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Master Thesis*

Heterogeneous Beliefs and Destabilization of Financial Markets

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Abstract

I develop a five-type heterogeneous agent model that provides a framework to empirically investigate the financial market described by De Long et al. (1990a,b). I show that it is feasible to construct a heterogeneous agent model where arbitrageurs can engage in momentum trading and noise traders leave the market due to short-sale constraints. The empirical results provide evidence that supports the theoretical framework by De Long et al. (1990a,b) in various respects. The analysis includes both simulation and estimation and the results show that arbitrageurs who are herding can have either a dampening or a destabilizing effect on asset prices. I find that a higher fraction of arbitrageurs that engage in momentum trading results in a higher price deviation from fundamental value. Similarly, a higher fraction of noise trader demand increases the misspricing and is associated with a more unstable market.

Keywords: Behavioral Finance; Market Efficiency; Heterogeneous agent models; Herd Behavior

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Preface and Acknowledgements

This thesis is the finishing line of both the Master programmes in Econometrics and Economics. The past years at the Erasmus School of Economics have given me the opportunity to follow courses on various interesting topics. Along the way, my interest for financial markets and behavioral finance grew. This thesis combines these topics and I rest assured that reading my thesis will give a better understanding of investor behavior and the effect on asset prices.

Since I started the Bachelor in Econometrics in 2009, Rotterdam and the Erasmus School of Economics have been an inspiring and enjoyable home. The Erasmus School of Economics is a place that triggers ambition, strengthens self-reliance, and allowed me to develop both my academical and practical skills. I could not have hoped for a better place.

I would like to thank Erik Kole and Maurizio Montone for their helpful comments guiding my efforts in the right way. As supervisors, you have shown me that Econometrics and Economics make a perfect combination. Each with its own quality.

The people I met and the friends I made during my time as a student in Rotterdam made my studies an even more pleasant experience, for which I would like to express my gratitude. Without going into further details, celebrations after a successful team effort were always the best. Without doubt, the people that have been the most supportive since the beginning are my parents. They have given me the opportunity, the perseverance, and the brains that were all essential during the course of my study program. I hope that I make you proud. Last but not least, I am indebted to my girlfriend Saskia for her encouragements, kind patience and understanding within the creation of this thesis. Thank you.

Contents

1 Introduction

It is well-known that in financial markets assets are not always priced at their fundamental value. Traditional economic theory, such as the efficient market hypothesis is however based on the rationality assumption and hence does not allow for prices to deviate from this fundamental value. Practice shows otherwise, as anomalies such as speculative prices, financial bubbles, and other stylized facts of asset prices are common in markets. Hence, research has shifted economic theory towards a broader and more social perspective which created amongst other things the vast field of behavioral finance (Shiller, 2003; Subrahmanyam, 2007).

Outstanding papers in this field show that investors are not completely rational as they can under- and overreact (Barberis et al., 1998; Daniel et al., 1998; Hong and Stein, 1999), or be overconfident (Odean, 1998; Daniel et al., 2001; Kyle and Wang, 1997). De Long et al. (1990a,b) present a heterogeneous market with noise traders and arbitrageurs and find that unpredictability of noise traders can result in high price deviations from the fundamental value (De Long et al., 1990a) and that the behavior of arbitrageurs can amplify this effect by trading ahead of noise traders' positive feedback strategies (De Long et al., 1990b). Their analyses provide a popular view on a micromarket that explains speculative prices, but this view is not fully tested empirically, however.

To this purpose, I model their dynamics using a heterogeneous agent model (Brock and Hommes, 1997, 1998) and estimate it to real price data. The methodology of this thesis builds on the asset pricing model by Boswijk et al. (2007). I refine their two-type HAM in order to include the more sophisticated framework by De Long et al. (1990a,b). Heterogeneous agents, divided into arbitrageurs and noise traders, switch between five different beliefs about next period's price. The market price is the weighted average of these five price expectations. The model follows De Long et al. (1990b) as arbitrageurs can herd by "jumping on the bandwagon" and it follows Chen et al. (2002) as noise traders do face short-sale constraints. As such, the model is used to further investigate the market described by De Long et al. (1990a,b).

I make two propositions, one that states that herding by arbitrageurs destabilizes the market and a second that states that noise trader demand has the same effect on prices. Using simulations I test whether the five-type model is able to give insight into these two statements. Next, I use annual data on the S&P500 index from 1950 to 2014 to empirically investigate the propositions.

Consistent with the first prediction, I find that a higher fraction of arbitrageurs that engage in momentum trading is associated with a higher deviation of the market price from the fundamental value. In addition, the time series of the fractions of the estimations show that arbitrageurs set a trend before noise traders do. The simulation results are mixed as the model considers both the dampening as well as the extrapolating effects of rational speculators' trading.

Consistent with the second proposition, I find that noise trader demand is associated with a higher deviation from the fundamental value. The fractions show that noise traders are responsible for a positive price shock. When dividing the time series using decile groups based on the fractions of noise traders I find a remarkable U-shape in volatility and market activity. Both seem to be higher when noise trader demand is at the extremes. This result and the graphs of the fractions show that a market with a high or low fraction of noise traders is a relatively unstable market according to the five-type model.

My analysis shows how heterogeneous agent models (in short HAMs) can be used to better understand a mircomarket. In this case, I manage to investigate the market by De Long et al. (1990a,b) using a five-type HAM. The simulation and estimations results provide better insights into the market dynamics and confirm my expectations in various respects.

I contribute to the academic debate in two ways. First, I present a new HAM that refines the simple two-type model by Boswijk et al. (2007). Most of the literature using HAMs does not examine herd behavior in particular and most models cannot be estimated using data. I present a model that is suitable for both purposes. Second, this HAM follows the market description provided by De Long et al. (1990a,b). Hence it allows for investigation of the impact of positive feedback trading by arbitrageurs and financial decisions by noise traders. As the model can also be estimated to real data, it enables us to further investigate empirically the popular theoretical framework of De Long et al. (1990a,b). In that sense, this thesis is an improvement on the understanding of financial markets from a behavioral perspective.

This thesis proceeds as follows. Section 2 reviews the literature on behavioral finance and herding in particular. Section 3 then explains the methodology and introduces the five-type model and the two propositions. Section 4 shows the simulation and estimation results and Section 5 concludes.

2 Literature Review

This thesis refers to three main strands of literature. First is the strand of research that explains how and why assumptions on rationality play a limited role in understanding behavioral issues in finance theory. Traditional economic theory suggests that investors are fully rational and prices reflect all obtainable information. This efficient market theory is a fertile medium for academics to develop finance models using these rational expectations. The research by psychologists Tversky and Kahneman (1973, 1974) shifted the academic discussion towards behavioral aspects of finance, creating an alternative view of financial markets that allows for suboptimal behavior among (groups of) investors. They explain how people rely on so-called heuristics that simplify decision making, leading to unpredictable or even irrational financial decisions. Barberis and Thaler (2003), Shiller (2003), and Subrahmanyam (2007) all review the growing amount of literature on behavioral finance and conclude that further studies should bear the weaknesses of the efficient markets theory in mind and focus on biases of agents and the effect on prices.

Examples that take this conclusion into account are Barberis et al. (1998), Daniel et al. (1998), and Hong and Stein (1999). They use heuristics to explain how investors form beliefs and relate it to empirical findings. Their main topic is the under- and overreaction of investors. Barberis and Shleifer (2003) study an economy with switching investors between different investment styles, also relating it to the heuristics. Other examples are Odean (1998) and Daniel et al. (2001) who argue that investors can be overconfident about the value of information and their own abilities, which could be an explanation for under- and overreaction. Daniel et al. (2001) conclude that so-called arbitrageurs can profit by trading against the mispricing caused by overconfidence. Kyle and Wang (1997) test overconfidence using game theory. They conclude that overconfidence indeed can exist in financial markets. Also asymmetric information among investors is a counterargument to efficient markets. Hirshleifer et al. (1994) explain that markets are not efficient as some investors receive information before others do. This leads to herding as the lately informed investors follow the trading strategy by the short-term investors who are informed first. Wang (1998) adds heterogeneity to the discussion, which they find in trader's disagreement on the precision of an informed trader's private signal. They show that their multi-period trading model corresponds to observed volume patterns and price volatility. The main objective of these papers is to find explanations for observed asset price anomalies that rational financial theories fail to explain until now. Behavioral finance provides excellent guidance for this search, as it has already discovered many new areas that could explain what is happening in financial markets (Shiller, 2003). This thesis is an addition to that.

The second strand of literature I address is a behavioral concept called herd behavior. This behavioral concept is among others described by Avery and Zemsky (1998) as "investors imitating prior actions of others, or following the crowd." These investors make their financial decisions based on a specific piece of information, ignoring other relevant parts of the puzzle. Keynes (1930) explains how agents rationally judge that the crowd is better informed, hence discount useful private information in favor of the actions of others. Baddeley et al. (2007) and Baddeley (2010) provide a more socio-psychological explanation for why people herd, relating it to the heuristics presented by Tversky and Kahneman (1974). De Long et al. (1990b) show that herd behavior can be rational as well as suboptimal behavior. They explain how rational speculators can follow a positive feedback investment strategy and "jump on the bandwagon" to trade ahead of demand by so-called noise traders. This herd behavior by rational investors can have a destabilizing effect on prices and they argue that it provides an explanation for overreaction. This approach is a different approach to market misvaluation as it focuses more on the consequences of trading behavior instead of the updating of expectations. It might be a different approach to investigate financial markets, but De Long et al. (1990b) provide a clear and popular framework that is used to model this trading behavior and it is therefore used as a guideline for this thesis.

Besides theoretical papers on herding, there is also a large number of papers that empirically investigate herd behavior. Two main approaches are statistical measures for herding and sequential trading models. As for the first approach, Lakonishok et al. (1992), Grinblatt et al. (1995), and Wermers (1999) use statistical measures for herding and focus on trading behavior of mutual funds. Popular is the measure by Lakonishok et al. (1992) that describes the "average tendency of funds to buy or sell a particular stock during a period, relative to what could be expected if these funds traded independently." Some papers find significant clustering by fund managers. But as Bikhchandani and Sharma (2000) and the previous mentioned authors themselves emphasize, these statistical measures do not provide for a clear measure of herding and need to be further refined.

The second approach is a more abstract way to model herd behavior as it describes an environment in which agents sequentially make decisions based on their private information and the actions of others. Avery and Zemsky (1998) investigate herd behavior in financial markets studying such a sequential trading financial market. They find that a sufficiently complex information structure makes price bubbles possible. Park and Sabourian (2011) reconsider this conclusion. They characterize different circumstances under which herding and contrarian behavior can occur in markets with efficient prices and that are consistent with large price fluctuations. Cipriani and Guarino (2014) also build upon the work of Avery and Zemsky (1998) by changing the model in order to estimate it to transaction data of a NYSE stock. They provide a structural estimation framework to investigate herding and find that herding is present during specific periods in a trading day. The main drawback of these models is that high-frequency data is needed in order to use these models for empirical analysis as trading signals play an important role.

Both empirical approaches have their cons and the main focus of these approaches is to find evidence for herd behavior instead of relating the behavioral concept to observed price anomalies. In this thesis a different approach is used, which is explained below.

The third strand of literature my work is related to is the literature on heterogeneous agent models or HAMs. This approach is introduced by Brock and Hommes (1997, 1998) who describe an asset pricing model with heterogeneous beliefs among agents who endogenously switch between different heuristics. According to LeBaron (2006) these agent-based models are especially suitable for verification of findings from behavioral finance. The following examples show that HAMs are used widely for different purposes. Boswijk et al. (2007) use a HAM to investigate behavioral heterogeneity and are the first to attempt to estimate a HAM to real market data. Similarly, De Jong et al. (2009) use a HAM to investigate a behavior called shift-contagion. They find evidence for a regime called the internationalist that allows to investigate the co-movement between home and foreign markets. Social interactions are studied using HAMs by Chiarella et al. (2003) and Chang (2007). Lux (1998), Lux and Marchesi (2000), Alfarano et al. (2005), and De Grauwe and Grimaldi (2006) all us the HAM to give an explanation of market dynamics such as volatility clustering, fat tails of the return distribution, and the exchange rate puzzle. The examples given here show that HAMs are indeed suitable for the research of findings from behavioral finance, hence the interest in this methodology.

3 Methodology

I develop a heterogeneous agent model that is used for investigation of the market dynamics described by De Long et al. (1990a,b). The heterogeneous agent model is introduced by Brock and Hommes (1997, 1998) and reformulated by Boswijk et al. (2007). Their ideas are a starting point for the model introduced in this section.

3.1 Heterogeneous Agent Models

Brock and Hommes (1998) present an asset pricing model with heterogeneous beliefs of the future price of a risky asset with agents choosing from the finite set of beliefs based on the past performance. They consider a model with one risky and one risk free asset with gross return $1 + r > 1$. The dynamics of wealth W_t are described by:

$$W_{t+1} = (1 + r)W_t + (P_{t+1} + Y_{t+1} - (1 + r)P_t)z_t\,, \tag{1}$$

where P_t denotes the (ex-dividend) price of the risky asset at time t, Y_t the stochastic cash flow (e.g. dividend) process, and z_t denotes the number of shares purchased at date t. The net cash flow from the risky asset is defined as:

$$R_{t+1} = P_{t+1} + Y_{t+1} - (1 + r)P_t\,, \tag{2}$$

meaning that I assume that agents borrow at the risk free rate r.

The market consists of H types of agents that follow a specific belief. As these beliefs are different per type, the market is heterogeneous. The model determines the fraction of investors per type ($n_{h,t}$). How these investors behave will be explained in coming sections. But first I discuss an essential assumption of the model by Brock and Hommes (1998), which is the type of utility function. It is assumed that each agent of type h has a utility function that exhibits constant absolute risk aversion (CARA). This means that they adopt a mean-variance efficient strategy. $E_{h,t}(.)$ and $V_{h,t}(.)$ denote the expectation of the conditional mean and variance operators of the wealth in next period, hence each agent solves:

$$z_{h,t} = \max_{z_{h,t}} E_{h,t}(W_{t+1}) - (a_h/2)V_{h,t}(W_{t+1}), \tag{3}$$

with risk aversion parameter a_h. Given equation (3) and the net cash flow from the risky asset (2), the mean-variance demand function of agent h is given by:

$$z_{h,t} = \frac{E_{h,t}(R_{t+1})}{a_h V_{h,t}(R_{t+1})}. \tag{4}$$

With zero net supply of the risky asset (Brock and Hommes, 1998) and assuming that all agents have the same risk aversion parameter (i.e. $a_h = a$) and homogeneous expectations about the conditional variance (i.e. $V_{h,t}(R_{t+1}) = V_t(R_{t+1})$), the market clearing equation is:

$$\sum_{h=1}^{H} n_{h,t} \frac{E_{h,t}(P_{t+1} + Y_{t+1}) - (1+r)P_t}{aV_t(R_{t+1})} = 0, \tag{5}$$

where $n_{h,t}$ is the fraction of investors in the economy of type h at the beginning of period t, before the actions of others are observed. The equilibrium pricing equation is now given by:

$$P_t = \frac{1}{1+r} \sum_{h=1}^{H} n_{h,t} E_{h,t}(P_{t+1} + Y_{t+1}). \tag{6}$$

The equilibrium pricing equation shows that the price at time t of the risky asset depends on the weighted average of the beliefs about next period payoffs. In addition, it shows that the equilibrium price will be high (low) when the fraction of investors expecting a high (low) next period payoff is large. As Boswijk et al. (2007) notice, equation (6) can be reformulated to:

$$r = \sum_{h=1}^{H} n_{h,t} \frac{E_{h,t}(P_{t+1} + Y_{t+1} - P_t)}{P_t}, \tag{7}$$

which shows that in equilibrium investors on average expect to earn the riskless discount rate r.

3.2 Deviation from Fundamental Value

A variation of the method by Brock and Hommes (1998) is the reformulation by Boswijk et al. (2007) of the pricing equation (6) in terms of price-to-cash-flow (PY) ratio, $d_t = P_t/Y_t$. This resembles the ideas of Campbell and Shiller (1988a,b) on stock valuation ratios. With agents having perfect knowledge of the underlying fundamental value, but heterogeneous beliefs about asset prices, it is useful to express their beliefs as a function of the deviation from fundamental value. This expression allows for a better interpretation of the model and a link to other research on the effect of different beliefs on the persistence of the deviation from the fundamental value. The deviation is often considered as it considers only the price changes not related to changes in fundamental value. First, the cash flow process and the fundamental PY-ratio are defined and then the model is specified in terms of deviation from this fundamental value.

Similar to Boswijk et al. (2007), I assume that the cash flow process follows a Gaussian random walk with drift, which means that:

$$\log Y_{t+1} = \mu + \log Y_t + \nu_{t+1}, \qquad \nu_{t+1} \sim \text{i.i.d. } N(0, \sigma_\nu^2)\,, \tag{8}$$

which implies:

$$\frac{Y_{t+1}}{Y_t} = e^{\mu+\nu_{t+1}} = e^{\mu+(1/2)\sigma_\nu^2} e^{\nu_{t+1}-(1/2)\sigma_\nu^2} = (1+g)\varepsilon_{t+1}\,, \tag{9}$$

where $\varepsilon_{t+1} = e^{\nu_{t+1}-(1/2)\sigma_\nu^2}$, hence $E_t(\varepsilon_{t+1}) = 1$. Here, g denotes the constant cash flow growth rate. As all agents are assumed to have the same belief about the exogenously given stochastic process of the cash flow, these beliefs are now given by:

$$E_{h,t}(Y_{t+1}) = E_t(Y_{t+1}) = (1+g)Y_t E_t(\varepsilon_{t+1}) = (1+g)Y_t\,. \tag{10}$$

In addition, the cash flow growth Y_{t+1}/Y_t is conditionally independent of d_{t+1} which implies:

$$E_{h,t}\left(\frac{P_{t+1}}{Y_t}\right) = E_{h,t}\left(\frac{Y_{t+1}}{Y_t}\right) E_{h,t}(d_{t+1}) = (1+g)E_{h,t}(d_{t+1})\,. \tag{11}$$

Using both two previous equations, the equilibrium pricing equation (6) can be reformulated in terms of the PY-ratio as follows:

$$\frac{P_t}{Y_t} = \frac{1}{1+r}\sum_{h=1}^{H} n_{h,t}\Big\{(1+g)E_{h,t}(d_{t+1}) + (1+g)\Big\} = \frac{1}{1+r}\sum_{h=1}^{H} n_{h,t}(1+g)\Big\{E_{h,t}(d_{t+1}) + 1\Big\}, \quad (12)$$

and as the fractions $n_{h,t}$ sum up to 1, the PY-ratio (d_t) is equal to:

$$d_t = \frac{1}{R^*}\Big\{1 + \sum_{h=1}^{H} n_{h,t}E_{h,t}(d_{t+1})\Big\}, \qquad R^* = \frac{1+r}{1+g}, \quad (13)$$

with R^* being the gross rate of return the equilibrium pricing equation.

I now specify the fundamental ratio. In contrast to the cash flows, prices are assumed to be affected by expectations. These heterogeneous beliefs about next period's PY-ratio ensure that the ratio can deviate from the fundamental ratio, which is not observable in an efficient market. Note that investors know the fundamental ratio (and price), but are unaware of the beliefs of other investors and of the current fractions of beliefs $n_{h,t}$. Hence, investors do not know the next period's deviation from the fundamental ratio. In case of a constant growth rate g the rational expectations fundamental price of the risky asset is given by:

$$P_t^* = \frac{1+g}{r-g}Y_t, \qquad r > g\,. \quad (14)$$

This equation is in line with the static Gordon growth model (Gordon, 1962) defining the fundamental value based on cash flows that grow at a constant rate (i.e. $1+g$) in perpetuity. The price-to-cash flow fundamental ratio now becomes:

$$d_t^* = \frac{P_t^*}{Y_t} = \frac{1+g}{r-g}\,. \quad (15)$$

Using the fundamental PY-ratio and the assumption of heterogeneity in expectations, the pricing equation can now be expressed as a function of the deviation from the fundamental ratio:

$$x_t = d_t - d_t^* = \frac{1}{R^*}\sum_{h=1}^{H} n_{h,t}E_{h,t}(x_{t+1})\,. \quad (16)$$

3.3 Heterogeneous Beliefs and Switching

The important part of the model is the way agents form their beliefs about next period's PY-ratio, $E_{h,t}(x_{t+1})$. As explained, agents have different beliefs about the expected transitory deviation of the (known) fundamental PY-ratio. First, the characteristics of belief functions are outlined and then the way in which investors can switch between these functions is explained.

