Impact of Tariff under Hecksher-Ohlin Comparative Advantage Setting and Firm Heterogeneity

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Abstract

This paper analyses the impact of tariff imposition on the industry and firm characteristics along with overall industry and consumer welfare under Hecksher - Ohlin comparative advantage setting with heterogeneous firms. The paper extends the paper of Bernard, Redding & Schott (2007) by adding tariffs and found that optimally, tariff should be imposed on the industry with comparative disadvantage in order to dampen the negative effect of trade liberalization on the firms operating in comparative disadvantage industry and the negative effect of trade liberalization on the real reward of the factor used intensively in the comparative disadvantage industry. The tariff not only dampens the negative effect but could also increase the intensity of the specialization effect due to trade liberalization.

I would like to express my sincere gratitude to Dr. J. Emami Namini for his valuable inputs and feedback, without which this thesis would not have been completed. Additionally, I would like to thank my father Ashok Danani, brother and sister-in-law Vishal and Piya Dasani whose support and faith in me allowed me to complete my thesis and master. Last, I would like to thank my close friends Banu Atav and Shreya Goel for providing me with constant support and encouragement throughout my studies. Without all of them, this accomplishment would not have been possible.

Contents

1 Introduction

The imposition of tariffs and the implementation of other trade restrictions has consistently been a topic of debate among policy makers. Recently, China has imposed 46% import duty on steel from UK, causing concerns in UK regarding possible job losses (Davies & Stewart, 2016). This move came after allegations that proposals from other EU members regarding dumping of steel by China into the EU was blocked by UK (Davies & Stewart, 2016). The US has also imposed temporary tariffs for steel in seven countries including China, due to dumping issues initiated by China (Rapoza, 2016). It has been argued that one of the reasons for the imposition of tariff is to protect the domestic industry (the so called infant-industry argument). The fact that China is dumping steel in the world market will cause issues in countries where steel is produced at a higher price than the selling price of Chinese steel. The less productive firms will face issues in surviving the competition and might be forced to close down, resulting in an increase in unemployment. Therefore, tariffs are imposed in order to protect the domestic industries. In this paper, the link between imposition of tariff and the consequence of the tariff on the firms' production activities, industry productivity, factor market and overall welfare will be analyzed.

This paper aims at extending the paper of Bernard, Redding & Schott (2007) regarding the analysis of comparative advantage in Melitz-setting of heterogeneous firms. Their paper investigated how, in equilibrium, countries, industries and firms interact in determining the impact of and response to trade liberalization. More specifically, they analyzed the interaction and impact of decreasing trade costs. Their model is characterized by firm heterogeneity and love of variety (similar to setting of Melitz (2007)) with two countries, two goods and two factors of production. This paper involves similar setting of comparative advantage, but with tariff imposition. This paper

will add to the existing stream of literature regarding the determination and imposition of tariffs by countries to protect home country/increase overall welfare (Venables, 1987; Krugman 1980; Gros 1987; Demidova & Rodriguez-Clare, 2009, Syropoulos 2002; Johnson 1953, Riezman 1982) and the stream of literature that extends the dynamic model analyses of Melitz (Demidova & Rodriguez-Clare, 2009; Felbermayr, Jung, & Larch, 2013; Baldwin & Forslid 2010; Costinot & Rodriguez-Clare, 2012).

The main findings of this paper are as follows: while tariff can be beneficial in terms of preventing severe deterioration of factor income and it is beneficial in order to allow domestic firms to operate, the impact on absolute domestic efficiency is negative as the average productivity decreases in the country imposing the tariff. Imposition of tariffs in comparative disadvantage industry increases the ratio of average industry productivity level in the comparative industry relative to comparative disadvantage industry (which magnifies the cross-country differences and increases welfare in terms of opportunity cost). The impact of tariff on consumers is ambiguous as it depends on the preferences of the consumers. Optimally, tariff should be imposed in the comparative disadvantage industry in order to allow specialization to still occur but reducing the negative side effect of trade liberalization in the comparative disadvantage industry.

The structure of the paper is as follows: Section 2 is literature review where related previous literature is discussed. The subsequent Section 3 explains the general setup of the model where the description and underlying assumptions of the model are explained and the model equilibrium is established. Section 4 presents the model equilibrium under trade liberalisation, after which Section 5 contains the discussion and implications of the main findings. Section 6 presents the overall conclusion, limitations and further suggestions, which is followed by appendix and the references.

