A comparison of growth convergence between the Gross Domestic Product and the Index of Sustainable Economic Welfare or Genuine Progress Indicator.

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for the completion of the study
Econometrics and Management Science in the
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October 27, 2015

Abstract

While the GDP measures economic output, it often gets treated as a proxy of welfare. To better reflect welfare the Index of Sustainable Economic Welfare and the Genuine Progress Indicator, united under the name of Green GDP, were created. This thesis tests for the β- and σ-convergence of the GDP and the Green GDP by using methods from the convergence growth literature. It is hypothesized that the GDP and Green GDP differ in their convergence. For the β-convergence section we consider unconditional and conditional β-convergence. To make the t-test in this section more reliable we bootstrap the t-distribution. We find both unconditional and conditional β-convergence for the GDP, but not for the Green GDP. In the σ-convergence section we see if the sample variance changes over time with an F-test of equality of variance and by regressing the sample variance on a trend. We also bootstrap the F-distribution. Furthermore we perform an adjusted Dickey-Fuller test to see if the sample variance is stationary. The results in this section are influenced by the unbalanced shape of the data. We do not conclude σ-convergence or divergence for either the GDP or Green GDP. Our final conclusion is that there is a difference between the GDP and the Green GDP in terms of their β-convergence, and thus that using the GDP as a proxy for welfare can be erroneous and propose more use of the Green GDP when the goal is to discuss welfare.

1 Introduction

Often times there is a lot of emphasis on the Gross Domestic Product, GDP. It is often used to explain how well a country is doing or to compare countries. In academics models are created and studies are done on the GDP. These include but are not limited to research on predicting the GDP growth and seeing how certain variables have effect on the GDP (Ang et al., 2006), predicting recessions (Wright, 2006) and research on the convergence between countries' GDP (Barro, 1989) (Mankiw et al., 1990). Policy makers might use the GDP to indicate the prosperousness of a country.

A key issue is whether it actually is a good indicator of welfare. The reason the GDP is deemed so important is that producing more, and therefore consuming more, is considered good. There are however a multitude of instances that increase the GDP that are not considered good for a countries' welfare, examples of that are more insurances, medical costs or oil spills. Problematically the GDP does not measure intangible assets. The GDP does not reflect when production has a bad effect on the environment or depletes limited resources. In addition the GDP does not change with a countries' wealth distribution. On the other hand increases in welfare from volunteer work or having more leisure time do not add to the GDP.

In light of these reasons the GDP might be a poor indicator for welfare. The GDP was created as an economic variable to measure growth. Attention has to be paid to the assumptions made in measuring GDP as oversimplification leads to wrongful use. The designer of the GDP Kuznets (1934) wrote "Economic welfare can not be measured unless the personal distribution of income is known. And no income measurement undertakes the reverse side of income, that is, the intensity and unpleasantness of effort going into the earnings of income. The welfare of a nation can, therefore, scarcely be inferred from a measurement of national income ..".

To better reflect welfare and to address the given issues with the GDP, first the Index of Sustainable Economic Welfare (ISEW) (Daly and Cobb, 1989) and later the Genuine Progress Indicator (GPI) (Cobb et al., 1995) were constructed. These indicators are sometimes called under the shared name of Green GDP, or GGDP as we call them in this thesis. After these indicators were designed independent researchers have constructed them for different countries.

We hypothesize that the GDP is a bad indicator of welfare. For that reason we compare the GDP and GGDP. There are many different areas in which they could be compared. In this thesis we study their growth convergence. Our research questions are first of all does the GGDP converge, and secondly do the GDP and the GGDP converge differently and should the GDP be used as a proxy for welfare.

In Section 2 we discuss the creation and shape of the data, and possible problems with the data. In Section 3 the models we use to study both β- and σ-convergence are shown. We follow Barro (1989) and Mankiw et al. (1990) for the β-convergence analyses, where we use bootstrap methods (Efron, 1979), using explanations from Cameron and Trivedi (2005). For the σ-convergence analyses Barro and Sala-i Martin (1992), Drennan et al. (2004) and Rapacki and Próchniak (2009) are followed. For one part of the σ-convergence a bootstrap method of Boos and Brownie (1989) is used. The results are shown and discussed in Section 4. We conclude that the GDP β-converges, while the GGDP does not. We do not find significant σ-convergence or σ-divergence for either variables. We conclude that the GDP and GGDP behave differently in their β-convergence and the GDP should not be used as an indicator for welfare when considering convergence.

2 Data

In this thesis we use GDP and GGDP data to study their convergence. For the β-convergence we use panel data of the variables being studied. In addition we expand the β-convergence section to conditional β-convergence, by adding extra variables to the regression that could help to further explain growth. We use variables that (Barro, 1989) also uses. The variables are enrolment rates for different levels of schooling and a composite measure that is government consumption minus spending on education or the military, as percentage of the GDP. Barro uses primary and secondary school enrolment rates, but we also allow the possibility of tertiary enrolment rates. For the σ-convergence section we study the sample variance of the GDP and GGDP variables, and thus only require the panel data of the GDP and GGDP.

Since we are comparing GDP and GGDP we only look at countries that have GGDP data. The data for the GGDP were made by independent researchers. Some of these have made ISEW and some GPI series. In this analysis we use these variables as if they were the same. The GPI grew out of the ISEW and is similar. To show how little they differ we include a graph from Nourry (2008) in Figure 1, who constructed both ISEW and GPI for France.

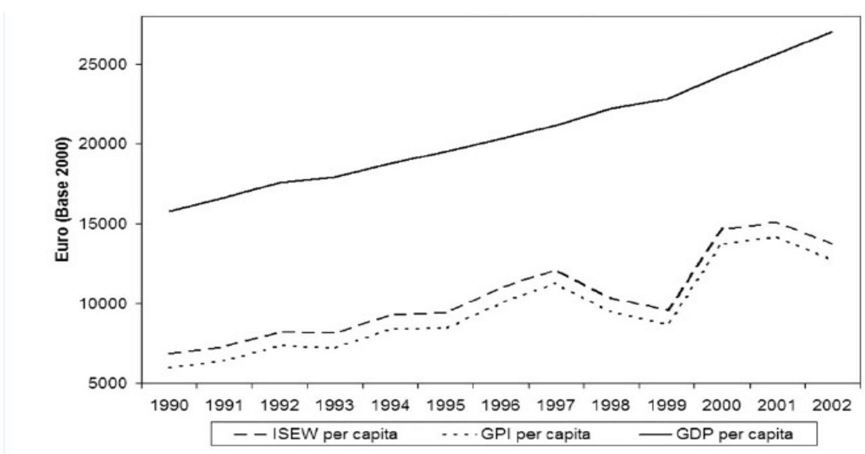


Figure 1: Difference of the ISEW and GPI for France. Graph was taken from Nourry (2008)

The countries for which there is GGDP data, as well as the authors who have created the data are shown in Table 1. The data of the GGDP that we use is from these articles. The data for the GDP is obtained from the World Bank Group. Because the GGDP time series were constructed by independent researchers the time for which there are data is different for the different countries. The GGDP data therefore has an unbalanced panel data structure. The data structure for the GDP is mostly balanced as the data for almost all countries run from 1961 to 2012. Only Thailand, New Zealand, Vietnam and Poland run from 1965, 1977, 1984 and 1990 respectively. To make the analyses for the GDP and GGDP more comparable we run every analysis twice for the GDP. Once in the full dataset and once with an unbalanced shape that has the same shape as the GGDP. This unbalanced shape is the shape of the GGDP available data, with two exceptions, as for New Zealand and Poland GDP data is available later than the GGDP data. For example, New Zealand has data for the GGDP from 1970 till 2005 and for the GDP from 1976 till 2012. The unbalanced data structure contains data for New Zealand from 1976 till 2005. The GDP data comes in 2005 USD currency. The GGDP data was in different currencies from different times for the different articles. We transformed all the GGDP data to 2005 USD as well.

How the GGDP is made is explained thoroughly in the 2007 report of the GPI (Talberth et al., 2007). Construction for the ISEW is explained in the book by Daly and Cobb (1989) and is very similar to that of the GPI. A brief summary of how the Green GDP is constructed is that it starts with personal consumption and then corrects for a list of intangible assets, including the shape of the wealth distribution and environmental damage among others. It is also corrected for amount of investment, use of natural resources and borrowing or lending from abroad.

The other variables that are used to research conditional β-convergence were obtained from UNESCO and the World Bank Group. The data of this is not regular and available at different times for different countries. To be able to do the conditional β-convergence analysis each country has to have a value for each of these variables. For each variable we use the value that is the average of the available data over the period that a country is in the data. This does make it that the conditional β-convergence analysis is no longer truly exogenous. However these values can be good proxies of how

Country	Author	Available data
Australia	Lawn (2008a)	1967-2006
Austria	Stockhammer et al. (1997)	1955-1992
Belgium	Bleys (2008)	1970-2000
Brazil	Torras (2005)	1965-1993
Chile	Castaneda (1999)	1965-1995
China	Wen et al. (2007)	1970-2005
Costa Rica	Torras (2005)	1970-1989
France	Nourry (2008)	1990-2002
India	Lawn (2008b)	1987–2003
Indonesia	Torras (2005)	1971-1984
Japan	Makino (2008)	1970-2003
Netherlands	Bleys (2007)	1971-2004
New Zealand	Forgie et al. (2008)	1970-2005
The Philippines	Torras (2005)	1970-1987
Poland	Gil and Sleszynski (2003)	1980-1997
Sweden	Stymne and Jackson (2000)	1950-1992
Thailand	Clarke and Shaw (2008)	1975-2004
UK	Stymne and Jackson (2000)	1950-1996
US	Talberth et al. (2007)	1950-2004
Vietnam	Hong et al. (2008)	1992-2004

Table 1: Countries that have available ISEW/GPI data, with authors and available time.

much countries are willing to invest in the future.

The shape of the data The GGDP grows slower, and differently than the GDP. In Table 2 the total and average growth is given for both the GDP and the GGDP. Because of the unbalanced dataset the average growth rate can be better used to compare countries. As can be seen the GGDP has a lower growth rate than the GDP with exception of Japan and Poland. For some countries the GGDP growth rate is even negative, although they are around zero. The GGDP is a composite index consisting of personal consumption, adjusted for wealth distribution, a list of intangible assets and corrections for in- or decreases in lending and investing. When the GDP increases, but the GGDP does not grow this indicates that the increase in consumption that comes with an increased GDP is completely offset by a decrease due to either changed intangible assets or corrections for lending or investing. For example, according to Talberth et al. (2007) for the USA the main reasons the GGDP grows less are depletion of and damage to natural capital. In the Appendix in Figures 10 through 12 there are graphs for each country's available GDP and Green GDP time series. As can be seen, for many countries the Green GDP does not grow more slowly, but instead grows equally and then stops growing. For example, in the USA the GGDP grows with 2.03% until 1977, after which it stops growing. Meanwhile the GDP grows with first 2.95% and after 1977 keeps growing with an average of 2.09% in those periods. After this period the growth in personal consumption that follows the growth in GDP is equalled by change in intangible assets that decrease the GGDP. In light of this Max-Neef (1995) hypothesized that there is a threshold to which countries can grow economically with an increase in well-being, after which all economic growth will lead to a deterioration of well-being, until technological advancement allows further growth in well-being.