Beliefs are presented by $f_{h,t}(.)$, a function depending on L past deviations:

$$E_{h,t}(x_{t+1}) = f_{h,t}(x_{t-1}, ..., x_{t-L}) \, . \tag{17}$$

Note that in this set up $f_{h,t}(.)$ represents the belief type at the beginning of period t. The equilibrium adaptive belief system can now be described as:

$$x_t = \frac{1}{R^*} \sum_{h=1}^{H} n_{h,t} f_{h,t}(x_{t-1}, ..., x_{t-L}) = \frac{1}{R^*} \sum_{h=1}^{H} n_{h,t} f_{h,t} \, . \tag{18}$$

Next, one has to define the belief function(s). In their paper Brock and Hommes present some simple belief types, all of the linear form:

$$f_{h,t} = \phi_h x_{t-1} + b_h \, , \tag{19}$$

where the beliefs are characterized by a trend (ϕ) and bias (b). Examples of types of investors presented by the paper are:

- Rational agents with perfect foresight who know not only all past prices and cash flows, but also the beliefs of other investors and the fractions. They are able to compute x_{t+1} perfectly, i.e. $f_{h,t} = x_{t+1}$.
- Fundamentalists who believe prices return to their fundamental solution i.e. $x_{t+1} = 0$ or similar $\phi_h = b_h = 0$. They know all the past prices and cash flows, but do not know the fractions $n_{h,t}$ of the other belief types. Note that the fundamental benchmark is a special case when $f_{h,t} = 0$ for all types h.
- Chartists who believe in a pure trend (without any bias) and extrapolate past deviations, i.e. $f_{h,t} = \phi_h x_{t-1}$ with $\phi_h > 1$. Contrarian investors also look for a trend, but move against it, hence $\phi_h < 1$.

- Biased traders with a belief that has an upward ($b > 0$) or downward ($b < 0$) bias parameter.

Boswijk et al. (2007) estimate a simple two-type dynamic asset pricing model to annual data of the S&P500 index from 1871 to 2003. They use a different definition than Brock and Hommes (1998) as they call investors with $\phi_h < 1$ fundamentalists and chartists or trend following investors have a parameter $\phi_h > 1$. I name their model the two-type HAM. In this thesis the micromarket described by De Long et al. (1990a,b) is used as a guideline for the model explained in next section.

There is sufficient proof that sentiment of investors varies over time (Boswijk et al., 2007). In order to model this variation, investors can switch between the different belief heuristics, where the choice of heuristic depends on a fitness function defined by the previous realized profits, publicly available at the beginning of period t. The fitness function of type h at the end of period t is denoted by:

$$\pi_{h,t} = R_t z_{h,t-1} = R_t \frac{E_{h,t-1}(R_t)}{aV_{t-1}(R_t)}\,, \tag{20}$$

with $z_{h,t-1}$ indicating the demand, formed in period $t-1$ of the risky asset by belief type h. As in Brock and Hommes (1998), previous realized profits can be taken into account by using the function $\Pi_{h,t} = \pi_{h,t} + \alpha\Pi_{h,t-1}$. Based on the fitness (or so-called performance) of the belief, an investor chooses between the different prediction strategies. The following discrete choice probability expresses the updated fractions:

$$n_{h,t} = \frac{\exp(\beta\Pi_{h,t-1})}{\sum_{h=1}^{H}\exp(\beta\Pi_{h,t-1})}\,, \tag{21}$$

where parameter β is the intensity of choice measuring how fast agents switch between different prediction strategies. The higher β, the more agents are willing to switch strategy. Hommes (2011) describes switching by a discrete choice model with asynchronous updating, which means that they add an additional parameter $\delta \in [0,1]$ to equation (21) that measures the fraction of investors who do not update their belief every period. This feature is consistent with experimental data (Hommes, 2011) and therefore added to the model, hence:

$$n_{h,t} = \delta n_{h,t-1} + (1-\delta)\frac{\exp(\beta\Pi_{h,t-1})}{\sum_{h=1}^{H}\exp(\beta\Pi_{h,t-1})}\,. \tag{22}$$

The timing of the economy is important in this model. At time t the previous realization (x_{t-1}) and especially the realized performance $\Pi_{h,t-1}$ are known. Then first, the fractions $n_{h,t}$ are determined by the previous performance of each trading strategy h, like equation (21) or (22). At the same time, traders revise their beliefs and based on the fractions and on the beliefs, x_t is determined. After this observation the realized profits for period t, $\Pi_{h,t}$ can be computed and will be used in the subsequent period.

3.4 The Model

A HAM is characterized by the types of agents and corresponding belief functions and switching mechanisms. In this section I define these characteristics of my model. The framework by De Long et al. (1990a,b) is used, who describe a market with two agents: arbitrageurs and noise traders. My HAM consists of five types of agents (hence I name it the five-type HAM). The five different beliefs have the same function:

$$f_{h,t} = \phi_h x_{t-1}, \qquad h = 1, \ldots, 5\,. \tag{23}$$

First, the arbitrageurs ($h = 1$) are rational agents who normally follow a mean reverting belief, i.e. $\phi_1 < 1$. These traders can both hold long as well as short positions and they and can be thought of as hedge funds. As De Long et al. (1990a) explain arbitrageurs look at a stock's fundamental value and take the actions of noise traders into account and trade accordingly. Note that the market settings are in line with Chen et al. (2002) who also assume that arbitrageurs can take long and short positions. Most of the time, such investors stabilize the deviations from fundamentals, but after a good news event they anticipate on the overreaction of positive feedback traders by trading ahead of these investors. Arbitrageurs will buy (more) today, as positive feedback traders will be buying tomorrow, destabilizing markets even more. This erratic behavior by rational speculators that triggers positive feedback trading is called "jumping on the bandwagon" (De Long et al., 1990b). It can be seen as herding as the investors defer their own belief and start following the noise traders beliefs. The fraction of arbitrageurs trading on the bandwagon is expressed as:

$$\zeta_t = \frac{\exp(\beta \Pi_{\mathrm{bw},t-1})}{\exp(\beta \Pi_{\mathrm{bw},t-1}) + \exp(\beta \Pi_{1,t-1})}\,, \tag{24}$$

where $\Pi_{\text{bw},t-1}$ is the average realized profit on the bandwagon. On the bandwagon, the arbitrageurs can choose between two types ($h = 3$ and $h = 4$), hence $\Pi_{\text{bw},t-1}$ is the average of these realized profits ($\Pi_{3,t-1}$ and $\Pi_{4,t-1}$). This enables arbitrageurs to herd as they can follow another belief that previously performed better. A situation in which this happens is when there is a positive trend picked up by noise traders and arbitrageurs can extrapolate this trend by triggering positive feedback trading. They are better off following this trend (on the bandwagon), than following their mean-reverting belief. Once on the bandwagon, the switching between the two beliefs $h = 3$ and $h = 4$ is determined using the discrete choice probability function as in equation (21):

$$\xi_{h,t} = \frac{\exp(\beta\Pi_{h,t-1})}{\sum_h \exp(\beta\Pi_{h,t-1})}, \qquad h = 3, 4\,. \tag{25}$$

Note that I have to specify a variable ($\xi_{h,t}$) that denotes the distribution between types 3 and 4 in order to make sure that the actual fractions $n_{h,t}$ sum up to one. In this case, the fractions $n_{3,t}$ and $n_{4,t}$ are determined by:

$$n_{h,t} = n_{1,t} \times \zeta_t \times \xi_{h,t}, \qquad h = 3, 4\,. \tag{26}$$

Also note that in equation (25) $\delta = 0$ as all traders on the bandwagon are likely to update their belief each time, in contrast to arbitrageurs not on bandwagon who are less likely to switch from their fundamental belief to a noise trader belief. This switching mechanism is defined below.

The second type of traders are the noise traders ($h = 2$) who typically follow a trend (i.e. $\phi_2 > 1$). These traders can be thought of as mutual funds who are less informed than the arbitrageurs. These traders misperceive signals about future prices they get from stockbrokers, technical analysts, or minor price movements, believing that they have special information. Irrelevant information about assets fundamentals is picked up as meaningful new information, leading prices to drift away from their fundamental value. The main restriction of these traders is that they cannot sell short (Chen et al., 2002), hence this feature is incorporated in the heterogeneous agent model as follows:

$$\lambda_t = -\min\left\{\frac{f_{2,t-1} - x_{t-1}}{3 + |f_{2,t-1} - x_{t-1}|}, 0\right\}, \tag{27}$$

where λ_t is the fraction of noise traders that leaves the market, which depends on the difference between the aggregate noise trader belief and the current market value. When noise trader's expectation about future prices is lower than current market price, part of the noise traders will leave the market as they cannot sell short. The equation (27) is used as it fits the situation well (a concave function between 0 and 1 that slowly converges to 1 if the belief $f_{2,t-1}$ is lower than the market value x_{t-1}).

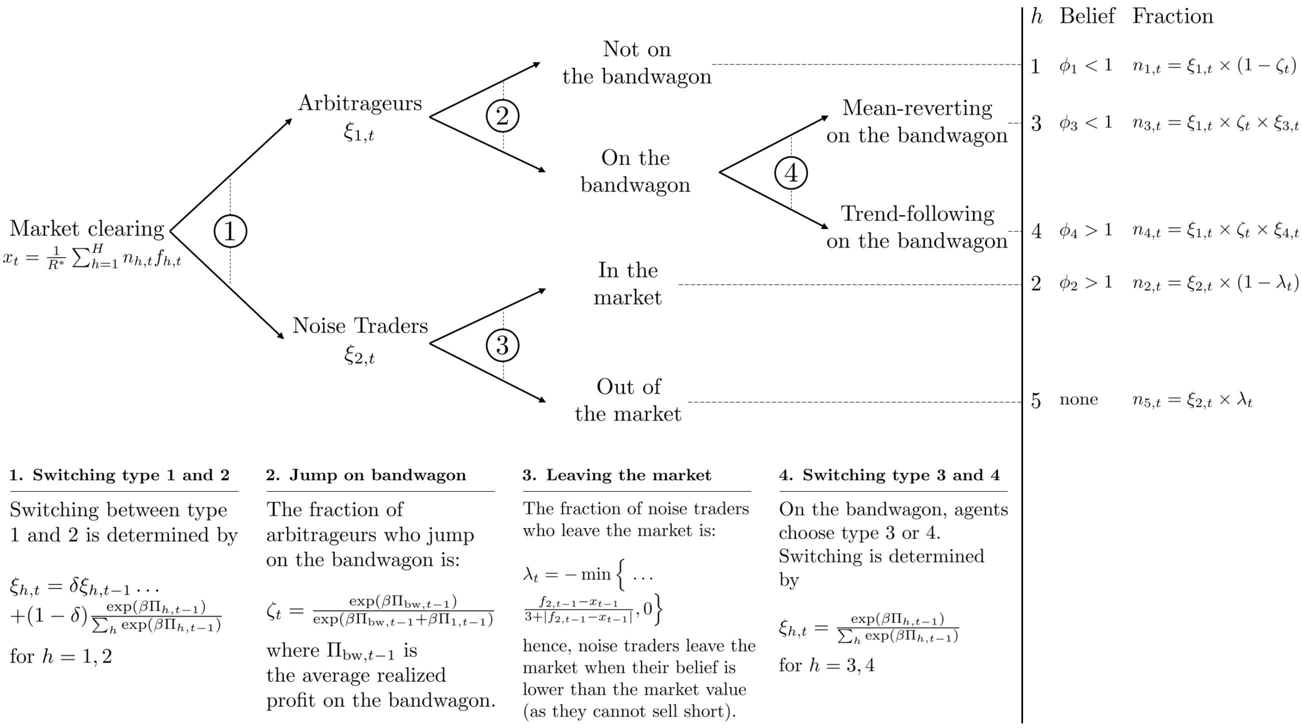
Finally, switching between arbitrageurs ($h = 1$) and noise traders ($h = 2$) is described by:

$$\xi_{h,t} = \delta \xi_{h,t-1} + (1-\delta) \frac{\exp(\beta \Pi_{h,t-1})}{\sum_h \exp(\beta \Pi_{h,t-1})}, \qquad h = 1, 2\,, \tag{28}$$

where the δ is now expected to be positive as these types of investors are more likely to continue following their own belief. Figure 1 summarizes the structure of the five-type HAM and gives an overview of the realizations of the fractions.

Figure 1: Structure of Five-Type HAM

Structure of the five-type heterogeneous agent model. There are five types of agents active in the market, all with different beliefs ($f_{h,t} = \phi_h x_{t-1}$). The tree presents the way the fraction of each type ($n_{h,t}$) is determined. Arbitrageurs ($h = 1$) can jump on the bandwagon (and once on the bandwagon they can follow two types, $h = 3$ or $h = 4$). Noise traders ($h = 2$) can leave the market ($h = 5$) if their belief is below the market value. Note that there are four switching mechanisms, which are explained at the bottom. All equations are explained in the text



3.5 The Propositions

In order to structure the approach, I establish two propositions that are investigated in this paper. The first proposition considers the behavior of the arbitrageurs:

> **Proposition 1.** *Arbitrageurs jumping on the bandwagon destabilize the market by strengthening positive feedback trading. An increase in the fraction of arbitrageurs on the bandwagon leads to higher volatility and higher deviation of the market price from the fundamental value.*

Following, De Long et al. (1990b) it is expected that arbitrageurs on the bandwagon will further destabilize the market. An increase in this fraction means that more arbitrageurs decided that following the crowd would be a better investment strategy.

The second proposition considers the noise trader demand:

> **Proposition 2.** *Noise traders destabilize the market by their misperception of price information. An increase in the fraction of noise traders leads to higher volatility and higher deviation of the market price from the fundamental value.*

Following De Long et al. (1990a,b) it is expected that noise trader demand creates instability in the market. Note that the fraction of noise traders includes the traders that left the market.

4 Empirical Results

In this section I discuss the empirical results of simulations as well as estimations. Especially, the focus is on the propositions presented in previous section and on the comparison between both the two-type HAM by Boswijk et al. (2007) and the five-type HAM established in this thesis.

First, I analyze the models using simulations. For that purpose, I introduce eight parameter sets and simulate 100 time series of 200 periods for both models across all sets. I first look at typical outcomes of these simulations. Then I check whether the models are able to match stylized facts of asset prices and compare this to the data. Finally, the simulated time series are divided into ten deciles based on either the fraction of investors on the bandwagon or the fraction of noise traders. Various metrics, such as the deviation from the fundamental value and volatility, are presented for each decile group and the differences between these metrics can tell whether my predictions in the propositions are supported by the simulations.

Second, I estimate the models using data on the S&P500 index. These estimation results are analyzed first. In particular the dynamics of the fractions are analyzed. Finally, the estimation results are analyzed using the decile group analysis.

4.1 Simulation Results

To run the simulations, I introduce stochastic disturbances to both models using the market clearing equation:

$$x_t = \frac{1}{R^*} \sum_{h=1}^{H} n_{h,t} f_{h,t} + \varepsilon_t \,, \tag{29}$$

where ε_t is iid with zero mean and standard deviation σ_ε. These disturbances represent the market uncertainty and random occasions that impact the aggregate belief.

I define eight parameter sets in order to investigate the impact of changes in (main) variables of the models. Table 1 shows the parameter sets. It can be seen that the belief parameters

(ϕ_h, $h = 1, \ldots, 4$) can be strong or weak i.e. the mean-reverting belief parameter is equal to 0.75 or 0.95 respectively. The trend-following belief is strong when $\phi_h = 1.25$ and weak when the parameter is fixed at 1.05. In addition, the parameter sets define the tendency to switch that is determined by β (the intensity of choice) and δ (the fraction of investors that update their belief). Again, a strong and weak version is introduced. Fixed variables are $\sigma_\varepsilon = 2.5$, $R^* = 1.05$, and $d^* = 25$ the fundamental value. These variables only have a minor impact on the simulation results. The simulations are initiated using $x_1 = x_2 = x_3 = 5$. If the simulations would be initiated with a specific trend in the deviations from fundamental value, the model would extrapolate this trend. Hence, simulations are initiated with no trend and all x_t's are equal to circa the average of the observed x_t's. In the first two periods, 50% of the agents are arbitrageurs and 50% are noise traders. No investors are on the bandwagon or out of the market initially. In addition, the cash flows are generated using a geometric Brownian motion with drift. The expected return of the cash flow process is 1.75% per year and annual volatility is 16.25%. Each simulation run consists of 100 samples of 200 periods for a specific parameter set.

Table 1: Parameter Sets

Parameter sets used for simulations. The beliefs parameters of investors on the bandwagon (ϕ_3 and ϕ_4) are equal to the belief parameters of arbitrageurs (ϕ_1) and noise traders (ϕ_2) respectively. The tendency to switch is determined by both β (the intensity of choice) and by δ (the fraction of investors that does not update beliefs each period)

	Belief parameters		Tendency to switch	
	$\phi_1 = \phi_3$	$\phi_2 = \phi_4$	β	δ
Set 1	0.95	1.05	0.10	50%
Set 2	0.95	1.05	0.50	0%
Set 3	0.75	1.25	0.10	50%
Set 4	0.75	1.25	0.50	0%
Set 5	0.95	1.25	0.10	50%
Set 6	0.95	1.25	0.50	0%
Set 7	0.75	1.05	0.10	50%
Set 8	0.75	1.05	0.50	0%

The figures 4 to 11 in the Appendix show graphs of typical simulation outcome of one run for each parameter set and for both models. These graphs show the dynamics of the fractions, logarithm of the simulated prices, deviation from fundamental value, and a volatility measure using the RiskMetrics approach[1] where the decay parameter is equal to 0.95. Note that these are

[1]For RiskMetrics approach see Morgan and Reuters (1994)

outcomes of one run, thus the graphs are not representative for all simulation samples. Table 2 shows the average fraction (and the average standard deviation of the fractions) of each investor type for each simulation run. From the findings in Table 2 and from the graphs in the Appendix, the following can be concluded.

First of all, when the noise traders have a strong belief parameter (i.e. $\phi_2 = 1.25$), the price (P_t) easily explodes due to an extreme increase in the belief (x_t). This especially happens if the mean reverting belief parameter of the other investors is weak and/or the tendency to switch is high (i.e. for S4, S5, and S6). This pattern can be seen in the simulated deviations from fundamental value in Figures 6 and 10 in the Appendix. Also, the average fraction of investors and standard deviations in Table 2 show the same pattern. It can be seen that the noise trader fraction is high (e.g. see five-type model S5) and/or the standard deviation is relatively high. There seems to be a critical value that makes a stable market unstable as price exploded. This seems to happen with a belief parameter for noise traders that is higher than 1.15 and it is left for future research to further investigate this characteristic of the HAM.

Second, the tendency to switch impacts the simulations as there are more and stronger fluctuations in the fractions and prices when the tendency to switch is higher (i.e. β is high, δ low, the even parameter sets). The impact is best shown by the graphs of the fractions. For example, compare the graphs of the two-type model and parameter sets 1 and 2 (see Figure 4). Using the first settings, the fraction of arbitrageurs does not exceeds 0.6 whereas in a market with a higher tendency to switch this fraction reaches almost 1.0 at some points in time. The same characteristic can also be observed from the standard deviations reported in Table 2. Higher standard deviations appear in simulations with a higher tendency to switch (e.g. compare outcomes of odd and even parameter sets). Note that the standard deviations of investor types 3, 4, and 5 are relatively small due to small ranges of the fractions compared to types 1 and 2.

Third, the simulated prices of the five-type model explode less often than the two-type model. The five-type model has a more detailed structure, as arbitrageurs can jump on the bandwagon and noise traders can leave the market. When looking at the graphs in the Appendix and Table 2 it can be seen that the extension seems to have a dampening effect on the simulated prices. The graphs in the Appendix show that the prices resulted from the two-type model explode more easily. In Table 2 it is most apparent when comparing outcomes of the models for S6. Simulated time series explode due to a high fraction of noise traders, hence the fraction for the two-type model is extremely high (0.935), whereas the five-type model has an average noise trader fraction of 0.452 for the same parameter set.