2 Literature Review

Johnson (1953) analyzed optimal tariff and retaliation and proposed that countries may still improve welfare by imposing tariff even though there might be retaliation. In the paper, retaliation is defined in the form of imposition of an optimum tariff under the assumption that the other country's tariff remains unchanged. The model is that of two-country two-good model, where one country exports one commodity in exchange for imports of the other commodity. The analyses concluded that there are specific circumstances when imposition of tariff at the risk of retaliation can still improve the welfare of the country: country will win bilateral tariff war if the relative monopoly or monopsony power is large (Johnson, 1953).

Venables (1987) analysed tariff and industrial policy under a different setting: the settings of increasing returns to scale, product differentiation and free entry of firms (endogenously determined) and demand for differentiated products of the Dixit-Stiglitz preferences, extending the paper of Krugman (1980). The paper aimed at : developing a theory of trade under differentiated products where it is possible that firms have different market shares in various markets that they operate in, and analyzing tariff and industrial policies in a model characterized by the attributes mentioned above. In the model, welfare is impacted through the changes in the average production costs of the firms and the changes in the mix and number of products that are available to the consumers. The analyses concluded that welfare level of the country is higher under free trade compared to autarky and that domestic welfare can be improved by by import tariffs (policies that tax foreign firms) or domestic subsidies (in the form of cost subsidies or export sales subsidies) (Venables, 1987).

Syropoulos (2002) extended the analyses of Johnson (1953) and investigated the determinants of outcomes in tariff wars using a game theoretic

approach. The paper aimed to identify the channel(s) through which the size effect occurs and investigate how country size affects the price elasticity of the demand function, market power and welfare. The model is characterized by constant returns to scale and comparative advantage with identical consumer tastes. The analyses concluded that a sufficient condition for a country to win a bilateral tariff war is that its relative size to the other country is sufficiently large. A small country is better off (has better terms of trade) under free trade than under retaliation because the small country lacks monopoly/monopsony power and therefore the price of its importables under retaliation is below autarky level. A large country will exploit its monopoly/monopsony power and retaliation enables a large country to do so(Syropoulos, 2002).

Gros (1987) analyzed the effects of uniform ad valorem tariff based on Krugman's (1980) model. The paper used model of trade described by Krugman (1980) with product differentiation and monopolistic competition in order to analyse the rate of optimal tariff under retaliation and without retaliation. The paper aimed to examine the determinants of optimal tariff rate and the effects of the tariff on welfare under particular condition of intra-industry trade. The analyses concluded that there exists an optimal tariff, even for a small country, which is increasing in the size of the economy and degree of heterogeneity. This is because under the conditions of intra-industry trade, each producer retains some monopoly power and that the home country would benefit if the producers exercise the monopoly power abroad but not at home. The optimal tariff can achieve this by introducing price discrimination between foreign and domestic sales. The value of the optimal tariff for a small country is equal to the proportional mark up used by monopolistically competitive producers (Gros, 1987).

Melitz (2003) analysed a dynamic industry model (open economy) where the industry is characterized by increasing returns to scale and productivity

differences with Dixit-Stiglitz preference form. There is only one sector and intra-industry trade arises due to love of variety preferences and increasing returns to scale technologies. The analyses concluded that the exposure to trade will cause the least productive firms to exit and simultaneously induce entry of the most productive firms into the export market. Moreover, the exposure to trade reallocates the resources and profits to the most productive firms. Exposure to trade increases the cut-off productivity level and this decreases the number of firms operating. Additionally, only some of the firms (the most efficient ones) will export. The two selection effects reallocate market shares to the most efficient firms and contribute to aggregate productivity gain without improving the individual productivity of the firms(Melitz, 2003).

Bernard, Redding & Schott (2007) extended the Melitz model to include another industry and another factor of production in order to analyze the interaction of country, industry and firm characteristics in general equilibrium under Hecksher-Ohlin settings. Their model consists of two countries, two factors of production and firm heterogeneity under comparative advantage situation. Their analyses showed that trade liberalization results in reallocation of resources, both intra-industry and inter-industry and raised the average industry productivity and average firm output in all the sectors, and that this impact is more pronounced in the industries with comparative advantage relative to industries with comparative disadvantage. Additionally, trade liberalization decreases the price of the goods and increases overall welfare (Bernard, Redding, & Schott, 2007).

