Erasmus University Rotterdam

Richer Cheap Talk

Extension of a signaling model

Bachelor thesis

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Abstract

Communication is crucial in every human life. It also is very versatile and widely studied in the economic discipline. One aspect of economics is the study of strategic communication, to which this thesis contributes by creating a model and an extension to this model. The model is a signaling game based on existing literature (Crawford and Sobel, 1982; Gibbons, 1992), whereas the extension is new theory: it consists of a communication signal prior to the signaling game. In this thesis, it is proved that equilibria exist in which such a signal is sent and it is proved that this signal enriches communication in the signaling game. A condition for such an equilibrium to exist is that the interest asymmetry between the players of the signaling game is limited.

Word of thanks

In the creation of my bachelor thesis, I was supported from many sides. First and foremost, I want to thank Professor Swank for accepting my request to supervise me writing a thesis in his discipline and for providing me with an idea to work out. I especially appreciated his fast replies to my e-mails and the frequent meetings at his office, where he provided feedback and inspired me to tackle the present problems. I also want to thank my fellow student Justin Vink for the substantive conversations that helped me gain valuable insights on intensive days on campus Woudestein of the Erasmus University. Finally, I want to thank my family and friends for their listening ears and encouragements.

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Introduction

Rick is owner of a company and wants to hire an employee. Rick invites Scottie for a job interview. Like every employee, Scottie has certain skills. Rick quantifies the skills of his employees relative to the market by assigning every employee a score that ranges from 0 to 1, where 0 is the score of the employee with the lowest skill and 1 is the score of the employee with the highest skill. The skill scores correspond to functions in Rick's company. Rick does not know Scottie's skills. Scottie knows her own skills and thus which score she should get, but she tends to exaggerate it to get a higher function. Rick knows this. In the job interview, Scottie will communicate to Rick, signaling her skills. After the job interview, Rick will assign Scottie a function in his company, depending on his knowledge about Scottie's skills. If Scottie's message cannot be trusted, Rick will expect her skill to be average. Prior to the job interview, Scottie considers sending her resume to Rick. From the resume, Rick can infer whether Scottie's skills are below or above average, thereby bisecting Scottie's possible skill scores. Will Scottie send her resume to Rick?

Ray owns a tennis club. To ensure fair opposition, Ray assigns all the members of the tennis club a score, based on their tennis performance relative to the other members of the club. The scores range from 0 to 1, where 0 is the score of the worst performing member and 1 is the score of the best performing member. Since the scores measure relative performance, the scores are recalculated when the club gains or loses members. One day, Susan becomes a member of the tennis club. Ray needs to assign Susan a score based on her tennis performance. However, Ray does not know Susan's tennis performance. Susan knows her own tennis performance and thus knows what score she should get, but she tends to exaggerate it to get to play against better opponents. If Susan's message cannot be trusted, Ray will expect her score to be average. Susan has a friend who is already a member of Ray's tennis club. Prior to her first contact with Ray, Susan can ask her friend to tell Ray about her tennis performance. The friend is no tennis expert and can only tell if Susan's performance is below or above average. Will Susan ask her friend to inform Ray about her performance prior to her contact with Ray?

The main objective of this thesis is exploring whether communication of partial information prior to a signaling game enriches the communication in the signaling game. To this end, a model is developed in which two players, with asymmetrical information and asymmetrical interests, communicate. The main feature of the model is a communication signal prior to the signaling game.

The results of this thesis reveal that it is optimal for the player with partial information to share this information and that there is no incentive to deceive the other player. Also, the results reveal that asymmetrical interests yield opaque communication, up to the point where the interests are so asymmetrical that communication is meaningless.

The results of this thesis provide two insights. The first insight is that, when interest asymmetry is limited, most of the available information will be shared through both the signaling game and the signal prior to the signaling game. The second insight is that, as the interest asymmetry becomes smaller, more specific information can be shared, that is, communication becomes richer.

Two findings of existing literature on the topic are proved to apply to the model and its extension. Namely the ally principle, which is claimed to be part of an ancient wisdom that was old before Rome (Bendor, Glazer and Hammond, 2001, p. 236) and the communication constraint (Swank, 2016).

There are more extensions to the cheap talk game à la Crawford and Sobel (1982). For example, Sharif (2016) developed a model in which the information of the Sender is relevant for multiple decisions of the Receiver.

This thesis continues by describing the model (below) and its equilibria (on page 4). Thereafter, a model extension is presented (on page 9) along with its corresponding equilibria. Finally, a conclusion is drawn (on page 17). References can be found on page 19 and appendices start on page 19.

Model

This model and its equilibria are based on a cheap talk model of Crawford and Sobel (1982) as described by Gibbons (1992). Consider a Sender, S, and a Receiver, R. R has to take action a_{i} from an infinite set of feasible actions $A = [0,1]$. R's optimal action, a_R^*, depends on the stochastic variable v, which is uniformly distributed on the interval [0,1]. v cannot be observed by R, but can be observed by S. S can inform R about the value of v by sending message m_i from an infinite set of feasible messages $M = [0,1]$. The interests of S and R differ in the nonnegative parameter b. The payoffs of S and R are given by respectively $U_S = -[a_{\mathrm{i}} - (v + b)]^2$ and $U_R = -(a_{\mathrm{i}} - v)^2$.

Timing:

1. Nature draws v from a uniform distribution with range [0,1].
2. S observes v, but R does not.

3. S sends message $m_{\mathrm{i}} \in M = [0,1]$.
4. R receives m_{i} and chooses action $a_{\mathrm{i}} \in A = [0,1]$.
5. Payoffs are given by $U_S = -[a_{\mathrm{i}} - (v + b)]^2$ and $U_R = -(a_{\mathrm{i}} - v)^2$.

Equilibria

The equilibria for this model are identified using backwards induction: firstly, the strategy of R will be identified, then the strategy of S will be identified.

This section continues by explaining two extreme equilibria of the signaling game. In the first equilibrium, a perfect Bayesian equilibrium, no information is lost in the communication. In the second equilibrium, a babbling equilibrium, communication contains no information at all.

Then, two partially pooling equilibria with respectively two and three message intervals are described. Interested readers can see Appendix E on page 34 for a description of a partially pooling equilibrium that is generalized to n steps.

Perfect Bayesian equilibrium

If $b = 0$, the interests of S and R are perfectly aligned and a perfect Bayesian equilibrium with perfect communication exists. In such an equilibrium, no information is lost in the communication: S chooses $m_{\mathrm{i}} = v$ and R chooses $a_{\mathrm{i}} = m_{\mathrm{i}}$. For all values of v, this results in $U_S = -[a_{\mathrm{i}} - (v + b)]^2 = -[v - (v + 0)]^2 = 0$ and $U_R = -(a_{\mathrm{i}} - v)^2 = -(v - v)^2 = 0$. In a perfect Bayesian equilibrium, the aggregate expected utility equals zero for both S and R on the entire interval $[0,1]$. No other equilibrium yields larger aggregate expected utility than this perfect Bayesian equilibrium.

Babbling equilibrium

A babbling equilibrium, in which communication is meaningless, always exists in a cheap talk game. In such an equilibrium, R always chooses a_{i} independently of m_{i}. In the absence of information infered from m_i, R's expected value of v equals $1/2$, so R will choose $a_i = E(v) = 1/2$ to maximize his own utility. Since m_i will be ignored, S's choice of m_i does not influence S's payoff. Therefore, it is optimal for S to send m_i containing no information, that is, sending m_i independently of v.

Each player's aggregate expected utility in a babbling equilibrium can be calculated using only one integral, since each player's actions are identical for all values of v.

R's aggregate expected utility equals $\int_0^1 -(a_i - v)^2 dv$, which can be rewritten as $\int_0^1 -\left(\frac{1}{2} - v\right)^2 dv$ and equals $-\frac{1}{12}$.

S's aggregate expected utility equals $\int_0^1 -[a_i - (v+b)]^2 dv$, which can be rewritten as $\int_0^1 -\left(\frac{1}{2} - v - b\right)^2 dv$ and equals $-b^2 - \frac{1}{12}$.

Note that in this babbling equilibrium, there are no restrictions to the size of b, other than the assumption of $b \geq 0$, because there is complete loss of information in the communication. Fortunately, when $b > 0$, partially pooling equilibria can exist.

Partially pooling equilibria

It has been shown that when $b = 0$, no information loss in communication can occur in a perfect Bayesian equilibrium. It has also been shown that, irrespective of the value of b, complete information loss can occur in a babbling equilibrium. There exists a medium, in which information loss in the communication occurs, but is limited.

When $b > 0$, informative communication cannot be achieved when S sends a message containing the exact value of v, since (per definition) S and R have asymmetrical interests. In other words, when $b > 0$, S's optimal action, a_S^*, does not equal R's optimal action, a_R^* (i.e. $a_S^* \neq a_R^*$). A message that equals S's optimal action (i.e. $m_{\mathrm{i}} = a_S^*$) cannot be an equilibrium, because R has an incentive to deviate from m_i, since $m_i = a_S^* \neq a_R^*$. Therefore, S has an incentive to deceive R. Thus, when $b > 0$ and S sends a message containing the exact value of v, this message is not trustworthy and an equilibrium cannot exist.

Instead, when $b > 0$, S can send a message that indicates that v lies on a certain interval. By doing so, some information is lost, but the communication becomes trustworthy. Based on a message that contains the interval of v, R can update his expected value of v, leading R to choose $a_i = E(v|m_i)$. The extend to which the communication is informative depends on the size of the message interval: a smaller message interval contains more information, because the maximum difference between the exact value of v and R's expected value of v is smaller.

This section continues by describing two partially pooling equilibria: one with two message intervals and one with three message intervals.

Two-step equilibrium

Suppose that the interval [0,1] is split into two message intervals, m_1 and m_2, that are separated by point x_1.

If R receives m_1, he infers that v lies on the interval $[0, x_1]$, thus updating his expected value of v to $x_1/2$. This leads R to choosing $a_i = a_1 = E(v|m_1) = x_1/2$ to maximize his own utility.

If R receives m_2, he infers that v lies on the interval $[x_1, 1]$, thus updating his expected value of v to $(x_1+1)/2$. This leads to R choosing $a_i = a_2 = E(v|m_2) = (x_1+1)/2$ to maximize his own utility.

Anticipating R's strategy, S sends m_1 for all values of v on the interval $[0, x_1)$ and m_2 for all values of v on the interval $(x_1, 1]$ to maximize her own utility. One condition regarding S's choice of m_i arises: if $v = x_1$, S must be indifferent between sending m_1 and m_2, that is, $U_S(m_1|v = x_1) = U_S(m_2|v = x_1)$, which can be rewritten as $-[a_1 - (v + b)]^2 = -[a_2 - (v + b)]^2$. Given that $0 < a_1 < v = x_1 < a_2$, the latter formula solves for $x_1 = 1/2 - 2b$ and $b < 1/4$ (see Appendix A.1 on page 19).

Recall that b is nonnegative. $b < 1/4$ is a necessary condition for a two-step equilibrium to exist on the interval [0,1]. If $b \geq 1/4$, the interests of R and S are too asymmetrical for a two-step equilibrium to exist on the interval [0,1]. In fact, if $b \geq 1/4$, the only equilibrium that exists on the interval [0,1], is a babbling equilibrium.

Given that $x_1 = 1/2 - 2b$, it can be proved that message interval 2, $[x_1, 1]$, is longer than message interval 1, $[0, x_1]$, that is: $1 - x_1 > x_1 - 0$, since this inequality yields $4b > 0$, which implicates that message interval 2 is $4b$ longer than message interval 1 (see Appendix A.2 on page 20). This result corresponds to findings of Gibbons (1992).

A player's aggregate expected utility for the two-step equilibrium on the interval [0,1] can be calculated using the sum of two integrals. The first integral covers the situation where $v = [0, x_1]$, S chooses $m_i = m_1$ and R chooses $a_i = a_1 = x_1/2$. The second integral covers the situation where $v = [x_1, 1]$, S chooses $m_i = m_2$ and R chooses $a_i = a_2 = (x_1 + 1)/2$.

