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FORECASTING US OUTPUT GROWTH WITH MIXED FREQUENCY DATA

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REPORT

Forecasting US Output Growth With Mixed Frequency Data[1]

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Abstract

This paper shows forecast results of the US output growth using the leading indicators. The output growth is sampled in quarterly data and the leading indicators are available in monthly frequency. Here I use a mixed frequency regression models to forecast the US output growth at forecast horizons up to one year. This approach is called MIDAS. Two kinds of data vintages are used in this research, the real-time and the end-of-sample vintages. My findings are that the forecast accuracy of MIDAS regression is significantly better than the autoregressive model and other competitors. In most cases, using the real-time vintage had better forecast performance.

keywords: MIDAS, leading indicator, US output growth, real-time vintage, end-of-sample vintage, RMSE.

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1   Introduction

The performance of leading indicators of the US output growth has been unreliable during the recession in 2001. The research by Stock and Watson (2003) of the performance of leading indicators around the 2001 recession concludes that indicators which might have been expected to perform well on the basis of past performance failed to do so and that some leading indicators performed above expectations. One solution as suggested in the paper of Clement and Galvaõ (2009) is to combine the forecasts from the individual-indicator models. For the period 1999Q1 to 2002Q3, Stock and Watson (2003) find a slight improvement of excess 5% on root mean square error (RMSE) from this strategy relative to an auto-regression in output growth.

Before I discuss about the forecasting of output growth, I explain what output growth is and what its frequency is. The output growth is a change in percentage of the gross domestic products (GDP). The macroeconomic variable GDP is a measurement of the total value of the produced goods and services in a period (in this research, we only consider quarterly GDP) in a country (in this case the US). The GDP does not have lower frequency than a quarter. This leads to models that forecast the US output growth prior to MIDAS, which will be further explained in this paper, that all variables on the right-hand-side (RHS) have also quarterly frequency. The RHS variables, which are the leading indicators, are often available higher frequency. Due to the quarterly data of output growth, one can not use higher frequency explanatory variables directly into the model. One can average each explanatory variable on every quarter such that the LHS and RHS have the same frequency. So for the first quarter of the year, you take the average of January, February and March. This average will be considered as Q1. The downside of this approach is that you lose information by averaging out the months into quarter and this can lead to worse forecast performance.

In this research, I will use the MIDAS (MIxed DAta Sampling) model used by Ghysels, Santa-Clara and Valkanov (2004). The MIDAS model is a solution to the above given problem. It allows the dependent variable and the explanatory variables to have different frequencies. For example, the output growth is quarterly and its explanatory variables are monthly. Using the MIDAS model has another advantage. That is, when the data of the leading indicator is released in the first month of the quarter, this information can be incorporated into the MIDAS model to have a more up-to-date information to forecast. This could be also the first and second months of the quarter, which will be done in this research. This approach will be called as Multiple-MIDAS-Current-Quarter (M-MIDAS-CQ). The MIDAS regression is a parsimonious way of allowing lags of explanatory variables, so that it is feasible to include multiple indicators in a single model. For my research, I use three variants of MIDAS as used in Clements and Galvaõ (2009). One is a multiple indicator model which is a referred as M-MIDAS and the other is the same as M-MIDAS with an additional auto-regressive term, called M-MIDAS-AR. The reason for including an auto-regressive term is that the AR model is competitive with the combined forecast of the single indicator model as said above. And the third is Multiple-MIDAS-Current-Quarter (M-MIDAS-CQ).

Different data vintages could effect the forecast performance. The conventional approach to real-time estimation and forecasting is the 'end-of-sample vintage data'. Koenig, Dolmas and Piger (2003) suggest to use 'real-time vintage data'. Like in the paper of Clements and Galvaõ (2009), I will also use these two data vintages. Using these two vintages, my interests for this research to replicate the paper of Clements and Galvaõ (2009) are the following: compare the forecast accuracy when using 'real-time vintage data' or the 'end-of-sample vintage data' for each model and compare the forecast accuracy with M-MIDAS model that uses current quarter information against the same model without current quarter information.

The forecast performance of MIDAS is tested against other models during the recession of 2001 (the 2001 recession is in my forecast sample). As an extension, the MIDAS model will be extended that each parameter and error term have different coefficients in different economic regimes which will be referred as Markov-Switching MIDAS. The reason for this is to see whether the Markov-Switching-

MIDAS benefits of the 2001 recession, given the different economic states. Eventually I used the Markov-Switching-Unrestricted-MIDAS (MS-U-MIDAS) as an extension for my thesis. This model will be compared with the ordinary Markov-Switching model (MS), where the LHS and RHS are in quarterly frequency. Further explanation of the MS-U-MIDAS will be given in the section Methodology.

The main purpose of this research is to test the forecast performance of the MIDAS model compared to the AR or ADL model. Hence the research question is:

Given that the GDP are quarterly and there are higher frequency data that can be incorporated as explanatory variable directly into the model, is the forecast performance of the MIDAS model better compared to the AR and ADL model, which do not incorporate the higher frequency data?

Knowing if the MIDAS approach is an effective tool to forecast time series, this model can be applied in other fields. A more accurate forecast can help business to gain more insight in the future. Business can therefore make better decisions for the future to remain competitive and to save cost. The MIDAS could be extended into a more complicated model and might result in better forecast accuracy. What I already have suggested is the Markov-Switching-Unrestricted-MIDAS (MS-U-MIDAS). And thus the second research question is:

Does the Markov-Switching-Unrestricted-MIDAS perform better forecasts of the 2001 recession than the other MIDAS models and the Markov-Switching model?

The plan of the rest of my paper is as follows. Section 2 describes the data that is used. Section 2.1 describes what leading indicators are used, from which vintages they are available and which data are also in real-time vintages. Section 2.2 describes what the end-of-sample and real-time vintages actually is. Section 3 begins how I measured the forecast accuracy. Section 3 is divided in seven subsections for each model that is used. These are the AR, basic MIDAS, Multiple Indicator MIDAS, Multiple Indicator MIDAS with an autoregressive term, Autoregressive Distributed Lag, Markov-Switching and the Markov Switching Unrestricted MIDAS model. Section 4 is divided into three subsections. The first subsection describes the difference between the real-time and end-of-sample vintages and also the differences between the M-MIDAS, M-MIDAS-AR and ADL models. The second subsection compares the M-MIDAS with use of current quarter information and the M-MIDAS without using current quarter information. The third subsection starts with reasoning why I incorporate regimes into the MIDAS model, then the comparison of the MS-U-MIDAS and the other MIDAS models. And at last the comparison of the MS-U-MIDAS and the Markov-Switching model. Section 5 concludes the paper and also discuss what improvement could be done.

