Bachelor Thesis

On the properties of a desired voting system

Are anonymity, neutrality, Pareto condition, Independence of Irrelevant Alternatives, monotonicity and strategy-proofness necessary conditions for a desired democratic system?

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Word count	15,671
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1. Introduction

The American presidential election of the year 2000 left the majority of the population with a feeling of unfair treatment when the final result showed that President George W. Bush had won with 30 states carried. On the contrary, his opponent, Al Gore, only carried 21 states. Now, why is the outcome by many people considered as unfair? Bush had 47.9 per cent of the inhabitants on his side, whereas Gore had 48.4 per cent of the inhabitants supporting him. Thus, we see that the majority of the population favoured Gore over Bush. Then, why on earth, was Gore not elected President in 2000? This question led to many vote recounts and the very essence of the district voting system even led to a court case. All these doubts about the institutional system were swept away by the Supreme Court in December of 2000: this election system had been used for nearly two centuries and one should not change a key foundation of the United States. This was the fourth time in American history such a paradox became true.

Voting systems have always been regarded as prawn to many paradoxes. The above case is just one example of dozens. One possible explanation as to why voting paradoxes can occur is the existence of the possibility to somehow alter the outcome of a procedure by voting strategically, rather than sincerely. What these terms mean will become clear at a later point. For now it suffices to say that voting for someone you do not want to win, might actually help your most favoured candidate (or more broadly: alternative) to win.

Simply put, the candidate that would always lose in a face-to-face procedure (that is: giving every voter the chance to say “I prefer X over Y” or the other way around) is called the Condorcet loser. Vice versa, the person that would always win in a face-to-face procedure is called the Condorcet winner. It suffices for now to give this basic distinction. Throughout the thesis, these terms will be elaborated on and the importance of Condorcet losers and winners will become more evident.

Referring back to the US example above, we can call Gore the Condorcet winner. After all, if we had asked every individual whether he or she preferred Bush or Gore, Gore would be the winner based on most votes count. Based on the district voting system the USA use, we can conclude that this system is inherently malfunctioning, because a Condorcet winner does not necessarily win the election. A keen eye might already have noticed that the votes for Gore and Bush as mentioned above do not add up to 100 per cent. This is vital for the theories and formulas that will be presented in this thesis. Condorcet winners can only lose in election systems with three or more alternatives (Brams & Fishburn, 2002). The key interfering factor here is the participation of, amongst others, Ralph Nader in the final election round. We will later see how voting (or not voting) for him might cause the Condorcet winner his head. In my view voting strategically should not pay in an ideal democratic voting system.

I suspect that a most-desired democratic system needs to fulfil a particular set of conditions. These conditions are: anonymity, neutrality, Pareto condition, Independence of Irrelevant Alternatives (IIA), monotonicity and strategy-proofness. Especially IIA and strategy-proofness will be elaborated on thoroughly throughout this thesis. I do so because these are, as I have found out during my research, the two conditions that are hardly met in any system and therefore require special attention to stress the importance of meeting these conditions.

To examine these conditions and the roles they fulfil in certain voting procedures, I will first introduce the definitions and axioms I will use in this thesis. After having given the relevant information to read and understand the rest of the thesis, I will put this information into perspectives with respect to common (and less common) voting procedures and test the conditions in terms of both formal logics and empirical data. The very last section will be focussed on answering whether all aforementioned conditions are necessary conditions for a most-desired (perfect?) democratic voting procedure.

2. Definitions

2.1 Condorcet winners and losers

In the previous section I concisely spoke about the Condorcet paradox. This paradox states that the wishes of majorities conflict with each other. In the above case we see that the wish of the majority of individuals is that Gore wins, but that the wish of the majority of the districts want Bush to win. The essence of this paradox lies in taking different majorities (here: individuals as well as districts). As I am of the opinion that in a just democratic system the Condorcet winner should also be the actual winner, I start this thesis by elaborating on this paradox. I hope that at the end of this section it has become more evident as to why it is important to analyse who the Condorcet winner is and who the Condorcet loser is and why a Condorcet winner should also be the actual winner.

Marie Jean Antoine Nicolas de Caritat, marquis de Condorcet is the key figure when it comes to assessing voting systems. As early as the eighteenth century, he already discovered that one can alter the outcome of an election with three or more alternatives, where the winner of a runner-off election would ultimately lose (Borda, 1781/1953). This might seem somewhat surreal. I will use the following example to illustrate the point in case.

Take the following three people as presidential runners of Imaginistan[1]: Wizard x, Wizard y and Wizard z. One's preference then can be ordered as followed:

Wizard	x	>	Wizard	y	>	Wizard	z
Wizard	x	>	Wizard	z	>	Wizard	y
Wizard	y	>	Wizard	x	>	Wizard	z
Wizard	y	>	Wizard	z	>	Wizard	x
Wizard	z	>	Wizard	x	>	Wizard	y
Wizard	z	>	Wizard	y	>	Wizard	x

Let ">" denote that one (strictly) prefers Wizard … over Wizard …

Now suppose that there are 31 *electing* inhabitants in this beautiful country. Suppose these are the numbers of people preferring a certain set of orderings:

Wizard	x	>	Wizard	y	>	Wizard	z	5	people
Wizard	x	>	Wizard	z	>	Wizard	y	7	people
Wizard	y	>	Wizard	x	>	Wizard	z	3	people
Wizard	y	>	Wizard	z	>	Wizard	x	7	people
Wizard	z	>	Wizard	x	>	Wizard	y	3	people
Wizard	z	>	Wizard	y	>	Wizard	x	6	people

Based on Majority voting we see that Wizard x is the winner with 12 people preferring him as the president.

[1] This is based on an example by Ad van Deemen (Swart, Van Deemen, Van der Hout, & Kop, 2003)

Now suppose that Wizard y withdraws from the elections due to illness. We then end up with the following result:

Wizard	x	>	Wizard	z	5	people
Wizard	x	>	Wizard	z	7	people
Wizard	x	>	Wizard	z	3	people
Wizard	z	>	Wizard	x	7	people
Wizard	z	>	Wizard	x	3	people
Wizard	z	>	Wizard	x	6	people

We now see, that 15 people prefer Wizard x, but there are 16 people who prefer Wizard z. Wizard x is called a *Condorcet loser* and Wizard z a *Condorcet winner*. This way of pairwise-comparison is also known as Majority rule, meaning that the Condorcet winner should in essence win the election based on a pairwise-comparison, being the most preferred candidate.

Naturally, one may now ask what happens when Wizard z withdraws from the election. This would yield the following structure:

Wizard	x	>	Wizard	y	5	people
Wizard	x	>	Wizard	y	7	people
Wizard	y	>	Wizard	x	3	people
Wizard	y	>	Wizard	x	7	people
Wizard	x	>	Wizard	y	3	people
Wizard	y	>	Wizard	x	6	people

We can now see, on a pairwise basis, that Wizard y is preferred over Wizard x by 16 to 15.

So, we see that Wizard x loses the election in both scenarios. Still, he wins if both other candidates, being Wizards y and z, participate in the election. Hence, this way of voting does not fulfil the requirement that the outcome should not be altered by adding irrelevant alternatives, being Wizards y and z respectively.

In this case, the Condorcet loser emerges as the winner by, as it seems, a mathematical 'error' that results from adding irrelevant alternatives, yielding a preference paradox. However, if this is possible by chance, it should also be possible by a way of voting strategically.

Now that we have a basic definition of what a Condorcet loser/winner is, it is time to move on to the next step: giving an explanation of the Condorcet paradox.

The Condorcet paradox is the event that the Condorcet loser indeed wins the election. Referring to the example displayed above, this means, that Wizard x indeed wins the election, leaving the Condorcet winners, Wizard y and z, behind. Hence, the Condorcet paradox addresses the event that the loser, pair-wisely compared to the other candidates, wins the election.

One may now think that this result might be justified on the basis of, for example, most votes count or some deep-held belief that certain voting systems are just, purely based on the fact that they are centuries old, like the Bush vs. Gore case. However, here, a Condorcet loser winning the election is inherently unjust, as this is proof that this system is prawn to the Condorcet paradox.

2.2 Conditions for a democracy

Now that we have a better picture regarding Condorcet winners and losers it is time to move on to the next step. My claim is that in an ideal democracy the Condorcet winner should always win. In this section I will go over the conditions I think of as essential for an ideal democracy. I will go over them one by one and explain their essentialities and why I see fit for them in an ideal democracy. These conditions are: anonymity, neutrality, Pareto condition, Independence of Irrelevant Alternatives (IIA), monotonicity and strategy-proofness. In this section I will elaborate on what these conditions mean. However, I will first explain the formal notation I will use in later paragraphs.

As should have become clear in the previous section, the Condorcet winner is the winner based on a pairwise comparison of one alternative against all the others, individually. This is called a Majority rule. It puts forward valuable properties for an ideal (democratic) voting system. Ad van Deemen draws the conclusion that an ideal voting system should fulfil the principles of anonymity, neutrality, the Pareto condition, IIA and monotonicity (Van Deemen, 1997). Since a Majority rule with only two alternatives is by definition strategy-proof (meaning that cheating does not pay) and the aforementioned conditions are derived from that Majority rule with only two alternatives, these form the basis for my definition of an ideal democratic voting system. I am by no means suggesting that in an ideal system only two candidates should be up for vote – this would be surrealistic (there is no system in which only two alternatives could cover all the wishes of the individuals), but I am using this as a starting point for this thesis.

