Opposite Effects of Meeting and Beating Competition

Wouter Hollenberg
401802

Abstract

Low Price Guarantees (LPG's) are adopted in a large variety of markets. Under the right conditions and form of LPG, this can lead to an increase in prices. This paper shows that automatically matching the competitor can lead to higher prices and loss of welfare. Beating a competitor by a specified amount or percentage will not be effective. When consumers incur hassle costs to claim a lower price, matching a competitor's price will be ineffective and competition enhancing effects are possible. However, beating a competitor by a fixed amount can lead to supra-competitive outcomes, while beating a competitor by a percentage of the difference will not result in higher prices.

Introduction

"Find a cheaper flight, vacation package, rental car, cruise or activity within 24 hours of booking and we'll refund the difference, plus give you a $50 travel coupon for future travel.", says Expedia, one of the world's largest websites for online bookings of hotels and flights. BestBuy claims: "*If you find a lower price online, in-store, or in print before you buy or within 30 days of your purchase we'll beat that price by 10% of the difference."*

Although previous literature, such as Salop (1982) and Belton (1986), has shown competition enhancing effects of low-price guarantees (LPG's), the main conclusion from economic literature is that LPG's raise equilibrium prices above standard Nash-Bertrand equilibrium. In various settings it has been proven that Low Price Guarantees will have a negative effect on social welfare and consumer surplus in particular. LPG's are defined as the guarantee that a firm has the lowest price available at that moment. An important distinction needs to be made between two different kinds of LPG's; Price Matchting Guarantees (PMG's) and Price Beating Guarantees (PBG's). A PMG is a commitment to meet the competitor's price. A PBG is the commitment to beat the price of any competitor by a specified amount, which is usually a percentage of the difference or a fixed amount. The focus in existing literature is on PMG's. For both kinds of LPG's equilibria with prices larger than Nash-Bertrand have been supported in equilibrium. However, no paper has shown conditions under which firms prefer a PBG over a PMG. The most traditional view regarding PMG's is from Belton (1986), which states that in equilibrium, low price guarantees will lead to collusive outcomes, as the incentive to undercut the competitor's price will be reduced when this competitor applies an LPG. In Bertrand competition, sequential undercutting will lead to prices equal to marginal costs, as firms will obtain full market demand by undercutting the competitor(s) by the smallest possible amount. However, when LPG's are adopted, undercutting the competitor will not necessarily lead to gains in profit, as the price will automatically be matched. As follows, in equilibrium, firms will adopt a low price guarantee and set their prices at collusive level.

Several factors may reduce this effect, such as hassle costs. Hassle costs is defined as the loss in utility from claiming the lower price. For example, loss in utility occurs when consumers have to argue with the salesman or manager to claim the lower price or when consumers need to obtain proof of a lower price another firm has set. When hassle costs are involved, a consumer's utility will be lower when it buys from the higher priced firm with low price guarantee. Therefore, the commitment to meet the price of a competitor will be less effective (Arbatskaya et al, 2006).

Even though there are a lot of similarities between these two commitments, the two different forms of LPG's have very different effects. The conclusion from existing literature, such as Salop (1982) and Belton (1986), state that, in absence of hassle costs, PMG's will lead to supra-competitive prices. Empirical evidence is found by Hess and Gerstner (1991). In the presence of hassle costs PMG's are less likely to facilitate tacit collusion. When a firm undercuts its competitor by a smaller amount than the hassle costs, the PMG will be ineffective (arbatskaya et al, 2006), which will also be shown by this paper. However, the conclusions concerning PBG's are different. Chen (1995) shows that outcomes with prices higher than Nash-Bertrand equilibrium are supported in equilibrium. However, this paper concludes that when the percentage by which the firms beat its competitor becomes smaller, the closer the equilibrium prices get to the Bertrand outcome. The paper shows the equilibrium prices for all values by which a firm can beat its competitor. However, this paper will show that when the percentage by which a firm beats its competitor is not considered as given, Bertrand outcome is the only Nash equilibrium and beating a competitor by a fixed amount is preferred over a strategy in which competitors are beaten by a percentage of the difference. Furthermore, this paper will show that the influence of hassle costs is different for PMG's and PBG's. While hassle costs will lower equilibrium prices when firms apply a PMG, the conclusion of this study is that the effect is exactly the opposite when PBG's are applied. For a PBG to be effective, firms must beat its competitor by an amount larger than the hassle costs, which does not always lead to higher profits. This paper provides conditions under which firms prefer a PBG over a PMG strategy. The effects of PMG's and PBG's will be analyzed in the presence and absence of hassle costs. This paper will show that when hassle costs are not at stake, firms prefer a Price Matching Guarantee and when hassle costs are at stake, firms will choose a PBG strategie. I also find that only a fixed compensation will reduce competition in presence of hassle costs. A proportional compensation does not, because a rival's collusive price can be undercut by such a small amount that consumers are not compensated for their hassle costs by proportional compensation of the rival firm.

Model

Consider a market with two firms. All Bertrand competition assumptions are applied and also;

1) $C_i = c_i q_i \qquad\qquad c_i = c_j \text{"}\ i\ ^1\ j$
2) $D(p) > 0 \forall p \in [0, \infty)$ and $D'(p) < 0$
3) $\exists p^m$ s.t. $\pi'(p) > 0 \forall p < p^m$ and $\pi'(p) < 0 \forall p > p^m$.

