Reference Dependent Preferences: Characterizing Demand and Optimal Pricing Rule

A thesis in accordance with the requirements for the degree of

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Abstract

This thesis studies consumer demand and pricing in a setting with reference-dependent preferences and loss aversion. We investigate the properties of individual demand and demand aggregation. We then study how monopolists set prices in such settings, and how this pricing behaviour depends on the specific variables related to reference dependence and loss-aversion. We also study optimal pricing from a regulator's perspective. We find that standard methods of demand aggregation are not necessarily feasible in this setting. Furthermore we see that the zero-sum result related to increased price mark-ups that we find in standard models do not necessarily hold when we consider loss-aversion and reference dependence. As a result, both consumers and producers can be better off in such cases. Lastly, we find that a regulator can always make consumers better off by setting prices appropriately in response to shifts in gain-loss valuation related variables. Our results offer a promising starting point for further research within behavioural industrial organization, particularly if our findings are integrated with other recent advances within the field.

Contents

You can't always get what you want, but if you try sometimes you just might find you get what you need

– *The Rolling Stones, 1969*

Preface

Rotterdam 18 September 2015

The quote on the previous page is not only intended to pay tribute to a great song and an amazing band, but also aims to capture some of my personal experiences over the last year. In many ways it is, appropriately enough, a tale of reference points. Sometimes you can demand too much of yourself, and try to push yourself beyond what is reasonable, especially in the face of adversity. It is in these moments that you have to look back to see that you already have those few precious things that really matter; you find you got what you need.

The last year has not been easy, and there are many people I would like to thank. First of all, my advisor, Dr. Dana Sisak. Your support in writing this thesis has been invaluable, and I am extremely grateful for your patience and effort over the last year in writing references and offering academic advise. Next, my many friends at the ESE. The guys in the 19th board, my committee, everyone in the Msc programme and all the IBEB'ers. Thank you for your understanding throughout the year. A group that deserves special attention is the Baarlo crew: Robert, Robin, David, Fons and Dayenne. Some of you I've known for 21 years, others for only 5, but your company and support have kept me going many times throughout the year.

En tenslotte, mijn ouders. Lieve pap en mam, we hebben een afschuwelijk jaar gehad. Het was een nachtmerrie, en we zijn er nog steeds niet helemaal doorheen. Maar we gaan het redden, met z'n drieën. Het is moeilijk voor mij om uit te drukken hoeveel alles wat jullie voor me gedaan hebben voor me betekent. Voor de mogelijkheden die jullie mij gegeven hebben ben ik jullie eeuwig dankbaar. Ik hoop dat jullie trots zijn op wat ik heb kunnen bereiken.

Niels

Chapter 1

Introduction

The field of industrial organization is in many ways one of the success stories of economic research. Building on the theory of the firm, industrial organization analyses market structures and competitive relations. The field of study has a long history, dating back to the earliest models of monopoly and perfect competition. An early example is Chamberlin's analysis of monopolistic competition (Chamberlin, 1933). With the advent of new game-theoretic tools, the field saw strong development throughout the 70's and 80's, as new methods of studying strategic interaction became available. The work of Fudenberg & Tirole in the early 80's in particular culminated in a long series of papers that fundamentally changed the way we think about markets (for example see Fudenberg & Tirole (1938a), (1938b), (1938c), (1984)). For an overview of their work around this time see Fudenberg (2015). Simultaneously, we saw rapid developments in the theory of competition policy, building on the new insights in the industrial organization literature. Regulators around the work took note, and much of their current work still builds on the foundations that were laid in this period. However by the end of the 80's development in theoretical industrial organization seemed to stagnate. As Tirole put it himself in his seminal 1988 textbook, industrial organization was "done" (Tirole, 1988). Naturally, this is an overexaggeration and since then we have seen many interesting advances in the fields of auction theory and mechanism design that directly relate to industrial organization and competition policy. But it is impossible to deny that the pase had somewhat stalled.

It required novel insights from new fields to once again invigorate research within IO. One of these fields is behavioural economics. This field, despite its young age, can already be named one of the other success stories of economic analysis. The advances within this field have greatly aided us in understanding the bounds of rationality and the nature of behavioural biases in economic processes. In particular, they have helped us understand the practical relevance of economic theories and, perhaps more importantly, the limits thereof. As such the field greatly adds to the legitimacy of economics as a social science that aims to describe human behaviour. One of the most comprehensive and influential theories within behavioural economics is prospect theory (Kahneman & Tversky, 1992). In a nutshell, prospect theory is a behavioural theory concering choices over risky alternatives. Two key features of the theory are reference dependence and loss aversion. Reference dependence implies that agents evaluate outcomes from a decision process not in terms of objective realizations but relative to a certain reference level. For example, if a consumer expects to consume a certain amount of a good (has this as a reference point) the consumer will be dissapointed if he/she consumes less than that level, regardless of the objective utility gained from consumption. In a way we can therefore think of reference dependence as formalizing the notion of a positive surprise or dissapointment. Loss aversion simply describes how such gains or losses are evaluated. Loosely speaking, it states that a loss is seen as more negative compared to the positive utility from a similar sized gain.

By introducing elements of prospect theory and other behavioural models into the analysis of markets the field of behavioural IO was born. This thesis falls into this category and introduces reference dependent preferences and loss-aversion into a model of a monopoly market. We will consider a continuum of consumers with different reference levels as well as a discrete model with two and n consumers (the motivation for this will be made clear in chapter 4). Our approach will be to characterize and derive individual as well as aggregated demand and see how it responds to changes in various parameters related to reference dependence and loss aversion. Next we analyze how a monopolist prices in such a market. In particular we will be investigating how price setting behaviour is affected by changes in the importance of gain-loss valuation and loss aversion as well as shifts in the reference level. We will also focus on the impact of these shifts on firm profits and consumer welfare. Next we turn towards price regulation. We look at how a regulator may optimally set prices to maximize welfare. Once again we focus on shifts in our variables of interest as well as consumer welfare and firm profits.

We note that this thesis is not the first to introduce reference dependence and loss aversion into a monopoly pricing model. In particular we identify two trends in the literature (see the next chapter for an in-depth discussion and overview). One branch of research has focussed on monopolist pricing when reference levels are endogenous and prices are non-deterministically set. This literature has typically analyzed models where one consumer makes a binary decision whether to buy or not to buy a single unit of a good. The second branch has focussed on monopoly pricing with a deterministic (fixed) reference point. These settings are most like the one considered in this paper. These papers however typically model consumer demand as being derived from one representative consumer. Whilst this approach is perfectly adequate when modelling standard consumer preferences, we argue that a lot of the interesting dynamics of the model are lost in this behavioural setting. In particular, the representative consumer approach implicitly assumes that all consumers in the economy have identical reference points as represented by the reference point of the representative consumer. The novelty of our model is that it introduces heterogeneity with respect to consumer reference points. We will see that this has crucial implications for the way demand is aggregated, which in turn greatly affects our pricing analysis. Our focus on price regulation also distinguishes our analysis from those typically found in the literature.

The structure of the thesis is as follows: chapter 2 contains an in-depth discussion of the relevant literature related to reference dependent preferences and its applications to industrial organization. Chapter 3 proceeds by describing the model that will be used throughout the thesis. We then start our analysis in chapter 4 by looking at individual as well as aggregated demand. In chapter 5 we will derive our main monopoly pricing results. This section contains the bulk of our analysis as much of our conclusions there can be readily generalized to our other settings. Chapter 6 considers the role of the regulator and price regulation before we conclude with a discussion in chapter 7.

Chapter 2

Literature Review

Reference dependence is a key feature of Kahneman & Tversky's original prospect theory formulation (1979). Later, improved and extended versions of the theory maintained this concept at its core (Kahneman & Tversky, 1992). Indeed, reference dependence can be interpreted as the outcome of a framing process: depending on how a purchase is presented, consumers adjust their reference level (Kahneman & Tversky 1986). As we can see, reference points are thus formed through a purely psychological process. In this view, reference points can be seen as the "status quo" or the "fair outcome" (Kahneman, Knetsch & Thaler, 1991). Over time, consumers may adjust their reference point once again to fit with their view of reaility. i.e. with what they perceive the new status quo to be (Kahneman et a;, 1986). Reference dependence alone however, cannot account for experimental and empirical tests of consumer rationality. Indeed, to be consistent with observed behaviour one needs to introduce the concept of loss-aversion (Tversky & Kahneman, 1991). As stated before, loss aversion refers to a situation in which a loss relative to a given refence point yields more disutility than the utility gained from an equal gain. As such, losses are weighted more heavily by consumers. Once such a behavioural bias is accounted for, experimental evidence can be shown to be consistent with our behavioural model. Kahneman,Knetsch and Thaler (1991) present a survey of such experiments.

Another area of research has investigated if loss aversion and reference dependence are applicable to a variety of market settings. An example of this is Blinder et al (1998), who study how reference dependence and loss aversion may be used to provide endogenous explanations for price stickiness. Their findings support the notion that such preferences may cause price stickiness to occur. This is particularly relevant for Keynesian models, in which price stickiness was typically exogenously imposed. Allowing this feature to be endogenous greatly improved the theoretical validity of such neo-Keynesian models. In the marketing literature, a survey of a panel of industry experts found that reference-based pricing practices was already occurring within many organizations, and many of these experts actively advised firms to take such behavioural effects into account when determining their pricing strategy (Marketing News, 1985). Experimental marketing research also confirmed the relevance of reference dependence in a variety of practical settings. Examples of such studies are Monroe (1990) and Rajendran (1999). The economics literature provides more empirical support in the form of Erickson and Johansson (1985), Kalwani and Yim (1992) and Winer (1986), who study reference dependence. Additional experiments by Hardie, Johnson, and Fader (1993) provide more evidence for the relevance of loss aversion to consumers, for both prices and quailty levels.

Based on these empirical findings, a literature applying behaviourally adjusted prefences to a variety of settings soon developed. Within the field of behavioural industrial organization one of these settings is monopoly pricing. DellaVigna and Malmendier (2004) investigate the impact of deviations from traditional preferences on contract design in a monopoly setting. Eliaz and Spiegler (2006) study pricing with heterogenously naive agents. Galperti (2014) studies the impact of time-inconsistency on monopoly pricing whereas Grubb (2009) studies overconfidence. Loss aversion was studied by Heidhaus & Koszegi (2008), amongst others. The relevance of these papers is that they managed to show how empirically observed relationships could be derived from a rigorous theoretical framework. In that sense, these papers were vital in consolidating theory and practice. This was a great step forward in the sense that theoretical industrial organization could now produce results that were similar to those found in practical fields such as marketing and strategy and could therefore be directly applied by firms and managers.

A key paper in the development of theoretical models of reference dependence and loss-aversion is Koszegi & Rabin (2006). This paper introduced two key concepts: first of all, it introduced a tractable and generalizable way of modelling reference dependence and loss-aversion by expressing gain-loss valuation in terms of a consumer's consumption valuation function. This formulation was shown to, without loss of generality, display all the features of prospect theory as described by Kahneman & Tversky (1979). Crucially, the simplicity of this modelling strategy allowed it to be easily introduced into a large number of existing models. The second key feature of the model is that it proposed a way of endogenizing reference points. In the earlier literature, reference points were assumed to be given, fixed and deterministic. In this perspective, reference points are subjective and therefore possibly incorrect assesments of reality or the status quo. In contrast to this view, Koszegi & Rabin propose that the reference point should be based on a consistent rule such that it can be endogenously explained based on the information available to the consumer. They suggest that reference point formation be therefore based upon rational expectations. This leads to the concepts of *personal equilibrium* and *preferred personal equilibrium.* In a personal equilibrium, an agent optimally chooses from a choice set given her probabilistic beliefs over this choice set, which is considered to consistute her reference point. In that sense, the model is consistent with Shalev's (2000) loss-aversion equilibrium. It may be the case that there are multiple personal equilbria given different feasible choice sets. Since consumers can always rank the outcomes of these personal equilibria, she can always select the choice set that realizes her preferred outcome. This is what Koszegi & Rabin define to be the preferred personal equilibrium. In a practical example at the end of the paper, Koszegi and Rabin show how the concepts of personal equilibrium and preferred personal equilibrium can be utilized to explain certain empirical phenomena. In particular, they apply the concepts to the behaviour of New York taxicab drivers, as studied earlier in Camerer et al. (1997) and Farber (2004), (2005).

The 2006 Koszegi and Rabin paper sparked two new brances of research. The first of these studies the impact of endogenous references on simple binary pricing models. In these models consumers choose to buy or not buy a good. Here consumers share an ex-ante stochastic reference point and evaluate each realization of stochastic consumption with each realization of the reference point. Examples are Karle (2014), Heidhues & Koszegi (2014) and Heidhues &

Koszegi (2008). The latter of these shows that their model can explain a variety of observed pricing behaviours. In particular, it explains the sticky and infrequent adjustment of prices as in Carlton (1986), Kashyap (1995) and Blinder (1998). Furthermore they manage to explain how prices can return to their previous levels even after an initial price change, as seen in Chevalier, Kashyap, and Rossi (2000). Next, their model shows how mark-ups are counter-cyclical as investigated by Bils (1987) and Chevalier and Scharfstein (1996), among others. Clearly then, models with endogenous reference points show great promise in providing theoretical foundations for frequently observed pricing behaviour. The disadvantage of these models is that due to their stochastic nature and the constant updating of endogenous references even very simple settings become highly complex. As a result the analysis of these models has typically been restricted to highly stylized settings, with single consumers making binary buy decisions about a single good. Another example of such a paper is Rosato (2014), who studies bait-and-switch tactics that manipulate reference points. Such tactics can raise profits even if consumers rationally expect them. Hahn et al. (2014) study non-linear pricing when consumers form ex-ante expectations based reference points before learning their valuation. Eisenhuth (2012) looks at optimal auctions for bidders with expectations based reference points.

Since models with expecations-based reference points are limited in scope by their inherent complexity, another branch of research has therefore resorted to applying the standard Koszegi & Rabin formulation to more complex settings by considering a fixed, deterministic reference point. Examples of such models include Sugden (2003) and Di Giorgi and Post (2011). A recent addition to the literature is Carbajal & Ely (2014), who study optimal menu setting and contracts for first-degree price discrimination with loss-averse consumers. Their initial set-up is similar to the one considered in this thesis, and we will present a complete information version of their incomplete information result in chapter 5. It is interesting to note that eventhough the incomplete information setting introduces a lot of complexity from a mechanism design standpoint, the use of price discrimination allows them to avoid having to aggregate individual consumer demands. As we will see in chapter 4 and 5, this means that many of the technical difficulties that arise from aggregation can be avoided. In particular, they show that contracts for individual consumers can be defined over different price intervals to allow for differing demand functions over such price intervals due to the kinks introduced by loss aversion.

A final paper that closely touches on our analysis is Sibly (2002). While using a exponential loss-aversion formulation that is different from the one considered in Koszegi & Rabin, it is one of the few papers to consider pure monopoly pricing as well as Ramsey pricing in a setting with reference dependence and loss-aversion. Sibly also elects to use a specific specification for the consuemr demand function and derives optimal pricing rules based on those. He finds that prices can be rigid over certain intervals due to the "holding on" effect of consumers trying to avoid disutility from loss-aversion. Whilst the analysis is sound, there is a strong caveat to his analysis: the model uses a representative consumer approach. This approach is well-established for standard preferences but is problematic when we consider loss-aversion. In particular, representative consumers are often interpreted to represent all consumers in a given economy. Thus, if a representative consumer is implemented in a model with reference dependence this implies that all consumers share the same reference point. Clearly this is not realistic and greatly reduces the generality of the model. Our innovation over the Sibly model will therefore be to introduce heterogeneity with respect to reference levels by allowing for a multitude of consumers. Simultaneously we consider a general specification of the consumer valuation schedule, instead of resorting to an exactly parameterized functional form.

Chapter 3

The Model

We now elaborate on the model we use to investigate our setting. We introduce the continuous version here, the discrete version follows directly from our discussion in chapter 4.

3.1 The Firm

We consider a profit-maximizing monopolist which produces a good whose quantity produced is given by $q \geq 0$. The costs of producing quantity q is given by $c(q) \geq 0$. We make the usual assumption on this cost function $c(\cdot)$, which is defined on $\mathbb{R}_+$. That is, we have that (F1) $c(q)$ is increasing in q, it is continuously twice differentiable (F2) it is strictly convex such that $c''(q) > 0$ for all $q > 0$ (F3). Without government intervention, the firm's objective is to set the price p such that profit is maximized. Profit is simply given by the following:

$$\Pi = p(q)q - c(q)$$

3.2 Consumers

To allow for differences in reference points between consumers, we assume that there is a continuum of consumers. Each consumer has differbent preferences and reference points depending on the type parameter $\theta \in \Theta = [\theta_L, \theta_H]$. We impose that $0 \leq \theta_L \leq \theta_H \leq \infty$. The distribution of consumer types is given by $F(\cdot)$ with support Θ and positive density $f(\cdot)$. As by Mussa & Rosen (1978), Maskin & Riley (1984), and Carbajal & Ely (2014) we assume that the inverse hazard rate:

$$h(\theta) = \frac{1 - F(\theta)}{f(\theta)}$$

is non increasing and continuously differentiable.

Consumption utility is a function $m : \mathbb{R}_+ \times \Theta \to \mathbb{R}$. We impose the standard assumptions on $m(\cdot, \cdot)$ as in Mussa & Rosen (1978), Maskin & Riley (1984), and Carbajal & Ely (2014): (C1) $m(\cdot, \cdot)$ is thrice continuously differentiable, (C2) $m(\cdot, \theta)$ is strictly increasing and concave for all $\theta \in \Theta$, $\theta > \theta_L$, (C3) $m(\cdot, \theta_L) = 0$ everywhere, (C4) for all $q \geq 0$ $m(q, \cdot)$ is increasing and concave,(C5) single crossing holds for all $q \geq 0, \theta \in \Theta$, such that $\frac{\partial^2 m(q,\theta)}{\partial q \partial \theta} > 0$, and (C6) $\frac{\partial^3 m(q^*,\theta)}{\partial {q^*}^3} < 0$.

3.3 Reference-Dependent Preferences

Of particular interest to our investigation is the impact of reference-dependent preferences on a regulator's optimal pricing rule. To consider this, we introduce a second component to utility. As in Carbajal & Ely (2014) and Koszegi & Rabin (2006) we introduce a gain-loss valuation component. Thus, a type θ consumer compares q to a type dependent reference point $r(\theta)$. Koszegi & Rabin (2006) introduce the concept of *personal equilibrium* (PE) and *preferred personal equilibrium* (PPE) to endogenously determine the reference point $r(\cdot)$. For simplicity however, as in Carbajal & Ely (2014), we take the reference point as given. Thus, we assume that $r(\cdot)$ is (possibly incorrectly) determined by past experiences or expectations of future consumption. For example, if we restrict $r(\cdot)$ to be based on rational expectations we reach the setting as described in Koszegi and Rabin. In particular, we consider a process $r : \Theta \to \mathbb{R}_+$. We assume the following: (R1) $r(\cdot)$ is increasing, (R2) is continuously differentiable everywhere.

Koszegi & Rabin (2006) show that given a set of basic assumptions reference dependent preferences can, without loss of generality, be represented in the following form:

$$\mu \times (m(q, \theta) - m(r(\theta), \theta))$$

Here we allow for loss aversion by defining μ as follows:

$$\mu = \begin{cases} \eta & : q > r(\theta) \\ \eta\lambda & : q \leq r(\theta) \end{cases}$$

Here $\eta > 0$ is the weight attached to gain-loss utility and $\lambda \geq 1$ measures the degree of loss aversion. Thus we can write the total valuation of a consumer of type θ as follows:

$$m(q, \theta) + \mu \times (m(q, \theta) - m(r(\theta), \theta))$$

If a consumer does not purchase any good, we have $q = 0$ and we thus get an outside utility equal to $-\lambda\eta m(r(\theta), \theta)$ (thus not buying any goods is considered to be a loss). We now define the sum of the consumption and gain loss valuation minus the outside utility as the net total valuation which is given by the following:

$$v(q, \theta) = (1 + \mu)m(q, \theta) + (\lambda\eta - \mu)m(r(\theta), \theta), \tag{3.1}$$

With:

$$\mu = \begin{cases} \eta & : q > r(\theta) \\ \eta\lambda & : q \leq r(\theta) \end{cases}$$

This concludes the description of our model.

Chapter 4

Demand

4.1 Individual Demand

We will now derive some basic properties of consumer demand that will be helpfull in all the following sections. A type θ consumer's total demand is given by the following:

$$q_D(\theta) = \text{argmax} v(q,\theta) - pq$$

for all θ for which $v(q_D(\theta),\theta) - pq_D(\theta) \geq 0$. For all θ for which $v(q_D(\theta),\theta) - pq_D(\theta) < 0$ we have $q_D(\theta) = 0$. Note that his specification implies the following:

Observation 1: *If for some $\underline{\theta} \in \Theta$ and for a given $\mu \in \{\eta, \eta\lambda\}$ we have $v(q_D(\underline{\theta}),\underline{\theta}) - pq_D(\underline{\theta}) = 0$ we have $q_D(\theta) > 0$ for all $\theta > \underline{\theta}$ and $q_D(\theta) = 0$ for all $\theta < \underline{\theta}$*

The observation follows since we have that $\frac{\partial v(q,\theta)}{\partial \theta} = (1+\mu)m_\theta(q,\theta) + (\lambda\eta - \mu)m_\theta(r(\theta),\theta) > 0$ for all $q \in \mathbb{R}_+$. Now consider the optimal consumption level of a type $\underline{\theta}$ consumer, given by $q_D(\theta)$. Then it holds that for all $\theta' > \underline{\theta} > \theta''$ we have $v(q_D(\underline{\theta}),\theta') > v(q_D(\underline{\theta}),\underline{\theta}) = 0 > v(q_D(\underline{\theta}),\theta'')$. Thus all consumers with a higher preference than $\underline{\theta}$ can gain positive utility through consumption of the good and thus will have positive demand, whereas all consumers with a lower preference will have negative consumption utility and thus zero demand. Note also that by continuity of the valuation function $m(.,\theta)$ and the reference schedule $r(\theta)$ in θ a value $\underline{\theta}$ exists if $q_D = 0$ for some θ.

