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*Evaluating Repair Policies: An analysis of repair  
shop policies for different planning horizons*

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# EVALUATING REPAIR POLICIES

An analysis of repair shop policies for different planning horizons

## Abstract

This paper builds on the research done by Liang et al. (2013). In their paper, they sketch a problem setting in which fleets of machines are used for production. All machines are subject to failure and sent to a repair shop if they break. Liang et al. (2013) propose several repair shop policies to process jobs in a repair shop. We implement some of these policies and replicate the results found in their paper. Furthermore, we extend the research by analyzing the short-term behavior. For this, we consider the model used by Liang et al. (2013) and relax some of the assumptions made by these authors.

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## I. Introduction

Machines are an important asset to most production facilities. Several fleets of machines may work on different stages of the production process, either in series or in parallel. Unfortunately, machines are prone to failure, which may cause downtime and hence production loss for the company. A sufficient number of spare parts and a sound repair policy are therefore needed to prevent the loss of profits due to machine failure. However, the optimal number of spare parts and the optimal repair policy, such that costs are minimized, depend on many factors of the production process.

In this paper, we consider the repair policies developed by Liang et al. (2013). They propose several ways to schedule repairs and determine the optimal number of spare parts for a given policy. Additionally, they develop a new repair policy, which they refer to as a Myopic(R) policy, and show that this policy gives results close to optimal.

The model studied by Liang et al. (2013) assumes however that a fleet has the same composition for an infinite horizon, which is often not the case in companies. The number of machines in a fleet is changing over time, as the demands for products and technologies for production are far from stationary. Additionally, the assumption that a failed machine can always be repaired is infeasible in reality, as no machine functions forever. As a consequence, fewer machines may be available at the end of the horizon, which should be taken into account when determining optimal base-stock levels.

The aim of this paper is twofold. First, we want to reproduce and analyse the results found by Liang et al. (2013) to see if we have mastered the repair policies. After that, we will analyse the short-term behaviour of some of these policies, and relax a number of assumptions made in the original model. More specifically, we will look into cases where the failure rate is influenced by the machine age and cases in which machines cannot always be repaired. With these insights, managers can then decide on a policy for a short span of time, which allows for easier adaptation in ever-changing markets.

This paper is structured in the following manner. Section II gives a more formal description of the problem setting. Section III discusses the literature relevant to this topic. The different repair policies considered in this paper are explained in more detail in Section IV. The mathematical models used to implement the policies are also described in this section. Sections V and VI respectively discuss the long-term performance and short-term performance of the implemented policies. These results will be discussed and will be compared to the results found by Liang et al. (2013) in Section VII. Finally, Section VIII gives a conclusion of the obtained results and suggestions for future research.

## II. Model description

In this section the model considered in this paper will be described. This is done extensively in Liang et al. (2013), but it will be rephrased here for clarity.

We consider  $r$  fleets of machines, indexed  $i = (1, 2, \dots, r)$ . Every machine is subject to failure if it is used for production. For a machine in fleet  $i$ , the time to failure follows an exponential distribution with rate  $\lambda_i$ . A broken machine can be repaired in a repair shop. Repair times for a machine in fleet  $i$  are exponentially distributed with rate  $\mu_i$ . The number of machines used for production in fleet  $i$  at time  $t$ , which will be called the number of working machines in this paper and is denoted by  $W_i(t)$ , is required to be  $N_i$  at every point in time. If less than  $N_i$  machines are working, a downtime penalty cost of  $b_i$  per time unit per machine is incurred.

Spare machines are kept in inventory to replace broken machines to reduce the risk of having downtime costs. Each fleet has a separate inventory, which follows a base-stock policy with level  $S_i$ . Let  $I_i(t)$  denote the number of functional machines in the inventory of fleet  $i$  at time  $t$ . The number of available machines in fleet  $i$  at time  $t$ , denoted by  $A_i(t)$ , is then defined as  $A_i(t) = W_i(t) + I_i(t)$ . Holding costs, denoted by  $h_i$ , are considered to be constant, so that the total holding costs for fleet  $i$  are equal to  $h_i \cdot S_i$  per time unit. When a machine breaks, a new machine is installed without any delay, provided  $I_i(t) > 0$ . If  $I_i(t) = 0$  when a machine fails, the fleet will have one less functional machine and incur downtime costs until a machine is sent from the repair shop.

The repair shop can follow several repair policies, which will be explained in Section IV. Given a policy, one can characterize the problem by a continuous-time Markov Chain, from which the steady-state probabilities can be derived. Let  $p_i(n)$  denote the steady-state probability of being in a state in which  $n$  machines are functional (both in fleet and in stock) in fleet  $i$ . Recall that the aim of the paper is to find optimal base-stock inventory levels for each policy, so that the results can be compared. As such, we take  $\mathbf{S} = (S_1, S_2, \dots, S_r)$  as the decision variable of the cost function under a policy  $X$ , which is given in Equation 1.

**Equation 1.**

$$C_X^*(\mathbf{S}) = \min_{\mathbf{S}} \left\{ \sum_{i=1}^r C_i(\mathbf{S}) \right\}$$

in which

$$C_i(\mathbf{S}) = h_i \cdot S_i + b_i \sum_{n=0}^{N_i} (N_i - n)p_i(n)$$

□

Given a policy  $X$ , one can find all  $p_i(n)$  by solving the underlying balance equations for a given set of parameters. The optimal solution  $C_X^*(\mathbf{S})$  can then be obtained by searching over different vectors of  $\mathbf{S}$ . More details on the optimization per repair policy are given in Section V.

### III. Literature Review

As mentioned in the introduction, this research extends previous research that has been done on this topic. This section aims to give a concise overview of the most relevant literature to this problem.

This research builds on the research done by Liang et al. (2013), who suggest several policies to schedule repairs in a repair shop. The main contribution of this paper is the development of the Myopic(R) scheduling policy, which was found to give results close to optimal while having a severely smaller computation time than the optimal policy. A closely related paper is that of Sahba et al. (2013), which was used extensively by Liang et al. (2013) in their research. Sahba et al. (2013) consider several solutions to deal with the repair shop problem sketched in Section II. More specifically, they look at a First-Come-First-Served (FCFS) repair policy with possible shared inventory. Especially relevant to this paper is the model without shared inventory, which is referred to as the RIF model in their paper. Under the assumption of no shared inventory, they formulate analytical expressions and numerical approximations to determine the performance of a repair shop following a First-Come-First-Served (FCFS) repair policy. These results were used by Liang et al. (2013) to evaluate the performance of

their policies. Since we also consider a FCFS repair policy in this paper, the expressions found by Sahba et al. (2013) are useful for evaluation of the policy. Both papers are therefore used extensively in this paper for gathering results.

Fleet composition is a thoroughly researched topic, as the fleet usually accounts for a large part of the operational costs (Etezadi & Beasley, 1983). Especially in transport, the fleet size is adjusted continuously to ensure that all demand is satisfied, while minimizing operational costs. This supports the idea to allow variations in the number of required machines and the number of machines on stock over time.

Relevant to our paper is the research on the relation between machine performance and age. These so-called hazard rate functions are a well-researched topic in reliability theory. Most papers, such as Wang et al. (2002) propose a bathtub shaped curve shown in Figure 1 (Lochbaum, 2015).

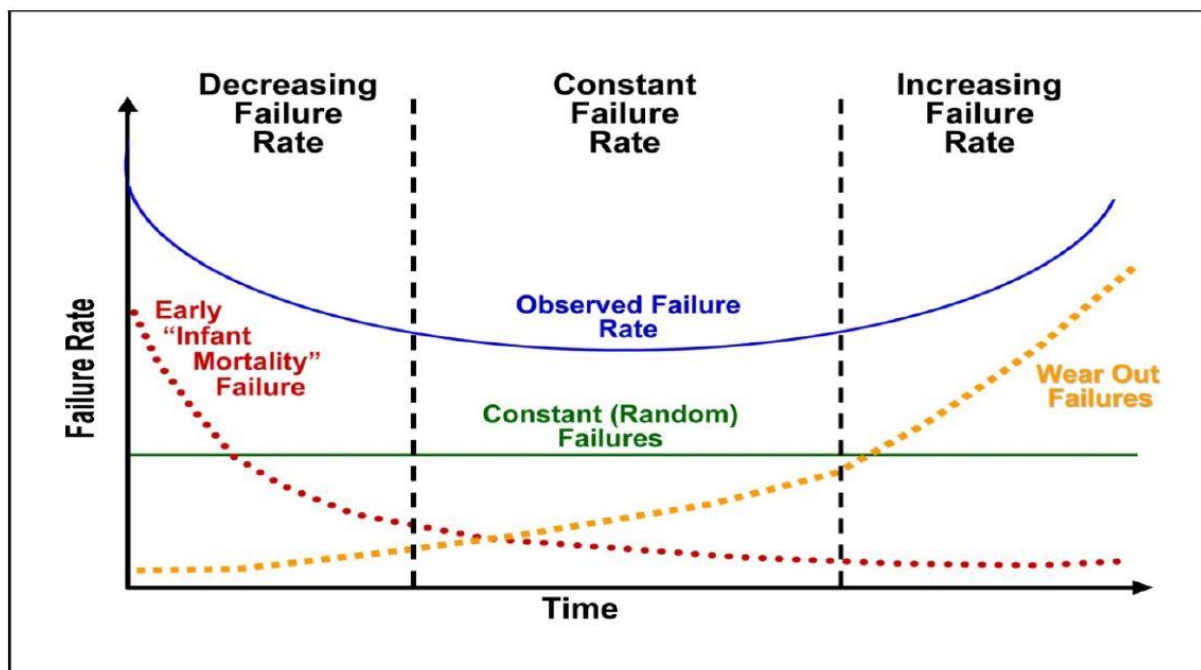


Figure 1 Bathtub curve

In the first stage, the machine experiences a lot of infant mortality failure, resulting in a relatively high failure rate. This gradually decreases over time to the point where the failure rate due to infant mortality is at a constant low. At the end of the lifetime, the machine will experience more wear-out failures, resulting in a higher overall failure rate. The result is a bathtub shaped curve, in which machine failure rates are relatively high at the start and end of the machine lifecycle. We will use this curve to model failure rates in our evaluation of the short-term performances of the policies.

## IV. Repair policy description

Orders in the repair shop can be processed in different ways depending on the policy. This section describes the different policies and their consequences for the model.

We consider three repair policies in this paper. The first is a Base Case policy, in which every fleet has a dedicated repair shop. The repair shop is then pooled in a Centralised Repair Shop (CRS) in a static First-Come-First-Served policy, which is the most basic policy considered with a shared repair shop.

Finally, we consider the dynamic Myopic(R) policy developed by Liang et al. (2013), which uses the repair time as a look-ahead time to determine which machine should be repaired next. In each of these cases, the repair shop is assumed to have a single server and have an infinite queue. Since both the departure times and interarrival times are exponentially distributed, we can model the server as an  $M_r/M/1$  multiclass queueing model.

It should be noted that Liang et al. (2013) also consider a static preemptive-resume priority policy, in which jobs can be halted temporarily in favour of more profitable jobs. Furthermore, they also formulate the problem as a Markov Decision Process, from which the optimal solution can be obtained. Due to time restrictions, these policies were not considered in this paper.

### i. Base Case

The Base Case policy is the most basic policy considered in this paper. As mentioned in the previous paragraph, each fleet has a dedicated repair shop available, which is modelled as a single server. Jobs are processed in the order they arrive and preemption, that is interrupting a current job to start on another, is not allowed. We denote the service rate of the repair shop of fleet  $i$  by  $\mu_i^{BC} \leq \mu_i$ .

### ii. First-Come-First-Served

The First-Come-First-Served policy is the most basic policy with a Centralised Repair Shop. The CRS is again modelled as a single server with infinite queue, but is now shared among all fleets. Jobs are processed in a FCFS fashion, which means that they are completed in the order they arrive. Liang et al. (2013) point out that we do not only need to know how many jobs the repair shop has from each class, but also their ordering. As a consequence, the problem becomes intractable when the size increases, because one needs to consider all possible permutations in the queue. Analysing the problem by a continuous-time Markov Chain (CTMC) is therefore infeasible, as analytical expressions are difficult to find. Sahba et al. (2013) however find exact expressions for cases where the repair rates are equal among all fleets. Therefore, we only consider problems with equal repair rates in this paper, so that we can use the formulae found in Sahba et al. (2013) for the evaluation of this policy.