2 Data

2.1 Leading indicators

This section explains what data will be used and from which sample will be taken. A leading indicator is a measurable economic factor that changes before the economy starts to follow a particular pattern or trend. Leading indicators are used to predict changes in the economy, but they are not always accurate. The source of the data is the Journal of Applied Econometrics Data Archive. Like in the paper of Clements and Galvaõ (2009) the ten included leading indicators are:

1. The term spread (10-year Federal Funds)

2. Stock price index (500 common stocks)

3. Average weekly hours in manufacturing

4. New claims for unemployment

5. Building permits

6. Vendor performance diffusion index

7. Orders - consumer goods and materials

8. Orders - non-defence capital goods

9. Consumer confidence index (Michigan)

10. Real money supply M2

And of course the dependent variable is the macroeconomic variable output growth. The sample of the leading indicator from the Conference Board's Composite Leading Indicator as used by Stock and Watson (2003) is between January 1959 and December 2003. These are available in monthly data. The output growth has the same sample but in quarterly frequency. Vintages of the output growth are from 1992Q2 to 2003Q1. The forecast sample is the period between 1992Q2 and 2003Q4, which incorporate the early 2000s recession. The following leading indicators are also available in real-time monthly vintage:

1. The term spread (10-year Federal Funds)

2. Stock price index (500 common stocks)

3. Average weekly hours in manufacturing

4. Building permits

5. Consumer confidence index (Michigan)

2.2 Real-time and end-of-sample vintages

Clements and Galvaõ (2009) suggested two approaches to use past data when estimating a regression model in real-time. Those are the real-time vintage data and the end-of-sample vintage data. Koenig *et al.* (2003) came up with the real-time vintage data, which will be referred as RTVint. It consist using only first-released data. Let y_t^{t+v} denote the US output growth from period t and vintage data $t + v$, where $v \geq 1$. Thus the first-released output growth in period t is when $v = 1$, that is y_t^{t+1}. For the AR model, the vector of first-released output growth or the LHS variable at time $t + 1$ is $[y_2^3, ..., y_{t-2}^{t-1}, y_{t-1}^t, y_t^{t+1}]'$. On the RHS the vector is $[y_1^2, ..., y_{t-3}^{t-2}, y_{t-2}^{t-1}, y_{t-1}^t]'$. Obviously the dimensions of these two vector are the same. But in the case of the MIDAS model, this works a bit differently. For the sake of exposition, suppose $K = 2$ (this K will be further explained in section Methodology) and consider a single monthly leading indicator, $x^{(3)}$, and forecast horizon of one quarter ahead, $h = 1$. Then the first-release data of the leading indicator is $x_t^{(3),t+1/3}$, that is one month after time t. The vectors on the RHS would be $[..., x_{t-2}^{(3),t-5/3}, x_{t-1}^{(3),t-2/3}]'$ and $[..., x_{t-7/3}^{(3),t-2}, x_{t-4/3}^{(3),t-1}]'$. For example, let t correspond to the fourth quarter of the year. Then we are regressing output growth in Q4 on the monthly indicators in September and August from the data vintage in October and September.

The traditional approach to regress a model with past data, is used with end-of-sample vintage data. This would be called as EndVint. At period t, the vintage data of the output growth for this period is at $t + 1$. So the vintage is the first month after the end of quarter t. For example, for $t = 1$ the vintage of this period is April. This sampling method however has it drawbacks as mentioned by Koenig *et al.* (2003). The data can be revised after it was first-released. When performing a regression model, this can lead to less accurate forecast.

3 Methodology

In this paper, I compare the forecast accuracy of different models. The models used for this research are the M-MIDAS, M-MIDAS-AR, M-MIDAS using current quarter information, Markov-Switching, Markov-Switching-Unrestricted-MIDAS and the Autoregressive Distributed Lag model. The benchmark is the AR model with one lag. The forecast accuracy is measured by means of root mean squared prediction error (RMSE). The formula is given below. All computations are done in Eviews 9.5.

$$RMSE = \sqrt{\frac{1}{k}\sum_{i=1}^{k}(y_{t+i} - \hat{y}_{t+i})^2}$$

Where k is the amount of observations in the forecast sample, y_{t+i} is the actual output growth and $\hat{y}_{t+i}$ the estimated output growth.

This section is divided as follows. First subsection describes the AR model with one lag. The three subsections thereafter describes about the MIDAS model and its variations. The next subsection describes the ADL model. And the last two subsections incorporate regimes as a forecast model. *Note: the following equations with numbering are the important ones.*

3.1 AR model

Suppose that the observation of the time series y_t depends on its p most recent lags. Then y_t is a linear function of $y_{t-1}, ..., y_{t-p}$. This can be shown as in Franses, van Dijk and Opschoor (2014):

$$y_t = c + \rho_1 y_{t-1} + \rho_2 y_{t-2} + ... + \rho_p y_{t-p} + \epsilon_t \tag{1}$$

where ϵ_t is unobservable. Since $p = 1$ in this research and in Clements and Galvaõ (2009), this formula becomes $y_t = \rho_1 y_{t-1} + \epsilon_t$.

3.2 Basic MIDAS

The MIDAS approach of Ghysels, Santa-Clara and Valkanov (2004) models the dependent variable with higher frequency explanatory variables that incorporate lags on a parsimonious way to prevent the proliferation of parameters. The basic MIDAS model for a single explanatory variable and h-step-ahead forecasting is given by

$$y_t = \beta_0 + \beta_1 B(L^{1/m}; \theta) x_{t-h}^{(m)} + \epsilon_t \tag{2}$$

where $B(L^{1/m}; \theta) = \sum_{j=1}^{K} b(j; \theta) L^{(j-1)/m}$, and $L^{s/m} x_{t-1}^{(m)} = x_{t-1-s/m}^{(m)}$. Here, t indexes the basic time unit (quarter) and m is the higher sampling frequency. Since the leading indicators are monthly, $m = 3$. In my research, I only consider $K = 12$. Clements and Galvaõ (2009) did for $K = 12$ and $K = 24$. Where I find that including 24 lags is too much, because including more lags adds computation time and including older observations is less relevant than more recent observations. The 'Exponential Almon Lag' of Ghysels *et al.* (2004) parametrizes $b(j; \theta)$ as

$$b(j; \theta) = \frac{exp(\theta_1 j + \theta_2 j^2)}{\sum_{j=1}^{K} exp(\theta_1 j + \theta_2 j^2)}$$

Unfortunately Eviews 9.5 has problems when I use this weight function for multiple leading indicators. An overflow occurs when I include more than two indicators into the MIDAS regression. The exact cause of the overflow is unknown. So I decided to use the 'Almon Lag Polynomial' as in the working paper of Pettenuzzo and Timmermann (2014). This gives:

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$$b(j;\theta) = \sum_{i=1}^{K} \theta_i j^i \tag{3}$$

This weight function did not cause any errors or overflow and worked for all ten leading indicators or the five real-time indicators. Due to this adjustment, the forecast results of my MIDAS models is not the same as in Clements and Galvaõ (2009).