For a type θ consumer we get that in the optimum we have the following optimal demand function (for types θ such that the particpation constraint is non-binding):

$$p = v_q(q,\theta) = (1+\mu)m_q(q,\theta)$$

Using the above condition we can derive the following lemma:

Lemma 1 *Define optimal consumption q^* as all q for which the above condition holds given p and θ. That is, $q^* = \{q : p = (1+\mu)m_q(q,\theta)\}$ for a given p and θ, $\mu \in \{\eta, \eta\lambda\}$. There exists a unique map q^* which defines optimal consumption such that $q^* = q^*(p,\theta,\mu)$ for all $p \in \mathbb{R}^+$ and $\theta \in \Theta$, $\mu \in \{\eta, \eta\lambda\}$ for $q^* \neq r(\theta)$. For this schedule it holds that the price elasticity of optimal consumption ε_* is given by $\varepsilon_* = \frac{q^*}{m_{q^*}(q^*,\theta)}\left[\frac{\partial^2 m(q^*,\theta)}{\partial q^{*2}}\right]^{-1}$ for $q^* \neq r(\theta)$. Furthermore it holds that $q^*(p,\theta,\eta\lambda) > q^*(p,\theta,\eta)$.*

Proof Given the defintion of q^* we can define the relation $G(\cdot)$ as follows (Note that it always holds by construction):

$$G(q^*,p,\theta) = p - (1+\mu)m_{q^*}(q^*,\theta) = 0$$

By our assumptions on $m(\cdot,\cdot)$ we have that $\frac{\partial G(q^*,p,\theta)}{\partial q^*} > 0$ for any fixed $\{q^*,p,\theta\}$. As such, by the implicit function theorem we can express q^* as some differentiable map q^*. Alternatively. there exists a unique function such that $q^* = g(p,\theta)$. Furthermore, we also know by the implicit function theorem that:

$$\frac{dq^*}{dp} = -\frac{\partial G/\partial p}{\partial G/\partial q^*} = -\frac{1}{-(1+\mu)\frac{\partial^2 m(q^*,\theta)}{\partial q^{*2}}} = \left[(1+\mu)\frac{\partial^2 m(q^*,\theta)}{\partial q^{*2}}\right]^{-1} < 0$$

Price elasticity of optimal consumption of a type θ consumer is thus given by:

$$\varepsilon_* = \frac{dq^*}{dp}\frac{p}{q^*} = \frac{q^*}{m_{q^*}(q^*, \theta)}\left[\frac{\partial^2 m(q^*, \theta)}{\partial q^{*2}}\right]^{-1}$$

Consider finally the first-order condition for optimal consumption:

$$p = (1 + \mu)m_{q^*}(q^*, \theta)$$

Since $\eta\lambda > \eta$ for $\lambda > 1$ it follows that for a fixed price p it must hold that $q^*(p, \theta, \eta\lambda) > q^*(p, \theta, \eta)$ for equality to hold. This follows from the fact that $m(q, \cdot)$ is concave and thus has a first derivative that is decreasing in q. This concludes the proof. ■

One interesting implication is that even though the level of demand is affected by loss aversion, elasticity is not. Before and after the reference level the elasticity is identical. This is because the demand curve before the reference level is simply the constant shifted demand curve after the reference level. From the first order condition for utility maximization it follows that we have a flat section in the demand schedule for $q^*(\theta) = r(\theta)$. That is, for some interval $[p^-, p^+]$ it holds that $q^* = r(\theta)$. We can easily derive these two levels of p. Consider the first order condition for utilty maximization as given above:

$$p = (1 + \mu)m_q(q, \theta)$$

Taking the left and right-side limits as q approaches $r(\theta)$ yields these two levels. We thus have:

$$p^+ = \lim_{q \to r(\theta)^-} (1 + \mu)m_q(q, \theta) = (1 + \eta\lambda)m_q(r(\theta), \theta)$$

$$p^- = \lim_{q \to r(\theta)^+} (1 + \mu)m_q(q, \theta) = (1 + \eta)m_q(r(\theta), \theta)$$

Note that by definition it holds that $\varepsilon_* = 0$ over the interval (p^-, p^+). Note that since the schedule is kinked at p^- and p^+ the elasticity is not defined here. If we combine all our information about individual demand we thus find the following:

$$q^* = \begin{cases} q^*(p, \theta, \eta) & \text{if } p < p^- \\ r(\theta) & \text{if } p^- \leq p \leq p^+ \\ q^*(p, \theta, \eta\lambda) & p > p^+ \end{cases} \tag{4.1}$$

For ease of interpretation such a demand schedule is provided in figure 1 below:

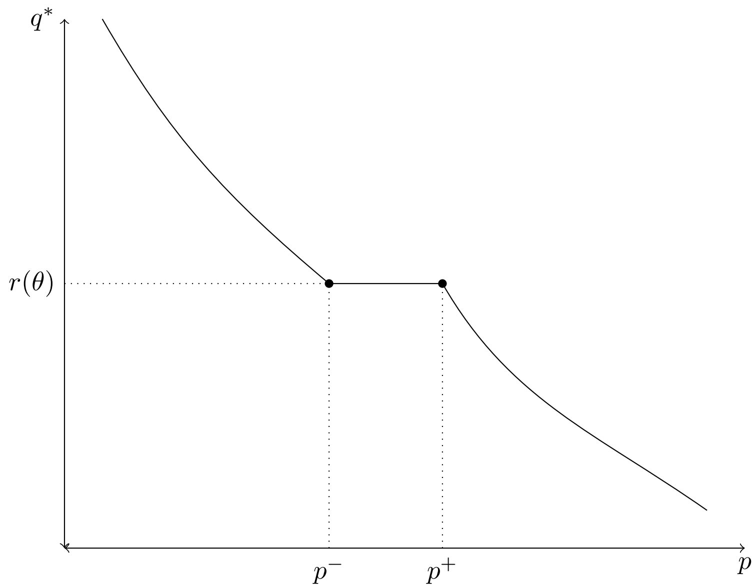

Figure 1: *A plot of q^* for an arbitrary consumer type θ,*

The demand schedule above was derived through a rather informal argument related to the first-order condition. We can make the argument more formal. For this, we introduce the concept of a subderivative. Let a subderivative of our consumer valuation $v(q, \theta)$ at a point q_0 be defined as a real number δ such that $v(q, \theta) - v(q_0, \theta) \geq \delta(q - q_0)$. Clearly, the subderivative is single-valued and equal to the derivative of $v(q, \theta)$ w.r.t q at all points at which $v(q, \theta)$ is differentiable, i.e. $q \neq r(\theta)$. However, at a kink or other non-differential point we have that there is a set of subderivatives, which is a nonempty closed interval $[a, b]$. Here a and b are:

$$a = \lim_{q \to r(\theta)^-} \frac{v(q,\theta) - v(r(\theta),\theta)}{q - r(\theta)}$$

$$a = \lim_{q \to r(\theta)^+} \frac{v(q,\theta) - v(r(\theta),\theta)}{q - r(\theta)}$$

This set $[a, b]$ is known as the subdifferential at point q. Note however that the boundaries of these subdifferentials are simply the left-and right side derivatives of the value function as q approaches $r(\theta)$. Thus we have:

$$a = \lim_{q \to r(\theta)^-} (1+\mu)m_q(q,\theta) = (1+\eta\lambda)m_q(r(\theta),\theta)$$

$$b = \lim_{q \to r(\theta)^+} (1+\mu)m_q(q,\theta) = (1+\eta)m_q(r(\theta),\theta)$$

Now we know that for a convex minimization problem we have that in general a point is a global minimum of a function if and only if zero is contained in the subdifferential. Since a convex minimization problem is equivalent to a concave maximization problem this implies that the maximum $v(q,\theta) - pq$ is achieved if and only if for $\delta \in [a, b]$ it holds that $\delta - p = 0$ which implies $\delta = p$. So as long as $p \in [a, b]$ we have that $q^* = r(\theta)$. This is thus the case if:

$$p \in [\lim_{q \to r(\theta)^-} (1+\mu)m_q(q,\theta) = (1+\eta\lambda)m_q(r(\theta),\theta), \lim_{q \to r(\theta)^+} (1+\mu)m_q(q,\theta) = (1+\eta)m_q(r(\theta),\theta)]$$

Note that this is exactly how we have defined p^- and p^+ above and therefore $q^* = r(\theta), \forall p \in [p^-, p^+]$, exactly as stated above.

It is once again illustrative to demonstrate the intuition behind this result graphically. Consider figure 2 below, which plots $v(q,\theta)$ as function of q. We have defined q^* to be the value of q which, for a given θ and p, maximizes the horizontal distance between the $v(q,\theta)$ and pq schedule. At all points, $q \neq r(\theta)$ this occurs where the two schedules have the same slope. At $q = r(\theta)$ however, the concept of slope is not well- defined for the $v(q,\theta)$ schedule. Instead, we think of a continuum of slopes, which is given by the shaded area in figure 2 below. This area is bounded by the pq schedule for $p = p^-$ and $p = p^+$. As you can see, the slope of these lines equals the slope of the $v(q,\theta)$ schedule just before and after the point $q = r(\theta)$ respectively. Note that we assume that he participation constraint is non-binding in the graph below (see further for a discussion).

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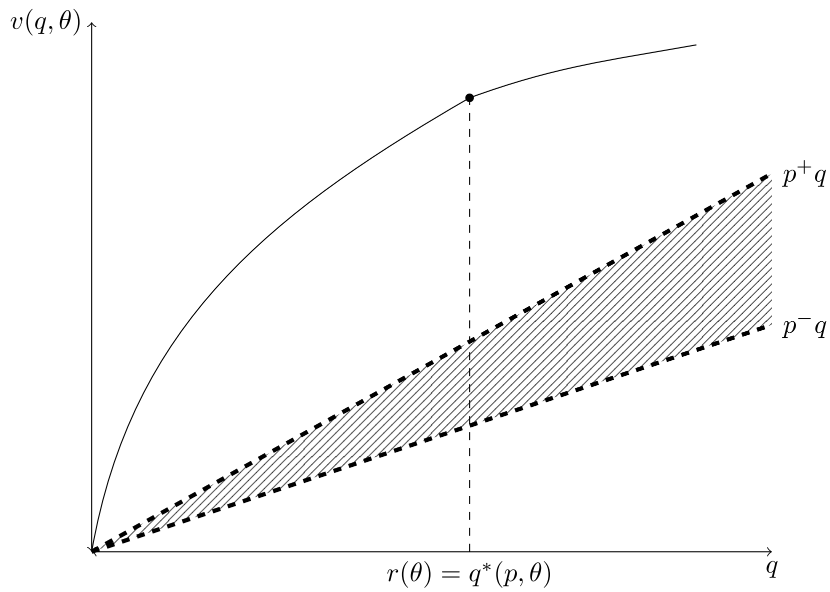

Figure 2: *A consumption level of* $q = r(\theta)$ *is optimal for all* $p \in [p^-, p^+]$.

So far we have considered demand only in the case the consumer has positive consumption in the form of q^*. We must however observe that optimal consumption q^* does not yet fully specify demand as it does not take into account the participation constraint. That is, q^* only specifies demand for all types θ for which $v(q,\theta) - pq \geq 0$.

We can once again make use of the implicit function theorem to find for which types θ this is the case. To do so, we derive $\underline{\theta}$. By our earlier observation we then also know that for all $\theta > \underline{\theta}$ the participation constraint is non-binding, whereas it is for all $\theta < \underline{\theta}$. Thus finding $\underline{\theta}$ allows us to fully characterize demand. This constitutes Lemma 2.

Lemma 2 *There exists a unique map $\underline{\theta}$ which defines the consumer type $\underline{\theta}$ for all $p \in \mathbb{R}^+$ and $q^* = g(p, \theta, \mu), \mu \in \{\eta, \lambda\eta\}$ for $q^*(\underline{\theta}) \neq r(\underline{\theta})$. Furthermore it holds that $\frac{d\underline{\theta}}{dp} > 0$.*

Proof By the participation constraint and the definition of $\underline{\theta}$ the folowing must hold for a type $\underline{\theta}$ consumer:

$$H(q^*, \underline{\theta}, p) = (1+\mu)m(q^*, \underline{\theta}) + (\lambda\eta - \mu)m(r(\underline{\theta}), \underline{\theta}) - pq^* = 0$$

Taking first-order derivatives yields:

$$\frac{\partial H(q^*, p, \underline{\theta})}{\partial \underline{\theta}} = (1+\mu)\left[m_{q^*}(q^*, \underline{\theta})\frac{\partial q^*}{\partial \underline{\theta}} + m_{\underline{\theta}}(q^*, \underline{\theta})\right] + (\lambda\eta - \mu)\left[m_{\underline{\theta}}(r(\underline{\theta}), \underline{\theta}) + m_{r(\underline{\theta})}(r(\underline{\theta}), \underline{\theta})\frac{\partial r(\underline{\theta})}{\partial \underline{\theta}}\right] - p\frac{\partial q^*}{\partial \underline{\theta}}$$

Now remember that we have defined q^* to be such that the first-order condition for utility maximization with respect to q holds. As a result it holds that $p = (1+\mu)m_{q^*}(q^*, \theta)$ for any fixed p. Plugging this into the derivative above and simplifying yields:

$$\frac{\partial H(q^*, p, \underline{\theta})}{\partial \underline{\theta}} = (1+\mu)m_{\underline{\theta}}(q^*, \underline{\theta}) + (\lambda\eta - \mu)\left[m_{\underline{\theta}}(r(\underline{\theta}), \underline{\theta}) + m_{r(\underline{\theta})}(r(\underline{\theta}), \underline{\theta})\frac{\partial r(\underline{\theta})}{\partial \underline{\theta}}\right]$$

This step also follows directly from the envelope theorem. By our assumptions on the valuation function $m(\cdot, \cdot)$ and the reference schedule $r(\cdot)$ we find that $\frac{\partial H(q^*, p, \underline{\theta})}{\partial \underline{\theta}} > 0$. As such, there exists a unique map $\underline{\theta}$ which defines the consumer type $\underline{\theta}$ for all $p \in \mathbb{R}^+$ and $q^* = g(p, \theta)$ by the implicit function theorem. That is, we have $\underline{\theta} = h(q^*, p)$. We can evaluate the derivative identically to lemma 1:

$$\frac{d\underline{\theta}}{dp} = -\frac{\partial H/\partial p}{\partial H/\partial \underline{\theta}} = \frac{q^*(p, \theta)}{(1+\mu)m_{\underline{\theta}}(q^*, \underline{\theta}) + (\lambda\eta - \mu)\left[m_{\underline{\theta}}(r(\underline{\theta}), \underline{\theta}) + m_{r(\underline{\theta})}(r(\underline{\theta}), \underline{\theta})\frac{\partial r(\underline{\theta})}{\partial \underline{\theta}}\right]}$$

This is strictly positive since $q^* \in \mathbb{R}^+$ and the denominator was proven to be strictly positive as an earlier part of this proof. ■

Figure 3 below shows graphically how $\underline{\theta}$ increases as p is increased for two arbitrary levels of θ and $r(\theta)$ (below the implications of the reference point for the selection of $\underline{\theta}$ will be discussed in great detail). By definition, q^* is the point at which the slope of the of the valuation function $v(q, \theta)$ equals that of the consumption expenditure schedule pq (this follows directly from the first order conditions). By the definition of $\underline{\theta}$ however, the two curves must also intersect. This can only be the case if tangency is achieved. We see that the point of tangency moves to a higher schedule for a higher type consumer as p increases. Intuitively this is the case since higher types have a higher marginal valuation of each good at each level of consumption. Note that in the figure $v(0, \theta) < 0$ since no consumption is seen as a loss.

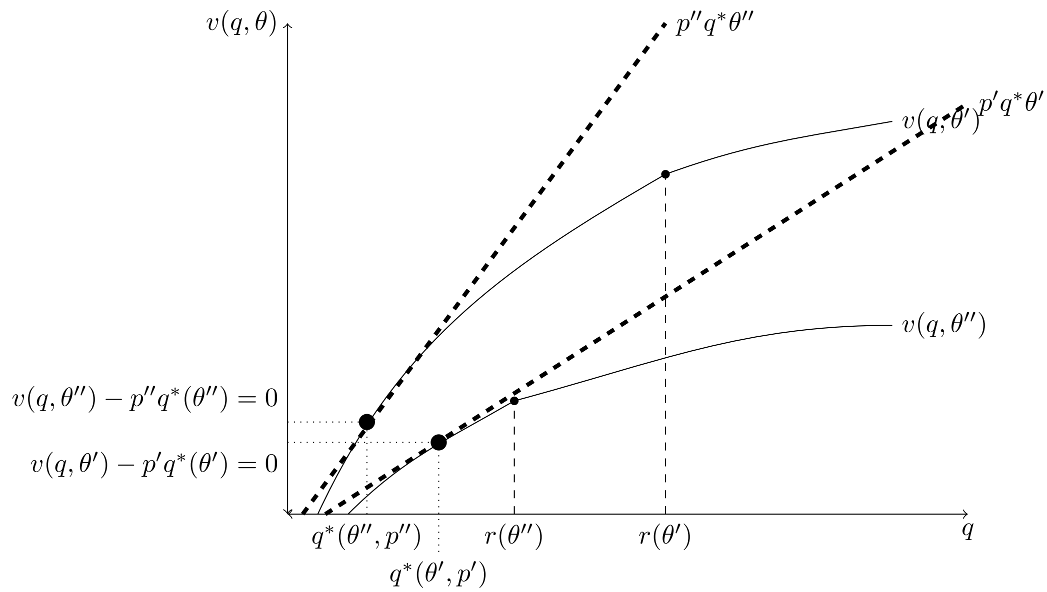

Figure 3: *The indifferent type consumer $\underline{\theta}$ shifts from the lower type θ' to the higher type θ'' as the price increases from p' to p''. Axes not drawn from origin*

We have once again excluded the case where $q^*(p, \underline{\theta}) = r(\underline{\theta})$ as the implicit function theorem does not apply due to the non-differentiability at this point. It is however easy to show that

$\frac{d\theta}{dp} > 0$ in this case as well. For optimal demand to be at this point it must hold that $p \in [p^-, p^+]$. For $\underline{\theta}$ this interval is given by the following:

$$p \in [\lim_{q \to r(\underline{\theta})^-}(1+\mu)m_q(q,\underline{\theta}) = (1+\eta\lambda)m_q(r(\underline{\theta}),\underline{\theta}), \lim_{q \to r(\underline{\theta})^+}(1+\mu)m_q(q,\underline{\theta}) = (1+\eta)m_q(r(\underline{\theta}),\underline{\theta})]$$

If $q^*(p,\underline{\theta}) = r(\underline{\theta})$ it thus holds that $v(q,\underline{\theta}) = p'q$ for some $p' \in [p^{-1}, p^+]$. Consider now a price increase from p' to $p'' < p^+$. Clearly we now get that $v(q,\underline{\theta}) < p''q$ as q is constant over this interval. This implies that the participation constraint is now violated for the original type $\underline{\theta}$ consumer and by assumption (C2) a higher type is now type $\underline{\theta}$. This finding has interesting implications for consumer behaviour. If a particular type consumer θ' is the type $\underline{\theta}$ consumer for some price $p' \in [p^{-1}, p^+]$, she will choose to consume $q^* = r(\theta')$ over the interval $[p^-, p']$ and consume zero units over the interval $[p', \infty)$, with her being indifferent between the two at p'. Thus, before halting consumption completely she chooses to "hang on" to consuming the reference amount as to avoid loss experienced by not consuming.

In general we see that $\underline{\theta}(q^*, p, \eta) \neq \underline{\theta}(q^*, p, \eta\lambda)$. We will now show that there is only one unique type $\underline{\theta}$ and that this consumerr optimally consumes $q^* = 0$ in equilibrium.This implies that $\underline{\theta} = \underline{\theta}(q^*, p, \eta)$.

Lemma 3 *The unique type $\underline{\theta}$ consumer always consumes $q^* = 0$ in an interior solution. For all $\theta'' \leq \underline{\theta}$ and the participation constraint is non-binding such that $q_D = q^* > 0$ for all $\theta' > \underline{\theta}$*

Proof A consumer $\underline{\theta}$ exists and is unique by Lemma 2. First we note that for a type $\underline{\theta}$ consumer we have an interior solution at $q^* = q_D = 0$ for a given price p if $p = v_q(0,\underline{\theta})$. This simply follows from the first-order condition for utility maximization. For all $\theta'' < \underline{\theta}$ it holds that $v_q(0,\theta'') < p$ by single crossing. As such, it is optimal for these types not to consume. Furhermore it holds that $v_q(q,\theta'') < p$ for all $q \in \mathbb{R}^+$ by concavity of the consumer valuation function since concavity implies that $v(q,\theta) \leq v(0,\theta)$. Therefore it follows that these consumers do not consume. For types $\theta' > \underline{\theta}$ it holds that $v_q(0,\theta') > p$, once again by single crossing.

Thus it is optimal to consume some quantity $q^* > 0$ for all these types. This concludes the proof. ■.