### iii. Myopic(R)

The Myopic(R) policy is the main contribution of Liang et al. (2013). Unlike the Base Case and FCFS policies, it is a dynamic policy, which means that it tries to anticipate on the future, rather than react to what happened in the past. The idea is to look a repair time ahead and see for every fleet how much it is better to repair a machine than to idle. This is done every time a machine fails or when a repair is completed. In this policy, preemption is allowed, which means the repair shop can decide to start a more profitable job in favour of the job they are currently working on.

To determine which machine should be repaired next, cost rate differences are calculated for each fleet. Let  $x_i$  denote the inventory position of fleet  $i$  when making the decision. For  $x_i > 0$ , or  $x_i = 0$  when  $A_i(t) = N_i$ , the cost rate difference for inventory position  $x_i$ , denoted by  $\Delta c_i^R(x_i)$ , is then defined as in Equation 2. A derivation of this formula is given in Liang et al. (2013).

#### Equation 2.

$$\Delta c_i^R(x_i) = -b_i \sum_{m=x_i+1}^{x_i+N_i} p_i^R(m)$$

□

Here  $p_i^R(m)$  is the probability of having  $m$  failures in fleet  $i$  during the look-ahead time. Furthermore, Liang et al. (2013) point out that for  $x_i = 0$  and  $A_i(t) < N_i$ , it holds that  $\Delta c_i^R(0) = -b_i$ , which is in line with similar policies mentioned in Liang et al. (2013) that are outside the scope of this paper.

Cost rate differences however are not the only factor that determine which machine is the most profitable to repair. Failure rates and service rates are of importance as well, as it might be more profitable for example to repair a machine that has a much smaller repair time than another, despite being less profitable overall. Therefore, Liang et al. (2013) propose a repair index  $\frac{\mu_i \Delta c_i^R(x_i)}{\lambda_i}$  for every fleet  $i$  to rank the jobs. The fleet with the lowest index is scheduled for repair next.

Liang et al. (2013) provide two expressions to find  $p_i^R(m)$  and prove these formulae in Proposition 1 of their paper. For this paper, we are only interested in the results, hence the proofs will be omitted. Equation 3 shows these expressions.

### Equation 3.

For  $x_i > 0$ , or  $x_i = 0$  when  $A_i(t) = N_i$ , then

- If  $0 \leq m \leq x_i$ , then

$$p_i^R(m) = \left( \frac{\lambda_i N_i}{\lambda_i N_i + \mu_i} \right)^m \left( 1 - \frac{\lambda_i N_i}{\lambda_i N_i + \mu_i} \right)$$

- If  $0 \leq x_i \leq m$ , then

$$p_i^R(m) = \frac{N_i!}{(N_i - m + x_i)!} \frac{\mu_i}{\lambda_i} \frac{(\lambda_i N_i)^{x_i}}{(N_i \lambda_i + \mu_i)^{x_i}} \prod_{j=0}^{m-x_i} (N_i + \frac{\mu_i}{\lambda_i} - j)$$

□

We are now able to compute repair index  $\frac{\mu_i \Delta c_i^R(x_i)}{\lambda_i}$  for every fleet  $i$  using Equation 3 and Equation 2. For a given vector  $\mathbf{S}$ , we can then construct a CTMC, from which the steady-state probabilities can be computed. Costs can then be obtained using Equation 1. Searching over different values of  $\mathbf{S}$  will then give the optimal objective value.

## V. Long-term analysis

In this section we aim to reproduce the results of Liang et al. (2013) to see if we have mastered the repair policies. Furthermore, reproduction of the results will give us valuable insights into the problem, which will improve the quality of the short-term analysis. All numerical results found by Liang et al. (2013) are given in the master thesis of the main author (Liang, 2011). The values obtained in this paper were compared to these figures. It should be noted a smaller set of parameters was used than in Liang et al. (2011), as our primary objective is to gain insights in the policies. More specifically, we considered all combinations of the following parameters:

- Fleet sizes:  $N \in \{(10, 5), (10, 10), (10, 15)\}$
- Repair rates:  $\mu_1 = 2, \frac{\mu_1}{\mu_2} \in \{2, 1, \frac{2}{3}\}$
- Holding costs:  $h_1 = 1, h_2 \in \{0.5, 0.7, 0.9\}$
- Downtime costs:  $\frac{b_1}{h_1} = \frac{b_2}{h_2} \in \{20, 80\}$

- Server rate utilization  $u = \frac{\lambda_1 N_1}{\lambda_1} = \frac{\lambda_2 N_2}{\lambda_2} \in \{0.45, 0.35, 0.25\}$

In total, this gives 162 combinations for the Base Case and the Myopic(R) policy and 54 combinations for the FCFS policy. Furthermore, we only consider problems with two fleets in this paper for simplification. A full list of numerical results for all the policies considered is shown in Appendix iii.

### i. Base Case

As mentioned in Section IV, the Base Case policy considers a separate repair shop for every fleet. As a consequence, all fleets can be modelled separately as a birth-death process.

We define the state spaces  $\tilde{\mathcal{S}}_i$  by all possible values for  $A_i(t)$ , so that  $\tilde{\mathcal{S}}_i = \{0, 1, \dots, N_i + S_i\}$  for  $i = 1, 2$ . Arrivals are obtained by a completed repair from the shop, whereas a failure of a machine signifies a departure. Since the repair shop only has a single server, the repair rates are equal in every state, except when  $A_i(t) = N_i + S_i$  when there are no broken machines. In this case, the repair rate is 0. Note that in the Base Case policy, we define  $2\mu_i^{BC} = \mu_i$ . Effective failure rates for fleet  $i$  in state  $s \in \tilde{\mathcal{S}}$  are given by  $\lambda_i(s) = \min\{s, N_i\}\lambda$ .

Much research has been done on solving birth-death processes, such as by Ross (2014). Steady-state probabilities can be found easily by expressing all probabilities in terms of  $P_0$  and using the condition that the sum of the probabilities must be equal to 1. In our approach, we chose to express all probabilities as a scalar multiple of  $P_0$ , ignoring that the probabilities must sum to 1 initially. We then incorporate this normalization constraint by dividing every scalar multiple by the sum of all scalars.

Optimization of an instance was done by searching over different values and combinations of  $S_i$ . Lower and upper bounds for  $S_i$  were obtained by looking at the results of Liang et al. (2011). More specifically, we looked at all combinations where  $0 \leq S_1 \leq 30$  and  $0 \leq S_2 \leq 30$ . The ranges are relatively large, so computation time can be saved if better bounds can be found. However, at this point there is no known method for this.

It was found that our results match the results of Liang et al (2011). Therefore, we conclude that we implemented the policy correctly.

### ii. First-Come-First-Served

Following the description in Section IV, the FCFS policy considers a centralised repair shop in which repair jobs are processed in the order they arrive. Using a CRS makes the problem harder to solve analytically as the size increases, as one has to consider all possible combinations of orders in the queue of the repair shop. As a consequence the problem can no longer be modelled as a birth-death process. This means that analytical expressions for the probabilities are unknown when the repair rates of the different fleets are not equal. Therefore, we only consider instances where the repair rates are the same.

Sahba et al. (2013) propose a method to find the steady-state probabilities for a more general problem with possible shared inventory and a certain number of required working machines to have a functional system. In their paper, they refer to this model as the RIF model, which will be applied to the problem considered in this paper. In particular, we consider the problem instance where only two fleets are used and there is no shared inventory. Furthermore, at least 1 machine must be working to have a functional system.

Following Sahba et al. (2013), we define  $\mathbf{y} = (y_1, y_2)$  as a vector with  $y_i$  type- $i$  orders in the repair shop for  $i = 1, 2$ . At each point in time, the state of the RIF model can be characterized using this vector  $\mathbf{y}$ . Effective failure rates per fleet are defined in the same way as for the Base Case policy.



However, since we have a CRS, we are interested in the minimum of failure rates of the two fleets, which gives us the state-dependent failure rates  $\Lambda(W_1(t), W_2(t)) = \lambda_1(W_1(t)) + \lambda_2(W_2(t))$ , where  $\lambda_i(W_i(t))$  denotes the failure rate of fleet  $i$  when  $W_i(t)$  machines are working. As mentioned before, repair rates are considered equal for the two fleets.

When solving the balance equations, it must be noted that a state with  $n$  orders in the repair shop (such that  $y_1 + y_2 = n$ ) can be a result of different values for  $y_1, y_2$ . Therefore, it is important to take into account all possible combinations of  $y_1, y_2$  in which this state is achieved. Sahba et al. (2013) use this observation to obtain equations to express all steady-state probabilities, denoted by  $p(W_1(t), W_2(t))$ , in terms of  $p(0,0)$ , which is the steady state probability of having all machines in repair. We express every steady-state probability as a scalar multiple of  $p(0,0)$  and normalize the probabilities afterwards, similar to the solution technique for the Base Case policy.

Since this section only considers problems with two fleets, it is possible to display the state space as a matrix. This makes expressing the probabilities in terms of  $p(0,0)$  much easier, as each cell in the matrix is a function of two neighbouring cells. Furthermore, by summing over the columns and rows, it is easy to obtain the marginal probabilities of having  $y_1$  and  $y_2$  orders respectively.

Using the same optimization technique as for the Base Case policy, we found the same numerical results as Liang et al. (2011). Hence, we conclude that we understood and implemented the policy correctly.

### iii. Myopic(R)

The Myopic(R) policy is the only dynamic policy considered in this paper. Under this policy, preemption is allowed, so that the repair shop can decide to postpone a job it is currently working on if a job with a higher priority arrives.

Prioritization is done based on the cost-rate indices defined in Section IV. As these indices only depend on the parameters of the model, they can be computed in advance before solving the underlying CTMC. That is, one can determine for every state a-priori from which fleet a machine will be repaired. Naturally, no machine will be repaired if there are no broken machines present. We will denote the effective repair rate of a state in which  $k$  machines are working in fleet 1 and  $l$  machines are working in fleet 2 by  $\mu_{(k,l)}$  for  $(k, l) \in \mathcal{S}$ .

Given that we only consider problems with 2 fleets in this paper, the states can be conveniently represented in matrix form. We use this notion to model the CTMC as a system of linear equations in the following manner.

Let  $Q$  denote the  $|\mathcal{S}| \times |\mathcal{S}|$  matrix with all instantaneous transition rates. For states  $(k, l) \in \mathcal{S}$  and  $(i, j) \in \mathcal{S}$ , let  $q_{(k,l)(i,j)}$  denote the instantaneous transition rate from state  $(k, l) \in \mathcal{S}$  to  $(i, j) \in \mathcal{S}$ . Furthermore, let  $d_{(k,l)} \in \{1, 2\}$  denote the fleet of which a machines is repaired in state  $(k, l) \in \mathcal{S}$ . We distinguish the following cases:

$$q_{(k,l)(i,j)} = \begin{cases} \lambda_1(k), & \text{if } k = i + 1 \text{ and } l = j \\ \lambda_2(l), & \text{if } k = i \text{ and } l = j + 1 \\ \mu_1, & \text{if } k = i - 1 \text{ and } l = j \text{ and } d_{(k,l)} = 1 \\ \mu_2, & \text{if } k = i \text{ and } l = j + 1 \text{ and } d_{(k,l)} = 2 \\ -v_{(k,l)}, & \text{if } (k, l) = (i, j) \end{cases}$$

where

$$v_{k,l} = \lambda_1(k) + \lambda_2(l) + \mu_{(k,l)}$$

Let  $P$  denote the  $|\mathcal{S}| \times 1$  vector with the steady-state probabilities  $p_{(k,l)}$ , where  $(k,l) \in \mathcal{S}$ . Furthermore, let  $\mathbf{0}$  be a  $|\mathcal{S}| \times 1$  zero vector. We can then represent the balance equations, along with the normalization constraint, in the following way:

$$Q'P = \mathbf{0}$$

$$\sum_{(k,l) \in \mathcal{S}} p_{(k,l)} = 1$$

This expression can be further simplified by noticing that the balance equations are linearly dependent. As such, we can incorporate the normalization constraint in the balance equations. Let  $Q_z$  be the matrix with the same elements as  $Q$ , except for the  $z$ 'th row which is a row of 1's. Furthermore let  $\mathbf{e}_z$  be a zero vector with a 1 on its  $z$ 'th element. The adjusted system of equations is then:

$$Q_z'P = \mathbf{e}_z$$

Solving this system of equations yields  $P$ , from which the marginal probabilities for each fleet can be obtained. Using Equation 1, one can then compute the costs for given values of  $S_1$  and  $S_2$ . Optimization is done similarly to the Base Case and First-Come-First-Served policies.

