

# **Bachelor Thesis**

# The Credit Ratings Game and its effect on firm investment

By

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**Abstract:** The spectacular collapse of structured financial products, which were given top ratings by the Credit Rating Agencies in the last financial crisis, brought back the debate regarding the importance and relevance of these agencies and their so-called "issuer pays" business model. This paper shows, from a game theoretical perspective, the influence of Credit Rating Agencies on firm investment. This is done through a signalling game involving cheap talk. The model outlines three key variables that affect firm investment: (1) the bias of the firm regarding its value, (2) the bias of the Credit Rating Agency regarding a firm's value, and (3) the cost of credit rating. These variables have a significant effect on communication between the players in this game and thus, by extensions, on firm investment.

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# I. Introduction

Corporations have several ways of raising capital. They can for example contract bank loans or issue new publicly traded securities. Another popular method used by companies to raise new capital is to issue fixed income securities to be traded on financial markets. When this is the case, companies will have an incentive to have these securities evaluated by Credit Rating Agencies (CRAs) in order to attract investors, especially banks and institutional investors who are limited in their investments to bonds holding an "investment grade" rating. Ratings can be seen as the creditworthiness of a firm.

The dramatic failure of many sophisticated structured finance product, despite their top ratings, has brought back the debate surrounding CRA's. The first published bond-rating dates back to the beginning of the 20<sup>th</sup> century, with the issue by John Moody in 1909 of ratings on railroad bonds (White, 2010). Moody would soon be followed by the entry in the ratings industry of Poor's Publishing Company in 1916, Standard Statistics Company in 1922<sup>1</sup>, and Fitch Publishing Company in 1924.

Two interesting features can be pointed out in the history of credit ratings. The first one is that CRAs precede the birth of the Securities and Exchange Commission (SEC), which was created in 1934 following the great depression. Second, over a century after their inception, these same agencies represent over 90% of the global market share in the credit rating industry. This does not mean however that the industry has remained unchanged in all its years of existence. Over the years, and through the implementation of several legislative and regulatory changes, the SEC has granted CRAs increasing importance. The first notable change occurred in 1936, when a federal regulation was implemented forbidding banks to invest in "speculative investment securities" and restricted their investments to bonds presenting "investment grade" ratings. This change meant that CRAs would now hold a key role in the regulatory environment of financial markets. A second important date for the credit rating industry was when, in the early 1970s, the agencies decided to shift their business model from an "investor pays" scheme to an "issuer pays" model. The former model saw banks and other institutional investors purchase rating manuals from the relevant CRAs that would allow them to follow guidelines to figure out which corporate bonds achieved the necessary "investment grades" to invest in. In the "issuer pays" scheme however, companies issuing bonds would now pay the CRAs directly to have their securities evaluated. This transfer in the payment mechanism did not cause significant protest at the time and CRAs would continue to operate successfully, or almost, for over 40 years.

Although several scandals such as Enron or Worldcom have led many to raise concerns on the efficiency of CRAs, it is the 2007 financial crisis that has seriously shattered the credit rating industry and hindered the CRAs reputation. Since then, extensive literature has been focusing on CRAs, covering grounds from regulatory issues to moral hazard.

<sup>&</sup>lt;sup>1</sup> The merger of Poor's Publishing Company with the Standard Statistics Company in 1941 gave birth to the Standard and Poor's Corporation

The purpose of this paper will to look at the effect of CRAs on firm investments. This research will be done from a game theoretical perspective through a signalling game involving cheap talk. This paper will thus focus on the following question:

# "Do credit rating agencies have a positive effect on the funding for firms in a cheap talk model?"

The intuition behind the use of cheap talk in this research is the existence of information asymmetry between the concerned agents. This model has three essential characteristics that fit in well in a model between CRAs, investors and issuers (firms): communication is 1) costless, 2) non-binding, and 3) unverifiable (Farrell, 1987).

In order to assess our research, we will focus on two different models, one involving a world without credit rating industries where only two agents exist: the issuing firm will send a message concerning the creditworthiness of its company to the investor. The investor will assess this message decide on an interest rate that the company will incur. In the second model, a third party will enter the world in the form of a CRA, whereby the issuing company will send the message to this third party at first, and the CRA will then communicate an advice on the interest rate the investor should charge to the firm. It is important to note that for reasons of simplicity, the second model will assume a world in which only one credit rating agency exists. The reason for such assumption is that it will avoid the situation in which the issuing firm will be looking for a CRA that will be the most willing to inflate ratings. In the credit rating industry, this issue is known as "issuer shopping" (Stolper, 2009) and is not relevant in the scope of this research.

The remainder of this paper will have the following structure. Section 2 will go through the existing literature on credit rating agencies, both empirical and theoretical. Section 3 will introduce the model in a market without the CRA, while section 4 will display the model in a model that includes this third party. Furthermore, the model that includes the agency will allow us to analyse the difference between issuer pays and the investor pays model. Section 5 will draw a comparison between the two models. Finally section 6 will conclude this study. The equations requiring proofs of calculation will be notified by a number and those proofs will be provided in the appendix.

# II. Literature review

Credit rating agencies, given their role and importance in the functioning of financial markets, have been the source of extensive literature. Since the financial crisis, the analysis has focused on the regulatory environment surrounding the credit rating industry.

CRAs have been particularly criticized for their role in the recent financial crisis. Baum, Karpava, Schäfer, and Stephan engaged in a study to assess the effects of CRA announcements on the value of the European currency and on the impact on sovereign bond yields of the major Eurozone countries. They found that a downgrade from a CRA reduces the value of the Euro and affects significantly sovereign bond yields. (Baum, Karpava, Schäfer, & Stephan, 2016).

The question of the regulations surrounding CRAs has been addressed extensively. Anno Stolper for instance argues collusive agreements to offer inflated ratings can be avoided by denying underperforming agencies the right to issue ratings in future periods and reward performing ones by limiting the number of agencies that are allowed to offer ratings (Stolper, 2009). Lawrence White on the other hand argues that less regulations on CRAs is a better response to the current issues that the industry faces. His intuition is that political pressures and extensive regulations raise the costs of providing ratings and will tend to discourage financial innovation. White takes the stand that regulations on CRAs will not serve financial markets, but that eliminating the reliance on ratings will alleviate the burden faced by CRAs and limit the incentive to release inflated ratings (White, 2010).

The switch to the issuer pays and credible alternatives to this model have also been discussed. Dion Bongaerts for instance discusses the potential improvement in social welfare that could arise with a return to the original investor-paid rating (the original system of CRAs). Alternative solutions such as the investor-produced rating (whereby investors have an incentive to produce their own credit assessment) or the implementation of a Franken rule (issuer pays and investor selects) options were also discussed (Bongaerts, 2014).

A similar study by Kashyap and Kovrijnykh discusses the different payment models for the CRAs and arrives to the conclusion that the investor pays-model gives more precise ratings than the issuer-pays model. They mention however that the free-rider problems associated with the investor-pays model causes inefficiencies (Kashyap & Kovrijnykh).

Closer to the approach of this paper, Patrick Bolton, Xavier Freixas and Joel Shapiro use a game theoretical approach to study the incentive of firms to give inflated ratings given a world with two types of investors: naïve and sophisticated investors. They find that there is asymmetry in the incentive to give inflated ratings given the state of the world. Indeed, in periods of economics and financial booms, when more naïve investors are present on the market, CRAs have a much higher incentive to give inflated ratings than in troublesome periods, characterized by a lower mass of naïve investors. Furthermore, they show that competition among CRAs may not be a good situation as it often leads to issuer shopping (Bolton, Freixas, & Shapiro, 2012).