Demidova & Rodriguez-Clare (2009) added to the Melitz model literature and investigated the effect of trade policy (export subsidies and import tariffs) on productivity and welfare based on the Melitz model for a small country. The country is said to be experiencing two distortions, namely: distortion in the allocation of consumer expenditures between foreign and

domestic varieties (there is too little spending on domestic varieties than in optimum); and there is a distortion associated with the fact that increase in the varieties of imports increases the consumer surplus, but the consumers do not take into account the impact of expenditure on imports on the number of imported varieties that is available for domestic consumption, resulting in the situation where the number of foreign varieties available to domestic consumers being less than optimum. They decomposed the welfare into four components, namely productivity, terms of trade, variety and curvature. The analysis concluded that an export subsidy causes an increase in the productivity but the overall welfare falls due to negative impact on the other three components of welfare. On the other hand, an import tariff increases welfare even though productivity falls (Demidova & Rodriguez-Clare, 2009).

Baldwin & Forslid (2010) extended the model and examined various aspects of trade liberalization under the setting of Melitz (2003) by adding another sector which is Walrasian, homogenous-goods sector with costless trade and exogenous wages. The analyses concluded that there is an anti-variety effect of trade liberalization (the goods traded become more homogeneous) and that this effect is more pronounced for small countries. However, trade liberalization under the model will always result in increase in welfare (Baldwin & Forslid, 2010).

Felbermayr, Jung & Larch (2013) added to the existing "tariff under Melitz-setting" literature by analyzing non-cooperative tariff policy in an asymmetric one-sector two-country (both large) model characterized by heterogeneity of firms as in Melitz (2003). They found that the optimal tariff consists of mark-up distortion, entry distortion and terms of trade effect; the optimal tariff increases in relative effect market size and degree of productivity dispersion; the response functions of the countries are negatively sloped; the tariffs are strategic substitutes and retaliation leads to an equilibrium tariff lower than optimal non-retaliation tariff; and that the Nash tariff is

increasing in relative average productivity, relative country size and the degree of productivity dispersion, while it is decreasing in variable trade costs (Felbermayr, Jung, & Larch, 2013).

The paper of Segerstrom & Sugita (2014) analysed the impact of tariffs on the industries, firms, consumers and countries under two-good, two-countries with potentially differing sizes and one factor of production setting where the industry is characterized by Melitz-type setting. In their model, they analysed the impact of unilateral and non-uniform trade liberalization. Their analyses showed that if the country being liberalized is relatively small in one of the industries, productivity falls in the liberalized industry and increases in the non-liberalized industry. If the liberalized country is relatively large, productivity rises in the liberalized industry as well as in the non-liberalized industry (the increase is greater in the non-liberalized industry). Both the scenarios result in an increase in the overall welfare of the liberalizing country (Segerstrom & Sugita, 2014).

The setting of this paper involves two-good, two-country, two-factors of production setting, just like that of Bernard, Redding & Schott (2007). This paper aims to analyse the impact of tariff on firms, consumers and industries under two-country, two-good and two-factor of production settings where the industries are characterized by heterogeneity of the firms under Melitz-type setting along with comparative advantage, the consumers are characterized by Cobb-Douglas preferences with a love for variety for each good (CES utility) and the firms are characterized by Cobb-Douglas production function which is impacted by the heterogeneity of the firms. The model follows the settings of Bernard, Redding & Schott (2007) and the specific contribution of this paper is to study the impact of tariffs under the setting employed in Bernard, Redding & Schott (2007).

3 Model

This model is characterized by two countries, Home and Foreign, with both countries having two factors of production, labor (L) and capital (K). There are two sectors of production, good A is capital-intensive and good B is labor-intensive. The standard assumptions of Hecksher-Ohlin model are applicable throughout this paper, which implies that both the countries are identical in terms of technologies and consumer preferences, but they differ in terms of factor endowments in a Hecksher-Ohlin setting. Home is labor-abundant and Foreign is capital-abundant. Factors of production are mobile between industries and within an industry, but they are not mobile between countries.

In the model, countries undergo trade liberalization and trade costs and tariff are incorporated into the model. Following Melitz (2003), the variable per-unit trade cost is modeled using the standard iceberg formulation, i.e. $\tau > 1$ units of good must be shipped in order for 1 unit to arrive at destination (Melitz, 2003). For analysis of comparative advantage, it is assumed that a country is trading with only one other country. Entry into an export market requires an investment cost and during production, the firms incur overhead production costs, both of which are higher than the costs incurred by firms operating only domestically. The presence of fixed entry and production costs mean that in equilibrium, some firms with relatively lower levels of productivity might still find it profitable to start production, but do not find it profitable to enter the export market (Bernard, Redding & Schott, 2007). The fixed export production cost uses both capital and labor with same intensities as the fixed domestic production costs. An additional variable of tariff will be included for imports; this means for a given price, the consumers pay higher price than the revenue received by the producers in the foreign country, the difference of which is the tariff revenue collected

by the government and redistributed.[1]