The results found in our study are the same as those found by Liang et al. (2013), which means we understood and implemented the policy correctly. It should be noted however that our optimization algorithm is relatively inefficient, especially for larger instances. We found running times up to 10 minutes, compared to the running times of less than a minute found by Liang et al. (2013). This is most likely due to the optimization method and relatively inefficient programming. Furthermore, we implemented all code in Matlab, which is notably slower than C++ used by Liang et al. (2013).

## VI. Short-term analysis

A problem with the policy evaluation in the previous section is that they all assume steady-state properties for the system. In reality however, the timespan a fleet remains the same may be relatively short, as companies have to adapt their assets to changing market situations. As such, assuming steady-state properties may not be appropriate to measure performance of the policies.

In this section, the Base Case and FCFS policies will be re-evaluated with a finite horizon to see if they show similar performances for a smaller time frame. To evaluate the costs, we will slightly redefine Equation 1, as we do not consider steady-state probabilities. Instead, we will use simulation as our main tool to analyse the short-term performance. The simulation setup is discussed in subsection i. Furthermore, we will relax some assumptions from the original model. The model scenarios will be described in the second subsection of this section. Finally, the results for the two policies are given in Subsections iii and iv respectively.

### i. Simulation setup

The most important element of the simulation are the fleets, which are sets of machines. A machine is modelled as a vector containing the fleet number, age, the time it will fail and the time it will be done repairing. When a machine is in repair, the time to fail will be set to infinity. Similarly, the time to repair will be set to infinity if a machine is not in repair. When a machine is functional, but not used in the system (that is, the machine is in inventory), both the time to repair and the time to fail will be set to infinity. Machine age is defined as the total time a machine has worked and is tracked so it can

be used as an input variable for the failure rate. We assume that machines do not age when they are in inventory.

Advancing the simulation clock is done based on events. We consider two types of events: machine failure and repair completion. When a machine fails, it is sent to the repair shop and cannot be used in the fleet until the repair is complete. The repair shop holds all machines that are in repair and is modelled as an M/M/1 server. Repair priority is done according to the policies discussed in Section IV. When a repair is completed, the machine will be removed from the repair shop and installed in the fleet (if  $W_i(t) < N_i$ ) or placed in the inventory (if  $W_i(t) \geq N_i$ ).

Counters are kept for the holding costs and downtime costs for every fleet and are updated every event. Equation 1 is not used here, as we no longer assume that the system reaches steady-state. The total costs are saved for every run, along with the optimal base-stock levels found for that run. Since the optimal base-stock levels and associated costs are largely influenced by the randomness of the simulation, we chose to run every parameter instance a number of times.

More specifically, we ran every simulation so that the half-CI length, denoted by  $\delta(\alpha, n)$ , is at most  $\gamma\%$  of the sample mean. That is:

$$\frac{\delta(\alpha, n)}{\bar{C}^X} = \frac{z_{\alpha/2} \cdot s_c^X / \sqrt{n}}{\bar{C}^X} < \frac{\gamma}{100}$$

where  $z_{\alpha/2}$  denotes the z-value for a standard normal distribution with significance level  $\alpha/2$ ,  $\bar{C}^X$  denotes the average costs over all runs and instances under policy X,  $s_c^X$  denotes the standard deviation in that same sample and  $n$  denotes the sample size, which is equal to the number of instances multiplied by the number of trial runs.  $\bar{C}^X$  and  $s_c^X$  were approximated by a trial simulation of 50 runs. Using values of  $\alpha = 0.05$  and  $\gamma = 5\%$  we then find the following expression for the required number of runs for a policy X, denoted by  $n_X^*$ :

$$n_X^* = 400 \cdot \left( \frac{1.96 \cdot \bar{s}_{50}^X}{\bar{C}_{50}^X} \right)^2$$

where  $\bar{s}_{50}^X$  and  $\bar{C}_{50}^X$  denote the average standard deviation and sample mean of the costs found in the trial simulation of 50 runs under policy X. It should be noted that in this approach, each instance under a policy X is run the same number of times. A different approach would be to determine  $n_X^*$  for every instance separately. This was not implemented in this paper due to time constraints. The minimum number of simulation runs, along with the sample means and standard deviations obtained from the trial runs, are given in Appendix i. It should be noted that a new random seed was used for every instance in every run. That is, no simulation was done with the same seed.

Optimization is done by enumerating all options. That is, we look at all possible combinations  $\mathbf{S} = (S_1, S_2)$ . We decided to set  $0 \leq S_1 \leq 20$  and  $0 \leq S_2 \leq 20$ , following the approach we used to determine the optimal base-stock levels for the long-term. After  $n_X^*$  runs, cost averages were taken for each combination of  $\mathbf{S}$ . The optimal solution is then the value for  $\mathbf{S}$  with the lowest mean costs.

### ii. Model scenarios

For every repair policy, we consider two cases. The first case is the most basic one and does not relax any assumptions made in the original model. It is used to compare the two policies under the same conditions as in the initial model. In the second case, we will relax the assumption that a machine can always be repaired. Upon failure, a machine has a probability to be permanently broken. This probability is a function of the machine age and is defined as  $P[\text{Machine unrepairable}] = \frac{a_{m,i}}{2T}$ , in

which  $a_{m,i}$  is the age of machine  $m$  in fleet  $i$  and  $T$  is the length of the simulation run. A linear function was chosen for simplicity, but we recommend using regression analysis to find a better fit, if data for machine failure is available. Permanent failure is checked before the machine goes into repair, so that the repair shop does not waste time on machines that cannot be repaired. We furthermore implement the bathtub-curve for failure rates in the following manner:

$$\lambda_{m,i}(a_{m,i}) = \begin{cases} (2 - a_{m,i})\lambda_i, & \text{if } 0 \leq a_{m,i} \leq T/10 \\ \lambda_i, & \text{if } T/10 \leq a_{m,i} \leq 8T/10 \\ \frac{4 + a_{m,i} - T}{2}\lambda_i, & \text{if } 8T/10 \leq a_{m,i} \leq T \end{cases}$$

The resulting function is a piece-wise linear function, in which the phase with infant mortality failure is half the length of the wear-out failure phase. The lengths of these phases are functions of  $T$ , which is feasible under the assumption that the simulation run is long enough to have both stages occurring. In reality however, a decision maker can decide on these intervals, as it largely depends on the problem setting and the horizon considered. Finally, we introduce a new decision rule for repair. A machine will only go into repair if the repair is finished before the end of the horizon. For this, we assume that the manager has knowledge of the timespan and that the employees in the repair shop can accurately predict when a repair is done.

### iii. Results Base Case

Two cases of the Base Case were run, as explained in the previous subsection. Table 1 shows the results of the simulations. The costs are given for a full simulation length of  $T = 10$ .

	Mean	Std. Dev.	Min	Max
BC 1	108.74	28.58	59.30	192.13
BC 2	181.52	53.68	89.85	334.91

Table 1 Short-run results Base Case

We can see that on average, the second scenario yields much higher costs than the first scenario. This will be analysed in more detail in Section VII.

### iv. Results First-Come-First-Served

Similar to the Base Case, we ran two scenarios using the model defined by Liang et al. (2013) and our own extended model. Unlike the long-term analysis, it is possible with simulation to analyse instances with different repair rates, hence 162 problems were considered. The results are shown in Table 2, again for a full simulation length of  $T = 10$ .

	Mean	Std. Dev.	Min	Max
FCFS 1	91.79	24.54	48.31	157.96
FCFS 2	174.21	52.40	90.21	318.14

Table 2 Short-run results First-Come-First-Served

Again, we see that the second scenario yields higher average costs than the first scenario. Furthermore, we see that the mean costs of the FCFS policy are smaller than the mean costs of the BC policy in both scenarios, which is in line with what we found for the long-term performance.

A full list of results for scenarios 1 and 2 can be found in Appendices iv and v respectively.

## VII. Discussion

This section aims to compare the results found for the different policies. We will first introduce some metrics for this purpose, after which we will evaluate the performance of the policies in long-term and short-term horizons respectively.

Liang et al. (2013) make use of relative cost savings for comparison, which will also be used in this paper. Let  $C_X^*$  denote the optimal costs for a given policy  $X$ . They then define the cost savings between a policy  $X$  and a policy  $Y$  in the following manner:

$$\Delta_X^Y \equiv \frac{C_X^* - C_Y^*}{C_X^*}$$

The results for pairwise comparison of the policies is shown in Table 3.

	Mean (%)	Std. Dev. (%)	Min (%)	Max (%)
$\Delta_{BC}^{FCFS}$	37.4%	1.5%	33.7%	39.5%
$\Delta_{BC}^{M(R)}$	44.6%	3.4%	38.1%	57.3%
$\Delta_{FCFS}^{M(R)}$	10.9%	4.2%	4.4%	22.9%

Table 3 Pairwise comparison repair policies (long-term)

The first column confirms the conclusion of Liang et al. (2013) that a Centralised Repair Shop with a larger capacity is more cost efficient than a separate repair shop for every fleet. Cost savings up to 57% were found when comparing the Myopic(R) policy to the Base Case policy for the instances considered in this paper. Furthermore, we confirm the conclusion that the Myopic(R) policy outperforms the FCFS policy in all cases, with a mean cost savings of nearly 11%.

A similar comparison was done with the results for the short-term policies. We compared the same policies for cases 1 and 2 (denoted by superscripts) to one another, and compared different policies in the same scenario. The results are shown in Tables 4 and 5 respectively.

	Mean (%)	Std. Dev. (%)	Min (%)	Max (%)
$\Delta_{BC^1}^{BC^2}$	-65.7%	10.2%	-88.3%	-37.5%
$\Delta_{FCFS^1}^{FCFS^2}$	-88.3%	12.1%	-114.5%	-58.8%

Table 4 Comparison Scenarios (short-term)

	Mean (%)	Std. Dev. (%)	Min (%)	Max (%)
$\Delta_{BC^1}^{FCFS^1}$	15.6%	3.8%	6.3%	25.6%
$\Delta_{BC^2}^{FCFS^2}$	4.1%	3.8%	-5.9%	15.2%

Table 5 Pairwise comparison repair policies (short-term)

The results from Table 4 show that by relaxing the model the costs increase. This is caused by the probability of machines being permanently broken, which requires companies to have more inventory, or to suffer more downtime costs. Furthermore, having an increased failure rate as a consequence of aging also reduces the average number of working machines. It is worth noting though that the differences in costs are higher when comparing the FCFS policy cases.

We see in Table 5 that repair shop pooling is, on average, also cheaper in the short-run. This holds for both policies, although the average cost savings are higher for the original model. In particular, we see in Scenario 2 that it is better in some cases not to pool the repair shop, but to use a dedicated repair

shop for every fleet. At this moment, it is unknown if there are common factors in these cases. Therefore, further research is necessary to investigate this phenomenon, as it might be useful for the decision maker to know when to use a CRS.

We can analyse the results more formally using statistical tools. Wackerly et al. (2008) argue that a t-test can be used to determine whether the means are significantly different from 0 under the assumption that the values come from a normal distribution. The QQ-plots shown in Appendix ii confirm a normal distribution, which means that we can use the t-test for statistical inference. In these QQ-plots, the values obtained from the comparisons were used, as we are interested in the statistical properties of these numbers.

More specifically, we are interested if the means given in Table 5 are significantly larger than 0 to confirm our hypothesis that a pooled repair shop is also more cost-effective for a finite horizon for both scenarios. Table 6 summarizes this and reports the null hypothesis, the associated t-statistic and the associated p-value for a significance level of  $\alpha = 0.05$  and  $\nu = 161$  degrees of freedom. In this table,  $\mu_c$  denotes the mean value (column 2 of Table 5) of comparison  $c$  (column 1 of Table 5).

