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MSc Economics & Business
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Does any model beat the GARCH (1.1)?

A Forecast comparison of volatility models through option prices

Author: M.M.S. Mathoera
Student number: 288430
Thesis supervisor: Dr. J. Lemmen
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PREFACE AND ACKNOWLEDGEMENTS

After 2 years of research and writing I finished this thesis. There were many ups and downs during this period, also in my personal life. With determination I finished this thesis. I want to thank everyone who motivated me through this period with special thanks to my parents and friends. I also want to thank my thesis supervisor for his patience, who not only motivated me but gave very insight in this subject, provided me his knowledge and very useful comments.

ABSTRACT

This paper studies different methods to estimate the volatility of index options. These volatilities are used as input to predict/forecast option prices with the Black-Scholes model. Option price predictions are compared using GARCH (1.1) model versus alternative models, namely, Historical volatility, EGARCH and TGARCH. As such I will calculate 4 different volatilities. The observations reveal significant results favouring GARCH (1.1) for the AEX index and TGARCH outperformed the other models for the S&P 500 index.

Keywords: volatility, implied volatility, forecasting, delta hedging

JEL Classification: G17

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CHAPTER 1 Introduction

Option pricing has been a challenging task ever since the foundation of the first financial options market, the Chicago Board Options Exchange (CBOE), in 1973. It was also in this year when Fisher Black and Myron Scholes constructed their famous option pricing formula in *"The Pricing of Options and Corporate Liabilities"* (1973). The insight that the expected return on a hedged position should be to equal the risk free rate, made their option pricing formula superior to the previously developed work. The Black & Scholes model builds on the work of Thorp and Kassouf (1967). They developed a hedged position which was constructed with a combination of shares and options on the same share. Black and Scholes made the connection that without arbitrage opportunities, in equilibrium; the expected return on such a hedged position should equal the return on a riskless asset. This was the missing link which made option pricing possible using the expected payoff of the hedged position, constructed by a long position the underlying stock and a short position in the option, and discounting it with the risk free rate. From that moment onwards, the Black & Scholes model has been the most applied option pricing technique (Hull, 2009).

The Black-Scholes differential equation does not involve any variables that are affected by the risk preferences of investors. This is a key property, which allows for risk neutral valuation as a tool for option pricing. The variables that do appear into the equation are current stock price (S_0), time (T), the risk free rate (r) and stock price volatility (σ). Of these variables only stock price volatility cannot be directly observed (Hull, 2009). Therefore a good estimation of this variable is essential for a good estimation of the option prices. This will be the main concern of my paper.

The volatility of a stock is a measure of uncertainty about the returns provided by the stock. Initially historical volatility was used to estimate this variable for the Black-Scholes option pricing formula. In practice, traders started to use implied volatility to estimate stock price volatilities. Nowadays more sophisticated approaches to estimating volatility are used involving GARCH models. These generalized autoregressive conditional heteroskedasticity (GARCH) techniques are the workhorse in volatility modeling. This paper studies the option pricing performance of Black & Scholes using GARCH (1,1) versus Black & Scholes using historical volatility and improved GARCH (1,1) models, like exponential GARCH (EGARCH and

Threshold GARCH (TGARCH), which captures asymmetric volatility. Further I will compute the errors between the model-determined prices and the real prices. In this way I can accurately compare these models and show, which one reflects the price in the best way. My data sample consists of the past recent years. It would be nice to validate these models with a recent time set.

In the following chapter 2 I will discuss the key papers on this topic (Literature review). Further, the data and the testing procedure that I will be using will be discussed in chapter 3 (Methodology and Data). Finally, I will show the results of the tests in chapter 4 (Results) and discuss these in chapter 5 (Conclusion).

CHAPTER 2 Literature Review

2.1 Introduction

In the past there has been much research about the GARCH (1.1) model and its modifications. This chapter will give a glimpse on the models used in the past and the empirical findings of papers in the past. Some of these models are still being used with some minor modifications. The backbone is the Black-Scholes model. Section 2.2 shows briefly the Black-Scholes model. Section 2.3 presents the historical volatility and the relation between implied volatility. Section 2.4 shows the history of the ARCH model. In section 2.5 the focus will be on the famous GARCH (1.1) model. Section 2.6 will focus on the modifications of the GARCH (1.1) model that captures asymmetric volatility and section 2.7 gives a summary of this chapter.

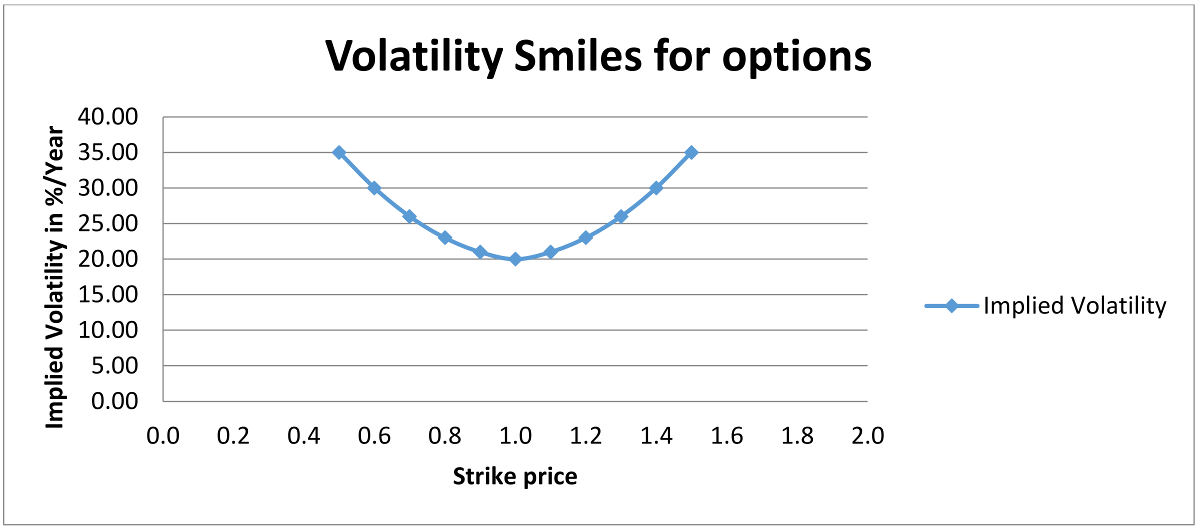
2.2 Black-Scholes

The Black-Scholes valuation method originally assumes stock return variance to be constant, meaning constant volatility (Black & Scholes, 1973). Since the derivation of their model, there has been speculation on the usefulness for option valuation. Most of the focus has been on the constant interest rate and constant volatility assumption. Already in 1973, Merton relaxed the interest rate assumption by allowing interest rates to be stochastic. However, it was until 1976 when Cox and Ross came up with their constant-elasticity-of-variance model, which was less counterfactual than the constant volatility assumption. Later models such as those of Bailey and Stulz (1989) and Wiggins (1987) allowed variance to be stochastic. Empirical evidence that shows that these efforts have indeed improved the Black-Scholes model is found in the paper of Lauterbach & Schultz (1990).

