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DEPARTMENT OF ECONOMICS

Master Thesis

Master of Economics & Business
MSc Economics of Management and Organizations

June 19, 2015

An Extension on the Standard Cheap-Talk Model of Crawford and Sobel: The Sender First Needs to Collect Information About his Type

Abstract

This paper examines a variation on the standard cheap-talk model of Crawford and Sobel. In Crawford and Sobel, the sender knows his type exactly. However, I make the extension that the sender does not know his type exactly. The sender first needs to collect information about his type, which could be done through determination in which interval his type lies, where an additional interval costs $c \geq 0$. So, the sender knows his type more precisely, if he adds more intervals. The main result that emerges from my model is that the sender does not take a lot of information, because less information improves communication. This is in line with the finding that the sender is less tempted to exaggerate with less information.

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1. Introduction

Communication is nowadays a very important aspect of society. Communication is used to convey information in market mechanisms, and in other ordinary situations as well. It is an activity in which meanings are exchanged by people by responding to each other's signals. Communication can occur, because information is shared with each other by means of sounds and/or shape. The essence of it is that senders and receivers transmit meanings to each other. A feature of communication is the duality or diversity of it. On a question follows a response, on which follows a counter reaction, etcetera. (Rebel, 2000)

This paper only concerns the simple talk form; costless, non-binding and not verifiable statements. Also called cheap-talk, in which information can be simply conveyed from a sender to a receiver (Gibbons, 1992). A very simple example of a cheap-talk statement could be: ''Hey, watch out for that car''. A common and social example of cheap-talk is the choice where two people will have dinner on a certain night. This example is elaborated in the section when cheap talk could be useful in the first place. But maybe more interesting is how cheap-talk can be used in an economic aspect, in the workplace for instance. An example is how an employee assesses the competence of an applicant on the basis of cheap-talk before hiring this candidate for the right job. Suppose you would like to apply for a job at your dream company, like Philips. Then the function Philips gives you depends on your ability:

		Job Philips offers to you	
		Demanding	Non-demanding
Your ability	High	3,1	0,0
	Low	0,0	1,2

Philips would like to offer you a job that suits your ability, so a demanding job if your ability is high and vice versa. However, Philips does not know your ability, in contrast to yourself. It is assumed that you only can enjoy the job that suits your ability. In the table above, your payoff/utility appears first, followed by the payoff of Philips. Through cheap-talk it is now possible to communicate the level of your ability to Philips, which is followed by Philips offering you a matching function. So, cheap-talk could be very useful in economic situations, but it does not guarantee efficiency (Farrel & Rabin, 1996).

I analyze a communication model which is a variation of the standard cheap-talk model.

This standard model is from Crawford and Sobel (1982) and describes the behavior of two players, a sender and a receiver. The sender is an informed agent who gives an advice or sends a message to the uninformed receiver, for example the decision maker. In the basic model of Crawford and Sobel, the sender knows his type exactly.

However, in this paper I seek to understand what happens if I make the extension that the sender does not know his type exactly. The sender first needs to collect some information about his type. He could do that through determination in which interval his type lies, where an additional interval costs $c \geq 0$. The sender knows his type more precisely, if he adds more intervals. Additional intervals must be, of course, between the interval [0,1]. Such an additional interval cannot exceed this range. So, the more intervals, the more precisely the information about the type of the sender. The timing in the model is then as follows. First, nature draws the type of the sender. Next, the sender can infer in which interval his type lies, through determination of the amount of intervals. Thereafter, the sender sends a messages to the receiver, who updates his beliefs on the basis of Bayes' rule and takes an action. And finally, the payoffs are realized.

In short, my game deviates from a standard cheap-talk game of Crawford and Sobel, in that the sender now first needs to collect some information about his type rather than he knows his type exactly. This could be done through determination in which interval his type lies at a cost c per additional interval. For the results I make the distinction between the case that $c = 0$ and the case that $c>0$.

The first thing that emerges from my model is that if $c = 0$ the sender can collect a lot of information for free or in other words that the sender can determine his type precisely. But, the sender does not do this, because he cheats with a higher probability if he could add as much intervals as he wants for free. So, adding more intervals leads to a lower chance on an equilibrium where the sender truthfully reveals his information. This is an interesting finding, because the sender is less tempted to exaggerate with less intervals, which means that he reports more honestly with less information. The same conclusion for this holds if $c>0$, which implies that the sender does not determine his type precisely, regardless of the costs.

The second finding if c = 0 is that better communication is always possible in my model (where each interval is of equal length) than in the standard model of Crawford and Sobel (where each interval is 4b bigger than its previous interval). This is because of the fact that the use of words becomes more efficient and due to a higher utility. Firstly, the standard deviation of the message is higher in the standard model than in my model, which implies better communication in my model. Secondly, the total utility is maximal if the intervals are of equal length, which implies a higher utility in my model than in the standard model. This holds for two intervals, but also for N intervals. The same conclusion for this finding holds if c>0 and c is not too big. The standard deviation is still higher in the standard model than in my model. And the utility is bigger in my model than in the standard model if the costs are not too high.

I then investigate what kind of equilibrium exists if the sender does not report honestly, but is going to exaggerate. For this situation to be an equilibrium I derive two conditions, which depends on the number of intervals and the parameter of the preference bias of the sender. With these two conditions it is possible to find out different communication strategies and to clarify for different situations how the sender will speak given the number of words. These two derived conditions make it possible to illustrate the minimal number of words. I also derive a formula to find the maximal number of words, which make this equilibrium complete. It is interesting to see that if there exists an equilibrium of three partitions in my model, then it does not mean that there exists by definition an equilibrium with two partitions as well, due to the binding conditions. This is contradictory to the standard model of Crawford and Sobel, which says that if there exists an equilibrium of ten partitions, then there also exists an equilibrium of nine partitions (as well as an equilibrium with eight partitions, seven partitions and so on). All these findings are again regardless of the costs, because the two derived conditions and the formula to derive the maximal number of words are the same if c=0 and if c>0.

The last finding of this paper illustrates that the utility is bigger if the number of intervals is three than if the number of intervals is nine (both with three words/partitions), which implies that unless c=0 and N can go to infinity for free, the sender does not do this. This is in line with the finding that the sender is less tempted to exaggerate with less intervals, which means that he reports more honestly with less information. There are two possible reasons why the sender does not take a lot of information:

1. The sender needs to make costs for it.
2. Or because less information improves communication.

The last finding illustrates that less information improves communication. So, even if the costs are zero and information is free, the sender does not take a lot of this information. This implies that if $c>0$, the sender does of course not take a lot of information as well. This makes the case $c>0$ less relevant and interesting, but to show that the same conclusion holds if $c>0$ I take the same steps as in the case that $c=0$, which results in the same conclusion.

The remainder of this paper is organized as follows. In the next section, I discuss the related literature. In section 3, I pay special attention to the situation when cheap talk could be useful in the first place. After that, in Section 4, I present the model. I then discuss the results in section 5, where I make the distinction between different cases of the costs. And finally, section 6 contains the conclusion.

2. Related Literature

In this section I discuss some literature in which cheap-talk plays a central role. The book ''A primer in Game Theory'', written by Robert Gibbons (1992), includes a part about cheap-talk. In this part Gibbons describes the meaning of cheap-talk, as well as the basics of a standard cheap-talk model. The book ''Game theory an introduction'' of Steven Tadelis (2013) has a section about cheap-talk too. This section describes the basics of cheap-talk.

In my previous work about cheap-talk I investigated the distinction between a strategic type and a non-strategic type of sender (Smallenburg, 2014). The assumption was made that a strategic type sends a message randomly in the interval [0,1]: $m \in [0,1]$, and that a non-strategic type chooses $m = t$. After receiving the message, the receiver must conclude if the sender is a strategic or a non-strategic type. I assumed that there was a probability ҵ that the sender is strategic and a probability 1-ҵ that he is non-strategic. The first finding shows that in a pure strategy equilibrium strategic players will sooner be believed if there are more of the non-strategic/honest players. The next findings concern a mixed strategy equilibrium. In these equilibria the influence of non-strategic types is essential. With only strategic types, the whole equilibrium would change. In addition, through the presence of non-strategic players, it becomes less likely that strategic players randomize their message, which was first assumed. Furthermore, the more non-strategic senders, the sooner the strategic players will be believed (same as in a pure strategy equilibrium).

In the paper ''When Words are not Enough'', by S.H. Bijkerk, V. Karamychev and O.H. Swank (2015), they extend a simple cheap-talk game by giving the receiver of information the option to investigate a sender's type. They have shown that the effects of the option to investigate communication depend on whether the receiver or the sender predominantly bears the cost of investigation. If the receiver primarily bears the cost of investigation, the option to investigate drives away all communication. If, by contrast, the sender predominantly bears the cost, the option to investigate disciplines the sender and improves communication. So, introducing an investigation device into a cheap-talk model demonstrates the vulnerability of cheap talk on the one hand, and shows how communication can be improved on the other. They have derived their results from an extended linear-uniform cheap-talk model, where some of the results carry over to a more general setting. For instance, the vulnerability of communication in the presence of an investigation device does not depend on the uniform distribution of the types or on the linearity of preferences. However, alternative assumptions about the distribution of the types or the players' payoff functions do affect the exact form of communication. In this respect, the results are only illustrative.

Most economists of cheap-talk models, like Crawford and Sobel (1982) and Benabou and Laroque (1992), assumed that a good advisor tells the truth and a bad adviser will sometimes tell the truth and sometimes lie. Stephen Morris (2001), however, endogenizes the behavior of the good advisor and showed that just as the bad advisor sometimes has an incentive to tell the truth, the good advisor may have an incentive to lie. In his paper ''Political Correctness'' wishes an informed advisor to convey her valuable information to an uninformed decision maker with identical preferences. Thus he has a current incentive to truthfully reveal his information. But if the decision maker thinks that the advisor might be biased in favor of one decision and the advisor does not wish to be thought to be biased, the advisor may have a reputational incentive to lie. This is due to political correctness, which refers to the following phenomenon: because certain statements will lead listeners to make adverse inferences about the type of the speaker, speakers have an incentive to alter what they say to avoid that inference. Speakers' attempts to avoid the adverse inference lead to the loss of real information. In the model of this paper, the information may be socially valuable; that is, all parties may lose from the suppression of information due to political correctness. There were three main insights from that model. First, in any informative equilibrium, certain statements will lower the reputation of the speaker independent of whether they turn out to be true. Second, if reputational concerns are sufficiently important, no information is conveyed in equilibrium and cheap-talk is useless. Third, while instrumental reputational concerns might arise for many reasons, a sufficient reason is that speakers wish to be listened to, simply because he wants her valuable and unbiased advice to have an impact on future decisions.