3.3 Multiple Indicators MIDAS

The MIDAS allows to include multiple leading indicators without having a lot of extra parameters, this model is called M-MIDAS. The M-MIDAS is given by

$$y_t = \beta_0 + \sum_{i=1}^{n} \beta_{1i} B_i(L^{1/3};\theta_i) x_{i,t-h}^{(3)} + \epsilon_t \tag{4}$$

The amount of monthly indicators is n, the indicators are indexed by $i = 1, ..., n$ and h is the forecast horizon. When using the M-MIDAS, it is possible to use current quarter information to forecast. The model needs small adjustment, that is changing the forecast horizon from steps of quarterly to steps of monthly. Or in mathematical terms, say s is the forecast horizon in months, change h to $s/3$. This formula can be written as:

$$y_t = \beta_0 + \sum_{i=1}^{n} \beta_{1i} B_i(L^{1/3};\theta_i) x_{i,t-s/3}^{(3)} + \epsilon_t \tag{5}$$

Here, $s = 2$. So two out of three months of the current quarter will be included to generate forecasts in section 4.2. This is the M-MIDAS using current quarter information (M-MIDAS-CQ).

3.4 Multiple Indicator MIDAS with an Autoregressive term

The autoregressive Multiple Indicator MIDAS or M-MIDAS-AR is the M-MIDAS model with an autoregressive term. One can not simply add an autoregressive term at equation (4). This is not in general appropriate, as mentioned by Ghysels *et al.* (2006). The reason why becomes clear if you rewrite the model as

$$y_t = \beta_0(1-\lambda)^{-1} + \sum_{i=1}^{n} \beta_{1i}(1-\lambda L^h)^{-1} B_i(L^{1/3};\theta_i) x_{i,t-h}^{(3)} + \tilde{\epsilon}_t$$

where $\tilde{\epsilon}_t = (1-\lambda L)^{-1}\epsilon_t$. It is apparent that the polynomial on $x_{t-h}^{(3)}$ is the product of a polynomial in $L^{1/3}$, $B(L^{1/m};\theta)$, and a polynomial in L, $\sum \lambda^j L^j$, which generates a 'seasonal' response of y to $x^{(3)}$. Clements and Galvaõ (2009) include autoregressive lags in MIDAS models via a common factor restriction to avoid seasonality. The model is specified as

$$y_t = \beta_0 + \lambda y_{t-h} + \sum_{i=1}^{n} \beta_{1i} B_i(L^{1/3};\theta_i)(1-\lambda L^h) x_{i,t-h}^{(3)} + \epsilon_t \tag{6}$$

3.5 ADL model

A competitor against the MIDAS model is the Autoregressive Distributed Lag model (ADL), like in the paper of Clements and Galvaõ (2009). Consider ADL(q,p) model and define $A(L) = 1 - \alpha_1 L - \alpha_2 L^2 - ... - \alpha_q L^q$ and $B(L) = \beta_0 + \beta_1 L + \beta_2 L^2 + ... + \beta_p L^p$, where L is the lag operator. For example, $L^p y_t = y_{t-p}$. The ADL(q,p) becomes:

$$A(L)y_t = c + B(L)x_t + \epsilon_t \tag{7}$$

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I include the autoregressive term with lag one, or $q = 1$. p is set to $K/3$, where K is the total lags that is included in the weight function that is used in the MIDAS models. So p becomes four, since $K = 12$.

3.6 Markov-Switching model

For the sake of a simple example why to have switching regimes in your model, consider an AR(1) model with a constant:

$$y_t = c_1 + \rho y_{t-1} + \epsilon_t$$

which I assume that this model adequately describes the observed data for $t = 1, 2, ..., t_0$. Suppose after t_0, there was a significant change in the average level of the series. So after t_0, the proper model in this situation is:

$$y_t = c_2 + \rho y_{t-1} + \epsilon_t$$

Where $c_1 \neq c_2$. It is possible to combine these two equation into one equation. That is:

$$y_t = c(S_t) + \rho y_{t-1} + \epsilon_t$$

where S_t is a random variable. S_t in general can have $1, 2, ..., M$ states. In this example $M = 2$, let say that $S_t = 1$ for $t = 1, 2, ..., t_0$ and $S_t = 2$ for $t = t_0 + 1, t_0 + 2, ...$ This is a simple example when a model switches from regime 1 to regime 2 at time t_0. It is possible that the model switch back to regime 1, let say at time $t_1 > t_0$ and at $t_2 > t_1$ it switches back to regime 2 and so on. This way of switching regimes can go back and forth many times until the end of the estimation sample. It is possible to specify S_t as a realization of a two-state Markov chain with

$$Pr(S_t = j | S_{t-1} = i, S_{t-2} = k, ..., y_{t-1}, y_{t-2}, ...) = Pr(S_t = j | S_{t-1} = i) = p_{ij}$$

This is the transition probability. This probability can be calculated with maximum likelihood estimation. The numerical optimization method to calculate the MLE is the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm . In general, the Markov-Switching model is the following regression model for n regressors:

$$y_t = \beta_0(S_t) + \sum_{i=1}^{n} \beta_i(S_t) x_{i,t} + \epsilon(S_t) \tag{8}$$

where t is the period in time and S_t is one of the M regimes for $S_t = \{1, ..., M\}$.

3.7 Markov-Switching-Unrestricted-MIDAS

Before I start with the Markov-Switching-Unrestricted-MIDAS (MS-U-MIDAS) model. I explain first what the Unrestricted-MIDAS is. It is basically a MIDAS model with $b(j; \theta) = 1$. So there is parameter for each explanatory variable and for each lag. This way, the amount of parameters increases considerably. Hence also the reason why I include only one leading indicator for this model. Too many parameters in a model with a small amount of observations result in bad forecast performance. The procedure to pick only one indicator was the 'general-to-specific' approach. This works as follows: first you start estimating with the ten indicators, then you drop one indicator that is the least significant and you estimate the regression model again but with the nine remaining leading indicators. Keep doing this procedure until you have one leading indicator left. In my case, this was the term spread. The Unrestricted-MIDAS model looks like this with n explanatory variables:

$$y_t = \beta_0 + \sum_{i=1}^{n} \sum_{j=0}^{K-1} \beta_{i,j+1} x_{i,t-h-j/3}^{(3)} + \epsilon_t$$

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The MS-U-MIDAS model is essentially the same as the above equation, except that the parameters and the error term differ at each regime. This becomes:

$$y_t = \beta_0(S_t) + \sum_{i=1}^{n} \sum_{j=0}^{K-1} \beta_{i,j+1}(S_t) x^{(3)}_{i,t-h-j/3} + \epsilon_t(S_t) \tag{9}$$

Where $S_t = \{1, ..., M\}$ stands for the M states. It allows the parameters to change in each state. In this research, M is set to two to keep the model simple and easier to interpret. Adding more regimes into the model strongly increases computation time when estimating the model. The regime probabilities are calculated in the same manner as in the Markov-Switching model.