$H_0$	$H_1$	t-statistic	p-value	Reject $H_0$ ?
$\mu_{\Delta_{BC^1}^{FCFS^1}} = 0$	$\mu_{\Delta_{BC^1}^{FCFS^1}} > 0$	52.25	0.0	Yes
$\mu_{\Delta_{BC^2}^{FCFS^2}} = 0$	$\mu_{\Delta_{BC^2}^{FCFS^2}} > 0$	12.73	0.0	Yes

Table 6 Testing hypothesis mean cost savings equal to 0

In both cases, we reject the null hypothesis. Therefore, we conclude that there is significant evidence to assume a mean cost savings larger than 0, which means that is more cost effective to have a centralised repair shop in both scenarios.

## VIII. Conclusion

In this paper we aimed to reproduce the results found by Liang et al. (2013) and check their validity. Furthermore, the insights obtained by implementing the policies were used to extend the model, and analyse the performance of the considered policies for a finite horizon.

With the results found in Sections V and VII we can confirm the conclusions of Liang et al. (2013). In particular, we find that using a pooled repair shop is cheaper than using a dedicated repair shop for every fleet. Furthermore, we confirm the notion that a First-Come-First-Served policy yields positive costs savings over a Base Case policy. The Myopic(R) policy developed in Liang et al. (2013) was found to outperform the FCFS policy in all cases.

We conclude from Sections VI and VII that the results found by Liang et al. (2013) are also valid for a finite horizon. This holds for the original model and our own extended model, in which some assumptions are relaxed. We find however that a FCFS policy does not necessarily outperform a Base Case policy in the extended model. Future research can build on these findings by searching for causes of this phenomenon.

Due to time constraints, we were unable to check the short-term performance of the Myopic(R) policy. This can be done in future research, to see if this policy shows similar behaviour to the two policies considered in this paper.

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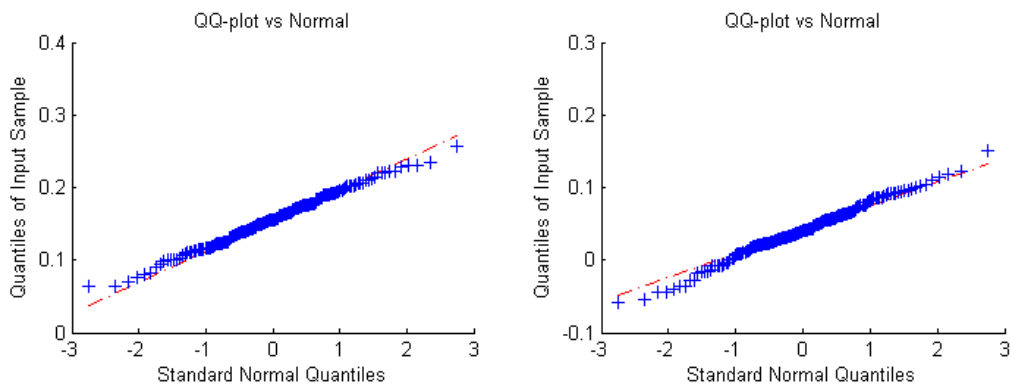
## X. Appendices

### i. Required number of simulations for 5% half-length CI

Policy	$\bar{C}_{50}^X$	$\bar{S}_{50}^X$	$n_X^*$
BC <sup>1</sup>	100.18	42.25	274
FCFS <sup>1</sup>	84.23	35.77	277
BC <sup>2</sup>	168.13	60.33	198
FCFS <sup>2</sup>	160.99	57.81	199

Appendix 1: Results trial simulation and minimal required runs

### ii. QQ-plots comparison values vs. normal distribution



Appendix 2: QQ-plots comparing values found in comparison to a normal distribution

In Appendix ii, the QQ-plots shows the comparison of the values for  $\Delta_{BC^1}^{FCFS^1}$  and  $\Delta_{BC^2}^{FCFS^2}$  respectively to a normal distribution.



iii. Full list of results long-term analysis

Appendix 3: Long-term results for  $N_1 = 10, N_2 = 5$

Parameters						Base Case			FCFS			Myopic(R)				
$\lambda_1$	$\lambda_2$	$\mu_1$	$\mu_2$	$h_1$	$h_2$	$b_1$	$b_2$	$S_1$	$S_2$	Costs	$S_1$	$S_2$	Costs	$S_1$	$S_2$	Costs
0.09	0.09	2	1	1	0.9	80	72	18	14	47.312				5	14	24.468
0.09	0.09	2	1	1	0.7	80	56	18	14	42.832				5	15	20.783
0.09	0.09	2	1	1	0.5	80	40	18	14	38.351				5	14	16.389
0.09	0.18	2	2	1	0.9	80	72	18	14	47.312	12	10	29.856	7	13	26.807
0.09	0.18	2	2	1	0.7	80	56	18	14	42.832	12	9	26.654	6	15	22.826
0.09	0.18	2	2	1	0.5	80	40	18	14	38.351	12	9	23.393	5	16	18.039
0.09	0.27	2	3	1	0.9	80	72	18	14	47.312				14	5	26.279
0.09	0.27	2	3	1	0.7	80	56	18	14	42.832				12	7	23.879
0.09	0.27	2	3	1	0.5	80	40	18	14	38.351				6	15	19.467
0.07	0.07	2	1	1	0.9	80	72	9	8	22.121				4	6	11.743
0.07	0.07	2	1	1	0.7	80	56	9	8	19.903				4	6	10.272
0.07	0.07	2	1	1	0.5	80	40	9	8	17.685				4	6	8.618
0.07	0.14	2	2	1	0.9	80	72	9	8	22.121	6	5	13.670	5	5	12.267
0.07	0.14	2	2	1	0.7	80	56	9	8	19.903	6	5	12.249	4	6	10.740
0.07	0.14	2	2	1	0.5	80	40	9	8	17.685	6	5	10.827	4	7	9.157
0.07	0.21	2	3	1	0.9	80	72	9	8	22.121				6	4	12.292
0.07	0.21	2	3	1	0.7	80	56	9	8	19.903				5	5	11.049
0.07	0.21	2	3	1	0.5	80	40	9	8	17.685				4	7	9.688
0.05	0.05	2	1	1	0.9	80	72	5	5	12.548				3	3	7.194
0.05	0.05	2	1	1	0.7	80	56	5	5	11.264				3	3	6.405
0.05	0.05	2	1	1	0.5	80	40	5	5	9.979				3	3	5.616
0.05	0.1	2	2	1	0.9	80	72	5	5	12.548	3	3	7.949	3	3	7.264
0.05	0.1	2	2	1	0.7	80	56	5	5	11.264	3	3	7.125	3	3	6.454
0.05	0.1	2	2	1	0.5	80	40	5	5	9.979	3	3	6.301	3	3	5.644
0.05	0.15	2	3	1	0.9	80	72	5	5	12.548				3	3	7.404
0.05	0.15	2	3	1	0.7	80	56	5	5	11.264				3	3	6.564
0.05	0.15	2	3	1	0.5	80	40	5	5	9.979				3	3	5.725
0.09	0.09	2	1	1	0.9	20	18	9	6	27.303				3	7	15.317
0.09	0.09	2	1	1	0.7	20	14	9	6	24.803				3	7	13.151
0.09	0.09	2	1	1	0.5	20	10	9	6	22.303				3	7	10.413
0.09	0.18	2	2	1	0.9	20	18	9	6	27.303	6	4	17.938	4	6	16.260
0.09	0.18	2	2	1	0.7	20	14	9	6	24.803	6	4	15.975	3	7	14.025
0.09	0.18	2	2	1	0.5	20	10	9	6	22.303	6	3	13.989	3	7	11.198
0.09	0.27	2	3	1	0.9	20	18	9	6	27.303				7	3	16.144
0.09	0.27	2	3	1	0.7	20	14	9	6	24.803				5	4	14.410
0.09	0.27	2	3	1	0.5	20	10	9	6	22.303				3	7	11.854
0.07	0.07	2	1	1	0.9	20	18	6	5	14.929				2	4	8.260
0.07	0.07	2	1	1	0.7	20	14	6	5	13.462				2	4	7.199
0.07	0.07	2	1	1	0.5	20	10	6	5	11.996				2	4	6.013
0.07	0.14	2	2	1	0.9	20	18	6	5	14.929	3	3	9.403	3	3	8.528
0.07	0.14	2	2	1	0.7	20	14	6	5	13.462	4	3	8.438	3	3	7.525
0.07	0.14	2	2	1	0.5	20	10	6	5	11.996	4	3	7.471	2	4	6.434
0.07	0.21	2	3	1	0.9	20	18	6	5	14.929				4	2	8.663
0.07	0.21	2	3	1	0.7	20	14	6	5	13.462				3	3	7.671
0.07	0.21	2	3	1	0.5	20	10	6	5	11.996				3	4	6.766
0.05	0.05	2	1	1	0.9	20	18	3	3	8.774				2	2	5.071
0.05	0.05	2	1	1	0.7	20	14	3	3	7.886				2	2	4.510
0.05	0.05	2	1	1	0.5	20	10	3	3	6.999				2	2	3.949
0.05	0.1	2	2	1	0.9	20	18	3	3	8.774	2	2	5.477	2	2	5.142
0.05	0.1	2	2	1	0.7	20	14	3	3	7.886	2	2	4.909	2	2	4.563
0.05	0.1	2	2	1	0.5	20	10	3	3	6.999	2	2	4.342	2	2	3.984
0.05	0.15	2	3	1	0.9	20	18	3	3	8.774				2	2	5.240
0.05	0.15	2	3	1	0.7	20	14	3	3	7.886				2	2	4.640
0.05	0.15	2	3	1	0.5	20	10	3	3	6.999				2	2	4.040

