Duopoly and Project Uncertainty

An analysis of the Bertrand and Cournot duopolies under uncertainty for the buyer

Draft Master Thesis

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Content

1 Introduction

In many projects there is information asymmetry between a buyer who would like to have a successful project and the suppliers on the market. The suppliers have more knowledge than the buyers and decision makers about the causes and underlying mechanisms that increases the probability of success. An ordinary car owner can observe that his car is broken and he can see whether a reparation is successful or not, but he cannot see the underlying problem. He therefore has to rely on the car dealers' advice. When there is only a single supplier who can supply a reparation, he has market power to exert his position. However, when there are more suppliers who are on a market of competition, the buyer can compare the given advice.

This thesis studies a signalling model. The Decision Maker representing the Demand Side (e.g. the car owner) has uncertainty in his project. The Suppliers who make an offer (e.g. the car dealers), have private information about this uncertainty. I will investigate whether the Suppliers will credible reveal the state of the world, what the transaction price will be and whether there is a surplus or a dead weight loss. I will compare the model situated in a monopoly and Bertrand and Cournot Duopoly.

Section 2 elaborates on the theoretical background on which the model is based. Section 3 explains the base model. In Section 4 I use this base model to show that a monopolistic Supplier can use his market position. Section 5 adds a second Supplier to the market and represents a Bertrand Duopoly. Finally, Section 6 will analyse a competition on supplied quantity representing a Cournot Duopoly.

2 Theoretical background

Before defining the signalling model, I will explain the background on which my model is based. The model is a refinement of the traditional competition models such as monopoly, Bertrand and Cournot duopoly. This refinement is based upon private information about required quality.

Competition is usually considered to be a mechanism that drives prices down. A monopolist can set the market price and thus the demand to optimize his profit. This leads to a higher price than the perfect competition price and therefore a smaller traded amount of goods. The reduction in consumer surplus that goes with this higher price and lower quantity is not fully compensated by the increase in producer surplus. This causes a dead weight loss for the total welfare (Martin, 2010).

Perfect competition on the other hand leads to equilibrium prices that are equal to the marginal costs which means that the suppliers cannot collect economic rent and the total welfare is maximal, there is no dead weight loss.

When a second supplier is active on the market, there is a duopoly. The suppliers do not have total market power because there is competition. There are two fundamental models: competition on the offered amount, where the demand side of the market causes the equilibrium price (Cournot model) and competition on price, where the traded amount arises comes from the asked price.

In the Cournot duopoly the suppliers optimize their offered quantity by setting their marginal benefit equal to the marginal production costs. In equilibrium this is a higher quantity than in a monopoly and a lower than in perfect competition. The price is also in between the monopoly and perfect competition price. Therefore, there still is a Dead Weight Loss, but it is smaller than in the Monopoly situation (Tirole, 1988).

When the suppliers compete on price as in the Bertrand duopoly, the addition of the second supplier is enough to prevent the suppliers from making any profit. The optimal strategy of the suppliers to undercut the other supplier drives prices to marginal costs. Therefore, there is no dead weight loss and the total aggregated producer surplus cannot be greater than the monopolistic supplier surplus (Tirole, 1988).

These (theoretical models) require certainty and common knowledge. Every player knows the possible actions and payoffs. The literature shows many different ways of uncertainty and the effects on the above competition models. The suppliers can be uncertain about their own production costs or about the market demand. Novshek and Sonnenschein describe the situation in which the suppliers are uncertain about the market demand. There is an exogenous parameter that influences demand. Their model sees in which situations a supplier is willing to buy or sell information about this exogenous parameter (Novshek & Sonnenschein, 1982). Grimm creates a more general model in which there is an uncertain demand. He shows that in a Cournot competition this leads to multiple symmetric equilibria as long as $p \geq 0$ (Grimm, 2008). Vives shows the situation that different suppliers have private information about the market demand (Vives, 1984). He investigates whether the suppliers share this information in the Bertrand and Cournot duopolies. A Cournot competition results in a sub-optimal equilibrium, the information is not shared. However, the Bertrand situation is more efficient and the information becomes public. Novshek and Sonnenschein also show that when there is an uncertainty about demand, a decrease in variance (so a decrease in the scope of uncertainty), leads to a decrease of consumer surplus (Novshek & Sonnenschein, 1982).

There can also be uncertainty at the demand side of the market. There are different models about the decision strategy of a Decision Maker (customer). There models are about signaling and verifiable information. Spence created a fundament about the strategy of an employer at the hiring process of a new employee (Spence, 1973). This Decision maker does not know the qualities of a new employee and he has to base his decision on some signals and signs. However, when these signals come with different costs for possible employees with different qualities (e.g. an education for which the costs or time consumption differs for various levels of intelligence), the employer can have a credible belief of the qualities of the employees. The equilibrium mechanism is that neither the employer nor the possible employees have an incentive to deviate in the process of:

- Possible employees decide on signal (influenced by the signal costs),
- Hiring the employee and observation of signals,
- Employer updates his beliefs based on project result,
- Employer decides about his wage scheme offer to employee.

Competition can also result in verifiable information revealed to the Decision Maker (Milgrom & Roberts, 1986). This Decision Maker has a sceptical approach accounting for the worst case scenarios. Unless there is verifiable information that the true state of the world is better than the worst, he bases his decision on the worst state. In a model with one supplier, this results in full disclosure. Whenever there are more than one supplier and not all the possible states are covered by the various suppliers, there is an incentive for the suppliers to abuse the information asymmetry, causing the transfer of information to be inefficient.

An example of the above described uncertainty for the demand side, with the Decision Maker is quality. It is not always possible to observe, before the transaction, what the quality of a good is. The Decision Maker may face difficulties with establishing the required quality. This may lead to a transaction that incurs either a (costly) oversupplied quality or a poor quality solution that does not meet the requirements.

Laffond and Tirole showed that there are two reasons for an unregulated supplier to supply quality (Laffont & Tirole, 1993). The first relates to entering the sales transaction. The quality needs to be observable before the transaction. This is called a search good. Delivering quality is than caused by a sales incentive. A higher quality would result in a higher demand or a higher willingness to pay. The other reason for suppling quality is the reputation concern. In a recurring transactional model suppling quality can raise the supplier s reputation. This incentive does also work when quality is revealed after transaction, for the experience. Whenever these two incentives are not sufficient, regulation is needed. Our model sees to such a case. The quality of the supplied product is public information (a search good), but the necessary quality is private information and will only be revealed after the transaction (experience good). The "relative purchased quality" is comparable to an experience good. The sales incentive fails short and because the transaction is not recurring, the reputation incentive does not work either.

In his economic analysis of the Dutch legislation Visscher examines these regulation solutions (Visscher, 2015). Tort Law and indemnification is not only a legal or moral concept to solve the discrepancy of who faces damage and who causes it. By paying the lesioned their damages, the negative externality of the action is internalized. This can cause someone who can act while having the disposal of private information, to use this information in favour of the dependants.

When I combine these different models and quality concepts, this results in my model in which the demand side, the Decision Maker, has uncertainty. The market demand depends on this uncertainty. However, the supply side possesses private information to solve this uncertainty. The market has a regulation solution that require the suppliers to pay a full refund on a project failure. So if there is a transaction, the suppliers are committed to the project result. This partially internalise the risk of a sub-optimal choice on quality and sharing little private information.

3 The base model

The model is situated on a market. This market consists of one customer who should make an implementation decision about a project and one or more Suppliers who can supply the goods needed for the execution of the project. The consumer is the Decision Maker, DM. The project can deliver the DM a positive outcome, this is a gain G. This gain does not depend on the amount that is produced or traded. The decision is represented by X. When there is one Supplier $X = 0$ is no execution and $X = 1$ is execution. In the extended models I will redefine X. An example project is repairing a broken car. A working car gives the DM the possibility to use it. The value of that use does not depend on the size of the reparation. The implementation decision in this case is whether he wants to have it repaired and who will supply the reparation.

In order to carry out the project he has to work with one or more Suppliers, S. These suppliers make an offer to the DM, consisting a certain quantity or quality represented by q, for a total price p.[1] Hence: $o_{Supplier} = \left(p_{Supplier}, q_{Supplier}\right)$. Any offer with $q_{Supplier} = 0$ is a refusal of participating in a transaction. When an offer is accepted and the Supplier actually produces and sells q_{sender}, he will incur a cost $c(q_{sender})$. For $c(q)$ is given that $c(q) > 0 \; \forall \; q > 0$ and $c'(q) > 0 \; \forall \; q \geq 0$ and $c''(q) \in \mathbb{R} \; \forall \; q \geq 0$.

The probability that the project is successful depends both on the produced q in the transaction and an exogenous q^*. So: $Pr(Success|q^*, q) = \pi(q^*, q)$, with $\pi \in [0,1]$. This q^* represents the extend of the problem represented by the project and it is drawn by Nature from any distribution $f(q^* = q)$. The project can demand a lot of effort or it can just be a little action that needs to be done in order to let the project be successful. So the project success depends on the q that is required by Nature and the actual produced q. A broken car can require a major repair or just the replacement of a small part. The probability that the car will work again depends both on how the car is broken and on the applied reparations.

This exogenous q^* is unknown to the DM, but is revealed to the Suppliers before they make their offers o. In the end it will be revealed for everyone whether the project is successful or not. For the car example: the DM does not understand the technique of is car, but the offering workshops do and the DM can recognise whether the car has been fixed in the end.

I will use a "step function" (Definition 1), $\pi_s(q^*, q)$, and a "threshold probability function" (Definition 2), $\pi_t(q^*, q)$, in many parts of this thesis. This $\pi_s(q, q^*)$ implies that: no production is no success, a solution that is too small incurs a low probability of success and a solution that is sufficient enough will give the high probability. So there is no gain in π by oversizing $q > q^*$. A workshop could give a car with only one blocked filter a major overhaul, but that has no added value compared to just replacing the filter.

Definition 1: Step Function

In the case of the step function there is no gain in probability by producing more q, except at producing anything instead of nothing, $q > 0$, and reaching a threshold, $q \geq q^*$. Its definition is:

$$\pi_s(q^*, q) \; = \begin{cases} 0, & q = 0 \\ \pi^L, & q < q^* \\ \pi^H, & q \geq q^* \end{cases}$$

$$0 < \pi^L < \pi^H$$

This implies that:

$$\frac{\partial \pi_s(q^*, q)}{\partial q} = 0, \qquad q > 0 \cup q \neq q^*$$

Definition 2: Threshold Probability Function

$\pi_t(q^*, q)$ is a non-continuous probability function. It is a generalisation of the step function π_s.

[1] Whether q represents a quantity or quality does not matter for the model. The nature of the project defines this. In the broken car example a high q can represent a thourough reparation and a low q a small fix.

In the case of the step function the only probability gain was in $q > 0$ and reaching the threshold of $q = q^*$. In $\pi_t(q^*, q)$ there may also be a probability gain between these steps.

$\pi_t(q^*, q)$ is therefore increasing in q and differentiable in $q > 0 \cup q \neq q^*$. Just like the step function, there is an increase in π as soon as the threshold of $q = q^*$ is reached.

Given that π is increasing in q, we can state:

$$\frac{\partial \pi_s(q^*, q)}{\partial q} \geq 0, \qquad q > 0 \cup q \neq q^*$$

Given that the threshold includes a probability gain we can state:

$$\lim_{q \downarrow 0} \pi(q^*, q) \geq \pi(q^*, 0) = 0$$
$$\lim_{q \downarrow x} \pi(q^*, q) \geq \lim_{q \uparrow x} \pi(q^*, q), \qquad x > 0$$

When the project is successful the DM receives his gain, G, and he pays the price p to the chosen Supplier. The producing Supplier has a result commitment. If the project fails, this is seen as a breach of contract. This leads to an indemnification: the DM does not have to pay the Suppliers and the Suppliers do not receive $p_{Supplier}$. Any Supplier that produced $q_{Supplier} > 0$ will still incur the production costs $c(q_{Supplier})$. So the DM has only to pay the workshop when the car is working again.

