Sequential Bargaining With Incomplete Information and Endogenous Reference Points

Anina Thiel

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Abstract

We study the role of reference-dependent preferences on the outcome of a negotiation between an uninformed seller and reference-dependent consumers. The results of our model show that loss aversion combined with an endogenous reference point causes more consumers to purchase in the first stage of the negotiation. The surplus for low-valuation consumers increases. As more consumers buy at higher prices, the profit of the seller increases. These results are driven by the ability of the seller to manipulate the consumer's reference point.

Declaration of Authorship

I, ANINA KATHARINA THIEL, declare that this thesis, titled *Sequential Bargaining With Incomplete Information and Endogenous Reference Points* and the work presented in it, is my own. I confirm that:

• This work was done wholly while in candidature for a Master's degree at the Erasmus University Rotterdam.

• Where I have consulted the published work of others, this is always clearly attributed.

• Where I have quoted from the work of others, the source is always given.

• With the exception of such quotations, this thesis is entirely my own work. I have acknowledged all main sources of help.

• Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

Date: December 16, 2015

Signed:

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1 Introduction

Consider a car seller who faces consumers with reference dependent preferences while not knowing the actual type of the consumers. Before making their purchasing decision, consumers are likely to have formed certain expectations about the price the seller will ask. Buyers might initially over- or underestimate the costs. They might be misinformed about the good or might not have gathered enough information. What happens if the price the seller asks is higher than what the consumer expected? Either, the consumer decides to abstain from buying a car altogether, or he will return at a later point in time to buy the car at the given asking price. The purpose is to model this intuitive reference-dependent behavior. In other words, we allow for consumer's expectations to change. The price consumers will face, provides additional information that the consumer will use to update his reference point. The seller is then able to use the price as an instrument to change the consumer's expectations. The question we are addressing is who will benefit in such a situation. Furthermore, we want to learn how the additional effect of having an endogenous reference point influences the equilibrium outcome compared to a situation in which reference points are static. Our model adds to literature by incorporating a rational and intuitive preference structure that has not yet been investigated. Our model provides insights to real-life situations where sellers understand the impact of consumer's preferences on the decision making process. The purpose of this paper is to analyze to what extent reference dependent preferences influence the efficiency of negotiations. Specifically, we will be looking at a two-period one-sided offer bargaining game with incomplete information on the seller's part. Consumers are assumed to have reference dependent preferences, whereby the reference point is endogenously determined in the negotiation process.

We expect the endogenous nature of the reference point to magnify the effects of loss aversion, previously analyzed in the literature. We find that sellers do indeed benefit from being able to influence the consumer's reference point and realize higher profits. In addition, low-valuation consumers are benefiting from a lower price as compared to the case without endogenous reference points. Our findings support those of Rosato (2013) with respect the the influence of consumer's loss aversion but shows that the effects are magnified under the given setup. Loss aversion and the updating process on the reference point are found to be the main drivers of our results.

The paper proceeds as follows: Section 2 provides a brief overview of related literature and the extensions in this paper. Section 3 presents the theoretical model and methodology. The results are presented and compared to the findings of predominant models in literature in Section 4. Section 5 discusses the limitations of our model and possible extensions. Section 6 provides a brief summary and conclusion of the paper.

2 Related Literature

The aim of this paper is to combine a sequential bargaining game with one-sided incomplete information with a model of reference- dependent preferences. To give the reader an idea of how these two topics have developed in literature over time and how they are linked, I briefly discuss the related literature in this section. The first paragraph discusses the main papers contributing to models of bargaining with one-sided incomplete information. The second paragraph elaborates on loss aversion and reference dependent preferences. The last paragraph covers endogenous reference- dependent preferences and models that are most similar to our research.

Bargaining with one-sided incomplete information is the main underlying framework of our research. Fudenberg and Tirole (1983) were among the first to adapt a simple sequential two-period, two-person bargaining game with incomplete information. In their model the seller makes all the offers and the buyer can either accept or reject them. Since bargaining is costly, both parties discount future payoffs with a discount factor that differs among them. Fudenberg and Tirole (1983) find surprising results regarding the efficiency of the equilibrium. Comparative statics show that an increase in the buyers discount factor (which differs from the sellers discount factor) might make him better off. Fudenberg and Tirole (1983) explain this result with the notion that the buyer cannot commit to an action that would compromise his strategy. Sobel and Takahashi (1983) extend the bargaining literature by analyzing a simple, multistage bargaining game with incomplete information. More specifically, they are looking into the seller-offer game in which the uninformed seller makes a take-it-or-leave-it offer to the buyer. If the buyer rejects, the game moves on to the next stage. Both Fudenberg and Tirole (1983), Sobel and Takahashi (1983) find that the single-price result only holds when the discount factor of the buyer and the seller are equal. Increasing uncertainty on the sellers part with respect to the buyers valuation for the good was found to hurt the seller. Increasing the length of the bargaining horizon is not beneficial to either one of the players. The results by Fudenberg and Tirole (1983) and Sobel and Takahashi (1983) are intuitive and simple. Our model extends this string of literature among many varying dimensions. Because of this, these papers provide us with the initial fundamental framework for bargaining models under one-sided incomplete information.

Building on these two papers, there are various papers and extensions that need to be discussed. For example, Kahneman and Tversky (1979) offer an alternative approach to expected utility theory. Most importantly, they include risk aversion into the decision making progress. They develop a decision model under risk, attaching more weight to losses than to gains. Peoples' expected utility is calculated as the product of a probability weighting function (decision weight) and the value function. The value function attaches a value to a specific outcome in which outcomes higher than the reference point are considered gains and outcomes lower than the reference point as losses. Important to note is that people are loss averse, so losses hurt more than gains bring pleasure. The probability weighting function captures that people tend to overestimate small probabilities and underestimate large ones. As a continuation of Kahneman and Tversky's prospect theory, Kőszegi and Rabin (2006) jointly model choice as well as the determination of the reference point. The basic idea is that a person's utility depends on a consumption bundle as well as a reference bundle. Overall utility is separable into what Kőszegi and Rabin (2006) call *intrinsic consumption utility*and *gain-loss utility*. *Intrinsic consumption utility* refers to the utility obtained from consumption whereas *gain-loss utility* compares the consumer's expectations with the actual realization. Hereby, the reference point is determined by a person's past beliefs about outcomes. The consumer assesses a price he would have to pay for a good as a combination of loss (spending money or not getting the good) and expected gain (getting the good or not spending money). The authors show that as the expected probability of purchase increases, the consumer's willingness to pay increases. Assuming that the consumer expects to buy the good, an increase in the expected price increases the consumers willingness to pay. This is due to the fact that high purchase expectations lead to an attachment effect which makes it more likely for the consumer to buy even at higher prices. A decrease in the price leads to a decrease in the consumer's willingness to pay because the consumer was initially willing to pay more for the good than the actual price demanded. Comparing that lower asking price to his initial willingness to pay leads to a comparison effect which decreases his willingness to pay. Working from a different perspective, Shalev (2002) presents a model in which players have reference- dependent preferences and are loss averse. Their reference point is exogenously given reflecting the players' expectations. Analyzing an extended bargaining model, Shalev (2002) finds that increasing loss aversion leads to worse outcomes for that player. By making reference points endogenous in a Rubinstein (1982) setting with complete information, he shows that this solution prevails. More recently, Compte and Jehiel (2003) adapt Rubinstein (1982) bargaining game under complete information and include reference point dependent preferences. They include an exogenously given updating process on the reference point, allowing it to shift from one period to the other. The reference point increases as a function of the best offer made until that bargaining stage. Due to this updating process, it was found that incentives for high offers were very low because it would lead to rejection in order to increase the reference point. Furthermore, they find that due to the updating process of the reference point, solutions were reached gradually rather than immediately. On the other hand, Li (2007) presents a model similar to that of Compte and Jehiel (2003), the main difference being that the update of the reference point occurs after every offer and not after breakdowns. In his model, a player always strictly prefers rejection than a discounted utility lower than the discounted utility of the most generous offer received up until that state. Therefore, in his model, offers start small and gradually increase. Following Compte and Jehiel (2003) and Li (2007), Hyndman (2010) extends the idea to repeated bargaining situations with one or both players having reference dependent preferences. In contrast to the other studies, Hyndman (2010) allows for an adjustment of the reference points only after actual consumption. If an offer is rejected both parties consume zero which leads to a decrease in the reference point in the second stage. This process is exogenously given. On the other hand, if both parties reached a conclusion and consumption c occurs at point in time t, the reference point for the next bargaining stage increases from previously r_t to $r_{t+1}=c_t$. He finds that in a model where both players have reference dependent preferences, and where after rejection the same reference point prevails ($r_t=r_{t+1}$), agreements are reached immediately rather than gradually. Furthermore, in repeated bargaining situations when a player with reference dependent preferences is matched with a rational player, the rational player will gradually give his share of the pie to the behavioral player. In the extreme case when reference points do not adjust, the rational player will cede the whole pie to the behavioral player. As the reader might have noticed, reference dependent models are numerous and various papers have tried to tackle and extend these findings. We add to the vast discussion on reference dependent preferences and we elaborate on the specific contributions in the next paragraph.

The model in this paper is closest to a recent finding by Rosato (2013). He analyzes a sequential bargaining model with reference dependent preferences. Based on the basic seller-offer game with one-sided incomplete information presented by Fudenberg and Tirole (1983), Rosato (2013) includes reference dependent preferences à la Kőszegi and Rabin (2006) in a simple two-period framework. In his model he differentiates between the two-type case in which a buyer's valuation can either be high or low and a continuous case in which the valuation is uniform on [0,1]. He assumes reference points to be fixed for every bargaining stage which is the main aspect in which our paper differs from his. He shows that the seller's payoff in equilibrium increases as the buyers loss aversion increases. Furthermore, he finds an increase in the overall trade efficiency since loss averse buyers are keen on buying in the first stage which is why the seller can serve a larger group of buyers at a higher initial price. In equilibrium, low-type buyers benefit since the second stage price is found to be lower than in the classical model. Our model extends that of Rosato (2013) by including an updating process on the reference point. We assume that the reference point shifts from one stage to another since the buyer might not hold realistic beliefs about the surplus he would receive from purchase. The reference point is to be understood as an expected surplus of the buyer. If the price he expected is lower than the one the seller is offering, it is logical to assume that the buyer updates his reference point by decreasing his expected surplus from one period to the other. Even though the seller is the uninformed party in this model, he has a strategic tool to influence the buyers willingness to pay. Overall, endogenous reference points are a relatively recent topic in current literature. Shalev (2000) notes that endogenous reference points seemed more realistic as experiences and anticipations shifted them in real life. In his model, the reference point is determined by the expected utility of the player in equilibrium whereas in Kőszegi and Rabin (2006) the reference point depends on a belief distribution based on past experiences. More recently, endogenous reference points have been applied to auctions (see Lange and Ratan (2010). The most recent of those studies has been presented by Ahmad (2015). He looks into first and second price auctions where the reference point is the expected price of the good for naive bidders or the updated (interim) expected price for sophisticated bidders. Even though an auction setting significantly differs from our bargaining model, it too underlines the importance of endogenous reference points to create more realistic models. With everything mind, our model shares certain commonality with various strands of literature. Our model touches areas, such as bargaining with one-sided incomplete information, loss aversion, reference dependent preferences and most importantly, endogenous reference dependent preferences. We add to the vast field of literature and discussion with respect to these topics.

3 Methodology and the Extended Bargaining Model

This model adopts the basic one-sided offer bargaining model under incomplete information by Fudenberg and Tirole (1983) and includes reference point-dependent preferences as presented by Kőszegi and Rabin (2006). The setup assumes one seller who makes all the offers and a unit mass of reference-dependent buyers. They bargain over an indivisible object. The sellers' production costs are assumed to be equal to zero. Buyers update their reference points based on the price the seller asks in the first stage. The bargaining process lasts for two periods, after which the game ends.

3.1 Methodology

We assume that that the references point in the first stage is equal to zero, i.e. $r_1 = 0$. This is a valid assumption because it implies that consumers do not have any prior beliefs on what price they will face when planning on purchasing the object. Initially, they are willing to purchase the object if the price the seller asks is lower than or equal to their valuation. The utility in the first stage is therefore given by the following:

$$U_1 = \begin{cases} (1-\alpha)(v-p_{1,i}) + \alpha(v-p_{1,i}), & \text{if } v-p_{1,i} \geq 0. \\ (1-\alpha)(v-p_{1,i}) + \alpha\lambda(v-p_{1,i}), & \text{otherwise.} \end{cases} \tag{1}$$

In the second stage, the buyer derives utility from purchasing the object whereby the utility function takes into account the *intrinsic consumption utility* as well as the *gain-loss utility* the buyer encounters from comparing the *intrinsic consumption utility* to his reference point, $r_2 \in [-1, 1]$. The utility function of buyers in $t = 2$ is therefore given by the following:

$$U_2 = \begin{cases} (1-\alpha)(v-p_{2,i}) + \alpha(v-p_{2,i}-r_2), & \text{if } v-p_{2,i} \geq r_2. \\ (1-\alpha)(v-p_{2,i}) + \alpha\lambda(v-p_{2,i}-r_2), & \text{otherwise.} \end{cases} \tag{2}$$

This function states that consumers derive utility not only from owning the object but also from comparing it to their reference point which is either a surplus or loss the consumer expects to make when purchasing the object. If the realized gain from purchasing is lower than the reference point, the consumer will experience a loss. Since we assume consumers to be loss averse, this loss will be felt stronger than a gain of equal amount which is captured by the coefficient of loss aversion, λ [1]. In this setup $\alpha \in [0, 1]$ measures the relative importance of the *gain-loss utility* compared to *intrinsic consumption utility*, $\lambda \in [1, 5]$ is the coefficient of loss aversion, $v \in [0, 1]$ gives the initial valuation of the good for the buyer (determines of which type the buyer is), p_t stands for the price the seller asks in period t, $r_2 \in [-1, 1]$ is the reference point which can be regarded as the buyers' expected surplus. In this setup, i denotes the cases resulting from different combinations of the utility functions and the outside option which can be either gains or losses. Our model differs from the one presented by Rosato (2013) and Kőszegi and Rabin (2006) in that we exclude the trade off between *intrinsic consumption utility* and *gain loss utility*[2]. Essentially, we are assuming that the *intrinsic consumption utility* does not depend on the preference parameter but is simply determined by the difference in the consumer's valuation and the price. Simply put, in Rosato's model the more buyers care about the reference point, the more they care about owning the good in general. In this model, how much consumers value owning the object is independent of how much they care about making a loss or a

[1] Please note that several empirical studes have analyzed the coefficient of loss aversion and found values ranging from 1.43 (Schmidt and Traub (2002)) to 4.8 (Fishburn and Kochenberger (1979)).

[2] We do this by multiplying the *intrinsic consumption utility* with (1-α) and the *gain- loss utility* with α. Simplifying our utility function, e.g. if it is a gain, then yields U_1 =v-p_t-αr_t, compared to the model of Rosato (2013) , where U_1= $(1 + \eta)$ $(v - p_t)$-η r.

surplus. This allows us to analyze the effect of the updated reference point in combination with the given preferences. The model will look at two periods only, $t = 1, 2$. We assume buyers learn their valuation, v, and therefore their type before the game starts. This valuation remains the same for both stages and will thus not be updated. The seller is the uninformed party and does not know the value buyers attach to the object. He assumes it to be drawn from a uniform distribution on [0,1]. The reference point r_1 in the first stage is exogenously given and initially equal for all types of buyers. This reference point will be updated in the second stage depending on the price asked by the seller in the first stage, given that the game moves on to the second stage. The updating process of the reference point is common knowledge and given by the following linear function:

$$r_2 = (1 - \beta)(v - p_1) + \beta r_1 \tag{3}$$

In this context, β is the updating parameter and determines how much the consumer's reference point is influenced by p_1. The seller maximizes expected profits. We assume that the seller can ask a price twice. Both players are impatient, discounting future payoffs with the same discount factor δ. We include an outside option for the buyer which can be either a gain or a loss depending on whether the buyer expects to make a loss or a surplus. In the first stage, since the consumer does not expect to make a surplus or loss from purchasing the object, the outside option of not buying is simply zero.

$$OO_1 = 0 \tag{4}$$

Since, $r_1 = 0$, the reference point is updated based on the difference between the valuation and the price in the first stage, $r_2 = (1 - \beta)(v - p_1)$. There will be a an outside option different from zero if $r_2 \neq 0$ for all consumers with $v \neq p_1$ and all $\beta \in [0, 1)$. This is stated in the equation (5) below:

$$OO_2 = \begin{cases} -\alpha r_2, & \text{if } r_2 \leq 0. \\ -\alpha\lambda r_2, & \text{otherwise.} \end{cases} \tag{5}$$

If the consumer expects to make a loss, which is stated in the first part of equation (5), not purchasing the object will feel like a gain. In other words, the consumer forgoes a loss which is a gain. Analogous, if the consumer expects to make a surplus, purchasing the object will feel like a loss. In the first stage the seller sets price p_1 and the buyer accepts or rejects that offer. If the offer is accepted, the game ends. If the offer is not accepted, the buyer first updates his reference point before the game moves on to the second stage. The seller suggests a second price which buyers can accept or reject. Irrespective of whether the second stage price is accepted or rejected, the game ends after the second period.

3.2 The Extended Bargaining Model

In the second stage, the seller can ask a price and the buyer decides whether to accept or reject the offer. The buyer will decide to purchase the object in period 2 if the utility he derives from buying is larger than the utility from not buying in period 2, as shown below:

$$U_2 \geq OO_2 \tag{6}$$

Since U_2 as well as OO_2 can be either a gain or a loss, the problem leads to four different sub-cases. The following cases state all possible combinations of these two utility functions. Whenever consumers expect to make a loss, this loss will be weighted by λ.

