Bachelor Thesis: Cargo Fare Class Mix Problem

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Abstract

This bachelor thesis replicated and extended upon previous research by Van Riessen et al. (2015a) on the Cargo Fare Class Mix (CFCM) Problem, a proposed implementation of revenue management on intermodal hinterland transport. Results of the replication confirmed the findings of the aforementioned paper. The solution method of the case of the CFCM Problem with one route, one destination, and two fare classes proposed by said paper was found to yield higher revenue than other approaches to the problem. Furthermore, this bachelor thesis proposed a solution method for a slightly more complicated case of the CFCM Problem with two routes differing in price and capacity, one destination, and two fare classes.

1 Introduction

The ports of the Hamburg-Le-Havre range are constantly challenged to increase the quality of its services. First of all, the eleven major ports and port groups in this range (Hamburg, Bremen, Wilhelmshaven, Amsterdam, Rotterdam, Zeeland Seaports, Antwerp, Gent, Zeebrugge, Dunkirk and Le-Havre) must compete for their share of 97% of throughput going to the common hinterland of North West Europe (De Langen, Van Meijeren, & Tavasszy, 2012). Secondly, all of these ports face congestion problems, partially caused by the handling of large container volumes from the port towards the hinterland (Van den Berg & De Langen, 2014). Faced with these realities, a modal split of hinterland transportation becomes a target of improvement for the port authorities in the range, e.g. in Rotterdam (Havenbedrijf Rotterdam, 2011), Antwerp and Hamburg (Van den Berg & De Langen, 2014).

Achieving these modal shift targets is a complicated task, as an integrated network approach is required (Veenstra, Zuidwijk, & van Asperen, 2012). As such, there is renewed interest of the study of planning models for intermodal hinterland transport in recent years (Van Riessen, Negenborn, & Dekker, 2015b). Van Riessen et al. (2015a) contributed to this growing interest by implementing revenue management as a means to optimize network logistics. They propose that hinterland transportation network operators should operate under a differentiated service portfolio, i.e. a portfolio where different services are provided at different prices. The Cargo Fare Class Mix (CFCM) problem was proposed, which seeks to decide the optimal limit of each offered transportation service as to maximize revenue. There should be two or more services offered,
each defined as a fare class. The classes differ only in price and lead time, and it is assumed higher-priced services have less lead time.

A capacity variable is defined as well, corresponding to vehicles belonging to the company. Additional costs are charged should accepted demand exceed the capacity. This corresponds to a case where the company has to outsource additional vehicles.

Here we introduce three dimensions of the CFCM problem. The problem concerns the transportation of all cargo from a single deep sea port through multiple possible routes \( r \) to destinations \( d \). The third dimension is the aforementioned fare classes \( p \). Based on these dimensions, a CFCM problem can be classified as a CFCM \( (r,d,p) \) problem.

Through our paper, we wish to reproduce and further extend the aforementioned study on the CFCM Problem (van Riessen, Negenborn, & Dekker, 2015a). Thus, we aim to answer the following research question:

“How should the optimal fare class sizes in a hinterland transport differentiated service portfolio with determined in the cases of one and two possible transport routes?”

2 Literature Review

2.1 Intermodal Networks

Transportation network operators are required to continuously optimize the transportation of containers to the hinterland. They have to allocate containers to available inland services (train, barge, or truck) to achieve the optimal transportation plan. A traditional approach in transportation is to optimize solely on costs. However, this approach is incomplete as customers also value quality of service (Crainic & Laporte, 1997).

As such, recent research restrict their optimization problems by a certain standard to quality of service. One example of these restrictions is the service-time requirements (Ishfaq & Sox, 2010; Ziliaskopoulos & Wardell, 2000). This type of constraint is generally added to model the time pressure present in freight transportation.

Van Riessen et al (2015b) noted the lack of literature in methods to create planning flexibility in inland container transportation. As of yet, customers are hesitant to delegate transport flexibility to network operators. One of the reasons behind this unwillingness is the current pricing method, which does not provide incentives for customers to delegate network operators. Thus, research into the implementation of revenue management in intermodal networks is required.

Nevertheless, several studies have been made on the pricing problem of intermodal transportation. One study proposed a programming model which jointly obtains the design of transportation network and the price of the transportation products offered (Ypsilantis & Zuidwijk, 2013). Another study (Li, Lin, Negenborn, & De Schutter, 2015) proposed a cost-plus-pricing strategy. Given the current costs of the transportation network, a customer will be offered a differentiated service portfolio to choose from, maximizing total profit.
2.2 Revenue Management in Aviation

One notable example of revenue management in operations management can be found in the aviation industry in the form of different fare classes for the same flight. It is standard practice for an airline to charge customers for different services on the same flight, such as for extra leg room or extra baggage. In the aviation industry, revenue management models are mainly concerned with the optimal fare mix, setting limits to the amount of demand of each fare class to be accepted (Barnhart, Belobaba, & Odoni, 2003).

Indeed, revenue management benefits airlines in two ways. Firstly, it is essentially a method for airlines to implement price discrimination on different consumer groups (Zeni, 2001). This pricing strategy allows airlines to accrue more revenue from customers with higher valuations of among other aspects, time (Stavins, 2001). Secondly, it increases utilizations by incentivizing customers to choose more flexible products at a lower price. Flexibility may help producers increase the utilization of their limited capacity to handle demand which is difficult to forecast (Petrick, Steinhardt, Gönsch, & Klein, 2012).

2.3 Revenue Management in Intermodal Networks

Prior to the study by Van Riessen et al (2015a), no revenue management strategy for intermodal hinterland transportation had yet been developed with the exception of one study concerning rail freight transportation (Bilegan, Brotcorne, Feillet, & Hayel, 2015). One explanation to this lack of literature is that the high number of stakeholders deciding the prices of hinterland transportation complicates the construction of revenue management models (van Riessen et al., 2015b). As such, Van Riessen et al (2015a) simplified the problem by considering the case of only one route and one destination.

However difficult, there is much promise to revenue management for hinterland transportation. The two characteristics of the aviation industry which benefit from revenue management is present in hinterland transportation as well. First of all, it is possible to segment the consumers of the hinterland transportation market into groups with different characteristics (van Riessen et al., 2015b). However, Van Riessen et al (2015b) notes that so far, only qualitative literature on the North-West European hinterland transportation market exists. It is possible that in the future, we will know more about the market segmentation of hinterland transportation, which enables the construction of better revenue management models. Secondly, transportation orders to the hinterland are agreed upon in long-term contracts. Thus, the decision of accepting demand is taken in advance, at the tactical level (van Riessen et al., 2015a). Meanwhile, the capacity of a network operator is fixed at the strategic level. Thus, network operators may benefit from the increased utilization revenue management offers.