The ex ante expected utility in the above described model are therefore:

$$U_{Sender} = X \cdot [\pi(q^*, q) \cdot p_{Sender} - c(q_{Supplier})]$$
$$U_{DM} = X \cdot [\pi(q^*, q) \cdot (G - p)]$$

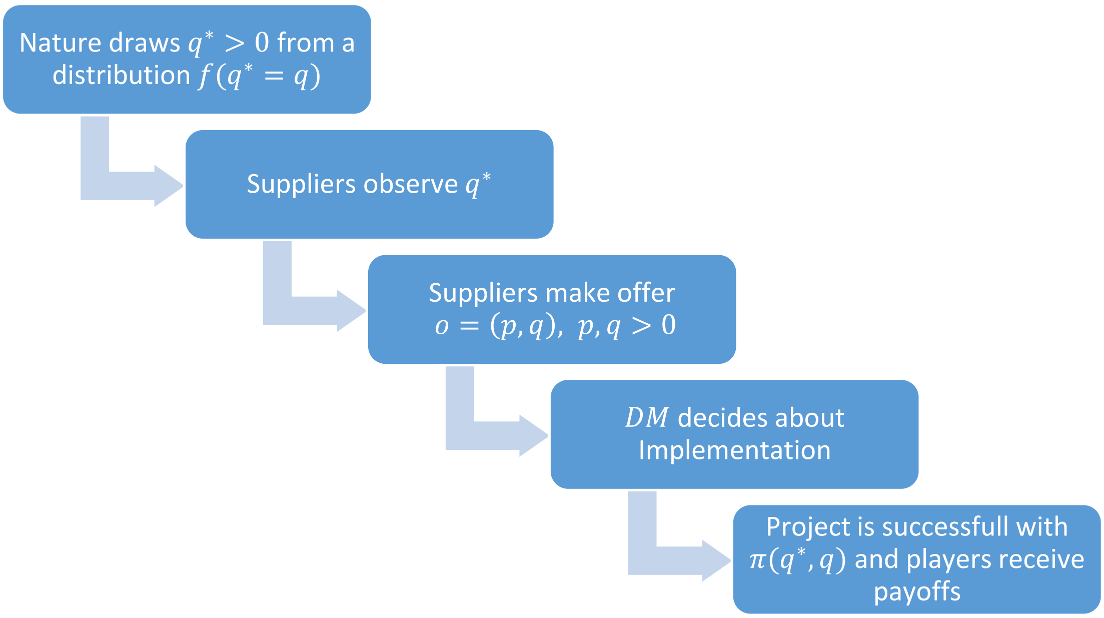

Figure 1: Timing of the models

The above described model can be seen as a dynamic Bayesian Game of Incomplete Information. In the analysis of the model I will look for Weak Perfect Bayesian Equilibria (WPBE). A strategy can only be part of a WPBE if it is optimal for the player given his beliefs about q^*. He cannot improve his expected utility by deviating of his strategy given the strategies of the other players. So a Supplier will not have an incentive to make any other offer given the Decision Maker's implementation decision and the other Supplier's offer. In the same way the Decision Maker has no incentive to make any other implementation decision given his beliefs about q^*. The beliefs about q^* must be consistent with the combined strategies of the Suppliers and the Decision Maker (thus: the Strategy Profile) (Tadelis, 2013).

I will refine this equilibrium concept with two additions. First, when the DM is indifferent between implementing an offer or implementing nothing at all, he will implement. This prevents the DM from entering the market and looking for a solution. For the car example: even if the expected net value of the reparation is equal to no reparation, the DM will have his car repaired. If this refinement were not assumed, there could be no ex ante incentive for Suppliers to be on the market, because there would be no customers.

The second refinement is that the indifferent Supplier prefers to be involved in a transaction over not being involved. That assures that there are transactions on a market where there is no economic rent for Suppliers due to competition. For the car example: if the workshops cannot make profit due to competition, they will prefer repairing cars as long as they are not facing an expected loss.

4 Analysis of a monopoly – One Supplier

In this section the model is simplified to one Supplier, two states and two possible offers. Further we use the Step Function (Definition 1) as function for $\pi(q^*, q)$. So in essence this is about a monopoly, with two possible products that are either sufficient or insufficient. The only option for the DM is to accept ($X = 1$) the offered amount, q, for the given price, p, or not to implement the solution at all ($X = 0$). So more formal:

The offer of the Supplier is either $o^H = (p^H, q^H)$ or $o^L = (p^L, q^L)$. The prices p^H and p^L are for the monopolistic Supplier a free choice.

The exogenous q^*, defined by nature, is given by:

$$q^* \in \{q_L^*, q_H^*\}, \ \ 0 < q_L^* < q_H^*$$

In order to align the possible offers and the states we define:

$$q^H = q_H^* \text{ and } q^L = q_L^*$$

The actions for the DM are to implement or not: $X \in \{0,1\}$.

This game can be seen as a Bayesian Game of incomplete information with two players (the Supplier being low type or high type), two actions for each player, two states. Its normal form including the payoffs is drawn in Figure 2.

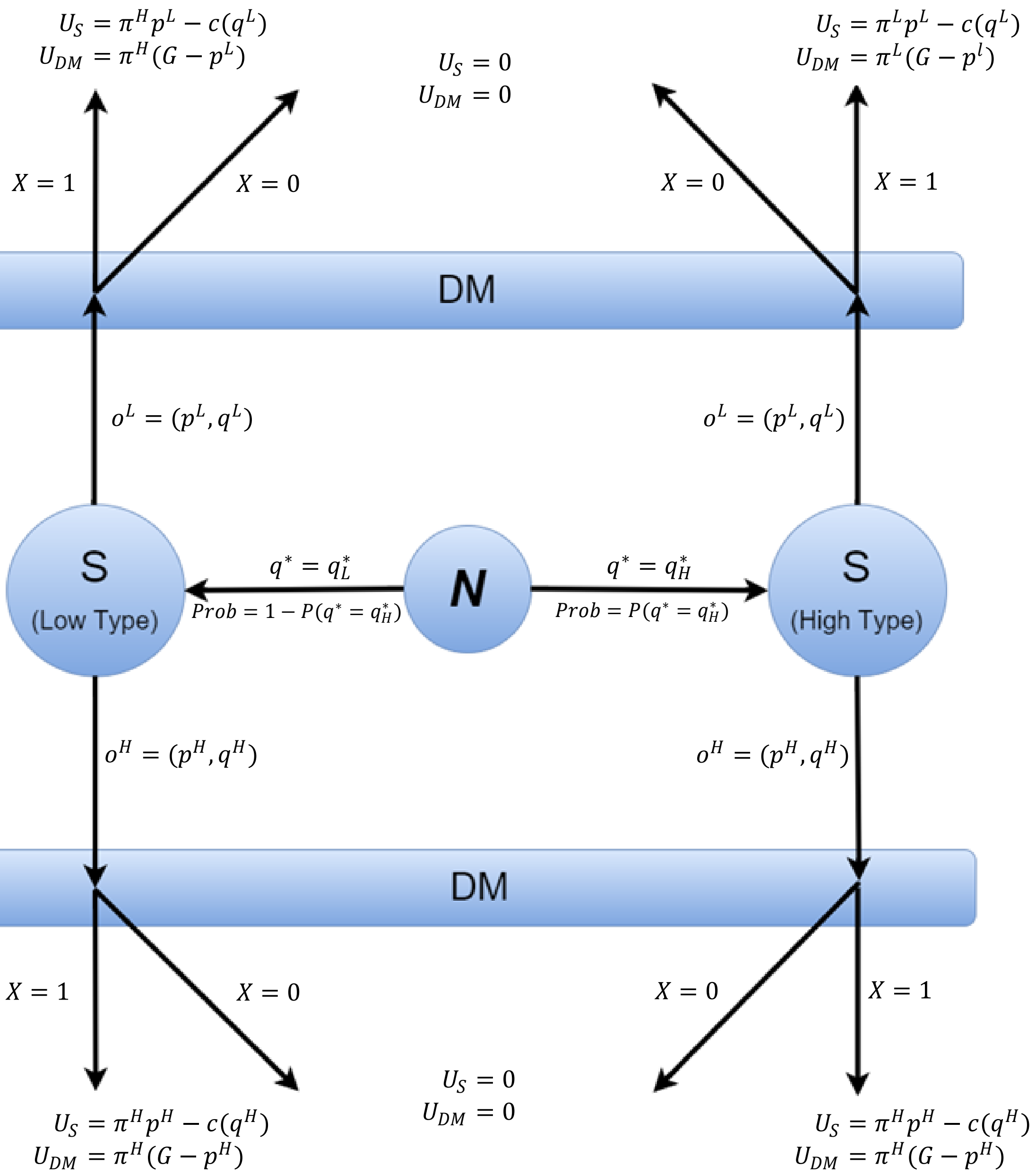

Figure 2: Extensive form of the monopoly game

The analysis starts with the DM's best response to the Supplier's offers for the different values of q^*. This defines his optimal consisting of the actions the player would do in both of the information sets. So his strategy must be an optimal response to both o^H and o^L.

The DM's prior belief about the required quality q^* is that the Supplier is high type ($q^* = q_H^*$) with probability $h = Prob(q^* = q_H^*) \in [0,1]$ and that the Supplier is low type ($q^* = q_L^*$) with probability $1 - h$. After receiving the offer o he will update his belief to $\hat{h}(o) = Prob(q^* = q_H^* | o)$.

Although the DM has the possibility to choose not to implement the offered solution, it is always optimal for him to implement as long as he has a possible resulting gain, $G - p \geq 0$ (Proposition 1). Therefore $p \leq G$ is the participation constraint for the DM. The risk of losing resources due to the product having $q < q^*$ is with the Supplier.

Proposition 1: Best responses for the DM: The optimal strategy for the DM is always implement the offered solution as long as $G - p \geq 0$, for any beliefs $\hat{h}(o) \in [0,1]$. [2]

Proof:

The DM will implement when:

$$U(Implement) \geq U(Not\ Implement)$$

For: $o = o^H$:

When DM is in information set where the high quality solution o^H is offered, it is optimal to implement when:

$$\hat{h}(o)\pi^H(G - p^H) + (1 - \hat{h}(o))\pi^H(G - p^H) \geq \hat{h}(o) \cdot 0 + (1 - \hat{h}(o)) \cdot 0 = 0$$
$$\pi^H(G - p^H) \geq 0$$

The probability of success always is $\pi(q^*, q^H) = \pi^H \geq 0$ (given). Hence as long as $G - p^H \geq 0$, he will choose $X = 1$, otherwise he will choose $X = 0$.

For $o = o^L$:

When DM is in information set where the low quality solution o^L is offered, it is optimal to implement when:

$$\hat{h}(o)\pi^L(G - p^L) + (1 - \hat{h}(o))\pi^H(G - p^L) \geq \hat{h}(o) \cdot 0 + (1 - \hat{h}(o)) \cdot 0 = 0$$
$$[\hat{h}(o)\pi^L + (1 - \hat{h}(o))\pi^H](G - p^L) \geq 0$$

Since it is given that $\hat{h}(o) \in [0,1]$, $\pi^L \geq 0$, $\pi^H \geq 0$, the DM's belief for the probability that he will receive his net gain, $G - p^L$, is:

$$\hat{h}(o)\pi^L + (1 - \hat{h}(o))\pi^H \geq 0$$

The Participation Constraint for the DM is therefore:

$$G - p^L \geq 0 \quad \textit{(PC-DM)}$$

So as long as $G - p^L \geq 0$, he will choose $X = 1$, otherwise he will choose $X = 0$. This is irrespective of the Supplier's type.

The Supplier on the other hand should make an offer o consisting of a price that gives him an expected positive payoff. This depends on the production costs and therefore on the offered q. His participation constraint, PC-S, is $p \geq p_{min}(q^*, q)$ (Lemma 1). The other part of the offer o, the quality q depends on his type, the production costs, the possible price difference between the low and high quality product, and the difference in the probability of success π (Lemma 2).

Lemma 1: Participation Constraint for the Supplier

The Supplier will only make an offer which would result in a positive expected payoff and therefore:

[2] Note that this proposition holds for any function $\pi(q^*, q)$, as long as $\pi(q^*, q) \in [0,1]$.

$$p \geq p_{min}(q^*, q) = \frac{c(q)}{\pi(q^*,q)} \qquad \textit{(PC-S)}$$

Proof:

The Supplier has a preference for the transaction not to take place (and thus to deviate and offer (0,0)) as soon as $E[U_S(p,q)] < 0$. Therefore, the Supplier will either raise the price p for any q so $p \geq p_{min}(q^*, q)$ or decide to offer (0,0).

$$E[U_S] = \pi(q^*, q)p - c(q), \qquad p = p_{min}(q^*, q)$$
$$E[U_S] = \frac{\pi(q^*, q)c(q)}{\pi(q^*, q)} - c(q) = 0$$

Corollary 1: Participation constraints for the Supplier and DM combined

Combining the participation constraints of both the DM and Supplier (Lemma 1) shows that a given q a transaction can occur if:

$$\frac{c(q)}{\pi(q^*,q)} \leq G \quad \textit{(PC)}$$

And the price range would be:

$$p(q) \in [\frac{c(q)}{\pi(q^*, q)}, G]$$

Now we have the constraints within which the Supplier and DM are willing to enter into a transaction (given by PC), we can state the conditions for the Supplier to offer the different solutions. This results in the following Incentive Compatibility Constraint (ICC-S, Lemma 2):

$$\pi(q^*, q^L)\Delta p + \Delta\pi p^L + \Delta\pi\delta p > c(q^H) - c(q^L)$$

In this:

Δp: The (possible) price difference between the products: $p^H - p^L$

The ICC-S compares the difference in production costs with:

$\pi(q^*, q^L)\Delta p$: The increase in profits due to the (possible) higher price for the high quality product, given the probability of success of the low quality product.

$\Delta\pi p^L$: The increase in profits due to the higher probability of success.

$\Delta\pi\Delta p$: The increase in profits due to the higher probability that the (possible) higher price will be paid.