Case 1: Marginal consumer in $t = 2$ determined by indifference between $U_2(loss)$ and $OO_2(loss)$

$$(1-\alpha)(y_1 - p_{2,1}) + \alpha\lambda(y_1 - p_{2,1} - (1-\beta)(y_1 - p_{1,1})) = -\alpha\lambda(1-\beta)(y_1 - p_{1,1}) \tag{7}$$

The left-hand side of the equation is the utility the buyer derives from purchasing in the second stage. In this case we assume that the reference point is larger than the difference in the consumers's valuation and the second stage price, which is why U_2 is denoted as a loss. Assuming that consumers are loss averse, we attach a weight $\lambda \in [1, 5]$ to the *gain-loss utility* of the utility function. The right-hand side of the equation states the outside option from not purchasing in the second stage. We assume that the valuation of the second stage marginal consumer, y, is larger than the first stage price, p_1, in which case not buying feels like a loss to the consumer since he forgoes an expected gain. Solving this equation yields a first possible indifferent second stage consumer:

$$y_1 = p_{2,1} \tag{8}$$

The valuation of the marginal consumer in the second stage equals the second stage price, a finding that replicates the initial model as presented by Fudenberg and Tirole (1983). However, assuming that OO_2 is a loss, indicates that $y - p_1 > 0$. More generally this means, that this consumer could have realized a gain in the first stage from purchasing the object but expects to make a gain in the second stage if he rejects p_1. In case of rejection, buying in $t = 2$ is only rational if the second stage price is low enough, such that the realized gain in the second stage outweighs the expected gain from the first stage. Only in such a case would the second stage utility be higher than the first stage utility and justify waiting. This seems a strong restriction to this case which will be further analyzed below.

Case 2: Marginal consumer in $t = 2$ determined by indifference between $U_2(loss)$ and $OO_2(gain)$

$$(1-\alpha)(y_2 - p_{2,2}) + \alpha\lambda(y_2 - p_{2,2} - (1-\beta)(y_2 - p_{1,2})) = -\alpha(1-\beta)(y_2 - p_{1,2}) \tag{9}$$

Solving this equation yields a second possible indifferent second stage consumer:

$$y_2 = \frac{p_{2,2}(1 + \alpha(\lambda - 1)) + p_{1,2}\alpha(\beta - 1)(\lambda - 1)}{a + \alpha\beta(\lambda - 1)} \tag{10}$$

The marginal consumer in the second stage increases in p_2 and decreases in p_1. [3] Since we assume U_2 to be a loss, we expect less people to purchase in the second stage the higher p_2. As p_1 increases it becomes less likely for U_2 to be a loss since consumers would expect to make a loss in the second period. It means that their reference point is more likely to be negative, which makes it less likely for the *gain-loss utility* part of the utility function to be a loss. This would lead to an increase in consumers in the second stage. There seems to be a contradiction to our initial assumptions that shall be discussed in the following section.

Case 3: Marginal consumer in $t = 2$ determined by indifference between $U_2(gain)$ and $OO_2(loss)$

$$(1-\alpha)(y_3 - p_{2,3}) + \alpha(y_3 - p_{2,3} - (1-\beta)(y_3 - p_{1,3})) = -\alpha\lambda(1-\beta)(y_3 - p_{1,3}) \tag{11}$$

Solving this equation yields a third possible indifferent second stage consumer:

$$y_3 = \frac{\alpha(\beta - 1)(\lambda - 1)p_{1,3} - p_{2,3}}{\alpha(\beta - 1)(\lambda - 1)} \tag{12}$$

This case predicts that the marginal consumer in the second stage would decrease in both p_1 and p_2. The higher p_1, the more likely it is that the consumer expects to make a loss in the second period which would be beneficial for him. It would lead to more buyers purchasing overall. However, if this phenomenon is driven by the expected loss of consumers, the outside option can never be a loss as this

[3] Since $\lambda \in [1, 5]$, (λ -1) will be positive for all λ. Since $\beta \in [0, 1]$, the term $(1 - \beta) \leq 0$ so that y_2 will decrease in p_1 for all $\beta \in [0, 1)$.

indicates that consumers expect a surplus. There seems to be a contradiction which will be discussed in the following section.

Case 4: Marginal consumer in $t = 2$ determined by indifference between $U_2(gain)$ and $OO_2(gain)$

$$(1-\alpha)(y_4 - p_{2,4}) + \alpha(y_4 - p_{2,4} - (1-\beta)(y_4 - p_{1,4})) = -\alpha(1-\beta)(y_4 - p_{1,4}) \tag{13}$$

Solving this equation yields a fourth possible indifferent second stage consumer:

$$y_4 = p_{2,4} \tag{14}$$

If both U_2 and OO_2 are a gain, we know that $y - p_1 < 0$. Given the first information, we then also know that the utility from purchasing in the first stage would have been negative for the consumer with a valuation of $v = y$. Therefore, the consumer does not have an incentive to purchase in the first stage. Furthermore, since he expects a loss in the second stage, there is an additional benefit from waiting until $t = 2$. It seems logical that as the price in the second stage increases, less people would want to purchase.

The seller simply maximizes his expected profit in the second stage. His belief in the second stage, resulting from a rejection of price p_1 in the first stage, should be that $v \leq x(p_1)$, whereby $x(p_1)$ represents the value of the buyer who is indifferent between buying in the first and in the second stage. Hence, in the second stage, the seller believes that v is uniform on [0,x]:

$$E[\pi_i | rejection\ of\ p_{1,i}] = p_{2,i}\frac{(x_i(p_{1,i}) - y_i)}{x_i(p_{1,i})} \tag{15}$$

Since there are four potential indifferent second stage consumers, there will be four different maximization problems. Based on this information, we can determine the indifferent buyer in the first stage, assuming he is indifferent between buying today and buying tomorrow whereby tomorrow's payoffs are discounted with δ. Please note, for the indifferent consumer in the first stage, the utility from purchasing, U_1, has to be positive. Otherwise, the consumer would never want to purchase in $t = 1$. Therefore, given U_1, we know that we can consider gain models only.[4] This means that $x_i \geq p_{1,i}$ for all four cases i. In the specific case when $r_1 = 0$, U_1 is always a gain if $v \geq p_1$. The consumer will

[4] Gain models are defined as models in which the gain-loss utility part of the utility function is positive. Hence, if $r_1 = 0$, $U_1 = (1-\alpha)(v - p_1) + \alpha(v - p1)$.

decide to purchase if the following condition holds:

$$U_1 \geq OO_1 + \delta U_2 \tag{16}$$

This equation states that the buyer will only accept the offer if his utility from doing so is larger than what he expects to receive in the case of rejection today. Not buying today will yield the utility of the outside option, which is zero for the special case in which $r_1 = 0$, and the discounted next period utility. Comparing the utilities of the buyer in $t = 1$ and $t = 2$, there are four possible outcomes which are analyzed in the following cases [5] :

Case 1: Marginal consumer in $t = 1$ determined by indifference between $U_1(gain)$ and $U_2(loss)$, given $OO_2(loss)$

$$(1-\alpha)(x_1(p_{1,1}) - p_{1,1}) + \alpha(x_1(p_{1,1}) - p_{1,1}) = \delta((1-\alpha)(x_1(p_{1,1}) - p_{2,1}(p_{1,1})) + \alpha\lambda(x_1(p_{1,1}) - p_{2,1}(p_{1,1}) - (1-\beta)(x_1(p_{1,1}) - p_{1,1})) \tag{17}$$

$$x_1(p_{1,1}) = \frac{2p_{1,1}(-1 + \alpha(\beta - 1)\delta\lambda)}{\delta(1 + \alpha(-1 + \beta\lambda - \lambda)) - 2} \tag{18}$$

This case takes the previously calculated second stage prices as given and compares the utility buyers derive from purchasing in the first stage with the utility they derive from purchasing in the second stage, given that the second stage utility will be discounted. We find that the valuation of the first stage marginal consumer is increasing in p_1 which seems plausible given that a higher p_1 renders purchasing in the first period more costly [6]. However, it also causes consumers to expect a loss in the second stage. Expecting a loss means that the reference point, r_2, is negative which makes it more likely for U_2 to be a gain rather than a loss because it increases the chances of the *gain-loss utility* to be positive. Whenever we refer to consumers expecting to make losses, we mean that consumers expect to have a negative reference point in the following bargaining period. Please keep in mind, that a negative reference point leads to an expected gain from purchasing in $t = 2$. Analogous, whenever consumers expect to make a gain, the reference point will be positive which increases the likelihood of U_2 being a loss. The outside option is always a gain, if consumers expect to make losses and a loss if consumers expect to make gains.

[5] Please note, these four cases correspond to the previous cases, i.e. Case 1 always refers to the *(loss,loss)* case etc.

[6] Please note, as x increases, less people purchase in $t = 1$ because x will be closer to the right boundary of the interval [0,1].

Case 2: Marginal consumer in $t = 1$ determined by indifference between $U_1(gain)$ and $U_2(loss)$, given $OO_2(gain)$

$$(1-\alpha)(x_2(p_{1,2})-p_{1,2})+\alpha(x_2(p_{1,2})-p_{1,2})=\delta((1-\alpha)(x_2(p_{1,2})-p_{2,2}(p_{1,2}))+\alpha\lambda(x_2(p_{1,2})-p_{2,2}(p_{1,2})-(1-\beta)(x_2(p_{1,2})-p_{1,2})) \tag{19}$$

$$x_2(p_{1,2}) = \frac{p_{1,2}(-2+\alpha(\beta-1)\delta(1+\lambda))}{-2+\delta(1+\alpha(-2+\beta+\beta\lambda))} \tag{20}$$

It seems as if the valuation of the marginal consumer is increasing in p_1. However, we are facing the same challenge as in *Case 1* when arguing in favor of this outcome, considering the expected loss in the second stage.

Case 3: Marginal consumer in $t = 1$ determined by indifference between $U_1(gain)$ and $U_2(gain)$, given $OO_2(loss)$

$$(1-\alpha)(x_3(p_{1,3})-p_{1,3})+\alpha(x_3(p_{1,3})-p_{1,3})=\delta((1-\alpha)(x_3(p_{1,3})-p_{2,3}(p_{1,3}))+\alpha(x_3(p_{1,3})-p_{2,3}(p_{1,3})-(1-\beta)(x_3(p_{1,3})-p_{1,3})) \tag{21}$$

$$x_3(p_{1,3}) = \frac{p_{1,3}(-2+\alpha(\beta-1)\delta(1+\lambda))}{-2+\delta(1+\alpha(\beta-1)(1+\lambda))} \tag{22}$$

Again, x is increasing in p_1. In terms of U_2 it makes sense to argue that a higher price has two effects in favor of an increase in x. First, as p_1 increases, buying becomes more expensive so less people will buy in the first stage. Secondly, a high p_1 induces an expected loss in the second stage which makes it more likely for U_2 to be a gain. However, if this was the case, then OO_2 could not be a loss but would rather have to be a gain.

Case 4: Marginal consumer in $t = 1$ determined by indifference between $U_1(gain)$ and $U_2(gain)$, given $OO_2(gain)$

$$(1-\alpha)(x_4(p_{1,4})-p_{1,4})+\alpha(x_4(p_{1,4})-p_{1,4})=\delta((1-\alpha)(x_4(p_{1,4})-p_{2,4}(p_{1,4}))+\alpha(x_4(p_{1,4})-p_{2,4}(p_{1,4})-(1-\beta)(x_4(p_{1,4})-p_{1,4})) \tag{23}$$

$$x_4(p_{1,4}) = \frac{2p_{1,4}(-1+\alpha(\beta-1)\delta)}{-2+(1+2\alpha(\beta-1))\delta} \tag{24}$$

In this particular case, all arguments in favor of an increase in the valuation of the marginal consumer due to an increase in p_1 are logical. Having analyzed our four potential marginal first stage buyers, we can now use this information to obtain the four potential second stage prices $p_{2,1}(p_{1,1})$, $p_{2,2}(p_{1,2})$, $p_{2,3}(p_{1,3})$ and $p_{2,4}(p_{1,4})$.

In the first stage, the seller maximizes his first - and second period profits. The profit function looks as follows:

$$E[\pi_i|y_i \leq x_i] = (1 - x_i)p_{1,i} + \delta((x_i - y_i)p_{2,i}) \tag{25}$$

The first term is the seller's profit from selling to consumers with a valuation higher than x_i at price p_1. The second term represents the discounted profit the seller realizes in the second stage by selling to consumers with valuation $y_i \leq v \leq x_i$. Since there are four possible values for x_i, y_i and $p_{2,i}$ each, we have to distinguish between the four cases more carefully. An optimal first stage price is calculated for each of the possible four values of the second stage indifferent buyer. The first model considers $x_1(p_{1,1})$, $p_{2,1}(p_{1,1})$ and y_1, the second model looks at $x_2(p_{1,2})$, $p_{2,2}(p_{1,2})$ and y_2, the third at $x_3(p_{1,3})$, $p_{2,3}(p_{1,3})$ and y_3 and the fourth at $x_4(p_{1,4})$, $p_{2,4}(p_{1,4})$ and y_4. Knowing the respective optimal prices, we can determine the four possible first stage indifferent consumers, the four possible second stage prices and the four possible second stage indifferent consumers[7].

3.2.1 Equilibrium Conditions

For each of the four models, we have to verify that the conditions hold in equilibrium given the strategy profiles of the two players and their corresponding beliefs. We have to check whether the following conditions hold simultaneously:

Condition 1: Assumption about U_1

The model assumes that U_1 is a gain for the indifferent consumer, x_i. Therefore, we have to check for each of our four outcomes whether $x \geq p_1 \forall \alpha \in [0,1], \beta \in [0,1]$ and $\delta \in [0,1]$. This condition is fulfilled for each model.

[7]For the formal proof see Appendix A1

Condition 2: Assumption about U_2

Each case makes a specific assumption about the nature of U_2. For the different cases we already noted the effects of p_1 and p_2 and we have stated why some of the assumptions are most likely not fulfilled in equilibrium. We now have to find out if this outcome is driven by mis-specifying the nature of U_2 or that of OO_2.

U_2 is a loss if:

$$(y_i - p_{2,i} - (1-\beta)(y_i - p_{1,i})) < 0 \tag{26}$$

U_2 is a gain if:

$$(y_i - p_{2,i} - (1-\beta)(y_i - p_{1,i})) \geq 0 \tag{27}$$

The equation states the *gain-loss utility* part of the utility function. If it is lower than zero, the consumer would expect to make a loss [8], if it is larger than zero, U_2 is a gain. For case 1 $(loss, loss)$ and case 2 $(loss, gain)$, U_2 is only a loss for a certain range of δ, α, λ and β. This is not surprising. As previously noted, for *Case 2*, it is very unlikely that U_2 can be a loss, given that OO_2 is a gain and vice versa. For *Case 3* and *Case 4*, the condition holds in general.

Condition 3: Assumption about OO_2

The first two conditions determine a range for which the cases are theoretically possible. Now, we have to check if the assumption we made about the outside option in the second stage holds for each of those cases.

OO_2 is a loss if:

$$y_i - p_{1,i} > 0 \tag{28}$$

OO_2 is a gain if:

$$y_i - p_{1,i} \leq 0 \tag{29}$$

This inequality states that if $r_2 > 0$, OO_2 is indeed a loss, and a gain otherwise.This condition holds for the previously determined range of parameters for *Case 1* and *Case 4* but not for *Case 2* and *Case 3*. This outcome is not surprising since whenever OO_2 is a loss, U_2 decreases in r_2. Analogous, if OO_2 is a gain, U_2 increases in r_2[9]. At this point, we already know that the equilibrium can only be determined by cases in which U_2 and OO_2 have the same property, i.e. both have to be either gains or losses.

[8]Please note, that the term *loss* does not refer to the reference point in this case but to the comparison between the reference point and the *intrinsic consumption utility* in the second stage, given by $y_i - p_{2,i}$. In fact, if the consumer expects a surplus (positive reference point, i.e. $y_i - p_{1,i} > 0$), he is more likely to feel a *loss* from purchasing because $(y_i - p_{2,i} - (1-\beta)(y_i - p_{1,i}))$ is more likely to be negative.

[9]Remember that OO_2 is a gain if $(v < p_1)$. This means that the reference point is negative and therefore U_2 increases in r_2.