Empirical research has also been done on CRAs. Alexander Matthies for instance made a literature review on the existing empirical research. The main conclusion of his review was that differences in rating classes do not correspond to equivalent differences in default probabilities Furthermore, he shows that the reactions to rating changes are asymmetrical, namely that downgrades exhibit stronger market reactions than upgrades (Matthies, 2013).

The question of the effect of increased competition in the credit rating industry was also addressed. Indeed, Becker and Milbourn show empirically that there is actually a negative relation between the number of agencies on the market and the overall quality of credit ratings. (Becker & Milbourn , 2010).

Eduardo Cavallo, Andrew Powell, and Roberto Rigobòn investigate whether credit rating agencies add value in terms of market information on fixed income securities other than what is already observed in bond spreads. Their tests focuses on sovereign debts and the effect of ratings changes, also analysing the effects of more or less anticipated events. The results show that there seems to be some additional information embedded in ratings that is not included in bond spreads, and that therefore, ratings seem to indeed add value (Cavallo, Powell, & Rigobòn , 2008).

In addition to the academic literature covering several topics regarding the credit rating industry, several reports are written every year from governmental or regulatory agencies to advocate for or against new policy implication or potential new regulations. For example, several reports were written following the 2007 financial crisis to assess the role of CRAs in rating sophisticated structured finance products. This shows the importance of the credit rating agencies and justifies the extensive interest that the agencies have spawned over the years

The cheap talk model that we will be using throughout this essay has also been subject to extensive academic research. In one of the first paper to treat a cheap talk model, Crawford and Sobel arrived to the conclusion that, in the scenario of costless communication, the more the agent's goals converge, the more direct communication is likely to play an important role. The authors also note that perfect communication is not to be expected unless the agent's interests are identical (Crawford & Sobel, 1982). In order to get a good insight in the cheap talk model, Sobel wrote an article, which gives an overview of its several implications and possible applications. Consistent with his earlier research with Crawford, he underlines that communication is more effective when preferences are similar. He also notes that although effective communication is difficult to achieve, effective cheap talk is possible but requires some common interest (Sobel, 2013).

# III. A model without Credit Rating Agencies

#### III. 1) Structure of the Game

Our cheap talk model is follows the framework developed by Crawford and Sobel (1982). There are two players, an informed sender (S) who sends an uninformed Receiver. In our particular application, the firm will act as the sender while the investor will take the place of the receiver. The firm sends a message "m" concerning the value of its company. The value of the company will be denoted as "v" and is assumed to be an accurate measure the present value of a company.

As pointed out by Gibson, the timing of a simple cheap-talk game is identical to that of a simple signalling game, only the payoffs between the agents will differ (Gibson, 1982). The timing of the game is as follows:

Nature draws a value "v<sub>i</sub>" for the issuing company (sender) from a set of possible values V ∈ [0,1] following a probability distribution p(v<sub>i</sub>), where p(v<sub>i</sub>) > 0, for each i and p(v<sub>1</sub> + … + v<sub>l</sub>) = 1 → V ~ u(0,1)

- 2. The sender observes its company value  $v_i$  and then chooses a message  $m_j$  from a set of feasible messages  $M = \{m_1, ..., m_l\}$
- 3. The investor (receiver) can observe  $m_i$  (but not  $v_i$ ) and chooses an investment level "i" (an action) which it will grant the issuing company from a set of feasible interest rates  $I \in [0,1]$

The payoffs for the players are given as follows:

- Payoff Firm:  $U^f = -[i (v + a)]^2$
- Payoff Investor:  $U^i = -[i v]^2$

These payoffs show the key feature surrounding cheap talk games: sending a message is costless, meaning the message will have no direct effect on the payoffs of the players. In the payoffs, parameter a > 0 is a stochastic term and can be considered as the measure of similarity between the players' preferences. When a = 0, these preferences are completely aligned.

The payoffs also give us the level of investment an investor is willing to make in a firm given its value. This level of investment is given by the expected value of the firm given the message (after his beliefs have been updated): i = E[v|m]. The level of investment made by the investor is thus positively related to the value of the firm.

The use of the cheap talk model here is justified by the fact that we are in a dynamic game. Indeed, dynamic games are characterized by a timing in which the different players do not choose an action simultaneously but sequentially, which is the case in our model as the investor will only choose an action after observing the first action (message) given by the firm. Furthermore, this is a game of incomplete information given the fact that the investor does not know the exact type of the firm. The purpose of a dynamic game of incomplete information and thus of our cheap talk model is to find a perfect Bayesian equilibrium (PBE).

In order to obtain a PBE in our model, the investor will need to have a prior belief about the type of the firm. In the case where only a single type exists, the investor's belief sets a probability of one on the single decision node. Furthermore, given their beliefs, the player's strategies (actions) need to be sequentially rational. This means that the strategies must be consistent and optimal with respect to the player's beliefs at each step of the game and the subsequent strategies of the other player. In a PBE, the investor, after observing the first action (message) from the firm, will update his beliefs regarding the firm's type according to Bayes rule. This is called the posterior belief.

Now that the conditions of the game have been defined, it is now possible to focus on the different possible equilibrium outcomes.

# III.2) Pooling equilibrium: "Babbling"

An interesting feature of the cheap-talk model compared to the signalling model is that it always contains a pooling equilibrium (Gibson, 1982). The intuition is that, since a message does not have any direct influence on the senders' payoff, should the receiver choose to ignore all messages, then pooling is the best answer for the Sender. This situation is formerly called "Babbling equilibrium". The intuition behind this equilibrium is that, regardless of the message sent by the issuing firm, the investor will choose to ignore it. In figure one, this means that no matter what value v a firm has, the message can stand anywhere on the interval, the investor will not take it into consideration. Since "babbling" discussions do not infer any information on the firm's value, his prior and posterior beliefs will be equal:

$$Prior = Posterior = E[v|m] = \frac{1}{2}$$

$$EU_i\left(E(v|m) = \frac{1}{2}\right) = -\int_0^1 \left(\frac{1}{2} - v\right)^2 dv = -\frac{1}{12}$$

$$EU_f\left(E(v|m) = \frac{1}{2}\right) = -\int_0^1 \left(\frac{1}{2} - v - a\right)^2 dv = -a^2 - \frac{1}{12}$$

Figure 1:



We can see from figure 1 that firms have no incentive to deviate from the message type they send to the investors, as there is only one step and thus no other message to deviate towards.

## III.2) Separating equilibrium: Firms reveal their true value

We have just examined the babbling equilibrium in this cheap-talk model. Given that all cheap-talk games have a possible pooling equilibrium, the interesting question is thus to ask whether a non-pooling equilibrium exists. A separating equilibrium occurs when different Sender-types have different preferences over the Receivers' actions. Let us first assume that issuing firms will send a message  $m_1 = v_i$  and thus indicate their true value to the investors. This means the investor will update his belief: E[v|m] = v.

This action from the decision maker (interest rate chosen by the investor) thus yields the following utility for the investor:

$$EU_{i} = -\int_{0}^{1} (v - v)^{2} dv$$
$$= -\int_{0}^{1} (0)^{2} dv$$
$$= 0$$

We can see that the situation in which firms communicate their true value is optimal for the investor (the expected utility is maximized at 0).