Lemma 2: Incentive Compatibility Constraint for offering a higher q

Both the high type Supplier and the low type Supplier will tend to offer the high quality product if the larger expected benefits are higher than the difference in production costs. In detail:

$$\pi(q^*, q^H)p^H - c(q^H) > \pi(q^*, q^L)p^L - c(q^L)$$

$$\pi(q^*, q^H)p^H - \pi(q^*, q^L)p^L > c(q^H) - c(q^L) \qquad \textit{(ICC-S)}$$

This “larger expected benefits” split out are:

$$\pi(q^*, q^H)p^H - \pi(q^*, q^L)p^L = \pi(q^*, q^L)\Delta p + \Delta\pi p^L + \Delta\pi\delta p$$
$$\Delta p = p^H - p^L; \Delta\pi = \pi(q^*, q^H) - \pi(q^*, q^L)$$

Note the Supplier cannot successfully offer q^H in equilibrium if the PC is not satisfied.

The two different type Suppliers will apply their ICC-S with respect to the true value of q^*.

For the high type Supplier, it holds that $\Delta\pi > 0$ (given). Since the PC dictates that for a successful transaction the price should not exceed G, it is optimal for the high type Supplier to set $p \to G$ irrespective of q. In that case the DM will always accept his offer. His own Participation Constraint, the minimum price p_{min}, may change when the offered amount of q. This constraint only is about the choice whether the Supplier wants to offer, the price is still $p \to G$, so Δp = 0. So it is optimal for the high type Supplier to offer q^H only if the gain in probability outweighs the rise in production costs. The Incentive Compatibility Constraint for the high type Supplier to offer o^H is therefore:

$$\Delta\pi G > c(q^H) - c(q^L) \qquad \textit{(ICC-HS)}$$

Since both $c(q)$ and $\pi(q^*, q)$ have an effect on the minimum price, the effect of offering a higher q depends on the parameter space. The parameter space can dictate that offering o^L or o^H is impossible given the Participation Constraints since either the production costs are too high, or the probability of success is too low to make an offer which creates any rent.

For the low type Supplier, it also is optimal to set $p \to G$, but there is no possible gain in probability $\Delta\pi$, since the Step Function defines that all $q \geq q^*$ have the same π and $q_L^* = q^L < q^H$. Since it is given that the marginal costs are $c'(q) > 0$, the minimum price for q^H is higher than the minimum price for q^L. So if $p_{min}(q^*, q^L)$ would violate the Participation Constraints, $p_{min}(q^*, q^H)$ would violate the Participation constraints as well.

This means that there is no reason for a low type Supplier to mimic a high type Supplier and offer a high type. The market power of the monopolistic Supplier is strong enough to reach a price of $p \to G$. The minimum price increases in q and is therefore no reason to deviate from this strategy.

Proposition 2: The optimal strategy of the monopolistic Supplier

The optimal strategy for the Supplier is given in the following table.

<table>
<tr><td colspan="4"></td><td colspan="2">Incentive Compatibility Constraint (Lemma 2; ICC-HS)</td></tr>
<tr><td rowspan="20">Participation Constraint ((Corollary 1; PC)</td><td>Low Type Supplier</td><td>High Type Suppler</td><td></td><td>$\boldsymbol{\Delta\pi G \geq c(q^H) - c(q^L)}$
ICC dictates higher quality product</td><td>$\boldsymbol{\Delta\pi G < c(q^H) - c(q^L)}$
ICC dictates higher quality product</td></tr>
<tr><td rowspan="8">$\boldsymbol{p_{min}(q_L^*, q^L) \leq G}$
PC for low type Supplier fulfilled.</td><td rowspan="2">$\boldsymbol{p_{min}(q_H^*, q^L) \leq G}$
$\boldsymbol{p_{min}(q_H^*, q^H) \leq G}$
PC for high type Supplier fulfilled</td><td>H</td><td>$o^H = (G, q^H)$
Separating EQ</td><td>$o^L = (G, q^L)$
Pooling EQ</td></tr>
<tr><td>L</td><td colspan="2">$o^L = (G, q^L)$</td></tr>
<tr><td rowspan="2">$\boldsymbol{p_{min}(q_H^*, q^L) \leq G}$
$\boldsymbol{p_{min}(q_H^*, q^H) > G}$
PC for high type Supplier fulfilled for q^L</td><td>H</td><td>Not existing combination[3]</td><td>$o^L = (G, q^L)$
Pooling EQ</td></tr>
<tr><td>L</td><td colspan="2">$o^L = (G, q^L)$</td></tr>
<tr><td rowspan="2">$\boldsymbol{p_{min}(q_H^*, q^L) > G}$
$\boldsymbol{p_{min}(q_H^*, q^H) \leq G}$
PC for high type Supplier fulfilled for q^H</td><td>H</td><td>$o^H = (G, q^H)$
Separating EQ</td><td>Not existing combination[3]</td></tr>
<tr><td>L</td><td colspan="2">$o^L = (G, q^L)$</td></tr>
<tr><td rowspan="2">$\boldsymbol{p_{min}(q_H^*, q^L) > G}$
$\boldsymbol{p_{min}(q_H^*, q^H) > G}$
PC for high type Supplier not fulfilled.</td><td>H</td><td>$(0{,}0)$
Separating EQ</td><td>$(0{,}0)$
Separating EQ</td></tr>
<tr><td>L</td><td colspan="2">$o^L = (G, q^L)$</td></tr>
<tr><td rowspan="5">$\boldsymbol{p_{min}(q_L^*, q^L) > G}$
PC for low type Supplier not fulfilled.</td><td rowspan="2">$\boldsymbol{p_{min}(q_H^*, q^L) > G}$
$\boldsymbol{p_{min}(q_H^*, q^H) \leq G}$
PC for high type Supplier fulfilled for q^H</td><td>H</td><td>$o^H = (G, q^H)$
Separating EQ</td><td>Not existing combination[3]</td></tr>
<tr><td>L</td><td colspan="2">$(0{,}0)$</td></tr>
<tr><td rowspan="2">$\boldsymbol{p_{min}(q_H^*, q^L) > G}$
$\boldsymbol{p_{min}(q_H^*, q^H) > G}$
PC for high type Supplier not fulfilled.</td><td>H</td><td>$(0{,}0)$
Pooling EQ</td><td>Not existing combination[3]</td></tr>
<tr><td>L</td><td colspan="2">$(0{,}0)$</td></tr>
<tr><td>$\boldsymbol{p_{min}(q_H^*, q^L) \leq G}$
PC for high type Supplier fulfilled for q^L</td><td colspan="3">Not existing combination since:
$p_{min}(q_L^*, q) \leq p_{min}(q_H^*, q)\ \forall\ q_L^* \leq q_H^*$</td></tr>
</table>

[3] For a proof see Appendix 1.

This optimal strategy for the monopolistic Supplier (Proposition 2) shows that in the Weak Perfect Bayesian Equilibrium:

- the DM will accept the offer, if he has a net surplus $G - p$ at a successful project
- the Supplier can make any profit, taken the risk of not getting paid due to an unsuccessful project into account
- NB: depending on the parameter space, the exogenous q^* will be revealed to the DM

or when:

- given the parameter space, the Supplier faces an expected loss and therefore asks a to high price
- the DM rejects this offer

When this is compared to First Best it can be shown that although the supplier has all the market power and can claim all the possible consumer surplus, there is no dead weight loss and the Supplier will make the same choice as the social planner (Proposition 3).

Proposition 3: The social optimum is the same as the WPBE and therefore there is no dead weight loss

Proof:

Where the Supplier in equilibrium maximizes his own utility $U_S = \pi(q^*, q)p - c(q)$ and this results in the strategy defined in Proposition 2, the social planner maximizes the total welfare in society and therefore induces the strategy profile that incurs the maximum total expected utility.

This optimization is:

$$\max W(q, p, X) = U_S + U_{DM} = X[\pi(q^*, q) \cdot p - c(q) + \pi(q^*, q) \cdot (G - p)]$$

This can be rewritten as:

$$\max W(q, p, X) = X \cdot [\pi(q^*, q) \cdot G - c(q)]$$

This means that if $\pi(q^*, q) \cdot G \geq c(q)$, it is social optimal to set $X = 1$. This constraint is equal to the Participation Constraint for the Supplier as shown in Lemma 1.

For the optimal choice of q this depends on the tradeoff between production costs $c(q)$ and probability to collect the gain G, this is the same choice as the Supplier faces in his Incentive Compatibility Constraint (Lemma 2).

The price p has no effect on the social optimum. If $p > G$ the market would choose $X = 0$. When there would be a possible surplus for the Supplier he would be able to set p any lower until $p = p_{min}$ in order to provoke a $X = 1$. Therefore, the Participation Constraint of the DM ($p \geq G$) is only needed to show his best response, not to find the social optimum.

This taken into account means that the social planner has the same outcome as the market.

The results of this model show that:

- The Supplier has all market power;
- This results in the DM just having to accept the offer and giving all of his surplus to the Supplier;
- The Supplier can decide about the transaction;

- The only surplus is at the Supplier;
- The outcome will be equal to first best and there is no dead weight loss, however the surplus is for the Supplier (Proposition 3).

Remarks on relaxing assumptions:

The above described model consists of two states and two possible offers representing both of the states. I further use the Step Function as a probability function. These assumptions result in the equilibrium offers as stated in Proposition 2. However, I will show that the Participation and Incentive Compatibility Constraints of the players are still valid without these assumptions. The conclusions about the distribution of market power, surplus and dead weight loss are therefore valid as well.

The ability for a Supplier to offer any q does not affect the Participation Constraints of the players. The Participation Constraint for the Supplier is defined as the minimum price that he needs to receive for the offered q in order to be willing to have a transaction (PC-S). The Incentive Compatibility Constraint as proposed in Lemma 2 (ICC-S) is still valid, but it needs to be expanded in order to be useful for deciding about $q_{Supplier} \geq 0$. Lemma 2 defines the preference between two different offers. Since the DM's best response is to accept any offer with $p \leq G$, the Supplier can set the price for $q > 0$ equal to G. The optimal offer is therefore $(G, q_{Supplier})$. This $q_{Supplier}$ is the optimal q to offer based on the order as stated in Lemma 2. The Supplier cannot raise production to $q^+ > q_{Supplier}$ in order to achieve a higher probability of success: $U_S(G, q_{Supplier}) > U_S(G, q^+)$. He also cannot lower his production to $q^- \in (0, q_{Supplier})$ in order to effectively cut costs: $U_S(G, q_{Supplier}) > U_S(G, q^-)$. Whenever this $q_{Supplier}$ violates the PC-S and $p_{min}(q^*, q_{Supplier}) > G$, he will offer (0,0).

In the above model, this would result in the Supplier offering $q_{Supplier} \in \{0, q^0, q^*\}$. The Suppliers optimal offer starts with $q^0 \downarrow 0$. This is the lowest q that the Supplier can offer without preventing the transaction from happening and offering (0,0). If raising the production and offer $q = q^*$resulting in an expected payoff by the higher probability of gaining G, he will offer q^*. However, if both of these violate the Participation Constraint, the Supplier will offer $(0,0)$.

The second assumption to relax is the two possible states: $q^* \in \{q_L^*, q_H^*\}$. When the exogenous q^* is drawn from any random distribution $f(q^* = q)$ with the only restriction of $f(q^* < 0) = 0$, the above conclusions are still valid. The Participation Constraint of the DM is based upon his updated belief $\hat{h}$ of q^* and therefore his perceived probability of success. Since he will always accept the offer as long as $G \geq p$, the distribution of q^* does not change this. The Participation Constraint of the Supplier also stays untouched. As long as the offered $o = (p, q)$ satisfies $p \geq \frac{c(q)}{\pi(q^*, q)}$, the Supplier is willing to offer $o \succcurlyeq (0,0)$. Choosing the optimal q is done by having his $q_{Supplier}$ optimized: $U_S(G, q_{Supplier}) > U_S(G, q^-)$ and $U_S(G, q_{Supplier}) > U_S(G, q^+)$. The optimal $q_{Supplier}$ is therefore still within $\{0, q^0, q^*\}$.

The third change is that $\pi(q^*, q)$ is not required to be the Step Function, but is defined as any function $\pi(q^*, q) = Pr(Success|q^*, q)$ for which holds that:

$\pi(q_L^*, q) \geq \pi(q_H^*, q)$ and $\pi(q^*, q^H) \geq \pi(q^*, q^L)$ for any $q^*, q_H^*, q_L^* > 0$ and $q_H^* > q_L^*, q^H > q^L$

In this case the Participation Constraint for DM stays the same, his choice to accept the offer as long as the project has a net gain of $G - p \geq 0$ does not depend on the different probabilities of success. The Participation Constraint for the Supplier does not change either. Since the function for $\pi(q^*, q)$ is common knowledge, the different function does not change the role of π in the Incentive Compatibility Constraint. However, relaxing the probability function being Step Function means that the optimal $q_{Supplier}$ does not needs to be $\{0, q^0, q^*\}$, the Supplier must set $q_{Supplier}$ as such that raising or decreasing the production does not give a higher expected payoff.

5 Analysis of the Bertrand Duopoly

The previous section described the situation where there is only one Supplier, who can supply the solution to the DM. This single Supplier had all the market power he needed in order to extract all the surplus. The situation was first best, but there is no consumer surplus. The next sections use two suppliers and investigate whether the situation still reaches first best and whether there is any consumer surplus for the DM.