Condition 4: $U_2 \geq U_1$

For the second stage indifferent consumer it has to hold that the utility he obtains from buying in the second stage is larger than from purchasing in the first stage. This condition is a rationality requirement rather than a technical requirement. If it is optimal for consumers with a valuation lower than x_i to postpone their purchasing decision to the second stage, then this behavior has to be optimal given the pricing strategy of the seller. The last consumer this condition has to hold for is the marginal consumer in the second stage. Since he is defined as being indifferent between purchasing in the second stage and never purchasing, based on the fact that it was optimal to reject the first stage price, the marginal consumer has to derive a higher utility from purchasing in the second stage as compared to the first stage. Since *Cases 2 and 3* have already been rejected, we have to test this condition for *Cases 1 and 4* only. It was found that this requirement is only fulfilled by *Case 4* and not *Case 1.* Let us illustrate this result in a case, where $\alpha = 0.5$, $\beta = 0.5$, $\lambda = 2$ [10] and $\delta = 1$. In *Case 1* and for the given parameters, the optimal values were found to be $p_{1,1} = 0.375$, $x_1 = 0.75$, $p_{2,1} = 0.375$, $y_1 = 0.375$. Since $v = y_1$ for the indifferent consumer in the second stage, the utility $U_{2,1} = 0$. The model predicts that only people with a very high valuation would decide to buy in the first stage since $x_1 = 0.75$. Furthermore, everybody with a valuation higher than 0.375 is predicted to buy in the second stage and consumers with a valuation lower than 0.375 never buy. But looking at a consumer with a valuation of e.g. $v = 0.6$, we find that the first stage utility $U_{1,1} = (0.6 - 0.375) = 0.225$, whereas the second stage utility is $U_{2,1} = 0.5(0.6 - 0.375) + 0.5(0.6 - 0.375 - 0.5(0.6 - 0.375)) = 0.17$. The second stage utility is clearly lower than the first stage utility which is why for a person with a valuation of 0.6, postponing the purchasing decision to the next period can never be a best response to the pricing strategy of the seller. The same holds true for anyone with a valuation higher than 0.375. This behavior is not sequentially rational, hence this model cannot predict the equilibrium outcome. Since only *Case 4* fulfills the technical conditions as well as the requirements of sequential rationality, we know that the seller will choose the prices predicted by this case. It was found that in the first stage only consumers with a valuation of $v \geq 0.71$ will purchase the object at a price of $p_1 = 0.425$. In the second stage, everybody with a valuation of $v \geq 0.354$ will buy at a price $p_2 = 0.354$. Given those prices, the seller is expected to realize a profit of $\pi = 0.3$. To show that this is sequentially rational and a best response to the pricing strategy of the seller, let us again consider a consumer with a valuation of $v = 0.6$. The consumer's utility in the first stage would be $U_{1,4} = (0.6 - 0.425) = 0.175$, compared

[10]We chose λ as the rounded weighted average of the empirical coefficients of loss aversion with respect to monetary losses. A value of 2 seems reasonable, given that most studies find values between 1.8 (e.g. Pennings and Smidts (2003)) and 2.25 (e.g.Kahneman and Tversky (1992))

to the utility in the second stage $U_{2,4} = 0.5(0.6 - 0.354) + 0.5(0.6 - 0.354 - 0.5(0.6 - 0.425)) = 0.202$. This is higher than the utility the consumer could obtain from purchasing in the first stage. More generally, we can show this by solving the following equation:

$$(v - 0.425) < 0.5(v - 0.354) + 0.5(v - 0.354 - 0.5(v - 0.425)) \tag{30}$$

This equation simply states that the utility from purchasing in the first stage at a price $p_{1,4} = 0.425$ has to be lower than the utility from purchasing in the second stage at a price $p_{2,4} = 0.354$.

$$v < 0.71 \tag{31}$$

Solving the above stated problem, we find that it holds for any valuation $v < 0.71$. Please note, 0.71 was exactly the value of the first stage indifferent buyer.

Summarizing this section leads to the following result:

Proposition 1: *In the two-period one-sided incomplete information model with consumers' types continuously distributed on [0,1] and where the seller makes all the offers there exists a unique Sequential PBE in which:*

(i) *buyers with* $v \in [x(p_1*), 1]$ *purchase in the first period at*

$$p_1^* = \frac{(2 - (1 + 2\alpha(-1 + \beta))\delta)^2}{2(1 - \alpha(1 - \beta)\delta)(4 + (3 + \alpha(-1 + \beta)(-4 + \delta))\delta)} \tag{32}$$

(ii) *consumer with* $v \in [p_2*, x(p_1*)]$ *buy in the second stage at*

$$p_2^* = \frac{2 - (1 + 2\alpha(1 - \beta))\delta}{2(-4 + (3 - \alpha(-1 + \beta)(-4 + \delta))\delta)} \tag{33}$$

(iii) *and consumer with* $v \in [0, p_2*)$ *never buy.*

Proof. For the formal proof, the reader is referred to Appendix $(A1)$.

■

4 Results

Our model yields the basic results found by Fudenberg and Tirole (1983) in the case when $\alpha = 0$ or when $\beta = 1$:

$$p_1^*|_{\alpha=0} = \frac{(2-\delta)^2}{8-6\delta} \tag{34}$$

$$p_2^*|_{\alpha=0} = \frac{\delta-2}{6\delta-8} \tag{35}$$

$$x^*|_{\alpha=0} = \frac{\delta-2}{3\delta-4} \tag{36}$$

$$y^*|_{\alpha=0} = \frac{\delta-2}{6\delta-8} \tag{37}$$

Comparing this base case price to the price when we include reference-dependent preferences, we find that for $\alpha \geq 0$ and $\beta \leq 1$, $p_1^* \geq p_1^*|_{\alpha=0}$ and $p_2^* \leq p_2^*|_{\alpha=0}$. Since the equilibrium is determined by the gain model, all of our optimal prices as well as the values for the marginal consumers are independent of λ. This is one of the main differences compared to the paper of Rosato (2013). The following figure plots the marginal consumers and relative prices of our model and the base model by Fudenberg and Tirole (1983), denoted by FT. For representation purposes, we chose to plot the optimal prices and valuations of the marginal consumers for $\alpha = 1$, $\beta = 0$ and $\delta = 0.5$. For the chosen values of α and β the differences between our model and the base model are highest.[11].

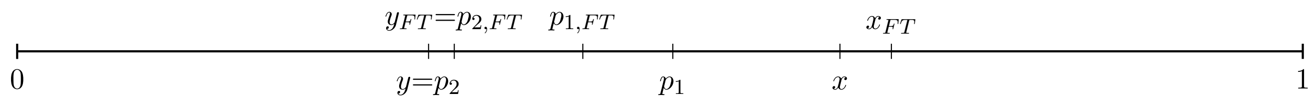

Figure 1: Location of Prices and Marginal Consumers

In our model, and as depicted in Figure 1, more consumers buy at a higher price p_1 in the first stage. In the second stage, our model predicts a lower price, p_2. More people will purchase at the given second stage price since $y < y_{FT}$. The following section discusses the influence of α, β and δ on the optimal prices and valuations of the marginal consumers.

4.1 First stage price p_1

The model of Fudenberg and Tirole (1983) predicts that for small values of δ, p_1 is relatively high but decreases as δ increases. Low values of δ indicate that the future is heavily discounted. Since con-

[11]This is due to the fact that we are representing the *full update* case. Here only the *gain-loss utility* is of importance and the reference point is fully determined by the difference in valuation and price in the first stage.

sumers and sellers discount future payoffs with the same discount factor, a low δ indicates that both players do not place much value on the future. In this case, p_1 is relatively high because both players are interested in reaching a conclusion in the first period. Consumers will thus accept higher prices as compared to a situation in which δ is higher; an effect that is magnified since consumers are loss averse. Therefore, the seller is able to set a higher price. As δ increases, there is an incentive for sellers to set p_1 lower. This is due to the fact that for buyers, waiting becomes relatively more attractive, partly because consumers anticipate that p_2 will be lower than p_1. Since more people are willing to buy in the second stage, the seller has an incentive to set p_1 lower to attract some buyers to purchase in the first stage. In addition, the seller also has an advantage of a higher discount factor since it makes selling in the second stage relatively more attractive as compared to a situation in which δ is very low. However, as δ increases further and reaches values close to 0.6, p_1 increases again. Now, it is beneficial for the seller to sell to high-value consumers first and sell to low-value consumers at a lower price in the second stage again. Our model supports these findings and replicates them for the case in which $\alpha = 0$ or $\beta = 1$. Therefore, plotting p_1 against δ leads to a U-shaped curve which can be seen in Figure 2 below.

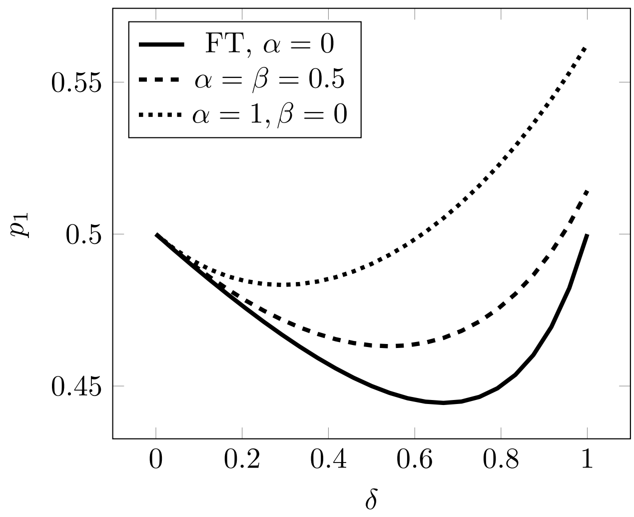

Figure 2: Plot of p_1 for different values of α and β

Given β, an increase in α partly offsets the effects of δ, as previously described. Since α only influences U_2, the higher α, the more important *gain-loss utility* becomes which puts a stronger focus on the reference point. As α increases, p_1 increases relatively more for given values of δ compared to the base model by Fudenberg and Tirole (1983). Consumers with $v \leq p_1$ expect to make a loss in the second stage which is advantageous since it will encourage them to purchase the object in the second stage [12]. In addition to the effect of loss aversion, this effect reinforces the higher price set by the seller. The p_1 found in our model is consistently higher for all values of δ, given that $\alpha \neq 0$ and $\beta \neq 1$.

[12] Please, remember that an expected loss leads to a gain in terms of U_2.

The price difference in the price found by Fudenberg and Tirole (1983) and the price our model predicts, is highest when $\beta = 0$ for any given δ and α. This case shall be referred to as *full update* since r_2 will be fully determined by the difference in the consumer's valuation and the market price in the first stage. Since β only influences r_2 and thereby U_2, the effect of β partly offsets the effect of δ. As β increases, the effect on the price is weakened since r_2 goes to 0 as β converges to 1. In this case, the effect of δ dominates the effect of β. These effects of α and β on the optimal price can be seen in Figure 1 where the solid line represents the base case[13], the dashed line shows the case in which $\alpha = \beta = 0.5$ and the dotted line which refers to the *full update* where $\alpha = 1$ and $\beta = 0$.

4.2 Second stage price p_2

In the base model by Fudenberg and Tirole (1983), p_2 increases exponentially in δ. In the extreme case when $\delta = 1$, $p_1 = p_2$. For all values of $\delta < 1$, we find that $p_1 > p_2$. For low values of δ, when the second period is relatively less important compared to the first period, p_2 is relatively low. The reason is that waiting has to be beneficial and since postponing the decision is relatively unattractive for the consumer, buyers can only be encouraged to wait and purchase in the second stage when the price is sufficiently low. In this case, some consumers would not decide to buy in the first stage due to a high price and would abstain from buying if p_2 was too high. Setting a lower price is advantageous for the seller since he will at least make positive profits in the second stage from the low-valuation consumers.

For high levels of δ, p_2 decreases in α. Since buyers faced a high first stage price for this range of δ and α, they expected to make a loss in the second period. Now, there are two effects that determine p_2. One would think that since consumers expect to make a loss in the second stage, they would accept a higher p_2 as compared to the base model. However, we observe that p_2 is actually lower. This is partly due to the fact that some consumers with a valuation $v \in (p_1, x)$ did not buy in the first stage but expect to make a surplus in the second stage. The second stage price thus has to accommodate not only consumers who expected a loss in the second stage but also consumers who expected to make a surplus. A relatively lower price ensures that these consumers buy in the second stage. Furthermore, a lower p_2 causes more people to buy in the second stage which increases the seller's profit. Moreover, if buyers expect to make a loss (if $v < p_1$), the outside option, OO_2 is a gain. In order to attract these consumers to buy, the price will have to be low enough so that the utility consumers get from purchasing outweighs the utility from the outside option. Combined, these two effects dominate the effect of setting a higher p_2. This relationship can be seen in Figure 3. For all values of $\delta \leq 0.3$ prices

[13] This is the same as $\alpha = 0$ or $\beta = 1$ in our model

of our model and the base model coincide.

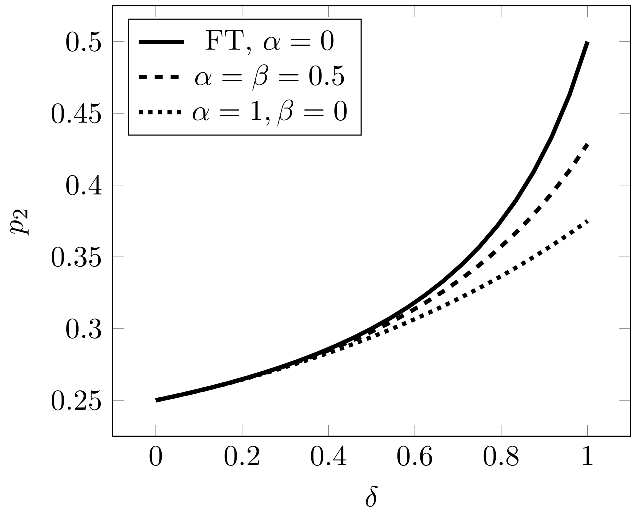

Figure 3: Plot of p_2 for different values of α and β

Given $\alpha \neq 0$, a low β causes the highest difference between our model and the base model. For this range our price is substantially lower. Keep in mind that a low β indicates a high update on the reference point. Not only does this lead to higher expected second stage losses for consumers but it also increases the value of the outside option which becomes a gain when the reference point is a loss. As δ increases, p_2 increases since the second stage becomes more important. However, if β is relatively low, the reference point becomes more important. This effect partly offsets the increase induced by a higher δ. The higher the losses that are expected, the higher the gains consumers could make from not buying the object (OO_2). Therefore, p_2 has to be low enough to invoke purchase in the second stage.

4.3 Marginal Consumer x in $t = 1$

Comparing the two models, we find that the valuation of the marginal consumer, x, differs from the base case only for relatively high levels of δ, i.e. $\delta \geq 0.4$. The marginal consumer in the first stage increases in δ. A higher δ counteracts the effect of loss aversion for high- valuation consumers. This indicates that less people are inclined to purchase in the first period the higher δ. This is due to three interrelated reasons. First, the price in the first stage is relatively high the higher δ, so that purchase becomes more expensive. Secondly, as δ increases, the second period becomes relatively more attractive which encourages consumers to wait. Thirdly, a higher p_1 causes more consumers to expect losses which is positive but causes less consumers to buy today. The first two reasons hold for our model as well as the base model. The third reason is specific to our case. Overall, the marginal buyer is lower in our model as compared to the base case (see Figure 4). This is due to the fact that in our model everybody with a valuation higher than the price would expect to make a gain in the second stage which puts them at a disadvantage in the second stage. This causes p_1 and x to be closer together. For $\beta \in [0, 1)$ and given δ, as α increases the valuation of marginal consumer in this model and the base case diverge. The higher α, the more important becomes the effect of the reference point through the *gain-loss utility*, and the lower x. This is due to the afore mentioned reason that everybody with $v \geq p_1$ would expect a gain in the second stage, so that more people will buy in the first stage. Again, α diminishes the effect of δ on x.

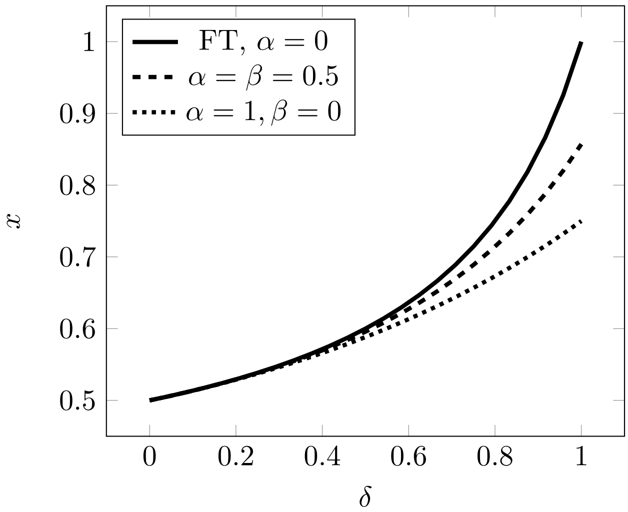

Figure 4: Plot of x for different values of α and β

For any $\alpha \neq 0$ and given δ, the the higher β, the less important the reference point. This leads to an increase in the valuation of the marginal consumer. In the case of a *full update*, $\beta = 0$, the difference between the valuations of the indifferent consumers between our model and the base model are maximal since the reference point has the strongest weight in this case.

4.4 Marginal consumer y in $t = 2$

For both models, the valuation of the second stage indifferent consumer,y, increases in δ. In our model the valuation of the marginal consumer lies strictly below that of the marginal consumer in the base model. This is due to two reasons. First, the more important the future becomes, the more people expect to buy in the second period, given that for high levels of δ, p_1 is relatively higher than in the base case. Secondly, since p_2 is lower, it is attractive for more consumers than in the base case to purchase in the second stage.

For $\beta \in [0, 1)$ and given δ the marginal consumers of the base model and our model diverge. This is due to the fact that more people are willing to buy in the second stage in our model since they want to realize their expected gains which resulted from a high first stage price[14]. Furthermore, since p_2 is lower, the gain that can be realized from purchasing outweighs the gain from not purchasing for every consumer with a valuation higher than the one of the indifferent consumer. For $\alpha \neq 0$ and a given δ, less consumers will buy in the second stage as β increases. This is based on the fact that the reference point becomes less important the higher β, so the closer we get to the base model. The positive effect we previously saw from expecting a loss in the second stage will decrease as β increases. These effects are summarized in Figure 5 below.