Assumption 1 implies equal and constant marginal costs for both firms. C denotes the total costs, c denotes the marginal costs and q is the quantity sold by a certain firm. Demand is positive and decreasing in p, according to assumption 2. Also, as assumption 3 implies, there exists a unique monopoly price for which industry profits are maximized. For all $p \neq p^m$, a value of p closer to p^m will lead to higher (industry) profits. Furthermore, all consumers have complete information about prices of both firms. The products sold by the firms are homogeneous, as LPG's typically apply to goods which are similar. However, one may argue that differentiation can also be defined as differences in service, location or horizontal differentiation. Therefore, a separate subsection in this paper will discuss to what extent the results of this paper will hold in a setting with heterogeneous goods. It is also assumed that firms will not act predatory. Firms will have 3 options; adopt a PMG, a PBG or choose to set no low price guarantee. When no price guarantee is adopted, prices will be set as in Nash-Bertrand equilibrium. In the situations where a low price guarantee is applied, firms have a posted price, which is the price they initially set and an effective price, which is the price consumers will actually pay. Also, when a PBG is adopted, firms have the option to beat its competitor by a percentage of the difference in posted price or by a fixed amount. In case a competitor is beaten by a percentage of the difference, the actual price of firm *i* will be $P_i = p_j - \gamma_i (p_i - p_j)$ iff $p_i \geq p_j$ with P as the actual price, while p denotes the posted price. For PMG's, $\gamma_i = 0$, as $P_i = p_j$. When a firm beats its competitor by a fixed amount $P_i = p_j - \lambda_i$, with $\lambda_i > 0$ iff $p_i > p_j$. The decision to use an LPG is usually part of the pricing strategy and therefore it is likely that firms will change their price in combination with the LPG, if an LPG is adopted. The firms will simultaneously choose a posted price and a LPG strategy. For PMG's, the level of $\gamma_i, \lambda_i = 0$ is given and equilibria can be found by finding a set of prices directly. To find such equilibria for PBG's, the optimal amount to beat the competitor will first be derived. Given the price of the competitor, there exists a posted price $P_i(p_i, \gamma_i) \in [0, p_j]$ and a $P_i(p_i, \lambda_i) \in [0, p_j]$, such that the level of PBG is set optimally.

Analysis

This section will provide an analysis of the model. This will be done in three different settings. In the first setting, there will be no hassle costs and the goods sold by the firms are homogeneous. In the second setting the firms will also sell homogeneous, while there are hassle costs for claiming a LPG. The last subsection it will be shown to what extent the results of the first two sections will hold in a market with heterogeneous goods.

Homogeneous goods in absence of hassle costs

This subsection provides an analysis of the model without any further restrictions. The equilibrium prices in PMG's and PBG's will be derived. As Bertrand competition implies and products are considered homogeneous, without low price guarantees, the price will be equal to marginal cost.

Theorem 1: When both firms adopt a PMG, the equilibria are all sets of prices for which $p_i = p_j \in [c, p^m]$ *or* $p_i = p^m, p_j \in (p^m, \infty)$.[1]

In Bertrand competition, sequential undercutting will lead to prices equal to marginal cost. However, prices will automatically be matched, due to the PMG, if one decides to undercut its competitor's price. Therefore, reduction in price will not lead to a larger market share, while industry profits will be diluted when prices are below monopoly level. Thus, for all prices between marginal costs and monopoly price, firms do not have an incentive to undercut the competitor's price. If one firm has a higher posted price, the other firm will raise its posted price and set it at the same level. If one firm has a posted price larger than the monopoly price and the other has its posted price at monopoly level, both firms have an actual price which is equal to the monopoly price. Therefore, neither firm has an incentive to set a different price, as industry profits are optimal. The most likely equilibrium outcome is that both firms set the monopoly price, as setting this price can only lead to higher profits and the monopoly price is always an element of the set of equilibrium prices. The results cannot be generalized to the case where

[1] Proof is provided in appendix 1

only one firm adopts a PMG and the other firm does not set a low price guarantee at all. The firm with the PMG can always undercut its competitor and obtain full market demand. As a best response, the competitor will set an equal price. For this new set of prices it is again optimal for the firm with the PMG to undercut the other firm. This sequence will go on, until $p_i = p_j = c$.

Lemma 1: When both firms adopt a PBG, it is always optimal for firm i to undercut the competitor by the smallest amount possible when $c < p_j \leq p^m$. *When* $p_j > p^m$, *optimally* $P_i = p^m$ *and* $\gamma_i = \frac{p_i - p^m}{p_i - p_j}$ *or* $\lambda_i = p_j - p^m$ [2].