The model is slightly different then in Section 4. There are two Suppliers, S_1 and S_2. Both of them observe the state q^*. They do their offer, $o_i = (p_i, q_i),\ i \in \{1,2\}$, independent of each other to the DM. The implementation decision of the DM is whether he chooses to implement one of the proposed solutions or to decline both of the offers: $X = \{0,1,2\}$. If he accepts an offer ($X \neq 0$), he will pay the chosen supplier his offered amount: $p = p_i$ for $q = q_i$. The timing is as shown in Figure 1. The model can therefore be seen as an extension to the Bertrand framework. There is competition between the Suppliers on price (traditional Bertrand situation) and the probability that the transaction is successful.

For the sake of simplicity, I still use the Step Function in this section and there are two possible states:

$$q^* \in \{q_L^*, q_H^*\},\ \ 0 < q_L^* < q_H^*$$

As we have seen in the previous section, when the Suppliers may offer any $q_i > 0$, they could have an incentive under the Step Function to offer $q^0 \downarrow 0$. This may result in an equilibrium in which the suppliers will not offer any useful solutions. I therefore refine the model and set the minimum q_i to q_L^*. The Suppliers will start their offers with the smallest solution that Nature could render useful.

So the formal model:

- Players: DM, S_1 and S_2
- Respective actions: $X \in \{0,1,2\}$, $o_1 = (p_1, q_1)$, $o_2 = (p_2, q_2)$; $q_1, q_2 > q_L^*$
- Payoffs:
  - $U_{DM}(X, o_1, o_2 | X \neq 0) = \pi(q^*, q) \cdot (G - p)$
  - $U_{DM}(X, o_1, o_2 | X = 0) = 0$
  - $U_{S_i}(X, o_1, o_2 | X = i) = \pi(q^*, q_i) \cdot (p_i) - c(q_i)$

- $U_{S_i}(X, o_1, o_2 | X \neq i) = 0$

In the monopoly model, we saw that the price in the actual transaction must be $p \leq G$, otherwise the DM would choose not to implement (Proposition 1). Since the only change for the DM is that he now has two offers to choose from, he still can deny paying more than G in order to secure a minimum expected utility of 0. Therefore, the Participation Constraint of the DM that $p_i \leq G$ is still valid.

With respect to the minimum price in the actual transaction, the Suppliers Participation Constraint has not changed either (Lemma 1). The addition of a second Supplier, does not lower the minimum price a Supplier needs to have in order to have an expected positive payoff.

Hence, we can say that Corollary 1 is valid as well. The transaction price p must be in the range $[\frac{c(q)}{\pi(q^*,q)}, G]$. When p is outside this range, either the DM ($p > G$) or the chosen Supplier ($p < \frac{c(q_i)}{\pi(q^*,q_i)}$) has an incentive to deviate.

Since the probability of success differs for the two Supplier types for any $q_i \in [q_L^*, q_H^*)$, the Supplier Participation Constraints and the minimum prices for a transaction are different as well for these values of q. I will define the difference in minimum prices between the low and high type Supplier as $\Delta p(q) = p_{min}(q_H^*, q) - p_{min}(q_L^*, q)$ (Lemma 3).

Lemma 3: Minimum price surplus when $q \in [q_L^*, q_H^*)$

The different supplier types have different minimum prices. The difference between the minimum price of the high type and low type supplier is $\Delta p(q) = \frac{c(q)}{\pi(q_H^*,q)} - \frac{c(q)}{\pi(q_L^*,q)}$. From this definition it follows that $\Delta p(q) = 0 \; \forall \; q \notin [q_L^*, q_H^*)$.[4]

Explanation:

The minimum price for which a Supplier is prepared to make an offer is (Lemma 1):

$$p_{min}(q^*, q) = \frac{c(q)}{\pi(q^*, q)}$$

When the offered amount is $q \in [q_L^*, q_H^*)$, there is a different value between the low type and high type Supplier with respect to their minimum prices. For this q, the low type Supplier faces a higher probability of the project being successful and thus receiving the price p than the high type Supplier, therefore he can afford to ask a lower price. When he would mimic a high type Supplier, he would be able to obtain this minimum price surplus of Δp.

When $q \notin [q_L^*, q_H^*)$, the values of $\pi(q^*, q)$ are equal for both the types.

The DM's optimal strategy consists of his best responses for the following situations on the different offers o_1 and o_2.

[4] Note that the statement of $\Delta p(q) = 0 \; \forall \; q \notin [\mathrm{q_L^*}, \mathrm{q_H^*})$ only holds when the Step Function is used. There may be an $\Delta p(q) \neq 0$ for other project success functions $\pi(q^*, q)$.

- $q_1 = q_2 \cup p_1 = p_2$
- $q_1 \neq q_2 \cup p_1 = p_2$
- $q_1 = q_2 \cup p_1 \neq p_2$
- $q_1 \neq q_2 \cup p_1 \neq p_2$

When $q_1 = q_2 \cup p_1 = p_2$, then $o_1 = o_2$. Therefore, the DM is indifferent between the two offered solutions, simply because they are the same. If his Participation Constraint is met and $p_1 \leq G$ then he would choose to implement one of the two solutions (so $X \neq 0$). His best response is therefore any mix with:

$$(0: X = 0, \alpha: X = 1, 1 - \alpha: X = 2), \qquad \alpha \in [0,1]$$

When both the prices are equal and the quality differs ($q_1 \neq q_2 \cup p_1 = p_2$), the DM will choose the Supplier with the best product as long as this price does not exceed the maximum price and violates his Participation Constraint (Lemma 4). By choosing the highest quality, he increases the project success as much as possible. Since he may collect any gain $G - p$ on project success, this is his best response. In the situation that $q_1 = q_2$ and the price differs, the DM will choose the cheapest solution as long as $\min[p_1, p_2] \leq G$ (Lemma 4).

Lemma 4: Equal prices or Equal q

If $p_1 = p_2$ and $q_i > q_j$, $i \in \{1,2\}$, then it is optimal to choose: $X = i$ if $p_i \leq G$ and $X = 0$ otherwise.[5]

Proof:

If $G - p \geq 0$ then the DM (weakly) prefers $X = i$ over $X = j$ if:

$$\pi(q^*, q_i)(G - p_i) \geq \pi(q^*, q_j)(G - p_j)$$

Since $p_i = p_j$, $G - p_i \geq 0$:

$$\pi(q^*, q_i) \geq \pi(q^*, q_j)$$

The same holds for equal $q_i = q_i$ and $p_i < p_j$, $i \in \{1,2\}$. It is optimal to choose: $X = i$ if $p_i \leq G$ and $X = 0$ otherwise.

Proof:

The DM prefers $X = i$ over $X = j$ if:

$$\pi(q^*, q_i)(G - p_i) > \pi(q^*, q_j)(G - p_j)$$

Since $q_i = q_j$ and $\pi(q^*, q) \geq 0$, he prefers $X = i$ over $X = j$ if $p_i < p_j$.

When the Suppliers make two total different offers, $q_1 \neq q_2 \cup p_1 \neq p_2$, the DM will evaluate his belief h about q^*. This is similar to the updating process in Section 4. The DM's prior belief about the required quality q^* is that the Supplier is high type ($q^* = q_H^*$) with probability $h =$

[5] When $\pi(q^*, q)$ is strictly increasing in q, choosing $X = i$ in this situation will be strictly dominant over $X = j$. However, when π is reflected by the Step Function and it does not hold that $q_i > q^* > q_j$, choosing $X = j$ is only weakly dominated by $X = i$ therefore it could also optimal to apply the following mix: $(0: X = 0, \alpha: X = 1, 1 - \alpha: X = 2)$, $\alpha \in [0,1]$.

$Prob(q^* = q_H^*) \in [0,1]$ and that the Supplier is low type ($q^* = q_L^*$) with probability $1 - h$. After receiving the offers o_1 and o_2 he will update his belief to $\hat{h}(o_1, o_2) = Prob(q^* = q_H^* | o_1, o_2)$ and $1 - \hat{h}(o_1, o_2)$.

The DM will choose o_i and set $X = i$ when the condition of Lemma 5 is met and p_i does not violate his Participation Constraint.

Lemma 5: DM's optimal action for different offers

All offers with $p > G$ will be rejected (Proposition 1), so if $p_1, p_2 > G$, choose $X = 0$.

If $p_i > G$ and $p_j \leq G$, choose $X = i$.

If $p_i, p_j \leq G$ choose $X = i$ if:

$$\left[\hat{h}(o_1, o_2) \cdot \pi(q_H^*, q_i) + \left(1 - \hat{h}\right) \cdot \pi(q_L^*, q_i)\right](G - p_i) \\ \geq [\hat{h}(o_1, o_2) \cdot \pi\left(q_H^*, q_j\right) + \left(1 - \hat{h}(o_1, o_2)\right) \cdot \pi\left(q_L^*, q_j\right)]\left(G - p_j\right)$$

Since I use the Step Function as project success probability function in this model, this Lemma can be simplified by identifying situations similar to Lemma 4 (equal q). See therefore Table 1.

Situation	Results in	Conclusion
$\boldsymbol{q_i, q_j > q_H^*}$ $\boldsymbol{p_i > p_j}$	$\pi(q_H^*, q_i) = \pi(q_L^*, q_i) = \pi\left(q_H^*, q_j\right) = \pi\left(q_L^*, q_j\right)] = \pi^H$	DM chooses S_j
$\boldsymbol{q_i, q_j \in (q_L^*, q_H^*)}$ $\boldsymbol{p_i > p_j}$	$\pi(q_H^*, q_i) = \pi\left(q_H^*, q_j\right) = \pi^L$ $\pi(q_L^*, q_i) = \pi\left(q_L^*, q_j\right)] = \pi^H$	DM chooses S_j
$\boldsymbol{q_i, q_j < q_L^*}$[6] $\boldsymbol{p_i > p_j}$	$\pi(q_H^*, q_i) = \pi(q_L^*, q_i) = \pi\left(q_H^*, q_j\right) = \pi\left(q_L^*, q_j\right)] = \pi^L$	DM chooses S_j

Table 1: Similarities in Lemma 4 and Lemma 5, when using the Step Function

Therefore Lemma 5 shows that there is only one situation that will not always roll back to a simple price competition: the choice between $q_i = q_H^*$ and $q_j = q_L^*$ (Corollary 2). As long as the DM has the belief that a high quality may be required by Nature ($\hat{h} \neq 0$), he will be prepared to pay a difference in price.

[6] This combination of offers is not reachable with $\min q_i = q_L^*$.

Corollary 2: Application of Lemma 5 for $q_i = q_H^*$ and $q_j = q_L^*$.

Since the DM will choose S_i over S_j when (given that $p_i, p_j \leq G$; Lemma 5):

$$\left[\hat{h}(o_1, o_2) \cdot \pi(q_H^*, q_H^*) + \left(1 - \hat{h}(o_1, o_2)\right) \cdot \pi(q_L^*, q_H^*)\right] \cdot (G - p_i) \geq \left[\hat{h}(o_1, o_2) \cdot \pi(q_H^*, q_L^*) + \left(1 - \hat{h}(o_1, o_2)\right) \cdot \pi(q_L^*, q_L^*)\right] \cdot \left(G - p_j\right)$$

This can be rewritten to:[7]

$$p_i - p_j \leq \hat{h}(o_1, o_2)\left(\frac{\pi^H - \pi^L}{\pi^H}\right)\left(G - p_j\right)$$

For $\hat{h}(o_1, o_2) = 1$ (belief that Suppliers are high type) this means optimizing payoff and the DM is prepared to pay for the relative growth of probability of gaining the project profit:

$$\pi^H \cdot (G - p_i) \geq \pi^L \cdot \left(G - p_j\right)$$

$$p_i - p_j \leq \left(\frac{\pi^H - \pi^L}{\pi^H}\right)\left(G - p_j\right)$$

For $\hat{h}(o_1, o_2) = 0$ (belief that Suppliers are low type) this means that the DM is not prepared to pay extra for $q_i = q_H^*$ and thus choosing the lowest p:

$$p_i - p_j \leq 0$$

$$p_j \geq p_i$$

For $0 < \hat{h}(o_1, o_2) < 1$, this effect is scaled to the updated belief $\hat{h}$ about q^*:

$$\pi^H \cdot (G - p_i) \geq \left[\hat{h}(o_1, o_2) \cdot \pi^L + \left(1 - \hat{h}(o_1, o_2)\right) \cdot \pi^H\right] \cdot \left(G - p_j\right)$$

$$p_i - p_j \leq \hat{h}(o_1, o_2)\left(\frac{\pi^H - \pi^L}{\pi^H}\right)\left(G - p_j\right)$$

So choose S_i when the increase in price does not exceed the relative growth of expected project profit.

The above Lemmas jointly determine into the optimal strategy of the DM (Proposition 4):

- Choosing not to implement whenever both of the solutions are too expensive;
- Choosing the highest q when the prices are equal;
- Choosing the lowest p when:
  - The quality is equal;
  - The quality does not result in a change of probability for project success;
  - The increase of project success does not justice the price difference;
- Mixing both Suppliers whenever indifferent.

[7] See Appendix 2.