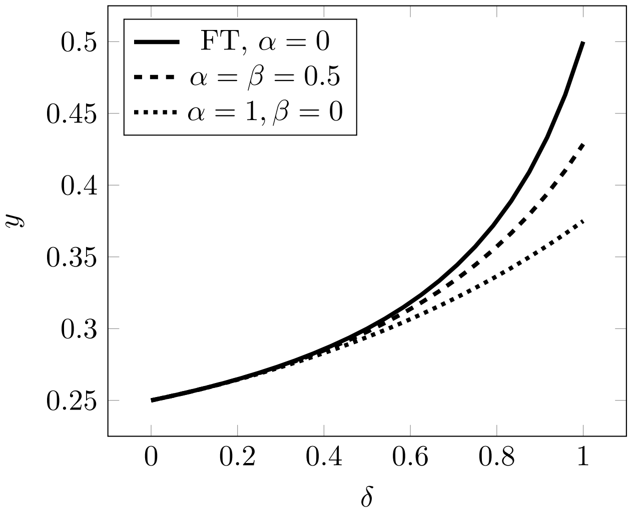

Figure 5: Plot of y for different values of α and β

The following Table 1 gives a summary of the effects of α, β and δ on the optimal prices, p_1 and p_2 as well as the valuations of the marginal consumers, x and y. For detailed comparative statics, the reader is referred to the Appendix ($A2$).

	p_1	p_2	x	y
α	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$
β	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$
δ	first $\downarrow$, then $\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$

Table 1: Summary effects of α, β and δ

All of our results are mainly driven by two forces. First, by the effect that a higher price has on the reference point. A high price causes consumers with $v < p_1$ to expect losses in the next stage which increases the likelihood of purchase in the next period. This leads to a higher first stage price. Moreover, loss aversion also causes those people to buy in the first stage who would expect to make a

[14]Consumers who expect to make a loss, feel a gain from purchasing if their realized loss is lower than their expected loss. This is why we refer to gains in this context.

gain in the second stage which increases the amount of people purchasing in the first stage. Secondly, it is driven by the effect of the pricing strategy on the outside option, OO_2, of consumers. For those consumers who expect to make a loss in the second stage, the comparatively low p_2, invokes purchase. This increases the number of buyers in the second stage. Since more people are purchasing the object at higher prices than in the base model, the profit of the seller, π, is higher under the model with endogenous reference points which can be seen in Figure 6. For the technical derivation of these properties, the reader is referred to Appendix ($A2$).

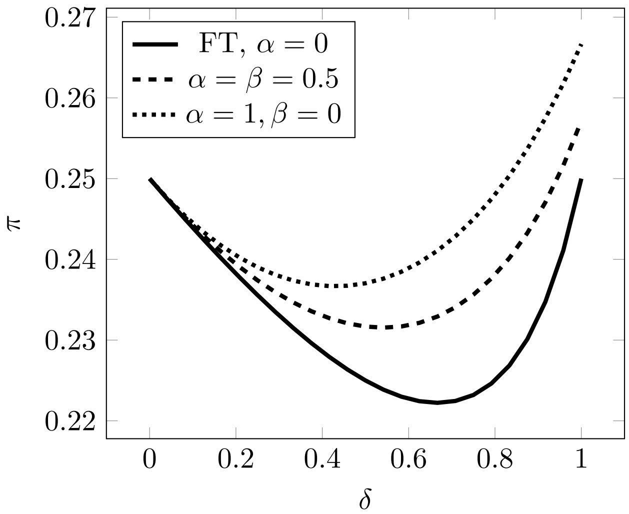

Figure 6: Plot of π for different values of α and β

4.5 Comparison with Rosato (2013)

There are some clear differences between the model presented by Rosato (2013) and this paper. However, due to the similar nature of both models this section will briefly discuss the main differences and parallels between our papers. In Figure 7, we plotted the optimal first stage price for Rosato (2013) and our model. For comparison reasons, we replaced η with α since both parameters are the weights of *gain loss utility* in Rosato's and this model, respectively.[15] Setting $\alpha = 0.8$ allows us to compare the essential parts of each model. Since our main contributions lie in the adjustment of *gain-loss utility*, weighing it relatively more seems logical. Please keep in mind, the *intrinsic consumption utility* has a higher weight in Rosato's model than in ours. However, this analysis is mainly concerned with comparing the *gain- loss utility* aspect of the two models.

[15] Please note, Rosato (2013) defines η to be larger than 0, whereas α lies in [0,1]. In order to compare the models, we restrict the analysis to values in the interval [0,1].

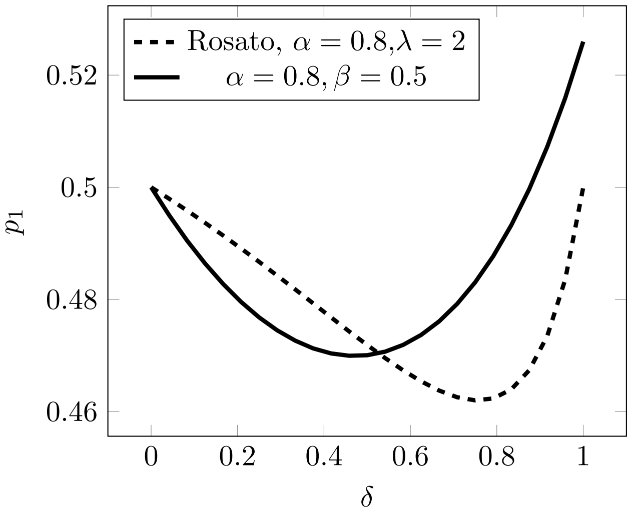

Figure 7: Comparison of p_1 for Rosato (2013) and this model for $\alpha = 0.8$, $\beta = 0.5$ and $\lambda = 2$

While this model's optimal price depends on β, Rosato's optimal price depends on λ. For low values of δ, Rosato's model predicts a higher first stage price, an effect that is magnified as λ increases. As consumers are more loss averse, they are willing to buy in the first period at a higher price. This reason holds for our model as well. In addition,in our model, consumers update their reference point which increases the relative attractiveness of the second stage as compared to Rosato's model. This additional effect in our model keeps the price lower. The effect of δ is the same for both models. A low δ renders the second stage relatively less attractive. Buyers and sellers are interested in reaching conclusions in the first stage which drives up the price.

Analyzing the second stage price p_2, Figure 8 shows that it is higher in Rosato's model than in our case. For Rosato (2013), p_2 decreases in λ. Being more loss averse leads consumers to buy in the first rather than the second stage.

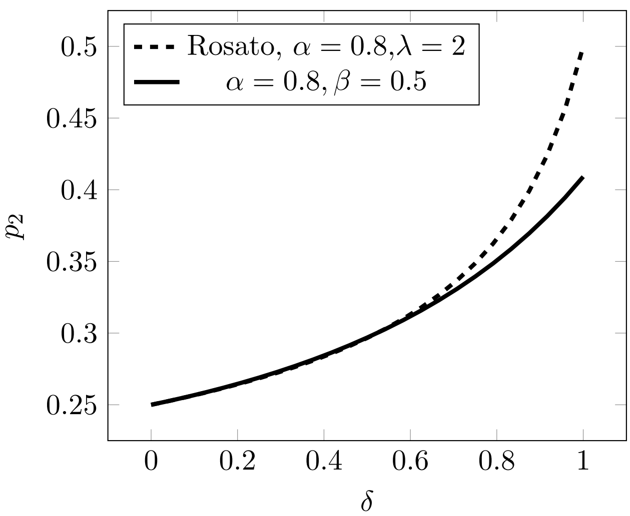

Figure 8: Comparison of p_2 for Rosato (2013) and this model for $\alpha = 0.8, \beta = 0.5$ and $\lambda = 2$

In our model, the additional outside option in the second stage keeps p_2 lower for high values of δ,i.e.

$\delta \geq 0.6$. For low values of δ, the second period is relatively unattractive which is why the effects of loss aversion and updated reference points do not significantly influence the price. As argued previously, consumers who expect to make a loss in $t = 2$ will realize a gain in not purchasing. Therefore, the optimal price has to be low enough for consumers to realize a gain higher than the expected gain from not purchasing. The difference on the price caused by this effect is large and increasing in δ. In the extreme case, when $\delta = 1$, Rosato's model predicts a price that is approximately 16 percent higher than ours.

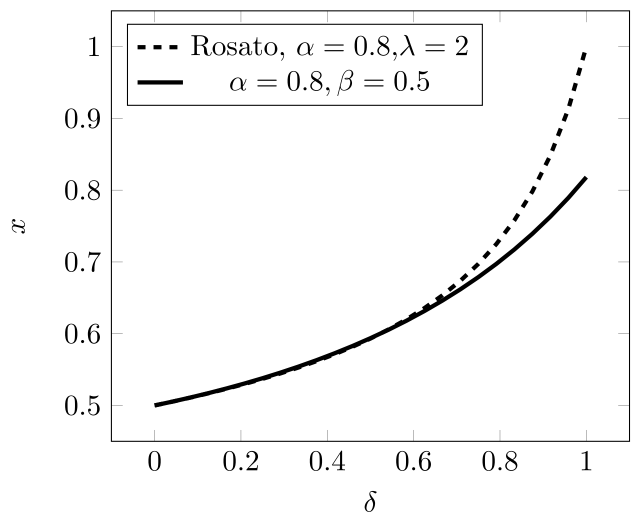

Figure 9: Comparison of x for Rosato (2013) and this model for $\alpha = 0.8, \beta = 0.5$ and $\lambda = 2$

Comparing the cut off value for the marginal consumer, x, in the two models, we find it is higher in Rosato's model which can be seen in Figure 9. Again, differences become visible only for values of $\delta \geq 0.6$. As δ increases, less consumers will buy in the first stage; an outcome predicted by both models. However, in Rosato (2013), less consumers will buy in the first stage as compared to our model. Loss aversion increases the number of consumers in both models. There is an additional, negative effect of the reference point update which keeps the valuation of the marginal consumer lower. Consumers with a very high valuation will expect a surplus in the second stage which invokes purchase in the first stage. For Rosato (2013), loss aversion increases the number of first stage consumers compared to Fudenberg and Tirole (1983). This is due to the fact that in Rosato's model consumers have a higher incentive to wait. However, since there is no update on the reference point, for high values of δ consumers have an incentive to postpone the purchasing decision to the second stage where they will face a lower price. In Rosaot's model, consider a scenario with $r > (v - p_1)$ which indicates that buyers would make a loss in the first stage. Since they will anticipate that $p_2 < p_1$, waiting is clearly advantageous. If $r < (v - p_1)$ and consumers expect a gain in the second period, they can increase their gain by waiting until $t = 2$, given that δ is high enough. In our model this effect is partly offset by the update of the reference point in combination with the outside option.

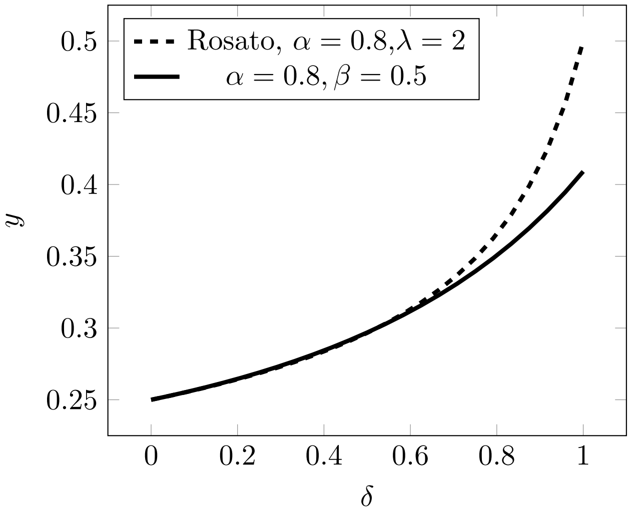

Figure 10: Comparison of y for Rosato (2013) and this model for $\alpha = 0.8, \beta = 0.5$ and $\lambda = 2$

Given that the price in the second stage is higher in Rosato's model, less consumers will decide to buy. Since the outside option in Rosato's model is zero, the seller can set a higher price, thereby addressing consumers with a higher valuation. This in turn, increases the cut off valuation of the marginal consumer in the second stage. Since our model allows for an outside option, consumers who expect to make a loss in the second stage, can realize a gain through not buying. Therefore, the seller has an incentive to set the price lower, addressing people with a lower valuation. This causes our p_2 to be lower than in Rosato's model, which can be seen in Figure 10.

The development of the seller's profits follow the movement of p_1 for Rosato's as well as our model. The update of the reference point weakens the effect of loss aversion for consumers with $v < p_1$ which is why in Rosato, for low values of δ, the same amount of people purchase the object as a higher price. This leads to higher profits in his model which can be seen in Figure 11. The additional price loss averse consumers pay in his model (for low values of δ) can be interpreted as a risk premium. This risk is substantially decreased in our model due to the endogenous reference point which increases the relative attractiveness of the second period as compared to a model without an update on the reference point. Even though Rosato's model predicts a higher profit for the seller for $\delta \leq 0.5$, our profits are significantly higher for $\delta > 0.5$ which is why the seller on average benefits more in our model.

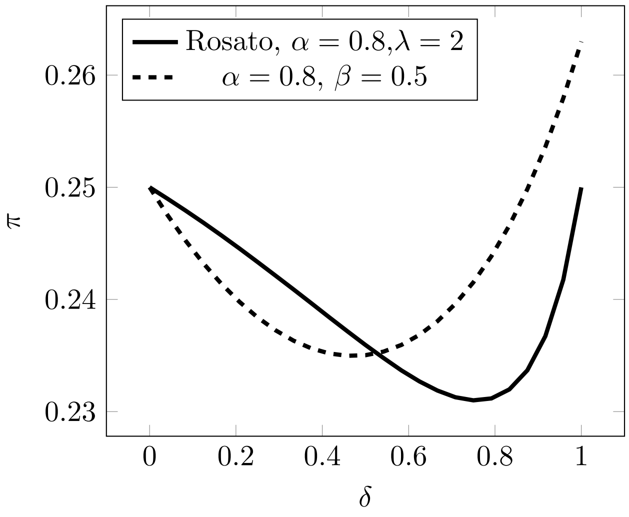

Figure 11: Plot of π for different values of α and β

All in all, we find that Rosato's as well as our model predict that more consumers will buy at higher prices, leading to higher profits for the seller. These effects are driven by loss aversion and for our model in particular, the update on the reference point. Furthermore, the inclusion of an outside option and the updated reference point have an additional effect on the optimal purchasing and pricing strategy of buyers and sellers which increases the divergence of the optimal values in our model and those of Fudenberg and Tirole (1983) as well as Rosato (2013). The additional explanation and main insight our model provides is that sellers can use the first stage price as an instrument to manipulate the consumer's reference point in such a manner that their own profits are increased.

5 Discussion

The reader should keep in mind that our model makes some crucial assumptions that are not necessarily fulfilled under real life circumstances. While it seems plausible for consumers not to have any prior expectations about making a surplus or a loss, i.e. $r_1 = 0$, it manages to only address a very stylized situation. Further research should include the option of initially expecting surpluses as well as losses. The assumption of a uniform distribution simplifies the analysis but might not properly capture real life distributions of consumer types. Incorporating additional distributions and analyzing the differences in effects might provide a deeper understanding to the mechanisms of the model. Moreover, limiting the negotiation to two periods does not allow us to properly state the effect of endogenous reference points on the length of the negotiation process which is another crucial part into understanding the effects of such preferences. Besides, it would be interesting to analyze what happens in situations where sellers do not fully anticipate the updating process or where sellers themselves have reference dependent preferences. Assuming the updating process to be linear is a good starting point but further research, of both, empirical and theoretical nature, needs to be conducted to fully understand how preferences are updated over time. For example, assuming a concave function representing the updating process might be a more realistic approach, especially if we are considering multiple bargaining rounds. For low prices, setting higher prices would lead to a stronger initial update but once the seller continues to increase the price from one bargaining stage to another, the update should increase at a decreasing rate. Another important aspect would be to include an additional punishment of the seller for setting the price too high. In our model, this punishment is captured by δ. However, δ is the same for both players. Including an additional punishment only related to the seller's pricing strategy would allow us to control the manipulation of the reference point.

6 Conclusion

The aim of this paper was to analyze the influence of reference dependence on the efficiency of a negotiation between a loss averse consumer and a rational seller. Allowing for an endogenous reference point and including an outside option, we find that the first stage price of the seller is higher than in the case without an updated reference point. This result is driven by consumer's loss aversion and the update on the reference point which creates an advantage for consumers who expect to make a loss in the second stage and a disadvantage for consumers who expect to make a surplus. In the former case, consumers have an additional incentive to postpone their purchasing decision to the next period because an expected loss will increase their utility in the second period. In the latter case, the effect is reversed and they will be invoked to purchase today rather than tomorrow. In the second period, the price has to accommodate both types of consumers and has to counteract the effect of the outside option. This leads to a lower second stage price. The fundamental result of our model is that the seller can use the first stage price to manipulate the consumer's reference point and increase his own profit.In our model, part of the consumer surplus is thereby ceded to the seller. By adding reference point-dependent preferences to a buyer's decision making process, the seller has a clear advantage. However, since the price in the second stage is consistently lower, the equilibrium payoff of low-valuation consumers is also higher than without given preferences.In conclusion, the seller and low-valuation consumers benefit as compared to the model without endogenous reference points.

References

Husnain Fateh Ahmad. Journal of economic behavior and organization. *The Journal of Finance*, 112: 46–63, 2015.

Olivier Compte and Philippe Jehiel. Bargaining with reference dependent preferences. *Working Paper*, 2003.

Peter Fishburn and Gary Kochenberger. Two-piece von neumann-morgenstern utility functions. *The Journal of the Decision Sciences*, 10(4):503–518, 1979.