For R, the aggregate expected utility can be expressed as follows: $U_R^T = \int_0^{x_1} U_R(v|m_1)\, dv + \int_{x_1}^1 U_R(v|m_2)\, dv$, which can be rewritten as $U_R^T = \int_0^{x_1} -\left[\frac{x_1}{2} - v\right]^2 dv + \int_{x_1}^1 -\left[\frac{x_1+1}{2} - v\right]^2 dv$, which equals $-b^2 - \frac{1}{48}$ (see Appendix A.3.i on page 20).

For S, the aggregate expected utility can be expressed as follows: $U_S^T = \int_0^{x_1} U_s(v|m_1)\, dv + \int_{x_1}^1 U_s(v|m_2)\, dv$, which can be rewritten as $U_S^T = \int_0^{x_1} -\left[\frac{x_1}{2} - (v + b)\right]^2 dv + \int_{x_1}^1 -\left[\frac{x_1+1}{2} - (v + b)\right]^2 dv$, which equals $-2b^2 - \frac{1}{48}$ (see Appendix A.3.ii on page 20).

Note that R's aggregate expected utility is b^2 larger than S's aggregate expected utility. This difference exists because b causes S to prefer action range $[0 + b, 1 + b]$, although both v and a lie on the interval [0,1]. Therefore, for all values of $a_S^* > 1$, the only possible action closest to a_S^* is $a = 1$. The larger b, the larger the difference between a and a_S^*. Since R's utility function is not dependent on b, R's optimal action always lies on the interval [0,1]. The quadratic character of the difference between R's aggregate expected utility and S's aggregate expected utility is caused by the quadratic character of b in the utility function of S.

It has been demonstrated that when the interest asymmetry of the players is limited, namely $b < 1/4$, the loss of information in the communication can also be limited. This result corresponds to findings of Gibbons (1992).

Moreover, a smaller value of b can make it possible to limit information loss in the communication even further, as will be illustrated in the following section.

Three-step equilibrium

Suppose that the interval [0,1] is split into three message intervals, m_i with $i = \{1, 2, 3\}$, where m_1 and m_2 are separated by point y_1 and m_2 and m_3 are seperated by point y_2.

If R receives m_1, he infers that v lies on the interval $[0, y_1]$, thus updating his expected value of v to $y_1/2$. This leads R to choosing $a_i = a_1 = E(v|m_1) = y_1/2$ to maximize his own utility.

If R receives m_2, he infers that v lies on the interval $[y_1, y_2]$, thus updating his expected value of v to $(y_1 + y_2)/2$. This leads to R choosing $a_i = a_2 = E(v|m_2) = (y_1 + y_2)/2$ to maximize his own utility.

If R receives m_3, he infers that v lies on the interval $[y_2, 1]$, thus updating his expected value of v to $(y_2 + 1)/2$. This leads to R choosing $a_i = a_3 = E(v|m_3) = (y_2 + 1)/2$ to maximize his own utility.

Anticipating R's strategy, S sends m_1 for all values of v on the interval $[0, y_1)$, m_2 for all values of v on the interval (y_1, y_2) and m_3 for all values of v on the interval $(y_2, 1]$ to maximize her own utility. Two conditions regarding S's choice of m_i arise.

Firstly, if $v = y_1$, S must be indifferent between sending m_1 and m_2, that is, $U_S(m_1|v = y_1) = U_S(m_2|v = y_1)$, which can be rewritten as $-[a_1 - (v + b)]^2 = -[a_2 - (v + b)]^2$. Given that $0 < a_1 < v = y_1 < a_2$, the latter formula solves for $y_1 = y_2/2 - 2b$ and $b < y_2/4$ (see Appendix B.1.i on page 21).

Secondly, if $v = y_2$, S must be indifferent between sending m_2 and m_3, that is, $U_S(m_2|v = y_2) = U_S(m_3|v = y_2)$, which can be rewritten as $-[a_2 - (v + b)]^2 = -[a_3 - (v + b)]^2$. The latter formula solves for $y_1 = 1/3 - 4b$, $y_2 = 2/3 - 4b$ and $b < 1/12$ (see Appendix B.1.ii on page 21).

Recall that b is nonnegative. $b < 1/12$ is a necessary condition for a three-step equilibrium to exist on the interval [0,1]. If $b \geq 1/12$, the interests of R and S are too asymmetrical for a three-step equilibrium to exist on the interval [0,1].

Given that $y_2 = 2/3 - 4b$, it can be proved that message interval 3, $[y_2, 1]$, is longer than message interval 2, $[y_1, y_2]$, that is: $1 - y_2 > y_2 - y_1$, since this inequality yields $4b > 0$, which implicates that message interval 3 is $4b$ longer than message interval 2 (see Appendix B.2.i on page 22). Note that the difference in interval length in the two-step equilibrium also equals $4b$ (recall Appendix A.2 on page 20).

Also, given that $y_1 = 1/3 - 4b$, it can be proved that message interval 2, $[y_1, y_2]$, is longer than message interval 1, $[0, y_1]$, that is: $y_2 - y_1 > y_1 - 0$, since this inequality yields $4b > 0$, which implicates that message interval 2 is $4b$ longer than message interval 1 (see Appendix B.2.ii on page 22). Note that the difference in interval length in both the two-step equilibrium and the three-step equilibrium equals $4b$ for all intervals (recall Appendix A.2 on page 20 and Appendix B.2.i on page 22).

A player's aggregate expected utility for the three-step equilibrium on the interval [0,1] can be calculated using the sum of three integrals. The first integral covers the situation where $v = [0, y_1]$, S chooses $m_i = m_1$ and R chooses $a_i = a_1 = y_1/2$. The second integral covers the situation where $v = [y_1, y_2]$, S chooses $m_i = m_2$ and R chooses $a_i = a_2 = (y_1 + y_2)/2$. The third integral covers the situation where $v = [y_2, 1]$, S chooses $m_i = m_3$ and R chooses $a_i = a_3 = (y_2 + 1)/2$.

For R, the aggregate expected utility can be expressed as follows: $U_R^T = \int_0^{y_1} U_R(v|m_1)\, dv + \int_{y_1}^{y_2} U_R(v|m_2)\, dv + \int_{y_2}^{1} U_R(v|m_3)\, dv$, which can be rewritten as $U_R^T = \int_0^{y_1} -\left(\frac{y_1}{2} - v\right)^2 dv + \int_{y_1}^{y_2} -\left(\frac{y_1+y_2}{2} - v\right)^2 dv + \int_{y_2}^{1} -\left(\frac{y_2+1}{2} - v\right)^2 dv$, which equals $-\frac{8}{3}b^2 - \frac{1}{108}$ (see Appendix B.3.i on page 22).

For S, the aggregate expected utility can be expressed as follows: $U_S^T = \int_0^{y_1} U_R(v|m_1)\, dv + \int_{y_1}^{y_2} U_R(v|m_2)\, dv + \int_{y_2}^{1} U_R(v|m_3)\, dv$, which can be rewritten as $U_S^T = \int_0^{y_1} -\left[\frac{y_1}{2} - (v+b)\right]^2 dv + \int_{y_1}^{y_2} -\left[\frac{y_1+y_2}{2} - (v+b)\right]^2 dv + \int_{y_2}^{1} -\left[\frac{y_2+1}{2} - (v+b)\right]^2 dv$, which equals $-\frac{11}{3}b^2 - \frac{1}{108}$ (see Appendix B.3.ii on page 23).

Note that R's aggregate expected utility is b^2 larger than S's aggregate expected utility. This is also observed in the two-step quilibrium, where it is explained (recall *Two-step equilibrium* on page 5).

Conclusion partially pooling equilibria

It has been proved that when $b < 1/4$, a two-step equilibrium exists and that when $b < 1/12$, a three-step equilibrium exist. In the three-step equilibrium, less information is lost in the communication than in the two-step equilibrium. Therefore, a three-step equilibrium is more efficient than a two-step equilibrium.

Examples

Scenario 1: Suppose that $b = 1/4$. The only equilibrium that exists is a babbling equilibrium. Communication contains no information and the aggregate expected utility equals $-\frac{1}{12}$ for R and $-b^2 - \frac{1}{12} = -\left(\frac{1}{4}\right)^2 - \frac{1}{12} = -\frac{7}{48}$ for S.

Scenario 2: Suppose that $b = 1/12$. Compared to Scenario 1, the interest asymmetry between R and S declined. Now, next to a babbling equilibrium, a two-step equilibrium exists. Since $b \nless 1/12$, a three-step equilibrium does not exist.

In the babbling equilibrium, the aggregate expected utility equals $-\frac{1}{12}$ for R and $-b^2-\frac{1}{12}=-\left(\frac{1}{12}\right)^2-\frac{1}{12}=-\frac{13}{144}$ for S.

In the two-step equilibrium, the aggregate expected utility equals $-b^2-\frac{1}{48}=-\left(\frac{1}{12}\right)^2-\frac{1}{48}=-\frac{1}{36}$ for R and $-2b^2-\frac{1}{48}=-2\left(\frac{1}{12}\right)^2-\frac{1}{48}=-\frac{5}{144}$.

Note that, compared to the babbling equilibrium, in the two-step equilibrium communication is richer and both players are better off.

Model extension

Assume that S gains partial information before the initial model is played. This information contains the certainty whether v lies on the interval $\left[0,\frac{1}{2}\right]$, interval I, or on the interval $\left[\frac{1}{2},1\right]$, interval II. Does S have an incentive to share this partial information? To answer this question, the utility of S in the situation of concealing the partial information will be compared to the utility of S in the situation of sharing the partial information.

For the model extension, an extra step in the timing is added, namely step 2 as described below.

Timing:

1. Nature draws v from a uniform distribution with range $[0,1]$.
2. S observes whether v lies on $\left[0,\frac{1}{2}\right]$ or $\left[\frac{1}{2},1\right]$ and chooses either to conceal or to share this information with R.
3. S observes the exact value of v, but R does not.
4. S sends message $m_{\mathrm{i}} \in M=[0,1]$.
5. R receives m_{i} and chooses action $a_{\mathrm{i}} \in A=[0,1]$.
6. Payoffs are given by $U_S=-[a_{\mathrm{i}}-(v+b)]^2$ and $U_R=-[a_{\mathrm{i}}-v]^2$.

Note that when S is credible and chooses to share the partial information, both players know whether v lies on interval I or on interval II. Therefore, although $m_{\mathrm{i}} \in M=[0,1]$ and $a_{\mathrm{i}} \in A=[0,1]$, in equilibrium, both m_i and a_i will lie on the interval v is known to lie on.

This thesis continues by describing the extended equilibria of two partially pooling equilibria with respectively two and three steps.

Extended two-step equilibrium

Suppose that S learns that v lies on interval I. If R receives m_1, he infers that v lies on the interval $[0,x_1]$, thus updating his expected value of v to $x_1/2$. This leads R to choosing $a_i=a_1=E(v|m_1)=x_1/2$ to maximize his own utility.

If R receives m_2, he infers that v lies on the interval $[x_1, 1/2]$, thus updating his expected value of v to $(x_1 + 1/2)/2$. This leads to R choosing $a_i = a_2 = E(v|m_2) = (x_1 + 1/2)/2$ to maximize his own utility.

Anticipating R's strategy, S sends m_1 for all values of v on the interval $[0, x_1)$ and m_2 for all values of v on the interval $(x_1, 1/2]$ to maximize her own utility. One condition regarding S's choice of m_i arises: if $v = x_1$, S must be indifferent between sending m_1 and m_2, that is, $U_S(m_1|v = x_1) = U_S(m_2|v = x_1)$, which can be rewritten as $-[a_1 - (v + b)]^2 = -[a_2 - (v + b)]^2$. Given that $0 < a_1 < v = x_1 < a_2$, the latter formula solves for $x_1 = 1/4 - 2b$ and $b < 1/8$ (see Appendix C.1 on page 24).