Let's now see what the consequences of a "true" message regarding company value will have on the firms' expected utility:

$$EU_f = -\int_0^1 (v - v - a)^2 dv$$
$$= -\int_0^1 -a^2 dv$$
$$= -a^2$$

Given the beliefs of the investor, would a firm want to deviate from this message? To see if this is the case, we can set a different message, which includes the issuing firms' bias:  $m_2 = v_i + a_i$ . The action that will be taken by the investor given his beliefs is  $E[v|m_2] = v + a_i$ , this leads the following utility to the firm:

$$EU_f = -\int_0^1 (v + a - v - a)^2 dv$$
$$= -\int_0^1 0$$
$$= 0$$

This shows us that it is optimal for a firm to send an inflated message to the investor concerning its value (Expected utility is maximized at 0)

$$EU_i = -\int_0^1 (v+a-v)^2 dv$$
$$= -a^2$$

We can see that  $0 > -a^2$ ,  $\forall a \in (0,1)$ . For the investor, this means that regardless of the value of a, all firms will have an incentive to deviate and thus no separating equilibrium can exist in this cheap-talk model. Formally, we can interpret this in our case as an incentive for each company to inflate its value in the message sent to the investor. An issuing firm will be better off inflating its true value given the negative payoff from communicating its exact value.

# III.3) Partially pooling equilibrium: Several message types (n=2,3)

Another interesting equilibrium to investigate is the partially pooling equilibrium. Crawford and Sobel defined the equilibrium as follows: the type space (in our case the range of company values  $v \in (0,1)$ ) is divided into "n" intervals. All the types in a given interval send the same message to the investor. The message differs however between the different types (Crawford & Sobel, 1982). It is interesting to note that the pooling (babbling) equilibrium defined above is actually a special type of partially pooling equilibrium where n = 1.

# • n=2:

In order to analyse a partially pooling equilibrium, we first consider an equilibrium where 2 message types are sent by different firms (figure 2).

Figure 2:



The size of message  $m_1$  is equal to  $d_1$ , and the size of message  $m_2$  is equal to  $(1 - d_1)$ .

$$E[v|m_1] = \frac{1}{2}d_1$$
$$E[v|m_2] = \frac{1}{2}(1+d_1)$$

The firm with a value  $d_1$  is indifferent between sending message  $m_1$  and sending message  $m_2$ . This firm will be indifferent when his expected utility is the same for sending message  $m_1$  or message  $m_2$ .

$$EU(m_{1}|v = d_{1}) = EU(m_{2}|v = d_{2})$$
  

$$\Leftrightarrow -\left(\frac{1}{2}d_{1} - (v + a)\right)^{2} = -\left(\frac{1}{2}(1 + d_{1}) - (v + a)\right)^{2}$$
  

$$\Leftrightarrow -\left(\frac{1}{2}d_{1} - (d_{1} + a)\right)^{2} = -\left(\frac{1}{2}(1 + d_{1}) - (d_{1} + a)\right)^{2}$$
  

$$\Rightarrow d_{1} = \frac{1}{2} - 2a$$
(1)

So, at a company value of  $v = d_1 = \frac{1}{2} - 2a$ , a firm will be indifferent between sending message 1 and message 2. Given the type space V = [0,1], a two-step equilibrium can only exist if:

$$d_1 > 0$$
  
$$\Leftrightarrow \frac{1}{2} - 2a > 0$$
  
$$\Rightarrow a < \frac{1}{4}$$

For values of a where  $a \ge 1/4$ , the preferences of the firm and the investor differ too much to allow for communication. Furthermore, we see that the distance between 0 and  $d_1$  is equal to 1/2 - 2a and the distance between  $d_1$  and 1 is 1/2 + 2a. The difference between message  $m_1$  and  $m_2$  is thus equal to 4a.

We can now observe, given this message from the indifferent company, what the expected utility will be for the investor:

$$EU_{i} = -\int_{0}^{\frac{1}{2}-2a} \left(\frac{1}{2}\left(\frac{1}{2}-2a\right)-v\right)^{2} dv - \int_{\frac{1}{2}-2a}^{1} \left(\frac{1}{2}\left(1+\frac{1}{2}-2a\right)-v\right)^{2} dv$$

$$= -\int_{0}^{\frac{1}{2}-2a} \left(\frac{1}{4}-a-v\right)^{2} dv - \int_{\frac{1}{2}-2a}^{1} \left(\frac{3}{4}-a-v\right)^{2} dv$$
$$= -a^{2} - \frac{1}{48}$$
(2)

Thus, in order for the investor not deviate from the 2-message equilibrium to the "babbling equilibrium":

$$-a^2 - \frac{1}{48} > -\frac{1}{12} \Longrightarrow a < \frac{1}{4}$$

We can also derive the expected utility for the firm:

$$EU_{f} = -\int_{0}^{\frac{1}{2}-2a} \left(\frac{1}{2}\left(\frac{1}{2}-2a\right)-(v+a)\right)^{2} dv - \int_{\frac{1}{2}-2a}^{1} \left(\frac{1}{2}\left(1+\frac{1}{2}-2a\right)-(v+a)\right)^{2} dv$$
$$= -\int_{0}^{\frac{1}{2}-2a} \left(\frac{1}{4}-v-2a\right)^{2} dv - \int_{\frac{1}{2}-2a}^{1} \left(\frac{3}{4}-v-2a\right)^{2}$$
$$= -2a^{2} - \frac{1}{48}$$
(3)

Just as for the investor, the expected utility for the firm will be higher in the case of a 2-message equilibrium compared to the "babbling outcome" when:

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$$-a^{2} - \frac{1}{12} < -2a^{2} - \frac{1}{48}$$
$$\implies a < \frac{1}{4}$$

• n=3:

Figure 3:

Now that we have found the 2-step partially pooling equilibrium, let's analyse what would happen when there are 3 possible message types.



Similarly to the situation with 2-steps, we analyse for which  $d_1$  a firm will be indifferent between sending message  $m_1$  and message  $m_2$ :

$$EU(m_1|v = d_1) = EU(m_2|v = d_1)$$

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$$\Leftrightarrow -\left(\frac{1}{2}d_{1} - (v+a)\right)^{2} = -\left(\frac{1}{2}(d_{2} + d_{1}) - (v+a)\right)^{2}$$
$$\Leftrightarrow -\left(\frac{1}{2}d_{1} - (d_{1} + a)\right)^{2} = -\left(\frac{1}{2}(d_{2} + d_{1}) - (d_{1} + a)\right)^{2}$$
$$\Rightarrow d_{1} = \frac{1}{2}d_{2} - 2a$$
(3)

Since we are looking at a 3-steps equilibrium, we are also interesting in finding which firm will be indifferent between sending message  $m_2$  and message  $m_3$ :

$$EU(m_2|v = d_2) = EU(m_3|v = d_2)$$
  

$$\Leftrightarrow -\left(\frac{1}{2}(d_2 + d_1) - (v + a)\right)^2 = -\left(\frac{1}{2}(1 + d_2) - (v + a)\right)^2$$
  

$$\Leftrightarrow -\left(\frac{1}{2}(d_2 + d_1) - (d_2 + a)\right)^2 = -\left(\frac{1}{2}(1 + d_2) - (d_2 + a)\right)^2$$
  

$$\Rightarrow d_2 = \frac{1}{2}d_1 - 2a + \frac{1}{2}$$
(4)