In Eviews 9.5 there was no built-in regression model to regress as the model above. Neither other computer software program like R, Matlab or Excel. Eviews has a built-in MIDAS regression and also has a built-in Markov-Switching regression. Since I could not use the combination of these two built-in regression models, I proposed to solve this problem as follows. We have the time series variable output growth, y_t, in quarterly frequency and the leading indicator in monthly frequency. Transform y_t into a three times larger vector, say $y_t^\star$. Each observation of y_t is in $y_t^\star$ three observations. In matrix notation, this looks like:

$$y_t = \begin{bmatrix} y_1 \\ y_2 \\ . \\ . \\ . \\ y_T \end{bmatrix} \implies \begin{bmatrix} y_1 \\ y_1 \\ y_1 \\ y_2 \\ y_2 \\ y_2 \\ . \\ . \\ . \\ y_T \\ y_T \\ y_T \end{bmatrix} = y_t^\star$$

Because of this transformation, the dimension of the time series variable output growth and the leading indicator are the same. This way I can use the built-in Markov-Switching regression in Eviews using $y_t^\star$ and the monthly indicators as an approach for MS-U-MIDAS.

4 Results

This section evaluates the forecast performance of the MIDAS regressions compared to the AR, ADL and Markov-Switching model using all ten indicators and five real-time indicators. Forecast accuracy is measured by means of root mean squared prediction error. This section is divided into three subsections. The first subsection is the comparison when using real-time vintage or end-of-sample vintage using M-MIDAS, M-MIDAS-AR and ADL models. The second subsection compares the ordinary M-MIDAS with the Multiple MIDAS model using current quarter information. The last subsection compares the Markov-Switching-Unrestricted-MIDAS model with the three MIDAS models and also compares with the ordinary Markov-Switching model.

4.1 Real-time vintage versus end-of-sample vintage

In this subsection, I discuss the difference in forecast performance using the real-time vintage (RTVint) and the end-of-sample vintage (EndVint). Table 1 shows the forecast performance of the models M-MIDAS, M-MIDAS-AR and ADL. The benchmark is the AR(1) model. The forecast accuracy is

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measured by the root mean squared prediction error (RMSE). The numbers represent the ratio of the RMSE of the model specified in the column header divided by the RMSE of the AR. For example, ratio greater than one means that the AR performed better than one of the other three models. Each model uses two different data vintages (RTVint and EndVint). Table 1(A) uses all the ten leading indicators and table 1(B) uses the five real-time leading indicators.

Table 1: performance of forecasting models with real-time vintages and end-of-sample vintages

Tabel 1(A)
10 indicators

	M-MIDAS		M-MIDAS-AR		ADL	
	RTVint	EndVint	RTVint	EndVint	RTVint	EndVint
h=1	0.769	0.781	0.773	0.790	1.023	1.010
h=2	0.860	0.864	0.904	0.884	1.219	1.029
h=4	0.995	1.032	1.034	1.081	1.272	1.065

Tabel 1(B)
5 indicators (full RT)

	M-MIDAS		M-MIDAS-AR		ADL	
	RTVint	EndVint	RTVint	EndVint	RTVint	EndVint
h=1	0.755	0.809	0.780	0.822	1.144	1.104
h=2	0.805	0.813	0.850	0.854	1.446	1.052
h=4	1.053	0.968	1.136	1.088	1.385	1.154

The forecast sample is between 1992Q2 and 2003Q4.

The numbers are ratios : $\frac{RMSE[X]}{RMSE[AR(1)]}$ where X = M-MIDAS, M-MIDAS-AR or ADL

RTVint is the real-time vintage

EndVint is the end-of-sample vintage

M-MIDAS: $y_t = \beta_0 + \sum_{i=1}^{n} \beta_{1i} B_i(L^{1/3}; \theta_i) x_{i,t-h}^{(3)} + \epsilon_t$

M-MIDAS-AR: $y_t = \beta_0 + \lambda y_{t-h} + \sum_{i=1}^{n} \beta_{1i} B_i(L^{1/3}; \theta_i)(1 - \lambda L^h) x_{i,t-h}^{(3)} + \epsilon_t$

where $B(L^{1/m}; \theta) = \sum_{j=1}^{K} b(j; \theta) L^{(j-1)/m}$ and $b(j; \theta) = \sum_{i=1}^{K} \theta_i j^i$

ADL: $A(L)y_t = c + B(L)x_t + \epsilon_t$

where $A(L) = 1 - \alpha_1 L - \alpha_2 L^2 - ... - \alpha_q L^q$ and $B(L) = \beta_0 + \beta_1 L + \beta_2 L^2 + ... + \beta_p L^p$

Looking at table 1(A), the M-MIDAS and M-MIDAS-AR both perform at least 13% better in terms of rmse than the AR model for the horizons one and two quarters ahead. They also perform better in both vintages (RTVint and EndVint). These ratios are superior compared to the results of Clements and Galvaõ (2009). The main reason of these differences is that I used different weight function in the MIDAS model as mentioned in the section Methodology. Both models perform almost equally with the AR model in the forecast horizon four quarters ahead. The ADL performs worse than the AR model for both vintages. The ADL seems to perform worse when using RTVint than EndVint.

Table 1(B) shows similar results as in table 1(A). Here, both MIDAS models outperform the AR model as well, especially in the shorter forecast horizons. The results of the ADL are worse than in table 1(A). And also the RTVint has worse performance than EndVint for ADL.

These results suggest that the both MIDAS models perform better than the ADL and AR. The M-MIDAS seems to perform better when using RTVint for both the ten leading indicators and the five real-time leading indicators. The M-MIDAS-AR performs about the same for both vintages. Overall when using five real-time indicators or all ten indicators, the performance are quite similar. Noticeably is that for both MIDAS models, the RTVint performs better than EndVint at short forecast horizon.