Table 2: Average Simulated Fraction of Investor Type

Average of the simulated fraction per investor type for each parameter set (S1-S8) and both models. Average standard deviation of the simulated fractions in parentheses. Refer to Table 1 for the parameter sets. Other variables: $T = 200$, $N = 100$, $\sigma_\varepsilon = 2.5$, $R^* = 1.05$, $d^* = 25$, initial observations $x_1 = x_2 = x_3 = 5$, and initial fractions are $n_1 = n_2 = 0.5$, $n_3 = n_4 = n_5 = 0$

Five-type model

	Arbitrageurs		Noise traders		Mean-revert on the bandwagon		Trend-following on the bandwagon		Noise traders that left the market	
	$h = 1$		$h = 2$		$h = 3$		$h = 4$		$h = 5$	
S1	0.279	(0.084)	0.405	(0.110)	0.115	(0.030)	0.115	(0.039)	0.086	(0.105)
S2	0.315	(0.168)	0.388	(0.163)	0.118	(0.030)	0.107	(0.038)	0.073	(0.096)
S3	0.318	(0.221)	0.417	(0.150)	0.086	(0.059)	0.106	(0.091)	0.072	(0.098)
S4	0.414	(0.342)	0.377	(0.277)	0.074	(0.055)	0.070	(0.056)	0.065	(0.114)
S5	0.146	(0.178)	0.717	(0.230)	0.044	(0.050)	0.057	(0.076)	0.036	(0.071)
S6	0.357	(0.325)	0.452	(0.283)	0.066	(0.052)	0.063	(0.053)	0.062	(0.107)
S7	0.306	(0.145)	0.395	(0.116)	0.105	(0.047)	0.111	(0.065)	0.082	(0.101)
S8	0.371	(0.269)	0.372	(0.217)	0.100	(0.046)	0.091	(0.050)	0.066	(0.108)

Two-type model

	Arbitrageurs		Noise traders	
	$h = 1$		$h = 2$	
S1	0.513	(0.026)	0.487	(0.026)
S2	0.549	(0.169)	0.451	(0.169)
S3	0.501	(0.116)	0.499	(0.116)
S4	0.200	(0.248)	0.800	(0.248)
S5	0.150	(0.217)	0.850	(0.217)
S6	0.065	(0.179)	0.935	(0.179)
S7	0.524	(0.046)	0.476	(0.046)
S8	0.569	(0.236)	0.431	(0.236)

4.2 Stylized Facts of Simulated Times Series

Next, I analyze the ability of the models to replicate empirical anomalies. The same simulations as in previous section are used to investigate three observed statistical properties of asset prices: non-normality of returns, the unit root property of asset prices, and volatility clustering.

But first, I analyze the stylized facts contained in the data. Table 3 shows statistical properties of the annual S&P500 data from 1950 until 2014. It is well-known that returns in financial markets are not normally distributed. Fat tails and excess kurtosis are a result of more extreme (often negative) returns that are observed more often than implied by a (standard) normal distribution. This anomaly was first presented by Mandelbrot (1963). In particular, returns are expected to have a kurtosis exceeding 3, reject the Jarque-Bera normality test, and have a Hill index ranging between 2 and 5 (a measure of fat tails)[2]. In Table 3 it can be seen that the kurtosis indeed exceeds 3 and that the normality test is rejected. The Hill indexes however are higher than expected, indicating that the fat tail property does not appear in the data.

Another stylized fact is the unit root property of asset prices. This property means that prices are non-stationary i.e. follow a random walk or martingale. This implies that the price process $P_t = \rho P_{t-1} + \epsilon_t$ has stationary increments ϵ_t and parameter $\rho = 1$ (Lux and Marchesi, 2000). This is tested using an augmented Dickey-Fuller test. Table 3 shows that $\rho = 1.02$ and that the augmented Dickey-Fuller test is not rejected. This suggests the presence of a unit root i.e. non-stationary prices.

Volatility clustering is the third stylized fact that is analyzed. As introduced by Mandelbrot (1963), volatility clustering is the phenomenon that extreme and small stock returns seem to cluster together, leading to significant autocorrelation in volatility. Volatility clustering is reflected in significant, slowly declining autocorrelation in squared and absolute returns. Moreover, it can be detected using a GARCH structure to show that there is time dependency in the volatility of asset returns (Hommes, 2001; De Grauwe and Grimaldi, 2006). In Table 3 it can be seen that the data shows no autocorrelation in squared and absolute returns, for all lags. The AR(1) coefficient is slightly positive (0.02). Moreover, the GARCH(1,1) results show that the ARCH term is not significant, but the GARCH term is significantly different from zero at 5% level. Due to these results and due to the fact that the sum of the ARCH and GARCH terms is not convincing enough close to one, I cannot conclude that there is evidence for volatility clustering in the annual return data.

Overall, the annual prices seem to be non-normally distributed and non-stationary. However, the tails are less fat than expected and volatility clustering seems not to appear in the data. I now analyze the ability of the simulated time series to generate these stylized facts.

[2] The Hill index is a measure of the tail exponent: the lower it is, the fatter the tail of the distribution. Real return data have a tail index ranging between 2 and 5. See De Grauwe and Grimaldi (2006)

Table 3: Statistical Properties of Data

Statistical properties of annual price data of the S&P500 index from 1950-2014. Analysis on the stylized facts: non-normality of returns, unit root property of asset prices, and volatility clustering. Multiple metrics are presented (Kurtosis, Hill index estimates, ρ coming from the price process $P_t = \rho P_{t-1} + \epsilon_t$, AR(1) coefficient, and GARCH(1,1) estimation results). In addition, multiple tests are performed (Jarque-Bera test for normality, augmented Dickey-Fuller test for stationarity, and Ljung-Box Q-tests for serial correlation)

Metric/Test	Value	Rejection of test (at 5% level)	Test statistic
Kurtosis	3.71		
Jarque-Bera test		Yes	8.40
Hill index estimates			
2.5% tail	11.54		
5% tail	6.70		
10% tail	6.66		
ρ	1.02		
Aug. Dickey-Fuller test		No	0.89
AR(1)	0.02		
Ljung-Box Q-test			
Squared returns			
Lag = 5		No	6.91
Lag = 10		No	8.31
Lag = 20		No	11.99
Absolute returns			
Lag = 5		No	9.08
Lag = 10		No	10.61
Lag = 20		No	15.73
ARCH term (t-statistic)	-0.16		(-1.47)
GARCH term (t-statistic)	1.10		(5.53)
Sum of terms	0.94		

Non-Normality of Returns

Table 4 presents results on the kurtosis, Jarque-Bera test and Hill index estimates of the simulated returns of both models. The simulated prices do not all meet this expectation as the median kurtosis is in most cases around (and even lower than) 3 and the Jarque-Bera test is not rejected (at maximum 49% of the 100 simulated samples). Comparing the two models, the five-type model seems to better match the stylized fact. The same can be concluded when looking at the Hill index estimates where three cut-off points of the tails are considered, 2.5%, 5%, and 10%. The median tail index exceeds 5 in less than 50% of the simulations of the five-type model. Using the other model, the tail index is even higher in most cases leading us to conclude that the simulated return distribution is not fat tailed especially when using the two-type model. Key conclusion is that both models do not consistently comply with the first stylized fact. But it can also be concluded that the five-type HAM better matches the non-normality of returns than the two-type HAM and that both models match the observed outcomes of the annual price data regarding the fat tails property.

Unit Root Property of Asset Prices

The ρ's and the results of the augmented Dickey-Fuller tests are presented in Table 5. The simulated ρ ranges between 0.91 and 1.51 for the five-type model and between 0.85 and 1.52 for the two-type model. The Dickey-Fuller test should not be rejected, which is the case with the data and in most simulations. The two-type model seems to meet expectations in more cases than the five-type model. The conclusion is that both HAMs generate non-stationary prices.

Volatility Clustering

Table 6 shows the results of AR(1) model estimations and Ljung-Box Q-tests for serial correlation in both squared and absolute returns. The simulated returns show similar statistics as the data: no significant autocorrelations in squared and absolute returns in the majority of the cases. This is shown by the test results, as the test for no serial correlation is rejected in only a few cases. For the five-type model, if parameter set 4 is used there seems to be some autocorrelation in squared and absolute returns (e.g. rejection in 42% and 38% of the simulated samples and 5 lags respectively), but still not in the majority of simulated times series.

Table 7 reports the results of GARCH(1,1) model estimations of the simulated returns. It can be observed that most of the ARCH terms are not significant and that most of the GARCH terms are significantly different from zero at 5% level. In addition the sum of both terms is in majority of the cases not close to one, which can also be observed from the median values ranging from 0.671 to 0.929 for both models. These results do not lead to the conclusion that the models are capable of reproducing clustering and persistence in volatility. The same appeared when looking at the GARCH(1,1) results from the actual price data.

To summarize, both HAMs seem incapable of reproducing non-normal returns and clustering volatility. They are however capable of reproducing the unit root property of prices. Notable, is the fact that most results seem to match with the annual S&P500 data (all but the non-normality of returns). This indicates that the models are able to simulate prices with observed anomalies that appear in yearly data. This means that the models should be adjusted when one tries to simulate data that matches periods of a lower frequency (e.g. daily or monthly). Here, I am interested in simulated time series from annual data and the models apparently fit for that purpose.

Table 4: Non-normality of Simulated Returns

Analysis of 100 samples of simulated times series of 200 periods using the five-type model and the two-type model by Boswijk et al. (2007). Refer to Table 1 for the parameter sets. Other variables: $\sigma_\varepsilon = 2.5$, $R^* = 1.05$, $d^* = 25$, initial observations $x_1 = x_2 = x_3 = 5$, and initial fractions are $n_1 = n_2 = 0.5$, $n_3 = n_4 = n_5 = 0$. Median (and range in parentheses) of average Kurtosis, Jarque-Bera test results at 5% level, and Hill index estimates with 2.5%, 5%, and 10% tail cut-off points

	Kurtosis	Jarque-Bera test (at 5% level)		Hill index estimates (median, range in parentheses)		
	Median, range in parentheses	No. of rejections	Median of test statistic	2.5% tail	5% tail	10% tail
			Five-type model			
Set 1	2.97	11	1.25	8.31	5.23	3.53
	(2.37-10.61)			(3.30-28.23)	(2.72-10.98)	(2.32-6.31)
Set 2	3.09	19	1.59	7.74	5.28	3.55
	(2.50-32.31)			(2.45-48.45)	(2.30-14.87)	(2.41-5.56)
Set 3	3.30	36	2.62	6.12	4.41	3.43
	(2.33-26.11)			(1.87-33.16)	(2.16-12.45)	(2.23-8.10)
Set 4	3.51	49	5.02	5.12	4.24	3.19
	(2.36-81.36)			(1.32-47.71)	(1.62-14.07)	(1.41-9.40)
Set 5	3.16	25	2.12	9.48	6.48	4.77
	(2.48-11.82)			(2.35-36.80)	(1.98-14.67)	(2.09-8.18)
Set 6	3.34	40	3.13	6.35	4.74	3.46
	(2.42-72.04)			(1.25-24.04)	(1.57-20.19)	(1.43-7.08)
Set 7	3.04	7	1.42	8.38	5.41	3.79
	(2.31-24.47)			(2.24-37.28)	(2.51-11.76)	(1.73-6.42)
Set 8	3.07	12	1.51	7.41	5.24	3.62
	(2.41-6.16)			(3.36-41.75)	(2.80-14.26)	(2.52-6.05)
			Two-type model			
Set 1	3.07	17	1.76	7.30	5.29	3.75
	(2.37-22.80)			(1.31-48.38)	(1.49-15.95)	(2.07-6.41)
Set 2	3.09	18	1.65	8.23	5.19	3.52
	(2.46-6.21)			(2.42-28.85)	(2.49-13.88)	(2.41-6.19)
Set 3	3.02	22	1.43	7.13	5.15	3.65
	(2.48-8.17)			(2.36-22.15)	(2.37-12.84)	(2.33-6.96)
Set 4	3.03	17	1.41	10.37	7.38	5.32
	(2.44-69.31)			(1.51-77.17)	(2.33-16.54)	(2.45-10.36)
Set 5	2.94	8	1.67	10.75	8.09	5.81
	(2.37-5.16)			(4.54-70.02)	(3.84-15.96)	(3.19-10.00)
Set 6	3.04	10	1.22	11.58	7.45	5.89
	(2.26-4.79)			(3.83-72.98)	(4.06-13.25)	(4.04-9.39)
Set 7	3.00	7	1.13	7.87	5.54	3.71
	(2.20-4.42)			(2.39-55.84)	(2.93-12.27)	(2.28-6.60)
Set 8	2.99	7	1.51	7.82	5.43	3.56
	(2.41-4.22)			(3.00-92.75)	(2.89-14.97)	(2.38-9.45)

Table 5: Unit Root Property of Simulated Prices

Analysis of 100 samples of simulated times series of 200 periods using the five-type model and the two-type model by Boswijk et al. (2007). Refer to Table 1 for the parameter sets. Other variables: $\sigma_\varepsilon = 2.5$, $R^* = 1.05$, $d^* = 25$, initial observations $x_1 = x_2 = x_3 = 5$, and initial fractions are $n_1 = n_2 = 0.5$, $n_3 = n_4 = n_5 = 0$. Median (and range in parentheses) of ρ coming from the price process $P_t = \rho P_{t-1} + \epsilon_t$ and augmented Dickey-Fuller test results at 5% level

	ρ	Augmented Dickey-Fuller test (at 5% level)	
	Median, range in parentheses	No. of rejections	Median of test statistic
Five-type model			
Set 1	0.983 (0.94-1.03)	17	-1.22
Set 2	0.983 (0.91-1.04)	12	-1.25
Set 3	0.984 (0.95-1.22)	7	-1.22
Set 4	0.983 (0.96-1.15)	8	-1.21
Set 5	1.165 (0.95-1.51)	2	14.86
Set 6	0.987 (0.93-1.38)	14	-0.94
Set 7	0.982 (0.93-1.01)	20	-1.33
Set 8	0.982 (0.92-1.06)	13	-1.28
Two-type model			
Set 1	0.983 (0.95-1.07)	12	-1.33
Set 2	0.985 (0.85-1.10)	12	-1.15
Set 3	0.986 (0.93-1.36)	9	-1.06
Set 4	1.173 (0.97-1.44)	2	13.41
Set 5	1.181 (0.99-1.44)	0	15.63
Set 6	1.169 (1.03-1.52)	0	14.64
Set 7	0.983 (0.92-1.07)	8	-1.30
Set 8	0.983 (0.93-1.13)	12	-1.27

Table 6: Serial Correlation in Simulated Returns

Analysis of 100 samples of simulated times series of 200 periods using the five-type model and the two-type model by Boswijk et al. (2007). Refer to Table 1 for the parameter sets. Other variables: $\sigma_\varepsilon = 2.5$, $R^* = 1.05$, $d^* = 25$, initial observations $x_1 = x_2 = x_3 = 5$, and initial fractions are $n_1 = n_2 = 0.5$, $n_3 = n_4 = n_5 = 0$. Analysis on returns using AR(1) estimates and Ljung-Box Q-tests at 5% level. Median of AR(1) coefficient and range of average in parentheses. For the Ljung-Box Q-test the number of rejections and median of test stastics (parentheses) are reported.

		Ljung-Box Q-test (at 5% level) No. of rejections (median of test statistic in parentheses)					
		Squared returns			Absolute returns		
	AR(1)	Lag = 5	Lag = 10	Lag = 20	Lag = 5	Lag = 10	Lag = 20
Five-type model							
Set 1	0.010 (-0.23-0.26)	15 (4.43)	11 (9.63)	9 (19.30)	11 (4.37)	9 (9.28)	9 (20.18)
Set 2	0.007 (-0.20-0.14)	18 (5.83)	18 (11.16)	14 (21.00)	14 (5.34)	9 (10.32)	8 (21.07)
Set 3	0.071 (-0.16-0.24)	33 (6.26)	30 (12.44)	23 (21.31)	27 (6.32)	27 (12.08)	23 (21.02)
Set 4	0.040 (-0.28-0.24)	42 (8.83)	38 (13.81)	31 (22.41)	38 (6.66)	42 (13.22)	37 (24.96)
Set 5	0.119 (-0.13-0.35)	20 (4.68)	16 (9.58)	15 (19.98)	17 (4.77)	19 (10.03)	14 (21.26)
Set 6	0.055 (-0.33-0.28)	35 (6.88)	32 (13.38)	33 (23.62)	34 (7.07)	40 (15.32)	36 (27.39)
Set 7	-0.023 (-0.17-0.23)	13 (4.72)	9 (10.29)	9 (19.79)	9 (4.77)	10 (10.14)	6 (20.85)
Set 8	-0.009 (-0.20-0.18)	9 (4.36)	10 (9.19)	10 (18.26)	6 (4.38)	6 (9.60)	8 (19.41)
Two-type model							
Set 1	-0.006 (-0.26-0.14)	16 (5.16)	19 (10.56)	21 (21.31)	10 (5.19)	13 (10.51)	14 (21.43)
Set 2	-0.001 (-0.15-0.15)	20 (4.66)	20 (8.87)	18 (20.42)	15 (5.04)	15 (10.18)	15 (19.43)
Set 3	0.040 (-0.15-0.22)	19 (4.67)	19 (10.35)	16 (20.79)	13 (4.62)	12 (9.97)	14 (19.95)
Set 4	0.113 (-0.14-0.36)	18 (5.14)	17 (9.91)	14 (19.60)	14 (5.04)	14 (10.96)	16 (20.17)
Set 5	0.109 (-0.06-0.32)	9 (4.44)	12 (10.10)	17 (21.87)	11 (4.81)	13 (10.17)	16 (22.03)
Set 6	0.071 (-0.13-0.28)	8 (4.58)	6 (9.61)	7 (19.18)	7 (4.80)	7 (9.94)	9 (19.85)
Set 7	-0.015 (-0.20-0.17)	5 (3.88)	2 (8.98)	3 (18.54)	3 (4.09)	2 (9.56)	3 (19.50)
Set 8	0.004 (-0.18-0.20)	10 (4.05)	7 (8.38)	10 (18.74)	5 (4.34)	7 (9.77)	8 (19.71)

Table 7: GARCH Results of Simulated Returns

Analysis of 100 samples of simulated times series of 200 periods using the five-type model and the two-type model by Boswijk et al. (2007). Refer to Table 1 for the parameter sets. Other variables: $\sigma_\varepsilon = 2.5$, $R^* = 1.05$, $d^* = 25$, initial observations $x_1 = x_2 = x_3 = 5$, and initial fractions are $n_1 = n_2 = 0.5$, $n_3 = n_4 = n_5 = 0$. Analysis on returns using GARCH(1,1) estimates. Median (range of average in parentheses) and number of significant estimates of ARCH and GARCH term, and the median of the sum of the ARCH and GARCH term estimates.