3.1 Consumers

Following Bernard, Redding & Schott (2007), the utility function of the consumers consists of two tiers: the upper tier of the utility which determines the consumption of the output of the two industries is characterized by a Cobb-Douglas utility function, the lower tier of the utility which determines the consumption of varieties takes the shape of CES (Bernard, Redding, & Schott, 2007).

$$C_i = [\int_{\omega \in \Omega_i} q_i(\omega)^\rho d\omega]^{\frac{1}{\rho}} \tag{1}$$

$$U = C_a^{\alpha_a} C_b^{\alpha_b} \tag{2}$$

where $\alpha_a + \alpha_b = 1$. Ω_i indicates the set of all available variety in a particular sector and that these goods are substitutes. This implies that ρ is a measure of the product differentiation, the elasticity of substitution between any two goods within the sector is $\sigma = \frac{1}{1-\rho}$ and that $\sigma > 1$ (Melitz, 2003). The above setup allows substitution between the industries outputs (upper tier) and allows for "love of variety" preferences (lower tier). Consequently, the dual price index within an industry is as follows:

$$P_i = [\int_{\omega \in \Omega_i} p_i(\omega)^{1-\sigma} d\omega]^{\frac{1}{1-\sigma}} \tag{3}$$

[1]The analysis will be done from the point of view of the home country. Subscript F will be used to denote foreign variables. No subscript for domestic variables

The consumers solve the within-sector consumer optimization problem:

$$max\ C_i = [\int_{\omega \in \Omega_i} q_i(\omega)^\rho d\omega]^{\frac{1}{\rho}} \tag{4}$$

subject to

$$\int_{\omega \in \Omega_i} p_i(\omega) q_i(\omega)(1+t) d\omega \tag{5}$$

Next, the consumers solve between-sector consumer optimization problem, formulated as follows:

$$max\ U = C_a^{\alpha_a} C_b^{\alpha_b} \tag{6}$$

subject to

$$E_a + E_b = E \tag{7}$$

where $E_i = P_i C_i$

Total factor income consists of the total wages paid to the total labor available in a country and the total rent income of the total capital available in the country. In the case of tariff imposition, the tariff revenue collected by government is then redistributed to the consumers and therefore also add to the income. The total income of both the factors of production and the tariff revenue is equal to the total consumer expenditure in both industries combined. Solving the optimization problem above, the total expenditure by the consumers on a particular sector equals $E_i = \alpha_i E$. The quantity of an imported good in the foreign country (and therefore the quantity of the exported good from the home country) is equal to:

$$q_{ix}(\varphi_{ix}) = \frac{p_i^{-\sigma} E_{Fi} (1+t_F)^{-\sigma}}{P_{Fi}^{1-\sigma}} = q_{Fim}(\varphi_{ix}) \tag{8}$$

The price index of imported goods produced at home country faced by the foreign consumers is higher than the price index of the same goods exported

from home due to the existence of the tariff [2].

$$P_{Fim} = (1 + t_F)P_{ix} \tag{9}$$

3.2 Producers

Following Bernard, Redding & Schott (2007), the production process consists of a fixed and variable cost which use both factors of production. The variable cost depends on the firm productivity level. Within an industry, all the firms share the same overhead cost and but their variable cost varies, depending on their productivity levels. The cost function is assumed to take Cobb-Douglas form.

$$\nu_i = [f_i + \frac{q_i(\varphi_i)}{\varphi_i}](w)^{\beta_i}(r)^{1-\beta_i} \tag{10}$$

For labor-intensive industry, $\beta_i > 0.5$. For capital intensive industry, $\beta_i < 0.5$

Due to fixed production costs, each firm chooses to produce a unique variety in equilibrium. The incorporation of differing factor intensities for different industries and differing factor endowments for different countries allows the incorporation of comparative advantage, where comparative advantage has a significant impact on how firms react and adjust to trade liberalization (Bernard, Redding & Schott, 2007).

There is a continuum of firms actively operating in the industry, with each firm having different productivity level. There is a residual demand curve with constant elasticity faced by all the firms and that they choose the same profit-maximizing markup of $\frac{1}{\rho}$ (Melitz, 2003).