Given that $x_1 = 1/4 - 2b$, it can be proved that message interval 2, $[x_1, 1/2]$, is longer than message interval 1, $[0, x_1]$, that is: $1/2 - x_1 > x_1 - 0$, since this inequality yields $4b > 0$, which, given that $b > 0$, implicates that message interval 2 is $4b$ longer than message interval 1 (see Appendix C.2 on page 24). Note that the difference in interval length of all initial equilibria equals $4b$ for all intervals (recall Equilibria on page 4).

A player's aggregate expected utility for the two-step equilibrium on the interval $[0,1/2]$ can be calculated using the sum of two integrals. The first integral covers the situation where $v = [0, x_1]$, S chooses $m_i = m_1$ and R chooses $a_i = a_1 = x_1/2$. The second integral covers the situation where $v = [x_1, 1/2]$, S chooses $m_i = m_2$ and R chooses $a_i = a_2 = (x_1 + 1/2)/2$.

For R, the aggregate expected utility can be expressed as follows: $U_R^I = \int_0^{x_1} U_R(v|m_1)\, dv + \int_{x_1}^{\frac{1}{2}} U_R(v|m_2)\, dv$, which can be rewritten as $U_R^I = \int_0^{x_1} -\left[\frac{x_1}{2} - v\right]^2 dv + \int_{x_1}^{\frac{1}{2}} -\left[\frac{x_1+1/2}{2} - v\right]^2 dv$, which equals $-\frac{1}{2}b^2 - \frac{1}{384}$ (see Appendix C.3.i on page 24).

For S, the aggregate expected utility can be expressed as follows: $U_S^I = \int_0^{x_1} U_s(v|m_1)\, dv + \int_{x_1}^{\frac{1}{2}} U_s(v|m_2)\, dv$, which can be rewritten as $U_S^I = \int_0^{x_1} -\left[\frac{x_1}{2} - (v + b)\right]^2 dv + \int_{x_1}^{\frac{1}{2}} -\left[\frac{x_1+1/2}{2} - (v + b)\right]^2 dv$, which equals $-b^2 - \frac{1}{384}$ (see Appendix C.3.ii on page 25).

Suppose that S learns that v lies on interval II. If R receives m_1, he infers that v lies on the interval $[1/2, x_1]$, thus updating his expected value of v to $(1/2 + x_1)/2$. This leads R to choosing $a_i = a_1 = E(v|m_1) = (1/2 + x_1)/2$ to maximize his own utility.

If R receives m_2, he infers that v lies on the interval $[x_1, 1]$, thus updating his expected value of v to $(x_1 + 1)/2$. This leads to R choosing $a_i = a_2 = E(v|m_2) = (x_1 + 1)/2$ to maximize his own utility.

Anticipating R's strategy, S sends m_1 for all values of v on the interval $[1/2, x_1)$ and m_2 for all values of v on the interval $(x_1, 1]$ to maximize her own utility. One condition regarding S's choice of m_i arises: if $v = x_1$, S must be indifferent between sending m_1 and m_2, that is, $U_S(m_1|v = x_1) = U_S(m_2|v = x_1)$, which can be rewritten as $-[a_1 - (v + b)]^2 = -[a_2 - (v + b)]^2$. Given that $0 < a_1 < v = x_1 < a_2$, the latter formula solves for $x_1 = 3/4 - 2b$ and $b < 3/8$ (see Appendix C.4 on page 25).

Given that $x_1 = 3/4 - 2b$, it can be proved that the interval $[x_1, 1]$ is longer than interval $[1/2, x_1]$, that is: $1 - x_1 > x_1 - 1/2$, since this inequality yields $4b > 0$, which, given that $b > 0$, implicates that the interval $[x_1, 1]$ is $4b$ longer than the interval $[1/2, x_1]$ (see Appendix C.5 on page 26). Note that the difference in interval length of all initial equilibria (recall Equilibria on page 4) and the extended two-step equilibrium on interval I equals $4b$ for all intervals.

A player's aggregate expected utility for the two-step equilibrium on the interval [1/2,1] can be calculated using the sum of two integrals. The first integral covers the situation where $v = [1/2, x_1]$, S chooses $m_i = m_1$ and R chooses $a_i = a_1 = (1/2 + x_1)/2$. The second integral covers the situation where $v = [x_1, 1]$, S chooses $m_i = m_2$ and R chooses $a_i = a_2 = (x_1 + 1)/2$.

For R, the aggregate expected utility can be expressed as follows: $U_R^{II} = \int_{\frac{1}{2}}^{x_1} U_R(v|m_1)\,dv + \int_{x_1}^{1} U_R(v|m_2)\,dv$, which can be rewritten as $U_R^{II} = \int_{\frac{1}{2}}^{x_1} -\left[\frac{x_1+1}{2} - v\right]^2 dv + \int_{x_1}^{1} -\left[\frac{x_1+1}{2} - v\right]^2 dv$, which equals $-\frac{1}{2}b^2 - \frac{1}{384}$ (see Appendix C.6.i on page 26).

For S, the aggregate expected utility can be expressed as follows: $U_S^{II} = \int_{\frac{1}{2}}^{x_1} U_s(v|m_1)\,dv + \int_{x_1}^{1} U_s(v|m_2)\,dv$, which can be rewritten as $U_S^{II} = \int_{\frac{1}{2}}^{x_1} -\left[\frac{\frac{1}{2}+x_1}{2} - (v+b)\right]^2 dv + \int_{x_1}^{1} -\left[\frac{x_1+1}{2} - (v+b)\right]^2 dv$, which equals $-b^2 - \frac{1}{384}$ (see Appendix C.6.ii on page 27).

Note that the aggregate expected utility outcomes are identical for the respective players on the intervals I and II. Also note that the difference in aggregate expected utility of R en S has bisected relative to the situation where v lies on the interval [0,1]. This occurs because the difference between the high end of the interval and the largest a_S^* has also bisected relative to the situation where v lies on the interval [0,1].

Irrespective of the value of v, for S, choosing to share the partial information with R yields an aggregate expected utility of $U_S^T = U_S^I + U_S^{II} = \left(-b^2 - \frac{1}{384}\right) + \left(-b^2 - \frac{1}{384}\right) = -2b^2 - \frac{1}{192}$, while choosing to conceal the partial information yields $-2b^2 - \frac{1}{48}$ (recall Appendix A.3.ii on page 20). Hence, by truthfully sharing the partial information, S has a gain in aggregate expected utility of $\left(-2b^2 - \frac{1}{192}\right) - \left(-2b^2 - \frac{1}{48}\right) = \frac{1}{64}$.

Extended two-step equilibrium - will S deviate?

In order to explore whether S has an incentive to deviate from the extended two-step equilibrium, this section explicates two scenarios in which S deceives R by sharing incorrect partial information. In both scenarios, R believes S and responds accordingly. Note that R's responses have already been described in the previous section (recall *Extended two-step equilibrium* on page 9).

In the first scenario, S observes that v lies on interval I and S deceives R by sharing partial information that indicates that v lies on interval II. In second scenario, S observes that v lies on interval II and S deceives R by sharing partial information that indicates that v lies on interval I. For both scenarios, S's optimal m_i will be determined. Each scenario concludes with comparing S's aggregate expected utility in that scenario with S's aggregate expected utility in a situation of always telling the truth, $U_S^I = U_S^{II} = -b^2 - \frac{1}{384}$ (recall Appendix C.6.ii on page 27).

Suppose that S observes that v lies on interval I and that S deceives R by sharing partial information that indicates that v lies on interval II. Note that $x_1 = \frac{3}{4} - 2b$ (recall Appendix C.4 on page 25)Given R's belief that v lies on interval II, S will choose a utility maximizing m_i. S's aggregate expected utility when always sending $m_i = m_1$ equals $U_S^{Ia_1} = \int_0^{\frac{1}{2}} - \left[\frac{1/2+x_1}{2} - (v+b)\right]^2 dv$, which equals $-2b^2 + \frac{3}{4}b - \frac{31}{384}$. S's aggregate expected utility when sending $m_i = m_2$ equals $U_S^{Ia_2} = \int_0^{\frac{1}{2}} - \left[\frac{x_1+1}{2} - (v+b)\right]^2 dv$, which equals $-2b^2 + \frac{5}{4}b - \frac{79}{384}$. For all values of $b < 1/4$, $U_S^{Ia_1} > U_S^{Ia_2}$, so S will send $m_i = m_1$ when she knows that v lies on interval I and R thinks that v lies on interval II. Yet, $U_S^{Ia_1} < U_S^I$, so S will not deceive R.

Suppose that S observes that v lies on interval II and that S deceives R by sharing partial information that indicates that v lies on interval I. Note that $x_1 = \frac{1}{4} - 2b$ (recall Appendix C.1 on page 24). Given R's belief that v lies on interval I, S will choose a utility maximizing m_i. S's aggregate expected utility when always sending $m_i = m_1$ equals $U_S^{IIa_1} = \int_{\frac{1}{2}}^{1} - \left[\frac{x_1}{2} - (v+b)\right]^2 dv$, which equals $-2b^2 - \frac{5}{4}b - \frac{79}{384}$. S's aggregate expected utility when sending $m_i = m_2$ equals $U_S^{IIa_2} = \int_{\frac{1}{2}}^{1} - \left[\frac{1/2+x_1}{2} - (v+b)\right]^2 dv$, which equals $-2b^2 - \frac{3}{4}b - \frac{31}{384}$. For all values of b, $U_S^{IIa_1} < U_S^{IIa_2}$, so S will send $m_i = m_1$ when she knows that v lies on interval II and R thinks that v lies on interval I. Yet, $U_S^{IIa_2} < U_S^I$, so S will not deceive R.

It has been proved that, in an extended two-step equilibrium on the interval [0,1], S will truthfully share the partial information with R.

Extended three-step equilibrium

Suppose that S learns that v lies on interval I. If R receives m_1, he infers that v lies on the interval $[0, y_1]$, thus updating his expected value of v to $y_1/2$. This leads R to choosing $a_i = a_1 = E(v|m_1) = y_1/2$ to maximize his own utility.

If R receives m_2, he infers that v lies on the interval $[y_1, y_2]$, thus updating his expected value of v to $(y_1 + y_2)/2$. This leads to R choosing $a_i = a_2 = E(v|m_2) = (y_1 + y_2)/2$ to maximize his own utility.

If R receives m_3, he infers that v lies on the interval $[y_2, 1/2]$, thus updating his expected value of v to $(y_2 + 1/2)/2$. This leads to R choosing $a_i = a_3 = E(v|m_3) = (y_2 + 1/2)/2$ to maximize his own utility.

Anticipating R's strategy, S sends m_1 for all values of v on the interval $[0, y_1)$, m_2 for all values of v on the interval (y_1, y_2) and m_3 for all values of v on the interval $(y_2, 1/2]$ to maximize her own utility. Two conditions regarding S's choice of m_i arise.

Firstly, if $v = y_1$, S must be indifferent between sending m_1 and m_2, that is, $U_S(m_1|v = y_1) = U_S(m_2|v = y_1)$, which can be rewritten as $-[a_1 - (v + b)]^2 = -[a_2 - (v + b)]^2$. Given that $0 < a_1 < v = y_1 < a_2$, the latter formula solves for $y_1 = y_2/2 - 2b$ and $b < y_2/4$ (see Appendix D.1.i on page 27).

Secondly, if $v = y_2$, S must be indifferent between sending m_2 and m_3, that is, $U_S(m_2|v = y_2) = U_S(m_3|v = y_2)$, which can be rewritten as $-[a_2 - (v + b)]^2 = -[a_3 - (v + b)]^2$. The latter formula solves for $y_1 = 1/6 - 4b$, $y_2 = 1/3 - 4b$ and $b < 1/24$ (see Appendix D.1.ii on page 28).

Given that $y_2 = 1/3 - 4b$, it can be proved that message interval 3, $[y_2, 1/2]$, is longer than message interval 2, $[y_1, y_2]$, that is: $1/2 - y_2 > y_2 - y_1$, since this inequality yields $4b > 0$, which, given that $b > 0$, implicates that message interval 3 is $4b$ longer than message interval 2 (see Appendix D.2.i on page 28).