Drew Fudenberg and Jean Tirole. Sequential bargaining with incomplete information. *Review of Economic Studies*, pages 221–247, 1983.

Kyle Hyndman. Repeated bargaining with reference-dependent preferences. *International Journal of Game Theory*, 40:527–549, 2010.

Daniel Kahneman and Amos Tversky. Prospect theory: An analysis of decision under risk. *Econometrica*, pages 263–292, 1979.

Daniel Kahneman and Amos Tversky. Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty*, 1992.

Botond Kőszegi and Matthew Rabin. A model of reference-dependent preferences. *The Quaterly Journal of Economics*, 121:1133–1165, 2006.

Andreas Lange and Anmol Ratan. Multi-dimensioanl reference-dependent preferences in sealed bid auctions - how (most) laboratory experiments differ from the field. *Journal of Financial Economics*, 68:634–645, 2010.

Duozhe Li. Bargaining with history-dependent preferences. *Journal of Economic Theory*, 136:695–708, 2007.

Joost M. E. Pennings and Ale Smidts. The shape of utility functions and organizational behavior. *Management Science*, pages 1251–1263, 2003.

Antonio Rosato. Sequential bargaining with reference-dependent preferences. 2013.

Ariel Rubinstein. Perfect equilibrium in a bargaining model. *Econometrica*, 50, 1982.

Ulrich Schmidt and Stefan Traub. An experimental test of loss aversion. *Journal of Risk and Uncertainty*, 25:233–249, 2002.

Jonathan Shalev. Loss aversion equilibrium. *International Journal of Game Theory*, 29:269–287, 2000.

Jonathan Shalev. Loss aversion and bargaining. *Theory and Decision*, 52:201–232, 2002.

Joel Sobel and Ichiro Takahashi. The multistage model of bargaining. *The Review of Economic Studies*, 50:411–426, 1983.

7 Appendix

A1: Proof of unique Sequential PBE

In the second stage, the consumer will decide to buy the object if the utility he obtains from purchase is larger than the utility from the outside option:

Case 1

$$(1-\alpha)(y-p2)+\alpha\lambda(p1-p2+\beta(y-p1)) = -\alpha\lambda(y-p1-(y-p1)) \tag{38}$$

$$y_1 = p_{2,1} \tag{39}$$

In the second stage, the seller will maximize expected profits from selling to consumers with valuation $y \leq v \leq x$:

$$\pi_{2,1} = \frac{(x_1 - p_{2,1})}{x_1} p_{2,1} \tag{40}$$

Maximizing $\pi_{2,1}$ with respect to $p_{2,1}$ and solving for $p_{2,1}$ yields the following:

$$\frac{\partial \pi_{2,1}}{\partial p_{2,1}} = 1 - \frac{2p_{2,1}}{x_1} \tag{41}$$

$$p_{2,1} = \frac{x_1}{2} \tag{42}$$

In the first stage, consumers will decide to purchase the object if the utility from buying today is larger than the utility from buying tomorrow which is represented by the following equation:

$$(1-\alpha)(x_1(p_{1,1})-p_{1,1})+\alpha(x_1(p_{1,1})-p_{1,1}) = \delta((1-\alpha)(x_1(p_{1,1})-p_{2,1}(p_{1,1}))+\alpha\lambda(x_1(p_{1,1})-p_{2,1}(p_{1,1})-(1-\beta)(x_1(p_{1,1})-p_{1,1})) \tag{43}$$

Solving for x_1 yields:

$$x_1(p_{1,1}) = \frac{2p_{1,1}(-1+\alpha(\beta-1)\delta\lambda)}{\delta(1+\alpha(-1+\beta\lambda-\lambda))-2} \tag{44}$$

The seller will maximize overall profits in the first stage:

$$\pi_{1,1} = (1 - \tfrac{2p_{1,1}(-1+\alpha(-1+\beta)\delta)}{-2+(1+2\alpha(-1+\beta))\delta})p_{1,1} + \delta(\tfrac{2p_{1,1}(-1+\alpha(-1+\beta)\delta)}{-2+(1+2\alpha(-1+\beta))\delta} - (\tfrac{p_{1,1}(-1+\alpha(-1+\beta)\delta)}{-2+(1+2\alpha(-1+\beta))\delta}))(\tfrac{p_{1,1}(-1+\alpha(-1+\beta)\delta)}{-2+(1+2\alpha(-1+\beta))\delta}) \tag{45}$$

$$\frac{\partial \pi_{1,1}}{\partial p_{1,1}} = 1 + \frac{2p_{1,1}\delta(-1+\alpha(-1+\beta)\delta\lambda)^2}{(-2+\delta(1+\alpha(-1-\lambda+2\beta\lambda)))^2} - \frac{4p_{1,1}(-1+\alpha(-1+\beta)\delta\lambda)}{-2+\delta(1+\alpha(-1-\lambda+2\beta\lambda))} \tag{46}$$

$$p_{1,1}^{*} = \frac{((2+\delta(-1+\alpha(1+\lambda-2\beta\lambda)))^2)}{(2(-1+(-1+))(4+(-3+(2+(2+(-4+)-)))))} \tag{47}$$

Given the optimal $p_{1,1}$, allows us to determine the optimal values for $p_{2,1}$, x_1 and y_1:

$$x_1^{*} = \frac{2+\delta(-1+\alpha(1+\lambda-2\beta\lambda))}{4+\alpha(-1+\beta)\delta^2\lambda+\delta(-3+\alpha(2+2\lambda-4\beta\lambda))} \tag{48}$$

$$p_{2,1}^{*} = \frac{2+\delta(-1+\alpha(1+\lambda-2\beta\lambda))}{2(4+\alpha(-1+\beta)\delta^2\lambda+\delta(-3+\alpha(2+2\lambda-4\beta\lambda)))} \tag{49}$$

$$y_1^{*} = \frac{2+\delta(-1+\alpha(1+\lambda-2\beta\lambda))}{2(4+\alpha(-1+\beta)\delta^2\lambda+\delta(-3+\alpha(2+2\lambda-4\beta\lambda)))} \tag{50}$$

Case 2

In the second stage, the consumer will decide to buy the object if the utility he obtains from purchase is larger than the utility from the outside option:

$$(1-\alpha)(y_2-p_{2,2})+\alpha\lambda(y_2-p_{2,2}-(1-\beta)(y_2-p_{1,2})) = -\alpha(1-\beta)(y_2-p_{1,2}) \tag{51}$$

$$y_2 = \frac{p_{2,2}(1+\alpha(\lambda-1))+p_{1,2}\alpha(\beta-1)(\lambda-1)}{1+\alpha\beta(\lambda-1)} \tag{52}$$

In the second stage, the seller will maximize expected profits from selling to consumers with valuation $y \leq v \leq x$:

$$\pi_{2,2} = \frac{(x_2-(\frac{p_{2,2}(1+\alpha(\lambda-1))+p_{1,2}\alpha(\beta-1)(\lambda-1)}{1+\alpha\beta(\lambda-1)}))}{x_2}p_{2,2} \tag{53}$$

Maximizing $\pi_{2,2}$ with respect to $p_{2,2}$ and solving for $p_{2,2}$ yields the following:

$$\frac{\partial\pi_{2,2}}{\partial p_{2,2}} = \frac{-2p_{2,2}(1+\alpha(-1+\lambda))+x_2(1+\alpha\beta(-1+\lambda))+p_{1,2}\alpha(-1+\beta+\lambda-\beta\lambda)}{x_2(1+\alpha\beta(-1+\lambda))} \tag{54}$$

$$p_{2,2} = \frac{x_2-p_{1,1}\alpha+p_{1,1}\alpha\beta-x_2\alpha\beta+p_{1,1}\alpha\lambda-p_{1,1}\alpha\beta\lambda+x_2\alpha\beta\lambda}{2(1-\alpha+\alpha\lambda)} \tag{55}$$

In the first stage, consumers will decide to purchase the object if the utility from buying today is larger than the utility from buying tomorrow which is represented by the following equation:

$$(1-\alpha)(x_2(p_{1,2})-p_{1,2})+\alpha(x_2(p_{1,2})-p_{1,2}) = \delta((1-\alpha)(x_2(p_{1,2})-p_{2,2}(p_{1,2}))+\alpha\lambda(x_2(p_{1,2})-p_{2,2}(p_{1,2})-(1-\beta)(x_2(p_{1,2})-p_{1,2})) \tag{56}$$

Solving for x_2 yields:

$$x_2(p_{1,2}) = \frac{p_{1,2}(-2+\alpha(\beta-1)\delta(1+\lambda))}{-2+\delta(1+\alpha(-2+\beta+\beta\lambda))} \tag{57}$$

The seller will maximize overall profits in the first stage:

$$\pi_{1,2} = (1-(\frac{p_{1,2(-2+\alpha(-1+\beta)\delta(1+\lambda))}}{-2+\delta(1+\alpha(-2+\beta+\beta\lambda))}))p_{1,2} + \delta((\frac{p_{1,2(-2+\alpha(-1+\beta)\delta(1+\lambda))}}{-2+\delta(1+\alpha(-2+\beta+\beta\lambda))}) - (\frac{(p_{1,2}\alpha(\beta-1)(\lambda-1)+\frac{p_{1,2}(-1+\alpha(-1+\beta)\delta)(1+\alpha(-1+\lambda))}{-2+\delta(1+\alpha(-2+\beta+\beta\lambda))})}{(1+\alpha\beta(-1+\lambda))}))((\frac{p1(-1+\alpha(-1+\beta)\delta)}{-2+\delta(1+\alpha(-2+\beta+\beta\lambda))}))) \tag{58}$$

$$\frac{\partial\pi_{1,2}}{\partial p_{1,2}} = 1+\frac{2p_{1,2}\delta(-1+\alpha(-1+\beta)\delta)^2(1+\alpha(-1+\lambda))}{(1+\alpha\beta(-1+\lambda))(-2+\delta(1+\alpha(-2+\beta+\beta\lambda)))^2} - \frac{2p_{1,2}(-2+\alpha(-1+\beta)\delta(1+\lambda))}{-2+\delta(1+\alpha(-2+\beta+\beta\lambda))} \tag{59}$$

$$p^*_{1,2} = \frac{((1+\alpha\beta(-1+\lambda))(-2+\delta(1+\alpha(-2+\beta+\beta\lambda)))^2)}{(2(4-3\delta+\alpha^3(\beta-1)\delta^2(\lambda-1)(\delta+\beta^2(1+\lambda)^2-\beta(2+\delta+2\lambda))-\alpha^2\delta((\delta-4)\delta+\beta(\delta^2-4+4\lambda^2-2\delta\lambda(1+\lambda))-2\beta(\delta^2+(\lambda-1)(3+\lambda)-\delta(2+\lambda+\lambda^2)))+\alpha(\delta(7+\lambda-\delta(3+\lambda))+\beta(4(\lambda-1)+\delta(-2-6\lambda+\delta(3+\lambda))))))} \tag{60}$$

Given the optimal $p_{1,2}$, allows us to determine the optimal values for $p_{2,2}$, x_2 and y_2:

$$x^*_2 = \frac{((1+\alpha\beta(-1+\lambda))(-2+\alpha(-1+\beta)\delta(1+\lambda))(-2+\delta(1+\alpha(-2+\beta+\beta\lambda))))}{(2(4-3\delta+\alpha^3(-1+\beta)\delta^2(-1+\lambda)(\delta+\beta^2(1+\lambda)^2-\beta(2+\delta+2\lambda))-\alpha^2\delta((-4+\delta)\delta+\beta^2(-4+\delta^2+4\lambda^2-2\delta\lambda(1+\lambda))-2\beta(-3+\delta^2+2\lambda+\lambda^2-\delta(2+\lambda+\lambda^2)))+\alpha(\delta(7+\lambda-\delta(3+\lambda))+\beta(4(-1+\lambda)+\delta(-2-6\lambda+\delta(3+\lambda))))))} \tag{61}$$

$$p^*_{2,2} = \frac{((-1+\alpha(-1+\beta)\delta)(1+\alpha\beta(-1+\lambda))(-2+\delta(1+\alpha(-2+\beta+\beta\lambda))))}{(2(4-3\delta+\alpha^3(-1+\beta)\delta^2(-1+\lambda)(\delta+\beta^2(1+\lambda)^2-\beta(2+\delta+2\lambda))-\alpha^2\delta((-4+\delta)\delta+\beta^2(-4+\delta^2+4\lambda^2-2\delta\lambda(1+\lambda))-2\beta(-3+\delta^2+2\lambda+\lambda^2-\delta(2+\lambda+\lambda^2)))+\alpha(\delta(7+\lambda-\delta(3+\lambda))+\beta(4(-1+\lambda)+\delta(-2-6\lambda+\delta(3+\lambda))))))} \tag{62}$$

$$y^*_2 = \frac{(2-\delta+\alpha^3(-1+\beta)\delta^2(-1+\lambda)(2-3\beta(1+\lambda)+\beta^2(1+\lambda)2)+\alpha(2+\delta-2\lambda+3\delta\lambda-\delta^2\lambda+\beta(-4+\delta)(1+(-1+\delta)\lambda))+\alpha^2\delta(4-4\lambda+\delta(-1+3\lambda)+\beta^2(1+\lambda)(4-4\lambda+\delta(-1+2\lambda))+\beta(3(-3+2\lambda+\lambda^2)-2\delta(-1+2\lambda+\lambda^2))))}{(2(4-3\delta+\alpha^3(-1+\beta)\delta^2(-1+\lambda))\delta+\beta^2(1+)2-\beta(2+\delta+2\lambda))+\alpha(\delta(7+\lambda-\delta(3+\lambda))+\beta(4(-1+\lambda)+\delta^2(3+)-2\delta(1+3\lambda)))-\alpha^2\delta((-4+\delta)\delta+\beta^2(\delta^2-2\delta\lambda(1+\lambda)+4(-1+\lambda^2))-2\beta(-3+\delta^2+2\lambda+\lambda^2-\delta(2+\lambda+\lambda^2)))))} \tag{63}$$

Case 3

$$(1-\alpha)(y_3-p_{2,3})+\alpha(y_3-p_{2,3}-(1-\beta)(y_3-p_{1,3})) = -\alpha\lambda(1-\beta)(y_3-p_{1,3}) \tag{64}$$

$$y_3 = \frac{\alpha(\beta-1)(\lambda-1)p_{1,3}-p_{2,3}}{\alpha(\beta-1)(\lambda-1)} \tag{65}$$

In the second stage the seller will maximize his expected profit:

$$\pi_{2,3} = \frac{(x_3 - (\frac{\alpha(\beta-1)(\lambda-1)p_{1,3}-p_{2,3}}{\alpha(\beta-1)(\lambda-1)}))}{x_3} p_{2,3} \tag{66}$$

Maximizing $\pi_{2,3}$ with respect to $p_{2,3}$ and solving for $p_{2,3}$ yields the following:

$$\frac{\partial \pi_{2,3}}{\partial p_{2,3}} = \frac{2p_{2,3} + x_3(-1+\alpha(-1+\beta)(-1+\lambda)) + p_{1,3}\alpha(-1+\beta+\lambda-\beta\lambda)}{x_3(-1+\alpha(-1+\beta)(-1+\lambda))} \tag{67}$$

$$p_{2,3} = \frac{1}{2}(p_{1,3}\alpha(\beta-1)(\lambda-1) + x_3(1+\alpha(-1+\beta+\lambda-\beta\lambda))) \tag{68}$$

In the first stage the consumer will decide to buy the object if the utility he obtains from buying the object in the first stage is larger than the utility from buying the object in the second stage:

$$(1-\alpha)(x_3(p_{1,3}) - p_{1,3}) + \alpha(x_3(p_{1,3}) - p_{1,3}) = \delta((1-\alpha)(x_3(p_{1,3}) - p_{2,3}(p_{1,3})) + \alpha(x_3(p_{1,3}) - p_{2,3}(p_{1,3}) - (1-\beta)(x_3(p_{1,3}) - p_{1,3})) \tag{69}$$

Solving for x_3 yields:

$$x_3(p_{1,3}) = \frac{p_{1,3}(-2+\alpha(\beta-1)\delta(1+\lambda))}{-2+\delta(1+\alpha(\beta-1)(1+\lambda))} \tag{70}$$

The seller will maximize overall profits in the first stage:

$$\pi_{1,3} = (1 - (\frac{p_{1,3}(-2+\alpha(-1+\beta)\delta(1+\lambda))}{-2+\delta(1+\alpha(-1+\beta)(1+\lambda))}))p_{1,3} + \delta((\frac{p_{1,3}(-2+\alpha(-1+\beta)\delta)(1+\lambda))}{-2+\delta(1+\alpha(-1+\beta)(1+\lambda))} - (\frac{p_{1,3}\alpha(-1+\beta)(-1+\lambda) + \frac{p_{1,3}(1-\alpha(-1+\beta)\delta\lambda)}{-2+\delta(1+\alpha(-1+\beta)(1+\lambda))}}{-1+\alpha(-1+\beta)(-1+\lambda)}))(\frac{p_{1,3}(-1+\alpha(-1+\beta)\delta\lambda)}{-2+\delta(1+\alpha(-1+\beta)(1+\lambda))})) \tag{71}$$

$$\frac{\partial \pi_{1,3}}{\partial p_{1,3}} = 1 - \frac{2p_{1,3}\delta(-1+\alpha(-1+\beta)\delta\lambda)^2}{(-1+\alpha(-1+\beta)(-1+\lambda))(-2+\delta(1+\alpha(-1+\beta)(1+\lambda)))^2} - \frac{2p_{1,3}(-2+\alpha(-1+\beta)\delta(1+\lambda))}{-2+\delta(1+\alpha(-1+\beta)(1+\lambda))} \tag{72}$$

$$p^*_{1,3} = \frac{((-1+\alpha(-1+\beta)(-1+\lambda))(-2+\delta(1+\alpha(-1+\beta)(1+\lambda)))^2)}{(2(-4+3\delta+\alpha(-1+\beta)(4(-1+\lambda)+\delta(6-\delta+2\lambda-3\delta\lambda+\alpha^2(-1+\beta)^2\delta(-1+\lambda)(1+\lambda)^2+\alpha(-1+\beta)(4-2\delta-2\delta\lambda+(-4+\delta^2)\lambda^2)))))} \tag{73}$$