The profit maximizing problem faced by the producers serving the do-

[2]The calculations for this section can be found in Appendix A and F.

mestic market is as follows:

$$\pi_i(\varphi_i) = p_i(\varphi_i)q_i(\varphi_i) - (f_i + \frac{q_i}{\varphi_i})w^{\beta_i}r^{1-\beta_i} \tag{11}$$

Since all the firms face the same elasticity of demand in the market, this yields a profit-maximizing equilibrium price which is equal to the constant mark-up over the marginal cost:

$$p_i(\varphi_i) = \frac{w^{\beta_i}r^{1-\beta_i}}{\rho\varphi_i} \tag{12}$$

The profit maximizing problem faced by the producers in the export market is as follows:

$$\pi_{ix}(\varphi_i) = p_{ix}(\varphi_i)q_{ix}(\varphi_i) - (f_{ix} + \frac{q_{ix}(\varphi_i)\tau}{\varphi_i})w^{\beta_i}r^{1-\beta_i} \tag{13}$$

Profit maximizing equilibrium price is equal to the constant mark-up over the marginal cost:

$$p_{ix}(\varphi_i) = \frac{\tau w^{\beta_i}r^{1-\beta_i}}{\rho\varphi_i} \tag{14}$$

The export price charged by the producers is a constant multiple of the domestic price due to the iceberg cost. The higher price reflects the higher marginal cost faced by the firms in the export market.

With the above pricing rule, the equilibrium revenue of the firms serving the domestic market is equal to:

$$r_i(\varphi_i) = \alpha_i E[\frac{\rho P_i\varphi_i}{w^{\beta_i}r^{1-\beta_i}}]^{\sigma-1} \tag{15}$$

The equilibrium revenue of the firms from serving the export market is equal to:

$$r_{ix}(\varphi_i) = \alpha_i E_F[\frac{\rho\varphi_i P_{Fi}}{\tau w^{\beta_i}r^{1-\beta_i}}]^{\sigma-1}(1+t_F)^{-\sigma} \tag{16}$$

Since the export price is a constant multiple of the domestic price, the revenue in the export market is proportional to the equilibrium in the domestic market (Bernard, Redding & Schott, 2007)

$$\frac{r_{ix}(\varphi_i)}{r_{id}(\varphi_i)} = \tau_i^{1-\sigma}(\frac{P_{Fi}}{P_i})^{\sigma-1}(\frac{E_{Fi}}{E_i})(1+t_F)^{-\sigma} \tag{17}$$

The total revenue earned by a firm active in production is as follows:

$$r_i(\varphi_i) = \begin{cases} r_{id}(\varphi_i) & \text{if it does not export,} \\ r_{id}(\varphi_i)[1+\tau^{1-\sigma}(\frac{P_{Fi}}{P_i})^{\sigma-1}(\frac{E_{Fi}}{E_i})(1+t_F)^{-\sigma} & \text{if it exports} \end{cases} \tag{18}$$

A firm that exports does not do so without also serving the domestic market due to the presence of fixed production costs and consumer love of variety (Bernard, Redding & Schott, 2007). The fact that not every firm exports mean that under free trade, the variety of goods available in a sector differ per country. The profits of the firm can be broken down into two components: profits earned from serving the domestic market and profits earned from serving the foreign market with respective fixed costs incurred.

$$\pi_{id}(\varphi_i) = \frac{r_{id}(\varphi_i)}{\sigma} - f_i w^{\beta_i} r^{1-\beta_i} \tag{19}$$

$$\pi_{ix}(\varphi_i) = \frac{r_{ix}(\varphi_i)}{\sigma} - f_{ix} w^{\beta_i} r^{1-\beta_i} \tag{20}$$

A firm will only engage in export if $\pi_{ix}(\varphi_i) > 0$. Therefore, the total firm profit is given by:

$$\pi_i(\varphi_i) = \pi_{id}(\varphi_i) + max\,[0, \pi_{ix}(\varphi_i)] \tag{21}$$

Post incurring the fixed-entry cost (a sunk cost) and entering the industry, the firms draw their productivity level from a distribution. A firm will only start production if the variable revenue is high enough to at least cover the

fixed costs of production. There are two zero profit cutoff (ZPC) conditions:

$$r_{id}(\varphi_{i^*}) = \sigma f_i (w)^{\beta_i} (r)^{1-\beta_i} \tag{22}$$

$$r_{ix}(\varphi_i^*) = \sigma f_{ix} (w)^{\beta_i} (r)^{1-\beta_i} \tag{23}$$

The above equation implies that the firms with a productivity level below the ZPC will not be able to generate enough revenues to cover the fixed cost and therefore immediately cease production and exit the market.