3. When could cheap-talk be useful in the first place?

This section of the paper describes when cheap-talk in general could be useful, before I start to apply math on cheap-talk. There are two very extreme insights in the meaning of cheap-talk. Firstly, some economists are wondering what the incentive is to tell the truth, because cheap-talk is costless and does not affect the payoffs in a direct way. Other economists are, however, more optimistic about the use of cheap-talk. They presume that cheap-talk lead to a Nash equilibrium which is Pareto-efficient. This two insights seems very extreme and a balance between both insights will be the most logical. This balanced middle way supposes that cheap-talk will affect the payoffs, due to the fact that people response to cheap-talk, despite the fact that cheap-talk is costless and that cheap-talk does not affect payoffs directly.

There always exists a babbling situation, because it is consistent with rational beliefs that cheap-talk could be regarded as meaningless. There is no reason that somebody's statements are correlated with those private information. So, one might think that the best way to response to cheap-talk is to ignore it, because there is no correlation. But, people will usually not assume that words do not mean what they always have meant. They take the literal meaning of such statements very serious and use these literal meaning as a base point to judge the credibility of a statement. Of course, one does not believe anything he hears, but one will definitely take the meaning of a statement into account. Beside the babbling situation, there also exists a situation in which everything is revealed. This situation could exist if players share a rich language. In this way, cheap-talk cannot be blocked through a lack of understanding, because one assumes that people who share a common language can express and understand a certain message without any troubles. This is called the ''rich language'' assumption. The only thing left that can block cheap-talk is now infidelity, such as lying.

Joseph Farrell and Matthew Rabin (1996) try to indicate when cheap-talk can convey private information in equilibria and the likelihood of these equilibria. They do this in a broader sense, without notations and theorem's. They find that cheap-talk could be useful and that this is often the case, but cheap-talk does not always lead to efficiency.

The important aspect of this paper is the incentive to lie. Without any incentive to lie, cheap-talk will convey full private information. The next table is used to clarify this, where the payoff of the employee is first showed, followed by the payoff of the employer:

		Job to employee	
		Demanding	Non-demanding
Employee's ability	High	2,1	0,0
	Low	0,0	1,2

The employer wants to offer the employee a demanding job if the ability is high and vice versa. The employer does not know the ability of the employee, in contrast to the employee. The payoffs show that there is no incentive to lie for the employee and the employee reveals his real ability. So, cheap-talk is useful in this case. However, if there is a strong incentive to lie, cheap-talk is not useful anymore:

		Job to employee	
		Demanding	Non-demanding
Employee's ability	High	2,1	0,0
	Low	2,0	1,2

The payoffs show in this case that there is an incentive to lie, because the employee always wants the demanding job, due to a higher wage or due to the fact that performance is hard to monitor. Thus, the employee always tries to convince the employer that his ability is high. His preferences are not correlated with the truth anymore. So, cheap-talk cannot convey private information in this example. But, cheap-talk could still be useful if there is some incentive to lie. Suppose that the employees' ability is continuous distributed among low and high and that the employer matches the wage and job with the ability of the employee; thus the higher the ability, the higher the wage. This implies that the employee wants to convince the employer that his ability is slightly higher (t+b) than what the employer expects (t), because the employee wants a higher wage. Extreme exaggeration is not tempting, because a too demanding job is hard to handle. In this situation, cheap-talk does not need to have a precise meaning to be useful. Inaccurate cheap-talk can still lead to an equilibrium, where the employee does not exaggerate too much (low values of b). If the preferences of both players are better aligned, more precise communication is possible, (see ''Model'').

Furthermore, Farrell and Rabin presume that cheap-talk is especially useful if it is ''self-signaling'' and ''self-committing''. The next table clarifies what these two terms mean, where the payoff of person 1 is first showed, followed by the payoff of person 2:

	Person 2				
	Cafe:	1	2	3	Station cafe
Person 1	Cafe 1	3,3	0,0	0,0	0,-2
	Cafe 2	0,0	3,3	0,0	0,-2
	Cafe 3	0,0	0,0	3,3	0,-2
	Station cafe	-2,0	-2,0	-2,0	1,1

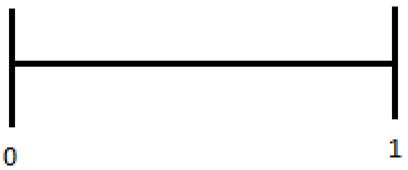
Person 1 and 2 want to have drinks together after work. The three cafes are of the same quality, while the station cafe is of lower quality. Most likely, both persons are going to talk with each other to solve their problem where to drink. Suppose person 1 says that he goes to cafe 2, how should person 2 evaluate the credibility of this statement? This credibility should be high, if the message is self-signaling: person 1 should only say this if, and only if, it is true; and if the message is self-committing: if person's 1 message will be believed, it creates incentives for person 1 to fulfill the message. Through this message both person 1 and person 2 realize a payoff of 3. So, cheap-talk solves this coordination problem efficiently. An alternative here could be the use of convention, which is, however, less efficient than cheap-talk. Suppose that the station is the midpoint of the city. Person 1 and 2 should go to the station cafe if they are not able to communicate to each other where to have drinks. This is better than no coordination (payoff of 1 instead of 0), but worse than if there are any cheap-talk possibilities (payoff of 3 instead of 1). In general current midpoints are rarely optimal, because these points are not customized for the given situation. Cheap-talk is then an excellent solution for such problems.

Lastly, cheap-talk could be also useful during conflicts in conversations, where conflicts means that both players could talk, instead of only one person. Take for example the ''battle of the sexes'', where both players could have the same preferences about what to do or they can have conflicting preferences. The problem of conflicting preferences could be solved if there is an opportunity to talk long enough. Negotiating about what to do through cheap-talk can decrease potential payoffs, but these reductions do never lead to payoffs lower than in the equilibrium where the other player's favorite is chosen. Cheap-talk could be meaningful in this example, because the worst a player could do, is to give up and to say that the other player may choose.

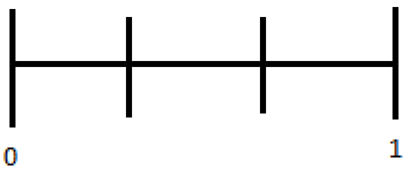
In short, cheap-talk could be very useful, but it does not guarantee efficiency.

4. Model

The communication part of my model is a variation of the cheap-talk model of Crawford and Sobel (1982), hereafter referred to as CS. Cheap-talk consists of simple talk; costless, non-binding and not verifiable statements. This model describes the behavior of two players, a S and a R. The sender is an informed agent who gives an advice or sends a message to the uninformed receiver, for example the decision maker. In the basic model of CS, the sender knows his type exactly. But, in this paper I make the extension that the sender does not know his type exactly. He first needs to collect some information about his type. He could do that through determination in which interval his type lies, where an additional interval costs $c \geq 0$. The sender knows his type more precisely, if he adds more intervals. Additional intervals must be, of course, between the interval [0,1]. Such an additional interval cannot exceed this range. In order to clarify, without adding any extra intervals, the sender knows that his type could be in the whole range of [0,1]:



However, if he adds two more intervals, which costs 2c, it becomes clear in which of the three intervals his type will be:



By adding extra intervals in the range [0,1], he could determine his type more precisely. So, the more intervals, the more precise the information about the type of the sender.

The timing and the moves in the model are then as follows:

- First, nature draws $t \in T$. Every sender is a particular kind of type, distributed among the set $T = [t_1,\dots,t_i]$. I make the assumption that T is uniformly distributed over the type space, which equals $T = [0,1]$.
- The range of T could be partitioned into intervals, where the sender determines the amount of intervals. So, the following step is, that through the determination of the

amount of intervals, the sender can infer in which interval ''t'' lies. An additional interval costs $c \geq 0$. Let $p = \{p_0,\ldots,p_N\}$ denote a partition of T = [0,1] in N intervals with $0 = p_0 < p_1 < \ldots < p_N = 1$. So, $p \in \{0, \frac{1}{N}, \frac{2}{N}, \ldots, 1\}$. If there are N intervals, the following holds: $t \in [\frac{k-1}{N}, \frac{k}{N}]$, with $k > 1$ and $k \leq N$.

- Thereafter, the sender chooses a message ''m'' from an infinite message set M, $M=[m_1,\ldots,m_j]$.
- Next, having received a message $m \in M$, the receiver updates his beliefs on the basis of Bayes' rule and takes an action ''a'' from a set of possible actions A, $A=\{a_1,\ldots,a_k\}$.
- Finally, the realized payoffs of both players are dependent of the type of the sender and the action of the receiver, $U_S(t_i,a_k)$ and $U_R(t_i,a_k)$:

$$U_S(t,a) = -|a-(t+b)| - c(N-1)$$
$$U_R(t,a) = -|a-t|$$

Where the parameter b (b>0) is the preference bias of the sender. This parameter measures the similarity among the preferences of both players. This means that the optimal action for the receiver equals a = t and for the sender a = t+b, when the sender is of type t. So, the lower the value of b (towards zero), the better the preferences of both players are aligned.

So, my game deviates from a standard cheap-talk game of CS, in that the sender now first needs to collect some information about his type rather than that he knows his type exactly. This could be done through determination in which interval t lies at a cost c. This is summarized in three stages, where in stage 1 and 2 the sender plays and in stage 3 the receiver will play:

- Stage 1: determination of the amount of intervals, to set the interval in which t lies.
- Stage 2: choosing a message, ''m'', which may indicate in which interval t lies.
- Stage 3: taking an action, ''a'', which leads (in combination with t) to the payoffs.