3 Cargo Fare Class Mix Problem

3.1 Modeling Framework

As mentioned in previous sections, this paper will not deviate from the definition of the CFCM problem proposed in Van Riessen et al (2015a). The general idea of this definition is already explained in the introduction of this paper.
Aside from the dimensions $r$, $d$, and $p$ previously introduced, the problem can also be classified based on the timespan of the problem into two problems. Firstly, the problem can be studied where demand is considered as long-term commitments. Secondly, the problem can be simplified by considering demand only for a specified planning horizon. Note that in the second case, the booking limits are still determined on a tactical level. However, the second case yields deterministic fare class limits as results to the problem. In both cases, demand is stochastic.

Three decisions must be taken in the CFCM $(r,d,p)$ problem: the balance of booking limits for each offered service, the routing of each cargo transported, and the schedule of when the goods will be sent to each destination.

In this paper, we will consider three cases of the CFCM problem with deterministic fare class limits. The planning horizon is defined to be daily for all three cases.

Figure 1: Schematic model of the CFCM(1,1,2) problem

We will first replicate the special case of CFCM (1,1,2), which was already solved in Van Riessen et al (2015a). That is, the case with only one route, one destination, and two fare classes, as described in Figure 1. We introduce the two fare classes as Express, where the cargo is guaranteed to arrive within one day, and Basic, where the cargo is guaranteed to arrive within two days. Thus, the decision variable will be the fixed booking limits of each fare class.

Figure 2: Schematic model of the CFCM(2,1,2) problem

Afterwards, our study wishes to build upon the findings of the original paper and investigate a more complicated case of the problem, namely a CFCM (2,1,2) Problem. That is, the case with two routes, one destination, and two fare classes, as can be seen in Figure 2. This case expands from the aforementioned CFCM(1,1,2) problem by an alternative route that the operator can take. Thus,
3.2 CFCM (1,1,2) Problem with Deterministic Daily Limits

Firstly, we will derive an analytical model for the CFCM (1,1,2) problem for the simplified case of daily planning horizons. The model will optimize revenue from two fare classes, Express and Basic, on one route to one destination as such that the capacity $C$ is not violated. Thus, there are only two decision variables: the deterministic daily booking limits for each fare class $L_E$ and $L_B$.

Next, we will discuss demand. Cargo of Express customers must be transported within one day, while cargo of Basic customers must be transported in two days or less. As mentioned in previous sections, demand is stochastic. The number of daily demand for each service type is described by distributions $N_E(t)$ and $N_B(t)$:

$$N_E(t) \ p_E(k) = P(N_E = k), k = 1, 2, ...$$
$$N_B(t) \ p_B(k) = P(N_B = k), k = 1, 2, ...$$

Demand of the same fare class for different days are assumed to be i.i.d. Moreover, $N_E(t)$ and $N_B(t)$ are assumed to be independent, but not identical, for all $t$.

Demand will always be accepted by the service provider as long as the booking limit for its respective fare class has not been reached. The service provider has a capacity $C$, which can be used to transport cargo for both fare classes. If the total number of accepted demand exceeds $C$, then the service provider will be forced to outsource additional vehicles.

Therefore, the distribution of daily accepted Express demand $D_E(t)$ is as follows:

$$D_E(t) = \min(N_E(t), L_E)$$ (1)
$$P(D_E(t) = k) = p_E(k), k = 1, 2, ..., L_E - 1$$ (2)
$$P(D_E(t) = L_E) = 1 - \sum_{k=0}^{L_E-1} p_E(k)$$ (3)

Similarly, the distribution of daily accepted Basic demand $D_B(t)$ is as follows:

$$D_B(t) = \min(N_B(t), L_B)$$ (4)
$$P(D_B(t) = k) = p_B(k), k = 1, 2, ..., L_B - 1$$ (5)
$$P(D_B(t) = L_B) = 1 - \sum_{k=0}^{L_B-1} p_B(k)$$ (6)

For the rest of this study, the time indicator $t$ is omitted, unless specifically required for clarity.

The assumptions made for this model is as follows. First, since Express demand cannot be postponed, it is prioritized over Basic Demand. Secondly,
there is no reason to accept more Express demand than the capacity, based on the same reasoning. Therefore, \( L_E \leq C \) is assumed.

Based on these assumptions, we can identify three types of demand being served on a particular day: First, there is the Express demand accepted that day, \( D_E \). Secondly, there is Basic demand accepted that day, \( D_B \). Thirdly, there is Basic demand accepted the day before today, which was not yet transported, \( R_B \). Likewise, the portion of \( D_B \) which is not transported is denoted as \( R_B(t + 1) \), and will be considered the following day. The portion of \( R_B(t + 1) \) which still cannot be transported the following day, is denoted as \( E_B \).

As such, three situations can arise in a particular day:

1. The service provider can only transport Express demand accepted that day and part of the Basic demand accepted the day before still remaining. All \( D_E \), no \( D_B \), and part of \( R_B \) are transported. All of \( D_B \) is assigned to \( R_B(t + 1) \), and the portion of \( R_B \) that is not transported is assigned to \( E_B \).

2. The service provider can only transport Express demand accepted that day, Basic demand accepted the day before still remaining, and part of the Basic demand accepted the day before. All \( D_E \), part of \( D_B \), and all \( R_B \) are transported. The portion of \( D_B \) not yet transported is assigned to \( R_B(t + 1) \).

3. The service provider can transport all demand. All \( D_E \), all \( D_B \), and all \( R_B \) are transported.

Finally, the objective function CFCM (1,1,2) problem can be formally defined as follows:

\[
\max_{L_E, L_B} J = f_E E(D_E) + f_B E(D_B) - p E(E_B)
\]  

Where \( f_E \) and \( f_B \) are fares received by the service provider for each accepted Express and Basic demand, respectively, and \( p \) is the penalty on having excess Basic demand not being transported using the available capacity.

This objective is subject to constraints for the expected accepted demand \( E(D_E) \) and \( E(D_B) \), and the expected excess demand \( E(E_B) \). These constraints depend on limits \( L_E, L_B \), and will be presented along with the solution method.

### 3.3 CFCM (2,1,2) Problem with Deterministic Daily Limits

The CFCM (2,1,2) model is an expansion to the CFCM (1,1,2) model described in previous sections. The CFCM (1,1,2) model only has a single route \( r_0 \) connecting the origin with the destination with capacity \( C \). We now add an alternative route \( r_A \) with capacity \( C_a \), which is costlier than \( r_0 \) to operate.