Also, given that $y_1 = 1/6 - 4b$, it can be proved that message interval 2 is longer than message interval 1, $[0, y_1]$, that is: $y_2 - y_1 > y_1 - 0$, since this inequality yields $4b > 0$, which, given that $b > 0$, implicates that message interval 2 is $4b$ longer than message interval 1 (see Appendix D.2.ii on page 29). Note that this outcome is identical to the outcome in the initial three-step equilibrium.

A player's aggregate expected utility for the three-step equilibrium on the interval [0,1/2] can be calculated using the sum of three integrals. The first integral covers the situation where $v = [0, y_1]$, S chooses $m_i = m_1$ and R chooses $a_i = a_1 = y_1/2$. The second integral covers the situation where $v = [y_1, y_2]$, S chooses $m_i = m_2$ and R chooses $a_i = a_2 = (y_1 + y_2)/2$. The third integral covers the situation where $v = [y_2, 1/2]$, S chooses $m_i = m_3$ and R chooses $a_i = a_3 = (y_2 + 1/2)/2$.

For R, the aggregate expected utility can be expressed as follows: $U_R^I = \int_0^{y_1} U_R(v|m_1)\, dv + \int_{y_1}^{y_2} U_R(v|m_2)\, dv + \int_{y_2}^{\frac{1}{2}} U_R(v|m_3)\, dv$, which can be rewritten as $U_R^I = \int_0^{y_1} -\left(\frac{y_1}{2} - v\right)^2 dv + \int_{y_1}^{y_2} -\left(\frac{y_1+y_2}{2} - v\right)^2 dv + \int_{y_2}^{\frac{1}{2}} -\left(\frac{y_2+1/2}{2} - v\right)^2 dv$, which equals $-\frac{4}{3}b^2 - \frac{1}{864}$ (see Appendix D.3.i on page 29).

For S, the aggregate expected utility can be expressed as follows: $U_S^I = \int_0^{y_1} U_R(v|m_1)\, dv + \int_{y_1}^{y_2} U_R(v|m_2)\, dv + \int_{y_2}^{\frac{1}{2}} U_R(v|m_3)\, dv$, which can be rewritten as $U_S^I = \int_0^{y_1} -\left[\frac{y_1}{2} - (v + b)\right]^2 dv + \int_{y_1}^{y_2} -\left[\frac{y_1+y_2}{2} - (v + b)\right]^2 dv + \int_{y_2}^{\frac{1}{2}} -\left[\frac{y_2+1/2}{2} - (v + b)\right]^2 dv$, which equals $-\frac{11}{6}b^2 - \frac{1}{864}$ (see Appendix D.3.ii on page 30).

Suppose that S learns that v lies on interval II. If R receives m_1, he infers that v lies on the interval $[1/2, y_1]$, thus updating his expected value of v to $(1/2 + y_1)/2$. This leads R to choosing $a_i = a_1 = E(v|m_1) = (1/2 + y_1)/2$ to maximize his own utility.

If R receives m_2, he infers that v lies on the interval $[y_1, y_2]$, thus updating his expected value of v to $(y_1 + y_2)/2$. This leads to R choosing $a_i = a_2 = E(v|m_2) = (y_1 + y_2)/2$ to maximize his own utility.

If R receives m_3, he infers that v lies on the interval $[y_2, 1]$, thus updating his expected value of v to $(y_2 + 1)/2$. This leads to R choosing $a_i = a_3 = E(v|m_3) = (y_2 + 1)/2$ to maximize his own utility.

Anticipating R's strategy, S sends m_1 for all values of v on the interval $[0, y_1)$, m_2 for all values of v on the interval (y_1, y_2) and m_3 for all values of v on the interval $(y_2, 1/2]$ to maximize her own utility. Two conditions regarding S's choice of m_i arise.

Firstly, if $v = y_1$, S must be indifferent between sending m_1 and m_2, that is, $U_S(m_1|v = y_1) = U_S(m_2|v = y_1)$, which can be rewritten as $-[a_1 - (v + b)]^2 = -[a_2 - (v + b)]^2$. Given that $0 < a_1 < v = y_1 < a_2$, the latter formula solves for $y_1 = y_2/2 - 2b$ and $b < y_2/4$ (see Appendix D.4.i on page 30).

Secondly, if $v = y_2$, S must be indifferent between sending m_2 and m_3, that is, $U_S(m_2|v = y_2) = U_S(m_3|v = y_2)$, which can be rewritten as $-[a_2 - (v + b)]^2 = -[a_3 - (v + b)]^2$. The latter formula solves for $y_1 = 2/3 - 4b$, $y_2 = 5/6 - 4b$ and $b < 1/6$ (see Appendix D.4.ii on page 31).

Given that $y_2 = \frac{5}{6} - 4b$, it can be proved that message interval 3, $[y_2, 1]$, is longer than message interval 2, $[y_1, y_2]$, that is: $1 - y_2 > y_2 - y_1$, since this inequality yields $4b > 0$, which, given that $b > 0$, implicates that message interval 3 is $4b$ longer than message interval 2 (see Appendix D.5.i on page 31).

Also, given that $y_1 = 2/3 - 4b$, it can be proved that message interval 2 is longer than message interval 1, $[1/2, y_1]$, that is: $y_2 - y_1 > y_1 - 1/2$, since this inequality yields $4b > 0$, which, given that $b > 0$, implicates that message interval 2 is $4b$ longer than message interval 1 (see Appendix D.5.ii on page 32). Note that this outcome is identical to the outcome in the initial three-step equilibrium and identical to the outcome of the extended three-step equilibrium on interval I.

A player's aggregate expected utility for the three-step equilibrium on the interval $\left[\frac{1}{2}, 1\right]$ equals the sum of three integrals. The first integral covers the situation where $v = \left[\frac{1}{2}, y_1\right]$, S chooses $m_i = m_1$ and R chooses $a_i = a_1 = \frac{1/2+y_1}{2}$. The second integral covers the situation where $v = [y_1, y_2]$, S chooses $m_i = m_2$ and R chooses $a_i = a_2 = \frac{y_1+y_2}{2}$. The third integral covers the situation where $v = [y_2, 1]$, S chooses $m_i = m_3$ and R chooses $a_i = a_3 = \frac{y_2+1}{2}$.

For R, this can be expressed as follows: $U_R^{II} = \int_{\frac{1}{2}}^{y_1} U_R(v|m_1)\,dv + \int_{y_1}^{y_2} U_R(v|m_2)\,dv + \int_{y_2}^{1} U_R(v|m_3)\,dv$, which can be rewritten as $U_R^{II} = \int_{\frac{1}{2}}^{y_1} -\left(\frac{1/2+y_1}{2} - v\right)^2 dv + \int_{y_1}^{y_2} -\left(\frac{y_1+y_2}{2} - v\right)^2 dv + \int_{y_2}^{1} -\left(\frac{y_2+1}{2} - v\right)^2 dv$, which equals $-\frac{4}{3}b^2 - \frac{1}{864}$ (see Appendix D.6.i on page 32).

For S, this can be expressed as follows: $U_S^{II} = \int_{\frac{1}{2}}^{y_1} U_R(v|m_1)\,dv + \int_{y_1}^{y_2} U_R(v|m_2)\,dv + \int_{y_2}^{1} U_R(v|m_3)\,dv$, which can be rewritten as

$U_S^{II} = \int_{\frac{1}{2}}^{y_1} -\left[\frac{1/2+y_1}{2} - (v+b)\right]^2 dv + \int_{y_1}^{y_2} -\left[\frac{y_1+y_2}{2} - (v+b)\right]^2 dv + \int_{y_2}^{1} -\left[\frac{y_2+1}{2} - (v+b)\right]^2 dv$, which equals $-\frac{11}{6}b^2 - \frac{1}{864}$ (see Appendix D.6.ii on page 33).

Note two things that are similar to the extended two-step equilibrium. Firstly, the aggregate expected utility outcomes are identical for the respective players on the intervals I and II. Secondly, the difference in aggregate expected utility of R en S has bisected relative to the situation where v lies on the interval [0,1]. This occurs because the difference between the high end of the interval and the largest a_S^* has also bisected relative to the situation where v lies on the interval [0,1].

Irrespective of the value of v, for S, choosing to share the partial information with R yields an aggregate expected utility of $U_S^T = U_S^I + U_S^{II} = \left(-\frac{11}{6}b^2 - \frac{1}{864}\right) + \left(-\frac{11}{6}b^2 - \frac{1}{864}\right) = -\frac{11}{3}b^2 - \frac{1}{432}$, while choosing to conceal the partial information yields $-\frac{11}{3}b^2 - \frac{1}{108}$ (recall Appendix B.3.ii on page 23). Hence, by sharing the partial information, S has a gain in aggregate expected utility of $\left(-\frac{11}{3}b^2 - \frac{1}{432}\right) - \left(-\frac{11}{3}b^2 - \frac{1}{108}\right) = \frac{1}{144}$.

Extended three-step equilibrium - will S deviate?

In order to explore whether S has an incentive to deviate from the extended three-step equilibrium, this section explicates two scenarios in which S deceives R by sharing incorrect information. In both scenarios, R believes S and responds accordingly. Note that R's responses have already been described in the previous section.

In the first scenario, S observes that v lies on interval I and S deceives R by sharing information that indicates that v lies on interval II. In second scenario, S observes that v lies on interval II and S deceives R by sharing information that indicates that v lies on interval I. For both scenarios, S's optimal m_i will be determined. Each scenario concludes with comparing S's aggregate expected utility in that scenario with S's aggregate expected utility in a situation of always telling the truth, $U_S^I = U_S^{II} = -\frac{11}{6}b^2 - \frac{1}{864}$.

Suppose that S observes that v lies on interval I and that S deceives R by sharing information that indicates that v lies on interval II. Note that $y_1 = \frac{2}{3} - 4b$ and $y_2 = \frac{5}{6} - 4b$. Given R's belief that v lies on interval II, S will choose a utility maximizing m_i. S's aggregate expected utility when always sending $m_i = m_1$ equals $U_S^{Ia_1} = \int_0^{\frac{1}{2}} -\left[\frac{1/2+y_1}{2} - (v+b)\right]^2 dv$, which equals $-\frac{9}{2}b^2 + \frac{1}{4}b - \frac{1}{72}$ (see Appendix D.7.i on page 33). S's aggregate expected utility when always sending $m_i = m_2$ equals $U_S^{Ia_2} = \int_0^{\frac{1}{2}} -\left[\frac{y_1+y_2}{2} - (v+b)\right]^2 dv$, which equals $-\frac{25}{2}b^2 + \frac{5}{2}b - \frac{13}{96}$. S's aggregate expected utility when always sending $m_i = m_3$ equals $U_S^{Ia_3} =$

$\int_0^{\frac{1}{2}} -\left[\frac{y_2+1}{2} - (v+b)\right]^2 dv$, which equals $-\frac{9}{2}b^2 + \frac{5}{4}b - \frac{7}{72}$. For all values of $b < 5/24$, $U_S^{Ia_1} > U_S^{Ia_2}$. For all values of $b < 1/12$, $U_S^{Ia_1} > U_S^{Ia_3}$. Given the restriction of $b < 1/24$ for an extended three-step equilibrium to exist, S will always send $m_i = m_1$ when she knows that v lies on interval I and R thinks that v lies on interval II. Yet, for all values of b, $U_S^{Ia_1} < U_S^I$, so S will not deceive R.