By combining both the ZPCs, the following expression linking productivity cut-off conditions is derived:

$$\frac{\varphi_{ix}^*}{\varphi_i^*} = \tau_i (1+t)^{\frac{\sigma}{\sigma-1}} (\frac{P_i}{P_{Fi}}) (\frac{E_{Fi} f_{ix}}{E_i f_i})^{\frac{1}{1-\sigma}} \tag{24}$$

As can be seen from the above equation, the ratio of the export cutoff productivity and zero-profit cutoff productivity depends on the relative price indices of the countries, the relative industry expenditure of the countries and the relative fixed costs of production, with the fixed cost of production for the exporting firms being higher than the fixed cost of production for the non-exporting firms. This implies only the relatively more productive firms will be able to serve the export market. When the value of R.H.S is greater than 1, only the relatively more productive firms will be able to serve the export market.

Following Melitz (2003), the distribution of firm productivity, ex-post, is conditional upon the successful entry and is cut off at the zero-profit productivity cut-off:

$$\mu(\varphi_i) = \begin{cases} \dfrac{g(\varphi)}{1 - G(\varphi_i^*)} & \text{if} \varphi_i > \varphi_i^*, \\ 0 & otherwise \end{cases} \tag{25}$$

Out of all the firms that enter into production, only a fraction of them draw a productivity level that is high enough in order to be profitable for them to enter the export market. The probability of a firm exporting, conditional upon successful entry is given by:

$$\chi_i = \frac{1 - G(\varphi_{ix}^*)}{1 - G(\varphi_i^*)} \tag{26}$$

The above equation implies that a fraction of firms draw a probability level that is not high enough to cover the fixed costs of production and therefore exit the industry; a fraction of firms $G(\varphi_{ix}^*) - G(\varphi_i^*)$ draw a productivity level that is enough to cover the fixed costs of production for the domestic market, but not high enough to cover the production costs for the export market; a fraction of firms $G(\varphi_{ix}^*)$ draw a productivity level that is high enough to serve the export market (and therefore the domestic market as well).

The steady state equilibrium regarding the firms is characterized by a constant mass of firms entering the industry and a constant mass of firms that are actively producing within the industry. Firms that are active in the industry also face a constant probability, δ, that they would exit the industry due to exogenous factors that would force the firm to exit (Melitz, 2003). The mass of firms that enter the industry and draws a productivity that is high enough to produce should be equal to the mass of firms that exit the industry (Bernard, Redding, & Schott, 2007).

$$[1 - G(\varphi_i^*)]M_{ei} = \delta M_i \tag{27}$$

The value of a firm is, therefore, equal to 0 if the firm draws a productivity below the domestic ZPC and is equal to the present value of the stream of future profits if the firm draws a productivity above the domestic ZPC. The present value of the stream of future profits must be discounted with the

probability of firm exit due to exogenous factors. The expected value of entry must be equal to the sunk cost of entry in the industry in order to ensure an equilibrium with positive production of the goods.

In equilibrium, the free entry condition states that the expected value of entry must be equal to the sunk cost of entry. Taking into account the export market, the free entry condition consists of product of the ex-ante probability of successful entry with the expected profitability of serving the domestic market, and the product of the ex-ante probability of successful entry with the expected probability of serving the foreign market (Bernard, Redding & Schott, 2007):

$$V_{ei} = \frac{1 - G(\varphi_i^*)}{\delta}[\overline{\pi}_{id} + \frac{1 - G(\varphi_{ix}^*)}{1 - G(\varphi_i^*)}\overline{\pi}_{ix}] = f_{ei} w^{\beta_i} r^{1-\beta_i} \tag{28}$$

The expected profits that is conditional upon successful entry of a firm is equal to:

$$\begin{aligned} \overline{\pi}_{id} &= f_i w^{\beta_i} r^{1-\beta_i} \int_{\varphi^*}^{\infty} [(\frac{\varphi_i}{\varphi_i^*})^{\sigma-1} - 1] \frac{g\varphi}{1 - G(\varphi_i^*)} d\varphi \\ \overline{\pi}_{ix} &= f_{ix} w^{\beta_i} r^{1-\beta_i} \int_{\varphi_x^*}^{\infty} [(\frac{\varphi_i}{\varphi_x^*})^{\sigma-1} - 1] \frac{g\varphi}{1 - G(\varphi_i^*)} d\varphi \end{aligned} \tag{29}$$