Nowadays traders still use the Black-Scholes model, but they allow the volatility used to price the option to depend on its maturity and strike price (Hull, 2009). Traders assume the probability distribution of an equity price to have a heavier left tail and less heavy right tail than the lognormal distribution assumed by Black-Scholes. This is because of significant differences in price observations in the market compared to the computed prices by the Black-Scholes model. These systematic valuation errors are documented in fact, also known as the smile effect.

The volatility smile has the general form illustrated in figure 1. The implied volatility is relatively low for at-the-money options. It becomes progressively higher as an option moves either into-the-money or out-of-the-money. (Hull, 2009)

Figure 1: Volatility Smile relationship between the implied volatilities and strike price for different option contracts with same underlying asset and maturity.



Volatility smiles are used to allow for non-log normality (Rubinstein, 1994). This volatility smile curve can be found when the implied volatility of an option is plotted as a function of its strike price. There are multiple models to account for this smile; two branches of volatility models are addressed by Derman (2003). A standard stylized fact in volatility theory is that the empirically observed 'smile' and 'skew' shapes in Black-Scholes implied volatilities contradict the model assumptions (Alexander 2004). Stochastic volatility models represent the spot volatility or variance as a diffusion or jump-diffusion process that is correlated with the underlying asset (e.g. Merton 1976, Hull & White 1987). Jump-diffusion contains two parts, a jump part and a diffusion part. The diffusion part is determined by a common Brownian motion (normal price variations) and the jump is determined by an impulse-function and a distribution function. In other words big shocks in stock prices are not incorporated in the basic Black and Scholes assumptions. Jump processes can be added as these allow big jumps in stock prices. Jump processes can solve the problem of non-normality found in stock returns as they allow stock prices to change in such a way found in empirical findings. The major drawback with the jump-diffusion models is that they cannot capture the volatility clustering effects.

Local volatility models derive spot and forward volatilities that are consistent with a 'snap-shot' of implied volatilities at a particular time (e.g. Dupire 1994, Derman and Kani 1994). Stochastic and local volatility models have thus been regarded as two alternative and competing approaches to the same unobservable quantity, the spot volatility of the underlying asset (Alexander, 2004). However multiple papers like Implied volatility functions: empirical tests (Dumas et al., 1998) indicate that the delta hedging performance of local volatility models is worse than the Black-Scholes models. Recall delta hedging is an options strategy that aims to reduce the risk associated with price movements in the underlying asset by offsetting long and short positions. For this reason the usual conclusion is that the assumption of a deterministic spot volatility is too restrictive and that stochastic volatility models are more realistic (Dumas et al., 1998). Key is that the stochastic volatility allows for the volatility to change periodically (Broadie and Detemple 2004). The next step is that jump processes are to be combined with stochastic volatility. Jump processes are the ability for the stock index or stock prices to decline or to increase by a magnitude which is very uncommon when the stock returns are normally distributed.

2.3 Historical Volatility

Implied volatility is related to historical volatility, however they are different. Implied volatility is set by the option price itself, whereas historical volatility looks at the recent history of the underlying stocks' price movements. The historical volatility is the volatility of a series of indices where we look back over the historical price path of the particular index. Historical simulation, and hence its implied volatility is widely used in practice. The main reasons are the ease with which it is implemented and its model-free nature (Hull & White, 1987). Firstly, the technique clearly is very easy to implement. No parameters have to be estimated by maximum likelihood or any other method. Therefore, no numerical optimization has to be performed (Hull & White, 1987).

The second advantage is more controversial since the argument is two sided. The technique is model-free in the sense that it does not rely on any particular parametric model such as GARCH for variance and a normal distribution for the standardized returns (Christoffersen and Diebold, 2000). Historical volatility lets the past m data points speak fully about the distribution of tomorrow's return without imposing any further assumptions. Hence this has its drawbacks but also its advantages such that it doesn't rely on modeling assumptions which can be biased once the model is poor (Christoffersen and Diebold, 2000). Nevertheless, the drawbacks of a historical volatility model are considerable; the choice of the data sample length, *m*, is one.

2.4 ARCH Model

Compared with the other types of volatility models, the historical volatility models are the easiest to manipulate and construct (Vitiello & Poon, 2008). Studies that find historical volatility models forecast better than ARCH models, include Taylor (1987) and Schert & Seguin (1990).

The GARCH model family started with the *Autoregressive Conditional Heteroscedasticity* (ARCH) model, which was initially introduced by Engle (1982). The key characteristic of this model is that its conditional variance varies over time. It is both logically inconsistent and statistically inefficient to use volatility measures that are based on the assumption of constant volatility over some period when the resulting series moves through time (Campbell, Lo, and MacKinlay 1997, p.481). Hence, the major contribution of the ARCH literature is the finding that apparent changes in the volatility of economic time series may be predictable and result from a specific type of nonlinear dependence rather than exogenous structural changes in variables (Bera and Higgins 1993). Advantages to ARCH models are simple and easy to handle, ARCH models take care of clustered errors, ARCH models take care of nonlinearities and ARCH models take care of changes in the econometrician's ability to forecast (Bera and Higgins 1993).

The ARCH model was generalized by Bollerslev (1986), who came up with the *Generalized Autoregressive Conditional Heteroskedasticity* (GARCH) model as an extension of the ARCH model. It is considered to be a financial time series-forecasting model, which enables researchers to resolve some of the Black and Scholes model flaws. It assumes that the randomness of the variance process varies with the variance and it is able to replicate a significant number of the conformed facts found in empirical financial time series. Since then, many researchers have tried to expand and use these models in several applications.