Suppose that S observes that v lies on interval II and that S deceives R by sharing information that indicates that v lies on interval I. Note that $y_1 = \frac{1}{6} - 4b$ and $y_2 = \frac{1}{3} - 4b$. Given R's belief that v lies on interval I, S will choose a utility maximizing m_i. S's aggregate expected utility when always sending $m_i = m_1$ equals $U_S^{IIa_1} = \int_{\frac{1}{2}}^1 -\left[\frac{y_1}{2} - (v+b)\right]^2 dv$, which equals $-\frac{9}{2}b^2 - \frac{5}{4}b - \frac{7}{72}$ (see Appendix D.7.ii on page 34). S's aggregate expected utility when sending $m_i = m_2$ equals $U_S^{IIa_2} = \int_{\frac{1}{2}}^1 -\left[\frac{y_1+y_2}{2} - (v+b)\right]^2 dv$, which equals $-\frac{25}{2}b^2 - \frac{5}{2}b - \frac{13}{96}$. For all values of b, $U_S^{Ia_3} > U_S^{Ia_2}$. For all values of $b < 1/12$, $U_S^{Ia_3} > U_S^{Ia_1}$. Therefore, S will always send $m_i = m_3$ when she knows that v lies on interval II and R thinks that v lies on interval I. Yet, for all values of b, $U_S^{Ia_3} < U_S^I$, so S will not deceive R.

It has been proved that, in an extended three-step equilibrium on the interval [0,1], S will always truthfully share the partial information with R.

Conclusion extended partially pooling equilibria

It has been proved that when $b < 1/8$, an extened two-step equilibrium exists and that when $b < 1/24$, an extended three-step equilibrium exist. In the extended three-step equilibrium, less information is lost in the communication than in the extended two-step equilibrium. Therefore, an extended three-step equilibrium is more efficient than an extended two-step equilibrium.

Examples

Scenario 1: Suppose that $b = 1/4$. The only equilibrium that exists is a babbling equilibrium. Communication contains no information and the aggregate expected utility equals $-\frac{1}{12}$ for R and $-b^2 - \frac{1}{12} = -\left(\frac{1}{4}\right)^2 - \frac{1}{12} = -\frac{7}{48}$ for S.

Scenario 2: Suppose that $b = 1/12$. Compared to Scenario 1, the interest asymmetry between R and S declined and now, next to a babbling equilibrium, a two-step equilibrium exists. Moreover, since b is small enough, an extended two-step equilibrium exists. However, b is too large for a three-step equilibrium to exist. The same goes for an extended three-step equilibrium.

In the babbling equilibrium, the aggregate expected utility equals $-\frac{1}{12}$ for R and $-b^2 - \frac{1}{12} = -\left(\frac{1}{12}\right)^2 - \frac{1}{12} = -\frac{13}{144}$ for S.

In the two-step equilibrium, the aggregate expected utility equals $-b^2 - \frac{1}{48} = -\left(\frac{1}{12}\right)^2 - \frac{1}{48} = -\frac{1}{36}$ for R and $-2b^2 - \frac{1}{48} = -2\left(\frac{1}{12}\right)^2 - \frac{1}{48} = -\frac{5}{144}$. Note that, compared to the babbling equilibrium, in the two-step equilibrium communication is richer and both players are better off.

In the extended two-step equilibrium, the aggregate expected utility equals $-\frac{1}{2}b^2 - \frac{1}{384} = -\frac{1}{2}\left(\frac{1}{12}\right)^2 - \frac{1}{384} = -\frac{7}{1152}$ for R and $-b^2 - \frac{1}{384} = -\left(\frac{1}{12}\right)^2 - \frac{1}{384} = -\frac{11}{1152}$ for S. Note that, compared tot the babbling equilibrium and the two-step equilibrium, in the extended two-step equilibrium communication is richer and both players are better off. This is the most efficient equilibrium when $b = 1/12$.

These examples illustrate that smaller interest asymmetry between S and R yields richer communication and makes both players better off.

Conclusion

In this thesis, it is explored whether communication of partial information prior to a signaling game enriches the communication in the signaling game. To this end, a model is developed in which two players, with asymmetrical information and asymmetrical interests, communicate. The main feature of the model is a communication signal prior to the signaling game. The model has three types of equilibria.

The first type is a perfect Bayesian equilbirium in which no information is lost in the communication. The restriction for such an equilibrium to exist is that the interests of the Sender and the Receiver are perfectly aligned, that is $b = 0$. This equilibrium yields the highest aggregate expected utility to both players.

The second type is a babbling equilibrium, in which communication is meaningless. Such an equilibrium has no restrictions to the asymmetricality of interests, because there is complete loss of information in the communication. This is the only equilibrium that exists when $b \geq 1/4$. This equilibrium yields the lowest aggregate expected utility to both players.

The third type is a partially pooling equilibrium, in which information loss in the communication occurs, but is limited. Such an equilibrium exists when $0 < b < 1/4$. A partially pooling equilibrium yields higher aggregate expected utility to both players than a babbling equilibrium, but lower aggregate expected utility to both players than a perfect Bayesian equilibrium.

There are two types of partially pooling equilibria described in this thesis. The first has two message intervals and is called a two-step equilibrium, and the second has three message intervals and is called a three-step equilibrium.

Both partially pooling equilibria are proved to have richer communication when partial information is shared prior to the signaling game. It has been proved that when $b < 1/8$, an extened two-step equilibrium exists and that when $b < 1/24$, an extended three-step equilibrium exist. Therefore, the main conclusion of this thesis is that when interest asymmetry is limited, sharing partial information via a communication signal prior to a signaling game yields richer communication.

The equilibria of the model and its extension can be summarized in two existing phenomena: the ally principle and the communication constraint.

The ally principle is proved to apply: it has been shown that the lower interest asymmetry between the Sender and the Receiver, the less information is lost in the communication, that is, the richer the information, and the better off both players are.

Also, the communication constraint is proved to apply: when $b \geq 1/4$, the only equilibrium that exists on the interval [0,1] is a babbling equilibrium. This restriction to b for informative communication to exist, corresponds findings of Gibbons (1992).

In terms of the examples in the introduction, this means the following.

When $b \geq 1/4$, Rick ignores what Scottie says about her score and Ray ignores what Susan sends of her resume. Scottie sends a resume independently of her own skill and Susan says a performance score independently of her actual score.

When $b < 1/4$, Scottie will tell her skill score to Rick and Susan will tell her performance score to Ray. When $b < 1/8$, Scottie will also send her resume to Ray and Susan will ask her friend to inform Ray about her performance.

When $b < 1/12$, communication between the ladies and the gentlemen can contain more information, since there exists an equilibrium with an extra, third, message interval. When $b < 1/24$, that equilibrium is extended and communication becomes even richer, because the partial information will be communicated to the Receiver through the resume or the friend.

The same holds, of course, for all Senders and Receivers in similar situations.

It has been found that the aggregate expected utility of the Receiver is always higher than the aggregate expected utility of the Sender. This difference equals b^2 in the initial equilibria and $\frac{1}{2}b^2$ in the extended equilibria.

This difference exists because b causes S to prefer action range $[0+b, 1+b]$, although both v and a lie on the interval [0,1]. Therefore, for all values of $a_S^* > 1$, the only possible actions closest to a_S^* are $a = 1$. The larger b, the larger the difference between a and the a_S^*. Since R's utility function does not depend on b, R's optimal action always lies on the interval [0,1].

Since the interval on which the signaling game is played bisects when the Sender sends a credible signal prior to the signaling game, the impact of b is smaller.

The quadratic character of the difference between R's aggregate expected utility and S's aggregate expected utility is caused by the quadratic character of b in the utility function of S.

As for the practical implications of this thesis, Sobel (2010, p. 33) justly stated: *"While the study of strategic communication supplies some powerful insights, the domain of plausible models is so rich that the most reliable intuition will fail in some simple environment."*

Future research can focus on further specifying the signal prior to the signaling game. For example, the signal can be expanded to three (or more) intervals. Future research can also focus on the impact of inacuracy: the impact of a difference between the action the Receiver wants to take and the action that is ultimately realized. The same can be done for inacuracy of the Sender: the impact of a difference between the message a Sender wants to send and the message that is actually sent.

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Appendix A.1

Two-step equilibrium

If $v = x_1$, S must be indifferent between sending m_1 and m_2:

$$U_S(m_1|v = x_1) = U_S(m_2|v = x_1) =$$

$$-[a_1 - (v + b)]^2 = -[a_2 - (v + b)]^2 =$$

$$|a_1 - (v + b)| = |a_2 - (v + b)|$$

which, given that $0 < a_1 < v = x_1 < a_2$, equals

$$(x_1 + b) - a_1 = a_2 - (x_1 + b) =$$

$$(x_1 + b) - \frac{x_1}{2} = \frac{x_1 + 1}{2} - (x_1 + b)$$

which yields

$$x_1 = \frac{1}{2} - 2b$$

which, given that $x_1 > 0$, yields

$$b < \frac{1}{4}$$

Appendix A.2

Two-step equilibrium

Message interval 2, $[x_1, 1]$, is longer than message interval 1, $[0, x_1]$, that is:

$$1 - x_1 > x_1 - 0$$

$$1 - 2x_1 > 0$$

$$1 - 2(1/2 - 2b) > 0$$

$$4b > 0$$

More specifically, message interval 2 is $4b$ longer than message interval 1.

Appendix A.3.i

Two-step equilibrium

R's aggregate expected utility equals:

$$U_R^T = \int_0^{x_1} U_R(v)\, dv + \int_{x_1}^1 U_R(v)\, dv =$$

$$\int_0^{x_1} -(a_1 - v)^2 dv + \int_{x_1}^1 -(a_2 - v)^2 dv =$$

$$\int_0^{x_1} -\left(\frac{x_1}{2} - v\right)^2 dv + \int_{x_1}^1 -\left(\frac{x_1 + 1}{2} - v\right)^2 dv =$$

$$\int_0^{\frac{1}{2}-2b} -\left(\frac{\frac{1}{2} - 2b}{2} - v\right)^2 dv + \int_{\frac{1}{2}-2b}^1 -\left[\frac{\left(\frac{1}{2} - 2b\right) + 1}{2} - v\right]^2 dv =$$

$$U_R^T = -b^2 - \frac{1}{48}$$

Appendix A.3.ii

Two-step equilibrium

S's aggregate expected utility equals:

$$U_S^T = \int_0^{x_1} U_S(v|a_1)\, dv + \int_{x_1}^1 U_S(v|a_2)\, dv =$$

$$\int_0^{x_1} -[E(a_1) - (v + b)]^2 dv + \int_{x_1}^1 -[E(a_2) - (v + b)]^2 dv$$

$$\int_0^{x_1} -\left(\frac{x_1}{2} - v - b\right)^2 dv + \int_{x_1}^1 -\left(\frac{x_1 + 1}{2} - v - b\right)^2 dv =$$

$$\int_0^{\frac{1}{2}-2b} -\left(\frac{\frac{1}{2}-2b}{2} - v - b\right)^2 dv + \int_{\frac{1}{2}-2b}^{1} -\left[\frac{\left(\frac{1}{2}-2b\right)+1}{2} - v - b\right]^2 dv =$$

$$U_S^T = -2b^2 - \frac{1}{48}$$

Appendix B.1.i

Three-step equilibrium

If $v = y_1$, S must be indifferent between sending m_1 and m_2:

$$U_S(m_1|v = y_1) = U_S(m_2|v = y_1) =$$

$$-[a_1 - (v+b)]^2 = -[a_2 - (v+b)]^2 =$$

$$|a_1 - (v+b)| = |a_2 - (v+b)|$$

which, given that $0 < a_1 < v = y_1 < a_2$, equals

$$(y_1 + b) - a_1 = a_2 - (y_1 + b) =$$

$$(y_1 + b) - \frac{y_1}{2} = \frac{y_1 + y_2}{2} - (y_1 + b)$$

which yields

$$y_1 = \frac{y_2}{2} - 2b$$

which, given that $y_1 > 0$, yields

$$b < \frac{y_2}{4}$$

Appendix B.1.ii

Three-step equilibrium

If $v = y_2$, S must be indifferent between sending m_2 and m_3:

$$U_S(m_2|v = y_2) = U_S(m_3|v = y_2) =$$

$$-[a_2 - (v+b)]^2 = -[a_3 - (v+b)]^2 =$$

$$|a_2 - (v+b)| = |a_3 - (v+b)|$$

which, given that $0 < a_2 < v = y_2 < a_3$, equals

$$(y_2 + b) - a_2 = a_3 - (y_2 + b) =$$

$$(y_2 + b) - \frac{y_1 + y_2}{2} = \frac{y_2 + 1}{2} - (y_2 + b) =$$

$$y_2 = \frac{1}{2} + \frac{y_1}{2} - 2b =$$

$$y_2 = \frac{2}{3} - 4b$$

$$y_1 = \frac{1}{3} - 4b$$

which, given that $y_1 > 0$, yields

$$b < \frac{1}{12}$$

Appendix B.2.i

Three-step equilibrium

Message interval 3, $[y_2, 1]$, is longer than message interval 2, $[y_1, y_2]$, that is:

$$1 - y_2 > y_2 - y_1$$

$$1 + y_1 - 2y_2 > 0$$

$$1 + \left(\frac{1}{3} - 4b\right) - 2\left(\frac{2}{3} - 4b\right) > 0$$

$$4b > 0$$

More specifically, message interval 3 is $4b$ longer than message interval 2.