The weighted average of firms' productivity reflects the relative output shares of the firms with different productivity levels, which also reflects the aggregate productivity. An industry, with M firms, with a certain distribution of productivity level will have an average productivity level which will generate the same level of aggregate output as an industry, with M firms that are identical to each other with the productivity level equal to the average productivity level. The average revenue/profit is equal to the revenue/profit of a firm with weighted average productivity level. The weighted average

productivity are as follows:

$$\overline{\varphi}_{ix} = [\int_{\varphi^*_{iix}}^{\infty} \varphi^{\sigma-1} \frac{g(\varphi)}{1 - G(\varphi^*_{ix})} d\varphi]^{\frac{1}{\sigma - 1}} \tag{30}$$

$$\overline{\varphi}_{i} = [\int_{\varphi^*_{i}}^{\infty} \varphi^{\sigma-1} \frac{g(\varphi)}{1 - G(\varphi^*_{i})} d\varphi]^{\frac{1}{\sigma - 1}} \tag{31}$$

There is a higher level of weighted productivity in the export market than the domestic market due to the existence of some firms with relatively lower levels of productivity that still enables them to operate in the domestic market but prevents them from entering the export market.

The equilibrium productivity cut-off level is the intersection between the free-entry condition and zero-profit condition, i.e. it is the productivity level at which the expected entry value is positive in order for the firms to enter the industry and at which firms will be able to generate just enough revenue to recover the fixed costs incurred during production to continue production. The free entry condition can be written as a function of the productivity cut-offs and parameters of the model:

$$V_{ei} = \frac{f_i}{\delta} \int_{\varphi^*_i}^{\infty} [(\frac{\varphi_i}{\varphi^*_i})^{\sigma-1} - 1] g(\varphi) d\varphi + \frac{f_{ix}}{\delta} \int_{\varphi^*_{ix}}^{\infty} [(\frac{\varphi_i}{\varphi^*_{ix}})^{\sigma-1} - 1] g(\varphi) d\varphi = f_{ei} \tag{32}$$

The above equation shows that the free entry condition (and therefore the expected value of entry) under free trade is the sum of the expected value of entry under autarky and the expected value of entry into the export market. The value of the second term implies that the expected value of entry into the export market increases as the ratio between the export productivity cut-off and the zero-profit productivity cutoff decreases. This in turn increases the additional value of opening up to trade (Bernard, Redding & Schott, 2007). The ratio of the productivities depends on equation (30) and therefore an

imposition of tariff would have an impact on this ratio. [3]

3.3 Goods Market

From the equilibrium price rule, it can be seen that the price charged by a firm for a particular variety is inversely related to the firm productivity. The price indices are the weighted averages of the prices charged by the firms according to their own productivity level.

Under open economy model, the overall dual price index can be written as a function of the mass of firms serving the domestic market and the mass of firms serving the export market in the foreign country and the price charged by firm with average productivity.

$$P_i = [M_i(p_{id}(\overline{\varphi}_i))^{1-\sigma} + \chi_F M_{Fi}[(1+t_H)^{1-\sigma}(p_{ix}(\overline{\varphi}_{ix}))]^{1-\sigma}]\frac{1}{1-\sigma} \tag{33}$$

From the equation above, it can be seen that the price index for an industry in an open economy varies between countries because of the differences in the firms operating and their productivities in the domestic and export markets of each country, the existence of the variable trade costs (in this case τ), differences in the proportion of the firms active in the export market and their productivities, and the level of tariff imposed.

The mass of firms active in the market is equal to[4]:

$$M = \frac{E_i}{\sigma(\frac{f_{ei}w^{\beta_i}r^{1-\beta_i}}{1-G(\underline{\varphi}_i^*)} + f_i w^{\beta_i} r^{1-\beta_i} + \chi f_{ix} w^{\beta_i} r^{1-\beta_i})} \tag{34}$$

[3]The calculations for this section can be found in Appendix B and G.
[4]The calculations for this section can be found in Appendix C and H.

3.4 Factor Markets

The equilibrium allocation of labor and capital of the two sectors depend upon the weighted average productivity in the two sectors, relative cost of the factors of production the firms that are active in production in the industries, the fixed costs of production in both the sectors the fixed entry cost in both the sectors.

Units of labor (capital) used for the variable part of production can be derived by taking the partial derivative of the marginal cost with respect to the wage (rent), as this will determine the use of labor and capital in the variable part of production. Units of labor (capital) used in the fixed part of production and the fixed entry cost can be determined by taking the partial derivative of wage (rent) and this will determine the use of labor and capital in the fixed part of production.