An individual order of preference of the Wizards, based on the previous example, can be rendered on the following (preference) relation *R*:

R = {<Wizard x, Wizard y>; <Wizard y, Wizard z>, <Wizard x, Wizard z>}

In this definition <*a*, *b*> is an ordered pair and <*a*, *b*> ϵ[2] *R* means that *a* is at least as good as *b*. <*a*, *b*> ϵ *R* can also be written as *aRb*. Now then, to show once preference we can write *aRb and not bRa*, meaning that *a* is at least as good as *b*, but *b* is not at least as good as *a* and hence *a* is preferred over *b*. If we write *aRb and bRa* this means that *a* is at least as good as *b* and vice versa and hence there exists indifference between the alternatives.

Now suppose that *A* is a set of alternatives. To stick to our example *A* = {Wizard x; Wizard y; Wizard z}. Also suppose that there is a set of *N* individuals, constituted by the voters: *N* = {Voter 1; Voter 2; …; Voter 31}. With this information we can identify the individual preference ordering of all individuals *i* with respect to the alternatives in *A* c.q. the Wizards by means of a relation *Ri* on *A*. This is called a preference relation on *A*. Hence, *R* is a preference relation on *A* if *R* is a set of ordered pairs {*a*, *b*} ϵ *A*.

Van Deemen then continues by stating that a preference relation needs to meet three fundamental requirements. They need to be 1) complete, 2) transitive and 3) anti-symmetric.

R is complete if and only if for all <*a*, *b*> ϵ *A*, *aRb* or *bRa*. This means that every alternative needs to be comparable to every other alternative in *A*, including itself.

R is transitive if and only if for every <*a*, *b*, *c*> ϵ *A* it is true that if *aRb* and *bRc* ⊃[3] *aRc*. This for example means that if Nader is preferred over Gore and Gore over Bush, he has to prefer Nader over Bush.

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[2] Meaning that the part on the left is an element of the set on the right of the ϵ-sign.
[3] Meaning that the part on the right follows from the part on the left of the ⊃-sign.

This can be logically proven[4], but this might go against a lot of intuitions in daily life. For example, one may prefer apple pie to cherry pie and cherry pie to chocolate pie. Then, when someone is asked to choose between chocolate pie and apple pie, he sometimes still chooses chocolate pie. However, based on transitivity, he should choose apple pie. Nonetheless, we see that people in practice might act against the logics and act irrational instead.

R is anti-symmetric if and only if for every <*a*, *b*> ϵ *A* with *a* ≠ *b* if *aRb*, then not *bRa*. This means that indifference (not even the slightest) does not occur. Hence, all individuals must have a certain preference between certain alternatives.

We will take the three conditions as explained above for granted while assessing the individual voting systems in the following sections, unless one is violated and this violation is relevant for an analysis with respect to the to be discussed conditions.

A preference relation *R* is a so-called weak ordering on *A* if *R* is complete and transitive. *R* is called a linear ordering on *A* if *R* is both complete and transitive and also fulfils the condition of being anti-symmetric. Consequently, there can be indifference in weak orderings, but not in linear orderings. We call *C(A)* the set of all complete relations on *A*, *W(A)* the set of all weak orderings on *A* and *L(A)* is the set of all linear orderings on *A*. It follows by definition that every linear ordering is also a weak ordering.

Next, a profile *p* associates with every individual *i* in *N* a linear ordering *Ri* on *A*. A profile *p* : *N* → *L(A)*. *p(i)* or *Ri* then is the individual linear ordering of a certain individual *i* in profile *p*. $L(A)^N$ then is the set of all relevant profiles for an election.

With this information we can constitute two relevant rules: the Choice rule and the Preference rule.

The Choice rule is used to choose one alternative from a set (e.g. a president, a mayor, etc.). A Choice rule then is *K* : $L(A)^N$ → *A*. Hence, *K* assigns to each profile *p* ϵ $L(A)^N$ a collective choice *K(p)* in *A*.

A Preference rule is used to establish a certain preference ordering of the alternatives. This information is needed in case, for example, the number one and two after an election win. A Preference rule then is *F* : $L(A)^N$ → *C(A)*. Hence, a Preference rule assigns to each profile *p* ϵ $L(A)^N$ a complete preference relation *F(a)* on *A*.

Now then, suppose that *N* is still a set of individuals and *A* is still a set of alternatives. Then, given a certain profile *p* and an alternative *a* in *A*, we define *t(a, p)* as the number of individuals *i* in *N* that have *a* as their first choice in their profile *p(i)*. This means that there does not exist an alternative *b* that is preferred to *a* by *i* in *p(i)*. Then, Most votes count as a Preference rule is rendered by the function *Pl* (standing for 'plurality') from $L(A)^N$ to *W(A)*, defined as *aPl(p)b* ≡[5] *t(a, p)* ≥ *t(b, y)*, meaning that *aPl(p)b* only holds if and only if the number of individuals that prefer *a* most in *p* is greater than or equal to the number of individuals that prefer *b* most in *p*. Please note that this specifically holds for Most votes count procedures with only two alternatives. In a later part of this thesis we will see that this is not necessarily the case if there are three or more alternatives.

After having extensively drawn the way of notation, we can now continue with an elaboration on the principles (being the principles of anonymity, neutrality, the Pareto condition, Independence of

[4] After all, if x is preferred over y, y is preferred over z, this yields the following formula: x > y > z. Then, by means of transitivity: x > z. Hence in the example: apple pie > cherry pie > chocolate pie, meaning apple pie > chocolate pie. However, in practice we might see apple pie < chocolate pie, which is an inverse change of preference.

[5] Meaning: if and only if.

Irrelevant Alternatives and monotonicity), based on the Majority rule and the (collective) Preference rule (the complete preference relation *F(a)* on *A*) that we have distilled thereof.

2.2.1 Anonymity

Anonymity states that individuals need to be treated equally. This means that no vote is worth more than another vote.

A collective preference rule *F :* $L(A)^N$ → *C(A)* is anonymous if, for all profiles *p* in $L(A)^N$ and for every permutation μ of *N*, $F(p \circ \mu) = F(p)$. In words: a voting procedure is anonymous if and only if a permutation of the preference rule still yields identical preference rule. Most voting procedures meet the anonymity requirement.

In my opinion this should be the case for an ideal democracy: every vote should be of equal worth as any other vote. However, in some cases, especially in indirect democracies (where individuals do not directly elect the winners of election, e.g. the election of the Dutch parliament) – where preferences might not be very clear, this condition should be tackled carefully. For example, the votes for the International Monetary Fund (IMF) General Assembly are based on the economic size of the countries, denominated in Special Drawing Rights (SDRs; a kind of universal 'currency'). One might think that every member should have an equal vote in the decision-making process. However, it would be odd if the votes of both Luxembourg and Belgium would outweigh the vote of the USA. Then countries could easily team up and vote against the proposals of very big countries. In this thesis I will primarily focus on direct democracies (where individuals elect the winner(s) of an election directly) and for this type of democracies I think anonymity is a necessary condition for an ideal democracy, since every voter is equal to another vote and hence their votes should be of equal value. A more detailed discussion regarding anonymity can be found in section 8.

2.2.2 Neutrality

Neutrality states that all the alternatives are treated equally. This is a more substantial rather than formal requirement, meaning that, for example, every presidential candidate counts (i.e. is relevant for the election), irrelevant of his or her opinions.

A collective preference rule *F :* $L(A)^N$ → *C(A)* is neutral if, for every permutation μ of *A* and for every profile *p*, $F(\mu p) = \mu(F(p))$. In words: a permutation of the profiles yields the same collective preference rule, but then with the permutation, meaning there is no change as compared to the original position.

Neutrality and anonymity are quite similar: anonymity basically states that all the individuals are of equal value, whereas neutrality states that all the alternatives are of equal value. It should not come as a surprise that in an ideal democracy all alternatives should be of equal value.

2.2.3 Pareto condition[6]

As mentioned earlier, the Pareto condition is met if and only if collectively alternative *a* is preferred over alternative *b*, if all individuals prefer *a* over *b*.

Formally, a collective preference rule *F :* $L(A)^N$ → *C(A)* meets the requirements for the Pareto condition if, for every profile *p* in $L(A)^N$ and for all alternatives *a* and *b* in *A*, if for every $i \in N$ $ap(i)b$ (and therefore not $bp(i)a$), then $aF(p)b$ and not $bF(p)a$. So, this means that if all of the individuals have a certain

[6] The following conditions are proved on the basis of the Majority rule with two alternatives, namely *a* and *b*.

alternative of the set *A* as their first choice in their profile *p*, then this should also collectively (the sum of the parts) be the most-preferred alternative (and hence be the winner).

In my opinion it should be evident that if all individuals prefer a certain alternative, that alternative should also be collectively preferred over the other alternatives. This is especially the case for the winner of the election: if all individuals have him or her as their first choice, then, collectively, he or she should be the winner. So, recall, that this is especially important for a Choice rule (picking one winner). If we would create a Preference rule, then we could easily see which alternative is preferred over which. Unfortunately, most elections only allow the individuals to give their vote to one alternative, disregarding their second, third, etc. preferences. We will come across voting systems that indeed do take into account more than just the first preferences and when discussing those voting systems it will become evident why it is important that *a* is collectively preferred over *b* when all individual profiles are added up.

Closely related to the Pareto condition as presented above, is the, what I would like to call, stronger Pareto condition. This conditions states that if the majority of the individuals prefer one alternative over the other, then also collectively that alternative should be preferred over the other. This way it also makes sense to call it the stronger Pareto condition: if it is fulfilled, it will also necessarily satisfy the weaker Pareto condition as presented above. It hardly ever occurs that all individuals in an election prefer one candidate over another. Therefore I chose to grant some attention to this stronger Pareto condition, since it is far more common that a majority of individuals prefer one alternative over another and, hence, that specific alternative should also collectively be preferred over another.