	ARCH term		GARCH term		Sum of terms
	Median, range in parentheses	No. of significant estimates	Median, range in parentheses	No. of significant estimates	Median
			Five-type model		
Set 1	-0.023	28	0.723	56	0.711
	(-0.13-0.26)		(-0.98-1.05)		
Set 2	-0.018	26	0.650	56	0.748
	(-0.17-0.37)		(-0.95-1.05)		
Set 3	0.039	34	0.599	51	0.722
	(-0.12-0.49)		(-1.04-1.05)		
Set 4	0.125	43	0.443	45	0.671
	(-0.13-1.19)		(-1.05-1.05)		
Set 5	-0.009	28	0.830	64	0.866
	(-0.12-0.30)		(-0.97-1.05)		
Set 6	0.078	38	0.600	59	0.792
	(-0.14-0.92)		(-1.05-1.04)		
Set 7	-0.020	22	0.712	56	0.753
	(-0.12-0.26)		(-1.05-1.05)		
Set 8	-0.033	31	0.711	51	0.755
	(-0.12-0.26)		(-1.05-1.05)		
			Two-type model		
Set 1	-0.024	30	0.811	66	0.857
	(-0.11-0.43)		(-0.91-1.05)		
Set 2	-0.008	27	0.731	58	0.804
	(-0.18-0.33)		(-1.05-1.05)		
Set 3	-0.011	31	0.706	58	0.765
	(-0.16-0.25)		(-1.05-1.05)		
Set 4	-0.037	20	0.867	67	0.899
	(-0.13-0.48)		(-0.92-1.05)		
Set 5	-0.031	9	0.925	68	0.929
	(-0.15-0.25)		(-1.05-1.05)		
Set 6	-0.046	29	0.910	73	0.916
	(-0.16-0.23)		(-1.04-1.05)		
Set 7	-0.058	35	0.833	65	0.815
	(-0.17-0.17)		(-0.94-1.05)		
Set 8	-0.034	26	0.754	58	0.756
	(-0.14-0.18)		(-1.05-1.05)		

4.3 Decile Group Analysis of Simulation Results

This section focuses on the two propositions presented in 3.5. These propositions provide a guideline to further analyze the models and the desired behavior of the associated agents. For each simulated sample, the fractions of the investor type(s) of interest are ranked in ascending order and divided into ten decile groups. Per decile group, the average return, average absolute, positive, and negative deviation from fundamental value, average volatility, and average of a measure for market activity is calculated. This results in averages per decile group, per sample. Then the median of these averages is calculated over the 100 samples per simulation setting. I perform this analysis for each of the eight parameter sets. For the five-type model, both propositions can be examined using these metrics per decile group. Only the effect of noise traders can be investigated for the two-type model, as the model does not specify a bandwagon.

I introduce a measure for market activity in order to better understand how much switching is taking place at each point in time. This measure is defined as the sum of the weighted average absolute change in each fraction. The higher this measure, the more agents switched belief, the higher the market activity. I expect this measure to be linked to volatility.

Table 8 reports the results of the decile groups based on the fraction of investors on the bandwagon (i.e. $n_{3,t} + n_{4,t}$). The top panel presents the median of the fraction of investors on the bandwagon per decile group. Not surprisingly the fraction increases each decile group. Note that the difference between the tenth and first deciles is higher when the tendency to switch is lower (compare results of odd and even parameter sets). This could be explained by more arbitrageurs switching to noise trading once the upward trend is picked up, hence a lower percentage of arbitrageurs on the bandwagon.

The second panel shows that returns are higher in periods with a lower bandwagon fraction. It is expected that more investors on the bandwagon cause more positive feedback trading, hence higher returns. The third, fourth, and fifth panel show mixed results when looking at the deviation from fundamental value per decile group. The expectation is that a higher fraction of investors on the bandwagon would lead to a higher deviation from fundamental value. Considering the absolute value of it, this is only the case for 3 of the 8 parameter sets. Note that the positive deviation from fundamental value is especially high for the lowest decile group and parameter set 5. Parameter set 5 is the odd one out which is caused by the fact that it has the strongest noise trader belief, the weakest belief of arbitrageurs, and the lowest tendency to switch. This means that the simulated x_t easily explodes. It also means that the dampening effect of other traders is low as switching back is more limited, leading to the highest average fraction of noise traders of all sets (see Table 2). Hence, in periods with the highest fraction of noise traders (and the lowest fraction of traders on the bandwagon), the positive deviation is extremely high is some of the simulated samples of parameter set 5.

The last two panels show the volatility and the median of the market activity measure per decile group. I see that in most cases volatility is lower when there are more investors on the bandwagon. Moreover, the market activity measure shows more or less the same pattern, i.e. when market activity is high, volatility is high and vice versa. The results go against my prediction in proposition 1, where I stated that volatility would be higher when more traders are on the bandwagon triggering positive feedback trading. However, I remark that these results on the volatility are not that strong, as the difference between the decile groups is relatively small.

The key conclusion from the simulated time series of investors on the bandwagon is that the findings show mixed results regarding the statement of the first proposition. The question remains whether arbitrageurs extrapolate or dampen price fluctuations. Note that this is however in line with the literature that finds evidence for both sides (De Long et al., 1990a,b).

In Table 9 the groups are based on the total fraction of noise traders (i.e. $n_{2,t} + n_{5,t}$). The results are more conclusive and support proposition 2. In the top panel of Table 9 it can be seen that parameter sets with a strong tendency to switch have a higher difference between the tenth and first decile, as the fraction reaches the maximum in some simulated samples. For parameter sets with a low tendency to switch, the difference between the median average fraction in d10 and d1 can be small (e.g. see results of parameter set 1).

The second panel shows the median of the average returns per decile group. For all but parameter set 5, it can be concluded that more noise trader demand leads to lower returns. De Long et al. (1990a) state that noise traders earning higher returns as they are compensated for taking more risk. Here, the opposite seems to be true, however note that I report the average market return and not the noise traders' return alone. In that sense, the results show that returns are lower when noise trader demand is higher which could be caused by arbitrageurs earning lower returns in a more risky market created by noise trader demand.

Again, parameter set 5 is the odd one out. Following the same reasoning, the high returns are caused by the result that noise traders are predominant during high positive return periods and that these periods frequently appear in the simulated samples of parameter set 5.

The three panels on the deviation all show a similar pattern: when there are more noise traders in the market the deviation from the fundamental is higher, both positive as well as negative. This is also the case when the noise trader belief is relatively weak. It indicates that noise trader demand causes destabilization of the market, in line with the second proposition and literature.

The sixth panel shows that volatility is slightly lower when more agents are noise traders. The difference between the median volatility of decile groups is actually quite low as they range between 18% and 22%. This is a remarkable finding, given the other results presented by Table 9. It contradicts the statement of proposition 2 as higher volatility would be associated with higher noise trader demand. Due to the use of continuously compounded returns of log prices, the returns do not fluctuate more during period characterized by steeply increasing prices compared to other periods. Moreover, if I consider the market activity measure it can be seen that it shows a similar pattern as the median volatility. Remarkable is the U-shaped pattern: high volatility periods appear when the noise trader demand is at the extreme decile groups (see differences between the highest, middle, and lowest decile groups). Noise trader demand is high during a bullish period, and low during periods when strong mean-reversion takes place. This could explain the U-shaped pattern, as an average fraction of noise traders depicts a relatively stable market period in time. Apparently, the downward periods are associated with even higher volatility (and market activity) than the upward periods. For some parameter sets, the U-shape pattern also appears in the deviation from fundamental value.

Table 14 in the Appendix shows the results of the simulated noise trader fractions of the two-type model. The same can be concluded about noise trader demand for this model.

Key conclusion is that the simulation results support the second proposition, as there is a strong indication that noise traders destabilize the market, which is mainly apparent in the higher deviation from fundamental value. This corresponds with the market view by De Long et al. (1990a,b). Moreover, the extreme fractions of noise traders are associated with higher volatility (and market activity). However, volatility in the lowest decile groups is the highest.

Overall, the HAMs show simulated results consistent with the second proposition on noise trader demand. In addition, the simulated results on the fraction of investors on the bandwagon are less conclusive. These investors seem both to dampen as well as to destabilize markets. This inconclusive result is supported by literature as there is evidence for both sides.

Table 8: Decile Group Analysis of Simulated Times Series - Fraction on the Bandwagon

Analysis of 100 samples of simulated times series of 200 periods using the five-type HAM. Refer to Table 1 for the parameter sets. Other variables: $\sigma_\varepsilon = 2.5$, $R^* = 1.05$, $d^* = 25$, initial observations $x_1 = x_2 = x_3 = 5$, and initial fractions are $n_1 = n_2 = 0.5$, $n_3 = n_4 = n_5 = 0$. Based on the fraction of arbitrageurs on the bandwagon (i.e. $n_{3,t} + n_{4,t}$) the simulated times series are divided into ten decile groups, for each of the 8 parameter sets (S1-S8). Some decile groups are combined into one group (e.g. the fourth, fifth, sixth, and seventh decile groups are combined in group d4/7). For each group I calculate the median of the average fraction in that group, the median of the average return, the median of the absolute, positive, and negative deviation from fundamental (i.e. x_t), the median of the average volatility, and the median of the average market activity. The volatility is computed using the RiskMetrics approach (where $\lambda = 0.95$). The market activity measure is the fraction weighted average change in fractions. I compare the decile groups by the difference between the highest and lowest, highest and middle, and middle and lowest fraction groups (i.e. d10-d1, d10-d4/7, d4/7-d1). "high" means that the deviation from fundamental value is higher than 1,000

(1) Median of average fraction

	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
S1	0.08	0.19	0.24	0.27	0.33	0.25	0.09	0.17
S2	0.10	0.21	0.25	0.25	0.25	0.15	0.00	0.14
S3	0.00	0.03	0.21	0.34	0.43	0.43	0.22	0.21
S4	0.00	0.03	0.17	0.24	0.25	0.25	0.08	0.17
S5	0.00	0.00	0.04	0.19	0.37	0.37	0.33	0.04
S6	0.00	0.03	0.17	0.24	0.25	0.25	0.08	0.17
S7	0.01	0.11	0.23	0.30	0.39	0.38	0.16	0.22
S8	0.02	0.12	0.22	0.25	0.25	0.23	0.03	0.21

(2) Median of average return

	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
S1	0.04	0.02	0.00	-0.01	-0.03	-0.07	-0.03	-0.04
S2	0.02	0.01	0.01	0.00	0.00	-0.02	0.00	-0.02
S3	0.03	0.01	0.01	-0.01	-0.02	-0.05	-0.03	-0.02
S4	0.02	0.00	0.00	0.00	0.00	-0.02	0.00	-0.02
S5	0.10	0.05	0.09	0.03	0.02	-0.08	-0.07	-0.01
S6	-0.01	0.00	0.01	0.01	0.02	0.03	0.01	0.02
S7	0.02	0.01	0.00	-0.01	-0.02	-0.05	-0.02	-0.03
S8	0.03	0.00	0.00	0.00	0.00	-0.03	-0.01	-0.02

(3) Median of average absolute deviation

	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
S1	4.19	3.66	3.26	4.42	6.31	2.12	3.06	-0.94
S2	5.68	5.22	4.06	3.46	3.12	-2.57	-0.94	-1.62
S3	5.16	3.95	3.97	6.17	6.19	1.03	2.22	-1.19
S4	6.05	5.22	4.07	3.30	3.15	-2.90	-0.91	-1.98
S5	77.30	30.85	high	14.91	11.59	-65.71	n.a.	n.a.
S6	9.13	8.01	6.24	4.76	3.87	-5.27	-2.37	-2.90
S7	3.21	2.87	2.89	3.54	4.67	1.47	1.78	-0.31
S8	3.77	3.57	3.30	3.00	2.76	-1.01	-0.55	-0.46

Table 8 continued

(4) Median of average positive deviation

	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
S1	4.15	3.51	3.23	3.91	5.23	1.08	2.00	-0.92
S2	3.90	3.22	3.24	2.96	2.71	-1.19	-0.53	-0.66
S3	5.77	4.45	4.33	5.91	6.03	0.26	1.70	-1.44
S4	3.78	4.01	3.54	3.14	3.07	-0.71	-0.48	-0.23
S5	110.51	39.30	high	17.19	12.76	-97.76	n.a.	n.a.
S6	7.07	6.17	5.67	4.82	4.18	-2.89	-1.50	-1.39
S7	3.32	2.64	2.67	3.04	3.69	0.38	1.03	-0.65
S8	3.36	2.71	2.73	2.67	2.66	-0.70	-0.07	-0.63

(5) Median of average negative deviation

	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
S1	-4.20	-3.69	-3.40	-4.63	-7.53	-3.33	-4.13	0.80
S2	-6.67	-6.11	-4.45	-3.72	-3.37	3.30	1.08	2.23
S3	-3.54	-2.93	-3.35	-5.51	-5.88	-2.34	-2.53	0.20
S4	-6.29	-5.91	-4.41	-3.37	-3.09	3.21	1.32	1.89
S5	-5.35	-4.77	-3.91	-4.60	-7.20	-1.85	-3.29	1.44
S6	-8.56	-8.35	-6.14	-4.28	-3.25	5.32	2.89	2.42
S7	-2.97	-3.03	-3.07	-3.79	-5.59	-2.63	-2.53	-0.10
S8	-4.17	-4.14	-3.65	-3.12	-2.85	1.32	0.80	0.52

(6) Median of average volatility

	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
S1	0.19	0.19	0.19	0.19	0.19	0.00	0.00	0.00
S2	0.20	0.20	0.19	0.19	0.19	-0.01	0.00	-0.01
S3	0.20	0.19	0.19	0.19	0.19	-0.01	0.00	-0.01
S4	0.23	0.21	0.20	0.20	0.20	-0.03	0.00	-0.03
S5	0.21	0.20	0.20	0.19	0.18	-0.03	-0.02	-0.01
S6	0.22	0.20	0.20	0.19	0.19	-0.03	-0.01	-0.02
S7	0.18	0.19	0.18	0.19	0.19	0.01	0.01	0.00
S8	0.19	0.19	0.19	0.19	0.19	-0.01	0.00	0.00

(7) Median of average market activity

	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
S1	0.11	0.09	0.06	0.08	0.14	0.03	0.07	-0.05
S2	0.27	0.18	0.11	0.09	0.08	-0.19	-0.03	-0.16
S3	0.20	0.16	0.11	0.12	0.20	0.00	0.09	-0.09
S4	0.32	0.39	0.28	0.18	0.16	-0.16	-0.12	-0.04
S5	0.14	0.13	0.06	0.10	0.14	0.00	0.07	-0.07
S6	0.38	0.38	0.27	0.17	0.14	-0.24	-0.13	-0.11
S7	0.17	0.13	0.08	0.11	0.18	0.02	0.10	-0.08
S8	0.38	0.34	0.19	0.13	0.12	-0.26	-0.07	-0.19

Table 9: Decile Group Analysis of Simulated Time Series - Fraction of Noise Traders

Analysis of 100 samples of simulated times series of 200 periods using the five-type HAM. Refer to Table 1 for the parameter sets. Other variables: $\sigma_\varepsilon = 2.5$, $R^* = 1.05$, $d^* = 25$, initial observations $x_1 = x_2 = x_3 = 5$, and initial fractions are $n_1 = n_2 = 0.5$, $n_3 = n_4 = n_5 = 0$. Based on the fraction of noise traders (i.e. $n_{2,t} + n_{5,t}$) the simulated times series are divided into ten decile groups, for each of the 8 parameter sets (S1-S8). Some decile groups are combined into one group (e.g. the fourth, fifth, sixth, and seventh decile groups are combined in group d4/7). For each group I calculate the median of the average fraction in that group, the median of the average return, the median of the absolute, positive, and negative deviation from fundamental (i.e. x_t), the median of the average volatility, and the median of the average market activity. The volatility is computed using the RiskMetrics approach (where $\lambda = 0.95$). The market activity measure is the fraction weighted average change in fractions. I compare the decile groups by the difference between the highest and lowest, highest and middle, and middle and lowest fraction groups (i.e. d10-d1, d10-d4/7, d4/7-d1). "high" means that the deviation from fundamental value is higher than 1,000

(1) Median of average fraction

	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
S1	0.45	0.48	0.49	0.50	0.52	0.07	0.02	0.05
S2	0.17	0.37	0.48	0.55	0.68	0.52	0.20	0.32
S3	0.25	0.40	0.48	0.52	0.61	0.36	0.14	0.23
S4	0.00	0.09	0.43	0.73	0.96	0.96	0.53	0.43
S5	0.39	0.50	0.90	1.00	1.00	0.61	0.10	0.51
S6	0.00	0.12	0.44	0.74	0.96	0.96	0.52	0.44
S7	0.38	0.45	0.49	0.51	0.54	0.16	0.05	0.11
S8	0.03	0.22	0.46	0.61	0.85	0.82	0.38	0.43

(2) Median of average return

	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
S1	0.03	0.01	0.00	-0.01	-0.01	-0.04	-0.01	-0.02
S2	0.04	0.02	0.01	-0.01	-0.03	-0.07	-0.04	-0.03
S3	0.00	0.01	0.01	-0.01	-0.02	-0.02	-0.03	0.01
S4	0.05	0.01	0.01	-0.01	-0.06	-0.11	-0.07	-0.05
S5	0.03	0.05	0.04	0.00	0.07	0.04	0.03	0.00
S6	0.02	0.02	0.01	-0.01	-0.07	-0.09	-0.08	-0.01
S7	0.02	0.00	0.00	-0.01	-0.01	-0.03	-0.01	-0.02
S8	0.03	0.02	0.00	-0.01	-0.03	-0.07	-0.04	-0.03

(3) Median of average absolute deviation

	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
S1	3.80	3.42	3.32	4.44	7.38	3.58	4.05	-0.48
S2	4.85	3.81	3.37	4.56	7.34	2.49	3.97	-1.48
S3	4.15	3.55	3.45	5.25	10.64	6.49	7.20	-0.70
S4	4.31	3.16	3.19	4.87	8.83	4.52	5.64	-1.12
S5	7.90	9.45	20.33	11.63	14.88	6.98	-5.45	12.43
S6	8.13	5.59	4.29	6.90	12.02	3.89	7.73	-3.84
S7	2.79	2.72	2.84	3.69	5.65	2.87	2.82	0.05
S8	3.02	2.86	2.95	3.50	5.17	2.15	2.22	-0.07

Table 9 continued

(4) Median of average positive deviation								
	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
S1	3.35	3.21	3.27	4.22	5.83	2.47	2.56	-0.09
S2	3.03	2.91	3.11	3.63	4.26	1.23	1.15	0.07
S3	4.78	4.02	3.67	5.26	9.57	4.79	5.91	-1.11
S4	3.06	2.97	3.19	4.12	5.67	2.60	2.48	0.13
S5	9.13	11.54	23.96	12.94	15.58	6.45	-8.37	14.82
S6	7.63	5.44	4.44	6.02	8.60	0.96	4.16	-3.20
S7	2.42	2.58	2.63	3.36	4.12	1.70	1.49	0.21
S8	2.37	2.56	2.83	2.66	3.44	1.06	0.61	0.45

(5) Median of average negative deviation								
	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
S1	-3.92	-3.46	-3.27	-4.61	-8.63	-4.71	-5.36	0.65
S2	-5.51	-4.14	-3.58	-5.18	-8.66	-3.16	-5.09	1.93
S3	-3.07	-3.01	-3.02	-4.92	-10.69	-7.62	-7.67	0.05
S4	-4.70	-3.26	-3.05	-5.20	-10.82	-6.12	-7.77	1.65
S5	-3.94	-4.00	-4.97	-8.59	-12.50	-8.56	-7.53	-1.03
S6	-7.49	-4.95	-4.09	-7.32	-13.52	-6.03	-9.43	3.40
S7	-2.82	-2.73	-2.98	-3.82	-6.62	-3.81	-3.64	-0.17
S8	-3.48	-2.98	-3.05	-4.02	-6.25	-2.77	-3.20	0.43

(6) Median of average volatility								
	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
S1	0.20	0.19	0.19	0.18	0.19	-0.01	0.00	-0.01
S2	0.21	0.19	0.19	0.19	0.20	-0.01	0.01	-0.02
S3	0.20	0.19	0.19	0.19	0.19	-0.01	0.00	-0.01
S4	0.22	0.21	0.20	0.20	0.21	-0.01	0.01	-0.02
S5	0.18	0.19	0.20	0.20	0.20	0.01	0.00	0.01
S6	0.21	0.20	0.20	0.20	0.21	0.00	0.01	-0.02
S7	0.20	0.19	0.18	0.18	0.19	-0.01	0.01	-0.01
S8	0.20	0.19	0.19	0.19	0.19	-0.01	0.01	-0.01

(7) Median of average market activity								
	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
S1	0.11	0.09	0.08	0.08	0.09	-0.01	0.02	-0.03
S2	0.27	0.13	0.09	0.12	0.24	-0.04	0.15	-0.19
S3	0.14	0.18	0.13	0.12	0.13	-0.01	0.00	-0.01
S4	0.35	0.37	0.18	0.27	0.40	0.05	0.22	-0.17
S5	0.14	0.12	0.10	0.11	0.10	-0.04	0.00	-0.04
S6	0.40	0.34	0.17	0.26	0.38	-0.02	0.21	-0.23
S7	0.15	0.14	0.10	0.09	0.13	-0.02	0.03	-0.05
S8	0.43	0.26	0.13	0.19	0.37	-0.06	0.24	-0.30

4.4 Estimation Results

Next, I estimate the models using S&P500 data, which is also done by Boswijk et al. (2007). First, the data is explained and then the estimation results are presented.