The existence of a costlier alternative route has two consequences. Firstly, the service provider can now accept more Express demand, as overall capacity has increased. This introduces a new tradeoff between receiving additional revenue from Express and incurring additional costs from the alternative route. Secondly, the remainder of Basic accepted demand \( R_B \) can be transported via \( r_A \), as opposed to being postponed to the next day. A second additional tradeoff
is therefore introduced, between additional costs from transporting the same day and the risk of the postponed demand not being transported.

To model these two tradeoffs, we introduce a new decision variable $L_A$ which is the limit to the amount of cargo that will be transported via $r_A$. Following the other two decision variables, we assume that the service provider determines $L_A$ at the tactical level. If the total amount of the three demand types being served on a particular day exceeds $C + L_A$, then only $C + L_A$ demand will be transported, even if it is still possible to send additional cargo via $r_A$.

Furthermore, we distinguish served demand transported via $r_A$ of a particular day as three different parameters: $A_E$, $A_P$, and $A_B$, for Express demand, postponed Basic demand from the previous day, and accepted Basic demand from the same day, respectively.

As such, unlike the CFCM (1,1,2) problem, the CFCM (2,1,2) problem is a profit maximization problem. The objective function of the CFCM(2,1,2) problem is as follows:

$$\max_{L_E, L_B, L_A} J = f_E E(D_E) + f_B E(D_B) - \rho (E(A_E) + E(A_P) + E(A_B)) - p E(E_B)$$  (8)

Instead of fares, $f_E$ and $f_B$ here denote the profits received by the service provider for each accepted Express and Basic demand, respectively if it was transported via $r_0$. The parameter $\rho$ denotes the additional cost incurred for transporting a unit of demand via $r_A$, and $p$ is the penalty on having excess Basic demand not being transported using the available capacity.

In addition to the expected accepted demand $E(D_E)$ and $E(D_B)$, and the expected excess demand $E(E_B)$, the objective function is also constrained by the expected amount of demand transported via $r_A$, $A_E$, $A_P$, and $A_B$. These constraints depend on limits $L_E$, $L_B$, $L_A$ and will be presented along with the solution method.

### 4 Solution Method for CFCM (1,1,2)

In this section we attempt to reproduce the solution method proposed in the original paper. The objective function (7) involves the expected values of $D_E$, $D_B$, and $E_B$. We are therefore required to describe these expected values as a function of the decision variables $L_E$ and $L_B$, and the fixed parameter $C$. The expected values of accepted demand $D_E$ and $D_B$ is derived in section 4.1, section 4.2 derives the expected value of $E_B$, and finally a formulation for the problem will be presented in section 4.3.

#### 4.1 Expected Accepted Demand

According to (1) and (4), the distribution of accepted demand $D_E$ and $D_B$ depend only on the unlimited demand distributions, $N_E$ and $N_B$, respectively. Since we assume that the demand distributions are known, we can derive explicit formulations of $E(D_E)$ and $E(D_B)$ from equations (1) - (7):

$$P(D_E(t) = L_E) = \sum_{k=0}^{L_E-1} k p_E(k) + L_E(1 - \sum_{k=0}^{L_E-1} p_E(k))$$  (9)
\[ P(D_B(t) = L_B) = \sum_{k=0}^{L_B-1} kp_B(k) + L_B(1 - \sum_{k=0}^{L_B-1} p_B(k)) \]

### 4.2 Expected Excess Accepted Demand

Derivation of the distribution of excess accepted demand \( E_B \) is more complicated than the distribution of accepted demand. This is because the distribution of \( E_B \) depends not only on \( D_E \), but also on basic demand which was accepted the previous day not yet transported, \( R_B \). As explained previously, \( R_B \) is determined by the situation which arose the previous day.

In this section, we will assume that the booking limits are fixed. Thus, given the fixed booking limits and the distribution of demand as described in previous sections, the situation which arises on a certain day can be fully described by \( R_B \). As such, \( R_B \) exhibits the following Markov property: for a given day \( t \), the state is fully described by \( R_B(t) \), and is independent from states other than \( t-1 \). For simplicity, the state \( R_B(t) \) will be further denoted as \( R_t^B \).

Based on the three situations presented in section 3.2, we can describe \( E_B(t) \) as a function of \( R_t^B \) as follows:

\[
E_B(t) = \max (R_t^B + D_E(t) - C, 0)
\]

Afterwards, we can formulate the probability distribution for excess demand \( E_B(t) \) based on the Markov state \( R_t^B \) as follows:

\[
P(E_B(t) = m) = \begin{cases} 
P(D_E(t) \leq C - R_t^B) & \text{if } m = 0 \\
P(D_E(t) = C + m - R_t^B) & \text{if } m > 0 \end{cases}
\]

Therefore, the probability of having excess demand can be summed up as follows:

\[
P(E_B > 0) = 1 - P(D_E(t) \leq C - R_t^B)
\]

\[
P(E_B > 0) = P(D_E(t) > C - R_t^B) \quad \text{(12)}
\]

Furthermore, the transition probabilities, that is the distribution of the remaining demand for the next day, \( R_{t+1}^B \), given the remaining demand of the current day \( R_t^B \), is denoted as \( p_R(i,j) \). We distinguish between the situation with excess demand (Situation 1 in section 3) and the situations without (Situation 2 and Situation 3). Thus, the transition probabilities is as follows:

\[
p_{R_{t+1}}(i,j) = P(R_{t+1}^B = j \mid R_t^B = i)
\]

As described previously, when Situation 1 occurs, all Basic demand accepted that day will be assigned to \( R_B \). Thus for \( E_B > 0 \):

\[
p_{R_{t+1}}(i,j) = P(D_B = j)
\]

In other words,

\[
P(R_{t+1}^B = j, E_B > 0 \mid R_t^B = i) = P(D_B = j)P(D_E > C - i) \quad \text{(13)}
\]
For Situation 2 \((j > 0)\) and Situation 3 \((j = 0)\), there are no excess accepted demand. Therefore for \(E_B = 0\) the following transition probability holds:

\[
p_{RB}(i,j) = \begin{cases} 
P(D_E + D_B + R_{B}^{t} - C = j) & \text{if } j > 0 \\
P(D_E + D_B + R_{B}^{t} < C) & \text{if } j = 0 \end{cases}
\]  

(14)

Note that for Situation 2 and Situation 3, all of \(R_{B}^{t}\) is transported. \(D_B\) is then assigned to the remaining slots \(S\), where:

\[
S = C - D_E - R_{B}^{t}
\]

\[
P(S = s) = P(D_E + R_{B}^{t} = C - s)
\]  

(15)

Where \(0 \leq s \leq C - R_{B}^{t}\). Note that for all cases where (15) is non-zero,

\[
D_E = C - R_{B}^{t} - s \leq C - R_{B}^{t}
\]