Formally, a collective preference rule *F :* $L(A)^N \rightarrow$ *C(A)* meets the requirements of the stronger Pareto condition if, for every profile *p* in $L(A)^N$ and for all alternatives *a* and *b* in *A*, if for the majority of the individuals *i ϵ N ap(i)b* (and therefore not *bp(i)a)*, then *aF(p)b* and not *bF(p)a*. So, this means that if most of the individuals have a certain alternative of the set *A* as their first choice in their profile *p*, then this should also collectively (the sum of the parts) be the most-preferred alternative (and hence be the winner).

In the next sections we will see that this might not always be the case. I think that meeting the stronger Pareto condtion is of utmost importance for an ideal democracy, since the winner of an election should cover most of the individuals' preferences.

2.2.4 Independence of irrelevant alternatives

When determining the winner between two alternatives *a* and *b* the outcome of this election should not be influenced by a (spurious, irrelevant) third alternative.

A collective preference rule *F :* $L(A)^N \rightarrow$ *C(A)* fulfils the condition of being independent of irrelevant alternatives if for all *a*, *b ϵ A* and for all profiles *p*, *q* ϵ $L(A)^N$, if *p* is limited to *a* and *b* is equal to *q* limited to *a* and *b*, then *F(p)* limited to *a* and *b* is equal to *F(q)* limited to *a* and *b*. *q* in this definition stands for a profile *p* with a certain permutation, allowing for other alternatives to enter the profile *p*. Hence, in words, this comes down to the following: if irrelevant alternatives (alternatives that have no chance of winning the election) are added to a profile *p*, making it another profile (i.e. *q*), then the (collective) preference ordering will not change with respect to alternatives *a* and *b*.

We have seen that election outcomes can be seriously altered by adding seemingly irrelevant alternatives. In my research I found out that most problems and paradoxes in voting procedures are given rise to by a problem regarding IIA and the closely-related problems regarding strategy-proofness, about which more in the following sections, where I will go over several voting systems. As for an ideal democratic voting procedure it should hold that it is paradox-free, these problems should be overcome and hence the requirements regarding IIA should be met.

2.2.5 Monotonicity

A candidate *a* should not be harmed if *a* is raised on some ballots without changing the orders of the other candidates (Woodall, 1996). This means two things (Smith, 2009):

1) If someone increases their vote for candidate *a* that should not worsen *a*'s chances of winning the election, ceteris paribus; and
2) If someone decreases their vote for candidate *b* that should not improve *b*'s chances of winning.

This might seem unreal for now, but we will see that it in some voting procedures an alternative might actually lose the election due to receiving more votes.

A collective preference rule *F :* $L(A)^N \rightarrow$ *C(A)* is monotonic if and only if, for all profiles *p* and *q* in $L(A)^N$ and for all alternatives *a* and *b* in *A*, if:

a. For all *i* ϵ *N*, if *ap(i)b* (and not *bp(i)a*), then *aq(i)b* (and not *bq(i)a*); and
b. There is an individual *k* ϵ *N* so that *bp(k)a* and *aq(k)b*, then *aF(p)b* implies that *aF(q)b* and not *bF(q)a*.

The first condition means that if individuals with profile *p* prefer *a* over *b*, then they should also prefer *a* over *b* in a permutated profile. The second condition shows that if there is an individual *k* in the collection of individuals *N* that prefers *b* over *a* in profile *p* and *a* over *b* in the adjusted profile *q* (recall that in that profile the ordering of *a* and *b* has not been altered), then according to the collective preference rule *F(p)*, in which *a* is still preferred over *b* implies that *a* should also be preferred over *b* in the collective preference rule *F(q)*.

I hope that it makes intuitive sense that in an ideal democratic system gaining votes should give you an advantageous position and losing votes a disadvantageous position.

2.2.6 Strategy-proofness

Regarding voting, there basically exist two types of voting: one can vote sincerely or one can vote strategically.

If one votes sincerely, this means that one votes for the alternative (e.g. election candidate) he or she wants to win. If one votes strategically (or: insincerely) he votes in such a way that does not represent his personal preference, but he votes for another alternative to amend the result in such a way that, let us call it, his number one ultimately wins or, more broadly, to prevent an undesired outcome.

The voting mechanism is called incentive-compatible once it is best for all voters to vote sincerely, which should be one of the goals of a democratic way of voting or electing (Vleugels, 1997). A mechanism is incentive-incompatible once it *could be* better for the voters to lie about their preference or to not vote for the alternative they, sincerely, regard as their number one. Hence, voting strategically/tactically (i.e. not for your number one) could ultimately yield a better outcome than if they had voted for their number one.

As there is no definite formal notation of strategy-proofness I will make clear what I mean with strategy-proofness in the following paragraph with the use of the Gibbard and Satterthwaite theorem.

Gibbard and Satterthwaite's theorem

When going over the voting procedures in the following sections it will become disturbingly clear that it is very hard to find a voting system that fulfils all the requirements that need to be met in order to speak of a voting procedure that is strategy-proof. This hunch finally resulted in extensive research. The two most relevant theorems formed consequently are Arrow's impossibility theorem (Arrow, 1950) and Gibbard and Satterthwaite's theorem (Gibbard 1973, Satterthwaite 1975). The first postulates that if there are three or more alternatives, it will be impossible to convert the ranked preferences of the individuals of that population into an aggregated ranking (that has to be complete and transitive) while also meeting a set of criteria specified for the ideal form of voting. This is a social choice rule topic I will not discuss. The other theorem, which I will abbreviate to GS, postulates a more interesting idea for this paper.

GS, just like Arrow's theorem, sets a set of basic criteria that need to be met in order for a voting system to be justified (Gibbard 1973, Satterthwaite 1975). I will not discuss the morals of a particular voting system here, so we will take them for granted for now. The criteria basically come down to the following:

1) There is no dictator[7];
2) The voting system needs to fulfil the Pareto condition, meaning that if all of the individuals prefer X, X is also preferred by the sum of the individuals. This condition is incorporated to makes sure that the collective choice will be minimally sensitive to the preference profile;
3) There are at least three candidates;
4) It does not pay for voters to lie / the voting system is not subject to strategic behaviour. This means that giving an insincere preference profile is not beneficial to those voters.

The theory basically comes down to the following. Suppose there is a choice rule $K : L(A)^N \rightarrow A$. Now then, the choice rule $K : L(A)^N \rightarrow A$ is dictatorial if there exists an individual $i \in N$ so that for each profile $P \in L(A)^N$ the collective choice *K(P)*, the best alternative is *P(i)*, meaning that *i* is the best alternative in all cases, making him the dictator.

$K : L(A)^N \rightarrow A$ is strategy-proof if for every profile *P* a unilateral deviation of an individual *i* from his original profile *P* to a transformed profile *Q* will not make *i* better off, meaning that giving a different profile will not result in a (subjectively) better outcome.

GS then states: suppose that there are at least three alternatives in *A,* and $K : L(A)^N \rightarrow A$ is strategy-proof, then *K* is by definition dictatorial. This makes intuitive sense looking back at the previous axioms, since if *K* always yields *i*, independent of the preference orderings, then *K* is by definition dictatorial[8].

We thus here find out that voting systems will never be at the same time strategy-proof and non-dictatorial (in case there are three or more alternatives), if the other required conditions as above are satisfied. Whether this is a problem is up to the social scientists. In my opinion a democracy is by definition not a dictatorship (not even in the broadest sense that GS put forward), so I will primarily focus on democratic voting systems we know. However, in the end I will briefly analyse a dictatorial voting procedure to shine some light on the problems it may contain.

To sum it all up: a voting procedure is strategy-proof if giving an insincere preference profile does not pay for an individual. I am of the opinion that this needs to be safeguarded as much as possible (primarily by safeguarding the other conditions that need to be met) in order to justify the voting procedure as

[7] This means a 'dictator' in the broadest possible sense. Meaning that if someone, for example, has the preference ordering P(n) = {A; B; C} this will not cause A to start losing or C to start winning.
[8] For more logical proof, please see (Swart, Van Deemen, Van der Hout, & Kop, 2003), pages 194-195.

much as possible in order to speak of an ideal democratic procedure. After all, a voting procedure in which insincere voting pays of, cannot be desired in an ideal democracy. I am saying as much as possible, since the previous formally proofs that every voting system with three or more alternatives cannot be both non-dictatorial and strategy-proof.

Be that as it may, the following counterargument in favour of strategic behaviour does deserve special notice. Van Hees and Dowding figured that manipulation is not all that bad (Van Hees & Dowding, 2006). They reasoned that it might be to the benefit of certain voters to use strategic behaviour in order to prevent an outcome that is worse than would occur had they voted sincerely, making them better off in the end. They called this sincere manipulation. In their eyes (sincere) manipulation is not all that bad, as it reflects what we see in real life and in some cases it might be to the advantage of the weaker parties of society to prevent a less-desired outcome. They do deserve credit for treating voting as a dynamic process in which voters need to be able to act according to their own benefit. However, I think that the possibility of sincere voting might only be to the advantage of the privileged voters who have access to all information and who are more actively involved in the procedure. It also gives rise to immensely complex mathematical and logical challenges, since voters are then allowed to change their preferences (i.e. reveal another preference than their original preference) during the election procedure.

For this thesis I will stick to the static presentation of one's preference ordering, unless I choose to amend the revealed preference ordering by means of example to test for strategy-proofness or other conditions.