Using a similar approach it is also simple to derive that there exists a unique reservation price $p_R(\theta)$ for all $\theta \in \Theta$ such that $q(p, \theta) > 0$ for all $p \leq p_R$ and 0 otherwise (see Appendix A for this derivation). By combining all the above we have fully characterized the demand of an individual consumer of type θ. It is given by:

$$q_D = \begin{cases} q^*(p, \theta, \mu) & \text{if } p \leq p_R(\theta) \\ 0 & \text{if } p > p_R(\theta) \end{cases} \tag{4.2}$$

If we look at the specification of q^* in equation (2) we see that we can have three specific cases: (1) $p_R > p^+$, (2) $p^+ > p_R > p^-$, (3) $p^- > p_R$. In the first case we have:

$$q_D = \begin{cases} q^*(p, \theta, \eta) & \text{if } p^- > p \\ r(\theta) & \text{if } p^+ \geq p \geq p^+ \\ q^*(p, \theta, \eta\lambda) & \text{if } p_R(\theta) \geq p > p^+ \\ 0 & \text{if } p > p_R(\theta) \end{cases}$$

In the second case:

$$q_D = \begin{cases} q^*(p, \theta, \eta) & \text{if } p^- > p \\ r(\theta) & \text{if } p_R(\theta) \geq p \geq p^- \\ 0 & \text{if } p > p_R(\theta) \end{cases}$$

In the third case:

$$q_D = \begin{cases} q^*(p, \theta, \eta) & \text{if } p_R(\theta) \geq p \\ 0 & \text{if } p > p_R(\theta) \end{cases}$$

Throughout the remainder of this thesis we will generally consider the first case, since it has the richest characteristics with respect to reference dependence and loss aversion. Note that by doing so we implicitly assume that $\underline{\theta}$ is of the type $\underline{\theta}(q^*, p, \eta\lambda)$. Since we have now fully specified individual demand, we can perform several comparative statics exercises. We are particularly interested in the impact of loss aversion, and therefore we derive some competitive statics results with respect to the valuation of gain-loss utilty η, the degree of loss-aversion λ and the reference schedule $r(\theta)$. The results of this exercise are summarized in proposition 1 below. We specifically focus here on the effects of reference dependence and loss aversion on the pricing intervals as determined in the demand function above. This is because we have shown in lemma 1 that elasticity (and thus the monopolist's pricing rule) is unaffected by loss aversion/gain-loss valuation outisde of the interval $[p^-, p^+]$. Note that for the remainder of this discussion we will assume the condition specified in point (4) in proposition 1 holds.

Proposition 1 *The following holds for the demand of loss-averse consumers ($\lambda > 1$):*

1. *An increase in the coefficient of loss aversion λ widens the interval $[p^-, p^+]$ over which $q_D = r(\theta)$. That is, for $\lambda'' > \lambda'$ it holds that $p^+(\lambda'') - p^-(\lambda'') > p^+(\lambda') - p^-(\lambda')$.*

2. *An increase in the coefficient of gain-loss utilty η widens the interval $[p^-, p^+]$ over which $q_D = r(\theta)$ and shifts p^+ and p^- to the right. That is, for $\eta'' > \eta'$ it holds that $p^+(\eta'') - p^-(\eta'') > p^+(\eta') - p^-(\eta')$.*

3. *An exogenous shift in the reference schedule $r(\theta)$ such that $r''(\theta) > r'(\theta)$ for all $\theta \in \Theta$ shifts p^+ and p^- to the left. Furthermore it decreases the width of the interval $[p^-, p^+]$ over which $q_D = r(\theta)$.*

4. *If $m_{q\theta}(r(\theta), \theta) > -m_{qq}(r(\theta), \theta)r'(\theta)$ it holds that $p^+(\theta'') > p^+(\theta')$ and $p^-(\theta'') > p^-(\theta')$ for $\theta'' > \theta'$. Furthermore the interval $[p^-, p^+]$ widens as θ increases. The reverse holds otherwise.*

Proof For the first part we recall that by the definition of p^+ and p^- we have the following:

$$p^+ - p^- = (\lambda - 1)\eta m_q(r(\theta), \theta)$$

with $m_q(r(\theta), \theta) > 0$ and $\lambda > 1$. We observe that the interval widens as λ increases from this expression. The second claim now follows directly from the argument above by observing that thexpression is also increasing in η for $m_q(r(\theta),\theta) > 0$ and $\lambda > 1$. The shift to the right follows by inspection of the expressions for p^+ and p^- since both are increasing in η, whereas only p^+ is increasing in λ. For the third claim, remember that $m(q,\theta)$ is concave in q, such that $m_q(r(\theta),\theta$ and therefore the entire expression above is decreasing in $r(\theta)$, as are the individual expressions for p^+ and p^-. The claim follows. Lastly, consider an increase in θ. We the interval is increasing if the following holds:

$$\frac{\partial}{\partial\theta}[p^+ - p^-] = (\lambda-1)\eta m_{qq}(r(\theta),\theta)r'(\theta) + (\lambda-1)\eta m_{q\theta}(r(\theta),\theta) > 0$$

Noting that we have that $m_{qq} < 0$ by concavity of the valuation function, $r'(\theta) > 0$ by assumption and $m_{q\theta} > 0$ by the single-crossing condition it is easy to see that the condition above can be rearranged to reach the condition stated in the proposition. By an identical argument the same condition applies to the individual expressions for p^+ and p^- and thus the claim follows. ■

Our findings in proposition 1 are readily interpreted. Let us consider the first result. Earlier on, we have seen that we can interpret the fact that the consumer stays at his reference level of consumption for a continuum as prices as the consumer "hanging on" to avoid incurring utility loss due to loss aversion. As the consumer becomes more loss averse, this motive becomes stronger, resulting in higher prices for which the consumer stays at his reference level. This is exactly what our first claim states formally.

Secondly, we look at the affect of an increase of the coefficient of gain-loss utility in claim 2. If this coefficient increases, gain-loss utility becomes more important. As a result, the marginal utility of consuming increases. As a result, the consumer will be willing to consume a higher level at a given price. As a result of this, the consumer only reaches his reference level of consumption at higher prices. This is exactly what is implied by the rightward shift of the interval. Note futhermore that the interval widens as in the first proposition. To see why this is the case, note that the losses from loss aversion are measured in units of gain loss utility. For levels of consumption higher than the reference level, η units of gain loss utility are lost if consumption is decreased by 1 unit. After, due to loss aversion, $\lambda\eta$ units are lost. We thus see that the two coefficients are complementary.

As a result we see that if gain-loss utility becomes more important we also find that the consumer becomes relatively more loss averse measured in units of gain-loss aversion. As such, the interval of prices over which the consumer hangs on to the reference level of consumption must widen by the exact same reasoning as in the discussion of claim 1.

Claim 3 follows a similar line of reasoning: as the reference amount increases for all consumer types we see that a typical consumer reaches her reference level at a higher level of consumption. Higher levels of consumption correspond to lower prices, which means that the interval of prices must shift to the left. At this lower level of prices and higher level of consumption the marginal utility of each unit of consumption is lower, and thus we see that the loss incurred by the consumer as a result of loss-aversion is relatively smaller. As such, the consumer has less of an incentive to hang on to its reference level of consumption, narrowing the interval of prices over which he does so.

Lastly, consider claim 4. Note that the price levels p^+ and p^- give the marginal utility of an extra unit of consumption at the reference level, which follows directly from utility maximizing behaviour. There are two counteracting effects to be considered when evaluation this marginal utility: at the one hand, a higher type consumer has a higher reference point, thus reducing marginal utility compared to a lower level, ceteris paribus. Simultaneously, higher type consumers have a higher marginal utility of consumption for a given quantity. Depending on whether these two effects dominates we see that the relevant pricing interval can shift either right or left. If the latter effects dominates, it shifts right and the interval widens. The intuition for this is identical to that of an increase in gain-loss valuation considered in claim 2. If the former dominates, we have the exact reverse. Note that throughout the rest of this paper we assume that the latter effect dominates and the interval shifts right. We do this in order to make aggregation feasible in the following section.

4.2 Aggregated Demand

For the case where there is no loss aversion (i.e. $\lambda = 1$) we see that the demand q_D is continuously differentiable in both p and θ and equal to the following:

$$q_D = \begin{cases} q^*(p, \theta, \eta) & \text{if } p_R(\theta) \geq p \\ 0 & \text{if } p > p_R(\theta) \end{cases}$$

To see this, note that for this case we have the following:

$$v(p, \theta) = (1 + \eta)m(q(p, \theta), \theta) \; \forall q \in \mathbb{R}^+$$

We thus see that for $\lambda = 1$ it holds that the consumer consumption valuation function is multiplied by a positve constant $1 + \eta$. Naturally, this does have a first-order effect on consumer behaviour but it does not introduce any discontinuities or points of indifferentiability. The properties of this particular valuation function are thus identical to those in a case without reference dependence. This implies that we can simply aggregate demand by integrating individual demand over our distribution $f(\theta)$ as we would do in a model without reference dependence. Thus total demand Q_D becomes:

$$Q_D = \int_{\underline{\theta}(p)}^{\theta_H} q^*(p, \theta) f(\theta) d\theta$$

For loss-aversion ($\lambda > 1$) we encounter several issues. First of all, we can no longer guarantee that individual demand is Riemann-integrable as we cannot generally claim that the set of points of discontinuity of q_D with respect to θ has zero measure. As such, our individual demand function does not generally meet Lebesgue's criterion. Note that points of discontinuity must occur when for some type θ it holds that we must have an intersection between demand and the reference schedule, that is $q(\theta, p) = r(\theta)$. So, for the our demand function to be continuous almost everywhere (i.e. the set of of discontinuities has zero measure) it is sufficient to have the set of such intersections be countable.

Intuitively, we need demand to increase smoothly as θ increases for aggregation to make sense. However, due to the fact that we have in no way restricted the reference level a certain consumer may have, demand may jump erratically. To solve this, we can impose that such jumps happen only occassionaly and not in rapid succession. The individual contributions of such jumps to the total are then sufficiently small such that they can be ignored.Unfortunately, it may not be possible to formulate a general set of sufficient conditions on $r(\theta)$ for this to be the case. Indeed, we require specific knowledge of both the reference schedule $r(\theta)$ as well as the valuation function $m(\cdot, \theta)$, thus sacrificing all the generality of the model.

A second issue is that with loss-aversion individual demand is non-differentiable and will need to be considered over specific intervals. This can be done for specific cases, but not generally. To see this, consider a discrete setting for two consumers. For simplicity we assume the first consumer is of type θ_L and the second consumer is of type θ_H. We know then that it must hold that $p_R(\theta_H) > p_R(\theta_L)$ as well as that $p^-(\theta_H) > p^-(\theta_L)$ and $p^+(\theta_H) > p^+(\theta_L)$. However, we do not generally know whether, for example, it holds that $p^-(\theta_H) > p^+(\theta_L)$. We can however impose a certain structure on this setting by limiting the degree of loss aversion and the importance of the gain-loss valuation component. In particular, let us impose the following:

$$p^+(\theta_H) > p^+(\theta_L) > p^-(\theta_H) > p^-(\theta_L)$$

Plugging in our expressions for these prices we get:

$$(1+\eta\lambda)m_q(r(\theta_H),\theta_H) > (1+\eta\lambda)m_q(r(\theta_L),\theta_L) > (1+\eta)m_q(r(\theta_H),\theta_H) > (1+\eta)m_q(r(\theta_L),\theta_L)$$

We know that the first and third inequalities must always hold so we only need to guarantee that the following holds:

$$(1+\eta\lambda)m_q(r(\theta_L),\theta_L) > (1+\eta)m_q(r(\theta_H),\theta_H) \tag{4.3}$$

Dividing both sides by $m_q(r(\theta_L),\theta_L)$ and $(1+\eta)$ yields:

$$\frac{(1+\eta\lambda)}{(1+\eta)} > \frac{m_q(r(\theta_H),\theta_H)}{m_q(r(\theta_L),\theta_L)}$$

The condition above is sufficient for the above structure to hold. Note that since $\frac{m_q(r(\theta_H),\theta_H)}{m_q(r(\theta_L),\theta_L)} \geq \frac{m_q(r(\theta''),\theta'')}{m_q(r(\theta'),\theta')}$ for all $\theta', \theta'' \in \Theta$ such that $\theta'' > \theta'$ this condition is sufficient for any two different consumer types. Based on the structure imposed by this condition total demand in the form of the sum of the two individual demands is given by the following:

$$Q_D = \begin{cases} q^*(p,\theta_H,\eta) + q^*(p,\theta_L,\eta) & \text{if } p^-(\theta_L) > p \\ q^*(p,\theta_H,\eta) + r(\theta_L) & \text{if } p^-(\theta_H) \geq p > p^-(\theta_L) \\ r(\theta_H) + r(\theta_L) & \text{if } p^+(\theta_L) \geq p > p^-(\theta_H) \\ r(\theta_H) + q^*(p,\theta_L,\eta\lambda) & \text{if } p^+(\theta_H) \geq p > p^+(\theta_L) \\ q^*(p,\theta_H,\eta\lambda) + q^*(p,\theta_L,\eta\lambda) & \text{if } p_R(\theta_L) \geq p > p^+(\theta_H) \\ q^*(p,\theta_H,\eta\lambda) & \text{if } p_R(\theta_H) \geq p > p_R(\theta_L) \\ 0 & \text{if } p > p_R(\theta_H) \end{cases}$$

By simply aggregating only two consumers we have moved from a maximum of four to seven possible price intervals that need to be considered seperately. In fact, for an n-consumer discrete setting there are $1+3n$ intervals to be considered (assuming condition (4) above holds). Since we can think of our continuous case as the limit of the discrete setting as n goes to infinity it is obvious that the continuous setting is almost everywhere non-differentiable and therefore largely unworkable. As such, we wil consider an n-person discrete setting in the following sections instead. One way of thinking of this setting is a having a mass of n consumer uniformly distributed over the interval $[\theta_L, \theta_H]$, with the firm only able to imperfectly identify consumer types over this interval. Our strategy will be to analyze the above two-consumer setting and generalize its findings to n consumers. The advantage of the discrete setting is that we can limit the number of points of non-differentiability whilst simulatenousy making it feasible to aggregate over all consumers. With demand fully described we can move on to the monopolist's problem.

Chapter 5

Monopoly Pricing

5.1 No Loss-Aversion Continuous Case

Let us, for sake of completeness, first analyze the continuous case with no loss aversion. As we have seen in the previous section demand is then simply given by the following:

$$Q_D(p) = \int_{\underline{\theta}(p)}^{\theta_H} q^*(p,\theta) f(\theta) d\theta$$

From there we can easily derive an expression for total profit of the monopolist, which is a function only of the price set by the monopolist:

$$\Pi(p) = Q_D(p)p - c(Q_D(p))$$

As we have seen in the previous section $Q_D(p)$ is everywhere differentiable, as is $c(\cdot)$ (by assumption). Therefore we can derive the following first-order condition for profit maximization (note that by the properties of our demand and cost function this first-order condition is sufficient for a maximum):

$$\frac{d\Pi(p)}{dp} = \frac{dQ_D(p)}{dp} p + Q_D(p) - c'(Q_D(p)) \frac{dQ_D(p)}{dp} = 0$$

Which can be rearranged to arive at a rather familiar expression:

$$\frac{p - c'(Q_D(p))}{p} = \frac{1}{\varepsilon_D}$$

Here $\varepsilon_D = -\frac{dQ_D(p)}{dp}\frac{p}{Q_D(p)}$ is the elastic of aggregated demand for the monopolist's good. Thus the monopolist prices according to the standard inverse elasticity monopoly pricing rule. We have already seen that the elasticity of individual demand iis not affected by gain loss aversion (see lemma 1). However, is not necessarily the case for aggregated elasticity. Let us therefore analyze the impact of the coefficient of gain-loss utility on aggregated elasticity in detail. The full expression for this elasticity is given by the following:

$$\varepsilon_D = -\frac{dQ_D(p)}{dp}\frac{p}{Q_D(p)} = \left(\frac{d}{dp}\left[\int_{\underline{\theta}(p)}^{\theta_H} q^*(p,\theta)f(\theta)d\theta\right]\right)\left[\int_{\underline{\theta}(p)}^{\theta_H} q^*(p,\theta)f(\theta)d\theta\right]^{-1} p$$

Working out the first term yields:

$$\frac{d}{dp}\left[\int_{\underline{\theta}(p)}^{\theta_H} q^*(p,\theta)f(\theta)d\theta\right] = \int_{\underline{\theta}(p)}^{\theta_H} \frac{dq^*(p,\theta)}{dp}f(\theta)d\theta - q^*(p,\underline{\theta}(p))f(\underline{\theta}(p))\frac{d\underline{\theta}(p)}{dp}$$

Plugging this into the expression given above:

$$\varepsilon_D = \frac{p\left[q^*(p,\underline{\theta}(p))f(\underline{\theta}(p))\frac{d\underline{\theta}(p)}{dp} - \int_{\underline{\theta}(p)}^{\theta_H}\frac{dq^*(p,\theta)}{dp}f(\theta)d\theta\right]}{\int_{\underline{\theta}(p)}^{\theta_H} q^*(p,\theta)f(\theta)d\theta}$$

Plugging in our expressions for $\frac{d\underline{\theta}(p)}{dp}$ and $\frac{dq^*(p,\theta)}{dp}$ from Lemma's 1 and 2 we get (noting that the second term in the denominator in $\frac{d\underline{\theta}(p)}{dp}$ drops out with no loss aversion since $\mu = \lambda\eta = \eta$):

$$\varepsilon_D = \frac{p}{\int_{\underline{\theta}(p)}^{\theta_H} q^*(p,\theta)f(\theta)d\theta}\left[\frac{q^*(p,\underline{\theta}(p))q^*(p,\theta)f(\underline{\theta}(p))}{(1+\eta)m_{\underline{\theta}}(q^*,\underline{\theta})} - \int_{\underline{\theta}(p)}^{\theta_H}\left[(1+\eta)\frac{\partial^2 m(q^*,\theta)}{\partial q^{*2}}\right]^{-1} f(\theta)d\theta\right]$$

To see what the impact of gain-loss utility is on the mark-up of the monopolist we must thus consider how the price elasticity of demand changes with the coefficient of valuation η. We arrive this in Appendix B. The result is summarized in the following proposition.

Proposition 2 *When gain-loss utility becomes more important (i.e. η increases), the elasticity of demand for the monopolist's product is reduced. We thus have that $\frac{d}{d\eta}\varepsilon_D < 0$. As a result, the firm has more market power and will therefore place a higher mark-up over marginal costs, resulting in higher prices, ceteris paribus.*

Proof The proposition follows directly from the derivation in Appendix B and the discussion above. ■

How can we explain the apparently contradictory result that aggregated elasticities are affected by gain-loss utility whereas individual elasticities are not? The key to this is the participation constraint. Total aggregated elasticity is determined by two effects: (1) the effect of prie changes on the elasticity of consumers that are already consuming and (2) the effect of price changes on consumers reentering or exiting the market (i.e changes in $\underline{\theta}$). We have seen before that the first effect is independent of gain-loss utility and thus aggregated elasticity is not affected. The second effect is however not independent. As η increases more consumers will choose to consume for a given price p. As a result, at every level of η more consumers will be active and fewer consumers will choose not to consume due to a price increase. Therefore, the second effect reduces overall elasticity and completely determines the effect on overall elasticity. We thus see that overall elasticity is reduced.

With this result established it is interesting to consider what the consequence of the above proposition is for firm profits and consumer welfare. It is well known that in a standard setting without gain-loss utility a higher mark-up increases firm profits and decreases consumer welfare. It is turns out that it is trivial to show that the same holds for firm profits in our setting, but that the effect on consumer welfare is ambiguous. This is summarized in Proposition 3 below

Proposition 3 *An increase in the coefficient of gain-loss utility η leads to higher profits for the monopolist. The effect of the increase on consumer welfare is ambiguous, and depends on the exact distribution of types $f(\theta)$ as well as the functional form of the valuation function $m(\cdot,\cdot)$.*

Proof We first adress firm profits. Consider an initial equilibrium at some price p' and resulting quantity $Q_D(p',\eta')$. Now we observe an increase in η from η' to η''. As a result, for the same given price p' we will find that we know have demand equal to $Q_D(p',\eta'') > Q_D(p',\eta')$. This follows from our derivations in Appendix B from which we concluded that $\frac{dq^*}{d\eta} > 0$ and $\frac{d\theta}{d\eta} < 0$. Let us now increase the price to $p'' > p'$ such that $Q_D(p'',\eta'') = Q_D(p',\eta')$. At this price we have the same costs, marginal costs and the same demand as before, but a higher price. It must thus hold that profits have increased, i.e $\Pi(p'',\eta'') > \Pi(p',\eta')$. Note that this price p'' will in general not be equal to the price that results from the increased mark-up as derived in proposition 2 (call this price p^*). Since p^* has been derived by maximizing profits it must hold that $\Pi(p^*, eta'') \geq \Pi(p'',\eta'')$. But we have just seen that shown that $\Pi(p'',\eta'') > \Pi(p',\eta')$ so it must also hold that $\Pi(p*,\eta'') > \Pi(p',\eta')$. So it follows directly that profits must increase as a result of an increase in η and the resulting increased mark-up over marginal costs. This proves the first part of the claim.