Brazil and Poland We disqualify the data of Brazil and Poland. For Brazil the currency in which the data was reported is the 1993 Cruzeiro real. The world bank group reports 968%, 2001% and 2251% inflation for

		Tot. growth		Avg. growth	
		GDP	GGDP	GDP	GGDP
Australia	1967 - 2006	117.6 %	-8.9 %	1.96 %	-0.23 %
Austria	1961 - 1992	154.4 %	60.3 %	2.96 %	1.49 %
Belgium	1970 - 2006	115.8 %	48.4 %	2.10 %	1.07 %
Brasil	1965 - 1993	116.9 %	101.5 %	2.71 %	2.45 %
Chile	1965 - 1995	118 %	-4.9 %	2.55 %	-0.16 %
China	1970 - 2005	1097.2 %	75.7 %	7.14 %	1.58 %
Costa Rica	1970 - 1989	29.6 %	41.4 %	1.31 %	1.75 %
France	1990 - 2002	18 %	114.3 %	1.28 %	6.04 %
India	1987 - 2003	84.7 %	22 %	3.68 %	1.18 %
Indonesia	1971 - 1984	86.3 %	26.1 %	4.54 %	1.67 %
Japan	1970 - 2003	127.7 %	171.3 %	2.45 %	2.98 %
Netherlands	1971 - 2004	85.4 %	-3.1 %	1.83 %	-0.09 %
New Zealand	1977 - 2005	50.5 %	-4.1 %	1.42 %	-0.14 %
Philippines	1970 - 1987	12.3 %	25.6 %	0.65 %	1.27 %
Poland	1990 - 1997	24.9 %	344 %	2.82 %	20.48 %
Sweden	1961 - 1992	89.6 %	13.4 %	2.02 %	0.39 %
Thailand	1975 - 2004	285.4 %	94.3 %	4.60 %	2.24 %
UK	1961 - 2002	157.6 %	41.7 %	2.28 %	0.83 %
US	1961 - 2004	178 %	50.3 %	2.35 %	0.93 %
Vietnam	1992 - 2004	96.7 %	26.4 %	5.34 %	1.82 %

Table 2: Comparing GDP and Green GDP growth rates.

1992 through 1994. Therefore the measuring error could be huge. After transforming the data to 2005 USD Brazil's Green GDP in 1993 is $126 USD(2005) per capita. This is the lowest amount per capita, where the second lowest is India with $268 USD(2005) per capita, and the highest is Australia with $17.859 USD(2005) per capita.

The GGDP data for Poland runs from 1980 to 1997. The GDP only runs from 1990 onward. We have plotted the two series in Figure 2. It can be seen that the GGDP makes a temporary dip with the minimum at 1990, where it is close to zero. After 1990 it grows back to an earlier reached level. As discussed, the unbalanced structure of the data for both the GDP and GGDP would use data for Poland from 1990 until 1997. However, this results into a false picture of massive growth in the GGDP of Poland. In Figure 3 we show a plot of the growth and level data of the GGDP and the regression estimated line, when Poland is included. As can be seen Poland is very much an outlier. Because in actuality the GGDP of Poland was only temporarily low and grows back to earlier reached heights, this huge growth is misleading and we do not use the data available for Poland in this thesis.

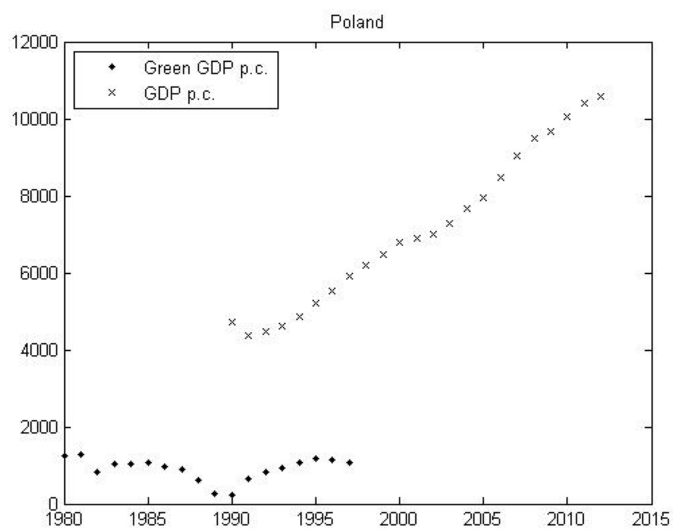


Figure 2: Poland's GDP and Green GDP time series

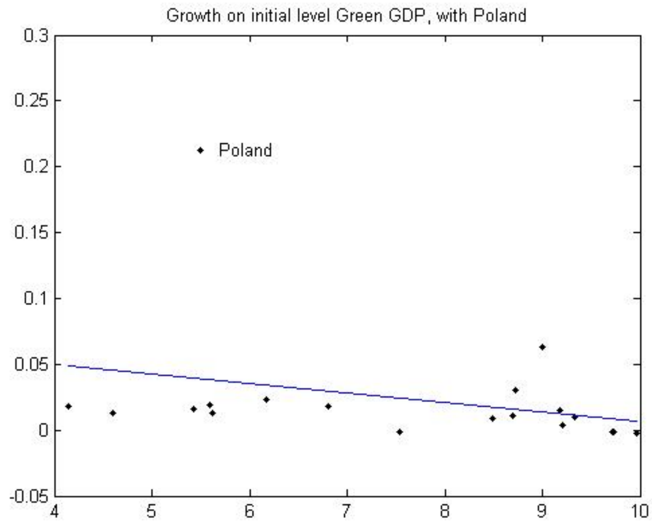


Figure 3: A plot of the growth and level data of the GGDP, with Poland as an outlier.

3 Research

In this section we discuss how we study the convergence growth of the GDP and GGDP. Two types of convergence research are distinguished. They are β- and σ-convergence. When the estimated parameter of a regression of the growth of a variable on its level is negative, i.e. that countries with a lower level on average grow faster there is β-convergence. When the sample variance of the cross-section of a variable goes down over time there is σ-convergence. We consider multiple analyses for both types of convergence.

To compare the GDP to the GGDP we perform analyses on both the variables. However, as discussed the data of the GGDP is unbalanced, which can have effects on the results of the analyses. To separate these effects from using a different variable, we use the same unbalanced data structure for the GDP. To see if there is a difference in results due to the unbalanced structure we also run the analyses for the GDP in its full dataset. From here on we call the three datasets the GDP(full) dataset, the GDP(Unbalanced) dataset and the GGDP dataset.

3.1 β-convergence

When the estimated parameter of a regression of growth on the level of a variable is negative, we say there is β-convergence. When this is the case countries with a low level have a higher growth. Although this is straightforward there are variations on the regression. It is possible to use the panel structure of the data or to instead regress the total growth on the initial level of the data. This is called a Barro regression in the literature. Barro (1989) and Mankiw et al. (1990) both use Barro regressions and expand the regression with extra variables to explain the growth. When the parameter for the level is negative in this regression it is called conditional β-convergence. Otherwise it is called unconditional β-convergence.

The hypothesis that there is β-convergence for Barro and Mankiw is based on neoclassical growth models. In these models growth is determined exogenously by a set of factors, namely capital, labour and level of technology. Through diminishing returns in growth the models suggest that countries with a lower level will grow faster.

Unconditional β-convergence Unconditional β-convergence research is done by regressing growth on the level and no other independent variables. We do this with some variations. First we use the panel data in three different panel regressions. The downside of this is that the short term movements in the data of the level and growth will influence the estimated parameter, which is not the β-convergence effect. The most common way in the literature therefore is to regress the total average growth on the initial level of the variable. This is called a Barro regression. Unlike Barro (1989) who had 98 countries, we only have data for 18 countries. By performing a Barro regression we only have 18 data points, which makes a t-test on the estimated parameter unreliable. To make the t-test more reliable, we bootstrap the t-distribution when we run the Barro regression.

We run three different panel regressions. A panel regression can differ in how much freedom parameters have to vary over time or individuals. Because the panel is unbalanced and there are 18 countries and 63 periods in the data, for some periods there are only a handful of data points. Because of that we do not consider time-varying parameters. We do consider panel regression where the parameters vary over individual countries. The three panel models we consider are:

$$ln(Y_{i,t}/Y_{i,t-1}) = \alpha_i + \beta_i * ln(Y_{i,t-1}) + \epsilon_{i,t} \tag{1}$$

$$ln(Y_{i,t}/Y_{i,t-1}) = \alpha_i + \beta * ln(Y_{i,t-1}) + \epsilon_{i,t} \tag{2}$$

$$ln(Y_{i,t}/Y_{i,t-1}) = \alpha + \beta * ln(Y_{i,t-1}) + \epsilon_{i,t} \tag{3}$$

where $ln(Y_{i,t}/Y_{i,t-1})$ is the growth and $ln(Y_{i,t-1})$ is the log level of the variable. Growth is defined as $ln(\frac{Y_{i,t}}{Y_{i,t-1}})$, which is in accordance with neo-classical growth models (Mankiw et al., 1990). The difference in the three panel regressions is whether their constant and parameter terms vary over the individual countries. The models are called the individual effect model, the individual constant model and the pooled model respectively.

As shown below the individual effect and the individual constant models are not able to use the difference in growth and level between countries, and so do not estimate β-convergence. What these models do instead is explain the movement of the data within countries. The pooled model also uses panel data and thus also explains the short term movement within countries. Explaining this can distract from finding the true effect of β-convergence. The results for the the individual effect and the individual constant models can be seen as an indicator for that distraction. This is exactly why the Barro regression does not use panel data structure. The advantage of the pooled model over the Barro model is that the panel structure supplies much more data.

Estimating the individual effect panel model (1), is done in 18 separate regression because the α_i and β_i are different for each country. This is why this model can not use the difference in growth or level between countries to estimate β. The individual constant model (2), has a different constant to explain growth for each country, but one shared parameter β. We do not assume this α_i is uncorrelated with the independent variable $ln(Y_{i,t-1})$. Therefore the fixed effect estimator, also called within estimator is used to estimate the model. The estimation is done by rewriting equation (2) into

$$ln(Y_{i,t}/Y_{i,t-1}) - \overline{ln(Y_t/Y_{t-1})}_i = \cancel{\alpha_i - \alpha_i} + \beta * (ln(Y_{i,t-1}) - \overline{ln(Y_{i,t-1})}_i) + \epsilon_{i,t} \quad (4)$$

where $\overline{ln(Y_t/Y_{t-1})}_i$ and $\overline{ln(Y_{i,t-1})}_i$ are the means of $ln(Y_{i,t}/Y_{i,t-1})$ and $ln(Y_{i,t-1})$ over time for individual i. This is used as an estimating equation for a linear regression. Because the mean for the growth and level for each country is subtracted, this model can also not use the differences in level and growth between countries to estimate β. What the individual effect model and the individual constant model do measure, is the effect the short run movements in the data have on the estimation process.

The β of the pooled panel model (3) is estimated by putting all data points from different countries and times into one vector and running an

OLS regression. This model, in contrast to the other two panel models, does use the difference in level and growth between countries. However, because of the panel shape of the data, the short-term movements within the data still influence the estimation. For this model only one constant parameter and one effect parameter are estimated.