3. Analysing different voting systems

Until now I have given a big introduction regarding the terms that will be used throughout this paper. This paragraph will be the last paragraph before analysing the specific voting systems. I will analyse the voting systems by first explaining their mechanics and then check whether they meet the drawn conditions as put forward above. Also, I will draw attention to what we see in practice with regards to the conditions for an ideal democratic voting procedure. My aim is to show whether or not the conditions necessary for an ideal democracy are actually desirable.

I will begin with the simplest and most fundamental way of voting: Most votes counts. Consecutively, I will cover the De Borda-system, the Alternative voting system and the Single transferable voting system. After each section I will briefly recap the findings from that section. After having assessed those voting systems, I will draw a more general conclusion as to whether anonymity, neutrality, Pareto condition, IIA, monotonicity and strategy-proofness are indeed necessary conditions for an ideal democratic voting system.

4. Most votes count

Since childhood almost all of us have kept the deep-held belief that a decision is made best based on which alternative has the most votes. This usually neglected, and mostly ever since neglects, the rankings of our second-best, third-best, etc. alternatives. Stunningly then, it must come as a surprise that Most votes count is not always by definition a majority choice. By means of example, take a look back at the imaginary country I sketched in the introduction of this paper. The majority of the voters, on a pairwise basis, prefer Wizard x the least. However, based on the principles of Most votes count, he still comes out as the winner of the election. We have seen that Wizard y is preferred over Wizard x by 16 to 15 votes; Wizard z is preferred over Wizard x, also, by 16 to 15 votes; and Wizard z is preferred over Wizard y, also, by 16 to 15 votes. Had the election been based on a pairwise voting system, meaning voters had to vote for either Wizard x or Wizard y; Wizard x or Wizard z; Wizard y or Wizard z, Wizard x would always lose. Unfortunately, adding all alternatives up to form one final election round, which might seem like a good idea based on budget reasons, leads to something which is horribly wrong. The point I want to make here is that even though someone might seem to be the justified winner of an election, that winner might not be the (ideal) Condorcet winner. Recall that this is a violation of the stronger Pareto condition. This already stresses the importance of that condition.

Take the pairwise comparison of Wizard z and Wizard y. We see that Wizard z wins with 16 votes to 15. However, adding Wizard x to the election round causes the inverse outcome. Analogously, take a look at the pairwise comparison of Wizard z and Wizard x. We see, that Wizard z wins, again, with 16 votes to 15. Additionally, as with the previous comparison, adding Wizard y leads to an inverse outcome. Respectively, Wizard x and Wizard y are irrelevant candidates, because in a pairwise comparison they both lose to Wizard z. Hence, we conclude that Most votes count does not fulfil the principle of Independence of Irrelevant Alternatives. It should be shocking that adding the Condorcet loser to the election round costs the Condorcet winner the win.

There are a couple of more pitfalls in the Most votes count procedure, but the important message for now is that the outcome of the election can be changed by strategic behaviour and that the outcome can be changed in such a way that the Condorcet loser will ultimately win. Please verify this by means of the following example.

If you recall the Imaginistan example, we see that there are seven voters with the following set:

P(n) = {Wizard y > Wizard z > Wizard x}

Clearly, these seven voters prefer Wizard z over Wizard x. By means of the Most votes count system, they can achieve this by giving the insincere (read: strategic) set:

P(n) = {Wizard z > Wizard y > Wizard x}

Now, based on the Most votes count system, Wizard z (recall, this is the 'original' Condorcet winner) wins the election.

It becomes evidently clear here that these voters have altered their preference ordering in such a way that they have prevented an undesired outcome. In this specific incident the Condorcet winner actually wins the election.

A last note I want to make here is that Most votes count does not necessarily fulfil the stronger Pareto condition, meaning that Most votes count does not necessarily elect the alternative with the most votes based on a pairwise comparison.

We see that for if the majority of individuals $i \in N$ $zp(i)x \wedge$[9] $\neg$[10]$xp(i)z$ and likewise for the majority of individuals $i \in N$ $zp(i)y \wedge \neg yp(i)z$ it should hold that $zF(p)xy$, meaning that Wizard z is collectively preferred if the majority of the individuals prefer Wizard z to Wizards x and y, individually. This clearly is not the case, since Wizard x wins the collection.

I will not analyse the flaws of a particular voting system, instead I want to analyse the voting procedure and so analyse the ideal conditions for a democracy. With Most votes count we have clearly seen that it does not meet the requirements of being IIA. I mentioned before that I already suspected that IIA is a key feature of an ideal democratic voting system. In this case we have clearly seen that not satisfying IIA can lead to serious changes in the outcome of an election. In this case we have also seen that Most votes count does not necessarily yield an outcome that is the majority choice. Consequently, we see

[9] Meaning "and"
[10] Meaning "not"

that the actual winner might not be the Condorcet winner, meaning that he cannot rely on the support of the majority of the voters. In my opinion this should be the case for an ideal democracy. We will later see that Most votes count procedures are also prawn to strategic behaviour. Since the conditions of IIA and the stronger Pareto condition can be so easily violated, my conclusion for this section is that the conditions of IIA and the stronger Pareto condition are very likely to be vital conditions for an ideal democratic voting procedure.

In the following section I will elaborate on a variant of Most votes count: the De Borda rule.

5. De Borda Rule

Being faced with this problem (that the outcome of Most votes count can be so easily manipulated), the French Jean-Charles, chevalier de Borda came up with a possible solution in order to try to overcome the problem that the Most votes count system did not always fulfil the stronger Pareto condition and, to a larger extent, did not always yield the Condorcet winner as the actual winner. He came up with a system that assigns values to the alternatives in one's preferences ordering. He created a formula which came down to the following (Borda, 1781/1953):

- Every voter has a certain preference ordering;
- These preference orderings form a set of alternatives;
- These alternatives are all assigned a particular value;
- The most-preferred alternative is assigned the value *N* (representing the total number of alternatives in one's preference ordering);
- The alternative after the most-preferred alternative is assigned the value *N-1*, the alternative after that *N-2*, and so on;
- The values are then multiplied by the amount of individuals per preference ordering;
- The alternative with the highest value wins the election.

To make the above reasoning clear, I will now briefly put it in perspective by using Imaginistan as an example. Recall, that the preference orderings were like this:

Wizard	x	>	Wizard	y	>	Wizard	z	5	people
Wizard	x	>	Wizard	z	>	Wizard	y	7	people
Wizard	y	>	Wizard	x	>	Wizard	z	3	people
Wizard	y	>	Wizard	z	>	Wizard	x	7	people
Wizard	z	>	Wizard	x	>	Wizard	y	3	people
Wizard	z	>	Wizard	y	>	Wizard	x	6	people

Wizard x gets the following score:

Bx (Borda score) = 5x3 + 7x3 + 3x2 + 7x1 + 3x2 + 6x1 = 61

Please check the following:

By = 62

Bz = 63

We now see that the Condorcet winner, Wizard z, is actually the winner of the election. However, it is more important to see whether this voting system will always yield the Condorcet winner as the winner of the election. Therefore, I will now draw special focus to IIA and strategy-proofness in order to further investigate if those are also necessary conditions for an ideal democracy.

For this I will use a province of Imaginistan, Pottercounty. Now, the seven voters of this small province need to vote for a mayor.

The preference orderings of the voters look like these:

P(1) = {Wizard b > Wizard c > Wizard a}

P(1) = {Wizard a > Wizard c > Wizard b}

P(2) = {Wizard a > Wizard b > Wizard c}

P(3) = {Wizard c > Wizard a > Wizard b}

The number between brackets represents the number of voters that have a particular preference ordering.

If we eliminate Wizard a from the election, we see that Wizard c wins from Wizard b with 4 to 3. If we eliminate Wizard b from the election, we see that Wizard c wins from Wizard a with 4 to 3. Hence, we can conclude that Wizard c is the Condorcet winner. However, based on the De Borda voting system, would he also be the winner of the election?

If we calculate the scores according to the De Borda rule, we find the following:

Ba = 16

Bb = 11

Bc = 15

So, we see that Wizard c is the Condorcet winner, but is not elected the winner if the De Borda rule is used. Hence, the De Borda-procedure violates the condition of IIA. To prove the previous (that De Borda-rule does not fulfil the property of IIA), please see that for all *a, b, c* ϵ *A* and for all profiles *p*, *q* ϵ $L(A)^N$, if *p* is limited to *a* and *c*, then *F(p)* limited to *a* and *c* is equal to *F(q)* limited to *a* and *c*. Likewise, if *p* is limited to *b* and *c*, then *F(p)* limited to *b* and *c* is equal to *F(q)* limited to *b,* and *c*. Note that the altered profile *q* here is the profile with alternative *b* and *a* added to respectively. In my opinion the Condorcet winner should be chosen in an ideal democracy and he should not lose due to adding alternatives that would have no chance of winning. Hence, for the moment, I emphasise the importance of IIA for an ideal democracy.

I will now draw a different example to show how a group can change the outcome of an election by strategic voting. Thereafter, I will draw yet another example to show that not only a group can change the outcome of an election by strategic voting, but also just one individual can.