For the case of Situation 2, we can also define \(S\) in terms of accepted basic demand of that day \(D_B\) and the state of \(R_{B}^{t+1}\) tomorrow, as follows:

\[
S = D_B - R_{B}^{t+1}
\]  

(16)

Meanwhile, situation 3 suggests that

\[
S \geq D_B
\]  

(17)

Using the expressions (15) – (17) we can rewrite (14) as follows:

\[
p_{RB}(i,j) = \begin{cases} 
\sum_{s=0}^{C-i} P(D_E + i = C - s)P(D_B = s + j) & \text{if } j > 0 \\
\sum_{s=0}^{C-i} P(D_E + D_B + R_{B}^{t} < C) & \text{if } j = 0 \end{cases}
\]  

(18)

Thus from (13) and (18) we can provide a general formulation of \(p_{RB}(i,j)\) as follows:

\[
p_{RB}(i,j) = P(D_B = j)P(D_E > C - i) + \sum_{s=0}^{C-i} P(D_E + i = C - s)P(D_B = s + j) \text{ if } j > 0 \\
p_{RB}(i,j) = P(D_B = 0)P(D_E > C - i) + \sum_{s=0}^{C-i} P(D_B + D_E + R_{B}^{t} < C) \text{ if } j = 0
\]  

(19)

Given \(p_{RB}(i,j)\) has been calculated, we can derive the steady-state distribution of the Markov state \(R_{B}\), \(\pi_j\) for fixed booking limits. In other words, \(\pi_j\) denotes the long-run probability of postponing \(j\) accepted Basic demand orders to the next day. The derivation can be found by finding a solution to the following Markov equilibrium equations.
\[ \pi_j = \sum_i \pi_i p_{RB}(i,j) \quad (20) \]
\[ \sum_i \pi_i = 1 \quad (21) \]

Finally, we can substitute the steady-state probabilities \( \pi_j \) to expression (11), yielding the distribution and expected value of excess accepted Basic demand as follows:

\[
P(E_B = m) = \begin{cases} 
\sum_{q=0}^{L_B} P(D_E \leq C - q) \pi_q & \text{if } m = 0 \\
\sum_{q=0}^{L_B} P(D_E = C + m - q) \pi_q & \text{if } m > 0
\end{cases}
\]

\[
E(E_B) = \sum_{m=0}^{L_B} m P(E_B = m) = \sum_{m=1}^{L_B} m P(E_B = m)
\]

\[
E(E_B) = \sum_{m=0}^{L_B} \sum_{q=0}^{L_B} P(D_E = C + m - q) \pi_q
\]

### 4.3 Formulation of the CFCM (1,1,2) Model

By using expressions (9) – (10) for expected accepted demand and expression (22) denoting the expected value of excess accepted Basic demand, we can describe the objective function (7) for given values of \( L_E \) and \( L_B \). Furthermore, the model is constrained by the Markov equilibrium equations provided in the previous section. As such, we can now provide a formulation of the CFCM (1,1,2) model.

\[
\max_{L_E, L_B} J = f_E E(D_E) + f_B E(D_B) - pE(E_B)
\]

Where

\[
E(D_E) = \sum_{k=0}^{L_E-1} k p_E(k) + L_E (1 - \sum_{k=0}^{L_E-1} p_E(k))
\]

\[
E(D_B) = \sum_{k=0}^{L_B-1} k p_B(k) + L_B (1 - \sum_{k=0}^{L_B-1} p_B(k))
\]

\[
E(E_B) = \sum_{m=0}^{L_B} \sum_{q=0}^{L_B} P(D_E = C + m - q) \pi_q
\]

Subject to:

\[
\pi_0 = \sum_{i=0}^{L_B} \pi_i [P(D_B = 0) P(D_E > C - i) + \sum_{s=0}^{C-i} P(D_E + i = C - s) P(D_B \leq s)]
\]

\[
\pi_{j, j>0} = \sum_{i=0}^{L_B} \pi_i [P(D_B = j) P(D_E > C - i) + \sum_{s=0}^{C-i} P(D_E + i = C - s) P(D_B = s+j)]
\]
\[ \sum_{i} \pi_i = 1 \]
\[ L_E, L_B \in \mathbb{N} \]
\[ \pi_q \geq 0 \]

### 4.4 Solution Method for the CFCM (1,1,2) Model

The CFCM (1,1,2) Model as formulated in the previous section cannot be solved in a straight-forward manner. Firstly, the objective function as a function of the decision variables \( L_E \) and \( L_B \) is in general not convex. The decision variables determine the probabilities of accepted demand, \( D_E \) and \( D_B \). Since the decision variables are present as parameters for the summation functions, the model becomes non-linear. Secondly, the decision variables themselves have integer values. Therefore, it is not possible to solve the problem with fast solution methods such as the simplex method.

However, it is possible to solve the problem iteratively. As shown in previous sections, fixing the values of \( L_E \) and \( L_B \) leaves only the Markov equilibrium equations to be solved. These equations are linear. As previously proposed by Van Riessen et al (2015a), the problem can be solved using a grid search over all possible combinations \((L_E, L_B)\).

The number of feasible combinations can be made finite using upper bounds for \( L_E \) and \( L_B \). When \( p > f_E \) is assumed, it is possible to conclude that \( L_E \leq C \) since accepting more Express demand than the capacity will result in a penalty. Moreover, when \( p > f_B \) is assumed, it is possible to conclude that \( L_B \leq 2C \). Since it is possible to transport Basic cargo within two days, there are at most twice as many cargo that can be accepted without a guaranteed penalty. These upper bounds ensures the grid search needs to examine \( 2C^2 \) combinations \((L_E, L_B)\). In each iteration, the model is reduced to an LP problem with fixed \((L_E, L_B)\) with a unique feasible solution.

### 5 Case Study for CFCM (1,1,2)

In this section, we aim to measure the performance of the CFCM (1,1,2). Specifically, we are interested in two comparisons. The first comparison is between the CFCM (1,1,2) model and other methods to determine the best feasible fare class mix, and the second comparison is between CFCM (1,1,2) with different parameters. To do so, three parameters was calculated apart from the expected revenue \((J)\): expected excess Basic demand \( E(E_B) \), expected capacity utilization \( \eta \), and computation time \( T \). Expected capacity utilization is calculated as follows:

\[
\eta = \frac{E(D_E) + E(D_B) - E(E_B)}{C}
\]

Computation time is documented based on execution of the solution method using a CPU with a clock speed of 3400 MHz.
5.1 Alternative Methods for the CFCM Problem

Van Riessen et al. (2015a) described an additional five methods to determine the best feasible fare class mix. These are:

1. Only offer Express service. In this approach Basic demand is ignored as Express service is not considered as a substitute for Basic service.