Appendix 4: Long-term results for  $N_1 = 10, N_2 = 10$

Parameters						Base Case					FCFS			Myopic(R)		
$\lambda_1$	$\lambda_2$	$\mu_1$	$\mu_2$	$h_1$	$h_2$	$b_1$	$b_2$	$S_1$	$S_2$	Costs	$S_1$	$S_2$	Costs	$S_1$	$S_2$	Costs
0.09	0.05	2	1	1	0.9	80	72	18	18	51.585				5	16	26.729
0.09	0.05	2	1	1	0.7	80	56	18	18	46.155				5	17	22.457
0.09	0.05	2	1	1	0.5	80	40	18	18	40.725				5	18	18.154
0.09	0.09	2	2	1	0.9	80	72	18	18	51.585	12	12	32.553	7	16	29.889
0.09	0.09	2	2	1	0.7	80	56	18	18	46.155	13	12	29.112	6	18	25.137
0.09	0.09	2	2	1	0.5	80	40	18	18	40.725	13	11	25.587	5	20	20.184
0.09	0.14	2	3	1	0.9	80	72	18	18	51.585				16	6	29.349
0.09	0.14	2	3	1	0.7	80	56	18	18	46.155				14	7	26.682
0.09	0.14	2	3	1	0.5	80	40	18	18	40.725				6	20	21.991
0.07	0.04	2	1	1	0.9	80	72	9	9	23.065				4	6	12.129
0.07	0.04	2	1	1	0.7	80	56	9	9	20.637				4	7	10.575
0.07	0.04	2	1	1	0.5	80	40	9	9	18.210				4	7	8.962
0.07	0.07	2	2	1	0.9	80	72	9	9	23.065	6	6	14.062	5	6	12.764
0.07	0.07	2	2	1	0.7	80	56	9	9	20.637	6	6	12.582	4	7	11.125
0.07	0.07	2	2	1	0.5	80	40	9	9	18.210	6	6	11.102	4	7	9.391
0.07	0.11	2	3	1	0.9	80	72	9	9	23.065				7	4	12.884
0.07	0.11	2	3	1	0.7	80	56	9	9	20.637				6	5	11.524
0.07	0.11	2	3	1	0.5	80	40	9	9	18.210				4	7	9.888
0.05	0.03	2	1	1	0.9	80	72	5	5	12.859				3	3	7.305
0.05	0.03	2	1	1	0.7	80	56	5	5	11.506				3	3	6.493
0.05	0.03	2	1	1	0.5	80	40	5	5	10.152				3	3	5.682
0.05	0.05	2	2	1	0.9	80	72	5	5	12.859	3	3	8.086	3	3	7.386
0.05	0.05	2	2	1	0.7	80	56	5	5	11.506	3	3	7.234	3	3	6.551
0.05	0.05	2	2	1	0.5	80	40	5	5	10.152	3	3	6.383	3	3	5.716
0.05	0.08	2	3	1	0.9	80	72	5	5	12.859				3	3	7.549
0.05	0.08	2	3	1	0.7	80	56	5	5	11.506				3	3	6.680
0.05	0.08	2	3	1	0.5	80	40	5	5	10.152				3	3	5.810
0.09	0.05	2	1	1	0.9	20	18	9	9	30.500				3	9	17.296
0.09	0.05	2	1	1	0.7	20	14	9	9	27.289				3	9	14.641
0.09	0.05	2	1	1	0.5	20	10	9	9	24.079				3	10	11.980
0.09	0.09	2	2	1	0.9	20	18	9	9	30.500	6	6	20.221	4	8	18.832
0.09	0.09	2	2	1	0.7	20	14	9	9	27.289	7	6	18.077	3	9	16.004
0.09	0.09	2	2	1	0.5	20	10	9	9	24.079	7	5	15.865	3	10	12.995
0.09	0.14	2	3	1	0.9	20	18	9	9	30.500				8	4	18.788
0.09	0.14	2	3	1	0.7	20	14	9	9	27.289				7	4	16.715
0.09	0.14	2	3	1	0.5	20	10	9	9	24.079				4	9	13.859
0.07	0.04	2	1	1	0.9	20	18	6	6	15.826				2	4	8.628
0.07	0.04	2	1	1	0.7	20	14	6	6	14.160				2	4	7.493
0.07	0.04	2	1	1	0.5	20	10	6	6	12.494				2	4	6.358
0.07	0.07	2	2	1	0.9	20	18	6	6	15.826	4	4	9.869	3	4	9.030
0.07	0.07	2	2	1	0.7	20	14	6	6	14.160	4	4	8.830	3	4	7.880
0.07	0.07	2	2	1	0.5	20	10	6	6	12.494	4	3	7.782	2	5	6.720
0.07	0.11	2	3	1	0.9	20	18	6	6	15.826				4	3	9.156
0.07	0.11	2	3	1	0.7	20	14	6	6	14.160				4	3	8.159
0.07	0.11	2	3	1	0.5	20	10	6	6	12.494				3	4	6.946
0.05	0.03	2	1	1	0.9	20	18	3	3	9.082				2	2	5.167
0.05	0.03	2	1	1	0.7	20	14	3	3	8.126				2	2	4.586
0.05	0.03	2	1	1	0.5	20	10	3	3	7.170				2	2	4.006
0.05	0.05	2	2	1	0.9	20	18	3	3	9.082	2	2	5.582	2	2	5.250
0.05	0.05	2	2	1	0.7	20	14	3	3	8.126	2	2	4.994	2	2	4.649
0.05	0.05	2	2	1	0.5	20	10	3	3	7.170	2	2	4.407	2	2	4.048
0.05	0.08	2	3	1	0.9	20	18	3	3	9.082				2	2	5.368
0.05	0.08	2	3	1	0.7	20	14	3	3	8.126				2	2	4.741
0.05	0.08	2	3	1	0.5	20	10	3	3	7.170				2	2	4.114

Appendix 5: Long-term results for  $N_1 = 10, N_2 = 15$

Parameters						Base Case					FCFS			Myopic(R)		
$\lambda_1$	$\lambda_2$	$\mu_1$	$\mu_2$	$h_1$	$h_2$	$b_1$	$b_2$	$S_1$	$S_2$	Costs	$S_1$	$S_2$	Costs	$S_1$	$S_2$	Costs
0.09	0.03	2	1	1	0.9	80	72	18	21	54.062				5	18	28.080
0.09	0.03	2	1	1	0.7	80	56	18	21	48.081				5	19	24.123
0.09	0.03	2	1	1	0.5	80	40	18	21	42.101				5	19	18.777
0.09	0.06	2	2	1	0.9	80	72	18	21	54.062	12	13	33.707	7	17	30.839
0.09	0.06	2	2	1	0.7	80	56	18	21	48.081	13	13	30.139	6	20	26.676
0.09	0.06	2	2	1	0.5	80	40	18	21	42.101	13	13	26.499	5	20	21.111
0.09	0.09	2	3	1	0.9	80	72	18	21	54.062				16	6	29.815
0.09	0.09	2	3	1	0.7	80	56	18	21	48.081				15	8	28.040
0.09	0.09	2	3	1	0.5	80	40	18	21	42.101				6	20	23.249
0.07	0.02	2	1	1	0.9	80	72	9	10	23.514				4	7	12.722
0.07	0.02	2	1	1	0.7	80	56	9	10	20.986				4	7	11.219
0.07	0.02	2	1	1	0.5	80	40	9	10	18.459				4	7	9.037
0.07	0.05	2	2	1	0.9	80	72	9	10	23.514	6	6	14.220	5	6	12.952
0.07	0.05	2	2	1	0.7	80	56	9	10	20.986	6	6	12.710	4	7	11.498
0.07	0.05	2	2	1	0.5	80	40	9	10	18.459	6	6	11.200	4	7	9.514
0.07	0.07	2	3	1	0.9	80	72	9	10	23.514				6	4	12.737
0.07	0.07	2	3	1	0.7	80	56	9	10	20.986				6	5	11.611
0.07	0.07	2	3	1	0.5	80	40	9	10	18.459				4	8	10.063
0.05	0.02	2	1	1	0.9	80	72	5	5	13.015				3	4	7.669
0.05	0.02	2	1	1	0.7	80	56	5	5	11.627				3	4	6.818
0.05	0.02	2	1	1	0.5	80	40	5	5	10.239				3	4	5.701
0.05	0.03	2	2	1	0.9	80	72	5	5	13.015	3	3	8.142	3	3	7.468
0.05	0.03	2	2	1	0.7	80	56	5	5	11.627	3	3	7.280	3	3	6.738
0.05	0.03	2	2	1	0.5	80	40	5	5	10.239	3	3	6.418	3	3	5.746
0.05	0.05	2	3	1	0.9	80	72	5	5	13.015				3	3	7.383
0.05	0.05	2	3	1	0.7	80	56	5	5	11.627				3	3	6.652
0.05	0.05	2	3	1	0.5	80	40	5	5	10.239				3	3	5.847
0.09	0.03	2	1	1	0.9	20	18	9	11	32.515				3	10	18.583
0.09	0.03	2	1	1	0.7	20	14	9	11	28.857				3	11	16.237
0.09	0.03	2	1	1	0.5	20	10	9	11	25.198				3	11	12.558
0.09	0.06	2	2	1	0.9	20	18	9	11	32.515	6	7	21.204	4	9	19.670
0.09	0.06	2	2	1	0.7	20	14	9	11	28.857	7	7	18.977	3	11	17.404
0.09	0.06	2	2	1	0.5	20	10	9	11	25.198	7	7	16.676	3	11	13.786
0.09	0.09	2	3	1	0.9	20	18	9	11	32.515				8	4	19.144
0.09	0.09	2	3	1	0.7	20	14	9	11	28.857				7	5	17.871
0.09	0.09	2	3	1	0.5	20	10	9	11	25.198				4	11	14.861
0.07	0.02	2	1	1	0.9	20	18	6	6	16.242				2	5	9.249
0.07	0.02	2	1	1	0.7	20	14	6	6	14.484				2	5	8.151
0.07	0.02	2	1	1	0.5	20	10	6	6	12.725				2	5	6.442
0.07	0.05	2	2	1	0.9	20	18	6	6	16.242	4	4	10.005	3	4	9.218
0.07	0.05	2	2	1	0.7	20	14	6	6	14.484	4	4	8.942	3	4	8.267
0.07	0.05	2	2	1	0.5	20	10	6	6	12.725	4	4	7.878	2	5	6.835
0.07	0.07	2	3	1	0.9	20	18	6	6	16.242				4	3	9.042
0.07	0.07	2	3	1	0.7	20	14	6	6	14.484				4	3	8.267
0.07	0.07	2	3	1	0.5	20	10	6	6	12.725				3	4	7.107
0.05	0.02	2	1	1	0.9	20	18	3	3	9.235				2	2	5.593
0.05	0.02	2	1	1	0.7	20	14	3	3	8.245				2	3	4.982
0.05	0.02	2	1	1	0.5	20	10	3	3	7.255				2	2	4.030
0.05	0.03	2	2	1	0.9	20	18	3	3	9.235	2	2	5.625	2	2	5.320
0.05	0.03	2	2	1	0.7	20	14	3	3	8.245	2	2	5.029	2	2	4.808
0.05	0.03	2	2	1	0.5	20	10	3	3	7.255	2	2	4.433	2	2	4.075
0.05	0.05	2	3	1	0.9	20	18	3	3	9.235				2	2	5.247
0.05	0.05	2	3	1	0.7	20	14	3	3	8.245				2	2	4.733
0.05	0.05	2	3	1	0.5	20	10	3	3	7.255				2	2	4.147