Currently, there is a huge debate going on in the just founded Imaginistan as to which city should become the capital of the country. For the sake of simplicity we will call the cities City 1, 2, 3 and 4. There are one hundred voters that have a say in this issue and the debate reveals the following preference orderings:

P(42) = {City 1 > City 2 > City 3 > City 4}

P(26) = {City 2 > City 3 > City 4 > City 1}

P(15) = {City 3 > City 4 > City 2 > City 1}

P(17) = {City 4 > City 3 > City 2 > City 1}

Based on the De Borda rule we can find the following scores:

B1 = 42x4 + 26x1 + 15x1 + 17x1 = 226

B2 = … = 294

B3 = … = 273

B4 = … = 233

We find that, according to the De Borda rule, City 2 should become the capital of this beautiful country. Please notice, that if a Most votes count procedure (e.g. a first-past-the-post procedure[11]) had been used, City 1 would have become the capital, since it got the most first preference votes.

One can easily verify that City 2 is also the Condorcet winner in this election. On a pairwise comparison City 2 beats City 1 with 58 to 42 votes, City 3 with 68 to 32 votes and, likewise, City 4 with 68 to 32 votes.

It should not come as a surprise that City 2 is not a very desired outcome for the voters who have the third or fourth preference ordering. Might there be a possibility for them to alter the outcome in such a way that it will be *less undesired* for them? One should already suspect that this is indeed possible. For example, if the voters with the third and fourth preference ordering would put City 3 first and City 2 last, we end up with the following insincere preference orderings:

P(42) = {City 1 > City 2 > City 3 > City 4}

P(26) = {City 2 > City 3 > City 4 > City 1}

P(15) = {City 3 > City 4 > City 1 > City 2}

P(17) = {City 4 > City 3 > City 1 > City 2}

This yields the following Borda values:

B1 = 258

B2 = 262

B3 = 290

B4 = 190

Now City 3 is the winner. Notice that City 3 is preferred over City 2 in the original position of the respective 32 voters. Even though City 3 was not the most favoured possible capital for the voters with the fourth preference, it was still better than City 2.

Like mentioned earlier, also a single strategic voter can change the outcome in his or her favour. Imagine that in Imaginistan there is a household of two parents and five children. One of the children needs to do the dishes. All of the family members have a say in who should do the dishes. Let us call the children C1, …, C5. The following preference orderings are revealed [12]:

[11] In a first-past-the-post procedure the alternative that reaches a certain threshold first is elected.

[12] This is based on an example by Ad van Deemen (de Swart, van Deemen, van der Hout & Kop, 2003, pp. 162-164).

P(3) = {C1 > C2 > C3 > C4 > C5}

P(2) = {C3 > C1 > C2 > C4 > C5}

P(2) = {C2 > C3 > C1 > C4 > C5}

This yields the following Borda values:

B1 = 29

B2 = 28

B3 = 27

B4 = 14

B5 = 7

Hence, the Borda preference ordering that follows is: P(Borda) = {C1 > C2 > C3 > C4 > C5}, making C1 the Borda dishwasher. Also check that C1 is not the Condorcet 'winner', since he beats all other children, except for C3.

Assume that one of the children with the third preference ordering wants to amend the outcome, because he made a deal with his big brother, C1. The deal is that C1 does not have to do the dishes and promises his little brother to do his homework.

The ingenious plan of the little brother is to give the following insincere preference ordering:

P(1) = {C2 > C3 > C4 > C5 > C1}

This creates the following amended set of preference orderings:

P(3) = {C1 > C2 > C3 > C4 > C5}

P(2) = {C3 > C1 > C2 > C4 > C5}

P(1) = {C2 > C3 > C1 > C4 > C5}

P(1) = {C2 > C3 > C4 > C5 > C1}

This yields the following Borda values:

B1 = 27

B2 = 28

B3 = 27

B4 = 15

B5 = 8

Now it is C2 who has to do the dishes rather than C1.

Based on the previous, we have to conclude this section with two statements: 1) De Borda rule does not fulfil the condition that it is independent or irrelevant alternatives and 2) based on the empirical examples drawn above, it should be clear that the outcome under the De Borda procedure can be changed by voting strategically.

Due to the fact that the De Borda-procedure takes into account all the preferences it becomes easier to cheat (that is: giving a false preference ordering in order to prevent an undesired outcome). For example, placing the most serious rival of an individual's favourite last, severely reduces his chances of winning. With this I also emphasise the importance of meeting the IIA condition: adding seemingly irrelevant alternatives (or Condorcet losers) is precisely the reason why the De Borda procedure gives way for strategic voting. After all, it would then be possible place an irrelevant alternative first in the preference ordering and the collectively regarded favourite last, diminishing his chances of winning.

Though the De Borda-procedure lacks the condition of being strategy-proof and IIA, Brams and Fishburn find it superior in many other aspects. Under the De Borda-rule many paradoxes do not occur which will be discussed in the following sections (Brams & Fishburn, 2002). Personally I am also quite fond of this voting procedure, because it would work if it would not allow for strategic behaviour. What I am trying to say is that if everyone would give their sincere preference, then the most preferred candidate would be chosen. Nevertheless, the fact that strategic voting pays would probably enable people to do so. Although Van Hees and Dowding might have a point that manipulation might actually be a good thing, I still think that in an ideal democracy strategic voting should not pay and hence strategy-proofness (and the previously discussed condition of IIA) should be a necessary condition for an ideal democracy.

In the next section I will analyse the properties IIA and strategy-proofness even more thoroughly.

6. Approval voting

Recall the Imaginistan example. In the beginning of this paper we assumed that one's preference ordering consisted of a set of alternatives and that every voter (strictly) preferred one Wizard over the other. Now suppose that a voter does not necessarily have to prefer one Wizard (strictly or weakly) over the other. Suppose that he can divide the alternatives by simply saying whether he would approve or disapprove of a particular alternative being the winner of the election. The alternative that is 'approved the most' (meaning the alternative that receives the most approvals) will ultimately win the election. Note that this might violate the anti-symmetricity condition, since Approval voting allows for indifferences between alternatives and hence the (collective) preference ordering might no longer be linear orderings, but become weak orderings instead, meaning that there is no evident preference between certain alternatives. If relevant I will state that the ordering in question is a weak ordering, but the difference between weak and linear orderings is not important at this stage.

Approval voting was first created by Guy Ottewell in 1977 (Ottewel, 1977). Later, Steven Brams and Peter Fishburn elaborated on it in 1978 (Fishburn & Brams, 1978). It thus is a fairly new system. We will later see, I apologize for spoiling this so early, that Approval voting is also a victim to strategic voting. However, according to Fishburn and Little, the possibility that the Condorcet winner might actually lose is a lot smaller when compared to other voting systems (Little & Fishburn, 1986). This makes intuitive sense, as under Approval voting the chance that the strongest alternative will also gain the most approvals is rather likely, maybe even more than under the previously discussed De Borda procedure.

I will now first use a small example to explain the basics of Approval voting. Afterwards I will show a possible way of strategic voting under Approval voting and then discuss this system a bit more in depth. At the end of this section I will briefly recap the findings regarding the necessary conditions of an ideal democratic voting procedure with respect to the research question.

For means of simplicity I will use the same example as the one I started this paper with. Only now I will use the '>>' symbol to indicate that the (set of) alternative(s) on the left of the symbol is approved by a (set of) voter(s) and that the (set of) alternative(s) on the right of the symbol is disapproved. For now it is irrelevant whether there exists either indifference or preferences between the alternatives on either the right or the left side of the approval symbol.

Wizard x		Wizard y	>>	Wizard z	5 voters
Wizard x		Wizard z	>>	Wizard y	7 voters
Wizard y	>>	Wizard x		Wizard z	3 voters
Wizard y	>>	Wizard z		Wizard x	7 voters
Wizard z		Wizard x	>>	Wizard y	3 voters
Wizard z		Wizard y	>>	Wizard x	6 voters

After having constructed this table one simply decides on the winner by counting the amount of approvals every alternative has and the one with the most approvals is the winner. Every 'vote' on the left of the approval symbol counts as an approval.

AVS(x) = 5 + 7 + 0 + 0 + 3 + 0 = 15

AVS(y) = 5 + 0 + 3 + 7 + 0 + 6 = 21

AVS(z) = 0 + 7 + 0 + 0 + 3 + 6 = 16

Here 'AVS()' stand for the Approval Voting Score of a certain alternative. So, under Approval voting Wizard y wins the election.

Like I mentioned before, this system, unfortunately, is also a victim of strategic voters. Let us assume that the individuals from the last group have the same information as we have. Suppose that they see Wizard y will win, but that they change their mind about him and rather see Wizard z win (for the sake of proving that strategic voting under Approval voting is possible). Can they pull it off to make Wizard y lose and, even move, Wizard z win?

Assume they give the strategic/insincere preference ordering:

Wizard z	>>	Wizard y		Wizard x	6 voters

Wizard y now gets 6 fewer votes than in the sincere position and consequently Wizard z wins the election. The previous thus illustrates that Approval voting does not meet the condition of being strategy-proof.

What does become evident here, is that Approval voting clearly fulfils the condition of monotonicity. We have seen that a collective preference rule *F* : $W(A)^N$[13] → *C(A)* is monotonic if, for all profiles *p* and *q* in $W(A)^N$ and for all alternatives *x, y* and *z* in *A*, if:

a. For all *i* ϵ *N*, if *yp(i)x* (and not *xp(i)y*), then *yq(i)x* (and not *xq(i)y*) (likewise for *y* over *z*); and
b. There is an individual *k* ϵ *N* so that *xp(k)y* and *yq(k)x* (likewise for *y* over *z*), then *yF(p)x* implies that *yF(q)x* and not *xF(q)y* (likewise for *y* over *z*).

Hence, under Approval voting, a winner cannot be made worse off if he collects more votes. Likewise, a loser cannot get an advantage by losing votes. This should be a basic principle of any voting procedure. For the remainder of this section I will further assess the possibility of strategic behaviour under Approval voting, given that this voting procedure does meet the requirement of being monotonic.