4.2 Using current quarter information

This subsection discusses the difference in forecast performance when using current quarter information and not using current quarter information. Here I also use the two vintages, the real-time vintage (RTVint) and the end-of-sample vintage (EndVint). Table 2 shows the forecast performance of the models M-MIDAS and M-MIDAS-CQ. M-MIDAS-CQ is the MIDAS model that uses current quarter information. I use two of the three months as current quarter information. The benchmark is the AR model as in previous subsection. The numbers in table 2 are also ratios like in table 1 and are computed in similar way. Table 2(A) uses all the ten leading indicators and table 2(B) uses the five real-time leading indicators. Table 3 shows the RMSE ratio of the M-MIDAS-CQ and M-MIDAS.

Table 2: Forecasting using current quarter information

Tabel 2(A) 10 indicators	M-MIDAS		M-MIDAS-CQ	
	RTVint	EndVint	RTVint	EndVint
h=1	0.769	0.781	0.791	0.840
h=2	0.860	0.864	0.771	0.784
h=4	0.995	1.032	0.961	1.135

Tabel 2(B) 5 indicators (full RT)	M-MIDAS		M-MIDAS-CQ	
	RTVint	EndVint	RTVint	EndVint
h=1	0.755	0.809	0.773	0.975
h=2	0.805	0.813	0.736	0.734
h=4	1.053	0.968	0.846	0.896

The forecast sample is between 1992Q2 and 2003Q4.
The numbers are ratios : $\frac{RMSE[X]}{RMSE[AR(1)]}$ where X = M-MIDAS, or M-MIDAS-CQ
RTVint is the real-time vintage
EndVint is the end-of-sample vintage
M-MIDAS: $y_t = \beta_0 + \sum_{i=1}^n \beta_{1i} B_i(L^{1/3}; \theta_i) x_{i,t-h}^{(3)} + \epsilon_t$
M-MIDAS-CQ: $y_t = \beta_0 + \sum_{i=1}^n \beta_{1i} B_i(L^{1/3}; \theta_i) x_{i,t-s/3}^{(3)} + \epsilon_t$
where $B(L^{1/m}; \theta) = \sum_{j=1}^K b(j; \theta) L^{(j-1)/m}$ and $b(j; \theta) = \sum_{i=1}^K \theta_i j^i$

Table 2(A) shows the results when using the 10 indicators. The result is that M-MIDAS-CQ performs better than the AR(1) except for forecast horizon four quarters ahead in EndVint. M-MIDAS-CQ performs better than M-MIDAS at forecast horizon two quarters ahead as showed in table 3. However M-MIDAS-CQ performs worse than M-MIDAS at forecast horizon one quarter ahead. In the paper of Clement and Galvaõ (2009), they do not have results of one quarter ahead forecast of this particular case. So I can not compare this results.

Table 2(B) has similar results as in table 2(A). Here the M-MIDAS-CQ also performs better than AR. But compared to the M-MIDAS, it actually performs better for forecast horizons two and four quarters ahead and for both vintages, especially four quarters ahead forecast horizon as showed in table 3.

These results show that M-MIDAS-CQ performs better than M-MIDAS at forecast horizon two and four quarters ahead like in Clements and Galvaõ (2009). But at one quarter ahead the M-MIDAS is superior to M-MIDAS-CQ. Like I already mentioned, I can not check if this last finding is correct.

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Table 3: M-MIDAS-CQ versus M-MIDAS

	10 indicators		5 indicators (full RT)	
	RTVint	EndVint	RTVint	EndVint
h=1	1.029	1.075	1.024	1.205
h=2	0.897	0.907	0.926	0.916
h=4	0.966	1.100	0.765	0.881

The forecast sample is between 1992Q2 and 2003Q4.

The numbers are ratios : $\frac{RMSE[M-MIDAS-CQ]}{RMSE[M-MIDAS]}$

RTVint is the real-time vintage

EndVint is the end-of-sample vintage

M-MIDAS: $y_t = \beta_0 + \sum_{i=1}^n \beta_{1i} B_i(L^{1/3}; \theta_i) x_{i,t-h}^{(3)} + \epsilon_t$

M-MIDAS-CQ: $y_t = \beta_0 + \sum_{i=1}^n \beta_{1i} B_i(L^{1/3}; \theta_i) x_{i,t-s/3}^{(3)} + \epsilon_t$

where $B(L^{1/m}; \theta) = \sum_{j=1}^K b(j; \theta) L^{(j-1)/m}$ and $b(j; \theta) = \sum_{i=1}^K \theta_i j^i$

4.3   Incorporating regimes into MIDAS

Figure 1 shows the US output growth between 1959Q2 and 2003Q4. You can see clearly that the output growth has been unstable. In the period mid seventies and eighties, the output growth has a more volatile movement. Meaning that the output growth was either larger or smaller than normal. After 1985 the output growth seems to stabilize around 0.8. What I suggested is to distinct the output growth series into two regimes. Each regime has its own parameter and disturbance error. This leads to the Markov-Switching model. I wanted to combine the Markov-Switching and MIDAS model. Eventually I came up with the Markov-Switching-Unrestricted-MIDAS model (MS-U-MIDAS) from the paper of Barsoum and Stankiewicz (2013). In this model, I used one leading indicator, because including multiple indicators causes too many parameters and this has a negative effect on the forecast performance due to overfitting. To solve this problem, 'general-to-specific' approach was used to incorporate only one leading indicator, that was the term spread.

This subsection is divided into two parts. The first part compares the MS-U-MIDAS to other MIDAS models. The second part compares the MS-U-MIDAS to the ordinary Markov-Switching model.
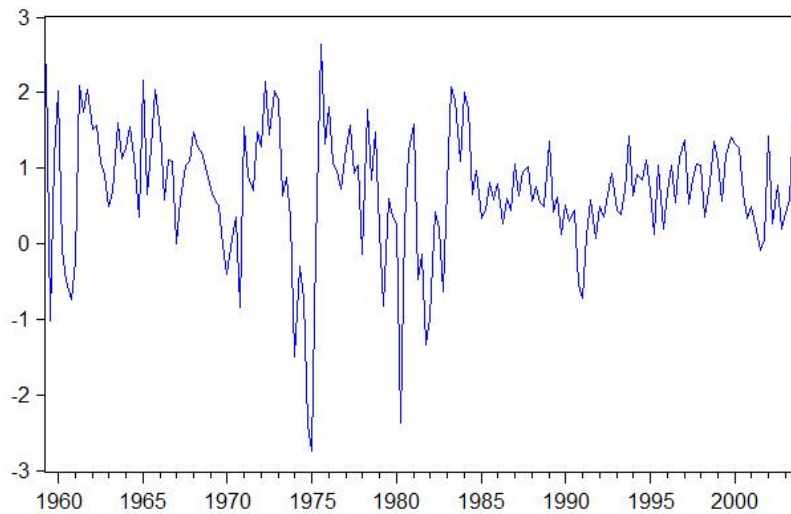

Figure 1: Output growth from 1959Q2 to 2003Q4

4.3.1 MS-U-MIDAS versus other MIDAS models

Table 4 compares the MS-U-MIDAS to the other MIDAS models. The numbers are ratios of the root mean squared prediction error. On the numerator the RMSE of the MS-U-MIDAS and on the denominator the RMSE of the model of the column header. The RMSE are calculated in both vintages (RTVint and EndVint). The three MIDAS models (M-MIDAS, M-MIDAS-AR and M-MIDAS-CQ) in table 4(A) use the ten leading indicators and five real-time leading indicators in table 4(B).

