Erasmus University Rotterdam

Contingent Capital Bonds

Master Thesis Quantitative Finance

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Abstract

Contingent capital bonds are introduced to automatically strengthen the capital structure of a bank when it gets into a state of possible non-viability. Through coupon cancellation and write-downs the equity increases and capital ratios are strengthened.

A new pricing model for a perpetual temporary write-down and coupon cancellation contingent capital bond (CoBo) is constructed, capturing a more complete set of CoBo characteristics than assessed in existing literature. The model allows for coupon cancellation, partial write-down, discretionary write-up and perpetual maturity.

The CoBo suffers a write-down when a trigger process breaches a trigger level. In addition to existing literature this research uses a double exponential jump diffusion trigger process. Furthermore, parameters are estimated through calibration on call option data. Using Monte Carlo the model is able to price a CoBo without using historical data on the CoBo itself. Due to the limited availability of historical data on CoBos this model solves the problem of determining an initial value at issuance.

The performance of the models is evaluated using 220 daily observations on a contingent capital bond issued by Deutsche Bank.

Key words: Contingent capital bond, double exponentiation jump diffusion, lognormal jump diffusion.

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1 Introduction

The recent financial crisis exposed some serious flaws in the regulation for banks. During this financial crisis the collapse of many large banks was prevented through bail outs by national governments, using taxpayer's money. These bail outs were necessary because the banks lacked sufficient capital to account for the liquidity margin calls for collateral and because these banks were considered 'too big to fail'. The financial crisis and these bail outs have triggered new regulatory standards by supervisory authorities around the world. These new requirements mainly focus on improving the quality and quantity of capital held by banks to lower the risks of a new financial crisis. In 2008 there were doubts about the solvency among banks themselves which resulted in a shortage of interbank funds. At the same time depositors and other lenders of the bank lost their confidence in banks. The new regulatory capital requirements should improve the confidence in banks. As a response to the new regulations, banks have developed financial products aimed to improve capital.

A contingent capital bond is a debt instrument that converts into shares or suffers a write-down when the issuing bank gets into a state of possible non-viability. They provide banks with a way of raising funds needed to adhere to the new capital requirements without directly issuing shares. Contingent capital bonds are hybrid capital that include a mandatory conversion or write-down when a trigger event occurs. The trigger mechanism provides an automatic strengthening of the capital structure of the bank, as debt is converted into equity or written down. A trigger event generally occurs when a capital ratio of the bank falls below a certain threshold value.

Research in the field of pricing contingent capital bonds has been growing. However, due to the wide variety of different features of contingent capital bonds issued in the past 7 years and given the fact that no consensus has been reached on which pricing method is best, there is still a great need for more research on different pricing models. Furthermore, new regulation has forced banks to adjust some of the features of contingent capital bonds in order to qualify as capital. There is also a lot of uncertainty surrounding contingent capital bonds. For example, the continuing change in regulation may prevent banks from using the redemption feature if the bank would than have to issue new bonds at higher costs (Hale and McCrum, February 9, 2016). Despite the uncertainty surrounding contingent capital bonds, the issuance of these bonds by banks has increased significantly in the past few years.

This research focuses on pricing a contingent capital bond that meets the new Basel III requirements for capital. This contingent capital bond does not convert into equity, but instead has the coupon cancellation feature and gets (temporarily) written down just enough to restore the capital ratio. These features are included in the debt instrument issued on November 18, 2014 by the German bank Deutsche Bank AG.

The contribution of this research entails the development of a pricing model for the perpetual temporary write-down and coupon cancellation contingent capital bond (hereafter CoBo). Existing literature focuses on some of the features of the CoBo that are necessary to meet the new Basel III requirements for capital, but not all together. Features of the CoBo already assessed in the literature are partial conversion (Glasserman and Nouri, 2012), coupon cancellation (Corcuera, 2014a), perpetual maturity (De Spiegeleer, 2014a) and a trigger process based on the lognormal jump diffusion model(Pennacchi, 2010). The pricing model in this research incorporates all of the above features together. In addition, the pricing model incorporates the discretionary write-up and optional redemption features. Furthermore, in this research a trigger process based on the double exponential jump diffusion model is developed (Kou, 2002). The literature and the features of the CoBo will be discussed in detail in Section 2 and 3, respectively.

Existing literature focuses on geometric Brownian motion and lognormal jump diffusion model (Merton, 1976) trigger processes. This research develops a double exponential jump diffusion model (Kou, 2002) trigger process. The double exponential jump diffusion trigger process has not yet been applied to contingent capital bonds. Jumps in the trigger process are important in order to capture the influential impact of sudden changes in the capital ratio on the price of the CoBo. The difference between the double exponential jump diffusion model and the lognormal jump diffusion model is the distinction between positive and negative jumps. This feature is interesting to evaluate, as the data shows differences between positive and negative outliers and positive jumps have a different impact on the CoBo price than negative jumps. Furthermore, existing literature did not estimate the parameters of the jump diffusion models. Instead, they assumed values that might be reasonable benchmark parameters (Glasserman and Nouri, 2012) (Pennacchi, 2010). Parameters for the models in this research are calibrated using share price and call option premium data.

Data on contingent convertible bonds is scarce, as this kind of products are only issued recently and contingent convertible bonds issued by different banks differ in characteristics. To determine a fair price at issuance a method is needed that does not need historical data on the bond itself. In contrast to existing literature on contingent capital bonds, the model in this research uses calibration techniques on call option premium data and share price data to estimate the parameters of the models. The model thus solves the problem of determining an initial value of the CoBo at issuance. The performance of the models is evaluated using 220 daily observations on a contingent capital bond issued by Deutsche Bank.

This research considers a pricing model based on two different processes: a model with a lognormal jump diffusion trigger process (Model 1) and one with a double exponential jump diffusion trigger process (Model 2). Model 2 has a MSE of 2.43 compared to a MSE of 5.34 for the Model 1. A Diebold-Mariano test concludes that Model 2 is more accurate in determining CoBo prices than Model 1. The prices of Model 2 have a correlation of 76% with the observed prices, compared to a correlation of 56% for Model 1. This research concludes that the model constructed in this research is able to capture important features of the CoBo. Furthermore, Model 1 and Model 2 are both able to accurately determine CoBo prices when historical data on the CoBo itself is unavailable, with Model 2 outperforming Model 1.

Regulators have set minimum capital levels that banks need to adhere to. In the Basel Accords regulators started proposing minimum capital requirements. Under Basel III, the CET1 ratio has been put forward as a key health indicator of a financial institution. The higher the ratio, the more common equity a bank has available for the bank's risk weighted assets. This ratio is an indication how the bank is able to deal with any unpleasant surprises in its investments and losses. The new minimum capital to risk weighted assets (RWA) requirements are a CET1 capital ratio of 4.5% and a Tier 1 capital ratio of 6.0%. This 1.5% difference between the two is called the Additional Tier 1 (AT1) capital, and this can be filled in by the contingent capital. According to the new regulatory Basel III framework, contingent capital bonds can be categorized as either belonging to the AT1 or Tier 2 capital category, depending on their specific features. In order to belong to the former category, a contingent capital bond must have certain features. It must have a coupon cancellation feature and no fixed maturity is to be imposed to the contract. Instead, the issuer is entitled to redeem the bond at any of the predetermined call dates.

Based on the trigger event two categories exist: (i) a going concern or high trigger contingent capital bond, which are intended to prevent an institution from entering into a recovery or resolution proceeding and (ii) a gone concern contingent capital bond, which may be triggered either before or during an official recovery and resolution proceeding. Contingent capital bonds that get written down have increased in popularity recent years (De Spiegeleer, 2014b). The main reason for this increased popularity is the growing interest of fixed-income investors, which are prohibited from holding shares.

The potential benefit of going concern contingent capital bonds is that when a bank's initial equity is depleted, the bank automatically recapitalizes. It enables raising capital at times when other options are impossible, either because of unfavorable market conditions or because they are unattractive to shareholders (Duffie, 2010). The 'recapitalization gridlock' reflects the unwillingness of shareholders to dilute their equity through share issuance or by 'fire sales' in unfavorable market conditions (Brunnermeier, 2009) (Adrian and Shin, 2010). From a regulators point of view this automatic recapitalization is a desirable feature, as it reduced the likelihood of financial distress and the need for a government bail out. From the banks point of view, there are also several advantages. Banks preparing for the upcoming European Central Bank stress test need to attract more capital in order to meet the stricter capital requirements. Contingent capital bonds are a relatively cheap way of raising capital. Since these type of bonds are only senior to the shareholder's equity, the costs are generally lower than the costs on equity demanded by the shareholders. Another advantage for banks is the tax deductibility of the interest paid on the bonds. In addition, they will count toward the bank's regulatory capital buffers and leverage ratios. This will allow the bank to meet its leverage and capital targets without having to raise Common Equity Tier 1 (CET1) capital, which is most expensive. At the same time, the higher yields relative to similarly-rated corporate debt make them attractive to investors.

Some important properties of contingent capital bonds make them somewhat riskier than typical senior debt issued by the same bank. The most important difference is that they are converted into shares or written down when the capital ratio of a bank gets below a certain threshold value. This conversion or write-down is mandatory, as opposed to a normal convertible bond where the holder has the right to convert. Another difference is the maturity, as AT1 contingent capital bonds are perpetual, while typical senior debt has a maturity of 10 years. Contingent capital bonds cannot be redeemed by the issuer within the first 5 years and there can be no incentive for redemption. Coupons are mandatory for senior debt, but for contingent capital bonds coupons are fully discretionary and noncumulative. Furthermore, contingent capital bonds are subordinated to all other debt and are only senior to shareholder's equity. The valuation boils down to the quantification of the trigger probability and the expected loss suffered by the investors if such a trigger event eventually takes place. The three most important value drivers of contingent capital bonds are the trigger, the write-down or share conversion and the maturity. These three drivers will be discussed in detail in Section 3.

2 Literature Review

In 2002, Flannery was the first to introduce a type of bond that would convert into equity when a company's capital ratio breached some predetermined threshold (Flannery, 2005). He called this bond a 'Reverse Convertible Debenture' and it would act as a buffer during financial distress situations. The crisis of 2008 and the new regulatory requirements set by supervisors led to the creation of new types of contingent capital bonds. A vast amount of recent literature focuses on the appropriate design of contingent capital bonds, whereas other literature propose new pricing models.

Pennacchi (2010) developed a structural credit risk model for the pricing of contingent bank capital (Pennacchi, 2010). Pennacchi models the bank's assets as a jump-diffusion process and where the default-free interest rates are stochastic. Using this process he constructs an accounting trigger as a ratio of the bank's current assets-to-deposits. Allowing for jumps in the asset value is more realistic and he finds that this has implications on the value of contingent capital. Pennacchi also investigates the bank's incentive to increase risk when it issues different forms of contingent capital as well as subordinated debt. He finds that banks that issue contingent capital face a moral hazard incentive to increase its assets jump risks. Increasing the risk of a bank's assets shifts risk from the original shareholders to contingent capital investors. Therefore, increasing the asset jump risk leads to a value transfer from contingent capital investors to the original bank shareholders. However, the incentive to transfer value from contingent capital investors to the bank's shareholders is smaller than when the bank has issued a similar amount of subordinated debt rather than contingent capital.

Glasserman and Nouri (2012), investigate the pricing of contingent capital using a capital ratio trigger and partial and ongoing conversion (Glasserman and Nouri, 2012). When the trigger is breached, just enough debt is converted into equity to meet the capital requirement. They derived closed-form expressions for yield spreads by adding a consistency requirement that market and book values of debt agree at issuance and at maturity. This partial and ongoing conversion/write-down feature is one of the requirements set by supervisors for contingent capital bonds in order to be categorized as AT1 capital.

De Spiegeleer and Schoutens (2012a) define a few disadvantages of using an accounting trigger, specifically the Tier 1 Capital ratio (De Spiegeleer, 2012). They state that investors have to rely on audited balance sheets that do not always provide enough details. Large US financial institutions that in 2008 failed or had to be bailed out by the government, were reporting capital ratios above the minimum requirement (Flannery, 2009). Another disadvantage is that capital ratios are not forward looking since they are published with delay. The information to determine the capital ratio is only released on quarterly or semi-annual basis, which leaves a lot of room for speculation regarding the fact whether the bond will be triggered or not. The use of a share price trigger also has a few disadvantages. Practitioners fear that share price could be subject to manipulation if the contingent capital certificates are close to conversion or write-down (Flannery, 2009).

De Spiegeleer and Schoutens (2012a) provided an in-depth analysis of pricing and structuring of contingent convertibles from a credit derivatives and an equity derivatives approach, where the trigger process is based on the share price (De Spiegeleer, 2012). They note that when there is a risk of conversion, investors will start delta hedging their position. When these investors start building a short position in the shares, they will push the share price down even further, causing even more delta hedging. They call this effect *death spiral risk*. The equity derivative approach in their paper does not take into account the fat tails and jump features incorporated in instruments like the contingent convertible bond. Corcuera et al. (2013) improve this equity derivative approach (Corcuera, 2013). They price contingent convertible bonds where the underlying risky asset dynamics are given by a smile conform model, more precisely, an exponential Levy process incorporating jumps and heavy tails. The model they derived better captures the intrinsic nature of contingent capital.

Corcuera et al. (2014) introduced the coupon cancellable contingent convertible bond, a type of contingent convertible bond where coupons can be canceled during the lifetime of the note (Corcuera, 2014a). They show that the *death spiral effect* can be significantly reduced by the coupon cancellation feature. Coupons get canceled when a coupon specific threshold is breached, lowering the debt burden of the bank and postponing possible conversion.

