \frac{(a - c)^2}{4(n + 1)^2}. \]

Suppose an arbitrary buyer \( j \) decides to become a free buyer and breach the contract. In this case, there is one less captive buyer than in the case where \( j \) signs. Since all buyers make their decision independently, their decisions will be the same as in the last case where \( v = \bar{v} \), so \( v = \bar{v} - 1 \). Substituting this in (25) and (26) gives us
\[ q_{jb} = \frac{(a - c)(2n - \bar{v} + 3)}{2(n + 1)(n - \bar{v} + 2)} \]
and
\[ p_b = \frac{3a + c + 2cn^2 + 2an + 4cn - \bar{v}(a + c + 2cn)}{2(n + 1)(n - \bar{v} + 2)}. \]
This gives
\[ \pi_{jb} = (p_b - c)q_{jb} - \tau = \frac{(a - c)^2(2n - \bar{v} + 3)^2}{4(n + 1)^2(n - \bar{v} + 2)^2} - \tau. \]

The monopolist earns \((w_h - c)q_i\) less because the buyer does not buy from him and also earns \((\bar{v} - 1)(w_h - w_b)q_i\) less on the buyers who remain captive because he had to decrease his wholesale price. Suppose he is entitled to compensation for his total decrease in profits, so \( \tau = (w_h - c)q_i + (\bar{v} - 1)(w_h - w_b)q_i \). We find that \( w_h = \frac{a + 3n + 2c(n - \bar{v})}{2(n - \bar{v} + 2)} \), so \( w_h - w_b = \frac{a - c}{2(n^2 - 2n\bar{v} + 3n + \bar{v}^2 - 3\bar{v} + 2)} \). This gives
\[ \tau = \frac{(a - c)^2}{4(n^2 - 2n\bar{v} + 3n + \bar{v}^2 - 3\bar{v} + 2)}. \]
The condition $\pi_{ih}^{v=v} > \pi_{jb}^{v=v=1}$ then comes down to
\[
\frac{(a-c)^2}{4(n+1)^2} > \frac{(a-c)^2(2n-v+3)^2}{4(n+1)^2(n-v+2)^2} - \frac{(a-c)^2}{4(n^2 - 2nv + 3n + v^2 - 3v + 2)}.
\] (27)
This holds for
\[
\left( n > 3, \bar{v} > n - \frac{\sqrt{2n+3} - 3}{2} \right).
\] (28)
Since this condition can both hold or not hold and the buyers play a simultaneous game, they have mixed strategies. Define $v^*$ as the threshold of $v$, i.e. the minimum number of captives for condition (28) to hold.
\[
v^* \pm n = \frac{\sqrt{2n+3} - 3}{2} \Rightarrow n - \frac{\sqrt{2n+3} - 3}{2}
\] (29)
\[
\frac{\partial v^*}{\partial n} = 1 - \frac{1}{2\sqrt{2n+3}}
\]
The ratio of the threshold of captive buyers to the total amount of buyers $\frac{v^*}{n}$ (the required fraction of cooperators) is increasing in $n$ when $n > 13$ (with the corresponding $v^* = 9$). Since $\lim_{n \to \infty} \frac{\partial v^*}{\partial n} = 1$, this ratio goes to 1. Condition (27) becomes increasingly unlikely in $n$ for $n \geq 14$, meaning the expected payoff of $h$ becomes increasingly smaller relative to the expected payoff of $b$, making $b$ a less attractive strategy.

**Proof.** The ratio $\frac{v^*}{n}$ increases in $n$ when $\frac{v^* + \partial v^*/\partial n}{n + \partial n/\partial n} > \frac{v^*}{n}$.

$v^* < n \frac{\partial v^*}{\partial n}$
$v^* < n - \frac{n}{2\sqrt{2n+3}}$
$v^* \simeq n - \frac{\sqrt{2n+3} - 3}{2} \Rightarrow n - \frac{n}{2\sqrt{2n+3}} > n - \frac{\sqrt{2n+3} - 3}{2}$
$6 - 3\sqrt{6} > n \text{ or } n > 6 + 3\sqrt{6}$
$n \in \mathbb{N} \Rightarrow \frac{v^* + \partial v^*/\partial n}{n + \partial n/\partial n} > \frac{v^*}{n} \forall n \geq 14$.