Proposition 4: Optimal DM strategy

Condition on p	Condition on q	Action	Based on
$\min[p_1, p_2] > G$	For all $q_1, q_2 \geq 0$	No transaction $X = 0$	Proposition 1
$p_i \leq G$ $p_j > G$	For all $q_1, q_2 \geq 0$	Satisfy PC-DM $X = i$	Proposition 1
$p_i = p_j$ $\max[p_1, p_2] \leq G$	For all $q_1, q_2 \geq 0$	Highest quality q	Lemma 4
$p_1 \neq p_2$ $\max[p_1, p_2] \leq G$	$q_i = q_j$	Lowest price p	Lemma 4
	$q_i \neq q_j$ $q_i, q_j \geq q_H^*$	Lowest price p	Table 1
	$q_i \neq q_j$ $q_i, q_j \in [q_L^*, q_H^*)$	Lowest price p	Table 1
	$q_i \in [q_L^*, q_H^*)$ and $q_j \geq q_H^*$ - and - The increase of believed project success does not justice the price difference	Lowest price p	Corollary 2
	$q_i \in [q_L^*, q_H^*)$ and $q_j \geq q_H^*$ - and - The increase of believed project success does justice the price difference	Highest quality q	Corollary 2
None of the above conditions are met		Mixing both Suppliers	

The rational Suppliers will take the DM's optimal strategy into account when they do their offer and make their offer o_i. The optimal strategy for the Supplier must maximize his expected payoff by offering the optimal combination of q_i and p_i. This shows that making any offer with $q_L^* < q_i < q_H^*$ or $q_i > q_H^*$ involves does not optimize the Supplier's payoff. (Lemma 6). So the optimal offer o_i consists of $q_i = \{q_L^*, q_H^*\}$.

Lemma 6: The optimal q_i to offer is either q_L^* or q_H^*.

Proof:

The optimal payoff for the Supplier S_i is achieved by the offer that is best response to the strategy of Supplier S_j and the DM's implementation strategy.

For the situation that $X = i$, the it must hold that (p, q_i) maximizes the Supplier's payoff. This optimisation is constraint by the implementation decision X. For any offers with (p, q_i) that give the DM an incentive to set $X \neq 0$, the payoff is not maximised, but reduced to 0.

For $X = i$ the Supplier optimizes his payoff by:

$$\max_{q_i, p_i} \pi(q^*, q_i) \cdot (p_i) - c(q_i)$$

Since it is given that $c(q)$ is increasing in q, and the Step Function defines that $\pi'(q^*, q) = 0$ for all $q \neq \{q_L^*, q_H^*\}$, it is optimal for the Supplier to keep the production costs as low as possible without touching the probability of success. Supplier S_i has therefore an incentive to lower his production until $q_i = \{q_L^*, q_H^*\}$, for the implementation decision is $X = i$.

This lowering production does not affect the DM's implementation decision and does not give him an incentive to choose $X \neq i$ (Proposition 4).

For $X \neq i$, the offer of the Supplier must allure the DM to choose $X = i$. The DM either chooses the lowest priced offer or the offer that assures him a higher expected payoff, given the believed probability of project success and the price difference.

The offered price can be lowered until $p = p_{min}(q^*, q) = \frac{c(q)}{\pi(q^*,q)}$ (Lemma 1). Therefore, lowering the offered q_i allows the Supplier to offer at a lower price which could cause the DM to set $X = i$.

The other possibility is that the Supplier changes from offering $q_i \in [q_L^*, q_H^*)$ and offers $q_i \geq q_H^*$ or vice versa. This changes the believed probability of project success for the DM. In the Step Function, the only change in probability can occur at $q_i = \{q_L^*, q_H^*\}$. Therefore, offering $q_i > q_H^*$ or $q_i \in (q_L^*, q_H^*)$ does not affect X any more, it only raises production costs. So the Supplier best offers $q_i \in (q_L^*, q_H^*)$.

The optimal Supplier strategy is composed of by the optimal actions of both the low type and the high type Supplier.

For the low type Supplier there are three possible offers while keeping $q_i = \{q_L^*, q_H^*\}$:

1. Credibly reveal that he is a low type Supplier by offering $q_i = q_L^*$ for a price below the high type minimum price for that amount. He therefore takes only a part of the price difference:
$$o_i = o^{LL} = (q_L^*, p), \qquad p_{min}(q_L^*) \leq p < p_{min}(q_L^*, q_L^*) + \Delta p(q_L^*)$$
2. Mimic a high type Supplier by offering $q_i = q_L^*$:
$$o_i = o^H = (q_H^*, p), \qquad p \geq p_{min}(q_L^*, q_H^*)$$
3. Mimic a high type Supplier who sends q_L^* and thus taking the full price difference $\Delta p(q_L^*)$ as extra profit:
$$o_i = o^L = (q_L^*, p), \qquad p \geq p_{min}(q_L^*, q_L^*) + \Delta p(q_L^*)$$

The first offer is a credible state revealing action since this offer consists of a price that is lower than the high type Supplier's minimum price and it therefore would violate his Participation Constraint (Corollary 3). This offer o^{LL} also is the only offer that leads to an equilibrium for the low type Suppliers (Lemma 7).

Corollary 3: Any offer $o^{LL} = (p_i, q_L^*)$ with $p_i < p_{min}(q_L^*, q_i) + \Delta p(q_i) \leq G$ reveals that Supplier S_i is a low type Supplier and therefore that the state is $q^* = q_L^*$.

Proof:

Given Lemma 1, a high type Supplier S_i will never make an offer with a q and $p < p_{min}(q_H^*, q)$ with the possibility of the DM deciding $X = i$, since he has an expected negative payoff. However, for a low type Supplier there are offers with $p_{min}(q_L^*, q) < p < p_{min}(q_H^*, q)$ that do comply to his Participation Constraint and results in a positive expected payoff for the Supplier.

The optimal strategy of the DM is to choose the lowest price if this $p \leq G$ unless the more expensive offer gives a higher expected payoff (Corollary 2). When the DM would choose $o^H = \left(q_H^*, p_{min}(q_H^*, q_H^*)\right)$ over o^L, it is optimal for the high type Supplier to offer o^H instead of o^L. However, when the DM would choose o^L over o^H or o^{LL} over o^H, he will choose the Supplier that offers the cheapest offer. Therefore, the high type Supplier violates his Participation Constraint (Lemma 1) by offering o^{LL}. So a high type Supplier cannot offer o^{LL} as part of his optimal strategy.

Therefore, offering o^{LL} causes the DM to update his belief to $h = 0$.

Lemma 7: A low type Supplier will always make an offer that reveals his type

$$o_i = o^{LL} = (p, q_L^*), \qquad p_{min}(q_L^*) \leq p < p_{min}(q_L^*, q_L^*) + \Delta p(q_L^*)$$

Proof:

Suppose Supplier S_i does not reveal his type and sends $o^H = (q_H^*, p_i),\ p_i \geq p_{min}(q_L^*, q_H^*)$.

In that case S_j could respond with a $o^H = \left(p_j, q_H^*\right),\ p_{min}(q_L^*, q_H^*) \leq p_j < p_i$ in order to ensure $X = j$. However, this gives S_i an incentive to lower his price as well until both their prices reach the minimum price for $q = q_H^*$: $p_i = p_j = p_{min}(q_L^*, q_H^*)$. This is a Bertrand duopoly situation. Both of the Suppliers have an expected utility of 0, and both will be included in the mixed strategy of the DM.

This situation can be no equilibrium because Supplier S_i can reveal his type and offer o^{LL}. This causes the DM to set $X = i$ and S_i will have an expected positive utility. As soon as o^{LL} is offered, the DM updates his belief to $\hat{h} = 0$ and his strategy is to choose the lowest price, so S_j will join in offering o^{LL} and the Bertrand duopoly will drive the price back to $p_i = p_j = p_{min}(q_L^*, q_L^*)$, which results in an equilibrium situation where both of the Suppliers have an expected utility of 0.

For the high type Supplier S_i offering o^{LL} is not possible, so there are only two possible offers while keeping $q_i = \{q_L^*, q_H^*\}$:

1. Offer the high quality solution and assure the DM of π^H by sending:
$$o_i = o^H = (q_H^*, p), \qquad p \geq p_{min}(q_H^*, q_H^*)$$
2. Offer the cheaper solution by offering q_L^*:
$$o_i = o^L = (q_L^*, p), \qquad p \geq p_{min}(q_H^*, q_L^*)$$

Whether the Supplier prefers o^H over o^L or o^L over o^H depends on the parameter space and the choice of the DM according to Corollary 2. A Bertrand competition will drive the price down to $p \geq p_{min}(q_H^*, q_i)$ and the optimal action for the high type Supplier is therefore to high type offer $o^H = (p_{min}(q_H^*, q_H^*), q_H^*)$ or $o_i = o^L = (p_{min}(q_H^*, q_L^*), q_L^*)$ based on the parameter space (Lemma 8). Since the low type Supplier will always offer o^{LL} and the high type Supplier will never do the same, the equilibrium is separating. Therefore, the DM will update his belief to $\hat{h} = 1$ after receiving any of the high type Suppliers offers.

Lemma 8: The high type Supplier will always offer that q_i that is optimal for the DM and sets his price to minimum.

In optimum the high type Suppliers will offer $o^H = (p_{min}(q_H^*, q_H^*), q_H^*)$ or $o^L = (p_{min}(q_H^*, q_L^*), q_L^*)$ depending on the parameter space as given in Corollary 2.

Proof:

The DM will choose $X = i$ when:

$$p_i - p_j \leq \left(\frac{\pi^H - \pi^L}{\pi^H}\right)(G - p_j)$$

Suppose S_i will offer $o^H = (p_i, q_H^*),\ p_i > p_{min}(q_H^*, q_H^*)$. The Bertrand duopoly predicts that S_j has an incentive to offer $o^H = \left(p_j, q_H^*\right),\ p_{min}(q_H^*, q_H^*) \leq p_j < p_i$. This is no best response until $p_i = p_j = p_{min}(q_H^*, q_H^*)$.

When the DM is prepared to pay the higher price for $q = q_H^*$, this is an equilibrium. When the DM would not be prepared to do so, both of the Suppliers have an incentive to deviate and offer $o^L = (p, q_L^*),\ p \geq p_{min}(q_H^*, q_L^*)$, which leads to another Bertrand situation, causing the equilibrium offers to be $o^L = (p_{min}(q_H^*, q_L^*), q_L^*)$.

The same result would be reached when this analyses is started with o^L. This o^L would unravel to $o^L = (p_{min}(q_H^*, q_L^*), q_L^*)$, which does or does not give an incentive to the Suppliers to upgrade their offer to o^H.

When the optimal offers for the low type and high type Suppliers are combined, we see that the Suppliers follow the preferences of the DM and are always in a situation of Bertrand competition (Proposition 5).

Proposition 5: The optimal Supplier strategy

When Lemma 7 and Lemma 8 are combined, the optimal Supplier strategy is that the low type Supplier will reveal his type and offer $q_i = q_L^*$ and that the high type Supplier will follow the DM's preferences regarding quality and price.

The price p_i for the offered amount of q_i is equal to the minimum price $p_{min}(q^*, q_i)$.

When equilibrium is compared to First Best it can be shown that the there is no dead weight loss and the Suppliers will produce the same q as the social planner and the transaction will take place in first best if and only if the Participation Constraints are satisfied (Proposition 6).

Proposition 6: The social optimum is the same as the WPBE and therefore there is no dead weight loss

Proof:

Where the players in equilibrium maximizes their own utility and this results in the strategy profile defined in Proposition 4 and Proposition 5, the social planner maximizes the total welfare in society and therefore induces the strategy profile that incurs the maximum total utility.

Since the two Suppliers have a symmetric utility function, the social optimum is indifferent between $X = 1$ and $X = 2$.

This optimization is therefore given by total payoff for $X \neq 0$ and a price p and quality q:

$$\max W(q, p | X \neq 0) = U_{S_1} + U_{S_2} + U_{DM} = \pi(q^*, q) \cdot p - c(q) + \pi(q^*, q) \cdot (G - p)$$

When this maximum results in a negative total payoff $W < 0$, it is social optimal to set $X = 0$.

This can be rewritten as:

$$\max W(q, p | X \neq 0) = \pi(q^*, q) \cdot G - c(q)$$

Note that this condition is equal to the social optimum in Proposition 3.

This means that if $\pi(q^*, q) \cdot G \geq c(q)$, it is social optimal to set $X = 1$ or $X = 2$, which is the case if and only if the Suppliers' Participation Constraints are met (Lemma 1).

The optimal q depends on the trade-off between production costs $c(q)$ and probability to have a successful project and achieve the gain G, this is the same choice as the Supplier faces when deciding on any offer with price G that meets the p_{min}. The Suppliers offer the optimal q for the DM's in order to be involved in the transaction (Lemma 7 and Lemma 8)

The price p has no effect on the social optimum. If $p > G$ the market would choose $X = 0$. When there is be a possible surplus for the DM, the Suppliers will be able to set p any lower until $p = p_{min}$ in order to have $X \neq 0$. Therefore, Proposition 4 and the participation constraint of the DM ($p \geq G$) is only needed to show the strategy of the DM and the choice between the Suppliers, not to find the social optimum.

This taken into account means that the social planner has the same outcome as the market.