2.5 GARCH (1.1) Model

Jin-Chuan Duan developed an option-pricing model using GARCH (1.1) asset return process in his paper "the GARCH option pricing model" (1995). Moreover, many empirical studies concerning the use of GARCH models for option prices have already taken place. For example Peter Christoffersen and Kris Jacobs in their paper "which GARCH model for option valuation" (2004), compare a number of GARCH models through a divergent dimension, using option prices and returns assuming the risk-neutral measure. Furthermore, Lars Stentoft in his paper "American Option Pricing using GARCH models and the Normal Inverse Gaussian distribution" (2008) used a GARCH (1.1) process to suggest a way of pricing American options in a model with time varying volatility and conditional skewness and leptokurtosis.

An advantage of this model is that tomorrow's variance is calculated as a weighted average of the long-run variance, today's squared return and today's variance. In this way, recent variances are given more weight. One minor disadvantage of the GARCH (1.1) model is that it does not allow for the leverage effect i.e. the conditional volatility goes up when the market goes down. More will follow on the leverage effect in section 2.6

2.6 Models that capture asymmetric volatility

Despite the success of the GARCH (1.1) model, it has therefore also been criticized since the framework fails to capture asymmetric volatility. Many researchers tried to adjust and improve the GARCH (1.1) model. The asymmetric effect or leverage effect occurs when bad news, negative shocks increases predictable volatility more than good news or positive shocks of similar magnitude (Engle and Ng, 2003). This phenomenon is called the leverage effect, because it assumes increased leverage increases volatility. More specifically, a negative return on an equity stock implies a lower equity value. When we assume that the debt level stays constant then the company becomes more highly levered and therefore more risky, which increases volatility (Christoffersen, 2003).

2.6.1 EGARCH

To deal with asymmetric effects of news that reflect the correlation between asset return and volatility, Nelson (1991) proposed *Exponential GARCH* (EGARCH). The EGARCH model differs on two aspects from the GARCH (1.1) model. First, according to EGARCH good news and bad news have a different impact on volatility and second, it allows big shocks to have a greater impact on volatility than in the GARCH (1.1) model.

2.6.2 NGARCH

Another asymmetric model that improves the GARCH (1.1) framework is the *Non Linear GARCH* model (NGARCH). The NGARCH model is a generalization of the Bera and Higgins (1993) model, a model that only contained ARCH lags. In this NGARCH model, the asymmetric effect depends upon the standard deviation (Longmore and Robinson, 2004).

2.6.3 GJR-GARCH

Moreover, another asymmetric model is the GJR-GARCH (GJR model) developed by Glosten, Jagannathan and Runkle (1993), which is based on the assumption that bad news has a higher impact than good news. This model is also designed to capture the leverage effect between asset return and volatility. However, the way this model is applied is not the same as EGARCH model. In the EGARCH model, the leverage coefficients are directly applied to the actual innovations while the leverage coefficients of the GJR model can connect to the model through an indicator variable. In this case the leverage coefficients should be negative for the EGARCH model and positive for the GJR model, when the asymmetric effect occurs.

2.6.4 TGARCH

In addition, another model that captures the stylized fact known as leverage effect is the Threshold GARCH model (TGARCH). Again, this implies that negative news tends to increase volatility by a larger amount than positive news (Francq and Zakoian 2010). Thus this model allows for negative asymmetric volatility, due to the extra weight that is given to the most recent negative returns. Zakoian introduced the TGARCH model in 1994. This model can be used as an effective tool to estimate the asymmetric relation between past returns and volatility. The TGARCH model is also referred to as the ZARCH or the ZGARCH model. The basic idea behind this model is closely related to that of the GJR model with the exception that it is the conditional standard deviation that is modeled and not the conditional variance (Longmore & Robinson, 2004). Due to this high similarity between GJR and TGARCH I will only use TGARCH in this paper. The TGARCH model does not draw on the underlying assumption of the Black & Scholes model that asset prices follow a geometric Brownian motion. Which states that it does not capture the stylized observations that asset returns are lognormal distributed. The TGARCH model uses a volatility function that is time varying and also incorporates the asymmetrical reaction of volatility on unanticipated returns.

Several empirical studies give an indication that of these EGARCH and TGARCH models, the TGARCH model gives the most accurate volatility forecasts. There is strong evidence that the modeling of asymmetric components is much more important than specifying the error distribution for improving the volatility forecasts of financial returns in the presence of fat-tails, leptokurtosis, skewness and the leverage effect. Furthermore, if asymmetric effects are neglected, the GARCH (1.1) model with normal distribution is more preferable than those models with more sophisticated error distributions. This suggests that when we allow for a flexible error distribution, it does not lead to significant improvements in volatility forecasts.

2.7 Summary

The backbone of my research is the Black-Scholes model that concentrates on the volatility. Further it explains the advantage and disadvantage of the historical volatility model. The ARCH models are easy to use and it deals with nonlinearities, which results in better forecasting. In my research I will only use historic volatility, GARCH (1.1), EGARCH and the TGARCH forecasting models.

CHAPTER 3 Methodology and Data

3.1 Introduction

In this chapter I will study different methods to estimate volatility and use those to predict option prices with the Black-Scholes model. The main aim is to compare these option price predictions using GARCH (1.1) model versus alternative models, namely, historical volatility, EGARCH and TGARCH. This chapter shows the hypothesis, formulas, explains and describes the models.

3.2 Hypothesis

H0: My observations show no significant improvements (over GARCH (1.1)) in option pricing due to extended models (namely Historical Volatility, EGARCH and TGARCH)

HA: The additional, more extensive models (over GARCH (1.1) have additional explanatory power and hence will be able to approximate prices significantly better.

My goal is to find the best volatility-forecasting model. Therefore I will evaluate the volatility forecasting performance through the Black-Scholes (B-S) model. As such, the model that predicts option prices most accurately will be regarded as preferable volatility-forecasting model.

3.3 Black-Scholes (B-S) model

To test my hypothesis, I will use different models to estimate the volatility of the underlying indices. I will investigate the market price of different European options on indices and its corresponding option prices computed through the B-S model for European call options discussed here:

$$c = S_0 N(d_1) - Ke^{-rT} N(d_2) \tag{1}$$

$$d_1 = \frac{\ln(\frac{S_0}{K}) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \tag{2}$$

$$d_2 = d_1 - \sigma\sqrt{T} \tag{3}$$

The B-S formula includes the term 'S' which is the price of the stock, 'K' the strike price op the option and 'T' the current time to maturity of the option. The terms '*N (d1)*' and '*N (d2)*' are the cumulative normal distribution functions and the term 'r' is the risk free interest rate of the option. As a proxy for the risk free rate I used daily one-week LIBOR rates.