Appendix B.2.ii

Three-step equilibrium

Message interval 2, $[y_1, y_2]$, is longer than message interval 1, $[0, y_1]$, that is:

$$y_2 - y_1 > y_1 - 0$$

$$y_2 - 2y_1 > 0$$

$$\left(\frac{2}{3} - 4b\right) - 2\left(\frac{1}{3} - 4b\right) > 0$$

$$4b > 0$$

More specifically, interval 2 is $4b$ longer than interval 1.

Appendix B.3.i

Three-step equilibrium

R's aggregate expected utility equals:

$$U_R^T = \int_0^{y_1} U_R(v)\, dv + \int_{y_1}^{y_2} U_R(v)\, dv + \int_{y_2}^{1} U_R(v)\, dv =$$

$$\int_0^{y_1} -(a_1 - v)^2\, dv + \int_{y_1}^{y_2} -(a_2 - v)^2\, dv + \int_{y_2}^{1} -(a_3 - v)^2\, dv =$$

$$\int_0^{y_1} -\left(\frac{y_1}{2} - v\right)^2 dv + \int_{y_1}^{y_2} -\left(\frac{y_1 + y_2}{2} - v\right)^2 dv + \int_{y_2}^{1} -\left(\frac{y_2 + 1}{2} - v\right)^2 dv =$$

$$\int_0^{\frac{1}{3}-4b} -\left[\frac{\left(\frac{1}{3} - 4b\right)}{2} - v\right]^2 dv + \int_{\frac{1}{3}-4b}^{\frac{2}{3}-4b} -\left[\frac{\left(\frac{1}{3} - 4b\right) + \left(\frac{2}{3} - 4b\right)}{2} - v\right]^2 dv$$

$$+ \int_{\frac{2}{3}-4b}^{1} -\left[\frac{\left(\frac{2}{3} - 4b\right) + 1}{2} - v\right]^2 dv =$$

$$U_R^T = -\frac{8}{3}b^2 - \frac{1}{108}$$

Appendix B.3.ii

Three-step equilibrium

S's aggregate expected utility equals:

$$U_S^T = \int_0^{y_1} U_S(v)\, dv + \int_{y_1}^{y_2} U_S(v)\, dv + \int_{y_2}^{1} U_S(v)\, dv =$$

$$\int_0^{y_1} -[a_1 - (v + b)]^2\, dv + \int_{y_1}^{y_2} -[a_2 - (v + b)]^2\, dv + \int_{y_2}^{1} -[a_3 - (v + b)]^2\, dv =$$

$$\int_0^{y_1} -\left(\frac{y_1}{2} - v - b\right)^2 dv + \int_{y_1}^{y_2} -\left(\frac{y_1 + y_2}{2} - v - b\right)^2 dv$$

$$+ \int_{y_2}^{1} -\left(\frac{y_2 + 1}{2} - v - b\right)^2 dv =$$

$$\int_0^{\frac{1}{3}-4b} -\left[\frac{\left(\frac{1}{3} - 4b\right)}{2} - v - b\right]^2 dv + \int_{\frac{1}{3}-4b}^{\frac{2}{3}-4b} -\left[\frac{\left(\frac{1}{3} - 4b\right) + \left(\frac{2}{3} - 4b\right)}{2} - v - b\right]^2 dv$$

$$+ \int_{\frac{2}{3}-4b}^{1} -\left[\frac{\left(\frac{2}{3} - 4b\right) + 1}{2} - v - b\right]^2 dv =$$

$$U_S^T = -\frac{11}{3}b^2 - \frac{1}{108}$$

Appendix C.1

Extended two-step equilibrium on interval I

If $v = x_1$, S must be indifferent between sending m_1 and m_2:

$$U_S(m_1|v = x_1) = U_S(m_2|v = x_1) =$$

$$-[a_1 - (v + b)]^2 = -[a_2 - (v + b)]^2 =$$

$$|a_1 - (v + b)| = |a_2 - (v + b)|$$

which, given that $0 < a_1 < v = x_1 < a_2$, equals

$$(x_1 + b) - a_1 = a_2 - (x_1 + b) =$$

$$(x_1 + b) - \frac{x_1}{2} = \frac{x_1 + 1/2}{2} - (x_1 + b)$$

which yields

$$x_1 = \frac{1}{4} - 2b$$

which, given that $x_1 > 0$, yields

$$b < \frac{1}{8}$$

Appendix C.2

Extended two-step equilibrium on interval I

Message interval 2, $[x_1, 1/2]$, is longer than message interval 1, $[0, x_1]$, that is:

$$1/2 - x_1 > x_1 - 0$$

$$1/2 - 2x_1 > 0$$

$$1/2 - 2(1/4 - 2b) > 0$$

$$4b > 0$$

More specifically, message interval 2 is $4b$ longer than message interval 1.

Appendix C.3.i

Extended two-step equilibrium on interval I

R's aggregate expected utility equals:

$$U_R^I = \int_0^{x_1} U_R(v|a_1)\, dv + \int_{x_1}^{\frac{1}{2}} U_R(v|a_2)\, dv =$$

$$\int_0^{x_1} -[E(a_1) - v]^2 dv + \int_{x_1}^{\frac{1}{2}} -[E(a_2) - v]^2 dv$$

$$\int_0^{x_1} -\left(\frac{x_1}{2} - v\right)^2 dv + \int_{x_1}^{\frac{1}{2}} -\left(\frac{x_1 + 1/2}{2} - v\right)^2 dv =$$

$$\int_0^{\frac{1}{4}-2b} -\left(\frac{\frac{1}{4} - 2b}{2} - v\right)^2 dv + \int_{\frac{1}{4}-2b}^{1} -\left[\frac{\left(\frac{1}{4} - 2b\right) + 1/2}{2} - v\right]^2 dv =$$

$$U_R^I = -\frac{1}{2}b^2 - \frac{1}{384}$$

Appendix C.3.ii

Extended two-step equilibrium on interval I

S's aggregate expected utility equals:

$$U_S^I = \int_0^{x_1} U_S(v|a_1)\, dv + \int_{x_1}^{\frac{1}{2}} U_S(v|a_2)\, dv =$$

$$\int_0^{x_1} -[E(a_1) - (v + b)]^2 dv + \int_{x_1}^{\frac{1}{2}} -[E(a_2) - (v + b)]^2 dv$$

$$\int_0^{x_1} -\left(\frac{x_1}{2} - v - b\right)^2 dv + \int_{x_1}^{\frac{1}{2}} -\left(\frac{x_1 + 1/2}{2} - v - b\right)^2 dv =$$

$$\int_0^{\frac{1}{4}-2b} -\left(\frac{\frac{1}{4} - 2b}{2} - v - b\right)^2 dv + \int_{\frac{1}{4}-2b}^{\frac{1}{2}} -\left[\frac{\left(\frac{1}{4} - 2b\right) + 1/2}{2} - v - b\right]^2 dv =$$

$$U_S^I = -b^2 - \frac{1}{384}$$

Appendix C.4

Extended two-step equilibrium on interval II

If $v = x_1$, S must be indifferent between sending m_1 and m_2:

$$U_S(m_1|v = x_1) = U_S(m_2|v = x_1) =$$

$$-[a_1 - (v + b)]^2 = -[a_2 - (v + b)]^2 =$$

$$|a_1 - (v + b)| = |a_2 - (v + b)|$$

which, given that $0 < a_1 < v = x_1 < a_2$, equals

$$(x_1 + b) - a_1 = a_2 - (x_1 + b) =$$

$$(x_1 + b) - \frac{1/2 + x_1}{2} = \frac{x_1 + 1}{2} - (x_1 + b)$$

which yields

$$x_1 = \frac{3}{4} - 2b$$

which, given that $x_1 > 0$, yields

$$b < \frac{3}{8}$$

Appendix C.5

Extended two-step equilibrium on interval II

The interval $[x_1, 1]$ is larger than the interval $[1/2, x_1]$, that is:

$$1 - x_1 > x_1 - 1/2$$

$$3/2 - 2x_1 > 0$$

$$3/2 - 2(3/4 - 2b) > 0$$

$$4b > 0$$

More specifically, the interval $[x_1, 1]$ is $4b$ larger than the interval $[1/2, x_1]$.

Appendix C.6.i

Extended two-step equilibrium on interval II

R's aggregate expected utility equals:

$$U_R^{II} = \int_{\frac{1}{2}}^{x_1} U_R(v|a_1)\, dv + \int_{x_1}^{1} U_R(v|a_2)\, dv =$$

$$\int_{\frac{1}{2}}^{x_1} -[E(a_1) - v]^2 dv + \int_{x_1}^{1} -[E(a_2) - v]^2 dv$$

$$\int_{\frac{1}{2}}^{x_1} -\left(\frac{1/2 + x_1}{2} - v\right)^2 dv + \int_{x_1}^{1} -\left(\frac{x_1 + 1}{2} - v\right)^2 dv =$$

$$\int_{\frac{1}{2}}^{\frac{3}{4}-2b} -\left[\frac{\frac{1}{2} + \left(\frac{3}{4} - 2b\right)}{2} - v\right]^2 dv + \int_{\frac{3}{4}-2b}^{1} -\left[\frac{\left(\frac{3}{4} - 2b\right) + 1}{2} - v\right]^2 dv =$$

$$U_R^{II} = -\frac{1}{2}b^2 - \frac{1}{384}$$

Appendix C.6.ii

Extended two-step equilibrium on interval II

S's aggregate expected utility equals:

$$U_S^{II} = \int_{\frac{1}{2}}^{x_1} U_S(v|a_1)\, dv + \int_{x_1}^{1} U_S(v|a_2)\, dv =$$

$$\int_{\frac{1}{2}}^{x_1} -[E(a_1) - (v+b)]^2 dv + \int_{x_1}^{1} -[E(a_2) - (v+B)]^2 dv$$

$$\int_{\frac{1}{2}}^{x_1} -\left(\frac{1/2 + x_1}{2} - v - b\right)^2 dv + \int_{x_1}^{1} -\left(\frac{x_1 + 1}{2} - v - b\right)^2 dv =$$

$$\int_{\frac{1}{2}}^{\frac{3}{4}-2b} -\left[\frac{\frac{1}{2} + \left(\frac{3}{4} - 2b\right)}{2} - v - b\right]^2 dv + \int_{\frac{3}{4}-2b}^{1} -\left[\frac{\left(\frac{3}{4} - 2b\right) + 1}{2} - v - b\right]^2 dv =$$

$$U_S^{II} = -b^2 - \frac{1}{384}$$

Appendix C.7.i

Extended two-step equilibrium on interval I

Note that R thinks that v lies on interval II, $\left[\frac{1}{2}, 1\right]$, so $a_1 = \frac{1/2 + x_1}{2}$ and $x_1 = \frac{3}{4} - 2b$ (recall Appendix C.1 on page 24). S's aggregate expected utility when always sending $m_i = m_1$ equals

$$U_S^{Ia_1} = \int_0^{\frac{1}{2}} -[a_1 - (v+b)]^2 dv =$$

$$U_S^{Ia_1} = \int_0^{\frac{1}{2}} -\left[\frac{1/2 + x_1}{2} - (v+b)\right]^2 dv =$$