The same year Corcuera et al. (2014) and Spiegeleer and Schoutens (2014) wrote a paper about contingent convertible bonds with extension risk, in terms of a structural and intensity approach, respectively (Corcuera, 2014b) (De Spiegeleer, 2014a). Extension risk is the possibility that the bond issuer does not redeem the bond at predetermined call dates but extends the maturity. They propose a recursive method to capture the risk that the maturity of a bond is increased and new coupon rates are established, a so called step-up. They also make a start for the case of an infinite horizon. Currently many issued contingent capital bonds have infinite maturity with predetermined dates upon which the issuer can redeem the bond.

The most interesting feature of a pricing model for contingent capital is the trigger process. Hilscher and Raviv (2014) consider a hypothetical bank that finances its assets through the issue of three types of claims as part of its capital structure: a single zero-coupon deposit, contingent convertible debt and a residual equity claim (Hilscher and Raviv, 2014). The pricing of the different liabilities which compose the capital structure of a bank depends on the assumed stochastic process of the underlying asset. They assume that the dynamics of the bank's assets follow a simple geometric Brownian motion. An advantage of assuming a Geometric Brownian Motion for the asset process is the availability of easy closed form solutions for European options. For the parameters of the geometric Brownian motion they choose base case parameter values and then perform sensitivity analysis around these values. Glasserman and Nouri (2012) also simulate the asset value process with a geometric Brownian motion (Glasserman and Nouri, 2012). The parameters used are not calibrated on any data but are intended to be representative for the end of 2006, before the financial crisis, based on data for the 20 largest (by assets) banks in the United States. The volatility and interest rate is taken as a fixed value.

Pennacchi (2010) considers a jump-diffusion formulation for the firm's underlying assets, were the trigger is stated as a ratio of assets to deposits (Pennacchi, 2010). Pennacchi states that a bank's assets are invested in a portfolio of loans, securities and off-balance sheet positions whose rate of return follow a mixed jump-diffusion process. For the parameters of the process he assumes values that might be reasonable benchmark parameters. He suggests to estimate parameters of the bank's asset return jump-diffusion process from information on a bank's stock returns, debt prices, and/or credit default swap spreads, but leaves this for future research.

In 2014, Pennacchi, Vermaelen and Wolff investigated so called Call Option Enhanced Reverse Convertibles (COERC) (Pennacchi and Wolff, 2014). In case of a trigger event, this kind of contingent capital bond gives existing shareholders the option to purchase the newly issued shares at the bond's par value. For their capital ratio simulation they consider the capital ratio described in (Pennacchi, 2010). In contrast to Pennacchi(2010), they use data on three large U.S. banks to estimate the model's parameters. They assume that the market return on the bank's total assets equals the market return of its total liabilities. After obtaining daily return estimates of the bank's total liabilities they used this data to determine the jump parameters of the jump-diffusion process. Parameters of the default free term structure are choses similar to Duan and Simonato (1999) and the initial short rate is assumed to equal 2% (Duan and Simonato, 1999).

Wilkens and Bethke (2014) assessed three different approaches for pricing contingent capital bonds (Wilkens, 2014). They assessed the credit derivative approach, equity derivative approach and the structural approach. They conclude that the equity derivatives model is the most practical for the pricing and risk management of contingent capital bonds. Erismann (2015) also conducted a theoretical and empirical analysis on these three pricing approaches (Erismann, 2015). The findings of his research conclude that future research should focus on the structural approach, as they provide the natural framework for capital ratio triggered contingent capital.

3 Contingent Capital Bond

In the past couple of years a large amount of different types of contingent capital bonds were issued by many banks. Contingent capital bonds issued by one bank can be significantly different from other contingent capital bonds issued by another bank. Differing characteristics of these bonds over the years are partly caused by changing regulatory requirements for capital set by supervisors. Different characteristics of these bonds between banks are partly caused by inconsistent national regulation. Most banks want to issue a contingent capital bond that meets the requirements set by regulators for AT1 capital. In the European Union, Basel III will be implemented through the Capital Requirements Regulation and Directive (CRD

IV), which includes the following requirements for AT1 capital.

- The instruments are not secured, or subject to a guarantee that enhances the seniority of the claims by the institution or its subsidiaries.
- It must be able to be written down or converted to CET1 capital upon the occurrence of a trigger event.
- The instruments are perpetual.
- The instruments may be called, redeemed or repurchased only where certain conditions are met, and not before five years after the date of issuance.
- The provisions governing the instruments give the institution full discretion at all times to cancel the distributions on the instruments for an unlimited period and on a non-cumulative basis, and the institution may use such canceled payments without restriction to meet its obligations as they fall due.

A complete list of criteria can be found in Volume 56, Official Journal of the European Union, 27 June 2013. The Bank Recovery and Resolution Directive (BRRD) contains some additional criteria for AT1 capital. The BRRD sets rules for all 28 Member States of the European Union to put an end to the old paradigm of bank bail outs. The BRRD provides that for capital to be categorized as AT1 capital instruments it must fully absorb losses at the so called point-of-non-viability (PONV) of the issuing institution. Furthermore, their terms must recognize that resolution authorities have the power to bail in (write-down or convert into equity) such instruments at the PONV and before any resolution action is taken. For these purposes, PONV is the point at which the relevant resolution authority determines that the institution meets the conditions for resolution or the point at which the authority decides that the institution would cease to be viable if the instruments were not written down or converted. These criteria make that contingent capital bonds are designed to work as a buffer that must fully absorb losses in distressed situations.

The value of a CoBo is mainly determined by the trigger process and the method of write-down and coupon cancellation. The trigger process determines when write-down or coupon cancellation occurs. The amount of coupon cancellation and write-down is such that the capital ratio stays above the minimum required ratio, incorporating the partial write-down feature. After a write-down occurred, future coupons are adjusted to a new principal value, called the Prevailing Principal.

The coupon rate is equal to a fixed percentage for the first five or ten years. After these first five or ten years, and every five years thereafter, a new coupon rate is set for five years. This new coupon rate is specified in the prospectus as the Reference Rate + x bps. These coupon rate adjustment dates are called Reset Dates. The Reference Rate is often coupled to a certain swap curve, such that at Reset Dates the new coupon rate is adjusted upwards or downwards depending on the current market situation.

After five or ten years, and every five years thereafter, the issuer can redeem the CoBo if this is more beneficial than extending the maturity. This redemption feature will be incorporated in the pricing model. A write-up is also possible, meaning that the Prevailing Principle amount is (partly) restored to initial principal value, but only at the sole discretion of the issuer. The dynamics of the CoBos are captured by incorporating these features in a simulation based pricing model. The price of the CoBo is determined by discounting the remaining coupons and Prevailing Principal.

As required by Basel III, the contingent capital bonds also incorporate a regulatory trigger. This means the regulator holds the discretion to trigger a write-down or equity conversion on the bond. This could occur before the minimum regulatory capital levels are breached. This research does not incorporate a regulatory trigger in the pricing model as it is uncertain when such a trigger might occur.

Trigger process

The trigger event is incorporated in the model by simulating a trigger process. An important feature of the trigger process is that it needs to improve upon coupon cancellation and write-down. The process should also be representative for the bank's financial health and realistically capture the dynamics of an accounting ratio. Although all issued contingent capital bonds feature some sort of an accounting trigger, academic literature has focused mainly on share price triggers. The dynamics of an accounting ratio and asset value are difficult to assess, as they are only reported (semi-) annually and the definition of different accounting ratios have changed over time due to changing regulations. Banks can calculate their accounting ratios on a daily basis and coupon cancellation or write-down can occur on any day, not only on accounting publication days. Some articles have develop a method to simulate an accounting trigger by choosing a certain process for the asset value. The bank's asset are simulated by geometric Brownian motion (Hilscher and Raviv, 2014) (Glasserman and Nouri, 2012) or the lognormal jump diffusion model (Pennacchi, 2010) (Pennacchi and Wolff, 2014). Parameter values of the asset process are based on time series data or set equal to values assumed to be representative. Due to the limited data availability it is not possible to calibrate the asset value process on bank's asset value data.

The new Basel III accord splits Tier 1 capital into two components: CET 1 capital and AT 1 capital. Under Basel III, a bank must maintain the following minimum capital requirements

- a Common Equity Tier 1 capital ratio of 4.5%
- a Tier 1 capital ratio of 6%

The above capital ratios are all expressed as a percentage of the total risk exposure amount, the risk weighted assets (RWA). See figure 2 in Appendix A for an overview of capital requirements as proposed by Basel for the coming years. The Tier 1 capital consists of the sum of the CET1 capital and AT 1 capital of the institution. CET1 capital consists mainly of common shares and retained earnings and AT1 capital consist mainly of preferred shares and contingent capital bonds. These two forms of capital are meant to absorb losses in the normal course of business. Most contingent capital bonds have one of these capital ratios as trigger process. The setting in this research focuses on a trigger process that allows for the possibility of sudden declines in the CET1 ratio.

A decline of the CET 1 ratio can be caused by decrease in equity or an increase in RWA. Both the numerator and denominator are sensitive to shocks, as typically occurs during periods of financial turmoil. In order to estimate parameters using sufficiently available data the trigger process is simulated by a share price process using two jump diffusion models; the lognormal jump diffusion model and the double exponential jump diffusion model. The lognormal jump diffusion model is the most widely used model when incorporating jumps and often fits the data well. However, as will be shown in Section 7, the data shows that negative and positive jumps have a different frequency and magnitude. Since the double exponential jump diffusion model can differ between negative and positive jumps it could be that this process better fits the data and could give more accurate CoBo prices. Furthermore, some empirical tests suggest that the double exponential jump diffusion model fits stock data better than the normal jump diffusion model (Kou, 2007). Parameters are estimated by means of calibration using call option premiums for different strikes and maturities and a closed form solution of the call option value.

Temporary partial write down

The simulated trigger process should improve upon partial write-down or coupon cancellation. Not all coupons are canceled at once. In case of a trigger event, only the first upcoming coupon is canceled. If after coupon cancellation the share price remains below its trigger level, the principal is written down. This write-down is such that the share price is restored to its trigger level. After a write-down, future coupon values are adjusted to the Prevailing Principal. The coupon rate stays constant for a predetermined period, but coupon values can change due to changing Prevailing Principal amounts.

The issuer of the CoBo has the full discretion to restore the Prevailing Principle after a write-down, called the write-up. The issuer has the right to execute this write-up multiple times, but only under certain conditions. The Prevailing Principle can not be written up to above its initial principal value and a write-up may not cause a trigger event. A write-up would not increase the capital buffer. A write-up goes at the expense of the net profit, such that effectively equity decreases in order to reinstate the principal amount of the CoBo. Although it does increase AT1 capital, it does at the cost of equity and thus not improving the total capital of the bank.

It is not specified when such a write-up occurs, but there are several reason for a bank to execute a write-up. If the Prevailing Principal is written down to a certain level and the coupons are low, investors may complain about the low coupon when the bank is already out of trouble. A write-up is a way to show investors that it is possible to participate in the recovery of distressed banks, which would add value to the CoBo from the investors point of view. Another reason is to maintain access to the capital market. If the bank does not restore the principal when the bank is out of trouble this sends a bad signal to the market. This bad signal could have a negative impact when the bank wants to issue new contingent capital in the future. After a write-up, the future coupon values are adjusted to the Prevailing Principal.

Even though a specific capital ratio level is not formally stated in the prospectus, the issuer would only decide to carry out such a write-up when the bank is clearly out of trouble. The write-up feature will be incorporated in the model by specifying a certain high share price trigger level for a write-up. This write-up is not mandatory, in contrast to the write-down. Therefore, in case of a upper threshold breach, the write-up will be linked to a parameter indicating the probability of a write-up.

Maturity

As Basel III requirements impose an infinite horizon feature for a contingent capital bond in order to qualify as AT1 capital, most issued contingent capital bonds are perpetual bonds. Issuers may redeem the CoBo, but in order for the CoBo to qualify as AT1 capital it can not be redeemed before five years after the date of issuance. Most contingent capital bonds are designed such that after five years the issuer can redeem the CoBo, on the so called First Call Date. After this First Call Date the issuer can redeem the CoBo on any fifth anniversary thereafter, on the so called Call Dates. The issuer can redeem all, but not some, of the CoBo capital. The redemption price is equal to the Prevailing Principal plus any accrued interest payment. Furthermore, no coupon step-up is allowed, as incentives for bonds to be called are not allowed by the Basel III regulations. A step-up means that future coupon payments are received at a higher, predetermined amount than previous or current periods.

The model in this research looks at the value of the CoBo on every Call Date and determines if, from an issuers point of view, it is more beneficial to redeem or to extend the maturity. This process is repeated until the CoBo is redeemed or completely written down. Every Call Date is also a Coupon Reset Date. If the bank decides not to redeem, the maturity is extended another five years and a new coupon rate is determined. This new coupon rate is set for another five years, with the new coupon rate denoted as a Reference Rate + x bps. The Reference Rate is often coupled to a certain swap curve, such that at Reset Dates the new coupon rate is adjusted upwards or downwards depending on the current market situation. The pricing model in this research incorporates these redemption and changing coupon rate features.

In certain situations the issuer has the right to early redeem the CoBo.

- If as a result of a tax law change the tax treatment of the CoBo changes, such that interest paid on CoBos are no longer deductible, the issuer has the right to redeem the CoBo.

- If as a result of a regulatory change the classification of the CoBo changes, such that it no longer qualifies as AT1 capital, the issuer has the right to redeem the CoBo.
- The issuer has the right to redeem the CoBo subject to certain conditions. These conditions mainly entail a consent of the regulator and that, after redemption, the capital ratio stays (sufficiently) above the regulatory minimum level.

The above situations are not incorporated in the model as the occurrences of these situations are uncertain.