Let us now consider consumer welfare. For a given consumer of type θ welfare is given by the following:

$$w(\theta,\eta) = v(q(p(\eta),\eta),\theta) - p(\eta)q(p(\eta),\eta) = (1+\eta)m(q(p(\eta),\eta),\theta) - p(\eta)q(p(\eta),\eta)$$

Taking the derivative with respect to η yields:

$$\frac{\partial w(\theta,\eta)}{\partial \eta} = m(q(p(\eta),\eta),\theta) + (1+\eta)\left(\frac{\partial m(q(p(\eta),\eta),\theta)}{\partial q}\left[\frac{\partial q(p(\eta),\eta)}{\partial p}\frac{dp(\eta)}{d\eta} + \frac{\partial q(p(\eta),\eta)}{\partial \eta}\right]\right) - q(p(\eta),\eta)\frac{dp(\eta)}{d\eta} - p(\eta)\left[\frac{\partial q(p(\eta),\eta)}{\partial p}\frac{dp(\eta)}{d\eta} + \frac{\partial q(p(\eta),\eta)}{\partial \eta}\right]$$

Which can be rearranged to:

$$m(q(p(\eta),\eta),\theta) + [(1+\eta)m_q(m(q(p(\eta),\eta),\theta) - p(\eta)]\left[\frac{\partial q(p(\eta),\eta)}{\partial p}\frac{dp(\eta)}{d\eta} + \frac{\partial q(p(\eta),\eta)}{\partial \eta}\right] - q(p(\eta),\eta)\frac{dp(\eta)}{d\eta}$$

As we observed in the previous section in an interior solution the following must hold for a consumer to be utility maximizing:

$$p(\eta) = (1+\eta)m_q(m(q(p(\eta),\eta),\theta)$$

As such the entire expression above reduces to the following:

$$\frac{\partial w(\theta,\eta)}{\partial \eta} = m(q(p(\eta),\eta),\theta) - q(p(\eta),\eta)\frac{dp(\eta)}{d\eta}$$

Note that the sign of this expression is ambiguous and is dependent on the level of η, and more importantly, the functional form of the valuation schedule $m(\cdot,\cdot)$. In particular welfare is increasing in η if $m(q(p(\eta),\eta),\theta) > q(p(\eta),\eta)\frac{dp(\eta)}{d\eta}$, unaffected if $m(q(p(\eta),\eta),\theta) = q(p(\eta),\eta)\frac{dp(\eta)}{d\eta}$ and decreasing if $m(q(p(\eta),\eta),\theta) < q(p(\eta),\eta)\frac{dp(\eta)}{d\eta}$. Crucially, the sign also varies with the level of θ. More specifically:

$$\frac{\partial^2 w(\theta,\eta)}{\partial \eta \partial \theta} = m_\theta(q(p(\eta),\eta),\theta) + \frac{\partial q(p(\eta),\eta)}{\partial \theta})\left[m_q(q(p(\eta),\eta),\theta) - \frac{dp(\eta)}{d\eta}\right]$$

Note that once again the sign of this derivative is ambigious and depends on the sign of the last term in square brackets. Due to single crossing we have that $m_{q\theta}(q(p(\eta),\eta),\theta) > 0$ whereas $\frac{dp(\eta)}{d\eta}$ does not vary with θ. As a result we see that if there exist types for which this last term is positive it will be positive for high types and negative for low types. In particular, define $\tilde{\theta}$ as the type for which $m_q(q(p(\eta),\eta),\theta) = \frac{dp(\eta)}{d\eta}$. Then for all $\theta > \tilde{\theta}$ it holds that $m_q(q(p(\eta),\eta),\theta) > \frac{dp(\eta)}{d\eta}$ and for all $\theta < \tilde{\theta}$ we have $m_q(q(p(\eta),\eta),\theta) < \frac{dp(\eta)}{d\eta}$. Note that whether such a level $\tilde{\theta}$ exists depends on the exact functional form of $m(\cdot,\cdot)$. Then for some types $\theta > \tilde{\theta}$ it holds that $\frac{\partial^2 w(\theta,\eta)}{\partial \eta \partial \theta} > 0$. Now, fix some function $m(\cdot,\cdot)$ such that there exists some type $\hat{\theta} > \tilde{\theta}$ for which $\frac{\partial w(\hat{\theta},\eta)}{\partial \eta} = 0$ (this is must be the case for some function $m(\cdot,\cdot)$ for which $m(q(p(\eta),\eta),\hat{\theta}) = q(p(\eta),\eta,\hat{\theta})\frac{dp(\eta)}{d\eta}$). Then for all types $\theta > \hat{\theta}$ it holds that $\frac{\partial w(\theta,\eta)}{\partial \eta} > 0$. We thus see that for valuation functions with the properties as described above high types may benefit from the increase in η whereas low types are potentially harmed by it. In particular, if $\hat{\theta} < \theta_L$ everyone is better off. For other valuation functions (e.g. when $\hat{\theta} > \theta_H$) everyone may be hurt. Without making further assumptions and restricting the generality of the model no further claims can be made. The same holds for overall welfare, which is given by the following:

$$W(\eta) = \int_{\underline{\theta}(p,\eta)}^{\theta_H} w(\theta, \eta) f(\theta) d\theta$$

If $\hat{\theta} < \theta_L$ all consumers are better off and thus total welfare increases. If $\hat{\theta} > \theta_H$ all consumers are worse of and total welfare decreases. For intermediate distributions i.e. $\theta_L < \hat{\theta} < \theta_H$ the effect on total welfare is ambigious and is clearly determined by the distribution over consumer types. The relative density concentrated in the types that gain versus the density concentrated in the types that lose together with the magnitude of these gains/losses determines overall welfare. Thus overall welfare depends on the exact functional form of the valuation function $m(\cdot, \cdot)$ and the distribution of types θ, which is what was claimed in the proposition. ■

The fact that profits of the monopolist unambiguously increase as a result of the higher mark-up is consistent with the standard monopoly model, and is easily understood intuitively. The fact that consumer welfare does not unambiguously decrease requires a little more thought. In the traditional monopoly model, increased mark-ups simply imply that, for any quantity demanded the price increases. As consumer utility is directly decreasing in prices we see that they must be worse off as a result. In our model, we see that increasing mark-ups are a result of increasing importance of gain-loss utility which also increases consumer utility, ceteris paribus. This increased gain-loss utility at each level of consumption introduces a second off-setting effect. The overall impact on consumer utility is thus determined by which of these two effects dominates. As indicated in the proposition, this crucially depends on the exact functional form of the valuation function as it is this function that determines the magnitude of the second effect.

At first sight, this conclusion is contradictory to standard economic theory in which higher mark-ups directly result in decreased consumer welfare.In our case we see that for some valuation functions and consumer distributions we have a positive sum-result. This implies that total surplus must have increased as a result of the increase in gain-loss valuation. To see why this occurs, note that (in this no loss-aversion scenario) we have defined total utility to be consumption utility + gain-loss utility, where gain-loss utility is some quantity dependent on the level of consumption and the reference level multiplied by the coefficient of gain-loss utility. As this coefficient increases, utility increases for any given reference level and consumption level and consumption utility decreases only in *relative* terms. Indeed, it is this modelling choice that directly leads to the results in proposition 2 and 3.

One can imagine alternative and equally suitable modelling choices that may not lead to this result. Consider for example the same model as we have considered above but this time consumption utility is weighted by a factor $(1 - \eta)$ such that overall utility stays constant and consumption utility decreases in *absolute* terms. In this case, the positive-sum welfare result can be expected to cease to exist and we return to the zero-sum result found in the traditional model. The disadvantage of this model choice is that any increase in gain-loss valuation implies a direct decrease in consumption valuation. This seems unreasonable, as there is no reason to assume that the utility of consuming a good would decrease in absolute terms as a result of such a change in gain-loss valuation. This is especially true if we consider the consumption of a good to have some intrinsic value which is to be considered separately from its gain-loss component.

A simple example illustrates this point (the example considers quality levels, but the argument direclty generalizes to quantity-bases reference points).Consider a consumer who wishes to purchase a good. The consumer can choose from two quality levels: A or B. In absolute terms, the consumer prefers quality A over quality B (that is, the consumer would pick quality level A if she had a free choice between the two quality levels). She reasonably assumes to be able to purchase a quality B product this is her reference point). Assume now that she is able to acquire a quality A product. By our model, we can decompose the utility she gets from this product into two parts: the first part is her consumption valuation, which covers the objective and subjective performance, convenience and the prestige from consuming a quality A product. The second component is gain-loss valuation, and consists of the utility she receives from having been able to consume a quality A product even though she expected to consume a quality B product. Now consider what happens if this second effect becomes more important to her. Under our modelling choice, the posiitve utility from being able to consume a quality

A product increases, whereas the intrinsic consumption valuation remains unchanged and the consumer is strictly better off. Under the alternative modelling choice, she has increased gain-loss valuation but decreased intrinsic consumption valuation such that the net effect is zero. Which of these is more reasonable? If we consider consumption valuation to be purely intrinsic (as we can in this case) it seems rather absurd to assume that an increase in gain-loss valuation would directly result in a decrease in the intrinsic valuation of consumption. After all the factors determining this valuation (performance, convenience, prestige etc) are not directly affected. After all, gain-loss valuation is a purely psychological phenomenon as is clear from its prospect theory origins. In this simple example, our current modelling choice seems to be more reasonable. On the other hand, our modelling choice makes direct comparisons between equilibria with different gain-loss valuation coefficients difficult. Clearly, both modelling choices have inherent disadvantages. For our purposes we stick with the first modelling choice, as it is genreally accepted in the literature(see the literature review for an overview). Further support for our modelling choice can be found by considering the orginal prospect theory formulation of reference-dependence (Kahneman & Tversky, 1992) in which gain-loss valuation is modelled similarly.

To summarize, in the continuous case without loss-aversion the introduction of gain-loss utility reduces the elasticity of aggregated demand, leading to a higher mark-up of prices over marginal costs. This higher mark-up unambigiously results in higher profits for the monopolist. We however see that the effect on consumer welfare is ambiguous and crucially depends on the functional form of the valuation function $m(\cdot, \cdot)$ and the distribution of consumer types $f(\theta)$.

5.2 Two-Consumer Discrete Case

We now turn to the discrete case with two consumers. As we have seen before, this setting introduces an additional layer of complexity as all the relevant price intervals need to be considered separately. Recalling our expression for aggregated demand for the two person case from the previous section, we now see that the monpolist's profit equals the following:

$$\Pi(p)=\begin{cases} p\left[q^*(p,\theta_H,\eta)+q^*(p,\theta_L,\eta)\right]-c\left(q^*(p,\theta_H,\eta)+q^*(p,\theta_L,\eta)\right) & \text{if } p^-(\theta_L)>p \\ p\left[q^*(p,\theta_H,\eta)+r(\theta_L)\right]-c\left(q^*(p,\theta_H,\eta)+r(\theta_L)\right) & \text{if } p^-(\theta_H)\geq p>p^-(\theta_L) \\ p\left[r(\theta_H)+r(\theta_L)\right]-c\left(r(\theta_H)+r(\theta_L)\right) & \text{if } p^+(\theta_L)\geq p>p^-(\theta_H) \\ p\left[r(\theta_H)+q^*(p,\theta_L,\eta\lambda)\right]-c\left(r(\theta_H)+q^*(p,\theta_L,\eta\lambda)\right) & \text{if } p^+(\theta_H)\geq p>p^+(\theta_L) \\ p\left[q^*(p,\theta_H,\eta\lambda)+q^*(p,\theta_L,\eta\lambda)\right]-c\left(q^*(p,\theta_H,\eta\lambda)+q^*(p,\theta_L,\eta\lambda)\right) & \text{if } p_R(\theta_L)\geq p>p^+(\theta_H) \\ pq^*(p,\theta_H,\eta\lambda)-c\left(q^*(p,\theta_H,\eta\lambda)\right) & \text{if } p_R(\theta_H)\geq p>p_R(\theta_L) \\ 0 & \text{if } p>p_R(\theta_H) \end{cases}$$

We can now proceed by finding the profit maximizing price within each of the price intervals. For an price arbitary interval $[p', p'']$we get:

$$\max \Pi(p)$$

Subject to:

$$p' \leq p \leq p''$$

Note that we can harmlessly replace the strict inequalities on the price intervals by weak inequalities for the purposes of this maximization problem due to continuity of demand. This maximization problem can be written in Karush-Kuhn-Tucker (KKT) form:

$$\max \Pi(p)$$

Subject to:

$$g_1(p) = p' - p \leq 0$$

$$g_2(p) = p - p'' \leq 0$$

Since our inequality constraints g_1 and g_2 are affine functions for all price intervals this constrained maximization problem meets the linear constraint qualification and thus satisfies all required regularity conditions. Furthermore our objective function $\Pi(p)$ is concave and our inequality constraints are both convex (note that strict convexity is not required so any linear function qualifes). As such the necessary KKT conditions are also sufficient. These conditions are given in a general form as well as for each of the relevant sub-intervals in appendix C. Note that it is not necessary to derive these conditions for the interval $p > p_r(\theta_H)$ since profit always equals 0 here. Similarly, for the interval $p^+(\theta_L) \geq p > p^-(\theta_H)$ we see that the unique maximum is found at $p = p^+(\theta_L)$. To see this note that over this entire interval quantities (and thus also production costs) are constant, so maximizing profits is equivalent to maxizing the price within this interval.

From the sets of conditions as outlined in Appendix C we can identify two scenarios:

1. $\mu_1 = \mu_2 = 0$. Here we have an interior solution and the first condition reduces to the standard first-order derivative for profit maximization.

2. $\mu_1 \neq 0$ and $\mu_2 = 0$ or $\mu_1 = 0$ and $\mu_2 \neq 0$. Here we have a corner solution and thus have $p = p'$ or$p = p''$, where p' and p'' are the relevant lower and upper price limit for the interval in question.

Note that we can also have situations in which we have a combination of both scenarios. This is happens if the profit function takes on a global maximum in one of the corner solutions. Since the analysis of such a situation is identical to that of the two scenarios above we do not discuss it seperately. As stated above, an interior solution implies that the KKT problem is in essence reduced to a standard optimization problem. It thus follows directly that the monopolist prices according to the standard inverse elasticity markup rules if it prices in the interior. Thus, for case 1 we once agian get the following pricing rule:

$$\frac{p - c'(Q_D(p))}{p} = \frac{1}{\varepsilon_D}$$

Thus we see that for interior solutions we can once again study the effects of gain-loss utility and loss aversion by studying the impact it has on the elasticity of demand. Before continuing with this analysis, let us consider a corner solution. A corner solution implies that the profit function considered over the interval reaches it global maximum outside of the price interval considered. As seen in proposition 1, changes in the degree of loss-aversion, the importance of gain-loss utility or shifts in the reference level cause the pricing interval for a certain mode of behaviour to widen/narrow or shift. Simultaneoulsy, such changes affect demand and therefore profit within the interval considered. We therefore need to consider the impact on the bounds of the interval as well as the impact on the profit function in such situations.

Before we continue with this analysis, we need to consolidate our findings from this section. After all, the conditions and pricing rule outlined above only identify a set of 6 maximums for the 6 non-trivial price intervals. Thus we have a set of prices $P = \{p_1, p_2, p_3), p_4, p_5, p_6\}$ (note that above we have already derived that $p_3 = p^+(\theta_L)$). This results in a set of profits $\Pi = \{\Pi_1, \Pi_2, \Pi_3, \Pi_4, \Pi_5, \Pi_6\}$. The firm prefers the maximum of this, which is equal to the maximal element of this set: $\Pi* = \{\Pi_i \in \Pi : \Pi_i > \Pi_k,\ \forall\ \Pi_k \in \Pi \setminus \Pi_i\}$. The optimal price is simply the price that yields the profit Π^*. Once again cannot make any general statements about the comparison of these profit levels over these intervals, but we can narrow down the search somewhat by eliminating several corner solutions.

To see why note that the left end point of the price interval i (assume that $i \notin \{1, 6\}$) is equal to the right endpoint of the price interval $i-1$. Similarly, the right end point equals the left end point of the interval $i+1$. This is because our demand function, and thus our profit function, is continuous everywhere. Consider now that the profit maximization problem over interval i yields a right side corner solution i.e $p_i^* = p_i''$. This implies that $\Pi_i(p_i'') \geq \Pi_i(p_i), \forall\ p_i \in [p_i', p_i'']$. Furthermore we have that $p_i'' = p_{i+1}'$ and $\Pi_i(p_i'') = \Pi_{i+1}(p_{i+1}')$ by continuity. Now consider $p_{i+1}^* \in [p_{i+}', p_{i+1}'']$. By definition it holds that $\Pi_i(p_{i+1}^*) \geq \Pi_{i+1}(p_{i+1}), \forall\ p_{i+1} \in [p_{i+1}', p_{i+1}'']$. In particular it holds that $\Pi_i(p_{i+1}^*) \geq \Pi_{i+1}(p_{i+1}') = \Pi_i(p_i'')$ with equality holding only if $p_{i+1}^* = p_{i+1}'$. From this it follows that if there is a right corner solution in interval i the maximized profit over i can only be as high as the maximized profit in interval $i+1$ which only occurs if there is a right corner solution in interval $i+1$. The exact same applies to left corner solutions: if there is a left corner solution in interval i the profit can at most be as high as the profit maximized over interval $i-1$ with equality holding only if there is a right corner solution for interval $i-1$. The direct consequence of this is that we can ignore any corner solutions in intervals i that do not match with the corner solutions of intervals i and $i+1$ respectively (depending on whether it is a left or right corner solution). Thus we only consider such matching corner solutions and interior solutions when looking for the optimal price.

Since the findings of this subsection have been rather trivial so far it will suffice to summarize them in the following observation:

Observation 2 *In the two-consumer discrete case with loss aversion we consider pricing separately over the 6 non-trivial price intervals. For interior optima over these price intervals the monopolist prices according to the inverse elasticity mark-up rule. Optimal profit and optimal prices can be derived by comparing the prices over these subintervals. We can ignore any corner solutions over price intervals that do not match with corner solutions in adjacent price intervals when looking for these optimal quantities.*

With this established we can turn our attention to the impact of gain-loss utility and loss-aversion on consumer pricing. Note that due to the generality of the model we cannot make any blanket statements about the price interval in which the firm optimally prices. This also implies that we cannot generally determine if the firm will price in a different interval as a result of a change in the degree of loss aversion/gain loss aversion or shifts in the reference schedule. We can however look at changes internal to the 5 non-trivial price intervals (for the interval $p^{-}(\theta_H) \leq p \leq p^{+}(\theta_L)$ the firm's pricing responses to changes in these parameters follow directly from proposition 1, whereas the other intervals are excluded by assumption). Note that we must analyze interior and corner solutions seperately. We consider interior solutions first. Since interior prices are set according to an inverse elasticity rule, it suffices to show how elasticities react to changes in η, λ and $r(\theta)$. We can consider movements from interior solutions to other interior solutions without loss of generality by considering only infinitesmal changes in these parameters. Appendix D derives explicit expressions for elasticities of demand over our non-trivial pricing intervals and considers their responses to the various parameters.

We find that when considering shifts in η for interior soutions we find the exact same result as for the continuous no loss-aversion case. Since this section simply considers a discrete version of that particular model this was to be expected. Note that the same caveats with respect to the interpretation of this result apply. Furthermore we find a very similar result with respect to the coefficient of loss-aversion over the relevant price intervals. This is once again to be expected since the coefficient of loss-aversion and the coefficient of gain-loss utility enter into the model multiplicatively. As such their first-order effect is identical. From an intuitive perspective the result makes a lot of sense. When the degree of loss-aversion increases consumers have an incentive to consume more at any given price, since consuming less than the reference level has a greater negative impact on utility. The monopolist responds by setting higher prices, increasing profits. Note that loss-aversion and gain-loss valuation are complementary, meaning that for higher levels of loss-aversion increases in gain-loss valuation lead to higher mark-ups by the monopolist and vice versa. As in proposition 1, this result can be understood by noting that loss-aversion is measured in units of gain-loss utility.

Shifts in the reference level of consumption can be understood in a similar manner. As the reference level of consumption increases, consumers find themselves in the loss-aversion domain at lower price levels (which follows from the leftward shift in p^- and p^+ as derived in proposition 1). Since the consumer has an incentive to avoid disutilility from loss-aversion it is optimal to consume more at any given price. The firm optimally responds by setting higher prices. Simultaneously the consumer experiences a higher level of disutility from gain-loss aversion at any level of consumption since the consumption level is now compared to a higher reference level of consumption. We summarize the above findings in Lemma 4:

Lemma 4 *In interior solutions, the monopolist sets its Lerner mark-up according to the standard inverse elasticity rule. Positive shifts in the gain-loss valuation coefficient η, the loss-aversioni coefficient λ and the reference level of consumption $r(\theta)$ all cause demand to be less elastic, ceteris paribus. As such, the monopolist increases its mark-up over marginal costs.*

Proof The lemma follows from the derivations in Appendix D. ■

As can be seen above, the analysis of interior solutions is highly similar to that of the continuous-loss aversion case. Indeed, we have seen that both cases follow the same elasticity mark-up rule. The two-person loss-aversion model however also introduces the possibility of corner solutions, which we will now analyze separately. We will analyze corner solutions for an arbitrary price interval such that our findings generalize to all intervals considered. Unfortunately, the following analysis does not offer much in terms of intuition or practical implications, since the existence of corner solutions is artificially imposed by the division of the overall profit-maximization problem into smaller profit-maximization problems over price intervals. The following is however critically important towards developing a full picture of firm price setting behaviour in our setting.To perform this analysis it is important to understand under which conditions a corner solution can occur. In a corner solution, the global maximum of the profit function as it is defined over the relevant price interval (denoted by $[p', p'']$ for a arbitrary price interval) is to be found outside of the boundaries of the interval. That is, the price optimal price p^* that is derived from the inverse elasticity pricing rule falls outside of the interval $[p', p'']$. In general we can have four scenarios:

1. We are in a right-hand corner solution such that $p = p''$ and the optimal price is p^* lies to the right of the interval such that $p^* > p''$.

2. We are in a right-hand corner solution such that $p = p''$ and the optimal price is p^* lies to the left of the interval such that $p^* < p'$.

3. We are in a left-hand corner solution such that $p = p'$ and the optimal price is p^* lies to the right of the interval such that $p^* > p''$.

4. We are in a left-hand corner solution such that $p = p'$ and the optimal price is p^* lies to the left of the interval such that $p^* < p'$.

By our assumptions the firm's profit function is concave over all price intervals. As a result, we can exclude cases 2 and 3. This is illustrated in figures 4a and 4b below. Note that any profit function that yields case 2 and 3 must be convex over some price interval. This can be seen from figure 4b. As such we conclude that these two cases cannot occur and therefore we restrict our analysis to corner solutions as described in cases 1 and 4. Note that cases 3 and 4 are simply mirror images of cases 1 and 2 and thus we omit their plots below.