[13] We now use the weak ordering, since indifference is allowed for in this model.

Please verify that Wizard z is the Condorcet winner for this system, defeating both Wizards x and y with 16 to 15 votes. Hence, Approval voting does not meet the requirements for the stronger Pareto condition. So, we have now assessed how strategic voting might pay in a voting system that is monotonic, but does not meet the requirements regarding the stronger Pareto condition. Approval voting is anonymous, neutral, meets the (weaker) Pareto condition and fulfils the requirements of IIA (Swart, Van Deemen, Van der Hout, & Kop, 2003).

Before concluding this section I would like to show that Approval voting might actually also have benefits regarding the Condorcet winner winning the election. I will try to show this using an experiment done by Fishburn and Little in 1986 (Little & Fishburn, 1986). The experiment took place after the annual 1985 TIMS (The Institute of Management Sciences) election.

There were three candidates running for presidency of the institute. Candidates A, B and C. Also, there were 1,828 voters. The presidency was based on Plurality voting (Pv). Hence, simply the candidate who got the most individual votes won the election. The Candidate that scored the most votes based on Pv was Candidate C with 835 votes, leaving the runner-up, Candidate B, behind with 827 votes. This is only 8 votes fewer than Candidate C.

Afterwards, Fishburn and Little asked the same voters to fill in another voting bill. This basically looked like this:

I approve of
○ Candidate A
○ Candidate B
○ Candidate C

They had thoughtfully chosen an anonymous and positively-formulated (meaning that it asked for the candidates they would approve rather than disapprove, which would be a form of Negative approval voting, which also happens, for example in Russia) way of asking the voters about their preferences. Consequently, they received an enormous feedback of about 85 per cent of the voters that also took part in the original voting for the presidency. For means of completeness Fishburn and Little extrapolated this 85 per cent to the full amount of 1,828 voters. They justified this by the finding that the votes based on both Plurality voting and Approval voting did not display a significantly different pattern. Although it must be said that there remains certain speculation about the missing 15 per cent, but we can certainly, though with a natural amount of restraint, draw conclusions based on the table that was drawn from the experiment.

Candidates	Official Pv	Actual Av	Extrapolated Av
A	166	417	486
B	827	1,038	1,224
C	835	908	1,054
Total	**1,828**	**2,363**	**2,764**
No. of Voters	1,828	1,567	1,828

Here ‘Official Pv’ stands for the official amount of votes each candidate received under Plurality voting. ‘Actual Av’ stands for the approval votes each candidate got based on the voting forms the experimenters received. ‘Extrapolated Av’ stands for the ‘Actual Av’ divided by $\frac{1}{percentage\ of\ feedback}$.

Under Plurality voting, as mentioned, Candidate C wins with only 8 more votes than Candidate B. This is only 0.4 per cent. Under Approval voting, Candidate B would win with a total of 170 votes as compared to Candidate C. This is 6.1 per cent.

Obviously, this is not enough information to justify the outcome of either voting system, concluding that either Candidate B or C is the only and absolute winner. Fishburn and Little also realised this. Therefore they added an extra question to the ballot they had sent to the voters: they asked them to rank the Candidates, or to stick with the previously used terminology: to reveal their preference orderings in a set of alternatives. This has two main points of interest for this paper:

1) With the acquainted information we can see who the Condorcet winner, if there is one, is by a pairwise comparison;
2) This information is profitable for an analysis as to whether Approval voting might result in the Condorcet winner winning the election.

I now specifically draw attention to the voters that voted for Candidate A, because, clearly, he or she is the loser of this election. Now then, how do the preference orderings of these 166 voters look like? The ballots that Fishburn and Little got returned revealed the following orderings:

- P(70) = {A; B; C};
- P(60) = {A; C; B};
- P(3) = {(AB)[14] >> C} (i.e. no preference between A and B, but approved those and disapproved C). These voters actually contradict themselves by saying they are indifferent between A and B, but choosing A instead;
- P(27) = {A; (BC)}

Now that it basically comes down to a battle between Candidates B and C, Fishburn and Little reasoned (Little & Fishburn, 1986), the votes for Candidate A should be redistributed to the other candidates.

Based on the preference orderings that the "A-voters" revealed above, it seems fair to credit the votes from P(70) and P(3) to Candidate B. Likewise, it is fair to credit the votes from P(60) to Candidate C. The votes from P(27) are too hard to credit to either of the two, because no clear preference is being revealed there. When we have credited these Pv votes to the table as drawn above, we see that Candidate C now has 901 votes and Candidate B has 900 votes. Moreover, if we would, hypothetically, split the votes from P(27), then, by ways of rounding off, both Candidate B and C have 914 votes and the election ends in a tie.

Based on the hypothetical redistribution as displayed above, we might conclude that for this election there might not even be a Condorcet winner. Even so, one should recall that Candidate C would win under Pv with only 8 more votes than Candidate B, whereas Candidate B would win under Av with a stunning 170 extra votes as compared to Candidate C.

According to Fishburn and Little, the (significant) discrepancy between the results under Av and Pv lies in the fact that C has more *stalwart* supporters (these are supporters that only vote for one candidate; these kind of voters are usually the voters considered in first-past-the-post procedures), whereas Candidate B gets more support from people approving (or even revealing preference between one another) more options. Additionally, more of Candidate C's supporters approved Candidate B than Candidate B's supporters approved Candidate C.

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[14] *If alternatives are presented between brackets, this (generally) means voters are indifferent between the two (or more) alternatives.*

Now then, based on Approval voting, Candidate B would have won rather than Candidate C. The question then comes up, whether that is actually a more desired outcome. Especially when there is no (clear) Condorcet winner, Fishburn and Little conclude that Approval voting yields the winner on the grounds of second choices. The winner than is the candidate (or alternative) with the broadest acceptance level with respect to the other alternatives. The requirement that needs to be fulfilled in order to draw conclusions like these is that there needs to be a sufficient amount of information on which claims can be made.

In practice, Fishburn and Little conclude, it is highly unlikely that there is a Condorcet winner in a certain set of alternatives that would not carry most of the approval votes. In theory, however, it is possible that the Condorcet winner does not carry the most approval votes. Moreover, please take a look back at the Imaginistan example as drawn in the beginning of this section. Wizard z here beats every other alternative in a pairwise comparison. Nonetheless, it is Wizard y that can count on most of the approval votes and hence win under Av.

In the first example we have seen that Approval voting thus does not fulfil the requirement of strategy-proofness. However, if a sufficient amount of information is gathered, combined with other voting systems a more justified[15] winner can be elected. I have three remarks I would like to make before concluding this section.

Firstly, regarding the additional information that needs to be gathered and analysed. The term "additional information", or even more vaguely "*sufficient* additional information" is a very arbitrary term and it will depend from situation to situation what amount of additional information is required and what amount will suffice to form a more constructed choice regarding the winner. Moreover, the willingness of people to give all the information required to elect the 'most justified' winner might in many situations lack amongst populations. This experiment took place in an academic setting, hence the population was more than willing to give the researchers the desired information they needed. This explains the 85 per cent return rate of the ballots sent to the voters.

This brings me to my second point. This experiment took place in a relatively small setting, consisting of a lot of academics. Asking approximately 1,800 people about their preferences is already quite a time-consuming task. For matters more important, would that also be possible? For instance, could you imagine that the, roughly, 100 million voters' preferences of the voters of the USA would have to be examined? Such a research would take years, if not decades. Also, there might be a huge hold-back amongst the citizens as to giving up all their preferences.

The last remark I want to draw attention to is actually in favour of Approval voting. Like mentioned above, this system opens ways for strategic voters to achieve their goals or, more generally, prevent an undesired outcome from occurring. The examples in which strategic voting actually paid off was an example with a small amount of voters. Hence, there should not be many trust issues amongst the strategic voters. Naturally, people who prefer a certain outcome or set of outcomes over the winning alternative would need all possible information in order to amend the outcome. Firstly, it is very hard to find out someone's (sincere) preference ordering. Secondly, every individual having a certain preference ordering has to put his faith in the other members (which might be 'members' of a different set of alternatives, but which also want to prevent (the same) undesired outcome) to do the same thing. What I want to make clear here, is that on a day-to-day basis it is not very likely that people will actually use strategic voting under Approval voting, because giving there sincere approvals should and will elect

[15] *Justified* in the sense that as much information as possible was taken into consideration (votes, preferences, etc.) before actually electing a winner.

the victor that is approved the most. This might not be the Condorcet winner (as shown before), but this is highly unlikely, especially in practice.

We have clearly seen that a voting procedure that does meet the requirements of anonymity, neutrality, IIA, monotonicity and the Pareto condition, but not the requirements in order to meet the stronger Pareto condition and is not strategy-proof is not ideal, since voters can manipulate the outcome of such a voting procedure. However, we have also seen that Approval voting offers great potential to elect the Condorcet winner and that Little and Fishburn even conclude that the theoretical flaws regarding voting paradoxes hardly form a problem in practice.