The dataset described in Shiller (1989) is used. It consists of annual data of the S&P500 index. I use annual data as it the most used data frequency in behavioral finance (Barberis and Thaler, 2003). A lower data frequency would be more affected by other effects than behavioral effects. As in Boswijk et al. (2007), both dividends and earnings are used as a measure of cash flow, but in this analysis I use the sample period 1950 to 2014. The constant growth rate g is the average growth rate of real cash flows and discount rate r is the sum of the average cash flow yield and g. Using these rates, the gross return R^* and fundamental PY-ratio d_t^* can be determined, see equations (13) and (15) respectively. Table 10 summarizes the values used for the fundamental process and reports a fundamental value for the PY-ratio of 29.98 when using dividends and 15.91 using earnings. Refer to Boswijk et al. (2007) for a more detailed explanation. Figure 2 shows the log of the stock price, the fundamental price, the PY-ratio, and fundamental ratio using the values of Table 10. The graphs show there is a spike in the PY-ratio around 2000 and that my sample also includes the drop in PY-ratio afterwards.

Table 10: Fundamental Value

Values used to determine the fundamental value (for the price-to-dividends and price-to-earnings ratio). S&P500 data from 1950-2014. *cpi* is the average inflation rate which is used to deflate nominal variables. y/p is the average cash flow yield (i.e. Y_t/P_{t-1}), g is the average growth rate of real cash flows (earnings are smoothed using 10-years moving-average), the discount rate is $r = y/p + g$, the gross rate of return is determined by $R^* = (1+r)/(1+g)$, and $d^* = (1+g)/(r-g)$ is the price-to-cash flow fundamental ratio. All numbers are expressed as percents, except R^* and the fundamental ratio d^*

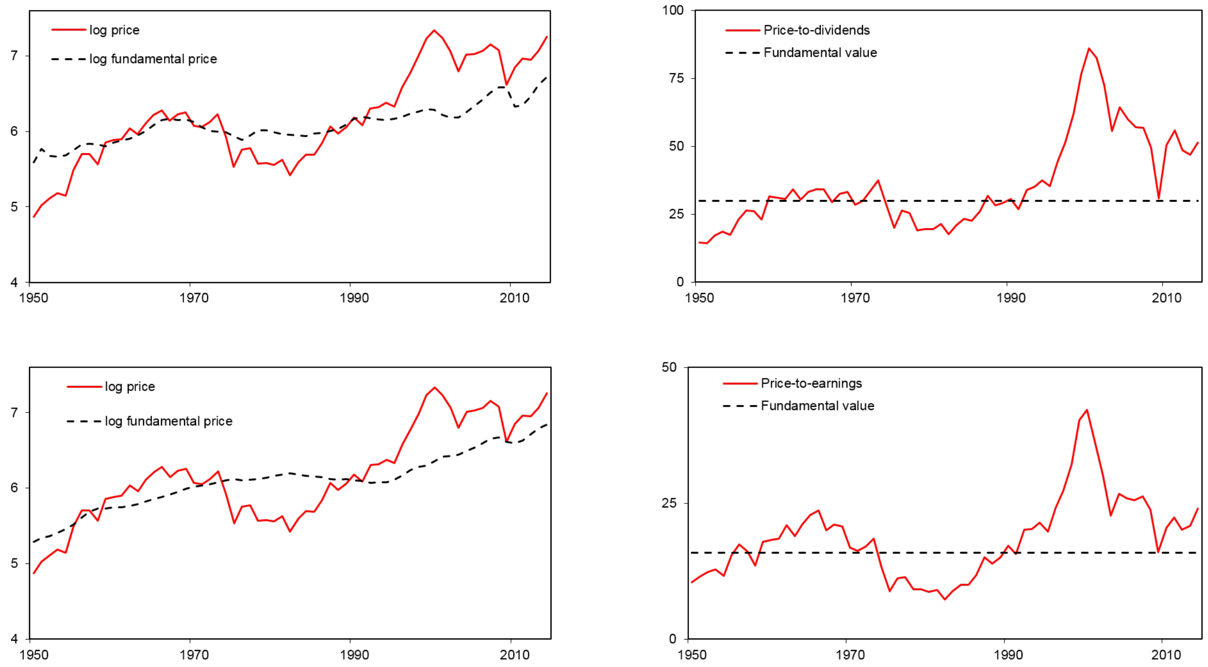
Cash flows	*cpi*	y/p	g	r	R^*	d^*
Dividends	3.61	3.40	1.97	5.37	1.033	29.98
Earnings	3.61	6.44	2.50	8.95	1.063	15.91

The belief parameters ϕ_h and the β are estimated using the annual data of the S&P500 index. Note that the δ is fixed at 0% and 50%, similar to the parameter sets. As with the simulation, equation (29) is used and the parameters are estimated by nonlinear least squares.

Table 11 reports the estimation output for both models. The results of the two-type model are similar to the results presented by Boswijk et al. (2007). The belief coefficients indicate that arbitrageurs follow a mean-reverting strategy (between 0.649 and 0.825) and that noise traders are trend followers (between 1.123 and 1.225). All estimated belief parameters are strongly significant, at a 1% level. The introduction of the δ is the main difference with the model by

Figure 2: Price and Fundamental Value

(left) Graphs of the log of the stock price of the S&P500 and of the fundamental value, and *(right)* the real and fundamental price-to-cash flow ratio. Dividends are used to measure cash flows for the top two graphs. The bottom two graphs use earnings as cash flows. Values of Table 10 are used to calculate the fundamental values



Boswijk et al. (2007). It makes sense that the beliefs are stronger and that β is higher when the δ is higher, as investors switch beliefs less often. Once they update their (strong) beliefs, they are more likely to switch. The β ranges from 0.154 to 0.214 and is not significantly different from zero. Boswijk et al. (2007) emphasize that this is a common result, as large changes in β cause only small variation of the fraction, hence β has a large standard deviation. In addition I report the R^2 of the regression, the Akaike selection criterion (AIC) value and corresponding log likelihood, the estimated coefficient of the AR(1) model, and the AIC and log likelihood of this AR(1) model. The two-type model achieves a lower AIC value than the linear AR(1) model, indicating that the model captures nonlinearity in the data. So far, I find no surprising results.

All results of the five-type model (except the results of the dividends with $\delta = 0$) show that also in this model the arbitrageurs can be classified as investors with a mean-reverting belief (ϕ_1 between 0.551 and 0.721) and that the noise traders are trend-following (ϕ_2 between 1.425 and 1.504). The beliefs are all significant and seem to be stronger than the estimated beliefs of the two-type model. This could be caused by the fact that investors have more options to choose (e.g. jump on the bandwagon), hence once following a belief that belief is strong. The β ranges from 0.002 to 0.023 and is again not significant and higher for the estimates with δ fixed to 50%. Note that the AIC values of the estimated HAM are now higher than the values of an AR(1) model. Only the estimated HAM in the third column has a log likelihood lower than the log likelihood of the AR(1) model. These results indicate that I did not gain explanatory power by adding complexity. And it also indicates that the model does not capture nonlinearity in the data. But, as the values are not that far apart (on average AIC is 6% higher than the $AIC_{AR(1)}$) and the structured, more complex model allows us to better understand the micromarket than a reduced form AR(1) model, I draw no further conclusions from the reported AIC values.

Note that the results of the estimated model to data using dividends as cash flows and with $\delta = 0$ are not considered here, as the β has a relatively high negative value that indicates that agents prefer to switch to a strategy that performs worse.

Figure 3 shows the fractions of the model estimated using earnings and $\delta = 50\%$. It can be seen that the fraction of the arbitrageurs and noise traders fluctuate over time, ranging from 0% to 80%. The fractions of investors on the bandwagon ranges between 0% and 20% and the percentage of noise traders that are out of the market is not higher than 32%. The pattern of the fraction is especially interesting in combination with the time series of the deviation from the fundamental value (see Figure 2). During a period with increasing deviation from fundamental (1995-2000), it can be seen that the fraction of noise traders increases to almost 60% just before the spike in 2000, with not many noise traders leaving the market. The increased fraction of noise traders is a pattern that complies with the proposition that noise traders' misperceptions can drive up prices leading to divergence of market prices and fundamental values. From Figure 3 it can be seen that arbitrageurs act in opposite direction of the noise traders (also concluded from estimated belief parameters). During the period 1995-2000 the fraction of arbitrageurs decreases and the fraction of investors on the bandwagon increases. Especially the fraction of trend-following investors on the bandwagon (with $\phi_4 = 1.359$) increases during this period. This could be explained by arbitrageurs detecting the upward trend caused by noise traders and that they decide to start herding, which in this case refers to trading ahead of the noise traders.

Another interesting dynamic is that during the drop in fundamental value (2000-2003), noise trading decreases and more arbitrageurs jump off the bandwagon (or join the market). Here, betting against the noise traders seems to pay-off, hence the increase in arbitrageurs. Note that when analyzing the fractions, it is almost impossible to conclude which type of investor is leading, but the reasoning of De Long et al. (1990b) can be applied as the agents behavior is in line with their reasoning (arbitrageurs following noise traders). Other estimation results show similar dynamics in fractions.

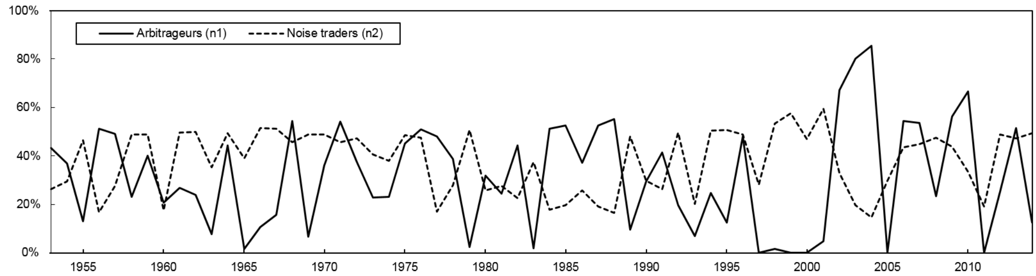
Table 11: Estimation Results

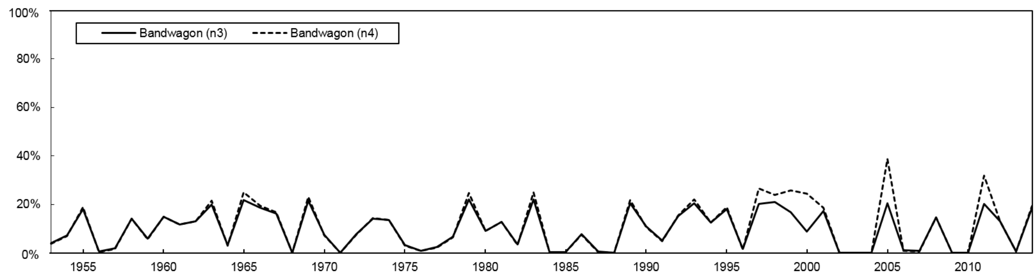
Estimation results of the five-type and two-type model. Estimated to annual data of the S&P500 index (1950-2014) using dividends and earnings as measure for cash flows, two different values for δ (0% and 50%), and using nonlinear least squared method. Estimated parameters of the beliefs parameters ($\phi's$) and the intensity of choice (β). * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parentheses. Reported statistics are the R^2, the Akaike information criterion (AIC), the associated value of the log likelihood, AR(1) coefficient fitted to the data, the AIC of the linear AR(1) model, and the log likelihood value

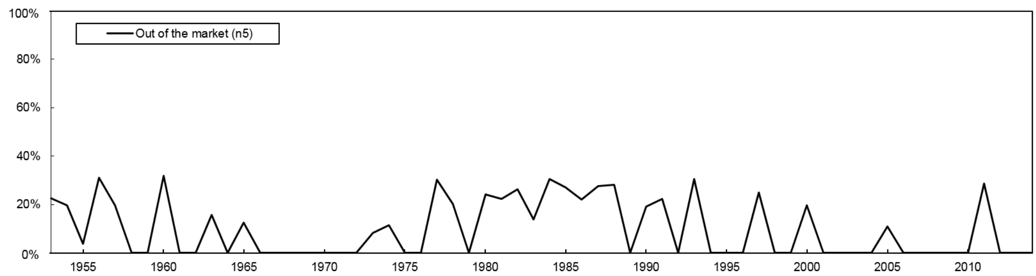
Five-type Model				
	Dividends		Earnings	
δ	0%	50%	0%	50%
ϕ_1	0.955***	0.551***	0.668***	0.721***
	(0.078)	(0.254)	(0.130)	(0.209)
ϕ_2	0.970***	1.504***	1.425***	1.425***
	(0.037)	(0.192)	(0.089)	(0.102)
ϕ_3	1.096	0.599	-0.286	0.726
	(1.067)	(4.716)	(0.890)	(1.022)
ϕ_4	0.530	0.796	1.738***	1.359
	(2.108)	(4.149)	(0.789)	(0.976)
β	-5.609	0.002	0.023	0.020
	(33.283)	(0.003)	(0.027)	(0.066)
R^2	0.83	0.84	0.85	0.82
AIC	4.00	3.97	2.23	2.41
$LogL$	3.85	3.82	2.08	2.26
$AR(1)$	0.92	0.92	0.91	0.91
$AIC_{AR(1)}$	3.71	3.71	2.22	2.22
$LogL_{AR(1)}$	3.65	3.65	2.16	2.16
Two-type Model				
ϕ_1	0.825***	0.737***	0.709***	0.649***
	(0.066)	(0.094)	(0.093)	(0.162)
ϕ_2	1.123***	1.157***	1.218***	1.225***
	(0.065)	(0.083)	(0.091)	(0.154)
β	0.176	0.214	0.163	0.154
	(0.388)	(0.983)	(0.188)	(0.298)
R^2	0.88	0.88	0.87	0.86
AIC	3.60	3.61	2.05	2.15
$LogL$	3.51	3.52	1.96	2.05
$AR(1)$	0.92	0.92	0.91	0.91
$AIC_{AR(1)}$	3.71	3.71	2.22	2.22
$LogL_{AR(1)}$	3.65	3.65	2.16	2.16

Figure 3: Resulted Fractions of Estimated Five-Type Model

Resulted fractions of the estimated five-type HAM (using earnings as cash flows and $\delta = 50\%$). Top Figure shows the times series of the fraction of arbitrageurs ($n_{1,t}$) and the fraction of noise traders in the market ($n_{2,t}$). The Figure in the middle shows the fractions of the arbitrageurs on the bandwagon (both $n_{3,t}$ and $n_{4,t}$). The bottom Figure shows the fraction of noise traders that left the market ($n_{5,t}$). See Table 11 (fourth column) for the estimates of the model







As a brief aside regarding the dynamics of the PY-ratio, I estimate the models to monthly data of the interesting period between January 1990 and December 2014. These monthly results show that I can use the HAM to understand the market, also at a higher frequency. Tables 15 and 16 in the Appendix report the statistics and the estimation results of the monthly analysis. The estimated belief parameters are less strong in this case. Moreover, in the models of the first three columns, the arbitrageurs are trend-following instead of mean-reverting as $\phi_1 > 1$ and even larger than ϕ_2. But, as the fourth column of Table 16 shows estimation results that meet my expectation, I investigate these outcomes in more detail. Figure 13 in the Appendix shows the dynamics of the fractions of the five-type model estimated using earnings as cash flows and with $\delta = 50\%$. Again, I find similar patterns as in the dynamics of the fractions when using annual data. That is to say, before the spike in 2000 noise trader demand increases and after the spike arbitrageurs take over the market. Moreover, the fraction of trend-following arbitrageurs on the bandwagon also spikes during the upward trend.

Key conclusion here is that the estimation results and especially the fractions support the statements in both proposition 1 and 2. As a final analysis, the estimation results will be divided in decile groups similar to the analysis on the simulation results.

4.5 Decile Group Analysis of Estimation Results

Table 12 reports the statistics per decile group of investors on the bandwagon. It can be seen that investors on the bandwagon are mainly present during periods with high returns and high positive deviations from fundamental value. This is already confirmed by the previous analysis on the fractions. In addition to this, I find a U-shaped pattern in the deviations from fundamental value. Again, this supports the idea that an equal distribution of the fractions over the different investors type depicts a stable period. Moreover, once more traders are on the bandwagon, the market activity seems to be lower. Note that the estimation results using earnings as cash flows are similar at some points, which is caused by the fact that the estimation results where $\delta = 50\%$ are an extrapolated version of the estimation results where $\delta = 0\%$. Also note that in some cases medians are not available in the highest or lowest decile. This happens when the fraction is at a maximum or minimum for more than 1/10 of the periods or if the metric is not observed in the decile group (e.g. negative deviations from fundamental value). In the simulated time series I excluded these observations, but here I only have one time series.

The findings in Table 13 show that noise trader demand increases the returns, increases the (positive) deviation from fundamental value, and decreases volatility and market activity. Again, a clear U-shaped pattern is present in the deviations from fundamental value. In opposite of the simulated results, I find that when noise trader dominate, they earn a higher return which better corresponds to the findings of De Long et al. (1990a). For the simple two-type model, the noise trader demand is also examined. The results are reported in Table 17 in the Appendix and the same can be concluded as with noise traders in the five-type HAM.

The main difference with the simulated times series is that I find more conclusive evidence for the first proposition and that for both fractions I find that the return is higher in the highest decile group (d10-d1 is positive for all estimation results). The fractions in Figure 3 give an explanation for this. Both the bandwagon fraction as well as noise trader demand is high during the steep price increase just before the year 2000. As a result of this data the returns in the tenth fraction deciles are the highest.

The estimation results show that the both models are able to find particular behavior for the types of traders examined by De Long et al. (1990a,b). I find that higher fractions of traders on the bandwagon and noise traders are associated with more unstable markets. Thus, the results of the decile group analysis of estimations are consistent with the predictions of propositions 1 and 2.

Table 12: Decile Group Analysis of Estimated Time Series - Fraction on the Bandwagon

Analysis of estimated times series using the five-type HAM. The belief parameters and β are estimated using both dividends (D) and earnings (E) as a proxy for cash flows. The δ is fixed at 0% or 50%. Table 11 shows the estimation results of each of the four combinations. Based on the fraction of arbitrageurs on the bandwagon (i.e. $n_{3,t} + n_{4,t}$) the times series are divided into ten decile groups. Some decile groups are combined into one group (e.g. the fourth, fifth, sixth, and seventh decile groups are combined in group d4/7). For each group I calculate the median of the average fraction in that group, the median of the average return, the median of the absolute, positive, and negative deviation from fundamental (i.e. x_t), the median of the average volatility, and the median of the average market activity. The volatility is computed using the RiskMetrics approach (where $\lambda = 0.95$). The market activity measure is the weighted average change in fractions. I compare the decile groups by the difference between the highest and lowest, highest and middle, and middle and lowest fraction groups (i.e. d10-d1, d10-d4/7, d4/7-d1). "high" means that the deviation from fundamental value is higher than 1,000. "n.a." if the fraction is at a maximum or minimum for more than 1/10 of the periods or if observed metric does not exist in the decile group (e.g. negative deviations from fundamental value)

(1) Median of average fraction

	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
D (0%)	0.00	0.18	0.63	1.00	1.00	1.00	0.37	0.63
D (50%)	0.00	0.00	0.37	0.82	1.00	1.00	0.63	0.37
E (0%)	0.05	0.33	0.47	0.58	0.85	0.80	0.38	0.42
E (50%)	0.00	0.03	0.38	0.83	0.99	0.99	0.62	0.38

(2) Median of average return

	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
D (0%)	0.09	0.09	0.02	0.03	n.a.	n.a.	n.a.	-0.08
D (50%)	-0.06	0.03	0.02	0.09	0.09	0.15	0.08	0.07
E (0%)	-0.06	0.09	-0.01	0.06	0.13	0.19	0.14	0.05
E (50%)	-0.06	0.09	-0.01	0.06	0.13	0.19	0.14	0.05

(3) Median of average absolute deviation

	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
D (0%)	38.75	8.28	5.06	11.07	n.a.	n.a.	n.a.	-33.69
D (50%)	23.48	11.07	5.06	8.28	38.75	15.27	33.69	-18.42
E (0%)	6.94	4.55	4.09	6.74	14.23	7.29	10.14	-2.85
E (50%)	6.94	4.55	4.09	6.74	14.23	7.29	10.14	-2.85

(4) Median of average positive deviation

	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
D (0%)	38.75	9.39	4.53	19.18	n.a.	n.a.	n.a.	-34.22
D (50%)	23.48	19.18	4.53	9.39	38.75	15.27	34.22	-18.95
E (0%)	6.94	5.04	3.46	8.05	14.23	7.29	10.77	-3.48
E (50%)	6.94	5.04	3.46	8.05	14.23	7.29	10.77	-3.48

(5) Median of average negative deviation

	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
D (0%)	n.a.	-6.99	-5.68	-6.00	n.a.	n.a.	n.a.	n.a.
D (50%)	n.a.	-6.00	-5.68	-6.99	n.a.	n.a.	n.a.	n.a.
E (0%)	n.a.	-3.98	-4.83	-3.80	n.a.	n.a.	n.a.	n.a.
E (50%)	n.a.	-3.98	-4.83	-3.80	n.a.	n.a.	n.a.	n.a.