3.4 Historical volatility model

The B-S model uses the historical volatility that is calculated by making use of the formula:

$$\sigma_t^2 = 1/(m-1)\sum_{i=1}^{m}(R_{t-i} - \bar{R})^2 \tag{4}$$

This formula presents the parameter 'm', which is the total number of observations included in the approximation and 'R' is the corresponding return, or the percentage change in the market variable, which can be calculated through:

$$R_t = (S_t - S_{t-1})/S_{t-1} \tag{5}$$

3.5 GARCH (1.1) model

The simplest GARCH model for computing dynamic variance is usually referred to as GARCH (1.1) and can be written as follows:

$$\sigma_{t+1}^2 = \omega + \alpha R_t^2 + \beta\sigma_t^2 \tag{6}$$

With,

$$\alpha + \beta < 1$$

$$\sigma^2 = \omega\,(1 - \alpha - \beta)$$

Where 'R_t' stands for the asset return and 'σ^2' for the long-run average variance, and α measures time-variation in conditional variances and β measures shocks. Certain advantages to this model are that tomorrows' variance is calculated as a weighted average of the long-run variance, today's squared return and today's variance. In this way, recent variances are given more weight. This can be seen from the following:

$$\sigma_{t+1}^2 = (1 - \alpha - \beta)\,\sigma^2 + \alpha R_t^2 + \beta\sigma_t^2 = \sigma^2 + \alpha\,(R_t^2 - \sigma^2) + \beta\,(\sigma_t^2 - \sigma^2) \qquad (7)$$

In order to forecast future variance for horizon 'k' I applied the following formula:

$$E_t\,(\sigma_{t+k}^2) - \sigma^2 = (\alpha + \beta)\,K^1\,(\sigma_{t-1}^2 - \sigma^2) \qquad (8)$$

The persistence of the model is *(α+β)*, and this is an indicator about the time a shock will persist. Hence, the closer (*α+β*) is to zero, the longer a shock will push variance away from its long-run average.

3.6 EGARCH model

The EGARCH model of Nelson (1991) provides an asymmetric model with the following equation:

$$\mathrm{Log}(\sigma_{t+1}^2) = \omega + \alpha\,(\emptyset R_t + \gamma[|R_t| - E|R_t|]) + \beta\log(\sigma_t^2) \qquad (9)$$

In the formula, the coefficient 'γ' captures the asymmetric impact of news with negative shocks having a greater impact than positive shocks of an equal magnitude. When $\gamma < 0$, than the volatility clustering effect will be captured by a significant 'α'. Using the log form allows the parameters to be negative without conditional variance becoming negative.

3.7 TGARCH model

The third asymmetric model we address is the TGARCH with the following formula:

$$\sigma_{t+1}^{1/2}1 = \omega + \alpha R_t^{1/2} + \gamma d_t R_t^{1/2} + \beta \sigma_t^{1/2} \tag{10}$$

Where d = 1 if R_t < 0 and d = 0 if $R_t \geq 0$

As it becomes apparent, the TGARCH formula incorporates a new component '$\gamma d_t R_t^{1/2}$' with a dummy variable 'd'. Rephrased, this improves the GARCH (1.1) model. The parameter 'γ' gives extra weight to the most recent returns '$R_t^{1/2}$ ', which depends on whether '$R_t^{1/2}$' is positive or negative. In short, TGARCH model allows for negative asymmetric volatility due to the extra weight that is given to the most recent negative returns.

3.8 Performance measure and Diebold Mariano test

Furthermore, I will measure the performance of these GARCH models by evaluating the pricing errors of the models used, which can be expressed through:

$$P_t = R_t - M_t \tag{11}$$

Where 'P_t' stands for the pricing error for an option at time 't', 'R_t ' is the real market price for this option and ' M_t' is the model-determined price.

I will implement this formula to all daily option prices. Given that the errors could be positive or negative, I will make use of the square amount of the pricing errors. The Root Mean Squared Error (RMSE) is then the square root of the average of the squared pricing errors of options in the whole sample. This is given by the formula:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{T} P_i^2}{T}} \tag{12}$$

The variable T is the lifetime of the options in our sample. The smaller RMSE gets, the smaller pricing errors are which subsequently means a better price forecasting. Looking only at the RMSE outcomes shows us only which model is more accurate, however, what I am actually interested in is whether there is a significant difference between the errors of the models. This will be done by the Diebold Mariano test:

$$D_t = P_j - P_h \tag{13}$$

Where 'P_j' denotes the pricing error of GARCH (1.1), 'P_h' denotes the other models (Historical, EGARCH and TGARCH). 'D_t' is the difference between the pricing errors of the two models that are tested.

The Diebold Mariano null hypothesis states that there is equal predictive accuracy between the models, in other words:

$$H_o: E[D_t] = 0 \tag{14}$$

Before testing the null hypothesis the average of the differences need to be calculated,

$$\overline{D} = \frac{\sum_{t=t_0}^{T} D_t}{T} \tag{15}$$

Than the null hypothesis will be tested with a normal T-test:

$$T_{test} = \frac{\overline{D}}{(\frac{Var(\overline{D})}{T})^{1/2}} \tag{16}$$

This means rejecting the null hypothesis of equal predicative accuracy at the 5% significance level if

$$|T_{test}| > 1.96$$

This test is essential and will allow me to validly test my hypothesis.

3.9 Data

In order to test my hypothesis and the models I have obtained the data. I will need extensive data in order to obtain the parameters in the models used. The first step of the analysis is to use real historical data. A logical choice for the data would be active options with transparent data prices and which are most liquid. For this reason I have chosen for the AEX index and the S&P 500 index. By making my choice I concentrated on option on indices instead of on single stock. The main reason for this particular choice is that these options will be less influenced by company specific information and hence more representative as a market. The data consists of daily closing values for a time horizon of 2480 trading days starting from July 1st 2002 to December 31st 2012. The reason for selecting this sample size, is the benefit of a more precise estimates due to a longer sample. After computing the optimal parameters for the GARCH-models with the help of the maximum likelihood estimation method, I will be heading to the second part of this paper: comparing the actual option prices with the prices calculated with the GARCH-models. For this out-of-the-sample test I have collected European call options prices on each of the two indices starting from January 1st, 2012 to December 31st, 2012. Call options are chosen for our analysis, but put options could have been used to the same extend. Given put-call-parity this choice is irrelevant since, for a given strike price and maturity the correct volatility for European put or call options should be the same, when used to predict option prices with the Black-Scholes model (Hull, 2009). Both the closing values as the option prices were obtained from DataStream. The options prices are observed daily and we used options maturing at June, July and August 2012. For these 3 maturities we chose 3 different strike prices, namely in-the-money (ITM), out-of-the money (OTM) and at-the-money (ATM). Similar to Hughen & Clark (2011), I choose options with moneyness between 85% and 115%. This is because this will cover the three classes and these are the options that are most traded. As such the following strike prices were obtained for the S&P500; 1035 (June), 1085 (July) and 1110 (August). For the AEX I choose 285 (June), 305 (July) and 320 (August). Following Heston & Nandi (2000) and Hughen (2011) we limited the maturities of the options to; greater than 10 days and less than 100 days. In total we gather 475 options on the S&P500 and 360 options on the AEX index. Moreover, we used the daily one week LIBOR rates as a proxy for the risk free rate, as is usually done by traders in practice (Hull, 2009).