The results of this model show that:

- The DM has all market power;
- This results in the Suppliers just having to offer the optimal for the DM and thus giving all of their surplus to the DM;
- Any deviation of this will result in losing the transaction
- The DM can decide about the transaction;
- The outcome will be equal to first best and there is no dead weight loss, however the surplus is for the DM (Proposition 6).

Remarks on relaxing assumptions:

The above described model consists of two states and offers limited to $\min q_i = \min q^*$. I further use the Step Function as a probability function. These assumptions result in the equilibrium offers as stated in Proposition 4 and Proposition 5. However, these assumptions are not necessary to describe the Strategy Profile of the players and therefore the distribution of market power, surplus and dead weight loss.

The minimum value was a mere equilibrium refinement in order to prevent players from offering $q \downarrow 0$. This could lead to non-realistic equilibrium situations. When the project success probability function is no Step Function, but is more realistic, the relation between the actually produced q, q^* and project success is also more realistic.

The outcome of the model does not change either when I further relax the Step Function and the two states, so that the exogenous q^* is drawn from any random distribution $f(q^* = q)$ with the only restriction of $f(q^* < 0) = 0$. As stated in the end of Section 4, the distribution has no impact on the Participation Constraints of the Suppliers and the DM. The constraints on whether $X = 0$ and $X \neq 0$ are therefore the same.

The optimal strategy of the DM regarding $X = \{1,2\}$ as stated in Proposition 4 does not change, but is defined differently. The choice between two different offers (Lemma 5) is still based on the expected probability of project success and the net surplus $G - p$ and therefore a trade-off between price and expected probability and thus quality. To be able to describe all possible values of q^*, the expected project success is rewritten as:

$$\hat{\pi}(q_i|o_1, o_2) = \sum_{q=0}^{\infty} \pi(q, q_i) \cdot \hat{h}(q^* = q)$$

In this the updated belief is: $\hat{h} = Prob(q^* = q|o_1, o_2)$.

The rewriting of the preference of the DM in Lemma 5 is therefore $X = i$ instead of $X = j$ if:

$$\hat{\pi}(q_i) \cdot (G - p_i) \geq \hat{\pi}(q_j) \cdot (G - p_j)$$

The application for $q_i > q_j$ in Corollary 2 changes therefore to choose $X = i$ if the price difference is smaller than the relative change in net probability gain:[8]

$$p_i - p_j \leq \frac{\hat{\pi}(q_i) - \hat{\pi}(q_j)}{\hat{\pi}(q_i)} \cdot (G - p_j)$$

[8] For a proof see: Appendix 3.

So the new DM strategy is given in Table 2:

Condition	Action	Based on
$\min[p_1, p_2] > G$	No transaction, $X = 0$	Proposition 1
$p_i = p_j$	Choose highest quality q	Lemma 4
$q_i = q_j$	Choose lowest price p	Lemma 4
$p_i - p_j > \frac{\hat{\pi}(q_i) - \hat{\pi}(q_j)}{\hat{\pi}(q_i)} \cdot (G - p_j)$ The increase of believed project success does not justice the price difference	Choose lowest price p	Rewritten Corollary 2
$p_i - p_j < \frac{\hat{\pi}(q_i) - \hat{\pi}(q_j)}{\hat{\pi}(q_i)} \cdot (G - p_j)$ The increase of believed project success does justice the price difference	Choose highest quality q	Rewritten Corollary 2
None of the above conditions are met	Mixing both Suppliers	

Table 2: Rewriting of Proposition 4

The optimal strategy for the Supplier S_i does not change either. This strategy consists of two parts and they do not change:

- Competition drives the Supplier S_i to reveal his type and not to mimic another type in order have the DM set $X = i$ (Lemma 7). There is no benefit in offering a solution for a price that is higher than $p_{min}(q^*, q)$. After receiving an offer of (p, q_i) the DM updates his belief and he can be sure about the true value of q^*.
- Competition drives the Supplier S_i to offer the amount of q_i, that is optimal given the DM's trade-off between a higher probability of success and a lower price (Lemma 8). Since the DM is sure about q^*, the Suppliers have to follow the DM's preferences in the trade-off between q and p.

6 Analysis of the Cournot Duopoly

This section will analyse the impact of uncertainty in the project's success on the traditional Cournot analysis. The model that is used here is slightly different than the model that is used the in the Bertrand situation (and the simplified, monopoly model). In the Bertrand analysis the Suppliers make their offers and the Decision Maker has to make a choice for one of the two offers (Bertrand) and whether the offer is accepted or not (Monopoly as well as Bertrand). The Suppliers compete on quantity in this section. The total available quantity is supplied by the Suppliers and the equilibrium price is defined by the preferences of the demand side.

In the traditional Cournot model there are two suppliers who produce homogenous and perfectly substitutable goods. They decide about the produced amount individually. In equilibrium all the goods are traded and market price is based on the available products and the preferences of the demand side. The optimal strategy of the suppliers is therefore based upon their own production costs, the demand function and the anticipated production of the other supplier. In equilibrium the optimal production of Supplier S_i equals what Supplier S_j anticipated that S_i would produce.

In the context of our model, the demand side is formed by the DM. For the implementation of the project he needs to buy goods from the Suppliers. In the broken car example, the Suppliers need to spend time on repairing the car, with hours of work reflected by q. These goods, or the time that the Suppliers spend, are substitutable. The DM will buy all of the offered q and the market price for the good (worked hour) is based upon his preferences and the amount of q that is traded. So there is no bargaining process or deciding about only buying part of the offered amount.

This would be the traditional Cournot model. However, similar to the previous sections, the price will only be paid to the Suppliers on project success. This success depends on the exogenous q^* and the total q that is used. So in the broken car example the Suppliers offer to work some hours on the car, which they both do. When the project is successful, the Suppliers will be paid the value of work for the DM and this will be divided based on the proportion of the work or the two suppliers.

The timing of this model is shown in Figure 3:

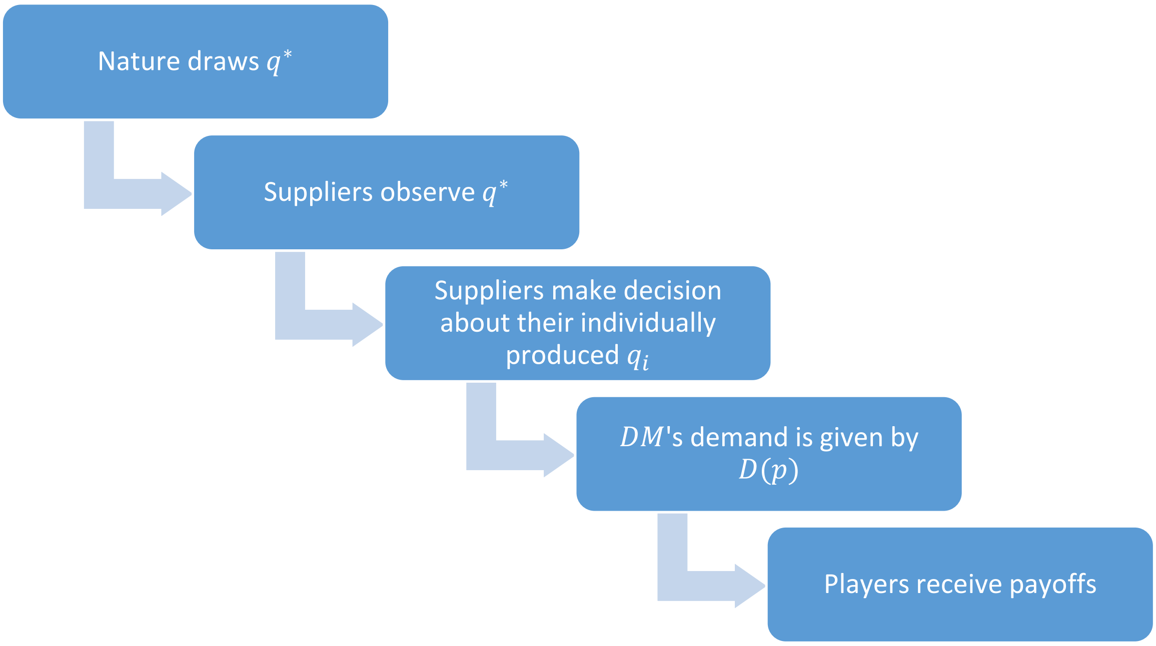

Figure 3 Timing of the Cournot Model

Before any actions are taken Nature draws $q^* \geq 0$ from any distribution $f(q^* = q)$. This q^* is observed by both Suppliers, S_1 and S_2. This is the same q^* as in the above models. However, in this section I will address it as quantity instead of quality in order to stay close to the original Cournot model. The project success function π represents the relationship between the total quantity that is used in the project and the exogenous q^*, $\pi(q^*, q_T)$. In the analysis I will first show the optimal production for any $\pi(q^*, q_T)$ that is increasing in q^* and q (as given in Section 3), and then apply the Step Function.

After observing q^* the supply side of the market, a Supplier, S_i, decides about the amount of q_i that he produces. He incurs a cost of $c(q_i)$ for producing q_i. The cost $c(q)$ is differentiable for all $q \geq 0$ and increasing in q, $c'(q) > 0$. The total produced amount is given by $q_T = \sum_{i=1}^{n} q_i$ where n denotes the number of Suppliers. So in the duopoly case this is: $q_T = q_1 + q_2$.

The demand side of the market, DM, observes the total produced amount q_T and purchases this amount at the market price of $p = D(q)$. On project success the DM collects the utility gain G and has to pay the Suppliers the market price p for supplying q_i. So q_T will be traded at $p = D(q_T)$. The only possibility that there is no transaction and therefore no project success is when $q_T = q_1 = q_2 = 0$.

So the payoffs are:

$$U_{S_i} = \pi(q^*, q_T) \cdot D(q_T) \cdot q_i - c(q_i)$$
$$U_{DM} = \pi(q^*, q_T) \cdot (G - D(q_T) \cdot q_T)$$

In order to demonstrate the analogy with the above models I will prove that when there is only one Supplier ($n = 1; q_T = q_1$), the addition of the Demand function will not change the results. Both the Supplier and DM will have the same optimal strategies and the First Best as the equilibrium outcomes are the same.

The model with one Supplier is a normal monopoly situation. The Supplier knows his profit and costs (π and q^* is known for the Supplier) and the Decision Maker has to buy from the Supplier or not at all. The Supplier can anticipate the demand for a given price. He therefore sets his price and produced quantity in such a way that he can extract the total consumer surplus and optimizes his profit. The price at which the goods are traded is therefore given by: $p = D(q_T) = \frac{G}{q_T}$ (Proposition 7).

The Supplier S_1 sets the quantity q_1 so that the marginal production costs are equal to the marginal gains of a higher probability. In case of the Step Function, he either invokes a high probability of project success at the production costs of reaching the q^* threshold or produces as little as possible (Proposition 8).

Proposition 7: Demand function and Market price is given by $p = D(q_T) = \frac{G}{q_T}$

Proof:

Demand is based on the optimal strategy of the DM. His budget constraint is given by:

$U_{DM} = \pi(q^*, q_T) \cdot (G - pq_T) \geq 0$. Since $\pi(q^*, q_T) > 0 \; \forall \; q_T > 0$ we can state $p \leq \frac{G}{q_T}$.

Any price lower than $\frac{G}{q_T}$ cannot be an equilibrium price since the consumer surplus is not fully extracted by the Supplier. That would be a reason for the supplier to raise the price and make more profit. Any price higher than $\frac{G}{q_T}$ would result in a negative expected payoff for the DM, causing him to lower the demand for q. So $p = \frac{G}{q_T}$.

Note that the total price paid $\frac{G}{q_T} \cdot q_T$ is equal to the equilibrium price of G in the monopoly model of section 4 for all the transactions.[9]

Proposition 8: Supply by the monopolistic Supplier[10]

Proposition: The optimal q_1 is reached when the marginal costs are equal to the marginal gains due to the increased probability of project success. For the Step Function, this means that he will produce q^* if his expected profit is at least as good as producing the lowest possible amount of q ($q_i \downarrow 0$).

Since the Supplier has complete market power and can extract the total consumer surplus ($G - \frac{p}{q_T} \cdot q_T$) for himself, the equilibrium outcome as stated in Proposition 8 is equal to the First Best Situation (Proposition 9).

Proposition 9: Since the monopolistic Supplier can extract the total consumer surplus, he has a social optimal strategy.

Proof:

First Best reflects the maximal total welfare:

$$\max_{q_1} W = U_{S_1} + U_{DM} = \pi(q^*, q_1)G - c(q_1)$$

The Supplier maximises:

$$\max_{q_1} U_{S_1} = \pi(q^*, q_1)\frac{G}{q_1}q_1 - c(q_1) = \pi(q^*, q_1)G - c(q_1)$$

Since the optimisation is the same ($\max_{q_1} \pi(q^*, q_1)G - c(q_1)$), the optimal q_1 is the same.

However, the traditional Cournot equilibrium dictates a different outcome when there are two Suppliers. I will show that the project uncertainty will not change this outcome and that the equilibrium situation is not equal to First Best.