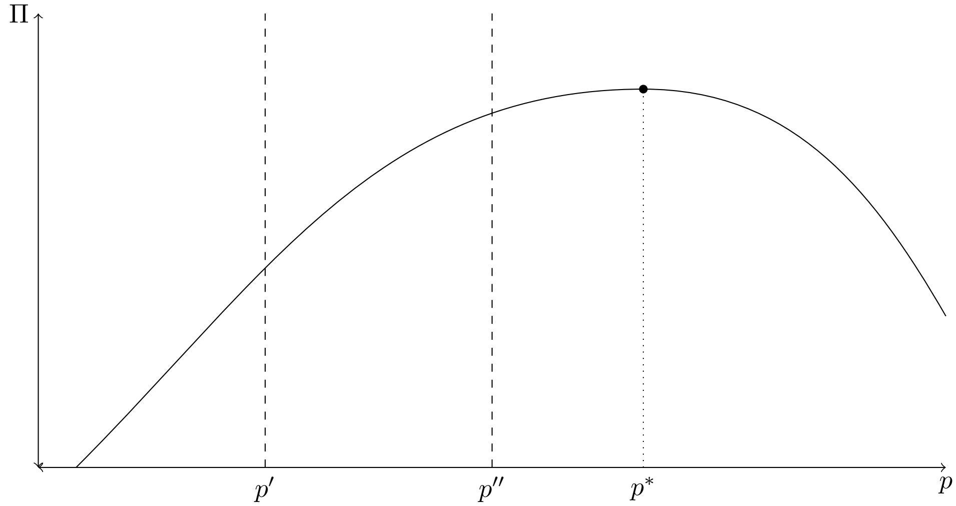

Figure 4a: *Case 1: Example of a right-hand corner solution with $p^* > p''$. Π is concave everywhere.*

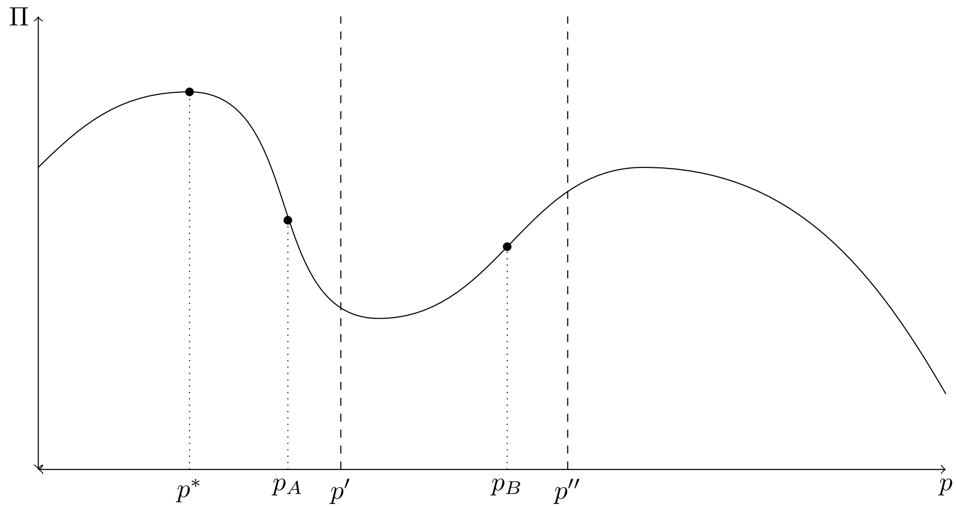

Figure 4b: *Case 2: Example of a right-hand corner solution with $p^* < p'$. Π is convex over $[p_A, p_B]$.*

Let us now first consider case 1. We once again examine shifts in our three main variables of interest: the degree of gain-loss valuation η, the degree of loss aversion λ, and the reference level of consumption $r(\theta)$. From proposition 1 we know that any shift in the limits of the price interval caused by an increase in the degree of gain-loss valuation must be a rightward shift. As such we have that $\frac{dp''}{d\eta} > 0$. Note that since we are in case 1 we only need to consider shifts in the right limit. From our discussion above we know that the same applies to the optimal interior solution such that $\frac{dp^*}{d\eta} > 0$[1]. This leaves us with three distinct outcomes: 1) $\frac{dp^*}{d\eta} > \frac{dp''}{d\eta}$ in which case we always stay in a corner solution, 2) $\frac{dp''}{d\eta} > \frac{dp^*}{d\eta}$ yet we stay in a corner solution, and 3) $\frac{dp''}{d\eta} > \frac{dp^*}{d\eta}$ and we move to an interior solution.

The first scenario follows trivially, as in this case the distance between p^* and p'' increases and we thus reach another corner solution. In the second scenario, the distance between p^* and p'' decreases, however we still have that $p^* > p''$. In the third scenario, the increase in p'' is sufficiently large such that $p'' > p*^*$ for the new level of η. For sufficiently small shifts in η we can derive a condition for whether scenario 2 or scenario 3 occurs based on a first-order Taylor approximation. In particular, we have the following (here pre-shift variables are indicated by a subscript 0 and post-shift variables are indicated by a subscript 1):

$$p_1''(\eta_1) \approx p_0'' + \frac{dp''}{d\eta}(\eta_0)(\eta_1 - \eta_0)$$

$$p_1^*(\eta_1) \approx p_0^* + \frac{dp^*}{d\eta}(\eta_0)(\eta_1 - \eta_0)$$

Here $\frac{dp''}{d\eta}(\eta_0) > \frac{dp^*}{d\eta}(\eta_0)$, $p_0^* > p_0''$, and $\eta_1 > \eta_0$. We are in scenario 2 if $p_1''(\eta_1) \leq p_1^*(\eta_1)$ and in scenario 3 if $p_1''(\eta_1) > p_1^*(\eta_1)$. By the above Taylor expansion this implies that we are in scenario 2 if:

$$\frac{dp''}{d\eta}(\eta_0) \leq \frac{p_0^* - p_0''}{\eta_1 - \eta_0} + \frac{dp^*}{d\eta}(\eta_0)$$

And in scenario 3 if:

[1] To see this simply apply the implicit function theorem to the inverse elasticity pricing rule for interior solutions. Implicitly differentiating the expression for p^* yields this result. The same applies for $\frac{dp^*}{d\lambda}$ and $\frac{dp^*}{r(\theta)}$.

$$\frac{dp''}{d\eta}(\eta_0) > \frac{p_0^* - p_0''}{\eta_1 - \eta_0} + \frac{dp^*}{d\eta}(\eta_0)$$

Which just states formally that we move to an interior solution only if the increase in the boundary of the price interval compensates for both the increase in the interior solution price p^* and the initial difference between the boundary price and the interior solution price $p^* - p''$. In conclusion, we thus observe the following:

Observation 3: *For right-hand corner solutions the following holds with respect to sufficiently small positive shifts in the coefficient of gain-loss valuation (i.e. for $\eta_1 - \eta_0 < \epsilon$):*

1. *If $\frac{dp''}{d\eta}(\eta_0) \leq \frac{p_0^*-p_0''}{\eta_1-\eta_0} + \frac{dp^*}{d\eta}(\eta_0)$ we remain in a corner solution. Note that this condition also covers the cases for which $\frac{dp^*}{d\eta} > \frac{dp''}{d\eta}$.*
2. *If $\frac{dp''}{d\eta}(\eta_0) > \frac{p_0^*-p_0''}{\eta_1-\eta_0} + \frac{dp^*}{d\eta}(\eta_0)$ we move from a corner solution to an interior solution.*

Crucially, in both cases we see that the positive shift in η has led to a price increase.

In the above statement we have not directly specified p''. From proposition 1 it clearly follows that, depending on the interval, p'' equals $p^+(\theta)$ for $\theta \in \{\theta_H, \theta_L\}$ and $p^-(\theta)$ for $\theta \in \{\theta_H, \theta_L\}$. All these prices shift to the right as a result of a shift in η and as such we can move to an interior solution or stay in a corner solution regardless of whether the endpoint equals p^+ or p^-. Next we can perform the exact same analysis for the parameter of loss-aversion, which yields observation 4:

Observation 4: *For right-hand corner solutions the following holds with respect to sufficiently small positive shifts in the coefficient of loss aversion (i.e. for $\lambda_1 - \lambda_0 < \epsilon$):*

1. *If $\frac{dp''}{d\lambda}(\lambda_0) \leq \frac{p_0^*-p_0''}{\lambda_1-\lambda_0} + \frac{dp^*}{d\lambda}(\lambda_0)$ we remain in a corner solution. Note that this condition also covers the cases for which $\frac{dp^*}{d\lambda} > \frac{dp''}{d\lambda}$.*
2. *If $\frac{dp''}{d\lambda}(\lambda_0) > \frac{p_0^*-p_0''}{\lambda_1-\lambda_0} + \frac{dp^*}{d\lambda}(\lambda_0)$ we move from a corner solution to an interior solution.*

Once again, p'' can equal $p^+(\theta)$ for $\theta \in \{\theta_H, \theta_L\}$ and $p^-(\theta)$ for $\theta \in \{\theta_H, \theta_L\}$ depending on the price interval under consideration. From proposition 1 we know that $\frac{dp^+}{d\lambda} > 0$ and $\frac{dp^-}{d\lambda} = 0$. As such we see that we can move from a corner solution to an interior solution only if $p'' = p^+$. For $p'' = p^-$ we always stay in a corner solution since the first condition in observation 4 always holds in this case. Note that as a result we do not necessarily have a price increase (as was the case for shifts in η), since for $p'' = p^-$ we stay in the exact same corner solution.

Finally, we turn to shifts in the reference schedule. By the exact same argument as for λ and η we reach the following observation:

Observation 5: *For right-hand corner solutions the following holds with respect to sufficiently small positive shifts in the reference schedule (i.e. for $r(\theta)_1 - r(\theta)_0 < \epsilon$):*

1. *If $\frac{dp''}{dr(\theta)}(r(\theta)_0) \leq \frac{p_0^* - p_0''}{r(\theta)_1 - r(\theta)_0} + \frac{dp^*}{dr(\theta)}(r(\theta)_0)$ we remain in a corner solution. Note that this condition also covers the cases for which $\frac{dp^*}{dr(\theta)} > \frac{dp''}{dr(\theta)}$.*
2. *If $\frac{dp''}{dr(\theta)}(r(\theta)_0) > \frac{p_0^* - p_0''}{r(\theta)_1 - r(\theta)_0} + \frac{dp^*}{dr(\theta)}(r(\theta)_0)$ we move from a corner solution to an interior solution.*

From proposition 1 we see that for an increase in the reference schedule we have a leftward shift in p'', which can once again be $p^+(\theta)$ or $p^-(\theta)$ for $\theta \in \{\theta_H, \theta_L\}$. Thus we have that $\frac{dp''}{dr(\theta)} < 0$ whilst simultaneously it holds that $\frac{dp^*}{dr(\theta)} > 0$. As a result condition 1 in observation 5 always holds and we will always stay in a corner solution. Crucially, the price set by the monopolist over this interval has decreased.

Having discussed the comparative statics of our three main parameters of interest for the right-hand corner solution case, we can now turn towards analyzing left-hand corner solutions. In many ways the following is simply the above analysis in a setting in which the real-line has been mirrored. We will therefore elect to keep our discussion brief, and choose to focus on the results of our analysis rather than repeating full analysis above in this marginally different setting.

In a left-hand corner solution, we have that $p' > p^*$. If we perform the same analysis based on a first-order Taylor expansion we reach the following observation with respect to shifts in η:

Observation 6: *For left-hand corner solutions the following holds with respect to sufficiently small positive shifts in the coefficient of gain-loss valuation (i.e. for $\eta_1 - \eta_0 < \epsilon$):*

1. *If $\frac{dp^*}{d\eta}(\eta_0) \leq \frac{p_0' - p_0^*}{\eta_1 - \eta_0} + \frac{dp'}{d\eta}(\eta_0)$ we remain in a corner solution. Note that this condition also covers the cases for which $\frac{dp'}{d\eta} > \frac{dp^*}{d\eta}$.*

2. *If $\frac{dp^*}{d\eta}(\eta_0) > \frac{p_0' - p_0^*}{\eta_1 - \eta_0} + \frac{dp'}{d\eta}(\eta_0)$ we move from a corner solution to an interior solution.*

Crucially, in both cases we see that the positive shift in η has led to a price increase.

The left limit of the interval can represent either $p^+(\theta)$ or $p^-(\theta)$ for $\theta \in \{\theta_H, \theta_L\}$. Like in observation 3 all of these prices shift left due to an increase in η and as such both scenarios in observation 6 are feasible for some valuation function $m(\cdot, \cdot)$ and some θ. Similarly, when considering shifts in λ:

Observation 7: *For left-hand corner solutions the following holds with respect to sufficiently small positive shifts in the coefficient of loss aversion (i.e. for $\lambda_1 - \lambda_0 < \epsilon$):*

1. *If $\frac{dp^*}{d\lambda}(\lambda_0) \leq \frac{p_0' - p_0^*}{\lambda_1 - \lambda_0} + \frac{dp'}{d\lambda}(\lambda_0)$ we remain in a corner solution. Note that this condition also covers the cases for which $\frac{dp'}{d\lambda} > \frac{dp^*}{d\lambda}$.*

2. *If $\frac{dp^*}{d\lambda}(\lambda_0) > \frac{p_0' - p_0^*}{\lambda_1 - \lambda_0} + \frac{dp'}{d\lambda}(\lambda_0)$ we move from a corner solution to an interior solution.*

Note that for $p' = p^-(\theta), \theta \in \{\theta_L, \theta_H\}$ it holds that $\frac{dp'}{d\lambda} = 0$. As such we get that we must move to an interior solution if the initial distance $p' - p^*$ is not too large for corner solutions of this type. Note that, unlike for right-hand corner solutions, both scenario's are therefore feasible for $p' = p^-(\theta)$. Just like in right-hand corner solutions it does however hold that prices set by the monopolist increase weakly. The same analysis for $r(\theta)$ yields:

Observation 8: *For left-hand corner solutions the following holds with respect to sufficiently small positive shifts in the reference schedule (i.e. for $r(\theta)_1 - r(\theta)_0 < \epsilon$):*

1. *If $\frac{dp^*}{dr(\theta)}(r(\theta)_0) \leq \frac{p_0' - p_0^*}{r(\theta)_1 - r(\theta)_0} + \frac{dp'}{dr(\theta)}(r(\theta)_0)$ we remain in a corner solution. Note that this condition also covers the cases for which $\frac{dp'}{dr(\theta)} > \frac{dp^*}{dr(\theta)}$.*

2. *If* $\frac{dp^*}{dr(\theta)}(r(\theta)_0) > \frac{p_0'-p_0^*}{r(\theta)_1-r(\theta)_0} + \frac{dp'}{dr(\theta)}(r(\theta)_0)$ *we move from a corner solution to an interior solution.*

We once again see that we get a different result here compared to right-hand corner solution since both scenarios are theoretically feasible (given $p' - p^*$ sufficiently small). If we stay in a corner solution prices decrease unambigiously. If we move to an interior solution however, the result on prices is ambiguous. It decreases if for $p_0' > p_1^* > p_1'$, it stays the same if $p_0' = p_1^* > p_1'$ and increases if $p_1^* > p_0' > p_1'$. The findings of observation 8 and our results from proposition 1 exclude none of these outcomes for interior solutions.

With this established we can conclude our analysis of corner solutions. So far we have derived results for shifts in each of the three parameters of interests in each of the two feasible corner solution scenarios, which we have summarized in a set of 6 observations to provide the analysis with some structure. We are now ready to summarize the analysis of corner solutions in one lemma, which is as follows:

Lemma 5 *There are two feasible corner solutions scenarios: right-hand corner solutions, and left-hand corner solutions. For right-hand corner solutions the following holds for shifts in* η*,* λ *and* $r(\theta)$*:*

1. *For shifts in* η *we can feasibly move to interior solutions or move to corner solutions, both when* $p'' = p^+$ *and* $p'' = p^-$*. In both cases the price set by the monopolist strictly increases.*

2. *For shifts in* λ *we can feasibly move to interior solutions or move to corner solutions for* $p'' = p^+$*, whereas we always stay in corner solutions for* $p'' = p^-$*. As a result the price set by the monopolist weakly increases.*

3. *For shifts in* $r(\theta)$ *we always stay in a corner solution, both when* $p'' = p^+$ *and* $p'' = p^-$*. As a result the price set by the monopolist strictly decreases.*

For left-hand corner solutions the following holds for shifts in η*,* λ *and* $r(\theta)$*:*

1. *For shifts in η we can feasibly move to interior solutions or move to corner solutions, both when $p' = p^+$ and $p' = p^-$. In both cases the price set by the monopolist strictly increases.*

2. *For shifts in λ we can feasibly move to interior solutions or move to corner solutions, both when $p' = p^+$ and $p' = p^-$. The price set by the monopolist weakly increases.*

3. *For shifts in $r(\theta)$ we can feasibly move to interior solutions or move to corner solutions, both when $p' = p^+$ and $p' = p^-$. The effect of the shift on the price set by the monopolist is ambiguous.*

Proof The result follows from combining the results of observations 3-8. $\blacksquare$.

Lemmata 4 and 5 together fully describe the monopolist's response to changes in our three key parameters of interest. As discussed in observation 2 we are unable to characterize equilbria in terms of closed-form functions of the various model parameters since this would involve making far-reaching assumptions regarding the exact functional form of the consumer valuation function, greatly reducing the generality of the model. We can however perform comparative statics exercises with respect to the two key equilibrium outcomes, firm profit and consumer welfare. In particular, we can determine if and how these two quantities are affected by shifts in our parameters of interest based on our general specfication. The results of this analysis are presented as proposition 3 below.

Proposition 3 *In the Two-Consumer Discrete Setting the following holds:*

1. *Positive shifts in η increase monopolist prices and profits, independent of whether we start in a interior or corner solution. Since this result applies to all price intervals, global maximum profits increase strictly.*

2. *Positive shifts in λ increase monopolist prices and profits strictly for interior solutions. For corner solutions prices increase weakly and profits increase strictly. Since this result only applies to the last four price intervals, global maximum profits increase weakly.*

3. *Positive shifts in $r(\theta)$ increase monopolist prices and profits strictly for interior solutions. For corner solutions the effect on prices and profits is ambiguous. Since profits may increase or decrease over different price intervals, effect on global maximum profits is ambigious.*

Consumer welfare may increase or decrease in all cases, depending on the functional form of $m(\cdot,\cdot)$.

Proof We first prove the profit results for interior solutions. The price increases for interior solutions follow direclty from Lemma 4. To get the profit results, consider an initial equilibrium at some price p_0 and resulting quantity $Q_D(p_0, \eta_0, \lambda_0, r(\theta)_0)$. Now we consider three separate increases in our parameters: an increase in η from η_0' to η_1, an increase in λ from λ_0 to λ_1, and in $r(\theta)$ from $r(\theta)_0$ to $r(\theta)_1$. Keeping prices fixed at p_0 the following now holds (which follows from Lemma 4):

$$Q_D(p_0, \eta_1, \lambda_0, r(\theta)_0) > Q_D(p_0, \eta_0, \lambda_0, r(\theta)_0)$$

$$Q_D(p_0, \eta_0, \lambda_1, r(\theta)_0) \geq Q_D(p_0, \eta_0, \lambda_0, r(\theta)_0)$$

$$Q_D(p_0, \eta_0, \lambda_0, r(\theta)_1) \geq Q_D(p_0, \eta_0, \lambda_0, r(\theta)_0)$$

Equality holds in the last two expressions for the price intervals for which $p < p^-(\theta_L)$. Let us now increase prices to p_1 in each of the three cases such that the following holds:

$$Q_D(p_1, \eta_1, \lambda_0, r(\theta)_0) = Q_D(p_0, \eta_0, \lambda_0, r(\theta)_0)$$

$$Q_D(p_1, \eta_0, \lambda_1, r(\theta)_0) = Q_D(p_0, \eta_0, \lambda_0, r(\theta)_0)$$

$$Q_D(p_1, \eta_0, \lambda_0, r(\theta)_1) = Q_D(p_0, \eta_0, \lambda_0, r(\theta)_0)$$

At this price we have the same costs, marginal costs and the same demand as before, but a higher price. It must thus hold that profits have increased, Note that this price p_1 will in general not be equal to the price p^* that results from the increased mark-up as derived in Lemma 4.

Since p^* has been derived by maximizing profits the following must hold:

$$\Pi(p^*, \eta_1, \lambda_0, r(\theta)_0) \geq \Pi(p_1, \eta_1, \lambda_0, r(\theta)_0)$$

$$\Pi(p^*, \eta_0, \lambda_1, r(\theta)_0) \geq \Pi(p_1, \eta_0, \lambda_0, r(\theta)_0)$$

$$\Pi(p^*, \eta_0, \lambda_0, r(\theta)_1) \geq \Pi(p_1, \eta_0, \lambda_0, r(\theta)_0)$$

But we have just seen that shown that the profits on the right-hand side of all three above inequalities where already higher than pre-shift profits. As such it holds that profits have increased. In particular, the above argument applies to the profit over each of the price intervals. I.e Π_i has increased for all $\Pi_i \in \Pi$. We have defined the profit realized by the monopolist as the maximum of the elements in Π. Since each of these elements has increased, the maximum of these elements has also increased. This proves the claim for interior solutions.

The proof for corner solutions is identical to that for interior solutions above, with the important distinction that the effects of prices are different for the three parameters. For shifts in η prices increase strictly over all intervals and the exact same reasoning as above applies. For λ prices increase only weakly by Lemma 5 and only over certain price intervals (intervals $\Pi_4, \Pi_5, \Pi_6, \Pi_7$), whereas by Lemma 4 quantities increase strictly. These effects combined constitute a strict increase in profits over the intervals affected, and a weak increase overall. For shifts in $r(\theta)$ corner solutions can yield both an increase or a decrease in prices by Lemma 5, whereas demand over all price intervals increases by Lemma 4. As such, the combined effect is ambigious. In the cases where prices increase, profits must striclty increase. In the cases where prices fall the overall outcome depends on if the price decrease dominates the demand increase and vice versa.

To prove the result with respect to consumer welfare, note that we can simply turn to our derivations of this result in Proposition 2 for individual welfare functions. There it was shown that the sign of $\frac{\partial w}{\partial \eta}$ is ambiguous and may vary with θ. Simpy replacing η by λ and $r(\theta)$ in the derivation in proposition 2 yields that the same holds for $\frac{\partial w}{\partial \lambda}$ and $\frac{\partial w}{\partial r(\theta)}$. As such, the overall effect on individual welfare is ambiguous and dependent on θ and the exact functional form of

$m(\cdot,\cdot)$. Clearly the same applies to total weflare, which is the sum of individual welfare. This concludes the proof. ■

The interpretation of the results with respect to the coefficient of gain-loss valuation is identical to the no loss-aversion continuous case. As it increases, it becomes more attractive for consumers to consume beyond their reference point, whereas consuming less than the reference level becomes more costly. As such, consumer demand increases. In this discrete case, we furthermore see that the price range over which each interval is defined shifts up as well. The combined result is that the monopolist is able to increase its profits. For consumers, we have two effects that counteract each other. First of all, at each level of consumption consumer welfare increases for a given price. However, the monopolist also increases prices . If the first effect dominates the second, consumer welfare increases. Otherwise, consumer welfare decreases. Which of these two scenarios applies is theoretically ambiguous without making further assumptions on the valuation schedule $m(\cdot,\cdot)$.