3.10 Summary

This chapter has described the models that are used and explains the variables in the models. Further it explains why there is the need to measure the performance through the Diebold Mariano test. It also explains why I choose the data sample for both the indices.

CHAPTER 4 Results

4.1 Introduction

In this chapter I will discuss the results that I have retrieved from E-views. First we will look at the estimated parameters for both indexes. Furthermore we will talk about the predicted volatilities and try to explain it and look at the distributions.

4.2 Estimated Parameters

As explained in the methodology part, we need the parameters for the GARCH models to forecast the volatilities. In the tables below, the estimated parameters of our within sample are shown for the GARCH models of both stock indices. One important point worth mentioning is that omega (ω_0) is negative for EGARCH. Since EGARCH doesn't put a restriction on ω_0 to be positive this is possible. Furthermore, the parameter estimations are shown in the tables 1A and 1B, and used as input for formulas 6-11.

Table 1A. Estimated parameters

AEX stock index			
Parameters	**GARCH (1.1)**	**EGARCH**	**TGARCH**
a_0	0.0986	0.2108	-0.0169
b_0	0.8945	0.9884	0.9325
w_0	1.62E-06	-0.2636	1.52E-06
g	NA	NA	0.1482
a*g	NA	NA	-0.0025
Persistence	9.93E-01	-	-
Daily Long-run volatility	0.02%	-	-
Annual Long-run volatility	0.2447	-	-
Long-run average variance	0.0037	-	-

Table 1B. Estimated parameters

S&P 500 stock index			
Parameters	**GARCH (1.1)**	**EGARCH**	**TGARCH**
a_0	0.0656	0.1325	0.0003
b_0	0.9203	0.9875	0.9256
w_0	2.07E-06	-0.2126	2.18E-06
g	NA	NA	0.1159
a*g	NA	NA	4.441E-05
Persistence	9.86E-01	-	-
Daily Long-run volatility	0.01%	-	-
Annual Long-run volatility	0.1921	-	-
Long-run average variance	0.0023	-	-

4.3 Predicted volatilities

The volatilities predicted with these parameters are shown in figures 1A and 1B. In figures 1A and 1B the GARCH models: GARCH (1.1), TGARCH and EGARCH follow a similar pattern. The historical volatility has in both figures the highest volatility, for this reason we can say that the historical volatility overestimates the volatility in comparison to the other models in our out of the sample period, which we expect to give the least accurate prices. Also worth mentioning is that the historical volatility in Figure 1A will also result in less accurate prices. In figure 1B we will expect only small minor differences in the pricing errors between the models, because they have a similar pattern and do not fluctuate far from each other.

Figure 2A. Out-of-the-sample forecasted volatilities for AEX index for the period 1/1/2012 to 31/12/2012