The Barro regression does not use panel data. Instead the estimation is done by regressing the total growth on the initial level. This means there is only one data point per country. The estimating equation is

$$\frac{1}{T_i} * ln(Y_{i,T}/Y_{i,0}) = \alpha + \beta * ln(Y_{i,0}) + \epsilon_i \tag{5}$$

where $Y_{i,0}$ is the initial level in the time series and $\frac{1}{T_i} * ln(Y_{i,T}/Y_{i,0})$ is the average log growth of country i over the entire series. We have to use the average total growth, because the total growths of the countries are measured over different lengths of periods. This also makes the results comparable to those of the pooled model. Only having one data point per country does have the advantage that it prevents the unbalanced structure of the data to allow some countries to have more weight in the estimation. In our data the counntries with the most data are western developed countries. The downside is that with only 18 data points, the t-test becomes unreliable. The equation for the Barro regression is

conditional β-convergence Conditional β-convergence is when the model is expanded to contain extra independent variables to explain the growth. These independent variables are proxies for investment in the future, that according to neo-classical growth models can further explain growth. The model equation is:

$$\frac{1}{T_i} * ln(Y_{i,T}/Y_{i,0}) = \alpha + \beta * ln(Y_{i,0}) + \sum_j \gamma_j * ln(X_{j,i}) + \epsilon_i \tag{6}$$

where $ln(X_{j,i})$ are the logs of the extra added variables. The estimation is done first with only one extra variable and then with combinations of multiple variables. The variables we use are variables that are also used by Barro (1989). They are primary, secondary and tertiary school enrolment rates and government consumption, government spending on education and government spending on defence as percentage of GDP. The last three Barro uses in a composite measure that is government consumption minus spending on education or defence. Barro treats this measure as a negative proxy for investment as more consumption means less investment. The data for these extra variables is not available for every period and not at the same time for each country. Therefore studying the panel models is not possible. For the Barro regression there has to be one value per country for every variable. However often the oldest available data for the variables is newer than the start of the GGDP data. Therefore we use the average of the available data of the variables for country i over the time that GGDP data is available for country i. This provides a value that can be seen as how much countries invest on average. First of all because of the way these values are picked, we only study this model with the unbalanced datasets. Secondly, using these values disrupts the exogeneity of the variables as they are an average of data that are newer than the initial levels of GGDP. This has to be kept in mind when conclusions are made from the results of this model. This also prevents use of this model in predicting growth. For Vietnam there is no available data of the government consumption in the time period of the data so we did not use the data of Vietnam in this model. Barro also used a variable that indicated the political unrest in a country to show when countries where out of equilibrium. This variable was the sum of political assassinations and coups over the time in the data. We lack data for a lot of the countries for this assassination variable and so do not use it in our research. This might have an impact if there are countries that have political unrest that could have been indicated by this variable and that political unrest has an effect.

3.2 Bootstrapping

Because the Barro regression has a small sample, the p-value of the t-test is not reliable. This is because the t-distribution is shaped as if the mean of the data is normal, which can be assumed because of asymptotic theory. With bootstrapping we estimate a t-distribution without needing asymptotic theory. Therefore the estimated p-values are more reliable. Below we explain how to bootstrap and some of the theory behind bootstrapping. A more complete explanation can be found in Cameron and Trivedi (2005).

Bootstrapping a distribution is done by sampling new datasets from the data. The data is treated as the distribution of your data, from which you sample. The new datasets that are sampled have the same size as the original data and are drawn with replacement. For example, assume a variable with five observations $Y = [Y(1)\ Y(2)\ Y(3)\ Y(4)\ Y(5)]$, then one such a draw could be $Y_b = [Y(1)\ Y(2)\ Y(1)\ Y(4)\ Y(2)]$. Sampling each possible variation would be to computationally intensive, so instead we sample randomly. When the number of samples, B, increases, the estimated distribution will be closer to the distribution with all samples. Cameron and Trivedi (2005) offer rules of thumb for a B of 200 to 1500 for different applications. This makes the probability more than 0.95 that the difference between the estimated distribution and the theoretical distribution with all possible samples is less than 10%. In the applications of the bootstrap in this thesis we sample 100.000 times, which the computer can handle.

Because we want to estimate the t-distribution we sample both the independent variable X and the dependent variable Y. To keep the effect that X has on Y, the data can not be sampled independently. There are two sampling methods we use to preserve this effect, namely pairwise sampling and residual sampling. With pairwise sampling the variables are sampled with corresponding data points. To go back to the earlier example if the b^{th} draw of Y is $Y_b = [Y(1)\ Y(2)\ Y(1)\ Y(4)\ Y(2)]$ then the b^{th} draw of X would be $X_b = [X(1)\ X(2)\ X(1)\ X(4)\ X(2)]$. For the residual sampling, first the original regression is run. Then B error vectors are sampled from the original error vector and added onto the original $\hat{Y} = \hat{\beta}_{Original} * X$, where $\hat{\beta}_{Original}$ is the $\hat{\beta}$ of the original regression. By adding the bootstrapped error vectors onto $\hat{Y}$ leads to B bootstraps Y_b of Y. This way of sampling does assume the errors to be i.i.d. which is a strong assumption so our preference goes to pairwise sampling. After sampling the data, B regressions with different samples are run and each time the t-statistic of $\hat{\beta}$ is estimated. To make the bootstrapped t-distribution centered around zero each of the bootstrapped t-statistics are estimated with the equation $\hat{t}_b^* = \frac{\hat{\beta}_b^* - \hat{\beta}_{original}}{\hat{\sigma}_b^*}$. After bootstrapping the t-distribution, the p-value is calculated by seeing how much percent of the bootstrapped distribution is lower or higher than the t-statistic of the original regression.

Bootstrapping is useful when distributions can not be estimated by standard theory, but it can also better estimate the distribution even when standard theory applies when the estimated statistic is asymptotically pivotal. Asymptotically pivotal statistics are statistic that are not dependent on parameters. This is proven by using Edgeworth expansions of the true distribution of the test-statistic $G_N(t, F_0) = Pr[T_N \leq t]$ and the bootstrapped distribution $G_N(t, F_N)$, where $F_0(x)$ is the cdf of the data, N is the amount of data and T_N the estimated test statistic. Then

$$G_N(t, F_0) = G_\infty(t, F_0) + \frac{g_1(t, F_0)}{\sqrt{N}} + O(N^{-1}) \tag{7}$$

and

$$G_N(t, F_N) = G_\infty(t, F_N) + \frac{g_1(t, F_N)}{\sqrt{N}} + O(N^{-1}) \tag{8}$$

where the difference terms $\frac{g_1(t,F_0)}{\sqrt{N}}$ and $\frac{g_1(t,F_N)}{\sqrt{N}}$ decrease with $\sqrt{N}$. These difference terms are hard to estimate. When the statistic is asymptotically pivotal the asymptotic distribution of both the bootstrapped distribution and the true distribution is the same, $G_\infty(t, F_0) = G_\infty(t, F_N)$. Furthermore $g_1(t, F_0) - g_1(t, F_N) = O(N^{-1/2})$, because as the amount of data increases the bootstrap estimate of the cdf of the data is closer to the true cdf. Combining equations (7) and (8) shows that for an asymptotically pivotal statistic

$G_N(t, F_0) - G_N(t, F_N) = \frac{O(N^{-1/2})}{\sqrt{N}} + O(N^{-1})$ or

$$G_N(t, F_0) = G_N(t, F_N) + O(N^{-1})$$

Comparing this with

$$G_N(t, F_0) = G_\infty(t, F_0) + O(N^{-1/2})$$

shows the bootstrap estimate of the distribution has an expected smaller difference from the true distribution when there is asymptotic refinement.

3.3 σ-convergence

We speak of σ-convergence when the variance of the cross-section decreases over time. When there is β-convergence countries with low level of a variable grow faster, but that does not necessarily mean the variance decreases(Quah, 1993). It might be the case that the variance of the GDP or GGDP is a stationary process even when there is β-convergence. Neoclassical growth models have no issue with increases in the variance. For example, consider two countries with an equal GDP, that invest differently. The country that invests more will grow more and thus the variance increases. Barro and Sala-i Martin (1992) acknowledge this idea and call β-convergence "Convergence in the sense that poor economies tend to grow faster than rich ones". σ-convergence is then considered to be convergence in the sense of variance of the cross section decreasing over time.

Young et al. (2008) show that the β-convergence is necessary for but does not imply σ-convergence by rewriting the β-convergence equation and taking the variance (3).

$$\sigma_t^2 \simeq (1+\beta)^2 \sigma_{t-1}^2 + \sigma_\epsilon^2 \tag{9}$$

For σ_t^2 to be stationary, β has to be between -1 and 0. Taking $\sigma_t^2 = \sigma_{t-1}^2$ leads to the equilibrium variance

$$\sigma_{equilibrium}^2 = \frac{\sigma_\epsilon^2}{1-(1+\beta)^2} \tag{10}$$

that increases with σ_ϵ^2 but decreases with a lower β.

We perform three analyses in the σ-convergence section. We do an F-test of equality of variance to compare the variance at the start and end of the data. We also run a regression of the variance on a constant and trend to see whether we can measure an effect of time on the sample variance. Finally we follow Drennan et al. (2004) in performing an adjusted Dickey-Fuller test to see whether the variance is stationary.

Data transformations For all three analyses we measure the sample variance of the cross section at time t as

$$S_t^2 = \frac{1}{n-1} \sum_{i=1}^{N} [Y_{i,t} - \mu_t]^2$$

where μ_t is the mean of $Y_{i,t}$. To further understand the dynamics of the sample variance and to be robust we report the results for a group of transformed datasets of $Y_{i,t}$. First of all we focus on the variance of the level of the GDP or GGDP, and also on the variance of the logs of those variables. The variance of the level might seem to make more sense, since we want to know whether the variable converges, however the derivation of the equations (9) and (10) are based on the β-convergence equations, which use the log of the variables. We also expect the mean of the variables to grow exponentially. Therefore the variance will grow as well and to adjust for this effect taking the log is a solution. Barro and Sala-i Martin (1992), Lee et al. (1997), Drennan et al. (2004) and Young et al. (2008) all consider the log of the variance of the GDP for their analyses of σ-convergence. Furthermore, we correct for the increase over time of the mean of the variables, by transforming the data to have the same mean, arbitrarily 100, in every period. The data is transformed by the formula

$$Y_{i,t}new = \frac{100 * Y_{i,t}}{\bar{Y}_t}$$

where $\bar{Y}_t$ is the average value of $Y_{i,t}$ in period t. We call the variance of $Y_{i,t}new$ the percentage variance. We call the variance of the untransformed data the absolute variance. The last variation of the data we consider is using different periods for the analyses. Because of the unbalanced shape we consider only periods between 1970 and 2003, as that is when there are at least 10 countries in the data, but to be more robust we also show results for start year 1972 or end year 2001. With all these variations and the three datasets the amount of results is quite large.

F-test For the F-test analysis we see if the variance at the start and end of the data are significantly different. The test-statistic is

$$F = \frac{S_{t_2}^2}{S_{t_1}^2} \tag{11}$$

where S_{t_i} is the sample variance from period t_i. To increase robustness we report the p-value for the standard asymptotic F-distribution as well as the bootstrapped distribution. Boos and Brownie (1989) shows how to bootstrap the F-distribution and tests the power. The distribution is bootstrapped by sampling data from the mixed dataset $S = \{Y_{i,t} - \mu_t\}$, where i is from every country in the data at time t and $t = t_1, t_2$. By drawing from S the distribution is centered around 1, as the two samples are each as likely to have a larger variance. Drawing from S assumes the hypothesis of equal variance is true, which allows it to be used as a test for that hypothesis. The data in S has its mean subtracted to prevent a change in the mean to influence the estimated F-statistics. Boos and Brownie (1989) shows the power of the F-test with the bootstrapped distribution can be better than the F-test with the asymptotic distribution, depending on the distribution of the data. We show both to be robust.