2. Only offer Basic service. In this approach Express demand is ignored as Basic service is not considered as a substitute for Express service.

3. Only offer Basic service with substitution. In this approach it is assumed that Express service can be perfectly substituted by Basic Service, i.e. when Express service is unavailable, all express demand arrivals become Basic demand arrivals.

4. Offer both services, but putting no limit on Express service. In this approach all Express up to capacity \( C \) is accepted.

5. Offer both services, but putting no limit on Basic service. In this approach all Basic demand up to capacity \( 2C \) is accepted.

An experiment was done to compare the CFCM (1,1,2) problem solution with the solutions from the aforementioned approaches under parameters described by Table 1.

<table>
<thead>
<tr>
<th>Case</th>
<th>( C )</th>
<th>( D_E )</th>
<th>( D_B )</th>
<th>( (f_E, f_B) )</th>
<th>Penalty</th>
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</thead>
<tbody>
<tr>
<td>CFCM (1,1,2)</td>
<td>20</td>
<td>Poisson(15)</td>
<td>Poisson(15)</td>
<td>110; 95</td>
<td>175</td>
</tr>
<tr>
<td>Express Only</td>
<td>20</td>
<td>Poisson(15)</td>
<td>0</td>
<td>110; 95</td>
<td>175</td>
</tr>
<tr>
<td>Basic Only</td>
<td>20</td>
<td>0</td>
<td>Poisson(15)</td>
<td>110; 95</td>
<td>175</td>
</tr>
<tr>
<td>Basic w/ Substitution</td>
<td>20</td>
<td>0</td>
<td>Poisson(30)</td>
<td>110; 95</td>
<td>175</td>
</tr>
<tr>
<td>No limit on Express</td>
<td>20</td>
<td>Poisson(15)</td>
<td>Poisson(15)</td>
<td>110; 95</td>
<td>175</td>
</tr>
<tr>
<td>No limit on Basic</td>
<td>20</td>
<td>Poisson(15)</td>
<td>Poisson(15)</td>
<td>110; 95</td>
<td>175</td>
</tr>
</tbody>
</table>

Table 1: Experiment parameters of the CFCM (1,1,2) Problem and its alternatives

The results of the experiments are as follows:

<table>
<thead>
<tr>
<th>Case</th>
<th>( (L_E, L_B) )</th>
<th>( J )</th>
<th>( \eta )</th>
<th>( E(E_B) )</th>
<th>( T ) s</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFCM (1,1,2)</td>
<td>14;7</td>
<td>2063</td>
<td>98.9%</td>
<td>0.13</td>
<td>20.5 s</td>
</tr>
<tr>
<td>Express Only</td>
<td>20;-</td>
<td>1627</td>
<td>73.9%</td>
<td>0</td>
<td>1.2 s</td>
</tr>
<tr>
<td>Basic Only</td>
<td>-;40</td>
<td>1425</td>
<td>75.0%</td>
<td>0</td>
<td>1.8 s</td>
</tr>
<tr>
<td>Basic w/ Substitution</td>
<td>-;20</td>
<td>1895</td>
<td>99.8%</td>
<td>0</td>
<td>1.8 s</td>
</tr>
<tr>
<td>No limit on Express</td>
<td>(20);6</td>
<td>2005</td>
<td>98.5%</td>
<td>1.09</td>
<td>2.0 s</td>
</tr>
<tr>
<td>No limit on Basic</td>
<td>5;(40)</td>
<td>1908</td>
<td>98.1%</td>
<td>0.38</td>
<td>1.4 s</td>
</tr>
</tbody>
</table>

Table 2: Results of the CFCM (1,1,2) Problem and its alternatives

These results confirm the findings of Van Riessen et al. (2015a). From these results, it is apparent that the results of CFCM (1,1,2) has the highest revenue. That is, revenue can be increased by combining Basic and Express services, and by setting limits on both services. From all alternatives, setting no limits on Express services has the closest results to the CFCM (1,1,2) problem.
5.2 Comparison of CFCM (1,1,2) Problem with Different Parameters

We are also interested in the behavior of CFCM (1,1,2) problem. While a complete parameter analysis is required to describe the effects of different parameters on the CFCM problem, a simple comparison of CFCM (1,1,2) problems with different parameters may still provide some insight on the performance of the model. Thus, we conducted an experiment using four sets of parameters, as described in Table 3:

<table>
<thead>
<tr>
<th>Case</th>
<th>C</th>
<th>$D_E$</th>
<th>$D_B$</th>
<th>$(f_E, f_B)$</th>
<th>Penalty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>50</td>
<td>Poisson(10)</td>
<td>Poisson(40)</td>
<td>110; 95</td>
<td>175</td>
</tr>
<tr>
<td>Case 2</td>
<td>50</td>
<td>Poisson(40)</td>
<td>Poisson(10)</td>
<td>110; 95</td>
<td>175</td>
</tr>
<tr>
<td>Case 3</td>
<td>50</td>
<td>Poisson(40)</td>
<td>Poisson(60)</td>
<td>110; 95</td>
<td>175</td>
</tr>
<tr>
<td>Case 4</td>
<td>80</td>
<td>Poisson(40)</td>
<td>Poisson(60)</td>
<td>110; 95</td>
<td>175</td>
</tr>
</tbody>
</table>

Table 3: Four sets of parameters for the CFCM (1,1,2) Problem

We will also compare the results of these cases with the alternative approach of setting no limits on Express, which was the best alternative for the CFCM (1,1,2) Problem in the previous experiment. The results are as follows:

<table>
<thead>
<tr>
<th>Case</th>
<th>($L_E$, $L_B$)</th>
<th>$J$</th>
<th>$\eta$</th>
<th>$E(E_B)$</th>
<th>$T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>(23;48)</td>
<td>4815.67</td>
<td>99.31%</td>
<td>0.29</td>
<td>238.1 s</td>
</tr>
<tr>
<td></td>
<td>(50;48)</td>
<td>4815.67</td>
<td>99.31%</td>
<td>0.29</td>
<td>5.6 s</td>
</tr>
<tr>
<td>Case 2</td>
<td>(44;15)</td>
<td>5146.29</td>
<td>97.71%</td>
<td>0.45</td>
<td>214.8 s</td>
</tr>
<tr>
<td></td>
<td>(50;13)</td>
<td>5126.40</td>
<td>98.99%</td>
<td>0.99</td>
<td>5.0 s</td>
</tr>
<tr>
<td>Case 3</td>
<td>(36;15)</td>
<td>5251.40</td>
<td>100%</td>
<td>0.15</td>
<td>218.9s</td>
</tr>
<tr>
<td></td>
<td>(50;11)</td>
<td>5167.86</td>
<td>100%</td>
<td>1.47</td>
<td>5.4 s</td>
</tr>
<tr>
<td>Case 4</td>
<td>(58;24)</td>
<td>8394.29</td>
<td>99.79%</td>
<td>0.15</td>
<td>1088.0s</td>
</tr>
<tr>
<td></td>
<td>(80;21)</td>
<td>8320.99</td>
<td>100%</td>
<td>1.55</td>
<td>15.8 s</td>
</tr>
</tbody>
</table>

Table 4: Results of the CFCM (1,1,2) Problems

These results also confirm the findings of Van Riessen et al (2015a). This section concludes the replication part of this paper.