The intuition behind lemma 1 is straightforward. The PBG will only be activated if a firm has a higher posted price. As this will automatically lead to a lower actual price for all $\gamma, \lambda \in \Re^+$. By Bertrand assumptions it is always optimal to undercut the competitor by the smallest possible amount if and only if the competitor's price is equal to or lower than the monopoly price, but higher than the marginal costs. If the competitor's price is larger than the monopoly price, it is best to set the effective price at monopoly level. When a firm beats its competitor by a fixed amount $\lambda_i = p_j - p^m$ will lead to $P_i = p^m$. In case a PBG contains a promise to beat the competitor by a percentage of the difference, $P_i = p^m$ if $\gamma_i = \frac{p_i - p^m}{p_i - p_j}$. According to lemma 1, for every $p_j > c$ there is a response for which firm *i* can successfully capture the full market and make a profit.

Theorem 2: If both firms adopt a PBG, there exists no supra-competitive equilibrium in PBG strategies and the only Nash-equilibrium is $p_i = p_j = c$.[3]

If one firm sets a posted price above marginal costs, the other firm can profitably set a higher posted price, which results in a lower actual price. With $p_i > c$, the best response of firm *j* is to set $p_j > p_i$ s.t. $\lim P_i = p_j^- \forall p_i \leq p^m$ and $p_j = p^m \forall p_i > p^m$, which is possible according to lemma 1. This results in $\pi_i = 0$. Consequently, there is a level of $p_i > p_j$ and $\gamma_i, \lambda_i > 0$ for which profits are higher than setting a price equal to the price of the competitor, so firms will keep raising posted prices and set γ_i or l_i such that $P_i = \min(p^m, p_j)$. By induction it follows that firms always have an incentive to raise its price when the price of the other firm is above marginal costs. Thus, the only Nash equilibrium is $p_i = p_j = c$, in which case neither of the firms has an incentive to set a higher posted price, as this will result in an actual price below marginal costs leading to a negative profit margin. Theorem 2 has shown that the Nash-Bertrand outcome is the only equilibrium when both firms adopt a PBG. The results can be generalized to a situation in which only one firm adopts a PBG. Consider the situation in which firm *i* sets a PBG and firm *j* does not. If firm *j* sets a price above marginal costs, firm *i* can set either $c < p_i < p_j$ or $p_i > p_j$ s.t. $c < P_i < p_j$, which is possible for all $p_j > c$ as shown in lemma 1. In this case, firm *i* will capture full market demand. Therefore, firm *j* has no incentive to raise prices.

[2] Proof is provided in appendix 2
[3] Proof is provided in appendix 3

To conclude, PBG's cannot be used to raise equilibrium prices in a standard Bertrand setting and the competitive outcome $p_i = p_j = c$ will prevail.

Homogeneous goods in presence of hassle costs

Theorem 1 and 2 have shown that the Nash-Bertrand outcome is the only equilibrium in PBG strategies with the standard Bertrand assumptions. However, in reality PBG's are adopted. When hassle costs are at stake, PBG's are supported in equilibrium, as theorem 3 and 4 will show. all the assumptions of setting 1 will apply in this setting. In addition:

4) $U_k = R_k - \min(P_i + h, P_j + h, p_i, p_j)$

Assumption 4 implies that consumers will only make use of the price beating guarantee when the difference between the actual price, the competitor's price and the posted price is larger than the hassle costs. h denotes the hassle costs from claiming the lower price and R_k denotes the reservation price of a certain consumer. For simplicity, there is a linear relationship between benefit in monetary terms and utility. The results in this subsection can be generalized to a setting with non-linear utility functions. The hassle costs are equal for all consumers.

Theorem 3: When both firms adopt a PMG, the only Nash-equilibrium is $p_i = p_j = c$.[4]

Without hassle costs, the Bertrand mechanism was affected by the PMG. Undercutting would not lead to a higher market share and was therefore not profitable. However, due to the hassle costs, consumers would still not buy from a firm with a higher posted price, even though it will match its competitor's price. The actual price of the firms are equal, but the hassle costs induce an additional loss in utility on top of the price of the product. As a result, the PMG will not be activated by any consumer. When PMG's are never used by consumers, the Bertrand mechanism is unaffected. Firms can obtain full market demand by undercutting its competitor and sequential undercutting will lead to the only Nash equilibrium in Bertrand competition, with prices equal to marginal costs. These results also apply to the situation in which only one firm adopts a PMG. The PMG is still ineffective as the competitor will be able to successfully undercut the firm setting the PMG. Furthermore, the firm setting the PMG has an incentive to cut prices as well, as it will also result in obtaining full market demand.

Lemma 2: When both firms adopt a PBG, the optimal amount to undercut the competitor is

$\lim h^+ \forall p_j \leq p^m + h$ *and, which is the case when* $\lambda_i = \lim h^+$ *or* $\lim \gamma_i = \dfrac{h}{p_i - p_j}^{+}$. $\forall p_j > p^m$,

optimally $p_i > p_j$ *s.t.* $P_i = p^m$, which is the case when $\gamma_i = \dfrac{p_i - p^m}{p_i - p_j}$ *or* $\lambda_i = p_j - p^m$.[5]

When firm *j* sets a price which is larger than the monopoly price, firm *i* can always set an actual price equal to the monopoly price, as shown in lemma 1. However, consumers will only buy at this monopoly price, when the posted price of firm *j* is larger than $p^m + h$. Therefore, it is only effective to set $p_i > p_j$