Table 12 continued

(6) Median of average volatility								
	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
D (0%)	0.16	0.16	0.15	0.16	n.a.	n.a.	n.a.	-0.01
D (50%)	0.17	0.16	0.15	0.16	0.16	0.00	0.01	-0.01
E (0%)	0.16	0.16	0.15	0.16	0.16	0.00	0.01	-0.01
E (50%)	0.16	0.16	0.15	0.16	0.16	0.00	0.01	-0.01

(7) Median of average market activity								
	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
D (0%)	0.48	0.36	0.21	0.29	n.a.	n.a.	n.a.	-0.27
D (50%)	0.13	0.14	0.14	0.20	0.15	0.02	0.01	0.00
E (0%)	0.24	0.09	0.09	0.12	0.18	-0.06	0.10	-0.16
E (50%)	0.21	0.15	0.13	0.14	0.17	-0.04	0.04	-0.07

Table 13: Decile Group Analysis of Estimated Time Series - Fraction of Noise Traders

Analysis of estimated times series using the five-type HAM. The belief parameters and β are estimated using both dividends (D) and earnings (E) as a proxy for cash flows. The δ is fixed at 0% or 50%. Table 11 shows the estimation results of each of the four combinations. Based on the fraction of noise traders (i.e. $n_{2,t} + n_{5,t}$) the times series are divided into ten decile groups. Some decile groups are combined into one group (e.g. the fourth, fifth, sixth, and seventh decile groups are combined in group d4/7). For each group I calculate the median of the average fraction in that group, the median of the average return, the median of the absolute, positive, and negative deviation from fundamental (i.e. x_t), the median of the average volatility, and the median of the average market activity. The volatility is computed using the RiskMetrics approach (where $\lambda = 0.95$). The market activity measure is the weighted average change in fractions. I compare the decile groups by the difference between the highest and lowest, highest and middle, and middle and lowest fraction groups (i.e. d10-d1, d10-d4/7, d4/7-d1). "high" means that the deviation from fundamental value is higher than 1,000. "n.a." if the fraction is at a maximum or minimum for more than 1/10 of the periods or if observed metric does not exist in the decile group (e.g. negative deviations from fundamental value)

(1) Median of average fraction								
	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
D (0%)	0.01	0.30	0.59	0.97	1.00	0.99	0.41	0.58
D (50%)	0.40	0.48	0.50	0.50	0.53	0.13	0.03	0.10
E (0%)	0.18	0.45	0.49	0.52	0.64	0.46	0.15	0.32
E (50%)	0.31	0.46	0.49	0.51	0.56	0.25	0.07	0.18

(2) Median of average return								
	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
D (0%)	0.10	0.08	0.02	0.03	n.a.	n.a.	n.a.	-0.08
D (50%)	-0.03	0.04	0.04	0.02	0.13	0.16	0.09	0.07
E (0%)	-0.06	0.09	0.00	0.05	0.14	0.20	0.14	0.06
E (50%)	0.01	0.01	0.03	0.05	0.12	0.11	0.09	0.02

(3) Median of average absolute deviation								
	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
D (0%)	35.94	7.87	6.21	11.07	n.a.	n.a.	n.a.	-29.73
D (50%)	22.55	11.03	6.17	7.78	35.74	13.19	29.57	-16.38
E (0%)	6.94	4.55	4.08	5.91	14.51	7.57	10.43	-2.86
E (50%)	9.35	3.70	4.05	4.81	16.13	6.78	12.08	-5.30

(4) Median of average positive deviation								
	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
D (0%)	41.32	8.68	4.60	19.18	n.a.	n.a.	n.a.	-36.72
D (50%)	22.55	21.96	6.38	6.76	39.08	16.53	32.70	-16.17
E (0%)	6.94	5.04	3.46	7.03	14.51	7.57	11.05	-3.48
E (50%)	9.35	3.78	3.96	4.18	16.13	6.78	12.17	-5.39

(5) Median of average negative deviation								
	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
D (0%)	-9.01	-6.58	-7.60	-6.00	n.a.	n.a.	n.a.	1.42
D (50%)	n.a.	-6.17	-5.89	-8.67	-15.30	n.a.	-9.41	n.a.
E (0%)	n.a.	-3.98	-4.69	-4.12	n.a.	n.a.	n.a.	n.a.
E (50%)	n.a.	-3.58	-4.17	-5.21	n.a.	n.a.	n.a.	n.a.

Table 13 continued

(6) Median of average volatility								
	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
D (0%)	0.16	0.16	0.15	0.16	n.a.	n.a.	n.a.	-0.01
D (50%)	0.16	0.16	0.15	0.16	0.16	0.00	0.01	-0.01
E (0%)	0.16	0.16	0.15	0.16	0.16	0.00	0.01	-0.01
E (50%)	0.17	0.16	0.16	0.15	0.16	-0.01	0.00	-0.01

(7) Median of average market activity								
	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
D (0%)	0.49	0.31	0.18	0.29	n.a.	n.a.	n.a.	-0.30
D (50%)	0.10	0.11	0.18	0.15	0.12	0.02	-0.06	0.08
E (0%)	0.24	0.09	0.08	0.10	0.18	-0.07	0.10	-0.16
E (50%)	0.23	0.10	0.16	0.11	0.12	-0.11	-0.04	-0.08

5 Conclusion

I develop a heterogeneous agent model that describes the market behavior of arbitrageurs and noise traders (De Long et al., 1990a,b). I show that the widely used theoretical framework introduced by these academics can be empirically investigated using the five-type model. The model is a refined version of the two-type HAM by Boswijk et al. (2007), in which arbitrageurs amplify mispricing rather than correct it and noise traders face short-sale constraints (Chen et al., 2002). I use the outcomes of simulations and estimations to examine the model dynamics and in particular I focus on two propositions of both types of investors. The two statements are supported by the main findings, as such the five-type model provides empirical evidence for the theoretical framework introduced by De Long et al. (1990a,b).

Consistent with the first proposition, I find that arbitrageurs can cause the market price to deviate further from the fundamental value. This is mainly shown by the decile analysis on the estimation results. The estimation results show that arbitrageurs are mean-reverting, in a normal situation. From the dynamics of the fractions, it can be seen that they jump on the bandwagon if there is a pronounced trend set by noise traders. This result supports the idea that arbitrageurs destabilize markets by triggering positive feedback strategies (De Long et al., 1990b). The simulation results are mixed as they also emphasize the mean-reverting effects of arbitrageurs.

Consistent with the second proposition, I report evidence that noise traders destabilize markets. From the simulation and estimation results it can be seen that the deviation from fundamental value is higher when the fraction of noise traders is higher. I find a U-shape pattern when looking at the volatility (and market activity) across deciles. Overall, this leads to the conclusion that indeed strong noise trader demand can be associated with a more unstable market.

This thesis shows how we can better understand micromarket dynamics using heterogeneous agent models. These models can specify a wide range of different agents and associated behavior. In addition, HAMs can be estimated to real data allowing for an empirical analysis of the proposed market dynamics. The model presented here is obviously relatively complex and stylized. The inputs and assumptions can be debated. Besides, many aspects impact asset prices and sure not all are take into account in my analyses. However I present a HAM that provides insightful findings from a behavioral finance perspective. As such, it is a step forward.

A suggestion for future research is to use the established five-type model for more detailed investigation of financial markets. For instance, one could focus on the critical value of the belief parameters that creates an unstable market. Or one could estimate the HAM to a different data set, focusing on different markets or specific time periods. A second suggestion is that one could use the insights of this thesis to extend or develop a different HAM. This methodology provides a flexible framework in which one can define countless market dynamics. Moreover, the model can be simulated and estimated and can empirically investigate financial markets and improve our understanding of them.

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A Appendix

Figure 4: Simulated Fractions - Two-type model

Typical outcome of simulation for each of the 8 parameter sets defined in Table 1 using the two-type model

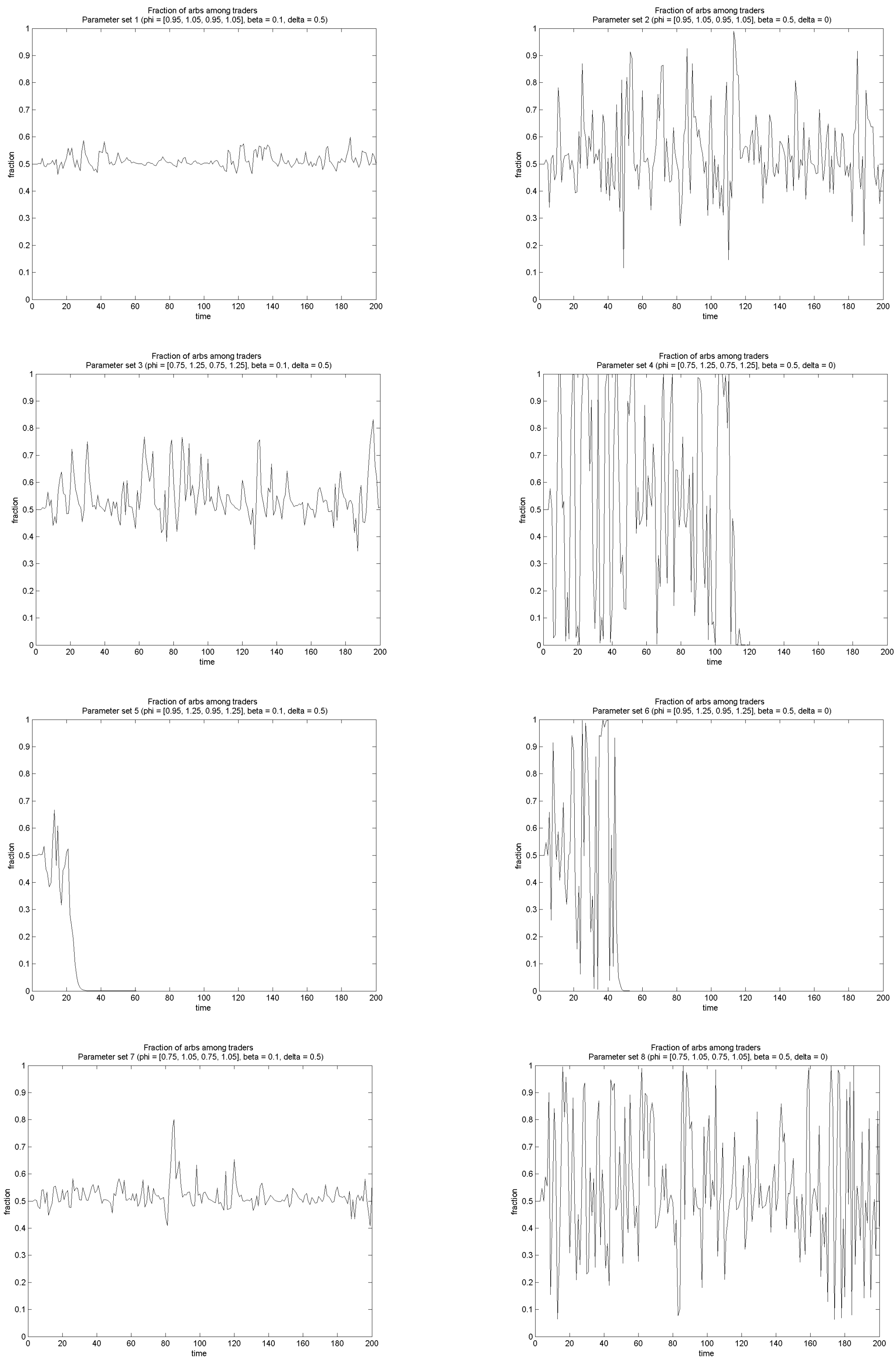

Figure 5: Simulated Log Price - Two-type model

Typical outcome of simulation for each of the 8 parameter sets defined in Table 1 using the two-type model

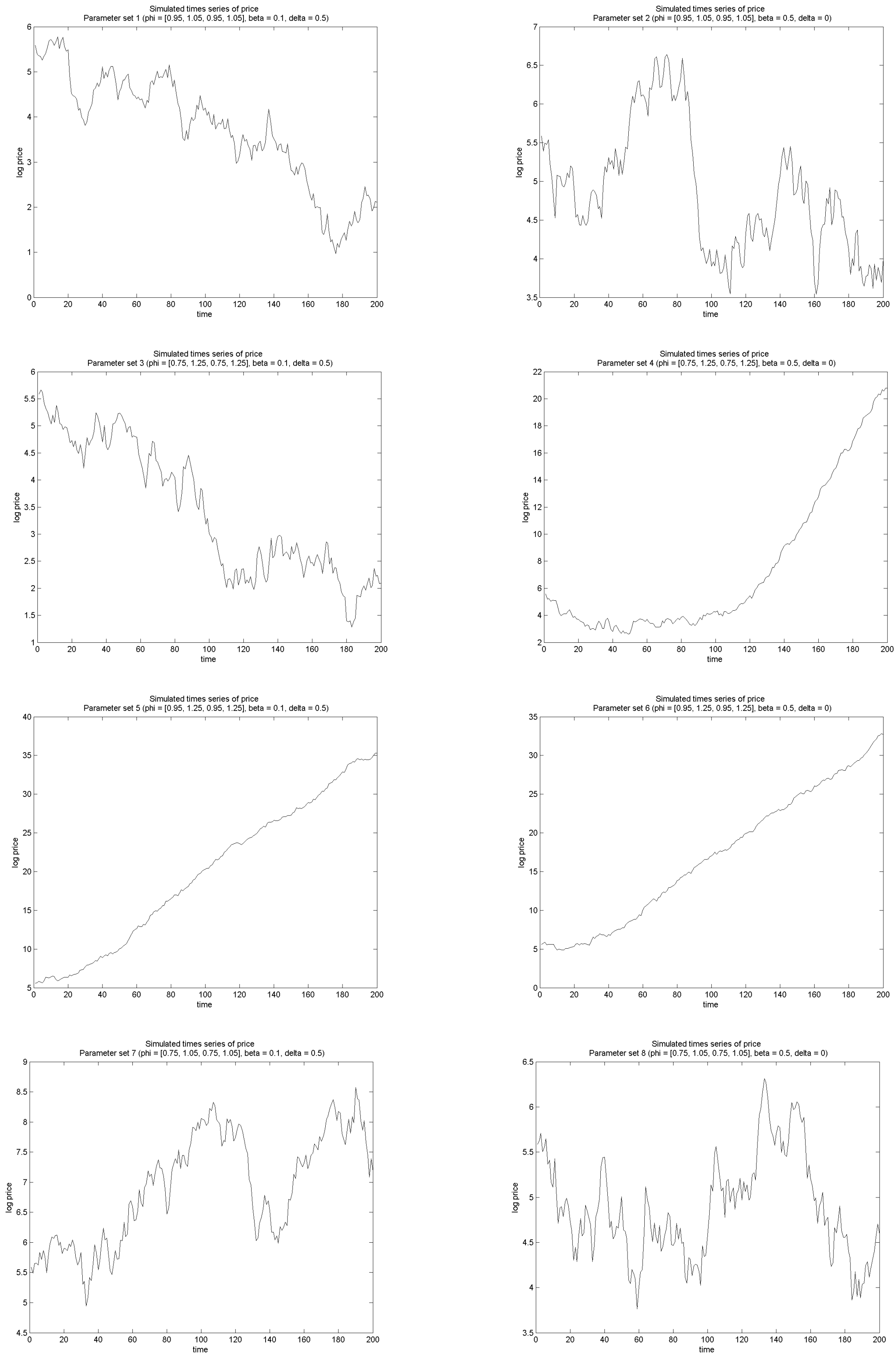

Figure 6: Simulated Deviation from Fundamental - Two-type model

Typical outcome of simulation for each of the 8 parameter sets defined in Table 1 using the two-type model

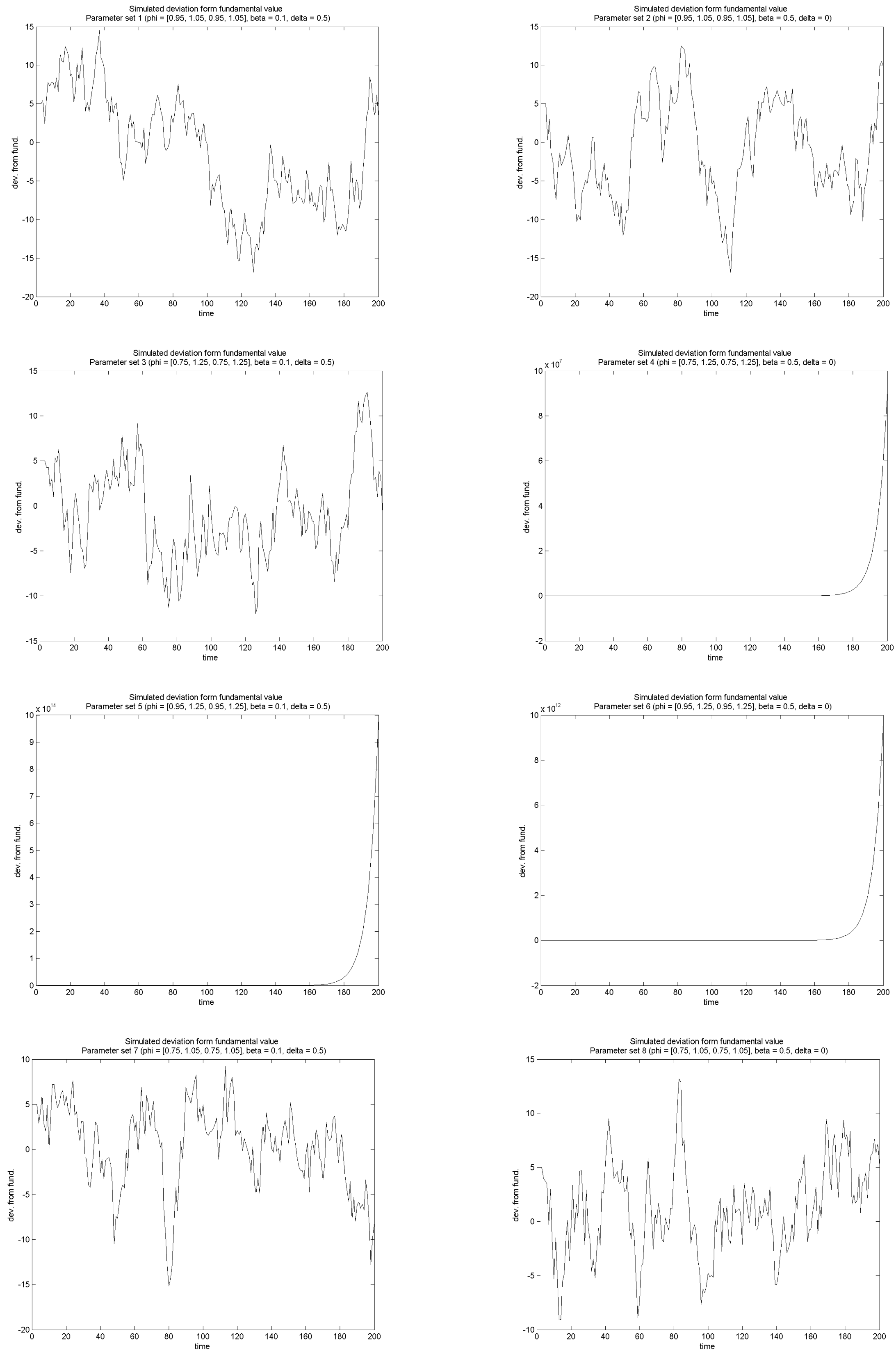

Figure 7: Simulated Volatility - Two-type model

Typical outcome of simulation for each of the 8 parameter sets defined in Table 1 using the two-type model. Volatility is computed following the RiskMetrics approach