6 Solution Method for CFCM (2,1,2)

The objective function (8) involves expected values of $D_E$, $D_B$, and $E_B$, which is present in the CFCM (1,1,2) model, as well as expected values of $A_E$, $A_P$, and $A_B$. In this section, we will describe these expected values as a function of the decision variables $L_E$, $L_B$, $L_A$, and the fixed parameter $C$.

Of the expected values already defined for the CFCM (1,1,2) model, $E(D_E)$ and $E(D_B)$ do not depend on the capacity. Thus, the additional capacity introduced along with the new route does not alter the formulation of the expected values described by equations (9) and (10). In contrast, the expected value of $E_B$ depends on the Markov steady-state probabilities which no longer holds in the CFCM (2,1,2) problem.
Thus, a new formulation of the Markov equations is derived in section 6.1 and the expected value of $E_B$ is derived in section 6.2. Afterwards, sections 6.3, 6.4, and 6.5 derive the expected values of $A_E, A_P,$ and $A_B$, respectively. Finally, a formulation for the CFCM (2,1,2) problem will be proposed in section 6.6.

### 6.1 Markov Steady-State Probabilities

In the CFCM (1,1,2) method, the entirety of the remainder of accepted Basic demand $R_{t+1}^B$ is postponed to the next day. As mentioned previously, an alternative route allows the service provider to send part of $R_{t+1}^B$ through the alternative route ($A_B^t$), and postponing the rest ($P_{t+1}^B$). In other words:

$$R_{t+1}^B = A_{t+1}^B + P_{t+1}^B$$

As such, $R_B$ no longer exhibits the Markov property. Instead, the property can be observed on $P_B$: for a given day $t$, the state is fully described by $P_B(t)$, and is independent from states other than $t - 1$. For simplicity, the state $P_B(t)$ will be further denoted as $P_B^t$.

The transition probabilities is still quite similar to that of $R_B$ in the CFCM (1,1,2). The only change made is the capacity, since now it is possible to transport at most an additional $L_A$ units of cargo. In the previous section we introduced the variable $S$ for the CFCM (1,1,2) problem. In the CFCM (2,1,2) problem, the variable $S$ denotes the remaining slots in $r_0$ to transport Basic Demand when all of yesterday’s postponed demand is transported via $r_0$. Thus, for all $D_E + P_B^t < C$ the variable $S$ is now calculated as:

$$S = C + L_A - D_E - P_B^t$$

$$P(S = s) = P(D_E + P_B^t = C - s)$$

Moreover, note that the maximum amount of slots $S$ given $P_B^t = i$ is $C + L_A - i$.

Therefore, the general formulation of the transitional probabilities $p_{P_B}(i,j)$ is as follows:

$$p_{P_B}(i,j) = P(D_B = j)P(D_E > C + L_A - i) + \sum_{s=0}^{C+L_A-i-1} P(D_E + i = C + L_A - s)P(D_B = s + j)$$

if $j > 0$

$$p_{P_B}(i,j) = P(D_B = 0)P(D_E > C + L_A - i) + \sum_{s=0}^{C+L_A-i-1} P(D_E + i = C + L_A - s)P(D_B \leq s)$$

if $j = 0$

As such, for fixed $L_E, L_B$, and $L_A$, it is possible to derive the long run probability of postponing $j$ accepted Basic demand orders to the next day, $\pi_j$, by finding a solution to the following Markov equilibrium equations:

$$\pi_j = \sum_i \pi_i p_{P_B}(i,j)$$
\[ \sum \pi_i = 1 \] (25)

6.2 Expected Excess Basic Demand

To derive the expected excess Basic demand, \( E(E_B) \), we describe \( E_B(t) \) as a function of \( P_B^t \) as follows:

\[
E_B(t) = \max (P_B^t + D_E(t) - C - L_A, 0)
\]

Based on the markov state \( P_B^t \), we can then define the probability distribution for excess demand \( E_B(t) \) as follows:

\[
P(E_B(t) = m) = \begin{cases} \sum_{q=0}^{L_B} P(D_E(t) \leq C + L_A - q) \pi_q & \text{if } m = 0 \\ \sum_{q=0}^{L_B} P(D_E(t) = C + L_A + m - q) \pi_q & \text{if } m > 0 \end{cases}
\]

Finally, we substitute the steady-state probabilities found by solving equations (24) – (25), \( \pi_j \) to expression (26) to obtain the distribution and expected value of excess accepted Basic demand:

\[
P(E_B(t) = m) = \sum_{m=0}^{L_B} mP(E_B = m) = \sum_{m=1}^{L_B} mP(E_B = m)
\]

\[
E(E_B) = \sum_{m=0}^{L_B} m \sum_{q=0}^{L_B} P(D_E = C + L_A + m - q) \pi_q
\]

6.3 Expected Express Demand Transported Via \( r_A \)

As mentioned previously, the introduction of an alternative route allows the service provider to accept more Express demand than \( C \) by paying additional costs \( \rho \) for each unit of demand that has to be transported via \( r_A \), \( A_E \). Thus to investigate the distribution of \( A_E \), we distinguish between two cases: where the service provider decides to set a limit on Express demand that is higher than \( C \) \((C < L_E < C + L_A)\), and where they do not \((L_E \leq C)\).

In the first case, two situations may occur based on the amount of Express demand accepted on a particular day:

1. \( A_E = 0 \), if \( D_E \leq C \)
2. \( A_E = D_E - C \), if \( D_E > C \)

Thus, given \( L_E \) where \( C < L_E < C + L_A \) holds, the expected Express demand transported via the alternative route is given as follows:

\[
E(A_E) = \sum_{k=C+1}^{L_E-1} (k - C)p_E(k) + (L_E - C)(1 - \sum_{k=0}^{L_E-1} p_E(k))
\]

In the first case, \( D_E \) never exceeds \( C \). Thus, \( A_E = 0 \) always holds.
Note that substituting a value of \( L_E \) lower than \( C \) will result in a negative number. We can exploit this outcome to derive a general formulation for the expected Express demand transported via \( r_A \):

\[
E(A_E) = \max \left[ \sum_{k=C+1}^{L_E-1} (k-C)p_E(k) + (L_E-C)(1-\sum_{k=0}^{L_E-1} p_E(k)), 0 \right] \quad (29)
\]

### 6.4 Expected Postponed Demand Transported Via \( r_A \)

Given that transporting through the alternative route is more expensive, a service provider may choose to postpone Basic demand to the next day in the hopes that said demand can be transported through the cheaper route. However, this is not always possible. Express demand holds priority over all other types of demand. Thus, the amount of postponed demand transported via the main route depends on accepted Express demand \( D_E \). By extension, the amount of postponed demand transported via \( r_A, A_P \), also depends on accepted Express demand \( D_E \).