We now have two equations with two unknowns. Substituting  $d_1$  into the above equation:

$$d_2 = \frac{1}{2} \left( \frac{1}{2} d_2 - 2a \right) - 2a + \frac{1}{2}$$

Which yields:

$$d_2 = \frac{2}{3} - 4a$$
$$d_1 = \frac{1}{3} - 4a$$

Let us observe the difference in size between the different messages:

d<sub>1</sub> - 0 = <sup>1</sup>/<sub>3</sub> - 4a
d<sub>2</sub> - d<sub>1</sub> = <sup>2</sup>/<sub>3</sub> - 4a - <sup>1</sup>/<sub>3</sub> + 4a = <sup>1</sup>/<sub>3</sub> → Difference in size between m<sub>1</sub> and m<sub>2</sub> = 4a
1 - d<sub>2</sub> = 1 + 4a - <sup>2</sup>/<sub>3</sub> = <sup>1</sup>/<sub>3</sub> + 4a → Difference in size between m<sub>2</sub> and m<sub>3</sub> = 4a

It is interesting to observe that the difference in the size of the messages is the same for n = 2and n = 3. Furthermore, one can see that in order for an equilibrium to occur, again we must have:

$$d_1 > 0$$
  
$$\Rightarrow \frac{1}{3} - 4a > 0$$
  
$$\Rightarrow a < \frac{1}{12}$$

For values  $a \ge 1/12$ , the preferences of the firm and the investor differ too much to allow for communication. We can now observe, from the companies indifferent between sending message  $m_1$  and  $m_2$ , what the expected utility will be for the investor:

$$EU_{i} = -\int_{0}^{\frac{1}{3}-4a} \left(\frac{1}{2}d_{1}-v\right)^{2} dv - \int_{\frac{1}{3}-4a}^{\frac{2}{3}-4a} \left(\frac{1}{2}(d_{1}+d_{2})-v\right)^{2} dv - \int_{\frac{2}{3}-4a}^{1} \left(\frac{1}{2}(1+d_{2})-v\right)^{2} dv$$

$$= -\int_{0}^{\frac{1}{3}-4a} \left(\frac{1}{6}-2a-v\right)^{2} dv - \int_{\frac{1}{3}-4a}^{\frac{2}{3}-4a} \left(\frac{1}{2}-4a-v\right)^{2} dv - \int_{\frac{2}{3}-4a}^{1} \left(\frac{5}{6}-2a-v\right)^{2} dv$$

$$= -\frac{8}{3}a^2 - \frac{1}{108}$$
 (5)

The investor will not deviate to a "babbling" outcome when:

$$-\frac{8}{3}a^2 - \frac{1}{108} \ge -\frac{1}{12}$$
$$\Rightarrow a \le \frac{1}{6}$$

The expected utility for the firm in this 3-step equilibrium:

$$EU_{f} = -\int_{0}^{\frac{1}{3}-4a} \left(\frac{1}{2}d_{1} - (v+a)\right)^{2} dv - \int_{\frac{1}{3}-4a}^{\frac{2}{3}-4a} \left(\frac{1}{2}(d_{1}+d_{2}) - (v+a)\right)^{2} dv - \int_{\frac{2}{3}-4a}^{1} \left(\frac{1}{2}(1+d_{2}) - (v+a)\right)^{2} dv$$

$$= -\int_{0}^{\frac{1}{3}-4a} \left(\frac{1}{6} - 2a - (v+a)\right)^{2} dv - \int_{\frac{1}{3}-4a}^{\frac{2}{3}-4a} \left(\frac{1}{2} - 4a - (v+a)\right)^{2} dv - \int_{\frac{2}{3}-4a}^{1} \left(\frac{5}{6} - 2a - (v+a)\right)^{2} dv$$

$$= -\int_{0}^{\frac{1}{3}-4a} \left(\frac{1}{6} - 3a - v\right)^{2} dv - \int_{\frac{1}{3}-4a}^{\frac{2}{3}-4a} \left(\frac{1}{2} - 5a - v\right)^{2} dv - \int_{\frac{2}{3}-4a}^{1} \left(\frac{5}{6} - 3a - v\right)^{2} dv$$

$$= -\frac{11}{3}a^{2} - \frac{1}{108}$$
(6)

Given this expected utility, we can now observe for which values of *a* a firm will not have an incentive to deviate to the "babbling" outcome:

$$-\frac{11}{3}a^{2} - \frac{1}{108} > -a^{2} - \frac{1}{12}$$
$$\Rightarrow a < \frac{1}{6}$$

# III.4) Takeaways from the model without CRA

This model shows the impact of the bias of the firm (stochastic term a) on the level of communication that occurs between the investor and the firm. Indeed, the impact of a can be summarized as follows:

- 1) a > 1/4: No communication occurs. The preferences between the investor and the firm differ too much to allow for it. This is the "babbling outcome".
- 2) 1/4 > a > 1/12: This is the case where we find a 2-message partially pooling equilibrium as well as a "babbling" equilibrium. The preferences between the investor and the firm differ, but not enough not to allow some communication to occur.
- 3) 1/12 > a: In this case, we have both a 2 and 3-message partially pooling equilibrium as well as the "babbling" equilibrium. Communication in this case is optimal within the scope of this model.

The intuition behind this model is now clear concerning the impact of *a*. Indeed, we can see that, the smaller the *a*, the more message types there are, and thus richer communication occurs. Furthermore, in the case where only one message type exists, the investors benefit the most from

richer communication, Indeed, expected utility is maximized for the investor when a = 0. Firms on the other hand have an incentive to inflate their value. This difference in preferences explains why no separating equilibrium can be found in this model.

It is also interesting to analyse if both players benefit from richer communication. In order to do this, we can compare the differences in expected utilities between the different equilibriums. First, let us compute the difference in expected utility for the firm:

$$EU_f^{n=2} - EU_f^{n=1} = \frac{1}{16} - a^2$$
$$EU_f^{n=3} - EU_f^{n=2} = \frac{5}{432} - \frac{5}{3}a^2$$

Now for the investor:

$$EU_i^{n=2} - EU_i^{n=1} = \frac{1}{16} - a^2$$
$$EU_i^{n=3} - EU_i^{n=2} = \frac{5}{432} - \frac{5}{3}a^2$$

There are two features that stand out from these differences. First, we can see that both players will benefit from richer communication. Indeed, in order to benefit from larger communication we must have:

$$EU_i^{n=2} - EU_i^{n=1} = \frac{1}{16} - a^2 > 0$$
$$\Rightarrow a < \frac{1}{4}$$

and,

$$EU_i^{n=3} - EU_i^{n=2} = \frac{5}{432} - \frac{5}{3}a^2 > 0$$
$$\Rightarrow a < \frac{1}{12}$$

We see that we find again the necessary conditions in order for the equilibrium at n = 2and n = 3 to hold. Hence, we see that indeed both players will benefit from larger communication. This is consistent with the intuition that richer communication offers higher expected utilities for the players. The second interesting feature is that, not only do both players benefit from larger communication, but do so in equal proportions. The differences in expected utilities between thus confirm that a low stochastic term *a* is thus optimal for both players. However, having seen that no separating equilibrium exists in this model, we can conclude from this model that firms will have an incentive to inflate their true value when communicating with the investor, but that this positive bias should remain as low as possible.

Now that we have seen the implications of this model in a world without a credit rating agency, we can now focus on a situation that includes this third party.

## IV. A model with Credit Rating Agencies

# IV.1) The model

Is the existence of CRAs justified are they beneficial to the investor and the firm? This question is an interesting one and one that is often asked. In order to analyse the impact of CRAs on firm investment, we will look at their influence on the expected utilities of the firm and the investor. Furthermore, this will give us the opportunity to compare the two business models presented in the introduction, namely the former "investor pays" and the current "issuer pays" system. Hence, we will be able to assess whether the shift in the business model of the CRAs that occurred in the 1970s was an optimal choice with respect to the level of communication that occurs between the involved players. The relevant game in this model will the same as in the previous one, with only a slight difference in the payoffs. Let us first look at the investor pays model that existed before the 1970s.