Effect on balance sheet

When the capital ratio of a bank drops below the trigger level a coupon cancellation and write-down of a CoBo has to restore this capital ratio. Most capital ratio have equity in the numerator. In order to increase this ratio, either the equity has to increase or the denominator has to decrease.

The CoBo is constructed such that it increases the equity in case of a trigger event. At first the upcoming coupon payment will be canceled, thereby lowering the interest expenses of the bank for that period. By lowering the costs, the retained earnings increase. Retained earnings are, through the profit and loss account, assigned to equity. If, after the coupon cancellation, the capital ratio is still below the trigger level, the principal of the CoBo is written down. By writing down the principal, the total debt due decreases. This will result in a positive result on the profit and loss account through the retained earnings. This again results in an increase in equity for that period. Therefore, both the cancellation of a coupon and the write-down increase the capital ratio.

4 Data

Data of call options on Deutsche Bank shares for different strikes and maturities and Deutsche bank share price data itself are used for the calibration of the jump diffusion models. For the simulation of CoBo prices the 30 year term structure of five year semi-annual euro swap curve. Data on the Deutsche Bank CoBo prices is used to compare the models prices to. The data ranges from November 18, 2014 to October 9, 2015.

Deutsche Bank daily log returns for the period November 18, 1994 to 21 December, 2015 are used to assess normality of the Deutsche Bank share returns.

All this data is gathered from Bloomberg L.P., an American multinational mass media corporation, which delivers a vast array of global financial information (e.g., data, analytics, news, communications, and charts). Accounting information is gathered from annual and financial reports of Deutsche Bank, which are openly accessible on the internet.

5 Pricing approaches

Due to the stochasticity of the trigger process, the payoff cannot be perfectly determined ex-ante and must therefore be approximated. The different pricing methods of contingent capital bonds can be divided into three approaches

- Credit derivative approach
- Equity derivative approach
- Structural approach

Credit derivative approach

In the credit derivative approach the contingent capital bond is priced from the viewpoint of a fixed income investor (De Spiegeleer, 2012). In fixed income mathematics corporate debt is often priced using a reduced form approach, also referred to as intensity based credit modeling. The intensity approach assumes an exogenously default event, which must not necessarily be correlated to the asset value of the company. The contingent capital bond is priced in terms of an extra yield on top of the risk-free rate to compensate for the possible conversion or write-down. The credit derivative approach uses the probability of default and the loss given default to determine the value of a financial product. Since the conversion or write-down is closely connected to a firms default probability, intensity based credit modeling lends itself as a pricing methodology. The probability of default and the loss given default are linked to the credit spread in a formula called the credit triangle

$$credit\ spread = \lambda(1 - R), \tag{1}$$

where λ is the default intensity and R is the recovery rate. In the event of default, R is the percentage amount of invested money one can expect to get recovered. Denote $S(t)$ as the cumulative probability of surviving to time t (no default by time t). Then the probability of default occurring within t and $t + \Delta t$ is equal to

$$S(t) - S(t + \Delta t) = S(t)\lambda(t)\Delta t, \tag{2}$$

which follows from

$$\frac{dS(t)}{S(t)} = -\lambda(t)dt\ with\ S(0) = 1, \tag{3}$$

when taking the limit $\Delta t \to 0$. This leads to the $Q(t)$ being equal to

$$Q(t) = 1 - S(t) = 1 - e^{-\int_0^t \lambda(u)du}, \tag{4}$$

where $Q(t)$ is the probability of default by time t. The credit derivative approach is a relatively simple 'rule of thumb' approach for which relevant pricing parameters are readily available in the market and is based on the standard Black-Scholes assumptions.

However, in order to accurately price contingent capital bonds the Black-Scholes assumptions are no longer feasible. Assumptions like constant and known interest rates and log normally distributed returns are not consistent with reality and have a influence on the valuation of the bond. Furthermore, this approach becomes difficult if the temporary and partial aspects of the write-down, the possibility of a write-up and the coupon adjustments need to be incorporated.

Equity derivative approach

The contingent capital bond is priced from the perspective of an equity derivative specialist, who sees the bond as combination of long and short positions in financial products (De Spiegeleer, 2012). The payoffs of a complete write-down contingent capital bond can be replicated by taking a long position in a corporate bond (CB) of the issuer and short positions in k binary down and in options (BDI_i) for each coupon payment C_i

$$\textit{Contingent capital bond value} \approx CB + \sum_{i=1}^{k} BDI_i, \tag{5}$$

with the shares of the issuer as underlying for the option. Before write-down the contingent capital bond behaves like the long position in the CB with a face value F and coupon payments C_i. When the trigger event occurs, the contingent capital bond is completely written down and the holder will not receive the coupon payments any more. The cancellation of the coupon payments C_i are captured by the BDI_i. The short positions in the BDI_i options are equal to the coupon values C_i and will cancel out these coupon payments after a trigger event occurs. A closed form solution is possible when all Black-Scholes assumptions are considered. The present value of the CB is equal to

$$PV_{CB} = Fe^{-rT} + \sum_{t=1}^{T} C_i e^{-rt}. \tag{6}$$

The present value of the BDI_i options is equal to (De Spiegeleer, 2012)

$$PV_{BDI} = \sum_{i=1}^{k} C_i e^{-rt_i}[\phi(-x_{1i} + \sigma\sqrt{t_i}) + (S^*/S)^{2\lambda-2}\phi(y_{1i} - \sigma\sqrt{t_i})] \tag{7}$$

with

$$x_{1i} = \frac{log(S/S^*)}{\sigma\sqrt{t_i} + \lambda\sigma\sqrt{t_i}} \tag{8}$$

$$y_{1i} = \frac{log(S^*/S)}{\sigma\sqrt{t_i} + \lambda\sigma\sqrt{t_i}} \tag{9}$$

$$\lambda = \frac{r - q + \sigma^2/2}{\sigma^2} \tag{10}$$

where S is the share price, S^* is the share price barrier, t_i is the maturity date of C_i, σ is the volatility, r is the risk-free continuous interest rate, q is the continuous dividend yield and ϕ denotes the standard cumulative normal distribution.

Relevant pricing parameters are readily available in the market, but this is a highly involved and complex implementation in the case of discontinuous returns. The jumps in the trigger process, which are an important aspect for determining contingent capital bond value, cause discontinuity. The BDI options rely on the standard Black-Scholes assumptions. However, in order to accurately price contingent capital bonds the Black-Scholes assumptions are no longer feasible. Furthermore, as for the credit derivative approach, this approach becomes difficult if the temporary and partial aspects of the write-down, the possibility of a write-up and the coupon adjustments need to be incorporated.

Structural approach

The structural model was pioneered by Merton in 1974 (Merton, 1974) and provides a natural pricing framework for capital ratio triggered contingent capital bonds. Under this approach the assets on the balance sheet are modeled using a stochastic model. This approach assumes an endogenous default event, as opposed to the exogenous default event in the credit derivative approach. In the structural approach the total value of the assets held by the banks is equal to the sum of all the capital instruments.

To incorporate the dynamics of the CoBo investigated in this research, a simulation based pricing method like the structural approach is used. A Monte Carlo simulation based pricing model is able to capture the contractual specifications of the CoBo. Through simulation, features like temporary and partial write-down and coupon adjustments can be incorporated. A trigger process will be simulated, which simulates possible trigger events. Every simulated path leads to different write-down and coupon cancellations moments and different write-down amounts. Based on the simulated path it is also determined if the CoBo is written up or redeemed at certain points in time.

The pricing approach consists of two stages: a simulation stage in which the trigger process determines which coupons are canceled and how much of the principal is written down. This stage ends when the CoBo is redeemed or is completely written down. In the second stage the value of the CoBo is determined by discounting the remaining coupons and principal. The price of the CoBo is approximated by taking the average of the simulations. The approach is described in detail in Section 6.

6 Methodology

This section starts with a motivation for, and a general description of, jump processes. Secondly the two jump processes used in this research are discussed. Thereafter the calibration technique is described and finally the pricing model is explained in detail.

Jump diffusion models

For the simulation part the trigger process is described by the share price simulated through a jump diffusion process. The first reason for choosing a jump diffusion model is that they capture some important empirical phenomena commonly seen in share prices. By using a jump-diffusion model for the asset return you can capture the leptokurtic features and volatility smile that asset return distributions tend to have (Kou, 2007). In other words, the return distribution differs from a normal distribution through the presence of heavier tails and higher peak than those of the normal distribution. The volatility smile revers to the implied volatility resembling a so called smile, meaning that it is a convex curve of the strike price. Both the lognormal jump diffusion model and the double exponential jump diffusion model capture these leptokurtic features, in contrast to the Black Scholes model.

Secondly, the jumps are important to capture the influential impact of sudden changes in the capital ratio on the price of the contingent capital bond. When the capital ratio only gradually declines, as in a geometric Brownian motion setting, the bank can anticipate by increasing the equity or decreasing the denominator of the capital ratio. Jumps cause a sudden, unanticipated change in the capital ratio, which could lead to sudden coupon cancellations and write-downs. The high peak and heavy tails of the distribution also enables it to not only capture overreactions by the market (heavy tails) but also under-reaction (high peak).

Another important reason is the existence of closed form solutions for standard call and put options. This enables calibration of the parameters on call option data.

Finally, jump diffusion models have some economical interpretation. The jumps resembles the market response, or the markets overreaction, caused by financial turmoil, released accounting information, default of assets and sudden changes in asset prices (Fama, 1998). A downside of using jump diffusion models is that these models are unable to capture volatility clustering, which is also commonly present in asset returns (Cont, 2005).

The share price under risk-neutral probability measure Q follows the jump diffusion process. Before the process is described in detail, some introduction to jump models is in order. The jump diffusion model consists of two parts; the diffusion part and the jump part. The diffusion part is modeled with a Brownian motion and the jump part with a Poisson process. The Brownian motion is a continuous time random process. The process $W = (W_t : t \geq 0)$ is a Q-Brownian motion (Q-BM) if and only if

1. W_t is continuous and $W_0 = 0$.

2. The value of W_t is distributed (under Q) as a normal random variable $N(0, t)$.

3. The increment $W_{s+t} - W_s$ is distributed as normal $N(0, t)$ (under Q) and is independent of $\mathcal{F}_s$, the history of what the process did up to time s.

The Brownian motion part of the model constructs the continuous path and the Poisson process accounts for the jumps. Denote a sequence $[\tau_i]_{i \geq 1}$ of independent exponential random variables with parameter λ. The cumulative distribution function is denoted as $P[\tau_i \geq y] = e^{-\lambda y}$. The Poisson process with parameter λ is equal to

$$N_t = \sum_{n \geq 1} 1_{t \geq T_n}, \tag{11}$$

with $T_n = \sum_{i=1}^{n} \tau_i$. The jumps occur at times T_i and the intervals between jumps are exponentially distributed. The jump size in this process is equal to 1. The Poisson process can be generalized to a process with arbitrary jump sizes. This is called the compound Poisson process, in which interval between jumps is still exponentially distributed, but jumps sizes can have an arbitrary distribution. Jump diffusion models are becoming increasingly popular in finance. The most important reason for this increased popularity for jump models is the presence of jumps observed in prices. These sudden large price movements can not be captured by the Brownian motion, unless a unrealistic high value of volatility is used. From the perspective of a risk manager, incorporating jumps in the model allows the risk manager to take into account the large price movements in small time intervals (Tankov, 2009).

Lognormal jump diffusion process

The lognormal jump diffusion model has three additional parameters compared to the traditional Black Scholes model, which only has r and σ. With these three additional parameters λ, μ_y and σ_y the lognormal jump diffusion model tries to capture the skewness and excess kurtosis. Jumps are lognormally distributed and occur following a Poisson process. Assuming $\lambda = 0$ and/or $\mu_y = \sigma_y = 0$ results in the normal Black Scholes model (Merton, 1976).

Given the risk-neutral share price process

$$S_t = S_0 \exp[(r - \frac{\sigma^2}{2} - \lambda k)t + \sigma Z_t + \sum_{k=1}^{N_t} Y_k] \tag{12}$$

derived from the following lognormal jump diffusion process for the share price

$$\frac{dS_t}{S_t} = (r - \lambda k)dt + \sigma dZ_t + (y_t - 1)dN_t, \tag{13}$$

where r is the instantaneous expected return on the share, σ is the instantaneous volatility of the share return conditional on the jumps does not occur, Z_t is a standard Brownian motion process and N_t is an Poisson process with intensity λ

$$dN_t = \begin{cases} 1 & \text{if jump occurs} \\ 0 & \text{if no jump occurs} \end{cases}$$

It is assumed that Z_t, N_t, and y_t are independent. The absolute jump size y_t is a nonnegative random variable drawn from lognormal distribution such that the relative price jump size, $y_t - 1$, is lognormally distributed with mean $E[y_t - 1] \equiv k = exp(\mu_y + \frac{1}{2}\sigma_y^2) - 1$,

$$(y_t - 1) \sim i.i.d.Lognormal(k, e^{2\mu_y + \sigma_y^2}(e^{\sigma_y^2} - 1)). \tag{14}$$

This is equivalent to saying that the log jump size $ln(y_t) \equiv Y_t$ is a normal random variable

$$Y_t = ln(y_t) \sim i.i.d.\mathcal{N}(\mu_y, \sigma_y^2). \tag{15}$$

Double exponential jump diffusion model

The double exponential jump diffusion model captures two important features of assets; the asymmetric leptokurtic feature and the volatility smile. The asymmetric leptokurtic feature captures the feature that return distributions of assets have a higher peak and two asymmetric heaver tails than assumed under a normal distribution. The volatility smile captures the feature that the implied volatility is not constant but resembles a convex curve of the strike price (Kou, 2002). Jump occurs following the same Poisson process as in the lognormal jump diffusion model, but the jumps follow a double exponential distribution instead of lognormal distribution. The difference with the lognormal jump diffusion model is the distinction between positive and negative jumps. The lognormal jump diffusion model only defines a mean of the jump, while the double exponential jump diffusion model contains parameters for the mean of positive jumps and negative jumps separately and a parameter for the probability of a positive jump. This feature is interesting to evaluate, as the data shows differences between positive and negative outliers. Again, assuming $\lambda = 0$ and/or $\mu_y = \sigma_y = 0$ will results in the normal Black Scholes model.