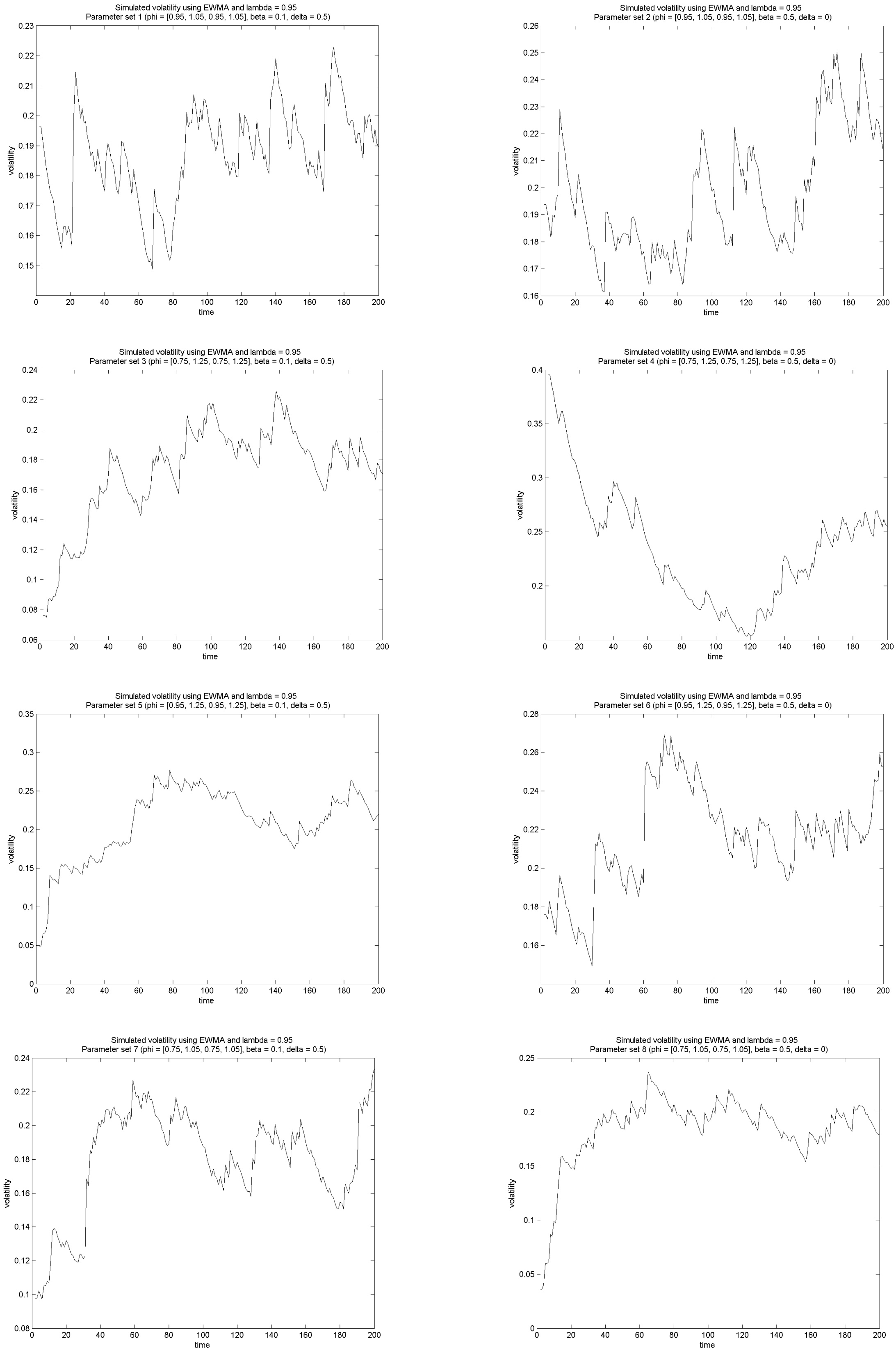

Figure 8: Simulated Fractions - Five-type model

Typical outcome of simulation for each of the 8 parameter sets defined in Table 1 using the five-type model

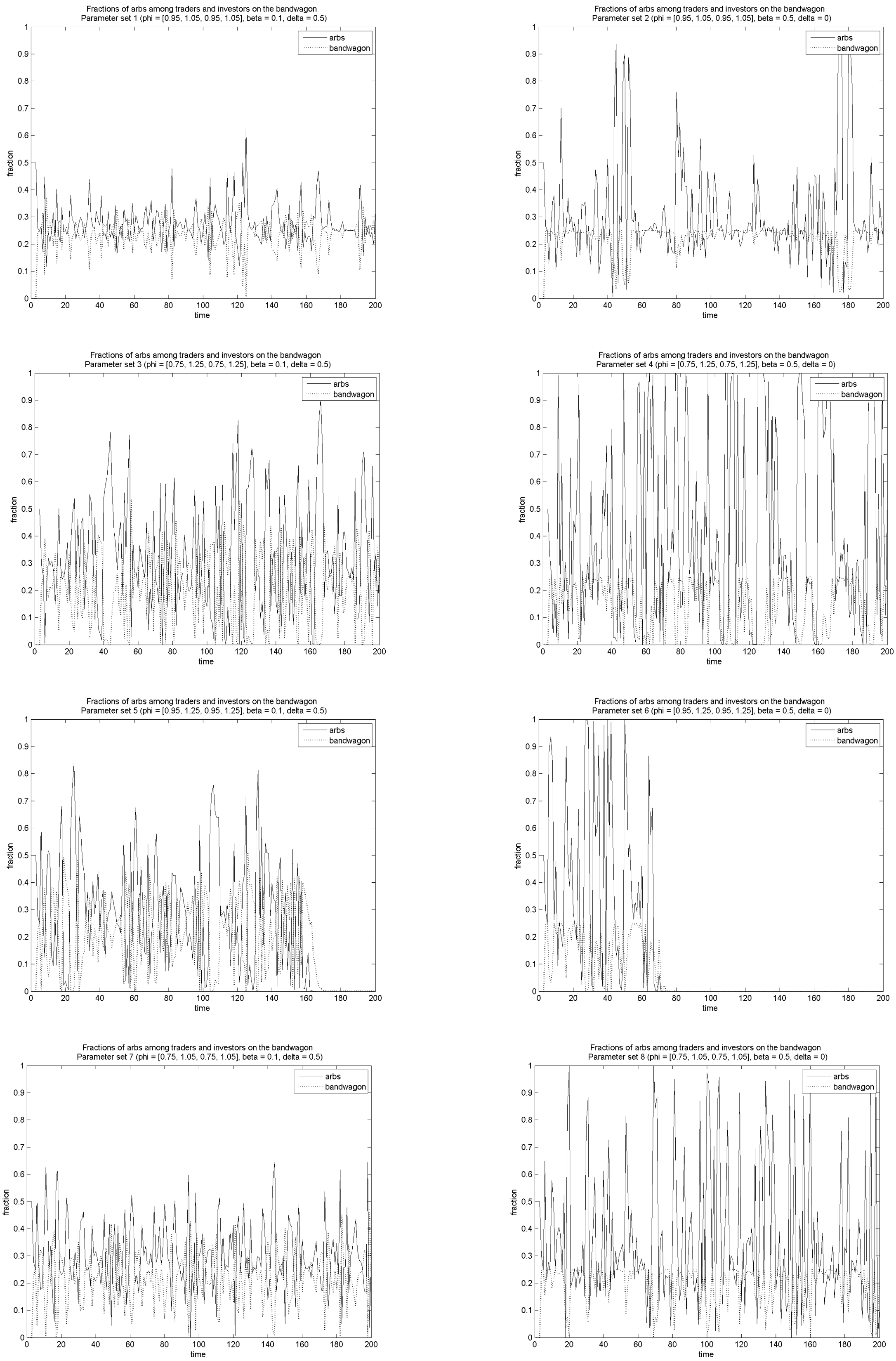

Figure 9: Simulated Log Price - Five-type model

Typical outcome of simulation for each of the 8 parameter sets defined in Table 1 using the five-type model

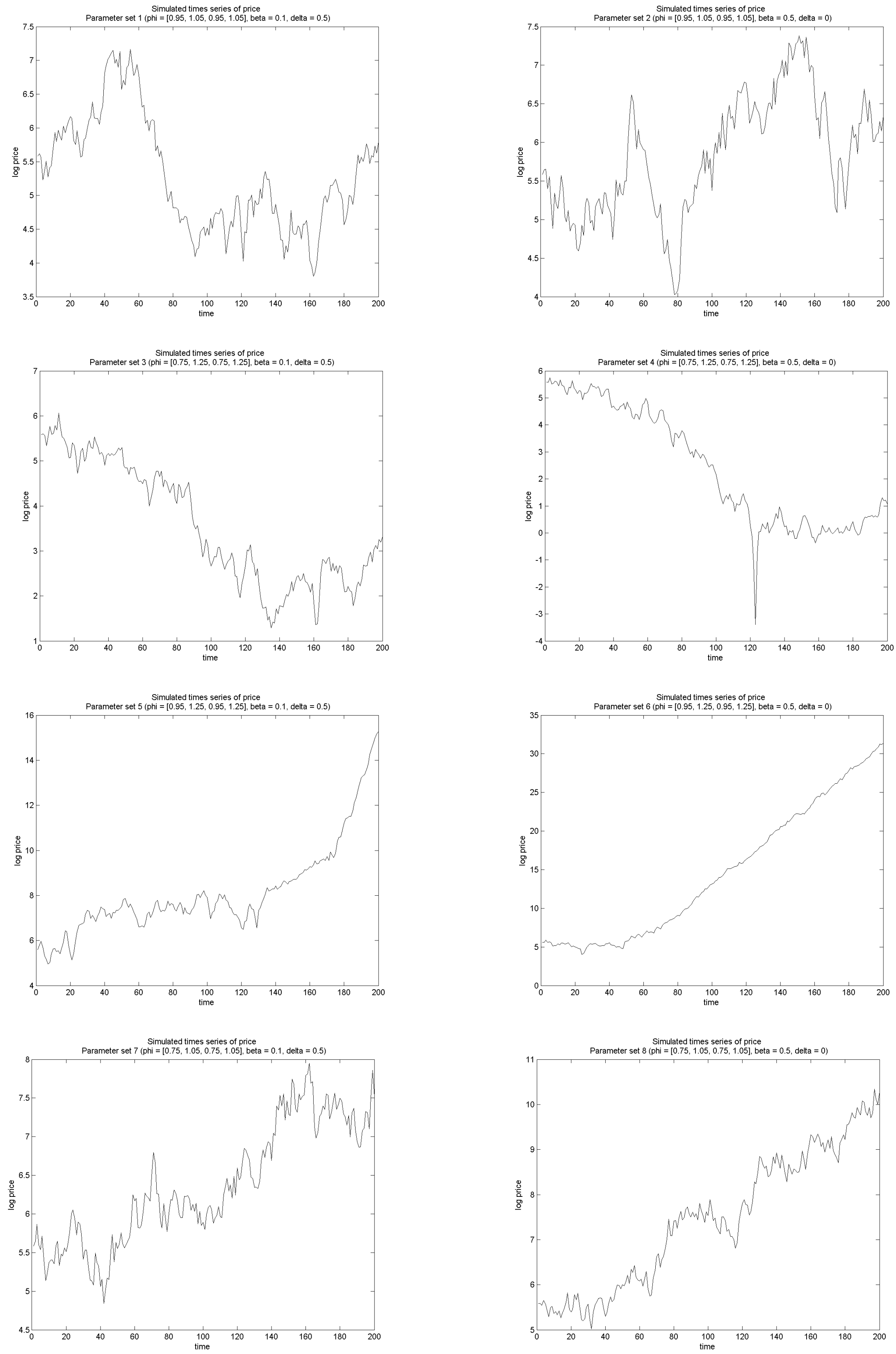

Figure 10: Simulated Deviation from Fundamental - Five-type model

Typical outcome of simulation for each of the 8 parameter sets defined in Table 1 using the five-type model

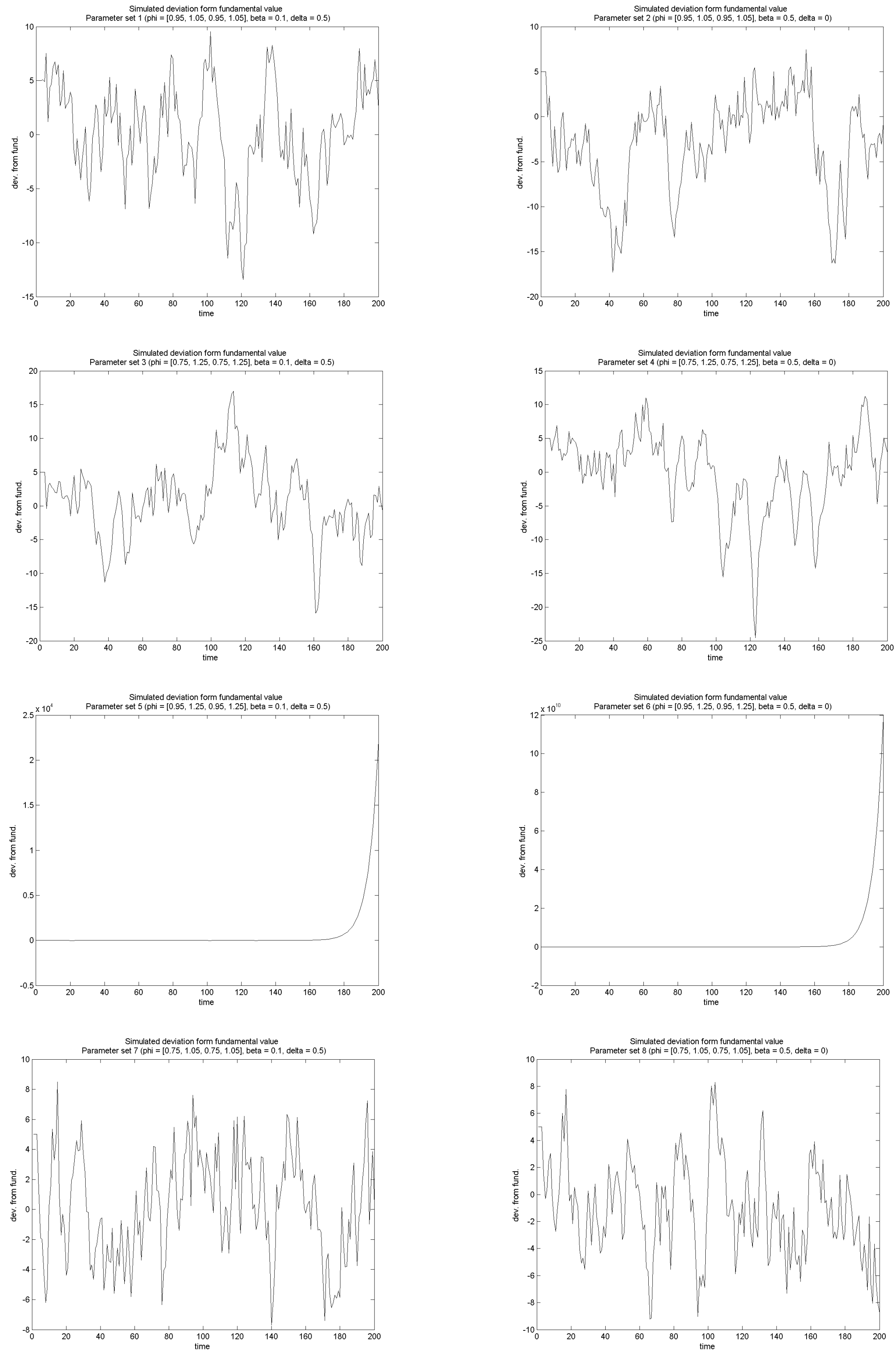

Figure 11: Simulated Volatility - Five-type model

Typical outcome of simulation for each of the 8 parameter sets defined in Table 1 using the five-type model. Volatility is computed following the RiskMetrics approach

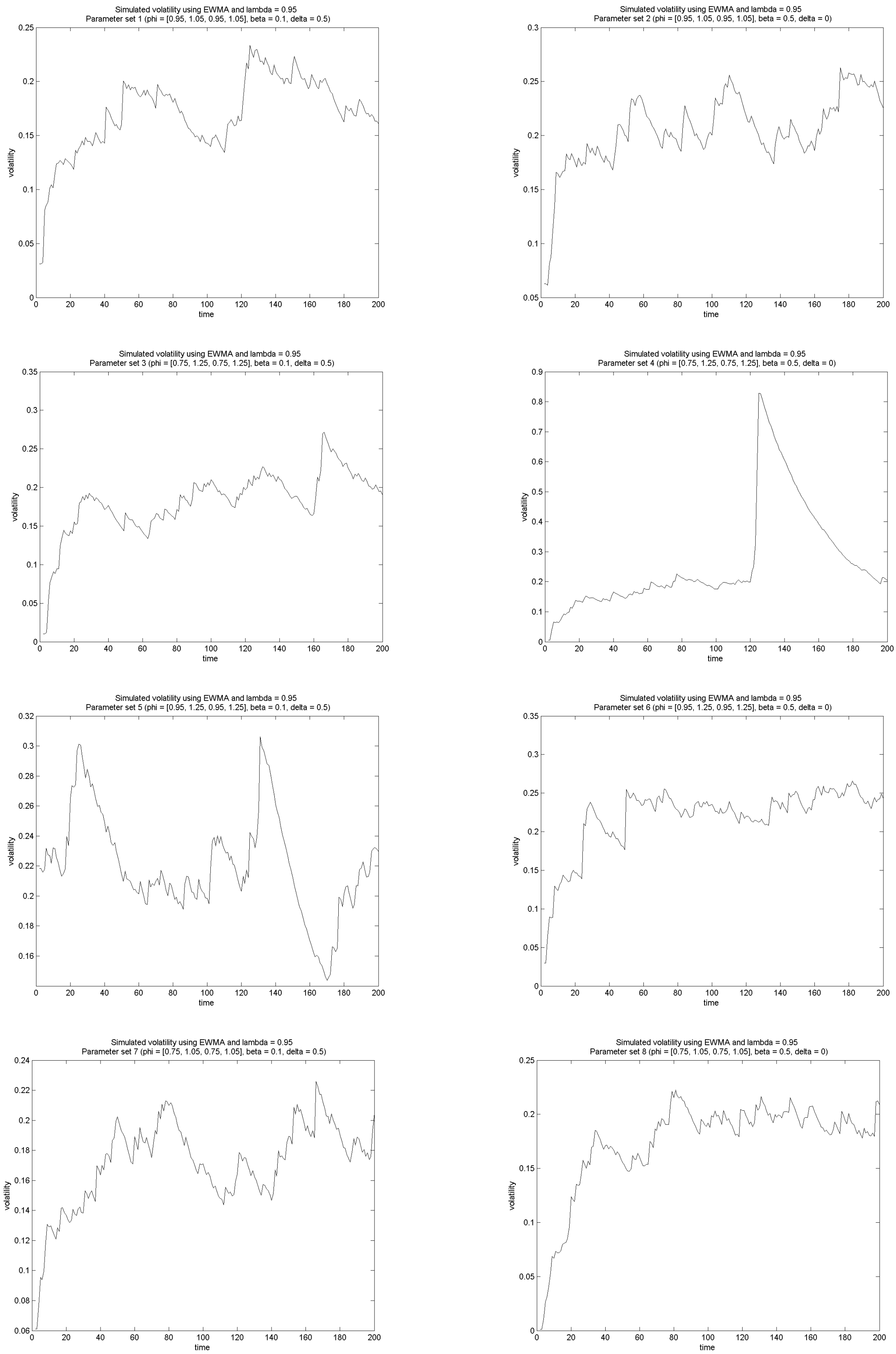

Table 14: Decile Group Analysis of Simulated Returns - Fraction of Noise Traders (Two-Type Model)