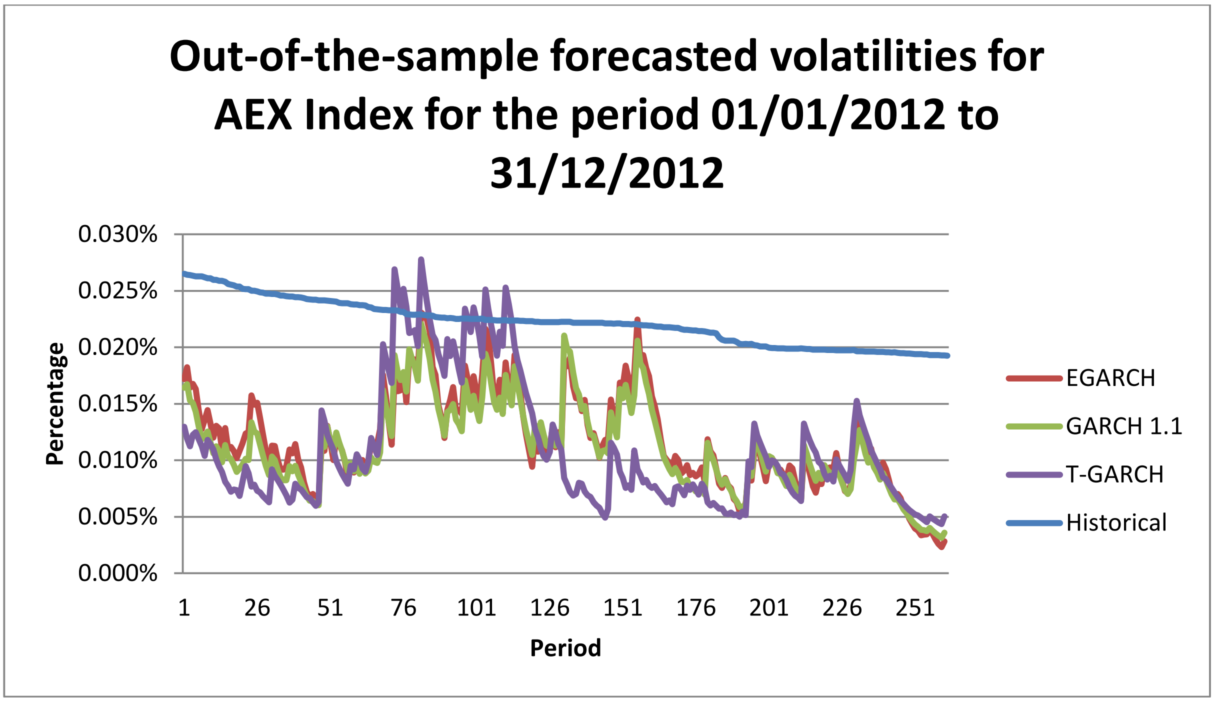
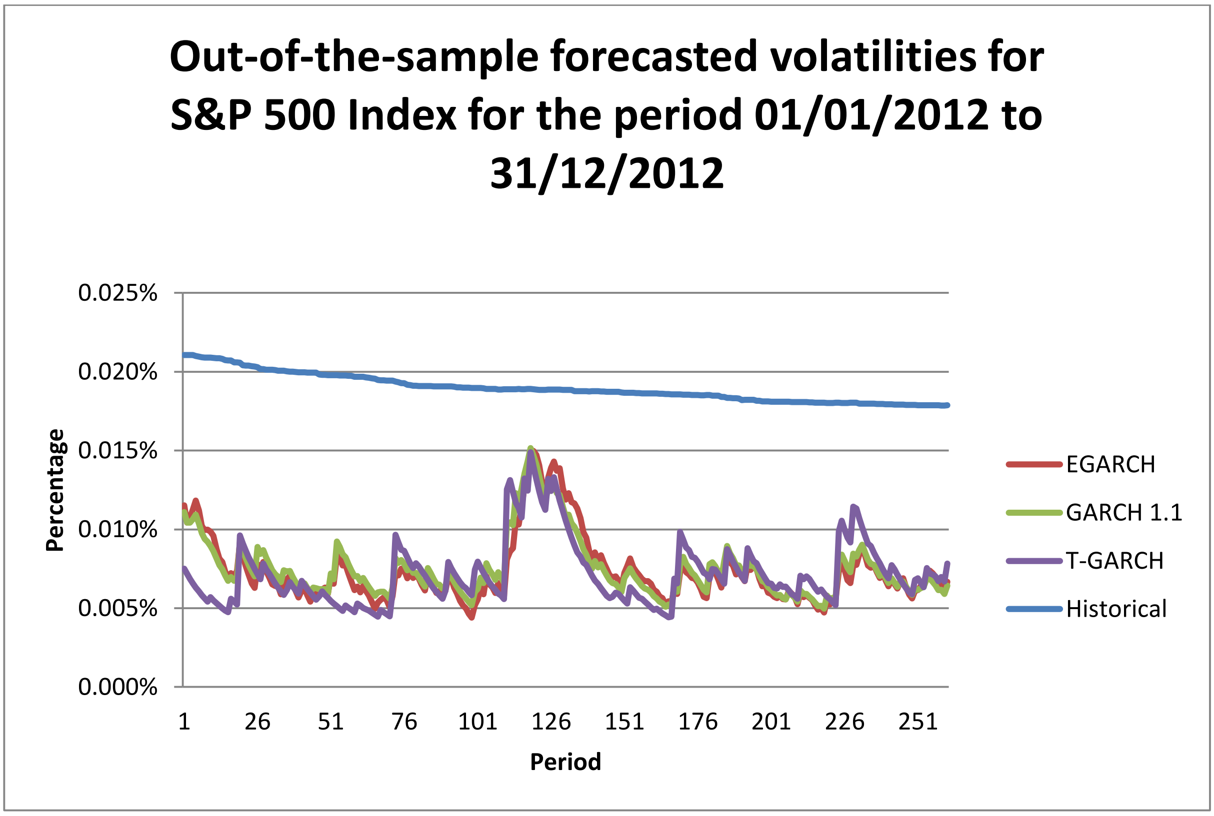
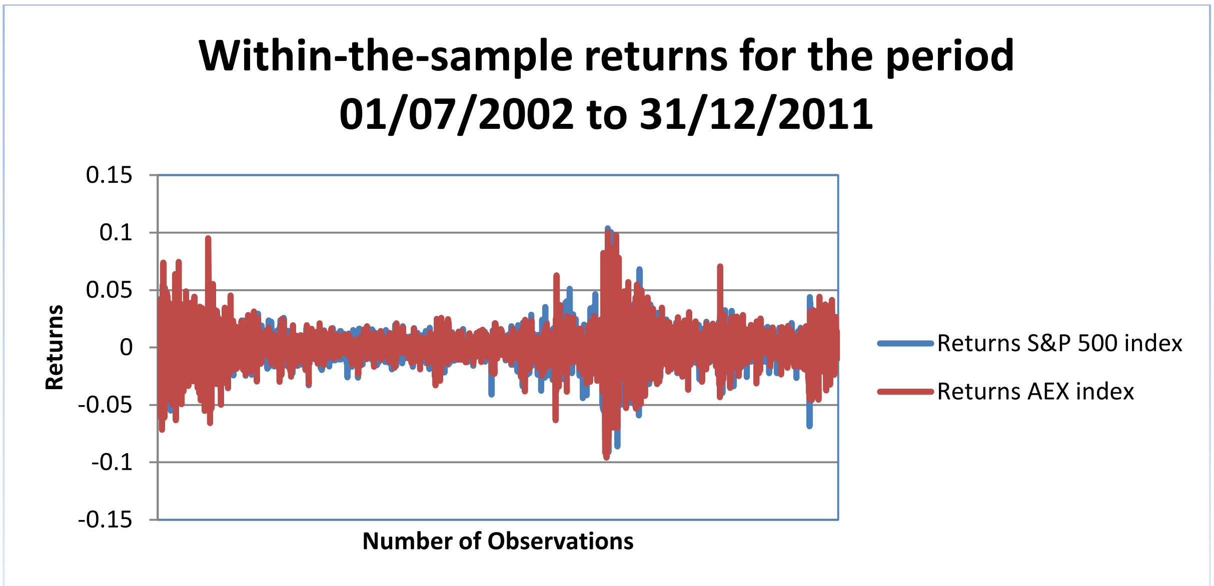


Figure 2B. Out-of-the-sample forecasted volatilities for S&P 500 index for the period 1/1/2012 to 31/12/2012



The forecasted volatilities are higher for the AEX index than for the S&P 500 index. One possible explanation is that the historical distribution of the returns of AEX index fluctuates more than the S&P 500 index, which is shown in figure 2.

Figure 3. Within-the-sample returns for the period 01/07/2002 to 31/12/2011



Furthermore, the parameters will help us calculate the option prices according to each model, which we then will compare with the real option prices. In the investigation on volatilities through different models we have conducted multiple tests as discussed in our methodology. From our analysis on the AEX index and S&P 500 index, the results in Tables 2A and 2B were obtained.

Table 2A. Root Mean Squared Errors for AEX index for all options, OTM, ATM and ITM

	Historical	GARCH (1.1)	EGARCH	TGARCH
All options	3.5884	***2.2576***	5.1854	2.8905
OTM	2.7786	***2.7905***	4.5482	3.3313
ATM	4.7602	2.8207	4.8685	***1.8412***
ITM	2.3212	2.2027	4.6767	***0.6128***

In the table above it is clearly shown that the Root Mean Squared Error (RMSE) is lowest for the GARCH (1.1) option prices. Hence, the GARCH (1.1) outperforms the other models.