Therefore, we distinguish between two cases: where accepted Express demand exceeds the capacity of the main route \((D_E > C)\), and where it does not \((D_E \leq C)\).

In the first case \((D_E \geq C)\), no postponed demand is transported through the main route. As such, two situations may occur based on the amount of postponed demand needs to be transported that particular day:

1. \( A_P = P^B_t \), all postponed demand is transported via \( r_A \). This occurs when there is enough capacity to transport all express demand and postponed demand. In other words,

\[
P^B_t < C + L_A - D_E
\]

2. \( A_P = L_A - C \), part of postponed demand is transported via \( r_A \), while the rest becomes excess demand. This situation occurs when there is exactly enough or not enough capacity to transport all express demand and postponed demand. In other words,

\[
P^B_t \geq C + L_A - D_E
\]

Therefore given \( D_E = i \geq C \), the expected postponed Basic demand transported through the alternative route is as follows:

\[
E(A_P \mid D_E = i, i \geq C) = \sum_{k=1}^{C+L_A-i-1} k\pi_i + \sum_{k=C+L_A-i}^{L_B} (C + L_A - i)\pi_i \quad (30)
\]

In the second case \((D_E \leq C)\), at least part of the postponed demand is transported through the main route. As such, three situations may occur based on the amount of postponed demand needs to be transported that particular day:
1. \( A_P = 0 \), all postponed demand is transported via \( r_0 \). This occurs when there is enough capacity in \( r_0 \) to transport all express demand and postponed demand. In other words,

\[
P_B^t \leq C - D_E
\]

2. \( A_P = D_E + P_B^t - C \), all postponed demand is transported, and part of it via \( r_A \). This occurs when there is enough overall capacity to transport all express demand. In other words,

\[
C - D_E < P_B^t < C + L_A - D_E
\]

3. \( A_P = L_A \), all cargo transported via \( r_A \) are postponed demand. This situation occurs when there is not enough capacity in \( r_A \) to transport all postponed demand. In other words,

\[
P_B^t \geq C + L_A - D_E
\]

Thus, given \( D_E = i < C \), the expected postponed demand transported through the alternative route is as follows:

\[
E(A_P | D_E = i, i \geq C) = \sum_{k=C-i+1}^{C+L_A-i-1} (i + k - C)\pi_i + \sum_{k=C+L_A-i}^{L_B} L_A\pi_i \quad (31)
\]

Based on equations (30) – (31), we can derive the expected postponed demand transported via \( r_A \) as follows:

\[
E(A_P) = \sum_{i=0}^{C-i} P(D_E = i)E(A_P | D_E = i, i < C) + \sum_{i=C}^{L_B} P(D_E = i)E(A_P | D_E = i, i \geq C)
\]

\[
E(A_P) = \sum_{i=0}^{C-i} P(D_E = i)(\sum_{k=C-i+1}^{C+L_A-i-1} (i + k - C)\pi_i + \sum_{k=C+L_A-i}^{L_B} L_A\pi_i) + \sum_{i=C}^{L_B} P(D_E = i)(\sum_{k=1}^{C+L_A-i-1} k\pi_i + \sum_{k=C+L_A-i}^{L_B} (C + L_A - i)\pi_i)
\]

(32)

### 6.5 Expected Basic Demand Transported Via the Alternative Route

Given the amount of postponed demand needs to be transported that particular day \( (P_B^t = i) \), three situations may occur based on the amount of slots left as described in equation (23) and the amount of accepted Basic demand \( D_B \):

In the first situation \( (A_B^t = 0) \), no Basic demand accepted in that particular day is transported via \( r_A \). This occurs in one of two cases:
1. No slots are available, the sum of accepted Express demand and postponed demand is at least equal to the overall capacity. In other words,

\[ D_E + P_B^t \geq C + L_A \]

2. There are slots available to transport Basic demand \((D_E + P_B^t = C + L_A - s, s > 0)\). However, no Basic demand is accepted \((D_B = 0)\)

Therefore, we can calculate the probability that no Basic demand accepted in that particular day is transported via \(r_A\) as follows:

\[
P(A_B^t = 0 \mid P_B^t = i) = P(D_E + i \geq C + L_A) \\
+ \sum_{s=1}^{C+L_A-i} P(D_E + i = C + L_A - s)P(D_B = 0)
\]

\[
P(A_B^t = 0) = \sum_{i=1}^{L_B} \pi_i [P(D_E + i \geq C + L_A) \\
+ \sum_{s=1}^{C+L_A-i} P(D_E + i = C + L_A - s)P(D_B = 0)]
\]

(33)

In the second situation \((A_B^t = j, 0 < j < L_A)\), some Basic demand accepted in that particular day is transported via \(r_A\). This occurs in one of two cases:

1. There are \(j\) slots available \((D_E + P_B^t = C + L_A - j)\) and at least \(j\) Basic demand is accepted \((D_B \geq j)\).
2. There are more than \(j\) slots available \((D_E + P_B^t \geq C + L_A - j)\) and exactly \(j\) Basic demand is accepted \((D_B = j)\).

Therefore, we can calculate the probability that \(j\) \((0 < j < L_A)\) accepted Basic demand is transported via \(r_A\) as follows:

\[
P(A_B^t = j \mid P_B^t = i) = P(D_E + i = C + L_A - j)P(D_B \geq j) \\
+ \sum_{s=1}^{C+L_A-i} P(D_E + i = C + L_A - s)P(D_B = j)
\]

\[
P(A_B^t = j) = \sum_{i=1}^{L_B} \pi_i [P(D_E + i = C + L_A - j)P(D_B \geq j) \\
+ \sum_{s=1}^{C+L_A-i} P(D_E + i = C + L_A - s)P(D_B = j)]
\]

(34)
In the third situation ($A_B = L_A$), all cargo transported via $r_A$ are accepted Basic demand from the same day. This occurs only when there are at least $L_A$ slots available, that is all Express demand and postponed demand can be transported via $r_0$ ($D_E + P_B^t \leq C$), and at least $L_A$ Basic demand is accepted ($D_B \geq L_A$).