Given the risk-neutral share price process

$$S_t = S_0 \exp[(r - \frac{\sigma^2}{2} - \lambda\xi)t + \sigma Z_t] \prod_{i=1}^{N_t} V_i \tag{16}$$

derived from the following Double Exponential Jump diffusion process for the share price

$$\frac{dS_t}{S_t} = (r - \lambda\xi)dt + \sigma dZ_t + d(\sum_{i=1}^{N_t}(V_i - 1)), \tag{17}$$

where r is the instantaneous expected return on the share, σ is the instantaneous volatility of the share return conditional on the jumps not occurring, Z_t is a standard Brownian motion process and $\xi = E(V)$. Furthermore, N_t is an Poisson process with intensity λ, with λ the average number of jumps in one period and N_t the total amount of jumps in period t. V_i is a sequence of independent identically distributed nonnegative random variables such that $Y = log(V)$ has an asymmetric double exponential distribution with density

$$f_Y(y) = p\eta_1 e^{-\eta_1 y} I_{y \geq 0} + q\eta_2 e^{\eta_2 y} I_{y<0}, \tag{18}$$

under the restrictions $\eta_1 > 1, \eta_2 > 0$ and where $p, q \geq 0$, $p + q = 1$ represent the probabilities of an upward or downward jump, respectively, and I_A denotes an indicator function equal to 1 if A is true. The requirement $\eta_1 > 1$ ensures that $\xi < \infty$ and $E(S_t) < \infty$, which essentially means that the upward jump cannot exceed 100%. This is a reasonable assumption, as this is not observed in the stock market. It is assumed that Z_t, N_t, and Y are independent. η_1 represents the upward jump size with jump magnitude equal to $\frac{1}{\eta_1 - 1} * 100\%$ and η_2 represents the downward jump size with jump magnitude equal to $\frac{-1}{\eta_2+1} * 100\%$.

Denote ξ^+ and ξ^- as exponential random variables with means $1/\eta_1$ and $1/\eta_2$, respectively. Then

$$log(V) = Y \sim \begin{cases} \xi^+, & \text{with probability p} \\ \xi^-, & \text{with probability q,} \end{cases}$$

with $\xi = E(V) = q\frac{\eta_2}{\eta_2+1} + p\frac{\eta_1}{\eta_1-1}$.

Calibration

The goal of calibration is to estimate the set of parameters $\theta_1 = (r, \sigma, \lambda, \mu_y, \sigma_y)$ for the lognormal jump diffusion model and to estimate the set of parameters $\theta_2 = (r, \sigma, \lambda, p, \eta_1, \eta_2)$ for the double exponential jump diffusion model. The parameters are estimated by configuring the model to European call option premiums for multiple maturities and strike prices.

There are multiple choices for calibration methodologies but, since this research works under the risk-neutral measure, consistency with current market prices is preferred. A so called inverse problem for calibration is employed. Through calibration parameters are estimated that fit observed market European call option premiums. The optimization problem is ill-posed, meaning that several parameter sets may be equally consistent with market prices. The calibration problem becomes an optimization problem with the aim to minimize the difference between market prices and the model prices. For the optimization problem a least squares approach is used. Taking M different observed call option premiums, the formulation of the optimization problem is given by

$$\min_{\theta_i} \sum_{j=1}^{M} \left(Call_j^{market} - Call_j^{model}(\theta_i) \right)^2, \; for \; i = 1, 2. \tag{19}$$

where $Call_j^{market}$ represents market observed call option premiums and $Call_j^{model}(\theta_i)$ represent call option premiums from Model i. Model 1 represents the model with the lognormal jump diffusion process for the share price and Model 2 the model with the double exponential jump diffusion process for the share price. The derivation of the formula for the closed-form call options price for Model 1 and Model 2 are given in Appendix F and G, respectively.

A problem with the optimization problem in Equation 19 is that it is a non-convex function with no particular structure. This means that most gradient-based optimization techniques can guarantee a local minimum, but unfortunately not a global one. The optimization techniques require an initial set of parameters to start the optimization. If this initial set of parameters is close enough to the global minimum the optimization will result in convergence towards the global minimum. For this reason the global optimization technique called Genetic Algorithm is used to find an initial parameter set close to the global minimum (Scrucca, 2013). The Genetic Algorithm technique takes an arbitrary population of candidate solutions of the optimization problem and evolve them towards better solutions. Statistically it guarantees an optimal solution. A drawback of this technique is the slow computational speed. When a good initial parameter set is found a local minimum is found using the Nelder-Mead optimization algorithm (Kelly, 1999). There may still be several, or non, solutions to the optimization problem. There have been proposed so called regularization methods to make the problem well-defined by introducing additional information. However, this is outside the scope of this research.

Another issue is the infinite sum in the call option formula for the lognormal jump diffusion model in Equation 42 and for the double exponential jump diffusion model in Equation 46. In order to calibrate the model on the call option premiums an estimation of this infinite sum is needed. To approximate the infinite sum with a finite sum a cut-off value is determined. By looking at the value of the Poisson loading ($\frac{e^{-\lambda T}(\lambda T)^i}{i!}$) and determining when this value becomes negligible, a cut-off value can be determined.

Pricing model

Here the features of the CoBo described in Section 3 are incorporated in the pricing model.

The Prevailing Principal amount at time t is denoted as $CoBo_t$ and is denoted in EUR million. $CoBo_t$ changes over time due to write-downs and write-ups.

One of the requirements for the CoBo to count towards AT1 capital is that there can be no incentive to redeem. This means there can be no step-up of the coupon. Therefore, the new coupon rate determined on Reset Date is described in the prospectus as the Reference Rate plus a fixed amount of x bps. The Reference Rate part of the coupon rate makes sure that the new coupon rate is adjusted upwards or downwards depending on the market situation. After five or ten years, and every five years thereafter, a new coupon rate is set for five years. These coupon rate adjustment dates are called Reset Dates, when $t = RD$. The Reference Rate and the fixed part are denoted as cs_{CoBo} and cr_{RD}, respectively. The CoBo coupon rate, denoted as v_{RD}, is constant between the Reset Dates and the coupons are paid annually. On each Reset Date the coupon rate is thus set equal to cr_{RD} plus some spread cs_{CoBo}

$$v_{RD} = cr_{RD} + cs_{CoBo}. \tag{20}$$

The coupon payments, denoted by C_t, are also denoted in EUR million and can change during the period up to Reset Date (and in between Reset Dates) as $CoBo_t$

changes over time such that the annual coupon at time t equals

$$C_t = v_{RD}\ CoBo_t. \tag{21}$$

In this research the trigger process is simulated by the share price described by Equation 12 or Equation 16. In order to simulate trigger events, a trigger level needs to be estimated. Therefore, a share price level that corresponds with the CET1 capital ratio trigger level is needed. This trigger level is denoted in the prospectus.

At each point in time the share price is evaluated. If the share price is below the trigger level, the CoBo takes action. A coupon cancellation or principal write-down increases equity, as described in Section 3. The coupon cancellation increases the equity with C_t and thus increases the share price with $C_t\ /\ \dot{C}$. If this does not increase the share price to above the trigger level, the principal is written down. This write-down value equals the amount necessary to increase the share price to its trigger level, with the Prevailing Principal amount as maximum write-down value.

$$\textit{write-down value} = min\left[\left(\dot{S} - (S_t + \frac{C_t}{\dot{C}})\right)\dot{C}, CoBo_t\right], \tag{22}$$

with $\dot{S}$ the trigger level of the share price. After a write-down, the share price increases with $min\left[\left(\dot{S} - (S_t + \frac{C_t}{\dot{C}})\right), \frac{CoBo_t}{\dot{C}}\right]$. In case of a write-down, the future coupon payment C_t are adjusted to account for the new Prevailing Principal amount.

When the process reaches the First Call Date, which is after five or ten years, the redemption option of the issuer is evaluated. On this First Call Date the issuer has the option to redeem the entire Prevailing Principle. The issuer will want to redeem the CoBo when the CoBo is expensive relative to the current market situation. Simultaneously with the First Call Date is the Reset Date. The fixed part cs_{CoBo} of the new coupon definition determines for a large part the coupon value. The level of this fixed part is determined at issuance. The safer the bank is at issuance, the lower the risk, the lower the demand for return and the lower cs_{CoBo}. When the capital ratio is high with respect to the capital ratio at issuance, the probability of a trigger event has decreased. When this probability decreases, the bank can lower

cs_{CoBo}. If the gain from a lower coupon exceeds the costs of issuing new (contingent) capital, the bank executes its option to redeem.

Another reason for a bank to redeem the CoBo when the capital ratio is high with respect to the capital ratio at issuance is because in this situation the bank has enough common equity. When the capital ratio is high, the bank must have a lot of common equity and therefore does not need the AT1 capital in order to maintain a sufficient capital ratio. In this situation, it would be better for the bank to issue normal bonds, as these are cheaper. The bank could redeem the CoBo in order to lower the cost of debt.

Therefore, the issuer will execute his redemption right when the capital ratio is high with respect to the capital ratio at issuance. This corresponds with the issuer redeeming when the share price is high with respect to the share price at issuance in this research. In the model it is assumed the bank redeems the CoBo when the share price has increased above a certain upper level relating to an increase in capital ratio of more than 4%.

When the bank redeems the CoBo, equity decreases with an amount equal to the Prevailing Principal amount. This causes the capital ratio to decrease. One of the requirements for a redemption is that the capital ratio stays above the trigger level with a certain margin. Before the redemption can occur, the model checks if the share price stays above a certain upper level after the redemption, which corresponds with the target CET1 ratio denoted in the annual report. If this requirement is not met the CoBo is not redeemed. This redemption option is thus evaluated on the First Call Date and every five years thereafter, on the Call Dates. This extension of the maturity on Call Dates is evaluated until around 99% of all simulated CoBos are redeemed or completely written down and further extension of the maturity has a negligible effect on the value of the CoBo.

The bank has the option to write-up the Prevailing Principle. A write-up can only occur after a write-down and the write-up value is limited to the *Current write-*

down amount. The *Current write-down amount* is the results of all write-downs and write-ups up to time t. There is no predetermined event after which a write-up occurs, in contrast to trigger level for the write-down. The issuer has full discretion to reinstate any portion of the *Current write-down amount.* The issuer will only decide to write-up the Prevailing Principle when the capital ratio has been restored well above the trigger level. The model assumes the issuer writes up when the share price breaches an upper level corresponding to the 10% target CET1 ratio.

An important requirement for a write-up to occur is that it will not cause a trigger event. A write-up goes at the expense of the equity capital, such that the capital ratio will decrease after a write-up. Therefore, the *write-up value* is limited to an amount such that the share price after write-up stays above an upper level corresponding again to the target CET1 ratio.

Furthermore, the model assumes this write-up occurs with a certain probability, as a write-up is uncertain. There is no way of determining the likelihood of a write-up occurring. The probability of a write-up, given that the above conditions are met, is set on 50%. The *write-up value* itself is also uncertain. Therefore, the value of the write-up is equal to a standard uniformly distributed variable $b \sim \mathcal{U}(0,1)$ times the *Current write-down amount*

$$\textit{Write-up value} = b \; * \; \textit{Current write-down amount}. \tag{23}$$

This way the model incorporates the uncertainty around the write-up occurrence and the *write-up value.* A write-up will not only increase the Prevailing Principle, but will also increase the coupon. After the write-up, future coupons will be equal to the coupon rate times the Prevailing Principal.

The second part of the simulation consist of determining the value of the CoBo given a certain share price path. In the first part of the simulation it is determined which coupons get canceled, how much of the principal is written down and written up and when the CoBo is redeemed or completely written down. The remaining coupons and principal are discounted to the issue date using rf_t, the point on the five year semi-annual euro swap curve at time t, which represents the risk-free rate.

Discounting the future cash flows with the risk-free rate would not be correct, as the CoBo is not a risk-free investment. Therefore, the discount factor needs to include some extra spread incorporating the risk of the CoBo and the credit risk of the issuer. In practice it is common to use the so called asset-swap (ASW) spread. This ASW spread is determined by looking at bonds issued by the bank. Looking at the difference in bond value when the bonds cash flows are discounted with the risk-free rate and the issue price, the ASW spread can be determined. The discount factor at which the CoBo coupons and CoBo principal are discounted is thus equal to

$$P_{t,T} = rf_t \; e^{-ASW(T-t)}. \tag{24}$$

The discounted cash flows result in a value for the CoBo. Looking at the average CoBo value over the simulated paths, a value for the CoBo is determined. In total m share price paths should be simulated such that the CoBo price is converged.

7 Results Deutsche Bank CoBo

This section applies the methodology of Section 6 to the contingent capital bond issued by Deutsche Bank Aktiengesellschaft on November 18, 2014 that satisfies the requirements for AT1 capital. The CoBo issued by Deutsche Bank is called the *Undated Non-cumulative Fixed to Reset Rate Additional Tier 1 Notes* and has the following features.

- CET 1 Capital ratio trigger of 5.125%
- Partial write-down
- Coupon cancellation
- Discretionary write-up

- Optional redemption
- Regulatory redemption
- Perpetual

Deutsche Bank issued a going concern CoBo, as their CET 1 trigger level is at 5.125%, while the minimum CET 1 capital ratio set by Basel equals 4.5%.