This is also consistent with my hypothesis. However, when dividing the options into different classes (OTM, ATM and ITM) I find that GARCH (1.1) only outperforms the others in the OTM class. For the classes ATM and ITM the results show that the TGARCH model gives the most accurate option prices. Moreover, the GARCH (1.1) and TGARCH included in my research always outperform the EGARCH and Historical model.

Table 2B. Root Mean Squared Errors for S&P 500 index for all options, OTM, ATM and ITM

	Historical	**GARCH (1.1)**	**EGARCH**	**TGARCH**
All options	7.5949	7.5200	8.2672	***7.2729***
OTM	7.4644	7.8800	7.2200	***6.9842***
ATM	11.056	8.2412	***6.2407***	10.7914
ITM	***4.1215***	5.5900	5.4712	6.2547

From my analysis for the S&P 500 index, the results in Table 2B were obtained. In this table we can observe that the TGARCH has the lowest RMSE for the option prices. As such, we regard TGARCH as the preferable volatility forecaster for the whole index. Once we divide the option classes TGARCH competes with Historical volatility and EGARCH for (OTM). For the class (ATM) the EGARCH model outperforms the rest of the models. Surprisingly Historical volatility outperforms the other models in the (ITM) class.

When comparing Table 2A and 2B, we can see that the RMSE for the S&P 500 index are overall higher than the AEX outcome. The reason for this is the bigger change in the pricing error.

In order to test my hypothesis, I performed a T-test. Table 3 below shows us that GARCH (1.1) differs significantly from the other volatility prediction models. Comparing the values from table 3 with the critical t-value of 1.96 (5% significance level) we could conclude that there is a significant difference between the two models. This is because the values are higher than the critical value. Starting with the AEX index, as I observed in the tables above GARCH (1.1) had outperformed the other volatility models. From table 3 we can conclude that the difference between GARCH (1.1) and the other models is significant. All the t-values are higher than 1.96. The same significant outcomes could be observer for the S&P 500 index. However here was the GARCH (1.1) model outperformed by TGARCH.

Table 3 T-test values calculated from the Diebold Mariano test

	GARCH (1,1)	Historical	EGARCH	TGARCH
AEX index	T-test	3.2999	6.1221	4.5686
S&P 500 Index	T-test	4.5450	5.2100	5.0561

4.4 Summary

This chapter has shown the results obtained from models. The graphs and tables illustrate and explains the results. GARCH (1.1) outperformed the other volatility models for the AEX index and TGARCH outperformed the other volatility models for the S&P 500 index.

CHAPTER 5 Conclusion

The goal of this paper was to find a volatility forecasting model which used in conjunction with the Black-Scholes model predicts option prices most accurately. Therefore, four different volatility models were chosen for our analysis: Historical volatility, GARCH (1.1), EGARCH and TGARCH. Based on our literature review we developed the null hypothesis which stated that GARCH (1.1) is the superior volatility model when used for Black-Scholes option pricing. To test this hypothesis European call options on the AEX index and S&P 500 index were chosen. As a timeframe, I choose daily closing values for a time horizon of 2480 trading days starting from July 1st 2002 to December 31st 2012. The within-the-sample period ranged from July 1st 2002 to December 31st 2011. The observed daily fluctuations in this period were used to find the necessary parameters of our GARCH models.

The analysis of AEX index options showed that the null hypothesis could not be rejected. The whole sample for the AEX index showed that the RMSE was the lowest for GARCH (1.1) model. The T-test results showed that GARCH (1.1) volatility forecast differed significantly from the other volatility models. Thus this implies that GARCH (1.1) model gives indeed the most accurate option prices when used in conjunction with the Black-Scholes model. However when options were divided into classes as ATM, ITM and OTM, there were some minor changes. GARCH (1.1) still performed as one of the best for the class OTM. The TGARCH was a slight better for the class ATM and much better for the class ITM. The better performance of the GARCH (1.1) model could be evidence that the returns were more symmetrical distributed.

The results found for the S&P 500 Index differed from those of the AEX index. Most importantly, I found the RMSE was lowest for the TGARCH once we aggregated all squared errors from different models. This finding indicates the null hypothesis should be rejected. This is completely the opposite of mine finding for the AEX index. The fact that the TGARCH volatilities resulted in the most accurate option prices for the S&P 500 index could indicate that the leverage effect is more pronounced for options on this index. As described in the literature review, this means that negative shocks increases predictable volatility more than good news or positive shocks of similar magnitude.

The EGARCH should have an advantage over TGARCH, that it has more flexibility. It does not impose any constraint on the coefficients in order for the variance to be positive. EGARCH formulates the conditional variance equation in terms of the log of the variance rather than the variance itself. Looking at my results TGARCH performed better than EGARCH.

In comparison to other authors like Awartani & Corradi (2005) who did a similar research for the S&P 500 index their findings were the same. They had a dataset from January 1990 to September 2001. According to their results the asymmetric models outperformed the GARCH (1.1) model. This is also the case in my research. Such a finding is rather robust to the choice of the forecast horizon. Several empirical studies give an indication that of these EGARCH and TGARCH models, the TGARCH model gives the most accurate volatility forecasts. There is strong evidence that the modeling of asymmetric components is much more important than specifying the error distribution for improving the volatility forecasts of financial returns in the presence of fat-tails, leptokurtosis, skewness and the leverage effect.

According the research of Kou (2002) the double exponential jump-diffusion model and the GARCH (1.1) should be the best model. However, this is not the case for my research. For the AEX index the GARCH (1.1) model is clearly the best performer compared to the other volatility models. For the S&P 500 index the inconsistency may be caused by poor volatility forecasting performance from the GARCH (1.1) model. The double exponential jump-diffusion could be better in forecasting. This is because, my dataset consists the global financial crisis of October 2008. There were indeed upward and downward jumps observed in the stock market for the S&P 500 index. These jumps are ignored by the Black-Scholes model.

To conclude, I only reject the null hypothesis for the S&P 500 index, it seems that this index has more exposure to the leverage effect. The TGARCH performed better for this index. The TGARCH model uses a volatility function that is time varying and also incorporates the asymmetrical reaction of volatility on unanticipated returns. Nevertheless, the null hypothesis was supported by the AEX index, where I found that the GARCH (1.1) had performed better than the other volatility models. This could be evidence that the returns are more symmetrical for this stock index, in other words positive and negative news are treated similar.