The intuition for the coefficient of loss aversion is identical, albeit over a limited range of prices. Furthermore, the impact on consumer welfare is one-sided. That is, consuming below the reference level becomes more costly, but consuming more does not yield more utility. Therefore, demand only increases over the price intervals for which this is relevant. These intervals do however shift like in the gain-loss aversion case. The result is that profits only weakly increase. To give a specific example of a case for which profits do not increase, consider an initial global equilibrium at a price for which we are beyond the reference level of consumption. In such a case, the maximum profit was realized in one of the seven price intervals for which $q^* > r(\theta)$ for both consumers. If loss-aversion becomes more important, profit increases in the intervals for which the coefficient is relevant (i.e. for higher price levels). However we may still observe that the global maximum is realized at the previous price level. In this case profits thus remain unchanged. We therefore see that profits only weakly increase.

For shifts in the reference level, we see that, similarly to increases in loss-aversion, consuming below the reference level becomes more costly. As a result, demand increases over the interior of the relevant price intervals. In response the firms have an incentive to set a higher price in the interior of these intervals. Simultaneously, the limits of these price intervals must shift left, depressing the maximum price that can be set in each interval. Depending on which of these two effects dominates profits either increase or decrease. Note that it may be the case that profits in some intervals decrease whereas profits for other intervals increase. Since the global maximum is the maximum from the profits over all these intervals the overall effects is therefore also ambiguous. [2]

This concludes our description of our two-person case. Our results with respect to gain-loss valuation are virtually identical to those derived for the no loss-aversion continuous case, as was to be expected. We can now generalize our findings to a more general n-consumer setting.

5.3 n-Consumer Discrete Case

Let us start by describing the setting in detail. We consider, without loss of generality, n consumers with different taste parameters, indexed by i. In particular, and once again without loss of generality, let us assume that $\theta_1 < \theta_2 < \ldots < \theta_{n-1} < \theta_n$ with $\theta_1 = \theta_L$ and $\theta_n = \theta_H$. The setting is now described by three sets of prices: a set of reservation prices $P_R = \{p_R(\theta_1), \ldots, p_R(\theta_n)\}$, a set of mininum reference level consumption prices $P^- = \{p^-(\theta_1), \ldots, p^-(\theta_n)\}$, and finally a set of maximum reference level consumption prices $P^+ = \{p^+(\theta_1), \ldots, p^+(\theta_n)\}$. For each set it holds that $p(\theta_i) < p(\theta_{i+1})$. These prices determine $3n + 1$ price intervals, $3n - 1$ of which are non-trivial. Demand over these intervals is derived identically to the two consumer case.

It should be clear that Lemmata 4 and 5 directly generalize to this n-person case. This follows directly for Lemma 5 since the results derived there were derived without making specific assumptions about the price intervals and thus generalize directly. For Lemma 4 this follows directly from our derivations in Appendix D. Since Lemmata 4 and 5 form the basis for Propo-

[2] Unfortunately, this explanation is far from intuitive, which is largely an artifact of the artificial imposition of the pricing intervals in the construction of our KKT problem.

sition 3, we see that this proposition also applies to the n-person setting. In conclusion we thus see that the discussion of our n person setting simply reduces to the analysis of the two consumer case as all our findings generalize.

5.4 Price Discrimination

Before we commence with the analysis of price regulation we will briefly discuss price discrimination. It turns out that the analysis of price discrimination under complete information is relatively simple to analyze, especially when compared to the rather contrived case-by-case analysis of the previous section[3]. Intuitively, this is because we can avoid a lot of the issues that result from having to consider aggregation of demand over distinct price intervals, since we can price to each consumer directly. To see this, consider the continuous loss-aversion setting introduced in our model specification. Now we will assume that q denotes the quality of the product, with $c(q)$ equaling the cost of producing one product of quality q. Let us define per consumer surplus as follows:

$$S(q(\theta), \theta) = (1 + \mu)m((q(\theta), \theta) + (\lambda\eta - \mu)m(r(\theta), \theta) - c((q(\theta)$$

Let us now define:

$$\bar{S}(q, \mu, \theta) = (1 + \mu)m((q(\theta), \theta) - c((q(\theta)$$

$$\bar{q}(\theta, \mu) = argmax\ \bar{S}(q, \mu, \theta)$$

It follows from Lemma 1 that $\bar{q}$ exists and equals $\bar{q}(\theta, \eta)$ for $\bar{q}(\theta, \mu) \geq r(\theta)$ and $\bar{q}(\theta, \eta\lambda)$ for $\bar{q}(\theta, \mu) \leq r(\theta)$. By lemma 1 we also know that $\bar{q}$ is strictly increasing and continuously differentiable for $\bar{q} \neq r(\theta)$. Furthermore we have that:

$$\bar{q}(\theta, \eta\lambda) > \bar{q}(\theta, \eta) > 0$$

[3]The discussion below is a complete-information adaption of the result for incomplete information setting as analyzed in Carbajal & Ely (2014). The original analysis focuses on maximizing firm profits, whereas our pricing rule aims to be welfare maximizing. Readers interested in comparative statics results with respect to the optimal contract in Proposition 4 are encouraged to read the original paper since their imperfect information results generalize directly to the complete information case.

We can now define the optimal contract menu under complete information:

Proposition 4 *The complete information optimal contract menu $\{q^*(\theta), p^*(\theta)\}$ for consumers with reference dependent preferences and loss aversion is given by:*

$$q^*(\theta) = \begin{cases} \bar{q}(\theta, \eta\lambda) & \text{for } \bar{q}(\theta, \eta\lambda) \leq r(\theta) \\ r(\theta) & \text{for } \bar{q}(\theta, \eta\lambda) \geq r(\theta) \geq \bar{q}(\theta, \eta) \\ \bar{q}(\theta, \eta) & \text{for } \bar{q}(\theta, \eta) \geq r(\theta) \end{cases}$$

With:

$$p^* = (1 + \mu)m((q^*(\theta), \theta) + (\lambda\eta - \mu)m(r(\theta), \theta) - c((q^*(\theta)$$

Proof For $\theta \in \Theta$ assume that $r(\theta) > \bar{q}(\theta, \eta\lambda)$ in which case $S(q(\theta), \theta) = \bar{S}(q, \mu, \theta)$. For all $q \in \mathbb{R}^+$ such that $r(\theta) > q$ the surplus function is unchanged with a unique maximum at $q = \bar{q}(\theta, \eta\lambda)$. Setting $q > r(\theta)$ changes the surplus function. However since $r(\theta) > \bar{q}(\theta, \eta)$ the only deviation that can increase surplus is $q = r(\theta)$. Surplus equals:

$$(1 + \eta\lambda)m(r(\theta), \theta) - c(r(\theta))$$

But by definition $\bar{q}(\theta, \eta\lambda)$ is the unique maximizer of consumer surplus for all $q \leq r(\theta)$ and therefore:

$$S(\bar{q}(\theta, \eta\lambda), \theta) \geq (1 + \eta\lambda)m(r(\theta), \theta) - c(r(\theta))$$

Thus there is no profitable deviation. Now assume $\bar{q}(\theta, \eta) \geq r(\theta)$. Note that for $q > r(\theta)$ it holds that $\bar{S}$ is simply S constant-shifted. As such, the unique maximizer of $\bar{S}$ equals the unique maximizer of S over this interval. By definition, $\bar{q}(\theta, \eta) \geq r(\theta)$ is thus the unique maximizer of S over all $q > r(\theta)$. Now assume a deviation to $q < r(\theta)$. The unique maximizer of S for this interval equals $\bar{q}(\theta, \eta\lambda)$. But it holds that $\bar{q}(\theta, \eta\lambda) > \bar{q}(\theta, \eta) \geq r(\theta)r(\theta)$. Clearly then, this deviation does not increase surplus since the unique maximizer over this interval was $\bar{q}(\theta, \eta)$. Now consider $q = r(\theta)$. By the exact same reasoning as before, this deviation can also not increase surplus since by definition we have that:

$$S(\bar{q}(\theta,\eta),\theta) \geq (1+\eta\lambda)m(r(\theta),\theta) - c(r(\theta))$$

So once again there is no profitable deviation. Finally consider $\bar{q}(\theta,\eta\lambda) \geq r(\theta) \geq \bar{q}(\theta,\eta)$.By choosing $q > r(\theta)$ we are in the strictly decreasing part of the total surplus. This follows from concavity of the profit function and the fact that $\bar{q}(\theta,\eta) \leq r(\theta)$. Similarly, by choosing $q < r(\theta)$ we are in the strictly increasing section of the total surplus, once again by concavity and the fact that $\bar{q}(\theta,\eta\lambda) \geq r(\theta)$. Therefore, surplus is uniquely maximized at $q^* = r(\theta)$.

To complete the proof note the firm can capture all surplus by simply setting its price equal to the surplus that is realized for a given price, which is exactly what the proposition states. ■

Proposition 4 aims to illustrate that contracts and the analysis of pricing is relatively simple when we can separate consumers (in this case through perfect price discrimination). The fact that we can look at consumers one at a time means we only need to concern ourselves with one reference point and therefore the analysis becomes significantly less involved. Before we continue with our analysis of price regulation for our continuous no loss-aversion and two persion loss-aversion setting we observe that the welfare maximizing contract menu trivially follows from proposition 4. As such we display it here instead of deferring it to the next chapter.

Corollary 1 *The complete information welfare maximizing optimal contract menu* $\{q^*(\theta), p^*(\theta)\}$ *for consumers with reference dependent preferences and loss aversion is as in proposition 4, except for the price which is given by* $p^* = c(q^*)$.

Note that by setting the price p^* we simply divide the maximized surplus between the consumer and monopolist. In proposition 4 all the surplus goes to the monopolist whereas in corollary 1 we set the price such that all surplus goes to the consumer whilst still ensuring the monopolist generates non-negative (zero) profits.

Chapter 6

Second-Best Ramsey Pricing

6.1 A Note on Price Regulation

In the above discussion we have seen that the introduction of reference dependence and loss-aversion in our monopoly model can lead to decreased consumer welfare under certain conditions. When this is the case it is interesting to consider how a government or other regulator can effectively implement price regulations in this new setting. The general goal of price regulation is to reach an optimal outcome in a market by setting prices. This requires us to define what we understand welfare to be. One option is to simply sum consumer welfare and firm profits. The problem with this approach is that it raises questions with regards to equity. Under this definition of welfare, for example, a perfectly discriminating firm capturing all surplus would be considered to be an efficient outcome. From a government's perspective this is clearly not desirable. Another alternative is to maximize consumer welfare. If we disregard the firm, this approach yields an efficient outcome in that prices equal marginal costs (Tirole, 1988). In the presence of fixed costs however, such an outcome would yield negative economic profit for the firm, which is of course not sustainable. Therefore, this first-best equilibrium is not practically feasible from the regulator's perspective.

Clearly we must look for a second-best alternative which can be feasibly implemented. One approach is to simply maximize consumer welfare under the constraint that the frm's economic profit should be non-negative (typically a zero-profit constraint is introduced for simplicity). This leads to full-information second-best prices, which can be shown to simply equal average costs for the monopolist, thus ensuring the zero-profit constraint holds. Alternatively, one can implement a two-part tariff scheme, in which prices are set equal to marginal costs and the firm receives a fixed access fee from the government that covers fixed costs. Under a weak set of regularity conditions both schemes will yield the same outcome (Tirole, 1988).

A final alternative, and the one we will consider in the following sections, is that of Ramsey pricing (named after Frank P. Ramsey's (1927) result). Although the terminology is typically reserved for a multi-good monopoly, the idea also applies to our one good setting. The idea here is to maximize the sum of consumer and producer utility, subject to a zero-profit constraint. Whilst this may seem counterintuitive, the approach has several interesting features. Most importantly, the optimal pricing rule can be expressed without explicitly specifying the cost structure of the firm. Secondly, the pricing rules resulting from this solution can be easily interpreted and analyzed in terms of comparative statics.

In the Ramsey framework, we maximize the sum of consumer welfare and firm profits subject to a profit constraint, which yields the following Lagrangian:

$$\mathcal{L}(p) = W(p) + \Pi(p) - \gamma(\Pi(p) - \Pi_0)$$

Commonly, the profit constraint is set such that firm profits are zero, i.e $\Pi_0 = 0$. In that case, we can rewrite as follows:

$$\mathcal{L}(p) = W(p) + (1 - \gamma)\Pi(p)$$

Note that γ, the Lagrange multiplier, must be negative. This follows from the fact that if we require higher profits for the firm prices must be set at a higher level, increasing deadweight loss. Therefore we can write $\mathcal{L}(p) = W(p) + \delta\Pi(p)$ with $\delta > 1$. It is this framework that we will use for our analysis in the following sections.

6.2 No Loss-Aversion Continuous Case

For the no loss-aversion continuous setting the objective function as derived above can be rewritten as follows:

$$\mathcal{L}(p, \eta) = W(p, \eta) + \delta \left[Q_D(p)p - c(Q_D(p))\right]$$

It holds that $\frac{\partial W(p,\eta)}{\partial p} = -Q_D(p)$. To see this, note that we can write individual welfare in surplus form as $w(p, \eta) = \int_p^{\infty} q(s, \eta)ds$. The derivative of this with respect to p equals $-q(p, \eta)$. Total welfare is simply the integral of this which equals $-Q_D(p)$ by definition. We can now write the first order condition as follows:

$$\frac{\partial \mathcal{L}(p, \eta)}{\partial p} = -Q_D(p) + \delta \left[p - c'(Q_D(p))\right] \frac{\partial Q_D(p)}{\partial p} + \delta Q_D(p)$$

This can be rearranged to yield:

$$\frac{\delta - 1}{\delta} Q_D(p) = -\left[p - c'(Q_D(p))\right] \frac{\partial Q_D(p)}{\partial p} \Rightarrow \frac{p - c'(Q_D(p))}{p} = \frac{\phi}{\varepsilon_D}$$

Where $\phi = \frac{\delta - 1}{\delta}$ and ε_D is the elasticity of demand as derived before. Note that $1 > \phi > 0$. This is the standard Ramsey mark-up expression. Note that since $\phi < 1$ the mark-up is naturally smaller than that derived under monopoly pricing. In particular, the mark-up is set such that the zero-profit condition is met. In the case for which there are no fixed costs (i.e $c(0) = 0$) the profit constraint is not binding and hence $\phi = 0$ such that price is set equal to marginal cost. If we have fixed costs the profit constraint binds and thus $\phi > 0$ such that we have a positive mark up covers fixed costs. If, for some reason, our profit requirement increases such that Π_0 becomes positive we also see an increase in mark-up to allow for this extra profit. Note that we always have a strictly smaller mark-up compared to the monopolist's own pricing rule. This follows by noting that $\phi \to 1$ as $\Pi_0 \to \infty$. The role of the elasticity of demand in the denominator also has a simple intuitive explanation. If demand for a good is more elastic increasing the mark-up to cover fixed costs more caused demand to fall by a relatively large amount. This has two effects: 1) the effect of increasing prices on revenue available to cover fixed costs is small for such goods, since part of the mark-up increase is cannibalized by the fall in demand, and 2) it has a larger impact on consumer welfare (i.e. a larger deadweight loss). As such, it is optimal to set the mark-up low compared to goods with a higher elasticity of demand.

We thus see that the Ramsay analysis yields an optimal pricing rule that is intuitively attractive, in the sense that it decomposes the price mark-up in terms of the stringency of the profit requirement (given by ϕ) and the inverse elasticity of demand. An advantage of this result is that the comparative statics results derived in the previous chapter for changes in the degree of gain-loss valuation directly generalize to the regulator's optimal pricing rule. Furthermore, we can make unambiguous statements about the development of consumer welfare and firm profit as a result of such shifts. This is captured in proposition 5 below:

Proposition 5 *In our continuous no-loss aversion setting an increase in η increases consumer welfare $W(p, \eta)$ whereas firm profits stay fixed at $\Pi = 0$. As a result of the shift the optimal mark-up over marginal-costs increases for $\phi \neq 0$. Furthermore, total revenue falls.*

Proof That firm profits stay fixed at zero follows trivially from the constraint imposed by the Lagrangian. To derive the result with respect to consumer welfare we note that individual consumer welfare $w(p, q, \eta)$ is increasing in η holding p and q fixed. That is, if η shifts from η' to η'' it holds that, for fixed p' and q' that:

$$w(p', q', \eta'') > w(p', q', \eta')$$

Next, we allow consumers to optimally choose their level of consumption q. Clearly, they do so to maximize their welfare. As such, they choose to consume a new quanity q'' of the good in response to the shift in η for which the following holds:

$$w(p', q'', \eta'') \geq w(p', q', \eta'')$$

With equality holding only for consumers for which $q' = q'' = 0$. Finally, we consider the government, that sets prices (subject to the zero-profit constraint) such that prices are maximized.

For this new price p'' consumers once again change their consumption pattern in response to this change in price. Assume this new consumptin level equals q'''. Cleary the regulator takes into account this effect when changing p such that it must hold that:

$$w(p'', q''', \eta'') > w(p', q'', \eta'')$$

Combining all three inequalities it thus follows that consumer welfare must have increased. The claim with respect to the price mark-up follows directly from the proof of proposition 2 and the derivations in appendix B. ■

The conclusion that consumer welfare increases unambiguously makes sense. In our analysis of monopoly pricing we saw that as a result of a shift in η overall welfare increases. In the pure monopoly case, the distribution of the welfare gain over consumers and monopolists was such that monopolist always benefit whereas consumers only benefit if the increase in overall welfare is sufficiently large. When the government sets prices it ensures that welfare accrues to consumers only.

6.3 Two-Consumer Discrete Case

In the two-consumer discrete case we can once again set up a KKT problem over the relevant price intervals. This yields a set of 7 price intervals, two of which are trivial: for $p > p_R(\theta_H)$ welfare and profits and equal to zero for all prices and the regulator is thus indifferent, for $p^+(\theta_L) > p > p^-(\theta_H)$ the regulator clearly sets the lowest price such that $p = p^+(\theta_L)$. The optimal prices corresponding to these price intervals are listed in $P = \{P_1, \ldots, P_7\}$ and are associated with welfare levels as contained in $W = \{W_1, \ldots, W_7\}$. Welfare in the optimum equals the maximal element of this set, label it W^*.Just as before, we can have interior or corner solutions. By continuity we can once again ignore cases for which corner solutions of adjacent intervals do not intersect. The general KKT set up for a given price interval $[p', p'']$ is as follows:

$$\max W(p) + \Pi(p)$$

Subject to:

$$\Pi(p) = 0$$

$$g_1(p) = p' - p \leq 0$$

$$g_2(p) = p - p'' \leq 0$$

This yields the following set of FOC's:

$$\frac{dW(p)}{dp} + \delta\frac{d\Pi(p)}{dp} = \mu_1\frac{dg_1(p)}{dp} + \mu_2\frac{dg_2(p)}{dp}$$

$$\Pi(p) = 0$$

$$g_1(p) \leq 0$$

$$g_2(p) \leq 0$$

$$\mu_1 \geq 0$$

$$\mu_2 \geq 0$$

$$\mu_1 g_1(p) = 0$$

$$\mu_2 g_2(p) = 0$$

It turns out that combining our analysis of regulation in the continuous case in the previous section with the analysis of the two-consumer discrete monopoly pricing case almost trivially yields the results we require. In an interior solution it holds that $\mu_1 = \mu_2 = 0$ and the whole problem thus reduces to the same elasticity mark-up rule as analyzed in the continuous case. and we thus directly arrive at a similar conclusion. For corner solutions we also see that we can directly extend the analysis of the monopolist pricing case with respect to price movements. Simultaneoulsy we see that, by the same argument as applied in proposition 5, we derive the required welfare results. We summarize these results in proposition 6 below:

Proposition 6 *In interior solutions, the regulator sets the Lerner price mark-up according to the Ramsey inverse elasticity rule. Positive shifts in the gain-loss valuation coefficient η, the loss-aversioni coefficient λ and the reference level of consumption $r(\theta)$ all cause demand to be less elastic, ceteris paribus. As such, these shifts lead to increased mark-ups over marginal costs. For corner solutions we have the following:*

1. *For shifts in η we can feasibly move to interior solutions or move to corner solutions, both when $p' = p^+$ and $p' = p^-$. In both cases the price mark-up set by the regulator strictly increases.*
2. *For shifts in λ we can feasibly move to interior solutions or move to corner solutions, both when $p' = p^+$ and $p' = p^-$. The price mark-up set by the regulator weakly increases.*
3. *For shifts in $r(\theta)$ we can feasibly move to interior solutions or move to corner solutions, both when $p' = p^+$ and $p' = p^-$. The effect of the shift on the price mark-up set by the regulator is ambiguous.*

In all cases firm profits remain constant at zero whereas consumer welfare increases.

Proof The proof for the interior case is identical to that of proposition 5 above. For the analysis of loss-aversion and the reference level simply replace η by λ and $r(\theta)$ in that proof. For corner solutions repeat the steps leading to observation 3 to 8 and lemma 4 in the previous chapter to reach the results outlined above. Note that the conclusion with respect to welfare applies to each of the 7 welfare levels in the set W. Welfare in the optimum equals the maximal element of these, so it must also increase in response to shifts. Firm profits equal zero by contruction, and thus the proof is complete. ■

It is important to remark that for certain costs functions there may be price intervals for which the profit of the firm is always strictly negative. In that case, there is no feasible solution over that price interval. Naturally this requires fixed costs to be high or the cost function to be extremely convex[1]. Note however that in such a case the first-order conditions from which our

[1] In particular, $c'(q) > v_q(q, \theta)$ for all $q \in \mathbb{R}^+$ is sufficient to ensure that no equilibrium is feasible

elasticity mark-up rule is derived does not hold, and therefore we must be in a corner solution. As such, these cases are not relevant for interior solutions, but may be relevant for corner solutions.