This section starts with a motivation for the jump processes for the Deutsche Bank CoBo. Secondly, the calibration results for the two jump processes are discussed together with an interpretation of the parameters. Thereafter the pricing model is applied to the Deutsche Bank CoBo and finally the results are discussed.

Motivation jump processes

Figure 3 in Appendix B shows the histogram of the Deutsche Bank daily log returns for the period November 18, 1994 to 21 December, 2015. Summary statistics for the data are given in Table 4 in Appendix B. The figure shows clear signs of heavy tails and peakedness which confirm that a normal distribution does not fit the data. A formal Jarque Bera for normality is denoted as

$$JB = \frac{n}{6}\left(Sk^2 + \frac{Ku^2}{4}\right) \sim \chi^2_{\nu=2}, \tag{25}$$

with $n = 4804$ the number of observations, $Sk = 0.03$ the skewness, $Ku = 9.00$ the kurtosis and $\chi^2_{\nu=2}$ a Chi-squared distribution wit ν degrees of freedom. With a JB value of 16,206 normality is rejected and therefore the Black Scholes model does not fit. It is hard to see in Figure 3, but the amount and magnitude of the positive outliers is somewhat higher compared to the negative outliers. This indicates that there are more and larger positive jumps than negative jumps in the data.

Calibration results

The goal of calibration is to estimate the set of parameters $\theta_1 = (r, \sigma, \lambda, \mu_y, \sigma_y)$ for the lognormal jump diffusion model and to estimate the set of parameters ($\theta_2 = r, \sigma, \lambda, p, \eta_1, \eta_2$) for the double exponential jump diffusion model. The parameters are estimated by configuring the model to Deutsche Bank European call option premiums for multiple maturities and strike prices given in Figure 4 in Appendix C.

As denoted in Section 6, first the issue of the infinite sum in the call option formula needs to be resolved. Table 5 in Appendix D shows the value of the Poisson loading ($\frac{e^{-\lambda T}(\lambda T)^i}{i!}$) for $\lambda = 3$ and $T = 4.5$. The values first increases and then decreases to a negligible value. The speed of decay depends on the values for λ and T. The initial increase is caused by λ and T values being larger than 1 for most estimations. Therefore, the infinite sum is approximated with a finite sum to the 50th item. The calibration resulted in the following values for the parameters of the lognormal jump diffusion model at 18 November, 2014.

Parameters	r	σ	λ	μ_y	σ_y
Values	-0.065	0.230	3.128	0.059	0.013

Table 1: Lognormal jump diffusion model parameters

The value of the optimization problem of Equation 19 for Model 1 is equal to 1.117. The instantaneous expected return on the share r includes the dividend paid and is large negative. This is due to the low interest environment in the last few years, while still paying dividend. The implied volatility σ is equal to 23.0%, taking into account the high volatility period. The model estimates that on average $\lambda = 3.128$ jumps occur per year. These jumps have an relative jump size of $k \equiv E^Q[Y_q - 1] = e^{\mu_y + \frac{1}{2}\sigma_y^2} - 1 = 6.086\%$. The volatility of the jumps is equal to $e^{2\mu_y + \sigma_y^2}(e^{\sigma_y^2} - 1) = 0.019\%$. Jumps in the prices are thus large positive, compensating partly for the large negative r.

An infinite sum for the Poisson loadings is also present in the call option formula for the double exponential jump diffusion model. To approximate the infinite sum with a finite sum a cut-off value is again determined. Due to λ being close to 1 for the double exponential jump diffusion model, the cut-off value is lower than for the lognormal jump diffusion model. Table 6 in Appendix D shows that the value of the Poisson loading first increases and then decreases to a negligible value for the 35th item. Therefore, the infinite sum is approximated with a finite sum to the 35th item. Due to the complexity of the closed form solution the computation time for the double exponential jump diffusion model is significantly larger than for the lognormal jump diffusion model. To lower the computation time only the call option premiums for the maturities 1M, 6M, 1Y, 2Y, 3Y, 4Y, 4.5Y and strikes €25 to €31 are used.

The calibration resulted in the following values for the parameters of double exponential jump diffusion model at 18 November, 2014.

Parameters	r	σ	λ	p	η_1	η_2
Values	-0.010	0.160	1.873	0.658	13.055	39.745

Table 2: Double exponential jump diffusion model parameters

The value of the optimization problem of Equation 19 for Model 2 is equal to 0.260. The instantaneous expected return on the share r includes the dividend paid and is again negative. The implied volatility σ is equal to 16.0%. The model estimates that on average $\lambda = 1.873$ jumps occur per year. It is more likely that a jump is positive, as the probability of an upward jump is 65.8%. Furthermore, the expected magnitude of an upward jump is $\frac{1}{\eta_1 - 1} * 100\% = 8.295\%$, which is higher than the expected magnitude of $\frac{-1}{\eta_2+1} * 100\% = -2.454\%$ of downward jump. Positive jumps are estimated to occur more often and have a larger magnitude than negative jumps, which is also indicated by the data.

The double exponential jump diffusion model parameters estimate less negative return r and lower implied volatility compared to the lognormal jump diffusion model. The double exponential jump diffusion model also expects less jumps than the lognormal jump diffusion model. However, the expected magnitude of (negative) positive jumps in the double exponential jump diffusion model is (lower) higher than the expected magnitude of jumps in the lognormal jump diffusion model.

Pricing model for Deutsche Bank CoBo

The values for the different variables of the pricing model for the Deutsche Bank CoBo and how they are determined are described next.

The starting value $CoBo_0$ is equal to the initial principal amount of €1200 million, as denoted in the prospectus. $CoBo_t$ changes over time due to write-downs and write-ups caused by trigger events.

The new coupon rate determined on Reset Date is described in the prospectus as the Reference Rate plus a fixed amount of x bps. After ten years, and every five years thereafter, a new coupon rate is set for five years. The Reference Rate cr_{RD} is equal to the five year semi-annual mid-swap rate for euro swap transactions at Reset Date. The fixed part cs_{CoBo} is equal to *500.3* bps. As described in the prospectus, v_{RD} is equal to *750* bps for the first ten years.

The trigger process described in the prospected is equal to the CET1 ratio, which is the CET1 capital divided by RWA. In this research the trigger process is simulated by the share price described by Equation 12 or Equation 16. In order to simulate trigger events, a trigger level needs to be estimated. Therefore, a share price level that corresponds with the 5.125% CET1 capital ratio trigger level is needed. Looking at the Deutsche Bank 3Q2014 interim report the CET1 capital was equal to €46.0 billion and the fully loaded CET1 ratio was 11.5% at the end of the third quarter of 2014. The share price at end 3Q2014 equaled €27.78. It is therefore assumed that €1.00 in share price corresponds with €1655.87 million in CET1 capital. Let $\dot{C}$ denote this €1655.87 million in CET1 capital. Taking RWA constant over time, a 1 percent point change in the CET1 capital ratio corresponds with a €4000.00 million change in CET1 capital. Hence, it is assumed that a 1 percent point change in the CET1 capital ratio corresponds with a €2.416 change in share price. The trigger level is thus equal to a share price of €12.38.

When the process reaches the First Call Date, which is after ten years, the redemption option of the issuer is evaluated. The issuer will execute his redemption right when when the share price is high with respect to the share price at issuance. In the model it is assumed the bank redeems the CoBo when the share price has increased more than €9.66 with respect to share price at issuance. This relates to an increase in capital ratio of more than 4%.

After redemption equity decreases with an amount equal to the Prevailing Principal amount. To assure that the share price stays above the trigger level the model checks if the share price stays above €24.16 after the redemption, which corresponds with the 10% target CET1 ratio denoted in the 3Q2014 report. If this requirement is not met the CoBo is not redeemed. This redemption option is thus evaluated on the First Call Date and every five years thereafter, on the Call Dates. This extension of the maturity on Call Dates is evaluated until the fourth Call Date. The model thus assumes that all CoBos not yet redeemed or completely written down are redeemed after 25 years. The reason for choosing 25 years is that the model has shown that around 99% of all simulated CoBos are redeemed or completely written down after 25 years and further extension of the maturity has a negligible effect on the value of the CoBo.

The issuer will only consider to write-up the Prevailing Principle when the share price has been restored well above the trigger level. The model assumes the issuer writes up when the share price breaches an upper level of €24.16, which corresponds with the 10% target CET1 ratio. The *write-up value* is limited to an amount such that the share price after write-up stays above €24.16, corresponding again to the target CET1 ratio of 10%. The model assumes this write-up occurs with a certain probability as described in Section 6.

The second part of the simulation consists of determining the value of the CoBo given a certain share price path. Looking up the CoBo of Deutsche Bank in Bloomberg the ASW spread is equal to 491 bps. The discount factor at which the CoBo coupons and CoBo principal are discounted is thus equal to

$$P_{t,T} = fr_t \; e^{-(491/10000)(T-t)}, \tag{26}$$

with rf_t, the point on the five year semi-annual euro swap curve at time t.

In total there are $m = 10,000$ share price paths simulated. The discounted cash flows result in a value for the CoBo. Looking at the average CoBo value over the simulated paths, a value for the CoBo is determined. As can be seen in Figures 5 and 6 in Appendix E the CoBo price is converged after $m = 10,000$ simulations. Computation time is approximately 4 minutes for Model 1 and approximately 5 minutes for Model 2. Furthermore, to assess the performance of the model over time the model is used to price the CoBo using almost one year of data. The results of the simulations are presented in the next section.

Results pricing model

Here the results of the simulation described in Section 7 are discussed using data from November 18, 2014 to October 9, 2015. At each point in time share price and call option premium data is used to determine parameter values for that same point in time. Given the parameters, share price paths are simulated with the two jump diffusion models. Finally, the methodology described in Section 6 leads to CoBo prices for the Deutsche Bank CoBo for every moment in time. Figure 1 shows the models CoBo prices together with the observed CoBo price. The prices are given as a percentage of par value. Model 1 again represents the model with the lognormal jump diffusion process for the share price and Model 2 the model with the double exponential jump diffusion process for the share price.

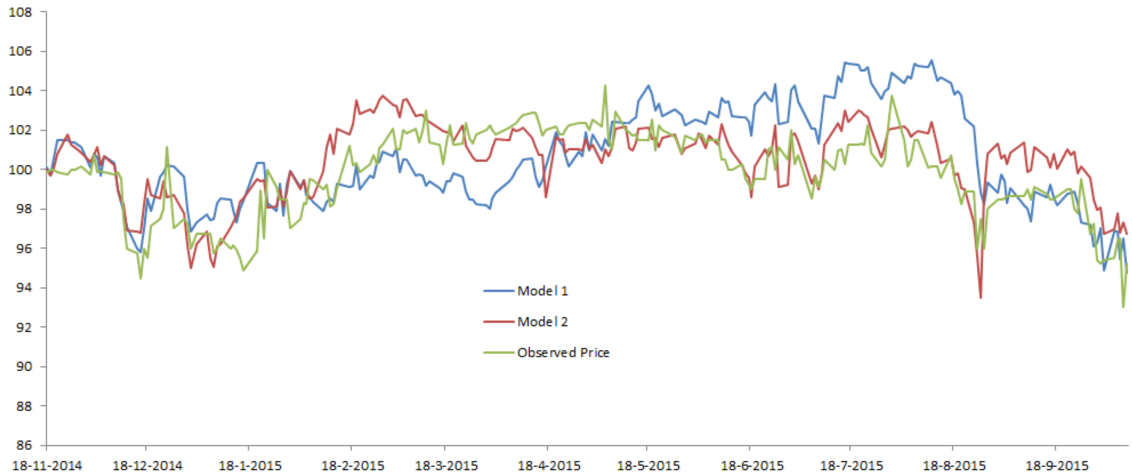

Figure 1: Model CoBo prices and observed CoBo prices for the period November 18, 2014 to October 9, 2015 (220 daily observations).

Denote P_t as the observed CoBo price at time t and $\hat{P}_{t|\mathcal{F}_t}$ the model price for time t, with $\mathcal{F}_t$ the information on share price and call option premiums up to time t. $\mathcal{F}_t$ does not contain P_t, as the model tries to determine a price without using information on the price of the CoBo itself. The pricing model thus determines a price of a CoBo on a certain day using information that is available on that same day. Pricing errors are computed as $e_{t|\mathcal{F}_t} = P_t - \hat{P}_{t|\mathcal{F}_t}$. The accuracy of the models is evaluated by the mean squared error (MSE) and mean absolute error (MAE), which should be as small as possible. Given a set of $N+1$ determined CoBo prices for $t = T, ..., T+N$ the MSE is computed as

$$MSE = \frac{1}{N+1} \sum_{t=T}^{t=T+N} e^2_{t|\mathcal{F}_t} \tag{27}$$

and the MAE is denoted as

$$MAE = \frac{1}{N+1} \sum_{t=T}^{t=T+N} |e_{t|\mathcal{F}_t}|. \tag{28}$$

Table 3 gives accuracy properties of Model 1 and Model 2.

	Model 1	Model 2
MSE	5.34	2.43
MAE	1.89	1.25

Table 3: Accuracy statistics

Model 1 has higher MSE and MAE compared to Model 2. To formally compare the MSE the Diebold-Mariano test is used. Denote the difference between the squared pricing errors between Model 1 and Model 2 as $d_t = e^2_{t|\mathcal{F}_t,1} - e^2_{t|\mathcal{F}_t,2}$. To test the null hypothesis of equal pricing accuracy, $E[d_t] = 0$, the Diebold-Mariano statistic (DM) is used to test if the sample mean of d_t is not significantly different from zero.

$$DM = \frac{\bar{d}}{\sqrt{\hat{V}(d_t)/N}} \overset{a}{\sim} \mathcal{N}(0,1), \tag{29}$$

with $\bar{d}$ the sample mean of d_t and $\hat{V}(d_t)$ an estimate of the variance of d_t computed as

$$\hat{V}(d_t) = \frac{1}{N-1} \sum_{t=T}^{t=T+N-1} (d_t - \bar{d})^2. \tag{30}$$

The DM statistic is equal to 6.06 and therefore Model 2 is more accurate based on a 99% critical value of 2.33.