Analysis of 100 samples of simulated times series of 200 periods using the two-type HAM. Refer to Table 1 for the parameter sets. Other variables: $\sigma_\varepsilon = 2.5$, $R^* = 1.05$, $d^* = 25$, initial observations $x_1 = x_2 = x_3 = 5$, and initial fractions are $n_1 = n_2 = 0.5$, $n_3 = n_4 = n_5 = 0$. Based on the fraction of noise traders (i.e. $n_{2,t}$) the simulated times series are divided into ten decile groups, for each of the 8 parameter sets (S1-S8). Some decile groups are combined into one group (e.g. the fourth, fifth, sixth, and seventh decile groups are combined in group d4/7). For each group I calculate the median of the average fraction in that group, the median of the average return, the median of the absolute, positive, and negative deviation from fundamental (i.e. x_t), the median of the average volatility, and the median of the average market activity. The volatility is computed using the RiskMetrics approach (where $\lambda = 0.95$). The market activity measure is the fraction weighted average change in fractions. I compare the decile groups by the difference between the highest and lowest, highest and middle, and middle and lowest fraction groups (i.e. d10-d1, d10-d4/7, d4/7-d1). "high" means that the deviation from fundamental value is higher than 1,000. "n.a." if the fraction is at a maximum or minimum for more than 1/10 of the periods or if observed metric does not exist in the decile group (e.g. negative deviations from fundamental value)

(1) Median of average fraction

	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
S1	0.43	0.47	0.49	0.50	0.52	0.09	0.03	0.06
S2	0.12	0.32	0.48	0.57	0.73	0.61	0.25	0.36
S3	0.25	0.39	0.47	0.52	0.61	0.36	0.14	0.22
S4	0.17	0.75	1.00	1.00	1.00	0.83	0.00	0.83
S5	0.41	0.71	1.00	1.00	1.00	0.59	0.00	0.59
S6	0.54	0.98	1.00	1.00	1.00	0.46	0.00	0.46
S7	0.37	0.45	0.48	0.51	0.54	0.17	0.06	0.11
S8	0.02	0.20	0.46	0.61	0.84	0.82	0.39	0.44

(2) Median of average return

	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
S1	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.00
S2	0.01	0.00	0.00	0.00	0.01	-0.01	0.01	-0.01
S3	0.00	0.01	0.00	0.01	0.00	0.00	-0.01	0.01
S4	0.03	0.03	0.02	-0.02	0.02	-0.01	0.00	-0.01
S5	0.00	0.08	0.12	0.05	0.04	0.04	-0.07	0.11
S6	0.04	0.10	0.03	n.a.	n.a.	n.a.	n.a.	-0.01
S7	0.00	0.00	0.00	0.01	0.02	0.02	0.02	0.00
S8	0.02	0.00	0.00	0.00	0.00	-0.02	0.00	-0.02

(3) Median of average absolute deviation

	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
S1	6.84	5.34	4.37	6.06	10.56	3.72	6.19	-2.47
S2	7.14	5.38	4.37	6.23	10.45	3.31	6.08	-2.77
S3	5.25	4.59	4.01	5.97	10.94	5.69	6.94	-1.24
S4	4.96	4.27	8.13	5.61	11.33	6.36	3.20	3.17
S5	12.71	61.53	high	13.27	19.98	7.27	n.a.	n.a.
S6	12.01	high	23.15	n.a.	n.a.	n.a.	n.a.	11.14
S7	3.26	3.20	3.12	4.07	6.54	3.28	3.42	-0.14
S8	3.42	3.00	3.03	3.96	6.63	3.22	3.61	-0.39

Table 14 continued

	(4) Median of average positive deviation							
	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
S1	5.95	4.93	4.48	5.71	10.27	4.32	5.79	-1.47
S2	7.28	5.55	4.37	6.04	10.20	2.93	5.84	-2.91
S3	5.23	4.55	3.99	5.77	10.49	5.27	6.50	-1.24
S4	5.04	4.66	10.72	5.85	12.55	7.51	1.84	5.68
S5	12.90	62.34	high	13.27	19.89	7.00	n.a.	n.a.
S6	12.12	high	23.15	n.a.	n.a.	n.a.	n.a.	11.02
S7	3.14	3.06	3.06	4.02	6.11	2.97	3.05	-0.08
S8	3.12	2.88	3.04	3.99	6.30	3.18	3.26	-0.08

	(5) Median of average negative deviation							
	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
S1	-6.26	-5.16	-3.99	-5.78	-9.97	-3.71	-5.98	2.27
S2	-6.22	-4.83	-4.16	-6.04	-9.59	-3.37	-5.43	2.06
S3	-4.62	-4.01	-3.73	-5.80	-10.49	-5.87	-6.76	0.90
S4	-2.80	-2.84	-3.68	-5.44	-10.38	-7.58	-6.70	-0.88
S5	-2.34	-2.45	-3.38	-10.23	-14.73	-12.39	-11.35	-1.04
S6	-1.76	-2.50	-4.90	n.a.	n.a.	n.a.	n.a.	-3.13
S7	-3.21	-3.14	-3.06	-3.97	-6.34	-3.13	-3.29	0.15
S8	-3.26	-2.98	-2.92	-3.83	-6.32	-3.06	-3.40	0.35

	(6) Median of average volatility							
	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
S1	0.20	0.19	0.19	0.19	0.19	-0.01	0.00	-0.01
S2	0.19	0.19	0.19	0.19	0.19	0.00	0.00	-0.01
S3	0.19	0.19	0.18	0.19	0.19	0.00	0.00	-0.01
S4	0.18	0.18	0.18	0.18	0.19	0.01	0.01	0.00
S5	0.16	0.18	0.19	0.20	0.19	0.04	0.01	0.03
S6	0.17	0.20	0.18	n.a.	n.a.	n.a.	n.a.	0.01
S7	0.20	0.19	0.19	0.18	0.19	-0.01	0.00	-0.01
S8	0.19	0.18	0.18	0.18	0.19	0.00	0.01	-0.01

	(7) Median of average market activity							
	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
S1	0.04	0.02	0.01	0.01	0.03	-0.01	0.01	-0.03
S2	0.27	0.18	0.09	0.14	0.33	0.06	0.24	-0.18
S3	0.10	0.09	0.04	0.05	0.09	-0.01	0.05	-0.06
S4	0.21	0.24	0.23	0.35	0.42	0.21	0.19	0.02
S5	0.07	0.04	0.03	0.04	0.10	0.04	0.07	-0.03
S6	0.18	0.12	0.24	n.a.	n.a.	n.a.	n.a.	0.06
S7	0.07	0.04	0.02	0.02	0.05	-0.03	0.03	-0.05
S8	0.29	0.27	0.15	0.22	0.45	0.16	0.30	-0.14

Figure 12: Deviation from Fundamental PY-Ratio (Monthly Data)

Graph of the deviation from fundamental PY-ratio of the monthly data from January 1990 to December 2014, using earnings as cash flows. Earnings are smoothed using 10 years moving-average.

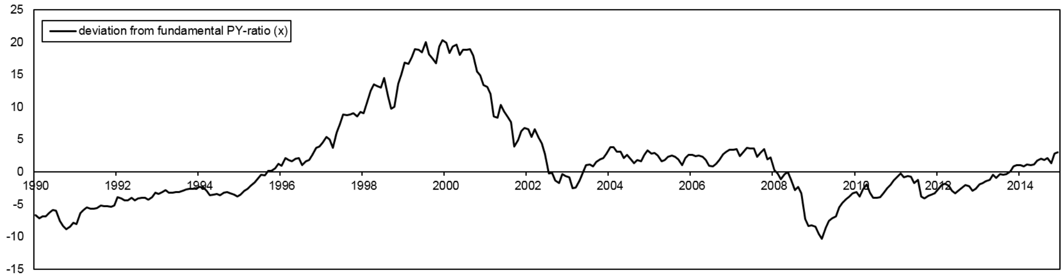

Table 15: Fundamental Value (Monthly Data)

Values used to determine the fundamental value (for the price-to-dividends and price-to-earnings ratio). Monthly S&P500 data from January 1990 until December 2014. *cpi* is the average inflation rate which is used to deflate nominal variables. y/p is the average cash flow yield (i.e. Y_t/P_{t-1}), g is the average growth rate of real cash flows (earnings are smoothed using 10 years moving-average), the discount rate is $r = y/p + g$, the gross rate of return is determined by $R^* = (1+r)/(1+g)$, and $d^* = (1+g)/(r-g)$ is the price-to-cash flow fundamental ratio. All numbers are expressed as percents, except R^* and the fundamental ratio d^*

Cash flows	cpi	y/p	g	r	R^*	d^*
Dividends	0.21	2.10	0.22	2.32	1.021	47.84
Earnings	0.21	4.23	0.25	4.48	1.042	23.68

Table 16: Estimation Results (Monthly Data)

Estimation results of the five-type and two-type model using monthly data. Estimated to monthly data of the S&P500 index (1990-2014) using dividends and earnings as measure for cash flows, two different values for δ (0% and 50%), and using nonlinear least squared method. Estimated parameters of the beliefs parameters ($\phi's$) and the intensity of choice (β). * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parentheses. Reported statistics are the R^2, the Akaike information criterion (AIC), the associated value of the log likelihood, AR(1) coefficient fitted to the data, the AIC of the linear AR(1) model, and the log likelihood value. More details on model and data in the text

Five-type Model				
	Dividends		Earnings	
δ	0%	50%	0%	50%
ϕ_1	1.138***	1.015***	1.133***	0.993***
	(0.023)	(0.022)	(0.038)	(0.075)
ϕ_2	1.012***	0.996***	1.025***	1.154***
	(0.006)	(0.008)	(0.012)	(0.015)
ϕ_3	0.566***	0.693***	1.966***	0.988***
	(0.332)	(0.367)	(0.820)	(0.154)
ϕ_4	1.720***	1.889***	0.367	1.055***
	(0.336)	(0.384)	(0.826)	(0.099)
β	0.026	0.011	0.035	1.324
	(0.032)	(0.017)	(0.077)	(1.108)
R^2	0.98	0.98	0.98	0.98
AIC	1.51	1.49	-0.12	-0.01
$LogL$	1.48	1.46	-0.15	-0.04
$AR(1)$	0.99	0.99	0.99	0.99
$AIC_{AR(1)}$	1.36	1.36	-0.16	-0.16
$LogL$	1.34	1.34	-0.17	-0.17
Two-type Model				
ϕ_1	0.988***	0.978***	1.017***	1.002***
	(0.013)	(0.021)	(0.011)	(0.017)
ϕ_2	1.038***	1.050***	1.056***	1.079***
	(0.015)	(0.024)	(0.016)	(0.026)
β	1.602	0.926	13.227	4.186
	(2.796)	(1.876)	(34.632)	(9.106)
R^2	0.98	0.98	0.98	0.98
AIC	1.34	1.35	-0.17	-0.17
$LogL$	1.32	1.33	-0.19	-0.19
$AR(1)$	0.99	0.99	0.99	0.99
$AIC_{AR(1)}$	1.36	1.36	-0.16	-0.16
$LogL$	1.34	1.34	-0.17	-0.17

Figure 13: Resulted Fractions of Estimated Five-Type Model (Monthly Data)

Resulted fractions of the estimated five-type HAM (using earnings as cash flows and $\delta = 50\%$). See Figure 12 for the time series of the deviation from fundamental value. The top Figure shows the times series of the fraction of arbitrageurs ($n_{1,t}$), the second Figure considers the fraction of noise traders in the market ($n_{2,t}$). The third and fourth Figures show the fractions of the arbitrageurs on the bandwagon (both $n_{3,t}$ and $n_{4,t}$). The bottom Figure shows the fraction of noise traders that left the market ($n_{5,t}$). The red line shows the 6 months moving-average. See Table 16 (fourth column) for the estimates of the model

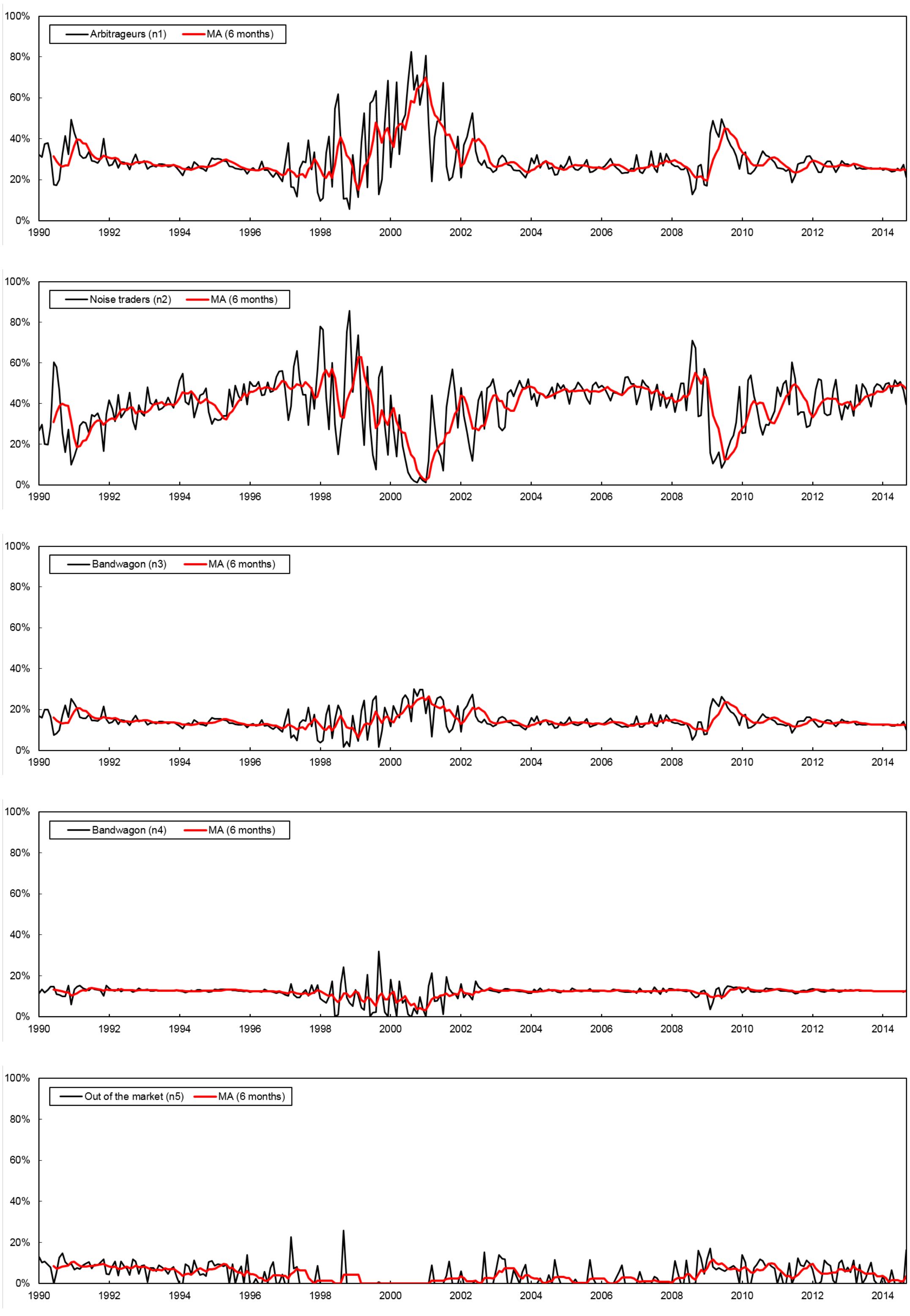

Table 17: Decile Group Analysis of Estimated Time Series - Fraction of Noise Traders (Two-Type Model)

Analysis of estimated times series using the two-type HAM. The belief parameters and β are estimated using both dividends (D) and earnings (E) as a proxy for cash flows (CF). The δ is fixed at 0% of 50%. Table 11 shows the estimation results of each of the four combinations. Based on the fraction of noise traders (i.e. $n_{2,t}$) the times series are divided into ten decile groups. Some decile groups are combined into one group (e.g. the fourth, fifth, sixth, and seventh decile groups are combined in group d4/7). For each group I calculate the median of the average fraction in that group, the median of the average return, the median of the absolute, positive, and negative deviation from fundamental (i.e. x_t), the median of the average volatility, and the median of the average market activity. The volatility is computed using the RiskMetrics approach (where $\lambda = 0.95$). The market activity measure is the fraction weighted average change in fractions. I compare the decile groups by the difference between the highest and lowest, highest and middle, and middle and lowest fraction groups (i.e. d10-d1, d10-d4/7, d4/7-d1). "high" means that the deviation from fundamental value is higher than 1,000. "n.a." if the fraction is at a maximum or minimum for more than 1/10 of the periods or if observed metric does not exist in the decile group (e.g. negative deviations from fundamental value)

(1) Median of average fraction								
	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
D (0%)	0.00	0.08	0.44	0.64	0.98	0.98	0.54	0.44
D (50%)	0.09	0.21	0.44	0.55	0.81	0.72	0.38	0.35
E (0%)	0.03	0.27	0.46	0.60	0.88	0.85	0.41	0.44
E (50%)	0.20	0.35	0.45	0.54	0.72	0.52	0.26	0.26
(2) Median of average return								
	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
D (0%)	-0.06	0.03	0.02	0.08	0.11	0.16	0.08	0.08
D (50%)	0.02	0.01	0.04	0.02	0.12	0.10	0.08	0.02
E (0%)	-0.06	0.09	-0.01	0.06	0.14	0.20	0.15	0.06
E (50%)	-0.07	0.06	0.03	0.03	0.12	0.18	0.09	0.10
(3) Median of average absolute deviation								
	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
D (0%)	23.48	11.07	6.21	7.87	34.50	11.02	28.29	-17.26
D (50%)	13.79	13.14	8.43	6.87	32.59	18.80	24.16	-5.36
E (0%)	6.94	4.55	4.12	5.82	14.51	7.57	10.39	-2.81
E (50%)	4.12	4.94	4.40	5.29	16.13	12.01	11.73	0.28
(4) Median of average positive deviation								
	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
D (0%)	23.48	19.18	4.60	8.68	38.75	15.27	34.15	-18.88
D (50%)	18.81	19.97	9.46	7.67	41.72	22.91	32.26	-9.35
E (0%)	6.94	5.04	3.40	6.72	14.51	7.57	11.11	-3.54
E (50%)	4.55	5.35	4.64	5.20	16.13	11.58	11.49	0.09
(5) Median of average negative deviation								
	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
D (0%)	n.a.	-6.00	-7.60	-6.58	-9.01	n.a.	-1.42	n.a.
D (50%)	-3.75	-7.29	-7.55	-4.20	-9.75	-6.00	-2.20	-3.80
E (0%)	n.a.	-3.98	-4.74	-3.80	n.a.	n.a.	n.a.	n.a.
E (50%)	-1.99	-4.47	-4.13	-5.44	n.a.	n.a.	n.a.	-2.14

Table 17 continued

(6) Median of average volatility								
	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
D (0%)	0.17	0.16	0.15	0.16	0.16	0.00	0.01	-0.01
D (50%)	0.16	0.16	0.15	0.16	0.16	-0.01	0.01	-0.01
E (0%)	0.16	0.16	0.15	0.16	0.16	0.00	0.01	-0.01
E (50%)	0.16	0.16	0.15	0.16	0.16	-0.01	0.00	-0.01

(7) Median of average market activity								
	d1	d2/3	d4/7	d8/9	d10	d10-d1	d10-d4/7	d4/7-d1
D (0%)	0.28	0.31	0.14	0.30	0.49	0.21	0.35	-0.15
D (50%)	0.08	0.18	0.15	0.12	0.16	0.08	0.01	0.08
E (0%)	0.27	0.21	0.12	0.22	0.46	0.20	0.34	-0.15
E (50%)	0.14	0.11	0.07	0.11	0.11	-0.04	0.04	-0.07