#### IV.2) Investor pays model:

Let us assume that the investor now has an outside option. Instead of communicating directly with the firm, he can now ask for advice from the CRA. However, should he ask for this advice, he will have to bear a cost *c*. Given the fact that the CRA can perfectly assess the type of the firm, this cost will allow the investor to perfectly learn this type. In this model, the investor does not communicate directly with the firm anymore, hence the message from the firm becomes irrelevant to the investor. This is a case where communication between both parties breaks down.

The payoff of the investor will be modified given the existence of a CRA:

• Payoff Investor:  $U_{CRA}^i = -[i-v]^2 - c$ 

The relevant question here is thus to ask when will the investor ask the credit rating agency for advice. Given that the investor now knows the exact value (type) of the firm through its communication with the CRA, this situation resembles the separating equilibrium from the first model, whereby the firms would reveal their true value. Note that we have seen that the separating equilibrium does not exist in the first model as the firm always has an incentive to inflate its value, and the investor anticipates this. In the case we are now confronted to however, the firm does not have the possibility to deviate as it is with the CRA that the investor is communicating. The expected utility for the investor when he pays for the advice from the CRA is thus:

$$EU_i^{CRA} = -\int_0^1 (v - v)^2 - c \, dv$$
$$= -\int_0^1 -c \, dv$$

As mentioned before, in the situation where it is the investor who pays for the advise from the CRA, he can also choose not to ask for this information. The conditions under which he will decide whether to ask for this advise will thus depend on the value of *c* compared to his expected utilities under the different equilibria from the first model.

First, we will see under which condition the investor will prefer the outside option of communicating with the CRA to the original situation of communicating directly with the firm in the situation of a "babbling" equilibrium:

$$EU_i^{n=1} = -\frac{1}{12} < -c$$
$$\Rightarrow c < \frac{1}{12}$$

We can see here that if the cost "c" is lower than 1/12, the investor will prefer to communicate with the CRA in order to obtain the true type of the firm. Should the cost exceed 1/12, we would be back to the "babbling" outcome whereby, regardless of the sender's message, the investor will choose to ignore it.

Likewise, let us observe the condition for "c" for which the investor will prefer communicating with the CRA rather than with the firm in the case where there are two different message types:

$$EU_i^{n=2} = -a^2 - \frac{1}{48} < -c$$
$$\Rightarrow c < a^2 + \frac{1}{48}$$

We can see that compared to before, the bias of the firm now comes into consideration. We must thus consider the condition for *a* under which the 2-message equilibrium can occur:

$$\begin{cases} \frac{1}{4} > a > \frac{1}{12} \\ \frac{1}{12} > a \end{cases}$$
$$\Rightarrow a < \frac{1}{4} \end{cases}$$

Let us see what happens to the value *c* when we plug in the largest value of a to allow for such equilibrium:

$$c < \left(\frac{1}{4}\right)^2 + \frac{1}{48}$$
$$\Rightarrow c < \frac{1}{12}$$

We can see that, as before, the investor will prefer to bear a cost "c" for the advice from the CRA rather than communicating directly with the firm when there are two possible message types.

Finally, the condition for "c" for which the investor will prefer to bear the cost in order to obtain the firm's true value compared to communicating directly with the firm in a 3-message equilibrium:

$$EU_i^{n=3} = -\frac{8}{3}a^2 - \frac{1}{108} < -c$$
$$\Rightarrow c < \frac{8}{3}a^2 + \frac{1}{108}$$

We know that in order for an equilibrium to occur in a situation where there are three different message types, *a* must hold the following condition:

$$\frac{1}{12} > a$$

Now, as for before, we can see under which condition of "c" the investor will prefer to bear the cost of the advice rather than communicating directly with the firm:

$$c < \frac{8}{3} * \left(\frac{1}{12}\right)^2 + \frac{1}{108}$$
$$\Rightarrow c < \frac{1}{36}$$

For a cost "c" lower than 1/36, the investor will prefer to bear the cost of advice in order to find out the firm's true type rather than communicating directly with it.

We can see from these different conditions that, as the condition for *a* decreases, the cost of advice *c* must also decrease. This is consistent with the intuition that the existence of a CRA becomes more and more obsolete as communication between the investor and the firm becomes more efficient.

#### IV.2) Issuer pays model:

We now consider the case where the investor must decide, before any communication, whether he prefers to communicate with the CRA or directly with the firm. This decision is called the commitment of the investor. Should he decide to communicate directly with the firm, the several equilibria calculated in the first model remain. On the other hand, if the investor chooses to communicate with the CRA, a different set of equilibria will arise. Once again we assume that the credit rating agency can perfectly assess the type of the firm. However, given that it is now the firm who pays for the CRA, the latter will have a bias regarding the firm's type. We will assume this bias to be  $a^c$ , where  $a^c < a$ , meaning that the bias of the CRA is smaller than the bias of the firm. Once again, this assumption is realistic as firms will know that if  $a^c > a$ , less communication will occur, which is not optimal for neither the firm, nor the investor. Therefore, we can consider  $a^c$  to take the value:

## $a^c = \beta * a$ , with $\beta \in (0,1)$

With respect to this model, we can consider  $\beta$  to be the worry a CRA will have regarding its reputation. In our case, a CRA will be extremely concerned with its reputation for values of  $\beta$  close to 0 (when  $\beta = 0$ , the CRA will reveal the firm's true value despite the fact that the issuing firm is paying for the rating:  $a^c = 0$ ). On the other hand, when  $\beta$  gets closer to 1, the CRA has less incentive to reveal the firm's true value (for  $\beta = 1$ , the CRA will show the same bias than the issuing firm:  $a^c = a$ ). Given that we have seen in the model without a CRA that a lower bias

allows for more efficient communication, one can easily see that better communication will occur if the investor believes that the CRA has a concern about its reputation. It is thus interesting to note that the SCE holds the right to designate the "nationally recognized statistical rating agencies" (NRSRO), and, although the conditions to approve or reject an agencies' request to become an NRSRO have always been quite opaque, this shows that, in theory, the "counter-bias" value  $\beta$  we are using in this model does exist to some extent (White, 2010).

We can now take a look at the implication the CRA has with respect to our previous conclusions on the model without this additional party. Let us first note that the payoff of the firm now differs slightly with the *a* now taking the form of the CRA's bias  $a^c$ :

• Payoff Firm:  $U_{CRA}^f = -[i - (v + a^c)]^2 = -[i - (v + \beta * a)]^2$ 

The first thing to note from this modified payoff is that the results obtained will only differ in the sense that the equilibrium condition will be given for  $a^c = \beta * a$ . We thus once again have the following equilibrium conditions:

1) 
$$a^c > \frac{1}{4} \Rightarrow \beta a > \frac{1}{4} \Rightarrow a > \frac{1}{4\beta}$$

As we have seen before, this condition for a leads to the ("babbling") pooling equilibrium (n=1). In this situation, the preferences of the players differ too much to allow for communication.

2) 
$$\frac{1}{4} > a^c > \frac{1}{12} \Rightarrow \frac{1}{4} > \beta a > \frac{1}{12} \Rightarrow \frac{1}{4\beta} > a > \frac{1}{12\beta}$$

Under these conditions, we find both a "babbling" equilibrium and a 2-message equilibrium. In this case, some communication occurs between the players.