The volatility of Model 1 is equal to 2.48%, compared to 1.90% for Model 2. Model 1 (Model 2) is somewhat more (less) volatile than the observed CoBo prices, which has a volatility equal to 2.15%. As can be seen in the figure 1, Model 1 is capable of following the upward and downward movements of the CoBo. The correlation between Model 1 prices and observed CoBo prices is equal to 56%. Model 2 is more capable of following the trend as the correlation between Model 2 prices and observed CoBo prices is equal to 76%.

Looking at the the accuracy and correlation between model prices and observed price the model using a double exponential jump diffusion share price process can better determine the price of a CoBo than the model with a lognormal jump diffusion share price process. Both models are able to accurately determine CoBo prices when data on the CoBo itself is unavailable.

8 Conclusion

In this research a method is developed for pricing a perpetual temporary write-down and coupon cancellation contingent capital bond without the availability of historical data on the bond itself. This particular contingent capital bond is designed to adhere to the new Basel III requirements for AT1 capital. Specific features are an accounting trigger, coupon cancellation, partial write-down, discretionary write-up, optional redemption and perpetual maturity. The pricing method is based on a structural approach using Monte Carlo simulation. An important aspect of the pricing method is the simulation of the trigger process. The trigger process is simulated by a share price process using jump diffusion models. The results show that a model with the double exponential jump diffusion process for the share price is more accurate in determining CoBo prices than a model with the lognormal jump diffusion process for the share price. The prices of the double exponential jump diffusion trigger process based model also has a significant higher correlation with the observed prices compared to the lognormal jump diffusion trigger process based model.

The purpose of contingent capital is to provide an automatic strengthening of the capital structure of the bank to prevent an institution from entering into a possible recovery or resolution proceeding. The CoBo described in this research is capable of increasing equity when the banks equity is low. The effect of the CoBo depends on the amount of CoBo capital issued relative to an issuer's total assets.

Currently, cost on equity is higher than the cost on CoBos, as CoBos are formally classified senior to equity. Nevertheless, CoBos get hit before (and are subordinated to) equity through coupon cancellations and write-downs. In case of a bankruptcy, CoBo holders would rank before equity holders and would get paid before equity holders. Nevertheless, going concern CoBos would already be completely written down before bankruptcy occurs. Furthermore, under CRD IV, bank management could legally turn off CoBo coupon payments but continue to pay common share dividends. This makes the seniority classification of CoBos with respect to equity a bit ambiguous.

Following the words of Donald Rumsfeld
"Reports that say that something hasn't happened are always interesting to me, because as we know, there are known knowns; there are things we know we know. We also know there are known unknowns; that is to say we know there are some things we do not know. But there are also unknown unknowns; the ones we do not know we do not know. And if one looks throughout the history of our country and other free countries, it is the latter category that tend to be the difficult ones."
and applying these to the CoBo, it can be stated that there are *known knowns*. The *known knows* are for example the trigger level. There are also some *know unknowns* driving the price of the CoBo. These are mainly the shocks to the capital ratio, the triggering of the capital ratio by regulatory authority and some other discretionary rights the issuer has on the write-up and the redemption of the CoBo. Aside from the *know unknowns* there are still some *unknown unknowns* surrounding the CoBo, as no trigger event has taken place so far and it is therefore unknown how the market would truly react to a trigger event. This make the CoBo a difficult financial product to price and investors should invest with caution. From a bank's perspective the CoBo is a product suitable for restoring the capital ratio during financially difficult times. From a regulator's point of view the CoBo could decrease the possibility of a bank bail out using tax payer's money. The issuance of contingent capital bonds are expected to grow significantly the coming years and different types of the contingent capital bonds are expected to enter the market. When time passes, and trigger events have occurred, future research could look at the effects and the impact of the

current *unknown unknowns*.

Even though this research incorporates most features of the CoBo, there are still some aspects not taken into account in this research. An example hereof is the discretion held by the regulator to trigger a write-down of the CoBo. The EU Bank Recovery and Resolution Directive (BRRD) establishes a comprehensive recovery and resolution regime. The BRRD measures give authorities power to bail in (write off or convert into equity) AT1 and Tier 2 instruments and is effective since January 2016. This could occur before the minimum regulatory capital levels are breached. Incorporating this feature in the pricing of the CoBo would lower the value from an investors perspective, as the possible loss increases. Future research could focus on implementing this regulatory trigger in a pricing model. It is difficult to model a trigger based on actions by a regulatory authority, since there is no predefined scenario which determines when a regulator would force a write-down. This could be included in the model by modeling a dual trigger, where the regulators power to convert/write-down is modeled by systematic risk of the banking sector as a whole. See Pennacchi (2010) for info on dual pricing trigger with financial index as extra trigger (Pennacchi, 2010). Furthermore, when more data on the CoBo prices comes available the validity of the pricing models in this research can be evaluated more thoroughly.

Appendices

A Capital requirements

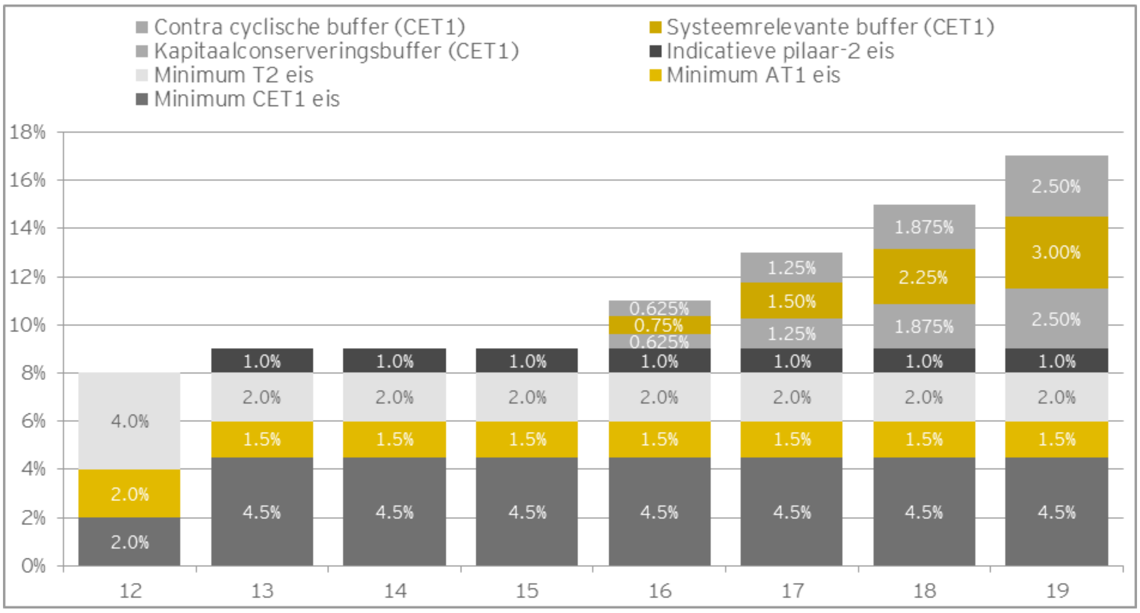

Figure 2: Capital requirements set by Basel III for 2012 to 2019

B Summary Statistic Deutsche Bank Stock

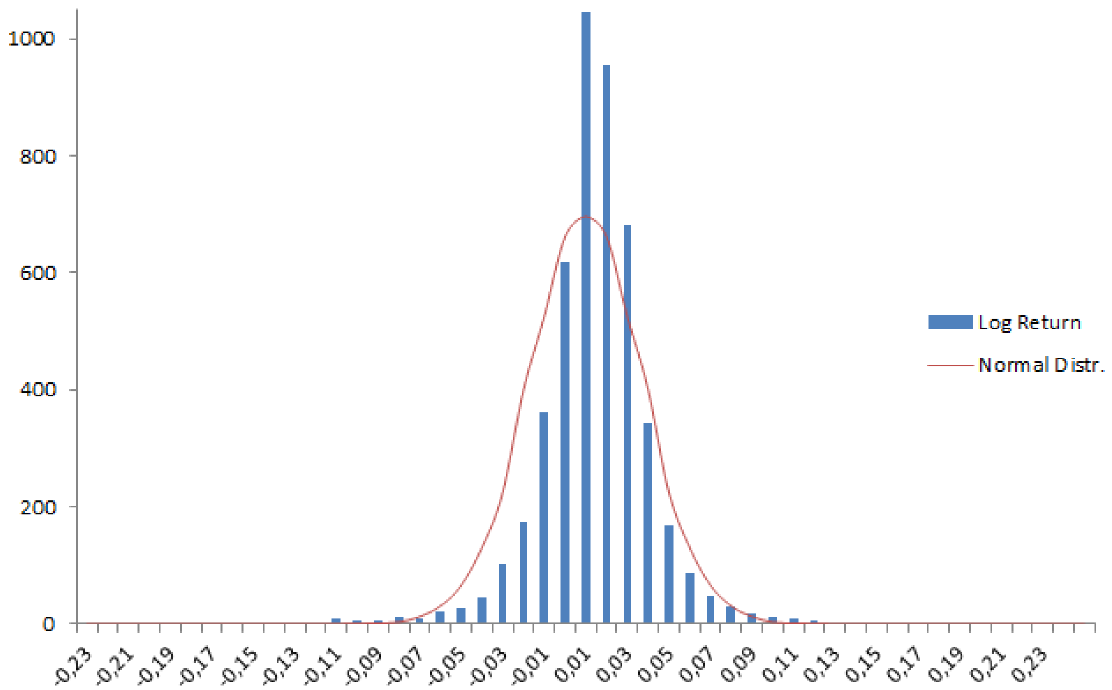

Figure 3: Histogram of daily log returns on Deutsche Bank shares for 18 November, 1994 to 21 December, 2015.

	Deutsche Bank shares
Nr. of obs.	4804
Mean	-0.0001
Std. dev.	0.0271
Skewness	0.0300
Kurtosis	8.9976
Jarque Bera	16206

Table 4: Summary statistics for daily log returns on Deutsche Bank shares for 18 November, 1994 to 21 December, 2015.

C Call option premiums

Strikes \ Maturities	1M	6M	1Y	1.5Y	2Y	2.5Y	3Y	3.5Y	4Y	4.5Y
24	0.91	1.53	2.23	2.26	2.68	2.43	2.76	2.36	2.63	2.27
25	0.42	1.15	1.82	1.91	2.33	2.13	2.45	2.10	2.37	2.04
26	0.17	0.83	1.47	1.61	2.02	1.86	2.17	1.87	2.13	1.84
27	0.07	0.60	1.18	1.34	1.74	1.62	1.93	1.66	1.91	1.65
28	0.04	0.43	0.94	1.12	1.50	1.41	1.70	1.47	1.71	1.49
29	0.02	0.30	0.75	0.94	1.29	1.23	1.50	1.31	1.54	1.34
30	0.01	0.21	0.59	0.78	1.11	1.07	1.33	1.16	1.38	1.20
31	0.01	0.15	0.46	0.65	0.95	0.93	1.18	1.03	1.24	1.09
32	0.00	0.10	0.36	0.54	0.81	0.80	1.04	0.92	1.12	0.98
33	0.00	0.08	0.28	0.45	0.70	0.70	0.92	0.81	1.00	0.88

Figure 4: Deutsche Bank call option premiums for 10 different maturities and 10 different strikes on 18 November, 2014.

D Poisson Loading

i	Poisson Loading	i	Poisson Loading	i	Poisson Loading	i	Poisson Loading	i	Poisson Loading
1	1,85E-05	11	9,32E-02	21	1,46E-02	31	1,83E-05	41	9,04E-10
2	1,25E-04	12	1,05E-01	22	8,99E-03	32	7,72E-06	42	2,91E-10
3	5,62E-04	13	1,09E-01	23	5,27E-03	33	3,16E-06	43	9,12E-11
4	1,90E-03	14	1,05E-01	24	2,97E-03	34	1,25E-06	44	2,80E-11
5	5,12E-03	15	9,45E-02	25	1,60E-03	35	4,84E-07	45	8,40E-12
6	1,15E-02	16	7,98E-02	26	8,32E-04	36	1,81E-07	46	2,46E-12
7	2,22E-02	17	6,33E-02	27	4,16E-04	37	6,62E-08	47	7,08E-13
8	3,75E-02	18	4,75E-02	28	2,01E-04	38	2,35E-08	48	1,99E-13
9	5,63E-02	19	3,37E-02	29	9,34E-05	39	8,14E-09	49	5,49E-14
10	7,60E-02	20	2,28E-02	30	4,20E-05	40	2,75E-09	50	1,48E-14

Table 5: Poisson loadings of the lognormal jump diffusion model with $\lambda = 3$ and $T = 4.5$.

i	Poisson Loading	i	Poisson Loading	i	Poisson Loading	i	Poisson Loading	i	Poisson Loading
1	7,90E-03	11	3,89E-02	21	5,96E-06	31	7,28E-12	41	3,51E-19
2	2,67E-02	12	2,19E-02	22	1,83E-06	32	1,53E-12	42	5,64E-20
3	6,00E-02	13	1,14E-02	23	5,37E-07	33	3,14E-13	43	8,86E-21
4	1,01E-01	14	5,47E-03	24	1,51E-07	34	6,23E-14	44	1,36E-21
5	1,37E-01	15	2,46E-03	25	4,08E-08	35	1,20E-14	45	2,04E-22
6	1,54E-01	16	1,04E-03	26	1,06E-08	36	2,25E-15	46	2,99E-23
7	1,48E-01	17	4,13E-04	27	2,65E-09	37	4,11E-16	47	4,30E-24
8	1,25E-01	18	1,55E-04	28	6,38E-10	38	7,30E-17	48	6,04E-25
9	9,39E-02	19	5,50E-05	29	1,49E-10	39	1,26E-17	49	8,32E-26
10	6,34E-02	20	1,86E-05	30	3,34E-11	40	2,13E-18	50	1,12E-26

Table 6: Poisson loadings of the double exponential jump diffusion model with $\lambda = 1.5$ and $T = 4.5$.