iv. Full list of results short-term analysis, scenario 1

Appendix 6: Short-term results scenario 1 for  $N_1 = 10, N_2 = 5$

Parameters								Base Case 1				FCFS 1			
$\lambda_1$	$\lambda_2$	$\mu_1$	$\mu_2$	$h_1$	$h_2$	$b_1$	$b_2$	$S_1$	$S_2$	Costs	$s_{costs}$	$S_1$	$S_2$	Costs	$s_{costs}$
0.09	0.09	2	1	1	0.9	80	72	9	6	152.052	51.144	8	5	134.640	53.543
0.09	0.09	2	1	1	0.7	80	56	8	4	137.179	94.502	8	4	116.699	38.972
0.09	0.09	2	1	1	0.5	80	40	7	6	117.483	93.806	7	5	106.916	53.306
0.09	0.18	2	2	1	0.9	80	72	8	8	165.360	65.892	7	6	144.721	105.285
0.09	0.18	2	2	1	0.7	80	56	7	9	150.092	72.842	6	6	125.973	135.884
0.09	0.18	2	2	1	0.5	80	40	7	6	131.657	96.690	6	6	111.274	95.329
0.09	0.27	2	3	1	0.9	80	72	7	8	188.285	158.864	6	9	155.387	71.566
0.09	0.27	2	3	1	0.7	80	56	8	9	163.340	110.827	7	9	144.758	60.181
0.09	0.27	2	3	1	0.5	80	40	8	9	141.956	81.784	6	10	116.569	42.175
0.07	0.07	2	1	1	0.9	80	72	7	5	119.744	36.054	6	4	105.013	69.633
0.07	0.07	2	1	1	0.7	80	56	6	5	107.501	86.696	5	4	95.484	105.350
0.07	0.07	2	1	1	0.5	80	40	7	4	97.610	49.826	5	4	86.783	79.039
0.07	0.14	2	2	1	0.9	80	72	6	6	135.607	87.732	4	5	104.612	108.851
0.07	0.14	2	2	1	0.7	80	56	7	6	122.850	60.310	5	5	103.710	90.239
0.07	0.14	2	2	1	0.5	80	40	5	6	102.409	106.633	5	5	89.602	70.386
0.07	0.21	2	3	1	0.9	80	72	6	8	148.881	86.289	6	6	126.769	68.453
0.07	0.21	2	3	1	0.7	80	56	6	7	138.183	128.583	5	6	109.763	88.107
0.07	0.21	2	3	1	0.5	80	40	6	7	113.062	80.065	5	6	88.037	43.897
0.05	0.05	2	1	1	0.9	80	72	4	4	98.804	120.772	4	3	81.269	78.131
0.05	0.05	2	1	1	0.7	80	56	5	4	87.859	75.266	5	3	74.291	33.677
0.05	0.05	2	1	1	0.5	80	40	4	4	71.515	54.116	4	2	64.307	63.547
0.05	0.10	2	2	1	0.9	80	72	5	4	101.803	94.192	4	4	89.796	84.589
0.05	0.10	2	2	1	0.7	80	56	5	5	93.766	59.549	4	4	77.311	80.812
0.05	0.10	2	2	1	0.5	80	40	5	5	80.577	39.831	4	4	61.671	18.318
0.05	0.15	2	3	1	0.9	80	72	5	6	110.866	54.347	4	4	87.262	60.961
0.05	0.15	2	3	1	0.7	80	56	4	5	97.039	111.018	3	6	76.310	41.603
0.05	0.15	2	3	1	0.5	80	40	5	7	89.256	36.271	4	4	68.740	48.948
0.09	0.09	2	1	1	0.9	20	18	5	4	115.627	66.558	4	3	102.275	74.646
0.09	0.09	2	1	1	0.7	20	14	5	3	105.516	80.346	4	3	95.004	80.804
0.09	0.09	2	1	1	0.5	20	10	5	5	93.366	52.979	4	4	83.208	56.543
0.09	0.18	2	2	1	0.9	20	18	6	5	136.049	71.144	4	5	113.905	72.334
0.09	0.18	2	2	1	0.7	20	14	5	7	123.134	57.999	5	4	102.934	56.520
0.09	0.18	2	2	1	0.5	20	10	6	5	106.541	54.902	4	5	88.372	54.428
0.09	0.27	2	3	1	0.9	20	18	7	7	152.138	51.332	5	6	126.815	60.710
0.09	0.27	2	3	1	0.7	20	14	5	6	127.395	63.725	6	6	110.457	25.119
0.09	0.27	2	3	1	0.5	20	10	6	7	112.467	55.425	5	6	96.927	44.710
0.07	0.07	2	1	1	0.9	20	18	5	3	91.383	42.036	4	3	79.010	39.543
0.07	0.07	2	1	1	0.7	20	14	4	3	87.540	65.579	4	3	73.938	39.002
0.07	0.07	2	1	1	0.5	20	10	4	3	78.123	65.382	4	4	68.337	33.589
0.07	0.14	2	2	1	0.9	20	18	5	5	107.529	46.941	4	4	90.109	54.055
0.07	0.14	2	2	1	0.7	20	14	5	5	96.969	38.477	4	4	83.426	44.654
0.07	0.14	2	2	1	0.5	20	10	4	4	86.159	52.515	5	4	74.509	20.643
0.07	0.21	2	3	1	0.9	20	18	5	6	118.401	43.151	4	5	94.068	37.100
0.07	0.21	2	3	1	0.7	20	14	5	5	102.519	47.493	4	4	81.053	40.451
0.07	0.21	2	3	1	0.5	20	10	5	5	90.389	39.511	4	6	76.387	27.230
0.05	0.05	2	1	1	0.9	20	18	3	2	71.382	61.807	3	2	66.324	48.578
0.05	0.05	2	1	1	0.7	20	14	3	2	67.829	59.549	3	2	52.784	37.465
0.05	0.05	2	1	1	0.5	20	10	4	2	62.089	30.500	3	2	52.080	35.776
0.05	0.10	2	2	1	0.9	20	18	4	3	84.704	47.925	3	3	68.404	43.570
0.05	0.10	2	2	1	0.7	20	14	3	4	75.277	58.769	3	3	60.616	37.840
0.05	0.10	2	2	1	0.5	20	10	4	4	67.326	32.505	3	3	54.209	31.723
0.05	0.15	2	3	1	0.9	20	18	3	4	86.216	55.382	3	4	73.152	27.917
0.05	0.15	2	3	1	0.7	20	14	4	4	80.761	39.722	3	3	62.113	35.918
0.05	0.15	2	3	1	0.5	20	10	3	5	67.701	37.224	3	3	54.669	36.944

Appendix 7: Short-term results scenario 1 for  $N_1 = 10, N_2 = 10$

Parameters								Base Case 1				FCFS 1			
$\lambda_1$	$\lambda_2$	$\mu_1$	$\mu_2$	$h_1$	$h_2$	$b_1$	$b_2$	$S_1$	$S_2$	Costs	$s_{costs}$	$S_1$	$S_2$	Costs	$s_{costs}$
0.09	0.05	2	1	1	0.9	80	72	7	6	149.163	132.569	7	4	127.460	93.544
0.09	0.05	2	1	1	0.7	80	56	8	6	135.064	75.115	8	5	124.785	60.834
0.09	0.05	2	1	1	0.5	80	40	7	6	119.223	83.494	7	4	104.407	80.352
0.09	0.09	2	2	1	0.9	80	72	7	8	163.375	89.968	7	6	137.782	67.564
0.09	0.09	2	2	1	0.7	80	56	6	8	148.295	136.953	6	7	126.069	79.500
0.09	0.09	2	2	1	0.5	80	40	6	6	135.395	142.317	6	8	116.411	88.154
0.09	0.14	2	3	1	0.9	80	72	7	9	184.658	160.039	6	8	153.706	99.298
0.09	0.14	2	3	1	0.7	80	56	8	9	162.087	84.843	7	9	143.422	72.132
0.09	0.14	2	3	1	0.5	80	40	7	9	138.871	100.322	5	7	122.158	131.657
0.07	0.04	2	1	1	0.9	80	72	7	5	116.954	22.756	6	5	109.455	45.324
0.07	0.04	2	1	1	0.7	80	56	7	5	110.444	42.032	5	4	95.185	99.112
0.07	0.04	2	1	1	0.5	80	40	6	4	99.505	83.362	5	5	84.429	78.647
0.07	0.07	2	2	1	0.9	80	72	6	6	129.453	70.439	6	5	111.704	46.971
0.07	0.07	2	2	1	0.7	80	56	6	7	119.111	69.514	5	5	100.523	84.537
0.07	0.07	2	2	1	0.5	80	40	6	6	102.435	58.736	5	5	87.909	69.926
0.07	0.11	2	3	1	0.9	80	72	6	7	151.270	109.534	5	7	122.505	72.809
0.07	0.11	2	3	1	0.7	80	56	6	7	133.068	102.361	5	5	105.062	103.411
0.07	0.11	2	3	1	0.5	80	40	6	9	112.102	52.923	5	5	95.132	108.724
0.05	0.03	2	1	1	0.9	80	72	6	4	96.928	10.913	5	3	80.102	42.252
0.05	0.03	2	1	1	0.7	80	56	5	4	88.686	74.527	4	2	73.132	87.093
0.05	0.03	2	1	1	0.5	80	40	5	4	77.506	49.812	5	3	69.501	46.148
0.05	0.05	2	2	1	0.9	80	72	5	5	104.374	67.127	3	3	91.884	122.809
0.05	0.05	2	2	1	0.7	80	56	4	4	89.155	86.566	4	4	77.096	64.552
0.05	0.05	2	2	1	0.5	80	40	5	5	80.860	46.554	4	3	70.447	61.445
0.05	0.08	2	3	1	0.9	80	72	5	5	112.829	87.815	4	5	91.143	48.563
0.05	0.08	2	3	1	0.7	80	56	4	5	96.368	97.558	4	4	81.935	72.688
0.05	0.08	2	3	1	0.5	80	40	5	6	87.807	48.227	4	5	71.091	46.545
0.09	0.05	2	1	1	0.9	20	18	5	5	122.690	66.813	5	3	101.494	61.575
0.09	0.05	2	1	1	0.7	20	14	6	4	109.646	60.365	6	3	96.804	50.825
0.09	0.05	2	1	1	0.5	20	10	5	3	95.555	56.182	4	4	87.602	66.042
0.09	0.09	2	2	1	0.9	20	18	5	7	139.482	62.498	6	5	115.518	35.915
0.09	0.09	2	2	1	0.7	20	14	6	6	121.115	48.037	5	5	100.301	48.207
0.09	0.09	2	2	1	0.5	20	10	6	6	104.497	37.324	5	4	92.963	58.425
0.09	0.14	2	3	1	0.9	20	18	6	7	151.272	67.491	5	7	128.844	47.286
0.09	0.14	2	3	1	0.7	20	14	5	6	135.893	86.680	5	6	116.679	77.136
0.09	0.14	2	3	1	0.5	20	10	6	8	115.625	42.400	4	7	95.429	50.375
0.07	0.04	2	1	1	0.9	20	18	5	4	99.564	41.143	3	2	79.379	74.079
0.07	0.04	2	1	1	0.7	20	14	5	3	86.389	48.000	4	3	76.391	51.165
0.07	0.04	2	1	1	0.5	20	10	5	3	81.635	52.307	4	3	69.032	49.743
0.07	0.07	2	2	1	0.9	20	18	4	5	111.413	63.341	4	4	92.398	48.859
0.07	0.07	2	2	1	0.7	20	14	5	5	95.736	35.799	4	4	77.429	32.806
0.07	0.07	2	2	1	0.5	20	10	4	5	87.488	59.140	4	3	71.490	42.759
0.07	0.11	2	3	1	0.9	20	18	5	5	118.120	61.342	4	4	97.082	64.738
0.07	0.11	2	3	1	0.7	20	14	4	6	101.365	51.892	4	5	85.783	37.007
0.07	0.11	2	3	1	0.5	20	10	5	6	92.606	48.316	4	5	77.352	42.251
0.05	0.03	2	1	1	0.9	20	18	4	3	76.180	36.965	3	2	65.687	55.153
0.05	0.03	2	1	1	0.7	20	14	4	2	68.984	44.320	3	2	60.984	52.019
0.05	0.03	2	1	1	0.5	20	10	3	3	59.302	42.438	3	3	48.311	17.567
0.05	0.05	2	2	1	0.9	20	18	3	3	85.491	66.247	3	3	68.532	41.475
0.05	0.05	2	2	1	0.7	20	14	4	3	78.218	51.316	3	3	63.292	41.178
0.05	0.05	2	2	1	0.5	20	10	3	4	66.055	43.446	3	2	55.244	39.051
0.05	0.08	2	3	1	0.9	20	18	3	4	89.153	63.133	3	3	76.438	60.225
0.05	0.08	2	3	1	0.7	20	14	3	4	74.854	47.408	3	3	66.500	47.599
0.05	0.08	2	3	1	0.5	20	10	3	4	65.187	45.439	3	4	56.655	27.292