$$U_S^{Ia_1} = -2b^2 + \frac{3}{4}b - \frac{31}{384}$$

Appendix D.1.i

Extended three-step equilibrium on interval I

If $v = y_1$, S must be indifferent between sending m_1 and m_2:

$$U_S(m_1|v = y_1) = U_S(m_2|v = y_1) =$$

$$-[a_1 - (v+b)]^2 = -[a_2 - (v+b)]^2 =$$

$$|a_1 - (v+b)| = |a_2 - (v+b)|$$

which, given that $0 < a_1 < v = y_1 < a_2$, equals

$$(y_1 + b) - a_1 = a_2 - (y_1 + b) =$$

$$(y_1 + b) - \frac{y_1}{2} = \frac{y_1 + y_2}{2} - (y_1 + b)$$

which yields

$$y_1 = \frac{y_2}{2} - 2b$$

which, given that $y_1 > 0$, yields

$$b < \frac{y_2}{4}$$

Appendix D.1.ii

Extended three-step equilibrium on interval I

If $v = y_2$, S must be indifferent between sending m_2 and m_3:

$$U_S(m_2|v = y_2) = U_S(m_3|v = y_2) =$$

$$-[a_2 - (v + b)]^2 = -[a_3 - (v + b)]^2 =$$

$$|a_2 - (v + b)| = |a_3 - (v + b)|$$

which, given that $0 < a_2 < v = y_2 < a_3$, equals

$$(y_2 + b) - a_2 = a_3 - (y_2 + b) =$$

$$(y_2 + b) - \frac{y_1 + y_2}{2} = \frac{y_2 + 1/2}{2} - (y_2 + b)$$

which, given that $y_1 = y_2/2 - 2b$, yields

$$y_2 = 1/3 - 4b$$

and

$$y_1 = 1/6 - 4b$$

which, given that $y_1 > 0$, yields

$$b < \frac{1}{24}$$

Appendix D.2.i

Extended three-step equilibrium on interval I

Message interval 3, $[y_2, 1/2]$, is longer than message interval 2, $[y_1, y_2]$, that is:

$$1/2 - y_2 > y_2 - y_1$$

$$1/2 + y_1 - 2y_2 > 0$$

$$1/2 + \left(\frac{1}{6} - 4b\right) - 2\left(\frac{1}{3} - 4b\right) > 0$$

$$4b > 0$$

More specifically, message interval 3 is $4b$ longer than message interval 2.

Appendix D.2.ii

Extended three-step equilibrium on interval I

Message interval 2, $[y_1, y_2]$, is longer than message interval 1, $[0, y_1]$, that is:

$$y_2 - y_1 > y_1 - 0$$

$$y_2 - 2y_1 > 0$$

$$\left(\frac{1}{3} - 4b\right) - 2\left(\frac{1}{6} - 4b\right) > 0$$

$$4b > 0$$

More specifically, interval 2 is $4b$ longer than interval 1.

Appendix D.3.i

Extended three-step equilibrium on interval I

R's aggregate expected utility equals:

$$U_R^I = \int_0^{y_1} U_R(v)\,dv + \int_{y_1}^{y_2} U_R(v)\,dv + \int_{y_2}^{\frac{1}{2}} U_R(v)\,dv =$$

$$\int_0^{y_1} -(a_1 - v)^2\,dv + \int_{y_1}^{y_2} -(a_2 - v)^2\,dv + \int_{y_2}^{\frac{1}{2}} -(a_3 - v)^2\,dv =$$

$$\int_0^{y_1} -\left(\frac{y_1}{2} - v\right)^2 dv + \int_{y_1}^{y_2} -\left(\frac{y_1 + y_2}{2} - v\right)^2 dv + \int_{y_2}^{\frac{1}{2}} -\left(\frac{y_2 + \frac{1}{2}}{2} - v\right)^2 dv =$$

$$\int_0^{\frac{1}{3}-4b} -\left[\frac{\left(\frac{1}{3} - 4b\right)}{2} - v\right]^2 dv + \int_{\frac{1}{3}-4b}^{\frac{2}{3}-4b} -\left[\frac{\left(\frac{1}{3} - 4b\right) + \left(\frac{2}{3} - 4b\right)}{2} - v\right]^2 dv$$

$$+ \int_{\frac{2}{3}-4b}^{\frac{1}{2}} -\left[\frac{\left(\frac{2}{3} - 4b\right) + \frac{1}{2}}{2} - v\right]^2 dv =$$

$$U_R^I = -\frac{4}{3}b^2 - \frac{1}{864}$$

Appendix D.3.ii

Extended three-step equilibrium on interval I

S's aggregate expected utility equals:

$$U_S^I = \int_0^{y_1} U_S(v)\,dv + \int_{y_1}^{y_2} U_S(v)\,dv + \int_{y_2}^{\frac{1}{2}} U_S(v)\,dv =$$

$$\int_0^{y_1} -[a_1 - (v+b)]^2\,dv + \int_{y_1}^{y_2} -[a_2 - (v+b)]^2\,dv + \int_{y_2}^{\frac{1}{2}} -[a_3 - (v+b)]^2\,dv =$$

$$\int_0^{y_1} -\left(\frac{y_1}{2} - v - b\right)^2 dv + \int_{y_1}^{y_2} -\left(\frac{y_1+y_2}{2} - v - b\right)^2 dv$$

$$+ \int_{y_2}^{\frac{1}{2}} -\left(\frac{y_2 + \frac{1}{2}}{2} - v - b\right)^2 dv =$$

$$\int_0^{\frac{1}{3}-4b} -\left[\frac{\left(\frac{1}{3} - 4b\right)}{2} - v - b\right]^2 dv + \int_{\frac{1}{3}-4b}^{\frac{2}{3}-4b} -\left[\frac{\left(\frac{1}{3} - 4b\right) + \left(\frac{2}{3} - 4b\right)}{2} - v - b\right]^2 dv$$

$$+ \int_{\frac{2}{3}-4b}^{\frac{1}{2}} -\left[\frac{\left(\frac{2}{3} - 4b\right) + \frac{1}{2}}{2} - v - b\right]^2 dv =$$

$$U_S^I = -\frac{11}{6}b^2 - \frac{1}{864}$$

Appendix D.4.i

Extended three-step equilibrium on interval II

If $v = y_1$, S must be indifferent between sending m_1 and m_2:

$$U_S(m_1|v = y_1) = U_S(m_2|v = y_1) =$$

$$-[a_1 - (v+b)]^2 = -[a_2 - (v+b)]^2 =$$

$$|a_1 - (v+b)| = |a_2 - (v+b)|$$

which, given that $0 < a_1 < v = y_1 < a_2$, equals

$$(y_1 + b) - a_1 = a_2 - (y_1 + b) =$$

$$(y_1 + b) - \frac{\frac{1}{2} + y_1}{2} = \frac{y_1 + y_2}{2} - (y_1 + b)$$

which yields

$$y_1 = \frac{y_2}{2} - 2b + \frac{1}{4}$$

which, given that $y_1 > 0$, yields

$$b < \frac{y_2}{4} + \frac{1}{8}$$

Appendix D.4.ii

Extended three-step equilibrium on interval II

If $v = y_2$, S must be indifferent between sending m_2 and m_3:

$$U_S(m_2|v = y_2) = U_S(m_3|v = y_2) =$$

$$-[a_2 - (v + b)]^2 = -[a_3 - (v + b)]^2 =$$

$$|a_2 - (v + b)| = |a_3 - (v + b)|$$

which, given that $0 < a_2 < v = y_2 < a_3$, equals

$$(y_2 + b) - a_2 = a_3 - (y_2 + b) =$$

$$(y_2 + b) - \frac{y_1 + y_2}{2} = \frac{y_2 + 1}{2} - (y_2 + b)$$

which, given that $y_1 = \frac{y_2}{2} - 2b + \frac{1}{4}$, yields

$$y_2 = \frac{5}{6} - 4b$$

and

$$y_1 = \frac{2}{3} - 4b$$

which, given that $y_1 > 0$, yields

$$b < \frac{1}{6}$$

Appendix D.5.i

Extended three-step equilibrium on interval II

Message interval 3, $[y_2, 1/2]$, is longer than message interval 2, $[y_1, y_2]$, that is:

$$1 - y_2 > y_2 - y_1$$

$$1 + y_1 - 2y_2 > 0$$

$$1 + \left(\frac{2}{3} - 4b\right) - 2\left(\frac{5}{6} - 4b\right) > 0$$

$$4b > 0$$

More specifically, message interval 3 is $4b$ longer than message interval 2.

Appendix D.5.ii

Extended three-step equilibrium on interval II

Message interval 2, $[y_1, y_2]$, is longer than message interval 1, $[0, y_1]$, that is:

$$y_2 - y_1 > y_1 - \frac{1}{2}$$

$$y_2 - 2y_1 + \frac{1}{2} > 0$$

$$\left(\frac{5}{6} - 4b\right) - 2\left(\frac{2}{3} - 4b\right) + \frac{1}{2} > 0$$

$$4b > 0$$

More specifically, interval 2 is $4b$ longer than interval 1.

Appendix D.6.i

Extended three-step equilibrium on interval II

R's aggregate expected utility equals:

$$U_R^{II} = \int_{\frac{1}{2}}^{y_1} U_R(v)\,dv + \int_{y_1}^{y_2} U_R(v)\,dv + \int_{y_2}^{1} U_R(v)\,dv =$$

$$\int_{\frac{1}{2}}^{y_1} -(a_1 - v)^2\,dv + \int_{y_1}^{y_2} -(a_2 - v)^2\,dv + \int_{y_2}^{1} -(a_3 - v)^2\,dv =$$

$$\int_{\frac{1}{2}}^{y_1} -\left(\frac{\frac{1}{2} + y_1}{2} - v\right)^2 dv + \int_{y_1}^{y_2} -\left(\frac{y_1 + y_2}{2} - v\right)^2 dv + \int_{y_2}^{1} -\left(\frac{y_2 + 1}{2} - v\right)^2 dv =$$

$$\int_{\frac{1}{2}}^{\frac{1}{3}-4b} -\left[\frac{\frac{1}{2} + \left(\frac{1}{3} - 4b\right)}{2} - v\right]^2 dv + \int_{\frac{1}{3}-4b}^{\frac{2}{3}-4b} -\left[\frac{\left(\frac{1}{3} - 4b\right) + \left(\frac{2}{3} - 4b\right)}{2} - v\right]^2 dv$$

$$+ \int_{\frac{2}{3}-4b}^{1} -\left[\frac{\left(\frac{2}{3} - 4b\right) + 1}{2} - v\right]^2 dv =$$

$$U_R^{II} = -\frac{4}{3}b^2 - \frac{1}{864}$$

Appendix D.6.ii

Extended three-step equilibrium on interval II

S's aggregate expected utility equals:

$$U_S^{II} = \int_{\frac{1}{2}}^{y_1} U_S(v)\,dv + \int_{y_1}^{y_2} U_S(v)\,dv + \int_{y_2}^{1} U_S(v)\,dv =$$

$$\int_{\frac{1}{2}}^{y_1} -[a_1 - (v+b)]^2\,dv + \int_{y_1}^{y_2} -[a_2 - (v+b)]^2\,dv + \int_{y_2}^{1} -[a_3 - (v+b)]^2\,dv =$$

$$\int_{\frac{1}{2}}^{y_1} -\left(\frac{\frac{1}{2}+y_1}{2} - v - b\right)^2 dv + \int_{y_1}^{y_2} -\left(\frac{y_1+y_2}{2} - v - b\right)^2 dv$$

$$+ \int_{y_2}^{1} -\left(\frac{y_2+1}{2} - v - b\right)^2 dv =$$

$$\int_{\frac{1}{2}}^{\frac{1}{3}-4b} -\left[\frac{\frac{1}{2}+\left(\frac{1}{3}-4b\right)}{2} - v - b\right]^2 dv$$

$$+ \int_{\frac{1}{3}-4b}^{\frac{2}{3}-4b} -\left[\frac{\left(\frac{1}{3}-4b\right)+\left(\frac{2}{3}-4b\right)}{2} - v - b\right]^2 dv$$

$$+ \int_{\frac{2}{3}-4b}^{1} -\left[\frac{\left(\frac{2}{3}-4b\right)+1}{2} - v - b\right]^2 dv =$$

$$U_S^{II} = -\frac{11}{6}b^2 - \frac{1}{864}$$

Appendix D.7.i

Extended three-step equilibrium on interval I

Note that R thinks that v lies on interval II, $\left[\frac{1}{2},1\right]$, so $a_1 = \frac{\frac{1}{2}+y_1}{2}$, $a_2 = \frac{y_1+y_2}{2}$, $a_3 = \frac{y_2+1}{2}$, $y_1 = \frac{1}{6} - 4b$ and $y_2 = \frac{1}{3} - 4b$ (recall Appendix D.1.ii on page 28). S's aggregate expected utility when always sending $m_i = m_1$ equals

$$U_S^{Ia_1} = \int_0^{\frac{1}{2}} -[a_1 - (v+b)]^2 dv =$$

$$U_S^{Ia_1} = \int_0^{\frac{1}{2}} -\left[\frac{1/2+y_1}{2} - (v+b)\right]^2 dv =$$

$$U_S^{Ia_1} = \int_0^{\frac{1}{2}} -\left[\frac{\frac{1}{2} + \left(\frac{1}{6} - 4b\right)}{2} - (v+b)\right]^2 dv =$$

$$U_S^{Ia_1} = -\frac{9}{2}b^2 + \frac{1}{4}b - \frac{1}{72}$$

The calculations for always sending $m_i = m_2$ and $m_i = m_3$ are analogous.