Given the optimal $p_{1,3}$, allows us to determine the optimal values for $p_{2,3}$, x_3 and y_3:

$$x^*_3 = \frac{((-1+\alpha(-1+\beta)(-1+\lambda))(-2+\alpha(-1+\beta)\delta(1+\lambda))(-2+\delta(1+\alpha(-1+\beta)(1+\lambda))))}{(2(-4+3\delta+\alpha(-1+\beta)(4(-1+\lambda)+\delta(6-\delta+2\lambda-3\delta\lambda+\alpha^2(-1+\beta)^2\delta(-1+\lambda)(1+\lambda)^2+\alpha(-1+\beta)(4-2\delta-2\delta\lambda+(-4+\delta^2)^2)))))} \tag{74}$$

$$p^*_{2,3} = \frac{((-1+\alpha(-1+\beta)(-1+\lambda))(-1+\alpha(-1+\beta)\delta\lambda)(-2+\delta(1+\alpha(-1+\beta)(1+\lambda))))}{(2(-4+3\delta+\alpha(-1+\beta)(4(-1+\lambda)+\delta(6-\delta+2\lambda-3\delta\lambda+\alpha^2(-1+\beta)^2\delta(-1+\lambda)(1+\lambda)^2+\alpha(-1+\beta)(4-2\delta-2\delta\lambda+(-4+\delta^2)\lambda^2)))))} \tag{75}$$

$$y^*_3 = \frac{((-2+\delta(1+\alpha(-1+\beta)(1+\lambda)))(1-\alpha(-1+\beta)\delta+\alpha(-1+\beta)(-1+\lambda)(-2+\delta(1+\alpha(-1+\beta)(1+\lambda)))))}{(2(-4+3\delta+\alpha(-1+\beta)(4(-1+\lambda)+\delta(6-\delta+2\lambda-3\delta\lambda+\alpha^2(-1+\beta)^2\delta(-1+\lambda)(1+\lambda)^2+\alpha(-1+\beta)(4-2\delta-2\delta\lambda+(-4+\delta^2)\lambda^2)))))} \tag{76}$$

Case 4

$$(1-\alpha)(y_4 - p_{2,4}) + \alpha(y_4 - p_{2,4} - (1-\beta)(y_4 - p_{1,4})) = -\alpha(1-\beta)(y_4 - p_{1,4}) \tag{77}$$

$$y_4 = p_{2,4} \tag{78}$$

In the second stage the seller will maximize his expected profit:

$$\pi_{2,4} = \frac{(x_4 - p_{2,4})}{x_4} p_{2,4} \tag{79}$$

Maximizing $\pi_{2,4}$ with respect to $p_{2,4}$ and solving for $p_{2,4}$ yields the following:

$$\frac{\partial \pi_{2,4}}{\partial p_{2,4}} = 1 - \frac{2p_{2,4}}{x_4} \tag{80}$$

$$p_{2,4} = \frac{x_4}{2} \tag{81}$$

In the first stage the consumer will decide to buy the object if the utility he obtains from buying the object in the first stage is larger than the utility from buying the object in the second stage:

$$(1-\alpha)(x_4(p_{1,4}) - p_{1,4}) + \alpha(x_4(p_{1,4}) - p_{1,4}) = \delta((1-\alpha)(x_4(p_{1,4}) - p_{2,4}(p_{1,4})) + \alpha(x_4(p_{1,4}) - p_{2,4}(p_{1,4}) - (1-\beta)(x_4(p_{1,4}) - p_{1,4})) \tag{82}$$

Solving for x_4 yields:

$$x_4(p_{1,4}) = \frac{2p_{,4}(-1+\alpha(\beta-1)\delta)}{-2+(1+2\alpha(\beta-1))\delta} \tag{83}$$

In the first stage the seller maximizes overall profits from the first and second stage:

$$\pi_{1,4} = (1 - (\tfrac{2p_{1,4}(-1+\alpha(-1+\beta)\delta)}{-2+(1+2\alpha(-1+\beta))\delta}))p_{1,4} + \delta((\tfrac{2p_{1,4}(-1+\alpha(-1+\beta)\delta)}{-2+(1+2\alpha(-1+\beta))\delta}) - \tfrac{p1(-1+\alpha(-1+\beta)\delta)}{-2+(1+2\alpha(-1+\beta))\delta}))(\tfrac{p_{1,4}(-1+\alpha(-1+\beta)\delta)}{-2+(1+2\alpha(-1+\beta))\delta})) \tag{84}$$

$$\frac{\partial \pi_{1,4}}{\partial p_{1,4}} = 1 - \frac{4p_{1,4}(-1+\alpha(-1+\beta)\delta)}{-2+(1+2\alpha(-1+\beta))\delta} + \frac{2p_{1,4}\delta(-1+\alpha(-1+\beta)\delta)2}{2p_{1,4}\delta(-1+\alpha(-1+\beta)/\delta)2} \tag{85}$$

$$p^*_{1,4} = \frac{(2-(1+2\alpha(-1+\beta))\delta)^2}{2(1-\alpha(1-\beta)\delta)(4+(-3+\alpha(-1+\beta)(-4+\delta))\delta)} \tag{86}$$

Given the optimal $p_{1,4}$, allows us to determine the optimal values for $p_{2,4}$, x_4 and y_4:

$$x^*_4 = \frac{2-(1+2\alpha(1-\beta))\delta}{-4+(3-\alpha(-1+\beta)(-4+\delta))\delta} \tag{87}$$

$$p^*_{2,4} = \frac{2-(1+2\alpha(1-\beta))\delta}{2(-4+(3-\alpha(-1+\beta)(-4+\delta))\delta)} \tag{88}$$

$$y^*_4 = \frac{2-(1+2\alpha(1-\beta))\delta}{2(-4+(3-\alpha(-1+\beta)(-4+\delta))\delta)} \tag{89}$$

For each of the four cases, we need to determine whether they fulfill the following conditions.

Condition 1:

As previously discussed, in equilibrium the valuation of the indifferent consumer x_i has to be larger than the first stage price $p_{1,i}$.

Condition 2:

In equilibrium, the assumption about whether U_2 is a loss or gain has to hold. To formally test this, we check if the following holds for all given values of α, β, λ and δ:

$$\begin{cases} (y_i - p_{2,i} - r_2) > 0, & U_2 \text{ is a gain.} \\ (y_i - p_{2,i} - r_2) < 0, & U_2 \text{ is a loss.} \end{cases} \tag{90}$$

Condition 3:

Since we assume that the outside option in the second stage, OO_2 can either be a gain or a loss, we have to verify if this assumption is fulfilled in equilibrium. Formally, we test the following [16]:

$$\begin{cases} (y_i - p_{1,i}) > 0, & OO_2 \text{ is a loss.} \\ (y_i - p_{1,i}) < 0, & OO_2 \text{ is a gain.} \end{cases} \tag{91}$$

Condition 4:

For the indifferent consumer in the second stage U_2 has to be larger than U_1:

$$\begin{cases} (1-\alpha)(y_i - p_{2,i}) + \alpha(y_i - p_{2,i} - (1-\beta)(y_i - p_{1,i})) > (1-\alpha)(y_i - p_{1,i}) + \alpha(y_i - p_{1,i}), & \text{if } U_2 \text{ is a gain} \\ (1-\alpha)(y_i - p_{2,i}) + \alpha\lambda(y_i - p_{2,i} - (1-\beta)(y_i - p_{1,i})) > (1-\alpha)(y_i - p_{1,i}) + \alpha(y_i - p_{1,i}), & \text{if } U_2 \text{ is a loss} \end{cases} \tag{92}$$

We find, that these conditions are only met by Case 4. Due to the very complex nature of testing these conditions simultaneously, I will attach only the raw output to the end of this Appendix.

A2: Comparative Statics

This section will present the comparative statics of our analysis in section 5. The optimal first stage price increases as α increases. It decreases in β and is convex in δ.

$$\frac{\partial p_1}{\partial \alpha} = \frac{((-1+\beta)\delta^2(-2+(1+2\alpha(-1+\beta))\delta)(2-2(1+\alpha(-1+\beta))\delta+\alpha(-1+\beta)\delta^2))}{((-1+\alpha(-1+\beta)\delta)^2(4+(-3-4\alpha(-1+\beta))\delta+\alpha(-1+\beta)\delta^2)^2)} > 0 \tag{93}$$

$$\frac{\partial p_1}{\partial \beta} = \frac{(\alpha\delta^2(-2+(1+2\alpha(-1+\beta))\delta)(2-2(1+\alpha(-1+\beta))\delta+\alpha(-1+\beta)\delta^2))}{((-1+\alpha(-1+\beta)\delta)^2(4+(-3-4\alpha(-1+\beta))\delta+\alpha(-1+\beta)\delta^2)^2)} < 0 \tag{94}$$

$$\frac{\partial p_1}{\partial \delta} = \frac{((-2+(1+2\alpha(-1+\beta))\delta)(2+(-3+2\alpha(-1+\beta))\delta-6\alpha^2(-1+\beta)^2\delta^2+\alpha^2(1+2\alpha(-1+\beta))(-1+\beta)^2\delta^3))}{2((-1+\alpha(-1+\beta)\delta)^2(4+(-3-4\alpha(-1+\beta))\delta+\alpha(-1+\beta)\delta^2)^2)} < 0 \tag{95}$$

[16] Please note, $OO_2 = -\alpha(1-\beta)(v-p_1)$ if the outsipde option is a gain and $OO_2 = -\alpha\lambda(1-\beta)(v-p_1)$ if the outside option is a loss. Simply looking at the difference between the consumer's valuation and the first stage price will therefore suffice to determine whether OO_2 is a gain or a loss.

$$\frac{\partial^2 p_1}{\partial \delta} = -\frac{((4+4\alpha^5(-1+\beta)^5(-6+\delta)\delta^5+4\alpha^6(-1+\beta)^6\delta^6-4\alpha^3(-1+\beta)^3\delta^2(24-10\delta+3\delta^2)+\alpha^4(-1+\beta)^4\delta^3(32+36\delta-12\delta^2+\delta^3)+4\alpha(-1+\beta)(-8+12\delta-12\delta^2+3\delta^3)-\alpha^2(-1+\beta)^2\delta(-96+108\delta-68\delta^2+9\delta^3))}{((-1+\alpha(-1+\beta)\delta)^3(4+(-3-4\alpha(-1+\beta))\delta+\alpha(-1+\beta)\delta^2)^3))} > 0 \tag{96}$$

The following shows that p_2 is decreasing in α but increasing in β and δ.

$$\frac{\partial p_2}{\partial \alpha} = \frac{(-1+\beta)\delta^3}{2(4+(-3-4\alpha(-1+\beta))\delta+\alpha(-1+\beta)\delta^2)^2} < 0 \tag{97}$$

$$\frac{\partial p_2}{\partial \beta} = \frac{\alpha\delta^3}{2(4+(-3-4\alpha(-1+\beta))\delta+\alpha(-1+\beta)\delta^2)^2} > 0 \tag{98}$$

$$\frac{\partial p_2}{\partial \delta} = \frac{2+\alpha(-1+\beta)(-4+\delta)\delta+2\alpha^2(-1+\beta)^2\delta^2}{2(4+(-3-4\alpha(-1+\beta))\delta+\alpha(-1+\beta)\delta^2)^2} > 0 \tag{99}$$

As we can see in the following, x is decreasing in α but increasing in δ and β .

$$\frac{\partial x}{\partial \alpha} = \frac{(-1+\beta)\delta^3}{(4+(-3-4\alpha(-1+\beta))\delta+\alpha(-1+\beta)\delta^2)^2} < 0 \tag{100}$$

$$\frac{\partial x}{\partial \beta} = \frac{\alpha\delta^3}{(4+(-3-4\alpha(-1+\beta))\delta+\alpha(-1+\beta)\delta^2)^2} > 0 \tag{101}$$

$$\frac{\partial x}{\partial \delta} = \frac{2+\alpha(-1+\beta)(-4+\delta)\delta+2\alpha^2(-1+\beta)^2\delta^2}{(4+(-3-4\alpha(-1+\beta))\delta+\alpha(-1+\beta)\delta^2)^2} > 0 \tag{102}$$

The following shows that y is decreasing in α but increasing in β and δ.

$$\frac{\partial y}{\partial \alpha} = \frac{(-1+\beta)\delta^3}{2(4+(-3-4\alpha(-1+\beta))\delta+\alpha(-1+\beta)\delta^2)^2} < 0 \tag{103}$$

$$\frac{\partial y}{\partial \beta} = \frac{\alpha\delta^3}{2(4+(-3-4\alpha(-1+\beta))\delta+\alpha(-1+\beta)\delta^2)^2} > 0 \tag{104}$$

$$\frac{\partial y}{\partial \delta} = \frac{2+\alpha(-1+\beta)(-4+\delta)\delta+2\alpha^2(-1+\beta)^2\delta^2}{2(4+(-3-4\alpha(-1+\beta))\delta+\alpha(-1+\beta)\delta^2)^2} > 0 \tag{105}$$

```
Reduce[(1 - α) (v2 - p1) + α (v2 - p1) <  (1 - α) (v2 - p2) + α λ (v2 - p2 - (1 - β) (v2 - p1)) &&
   (v2 - p2 - (1 - β) (v2 - p1)) > 0 && (v2 - p1) > 0 && x > p1 &&
   0 < δ < 1 && 0 < α < 1 && 0 < β < 1 && 1 < λ < 5, Reals]
```

Case 1 :

$$
\begin{aligned}
\text{Reduce}\Bigg[& (1-\alpha)\left(\frac{2+\delta(-1+\alpha(1+\lambda-2\beta\lambda))}{8+2\alpha(-1+\beta)\delta^2\lambda+\delta(-6+\alpha(4+4\lambda-8\beta\lambda))} - \right. \\
& \left.\left(-\frac{(2+\delta(-1+\alpha(1+\lambda-2\beta\lambda)))^2}{2(-1+\alpha(-1+\beta)\delta\lambda)(4+\delta(-3+\alpha(2+(2+\beta(-4+\delta)-\delta)\lambda)))}\right)\right) + \\
& \alpha\left(\frac{2+\delta(-1+\alpha(1+\lambda-2\beta\lambda))}{8+2\alpha(-1+\beta)\delta^2\lambda+\delta(-6+\alpha(4+4\lambda-8\beta\lambda))} - \right. \\
& \left.\left(-\frac{(2+\delta(-1+\alpha(1+\lambda-2\beta\lambda)))^2}{2(-1+\alpha(-1+\beta)\delta\lambda)(4+\delta(-3+\alpha(2+(2+\beta(-4+\delta)-\delta)\lambda)))}\right)\right) < \\
& (1-\alpha)\left(\frac{2+\delta(-1+\alpha(1+\lambda-2\beta\lambda))}{8+2\alpha(-1+\beta)\delta^2\lambda+\delta(-6+\alpha(4+4\lambda-8\beta\lambda))} - \right. \\
& \left.\left(\frac{2+\delta(-1+\alpha(1+\lambda-2\beta\lambda))}{2\left(4+\alpha(-1+\beta)\delta^2\lambda+\delta(-3+\alpha(2+2\lambda-4\beta\lambda))\right)}\right)\right) + \\
& \alpha\lambda\left(\frac{2+\delta(-1+\alpha(1+\lambda-2\beta\lambda))}{8+2\alpha(-1+\beta)\delta^2\lambda+\delta(-6+\alpha(4+4\lambda-8\beta\lambda))} - \right. \\
& \left(\frac{2+\delta(-1+\alpha(1+\lambda-2\beta\lambda))}{2\left(4+\alpha(-1+\beta)\delta^2\lambda+\delta(-3+\alpha(2+2\lambda-4\beta\lambda))\right)}\right) - \\
& (1-\beta)\left(\frac{2+\delta(-1+\alpha(1+\lambda-2\beta\lambda))}{8+2\alpha(-1+\beta)\delta^2\lambda+\delta(-6+\alpha(4+4\lambda-8\beta\lambda))} - \right. \\
& \left.\left.\left(-\frac{(2+\delta(-1+\alpha(1+\lambda-2\beta\lambda)))^2}{2(-1+\alpha(-1+\beta)\delta\lambda)(4+\delta(-3+\alpha(2+(2+\beta(-4+\delta)-\delta)\lambda)))}\right)\right)\right) \mathbin{\&\&} \\
& \left(\frac{2+\delta(-1+\alpha(1+\lambda-2\beta\lambda))}{8+2\alpha(-1+\beta)\delta^2\lambda+\delta(-6+\alpha(4+4\lambda-8\beta\lambda))} - \right. \\
& \left(\frac{2+\delta(-1+\alpha(1+\lambda-2\beta\lambda))}{2\left(4+\alpha(-1+\beta)\delta^2\lambda+\delta(-3+\alpha(2+2\lambda-4\beta\lambda))\right)}\right) - \\
& (1-\beta)\left(\frac{2+\delta(-1+\alpha(1+\lambda-2\beta\lambda))}{8+2\alpha(-1+\beta)\delta^2\lambda+\delta(-6+\alpha(4+4\lambda-8\beta\lambda))} - \right. \\
& \left.\left.\left(-\frac{(2+\delta(-1+\alpha(1+\lambda-2\beta\lambda)))^2}{2(-1+\alpha(-1+\beta)\delta\lambda)(4+\delta(-3+\alpha(2+(2+\beta(-4+\delta)-\delta)\lambda)))}\right)\right)\right) < \\
& 0 \mathbin{\&\&} \left(\frac{2+\delta(-1+\alpha(1+\lambda-2\beta\lambda))}{8+2\alpha(-1+\beta)\delta^2\lambda+\delta(-6+\alpha(4+4\lambda-8\beta\lambda))} - \right. \\
& \left.\left(-\frac{(2+\delta(-1+\alpha(1+\lambda-2\beta\lambda)))^2}{2(-1+\alpha(-1+\beta)\delta\lambda)(4+\delta(-3+\alpha(2+(2+\beta(-4+\delta)-\delta)\lambda)))}\right)\right) > 0 \mathbin{\&\&} \\
& \frac{2+\delta(-1+\alpha(1+\lambda-2\beta\lambda))}{4+\alpha(-1+\beta)\delta^2\lambda+\delta(-3+\alpha(2+2\lambda-4\beta\lambda))} > \\
& \left(-\frac{(2+\delta(-1+\alpha(1+\lambda-2\beta\lambda)))^2}{2(-1+\alpha(-1+\beta)\delta\lambda)(4+\delta(-3+\alpha(2+(2+\beta(-4+\delta)-\delta)\lambda)))}\right) \mathbin{\&\&} \\
& 0<\delta<1 \mathbin{\&\&} 0<\alpha<1 \mathbin{\&\&} 0<\beta<1 \mathbin{\&\&} \\
& 1<\lambda<5,\ \text{Reals}\Bigg]
\end{aligned}
$$

```
False
```