The intuition of the above result is identical to that derived for the continuous case. As a result of the shifts in the three coefficients of interest total welfare expands. The regulator now sets prices such that any additional welfare must accrue to consumers. The overall result is that consumers are better off as a result of the shifts in these variables. Note that the pricing rule must, by definition of second-best, introduce some small distortions. However, by the government's objective function any such distortions must be minimized. In fact, we can interpret the inverse elasticity Ramsey rule as being distortion minimizing. Larger elasticities of demand imply larger distortions, and thus mark-ups are small for such demand functions to minimize these distortions.

It is clear from the discussion above that the comparative statics of Ramsey pricing are very similar to those of monopolist pricing as derived in the previous chapter. It is this property that makes the Ramsey mark-up formulation of optimal prices extremely attractive in our setting, especially given the rather tedious case-by-case analysis in which we were forced to engage in the last chapter. This also implies that, as in the previous chapter, the results of our analysis generalize directly to an n-person setting. We therefore omit any further discussion of this setting and move on to the next chapter, in which we discuss our results.

Chapter 7

Discussion & Conclusion

The aim of this paper has been to extend a model of consumer demand with reference-dependent preferences and loss-aversion to a multi-consumer setting. Earlier models have typically considered settings with only one consumer, which is often justified as being a representative consumer. Whilst this approach is valid for typical models of consumer behaviour and demand, this justification breaks down in the case of reference dependence. This is primarily due to the fact that such models of demand can, by constructing, only consider one reference level. From a representative consumer point of view this can be thought of as being equivalent to a setting in which all consumers have the exact same reference point, which greatly reduces generality. Models that have accounted for heterogeneity in reference levels have been applied to study price discrimination specifically, but this analysis has not been extended to uniform monopoly pricing.

From this perspective, the analysis in this paper should be considered to be exploratory in nature, and in many ways its aim is to show if standard methods of economic analysis extend to unfamiliar settings. In this respect we have reached some interesting conclusions. First of all, we see that deriving single consumer demand in a general setting is only marginally more involved when compared to models without reference dependence and loss-aversion. It is once we start aggregating demand that we run into significant complications. In particular, aggregation would involve dealing with an infinite number of points of indifferentiability, which means a standard analysis based on first-order conditions fails. As shown in the text, this issue does not arise in a no loss-aversion case. It is for this reason that we have considered a two-person discrete case. As we have seen the conclusions of this analysis generalize directly to an n-person setting.

In our analysis of the continuous setting with no loss-aversion we find that we can express the optimal pricing rule as a standard inverse elasticity rule with respect to the Lerner index. In this setting our parameter of interest is the coefficient of gain-loss aversion. We find that shifts in the gain-loss aversion strictly increase firm profits, whereas consumers may gain or lose, depending on their specific valuation function and distribution. We thus find that it is possible to have a positive-sum result in this behavioural setting. This strongly contrasts with the monopoly analysis in a standard setting, in which any increased price mark-up strictly decreases welfare. The intuition behind this result is simple: for a given price an increase in gain-loss utilty increases consumer utility. The firm is able to increase its price mark-up to capture some of this additional surplus. The direct effect of a shift in the coefficient of gain-loss aversion thus increases consumer utility, the second indirect pricing effect reduces it. It is the net of these two effects that determines whether or not a consumer is better off.

In our two-person discrete setting we find very similar results with respect to shifts in the coefficient of gain-loss aversion, loss-aversion, and the reference schedule. The main difference is that the analysis here was performed on an interval-by-interval basis to account for the non-differentiabilities introduced by loss-aversion. This introduces some artificial complexity to the analysis, particularly with respect to the analysis of corner solutions. These factors do however not alter the intuition behind the results, which follows the same line of argument as that presented in the continuous no loss-aversion setting. We also briefly presented some results about complete information price discrimination in this setting. The discussion there mainly illustrates how much of the complexity in our other analyses hails from the inherent problems with aggregation that we identified in our discussion of demand. With complete information first-degree price discrimination we are able to analyze optimal contracts on the level of an individual consumer. This greatly simplifies the analysis.

It is to be noted, as we did in the main text, that most of the results are not robust with respect to the applied modelling strategy. In particular, we have elected to let the consumption valuation of a certain consumer be unaffected by any changes in gain-loss valuation. One might propose an alternative strategy in which overall utility is a weighted sum of consumption valuation and gain-loss valuation, with weights adjusting to reflect the relative importance of these two factors but always summing to one. The downside of this modelling approach from an intuitive point of view is that it would imply that consumption valuation must decrease proportionally to the relevance of loss-aversion and gain-loss utility. This is inconsistent with the notion that the consumption valuation represents a set of objective and primarily non-psychological factors. It is also inconsisent with gain-loss valuation modelling as typically applied in the literature.

When considering price regulation, we find that the optimal pricing rule can be easily expressed in terms of a standard Ramsey inverse-elasticity form. As such, the standard Ramsey intuition also extends readily to our model: mark-ups over marginal costs are set such that fixed costs are covered and the firm makes zero profit. If the firm's profit constraint is more stringent (i.e. fixed costs are higher) mark-ups are higher. Furthermore, the same elasticity argument holds as in the previous cases. This intuition holds for both the continuous as well as the discrete case. As a result of price regulation we see that consumer welfare strictly increases due to positive shifts in our variables of interest. As discussed before, shifts in these variables increase overall welfare. Through price regulation the regulator can now assure that all of this additional surplus accrues to consumers, keeping firm profits constant.

Our analysis has thus shown that consumer demand, monopoly pricing and price regulation with reference-dependence and loss-aversion share many of the features and insights that we are used to from the standard model of demand. There are however some important differences. First of all, loss-aversion and reference dependence tend to increase monopolist profits when compared to a model without them. This is because the monopolist is able to capitalize on the fact that consumers have an incentive to "hang on" to a level of consumption above their reference level. This has a tendency to reduce the elasticity of demand and therefore also increase the monopolist's mark-up and profits. It is however not the case that consumer welfare is necessarily negatively affected by this pricing behaviour, as we have discussed before. The zero-sum nature of standard monopoly pricing models thus does not generalize. Thus we get the initially paradoxical but logically intuitive result that increased loss-aversion and a higher reference level may positively affect consumer welfare. When a regulator sets prices it can exploit this positive-sum result to always ensure that consumers benefit.

Naturally, there are some limitations to our analysis. First of all, we have chosen a rather general functional form in order to make the results of this analysis broadly generalizable. This however limits the precision of the claims we have been able to make. For example, our claims about consumer welfare under monopoly pricing have generally stated that the results are ambiguous. Allowing a more strictly specified functional form of the valuation function would allow one to make more concrete claims, as well as derive exact conditions under which such results may hold. Naturally this goes at the cost of a certain degree of generalizability. As discussed above, we have also seen that many of our results are not robust to a choice of modelling methodology. We have defended this choice based on an intuitive argument as well as an appeal to the literature, but it may be fruitful to investigate model specificaions that are robust to such considerations.

It would furthermore be useful to derive some more general results with respect to continuous aggregation in the loss-aversion case. We have seen that continuous aggregation in this case leads to infinitely many points of indifferentiability. It may however be the case that in the limit the demand function is sufficiently smooth such that it permits some form of differentiability. At the time of writing, the author is not aware of such a result. Furthermore, it may be fruitful to repeat the analysis in this case with methods that do not depend on differentiability. Several monotone comparative statics techniques may be particularly useful here. Next to these issues, it may be interesting to extend our model to allow for endogenous reference points. As mentioned in our literature review, models with endogenous reference points have risen to prominence within behavioural industrial organization. Typically however, such models have studied extremely basic settings. A common example is a consumer purchasing a single good, facing a binary decision to buy or not to buy. Even in such simple settings, these models have been shown to be highy advanced. As such, we have chosen to limit our study to fixed reference points.

In many ways, the analysis in this paper has been exploratory, and has hopefully paved the way for future research within this field. In particular, our current model and our findings could feasibly be integrated with some of the other developments within behavioural industrial organization. In that sense, the work presented in this thesis has aimed to extend models of reference dependence to a basic multi-consumer setting whilst exploring some of the basic properties of these settings. Simultaneously we have aimed to identify some key issues that arise when making this extensions, providing a starting point for a hopefully fruitful line of further research.

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Appendices

Appendix A: Derivation of the Reservation Price

We can derive the reservation price by using our definition of $\underline{\theta}$. For a type θ consumer to be the lowest type to still consume it must hold that:

$$\theta = \underline{\theta}(q^*, p)$$

Given that demand is a function of price as well we can simply write:

$$\theta = \underline{\theta}(p)$$

By the properties of $\underline{\theta}$ derived in lemma 2 we know that it is monotonically increasing in p. Since the relation is also continuous in p it is invertible. This inverse provides us with the price for which a given θ is the lowest valuation consumer. This is exactly the definition of a reservation price. As such, there exist some function p_R of θ:

$$p_R(\theta)$$

We can derive the derivative of this function easily:

$$\frac{dp_R}{d\theta} = \frac{1}{\underline{\theta}'(p_R(\theta))}$$

From lemma 2 we know that $\underline{\theta}'(p) > 0$ such that we also have that $\frac{dp_R}{d\theta} > 0$. This is exactly what was claimed in the text.

Appendix B: Derivation of Elasticity Response to Gain-Loss Coefficient

Remember from the main text that:

$$\varepsilon_D = \frac{p}{\int_{\underline{\theta}(p)}^{\theta_H} q^*(p,\theta) f(\theta) d\theta} \left[\frac{q^*(p,\underline{\theta}(p)) q^*(p,\theta) f(\underline{\theta}(p))}{(1+\eta) m_{\underline{\theta}}(q^*,\underline{\theta})} - \int_{\underline{\theta}(p)}^{\theta_H} \left[(1+\eta) \frac{\partial^2 m(q^*,\theta)}{\partial q^{*2}} \right]^{-1} f(\theta) d\theta \right]$$

We can now compute the following:

$$\frac{d}{d\eta}\varepsilon_D = \left(\frac{d}{d\eta} \left[\frac{p}{\int_{\underline{\theta}(p)}^{\theta_H} q^*(p,\theta) f(\theta) d\theta} \right] \right) \left[\frac{q^*(p,\underline{\theta}(p)) q^*(p,\theta) f(\underline{\theta}(p))}{(1+\eta) m_{\underline{\theta}}(q^*,\underline{\theta})} - \int_{\underline{\theta}(p)}^{\theta_H} \left[(1+\eta) \frac{\partial^2 m(q^*,\theta)}{\partial q^{*2}} \right]^{-1} f(\theta) d\theta \right] +$$
$$\left(\frac{d}{d\eta} \left[\frac{q^*(p,\underline{\theta}(p)) q^*(p,\theta) f(\underline{\theta}(p))}{(1+\eta) m_{\underline{\theta}}(q^*,\underline{\theta})} - \int_{\underline{\theta}(p)}^{\theta_H} \left[(1+\eta) \frac{\partial^2 m(q^*,\theta)}{\partial q^{*2}} \right]^{-1} f(\theta) d\theta \right] \right) \left[\frac{p}{\int_{\underline{\theta}(p)}^{\theta_H} q^*(p,\theta) f(\theta) d\theta} \right]$$

For the remainder consider that both q and $\underline{\theta}$ are functions of η (which we first assumed to be constant). We can use the methods from lemma 1 and 2 to find expressions for $\frac{d\underline{\theta}(p,\eta)}{d\eta}$ and $\frac{dq^*(p,\theta,\eta)}{d\eta}$. In particular we have:

$$\frac{dq^*}{d\eta} = -\frac{\partial G/\partial \eta}{\partial G/\partial q^*} = -\frac{m_{q^*}(q^*,\theta)}{(1+\eta)\frac{\partial^2 m(q^*,\theta)}{\partial q^{*2}}} > 0$$

And:

$$\frac{d\underline{\theta}}{d\eta} = -\frac{\partial H/\partial \eta}{\partial H/\partial \underline{\theta}} = -\frac{m(q^*,\theta)}{(1+\eta) m_{\underline{\theta}}(q^*,\underline{\theta})} < 0$$

Evaluating the above expression part-by-part we get for the first bracketed expression:

$$\frac{d}{d\eta} \left[\frac{p}{\int_{\underline{\theta}(p,\eta)}^{\theta_H} q^*(p,\theta,\eta) f(\theta) d\theta} \right] = \frac{p \left[\int_{\underline{\theta}(p,\eta)}^{\theta_H} \frac{m_{q^*}(q^*,\theta)}{(1+\eta)\frac{\partial^2 m(q^*,\theta)}{\partial q^{*2}}} f(\theta) d\theta - \frac{m(q^*,\theta)}{(1+\mu) m_{\underline{\theta}}(q^*,\underline{\theta})} q^*(p,\underline{\theta}(p,\eta),\eta) f(\underline{\theta}(p,\eta)) \right]}{\left[\int_{\underline{\theta}(p,\eta)}^{\theta_H} q^*(p,\theta,\eta) f(\theta) d\theta \right]^2} < 0$$

We also know the following about the second bracketed term above:

$$\left[\frac{q^*(p,\underline{\theta}(p,\eta),\eta)q^*(p,\theta,\eta)f(\underline{\theta}(p,\eta))}{(1+\eta)m_{\underline{\theta}}(q^*(\eta),\underline{\theta}(\eta))}-\int_{\underline{\theta}(p,\eta)}^{\theta_H}\left[(1+\eta)\frac{\partial^2 m(q^*(\eta),\theta)}{\partial {q^*}^2}\right]^{-1}f(\theta)d\theta\right]>0$$

Next we evaluate the third bracketed term. We split up the derivative to make its derivation tractable:

$$\begin{aligned}\frac{d}{d\eta}&\left[\frac{q^*(p,\underline{\theta}(p,\eta),\eta)q^*(p,\theta,\eta)f(\underline{\theta}(p,\eta))}{(1+\eta)m_{\underline{\theta}}(q^*(\eta),\underline{\theta}(\eta))}-\int_{\underline{\theta}(p,\eta)}^{\theta_H}\left[(1+\eta)\frac{\partial^2 m(q^*(\eta),\theta)}{\partial {q^*}^2}\right]^{-1}f(\theta)d\theta\right]=\\ &\frac{d}{d\eta}\left[\frac{q^*(p,\underline{\theta}(p,\eta),\eta)q^*(p,\theta,\eta)f(\underline{\theta}(p,\eta))}{(1+\eta)m_{\underline{\theta}}(q^*(\eta),\underline{\theta}(\eta))}\right]-\frac{d}{d\eta}\left[\int_{\underline{\theta}(p,\eta)}^{\theta_H}\left[(1+\eta)\frac{\partial^2 m(q^*(\eta),\theta)}{\partial {q^*}^2}\right]^{-1}f(\theta)d\theta\right]\end{aligned}$$

Splitting this derivative up further:

$$\begin{aligned}\frac{d}{d\eta}\left[\frac{q^*(p,\underline{\theta}(p,\eta),\eta)q^*(p,\theta,\eta)f(\underline{\theta}(p,\eta))}{(1+\eta)m_{\underline{\theta}}(q^*(\eta),\underline{\theta}(\eta))}\right]&=\frac{\frac{d}{d\eta}\left[q^*(p,\underline{\theta}(p,\eta),\eta)q^*(p,\theta,\eta)f(\underline{\theta}(p,\eta))\right](1+\eta)m_{\underline{\theta}}(q^*(\eta),\underline{\theta}(\eta))}{\left[(1+\eta)m_{\underline{\theta}}(q^*(\eta),\underline{\theta}(\eta))\right]^2}-\\ &\frac{\frac{d}{d\eta}\left[(1+\eta)m_{\underline{\theta}}(q^*(\eta),\underline{\theta}(\eta))\right]q^*(p,\underline{\theta}(p,\eta),\eta)q^*(p,\theta,\eta)f(\underline{\theta}(p,\eta))}{\left[(1+\eta)m_{\underline{\theta}}(q^*(\eta),\underline{\theta}(\eta))\right]^2}\end{aligned}$$

And using our earlier definitions we get:

$$\frac{d}{d\eta}\left[q^*(p,\underline{\theta}(p,\eta),\eta)q^*(p,\theta,\eta)f(\underline{\theta}(p,\eta))\right]<0$$

$$\frac{d}{d\eta}\left[(1+\eta)m_{\underline{\theta}}(q^*(\eta),\underline{\theta}(\eta))\right]>0$$

$$q^*(p,\underline{\theta}(p,\eta),\eta)q^*(p,\theta,\eta)f(\underline{\theta}(p,\eta))>0$$

$$(1+\eta)m_{\underline{\theta}}(q^*(\eta),\underline{\theta}(\eta))>0$$

From which it follows that:

$$\frac{d}{d\eta}\left[\frac{q^*(p,\underline{\theta}(p,\eta),\eta)q^*(p,\theta,\eta)f(\underline{\theta}(p,\eta))}{(1+\eta)m_{\underline{\theta}}(q^*(\eta),\underline{\theta}(\eta))}\right] < 0$$

It is also easily seen that for the fourth bracketed term it must hold that:

$$\frac{p}{\int_{\underline{\theta}(p,\eta)}^{\theta_H} q^*(p,\theta,\eta)f(\theta)d\theta} > 0$$

Combining this information yields the following overall result:

$$\frac{d}{d\eta}\varepsilon_D = \underbrace{\left(\frac{d}{d\eta}\left[\frac{p}{\int_{\underline{\theta}(p)}^{\theta_H} q^*(p,\theta)f(\theta)d\theta}\right]\right)}_{<0}\underbrace{\left[\frac{q^*(p,\underline{\theta}(p))q^*(p,\theta)f(\underline{\theta}(p))}{(1+\eta)m_{\underline{\theta}}(q^*,\underline{\theta})} - \int_{\underline{\theta}(p)}^{\theta_H}\left[(1+\eta)\frac{\partial^2 m(q^*,\theta)}{\partial q^{*2}}\right]^{-1} f(\theta)d\theta\right]}_{>0} +$$

$$\underbrace{\left(\frac{d}{d\eta}\left[\frac{q^*(p,\underline{\theta}(p))q^*(p,\theta)f(\underline{\theta}(p))}{(1+\eta)m_{\underline{\theta}}(q^*,\underline{\theta})} - \int_{\underline{\theta}(p)}^{\theta_H}\left[(1+\eta)\frac{\partial^2 m(q^*,\theta)}{\partial q^{*2}}\right]^{-1} f(\theta)d\theta\right]\right)}_{<0}\underbrace{\left[\frac{p}{\int_{\underline{\theta}(p)}^{\theta_H} q^*(p,\theta)f(\theta)d\theta}\right]}_{>0}$$

Or in short:

$$\frac{d}{d\eta}\varepsilon_D < 0$$

Which is what we wanted to show.

Appendix C: KKT conditions

In its most general form the KKT conditions for our maximization problem are given below:

$$\frac{d\Pi(p)}{dp} = \mu_1 \frac{dg_1(p)}{dp} + \mu_2 \frac{dg_2(p)}{dp}$$

$$g_1(p) \leq 0$$

$$g_2(p) \leq 0$$

$$\mu_1 \geq 0$$

$$\mu_2 \geq 0$$

$$\mu_1 g_1(p) = 0$$

$$\mu_2 g_2(p) = 0$$

We now consider these conditions for our intervals. First of all, consider $p < p^{-}(\theta_L)$. Note that we can ignore the $p > 0$ constraint since any positive price will yield a non-negative profit given a suitable cost function $c(\cdot)$.

(1) For $p \leq p^{-}(\theta_L)$:

$$[p - c'(Q_D)] \left[\frac{\partial q^*(p, \theta_H, \eta)}{\partial p} + \frac{\partial q^*(p, \theta_L, \eta)}{\partial p} \right] + [q^*(p, \theta_H, \eta) + q^*(p, \theta_L, \eta)] = \mu_1$$

$$p - p^{-}(\theta_L) \leq 0$$

$$\mu_1 \geq 0$$

$$\mu_1 \left[p - p^{-}(\theta_L) \right] = 0$$

For the other intervals we need to consider both the upper and the lower price constraint.