Appendix 8: Short-term results scenario 1 for  $N_1 = 10, N_2 = 15$

Parameters								Base Case 1				FCFS 1			
$\lambda_1$	$\lambda_2$	$\mu_1$	$\mu_2$	$h_1$	$h_2$	$b_1$	$b_2$	$S_1$	$S_2$	Costs	$s_{costs}$	$S_1$	$S_2$	Costs	$s_{costs}$
0.09	0.03	2	1	1	0.9	80	72	9	5	149.713	93.116	7	5	132.203	92.091
0.09	0.03	2	1	1	0.7	80	56	8	5	134.606	110.519	8	4	121.105	60.327
0.09	0.03	2	1	1	0.5	80	40	8	5	120.343	66.950	7	4	108.504	75.267
0.09	0.06	2	2	1	0.9	80	72	8	8	172.768	100.852	7	7	143.047	59.850
0.09	0.06	2	2	1	0.7	80	56	8	8	152.398	78.855	7	6	131.301	92.832
0.09	0.06	2	2	1	0.5	80	40	8	8	130.841	61.186	7	6	114.688	85.187
0.09	0.09	2	3	1	0.9	80	72	9	9	192.128	94.185	6	9	157.956	101.424
0.09	0.09	2	3	1	0.7	80	56	7	11	166.403	115.225	6	10	139.616	64.114
0.09	0.09	2	3	1	0.5	80	40	7	10	142.218	87.849	7	8	120.368	54.084
0.07	0.02	2	1	1	0.9	80	72	7	5	120.808	61.774	6	4	103.498	57.476
0.07	0.02	2	1	1	0.7	80	56	7	4	108.377	62.530	5	3	88.543	79.345
0.07	0.02	2	1	1	0.5	80	40	6	5	97.692	65.231	4	5	85.741	88.561
0.07	0.05	2	2	1	0.9	80	72	7	6	135.327	81.419	5	7	120.005	53.564
0.07	0.05	2	2	1	0.7	80	56	6	6	119.568	97.566	5	5	93.199	48.738
0.07	0.05	2	2	1	0.5	80	40	7	6	112.463	65.843	5	4	89.538	84.043
0.07	0.07	2	3	1	0.9	80	72	5	6	139.279	126.232	5	6	116.364	76.341
0.07	0.07	2	3	1	0.7	80	56	7	8	139.516	77.113	5	6	103.744	60.466
0.07	0.07	2	3	1	0.5	80	40	6	7	114.201	83.181	5	8	90.624	10.387
0.05	0.02	2	1	1	0.9	80	72	5	3	95.791	88.455	4	4	79.459	23.385
0.05	0.02	2	1	1	0.7	80	56	4	5	80.974	40.136	4	3	74.402	82.636
0.05	0.02	2	1	1	0.5	80	40	5	4	75.393	48.127	5	3	67.430	24.952
0.05	0.03	2	2	1	0.9	80	72	5	6	108.274	49.468	4	4	84.406	57.688
0.05	0.03	2	2	1	0.7	80	56	5	4	89.976	87.746	4	4	75.114	53.489
0.05	0.03	2	2	1	0.5	80	40	5	5	80.914	62.568	5	3	69.382	30.317
0.05	0.05	2	3	1	0.9	80	72	5	6	109.319	51.154	4	4	87.289	63.733
0.05	0.05	2	3	1	0.7	80	56	4	6	95.891	84.643	4	4	78.198	57.629
0.05	0.05	2	3	1	0.5	80	40	5	6	83.782	33.258	3	4	65.363	84.505
0.09	0.03	2	1	1	0.9	20	18	5	5	119.577	63.894	6	3	100.145	41.000
0.09	0.03	2	1	1	0.7	20	14	5	5	106.638	59.784	5	4	96.549	59.815
0.09	0.03	2	1	1	0.5	20	10	5	5	97.523	57.053	5	3	83.432	45.410
0.09	0.06	2	2	1	0.9	20	18	6	5	135.684	84.506	6	5	117.805	42.122
0.09	0.06	2	2	1	0.7	20	14	6	5	124.732	71.993	5	5	102.797	59.540
0.09	0.06	2	2	1	0.5	20	10	6	6	104.968	42.678	5	5	92.054	46.338
0.09	0.09	2	3	1	0.9	20	18	6	7	147.634	71.908	4	6	126.281	78.656
0.09	0.09	2	3	1	0.7	20	14	6	7	136.650	70.179	5	6	110.805	53.074
0.09	0.09	2	3	1	0.5	20	10	5	7	115.073	64.315	5	6	97.039	56.848
0.07	0.02	2	1	1	0.9	20	18	5	4	95.138	31.672	4	3	80.186	43.895
0.07	0.02	2	1	1	0.7	20	14	5	4	87.441	38.944	4	4	78.577	38.280
0.07	0.02	2	1	1	0.5	20	10	4	3	72.789	41.050	4	3	68.182	43.357
0.07	0.05	2	2	1	0.9	20	18	5	5	106.451	40.242	4	4	92.348	56.233
0.07	0.05	2	2	1	0.7	20	14	5	5	98.251	44.283	4	3	80.061	51.500
0.07	0.05	2	2	1	0.5	20	10	4	5	81.062	48.290	4	4	69.733	35.327
0.07	0.07	2	3	1	0.9	20	18	4	6	117.891	62.115	4	4	94.716	47.556
0.07	0.07	2	3	1	0.7	20	14	5	6	105.024	42.627	4	5	86.585	42.983
0.07	0.07	2	3	1	0.5	20	10	5	5	88.996	41.992	3	5	71.210	43.590
0.05	0.02	2	1	1	0.9	20	18	3	3	74.722	47.509	3	2	63.656	49.251
0.05	0.02	2	1	1	0.7	20	14	3	2	67.589	61.287	3	2	57.285	46.676
0.05	0.02	2	1	1	0.5	20	10	3	3	60.103	39.663	3	3	52.696	35.180
0.05	0.03	2	2	1	0.9	20	18	4	4	85.771	35.846	3	3	69.424	44.786
0.05	0.03	2	2	1	0.7	20	14	3	4	73.901	45.918	3	3	60.715	37.000
0.05	0.03	2	2	1	0.5	20	10	3	3	63.807	46.626	3	3	52.648	29.393
0.05	0.05	2	3	1	0.9	20	18	4	3	91.963	66.057	3	4	73.945	36.270
0.05	0.05	2	3	1	0.7	20	14	3	4	80.545	62.608	3	4	64.256	32.238
0.05	0.05	2	3	1	0.5	20	10	3	3	69.155	48.901	3	4	55.218	25.268

v. Full list of results short-term analysis, scenario 2

Appendix 9: Short-term results scenario 2 for  $N_1 = 10, N_2 = 5$

Parameters								Base Case 2				FCFS 2			
$\lambda_1$	$\lambda_2$	$\mu_1$	$\mu_2$	$h_1$	$h_2$	$b_1$	$b_2$	$S_1$	$S_2$	Costs	$s_{costs}$	$S_1$	$S_2$	Costs	$s_{costs}$
0.09	0.09	2	1	1	0.9	80	72	12	9	239.591	157.039	14	9	243.395	142.840
0.09	0.09	2	1	1	0.7	80	56	13	10	219.307	114.227	15	8	228.857	85.484
0.09	0.09	2	1	1	0.5	80	40	14	10	202.467	64.899	14	8	205.957	124.435
0.09	0.18	2	2	1	0.9	80	72	12	14	282.595	135.509	15	13	276.266	41.649
0.09	0.18	2	2	1	0.7	80	56	14	13	256.332	93.899	12	13	243.137	122.790
0.09	0.18	2	2	1	0.5	80	40	15	15	232.332	43.987	13	14	218.818	91.907
0.09	0.27	2	3	1	0.9	80	72	13	17	324.556	173.522	14	14	304.995	141.421
0.09	0.27	2	3	1	0.7	80	56	14	19	293.433	87.621	13	16	270.956	120.104
0.09	0.27	2	3	1	0.5	80	40	12	18	239.066	132.409	12	18	235.393	111.023
0.07	0.07	2	1	1	0.9	80	72	12	6	187.121	83.931	10	7	188.658	118.017
0.07	0.07	2	1	1	0.7	80	56	11	7	171.634	63.234	9	6	172.995	144.726
0.07	0.07	2	1	1	0.5	80	40	10	9	158.807	79.038	10	7	150.084	88.424
0.07	0.14	2	2	1	0.9	80	72	11	10	219.595	93.305	11	10	213.625	78.444
0.07	0.14	2	2	1	0.7	80	56	11	10	200.034	80.964	11	9	189.209	91.042
0.07	0.14	2	2	1	0.5	80	40	11	10	168.347	54.505	10	10	164.278	58.960
0.07	0.21	2	3	1	0.9	80	72	13	12	251.731	89.631	9	12	235.813	137.206
0.07	0.21	2	3	1	0.7	80	56	12	13	220.917	52.065	8	16	214.048	105.336
0.07	0.21	2	3	1	0.5	80	40	11	13	193.266	76.211	9	14	176.150	93.922
0.05	0.05	2	1	1	0.9	80	72	7	6	135.819	74.426	8	4	133.285	76.802
0.05	0.05	2	1	1	0.7	80	56	9	4	126.153	45.073	7	5	119.607	65.495
0.05	0.05	2	1	1	0.5	80	40	8	4	112.506	57.281	8	5	109.275	31.766
0.05	0.10	2	2	1	0.9	80	72	10	7	165.777	23.682	8	7	151.616	66.888
0.05	0.10	2	2	1	0.7	80	56	7	6	143.197	111.144	7	7	133.053	80.636
0.05	0.10	2	2	1	0.5	80	40	7	6	134.247	115.393	8	6	117.803	46.173
0.05	0.15	2	3	1	0.9	80	72	9	8	177.290	92.148	8	9	163.712	27.317
0.05	0.15	2	3	1	0.7	80	56	7	8	154.135	114.707	7	8	139.310	82.926
0.05	0.15	2	3	1	0.5	80	40	7	9	135.917	102.717	6	9	120.523	66.606
0.09	0.09	2	1	1	0.9	20	18	11	6	201.866	83.411	11	7	201.885	63.783
0.09	0.09	2	1	1	0.7	20	14	11	7	181.501	61.104	12	7	182.273	39.633
0.09	0.09	2	1	1	0.5	20	10	10	9	171.817	76.489	10	6	163.800	84.946
0.09	0.18	2	2	1	0.9	20	18	9	10	235.289	107.024	11	10	223.821	57.153
0.09	0.18	2	2	1	0.7	20	14	11	10	211.781	68.679	10	11	203.075	64.887
0.09	0.18	2	2	1	0.5	20	10	9	11	185.561	94.022	12	8	179.827	44.761
0.09	0.27	2	3	1	0.9	20	18	10	11	260.249	103.827	11	14	254.062	64.125
0.09	0.27	2	3	1	0.7	20	14	10	11	222.829	86.299	9	12	214.699	80.567
0.09	0.27	2	3	1	0.5	20	10	11	13	208.909	77.973	9	13	189.722	68.020
0.07	0.07	2	1	1	0.9	20	18	8	5	154.447	71.250	7	5	152.601	73.058
0.07	0.07	2	1	1	0.7	20	14	7	5	133.850	65.754	8	5	138.809	69.681
0.07	0.07	2	1	1	0.5	20	10	8	4	127.824	59.364	8	5	128.747	74.495
0.07	0.14	2	2	1	0.9	20	18	9	7	176.888	67.194	7	8	172.814	80.307
0.07	0.14	2	2	1	0.7	20	14	8	8	160.806	59.358	7	6	151.932	79.859
0.07	0.14	2	2	1	0.5	20	10	8	9	144.890	54.112	7	6	135.224	71.986
0.07	0.21	2	3	1	0.9	20	18	8	9	195.669	91.312	7	11	192.353	66.934
0.07	0.21	2	3	1	0.7	20	14	8	12	176.926	49.405	7	8	165.742	89.200
0.07	0.21	2	3	1	0.5	20	10	8	8	156.321	75.958	7	8	139.102	67.228
0.05	0.05	2	1	1	0.9	20	18	5	4	113.560	87.323	6	4	112.781	53.646
0.05	0.05	2	1	1	0.7	20	14	6	3	103.639	59.567	6	4	99.967	42.451
0.05	0.05	2	1	1	0.5	20	10	6	4	92.990	36.530	6	4	90.211	42.953
0.05	0.10	2	2	1	0.9	20	18	6	4	129.720	80.220	5	4	119.149	66.518
0.05	0.10	2	2	1	0.7	20	14	6	5	114.117	56.964	6	5	108.427	50.585
0.05	0.10	2	2	1	0.5	20	10	6	7	104.470	42.640	6	5	95.499	27.885
0.05	0.15	2	3	1	0.9	20	18	6	6	137.676	64.122	5	5	130.918	84.379
0.05	0.15	2	3	1	0.7	20	14	6	5	119.292	64.397	6	6	116.756	49.780
0.05	0.15	2	3	1	0.5	20	10	5	7	111.160	65.808	5	7	100.976	58.799