A perfect Bayesian equilibrium consists of a sending strategy and a partition strategy of the sender and an action strategy and beliefs of the receiver.

Each type t of the sender sends a message that maximizes his expected utility. The sender follows a partition strategy if σ (m|t) (the probability with which the sender sends m conditional on t) is uniform, supported on $[p_i(N), p_{i+1}(N)]$ if t ϵ (p_i, p_{i+1}) for i ϵ {0,…,N-1} and N > 1. A sender's message may indicate in which interval t lies. The sender knows his type more precisely, if he adds more intervals. In equilibrium, the receiver responses to the sender's message with an action which maximizes his expected utility. He updates his beliefs about the type of the sender by means of Bayes' rule. Further, his action strategy is an optimal response to the sender's message. This optimal response would be in this case always the midpoint of the interval in which t lies. The game can be solved by backward induction.

In cheap-talk games a pooling equilibrium always exists. In this equilibrium the message of the sender does not contain any information about the type of the sender. Therefore, it is much more interesting to investigate if there also an equilibrium exists in which the sender follows a partially pooling/semi-separating communication strategy.

I first take a look at the case where c = 0 and N = ∞. So, an additional interval does not cost anything (c = 0) and there are so many intervals (N = ∞) that the sender knows his type exactly. This case is similar to the standard cheap-talk model of CS where the sender knows his type exactly as well and admits all usual cheap-talk equilibria. CS say that the space in which a type lies is divided into N-numbers of intervals: $[0,t_1)$, $[t_1,t_2)$,…,$[t_{i-1},1)$. All types of senders in the same interval will send the same message, but types of different intervals will send different messages. Suppose an equilibrium in which the sender follows partition strategy p(N) as described earlier. m ϵ $[t_i , t_{i+1}]$ leads to an expected value of t equal to $\frac{1}{2}(t_i + t_{i+1})$. At t = t_i the sender is indifferent between sending m ϵ $[t_{i-1} , t_i]$ and sending m ϵ $[t_i , t_{i+1}]$, implying:

$$-\left|\frac{1}{2}\ (t_{i-1} + t_i\ ) - (t_i + b)\right| = -\left|\frac{1}{2}(t_i + t_{i+1}) - (t_i + b)\right|$$

Rewriting yields: $(t_{i+1} - t_i\ )$ - $(t_i - t_{i-1}\ )$ = 4b. This means that the length of interval $(t_{i+1} - t_i\ )$ is 4b larger than the length of interval $(t_i - t_{i-1}\ )$. The reason for this is well-known and is described below.

So, all the intervals cannot be of exactly equal length. This stems from the fact that, given the type of sender (t), the optimal action for the sender (t+b) will exceed the optimal action for the receiver (t) with the value b. If both intervals are of equal length, then the type of sender that exactly lies on the border will prefer the message that is linked to the last interval. The types of senders that are just below this border, will prefer the message that is linked to the last interval as well. So, there is only one way to make these ''border types'' indifferent between a certain interval and the previous interval: a certain interval ($[t_i , t_{i+1}]$) must be 4b bigger than its previous interval ($[t_{i-1} , t_i)$) (and the next interval must be 8b bigger than the first interval), where the N[th] interval must be exact t=1. 4b comes from the following equation: $t_i + b = \frac{1}{2}\left[\frac{(t_{i-1} + t_i)}{2} + \frac{(t_i + t_{i+1})}{2}\right]$. Rewriting this leads to:

$(t_{i+1} - t_i) = (t_i - t_{i-1}) + 4b$. Another implication is that the maximum number of intervals ($N^*(b)$), which is $\frac{1}{2}[1 + \sqrt{1 + (\frac{2}{b})}\ ]$, depends negatively on b. Thus, the maximum amount of intervals increases if b decreases. This is because a lower b will lead to more perfect communication. Notice that both the number of intervals and their boundaries depend on b! In short, this is the general case of CS and the case in my model if c = 0 and N = ∞.

To clarify such a partially pooling equilibrium of CS and to show which communication is possible if b takes a particular value, I look at an equilibrium with two intervals (N=2), $[0,t_1)$ and $[t_1,1]$. The receiver believes that the sender of $[0,t_1)$ is uniformly distributed among this interval and the optimal action for him is then $(t_1)/2$. This holds for an message of the other interval too. The optimal action (response) for the receiver is here $(t_1+1)/2$. Types of interval $[0,t_1)$ must prefer action $(t_1)/2$ and types of interval $[t_1,1]$ must prefer action $(t_1+1)/2$. This equilibrium only holds if t_1 is the type t, whose optimal action t+b is equal to the midpoint of these two actions: $t_1 + b = \frac{1}{2}[\frac{(t_1)}{2} + \frac{(t_1 + 1)}{2}]$. This leads to t_1 = ½ - 2b. Because T = [0,1], b must be positive here. So, this equilibrium with two words could only exist if b < ¼, otherwise the preferences of both players are too weakly aligned. This implies further that if b < 1/12 communication of three words could occur, if b < 1/24 communication of four worlds could occur and so on. So, there is no communication possible when the preferences of both players are not aligned and there is more precise communication possible when the preferences of both players are more aligned.

To give a better insight in the model described above, it could be useful to take some interpretations of the model into account:

- The decision maker is a public official who has to maximize a social welfare function by using a particular policy which creates transfers to a special interest. The socially optimal level of the policy depends on the state of the world. The public official is advised by an expert who certainly has some information about the state.
- The decision maker is an editor of a journal who must decide on a response to a submitted paper, where the editor would like to give a more positive response if the quality of the paper is higher. The editor is advised by a referee who is better able to assess the quality of the paper.
- The decision maker is a personnel officer allocating a salary budget between a male employee and a female employee, where the personnel officer wants to allocate a larger share to the more productive employee. The personnel officer is advised by a supervisor who certainly has information about which employee is more productive.

5. Results

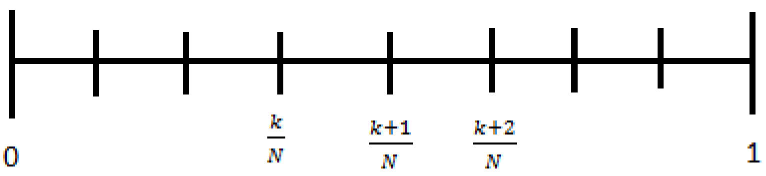
In this section I present the results. I make the distinction between different cases of the costs c. The case that c = 0 and the case that c>0.

Case 1: c = 0

The first step is to look what happens if I make the assumption that c = 0. This implies that the sender can collect a lot of information for free or in other words that the sender can determine his type precisely. But I will show that the sender does not do this, because he cheats with a higher probability if he could add as much intervals as he wants for free.

Proposition 1: *If* $b < \frac{1}{2N}$ *there exists an equilibrium where the sender truthfully reveals his information. However, if* $b > \frac{1}{2N}$ *there is no equilibrium where the sender reports honestly and where he is going to exaggerate.*

Proof. Suppose that there are N intervals which are of equal length and that the sender has determined that his type is in the interval t ϵ [$\frac{k}{N}$, $\frac{k+1}{N}$]:



Honestly reporting yields then: m ϵ [$\frac{k}{N}$, $\frac{k+1}{N}$]. But maybe the sender tends to exaggerate his report, which yields: m ϵ [$\frac{k+1}{N}$, $\frac{k+2}{N}$]. Under certain conditions the sender does not report honestly, due to a relatively high value of b. Reporting honestly leads to an action of the receiver which is a = $\frac{2k+1}{2N}$ and an exaggerated report leads here to an action of the receiver which is a = $\frac{2k+3}{2N}$. If b takes a relatively high value then the sender probably prefers the action a = $\frac{2k+3}{2N}$ to the action a = $\frac{2k+1}{2N}$. These actions are necessary for the utility functions and to make a distinction between the situation with and without an equilibrium. The utility function of the sender is $U_S(t, a) = -|a - (t + b)| - c$(N-1). But in this case the costs are zero, which leads to an utility of $U_S(t,a) = -|a-(t+b)|$. Honestly reporting yields then an utility of:

$$\int_{\frac{k}{N}}^{\frac{2k+1}{2N}-b} -\left(\frac{2k+1}{2N}-t-b\right)dt + \int_{\frac{2k+1}{2N}-b}^{\frac{k+1}{N}} -(t+b-\frac{2k+1}{2N})dt = -\frac{1}{4N^2}(4b^2N^2+1)$$

And an exaggerated report results then in an utility of:

$$\int_{\frac{k}{N}}^{\frac{k+1}{N}} -\left(\frac{2k+3}{2N}-t-b\right)dt = \frac{1}{N^2}(bN-1)$$

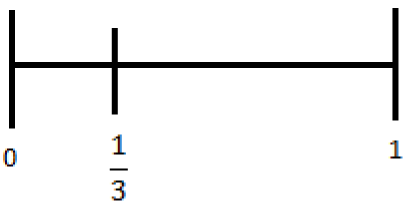
So, there exists an equilibrium where the sender truthfully reveals his information if the following condition holds: $-\frac{1}{4N^2}(4b^2N^2+1) > \frac{1}{N^2}(bN-1)$. Rewriting this leads to the condition of b: $b < \frac{1}{2N}$, which results in an equilibrium with honestly reporting. However, there is no equilibrium with honestly reporting if $-\frac{1}{4N^2}(4b^2N^2+1) < \frac{1}{N^2}(bN-1)$. Rewriting this yields $b > \frac{1}{2N}$. Thus, if $b < \frac{1}{2N}$ then there is an equilibrium where the sender truthfully reveals his information. And if $b > \frac{1}{2N}$ then there is no equilibrium where the sender reports honestly and where he is going to exaggerate.

If there are, for example, 5 intervals then b < $\frac{1}{10}$ and if there are 6 intervals then b < $\frac{1}{12}$ for the equilibrium with honestly reporting to hold and so on. This means that adding more intervals leads to a lower chance on an equilibrium where the sender truthfully reveals his information. This is because the values of b for which this equilibrium exists become lower and lower if the amount of intervals increase. So, this implies that the sender does not determine his type precisely. Thus, although information is free and he can collect a lot of information, the sender does not do this, due to the finding above that the sender cheats with a higher probability if he adds more and more intervals. This is an interesting finding, because the sender is less tempted to exaggerate with less intervals, which means that he reports more honestly with less information.