Regression of the variance on a trend Another method to see whether the variance changes over time is by running a regression of the sample variance on a constant and a trend. We estimate a regression with the formula

$$\sigma_t^2 = \alpha_0 + \alpha_1 * t + u_t \tag{12}$$

where u_t is a noise term. If the estimated $\hat{\alpha_1}$ is significant we can also speak of σ-con or divergence. This result can not be extrapolated as that could lead to a negative variance. We note that the results for this analysis and the F-test analysis does not have to be the same as this analysis considers average change in the variance over time, while the F-test considers the absolute difference.

Adjusted Dickey-Fuller test The last analysis is performing an adjusted Dickey-Fuller(ADF) test to see whether the variance is stationary. When there is β-convergence there should be a stationary variance as Young et al. (2008) shows. The ADF-test is done by regressing the difference of the sample variance ΔS_t on a possible constant and trend if these are significant, on the sample variance of the last period S_{t-1} and a summation of difference lags. The equation becomes

$$\Delta S_t = \alpha_0 + \alpha_1 * t + \gamma * S_{t-1} + \delta_1 * \Delta S_{t-1} + ... + \delta_h * \Delta S_{t-h} \tag{13}$$

where h is decided on by maximizing the Schwarz information criterion. The test is finished by taking the t-statistic of $\hat{\gamma}$ and seeing what the corresponding p-value is with the Dickey-Fuller distribution.

4 Results

In this section we discuss the results we have obtained for the β-convergence and the σ-convergence analyses and the conclusions we make with those results. We first go over the results for the β-convergence study and then for those of the σ-convergence study. The two sections are subdivided into smaller parts.

4.1 β-convergence

For the β-convergence section there is the unconditional and conditional part. We first discuss the results for the unconditional β-convergence, which are the results for the three panel models and the Barro regression model for the unconditional β-convergence. Following we show the results for when we expand the research to the conditional β-convergence.

unconditional β-convergence As discussed, we studied three panel models and the Barro regression model. We first dicuss the results for the three panel models, which are the individual effect, the individual constant and the pooled model.

As discussed, significant results for the individual effect and individual constant model indicate a distraction from estimating the β-converge effect for the pooled panel model. The results for the individual effect model are shown in Table 3. The results are very mixed for different countries and the different datasets. There are positive significant and negative significant and non significant effects measured. It is not clear if the estimation of the β-convergence effect with the pooled model is influenced by the panel structure.

Similarly in Table 4 the results for the individual constant model, following formula (2), are shown. This model also does not use the difference of the growth and level between countries to estimate its parameter. Unlike the individual effect model, now only one β is estimated for all countries. The β is significantly positive, at the 5% level for all datasets. Countries below their own average level seem to grow slower than their average growth, both for GDP and GGDP. These results do indicate that the estimated parameter of the pooled model is probably influenced by the short term movements in the data.

In the pooled model, equation (3), and the Barro regression model, equation (5), the difference between countries in growth and level does get used. The results for the pooled model are in Table 5 and the results for the Barro regression are in Table 6. The results for both models are significant for the GDP, both the full and unbalanced dataset, and not significant for the GGDP. For the pooled model the estimates of β are all a little less negative,

	GDP_{Full}			GDP_{Unb}			GGDP		
	$\hat{\beta}$	p-value	R^2	$\hat{\beta}$	p-value	R^2	$\hat{\beta}$	p-value	R^2
Australia	-0.685	0.210	0.013	0.471	0.357	0.004	1.255	0.365	0.003
Austria	-2.397	0.000	0.211	-2.336	0.021	0.131	-6.831	0.004	0.217
Belgium	-3.277	0.000	0.293	-2.004	0.082	0.055	-1.179	0.438	0.001
Chile	2.526	0.045	0.562	12.295	0.004	0.213	24.546	0.0275	0.121
China	2.544	0.002	0.152	2.061	0.002	0.213	2.908	0.127	0.038
Costa Rica	0.892	0.274	0.007	-1.789	0.447	0.001	6.448	0.301	0.016
France	-4.115	0.000	0.418	7.209	0.106	0.138	12.733	0.250	0.042
India	3.640	0.000	0.267	1.140	0.353	0.010	31.957	0.027	0.224
Indonesia	0.738	0.204	0.013	-2.084	0.261	0.0348	94.072	0.002	0.512
Japan	-5.140	0.000	0.389	-3.264	0.017	0.133	-7.639	0.018	0.129
Netherlands	-3.202	0.000	0.217	0.640	0.335	0.006	12.535	0.083	0.059
New Zealand	0.737	0.390	0.002	5.737	0.055	0.092	39.624	0.008	0.194
Philippines	2.847	0.114	0.028	1.436	0.451	0.001	-8.621	0.150	0.067
Sweden	-1.597	0.081	0.039	-5.075	0.002	0.239	1.0473	0.434	0.001
Thailand	-0.849	0.173	0.019	-1.553	0.208	0.0237	-0.348	0.471	0.000
UK	-0.776	0.202	0.014	1.102	0.161	0.024	11.704	0.044	0.073
USA	-2.159	0.009	0.106	-1.433	0.092	0.042	-9.516	0.004	0.152
Vietnam	1.589	0.0192	0.149	-2.913	0.068	0.190	10.745	0.298	0.026

Table 3: The statistics for the 18 separate individual effect model, shown in equation (1).

	GDP_{Full}	GDP_{Unb}	GGDP
$\hat{\beta}$	0.536	0.701	3.405
p-value	0.987	0.957	0.984
R^2	0.006	0.006	0.009

Table 4: Different statistics for the individual constant model (2).

	GDP_{Full}	GDP_{Unb}	GGDP
$\hat{\beta}$	-0.391	-0.658	-0.108
p-value	0.000	0.000	0.281
R^2	0.039	0.111	0.001

Table 5: Different statistics for the pooled model (3).

which may be explained by the fact that the pooled model estimates the β to also explain the short term movements in the panel data. The Barro regression has a much higher R^2 that may also be explained by the lack of the short term movements in the data. Because the sample is so small for the Barro regression we bootstrapped the t-distributions for the tests. In Table 6 the p-values are shown for the two bootstrapped distributions that use different sampling methods and for the standard asymptotic t(17) distribution. The p-values are hardly different from the standard t(17) estimated p-values. This allows us to be more certain that they are accurate. In Figure 4 we show a graph of the pdf of the bootstrapped distributions and the standard t(17) distribution for the GGDP. For the GDPs datasets the bootstrapped distributions were similar and are shown in the Appendix in Figures 13 and 14. In the Appendix in Figure 15 we show the data for the Barro regression for the different datasets with the regression estimated line. Because of the results of the pooled model as well as of the Barro regression model, with robust p-values, we conclude there is unconditional β-convergence for the GDP, but not for the GGDP.

The fact that we find unconditional β-convergence and Barro does not may be explained by the set of countries in our data. There are a lot of

	GDP_{Full}	GDP_{Unb}	GGDP
$\hat{\beta}$	-0.523	-0.688	-0.134
$\text{p-value}_{Pairwise}$	0.000	0.000	0.258
$\text{p-value}_{Residual}$	0.000	0.000	0.226
$\text{p-value}_{Asymptotic}$	0.000	0.000	0.251
R^2	0.560	0.549	0.029

Table 6: Different statistics for the Barro regressions (5).

countries with low level and low gowth of GDP in Barro's data. Our data exists of only the countries with GGDP variables, which are either developing or developed countries. This means low level high growth or high level "low" growth countries. We believe this self selection of countries is the reason we find unconditional β-convergence for the GDP where Barro does not.

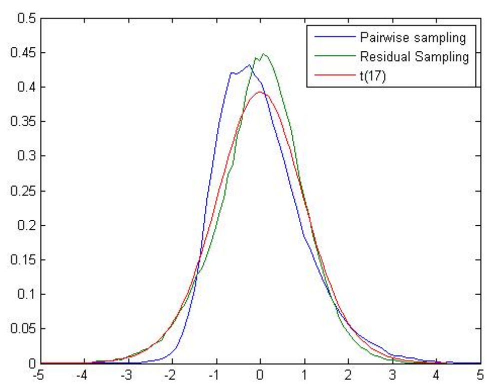


Figure 4: The bootstrapped and asymptotic t(17) t-distributions for the GGDP dataset.

Conditional β-convergence For the conditional β-convergence analysis we show the results of the regression of the equation (6) with different combinations of variables. Because of the scarcity of the data of the extra variables, we only look at the unbalanced datasets. We first show regressions with only one added variable. In Tables 7 and 8 are the results for the Barro regressions for the GDP(Unbalanced) and GGDP datasets. In the table the regressions are named (1) through (8), which are the regressions with each one different added variable. For the GDP dataset the estimated $\hat{\beta}$s are all significant, except for when tertiary enrolment rate is added. None of the $\hat{\gamma}$s are. For the GGDP regressions none of the $\hat{\beta}$s nor any $\hat{\gamma}$s are significant. The results are very similar to those of the unconditional β-convergence section. This is also true for the level of R^2.

In Table 9 the results of the convergence with combinations of variables are shown for both GDP and GGDP. The variables Barro took were primary and secondary enrolment rate and the composite measure of government consumption minus government spending on education and defence. We show the results for this regression and try to find significance for the β by changing these variables. For the GGDP the tertiary enrolment rate, government spending on education and government consumption had the lowest p-value for their respective t-tests. Therefore we also show the results for regressions with tertiary instead of primary enrolment rate and government consumption or spending on education instead of the composite measure. This results into 6 regressions named (1) though (6) in the table. Regression (1) is with the variables Barro used. The GGDP parameters have p-values closer to the critical value, but are still not significant.

Changing the regressions in order to find significance is p-hacking. We do this to disprove our hypothesis that the GDP and GGDP converge differently. We do not find those results however, as the estimated β for the GGDP never is significant.

Summary β-convergence We find unconditional β-convergence with both the pooled model and the Barro regression model for the GDP. The results of individual effect and individual constant models indicated that the estimated parameter of the pooled model is influenced by the short term movements within the data. For the GGDP there was no significant β-convergence.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)[a]	(8)
Estimated $\hat{\beta}$	-0.640	-0.921	-0.078	-0.831	-1.036	-0.924	-0.651	-0.826
	0.001	0.011	0.418	0.021	0.001	0.007	0.001	0.001
PRIM	-0.998	–	–	–	–	–	–	–
	0.431	–	–	–	–	–	–	–
SEC	–	1.384	–	–	–	–	–	–
	–	0.191	–	–	–	–	–	–
TERT	–	–	-1.528	–	–	–	–	–
	–	–	0.061	–	–	–	–	–
average enrollment	–	–	–	1.883	–	–	–	–
	–	–	–	0.280	–	–	–	
Gov. consumption	–	–	–	–	2.836	–	–	–
	–	–	–	–	0.411	–	–	
Gov. spending education	–	–	–	–	–	1.564	–	–
	–	–	–	–	–	0.164	–	–
Gov. spending military	–	–	–	–	–	–	-0.160	–
	–	–	–	–	–	–	0.410	–
Govcons. - educ - military	–	–	–	–	–	–	–	1.256
	–	–	–	–	–	–	–	0.135
R^2	0.494	0.521	0.574	0.506	0.587	0.528	0.553	0.537

Table 7: Results GDP(unbalanced) regressions with one extra regressor.

[a] Because Costa Rica does not have a military, we discard the Costa Rica data for regression (7).