Like I mentioned before it should not pay to cheat in an ideal democracy. Under Approval voting it is easy to cheat, as preventing an undesired outcome could easily be accomplished by simply not approving of an alternative. In my opinion strategy-proofness as well as the stronger Pareto condition still are very important conditions to be met, but not necessary conditions to be met in an ideal democracy, because Fishburn and Little have convincingly shown that the outcome under Approval voting might be rather justified (see footnote 15). I do want to stress that for voting procedures with more than three alternatives special attention should be given to all the results. Here the actual winner of the election won by just a few votes and a clear Condorcet winner could not be found. In my opinion it might be a good idea to run a second round of election if there is a clear tie between two voters. The empirical impracticality then unfortunately might be that there might be voters who do not want to vote for a second time. However, in theory this might actually be a good idea, since, recall from the introduction, Majority voting with just two alternatives fulfils the requirements regarding anonymity, neutrality, Pareto condition, IIA, monotonicity and strategy-proofness and will consequently always yield the Condorcet winner as the actual winner of the election. We have seen that Approval voting in practice offers ways of a preventing paradoxes to occur, but this requires special audit and control.

7. The Single-transferrable voting system

The Single-transferrable voting system is a voting system that is both proportional and ordinal (Doron & Kronick, 1977). The first meaning that candidates are elected pro ratio with respect to the voters. Moreover, candidates do not need a certain amount of votes to win, but they merely need to reach a certain quorum or threshold in order to win (or be elected). The second meaning that the voters have to reveal their preference orderings, because all their preferences are taken into account rather than only their first (two, three, etc.) preference(s). What this means for us will become clear with an example.

Suppose that there is a neighbouring country to Imaginistan, Lotri. Lotri has 27 citizens. Those citizens also have to decide who will carry the burden of leadership of that country. Once someone presents himself or herself as a candidate for leadership, he or she can no longer take part in the voting procedure. Furthermore, as opposed to many countries, Lotri does not rely on a sole and supreme leader, but it annually elects two consuls. There are four consul candidates, Aragorn, Bilbo, Cirion and Dwalin, Aragorn and Bilbo are very liberal thinkers, whereas Cirion and Dwalin are rather conservative. Now, suppose that we have the following preference orderings[16]:

$P(7) = \{A > B > C > D\}$

$P(6) = \{B > A > C > D\}$

$P(6) = \{C > D > B > A\}$

[16] This example is based on an example by P. Hoffman (Hoffman, 1988).

P(4) = {D > C > B > A}

If the election would allow every voter to only vote for two alternatives, Aragorn and Bilbo would win, both having 13 voters on their side. Hence, the consulate would be composed fully of liberal politicians. Please note that the conservative candidates also represent roughly 43 per cent of the electorate.

Now then, what would the outcome look like if we had used Single-transferable voting? For a start, I will briefly outline the mechanics of Stv. First we will need an election threshold before explaining the basics of this voting system. The election threshold is the smallest number of votes of first choice so that the maximum amount of candidates that can reach the election threshold corresponds to the available number of seats (Hoffman, 1988). Mathematically, assuming that there are N voters and K available seats, the election threshold T is then the smallest natural number so that $KT \leq N$ and $(K + 1)T > N$. Hence, $T = \frac{N}{K+1} + 1$ where $\frac{N}{K+1}$ is the natural number obtained by rounding down $\frac{N}{K+1}$. So, the election threshold in our example is $T = \frac{23}{2+1} + 1 = 8$.

The answer acquired above makes intuitive sense. After all, for 23 and 4 candidates, there can be no more than 2 candidates that get eight votes of first choice: 3 candidates with eight votes of first choice would require 3 x 8 = 24 voters. Moreover, with an election threshold of only seven, 3 instead of 2 candidates could get seven votes of first choice, which would come down to 3 x 7 = 21 voters.

Now we can continue with the general Stv theory.

Stv is based on (imaginary) voting rounds. All the preference orderings are taken into account. Then, one analyses whether there exists an alternative that reaches the election threshold (or a multiple thereof). Then, the least-favourite candidate (or alternative) is eliminated from the election. The votes that he or she would have received are then transferred to the second choice of the candidate with the respective preference ordering. This process continues until all the seats are occupied. I will now apply Stv to the example drawn above, as it will then become clearer how the system works.

We see that in the first imaginary voting round no candidate reaches the election threshold. We also see that Dwalin is the least-preferred candidate in this election. Hence, we eliminate him from the election and give the votes of the voters with this preference ordering to their second choice, Cirion. This yields the following:

P(7) = {A > B > C > D}

P(6) = {B > A > C > D}

P(10) = {C > D > B > A}

We see that Cirion now passes the election threshold and is elected as one of the consuls of Lotri. Since there are 10 voters having Cirion as their first choice and the threshold is only eight votes, we are left with two 'residual' votes. These two votes are then given to their second choice, Bilbo. Also, since Cirion now already gained a seat, he is 'eliminated' from the preference orderings of the other voters, just like Dwalin. This yields yet again a new figure:

P(7) = {A > B}

P(8) = {B > A}

Ultimately, we see that now also Bilbo reaches the election threshold. Now that both consul positions have been granted amongst the candidates, there is no need for another elimination round.

So, what do we end up with? By means of Stv Bilbo and Cirion are elected as the winners of the election. More strikingly, Cirion, who would not have won under Pv, is the first candidate to reach a consul position. It also becomes clear at this stage why Stv is considered a proportional voting system. We have seen that under Pv two liberal consuls would have seized leadership, although the conservatives contributed to 43 per cent of the electorate. Hence, this system is really suitable for district systems with (linear) proportional representation. Moreover, we now have analysed a voting system in which two candidates had to be elected, which is something that does not find a lot of support around the globe.

It should not be hard for someone to see that Dwalin is definitely not the Condorcet winner (i.e. the alternative that would defeat any other alternative in a face-to-face election procedure). One should see that the voters with the first and second profile 'overrule' the third and fourth profile with respect to the preference of those voters for Dwalin, simply by outnumbering them. Hence, we eliminate Dwalin from the election. We are then left with the following set of preference orderings:

P(7) = {A > B > C}

P(6) = {B > A > C}

P(6) = {C > B > A}

P(4) = {C > B > A}

If we run a pairwise comparison we see that Bilbo defeats Aragorn by 16 to 7 votes and Cirion by 13 to 10 votes. Hence, Bilbo is the Condorcet winner. We have seen that Bilbo was indeed elected to be one of the consuls. So, it looks like Stv does indeed elect the Condorcet winner. However, we also see that Cirion is the Condorcet loser, losing to both Bilbo and Aragorn 13 to 10 votes. Stunningly, Cirion is the first one to be elected as one of the consuls.

Here is when I wondered: what would happen if not two consuls, but one supreme leader would have to be chosen amongst the candidates. What would happen then? At first I almost instantly concluded that then Cirion, the Condorcet loser, would win. However, this is a pitfall. After all, if not two, but one candidate has to be elected, the election threshold also alters. If one candidate has to be elected, the election threshold becomes: $\frac{23}{1+1} + 1 = 12.$ Now then, what mechanics will we see taking place if not two leaders, but one leader would have had to be chosen? It results the following:

Again, we start in the original position:

P(7) = {A > B > C > D}

P(6) = {B > A > C > D}

P(6) = {C > D > B > A}

P(4) = {D > C > B > A}

Dwalin has the least votes, thus he will be eliminated and his voters' second choice gains those votes:

P(7) = {A > B > C}

P(6) = {B > A > C}

P(10) = {C > B > A}

Still, the election threshold is not reached. Bilbo has the least votes, so his voters' votes go to their second choice and Bilbo is eliminated:

P(13) = {A > C}

P(10) = {C > A}

Finally, the election threshold is reached. Surprisingly, not the Condorcet loser, Dwalin, not the Condorcet winner, Bilbo, the seemingly strong Cirion, but the silent Aragorn wins the election. P. Hoffman comprehensibly explains how Stv works, but does not check whether this system does indeed yield the Condorcet winner, as I have done in this section. The system itself is rather new and is used in Ireland, Malta, South Africa and a few other countries. Besides, it has been implemented for a lot of municipality elections and other lower government affairs. As probably has become clear, it is really time-consuming to apply this procedure, especially when more voters, more candidates and more preference orderings are involved. The essence of this voting system is, basically, that no vote is wasted during the procedure. In itself that is a very noble goal, but it is hard to apply in reality.

For the remainder of this section I would like to pay more attention to the Additional Support (or Lack of Monotonicity or Negative Responsiveness) paradox (Smith, 1973): If candidate x is elected under a given distribution of voters' preferences amongst the competing candidates, it is possible that, ceteris paribus, x may not be elected if some voter(s) increase(s) his (their) support for x by moving x to a higher position in his (their) preference ordering. Hence, this would mean a violation of the Monotonicity principle. At the end of this section it will become clear how a voting system operates if it fails to be monotonic.

To analyse this I will use an example by Steven Brams which he used to show the American Mathematical Society that Stv is subject to insincere behaviour if (a) voter(s) mentions fewer candidates in his or her preference orderings. He used an example with 4 candidates, 17 voters, 3 preference orderings and 2 winners that need to be elected (Brams & Fishburn, 2002).

Please take a look at the following set of preference orderings:

P(6) = {X > A > B > C}

P(6) = {X > B > C > A}

P(5) = {X > C > A > B}

Based on the previous the election threshold is $\frac{17}{2+1} + 1 = 6$.

In the first round X wins, as he is the first choice of all voters. Obviously, X will not need all 17 votes in order to become a winner. We are left with a residual of 11 votes. Then, X is eliminated from the preference orderings. Since X is the first choice for all voters, the 11 residual votes will be given to the respective second choices on a proportional basis. This yields the following scheme:

P($\frac{66}{17}$) = {A > B > C)

P($\frac{66}{17}$) = {B > C > A)

P($\frac{55}{17}$) = {C > A > B)

Since none of the candidates reaches the threshold, the candidate with the lowest votes, C, is eliminated from the election:

P($\frac{66}{17}$) = {A > B)

P($\frac{66}{17}$) = {B > A)

P($\frac{55}{17}$) = {A > B)

Now A reaches the threshold and is selected as one of the leaders, together with X.