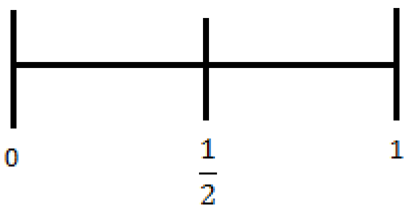
The second step is to compare the situation of the standard cheap talk model (where each interval is 4b bigger than its previous interval) and my extension of this model (where each interval is of equal length). I will show that better communication is possible in my extension of the model than in the standard model, due to the fact that the use of words becomes more efficient and due to a higher utility.

Proposition 2: *The utility is maximized if the intervals are of equal length, which implies that better communication is possible in my model than in the standard model of CS.*

Proof. First, I show this for two intervals and afterwards I show that this also holds for N intervals. So, in the standard model the length of an interval increases with 4b in comparison with its previous interval:



But, in my model the intervals are of equal length:



I first take a look into the average deviation of the message, which is higher for the standard model than in my model. This deviation is calculated for the standard model and my model, respectively, as follows: $\frac{1}{12} * \frac{1}{3} + \frac{1}{6} * \frac{2}{3} = \frac{5}{36}$ and $\frac{1}{8} * \frac{1}{2} + \frac{1}{8} * \frac{1}{2} = \frac{1}{8}$. The standard deviation is thus higher for the standard model than my model, which implies that better communication is possible in my model than in the standard model.

Secondly, the utility function of the sender is still $U_S(t, a) = -|a - (t + b)|$. The total utility in the former case is then:

$$\sum_{i=1}^{2} \frac{1}{3} \left[\frac{\int_0^{\frac{1}{6}-b} -\left(\frac{1}{6}-t-b\right) dt}{\frac{1}{6}-b} + \frac{\int_{\frac{1}{6}-b}^{\frac{1}{3}} -(t+b-\frac{1}{6}) dt}{\frac{1}{6}+b} \right] + \frac{2}{3} \left[\frac{\int_{\frac{1}{3}}^{\frac{2}{3}-b} -\left(\frac{2}{3}-t-b\right) dt}{\frac{1}{3}-b} + \frac{\int_{\frac{2}{3}-b}^{1} -(t+b-\frac{2}{3}) dt}{\frac{1}{3}+b} \right]$$

$= -\frac{1}{18} + -\frac{2}{9} = -\frac{5}{18}$.

And the utility in my model is:

$$\sum_{i=1}^{2} \frac{1}{2} \left[\frac{\int_0^{\frac{1}{4}-b} -\left(\frac{1}{4}-t-b\right)dt}{\frac{1}{4}-b} + \frac{\int_{\frac{1}{4}-b}^{\frac{1}{2}} -(t+b-\frac{1}{4})dt}{\frac{1}{4}+b} \right] + \frac{1}{2} \left[\frac{\int_{\frac{1}{2}}^{\frac{3}{4}-b} -\left(\frac{3}{4}-t-b\right)dt}{\frac{1}{4}-b} + \frac{\int_{\frac{3}{4}-b}^{1} -(t+b-\frac{3}{4})dt}{\frac{1}{4}+b} \right]$$

$= -\frac{1}{8} + -\frac{1}{8} = -\frac{1}{4}$.

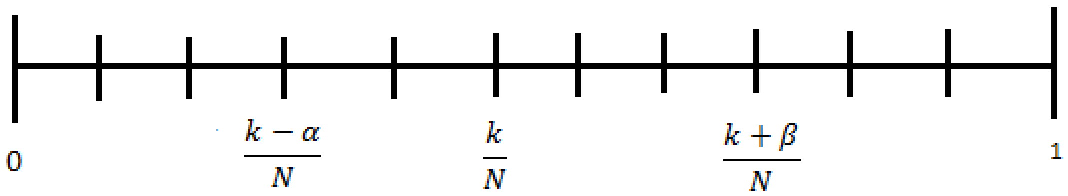
The utility in my model is the same for the two intervals, while the utility in the standard model increases for each interval. This implies that in my model the tendency to exaggerate is lower in comparison with the standard model, because of the equal utilities compared to the increasing utilities. So, the use of words becomes more efficient in my model. Furthermore, the total utility is higher in my model than in the standard model, which implies that the utility is maximal if the intervals are of equal length.

The same holds for N intervals, because N intervals are divisible in several parts of two intervals. Thus, if it holds for two intervals, then it holds for all parts of two intervals on the total of N intervals. This implies that utility is maximized if the intervals are of equal length, which is due to the more efficient way words are used and due to the higher utility.

The following step is to investigate what kind of equilibrium exists if $b > \frac{1}{2N}$, so what happens if the sender does not report honestly, but is going to exaggerate.

Proposition 3: *If* $b > \frac{1}{2N}$ *there exists an equilibrium with a minimal number of words which depends on the conditions β-α > 4bN-2 and β-α < 4bN+2, and with a maximal number of words which depends on the formula p =* $-\frac{1}{2z}(z \pm \sqrt{-4z + 8Nz + z^2 + 4} + 2)$.

Proof. Firstly, to illustrate this equilibrium I make use of the following figure:



Let α be the intervals to the left and let β be the intervals to the right. In the standard cheap-talk model of Crawford and Sobel the sender of type $\frac{k}{N}$ must be indifferent between sending m ϵ [$\frac{k-\alpha}{N}$, $\frac{k}{N}$] and m ϵ [$\frac{k}{N}$, $\frac{k+\beta}{N}$]. However, in my extension of this model the utility of sending m ϵ [$\frac{k}{N}$, $\frac{k+\beta}{N}$] must be bigger than m ϵ [$\frac{k-\alpha}{N}$, $\frac{k}{N}$] if the type of the sender lies in an interval in [$\frac{k}{N}$, $\frac{k+\beta}{N}$] and the utility of sending m ϵ [$\frac{k-\alpha}{N}$, $\frac{k}{N}$] must be bigger than m ϵ [$\frac{k}{N}$, $\frac{k+\beta}{N}$] if the type of the sender lies in an interval between [$\frac{k-\alpha}{N}$, $\frac{k}{N}$]. This must especially hold for the types of sender close to $\frac{k}{N}$, because at these places the sender is more tempted to exaggerate. So, for this situation to be an equilibrium it must hold that the utility of sending m ϵ [$\frac{k-\alpha}{N}$, $\frac{k}{N}$] must be bigger than m ϵ [$\frac{k}{N}$, $\frac{k+\beta}{N}$] if t ϵ [$\frac{k-1}{N}$, $\frac{k}{N}$] and that the utility of sending m ϵ [$\frac{k}{N}$, $\frac{k+\beta}{N}$] must me bigger than m ϵ [$\frac{k-\alpha}{N}$, $\frac{k}{N}$] if t ϵ [$\frac{k}{N}$, $\frac{k+1}{N}$]. Otherwise there might be some senders who are tempted to deviate.

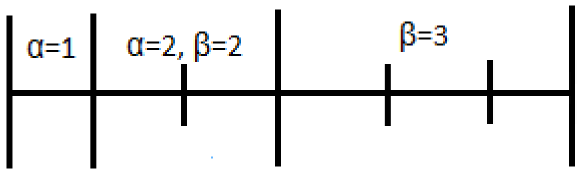
The utilities for both intervals are as follows if t ϵ [$\frac{k-1}{N}$, $\frac{k}{N}$]:

- Utility m ϵ [$\frac{k-\alpha}{N}$, $\frac{k}{N}$]: $\frac{\int_{\frac{k-1}{N}}^{\frac{k}{N}} -\left(t+b-\frac{2k-\alpha}{2N}\right) dt}{\frac{k}{N}-\frac{k-1}{N}} = -\frac{1}{2N}(\alpha + 2bN - 1)$
- Utility m ϵ [$\frac{k}{N}$, $\frac{k+\beta}{N}$]: $\frac{\int_{\frac{k-1}{N}}^{\frac{k}{N}} -\left(\frac{2k+\beta}{2N}-t-b\right) dt}{\frac{k}{N}-\frac{k-1}{N}} = -\frac{1}{2N}(\beta - 2bN + 1)$

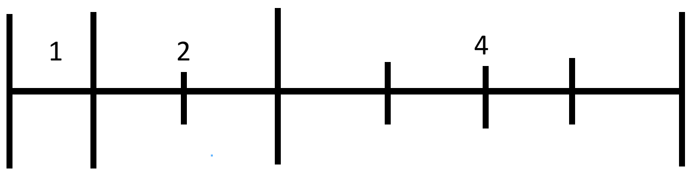
The former one must be bigger than the latter one:
$-\frac{1}{2N}(\alpha + 2bN - 1) > -\frac{1}{2N}(2bN - \beta + 1)$. Rewriting this yields β-α > 4bN-2.

If β and α are equal to each other then $b < \frac{1}{2N}$, which is logical, because if b is too high the sender is more tempted to exaggerate. And if 4bN-2>0 then β>α, which implies that there must be more intervals to the right of $\frac{k}{N}$ then to the left of $\frac{k}{N}$. Otherwise there does not exist an equilibrium. The larger b, the larger the difference between β and α will be. So, it is important that b is small enough to prevent exaggerated reports. The same holds for N: the larger N, the larger the difference between β and α will be.