Looking at table 4(A) the MS-U-MIDAS performs at least 18% worse than the three MIDAS models at forecast horizon one quarter ahead for both vintages. At forecast horizon two quarters ahead, M-U-MIDAS performs slightly better than M-MIDAS and M-MIDAS-AR and it performs slightly worse than M-MIDAS-CQ. At forecast horizon four quarters ahead, the MS-U-MIDAS outperforms the three MIDAS models by at least 15% in terms of RMSE. For each forecast horizon, the two vintages does not change the conclusion about the performance of the MS-U-MIDAS.

Table 4(B) has similar results as table 4(A). Based on the results, it seems that at forecast horizon two quarters ahead, the MS-U-MIDAS performs worse than in table 4(A). Like in table 4(A), the MS-U-MIDAS here also outperforms the three MIDAS models at forecast horizon four quarters ahead.

Table 4: Forecast performance of MS-U-MIDAS against other MIDAS models

Table 4(A) 10 indicators						
	M-MIDAS		M-MIDAS-AR		M-MIDAS-CQ	
	RTVint	EndVint	RTVint	EndVint	RTVint	EndVint
h=1	1.268	1.269	1.260	1.255	1.232	1.181
h=2	0.985	0.985	0.937	0.962	1.099	1.085
h=4	0.820	0.770	0.789	0.735	0.849	0.700

Table 4(B) 5 indicators (full RT)						
	M-MIDAS		M-MIDAS-AR		M-MIDAS-CQ	
	RTVint	EndVint	RTVint	EndVint	RTVint	EndVint
h=1	1.290	1.225	1.250	1.207	1.261	1.017
h=2	1.066	1.061	0.996	0.996	1.151	1.158
h=4	0.738	0.782	0.718	0.730	0.964	0.887

The forecast sample is between 1992Q2 and 2003Q4.

The numbers are ratios : $\frac{RMSE[MS-U-MIDAS]}{RMSE[X]}$ where X = M-MIDAS, M-MIDAS-AR or M-MIDAS-CQ

RTVint is the real-time vintage

EndVint is the end-of-sample vintage

M-MIDAS: $y_t = \beta_0 + \sum_{i=1}^{n} \beta_{1i} B_i(L^{1/3}; \theta_i) x_{i,t-h}^{(3)} + \epsilon_t$

M-MIDAS-AR: $y_t = \beta_0 + \lambda y_{t-h} + \sum_{i=1}^{n} \beta_{1i} B_i(L^{1/3}; \theta_i)(1 - \lambda L^h) x_{i,t-h}^{(3)} + \epsilon_t$

M-MIDAS-CQ: $y_t = \beta_0 + \sum_{i=1}^{n} \beta_{1i} B_i(L^{1/3}; \theta_i) x_{i,t-s/3}^{(3)} + \epsilon_t$

where $B(L^{1/m}; \theta) = \sum_{j=1}^{K} b(j; \theta) L^{(j-1)/m}$ and $b(j; \theta) = \sum_{i=1}^{K} \theta_i j^i$

MS-U-MIDAS: $y_t = \beta_0(S_t) + \sum_{i=1}^{n} \sum_{j=0}^{K-1} \beta_{i,j+1}(S_t) x_{i,t-h-j/3}^{(3)} + \epsilon_t(S_t)$

Overall the MS-U-MIDAS performs worse at forecast horizon one quarter ahead than the three MIDAS models. At forecast horizon two quarters ahead, it performs about equally as the three models. And it is superior to the three models at forecast horizon four quarters ahead. Using the RTVint or the EndVint does not make a difference in conclusion.

4.3.2 MS-U-MIDAS versus Markov-Switching model

My finding of the Markov-Switching-Unrestricted-MIDAS is that it performs better at longer forecast horizons than the other three MIDAS models. The next step is to compare the MS-U-MIDAS to the Markov-Switching model to see if the Markov-Switching model actually gain forecast performance when using higher frequency variable. Both models use only one leading indicator, the term spread. And both models include lags up to one year.

The results of table 5 are ratios of the RMSE with AR as benchmark. Both real-time vintage and end-of-sample vintage were used to examine if there are any differences when using different vintages. From table 5 the Markov-Switching-Unrestricted-MIDAS performs better than the AR(1) for both vintages and each forecast horizon. Improvements of at least 13%. The Markov-Switching model forecast performance is slightly better compared to AR(1). At forecast horizon four quarters ahead, the MS model seems to be inconsistent, because it suddenly performs more than 40% worse than the AR model when using real-time vintage. The MS-U-MIDAS on the other hand seems to be more consistent. There are no ratios that changes dramatically. Like in previous subsection, the MS-U-MIDAS is superior at longer forecast horizon. It also outperforms the Markov-Switching model at any forecast horizon and for both vintages. The Markov-Switching model can gain forecast performance when using higher frequency explanatory variable based on table 5.

Table 5: Forecast performance of the two Markov models

	MS-U-MIDAS			MS	
	RTVint	EndVint		RTVint	EndVint
h=1	0.840	0.854		0.958	0.958
h=2	0.859	0.863		0.921	0.987
h=4	0.777	0.757		1.487	0.857

The forecast sample is between 1992Q2 and 2003Q4.

The numbers are ratios : $\frac{RMSE[X]}{RMSE[AR(1)]}$ where X = MS-U-MIDAS or MS

RTVint is the real-time vintage

EndVint is the end-of-sample vintage

MS-U-MIDAS: $y_t = \beta_0(S_t) + \sum_{i=1}^{n}\sum_{j=0}^{K-1} \beta_{i,j+1}(S_t)x_{i,t-h-j/3}^{(3)} + \epsilon_t(S_t)$

MS: $y_t = \beta_0(S_t) + \sum_{i=1}^{n} \beta_i(S_t)x_{i,t} + \epsilon(S_t)$

Now I analyze the regime probabilities for both Markov-Switching models. Figure 2 shows four graphs of the regime probabilities for the Markov-Switching-Unrestricted-MIDAS model. Figure 2a (regime 1) and figure 2b (regime 2) use the real-time vintage, figure 2c (regime 1) and figure 2d (regime 2) use the end-of-sample data. The gray shades are the periods of US recessions. The sample is between 1959Q2 and 1992Q1. Figure 3 has the same structure as figure 2, but these probabilities are estimated by the Markov-Switching model.