[4] Proof is provided in appendix 4
[5] Proof is provided in appendix 5

s.t. $P_i = p^m$ when $p_j > p^m + h$. When $p_j \in (p^m, p^m + h)$, it is still possible for firm *i* to set the actual price at monopoly level, but the utility of consumers will still be lower if these consumers buy from firm *i,* due to the hassle of getting their refund. Thus, for all $p_j < p^m + h$ firm *i* must set a price below the monopoly price to successfully undercut the posted price of its competitor. As the difference must always be larger than the hassle costs, optimal undercutting is setting an actual price slightly below $p_j - h$. The two forms of PBG's have an important distinctive property, which is key in theorem 4. When a competitor is beaten by a fixed amount, the competitor can always set its posted price, such that $P_i \in [0, p_j - \lambda_i]$ for all sets of p_i, p_j. Beating competitors by a percentage of the difference in price, the competitor can always set its posted price, such that $P_i \in [0, p_j]$.

Theorem 4: If both firms adopt a PBG, $p_i = p_j = c$ *and all values* $p = p_i = p_j$ *on the interval* $p \in [c + h, p^*]$ *are Nash-equilibria, with* $p^* = p_i$ *s.t.* $\frac{\pi_i(p_i = p_j)}{2} = \pi(p_i = p_j - h)$ *iff* $P_i = p_j - \lambda_i \forall p_i > p_j$. *When* $P_i = p_j - \gamma_i(p_i - p_j) \forall p_i \geq p_j$ *the only equilibrium is* $p_i = p_j = c$.[6]

As shown in lemma 2, firm *i* can successfully capture full market demand and set the monopoly price as actual price when $p_j > p^m + h$. Therefore, prices above this value cannot be supported in equilibrium, as firms can always set the monopoly price, given the price of the other firm. Lemma 2 has shown that on the interval $p_j \in (c, p^m + h]$ firm *i* can capture full market demand by setting its actual price $\lim P_i = p_j - h^-$. This leads to positive profits as long as the effective price is larger than the marginal cost. Thus, when $p_j < c + h$ firm *i* can undercut firm *j,* but automatic undercutting, such that firm *i* obtains market demand, will lead to a negative profit margin. However, it can set $p_i < p_j$ and firm *j* will not be able to set its PBG in a way that it automatically captures the market and makes a profit. As a result, sequential undercutting will lead to prices equal to marginal costs. Setting an effective price lower than the price of the competitor is only effective when the effective price also compensates for the hassle costs. When $p_j = c + h$, undercutting will lead to zero profits and setting an equal price is always more profitable. The profit of undercutting gets larger when the price increases and when $p_j = p^m + h$, firm *i* can set the monopoly price and obtain full market demand. As the profit function is strictly increasing in price and continuous, there exists a unique value p^* such that the profits from undercutting are equal to the profits of setting the same price. As long as the price does not exceed this threshold value, firms do not have an incentive to undercut its competitor. Therefore, all values of $p_i = p_j \in (c + h, p^*]$ are equilibria. However, this only applies when the PBG contains a fixed amount by which a competitor will be beaten. When the effective price is a percentage of the difference in price, firm *j* can set its posted price such that $P_i \in [0, p_i]$ for every combination of p_i and γ_i. As a result, it is always possible to undercut the competitor by such a small amount s.t. $P_i + h > p_i > p_j$ and firm *j* captures full market demand. More formally:

[6] Proof is provided in appendix 6

$$\exists p_j \in (c, p_i)\forall p_i, \gamma_i \text{ s.t. } p_i < P_i + h$$

Therefore, undercutting and obtaining full market demand is always possible, by setting a posted price just below the posted price of the competitor, when PBG's are defined as a percentage of the difference in price. When a firm beats its competitor by a fixed amount, setting a lower posted price will always result in the other firm capturing full market demand when $\lim \lambda = h^+$ and the equilibria in theorem 4 will hold.

Heterogeneity

Even though LPG's apply to similar products with similar specifications, one might state that heterogeneity is still at stake in the form of locational differences, differences in service or horizontal differentiation. This subsection will show to what extent the results of this paper can be generalized to a setting with heterogeneous products. To do so, the following assumptions are added to the model:

5) $D_i = a - \min(p_i, P_i + h) + \beta(p_j, P_j + h) \qquad \beta \in (0,1)$

As a direct implication of assumption 5, an equal increase in $\min(P_i, P_i + h)$ and $\min(P_j, P_j + h)$ will lead to an increase in profits for both firms on the interval $\min(P, P+h) \in [0, P^m)$. Also, assumptions (1)-(3) are applied in this subsection. Assumption 4 no longer applies, as the hassle costs are accounted for in the demand function. Due to the heterogeneity, the Nash-Bertrand equilibrium is no longer equal to marginal costs. The equilibrium price in absence of LPG's will be denoted as p^{NB}.

Theorem 5: When firms adopt a PMG in absence of hassle costs, $p_i = p_j \in [c, P^m)$ *are equilibrium values*[7]. $p_i = p_j = p^{NB}$ *when firms adopt a PMG in the presence of hassle costs*[8].