	(1)	(2)	(3)	(4)	(5)	(6)	(7)[a]	(8)
Estimated $\hat{\beta}$	-0.151	-0.212	-0.388	-0.317	-0.462	-0.427	-0.116	-0.316
	0.256	0.313	0.139	0.226	0.128	0.192	0.309	0.147
PRIM	-3.068	–	–	–	–	–	–	–
	0.351	–	–	–	–	–	–	–
SEC	–	0.474	–	–	–	–	–	–
	–	0.407	–	–	–	–	–	–
TERT	–	–	0.911	–	–	–	–	–
	–	–	0.175	–	–	–	–	–
average enrollment	–	–	–	2.084	–	–	–	–
	–	–	–	0.296	–	–	–	
Gov. consumption	–	–	–	–	2.532	–	–	–
	–	–	–	–	0.163	–	–	
Gov. spending education	–	–	–	–	–	1.715	–	–
	–	–	–	–	–	0.245	–	–
Gov. spending military	–	–	–	–	–	–	-0.081	–
	–	–	–	–	–	–	0.470	–
Govcons. - educ - military	–	–	–	–	–	–	–	1.402
	–	–	–	–	–	–	–	0.180
R^2	0.34	0.027	0.084	0.044	0.090	0.057	0.021	0.081

Table 8: Results Green GDP regressions with one extra regressor.

[a]Because Costa Rica does not have a military, we discard the Costa Rica data for regression (7).

	(1)	(2)	(3)	(4)	(5)	(6)
Estimated $\hat{\beta}$	-1.377	-0.520	-1.729	-1.106	-1.467	-0.927
	0.005	0.237	0.005	0.104	0.003	0.012
Estimated constant	31.938	4.947	-13.047	3.288	17.590	1.688
	0.137	0.144	0.357	0.237	0.256	0.378
PRIM	-6.526	–	3.280	–	-3.981	–
	0.161	–	0.336	–	0.253	–
SEC	2.107	1.064	2.431	2.054	1.877	1.204
	0.117	0.251	0.087	0.125	0.132	0.216
TERT	–	-1.157	–	-0.963	–	-0.736
	–	0.170	–	0.182	–	0.263
Govcons. - educ - military	1.583	0.549	–	–	–	–
	0.096	0.340	–	–	–	–
Gov. spending education	–	–	3.300	2.099	–	–
	–	–	0.073	0.128	–	–
Government consumption	–	–	–	–	2.852	2.170
	–	–	–	–	0.050	0.134
R^2	0.597	0.595	0.611	0.632	0.629	0.139

(a) Parameters and p-values of conditional β-convergence regressions for the GDP(unbalanced) dataset.

	(1)	(2)	(3)	(4)	(5)	(6)
Estimated $\hat{\beta}$	-0.597	-0.592	-0.849	-0.843	-0.707	-0.812
	0.151	0.131	0.176	0.162	0.138	0.097
Estimated constant	31.365	0.665	-5.246	-0.653	19.228	-3.260
	0.227	0.457	0.463	0.464	0.318	0.332
PRIM	-7.160	–	0.759	–	-5.146	–
	0.224	–	0.475	–	0.287	–
SEC	0.946	-0.440	1.391	0.804	1.014	-0.167
	0.339	0.419	0.283	0.375	0.329	0.468
TERT	–	1.107	–	0.733	–	1.107
	–	0.151	–	0.254	–	0.148
Govcons. - educ - military	1.706	1.666	–	–	–	–
	0.156	0.152	–	–	–	–
Gov. spending education	–	–	2.677	2.021	–	–
	–	–	0.247	0.251	–	–
Government consumption	–	–	–	–	2.612	2.981
	–	–	–	–	0.270	0.134
R^2	0.126	0.162	0.085	0.118	0.118	0.174

(b) Parameters and p-values of conditional β-convergence regressions for the GGDP dataset.

Table 9: Results for the conditional β-convergence regressions with multiple variables.

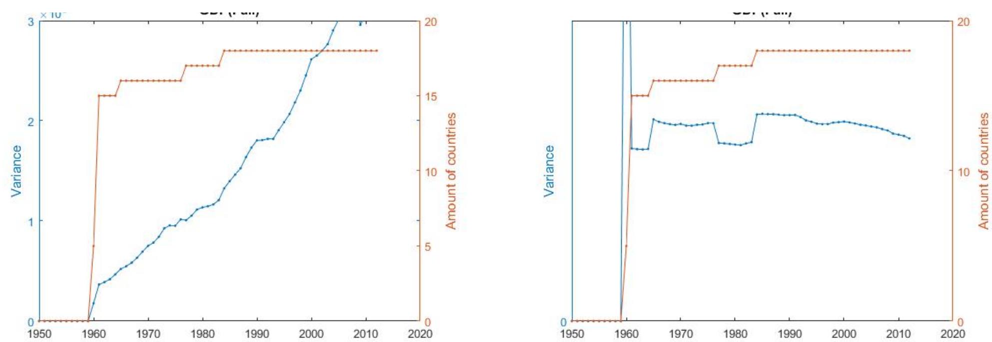
For the conditional β-convergence the extra variables' data was not readily available which forced us to use values that were not exogenous. We also ran multiple regressions with different combinations of variables, but still there is no significant β-convergence for the GGDP. Our conclusion for the β-convergence research is that the GDP β-converges and the GGDP does not.

4.2 σ-convergence

In this section we cover the results for the σ-convergence analysis. We show the development of the variance of the cross-section over time. Following we show the results for the equality of variance F-test, for the regression of the sample variance on a trend and for the adjusted Dickey-Fuller(ADF) test. We show the results for the three datasets, using the variance of the levels and the logs of the variables, we consider absolute and percentage variance and run the analyses for different periods. Because of the multitude of different results we put some graphs in the appendix.

Sample variance over time The sample variance of the cross-section is measured over time and changes value. In Figures 5 and 6 the development of the sample variance is shown for the GDP(Full) dataset, for different variations of the data. In the appendix in Figures 16 through 19 are the same graphs for the GDP(Unbalanced) and GGDP datasets. The size of the variance is different for the different variations, but that is not important as for all the analyses the size of the variance is corrected for. All that is important is the shape, i.e. the increase or decrease over time. Because the mean of the GDP grows over time, the percentage variance is relatively lower at the end of the measured period than at the start. We note that the percentage variance of the level of the variable and the log of the variable are almost the same. In the results to follow there will also not be a large difference.

In the figures we also show the amount of countries in the data over time. For the full dataset it can be seen that Thailand, New Zealand and Vietnam are available at later points. As can be seen the sample variance of the GDP(Full) dataset jumps in 1977 and 1984 when New Zealand and Vietnam enter the data. In the figures in the appendix it is also clear that changes of what countries are in the data have an effect on the sample variance. This presents a problem for making inferences on the unbalanced datasets.



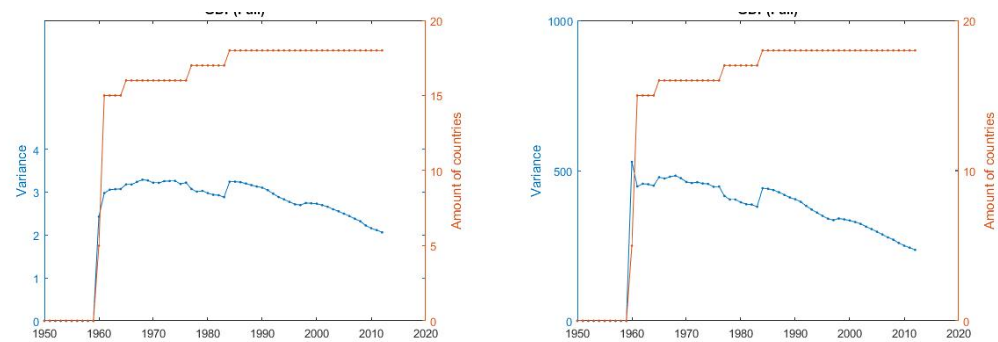
(a) Absolute Variance GDP$_{Full}$

(b) Percentage Variance GDP$_{Full}$

Figure 5: Sample variance of the level of the GDP$_{Full}$ over time.

F-test For the F-test we look at only the variance of the start and end of the data. The F-statistic is significant when the two compared sample variances differ enough from each other. The null hypothesis is that the two variances are equal and the test is two-sided so the null is rejected for $p \leq 0.05, p \geq 0.95$. We compare the test-statistic to the standard asymptotic and the bootstrapped F-distribution. In Figure 7 the difference is shown between the standard F distribution with 10 and 9 degrees of freedom and the bootstrapped F distribution for the GGDPs percentage variance for years 1970 and 2003 and in Figure 8 the bootstrapped distribution for the percentage variance of the GDP(unbalanced is shown. All the bootstrapped distributions were like these ones, shaped smoothly with often times thinner tails. The only exception is the bootstrapped F distribution for the percentage variance of the log of the GDP(Full) data. This distribution can be seen in Figure 9. This shape with two dips left and right to 1 are seen for this data for all combinations of periods. These are the only distributions with fatter tails. The p-values for the F-test for every variation on log or level, absolute or percentage variance and different time period are in Tables 10 and 11.

For the absolute variance of the GDP(Full) and GDP(Unbalanced) there is significant σ-divergence. This results is expected when considering Figures



(a) Absolute Variance GDP$_{Full}$ (b) Percentage Variance GDP$_{Full}$

Figure 6: Sample variance of the log of the GDP$_{Full}$ over time.

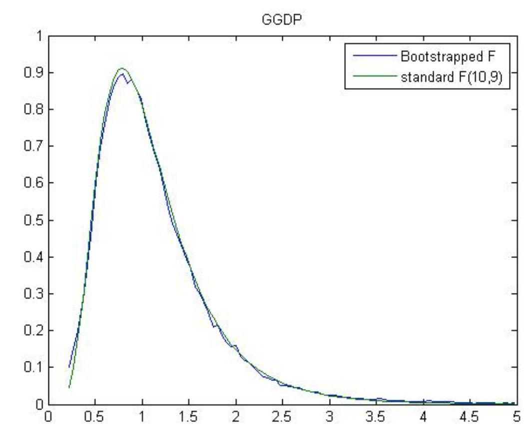


Figure 7: Bootstrapped F distribution for the percentage variance of the GGDP.

5 and 16, that show a massive increase in the sample variance over time. It can be concluded that this effect is due to the increase of the mean of the GDP, as for the percentage variance of the level of the GDP there is no significant σ-con- or divergence. In fact for all variations on this analysis except for the absolute variance of the level of GDP there were no significant results. For the GGDP there was also no significant σ-convergence or divergence. The p-values for the bootstrapped and the standard distribu-

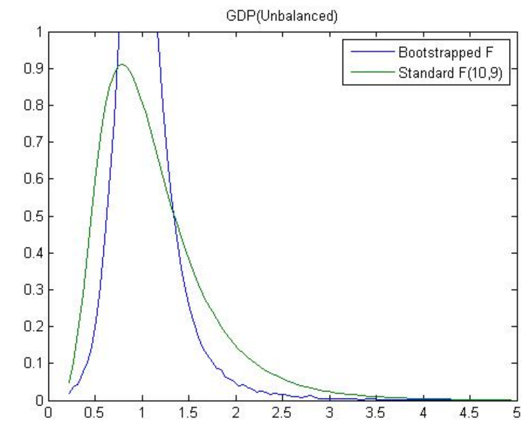


Figure 8: Bootstrapped F distribution for the percentage variance of the $\text{GDP}_{Unbalanced}$.