Therefore, we can calculate the probability that $j$ ($0 < j < L_A$) accepted Basic demand is transported via $r_A$ as follows:

$$P(A_B = L_A \mid P_B^t = i) = \sum_{s=L_A}^{C+L_A-i} (D_E + i = C + L_A - s)P(D_B \geq L_A)$$

$$P(A_B = L_A) = \sum_{i=1}^{L_B} \sum_{s=L_A}^{C+L_A-i} (D_E + i = C + L_A - s)P(D_B = j)$$

Based on equations (33) – (35), we can derive the expected Accepted Basic demand transported via $r_A$ as follows:

$$E(A_B) = \sum_{j=1}^{L_A} \sum_{i=0}^{L_B} \pi_i [P(D_E + i = C + L_A - j)P(D_B \geq j)$$

$$\quad + \sum_{s=1}^{C+L_A-i} P(D_E + i = C + L_A - s)P(D_B = j)]$$

$$\quad + L_A \sum_{i=1}^{L_B} \pi_i [\sum_{s=L_A}^{C+L_A-i} (D_E + i = C + L_A - s)P(D_B \geq L_A)]$$

(36)

### 6.6 Formulation of the CFCM (2,1,2) Model

In previous sections we have derived the expressions for expected accepted demand (9) – (10), expected excess Basic demand (27), expected Express (29), postponed (32), and Basic (36) demand transported via $r_A$. These expressions allow us to describe the objective function (8) as a function of the decision variables $L_E$, $L_B$, $L_A$ and the parameter $C$. Furthermore, we have defined the Markov equilibrium equations (24) – (25) which constrains the model. Therefore, we can now provide a formulation of the CFCM (2,1,2) Model.

$$\max_{L_E, L_B, L_A} J = f_E E(D_E) + f_B E(D_B) - \rho (E(A_E) + E(A_F) + E(A_B)) - pE(B_E)$$

Where

$$E(D_E) = \sum_{k=0}^{L_E-1} kp_E(k) + L_E(1 - \sum_{k=0}^{L_E-1} p_E(k))$$

$$E(D_B) = \sum_{k=0}^{L_B-1} kp_B(k) + L_B(1 - \sum_{k=0}^{L_B-1} p_B(k))$$
\[ E(E_B) = \sum_{m=0}^{L_B} m \sum_{q=0}^{L_B} P(D_E = C + L_A + m - q) \pi_q \]

\[ E(A_E) = \max \left\{ \sum_{k=C+1}^{L_E} (k - C)p_E(k) + (L_E - C)(1 - \sum_{k=0}^{L_E-1} p_E(k)), 0 \right\} \]

\[ E(A_P) = \sum_{i=0}^{C-i} P(D_E = i)( \sum_{k=C+i+1}^{C+L_A-i-1} (i + k - C) \pi_i + \sum_{k=C+L_A-i}^{L_R} L_A \pi_i ) \]

\[ + \sum_{i=C}^{L_B} P(D_E = i)( \sum_{k=1}^{C+L_A-i-1} k \pi_i + \sum_{k=C+L_A-i}^{L_B} (C + L_A - i) \pi_i ) \]

\[ E(A_B) = \sum_{j=1}^{L_B} \sum_{i=0}^{L_B} \pi_i [P(D_E + i = C + L_A - j) P(D_B \geq j) \]

\[ + \sum_{s=1}^{C+L_A-i} P(D_E + i = C + L_A - s) P(D_B = j) ] \]

\[ + L_A \sum_{i=1}^{L_B} \pi_i [ \sum_{s=L_A}^{C+L_A-i} (D_E + i = C + L_A - s) P(D_B \geq L_A) ] \]

Subject to:

\[ \pi_0 = \sum_{i=0}^{L_B} \pi_i [P(D_B = 0) P(D_E > C + L_A - i) \]

\[ + \sum_{s=0}^{C+L_A-i} P(D_E + i = C + L_A - s) P(D_B \leq s) ] \]

\[ \pi_{j,j>0} = \sum_{i=0}^{L_B} \pi_i [P(D_B = j) P(D_E > C + L_A - i) \]

\[ + \sum_{s=0}^{C+L_A-i} P(D_E + i = C + L_A - s) P(D_B = s + j) ] \]

\[ \sum_{i}^{L_B} \pi_i = 1 \]

\[ L_E, L_B, L_A \in \mathbb{N} \]

\[ \pi_q \geq 0 \]
6.7 Solution Method for the CFCM (2,1,2) Model

The CFCM (2,1,2) Model as formulated in the previous section suffers from the same drawbacks as the CFCM (1,1,2) Model: it is non-linear, generally not convex, and has integer decision variables. Therefore, the CFCM (2,1,2) should also be solved iteratively using a grid-search over all possible combinations \((L_E, L_B, L_A)\).

It is important to note that the added decision variables increases the number of combinations to search over. To be precise, for given capacities \(C\) and \(C_A\), a total of \(2 \sum_{L_A=0}^{C_A} (C + L_A)^2\) combinations, significantly more than \(2C^2\). As such, it may be advisable to resort to heuristics. For example, instead of a grid search, the problem can search over most, instead of all combinations in a systematic manner to determine the feasible solution.

7 Conclusion

In this study, we replicated the CFCM Problem as proposed by Van Riessen et al. (2015a). The results of our replication confirmed the findings of said paper. We found that the solution method proposed in the original work yields higher revenue than other approaches towards solving the CFCM (1,1,2) Problem. As such, we have confirmed the key insight of the aforementioned study that revenue is best increased by finding the optimal balance between offered services.

We then extended the study by analyzing the CFCM (2,1,2) Problem. That is, a case of the CFCM Problem with two one destination, two fare classes, and two routes which differ only in cost and capacity. This study succeeded in finding a solution method for the CFCM (2,1,2) Problem.

7.1 Limitations and Future Research

There are three limitations to our study. Firstly, we are limited in our ability to implement the findings of our extension by solving a case study. Therefore, we wish to see our findings implemented and tested in a future study. Secondly, we assumed that the two routes differ only in capacity and price. However, this is not the only way to model a CFCM (2,1,2) problem. Thus, we hope that a future study will attempt to model and solve other forms of the CFCM (2,1,2) Problem. Thirdly, we assumed that \(L_A\) is decided at the tactical level. This assumption made it possible for the CFCM (2,1,2) Problem to be described by a single optimization problem. However, by not allowing flexibility at the operational level, there might be lost revenue.

Furthermore, we hope to see the findings of our research employed for the formulation of a general CFCM \((r,1,2)\) Problem. In general, the CFCM Problem is still a young topic. The four topics of future research that we have proposed will shed even more light on the potential of revenue management in hinterland transport.

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References