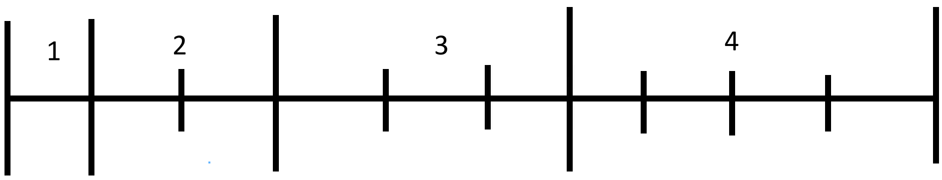
With this condition it is now possible to find out different communication strategies. These strategies depend of course on b and N. Suppose that b and N take values that 4bN-2 lies between 0 and 1. This means that β-α must be bigger than 0, for example 1. To clarify such strategies I show for different situations how the sender will speak given the number of words. If N=6 there exist different equilibria, which are indicated with the longer lines:



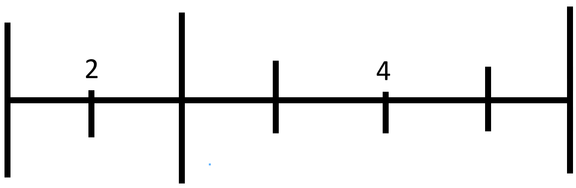
So, here communication of three words is possible where each partition/word is one interval bigger than its previous one, because β-α=1. If N=7, there will be an extra interval on one of the partitions. This must be on the last partition, because otherwise β-α>1 will not hold:



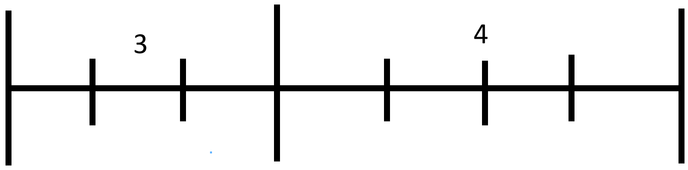
If N=10 communication of four words is possible now, where each partition/word is one interval bigger than its previous partition again:



So, given β-α=1 it is now possible to find out the communication strategy. But, it is possible that there are inefficiencies. If N=6 the following is also possible:

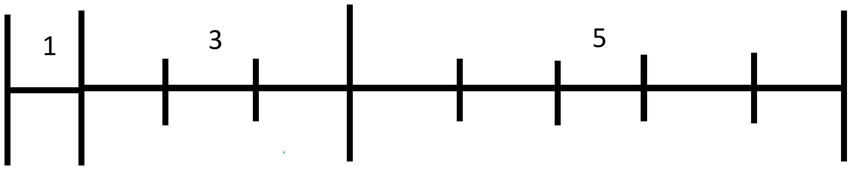


And the same holds for N=7:

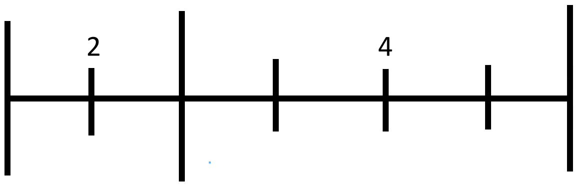


However, the former ones of N=6 and N=7 are more efficient, because communication of more words is possible then. For this paper I will only focus on equilibria which are efficient if there are two or more equilibria possible.

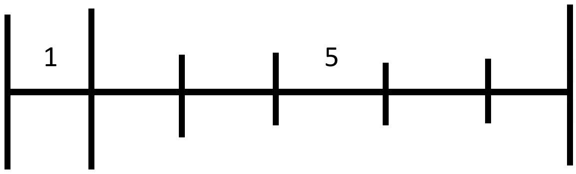
Suppose now that β-α=2 and that N=6. If the first partition is one, than the second one must be three and the last partition must be five; which means that N=9 and is thus not possible:



However, the following one is possible, where communication of two words is possible:



The following one is also possible, but this one is inefficient and is not interesting:



The utilities for both intervals are as follows if t ϵ $[\frac{k}{N}, \frac{k+1}{N}]$:

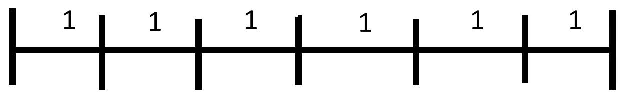
- Utility m ϵ $[\frac{k-\alpha}{N}, \frac{k}{N}]$: $\frac{\int_{\frac{k}{N}}^{\frac{k+1}{N}} -\left(t+b-\frac{2k-\alpha}{2N}\right) dt}{\frac{k+1}{N}-\frac{k}{N}} = -\frac{1}{2N}(\alpha + 2bN + 1)$
- Utility m ϵ $[\frac{k}{N}, \frac{k+\beta}{N}]$: $\frac{\int_{\frac{k}{N}}^{\frac{k+1}{N}} -\left(\frac{2k+\beta}{2N}-t-b\right) dt}{\frac{k+1}{N}-\frac{k}{N}} = \frac{1}{2N}(2bN - \beta + 1)$

The latter one must be bigger than the former one now:

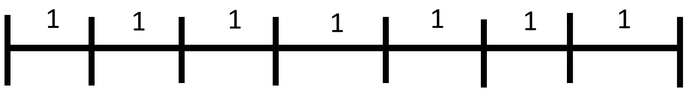
$\frac{1}{2N}(2bN - \beta + 1) > -\frac{1}{2N}(\alpha + 2bN + 1)$. Rewriting this yields β-α < 4bN+2.

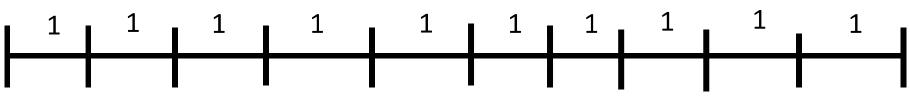
If β and α are equal to each other then $b > -\frac{1}{2N}$. And if 4bN+2>0 then it could be that β>α, which implies that there could be more intervals to the right of $\frac{k}{N}$ then to the left of $\frac{k}{N}$. Otherwise there does not exist an equilibrium. Here holds that the larger b and the larger the N, the larger the difference between β and α could be (it is not necessary).

With this condition it is now again possible to find out different communication strategies. These strategies depend of course on b and N. Suppose that b and N take values that 4bN+2 lies between 0 and 1. This means that β-α must be smaller than 1, so for example β-α=0. This leads to other equilibria then when β-α > 4bN-2 and where 4bN-2 lies between 0 and 1, which means that β-α must be bigger than 0, for example 1. If β-α=0 communication of six words is possible if N=6, because each interval could be equal to each other:

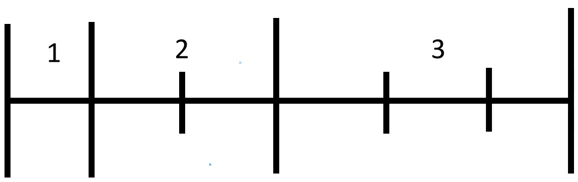


The same hold for N=7 and N=10. So, communication of seven words is possible if N=7 and communications of ten words if N=10:

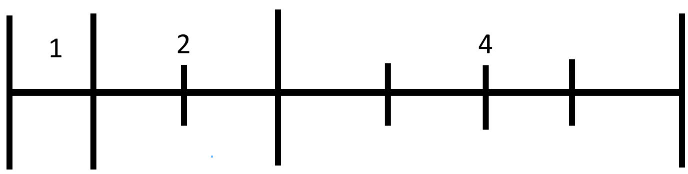




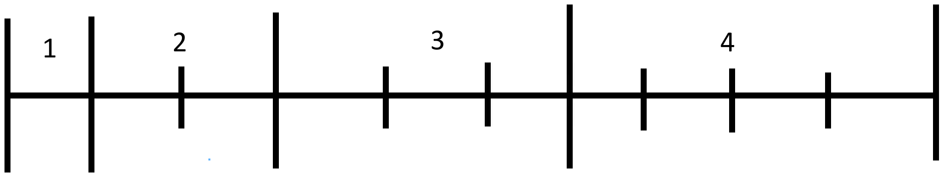
Suppose now that β-α must be smaller than 2, for example β-α=1. This gives the same strategies as β-α > 4bN-2 and where 4bN-2 lies between 0 and 1, which means that β-α must be bigger than 0, for example 1. So, if N=6:



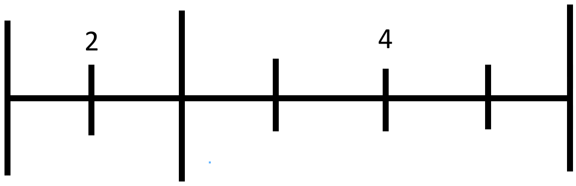
If N=7:



And if N=10:



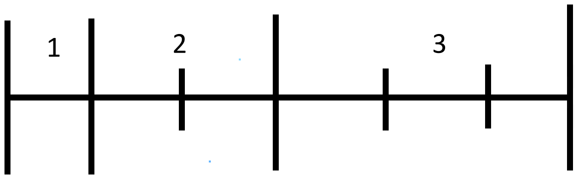
Suppose now that β-α must be smaller than 3, for example β-α=2. This gives the same strategies as β-α > 4bN-2 and where 4bN-2 lies between 1 and 2, which means that β-α must be bigger than 1, for example 2. So, if N=6:



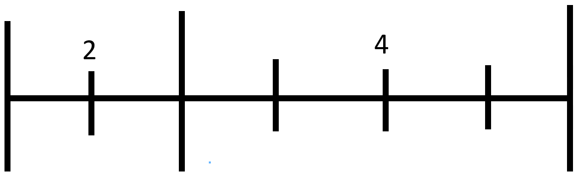
What I have seen so far is that β-α must be bigger than 0 (for example 1) if 4bN-2 lies between 0 and 1 due to the condition β-α > 4bN-2 and that β-α must be smaller than 2 (for example 1) if 4bN+2 lies between 1 and 2 due to the condition β-α < 4bN+2. And also that β-α must be bigger than 1 (for example 2) if 4bN-2 lies between 1 and 2 due to the condition β-α > 4bN-2 and that β-α must be smaller than 3 (for example 2) if 4bN+2 lies between 2 and 3 due to the condition β-α < 4bN+2. So, this implies that the second condition (β-α < 4bN+2) is also binding, due to the opposite signs, < and +2, compared to the first condition (β-α > 4bN-2), > and -2. Thus, the difference between β and α must be not too big, but also not too small.