The demand side does not change when there are two Suppliers, therefore Proposition 7 still holds and the equilibrium price must be $p = D(q_T = q_1 + q_2) = \frac{G}{q_T} = \frac{G}{q_1+q_2}$. However, the addition of a Supplier S_j changes the optimal supplied amount for a Supplier S_i ($i, j \in \{1,2\}$). The price and expected project success does not only depend on his produced quantity q_i, but also on q_j. The optimal production is given by the marginal benefits of increasing production and thus increasing the probability of success as well as market share being equal to the marginal production costs (Proposition 10).

[9] When $q_T = 0$, there is no production and therefore no transaction. In the monopoly section, this was reflected by a Supplier offering $q = 0$ or the DM deciding not to implement ($X = 0$).

[10] Proof and details in Appendix 4.

The First Best optimal production is only defined by the marginal benefits of increasing project success, though (Proposition 11). The diversion of production does only depend on the cost structure (Corollary 4).

When the First Order Conditions of Proposition 10 and Proposition 11 are compared, it is clear that the Suppliers have an incentive to raise production above First Best until they cannot increase profit by increasing market share.

Proposition 10: Supply by the Cournot Supplier[11]

Proposition: In equilibrium, the marginal costs are equal to the marginal benefits of sum of the increased Gain due to the increase in probability π (given the market share in the optimum) and of the increased market share (given the probability in the optimum).

Proposition 11: First Best production in the Cournot Duopoly[12]

Proposition: For First Best, the marginal costs equal to the marginal benefits of sum of the increased Gain due to the increase in probability π (given the market share in the optimum).

Corollary 4: Optimal Distribution of Production in First Best of the Cournot Duopoly with a Step Function

The production of producing q_T in First Best depends on the production cost function.

When the cost function is convex (the marginal costs increase), $c''(q) > 0$ the production in optimum must be equally spread among the Suppliers to keep them as low as possible:

$$q_1 = q_2 = \frac{q_T}{2}$$

However, if marginal costs decrease and the cost function is concave, $c''(q) < 0$, the production must be concentrated with one Supplier:

$$q_i > q_j, \qquad i \neq j; i, j \in \{1,2\}$$

If the production cost is linear and the marginal production costs are therefore constant, $c''(q) = 0$, then there is an optimal q_T and any combination is correct.

$$q_1 = q_T - q_2, \;\; q_1, q_2 \geq 0$$

The results of this model show that:

- The Suppliers have all market power;
- This results in the Suppliers having an incentive to produce more than is the optimal for the total welfare and a dead weight loss arises;
- The Suppliers have an expected surplus.
- The equilibrium is separating, since the optimal production depends on $\pi(q^*, q)$. A low type Supplier cannot mimic a high type Supplier by producing a different amount.

[11] Proof and details in Appendix 5.
[12] Proof and details in Appendix 6.

7 Conclusion

In this thesis I developed a model that provides a solution for the lack of incentives to offer a quality product. When there are no reputation concerns and quality is not verifiable upfront, the market will not always supply quality. I showed that the right to compensation on failure can change this. In my framework the object of the transaction is a project with a fixed value on project success. The quality or quantity of produced goods does not matter for the value of project success. It only influences the probability of success. The supply side will only receive their payment on project success. When the demand side, the DM has to choose for quality and the suppliers have private information about the required quality, this right to compensation causes the suppliers to share (parts of) the private information.

A summary of the results is shown in Table 3:

	Monopoly	Bertrand Duopoly	Cournot Duopoly
State of world revealed	Yes	Yes	Yes
Transaction price	$p \to G$	$p = \dfrac{c(q)}{\pi(q^*, q)}$	$p = \dfrac{G}{q_T}$
Surplus	Producer Surplus	Consumer Surplus	Producer Surplus (Total is smaller than Monopoly Surplus)
Dead weight loss	No	No	Yes, the Suppliers overproduce in EQ

Table 3: Summary of model Conclusions

In the previous sections I have shown that a monopolistic Supplier will credibly incorporate the different constraints of the model. He only produces and offers a project participation whenever this is socially optimal and the costs of producing are covered by the value of project success. This Supplier will also produce the optimal amount of q. This supplier will have enough market power to exert all of the value of project success, but in combination with the right of compensation this incorporates the incentives to have an optimal production.

The outcome of the monopoly model is therefore not a traditional monopoly outcome in which there is a dead weight loss. The monopolist collects all the rent and therefore acts as a social planner. Whether the DM will now reveal the state of the world, depends on the parameter space. Whenever the Incentive Compatibility Constraint dictates that the Supplier should supply different products for different states, the equilibrium is separating. However, for all of the states that the same q is offered, there would be a pooling equilibrium. In that case the information asymmetry is irrelevant, as the offered product always is the best.

The addition of a second supplier and the possibility for the DM to choose between the options has similar results. This model is comparable to the Bertrand Duopoly. In case of the traditional Bertrand duopoly with no information asymmetry or quality concerns, is the market price equal to marginal costs. The demand side will always choose the lowest price and therefore it is a dominant strategy to lower the price until reaching marginal costs. The project uncertainty is known to the suppliers and the expected value of the risk of project failure can be seen as a production cost for the supplier.

The addition of the quality and the right of compensation does not change this strategy. The Suppliers will account for the probability of project failure and thus not being paid. Therefore, the Supplier will not make an offer that results in a negative expected payoff. However, the DM will still choose the lowest price for comparable offers. The Suppliers have an incentive to credibly reveal the lower required qualities (lower values of the exogenous q^*) in order to communicate a high probability of success for a low price. In the traditional Bertrand Duopoly the price is equal to the marginal costs. In my model with project uncertainty this is exactly the same. The marginal expected value of the project revenue (the marginal value of increasing the probability of project success) is equal to the marginal costs. Therefore, the competition will force the Suppliers offer a social optimal value of q. The total welfare in equilibrium is therefore optimal.

However, whenever the two Suppliers have to cooperate on the project, the optimal total welfare is no longer reached. When both of the Suppliers need to supply a part of the total quantity of produced goods and are paid according to the ratio of supply, the Suppliers have not only an incentive to provide an optimal quantity, but also to reach a production share that is as high as possible. This is comparable to the traditional Cournot Duopoly in which Suppliers optimize their marginal revenue against the marginal costs.

In the traditional model the total production is lower than optimal because the market price declines as the total production grows. The Suppliers will only produce an extra good, if the marginal revenue outweigh the marginal costs. In my model, this is the other way round. The value of half of the production stays the same ($\frac{G}{2}$). Therefore, the Suppliers have a higher production than optimal by pursuing a higher market share. Since the cost structure is symmetric, this can result in the (symmetric) equilibrium, where both of the Suppliers supply half of the production and receive half of the value, but produce more than first best. If they would be able to make a (prohibited) enforceable commitment to produce less or even the first best production, they would receive the same $\frac{G}{2}$ on project success, but their expected payoff is better. When the game is no longer a single shot, this can lead to a cartel.

For the Decision Maker the situation of the Bertrand Duopoly is the best. When different suppliers may make an offer and only one has to execute the project, the price is equal to marginal costs and the offers will reflect the optimal decision of q. In this case there is a surplus for the demand side. From total welfare perspective this is an optimal market, just as the monopoly situation. This results in a first best solution in which the total surplus is with the supply side. The Cournot Duopoly situation is not first best, and involves a surplus for the suppliers. However, the total surplus is smaller than in first best.

In new research the used framework may be expanded. First, the project value is uncorrelated to the transactional q. However, there may be situations when the value of q does matter for the project outcome. For the example of the broken car the value of the project depends on whether it works (project success) and the expected lifetime. In that case would a big repair prolong the lifetime and this raises the value of the car.

Another extension on the framework is that reclaiming the paid transaction price for the DM incurs a transaction cost, such as legal costs. In that case the DM still has a risk by buying a lower q. As already described about the Cournot situation, making the game a repeated game can be interesting as well. This can cause reputational and competition concerns.

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9 Appendices

Appendix 1: Proof of the relationship between $p_{min}(q)$ and high type probability gain

In table reflecting the optimal monopolistic Supplier strategy (Proposition 2) is shown that there is no possibility that:

- $G\big(\pi(q^*,q^H)-\pi(q^*,q^L)\big) > c(q^H)-c(q^L)$ and $p^L_{min} < G$ and $p^H_{min} > G$
- $G\big(\pi(q^*,q^H)-\pi(q^*,q^L)\big) < c(q^H)-c(q^L)$ and $p^L_{min} > G$ and $p^H_{min} < G$

This seems intuitive: if it is not possible to supply at the maximum price of G at high quality, it cannot hold that high quality gives a higher utility to the Supplier and the Supplier would be prepared to supply low quality at a lower price.

A more formal proof.

Suppose it would be true that:

- $G\big(\pi(q^*,q^H)-\pi(q^*,q^L)\big) > c(q^H)-c(q^L)$
- $p^L_{min} < G$
- $p^H_{min} > G$

Then:

$$p^L_{min}\big(\pi(q^*,q^H)-\pi(q^*,q^L)\big) < p^H_{min}\big(\pi(q^*,q^H)-\pi(q^*,q^L)\big)$$

Given (Lemma 1):

$$p_{min}(q) = \frac{c(q)}{\pi(q^*,q)}$$
$$p^L_{min}(q) = \frac{c(q^L)}{\pi(q^*,q^L)}$$
$$p^H_{min}(q) = \frac{c(q^H)}{\pi(q^*,q^H)}$$

If the minimum prices are substituted by their definitions this is:

$$\frac{c(q^L)}{\pi(q^*,q^L)}\pi(q^*,q^H) - \frac{c(q^L)}{\pi(q^*,q^L)}\pi(q^*,q^L) < \frac{c(q^H)}{\pi(q^*,q^H)}\pi(q^*,q^H) - \frac{c(q^H)}{\pi(q^*,q^H)}\pi(q^*,q^L)$$

This can be rewritten to:

$$\frac{c(q^L)}{\pi(q^*,q^L)}\pi(q^*,q^H) - c(q^L) < c(q^H) - \frac{c(q^H)}{\pi(q^*,q^H)}\pi(q^*,q^L)$$
$$\frac{\pi(q^*,q^H)}{\pi(q^*,q^L)}c(q^L) - c(q^L) < c(q^H) - \frac{\pi(q^*,q^L)}{\pi(q^*,q^H)}c(q^H)$$
$$\left(\frac{\pi(q^*,q^H)}{\pi(q^*,q^L)} - 1\right)c(q^L) < \left(1 - \frac{\pi(q^*,q^L)}{\pi(q^*,q^H)}\right)c(q^H)$$

The definition of $\pi(q^*,q)$ says that if q: $\pi(q^*,q^H) \geq \pi(q^*,q^L)\ \forall\ q^H > q^L$. Therefore:

$$\frac{\pi(q^*,q^H)}{\pi(q^*,q^L)} \geq 1 \rightarrow \frac{\pi(q^*,q^H)}{\pi(q^*,q^L)} - 1 \geq 0$$

$$\frac{\pi(q^*,q^L)}{\pi(q^*,q^H)} \leq 1 \rightarrow 1 - \frac{\pi(q^*,q^L)}{\pi(q^*,q^H)} \leq 0$$

Also it is given that $c(q) \geq 0$ so it cannot hold that:

$$\left(\frac{\pi(q^*,q^H)}{\pi(q^*,q^L)} - 1\right) c(q^L) \geq 0 \geq \left(1 - \frac{\pi(q^*,q^L)}{\pi(q^*,q^H)}\right) c(q^H)$$

And:

$$\left(\frac{\pi(q^*,q^H)}{\pi(q^*,q^L)} - 1\right) c(q^L) < \left(1 - \frac{\pi(q^*,q^L)}{\pi(q^*,q^H)}\right) c(q^H)$$

This proof is symmetric for

$G\big(\pi(q^*,q^H) - \pi(q^*,q^L)\big) < c(q^H) - c(q^L)$ and $p^L_{min} > G$ and $p^H_{min} < G$

Appendix 2: Rewriting in Corollary 2

Simple algebra shows that:

$$\begin{aligned}
\pi^H \cdot (G - p_i) &\geq \left[\hat{h} \cdot \pi^L + \left(1 - \hat{h}\right) \cdot \pi^H\right] \cdot \left(G - p_j\right) \\
\pi^H G - \pi^H p_i &\geq \hat{h}\pi^L G - \hat{h} \cdot \pi^L p_j + \left(1 - \hat{h}\right)\pi^H G - \left(1 - \hat{h}\right)\pi^H p_j \\
-\pi^H p_i &\geq \hat{h}\pi^L \cdot G - \hat{h}\pi^H G - \hat{h} \cdot \pi^L p_j - \left(1 - \hat{h}\right)\pi^H p_j \\
0 &\geq \hat{h}\pi^L \cdot G - \hat{h}\pi^H G - \hat{h}\pi^L p_j - \pi^H p_j + \hat{h}\pi^H p_j + \pi^H p_i \\
\hat{h}\pi^H G - \hat{h}\pi^L \cdot G &\geq \pi^H p_i - \pi^H p_j + \hat{h}\pi^H p_j - \hat{h}\pi^L p_j
\end{aligned}$$

The expected gain of the high quality product is greater than the expected price to pay.

$$\begin{aligned}
\hat{h}\pi^H G - \hat{h}\pi^L \cdot G &\geq \pi^H p_i - \pi^H p_j + \hat{h}\pi^H p_j - \hat{h}\pi^L p_j \\
\hat{h}\pi^H G - \hat{h}\pi^L \cdot G - \hat{h}\pi^H p_j + \hat{h}\pi^L p_j &\geq \pi^H (p_i - p_j) \\
\hat{h}\pi^H (G - p_j) - \hat{h}\pi^L \cdot (G - p_j) &\geq \pi^H (p_i - p_j) \\
\hat{h}(\pi^H - \pi^L)(G - p_j) &\geq \pi^H (p_i - p_j) \\
\hat{h}(\frac{\pi^H - \pi^L}{\pi^H})(G - p_j) &\geq (p_i - p_j) \\
\hat{h}\left(\frac{\pi^H - \pi^L}{\pi^H}\right)\left(G - p_j\right) &\geq p_i - p_j \\
p_i - p_j &\leq \hat{h}(\frac{\pi^H - \pi^L}{\pi^H})(G - p_j)
\end{aligned}$$

So the price difference may not exceed the updated belief ($\hat{h}$) about the relative change in probability ($\frac{\pi^H - \pi^L}{\pi^H}$) of gaining the expected project profit $G - p_j$.