E Price Conversion

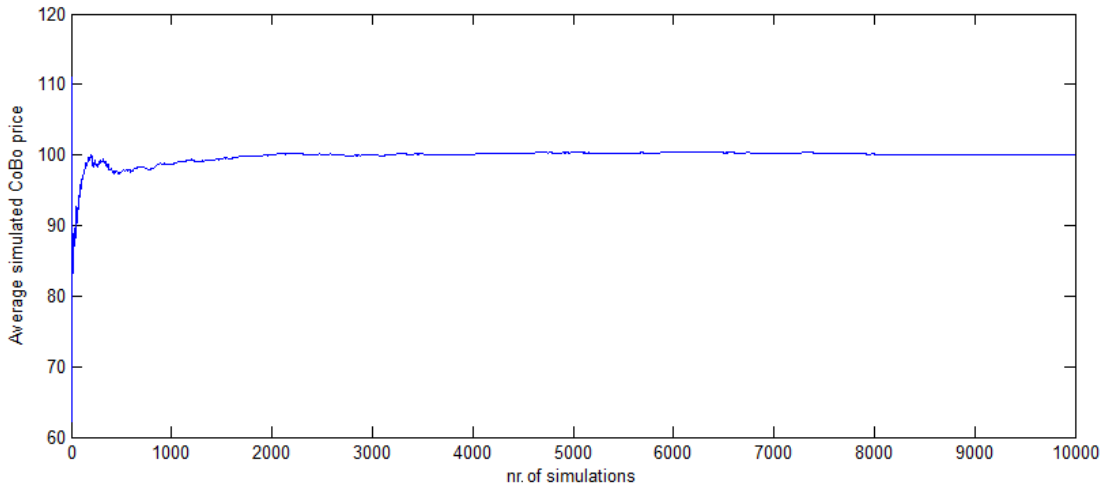

Figure 5: Conversion of the CoBo price after m=10,000 simulations for model with lognormal jump diffusion process for the share price.

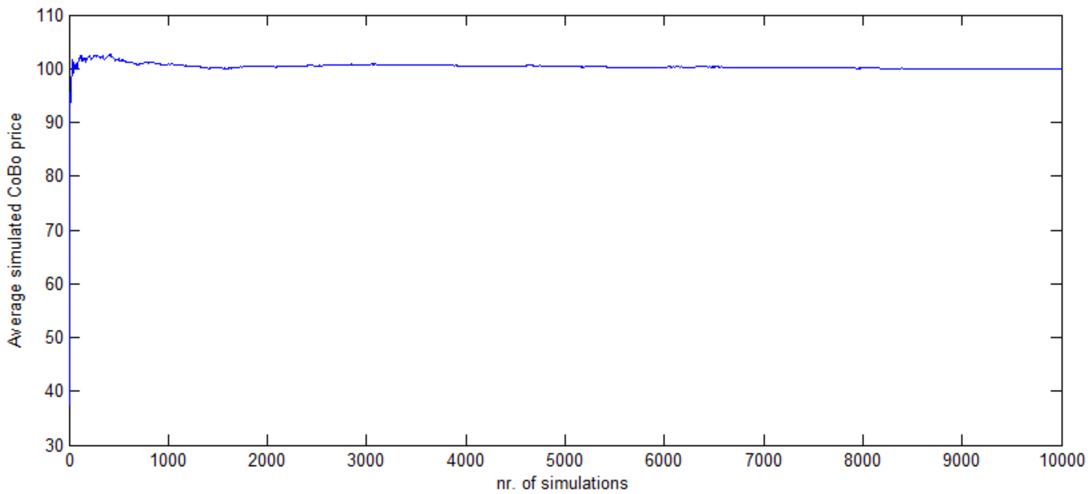

Figure 6: Conversion of the CoBo price after m=10,000 simulations for model with double exponential jump diffusion process for the share price.

F Closed form call option formula lognormal jump diffusion model

This appendix shows the derivation of the closed form formula for call option premiums with a lognormal jump diffusion share process. The European call option premium is equal to

$$C^j = e^{-rT} E^Q[(S_T - K)^+] \tag{31}$$

with S_T described by the lognormal jump diffusion model as in Section 6

$$S_T = S_0 \exp\left[(r - \frac{\sigma^2}{2} - \lambda k)T + \sigma Z_T + \sum_{k=1}^{N_T} Y_k\right] \tag{32}$$

with Poisson counter $N_T = 0, 1, 2, ..., i$ and $\sum_{k=1}^{i} Y_k \sim \mathcal{N}(i\mu_y, i\sigma_y^2)$. Therefore

$$C^j = e^{-rT} \sum_{i=0}^{\infty} \frac{e^{-\lambda T}(\lambda T)^i}{i!} E^Q[(S_T - K)^+ | N_T = i] \tag{33}$$

which is roughly said the expected call pay-off split to the possible number of jumps. The part inside the exponential in Equation 32 is normally distributed

$$(r - \frac{\sigma^2}{2} - \lambda k)T + \sigma Z_T + \sum_{k=1}^{i} Y_k \sim \mathcal{N}[(r - \frac{\sigma^2}{2} - \lambda k)T + i\mu_y, \sigma^2 T + i\sigma_y^2] \tag{34}$$

since $Z_T \sim \mathcal{N}(0, T) \implies \sigma Z_T \sim \mathcal{N}(0, T\sigma^2)$ and $Y_k \sim \mathcal{N}(\mu_y, \sigma_y^2) \implies \sum_{k=1}^{i} Y_k \sim \mathcal{N}(i\mu_y, i\sigma_y^2)$. Rewrite $(r - \frac{\sigma^2}{2} - \lambda k)T + \sigma Z_T + \sum_{k=1}^{i} Y_k$

$$(r - \frac{\sigma^2}{2} - \lambda k)T + i\mu_y + \sqrt{\frac{\sigma^2 T + i\sigma_y^2}{T}} B_T \sim \mathcal{N}[(r - \frac{\sigma^2}{2} - \lambda k)T + i\mu_y, \sigma^2 T + i\sigma_y^2], \tag{35}$$

with B_T a standard Brownian motion process. This is convenient because the summation term is gone. Equation 33 can thus be rewritten as

$$C^j = e^{-rT} \sum_{i=0}^{\infty} \frac{e^{-\lambda T}(\lambda T)^i}{i!} E^Q\left[\left(S_0 \exp\left((r - \frac{\sigma^2}{2} - \lambda k)T + i\mu_y + \sqrt{\frac{\sigma^2 T + i\sigma_y^2}{T}} B_T\right) - K\right)^+ | N_T = i\right] \tag{36}$$

because a normal distribution is uniquely determined by its mean and variance. Now add $\frac{i\sigma_y^2}{2} - \frac{i\sigma_y^2}{2} = 0$ inside the exponential of Equation 36.

$$
\begin{aligned}
C^j &= e^{-rT}\sum_{i=0}^{\infty}\frac{e^{-\lambda T}(\lambda T)^i}{i!} \\
&\quad E^Q\left[\left(S_0\exp\left((r-\frac{\sigma^2}{2}-\lambda k+\frac{i\sigma_y^2}{2T}-\frac{i\sigma_y^2}{2T})T+i\mu_y+\sqrt{\frac{\sigma^2T+i\sigma_y^2}{T}}B_T\right)-K\right)^+|N_T=i\right] \\
&= e^{-rT}\sum_{i=0}^{\infty}\frac{e^{-\lambda T}(\lambda T)^i}{i!} \\
&\quad E^Q\left[\left(S_0\exp\left((r-\frac{\sigma^2+\frac{i\sigma_y^2}{T}}{2}-\lambda k-\frac{i\sigma_y^2}{2T})T+i\mu_y+\sqrt{\frac{\sigma^2T+i\sigma_y^2}{T}}B_T\right)-K\right)^+|N_T=i\right]
\end{aligned}
\tag{37}
$$

Take $\sigma_i^{jump}=\sqrt{\sigma^2+\frac{i\sigma_y^2}{T}}$. Then Equation 37 becomes

$$
\begin{aligned}
C^j &= e^{-rT}\sum_{i=0}^{\infty}\frac{e^{-\lambda T}(\lambda T)^i}{i!} \\
&\quad E^Q\left[\left(S_0\exp\left((r-\frac{(\sigma_i^{jump})^2}{2}-\lambda k-\frac{i\sigma_y^2}{2T})T+i\mu_y+\sigma_i^{jump}B_T\right)-K\right)^+|N_T=i\right] \\
&= e^{-rT}\sum_{i=0}^{\infty}\frac{e^{-\lambda T}(\lambda T)^i}{i!} \\
&\quad E^Q\left[\left(S_0\exp\left((r-\lambda k+\frac{i\mu_y+i\sigma_y^2/2}{T}-\frac{1}{2}(\sigma_i^{jump})^2)T+\sigma_i^{jump}B_T\right)-K\right)^+|N_T=i\right].
\end{aligned}
\tag{38}
$$

Take

$$
\begin{aligned}
r^{jump} &= r-\lambda k+\frac{i\mu_y+i\sigma_y^2/2}{T} \\
&= r-\lambda k+\frac{i(\mu_y+\sigma_y^2/2)}{T} \\
&= r-\lambda k+\frac{i\ln(e^{\mu_y+i\sigma_y^2/2}-1+1)}{T} \\
&= r-\lambda k+\frac{i\ln(1+k)}{T}
\end{aligned}
\tag{39}
$$

Then Equation 38 becomes

$$C^j = e^{-rT}\sum_{i=0}^{\infty}\frac{e^{-\lambda T}(\lambda T)^i}{i!}$$
$$E^Q\left[\left(S_0\exp\left((r^{jump}-\frac{(\sigma_i^{jump})^2}{2})T+\sigma_i^{jump}B_T\right)-K\right)^+|N_T=i\right]. \tag{40}$$

The Black Scholes call option formula is

$$C^{BS}(S_0,r,\sigma,K,T) = e^{-rT}E^Q\left[\left(S_0\exp\left((r-\frac{\sigma^2}{2})T+\sigma B_T\right)-K\right)^+\right]. \tag{41}$$

Using Equation 39 it can be derived that $e^{-rT} = e^{-r^{jump}T-\lambda kT+i\ln(1+k)} = e^{-r^{jump}T}e^{-\lambda kT}(1+k)^i$. Equation 40 can therefore be written as

$$\begin{aligned} C^j &= \sum_{i=0}^{\infty} e^{-\lambda kT}(1+k)^i\frac{e^{-\lambda T}(\lambda T)^i}{i!} \\ &\quad e^{-r^{jump}T}E^Q\left[\left(S_0\exp\left((r^{jump}-\frac{(\sigma_i^{jump})^2}{2})T+\sigma_i^{jump}B_T\right)-K\right)^+|N_T=i\right] \\ &= \sum_{i=0}^{\infty}\frac{e^{-\lambda^{jump}T}(\lambda^{jump}T)^i}{i!}C^{BS}(S_0,r^{jump},\sigma_i^{jump},K,T) \end{aligned} \tag{42}$$

with $\lambda^{jump}=\lambda(1+k),\ r^{jump}=r-\lambda k+\frac{i\ln(1+k)}{T},\ \sigma_i^{jump}=\sqrt{\sigma^2+\frac{i\sigma_y^2}{T}}$.

R code

The code that is used for the closed form call option formula in the calibration is given below. The correctness of the code is checked with the example given in the Financial Derivatives course of Michel van der Wel at the Erasmus University Rotterdam. Taking S_0=K=20, r=2%, σ = 20%, λ = 1, k=5%, σ_y=25% and T=1 as parameter values the outcome is equal to the expected outcome of €2.65.

```
# Load libraries
library(stats)
library(grDevices)
library(RQuantLib)
#Lognormal jump diffusion model call option formula
jump.price<-function(strike,maturity,x){
  jumpprice<-0
  lambda <-x[1]; r <-x[2];  k <-x[3];  sigma_y <-x[4];  sigma<-x[5]
  lambda_jump <- lambda*(1+k)
  r_jump <- r-lambda*k
  sigma_jump <- sqrt(sigma^2)

  poisson <- function(t,i) {
    (exp(-lambda_jump*t)*(lambda_jump*t)^i)/factorial(i)
  }
  bs<- function(K,t){
    S_0*pnorm((log(S_0/K) +(r_jump + (sigma_jump^2)/2)*t)/(sigma_jump*sqrt(t))) -
      K*exp(-r_jump*t)*pnorm((log(S_0/K) +(r_jump - (sigma_jump^2)/2)*t)/(sigma_jump*sqrt(t)))
  }
  jumpprice <- jumpprice + poisson(maturity,0)*bs(strike,maturity)

  for(i in 1:n){
    r_jump <- r-lambda*k+(i*log(1+k))/maturity
    sigma_jump <- sqrt(sigma^2 + (i*sigma_y^2)/maturity)
    jumpprice = jumpprice + poisson(maturity,i)*bs(strike,maturity)
  }
  return(jumpprice)
}

S_0 <- 20 ;n<-100; K <- 20 ;Tt <- 1
x<- c(1,0.02,0.05,0.25,0.20 )
```

G Closed form call option formula double exponential jump model

This appendix shows the closed form formula for call option premiums with a double exponential jump diffusion share process. For the complete derivation and proof see (Kou, 2002).