An interesting next question is to what extent my model deviates at this point compared to the standard model of Crawford and Sobel. If there exists an equilibrium of ten partitions in the standard model, then there also exists an equilibrium of nine partitions (as well as an equilibrium with eight partitions, seven partitions and so on), according to CS. In my model this is not necessarily the case, due to the derived conditions of above. This is an important issue, because if N goes to infinity, my model is the same as the model of CS. But, apparently something will change; the second condition (β-α < 4bN+2) falls away if N goes to infinity. This is because for CS holds that if there is an equilibrium of ten partitions, there exists an equilibrium of nine partitions as well. However, in my model this does not necessarily holds. Suppose there exists an equilibrium with three partitions, then it does not mean that there exists by definition an equilibrium with two partitions, because the second condition binds. The equilibrium of two partitions is inefficient; but it is an equilibrium. The selection problem of equilibria is bigger in the model of CS than in my model due to the derived conditions. This is because some equilibria fall away in my model which cannot be an equilibrium at all.

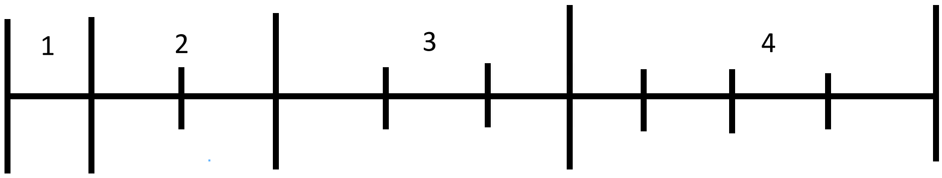
To clarify that in my model it does not necessarily hold that there exists by definition an equilibrium with two partitions if there exists an equilibrium with three partitions, I make use of the following example. This example shows that there will not be an equilibrium with two partitions. Suppose that due to the second condition β-α must be smaller than two, β-α<2. An equilibrium with communication of three words (and N=6) will exist:



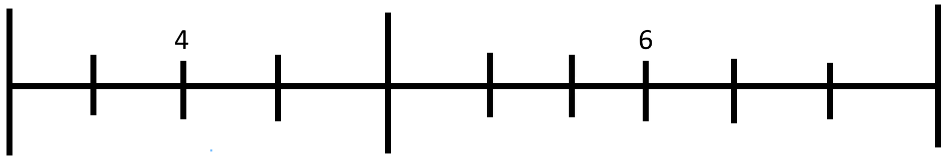
However, an equilibrium with two words (and N=6) cannot exist, because of the binding second condition (the difference between 4 and 2 is namely not smaller than 2):



The same holds for an equilibrium of four partitions (and N=10). An equilibrium with communication of four words will exist if β-α<2:

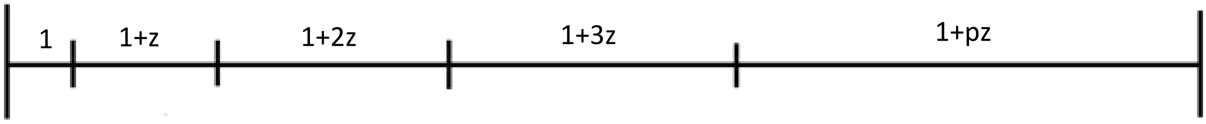


But, an equilibrium with two words (and N=10) cannot exist, again due to the second condition which binds (the difference between 6 and 4 is namely not smaller than 2):



So, this example shows that there exists an equilibrium with more than two partitions, but not with two partitions. This is contradictory to the standard model of CS.

Next, to complete the equilibrium where $b > \frac{1}{2N}$ I need to investigate the maximal number of words. I will show the maximal number of words/partitions given the number of intervals. Suppose that B-α=z and that the first interval is equal to 1. In figure this looks like as follows (where p stands for partitions):



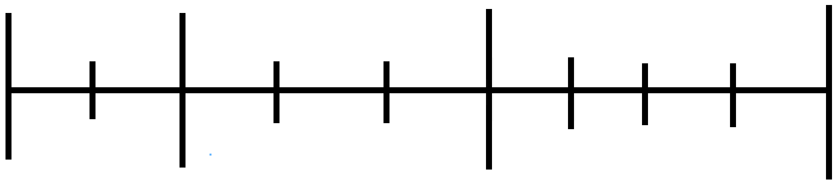
The total length is equal to 1 or N. In an N-step equilibrium, if the first step is of length 1, then the second step must be of length 1+z, the third of length 1+2z, and so on. The N^{th} step must end exactly at 1, so I must have: 1 + (1+z) + (1+2z) + … + (1+pz) = N. Rewriting this yields $1 + p + \frac{1}{2}p(1+p)z = N$. With this formula it is now possible to find the maximal number of words/partitions given the number of intervals N and the difference between B-α (z). Rewriting $1 + p + \frac{1}{2}p(1+p)z = N$ to p results in p = N-1 if z = 0 and in $p = -\frac{1}{2z}(z \pm \sqrt{-4z + 8Nz + z^2 + 4} + 2)$ if z ≠ 0. So, due to this formula it is now possible to derive the maximal number of words/partitions. And it was already possible to illustrate the minimal number of words/partitions due to the earlier derived conditions (β-α > 4bN-2, β-α < 4bN+2), which completes the equilibrium.

The final step is to examine if less information improves communication. The first step of the results already showed that the sender is less tempted to exaggerate if the amount of intervals is lower. Till now I investigated how communication looks like for a given N.

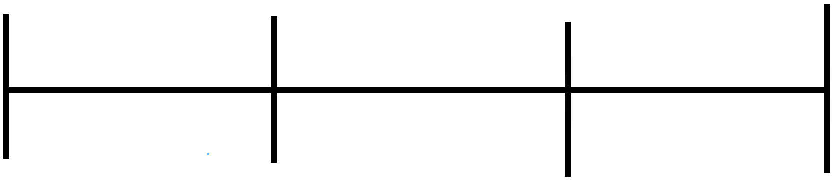
Proposition 4: Less information improves communication, regardless of the costs. So, the sender does not take a lot of information.

Proof. At this point, I investigate what is more useful if there are three partitions/words:

- and if N>3 (for example N=9) with three partitions/words:



- or if N=3 with also three partitions/words, but without exaggerated reporting:



I will show that the latter one leads to a bigger utility for the sender. Furthermore, the latter one is more efficient than the former one. The utility if N=3 is:

$$\sum_{i=1}^{3}\frac{1}{3}\left[\frac{\int_0^{\frac{1}{6}-b}-\left(\frac{1}{6}-t-b\right)dt}{\frac{1}{6}-b}+\frac{\int_{\frac{1}{6}-b}^{\frac{1}{3}}-(t+b-\frac{1}{6})dt}{\frac{1}{6}+b}\right]+\frac{1}{3}\left[\frac{\int_{\frac{1}{3}}^{\frac{3}{6}-b}-\left(\frac{3}{6}-t-b\right)dt}{\frac{1}{6}-b}+\frac{\int_{\frac{3}{6}-b}^{\frac{2}{3}}-(t+b-\frac{3}{6})dt}{\frac{1}{6}+b}\right]+$$

$$\frac{1}{3}\left[\frac{\int_{\frac{2}{3}}^{\frac{5}{6}-b}-\left(\frac{5}{6}-t-b\right)dt}{\frac{1}{6}-b}+\frac{\int_{\frac{5}{6}-b}^{1}-(t+b-\frac{5}{6})dt}{\frac{1}{6}+b}\right]=-\frac{1}{18}+-\frac{1}{18}+-\frac{1}{18}=\ -\frac{1}{6}.$$

And the utility if N=9 is:

$$\sum_{i=1}^{3}\frac{2}{9}\left[\frac{\int_0^{\frac{1}{9}-b}-\left(\frac{1}{9}-t-b\right)dt}{\frac{1}{9}-b}+\frac{\int_{\frac{1}{9}-b}^{\frac{2}{9}}-(t+b-\frac{1}{9})dt}{\frac{1}{9}+b}\right]+\frac{3}{9}\left[\frac{\int_{\frac{2}{9}}^{\frac{7}{18}-b}-\left(\frac{7}{18}-t-b\right)dt}{\frac{3}{18}-b}+\frac{\int_{\frac{7}{18}-b}^{\frac{5}{9}}-(t+b-\frac{7}{18})dt}{\frac{3}{18}+b}\right]+$$

$$\frac{4}{9}\left[\frac{\int_{\frac{5}{9}}^{\frac{7}{9}-b}-\left(\frac{7}{9}-t-b\right)dt}{\frac{2}{9}-b}+\frac{\int_{\frac{7}{9}-b}^{1}-(t+b-\frac{7}{9})dt}{\frac{2}{9}+b}\right]=-\frac{2}{81}+-\frac{1}{18}+-\frac{8}{81}=\ -\frac{29}{162}.$$

So, the utility is bigger if N=3 than if N=9 (both with three words/partitions), which implies that unless c=0 and N can go to infinity for free, the sender does not do this. This is in line with the finding that the sender is less tempted to exaggerate with less intervals, which means that he reports more honestly with less information.

Thus, there are two possible reasons why the sender does not take a lot of information:

1. The sender needs to make costs for it.
2. Or because less information improves communication.

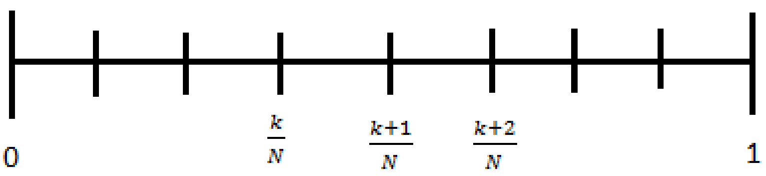
The example of above illustrates that less information improves communication. So, even if the costs are zero and information is free, the sender does not take a lot of this information. This implies that if c>0, the sender does of course not take a lot of information as well. This makes the following case (c>0) less relevant and interesting, but to show that the same conclusion holds if c>0, I refer to the following section where I will cover the case c>0.

Case 2: c > 0

So, even if the costs are zero and information is free, the sender does not take a lot of this information. This implies that if c>0, the sender does of course not take a lot of information as well. This makes this case (c>0) less relevant and interesting, but to show that the same conclusion holds, I take the same steps as in the case that c=0.