Appendix D.7.ii

Extended three-step equilibrium on interval II

Note that R thinks that v lies on interval I, $\left[0, \frac{1}{2}\right]$, so $a_1 = \frac{y_1}{2}$, $a_2 = \frac{y_1 + y_2}{2}$, $y_1 = \frac{1}{6} - 4b$ and $y_2 = \frac{1}{3} - 4b$ (recall Appendix D.1.ii on page 28). S's aggregate expected utility when always sending $m_i = m_1$ equals

$$U_S^{Ia_1} = \int_0^{\frac{1}{2}} -[a_1 - (v+b)]^2 dv =$$

$$U_S^{Ia_1} = \int_0^{\frac{1}{2}} -\left[\frac{1/2 + y_1}{2} - (v+b)\right]^2 dv =$$

$$U_S^{Ia_1} = \int_0^{\frac{1}{2}} -\left[\frac{\frac{1}{2} + \left(\frac{1}{6} - 4b\right)}{2} - (v+b)\right]^2 dv =$$

$$U_S^{Ia_1} = -\frac{9}{2}b^2 + \frac{1}{4}b - \frac{1}{72}$$

The calculations for always sending $m_i = m_2$ and $m_i = m_3$ are analogous.

Appendix E

n-step equilibrium

Suppose that the interval [0,1] is split into n message intervals, m_i with $i = \{1, 2, 3, \dots, n\}$, where m_1 and m_2 are seperated by point z_1, where, with $k = \{2, 3, 4, \dots, n-1\}$, m_k and m_{k+1} are separated by point z_k and where m_{n-1} and m_n are seperated by point z_{n-1}.

If R receives m_1, he infers that v lies on the interval $[0, z_1]$, thus updating his expected value of v to $z_1/2$. This leads R to choosing $a_i = a_1 = E(v|m_1) = z_1/2$ to maximize his own utility.

If R receives m_k, he infers that v lies on the interval $[z_{k-1}, z_k]$, thus updating his expected value of v to $(z_{k-1} + z_k)/2$. This leads R to choosing $a_i = a_k = E(v|m_k) = (z_{k-1} + z_k)/2$ to maximize his own utility.

If R receives m_n, he infers that v lies on the interval $[z_{n-1}, 1]$, thus updating his expected value of v to $(z_{n-1}+1)/2$. This leads R to choosing $a_i = a_n = E(v|m_n) = (z_{n-1}+1)/2$ to maximize his own utility.

Anticipating R's strategy, S sends m_1 for all values of v on the interval $[0, z_1)$, m_k for all values of v on the interval (z_{k-1}, z_k) and m_n for all values of v on the interval $(z_{n-1}, 1)$.

If $v = z_k$, S must be indifferent between sending m_k and m_{k+1}, that is: $U_S(m_k|v = z_k) = U_S(m_{k+1}|v = z_k)$, which equals $-[a_k - (v+b)]^2 = -[a_{k+1} - (v+b)]^2$, which equals $-\left[\frac{z_{k-1}+z_k}{2} - (v+b)\right]^2 = -\left[\frac{z_k+z_{k+1}}{2} - (v+b)\right]^2$, which yields $z_{k+1} - z_k = z_k - z_{k-1} + 4b$, which proves that the length of an interval equals the length of the previous interval plus $4b$ (see Appendix F.1 on page 37). This corresponds to findings of Gibbons (1992).

Suppose that interval 1 is of length c. Given that each interval is $4b$ longer than the previous interval, interval 2 is of length $c + 4b$, interval 3 is of length $c + 4b * 2$ etcetera. This series, consisting of n steps, must equal 1 when v lies on the interval [0,1], that is, $nc + 4b\sum_{i=1}^{n-1} i = 1$, which equals $nc + 4b\frac{n(n-1)}{2} = 1$ and can be simplified to

$$nc + 2bn(n-1) = 1 \tag{1}$$

which can be rewritten as the definition of the first interval, which equals the first indifference point: $c = z_1 = \frac{1-bn(n-1)}{n}$. Every following indifference point equals $z_i = i * z_1 + 2bn(n-1)$. Substituting z_1 and simplifying yields $z_i = \frac{i-2bn(i-n)(n-1)}{n}$. Recall that n and i are integers and note that when $i = n$, z_i always equals 1, corresponding to the right boundary of the interval of v. Also, note that $n \neq 0$.

In (1), the value of endogenous variable c is formed such that the equation holds. Stated differently, given the value of exogenous parameter b, the value of integer n is set so that $2bn(n-1) < 1$, after which the value of c ensures that $nc = 1 - 2bn(n-1)$. Therefore, the value of n is not dependent on c, but rather the other way around. Consequently, the value of n is subject to $2bn(n-1) < 1$, which solves for $n < \frac{1}{2}\left(1 + \sqrt{1 + \frac{2}{b}}\right)$ (see Appendix F.2 on page 37).

From (1), it can be infered that when the interest asymmetricality of R and S is infinitely small, that is, b approaches zero, the maximum number of intervals becomes infinitely large, that is, n approaches infinity and the length of interval 1 becomes infinitely small, that is, c approaches zero.

A player's aggregate expected utility for the n-step equilibrium on the interval [0,1] can be calculated using the sum of n integrals. The first integral covers the situation where $v = [0, z_1]$, S chooses $m_i = m_1$ and R chooses $a_i = a_1 = z_1/2$. The k-th integral covers the situation where $v = [z_{k-1}, z_k]$, S chooses $m_i = m_k$

and R chooses $a_i = a_k = (z_{k-1} + z_k)/2$. The n-th integral covers the situation where $v = [z_{n-1}, 1]$, S chooses $m_i = m_n$ and R chooses $a_i = a_n = (z_{n-1} + 1)/2$.

Extended n-step equilibrium

Suppose that the interval $\left[0, \frac{1}{2}\right]$ is split into n message intervals, m_i with $i = \{1, 2, 3, \dots, n\}$, where m_1 and m_2 are seperated by point z_1, where, with $k = \{2, 3, 4, \dots, n-1\}$, m_k and m_{k+1} are separated by point z_k and where m_{n-1} and m_n are seperated by point z_{n-1}.

If R receives m_1, he infers that v lies on the interval $[0, z_1]$, thus updating his expected value of v to $z_1/2$. This leads R to choosing $a_i = a_1 = E(v|m_1) = z_1/2$ to maximize his own utility.

If R receives m_k, he infers that v lies on the interval $[z_{k-1}, z_k]$, thus updating his expected value of v to $(z_{k-1} + z_k)/2$. This leads R to choosing $a_i = a_k = E(v|m_k) = (z_{k-1} + z_k)/2$ to maximize his own utility.

If R receives m_n, he infers that v lies on the interval $[z_{n-1}, 1/2]$, thus updating his expected value of v to $(z_{n-1} + 1/2)/2$. This leads R to choosing $a_i = a_n = E(v|m_n) = (z_{n-1} + 1/2)/2$ to maximize his own utility.

Anticipating R's strategy, S sends m_1 for all values of v on the interval $[0, z_1)$, m_k for all values of v on the interval (z_{k-1}, z_k) and m_n for all values of v on the interval $(z_{n-1}, 1/2)$.

If $v = z_k$, S must be indifferent between sending m_k and m_{k+1}, that is: $U_S(m_k|v = z_k) = U_S(m_{k+1}|v = z_k)$, which equals $-[a_k - (v + b)]^2 = -[a_{k+1} - (v + b)]^2$, which equals $-\left[\frac{z_{k-1}+z_k}{2} - (v + b)\right]^2 = -\left[\frac{z_k+z_{k+1}}{2} - (v + b)\right]^2$, which yields $z_{k+1} - z_k = z_k - z_{k-1} + 4b$, which proves that the length of an interval equals the length of the previous interval plus $4b$ (recall Appendix F.1 on page 37).

Suppose that interval 1 is of length c. Given that each interval is $4b$ longer than the previous interval, interval 2 is of length $c + 4b$, interval 3 is of length $c + 4b * 2$ etcetera. This series, consisting of n steps, must equal 1/2 when v lies on the interval $\left[0, \frac{1}{2}\right]$, that is, $nc + 4b \sum_{i=1}^{n-1} i = \frac{1}{2}$, which equals $nc + 4b\frac{n(n-1)}{2} = \frac{1}{2}$ and can be simplified to

$$nc + 2bn(n-1) = 1/2 \tag{2}$$

which can be rewritten as the definition of the first interval, which equals the first indifference point: $c = z_1 = \frac{1/2 - 2bn(n-1)}{n}$. Every following indifference point equals $z_i = i * z_1 + 2bn(n-1)$. Substituting z_1 and simplifying yields $z_i = \frac{i - 2bn(i-n)(n-1)}{n}$. Recall that n and i are integers and note that when $i = n$, z_i always equals 1. Also, note that $n \neq 0$.

In (2), the value of endogenous variable c is formed such that the equation holds. Stated differently, given the value of exogenous parameter b, the value of integer n is set so that $2bn(n-1) < 1/2$, after which the value of c ensures that $nc = 1/2 - 2bn(n-1)$. Therefore, the value of n is not dependent on c, but rather the other way around. Consequently, the value of n is subject to $2bn(n-1) < 1/2$, which solves for $n < \frac{1}{2}\sqrt{1+\frac{2}{b}}$ (see Appendix F.2 on page 37).

Appendix F.1

n-step equilibrium

If $v = z_k$, S must be indifferent between sending m_k and m_{k+1}:

$$U_S(m_k|v = z_k) = U_S(m_{k+1}|v = z_k) =$$

$$-[a_k - (v+b)]^2 = -[a_{k+1} - (v+b)]^2 =$$

$$|a_k - (v+b)| = |a_{k+1} - (v+b)|$$

Which, given that $0 < a_k < v = z_k < a_{k+1}$, equals

$$(z_k + b) - a_k = a_{k+1} - (z_k + b) =$$

$$(z_k + b) - \frac{z_{k-1} + z_k}{2} = \frac{z_k + z_{k+1}}{2} - (z_k + b) =$$

$$z_k - z_{k-1} + 4b = z_{k+1} - z_k$$

This proves that the length of an interval equals the length of the previous interval plus $4b$.

Appendix F.2

n-step equilibrium

$$2bn(n-1) < 1$$

$$n^2 - n - \frac{1}{2b} < 0$$

Given that both n and b are positive, this yields

$$n < \frac{1}{2}\frac{b + \sqrt{b^2 + 2b}}{b}$$

which can be simplified to

$$n < \frac{1}{2}\left(1 + \sqrt{1 + \frac{2}{b}}\right)$$