Case 2 :

```
Reduce[
 (1 - α) ((2 - δ + α^3 (-1 + β) δ^2 (-1 + λ) (2 - 3 β (1 + λ) + β^2 (1 + λ)^2) + α (2 + δ - 2 λ + 3 δ λ -
            δ^2 λ + β (-4 + δ) (1 + (-1 + δ) λ)) + α^2 δ (4 - 4 λ + δ (-1 + 3 λ) + β^2 (1 + λ)
             (4 - 4 λ + δ (-1 + 2 λ)) + β (3 (-3 + 2 λ + λ^2) - 2 δ (-1 + 2 λ + λ^2))))) /
        (2 (4 - 3 δ + α^3 (-1 + β) δ^2 (-1 + λ) (δ + β^2 (1 + λ)^2 - β (2 + δ + 2 λ)) +
           α (δ (7 + λ - δ (3 + λ)) + β (4 (-1 + λ) + δ^2 (3 + λ) - 2 δ (1 + 3 λ))) -
           α^2 δ ((-4 + δ) δ + β^2 (δ^2 - 2 δ λ (1 + λ) + 4 (-1 + λ^2)) -
              2 β (-3 + δ^2 + 2 λ + λ^2 - δ (2 + λ + λ^2))))) -
      ((1 + α β (-1 + λ)) (-2 + δ (1 + α (-2 + β + β λ)))^2) /
       (2 (4 - 3 δ + α^3 (-1 + β) δ^2 (-1 + λ) (δ + β^2 (1 + λ)^2 - β (2 + δ + 2 λ)) -
           α^2 δ ((-4 + δ) δ + β^2 (-4 + δ^2 + 4 λ^2 - 2 δ λ (1 + λ)) -
              2 β (δ^2 + (-1 + λ) (3 + λ) - δ (2 + λ + λ^2))) +
           α (δ (7 + λ - δ (3 + λ)) + β (4 (-1 + λ) + δ (-2 - 6 λ + δ (3 + λ))))))) +
   α ((2 - δ + α^3 (-1 + β) δ^2 (-1 + λ) (2 - 3 β (1 + λ) + β^2 (1 + λ)^2) +
          α (2 + δ - 2 λ + 3 δ λ - δ^2 λ + β (-4 + δ) (1 + (-1 + δ) λ)) +
          α^2 δ (4 - 4 λ + δ (-1 + 3 λ) + β^2 (1 + λ) (4 - 4 λ + δ (-1 + 2 λ)) +
             β (3 (-3 + 2 λ + λ^2) - 2 δ (-1 + 2 λ + λ^2)))) /
        (2 (4 - 3 δ + α^3 (-1 + β) δ^2 (-1 + λ) (δ + β^2 (1 + λ)^2 - β (2 + δ + 2 λ)) +
           α (δ (7 + λ - δ (3 + λ)) + β (4 (-1 + λ) + δ^2 (3 + λ) - 2 δ (1 + 3 λ))) -
           α^2 δ ((-4 + δ) δ + β^2 (δ^2 - 2 δ λ (1 + λ) + 4 (-1 + λ^2)) -
              2 β (-3 + δ^2 + 2 λ + λ^2 - δ (2 + λ + λ^2))))) -
      ((1 + α β (-1 + λ)) (-2 + δ (1 + α (-2 + β + β λ)))^2) /
       (2 (4 - 3 δ + α^3 (-1 + β) δ^2 (-1 + λ) (δ + β^2 (1 + λ)^2 - β (2 + δ + 2 λ)) -
           α^2 δ ((-4 + δ) δ + β^2 (-4 + δ^2 + 4 λ^2 - 2 δ λ (1 + λ)) -
              2 β (δ^2 + (-1 + λ) (3 + λ) - δ (2 + λ + λ^2))) +
           α (δ (7 + λ - δ (3 + λ)) + β (4 (-1 + λ) + δ (-2 - 6 λ + δ (3 + λ))))))) <
   (1 - α) ((2 - δ + α^3 (-1 + β) δ^2 (-1 + λ) (2 - 3 β (1 + λ) + β^2 (1 + λ)^2) +
          α (2 + δ - 2 λ + 3 δ λ - δ^2 λ + β (-4 + δ) (1 + (-1 + δ) λ)) +
          α^2 δ (4 - 4 λ + δ (-1 + 3 λ) + β^2 (1 + λ) (4 - 4 λ + δ (-1 + 2 λ)) +
             β (3 (-3 + 2 λ + λ^2) - 2 δ (-1 + 2 λ + λ^2)))) /
        (2 (4 - 3 δ + α^3 (-1 + β) δ^2 (-1 + λ) (δ + β^2 (1 + λ)^2 - β (2 + δ + 2 λ)) +
           α (δ (7 + λ - δ (3 + λ)) + β (4 (-1 + λ) + δ^2 (3 + λ) - 2 δ (1 + 3 λ))) -
           α^2 δ ((-4 + δ) δ + β^2 (δ^2 - 2 δ λ (1 + λ) + 4 (-1 + λ^2)) -
              2 β (-3 + δ^2 + 2 λ + λ^2 - δ (2 + λ + λ^2))))) -
      ((-1 + α (-1 + β) δ) (1 + α β (-1 + λ)) (-2 + δ (1 + α (-2 + β + β λ)))) /
       (2 (4 - 3 δ + α^3 (-1 + β) δ^2 (-1 + λ) (δ + β^2 (1 + λ)^2 - β (2 + δ + 2 λ)) -
           α^2 δ ((-4 + δ) δ + β^2 (-4 + δ^2 + 4 λ^2 - 2 δ λ (1 + λ)) -
              2 β (-3 + δ^2 + 2 λ + λ^2 - δ (2 + λ + λ^2))) +
           α (δ (7 + λ - δ (3 + λ)) + β (4 (-1 + λ) + δ (-2 - 6 λ + δ (3 + λ))))))) +
   α λ ((2 - δ + α^3 (-1 + β) δ^2 (-1 + λ) (2 - 3 β (1 + λ) + β^2 (1 + λ)^2) +
          α (2 + δ - 2 λ + 3 δ λ - δ^2 λ + β (-4 + δ) (1 + (-1 + δ) λ)) +
          α^2 δ (4 - 4 λ + δ (-1 + 3 λ) + β^2 (1 + λ) (4 - 4 λ + δ (-1 + 2 λ)) +
             β (3 (-3 + 2 λ + λ^2) - 2 δ (-1 + 2 λ + λ^2)))) /
```

```
(2 (4 - 3 δ + α^3 (-1 + β) δ^2 (-1 + λ) (δ + β^2 (1 + λ)^2 - β (2 + δ + 2 λ)) +
     α (δ (7 + λ - δ (3 + λ)) + β (4 (-1 + λ) + δ^2 (3 + λ) - 2 δ (1 + 3 λ))) -
     α^2 δ ((-4 + δ) δ + β^2 (δ^2 - 2 δ λ (1 + λ) + 4 (-1 + λ^2)) -
        2 β (-3 + δ^2 + 2 λ + λ^2 - δ (2 + λ + λ^2))))) -
((-1 + α (-1 + β) δ) (1 + α β (-1 + λ)) (-2 + δ (1 + α (-2 + β + β λ)))) /
(2 (4 - 3 δ + α^3 (-1 + β) δ^2 (-1 + λ) (δ + β^2 (1 + λ)^2 - β (2 + δ + 2 λ)) -
     α^2 δ ((-4 + δ) δ + β^2 (-4 + δ^2 + 4 λ^2 - 2 δ λ (1 + λ)) -
        2 β (-3 + δ^2 + 2 λ + λ^2 - δ (2 + λ + λ^2))) +
     α (δ (7 + λ - δ (3 + λ)) + β (4 (-1 + λ) + δ (-2 - 6 λ + δ (3 + λ)))))) -
(1 - β) ((2 - δ + α^3 (-1 + β) δ^2 (-1 + λ) (2 - 3 β (1 + λ) + β^2 (1 + λ)^2) +
       α (2 + δ - 2 λ + 3 δ λ - δ^2 λ + β (-4 + δ) (1 + (-1 + δ) λ)) +
       α^2 δ (4 - 4 λ + δ (-1 + 3 λ) + β^2 (1 + λ) (4 - 4 λ + δ (-1 + 2 λ)) +
          β (3 (-3 + 2 λ + λ^2) - 2 δ (-1 + 2 λ + λ^2)))) /
    (2 (4 - 3 δ + α^3 (-1 + β) δ^2 (-1 + λ) (δ + β^2 (1 + λ)^2 - β (2 + δ + 2 λ)) +
          α (δ (7 + λ - δ (3 + λ)) + β (4 (-1 + λ) + δ^2 (3 + λ) - 2 δ (1 + 3 λ))) -
          α^2 δ ((-4 + δ) δ + β^2 (δ^2 - 2 δ λ (1 + λ) + 4 (-1 + λ^2)) -
             2 β (-3 + δ^2 + 2 λ + λ^2 - δ (2 + λ + λ^2))))) -
    ((1 + α β (-1 + λ)) (-2 + δ (1 + α (-2 + β + β λ)))^2) /
    (2 (4 - 3 δ + α^3 (-1 + β) δ^2 (-1 + λ) (δ + β^2 (1 + λ)^2 - β (2 + δ + 2 λ)) -
          α^2 δ ((-4 + δ) δ + β^2 (-4 + δ^2 + 4 λ^2 - 2 δ λ (1 + λ)) -
             2 β (δ^2 + (-1 + λ) (3 + λ) - δ (2 + λ + λ^2))) +
          α (δ (7 + λ - δ (3 + λ)) + β (4 (-1 + λ) + δ (-2 - 6 λ + δ (3 + λ)))))))) &&
((2 - δ + α^3 (-1 + β) δ^2 (-1 + λ) (2 - 3 β (1 + λ) + β^2 (1 + λ)^2) +
     α (2 + δ - 2 λ + 3 δ λ - δ^2 λ + β (-4 + δ) (1 + (-1 + δ) λ)) +
     α^2 δ (4 - 4 λ + δ (-1 + 3 λ) + β^2 (1 + λ) (4 - 4 λ + δ (-1 + 2 λ)) +
        β (3 (-3 + 2 λ + λ^2) - 2 δ (-1 + 2 λ + λ^2)))) /
  (2 (4 - 3 δ + α^3 (-1 + β) δ^2 (-1 + λ) (δ + β^2 (1 + λ)^2 - β (2 + δ + 2 λ)) +
        α (δ (7 + λ - δ (3 + λ)) + β (4 (-1 + λ) + δ^2 (3 + λ) - 2 δ (1 + 3 λ))) -
        α^2 δ ((-4 + δ) δ + β^2 (δ^2 - 2 δ λ (1 + λ) + 4 (-1 + λ^2)) -
           2 β (-3 + δ^2 + 2 λ + λ^2 - δ (2 + λ + λ^2))))) -
  ((-1 + α (-1 + β) δ) (1 + α β (-1 + λ)) (-2 + δ (1 + α (-2 + β + β λ)))) /
  (2 (4 - 3 δ + α^3 (-1 + β) δ^2 (-1 + λ) (δ + β^2 (1 + λ)^2 - β (2 + δ + 2 λ)) -
        α^2 δ ((-4 + δ) δ + β^2 (-4 + δ^2 + 4 λ^2 - 2 δ λ (1 + λ)) -
           2 β (-3 + δ^2 + 2 λ + λ^2 - δ (2 + λ + λ^2))) +
        α (δ (7 + λ - δ (3 + λ)) + β (4 (-1 + λ) + δ (-2 - 6 λ + δ (3 + λ)))))) -
  (1 - β) ((2 - δ + α^3 (-1 + β) δ^2 (-1 + λ) (2 - 3 β (1 + λ) + β^2 (1 + λ)^2) +
        α (2 + δ - 2 λ + 3 δ λ - δ^2 λ + β (-4 + δ) (1 + (-1 + δ) λ)) +
        α^2 δ (4 - 4 λ + δ (-1 + 3 λ) + β^2 (1 + λ) (4 - 4 λ + δ (-1 + 2 λ)) + β
           (3 (-3 + 2 λ + λ^2) - 2 δ (-1 + 2 λ + λ^2)))) /
     (2 (4 - 3 δ + α^3 (-1 + β) δ^2 (-1 + λ) (δ + β^2 (1 + λ)^2 - β (2 + δ + 2 λ)) +
           α (δ (7 + λ - δ (3 + λ)) + β (4 (-1 + λ) + δ^2 (3 + λ) - 2 δ (1 + 3 λ))) -
           α^2 δ ((-4 + δ) δ + β^2 (δ^2 - 2 δ λ (1 + λ) + 4 (-1 + λ^2)) -
              2 β (-3 + δ^2 + 2 λ + λ^2 - δ (2 + λ + λ^2))))) -
     ((1 + α β (-1 + λ)) (-2 + δ (1 + α (-2 + β + β λ)))^2) /
```

```
          (2 (4 - 3 δ + α^3 (-1 + β) δ^2 (-1 + λ) (δ + β^2 (1 + λ)^2 - β (2 + δ + 2 λ)) -
               α^2 δ ((-4 + δ) δ + β^2 (-4 + δ^2 + 4 λ^2 - 2 δ λ (1 + λ)) -
                  2 β (δ^2 + (-1 + λ) (3 + λ) - δ (2 + λ + λ^2))) +
               α (δ (7 + λ - δ (3 + λ)) + β (4 (-1 + λ) + δ (-2 - 6 λ + δ (3 + λ))))))) <
   0 && ((2 - δ + α^3 (-1 + β) δ^2 (-1 + λ) (2 - 3 β (1 + λ) + β^2 (1 + λ)^2) +
          α (2 + δ - 2 λ + 3 δ λ - δ^2 λ + β (-4 + δ) (1 + (-1 + δ) λ)) +
          α^2 δ (4 - 4 λ + δ (-1 + 3 λ) + β^2 (1 + λ) (4 - 4 λ + δ (-1 + 2 λ)) +
             β (3 (-3 + 2 λ + λ^2) - 2 δ (-1 + 2 λ + λ^2))))) /
       (2 (4 - 3 δ + α^3 (-1 + β) δ^2 (-1 + λ) (δ + β^2 (1 + λ)^2 - β (2 + δ + 2 λ)) +
             α (δ (7 + λ - δ (3 + λ)) + β (4 (-1 + λ) + δ^2 (3 + λ) - 2 δ (1 + 3 λ))) -
             α^2 δ ((-4 + δ) δ + β^2 (δ^2 - 2 δ λ (1 + λ) + 4 (-1 + λ^2)) -
                2 β (-3 + δ^2 + 2 λ + λ^2 - δ (2 + λ + λ^2))))) -
       ((1 + α β (-1 + λ)) (-2 + δ (1 + α (-2 + β + β λ)))^2) /
       (2 (4 - 3 δ + α^3 (-1 + β) δ^2 (-1 + λ) (δ + β^2 (1 + λ)^2 - β (2 + δ + 2 λ)) -
             α^2 δ ((-4 + δ) δ + β^2 (-4 + δ^2 + 4 λ^2 - 2 δ λ (1 + λ)) -
                2 β (δ^2 + (-1 + λ) (3 + λ) - δ (2 + λ + λ^2))) +
             α (δ (7 + λ - δ (3 + λ)) + β (4 (-1 + λ) + δ (-2 - 6 λ + δ (3 + λ)))))) < 0 &&
   ((1 + α β (-1 + λ)) (-2 + α (-1 + β) δ (1 + λ)) (-2 + δ (1 + α (-2 + β + β λ)))) /
     (2
        (4 - 3 δ + α^3 (-1 + β) δ^2 (-1 + λ) (δ + β^2 (1 + λ)^2 - β (2 + δ + 2 λ)) -
          α^2 δ ((-4 + δ) δ + β^2 (-4 + δ^2 + 4 λ^2 - 2 δ λ (1 + λ)) -
             2 β (-3 + δ^2 + 2 λ + λ^2 - δ (2 + λ + λ^2))) +
          α (δ (7 + λ - δ (3 + λ)) + β (4 (-1 + λ) + δ (-2 - 6 λ + δ (3 + λ)))))) >
    ((1 + α β (-1 + λ)) (-2 + δ (1 + α (-2 + β + β λ)))^2) /
     (2
        (4 - 3 δ + α^3 (-1 + β) δ^2 (-1 + λ) (δ + β^2 (1 + λ)^2 - β (2 + δ + 2 λ)) -
          α^2 δ ((-4 + δ) δ + β^2 (-4 + δ^2 + 4 λ^2 - 2 δ λ (1 + λ)) -
             2 β (δ^2 + (-1 + λ) (3 + λ) - δ (2 + λ + λ^2))) +
          α (δ (7 + λ - δ (3 + λ)) + β (4 (-1 + λ) + δ (-2 - 6 λ + δ (3 + λ)))))) &&
   0 < δ < 1 && 0 < α < 1 && 0 < β <
    1 &&
   1 <
    λ <
    5,
  Reals] // Simplify
False
```