3)  $\frac{1}{12} > a^c \Rightarrow \frac{1}{12} > \beta a \Rightarrow \frac{1}{12\beta} > a$ 

This is the case where all three equilibrium exist ("babbling", 2-message, and 3-message equilibrium). For these values of *a*, communication is optimal as far as our model is concerned.

As we have seen, a CRA with a high concern over its reputation ( $\beta \rightarrow 0$ ) will reveal the firm's real type to the investor and allows for better communication. This is consistent with the results we find here. Indeed, the lower  $\beta$  implies a higher value for  $1/4\beta$  and  $1/12\beta$  and thus makes the third condition of equilibrium more likely to occur (the case where all three equilibrium exist). As mentioned above, the third equilibrium condition allows for optimal communication with respect to our model.

## **V. Discussion**

We have now analysed the three possible models under investigation, one without the existence of CRA, one where the CRA is paid by the investor, and where it is the firm who pays for the CRA. From these models, we have been able to see that the expected utility for the investor depends on three variables:

- The bias of the firm *a*,
- The bias of the CRA  $\beta$ ,

### • The cost of the CRA *c*.

It is interesting to investigate how the model behaves with different values for these variables and what it means for our equilibrium. This will allow us to see under which circumstances of all three variables the investor would prefer the investor pays model over the issuer pays model, or when he would rather have a world without CRA.

We can thus first assume we are in a world with a low bias of the firm *a*, a bias of the CRA superior to that of the firm ( $\beta > 1$ ), and a high cost of CRA. Note that  $\beta > 1$  implies  $a^c > a$ . The expected utility of the investor would then be the following:

- $EU_i(No\ credit\ rating\ ) = -\frac{8}{3}a^2 \frac{1}{108} = -\frac{1}{108}^+$
- $EU_i(Investor pays) = -c$ , where c is high
- $EU_i(Issuer pays) = -\frac{8}{3}\beta * a^2 \frac{1}{108} = -\frac{1}{108}$

If we assume:

$$-c < -\frac{8}{3}a^2 - \frac{1}{108} \Rightarrow c > \frac{8}{3}a^2 + \frac{1}{108}$$

or,

$$\frac{8}{3}a^2 < c - \frac{1}{108} \Rightarrow a^2 < \frac{3}{8}c - \frac{1}{288}$$

then:

 $EU_i(No\ credit\ rating) > EU_i(Issuer\ pays) > EU_i(Investor\ pays)$ 

This is thus valid if the following assumptions are met:

1) The bias of the firm is low:  $a < \sqrt{\frac{8}{3}c - 1/288}$ 

2) The bias of the CRA is higher than that of the firm:  $\beta > 1$ 

3) The cost for the credit rating is sufficiently high:  $c > \frac{8}{3}a^2 + \frac{1}{108}a^2$ 

In this case, the investor will prefer not to have the CRA investigate. This intuition is logical as, should a CRA be used, the investor would either have to pay to high of a price for very little information (in the case of the investor pays model), or would receive information on the firm's value that is more inflated than what the firm itself would communicate (under the issuer pays model).

Let us now look at a set of assumption completely at the opposite of the first case, namely a high firm bias, an "efficient" credit rating agency (the bias of the CRA is lower than that of the firm,  $\beta < 1$ ), and a low cost of rating. Note that by "high *a*" we assume that the bias is such that we land in the "babbling" outcome (a > 1/4), furthermore we assume on the other hand that  $1/12 < a^c < 1/4$ . Once again we investigate what would happen with respect to the expected utility of the investor:

- $EU_i$ (No credit rating) =  $-\frac{1}{12}$
- $EU_i(Investor pays) = -c$
- $EU_i(Issuer \ pays) = -\beta * a^2 \frac{1}{48} = -\frac{7}{192} \cong -0.0365$

 $EU_i(Investor pays) > EU_i(Issuer pays) > EU_i(No credit rating)$ 

This is thus valid if the following assumptions are met:

- 1) A high bias of the firm:  $a > \frac{1}{4}$
- 2) A bias of the of the CRA:  $\frac{1}{12} < a^c < \frac{1}{4}$ , which yields for  $\beta: \frac{1}{12a} < \beta < \frac{1}{4a}$
- 3) A lower cost for credit:  $-c > -\beta a^2 \frac{1}{48} \Rightarrow c < \beta a^2 + \frac{1}{48}$

It is interesting to note that this set of assumption leads to the opposite order of preferences for the investor. The reason for this reversal using an exactly opposite set of assumption is that in this case, given the fact that the bias of the firm is large (and larger than that of the CRA), the investor will prefer to communicate with the CRA. Furthermore, the low cost of credit offered by the CRA induces the investor to pay himself for the credit rating in order to enjoy the true value of the firm. The investor, in this case, will always choose to work with a CRA as he will want to avoid the babbling outcome that would result given this set of assumptions.

For the third and final set of values for the three key variables of the model we will keep the assumptions regarding the biases of the firm and of the CRA unchanged compared to the previous set of assumptions. The difference here will be to assume a high cost of credit rating. This yields the following expected utilities for the investor:

- $EU_i$ (No credit rating) =  $-\frac{1}{12}$
- $EU_i(Investor pays) = -c$ , where c is high
- $EU_i(Issuer \ pays) = -\beta * a^2 \frac{1}{48} = -\frac{7}{192} \cong -0.0365$

Thus, if we assume:

- 1) A high bias of the firm:  $a > \frac{1}{4}$
- 2) A bias of the of the CRA:  $\frac{1}{12} < a^c < \frac{1}{4}$ , which yields for  $\beta: \frac{1}{12a} < \beta < \frac{1}{4a}$
- 3) A higher cost of credit rating:  $-\beta a^2 \frac{1}{48} > -c \Rightarrow c > \beta a^2 + \frac{1}{48}$

We will have the following order of preference for the investor:

 $EU_i$ (Issuer pays) >  $EU_i$ (No credit rating) >  $EU_i$ (Investor pays)

As we can see, we are now in a case where the investor will prefer the firm (issuer) to pay for the credit rating of its own security. The intuition behind this is that the high cost of rating refrains the investor to pay for the credit rating, as it would increase the cost of investment. The investor thus prefers to suffer the lower bias of the CRA in the valuation of the firm as it will consider the cost of this bias to be lower than the actual cost of paying for the credit rating himself.

The three different set of assumptions regarding the values of the key variables of our models show us the three different preferences with respect to the business models of the CRAs. This is summarized in the following table:

Years	Assumptions on variables	Business Model
	• $a < \sqrt{\frac{8}{3}c - \frac{1}{288}}$	
Before 1909	• $\beta > 1$	No CRAs
	• $c > \frac{8}{3}a^2 + \frac{1}{108}$	
	• $a > \frac{1}{4}$	
1909-1970's	• $\frac{1}{12a} < \beta < \frac{1}{4a}$	Investor pays
	• $c < \beta a^2 + \frac{1}{48}$	
	• $a > \frac{1}{4}$	
1970's-Today	• $\frac{1}{12a} < \beta < \frac{1}{4a}$	Issuer pays
	• $c > \beta a^2 + \frac{1}{48}$	

In the case before 1909, credit ratings had not yet been introduced on the market. The high cost of rating in this case may come from the fact that, given the very low accounting rules and standards at that time, it was complicated and timely for a potential CRA to give an accurate rating. Furthermore, it is possible that, given the lower access to the stock market at that time, it was more difficult for firms to find investors. Therefore, firms had an incentive to ensure near perfect communication by not inflating their value. This may thus explain the lower added value of a CRA in such a system as it would not be able to give additional information regarding the firm's type.