Appendix 10: Short-term results scenario 2 for  $N_1 = 10, N_2 = 10$

Parameters						Base Case 2						FCFS 2			
$\lambda_1$	$\lambda_2$	$\mu_1$	$\mu_2$	$h_1$	$h_2$	$b_1$	$b_2$	$S_1$	$S_2$	Costs	$s_{costs}$	$S_1$	$S_2$	Costs	$s_{costs}$
0.09	0.05	2	1	1	0.9	80	72	14	9	241.431	96.889	14	9	231.629	55.236
0.09	0.05	2	1	1	0.7	80	56	15	8	226.515	110.127	13	8	225.913	134.852
0.09	0.05	2	1	1	0.5	80	40	14	8	202.372	121.005	12	10	200.510	120.899
0.09	0.09	2	2	1	0.9	80	72	14	15	288.561	118.827	15	13	276.905	45.492
0.09	0.09	2	2	1	0.7	80	56	13	14	252.379	124.664	14	14	256.298	85.560
0.09	0.09	2	2	1	0.5	80	40	15	13	229.065	58.580	14	13	224.333	74.184
0.09	0.14	2	3	1	0.9	80	72	13	19	334.908	149.672	13	17	318.140	151.120
0.09	0.14	2	3	1	0.7	80	56	14	16	285.028	152.304	15	17	282.455	64.361
0.09	0.14	2	3	1	0.5	80	40	13	17	242.361	101.125	13	18	236.859	103.832
0.07	0.04	2	1	1	0.9	80	72	10	9	184.217	29.710	10	8	182.772	68.271
0.07	0.04	2	1	1	0.7	80	56	11	7	173.098	66.206	10	9	175.352	56.861
0.07	0.04	2	1	1	0.5	80	40	10	8	157.109	96.591	8	7	150.229	115.735
0.07	0.07	2	2	1	0.9	80	72	10	10	222.554	162.921	9	10	208.801	120.777
0.07	0.07	2	2	1	0.7	80	56	12	9	205.687	122.765	9	12	188.190	79.353
0.07	0.07	2	2	1	0.5	80	40	10	12	166.176	41.572	9	11	166.758	105.110
0.07	0.11	2	3	1	0.9	80	72	10	12	249.978	170.127	10	12	230.870	117.160
0.07	0.11	2	3	1	0.7	80	56	11	13	226.789	120.231	9	12	207.266	139.782
0.07	0.11	2	3	1	0.5	80	40	11	13	190.642	68.594	9	12	184.183	145.895
0.05	0.03	2	1	1	0.9	80	72	7	4	139.231	121.175	8	5	134.660	74.368
0.05	0.03	2	1	1	0.7	80	56	7	6	131.385	99.572	7	6	125.792	62.384
0.05	0.03	2	1	1	0.5	80	40	7	4	109.272	77.367	7	7	113.753	65.617
0.05	0.05	2	2	1	0.9	80	72	8	7	159.541	87.013	7	7	153.810	95.787
0.05	0.05	2	2	1	0.7	80	56	8	7	145.793	113.552	7	6	135.802	89.591
0.05	0.05	2	2	1	0.5	80	40	8	9	132.795	57.474	7	9	119.263	27.309
0.05	0.08	2	3	1	0.9	80	72	8	9	183.749	122.545	6	8	166.676	129.924
0.05	0.08	2	3	1	0.7	80	56	8	8	159.458	106.288	8	8	146.222	79.382
0.05	0.08	2	3	1	0.5	80	40	7	8	134.512	113.123	6	9	125.624	91.650
0.09	0.05	2	1	1	0.9	20	18	11	6	203.708	97.787	12	7	197.651	41.537
0.09	0.05	2	1	1	0.7	20	14	10	7	181.968	89.155	11	7	183.213	54.243
0.09	0.05	2	1	1	0.5	20	10	11	6	170.578	64.624	11	8	170.158	64.005
0.09	0.09	2	2	1	0.9	20	18	9	11	242.209	123.684	9	12	232.434	86.290
0.09	0.09	2	2	1	0.7	20	14	11	9	213.631	82.022	11	10	207.100	62.749
0.09	0.09	2	2	1	0.5	20	10	11	9	185.326	65.164	10	11	181.666	65.889
0.09	0.14	2	3	1	0.9	20	18	12	13	269.406	78.518	10	13	260.283	103.629
0.09	0.14	2	3	1	0.7	20	14	10	16	245.543	90.599	9	14	236.232	94.079
0.09	0.14	2	3	1	0.5	20	10	12	13	209.738	54.268	10	14	200.809	74.037
0.07	0.04	2	1	1	0.9	20	18	9	6	159.598	43.818	8	5	148.824	63.322
0.07	0.04	2	1	1	0.7	20	14	8	6	143.022	58.238	9	4	141.454	51.733
0.07	0.04	2	1	1	0.5	20	10	7	4	130.819	80.376	8	5	122.284	46.692
0.07	0.07	2	2	1	0.9	20	18	8	8	179.315	75.488	8	7	171.495	65.050
0.07	0.07	2	2	1	0.7	20	14	6	9	162.840	90.048	8	7	153.774	54.811
0.07	0.07	2	2	1	0.5	20	10	8	8	147.771	88.756	7	9	139.852	70.337
0.07	0.11	2	3	1	0.9	20	18	7	11	203.996	88.308	7	10	192.905	80.381
0.07	0.11	2	3	1	0.7	20	14	9	10	182.881	54.209	7	11	171.502	65.432
0.07	0.11	2	3	1	0.5	20	10	9	10	158.379	66.811	7	10	146.385	78.784
0.05	0.03	2	1	1	0.9	20	18	6	4	113.219	48.391	6	4	113.809	51.874
0.05	0.03	2	1	1	0.7	20	14	6	4	100.889	42.352	6	4	96.867	37.416
0.05	0.03	2	1	1	0.5	20	10	5	4	89.849	51.830	6	5	94.675	40.880
0.05	0.05	2	2	1	0.9	20	18	6	5	133.382	74.651	5	5	120.282	62.601
0.05	0.05	2	2	1	0.7	20	14	5	6	118.203	80.216	6	7	115.479	25.787
0.05	0.05	2	2	1	0.5	20	10	5	5	106.823	70.189	5	6	97.584	48.173
0.05	0.08	2	3	1	0.9	20	18	5	7	144.235	78.989	6	7	133.490	36.851
0.05	0.08	2	3	1	0.7	20	14	5	7	129.330	71.324	6	7	121.511	47.149
0.05	0.08	2	3	1	0.5	20	10	5	7	111.495	68.268	4	8	101.551	52.797



Appendix 11: Short-term results scenario 2 for  $N_1 = 10, N_2 = 15$

Parameters								Base Case 2				FCFS 2			
$\lambda_1$	$\lambda_2$	$\mu_1$	$\mu_2$	$h_1$	$h_2$	$b_1$	$b_2$	$S_1$	$S_2$	Costs	$s_{costs}$	$S_1$	$S_2$	Costs	$s_{costs}$
0.09	0.03	2	1	1	0.9	80	0.09	16	10	250.893	9.518	16	7	248.278	97.503
0.09	0.03	2	1	1	0.7	80	0.09	13	7	222.448	165.558	15	9	224.335	59.145
0.09	0.03	2	1	1	0.5	80	0.09	14	10	204.600	116.601	14	10	199.894	61.704
0.09	0.06	2	2	1	0.9	80	0.09	12	16	289.528	129.436	13	12	282.394	150.042
0.09	0.06	2	2	1	0.7	80	0.09	14	13	266.837	167.066	14	13	251.747	75.684
0.09	0.06	2	2	1	0.5	80	0.09	12	17	230.994	101.601	13	14	224.440	127.330
0.09	0.09	2	3	1	0.9	80	0.09	13	18	325.205	155.102	13	17	316.686	143.711
0.09	0.09	2	3	1	0.7	80	0.09	14	18	283.639	112.706	13	18	274.666	93.630
0.09	0.09	2	3	1	0.5	80	0.09	13	19	252.533	120.553	12	18	244.632	171.164
0.07	0.02	2	1	1	0.9	80	0.07	9	8	187.155	135.543	10	7	192.422	105.009
0.07	0.02	2	1	1	0.7	80	0.07	12	7	179.745	71.671	10	7	172.307	122.070
0.07	0.02	2	1	1	0.5	80	0.07	10	8	157.823	104.177	10	8	153.822	70.021
0.07	0.05	2	2	1	0.9	80	0.07	10	11	228.485	155.253	10	11	204.831	41.875
0.07	0.05	2	2	1	0.7	80	0.07	10	10	203.501	165.207	11	12	199.935	50.638
0.07	0.05	2	2	1	0.5	80	0.07	9	10	183.529	161.117	10	11	171.621	88.181
0.07	0.07	2	3	1	0.9	80	0.07	11	11	258.911	160.436	9	14	234.749	113.039
0.07	0.07	2	3	1	0.7	80	0.07	9	14	223.674	136.410	8	14	211.267	131.714
0.07	0.07	2	3	1	0.5	80	0.07	9	12	194.788	181.118	11	13	182.304	43.662
0.05	0.02	2	1	1	0.9	80	0.05	8	5	139.010	75.190	8	5	134.043	41.325
0.05	0.02	2	1	1	0.7	80	0.05	8	5	131.975	79.949	7	5	122.365	65.135
0.05	0.02	2	1	1	0.5	80	0.05	8	6	112.845	27.580	8	5	114.534	57.092
0.05	0.03	2	2	1	0.9	80	0.05	8	7	167.564	122.115	7	7	150.798	74.133
0.05	0.03	2	2	1	0.7	80	0.05	8	8	141.701	33.479	8	7	134.938	42.369
0.05	0.03	2	2	1	0.5	80	0.05	6	8	136.749	120.286	7	7	116.028	73.189
0.05	0.05	2	3	1	0.9	80	0.05	8	10	180.049	81.771	7	9	158.873	44.432
0.05	0.05	2	3	1	0.7	80	0.05	6	9	155.407	154.398	6	10	146.909	78.745
0.05	0.05	2	3	1	0.5	80	0.05	8	9	141.273	88.880	7	10	128.165	49.521
0.09	0.03	2	1	1	0.9	20	0.09	10	7	193.308	79.410	10	7	201.656	96.999
0.09	0.03	2	1	1	0.7	20	0.09	12	7	180.909	35.955	10	7	191.608	86.746
0.09	0.03	2	1	1	0.5	20	0.09	12	6	169.558	48.459	11	7	162.724	47.833
0.09	0.06	2	2	1	0.9	20	0.09	12	11	243.745	79.774	10	10	234.243	97.443
0.09	0.06	2	2	1	0.7	20	0.09	11	13	218.933	65.362	9	11	206.238	86.935
0.09	0.06	2	2	1	0.5	20	0.09	9	10	190.965	108.995	9	9	184.750	90.570
0.09	0.09	2	3	1	0.9	20	0.09	10	16	277.928	82.698	10	13	270.594	112.789
0.09	0.09	2	3	1	0.7	20	0.09	11	13	248.595	84.218	10	12	232.748	90.893
0.09	0.09	2	3	1	0.5	20	0.09	10	13	213.974	98.185	10	15	204.600	66.257
0.07	0.02	2	1	1	0.9	20	0.07	9	5	157.560	72.602	8	5	155.600	84.037
0.07	0.02	2	1	1	0.7	20	0.07	8	6	136.208	45.953	8	6	139.981	56.713
0.07	0.02	2	1	1	0.5	20	0.07	7	4	129.750	79.708	8	5	127.141	54.531
0.07	0.05	2	2	1	0.9	20	0.07	6	8	187.257	101.814	8	7	182.938	91.231
0.07	0.05	2	2	1	0.7	20	0.07	7	7	164.585	88.607	8	7	158.802	78.265
0.07	0.05	2	2	1	0.5	20	0.07	8	9	146.811	68.775	8	7	143.633	73.156
0.07	0.07	2	3	1	0.9	20	0.07	7	10	204.870	86.188	7	10	201.033	91.177
0.07	0.07	2	3	1	0.7	20	0.07	9	10	190.500	88.766	8	11	173.065	46.857
0.07	0.07	2	3	1	0.5	20	0.07	9	10	160.438	50.874	7	11	150.857	58.465
0.05	0.02	2	1	1	0.9	20	0.05	5	4	111.574	66.992	5	5	110.675	45.123
0.05	0.02	2	1	1	0.7	20	0.05	6	3	109.227	66.591	5	4	102.595	60.796
0.05	0.02	2	1	1	0.5	20	0.05	6	4	91.083	39.959	5	4	94.382	61.118
0.05	0.03	2	2	1	0.9	20	0.05	5	5	129.792	72.474	6	5	124.838	56.766
0.05	0.03	2	2	1	0.7	20	0.05	6	6	118.169	56.717	6	5	111.473	44.638
0.05	0.03	2	2	1	0.5	20	0.05	6	6	105.190	51.681	5	5	96.649	55.405
0.05	0.05	2	3	1	0.9	20	0.05	5	8	144.006	58.626	5	7	132.611	46.235
0.05	0.05	2	3	1	0.7	20	0.05	6	7	124.481	44.631	6	7	119.789	43.306
0.05	0.05	2	3	1	0.5	20	0.05	6	8	112.783	44.949	5	8	102.017	41.734