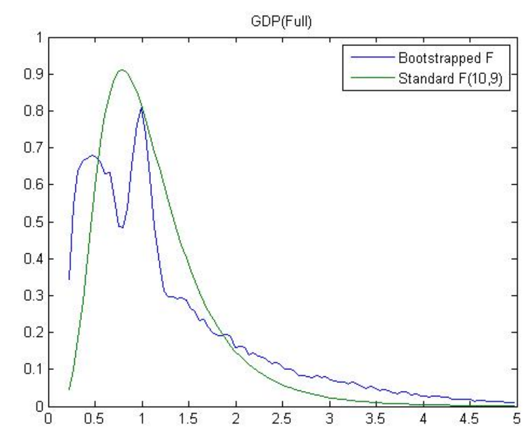


Figure 9: Bootstrapped F distribution for the percentage variance of the log of the GDP_{Full}.

		GDP_{Full}	GDP_{Unb}	GGDP
1970-2003	$p_{asymptotic}$	0.99	0.98	0.65
	$p_{bootstrapped}$	1.00	1.00	0.72
1972-2003	$p_{asymptotic}$	0.99	0.98	0.61
	$p_{bootstrapped}$	1.00	1.00	0.66
1970-2001	$p_{asymptotic}$	0.99	0.98	0.62
	$p_{bootstrapped}$	1.00	1.00	0.70
1972-2001	$p_{asymptotic}$	0.98	0.97	0.57
	$p_{bootstrapped}$	1.00	1.00	0.63

(a) Absolute variance.

		GDP_{Full}	GDP_{Unb}	GGDP
1970-2003	$p_{asymptotic}$	0.49	0.68	0.36
	$p_{bootstrapped}$	0.49	0.86	0.31
1972-2003	$p_{asymptotic}$	0.50	0.63	0.39
	$p_{bootstrapped}$	0.51	0.69	0.31
1970-2001	$p_{asymptotic}$	0.50	0.52	0.25
	$p_{bootstrapped}$	0.52	0.58	0.19
1972-2001	$p_{asymptotic}$	0.51	0.46	0.27
	$p_{bootstrapped}$	0.54	0.33	0.17

(b) Percentage variance.

Table 10: Different p-values for the F-test for the variation of the level of the datasets.

		GDP_{Full}	GDP_{Unb}	GGDP
1970-2003	$p_{asymptotic}$	0.33	0.61	0.55
	$p_{bootstrapped}$	0.37	0.83	0.64
1972-2003	$p_{asymptotic}$	0.33	0.54	0.36
	$p_{bootstrapped}$	0.36	0.58	0.16
1970-2001	$p_{asymptotic}$	0.36	0.59	0.49
	$p_{bootstrapped}$	0.39	0.76	0.47
1972-2001	$p_{asymptotic}$	0.30	0.43	0.25
	$p_{bootstrapped}$	0.38	0.53	0.11

(a) Absolute variance.

		GDP_{Full}	GDP_{Unb}	GGDP
1970-2003	$p_{asymptotic}$	0.22	0.57	0.52
	$p_{bootstrapped}$	0.30	0.71	0.57
1972-2003	$p_{asymptotic}$	0.22	0.49	0.33
	$p_{bootstrapped}$	0.31	0.46	0.10
1970-2001	$p_{asymptotic}$	0.25	0.52	0.43
	$p_{bootstrapped}$e	0.32	0.56	0.32
1972-2001	$p_{asymptotic}$	0.25	0.43	0.24
	$p_{bootstrapped}$	0.33	0.33	0.06

(b) Percentage variance.

Table 11: Different p-values for the F-test for the variation of the log of the datasets.

tion do not differ greatly. The difference in which period is considered is almost non-existent for the GDP(full) dataset. For the unbalanced datasets the p-value does change a bit when looking at 1972 instead of 1970 for the start of the data. Because for the full dataset this jump in variation is not present, we conclude this is because from 1971 onwards the unbalanced data also includes Indonesia and the Netherlands and not because there was a recession in 1970. We conclude there is σ-divergence for the GDP due to an increased mean, but correcting for the increase of the mean there is no σ-con- or divergence for either GDP or GGDP.

Regression test Instead of testing the equality of variance it is also possible to test for σ-convergence by regressing the sample variance over time on a constant and a trend, as seen in equation (12). To see whether the parameter α_1 is significant only takes a simple t-test. In Tables 12 through 13 the estimates $\hat{\alpha}_1$ and the p-values of the t-test are shown for the different variations of the data. Note that the difference in seize of the $\hat{\alpha}_1$ comes from the difference in transformations, namely transforming into percentage variance or taking the log. The $\hat{\alpha}_1$ are reported to show the sign of the estimated parameter.

		GDP_{Full}	GDP_{Unb}	GGDP
1970-2003	$\hat{\alpha}_1$	$6.02 * 10^6$	$5.73 * 10^6$	$-6.84 * 10^4$
	p_{α_1}	0.0000	0.0000	0.4378
1972-2003	$\hat{\alpha}_1$	$6.19 * 10^6$	$5.87 * 10^6$	$-1.03 * 10^5$
	p_{α_1}	0.0000	0.0000	0.2955
1970-2001	$\hat{\alpha}_1$	$5.79 * 10^6$	$5.41 * 10^6$	$-1.67 * 10^5$
	p_{α_1}	0.0000	0.0000	0.0662
1972-2001	$\hat{\alpha}_1$	$5.96 * 10^6$	$5.23 * 10^6$	$-2.2 * 10^5$
	p_{α_1}	0.0000	0.0000	0.0294

(a) Absolute variance.

		GDP_{Full}	GDP_{Unb}	GGDP
1970-2003	$\hat{\alpha}_1$	12.0	-18.2	-88.9
	p_{α_1}	0.0423	0.0928	0.0000
1972-2003	$\hat{\alpha}_1$	14.6	-25.1	-87.2
	p_{α_1}	0.0273	0.0350	0.0000
1970-2001	$\hat{\alpha}_1$	13.9	-26.4	-99.0
	p_{α_1}	0.0367	0.0194	0.0000
1972-2001	$\hat{\alpha}_1$	17.2	-35.5	-98.5
	p_{α_1}	0.0222	0.0041	0.0000

(b) Percentage variance.

Table 12: The results for the σ-convergence regression analysis, using the variance of the level of the variable.

Again there is strong evidence for σ-divergence for the GDP, when considering absolute variance of the level. When considering the percentage variance there is an increase in variance for the GDP(Full) dataset and a decrease for the GDP(Unbalanced) dataset that dependent on the period studied is significant. When considering the variance of the log of the GDP, this is switched and there is a significant decrease for the full dataset and a significant increase for the unbalanced dataset. The results are thus very much influenced by the unbalanced dataset. Therefore the results for the GGDP are also less reliable. There is a significant decrease of the variance over time for the GGDP when considering either the log or the percentage variance, but as the GGDP also has an unbalanced dataset we only consider

		GDP_{Full}	GDP_{Unb}	GGDP
1970-2003	$\hat{\alpha}_1$	-0.018	0.007	-0.041
	p_{α_1}	0.0000	0.0585	0.0000
1972-2003	$\hat{\alpha}_1$	-0.018	0.008	-0.046
	p_{α_1}	0.0000	0.0708	0.0000
1970-2001	$\hat{\alpha}_1$	-0.017	0.007	-0.047
	p_{α_1}	0.0000	0.1234	0.0000
1972-2001	$\hat{\alpha}_1$	-0.017	0.007	-0.054
	p_{α_1}	0.0000	0.1445	0.000

(a) Absolute variance.

		GDP_{Full}	GDP_{Unb}	GGDP
1970-2003	$\hat{\alpha}_1$	-4.18	-0.651	-7.920
	p_{α_1}	0.0000	0.2661	0.0000
1972-2003	$\hat{\alpha}_1$	-4.21	-0.658	-8.85
	p_{α_1}	0.0000	0.3098	0.0000
1970-2001	$\hat{\alpha}_1$	-4.05	-0.834	-8.95
	p_{α_1}	0.0000	0.2029	0.0000
1972-2001	$\hat{\alpha}_1$	-4.05	-0.869	-10.15
	p_{α_1}	0.0000	0.2339	0.0000

(b) Percentage variance.

Table 13: The results for the σ-convergence regression analysis, using the variance of the log of the variable.

the results of the F-test for our conclusion.

Unit root test We also do an Adjusted Dickey-Fuller test to see whether the sample variance is stationary. Since there was unconditional β-convergence for the GDP the variance should be stationary. In Tables 14 through 15 are the p-values for the ADF-test. The ADF-test has the null hypothesis that the data is non-stationary. The ADF-test is one-sided and the null is only rejected at p-values less than 0.10.

We see that the null is not rejected for the data when it is not corrected for the increase of the mean. When we consider percentage variance, the results are dependent on the exact transformation, but mostly on which period studied. The reason the results change so much is that for some of the transformations suddenly a constant, trend or lagged difference is significant. When these extra regressors are in the regression the lag of the variance loses its significance. We see the results are very dependent on the exact time period studied. For the absolute variance we see non of the datasets reject the null, and thus are deemed non-stationary. However when considering the percentage variance, we reject the null for 1970-2001 or 1970-2003, but not for 1972-2001 or 1972-2003. We also do not reject the null for the full dataset even for 1970-2001 or 1970-2003. We again see that the results are different because of the unbalanced shape of the data as well as the difference of studied periods. Because of this we declare the results for this test inconclusive for the GGDP.

	GDP_{Full}	GDP_{Unb}	GGDP
1970-2003	0.9987	1.0000	0.3034
1972-2003	0.9984	1.0000	0.8338
1970-2001	1.000	1.0000	0.2627
1972-2001	1.000	1.0000	0.7959

(a) Absolute variance.

	GDP_{Full}	GDP_{Unb}	GGDP
1970-2003	0.6474	0.0000	0.0000
1972-2003	0.6545	0.1249	0.3829
1970-2001	0.6693	0.0000	0.0000
1972-2001	0.6762	0.1456	0.0378

(b) Percentage variance.

Table 14: Different p-values for the ADF-test, for the variance of the level of the variable.

	GDP_{Full}	GDP_{Unb}	GGDP
1970-2003	0.1158	0.0000	0.0000
1972-2003	0.0000	1.0000	0.3276
1970-2001	0.1636	0.6899	0.0000
1972-2001	0.2006	0.6512	0.2389

(a) Absolute variance.

	GDP_{Full}	GDP_{Unb}	GGDP
1970-2003	0.0323	0.0000	0.0000
1972-2003	0.5195	0.1857	0.2883
1970-2001	0.0479	0.0000	0.0000
1972-2001	0.0752	0.2164	0.1984

(b) Percentage variance.

Table 15: Different p-values for the ADF-test, for the variance of the log of the variable.

Summary σ-convergence We showed the sample variances of the variables and of the transformed variables. We see that the sample variance is very sensitive to the unbalanced shape of the data. We did three analyses, namely an F-test of equality of variance, a regression of the sample variance on a constant and trend and an Adjusted Dickey-Fuller-test to test for stationary data. The F-statistic is robust against the unbalanced data structure, the transformations and considering different periods. With the F-test we conclude the GDP σ-diverges. This greater variance is coming with a greater mean. When considering percentage variance, or the variance of the log of the GDP there is neither convergence or divergence. For the GGDP as well there is neither convergence or divergence. The other two analyses had changing results for the unbalanced shape of the data, the used transformations and the different periods studied. Because of that we deem the results unreliable. That is why we only use the results of the F-test in our final conclusion about σ-convergence.

5 Conclusion

In this thesis we examine the convergence of both the GDP and the GGDP. Because the shape of the data for the GGDP is unbalanced, to compare we perform the analyses on the GDP with the same unbalanced dataset and with a balanced one as well, to show the effect of the shape of the data.