The release of the first corporate bond rating by John Moody in 1909 marked the first change in these assumptions. Indeed, between 1909 and the 1970's, the additional regulations on bookkeeping and compliance requirements that were implemented throughout the years made it easier for CRA's to correctly assess the firm's value and thus have a bias that would be lower than that of the firm. In that period, the assumption that firms had an incentive to inflate their value at a high level may be motivated by the fact that firms became increasingly about profit maximization. Perhaps the reason for this is the fact that the more and more investors were introduced to the stock market. One of the means to achieve this was to optimize firm investment through a lower interest rate provided by the investor. Given that a higher firm value means a lower interest rate, firms thus had an incentive to strongly inflate their true value. The low cost of CRAs however breaks down the communication between the firm and the investor, as the latter would now be happy to pay a CRA in order to obtain a firms' true type.

The 1970's were marked by the switch from an investor pays to an issuer pays model. This change has suffered extensive criticism throughout the years, especially in recent years following the 2007-2008 financial crisis, where the CRAs were blamed for having inflated the value of several toxic financial products. In our model, this shift may be justified by an increase in the cost of credit ratings. The increase in the cost of rating can be justified by the rise of structured finance. The role of CRAs in structured finance market has been the main topic of an extensive report by the International Organization of Securities Commissions. The report suggests that the increase in financial structured products and the role of CRAs in rating them should lead to the revision of the code of conduct that governs CRAs (Peterson, 2008). This shift in the financial sector may also explain why firms manage to keep a high bias when communicating on their value. Indeed, although more information concerning companies became available to investors, the complexity of structured financial products allowed firms to continue inflating their true value. Regarding the rise in the cost of advice, the increasing complexity of financial products meant CRAs started to spend more time and resources in assessing a firm's true value, thus increasing the cost of the advice. This increase in costs meant that investors would now want to receive information from CRAs, but they prefer the issuing firm to pay for it, at a cost of obtaining a value that does not fully reveal a firm's type. The assumptions leading to the issuer pays model thus show the risk of an inflated firm type even though communication occurs with a CRA.

It would be interesting to assess the conclusions of our cheap talk model empirically. However, the difficulty for this empirical approach would be to find efficient proxies for the key variables of our model, especially the variables concerning the bias of the firm and of the CRA.

# **VI.** Conclusion

This paper evaluates from a game theoretical perspective, using a cheap-talk model, whether CRAs have a positive effect on the funding of firms. As we have seen, the funding of firm in our model is affected by the interest rate given by the investor. The investor has the choice to assess the value of a firm himself or to ask for advice from a CRA. Our model shows that the impact of CRAs on firm investment depends on three key variables. The cost of the advice from the CRA, the bias of the firm who will want to inflate its true value, and the bias of the CRA should it be paid for by the firm. The first conclusion is that if a firm has a low bias, communication will perfectly occur between the firm and the investor and thus, a CRA would have a negative effect on firm investment. When firms exhibit a high bias on the other hand, the presence of a CRA will indeed have a positive effect on the funding of a firm. The cost of advice from the CRA will only be relevant in knowing who will pay for the credit rating. Indeed, a large cost for advice will induce the investor to prefer that the issuer pays for the rating, despite knowing that the rating will not perfectly reflect reality. When the cost of advice is low on the other hand, the investors will prefer to pay for the ratings themselves and thus enjoy the benefit of discovering the true value of the firm, and thus offer an optimal interest rate.

Further research could attempt to include several other payment systems to the CRAs to assess whether these new suggestions could positively influence the funding of firms. An interesting addition to this model would be to add the Franken rule suggested by Bongaerts. The Franken rule (named after US senator Al Franken, who first proposed this method) has the issuers take part in a market wide fund, which would then be used in order to finance ratings by the CRAs. A committee elected mainly by the investment community would then select the CRAs (Bongaerts, 2014). This type of business model in which issuers pay and investors select would thus offer an interesting middle ground between the issuer pays and investor pays model. It could also be interesting to see how another game theoretical model would change our conclusions. A persuasion game for example could accurately describe the type of communication which occurs between the different agents of our model. In this particular game setup, a speaker attempts to convince the listener to undertake a specific action. Glazer and Rubinstein define the characteristics of a persuasion game and notify that whether the listener decides to accept the request of the speaker depends the information possessed by the speaker. However, communication is limited and the speaker is restricted in the amount of evidence it can show to the listener (Glazer & Rubinstein, 2006). This specific type of game illustrates well the restrictions that may exist in the amount of communication that is possible between a firm, an investor, and a CRA.

# **VII.** Appendix

(1):

$$-\left(\frac{1}{2}d_{1}-(d_{1}+a)\right)^{2} = -\left(\frac{1}{2}(1+d_{1})-(d_{1}+a)\right)^{2}$$
  

$$\Rightarrow -\left(-a-\frac{1}{2}d_{1}\right)^{2} = -\left(-a-d_{1}+\frac{1}{2}(d_{1}+1)\right)^{2}$$
  

$$\Rightarrow -\left(-a-\frac{1}{2}d_{1}\right)^{2} = -\frac{1}{4}+a-a^{2}+d_{1}\left(\frac{1}{2}-a\right)-\frac{1}{4}d_{1}^{2}$$
  

$$\Rightarrow \frac{1}{4}-a+a^{2}-\left(-a-\frac{1}{2}d_{1}\right)^{2}-d_{1}\left(\frac{1}{2}-a\right)+\frac{1}{4}d_{1}^{2} = 0$$
  

$$\Rightarrow \frac{1}{4}-a-\frac{1}{2}d_{1} = 0 \Rightarrow 1-4a-2d_{1} = 0$$
  

$$\Rightarrow d_{1} = \frac{1}{2}-2a$$

$$EU_{i} = -\int_{0}^{\frac{1}{2}-2a} \left(\frac{1}{4}-a-v\right)^{2} dv - \int_{\frac{1}{2}-2a}^{1} \left(\frac{3}{4}-a-v\right)^{2} dv$$

First, let us solve the first integral independently:

$$-\int_{0}^{\frac{1}{2}-2a} \left(\frac{1}{4}-a-v\right)^{2} dv$$

Substitute:  $u = -a - v + \frac{1}{4}$  and du = -dv

$$= \int_{\frac{1}{4}-a}^{a-\frac{1}{4}} u^2 du = \left(\frac{u^3}{3}\right)_{\frac{1}{4}-a}^{a-\frac{1}{4}} = \frac{1}{96}(4a-1)^3$$

Now, let us solve the second integral independently:

$$-\int_{\frac{1}{2}-2a}^{1} \left(\frac{3}{4}-a-v\right)^2 dv$$

Substitute:  $u = -a - v + \frac{3}{4}$  and du = -dv

$$= \int_{a+\frac{1}{4}}^{-a-\frac{1}{4}} u^2 du = \left(\frac{u^3}{3}\right)_{a+\frac{1}{4}}^{-a-\frac{1}{4}} = -\frac{1}{96}(4a+1)^3$$

We thus solve for the expected utility of the investor:

$$EU_i = \frac{1}{96}(4a-1)^3 - \frac{1}{96}(4a+1)^3$$
  
=  $\frac{(4a-1)^3 - (4a+1)^3}{96} = \frac{-64a^3 - 48a^2 - 12a + 1 + 64a^3 - 48a^2 + 12a - 1}{96}$   
=  $\frac{-96a^2 - 2}{96} = -a^2 - \frac{1}{48}$ 