Case 3 :

```
Reduce[(1 - α) (((-2 + δ (1 + α (-1 + β) (1 + λ)))
           (1 - α (-1 + β) δ λ + α (-1 + β) (-1 + λ) (-2 + δ (1 + α (-1 + β) (1 + λ))))) /
        (2 (-4 + 3 δ + α (-1 + β) (4 (-1 + λ) + δ (6 - δ + 2 λ - 3 δ λ + α^2 (-1 + β)^2 δ
                 (-1 + λ) (1 + λ)^2 + α (-1 + β) (4 - 2 δ - 2 δ λ + (-4 + δ^2) λ^2))))) -
       ((-1 + α (-1 + β) (-1 + λ)) (-2 + δ (1 + α (-1 + β) (1 + λ)))^2) /
        (2 (-4 + 3 δ + α (-1 + β) (4 (-1 + λ) + δ (6 - δ + 2 λ - 3 δ λ + α^2 (-1 + β)^2 δ (-1 + λ)
                 (1 + λ)^2 + α (-1 + β) (4 - 2 δ - 2 δ λ + (-4 + δ^2) λ^2)))))) +
```

$$\alpha\,(((-2+\delta\,(1+\alpha\,(-1+\beta)\,(1+\lambda)))\,(1-\alpha\,(-1+\beta)\,\delta\,\lambda+\alpha\,(-1+\beta)\,(-1+\lambda)$$
$$(-2+\delta\,(1+\alpha\,(-1+\beta)\,(1+\lambda)))))\,/$$
$$(2\,(-4+3\,\delta+\alpha\,(-1+\beta)\,(4\,(-1+\lambda)+\delta\,(6-\delta+2\,\lambda-3\,\delta\,\lambda+\alpha^2\,(-1+\beta)^2\,\delta$$
$$(-1+\lambda)\,(1+\lambda)^2+\alpha\,(-1+\beta)\,(4-2\,\delta-2\,\delta\,\lambda+(-4+\delta^2)\,\lambda^2)))))-$$
$$((-1+\alpha\,(-1+\beta)\,(-1+\lambda))\,(-2+\delta\,(1+\alpha\,(-1+\beta)\,(1+\lambda)))^2)\,/$$
$$(2\,(-4+3\,\delta+\alpha\,(-1+\beta)\,(4\,(-1+\lambda)+\delta\,(6-\delta+2\,\lambda-3\,\delta\,\lambda+\alpha^2\,(-1+\beta)^2\,\delta\,(-1+\lambda)$$
$$(1+\lambda)^2+\alpha\,(-1+\beta)\,(4-2\,\delta-2\,\delta\,\lambda+(-4+\delta^2)\,\lambda^2))))))<$$
$$(1-\alpha)\,(((-2+\delta\,(1+\alpha\,(-1+\beta)\,(1+\lambda)))\,(1-\alpha\,(-1+\beta)\,\delta\,\lambda+\alpha\,(-1+\beta)$$
$$(-1+\lambda)\,(-2+\delta\,(1+\alpha\,(-1+\beta)\,(1+\lambda)))))\,/$$
$$(2\,(-4+3\,\delta+\alpha\,(-1+\beta)\,(4\,(-1+\lambda)+\delta\,(6-\delta+2\,\lambda-3\,\delta\,\lambda+\alpha^2\,(-1+\beta)^2\,\delta$$
$$(-1+\lambda)\,(1+\lambda)^2+\alpha\,(-1+\beta)\,(4-2\,\delta-2\,\delta\,\lambda+(-4+\delta^2)\,\lambda^2)))))-$$
$$((-1+\alpha\,(-1+\beta)\,(-1+\lambda))\,(-1+\alpha\,(-1+\beta)\,\delta\,\lambda)\,(-2+\delta\,(1+\alpha\,(-1+\beta)\,(1+\lambda))))\,/$$
$$(2\,(-4+3\,\delta+\alpha\,(-1+\beta)\,(4\,(-1+\lambda)+\delta\,(6-\delta+2\,\lambda-3\,\delta\,\lambda+\alpha^2\,(-1+\beta)^2\,\delta\,(-1+\lambda)$$
$$(1+\lambda)^2+\alpha\,(-1+\beta)\,(4-2\,\delta-2\,\delta\,\lambda+(-4+\delta^2)\,\lambda^2))))))+$$
$$\alpha\,(((-2+\delta\,(1+\alpha\,(-1+\beta)\,(1+\lambda)))\,(1-\alpha\,(-1+\beta)\,\delta\,\lambda+\alpha\,(-1+\beta)\,(-1+\lambda)$$
$$(-2+\delta\,(1+\alpha\,(-1+\beta)\,(1+\lambda)))))\,/$$
$$(2\,(-4+3\,\delta+\alpha\,(-1+\beta)\,(4\,(-1+\lambda)+\delta\,(6-\delta+2\,\lambda-3\,\delta\,\lambda+\alpha^2\,(-1+\beta)^2\,\delta$$
$$(-1+\lambda)\,(1+\lambda)^2+\alpha\,(-1+\beta)\,(4-2\,\delta-2\,\delta\,\lambda+(-4+\delta^2)\,\lambda^2)))))-$$
$$((-1+\alpha\,(-1+\beta)\,(-1+\lambda))\,(-1+\alpha\,(-1+\beta)\,\delta\,\lambda)\,(-2+\delta\,(1+\alpha\,(-1+\beta)\,(1+\lambda))))\,/$$
$$(2\,(-4+3\,\delta+\alpha\,(-1+\beta)\,(4\,(-1+\lambda)+\delta\,(6-\delta+2\,\lambda-3\,\delta\,\lambda+\alpha^2\,(-1+\beta)^2\,\delta$$
$$(-1+\lambda)\,(1+\lambda)^2+\alpha\,(-1+\beta)\,(4-2\,\delta-2\,\delta\,\lambda+(-4+\delta^2)\,\lambda^2)))))-$$
$$(1-\beta)\,(((-2+\delta\,(1+\alpha\,(-1+\beta)\,(1+\lambda)))\,(1-\alpha\,(-1+\beta)\,\delta\,\lambda+\alpha\,(-1+\beta)$$
$$(-1+\lambda)\,(-2+\delta\,(1+\alpha\,(-1+\beta)\,(1+\lambda)))))\,/$$
$$(2\,(-4+3\,\delta+\alpha\,(-1+\beta)\,(4\,(-1+\lambda)+\delta\,(6-\delta+2\,\lambda-3\,\delta\,\lambda+\alpha^2\,(-1+\beta)^2\,\delta$$
$$(-1+\lambda)\,(1+\lambda)^2+\alpha\,(-1+\beta)\,(4-2\,\delta-2\,\delta\,\lambda+(-4+\delta^2)\,\lambda^2)))))-$$
$$((-1+\alpha\,(-1+\beta)\,(-1+\lambda))\,(-2+\delta\,(1+\alpha\,(-1+\beta)\,(1+\lambda)))^2)\,/$$
$$(2\,(-4+3\,\delta+\alpha\,(-1+\beta)\,(4\,(-1+\lambda)+\delta\,(6-\delta+2\,\lambda-3\,\delta\,\lambda+\alpha^2\,(-1+\beta)^2\,\delta\,(-1+$$
$$\lambda)\,(1+\lambda)^2+\alpha\,(-1+\beta)\,(4-2\,\delta-2\,\delta\,\lambda+(-4+\delta^2)\,\lambda^2)))))))\ \&\&$$
$$(((-2+\delta\,(1+\alpha\,(-1+\beta)\,(1+\lambda)))\,(1-\alpha\,(-1+\beta)\,\delta\,\lambda+\alpha\,(-1+\beta)\,(-1+\lambda)$$
$$(-2+\delta\,(1+\alpha\,(-1+\beta)\,(1+\lambda)))))\,/$$
$$(2\,(-4+3\,\delta+\alpha\,(-1+\beta)\,(4\,(-1+\lambda)+\delta\,(6-\delta+2\,\lambda-3\,\delta\,\lambda+\alpha^2\,(-1+\beta)^2\,\delta$$
$$(-1+\lambda)\,(1+\lambda)^2+\alpha\,(-1+\beta)\,(4-2\,\delta-2\,\delta\,\lambda+(-4+\delta^2)\,\lambda^2)))))-$$
$$((-1+\alpha\,(-1+\beta)\,(-1+\lambda))\,(-1+\alpha\,(-1+\beta)\,\delta\,\lambda)\,(-2+\delta\,(1+\alpha\,(-1+\beta)\,(1+\lambda))))\,/$$
$$(2\,(-4+3\,\delta+\alpha\,(-1+\beta)\,(4\,(-1+\lambda)+\delta\,(6-\delta+2\,\lambda-3\,\delta\,\lambda+\alpha^2\,(-1+\beta)^2\,\delta$$
$$(-1+\lambda)\,(1+\lambda)^2+\alpha\,(-1+\beta)\,(4-2\,\delta-2\,\delta\,\lambda+(-4+\delta^2)\,\lambda^2)))))-$$
$$(1-\beta)\,(((-2+\delta\,(1+\alpha\,(-1+\beta)\,(1+\lambda)))\,(1-\alpha\,(-1+\beta)\,\delta\,\lambda+\alpha\,(-1+\beta)$$
$$(-1+\lambda)\,(-2+\delta\,(1+\alpha\,(-1+\beta)\,(1+\lambda)))))\,/$$
$$(2\,(-4+3\,\delta+\alpha\,(-1+\beta)\,(4\,(-1+\lambda)+\delta\,(6-\delta+2\,\lambda-3\,\delta\,\lambda+\alpha^2\,(-1+\beta)^2$$
$$\delta\,(-1+\lambda)\,(1+\lambda)^2+\alpha\,(-1+\beta)\,(4-2\,\delta-2\,\delta\,\lambda+(-4+\delta^2)\,\lambda^2)))))-$$
$$((-1+\alpha\,(-1+\beta)\,(-1+\lambda))\,(-2+\delta\,(1+\alpha\,(-1+\beta)\,(1+\lambda)))^2)\,/$$
$$(2\,(-4+3\,\delta+\alpha\,(-1+\beta)\,(4\,(-1+\lambda)+\delta\,(6-\delta+2\,\lambda-3\,\delta\,\lambda+\alpha^2\,(-1+\beta)^2\,\delta\,(-1+\lambda)$$
$$(1+\lambda)^2+\alpha\,(-1+\beta)\,(4-2\,\delta-2\,\delta\,\lambda+(-4+\delta^2)\,\lambda^2)))))))>0\ \&\&$$
$$(((-2+\delta\,(1+\alpha\,(-1+\beta)\,(1+\lambda)))\,(1-\alpha\,(-1+\beta)\,\delta\,\lambda+\alpha\,(-1+\beta)\,(-1+\lambda)$$
$$(-2+\delta\,(1+\alpha\,(-1+\beta)\,(1+\lambda)))))\,/$$
$$(2\,(-4+3\,\delta+\alpha\,(-1+\beta)\,(4\,(-1+\lambda)+\delta\,(6-\delta+2\,\lambda-3\,\delta\,\lambda+\alpha^2\,(-1+\beta)^2\,\delta$$

```
        (-1 + λ) (1 + λ)^2 + α (-1 + β) (4 - 2 δ - 2 δ λ + (-4 + δ^2) λ^2))))) -
  ((-1 + α (-1 + β) (-1 + λ)) (-2 + δ (1 + α (-1 + β) (1 + λ)))^2) /
   (2 (-4 + 3 δ + α (-1 + β) (4 (-1 + λ) + δ (6 - δ + 2 λ - 3 δ λ + α^2 (-1 + β)^2 δ (-1 + λ)
          (1 + λ)^2 + α (-1 + β) (4 - 2 δ - 2 δ λ + (-4 + δ^2) λ^2)))))) > 0 &&
((-1 + α (-1 + β) (-1 + λ)) (-2 + α (-1 + β) δ (1 + λ)) (-2 + δ (1 + α (-1 + β) (1 + λ)))) /
  (2
    (-4 + 3 δ + α (-1 + β) (4 (-1 + λ) + δ (6 - δ + 2 λ - 3 δ λ + α^2 (-1 + β)^2 δ
          (-1 + λ) (1 + λ)^2 + α (-1 + β) (4 - 2 δ - 2 δ λ + (-4 + δ^2) λ^2))))) >
 ((-1 + α (-1 + β) (-1 + λ)) (-2 + δ (1 + α (-1 + β) (1 + λ)))^2) /
  (2
    (-4 + 3 δ + α (-1 + β) (4 (-1 + λ) + δ (6 - δ + 2 λ - 3 δ λ + α^2 (-1 + β)^2 δ
          (-1 + λ) (1 + λ)^2 + α (-1 + β) (4 - 2 δ - 2 δ λ + (-4 + δ^2) λ^2))))) &&
 0 < δ < 1 && 0 < α < 1 && 0 < β < 1 && 1 < λ < 5, Reals]
```

```
False
```

Case 4 :

```
Reduce[
 (1 - α) (-(-2 + (1 + 2 α (-1 + β)) δ)/(2 (4 + (-3 + α (-1 + β) (-4 + δ)) δ)) - (-((-2 + (1 + 2 α (-1 + β)) δ)^2 / (2 (-1 + α
            (-1 + β) δ) (4 + (-3 + α (-1 + β) (-4 + δ)) δ))))) +
   α (-(-2 + (1 + 2 α (-1 + β)) δ)/(2 (4 + (-3 + α (-1 + β) (-4 + δ)) δ)) - (-((-2 + (1 + 2 α (-1 + β)) δ)^2 /
           (2 (-1 + α (-1 + β) δ) (4 + (-3 + α (-1 + β) (-4 + δ)) δ))))) <
   (1 - α) (-(-2 + (1 + 2 α (-1 + β)) δ)/(2 (4 + (-3 + α (-1 + β) (-4 + δ)) δ)) - (-(-2 + (1 + 2 α (-1 + β)) δ)/(2 (4 + (-3 + α (-1 + β) (-4 + δ)) δ)))) +
   α (-(-2 + (1 + 2 α (-1 + β)) δ)/(2 (4 + (-3 + α (-1 + β) (-4 + δ)) δ)) - (-(-2 + (1 + 2 α (-1 + β)) δ)/(2 (4 + (-3 + α (-1 + β) (-4 + δ)) δ))) -
       (1 - β) (-(-2 + (1 + 2 α (-1 + β)) δ)/(2 (4 + (-3 + α (-1 + β) (-4 + δ)) δ)) - (-((-2 + (1 + 2 α (-1 + β)) δ)^2 /
              (2 (-1 + α (-1 + β) δ) (4 + (-3 + α (-1 + β) (-4 + δ)) δ)))))) &&
 (-(-2 + (1 + 2 α (-1 + β)) δ)/(2 (4 + (-3 + α (-1 + β) (-4 + δ)) δ)) - (-(-2 + (1 + 2 α (-1 + β)) δ)/(2 (4 + (-3 + α (-1 + β) (-4 + δ)) δ))) -
    (1 - β) (-(-2 + (1 + 2 α (-1 + β)) δ)/(2 (4 + (-3 + α (-1 + β) (-4 + δ)) δ)) - (-((-2 + (1 + 2 α (-1 + β)) δ)^2 /
            (2 (-1 + α (-1 + β) δ) (4 + (-3 + α (-1 + β) (-4 + δ)) δ)))))) > 0 &&
 (-(-2 + (1 + 2 α (-1 + β)) δ)/(2 (4 + (-3 + α (-1 + β) (-4 + δ)) δ)) - (-((-2 + (1 + 2 α (-1 + β)) δ)^2 /
         (2 (-1 + α (-1 + β) δ) (4 + (-3 + α (-1 + β) (-4 + δ)) δ))))) < 0 &&
 (-(-2 + (1 + 2 α (-1 + β)) δ)/(4 + (-3 + α (-1 + β) (-4 + δ)) δ)) > (-((-2 + (1 + 2 α (-1 + β)) δ)^2 /
       (2 (-1 + α (-1 + β) δ) (4 + (-3 + α (-1 + β) (-4 + δ)) δ)))) &&
 0 < δ < 1 && 0 < α < 1 && 0 < β < 1 && 1 < λ < 5, Reals]
```

```
1 < λ < 5 && 0 < β < 1 && 0 < δ < 1 && 0 < α < 1
```