Similar to theorem 1, in this setting, PMG's can lead to supra-competitive outcomes in the absence of hassle costs, but can also have competition enhancing effects. Assumption 5 implies that an equal increase in both prices leads to larger profits for all prices between Bertrand and monopoly prices. It is not possible to undercut the competitor, due to the PMG, which means that any attempt of undercutting will automatically lead to lower prices for the competitor as well. A decrease in both prices will lead to lower profits for both firms. Thus, there is no incentive to cut prices. Raising prices is only effective when a firms' posted price is lower than the posted price of its competitor. This will lead to an increase in the effective price of both firms, which results in higher profits for all prices below monopoly price. As firms will not sell any products when prices are below marginal costs, all prices between the marginal costs and the monopoly price are equilibria. In a setting with differentiated products, the equilibrium value without LPG's is above marginal costs. Therefore, all equilibria in which $p_i = p_j \in [c, P^{NB})$ are actually pro-competitive. In the presence of hassle costs, the outcome is no different than theorem 3 and the only equilibrium is the Nash-Bertrand equilibrium. The intuition behind this result is exactly the same. Increases and decreases in price will have the same effect as when PMG's are not applied, as long as $p_i \in [p_j - h, p_j + h)$. When prices are larger (smaller) than in Bertrand setting, a small decrease (increase) in price will lead to a larger profit, given the price of the other firm.

---

[7] Proof is provided in appendix 7
[8] Proof is provided in appendix 8

Theorem 6: When firms adopt a PBG strategy in absence of hassle costs $p_i = p_j \in [c, p^{NB}]$ *are equilibria*[9]. $p_i = p_j \in [c, p^*]$ *when firms adopt a PBG in the presence of hassle costs*[10].

The equilibria are no different when firms adopt a PBG in absence of hassle costs. When firm *i* sets a price above Nash-Bertrand level, the optimal response of the firm *j* is to set its effective price below the posted price of the firm *i*. The highest effective price a firm can set is the posted price of the other firm. The best response of firm *i* is to set its price equal to the price of firm *j,* as the profits are increasing in effective price, for all prices of firm *j* below Nash-Bertrand level. When the maximum price is below marginal costs, firms will not sell anything and these prices are no equilibrium values. In case a PBG is adopted in the presence of hassle costs, there also exists a threshold value for the price, larger than p^{NB}, such that neither firm has an incentive to change its price. Similar to the situation with homogeneous goods, in this case a PBG as a percentage of the price of the competitor will not lead to supra-competitive. The intuition is exactly the same as in theorem 4. Even though the effective price with PBG must be lower than the posted price of the competitor, prices below Nash-Bertrand are no equilibrium values. It is still possible to set a posted price higher than the posted price of the competitor and set the PBG such that $P_i + h > p_i$. As a result, consumers will not use the option to claim the lower price.

Concluding remarks

This paper shows that LPG's can lead to supra-competitive outcomes under the right conditions. Also, it shows that for every situation, an LPG strategy can be adopted in which prices can rise above Nash-Bertrand equilibrium. When there are no hassle costs involved, a PMG is most likely to raise equilibrium prices. However, when goods are differentiated this may also lead to a reduction in profits as firms are unable to play their best response, due to their PMG when prices are below Nash- Bertrand equilibrium. A PBG strategy will not lead to higher profits. In fact, with heterogeneous goods it can only reduce profits in the industry, as firms cannot set a higher price, due to their PBG. The situation in presence of hassle costs is the opposite. A PMG cannot lead to higher profits, as firms can undercut their competitor by setting a lower posted price and the competitor cannot effectively meet this price. Consumers need to be compensated for the hassle costs and a PBG is necessary to make up for the loss in utility due to the hassle of claiming a lower price. Therefore, a PBG can be effective when hassle costs are involved. In this case it also applies that equilibria below Nash-Bertrand can hold when firms are differentiated, as firms are unable to set a higher price when prices are below Nash-Bertrand equilibrium. Also, this paper shows that beating a competitor by a fixed amount, weakly dominates a strategy in which a competitor is beaten by a percentage of the difference. When a competitor sets a PBG by beating others by a percentage of the difference in the presence of hassle costs, it is able to undercut this competitor by such a small amount, that it will not compensate its customers for the hassle to claim the lower price.

[9] Proof is provided in appendix 9

[10] Proof is provided in appendix 10

Appendix

Appendix 1

$\gamma, \lambda = 0 \Rightarrow P_i = \min(\mathrm{p_i}, p_j)$. Symmetry implies that also $P_j = \min(p_j, p_i)$. As follows, $P_i = P_j = \min(p_i, p_j)$. By Bertrand assumptions, $s_i = s_j = \frac{1}{2} \forall P_i, P_j$, where s denotes the market share. $\pi' \geq 0 \forall p \in [0, p^m]$ and $P_i \in [0, p_j]$. Thus, $\max \pi_i(P_i) = \pi_i(P_i = p_j) \forall p_j \leq p^m$.
$p_i \geq p_j \Rightarrow P_i = p_j$. Symmetry implies that $\max \pi_j(P_j) = \pi_j(P_j = p_i) \forall p_i \leq p^m$ and
$p_j \geq p_i \Rightarrow P_j = p_i$. The only way both conditions can hold is when $p_i = p_j \forall p_i, p_j \leq p^m$.
When $p_j > p^m$, optimally $p_i = p^m$, s.t. $P_j = P_i = \min(p_i, p_j) = p^m$, which maximizes industry profits and firm *j* will be indifferent between all $p_j \in [p^m, \infty)$.