For the unconditional β-convergence the results lead to the conclusion that the GDP does β-converge, while the GGDP does not. For conditional β-convergence the same results were there. The added variables are not fully available so we used values that compromises the exogeneity of those variables. Also for the conditional β-convergence there is no significant convergence for the GGDP.

Where we find unconditional and conditional convergence for the GDP, Barro only finds conditional convergence. This may be explained by that his data also contains undeveloped countries. In our data only countries with a GGDP variable are present, which is a selection of countries that are either developing countries or developed countries, i.e. low level high growth and high level low growth countries. This allows us to estimate a significant negative parameter.

In our study of the σ-convergence it is clear that the unbalanced shape of the data has a large effect on the sample variance and thus on the results. We do an F-test of equality of variance, a regression of the variance on a trend, and an ADF-test. The latter two are inconclusive because the unbalanced data structure and considering slightly different periods leads to different results. To improve these analyses there has to be more and balanced data for the GGDP. The F-statistic is robust against studying different periods and the unbalanced data structure. It concludes there is significant σ-divergence for the GDP, however when we correct for the increase of the mean this disappears. For the GGDP there is no significant σ-convergence or divergence. We do not reject the null hypothesis of the F-test that the variance at the start and end of the data, 1970 and 2003, is different from each other for either GDP or GGDP.

Our research questions in this thesis are whether the GGDP converges and if the GDP and GGDP differ in their convergence. We conclude first of all that the GGDP does neither β-converge nor σ-converge and secondly that, as the GDP does β-converge, the two variables differ in their convergence. We conclude that convergence of economic production does not necessarily mean convergence of welfare. Therefore we conclude that the GDP can not be used as a proxy for welfare, when discussing convergence. This also indicates that the GDP should not be used as a proxy for welfare in other areas. We leave comparisons between the GDP and GGDP in those other areas, and research of the GGDP in general to other studies. We recommend more standardised measurement of the GGDP by statistics bureaus and more use of the GGDP by policy makers.

"This planet has - or rather had - a problem, which was this: most of the people living on it were unhappy for pretty much all of the time. Many solutions were suggested for this problem, but most of these were largely concerned with the movement of small green pieces of paper, which was odd because on the whole it wasn't the small green pieces of paper that were unhappy."

- Douglas Adams, *The Hitchhiker's Guide to the Galaxy*

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6 Appendix

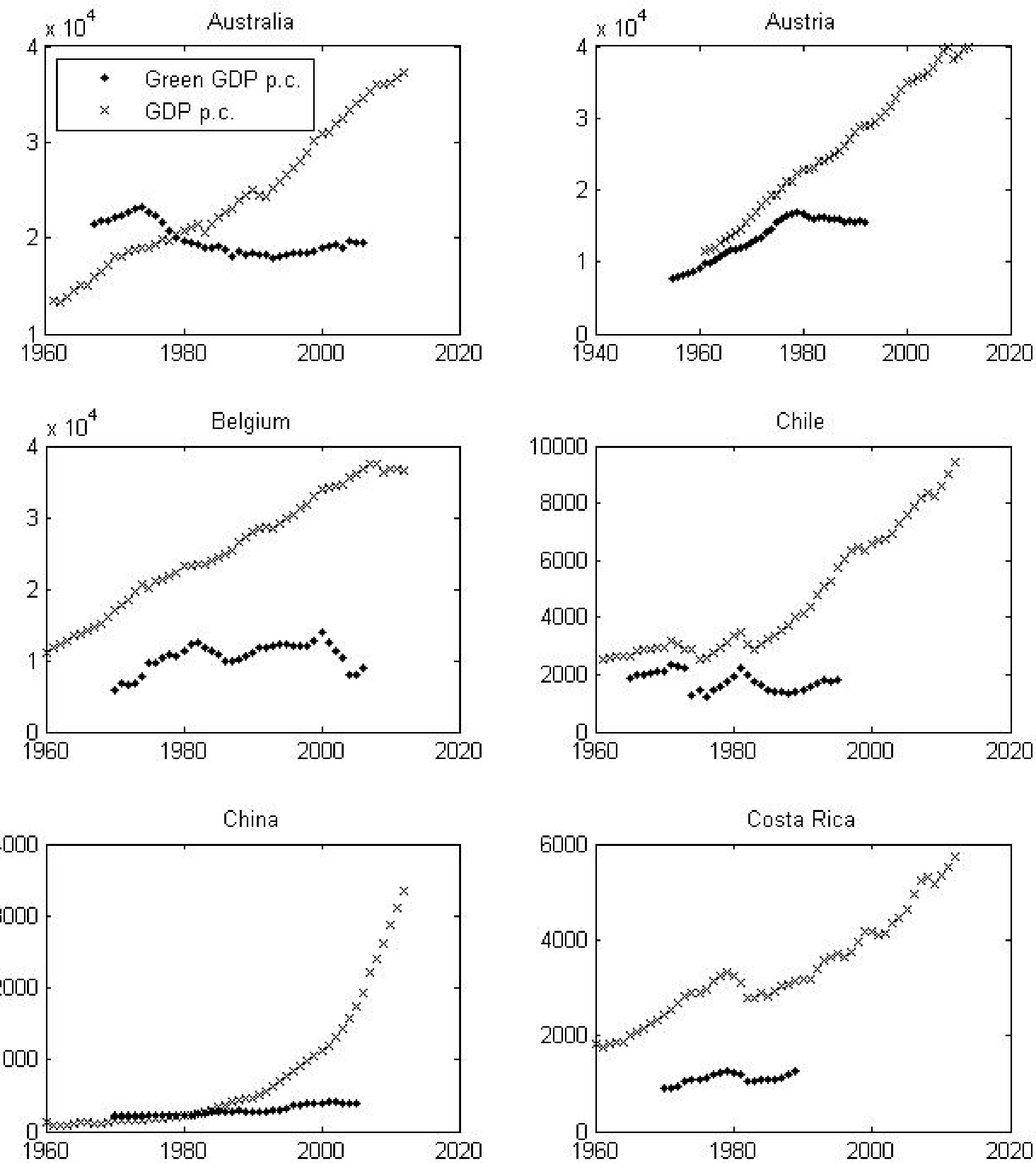
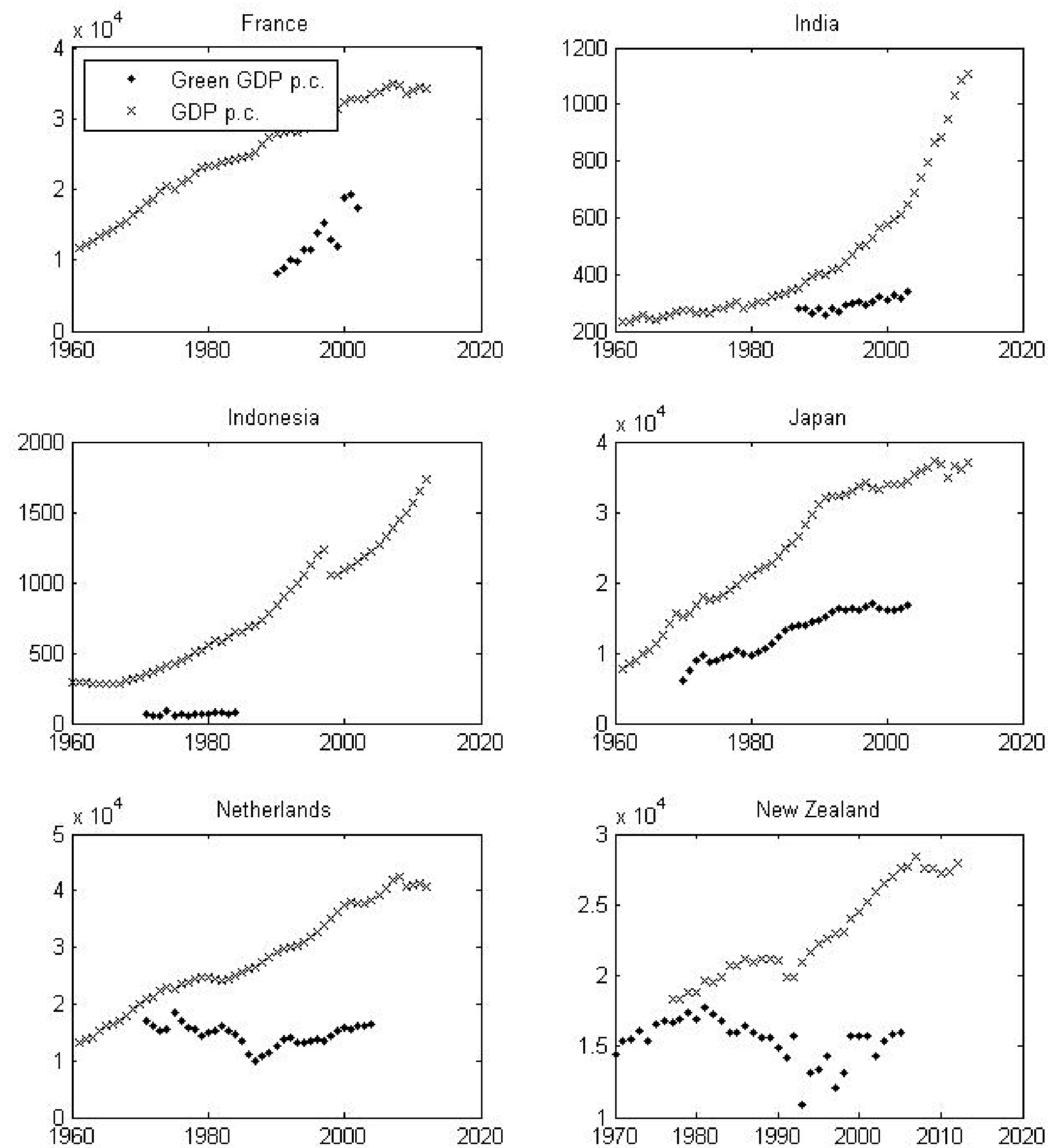