(3):  

$$-\left(\frac{1}{2}d_{1}-(d_{1}+a)\right)^{2} = -\left(\frac{1}{2}(d_{2}+d_{1})-(d_{1}+a)\right)^{2}$$

$$\Rightarrow -\left(-a-\frac{1}{2}d_{1}\right)^{2} = -a^{2}-\frac{1}{4}d_{1}^{2}+d_{1}\left(\frac{1}{2}d_{2}-a\right)+ad_{2}-\frac{1}{4}d_{2}$$

$$\Rightarrow a^{2}-a-\frac{1}{2}d_{1}2+\frac{1}{4}d_{1}^{2}-d_{1}\left(\frac{1}{2}d_{2}-a\right)-ad_{2}+\frac{1}{4}d_{2}=0$$

$$\Rightarrow -ad_{2}-\frac{1}{2}d_{1}d_{2}+\frac{1}{4}d_{2}=0$$

$$\Rightarrow 4a+2d_{1}-d_{2}=0 \Rightarrow 2d_{1}=d_{2}-4a$$

$$\Rightarrow d_{1}=\frac{1}{2}d_{2}-4a$$

(4):

$$-\left(\frac{1}{2}(d_{2}+d_{1})-(d_{2}+a)\right)^{2} = -\left(\frac{1}{2}(1+d_{2})-(d_{2}+a)\right)^{2}$$

$$\Rightarrow -a^{2}+ad_{1}-\frac{1}{4}d_{1}^{2}+d_{2}\left(\frac{1}{2}d_{1}-a\right)-\frac{1}{4}d_{2}^{2} = -\frac{1}{4}+a-a^{2}+d_{2}\left(\frac{1}{2}-a\right)-\frac{1}{4}d_{2}^{2}$$

$$\Rightarrow \frac{1}{4}-a+ad_{1}-\frac{1}{4}d_{1}^{2}+d_{2}\left(\frac{1}{2}d_{1}-\frac{1}{2}\right)=0$$

$$\Rightarrow -1+4a-d_{1}+2d_{2}=0 \Rightarrow 2d_{2}=1-4a+d_{1}$$

$$\Rightarrow d_{2} = \frac{1}{2}d_{1}-2a+\frac{1}{2}$$

$$EU_{i} = -\int_{0}^{\frac{1}{3}-4a} \left(\frac{1}{6}-2a-v\right)^{2} dv - \int_{\frac{1}{3}-4a}^{\frac{2}{3}-4a} \left(\frac{1}{2}-4a-v\right)^{2} dv - \int_{\frac{2}{3}-4a}^{1} \left(\frac{5}{6}-2a-v\right)^{2} dv$$

First, let us solve the first integral independently:

$$-\int_{0}^{\frac{1}{3}-4a} \left(\frac{1}{6}-2a-v\right)^{2} dv$$

Substitute  $u = -2a - v + \frac{1}{6}$  and du = -dv

$$= \int_{\frac{1}{6}-2a}^{2a-\frac{1}{6}} u^2 du = \left(\frac{u^3}{3}\right)_{\frac{1}{6}-2a}^{2a-\frac{1}{6}} = \frac{1}{324}(12a-1)^3$$

Now, let us solve the second component independently:

$$-\int_{\frac{1}{3}-4a}^{\frac{2}{3}-4a} \left(\frac{1}{2}-4a-v\right)^2 dv$$

Substitute  $u = -4a - v + \frac{1}{2}$  and du = -dv

$$= \int_{-\frac{1}{6}}^{\frac{1}{6}} u^2 du = \left(\frac{u^3}{3}\right)_{-\frac{1}{6}}^{\frac{1}{6}} = -\frac{1}{324}$$

Finally, let us solve the third and final component independently:

$$-\int_{\frac{2}{3}-4a}^{1} \left(\frac{5}{6}-2a-v\right)^2 dv$$

Substitute  $u = -2a - v + \frac{5}{2}$  and du = -dv

$$= \int_{2a+\frac{11}{6}}^{\frac{3}{2}-2a} u^2 du = \left(\frac{u^3}{3}\right)_{2a+\frac{11}{6}}^{\frac{3}{2}-2a} = \frac{1}{3} \left(\left(\frac{3}{2}-2a\right)^3 - \left(2a+\frac{11}{6}\right)^3\right)$$
$$= -\frac{16}{3}a^3 - \frac{4}{3}a^2 - \frac{101}{9}a - \frac{301}{324}$$

We thus solve for the expected utility of the investor:

$$EU_i = \frac{1}{324}(12a - 1)^3 - \frac{1}{324} - \frac{16}{3}a^3 - \frac{4}{3}a^2 - \frac{101}{9}a - \frac{301}{324}$$
$$= -\frac{8}{3}a^2 - \frac{1}{108}$$

(6):

$$EU_f = -\int_0^{\frac{1}{3}-4a} \left(\frac{1}{6} - 3a - v\right)^2 dv - \int_{\frac{1}{3}-4a}^{\frac{2}{3}-4a} \left(\frac{1}{2} - 5a - v\right)^2 dv - \int_{\frac{2}{3}-4a}^{1} \left(\frac{5}{6} - 3a - v\right)^2 dv$$

First, let us solve the first integral independently:

$$-\int_{0}^{\frac{1}{3}-4a} \left(\frac{1}{6}-3a-v\right)^{2} dv$$

Substitute  $u = -3a - v + \frac{1}{6}$  and du = -dv

$$\begin{aligned} &\int_{\frac{1}{6}-3a}^{a-\frac{1}{6}} u^2 du = \left(\frac{u^3}{3}\right)_{\frac{1}{6}-3a}^{a-\frac{1}{6}} = \frac{1}{3} \left( \left(a-\frac{1}{6}\right)^3 - \left(\frac{1}{6}-3a\right)^3 \right) \\ &= \frac{28}{3}a^3 - \frac{5}{3}a^2 + \frac{1}{9}a - \frac{1}{324} \end{aligned}$$

Now, let us solve the second component independently:

$$-\int_{\frac{1}{3}-4a}^{\frac{2}{3}-4a} \left(\frac{1}{2}-5a-v\right)^2 dv$$

Substitute  $u = -5a - v + \frac{1}{2}$  and du = -dv

$$= \int_{\frac{1}{6}-a}^{-a-\frac{1}{6}} u^2 du = \left(\frac{u^3}{3}\right)_{\frac{1}{6}-a}^{-a-\frac{1}{6}} = -\frac{1}{3}a^2 - \frac{1}{324}$$

Finally, let us solve the third and final component independently:

$$-\int_{\frac{2}{3}-4a}^{1} \left(\frac{5}{6}-3a-v\right)^2 dv$$

Substitute  $u = -3a - v + \frac{5}{6}$  and du = -dv

$$= \int_{a+\frac{1}{6}}^{-3a-\frac{1}{6}} u^2 du = \left(\frac{u^3}{3}\right)_{a+\frac{1}{6}}^{-3a-\frac{1}{6}} = \frac{1}{3} \left( \left(-3a-\frac{1}{6}\right)^3 - \left(a+\frac{1}{6}\right)^3 \right)$$
$$= -\frac{28}{3}a^3 - \frac{5}{3}a^2 - \frac{1}{9}a - \frac{1}{324}$$

We thus solve for the expected utility of the investor:

$$EU_f = \frac{28}{3}a^3 - \frac{5}{3}a^2 + \frac{1}{9}a - \frac{1}{324} - \frac{1}{3}a^2 - \frac{1}{324} - \frac{28}{3}a^3 - \frac{5}{3}a^2 - \frac{1}{9}a - \frac{1}{324}$$
$$= -\frac{11}{3}a^2 - \frac{3}{324} = -\frac{11}{3}a^2 - \frac{3}{324}$$

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