We can consider that regime 1 represents the recession period, because when a recession occurs the probability is high in regime 1. This holds for both vintages and for both models. In figure 2a the regime probability seems to capture all US recessions, but the probability is also high when there is no recession. This makes the regime probability unreliable if we want to know if we are in recession or not. The regime probability is not suitable as an indicator for a recession. This is used with real-time vintage data. If we use end-of-sample data, the regime probability over time is yet again unreliable, see figure 2c. The Markov-Switching model does not make a difference in terms of reliability of the regime probabilities. In figure 3a and figure 3c, the regime probabilities are also high when there is no recession. There are differences of the regime probabilities behavior over time between the MS-U-MIDAS and Markov-Switching model. Between 1985 and 1990 the probabilities in regime 1 were high for the MS-U-MIDAS model and low for the MS model, which is contradicting. Also the MS-U-MIDAS
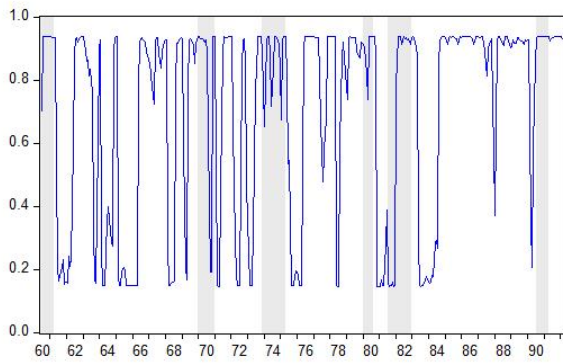
has more frequent 'up-and-down' movement than the MS model. This is caused of the higher frequency data in the MS-U-MIDAS. The regime probability seems to react more sensitively based on figure 2.
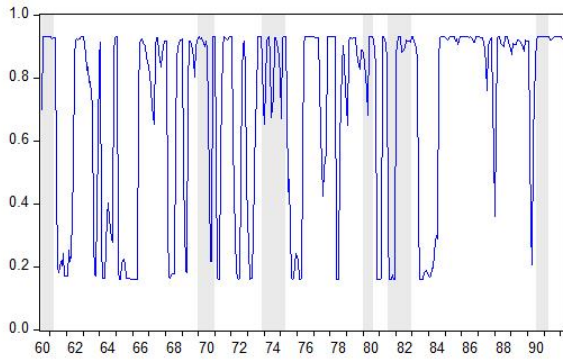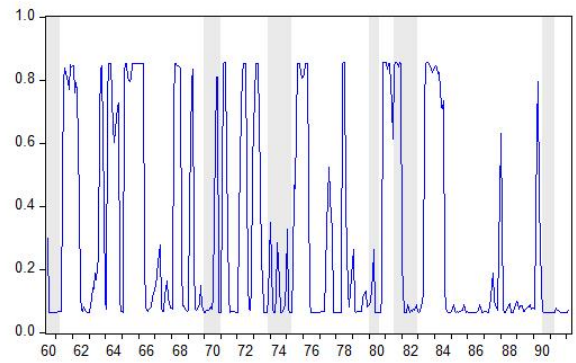

(a) regime 1 using real-time vintage
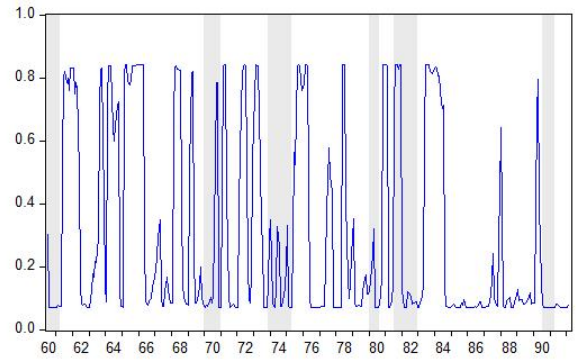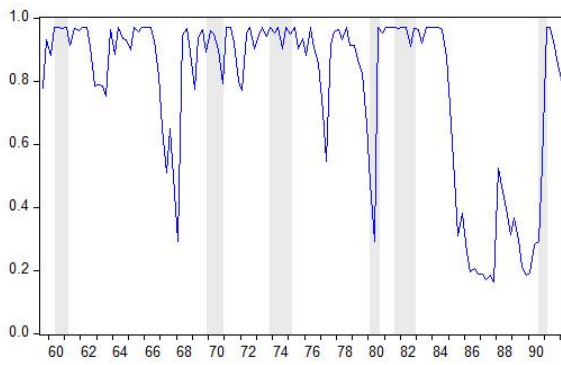
(b) regime 2 using real-time vintage
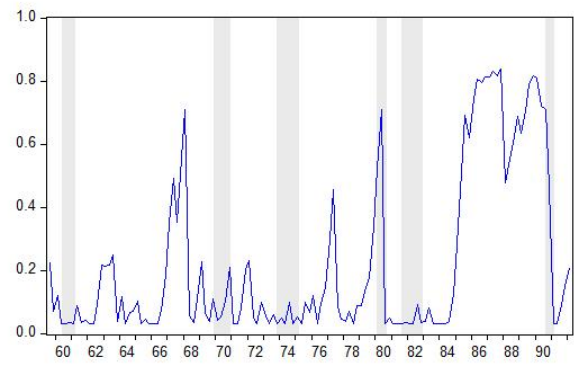
(c) regime 1 using end-of-sample vintage
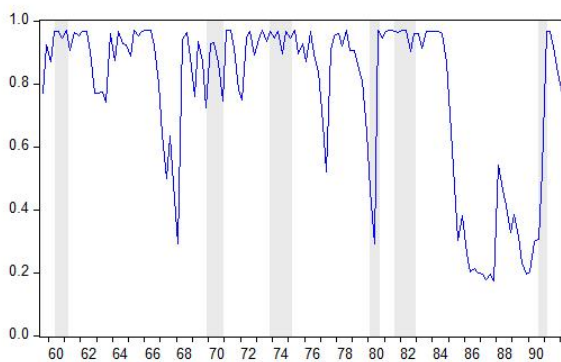
(d) regime 2 using end-of-sample vintage

Figure 2: Regime probabilities between 1959Q2 and 1992Q1 of the MS-U-MIDAS model
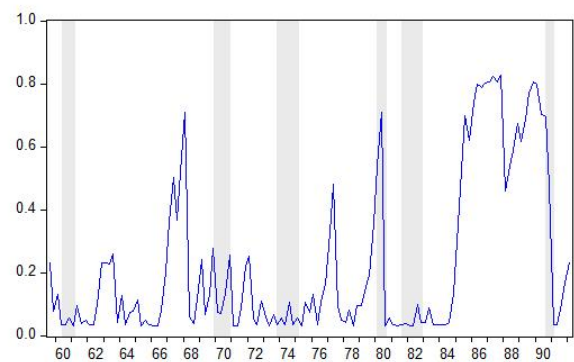
(a) regime 1 using real-time vintage