The European call option premium is equal to

$$C^{dejd} = e^{-rT} E^Q[(S_T - K)^+] \tag{43}$$

with S_T described by the double exponential jump diffusion model as in Section 6

$$S_T = S_0 \exp\left[(r - \frac{\sigma^2}{2} - \lambda\xi)T + \sigma Z_T\right] \prod_{i=1}^{N_T} V_i. \tag{44}$$

Following the proof in (Kou, 2002) it can be shown that the price of a call option is given by

$$\begin{aligned} C^{dejd} &= S_0 \Upsilon(r + \frac{\sigma^2}{2} - \lambda\xi, \sigma, \lambda^*, p^*, \eta_1^*, \eta_2^*; log(K/S_0), T) \\ &+ Ke^{-rT}\Upsilon(r - \frac{\sigma^2}{2} - \lambda\xi, \sigma, \lambda, p, \eta_1, \eta_2; log(K/S_0), T) \end{aligned} \tag{45}$$

with K the strike price, $\lambda^* = \lambda(1+\xi), p^* = \frac{p}{1+\xi}\frac{\eta_1}{\eta_1 - 1}, \eta_1^* = \eta_1 - 1, \eta_2^* = \eta_2 + 1$ and $\xi = q\frac{\eta_2}{\eta_2+1} + p\frac{\eta_1}{\eta_1-1} - 1$. The function Υ is given by

$$\begin{aligned} \Upsilon(\mu, \sigma, \lambda, p, \eta_1, \eta_2; a, T) = &\frac{e^{(\sigma\eta_1)^2 T/2}}{\sigma\sqrt{2\pi T}} \sum_{n=1}^{\infty} \pi_n \sum_{k=1}^{n} P_{n,k}(\sigma\sqrt{T}\eta_1)^k \\ &* I_{k-1}\Big(a - \mu T; -\eta_1, -\frac{1}{\sigma\sqrt{T}}, -\sigma\eta_1\sqrt{T}\Big) \\ &+ \frac{e^{(\sigma\eta_2)^2 T/2}}{\sigma\sqrt{2\pi T}} \sum_{n=1}^{\infty} \pi_n \sum_{k=1}^{n} Q_{n,k}(\sigma\sqrt{T}\eta_2)^k \\ &* I_{k-1}\Big(a - \mu T; \eta_2, \frac{1}{\sigma\sqrt{T}}, -\sigma\eta_2\sqrt{T}\Big) \\ &+ \pi_0 \Phi(-\frac{a - \mu T}{\sigma\sqrt{T}}), \end{aligned} \tag{46}$$

with $\pi_n = P(N_T = n) = e^{-\lambda T}(\lambda T)^n/n!$, function I_n as defined in Equation 51 and the functions $P_{n,k}$ and $Q_{n,k}$ as defined in Equation 58 and 59, respectively.

The proof in (Kou, 2002) makes use of a special function, the so-called Hh function:
For every $n \geq 0$, the Hh function is a non-increasing function defined by

$$Hh_n(x) = \int_x^{\infty} Hh_{n-1}(y)dy = \frac{1}{n!}\int_x^{\infty}(t-x)^n e^{-t^2/2}dt \geq 0,\ for\ n = 0, 1, 2, ... \quad (47)$$

$$Hh_{-1}(x) = e^{-\frac{x^2}{2}} = \sqrt{2\pi}\phi(x) \quad (48)$$

$$Hh_0(x) = \sqrt{2\pi}\Phi(-x) \quad (49)$$

The Hh function can be seen as a generalization of the cumulative normal distribution function. To compute Hh functions the following recursive relation can be used

$$nHh_n(x) = Hh_{n-2}(x) - xHh_{n-1}(x),\ for\ n \geq 1, \quad (50)$$

using the normal density function and the normal distribution function in Equation 48 and Equation 49 to determine Hh_1. For more details, see page 971 (Abramowitz and Stegun, 1972)

The Hh function is used to evaluate the following integral

$$I_n(c;\alpha,\beta,\delta) = \int_c^{+\infty} e^{\alpha x} Hh_n(\beta c - \delta)dx,\ \forall\ n \geq -1. \quad (51)$$

For $\forall\ n \geq -1$ the integral in Equation 51 is equal to:

$$I_n(c;\alpha,\beta,\delta) = -\frac{e^{\alpha c}}{\alpha}\sum_{i=0}^{n}(\frac{\beta}{\alpha})^{n-i}Hh_i(\beta c - \delta) \quad (52)$$

$$+(\frac{\beta}{\alpha})^{n+1}\frac{\sqrt{2\pi}}{\beta}e^{\frac{\alpha\delta}{\beta}+\frac{\alpha^2}{2\beta^2}}\Phi(-\beta c + \delta + \frac{\alpha}{\beta}) \quad (53)$$

if $\beta > 0$ and $\alpha \neq 0$ and equal to

$$I_n(c; \alpha, \beta, \delta) = -\frac{e^{\alpha c}}{\alpha} \sum_{i=0}^{n} (\frac{\beta}{\alpha})^{n-i} Hh_i(\beta c - \delta) \tag{54}$$

$$- (\frac{\beta}{\alpha})^{n+1} \frac{\sqrt{2\pi}}{\beta} e^{\frac{\alpha\delta}{\beta} + \frac{\alpha^2}{2\beta^2}} \Phi(\beta c - \delta - \frac{\alpha}{\beta}) \tag{55}$$

if $\beta < 0$ and $\alpha < 0$.

Proof of this statement can be found on page 1099 of (Kou, 2002).

The memoryless property of exponential random variables yields to the conclusion that

$$\xi^+ - \xi^- \sim \begin{cases} \xi^+, & \text{with probability } \eta_2/(\eta_1 + \eta_2) \\ -\xi^-, & \text{with probability } \eta_1/(\eta_1 + \eta_2) \end{cases} \tag{56}$$

because the probabilities of the events $\xi^+ > \xi^-$ and $\xi^+ < \xi^-$ are $\eta_2/(\eta_1 + \eta_2)$ and $\eta_1/(\eta_1 + \eta_2)$, respectively. This extends to the following equation

$$\sum_{i=1}^{n} Y_i \sim \begin{cases} \sum_{i=1}^{k} \xi_i^+, & \text{with probability } P_{n,k},\ k = 1, 2, ..., n \\ -\sum_{i=1}^{k} \xi_i^-, & \text{with probability } Q_{n,k},\ k = 1, 2, ..., n \end{cases} \tag{57}$$

with $P_{n,k}$ and $Q_{n,k}$ given by

$$P_{n,k} = \sum_{i=k}^{n-1} \binom{n-k-1}{i-k} \binom{n}{i} (\frac{\eta_1}{\eta_1 + \eta_2})^{i-k} (\frac{\eta_2}{\eta_1 + \eta_2})^{n-i} p^i q^{n-i} \tag{58}$$

and $P_{n,n} = p^n$.

$$Q_{n,k} = \sum_{i=k}^{n-1} \binom{n-k-1}{i-k} \binom{n}{i} (\frac{\eta_1}{\eta_1 + \eta_2})^{n-i} (\frac{\eta_2}{\eta_1 + \eta_2})^{i-k} p^{n-i} q^i \tag{59}$$

and $Q_{n,n} = q^n$.

R code

The code that is used for the closed form call option formula in the calibration is given below. The correctness of the code is checked with the example given in the paper by Kou (Kou, 2002). Taking η_1=10, η_2=5, λ=1, p=0.4, σ=16%, r=5%, S_0=100, K=98 and T=0.5 the outcome is equal to the expected outcome of €9.14732.

```r
# Load libraries
library(stats)
library(grDevices)
library(RQuantLib)

# Double exponential jump diffusion model call option formula
phi<-function(x){
  phifunction<-function(t){
    exp(-t^2)
  }
  int<-integrate(phifunction,0,x/sqrt(2))
  phivalue<-((2/sqrt(pi))*int$value+1)/2
  return(phivalue)
}

Hh <- function(n,x) {
  if (x>=-6){
    if(x<10){
      Hhfunct <- function(t){
        (t-x)^n*exp(-t^2/2)
      }
      int<- integrate(Hhfunct,x,Inf)
      Hvalue <- (1/factorial(n))*int$value
    }
    else{
      Hvalue<-0
    }
  }
  else{
    temp<-(x+sqrt((x^2)+4*n))/2
    Hhfunct <- function(t){
      (t-x)^n*exp(-t^2/2)
    }
    int1<-integrate(Hhfunct,x,temp-3)
    int2<-integrate(Hhfunct,temp-3,temp-1)
    int3<-integrate(Hhfunct,temp-1,temp)
    int4<-integrate(Hhfunct,temp,temp+1)
    int5<-integrate(Hhfunct,temp+1,temp+3)
    int6<-integrate(Hhfunct,temp+3,Inf)
    Hvalue<-(1/factorial(n))*(int1$value+int2$value+int3$value+int4$value+int5$value+int6$value)
  }

  return(Hvalue)
}

I <- function(j,l,a,b,d) {

  if (b>0 && a!=0){
    sumh<-0
    for (i in 0:j){
      sumh<-sumh + (b/a)^(j-i)*Hh(i,b*l-d)
    }
    Ivalue <- -(exp(a*l)/a)*sumh + ((b/a)^(j+1))*(sqrt(2*pi)/b)*
                exp(a*d/b + (1/2)*(a/b)^2) * phi(-b*l+d+a/b)
  }
  else if (b<0 && a<0){
    sumh<-0
    for (i in 0:j){
      sumh<-sumh + (b/a)^(j-i)*Hh(i,b*l-d)
    }
    Ivalue <- -(exp(a*l)/a)*sumh - ((b/a)^(j+1))*(sqrt(2*pi)/b)*
                  exp(a*d/b + (1/2)*(a/b)^2) * phi(b*l-d-a/b)
  }
  else if (b>0 && a==0){
    Ivalue <- Hh(j+1,b*l-d)/b
  }
  return(Ivalue)
}
```

```
Pni <- function (n,i,p,n1,n2){
  if (i<n){
    Pnivalue<-0
    for (j in i:n-1){
      Pnivalue<-Pnivalue + choose(n,j)*(p^j)*((1-p)^(n-j))*
                  choose(n-i-1,j-i)*((n1/(n1+n2))^(j-i))*((n2/(n1+n2))^(n-j))
    }
  }
  if (i==n){
    Pnivalue<-p^n
  }
  return(Pnivalue)
}

Qni <- function (n,i,p,n1,n2){
  if (i<n){
    Qnivalue<-0
    for (j in i:n-1){
      Qnivalue<-Qnivalue + choose(n,j)*((1-p)^j)*(p^(n-j))*
                  choose(n-i-1,j-i)*((n2/(n1+n2))^(j-i))*((n1/(n1+n2))^(n-j))
    }
  }
  if (i==n){
    Qnivalue<-(1-p)^n
  }
  return(Qnivalue)
}

cprob<-function(mu,n1,n2,la,p,sig,a,Tt,nStep){
  Itwo<-rep(NA,nStep)
  for (k in 1:nStep){
    Itwo[k]<-I(k-1,a-mu*Tt,-n1,-1/(sig*sqrt(Tt)),-(sig*sqrt(Tt)*n1))
  }

  Ifour<-rep(NA,nStep)
  for (k in 1:nStep){
    Ifour[k]<-I(k-1,a-mu*Tt,n2,1/(sig*sqrt(Tt)),-(sig*sqrt(Tt)*n2))
  }

  PiN <- function(n){
    PiNvalue<- exp(-la*Tt)*((la*Tt)^n)/factorial(n)
    return(PiNvalue)
  }

  PiNPni<-matrix(NA, nrow = nStep, ncol = nStep)
  for (n in 1:nStep){
    for (k in 1:n){
      PiNPni[n,k]<-PiN(n)*Pni(n,k,p,n1,n2)*((sig*sqrt(Tt)*n1)^k)
    }
  }

  PiNQni<-matrix(NA, nrow = nStep, ncol = nStep)
  for (n in 1:nStep){
    for (k in 1:n){
      PiNQni[n,k]<-PiN(n)*Qni(n,k,p,n1,n2)*((sig*sqrt(Tt)*n2)^k)
    }
  }

  sec<-0
  for (n in 1:nStep){
    for (k in 1:n){
      sec<-sec+ PiNPni[n,k]*Itwo[k]

    }
  }

  fourth<-0
  for (n in 1:nStep){
    for (k in 1:n){
      fourth<-fourth+ PiNQni[n,k]*Ifour[k]
```

```
    }
  }
  cprobvalue<-(sec*exp(((sig*n1)^2)*Tt/2) + fourth * exp(((sig*n2)^2)*Tt/2)) /
               (sqrt(2*pi)* sig*sqrt(Tt)) + exp(-la*Tt)*phi(-(a-mu*Tt)/(sig*sqrt(Tt)))
  return(cprobvalue)

}

call<-function(K,Tt,x){
  n1<-x[1]; n2<-x[2]; la<-x[3]; p<-x[4]; sig<-x[5]; r<-x[6]

  E<-p*n1/(n1-1)+(1-p)*n2/(n2+1)-1
  temp1r<-r+sig*sig/2 - la*E
  temp2r<-temp1r-sig*sig

  callvalue<- S_0*cprob(temp1r,(n1-1),(n2+1),(la*(1+E)),(p*n1/((1+E)*(n1-1))),sig,log(K/S_0),Tt,nStep) -
              K*exp(-r*Tt)*cprob(temp2r,n1,n2,la,p,sig,log(K/S_0),Tt,nStep)
  return(callvalue)
}

S_0 <- 100; K <- 98; Tt <- 0.5; nStep<-15
x<- c(10,5,1,0.4,0.16,0.05)

call(K,Tt,x)
```

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