Appendix 2

Say $p_j > p^m$. Optimally $p_i = p^m$ as $\pi_i(P_i = p^m) = \pi^m$. Thus, optimally, γ_i or λ_i and p_i are set, such that $P_i = p^m < p_j$. In case of beating a competitor by a percentage of the difference:
$P_i = p_j - \gamma_i(p_i - p_j) = p^m$. $p_j - \gamma_i(p_i - p_j) = p^m \Leftrightarrow \gamma_i = \dfrac{p_j - p^m}{p_i - p_j}$. In case of beating a competitor by a fixed amount: $P_i = p_j - \lambda_i = p^m \Rightarrow \lambda_i = p_j - p^m$. Thus, $p_j > p^m \Rightarrow \gamma_i = \dfrac{p_j - p^m}{p_i - p_j}$ or $\lambda_i = p_j - p^m$ will lead to $P_i = p^m \forall p_j > p^m$. When $p_j \leq p^m \Rightarrow \max P_i = \lim p_j^-$ as $\pi'(p) \geq 0 \forall p \leq p^m$. As follows, $\max P_i \mid (c < p_j < p^m) = \lim p_j^-$ implying that optimally, firm *i* undercuts p_j by the smallest possible amount.

Appendix 3

$p_i < p_j \Rightarrow P_j < P_i \Rightarrow \pi_i = 0$. As a lower posted price results in $\pi_i = 0$, the following conditions must hold for all $p_i, p_j \in (c, \infty)$:

$$P_i \leq p_j \Rightarrow p_i \geq p_j$$
$$P_j \leq p_i \Rightarrow p_j \geq p_i$$

The only way both equations can hold is when $p_j = p_i$. Now say $c < p_j = p_i = p$. Profits for each firm are $\frac{1}{2} pQ(p)$. For firm *i,* raising prices, s.t. $P_i < p_j$, with γ_i, λ_i at the optimal level derived in lemma 1, would result in:

$$c < p \leq p^m \Rightarrow \pi_i = pQ(p) > \frac{1}{2} pQ(p)$$
$$p > p^m \Rightarrow \pi_i = \pi^m > \frac{1}{2} pQ(p)$$

As a result, $\pi_j = 0$ as $p_j < p_i \Rightarrow p_j > P_i$. As a reaction, optimally firm *j* sets p_j s.t. $\pi_j = \max(\pi(p_i), \pi(p^m))$. Thus, $\forall p_j > c \exists p_i > p_j$ s.t. $\pi_i(P_i) > \frac{1}{2} p_j Q(p_j)$. As follows, any $p > c$ cannot be a Nash equilibrium. When $p = c \Rightarrow \pi = 0$. Raising prices by firm *i* would result in:

$P_i < c \Rightarrow \pi_i < 0$, which implies that neither firm has an incentive to deviate and $p_i = p_j = c$ is the only Nash equilibrium

Appendix 4

Say that $p_i > p_j$. $\max U_k = R_k - \min(P_i + h, P_j + h, p_i, p_j) = R_k - \min(P_i + h, p_j)$.
$p_i > p_j \Rightarrow P_i = p_j \Rightarrow \min(P_i + h, p_j) = p_j \Rightarrow \max U_k = R_k - p_j$. Thus, $\pi_i = 0 \forall p_i > p_j$.
Symmetry implies that $\pi_j = 0 \forall p_j > p_i$ and thus, in Nash-equilibrium, $p_i = p_j$. Say that $p_i = p_j > c$.
$\lim_{p_i \to p_j^-} p_i = p_j \Rightarrow \max U_k = R_k - \min(p_i, P_j + h) = R_k - p_i$. As undercutting leads to higher profits, the Bertrand mechanism is unaffected by the PMG and $p_i = p_j = c$.

Appendix 5

$(U_k \mid p_i > p_j) = \max(R_k - \min(P_i + h, p_j)$. $R_k - (P_i + h) > R_k - p_j$ iff $P_i + h < p_j$. Thus, firm *i* undercuts the price of firm *j* successful iff $P_i + h < p_j$. As $P_i = p^m \Rightarrow \pi_i = 0 \forall p_j < p^m + h$ it is optimal to set γ s.t. $\lim P_i = p_j - h^-$ as $\pi' \geq 0 \forall p \leq p^m$.

$P_i = p_j - \gamma(p_i - p_j) = p_j - h \Rightarrow \gamma = \dfrac{h}{p_i - p_j}$. When $p_j \geq p^m + h$, $P_i = p^m \Rightarrow \pi_i = \pi^m$ as

$P_i + h < p_j$. $P_i = p_j - \gamma(p_i - p_j) = p^m \Rightarrow \gamma = \dfrac{p_i - p^m}{p_i - p_j}$. $P_i = p_j - \lambda_i = p^m \Rightarrow \lambda_i = p_j - p^m$.

$P_i = p_j - \lambda_i = \lim p_j - h^- \Rightarrow \lambda_i = \lim h^+$.