Furthermore for future research, it would be interesting to take a shorter time horizon. My sample size includes the financial crisis. Moreover, out-of-the-sample forecasts were predicted in a stable volatility period, the actual volatility did not fluctuate that much. Therefore it might be interesting to do a similar research with data without a financial crisis. For instance one could assume that the leverage effect might be less pronounced, when there is no financial crisis in the within-the-sample period. It would be interesting to do the same research with the double exponential jump-diffusion model of Kou. In this way we could validate the performance of this model during a crisis period with upward and downward jumps.

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APPENDIX A S&P 500 EGARCH

Dependent Variable: RSP500 egarch
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 02/10/15 Time: 19:00
Sample: 2 2480
Included observations: 2479
Convergence achieved after 19 iterations
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(1) + C(2)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(3)
*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
	Variance Equation			
C(1)	-0.212616	0.025004	-8.503310	0.0000
C(2)	0.132501	0.009102	14.55711	0.0000
C(3)	0.987500	0.002151	458.9997	0.0000

R-squared	-0.000000	Mean dependent var	-5.09E-06
Adjusted R-squared	0.000403	S.D. dependent var	0.014519
S.E. of regression	0.014516	Akaike info criterion	-6.025278
Sum squared resid	0.522357	Schwarz criterion	-6.018240
Log likelihood	7471.332	Hannan-Quinn criter.	-6.022722
Durbin-Watson stat	2.308650		

APPENDIX B S&P 500 GARCH (1.1)

Dependent Variable: RSP5001.1 garch
Method: ML - ARCH
Date: 02/10/15 Time: 18:52
Sample: 2 2480
Included observations: 2479
Convergence achieved after 11 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(1) + C(2)*RESID(-1)^2 + C(3)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
	Variance Equation			
C	2.07E-06	3.11E-07	6.647496	0.0000
RESID(-1)^2	0.065614	0.006185	10.60864	0.0000
GARCH(-1)	0.920366	0.007559	121.7575	0.0000

R-squared	-0.000000	Mean dependent var	-5.09E-06
Adjusted R-squared	0.000403	S.D. dependent var	0.014519
S.E. of regression	0.014516	Akaike info criterion	-6.044035
Sum squared resid	0.522357	Schwarz criterion	-6.036997
Log likelihood	7494.581	Hannan-Quinn criter.	-6.041479
Durbin-Watson stat	2.308650		

APPENDIX C S&P 500 TGARCH

Dependent Variable: RSP500 tgarch
Method: ML - ARCH
Date: 02/10/15 Time: 18:50
Sample: 2 2480
Included observations: 2479
Convergence achieved after 12 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(1) + C(2)*RESID(-1)^2 + C(3)*RESID(-1)^2*(RESID(-1)<0) + C(4)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
		Variance Equation		
C	2.18E-06	3.30E-07	6.608772	0.0000
RESID(-1)^2	0.000383	0.006839	0.056054	0.9553
RESID(-1)^2*(RESID(-1)<0)	0.115953	0.012613	9.193190	0.0000
GARCH(-1)	0.925673	0.008439	109.6909	0.0000

R-squared	-0.000000	Mean dependent var	-5.09E-06
Adjusted R-squared	0.000403	S.D. dependent var	0.014519
S.E. of regression	0.014516	Akaike info criterion	-6.068122
Sum squared resid	0.522357	Schwarz criterion	-6.058738
Log likelihood	7525.437	Hannan-Quinn criter.	-6.064714
Durbin-Watson stat	2.308650		

APPENDIX D AEX EGARCH

Dependent Variable: RAEX egarch
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 02/10/15 Time: 18:49
Sample: 2 2480
Included observations: 2479
Convergence achieved after 8 iterations
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(1) + C(2)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(3)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
		Variance Equation		
C(1)	-0.263634	0.024645	-10.69715	0.0000
C(2)	0.210894	0.012730	16.56676	0.0000
C(3)	0.988456	0.002362	418.4693	0.0000

R-squared	-0.000071	Mean dependent var	-0.000137
Adjusted R-squared	0.000332	S.D. dependent var	0.016278
S.E. of regression	0.016276	Akaike info criterion	-5.964279
Sum squared resid	0.656684	Schwarz criterion	-5.957241
Log likelihood	7395.724	Hannan-Quinn criter.	-5.961723
Durbin-Watson stat	2.024699		

APPENDIX E AEX GARCH (1.1)

Dependent Variable: RAEX 1,1 garch
Method: ML - ARCH
Date: 02/10/15 Time: 18:51
Sample: 2 2480
Included observations: 2479
Convergence achieved after 10 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(1) + C(2)*RESID(-1)^2 + C(3)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
		Variance Equation		
C	1.62E-06	3.43E-07	4.734468	0.0000
RESID(-1)^2	0.098647	0.007843	12.57717	0.0000
GARCH(-1)	0.894594	0.007774	115.0713	0.0000

R-squared	-0.000071	Mean dependent var	-0.000137
Adjusted R-squared	0.000332	S.D. dependent var	0.016278
S.E. of regression	0.016276	Akaike info criterion	-5.969403
Sum squared resid	0.656684	Schwarz criterion	-5.962365
Log likelihood	7402.075	Hannan-Quinn criter.	-5.966847
Durbin-Watson stat	2.024699		

APPENDIX F AEX TGARCH

Dependent Variable: RAEX t garch
Method: ML - ARCH
Date: 02/10/15 Time: 18:51
Sample: 2 2480
Included observations: 2479
Convergence achieved after 11 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(1) + C(2)*RESID(-1)^2 + C(3)*RESID(-1)^2*(RESID(-1)<0) +
C(4)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
		Variance Equation		
C	1.52E-06	2.06E-07	7.369355	0.0000
RESID(-1)^2	-0.016990	0.007788	-2.181516	0.0291
RESID(-1)^2*(RESID(-1)<0)	0.148283	0.012075	12.27987	0.0000
GARCH(-1)	0.932565	0.006436	144.8958	0.0000

R-squared	-0.000071	Mean dependent var	-0.000137
Adjusted R-squared	0.000332	S.D. dependent var	0.016278
S.E. of regression	0.016276	Akaike info criterion	-6.017020
Sum squared resid	0.656684	Schwarz criterion	-6.007636
Log likelihood	7462.097	Hannan-Quinn criter.	-6.013612
Durbin-Watson stat	2.024699		

APPENDIX G Volatility smiles

Figure 18.1 Volatility smile for foreign currency options.