Proposition 5: *If c>0 the conclusions of the case c=0 do not change. So, with the main result that the sender does not take a lot of information, because less information improves communication.*

Proof. Thus, I investigate what happens if c>0. When c was zero, the sender could collect a lot of information for free or in other words, he could determine his type precisely. However, I showed that the sender does not do this due to the finding that the sender cheats with a higher probability if he adds more and more intervals. So, if c=0 the sender was less tempted to exaggerate with less intervals, which means that he reports more honestly with less information. If c>0 this information is not free anymore. To see what happens I assume again that there are N intervals which are of equal length and that the sender has determined that his type is in the interval t ϵ [$\frac{k}{N}$, $\frac{k+1}{N}$] again:



Honestly reporting yields: m ϵ [$\frac{k}{N}$, $\frac{k+1}{N}$], while an exaggerated report yields: m ϵ [$\frac{k+1}{N}$, $\frac{k+2}{N}$]. Reporting honestly leads to an action of the receiver which is a = $\frac{2k+1}{2N}$ and an exaggerated report leads here to an action of the receiver which is a = $\frac{2k+3}{2N}$. The utility function of the sender is now $U_S(t,a) = -|a-(t+b)| - c$(N-1). Honestly reporting leads then to an utility of:

$$\int_{\frac{k}{N}}^{\frac{2k+1}{2N}-b} -\left(\frac{2k+1}{2N} - t - b\right)dt + \int_{\frac{2k+1}{2N}-b}^{\frac{k+1}{N}} -(t + b - \frac{2k+1}{2N})dt - c(N-1)$$

$= -\frac{1}{4N^2}(4b^2N^2 + 4cN^3 - 4cN^2 + 1)$

And an exaggerated report results then in an utility of:

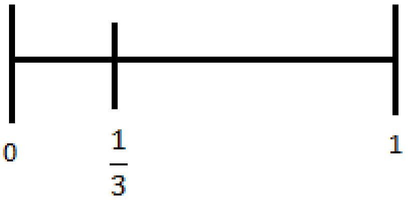
$$\int_{\frac{k}{N}}^{\frac{k+1}{N}} -\left(\frac{2k+3}{2N} - t - b\right) dt - c(N-1) = \frac{1}{N^2}(-cN^3 + cN^2 + bN - 1)$$

So, there exists an equilibrium where the sender truthfully reveals his information if the following condition holds: $-\frac{1}{4N^2}(4b^2N^2 + 4cN^3 - 4cN^2 + 1) > \frac{1}{N^2}(-cN^3 + cN^2 + bN - 1)$. Rewriting this yields to the same condition as earlier: $b < \frac{1}{2N}$, which results in an equilibrium with honestly reporting. However, there is no equilibrium if $-\frac{1}{4N^2}(4b^2N^2 + 4cN^3 - 4cN^2 + 1) < \frac{1}{N^2}(-cN^3 + cN^2 + bN - 1)$. Rewriting this yields $b > \frac{1}{2N}$, which results in no equilibrium, because the sender is going to exaggerate then.

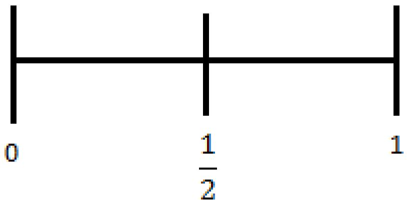
So, because of the same condition ($b < \frac{1}{2N}$), the conclusion is the same here. This means that the sender does not determine his type precisely, regardless of the costs. If there are, for example, 5 intervals then b < $\frac{1}{10}$ and if there are 6 intervals then b < $\frac{1}{12}$ for the equilibrium with honestly reporting to hold and so on. This means that adding more intervals leads to a lower chance on an equilibrium where the sender truthfully reveals his information. This is because the values of b for which this equilibrium exists become lower and lower if the amount of intervals increases. Thus, for both cases (c=0 and c>0), the sender does not determine his type precisely, because of the finding that he cheats with a higher probability if he adds more and more intervals.

The second step was to compare the situation of the standard cheap talk model (where each interval is 4b bigger than its previous interval) and my extension of this model (where each interval is of equal length). With c=0 I showed that better communication is possible in my extension of the model than in the standard model regardless of the number of intervals, due to the fact that the use of words becomes more efficient and due to a higher utility. To investigate if this is still the case I first check this for two intervals again.

So, in the standard model the length of an interval increases with 4b in comparison with its previous interval:



And in my model the intervals are of equal length:



The average deviation of the message remains the same if c>0, which is $\frac{1}{12} * \frac{1}{3} + \frac{1}{6} * \frac{2}{3} = \frac{5}{36}$ in the standard model and $\frac{1}{8} * \frac{1}{2} + \frac{1}{8} * \frac{1}{2} = \frac{1}{8}$ in my model. The standard deviation is thus higher for the standard model than my model, which implies that better communication is possible in my model than in the standard model.

Secondly, the utility function of the sender is now $U_S(t,a) = -|a-(t+b)| - c(N-1)$. Because N=2, the utility in my model is $U_S(t,a) = -|a-(t+b)| - c$. The utility in the standard model is still the same:

$$\sum_{i=1}^{2} \frac{1}{3}\left[\frac{\int_0^{\frac{1}{6}-b} -\left(\frac{1}{6}-t-b\right)dt}{\frac{1}{6}-b} + \frac{\int_{\frac{1}{6}-b}^{\frac{1}{3}} -(t+b-\frac{1}{6})dt}{\frac{1}{6}+b}\right] + \frac{2}{3}\left[\frac{\int_{\frac{1}{3}}^{\frac{2}{3}-b} -\left(\frac{2}{3}-t-b\right)dt}{\frac{1}{3}-b} + \frac{\int_{\frac{2}{3}-b}^{1} -(t+b-\frac{2}{3})dt}{\frac{1}{3}+b}\right]$$

$$= -\frac{1}{18} + -\frac{2}{9} = -\frac{5}{18}.$$

And the utility in my model is:

$$\sum_{i=1}^{2} \frac{1}{2}\left[\frac{\int_0^{\frac{1}{4}-b} -\left(\frac{1}{4}-t-b\right)dt}{\frac{1}{4}-b} + \frac{\int_{\frac{1}{4}-b}^{\frac{1}{2}} -(t+b-\frac{1}{4})dt}{\frac{1}{4}+b} - c\right] + \frac{1}{2}\left[\frac{\int_{\frac{1}{2}}^{\frac{3}{4}-b} -\left(\frac{3}{4}-t-b\right)dt}{\frac{1}{4}-b} + \frac{\int_{\frac{3}{4}-b}^{1} -(t+b-\frac{3}{4})dt}{\frac{1}{4}+b} - c\right]$$

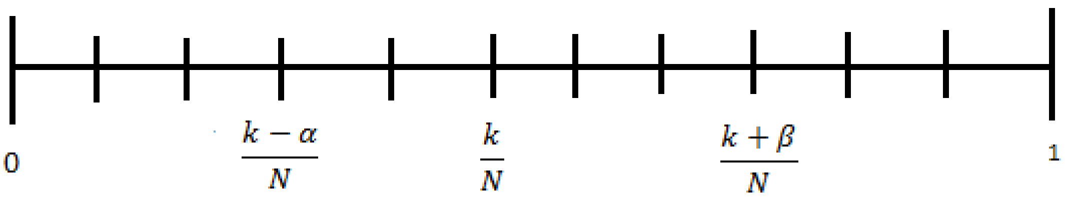
$$= -\frac{1}{2}c - \frac{1}{8} + -\frac{1}{2}c - \frac{1}{8} = -c - \frac{1}{4}.$$

The utility in my model is bigger than the utility in the standard model if c is not too big: $-c - \frac{1}{4} > -\frac{5}{18}$. Rewriting this leads to c < $\frac{1}{36}$. The utility in my model is still the same for the two intervals, while the utility in the standard model increases for each interval. This implies that in my model the tendency to exaggerate is lower in comparison with the standard model, because of the equal utilities compared to the increasing utilities. So, the use of words becomes more efficient in my model. And furthermore, if c is not that big the total utility is higher in my model than in the standard model, which implies that the utility is maximal if the intervals are of equal length.

The same holds for N intervals, because N intervals are divisible in several parts of two intervals. Thus, if it holds for two intervals, then it holds for all parts of two intervals on the total of N intervals. This implies that utility is maximized if the intervals are of equal length and the costs are not too high, which is due to the more efficient way words are used and due to the higher utility.

Then, the following step was to investigate what kind of equilibrium exists if $b > \frac{1}{2N}$, so what happens if the sender does not report honestly, but is going to exaggerate.

Again, I make use of the following figure to look more deeply into this equilibrium:



In my extension of this model the utility of sending m ϵ $[\frac{k}{N}, \frac{k+\beta}{N}]$ must be still bigger than m ϵ $[\frac{k-\alpha}{N}, \frac{k}{N}]$ if the type of the sender lies in an interval in $[\frac{k}{N}, \frac{k+\beta}{N}]$ and the utility of sending m ϵ $[\frac{k-\alpha}{N}, \frac{k}{N}]$ must be still bigger than m ϵ $[\frac{k}{N}, \frac{k+\beta}{N}]$ if the type of the sender lies in an interval between $[\frac{k-\alpha}{N}, \frac{k}{N}]$. This must especially hold for the types of sender close to $\frac{k}{N}$, because at these places the sender is more tempted to exaggerate. So, for this situation to be an equilibrium it must hold that the utility of sending m ϵ $[\frac{k-\alpha}{N}, \frac{k}{N}]$ must be bigger than m ϵ $[\frac{k}{N}, \frac{k+\beta}{N}]$ if t ϵ $[\frac{k-1}{N}, \frac{k}{N}]$ and that the utility of sending m ϵ $[\frac{k}{N}, \frac{k+\beta}{N}]$ must me bigger than m ϵ $[\frac{k-\alpha}{N}, \frac{k}{N}]$ if t ϵ $[\frac{k}{N}, \frac{k+1}{N}]$. Otherwise there might be some senders who are tempted to deviate.