Appendix 6

Say that $p_j = c \Rightarrow P_i \in [0, p_j]$. $P_i < p_j \Rightarrow \pi_i \leq 0$. $P_i = p_j \Rightarrow \pi_i = 0$, Thus, $p_i = p_j = c$ is a Nash-equilibrium. Say that $p_j \in (c, c + h)$. $p_i > p_j$ s.t. $P_i \in [c, p_j) \Rightarrow s_i = 0 \Rightarrow \pi_i = 0$.
$p_i > p_j$ s.t. $P_i \in [0, c) \Rightarrow P_i - c < 0 \Rightarrow \pi < 0$
$p_i < p_j$ s.t. $P_i \in (c, p_j) \Rightarrow P_i - c > 0, s_i = 1$

$\pi_i' > 0 \forall p_i \in [0, p_j) \Rightarrow \max \pi(p_i) = \lim p_j^-$. Sequential undercutting will lead to $p_i = p_j = c$.
When $p_j \in [p^m + h, \infty)$, optimally $p_i > p_j$ *s.t.* $P_i = p^m$. Thus, $\exists p_i > p_j \forall p_j \in (p^m + h, \infty)$ s.t. $P_i = p^m$ and no set of $p_i, p_j \in (p^m + h, \infty)$ is a Nash-equilibrium.

Say that $p_j \in (c + h, p^m + h)$ and firm *i* has a PBG in the form of $P_i = p_j - \gamma_i(p_i - p_j)$. $\exists p_i < p_j \forall p_j, \gamma_i$ s.t. $p_i < p_j < P_j + h$. Therefore, undercutting is always possible when a PBG is defined as a percentage of the difference in price. When $P_i = p_j - \lambda \forall p_i > p_j$, Setting $p_j < p_i \Rightarrow P_i = p_j - \lambda$. As proven in lemma 2, optimally $\lambda_i = \lim h^+$ and undercutting by setting $p_i < p_j$ is not possible. Undercutting by setting $p_i > p_j$ s.t. $P_i + h < p_j$, is the only option.

Therefore, if and only if $\frac{\pi_i(p_i = p_j)}{2} \geq \pi(p_i = p_j - h)$ holds, $p_i = p_j$ can hold in equilibrium iff the LPG is defined as $P_i = p_j - \lambda \mid p_i > p_j$.

When $\lim P_i = p_j - h$ $\pi_i < 0 \forall p_j \in (c, c+h) \Rightarrow \frac{\pi_i(p_i = p_j)}{2} > \pi(p_i = p_j - h) \forall p_j \in c, c+h$

When $p_j = p^m + h \Rightarrow \frac{\pi_i(p_i = p_j)}{2} < p^m = \pi(p_i = p_j - h)$, as $\pi' < 0 \forall p > p^m$ by assumption 3.

Assumption 3 also implies that π is strictly increasing on the interval $P \in [c, p^m)$. By continuity, it follows that on the interval $P \in [c, p^m)$ there exists a unique value p^* s.t. $\frac{\pi_i(p_i = p_j)}{2} = \pi(p_i = p_j - h)$. $p < p^m \Rightarrow \frac{\pi_i(p_i = p_j)}{2} \geq \pi(p_i = p_j - h)$, which means that all values of $p \in [c+h, p^*]$ are Nash-equilibria.

Appendix 7

$D_i = a - \min(p_i, P_i + h) + \beta(p_j, P_j + h)$. $P_i \in \{p_i, p_j\} \Rightarrow D_i = a - \min(p_i, p_j) + \beta(p_i, p_j)$. $\pi(p_i = p_j) = \pi(p_i > p_j)$ as $P_i = p_j \forall p \in [p_j, \infty)$. Therefore, an increase will be ineffective. By assumption 3 it follows that an equal decrease in P_i, P_j will lead to lower profits for all $P_i, P_j < p^m$. As $P_i = P_j \forall p_i, p_j$ undercutting will lead to lower profits for all $P_i, P_j < p^m$. $P_i, P_j < c \Rightarrow \pi < 0$, thus $P_i, P_j \in [c, p^m]$ are equilibria value.

Appendix 8

$D_i = a - \min(p_i, P_i + h) + \beta(p_j, P_j + h)$. $P_i + h < p_i \Rightarrow D_i = a - P_i + h + \beta p_j$.
$P_i = p_j \Rightarrow \pi_i = (a - (p_j + h) + \beta p_j)(p_j - c)$. $\pi_i(p_i = p_j) = (a - p_j + \beta p_j)(p_j - c)$
$\pi_i(p_i = p_j) = (a - p_j + \beta p_j)(p_j - c) > (a - (p_i + h) + \beta p_j)(p_j - c) \forall p_j > c$, which means that firms will always be better off, by setting $p_i = p_j$. $P_i + h > p_i \Rightarrow \min(p_i, P_i + h) = p_i$ and thus the LPG is an empty set. As follows, in equilibrium $p_i = p_j$. $\pi(p_i = p_j - \varepsilon) > \pi(p_i = p_j) \forall p_j > p^{NB}$, where ε is defined as in the ε-∂ definition, which means that prices larger than Bertrand outcome are not supported in equilibrium. $\pi(p_i = p_j + \varepsilon) > \pi(p_i = p_j) \forall p_j < p^{NB}$, thus prices below Bertrand equilibrium are not supported in equilibrium and the only equilibrium is $p_i = p_j = p^{NB}$.