Figure 10: Graphs of GDP and Green GDP time series for different countries



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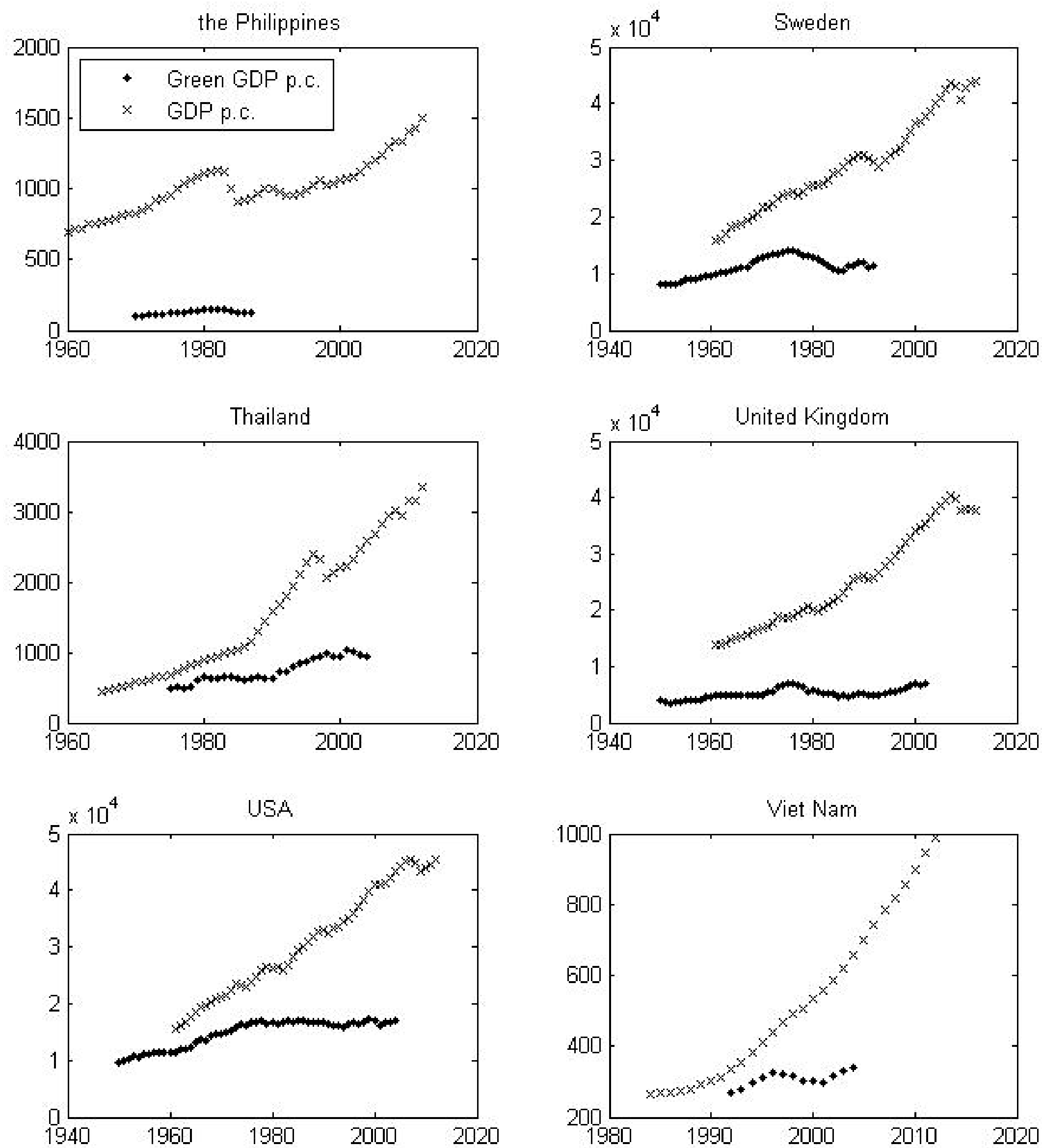


Figure 12: Graphs of GDP and Green GDP time series for different countries

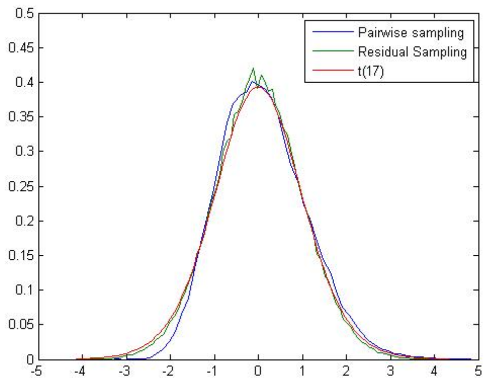


Figure 13: The bootstrapped and asymptotic t(17) t-distributions for the GDP(Full) dataset.

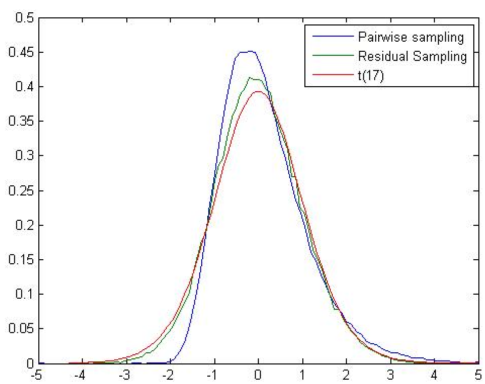
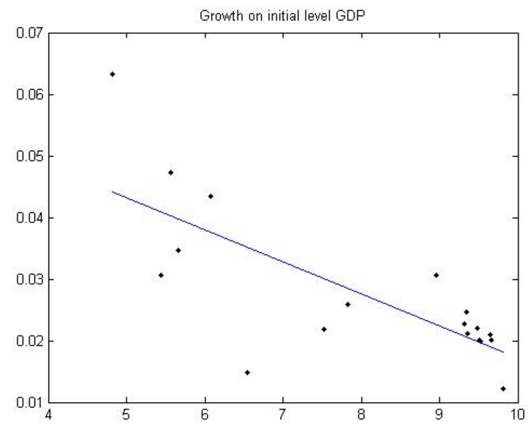
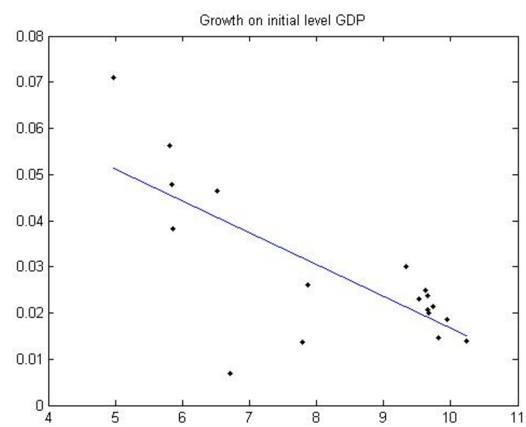


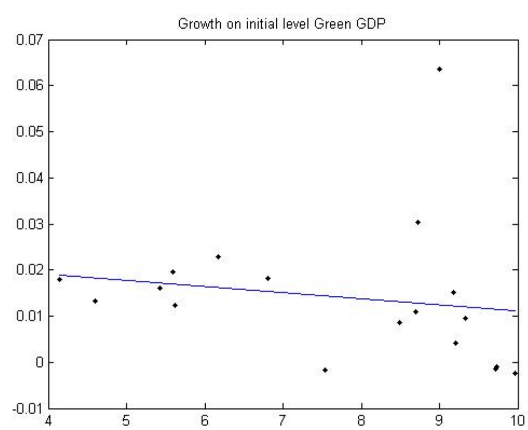
Figure 14: The bootstrapped and asymptotic t(17) t-distributions for the GDP(Unbalanced) dataset.



(a) GDP_{Full}

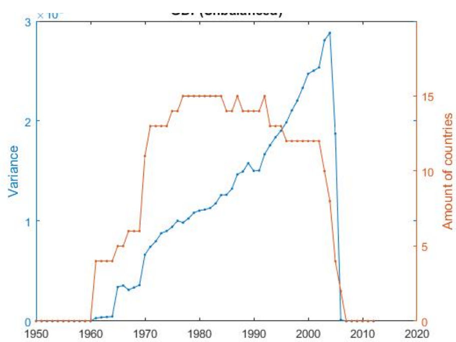


(b) $\text{GDP}_{Unbalanced}$

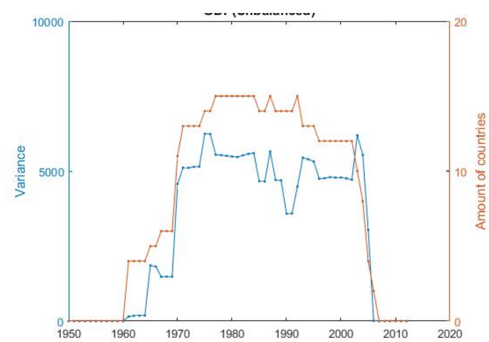


(c) GGDP

Figure 15: The data points and estimated slopes for the Barro regressions with different datasets.

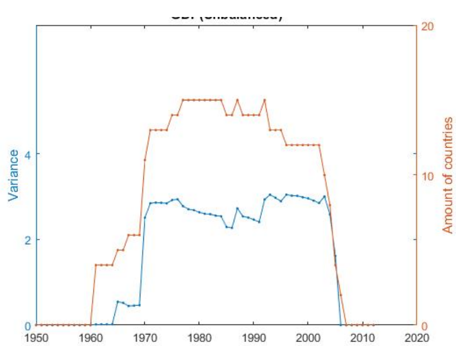


(a) Absolute Variance GDP$_{Unbalanced}$

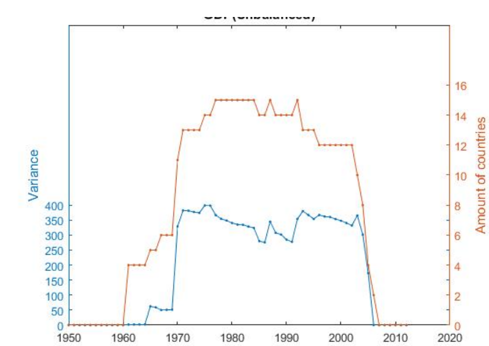


(b) Percentage Variance GDP$_{Unbalanced}$

Figure 16: Sample variance of the level of the GDP$_{Unbalanced}$ over time.

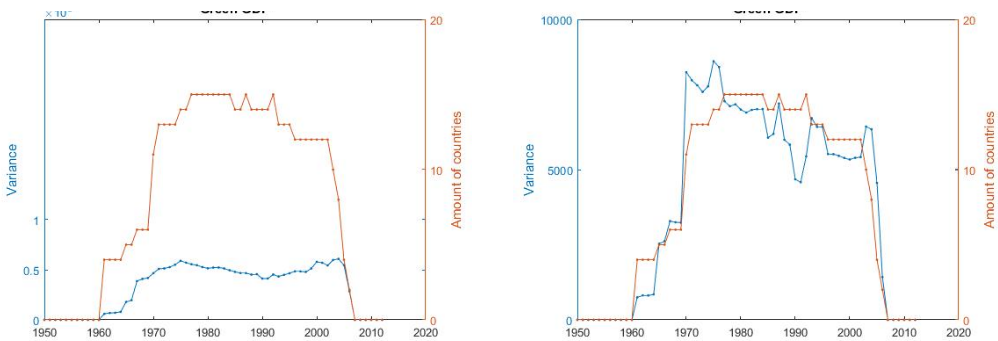


(a) Absolute Variance GDP$_{Unbalanced}$



(b) Percentage Variance GDP$_{Unbalanced}$

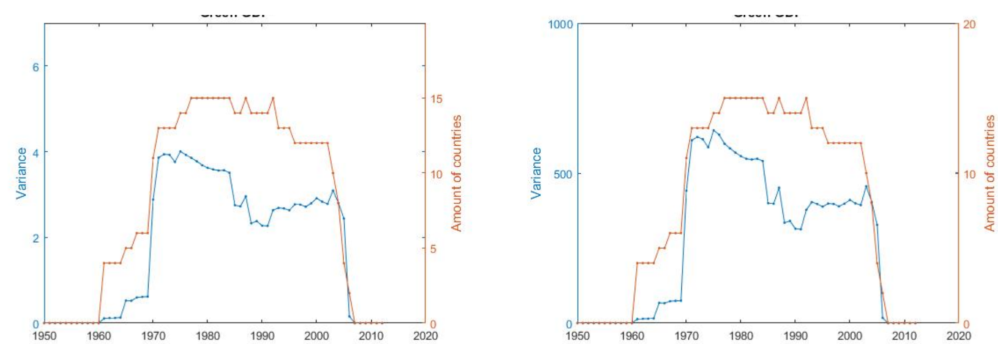
Figure 17: Sample variance of the log of the GDP$_{Unbalanced}$ over time.



(a) Absolute Variance GGDP

(b) Percentage Variance GGDP

Figure 18: Sample variance of the level of the GGDP over time.



(a) Absolute Variance GGDP

(b) Percentage Variance GGDP

Figure 19: Sample variance of the log of the GGDP over time.