The utilities for both intervals are as follows if t ϵ $[\frac{k-1}{N}, \frac{k}{N}]$:

- Utility m ϵ $[\frac{k-\alpha}{N}, \frac{k}{N}]$: $\frac{\int_{\frac{k-1}{N}}^{\frac{k}{N}} -(t+b-\frac{2k-\alpha}{2N})\,dt}{\frac{k}{N}-\frac{k-1}{N}} - c(N-1)$

  $= -\frac{1}{2N}(\alpha + 2cN^2 + 2bN - 2cN - 1)$
- Utility m ϵ $[\frac{k}{N}, \frac{k+\beta}{N}]$: $\frac{\int_{\frac{k-1}{N}}^{\frac{k}{N}} -\left(\frac{2k+\beta}{2N}-t-b\right)dt}{\frac{k}{N}-\frac{k-1}{N}} - c(N-1)$

  $= -\frac{1}{2N}(\beta + 2cN^2 - 2bN - 2cN + 1)$

The former one must be bigger than the latter one:

$-\frac{1}{2N}(\alpha + 2cN^2 + 2bN - 2cN - 1) > -\frac{1}{2N}(\beta + 2cN^2 - 2bN - 2cN + 1)$. Rewriting this leads again to the condition β-α > 4bN-2.

The utilities for both intervals are as follows if t ϵ $[\frac{k}{N}, \frac{k+1}{N}]$:

- Utility m ϵ $[\frac{k-\alpha}{N}, \frac{k}{N}]$: $\frac{\int_{\frac{k}{N}}^{\frac{k+1}{N}} -(t+b-\frac{2k-\alpha}{2N})\,dt}{\frac{k+1}{N}-\frac{k}{N}} - c(N-1)$

  $= -\frac{1}{2N}(\alpha + 2cN^2 + 2bN - 2cN + 1)$
- Utility m ϵ $[\frac{k}{N}, \frac{k+\beta}{N}]$: $\frac{\int_{\frac{k}{N}}^{\frac{k+1}{N}} -\left(\frac{2k+\beta}{2N}-t-b\right)dt}{\frac{k+1}{N}-\frac{k}{N}} - c(N-1)$

  $= \frac{1}{2N}(2bN - 2cN^2 - \beta + 2cN + 1)$

The latter one must be bigger than the former one now:

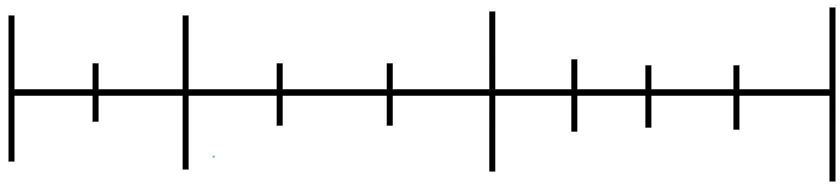
$\frac{1}{2N}(2bN - 2cN^2 - \beta + 2cN + 1) > -\frac{1}{2N}(\alpha + 2cN^2 + 2bN - 2cN + 1)$. Rewriting this results again to the condition β-α < 4bN+2.

The conditions are here the same as at the case that c=0, which implies that the communication strategies depends of course on b and N again. So, I refer to the clarification of such strategies at the case that c=0, to see how the sender will speak in different situations given the number of words. Thus, it is not possible to make a distinction between c=0 and c>0 in these strategies, because the conditions do not depend on the costs. I also refer to the case that c=0 for the explanation of the difference between the standard model of CS and my model at this point, because this is the same for the case that c>0 due to the same derived conditions. This implies that if there exists an equilibrium of three partitions in my model, then it does not mean that there exists by definition an equilibrium with two partitions as well (regardless of the costs), due to the second binding condition. This is contradictory to the standard model of Crawford and Sobel, which says that if there exists an equilibrium of ten partitions, then there also exists an equilibrium of nine partitions (as well as an equilibrium with eight partitions, seven partitions and so on).

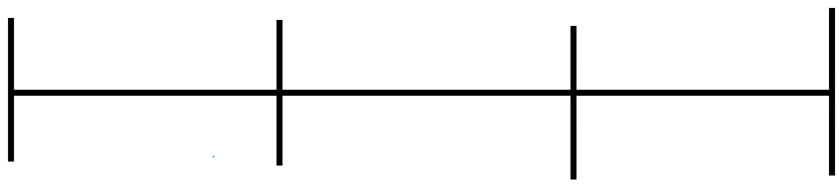
So, these conditions make it possible to illustrate the minimal number of words/partitions. To derive the maximal number of words/partitions the same formulas holds as if c=0, because these formulas does not depend on the costs. Thus, the formulas p = N-1 (if z = 0) and p = $-\frac{1}{2z}(z \pm \sqrt{-4z+8Nz+z^2+4}+2)$ (if z ≠ 0) are again used to calculate the maximal number of words.

Finally, I investigated what is more useful if there are three partitions/words:

- and if N>3 (for example N=9) with three partitions/words:



- or if N=3 with also three partitions/words, but without exaggerated reporting:



If c=0 I showed that the latter one leads to a bigger utility for the sender. Furthermore the latter one is more efficient than the former one. Now, the utility if N=3 is:

$$\sum_{i=1}^{3}\frac{1}{3}\left[\frac{\int_{0}^{\frac{1}{6}-b}-\left(\frac{1}{6}-t-b\right)dt}{\frac{1}{6}-b}+\frac{\int_{\frac{1}{6}-b}^{\frac{1}{3}}-(t+b-\frac{1}{6})dt}{\frac{1}{6}+b}-2c\right]+\frac{1}{3}\left[\frac{\int_{\frac{1}{3}}^{\frac{3}{6}-b}-\left(\frac{3}{6}-t-b\right)dt}{\frac{1}{6}-b}+\frac{\int_{\frac{3}{6}-b}^{\frac{2}{3}}-(t+b-\frac{3}{6})dt}{\frac{1}{6}+b}-2c\right]+$$

$$\frac{1}{3}\left[\frac{\int_{\frac{2}{3}}^{\frac{5}{6}-b}-\left(\frac{5}{6}-t-b\right)dt}{\frac{1}{6}-b}+\frac{\int_{\frac{5}{6}-b}^{1}-(t+b-\frac{5}{6})dt}{\frac{1}{6}+b}-2c\right]$$

$$=-\frac{2}{3}c-\frac{1}{18}+-\frac{2}{3}c-\frac{1}{18}+-\frac{2}{3}c-\frac{1}{18}=\ -2c-\frac{1}{6}.$$

And the utility if N=9 is:

$$\sum_{i=1}^{3}\frac{2}{9}\left[\frac{\int_{0}^{\frac{1}{9}-b}-\left(\frac{1}{9}-t-b\right)dt}{\frac{1}{9}-b}+\frac{\int_{\frac{1}{9}-b}^{\frac{2}{9}}-(t+b-\frac{1}{9})dt}{\frac{1}{9}+b}-8c\right]+\frac{3}{9}\left[\frac{\int_{\frac{2}{9}}^{\frac{7}{18}-b}-\left(\frac{7}{18}-t-b\right)dt}{\frac{3}{18}-b}+\frac{\int_{\frac{7}{18}-b}^{\frac{5}{9}}-(t+b-\frac{7}{18})dt}{\frac{3}{18}+b}-\right.$$

$$\left.8c\right]+\frac{4}{9}\left[\frac{\int_{\frac{5}{9}}^{\frac{7}{9}-b}-\left(\frac{7}{9}-t-b\right)dt}{\frac{2}{9}-b}+\frac{\int_{\frac{7}{9}-b}^{1}-(t+b-\frac{7}{9})dt}{\frac{2}{9}+b}-8c\right]$$

$$=-\frac{16}{9}c-\frac{2}{81}+-\frac{24}{9}c-\frac{1}{18}+-\frac{32}{9}c-\frac{8}{81}=\ -8c-\frac{29}{162}.$$

Rewriting $-2c-\frac{1}{6}>-8c-\frac{29}{162}$ results in c > $-\frac{1}{486}$. This is a logical result, because at the case c=0 the utility of N=3 was bigger than if N=9. So, the utility is of course bigger for N=3 than for N=9 (both with three words/partitions) if c>0, which implies again that the sender does not take a lot of information.

Thus, there were two possible reasons why the sender does not take a lot of information:

1. The sender needs to make costs for it.
2. Or because less information improves communication.

The example of above illustrates that less information improves communication for both cases (c=0 and c>0). So, even if the costs are zero and information is free, the sender does not take a lot of this information. This implies that if c>0, the sender does of course not take a lot of information as well.

6. Conclusion

In this paper I have analyzed a variation of the standard cheap-talk model. In the basic model of Crawford and Sobel, the sender knows his type exactly. However, in this paper I have made the extension that the sender does not know his type exactly. The sender first needs to collect some information about his type. He could do that through determination in which interval his type lies, where an additional interval costs $c \geq 0$. The sender knows his type more precisely, if he adds more intervals. For the results I make the distinction between the case that $c = 0$ and $c>0$.

The main finding that emerged from my model is that less information improves communication. So, even if the costs are zero and information is free, the sender does not take a lot of this information. This implies that if $c>0$, the sender does of course not take a lot of information as well. Thus, the sender does not take a lot of information regardless of the costs. This is in line with the finding that the sender is less tempted to exaggerate with less information. This finding showed that adding more intervals leads to a lower chance on an equilibrium where the sender truthfully reveals his information. I also showed that the utility is maximized if the intervals are of equal length, which implies that better communication is possible in my model than in the standard model of Crawford and Sobel. Further, I investigated what kind of equilibrium exists if the sender does not report honestly, but is going to exaggerate. I derived two conditions for the minimal number of words, which make it possible to find out different communication strategies. And I derived a formula for the maximal number of words, which make this equilibrium complete.

In this paper I have made the assumption that the sender does not know his type exactly, but that he first needs to collect information about his type, which could be done through determination in which interval his type lies at a cost c per interval. But, what happens with the results if for one certain cost the sender can determine his type precisely? Does he still not take information or does the conclusion change now? And, what happens if there are only a few senders who could determine their type through determination in which interval their type lies at a cost c per interval? Does the results change now since those few senders might be privileged compared to the others? I let these questions for further research.

7. References

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