(b) regime 2 using real-time vintage

(c) regime 1 using end-of-sample vintage

(d) regime 2 using end-of-sample vintage

Figure 3: Regime probabilities between 1959Q2 and 1992Q1 of the Markov-Switching model

Table 6 represents the count table of the regime probabilities and the US recessions. This table is separated into two panels. Table 6(A) uses the regime probabilities of the MS-U-MIDAS and table 6(B) uses the regime probabilities of the MS. Each model regime probability is calculated with two different vintages, RTVint and EndVint. I consider regime 1 probability higher than 50% as an indication that a recession might occur. To add one count at cell 'yes and yes', the regime probability must be higher than 50% and a recession has to be occurred at that time. To add one count at cell 'yes and no', the regime probability must be higher than 50% and a economic growth has to be occurred at that time. Similar manner for counting other cells. The hit rate is the sum of the cells 'yes and yes' and 'no and no' divided by total counts. This percentage tells us how accurate the regime probability actually is as an indicator for recession and economic growth occurrence.

Looking at table 6(A), the hit rate is lower than %50 in both vintages. This already confirmed my interpretation of figure 2, that the regime probability is not suitable as an indicator for a recession. It tends to indicate very quickly that a recession occur even at times when there is economic growth. This can be seen from table 6(A) in both vintages. At some moments in time the regime probability failed to indicate a recession, but this number is rather small relatively to the numbers in other cells. There are not many differences between both vintages.

Table 6(B) has similar results as table 6(A). Likewise the hit rate is low, there are small differences between the two vintages. The regime probability also tends to indicate a recession quickly. Overall the regime probabilities of the MS is slightly worse than in MS-U-MIDAS based on the hit rate results. Despite the inaccurate regime probabilities of both models, the forecast performance does not suffer especially the MS-U-MIDAS model.

Table 6: Count table of the US recession with hit rate

Table 6(A) MS-U-MIDAS

		Real-Time					End-Of-Sample	
		Recession					Recession	
		yes	no				yes	no
Regime	yes	51	224		Regime	yes	48	227
Probability	no	4	103		Probability	no	7	100
	hit rate	0.40				hit rate	0.39	

Table 6(B) MS

		Real-Time					End-Of-Sample	
		Recession					Recession	
		yes	no				yes	no
Regime	yes	19	87		Regime	yes	19	86
Probability	no	1	24		Probability	no	1	25
	hit rate	0.33				hit rate	0.34	

Regime probability larger than 0.5 indicates that a recession occur. To count the 'yes and yes' cell one up, the regime probability indicate that a recession occur and an actual recession has occurred. Hit rate is calculated as the sum of 'yes and yes' and 'no and no' divided by the sum of total count.

MS-U-MIDAS: $y_t = \beta_0(S_t) + \sum_{i=1}^{n} \sum_{j=0}^{K-1} \beta_{i,j+1}(S_t) x_{i,t-h-j/3}^{(3)} + \epsilon_t(S_t)$

MS: $y_t = \beta_0(S_t) + \sum_{i=1}^{n} \beta_i(S_t) x_{i,t} + \epsilon(S_t)$

5 Conclusion and discussion

In other papers such as Ghysels *et al.* (2006) and Clements and Galvaõ (2009), monthly indicators were used to forecast the US output growth. I investigated the MIDAS approach to compare with other models, whether the use of higher frequency data can lead to better forecast performance. A typical method to forecast the output growth growth series is to use an autoregressive process. This model is the benchmark for my MIDAS models. In this paper, I used different sorts of MIDAS approaches. The first one is the Multiple Indicators MIDAS, second is Multiple Indicators MIDAS with an autoregressive term, third is the Multiple Indicators MIDAS using current quarter information and the last one is Markov-Switching-Unrestricted-MIDAS model. For each model, two different vintage data were used, RTVint and EndVint. Also for each model, forecast were made at different horizons. Forecast horizons of one, two and four quarters ahead. The estimation sample is from 1959Q2 to 1992Q1 and the forecast sample is from 1992Q2 to 2003Q4. Notice that the forecast sample include the 2001 recession.

My findings are that the all MIDAS models had better forecast performance than the autoregressive model. The M-MIDAS and M-MIDAS-CQ seem to perform better when using RTVint for both the ten leading indicators and the five real-time leading indicators. Comparing the M-MIDAS to the M-MIDAS-CQ, the latter performs better at longer forecast horizons, especially when using the five real-time leading indicators. At shorter horizon, the M-MIDAS-AR performed better when using RTVint in stead of EndVint. The MS-U-MIDAS performs worse at shorter forecast horizon than the three MIDAS models. And it is superior to the three models at longer forecast horizon. This holds for both vintages. Comparing this model to the Markov-Switching model, it clearly shows that the MS-U-MIDAS outperforms the Markov-Switching model at any forecast horizon and for both vintages. If we go back to the section Introduction, where I propose two research question. The answer to the first question is that overall using higher frequency variables can lead to better forecast performance

in terms of root mean squared prediction error. It is shown that the MIDAS approach is an effective method to use to forecast. This is partly due to the prevention of proliferation of the parameters, because of its weight function $B(L^{1/m}; \theta)$. As I experienced with the MS-U-MIDAS including ten indicators, there were significantly more parameters relatively to the other MIDAS models and it resulted in far worse RMSE. The answer to the second question is that the MS-U-MIDAS performs better than any models at longer forecast horizon for both vintages. And it is superior to the Markov-Switching model. At shorter horizon however, it is not performing well.

The MIDAS results are in most cases much better than the AR and ADL model. If we compare this results with Clement and Galvaõ (2009), my MIDAS results are superior to their paper. The main reason of the difference in results is that I used different weight function. Clements and Galvaõ (2009) and Ghysels *et al.* (2006) suggested to use the 'Exponential Almon Lag'. This function caused overflow error in Eviews, when I include more than two leading indicators. Since all my results use ten indicators or five real-time indicators, I could not use this particular weight function. Due to the overflow problem, I decided to use other weight function the 'Almon Lag Polynomial'.

For my Markov-Switching-Unrestricted-MIDAS approach, the output growth vector had been transformed as I already explained in the section Methodology. This transformation was needed to perform a Markov-Switching regression with higher frequency data. A combination of mixed frequency data and Markov-switching program is not available. The idea of the output growth vector transformation was my idea to improvise to use a Markov-Switching model with higher frequency regressor.

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