Appendix 9

$D_i = a - \min(p_i, P_i) + \beta(\min p_j, P_j)$, $P_i \in [0, p_j]$. $p_j > p^m \Rightarrow \max \pi(P_i) = \pi(p^m)$ and $p_i > p_j > p^m$. When $p_i > p^m \Rightarrow \max \pi(P_j) = \pi(p^m)$. Thus, all prices above monopoly price cannot hold in equilibrium, as both firms always have an incentive to set $p_i > p_j$ s.t. $P_i = p^m$. When $p_i \in (p^{NB}, p^m)$, optimally $P_j < p_i$. Therefore, all equilibria larger than Nash Bertrand are no equilibria values of p_i, p_j. When $p_i \in [c, p^{NB})$, Optimally $P_j > p_i$, which is not possible since $P_j \in [0, p_i]$. As, $\pi'(P_j) > 0 \forall p_i < p^{NB}$, the best response is to set $P_j = p_j \forall p_j$ which implies that

$p_j = p_i \forall p_i, p_j \le p^{NB}$. When $p_i < c \Rightarrow \pi(p_j = p_i) < 0$, so the firm will not sell any products. Thus, $p_i = p_j \in [c, p^{NB}]$ are equilibria values.

Appendix 10

$D_i = a - \min(p_i, P_i + h) + \beta(\min p_j, P_j + h)$, $h > 0$. PBG only effective if $P_i < p_i + h$.
$D_i = a - (P_i + h) + \beta p_j \forall (P_i < p_i + h)$. Undercutting takes place iff $\pi(P_i = p_j - h) > \pi(p_i = p_j)$ or $\pi(p_i < p_j) > \pi(p_i = p_j)$. When $p_j = p^{NB}$ undercutting cannot be more profitable, as $\max \pi_i = p^{NB}$ when $(p_j = p^{NB})$ by definition. When $p_j < p^{NB}$, setting $p_i > 0$ s.t. $P_i + h > p_i$ will results in $\min(p_i, P_i + h) = p_i$. $\pi_i'(p_i) > 0 \forall p_i \in [p_j, p^{NB})$ and thus, prices below Nash-Bertrand are not supported in equilibrium. $p_j = p^{NB} \Rightarrow \max \pi_i(p_i, p_j) = \pi(p^{NB})$ by definition. Therefore, $p_i = p_j = p^{NB}$ is an equilibrium value. For $p_j > p^{NB}$, optimally $P_i \in [p^{NB}, p_j)$. Undercutting takes place iff $\pi(p_i < p_j) > \pi(p_i = p_j)$ or $\pi(P_i = p_j - h) > \pi(p_i = p_j)$. Setting $p_i < p_j$ will lead to $P_j < p_i$. An equal decrease in p_i and p_j will lead to lower profits for all $p_i, p_j \in [p^{NB}, p^m]$. When $P_j = p_i - \gamma(p_j - p_i)$ than $\exists p_i < p_j$ s.t. $P_j > p_i + h$ with $\pi_i(p_i < p_j) > \pi_i(p_i = p_j)$, since $\pi_i'(p_i) < 0 \forall p_j > p^{NB}$. Therefore, $p_i, p_i > p^{NB}$ cannot be supported in equilibrium when the PBG is defined as $P_i = p_j - \gamma_i(p_i - p_j)$. When $P_i = p_j - \lambda_i$, $\exists p_i < p_j$ s.t. $P_j > p_i + h$ iff $\lambda_j < h$. As $\pi_j'(P_j) \le 0 \forall P_j \le p^m$, $\pi_j'(\lambda_j) < 0 \forall \lambda_j > h$ and optimally $\lambda_j = h$. The best response curve for firm *i* is given by $P_i = \dfrac{a + c + \beta p_j + h}{2}$. The optimal value of P_i is weakly increasing in P_j.
$\pi''(p) < 0 \forall p \in [0, \infty)$ and by continuity it follows that $\pi'(P_i = p_j - h) - \pi'(p_i = p_j) > 0$. As follows, $\exists p^* > p^{NB}$ s.t. $\pi(P_i = p_j - h) = \pi(p_i = p_j)$.
$p_j \le p^* \Rightarrow \pi(p_i = p_j) > \pi(P_i \in [c, p_j - h]$

Bibliography

Arbatskaya, M. (2001). Can low-price guarantees deter entry? *International Journal of Industrial Organization*, 1387-1406.

Arbatskaya, M., Hviid, M., & Schaffer, G. (2006). On the Incidence and Variety of Low-Price Guarantees. *The Journal of Law & Economics*, 307-332.

Belton, T. M. (1986). A model of duopoly and meeting or beating competition. *International Journal of Industrial Organization*, 399-417.

Chen, Z. (1995). How low is a guaranteed-Lowest-Price. *The Canadian Journal of Economics*, 683-701.

Gerstner, E., & Hess, J. (1991). Price-matching policies: an empirical case. *Managerial and Decision Economics*, 305-315.

Salop, S. (1982). *Practices that (credibly) facilitate oligopoly coordination.* Washington: Federal Trade Commission.