(2) For $p^-(\theta_L) \leq p \leq p^-\theta_H$:

$$[p - c'(Q_D)] \frac{\partial q^*(p, \theta_H, \eta)}{\partial p} + [q^*(p, \theta_H, \eta) + r(\theta_L)] = \mu_1 - \mu_2$$

$$p - p^-(\theta_H) \leq 0$$

$$p^-(\theta_L) - p \leq 0$$

$$\mu_1 \geq 0$$

$$\mu_2 \geq 0$$

$$\mu_1 \left[p - p^-(\theta_L)\right] = 0$$

$$\mu_2 \left[p^-(\theta_L) - p\right] = 0$$

(3) For $p^+(\theta_L) \leq p \leq p^+(\theta_H)$:

$$[p - c'(Q_D)] \frac{\partial q^*(p, \theta_L, \eta\lambda)}{\partial p} + [q^*(p, \theta_L, \eta\lambda) + r(\theta_H)] = \mu_1 - \mu_2$$

$$p - p^+(\theta_H) \leq 0$$

$$p^+(\theta_L) - p \leq 0$$

$$\mu_1 \geq 0$$

$$\mu_2 \geq 0$$

$$\mu_1 \left[p - p^+(\theta_H)\right] = 0$$

$$\mu_2 \left[p^+(\theta_L) - p\right] = 0$$

(4) For $p^+(\theta_H) \leq p \leq p_R(\theta_L)$:

$$[p - c'(Q_D)]\left[\frac{\partial q^*(p, \theta_L, \eta\lambda)}{\partial p} + \frac{\partial q^*(p, \theta_H, \eta\lambda)}{\partial p}\right] + [q^*(p, \theta_L, \eta\lambda) + q^*(p, \theta_H, \eta\lambda)] = \mu_1 - \mu_2$$

$$p - p_R(\theta_L) \leq 0$$

$$p^+(\theta_H) - p \leq 0$$

$$\mu_1 \geq 0$$

$$\mu_2 \geq 0$$

$$\mu_1 [p - p_R(\theta_L)] = 0$$

$$\mu_2 \left[p^+(\theta_H) - p\right] = 0$$

(5) For $p_R(\theta_L) \leq p \leq p_R(\theta_H)$:

$$[p - c'(Q_D)] \frac{\partial q^*(p, \theta_H, \eta\lambda)}{\partial p} + q^*(p, \theta_H, \eta\lambda) = \mu_1 - \mu_2$$

$$p - p_R(\theta_H) \leq 0$$

$$p_R(\theta_L) - p \leq 0$$

$$\mu_1 \geq 0$$

$$\mu_2 \geq 0$$

$$\mu_1 [p - p_R(\theta_H)] = 0$$

$$\mu_2 [p_R(\theta_L) - p] = 0$$

Appendix D: Elasticities over Non-Trivial Pricing Intervals

D1: Derivation of Elasticities

(1) For $p \leq p^-(\theta_L)$ we have that:

$$\varepsilon_D = -\frac{\partial}{\partial p}\left[q^*(p,\theta_H,\eta) + q^*(p,\theta_H,\eta\lambda)\right]\frac{p}{q^*(p,\theta_H,\eta) + q^*(p,\theta_L,\eta)}$$

By lemma 1 this equals:

$$\varepsilon_D = -\frac{1}{1+\eta}\left[\frac{1}{\frac{\partial^2 m(q^*,\theta_H)}{\partial q^{*2}}} + \frac{1}{\frac{\partial^2 m(q^*,\theta_L)}{\partial q^{*2}}}\right]\frac{p}{q^*(p,\theta_H,\eta) + q^*(p,\theta_L,\eta)}$$

(2) For $p^-(\theta_L) \leq p \leq p^-(\theta_H)$ we have:

$$\varepsilon_D = -\frac{\partial q^*(p,\theta_H,\eta)}{\partial p}\frac{p}{q^*(p,\theta_H,\eta) + r(\theta_L)}$$

Which can be written as:

$$-\frac{1}{(1+\eta)\frac{\partial^2 m(q^*,\theta_H)}{\partial q^{*2}}}\frac{p}{q^*(p,\theta_H,\eta) + r(\theta_L)}$$

(3) For $p^+(\theta_L) \leq p \leq p^+(\theta_H)$ we have:

$$\varepsilon_D = -\frac{\partial q^*(p,\theta_L,\eta\lambda)}{\partial p}\frac{p}{q^*(p,\theta_L,\eta\lambda) + r(\theta_H)}$$

Rewriting as above:

$$-\frac{1}{(1+\eta\lambda)\frac{\partial^2 m(q^*,\theta_L)}{\partial q^{*2}}}\frac{p}{q^*(p,\theta_L,\eta\lambda) + r(\theta_H)}$$

(4) For $p^+(\theta_H) \leq p \leq p_R(\theta_L)$ we have:

$$\varepsilon_D = -\frac{\partial}{\partial p}\left[q^*(p,\theta_H,\eta\lambda) + q^*(p,\theta_H,\eta\lambda)\right]\frac{p}{q^*(p,\theta_H,\eta\lambda) + q^*(p,\theta_L,\eta\lambda)}$$

By lemma 1 this equals:

$$\varepsilon_D = -\frac{1}{1+\eta\lambda}\left[\frac{1}{\frac{\partial^2 m(q^*,\theta_H)}{\partial q^{*2}}} + \frac{1}{\frac{\partial^2 m(q^*,\theta_L)}{\partial q^{*2}}}\right]\frac{p}{q^*(p,\theta_H,\eta\lambda) + q^*(p,\theta_L,\eta\lambda)}$$

(5) Finally for $p_R(\theta_L) \leq p \leq p_R(\theta_H)$ we have the elasticity for a type θ_H consumer as defined in lemma 1:

$$\varepsilon_D = \frac{q^*(p,\theta_H,\eta\lambda)}{m_{q^*}(q^*,\theta_H)}\left[\frac{\partial^2 m(q^*,\theta_H)}{\partial q^{*2}}\right]^{-1}$$

D2: Elasticity Responses to Gain-Loss Coefficient

We once again consider the 5 individual price interval. Note that the limits of these intervals shift in response to changes in the gain-loss coefficient as derived in Proposition 1. This discussion concerns elasticities in the interior of both intervals.

(1) For $p \leq p^-(\theta_L)$:

$$\frac{\partial}{\partial\eta}\varepsilon_D = \frac{\partial}{\partial\eta}\left(-\frac{1}{1+\eta}\left[\frac{1}{\frac{\partial^2 m(q^*,\theta_H)}{\partial q^{*2}}} + \frac{1}{\frac{\partial^2 m(q^*,\theta_L)}{\partial q^{*2}}}\right]\frac{p}{q^*(p,\theta_H,\eta) + q^*(p,\theta_L,\eta)}\right)$$

$$= \frac{1}{(1+\eta)^2}\left[\frac{1}{\frac{\partial^2 m(q^*,\theta_H)}{\partial q^{*2}}} + \frac{1}{\frac{\partial^2 m(q^*,\theta_L)}{\partial q^{*2}}}\right]\frac{p}{q^*(p,\theta_H,\eta) + q^*(p,\theta_L,\eta)} +$$
$$\frac{1}{1+\eta}\left[\frac{\frac{\partial^3 m(q^*,\theta_H)}{\partial q^{*3}}\frac{dq^*(p,\theta_H,\eta)}{d\eta}}{\left(\frac{\partial^2 m(q^*,\theta_H)}{\partial q^{*2}}\right)^2} + \frac{\frac{\partial^3 m(q^*,\theta_L)}{\partial q^{*3}}\frac{dq^*(p,\theta_L,\eta)}{d\eta}}{\left(\frac{\partial^2 m(q^*,\theta_L)}{\partial q^{*2}}\right)^2}\right]\frac{p}{q^*(p,\theta_H,\eta) + q^*(p,\theta_L,\eta)} +$$
$$\frac{1}{1+\eta}\left[\frac{1}{\frac{\partial^2 m(q^*,\theta_H)}{\partial q^{*2}}} + \frac{1}{\frac{\partial^2 m(q^*,\theta_L)}{\partial q^{*2}}}\right]\frac{p\left[\frac{dq^*(p,\theta_H,\eta)}{d\eta} + \frac{dq^*(p,\theta_L,\eta)}{d\eta}\right]}{[q^*(p,\theta_H,\eta) + q^*(p,\theta_L,\eta)]^2}$$

From our previous discussions we know:

$$\left[\frac{1}{\frac{\partial^2 m(q^*,\theta_H)}{\partial q^{*2}}} + \frac{1}{\frac{\partial^2 m(q^*,\theta_L)}{\partial q^{*2}}}\right] < 0$$

$$\frac{p}{q^*(p,\theta_H,\eta) + q^*(p,\theta_L,\eta)} > 0$$

$$\left[\frac{\frac{\partial^3 m(q^*,\theta_H)}{\partial q^{*3}}\frac{dq^*(p,\theta_H,\eta)}{d\eta}}{\left(\frac{\partial^2 m(q^*,\theta_H)}{\partial q^{*2}}\right)^2}+\frac{\frac{\partial^3 m(q^*,\theta_L)}{\partial q^{*3}}\frac{dq^*(p,\theta_L,\eta)}{d\eta}}{\left(\frac{\partial^2 m(q^*,\theta_L)}{\partial q^{*2}}\right)^2}\right]<0$$

$$\frac{p\left[\frac{dq^*(p,\theta_H,\eta)}{d\eta}+\frac{dq^*(p,\theta_L,\eta)}{d\eta}\right]}{\left[q^*(p,\theta_H,\eta)+q^*(p,\theta_L,\eta)\right]^2}>0$$

Plugging these inequalities into our expression for the overall elasticity above yields:

$$\frac{\partial}{\partial\eta}\varepsilon_D<0$$

(2) For $p^-(\theta_L)\leq p\leq p^-(\theta_H)$:

$$\frac{\partial}{\partial\eta}\varepsilon_D=\frac{\partial}{\partial\eta}\left(-\frac{1}{(1+\eta)\frac{\partial^2 m(q^*,\theta_H)}{\partial q^{*2}}}\frac{p}{q^*(p,\theta_H,\eta)+r(\theta_L)}\right)$$

$$=\left[\frac{\frac{\partial^2 m(q^*,\theta_H)}{\partial q^{*2}}+(1+\eta)\frac{\partial^3 m(q^*,\theta_H)}{\partial q^{*3}}\frac{dq^*(p,\theta_H,\eta)}{d\eta}}{\left((1+\eta)\frac{\partial^2 m(q^*,\theta_H)}{\partial q^{*2}}\right)^2}\right]\left[\frac{p}{q^*(p,\theta_H,\eta)+r(\theta_L)}\right]+$$
$$\left[\frac{1}{(1+\eta)\frac{\partial^2 m(q^*,\theta_H)}{\partial q^{*2}}}\right]\left[\frac{p\frac{dq^*(p,\theta_H,\eta)}{d\eta}}{\left[q^*(p,\theta_H,\eta)+r(\theta_L)\right]^2}\right]$$

As earlier derived it holds that:

$$\frac{\partial^2 m(q^*,\theta_H)}{\partial q^{*2}}<0$$

$$\frac{\partial^3 m(q^*,\theta_H)}{\partial q^{*3}}<0$$

$$\frac{dq^*(p,\theta_H,\eta)}{d\eta}>0$$

From which it follows that $\frac{\partial}{\partial\eta}\varepsilon_D<0$.

(3) For $p^+(\theta_L)\leq p\leq p^+(\theta_H)$:

$$\frac{\partial}{\partial\eta}\varepsilon_D = \frac{\partial}{\partial\eta}\left(-\frac{1}{(1+\eta\lambda)\frac{\partial^2 m(q^*,\theta_L)}{\partial q^{*2}}}\frac{p}{q^*(p,\theta_L,\eta\lambda)+r(\theta_H)}\right)$$

$$= \left[\lambda\frac{\frac{\partial^2 m(q^*,\theta_L)}{\partial q^{*2}} + (1+\eta\lambda)\frac{\partial^3 m(q^*,\theta_L)}{\partial q^{*3}}\frac{dq^*(p,\theta_L,\eta\lambda)}{d\eta\lambda}}{\left((1+\eta\lambda)\frac{\partial^2 m(q^*,\theta_L)}{\partial q^{*2}}\right)^2}\right]\left[\frac{p}{q^*(p,\theta_L,\eta\lambda)+r(\theta_H)}\right] + \left[\frac{1}{(1+\eta\lambda)\frac{\partial^2 m(q^*,\theta_L)}{\partial q^{*2}}}\right]\left[\frac{\lambda p\frac{dq^*(p,\theta_L,\eta\lambda)}{d\eta\lambda}}{[q^*(p,\theta_L,\eta\lambda)+r(\theta_H)]^2}\right]$$

By the same inequalities as considered for interval (2) it follows that $\frac{\partial}{\partial\eta}\varepsilon_D < 0$.

(4) For $p^+(\theta_H) \leq p \leq p_R(\theta_L)$:

$$\frac{\partial}{\partial\eta}\varepsilon_D = \frac{\partial}{\partial\eta}\left(-\frac{1}{1+\eta\lambda}\left[\frac{1}{\frac{\partial^2 m(q^*,\theta_H)}{\partial q^{*2}}} + \frac{1}{\frac{\partial^2 m(q^*,\theta_L)}{\partial q^{*2}}}\right]\frac{p}{q^*(p,\theta_H,\eta\lambda)+q^*(p,\theta_L,\eta\lambda)}\right)$$

$$= \frac{\lambda}{(1+\eta\lambda)^2}\left[\frac{1}{\frac{\partial^2 m(q^*,\theta_H)}{\partial q^{*2}}} + \frac{1}{\frac{\partial^2 m(q^*,\theta_L)}{\partial q^{*2}}}\right]\frac{p}{q^*(p,\theta_H,\eta\lambda)+q^*(p,\theta_L,\eta\lambda)} + \frac{\lambda}{1+\eta\lambda}\left[\frac{\frac{\partial^3 m(q^*,\theta_H)}{\partial q^{*3}}\frac{dq^*(p,\theta_H,\eta\lambda)}{d\eta\lambda}}{\left(\frac{\partial^2 m(q^*,\theta_H)}{\partial q^{*2}}\right)^2} + \frac{\frac{\partial^3 m(q^*,\theta_L)}{\partial q^{*3}}\frac{dq^*(p,\theta_L,\eta\lambda)}{d\eta\lambda}}{\left(\frac{\partial^2 m(q^*,\theta_L)}{\partial q^{*2}}\right)^2}\right]\frac{p}{q^*(p,\theta_H,\eta\lambda)+q^*(p,\theta_L,\eta\lambda)} + \frac{\lambda}{1+\eta\lambda}\left[\frac{1}{\frac{\partial^2 m(q^*,\theta_H)}{\partial q^{*2}}} + \frac{1}{\frac{\partial^2 m(q^*,\theta_L)}{\partial q^{*2}}}\right]\frac{p\left[\frac{dq^*(p,\theta_H,\eta\lambda)}{d\eta\lambda} + \frac{dq^*(p,\theta_L,\eta\lambda)}{d\eta\lambda}\right]}{[q^*(p,\theta_H,\eta\lambda)+q^*(p,\theta_L,\eta\lambda)]^2}$$

Referring to the inequalities derived for the first price interval it follows that also here it holds that $\frac{\partial}{\partial\eta}\varepsilon_D < 0$.

(5) Finally for $p_R(\theta_L) \leq p \leq p_R(\theta_H)$:

$$\frac{\partial}{\partial\eta}\varepsilon_D = \frac{\partial}{\partial\eta}\left(\frac{q^*(p,\theta_H,\eta\lambda)}{m_{q^*}(q^*,\theta_H)}\left[\frac{\partial^2 m(q^*,\theta_H)}{\partial q^{*2}}\right]^{-1}\right)$$

$$= \frac{\lambda \frac{dq^*(p,\theta_H,\eta\lambda)}{d\eta\lambda}\left[m_{q^*}(q^*,\theta_H) - q^*(p,\theta_H,\eta\lambda)\frac{\partial^2 m(q^*,\theta_H)}{\partial q^{*2}}\right]}{[m_{q^*}(q^*,\theta_H)]^2}\left[\frac{\partial^2 m(q^*,\theta_H)}{\partial q^{*2}}\right]^{-1} + \frac{q^*(p,\theta_H,\eta\lambda)}{m_{q^*}(q^*,\theta_H)}\frac{\lambda\frac{dq^*(p,\theta_H,\eta\lambda)}{d\eta\lambda}\frac{\partial^3 m(q^*,\theta_H)}{\partial q^{*3}}}{\left[\frac{\partial^2 m(q^*,\theta_H)}{\partial q^{*2}}\right]^2}$$

It follows directly from the inequalities derived earlier that $\frac{\partial}{\partial \eta}\varepsilon_D < 0$.

D3: Elasticity Responses to Loss-Aversion Coefficient

Next we consider the loss aversion coefficient. For this case we only need to look at the last three intervals. First consider the following:

(3) For $p^+(\theta_L) \leq p \leq p^+(\theta_H)$:

$$\frac{\partial}{\partial \lambda}\varepsilon_D = \frac{\partial}{\partial \lambda}\left(-\frac{1}{(1+\eta\lambda)\frac{\partial^2 m(q^*,\theta_L)}{\partial q^{*2}}}\frac{p}{q^*(p,\theta_L,\eta\lambda)+r(\theta_H)}\right)$$

$$= \left[\eta\frac{\frac{\partial^2 m(q^*,\theta_L)}{\partial q^{*2}} + (1+\eta\lambda)\frac{\partial^3 m(q^*,\theta_L)}{\partial q^{*3}}\frac{dq^*(p,\theta_L,\eta\lambda)}{d\eta\lambda}}{\left((1+\eta\lambda)\frac{\partial^2 m(q^*,\theta_L)}{\partial q^{*2}}\right)^2}\right]\left[\frac{p}{q^*(p,\theta_L,\eta\lambda)+r(\theta_H)}\right] + \left[\frac{1}{(1+\eta\lambda)\frac{\partial^2 m(q^*,\theta_L)}{\partial q^{*2}}}\right]\left[\frac{\eta p\frac{dq^*(p,\theta_L,\eta\lambda)}{d\eta\lambda}}{[q^*(p,\theta_L,\eta\lambda)+r(\theta_H)]^2}\right]$$

Note that this expression is functionally identical to the one derived for this interval in section D2. As such it follows that $\frac{\partial}{\partial \lambda}\varepsilon_D < 0$.

(4) For $p^+(\theta_H) \leq p \leq p_R(\theta_L)$:

$$\frac{\partial}{\partial \lambda}\varepsilon_D = \frac{\partial}{\partial \lambda}\left(-\frac{1}{1+\eta\lambda}\left[\frac{1}{\frac{\partial^2 m(q^*,\theta_H)}{\partial q^{*2}}} + \frac{1}{\frac{\partial^2 m(q^*,\theta_L)}{\partial q^{*2}}}\right]\frac{p}{q^*(p,\theta_H,\eta\lambda)+q^*(p,\theta_L,\eta\lambda)}\right)$$

$$= \frac{\eta}{(1+\eta\lambda)^2}\left[\frac{1}{\frac{\partial^2 m(q^*,\theta_H)}{\partial q^{*2}}}+\frac{1}{\frac{\partial^2 m(q^*,\theta_L)}{\partial q^{*2}}}\right]\frac{p}{q^*(p,\theta_H,\eta\lambda)+q^*(p,\theta_L,\eta\lambda)}+$$

$$\frac{\eta}{1+\eta\lambda}\left[\frac{\frac{\partial^3 m(q^*,\theta_H)}{\partial q^{*3}}\frac{dq^*(p,\theta_H,\eta\lambda)}{d\eta\lambda}}{\left(\frac{\partial^2 m(q^*,\theta_H)}{\partial q^{*2}}\right)^2}+\frac{\frac{\partial^3 m(q^*,\theta_L)}{\partial q^{*3}}\frac{dq^*(p,\theta_L,\eta\lambda)}{d\eta\lambda}}{\left(\frac{\partial^2 m(q^*,\theta_L)}{\partial q^{*2}}\right)^2}\right]\frac{p}{q^*(p,\theta_H,\eta\lambda)+q^*(p,\theta_L,\eta\lambda)}+$$

$$\frac{\eta}{1+\eta\lambda}\left[\frac{1}{\frac{\partial^2 m(q^*,\theta_H)}{\partial q^{*2}}}+\frac{1}{\frac{\partial^2 m(q^*,\theta_L)}{\partial q^{*2}}}\right]\frac{p\left[\frac{dq^*(p,\theta_H,\eta\lambda)}{d\eta\lambda}+\frac{dq^*(p,\theta_L,\eta\lambda)}{d\eta\lambda}\right]}{\left[q^*(p,\theta_H,\eta\lambda)+q^*(p,\theta_L,\eta\lambda)\right]^2}$$

Once again this expression is identical to that as derived in for interval (4) in section D2, with all terms pre-multiplied with η instead of λ. As such it once again follows that $\frac{\partial}{\partial\lambda}\varepsilon_D < 0$.

(5) For $p_R(\theta_L) \leq p \leq p_R(\theta_H)$:

$$\frac{\partial}{\partial\lambda}\varepsilon_D = \frac{\partial}{\partial\lambda}\left(\frac{q^*(p,\theta_H,\eta\lambda)}{m_{q^*}(q^*,\theta_H)}\left[\frac{\partial^2 m(q^*,\theta_H)}{\partial q^{*2}}\right]^{-1}\right)$$

$$= \frac{\eta\frac{dq^*(p,\theta_H,\eta\lambda)}{d\eta\lambda}\left[m_{q^*}(q^*,\theta_H)-q^*(p,\theta_H,\eta\lambda)\frac{\partial^2 m(q^*,\theta_H)}{\partial q^{*2}}\right]}{[m_{q^*}(q^*,\theta_H)]^2}\left[\frac{\partial^2 m(q^*,\theta_H)}{\partial q^{*2}}\right]^{-1}+\frac{q^*(p,\theta_H,\eta\lambda)}{m_{q^*}(q^*,\theta_H)}\frac{\eta\frac{dq^*(p,\theta_H,\eta\lambda)}{d\eta\lambda}\frac{\partial^3 m(q^*,\theta_H)}{\partial q^{*3}}}{\left[\frac{\partial^2 m(q^*,\theta_H)}{\partial q^{*2}}\right]^2}$$

By the same comparison to the expression for this interval in D2 it once again follows that $\frac{\partial}{\partial\lambda}\varepsilon_D < 0$.

D4: Elasticity Responses to Reference Schedule Shift

We consider here a uniform shift in the reference schedule. That is, for each θ we assume that $r(\theta)$ shifts the same amount. We only need to consider intervals (2) and (3).

(2) For $p^-(\theta_L) \leq p \leq p^-(\theta_H)$:

$$\frac{\partial}{\partial r(\theta_L)}\varepsilon_D = \frac{\partial}{\partial r(\theta_L)}\left(-\frac{1}{(1+\eta)\frac{\partial^2 m(q^*,\theta_H)}{\partial q^{*2}}}\frac{p}{q^*(p,\theta_H,\eta)+r(\theta_L)}\right)$$

$$= \frac{1}{(1+\eta)\frac{\partial^2 m(q^*,\theta_H)}{\partial q^{*2}}}\frac{p}{\left[q^*(p,\theta_H,\eta)+r(\theta_L)\right]^2}$$

Since we have that $\frac{\partial^2 m(q^*,\theta_H)}{\partial q^{*2}} < 0$ we have that $\frac{\partial}{\partial r(\theta_L)}\varepsilon_D < 0$.

(3) For $p^+(\theta_L) \leq p \leq p^+(\theta_H)$:

$$\frac{\partial}{\partial r(\theta_H)}\varepsilon_D = \frac{\partial}{\partial r(\theta_H)}\left(-\frac{1}{(1+\eta\lambda)\frac{\partial^2 m(q^*,\theta_L)}{\partial q^{*2}}}\frac{p}{q^*(p,\theta_L,\eta\lambda)+r(\theta_H)}\right)$$

$$= \frac{1}{(1+\eta\lambda)\frac{\partial^2 m(q^*,\theta_L)}{\partial q^{*2}}}\frac{p}{\left[q^*(p,\theta_L,\eta\lambda)+r(\theta_H)\right]^2}$$

By the same argument as above we have that $\frac{\partial}{\partial r(\theta_H)}\varepsilon_D < 0$.