## B Exclusive contracts in a sequential game

Define $q_i$ ($q_j$) as the quantity that a buyer who signed with seller $\alpha$ ($\beta$) will choose and $u$ as the number of buyers who sign with $\alpha$.

We get $q_i = \arg \max q_i \pi_i$, so $q_i = \frac{a - \sum_{u+1} (q_j - w_\alpha)u + 1}{u+1} = \frac{a - (n-u)q_j - w_\beta}{u+1}$. Similarly, $q_j = \arg \max q_j \pi_j$.

Combining these yields
\[
q_i = \frac{a - (n-u+1)w_\alpha + (n-u)w_\beta}{n+1}.
\]

We use the optimal quantity to calculate the profit maximizing wholesale price $w_\alpha = \arg \max w_\alpha \sum_u (w_\alpha - c)q_i$. 

28
\[
\frac{\partial \pi_\alpha}{\partial w_\alpha} = a - 2(n - u + 1)w_\alpha + (n - u)w_\beta + (n - u + 1)c = 0
\]

\[
w_\alpha = \frac{a + c + (n - u)(w_\beta + c)}{2(n - u + 1)}
\]

Similarly, we get

\[
w_\beta = \frac{a + c + u(w_\alpha + c)}{2u + 2}.
\]

Substituting gives

\[
w_\alpha = \frac{2a + 2c - 3cu^2 + au + an - cu + 3cn + 3cuu}{4n + 3nu - 3u^2 + 4}.
\]

Substituting \(w_\alpha\) in \(\alpha\)'s profit function yields

\[
\pi_\alpha = \frac{u(a - c)^2(n - u + 1)(u + n + 2)^2}{(n + 1)(-3u^2 + 3nu + 4n + 4)^2}.
\]

To test the existence of an equilibrium where the incumbents pay a signing fee to each buyer, we check if capturing a marginal buyer is a profitable strategy. Ultimately all buyers receive the same signing bonus, i.e. that of the marginal buyer. Bertrand competition leads us to assume that this bonus is equal to the profit differential, which must obviously be positive in order for this equilibrium to exist. Suppose firm \(\alpha\) has captured \(u\) firms and needs to decide whether it will capture another one. It is willing to pay its positive profit differential, which is given by \(\partial \pi_\alpha = \pi_\alpha^{u=\bar{u}+1} - \pi_\alpha^{u=\bar{u}}\). Using the profit function 30 this renders

\[
\frac{(a - c)^2}{n + 1} \frac{(\bar{u} + 1)(n - \bar{u})(n + \bar{u} + 3)^2}{(7n - 6\bar{u} + 3n\bar{u} - 3\bar{u}^2 + 1)^2} - \frac{u(n - \bar{u} + 1)(\bar{u} + n + 2)^2}{(-3u^2 + 3nu + 4n + 4)^2}.
\]

Since the firms are symmetrical and we assume that in case of equal offers the buyer follows a mixed signing strategy, the firms will both capture half of the buyers if they use exclusive contracts, so we write \(\bar{u} = n/2\). This gives us

\[
w_{\alpha, \beta} = \frac{2a + 2c + cn}{n + 4}
\]

and

\[
q_{i,j} = \frac{(a - c)(n + 2)}{(n + 1)(n + 4)}.
\]

Then we calculate

\[
F = \partial \pi_\alpha(\bar{u} = n/2)
\]
\[ \pi_a = \frac{(a-c)^2 (3n^4 + 53n^3 + 278n^2 + 508n + 280)}{(n+1)(3n^3 + 28n^2 + 68n + 16)^2}. \]

Since \( F > 0 \) we know that capturing a marginal buyer is profitable when \( \hat{u} = n/2 \). A capturing equilibrium exists if and only if both firms make zero or positive profits. We write

\[ \pi_a = \frac{((w_a - c)q_i - F) u}{n + 1} = -n \frac{(a-c)^2 (n+2)^2}{(3n^3 + 28n^2 + 68n + 16)^2} - 3n^3 + 4n^2 + 80n - 8 \]

\[ \geq 0 \forall n \geq 6 \]

So there exists an equilibrium where both firms capture half of the buyers if and only if \( n \geq 6 \). Each buyer signs with either firm and receives a signing bonus \( F \). The firms are able to make a profit, despite being Bertrand competitors.

References