Appendix 3: Rewriting Corollary 2 with $\hat{\pi}$

$$\hat{\pi}(q_i) \cdot (G - p_i) \geq \hat{\pi}(q_j) \cdot (G - p_j)$$

$$\hat{\pi}(q_i) \cdot G - \hat{\pi}(q_i) \cdot p_i \geq \hat{\pi}(q_j) \cdot G - \hat{\pi}(q_j) \cdot p_j$$

$$\hat{\pi}(q_j) \cdot p_j - \hat{\pi}(q_i) \cdot p_i \geq (\hat{\pi}(q_j) - \hat{\pi}(q_i)) \cdot G$$

$$\frac{\hat{\pi}(q_j)}{\hat{\pi}(q_i)} \cdot p_j - p_i \geq \frac{\hat{\pi}(q_j) - \hat{\pi}(q_i)}{\hat{\pi}(q_i)} \cdot G$$

$$(1 - \frac{\hat{\pi}(q_i) - \hat{\pi}(q_j)}{\hat{\pi}(q_i)}) \cdot p_j - p_i \geq \frac{\hat{\pi}(q_j) - \hat{\pi}(q_i)}{\hat{\pi}(q_i)} \cdot G$$

$$p_j - p_i \geq \frac{\hat{\pi}(q_j) - \hat{\pi}(q_i)}{\hat{\pi}(q_i)} \cdot G + \frac{\hat{\pi}(q_i) - \hat{\pi}(q_j)}{\hat{\pi}(q_i)} \cdot p_j$$

$$p_j - p_i \geq \frac{\hat{\pi}(q_j) - \hat{\pi}(q_i)}{\hat{\pi}(q_i)} \cdot G - \frac{\hat{\pi}(q_j) - \hat{\pi}(q_i)}{\hat{\pi}(q_i)} \cdot p_j$$

$$p_j - p_i \geq \frac{\hat{\pi}(q_j) - \hat{\pi}(q_i)}{\hat{\pi}(q_i)} \cdot (G - p_j)$$

$$p_i - p_j \leq \frac{\hat{\pi}(q_i) - \hat{\pi}(q_j)}{\hat{\pi}(q_i)} \cdot (G - p_j)$$

Appendix 4: Proof of Proposition 8

Proposition: The optimal q_1 is reached by when the marginal costs are equal to the marginal gains due to the increased probability of project success (1). For the Step Function, this means that he will produce q^* if his expected profit is at least as good as producing the lowest possible amount of q ($q_i \downarrow 0$) (2).

The Supplier optimizes:

$$\max_{q_1} U_{S_1} = \pi(q^*, q_i)pq_1 - c(q_1)$$

$$\max_{q_1} U_{S_1} = \pi(q^*, q_1)\frac{G}{q_1}q_1 - c(q_1) = \pi(q^*, q_1)G - c(q_1)$$

So his First Order Constraint is:

$$\frac{\partial U_{S_1}}{\partial q_1} = \frac{\partial \pi(q^*, q_1)}{\partial q_1}G - c'(q_1) = 0$$

Equilibrium condition (when π is differentiable in q); proof of part (1):

$$\frac{\partial \pi(q^*, q_1)}{\partial q_1}G = c'(q_1)$$

However, when $\pi(q^*, q)$ is a Threshold Probability Function, the utility function not differentiable in q for $q = q^*$, so the optimal amount is given by: the optimum is equal to the maximum utility achievable in the lower range, the upper range or by just reaching $q_1 = q^*$.

$$q_{S_1} = \operatorname*{argmax}_{q_1}\left[U_{s_1,lower}, U_{s_1,upper}, U_{S_1^*}\right]$$

$$\begin{aligned} U_{s_1,lower} &= \max_{q_1} \pi(q^*, q_1)G - c(q_1), \qquad 0 < q_1 < q^* \\ U_{s_1,upper} &= \max_{q_1} \pi(q^*, q_1)G - c(q_1), \qquad q^* \\ U_{S_1^*} &= \lim_{q_1 \downarrow q^*} \pi(q^*, q)\, G - c(q) \end{aligned}$$

When the function $\pi(q^*, q)$ is the Step Function it is given that the probability does not change between the boundaries and thus:

$$\frac{\partial \pi(q^*, q_1)}{\partial q_1} = 0, \qquad q_1 \neq q^* \cup q_1 > 0$$

So the optimal $U_{s_1,lower}, U_{s_1,upper}$ is defined by:

$$\max_{q_1} \pi(q^*, q_1)G - c(q_1)$$

$$\frac{\partial \pi(q^*, q_1)}{\partial q_1}G - c'(q_1) = -c'(q_1)$$

Since $c'(q) > 0$ the optimal q_1 between the thresholds is the smallest that is within the boundary. So:

$$\underset{q_1}{\operatorname{argmax}} U_{T_{lower}} \downarrow 0$$

And

$$\underset{q_1}{\operatorname{argmax}} U_{T_{upper}} \downarrow q^*$$

In this situation we can state:

$$U_{T_{upper}} = U_{T^*}$$

So the produced quantity is in optimum just above the threshold of starting the project, $q_i \downarrow 0$ or the quantity necessary for the high probability of success, $q = q^*$ given by:

$$q_1 = \underset{q_1}{\operatorname{argmax}} [U_{T_{lower}}, U_{T^*}]$$

So the choice is either to face the cost of reaching the q^* threshold gaining the higher probability of success or producing as little as possible and thus proving (2):

$q_1 = q^*$ if:

$$\pi^L G - \lim_{q \downarrow 0} c(q) < \pi^H G - c(q^*)$$

which can be written as:

$$(\pi^H - \pi^L)G > c(q^*) + \lim_{q \downarrow 0} c(q)$$

Appendix 5: Proof of Proposition 10

Proposition: In optimum, the marginal costs equal to the marginal benefits of sum of the increased Gain due to the increase in probability π (given the market share in the optimum) and of the increased market share (given the probability in the optimum).

So each of the Suppliers maximizes his profit (the fraction of the gain, G, that he claims by) based on the probability of success and the production costs.

$$\max_{q_i} U_{S_i} = \pi(q^*, q_T)pq_i - c(q_i)$$

It is given that (Proposition 7):

$$p = D(q_T) = \frac{G}{q_T} = q_i + q_j$$

So the optimal supplied amount, given the other players strategies is given by:

$$\max_{q_i} U_{S_i} = \pi\left(q^*, q_i + q_j\right) \cdot \frac{G}{q_i + q_j} q_i - c(q_i)$$

As Cournot pointed out, there is an equilibrium where $q_1 = q_2$ when the cost function $c(q)$ is symmetric (Tirole, 1988). So therefore we can state that $q_i = q_j$ can lead to an equilibrium.

So the First Order Constraint in this equilibrium is:

$$G \cdot \frac{\partial \pi(q^*, 2q_i)}{\partial q_i} \cdot \frac{q_i}{2q_i} + G \cdot \pi(q^*, 2q_i) \cdot \frac{1}{2q_i} \cdot \frac{q_i}{2q_i} = c'(q_i)$$
$$G \cdot \frac{\partial \pi(q^*, 2q_i)}{\partial q_i} \cdot \frac{1}{2} + G \cdot \pi(q^*, 2q_i) \cdot \frac{1}{4q_i} = c'(q_i)$$
$$G \cdot \frac{\partial \pi(q^*, 2q_i)}{\partial q_i} \cdot \frac{1}{2} + G \cdot \pi(q^*, 2q_i) \cdot \frac{1}{4q_i} = c'(q_i)$$

However, when $\pi(q^*, q)$ is a Threshold Probability Function, the utility function not differentiable in q for $q = q^*$, so the optimal amount is given by the reaction function:

$$q_i^*(q^*, q_i) = \underset{q_i}{\operatorname{argmax}}[U_{T_{lower}}, U_{T_{upper}}, U_{T^*}]$$

Where:

$$U_{T_{lower}} = \max_{q_i} \pi(q^*, q_T)G - c(q_i), \qquad 0 > q_i > (q^* - q_j)$$
$$U_{T_{upper}} = \max_{q_i} \pi(q^*, q_T)G - c(q_i), \qquad (q^* - q_j) > q_i$$
$$U_{T^*} = \lim_{q \downarrow q^* - q_j^*} \pi(q^*, q)\, G - c(q)$$

When the function $\pi(q^*, q)$ is the Step Function it is given that the probability does not change between the boundaries and thus:

$$\frac{\partial \pi(q^*, q_T)}{\partial q_i} = 0, \qquad q_i \neq q^* \cup q_j > 0$$

So optimal q_i for $q_i \neq q^* - q_j \cup q_i > 0$ is only defined by the battle for the market share:

$$G \cdot \pi(q^*, q_T) \cdot \frac{1}{q_T} \cdot \frac{q_j}{q_T} = c'(q_i)$$

Which is in the Symmetric equilibrium would be:

$$G \cdot \pi(q^*, 2q_i) \cdot \frac{1}{4q_i} = c'(q_i)$$

So the optimal response production is when the marginal costs are equal to the marginal gains of increasing the market share given the total amount produced and the probability of project success cannot be changed profitable by increasing or decreasing the production.

Appendix 6: Proof of Proposition 11

Proposition: For First Best, the marginal costs equal to the marginal benefits of sum of the increased Gain due to the increase in probability π.

First best Maximizes $W = U_{S_1} + U_{S_2} + U_{DM}$.

So the q_1^{FB} and q_2^{FB} is given by:

$$\max_{q_1, q_2} W \ = U_{S_1} + U_{S_2} + U_{DM} = \pi(q^*, q_1 + q_2) \cdot G - c(q_1) - c(q_2)$$

If $\pi(q^*, q)$ is differentiable in $q > 0$ then:

$$\max_{q_i} \ \pi(q^*, q_T)G - c(q_1) - c(q_2)\,, \qquad i \in \{1,2\}$$

The FOCs are:

$$\frac{\partial W}{\partial q_i} = \frac{\partial \pi(q^*, q_T)}{\partial q_i} G - c'(q_i) = 0, \qquad i \in \{1,2\}$$

$$\frac{\partial \pi(q^*, q_T)}{\partial q_i} G = c'(q_i), \qquad i \in \{1,2\}$$

So FOC gives that the production is optimal when all the Supplier's marginal costs are equal to the marginal gain due to an increase in the probability of success, π. Therefore, no supplier can raise production and achieve a profitable gain in project success.

If $\pi(q^*, q)$ is a Threshold Probability Function, then the analysis is similar to the one in Proposition 8. The optimum is determined by:

$$\max_{q_i}[W_{lower}, W_{upper}, W^*]\,, \qquad i \in \{1,2\}$$

$$W_{lower} = \max_{q_i} \pi(q^*, q_i)G - c(q_i)\,, \qquad 0 < q_i < q^* - q_j$$

$$W_{upper} = \max_{q_i} \pi(q^*, q_i)G - c(q_i)\,, \qquad q_i > q^* - q_j$$

$$W^* = \lim_{q_i \downarrow q^*} \pi(q^*, q)\, G - c(q)$$

When the function $\pi(q^*, q)$ is the Step Function it was given that:

$$\frac{\partial \pi(q^*, q_T)}{\partial q_i} = 0, \qquad q_i \neq q^* \cup q_j > 0$$

So the optimal W_{lower} and W_{upper} are defined by:

$$\frac{\partial W}{\partial q_i} = -c'(q_i), \qquad i \in \{1,2\}$$

Since $c'(q) > 0$ the optimal q_1 and q_2 between the thresholds are the smallest that is within the boundary. So:

$$q_1 = \underset{q_1, q_2}{\operatorname{argmax}}[W_{lower}, W^*]$$

So the choice is either to face the cost of reaching the $q_T = q^*$ threshold and gaining the higher probability of success or producing as little as possible. $q_T = q^*$ if (for any $0 \leq q_1 \leq q^*$):

$$\pi^L G - 2 \cdot \lim_{q \downarrow 0} c(q) < \pi^H G - c(q_1) - c(q^* - q_1)$$

$$(\pi^H - \pi^L) G > c(q_1) + c(q^* - q_1) - 2 \cdot \lim_{q \downarrow 0} c(q)$$