Now suppose that two of the voters with the preference ordering *P(n) = {X > B > C > A}* only reveal their preference for X, basically meaning that they do not approve of the other alternatives. This yields the new set of preference orderings:

P(6) = {X > A > B > C}

P(4) = {X > B > C > A}

P(2) = {X}

P(5) = {X > C > A > B}

Now, X and C will be elected as the leaders. Recall that C is preferred over A by the two insincere voters in the original position. There is one final note I want to make about this example. X is obviously a (sole) winner, independent of how many leaders need to be elected. However, once X is eliminated from the procedure, there does not exist a Condorcet winner anymore. Even more so, A, B and C violate the principle of transitivity, because, in a pairwise comparison, A defeats B, B defeats C and C defeats A.

I have mentioned that more votes might actually cost a candidate his or her head. It is relevant to show such a paradox, because paradoxes are the roots of successful strategic behaviour. At this point we can conclude that Stv clearly violates the monotonicity condition. We have also seen that that easily gives rise to strategic behaviour.

Doron and Kronick have elicited the previous with the following example. There are 4 candidates, 26 voters, 5 preference orderings and 2 candidates need to be elected (Doron & Kronick, 1977).

Please take a look at the following set of preference orderings:

P(9) = {A > B > C > D}

P(6) = {C > D > B > A}

P(2) = {D > C > B A}

P(4) = {D > B > C > A}

P(5) = {B > C > D > A}

The election threshold is $\frac{26}{2+1} + 1 = 9$.

Thus, in round one A is elected as one of the winners. Then, in the next round B is eliminated from the procedure, as he has the least votes and no other candidate reaches the election threshold. The votes he would have had are then transferred to candidate C. C then reaches the election threshold and is elected as the other winner.

Now it becomes interesting. Consider that the voters with the preference ordering *P(n) = {D > C > B > A}* no longer prefer D over C, but that they prefer C over D, creating the new set of preference orderings with two extra votes for C:

P(9) = {A > B > C > D}

P(6) = {C > D > B > A}

P(2)* = {C > D > B A}

P(4) = {D > B > C > A}

P(5) = {B > C > D > A}

Again, A is elected in round one. Like before, D is eliminated in the second round. Then, those votes are given to B, which now gets 2 extra votes than in the original procedure. He then reaches the election threshold of 9 votes and is elected as the second winner. So, C now fails to become the second winner due to the extra support of the two previously mentioned voters.

I would like to conclude that a voting procedure that violates the monotonicity criterion (in this case Stv) easily gives way to strategic voting. We have even seen that extra support for a winner, might in the end make him worse off. To recap I want to stress the importance of meeting the monotonicity condition in order to create an ideal democratic voting system.

Before I draw my main conclusion regarding the research question I will go over one last voting system: a dictatorship.

8. Dictatorship

Although a dictatorship remains to have a very controversial status, it does provide a lot of efficiency when it comes to decision making. After all, if one person decides what the outcome will be, then the entire procedure will always yield a clear-cut answer. I will now devote a very brief section to dictatorship to conclude the material part of this paper.

Gibbard and Satterthwaite define a Choice rule *K :* $L(A)^N$ dictatorial if there exists an individual *i* ϵ *N* such that for each profile *p* ϵ $L(A)^N$ the collective choice *K(p)* is the most preferred alternative in *p(i)*. Meaning that the collective choice will be the most preferred alternative by the individual *i*. Therefore *i* will be called the dictator.

K : $L(A)^N$ → *A* is strategy-proof if for every profile *P* a unilateral deviation of an individual *i* from his original profile *P* to a transformed profile *Q* will not make *i* better off, meaning that giving a different profile will not result in a (subjectively) better outcome[17]. So, what does that mean with respect to the paragraph above? This means that strategic behaviour of an individual is not beneficiary to that individual, independent of what the other individuals do. Hence, the first preference of dictator *i* is the only relevant alternative for the election, as that will be the winner of the election, no matter what (Storcken & Swart, 1992).

One should notice that a dictatorship severely violates the condition of anonymity. After all, only the votes for the dictator's first preference are the votes that count and hence not all votes are worth the same or, to stick with the common definition, 'anonymous'. My first intention would be to state that

[17] Formally this comes down to the following: a Choice rule *K :* $L(A)^N$ -> *A* is strategy-proof if and only if, for all *i* ϵ *N* and for all *p*, *q* ϵ $L(A)^N$, if *p(j)* = *q(j)* for all *j* ≠ *i*, then *K(p) p(i) K(q)*. Therefore, *K* is strategy-proof if, for every profile *p*, unilateral deviation of an individual *i* from *p* to *q* is not beneficiary to *i* (*K*(*p*) is at least as good as *K*(*q*) for *i*).

anonymity is thus a necessary condition for an ideal democratic voting procedure, since in a dictatorial voting procedure the only condition that is being violated in the procedure is anonymity (Swart, Van Deemen, Van der Hout, & Kop, 2003). However, we should leave the dictatorship as we know it and look at the broader sense of the principle of anonymity. The International Monetary Fund (IMF) also uses a voting procedure that does not meet the anonymity requirement. Voting power in the IMF is based on a quota system. Each member has a number of basic votes (each member's number of basic votes equals 5.502% of the total votes) (IMF, Retrieved 18 March 2012), plus one additional vote for each Special Drawing Right (SDR) of 100,000 of a member country's quota (Blomberg & Broz, 2006). The Special Drawing Right is the unit of account of the IMF and represents a claim to currency. It is based on a basket of key international currencies. Hence, some votes are worth more than other votes. But is this by definition a bad thing? Imagine that a small country like Luxembourg would have a vote worth the same power as the vote of neighbouring France? Or even China? The voting procedure has been purposefully designed to even out the voting power amongst all members (Blomberg & Broz, 2006).

Although the above might be a counterargument against anonymity, since in some cases the votes of some voters (for the IMF: countries) should not be of equal value, we must realise that for a democratic voting procedure (e.g. the US presidential election) it is essential that every citizen has the right to vote and that that vote is worth as much as any other vote. This is a keystone of a democracy.

9. General conclusion

We have analysed the most common and not so common voting systems around the globe in order to find out whether anonymity, neutrality, Pareto condition, IIA, monotonicity and strategy-proofness are necessary (or desirable) conditions for an ideal democratic voting procedure. We have done so by assessing how these conditions are met and what happens if one (or more) condition(s) are not met. We have given special attention to the condition of IIA and strategy-proofness. We have seen that many voting procedures are prawn to strategic behaviour (e.g. de Borda rule and Stv), especially when more conditions are violated (e.g. recall the Most votes count procedure). Hence we tried to cope with strategic behaviour and went over several voting procedures which, in my opinion, did a relatively good job with respect to preventing strategic voters from strategic voting that would result in a less undesired outcome in their eyes (e.g. Av and Stv). We have also seen that IIA and strategic voting are closely related (recall the Imaginistan examples). We have even seen that a procedure that violates the stronger Pareto condition – which I thought to be necessary for any democratic procedure – does not necessarily be met in order to rather effectively prevent strategic behaviour if the procedure is both monotonic and satisfies IIA (in our case this was the Approval voting procedure). Lastly, we have seen that a dictatorial voting procedure is the only voting procedure that satisfies the conditions regarding neutrality, Pareto condition, IIA, monotonicity and (most important) strategy-proofness. However, a dictatorial voting procedure is already by definition not a democratic voting procedure. A dictatorial voting procedure violates the principles of anonymity, since only the votes for the dictator are worth anything. However, we have also seen that that violation can be relativized (recall the IMF example).

On the basis of the researches this thesis is built upon I will now draw several conclusions regarding the research question which formed the start of this thesis. Gibbard and Satterthwaite have convincingly proven that strategy-proofness in an election of three or more alternatives without being a dictatorial rule is out of the question. My main task was to find what conditions have to be met in order to create an ideal democratic voting system, since a ‘true’ democratic voting system with more than two alternatives does not exist if we want a voting procedure that is strategy-proof. Based on the previous I conclude that especially IIA and monotonicity are conditions that ought to be protected while searching for an ideal democratic voting system. Anonymity, like I said before, also is a keystone of a pure democracy. Neutrality has not been discussed thoroughly enough in this thesis to draw any conclusions about[18]. Strategy-proofness is, still, a necessary condition for an ideal democratic voting procedure. However, ironically, the – according to my personal belief – most important condition of a democratic voting procedure can only be met in a dictatorial voting procedure. Hence, strategy-proofness cannot, by definition, be a necessary condition for a democratic voting procedure (with three or more alternatives), but should be secured as much as possible and be *de facto* met (for example by assessing as much data as possible, like Little and Fishburn did with their research regarding Approval voting). The Pareto condition, like the neutrality condition is not researched thoroughly enough to make decisive claims about. The last condition I discussed in this thesis was the stronger Pareto condition. I initially reasoned that the stronger Pareto condition is of utmost importance for a just voting procedure. However, we have seen that this condition can be violated and such a voting system might still come close to being ideal (I am referring to Approval voting). I therefore conclude that meeting the stronger Pareto condition is an important requirement, but not a necessary requirement for a democratic voting procedure.

[18] Neutrality is closely linked to anonymity: it basically is anonymity, but then on the alternatives' side. Hence, there exists a legitimate reason to suspect that a violation of neutrality might result in a dictatorial form of voting in the sense that has been put forward by Gibbard and Satterthwaite.

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