The total labor and capital used is equal to the labor and capital used by firms with weighted average productivity multiplied by the number of active firms:

$$[f_i\beta_i(\frac{r}{w})^{1-\beta_i}M_i+\frac{q_i(\overline{\varphi}_i)}{\overline{\varphi}_i}\beta_i(\frac{r}{w})^{1-\beta_i}\frac{1}{\overline{\varphi}_i}M_i]+[f_{ix}\beta_i(\frac{r}{w})^{1-\beta_i}M_{ix}+\frac{q_{ix}(\overline{\varphi}_{ix})}{\varphi_{ix}}\beta_i(\frac{r}{w})^{1-\beta_i}\frac{1}{\varphi_{ix}}M_{ix}] + [f_{ei}\beta_i(\frac{r}{w})^{(1-\beta_i)}M_{ei}] \tag{35}$$

$$
\begin{aligned}
f_i[1-\beta_i](\frac{w}{r})^{\beta_i}M_i + \frac{q_i(\overline{\varphi}_{ix})}{\overline{\varphi}_i}[1-\beta_i](\frac{w}{r})^{\beta_i}\frac{1}{\overline{\varphi}_i}M_i] \\
+ [f_{ix}[1-\beta_i](\frac{w}{r})^{\beta_i}M_{ix} + \frac{q_{ix}(\overline{\varphi}_{ix})}{\overline{\varphi}_{ix}}[1-\beta_i](\frac{w}{r})^{\beta_i}\frac{1}{\varphi_{ix}}M_{ix}] \\
+ f_{ei}[1-\beta_i](\frac{w}{r})^{\beta_i}M_{ei}
\end{aligned}
\tag{36}
$$

The terms in the first set of brackets represent the amount of factors used in serving the domestic market, the terms in the second set of brackets represent the amount of factors used in serving the export market and the third set of terms indicate the amount of factors used in the entry process into the industry.

The Labor to Capital ratio in a sector is equal to [5] :

$$
\frac{L_i}{K_i} = \frac{\beta_i}{1-\beta_i}\frac{r}{w} \tag{37}
$$

3.5 Country Revenue and Welfare

Country welfare is analysed through the level of productivity of the industries (with additional analyses regarding magnification effect which is discussed below), the variety available to consumers (which depends on the mass of firms operating in the industry) and the real factor rewards of the country.

[5] The calculations for this section can be found in Appendix D and I

The revenue from the industry is equal to:

$$R_i = E_i M_i[\frac{p_i(\overline{\varphi}_i)}{P_i}]^{1-\sigma} + \chi_i E_{Fi} M_i[\frac{\tau p_{id}(\overline{\varphi}_{ix})}{P_{Fi}}]^{1-\sigma}(1+t_F)^{-\sigma} + t_H[E_i \chi_{Fi} M_{Fi}(1+t_H)^{-\sigma}(\frac{\tau p_{Fid}(\overline{\varphi}_{Fix})}{P_i})^{1-\sigma}] \tag{38}$$

The first term indicates the revenue from the domestic market in the industry, the second term indicates the revenue of the exporting firms from exporting their goods overseas, and the third term is equal to the tariff revenue from industry imports.

The total factor income of the country is equal to the total wage earned by labor and the total rent earned by the capital. It is assumed that all units of labor and capital are used in production. The revenue earned by labor and capital is equal to :

$$w[f_i\beta_i(\frac{r}{w})^{1-\beta_i}M_i + \frac{q_i(\overline{\varphi}_i)}{\overline{\varphi}_i}\beta_i(\frac{r}{w})^{1-\beta_i}\frac{1}{\overline{\varphi}_i}M_i] + w[f_{ix}\beta_i(\frac{r}{w})^{1-\beta_i}M_{ix} + \frac{q_{ix}(\overline{\varphi}_{ix})}{\varphi_{ix}}\beta_i(\frac{r}{w})^{1-\beta_i}\frac{1}{\varphi_{ix}}M_{ix}] + w[f_{ei}\beta_i(\frac{r}{w})^{(1-\beta_i)}M_{ei}] \tag{39}$$

$$r[f_i[1-\beta_i](\frac{w}{r})^{\beta_i}M_i + \frac{q_i(\overline{\varphi}_{ix})}{\overline{\varphi}_i}[1-\beta_i](\frac{w}{r})^{\beta_i}\frac{1}{\overline{\varphi}_i}M_i]$$
$$+$$
$$r[f_{ix}[1-\beta_i](\frac{w}{r})^{\beta_i}M_{ix} + \frac{q_{ix}(\overline{\varphi}_{ix})}{\overline{\varphi}_{ix}}[1-\beta_i](\frac{w}{r})^{\beta_i}\frac{1}{\varphi_{ix}}M_{ix}]$$
$$+$$
$$r[f_{ei}[1-\beta_i](\frac{w}{r})^{\beta_i}M_{ei}] \tag{40}$$

The tariff revenue is equal to the per unit tariff imposed multiplied by the overall export revenue of the foreign firms (as this denotes the overall value of imports to the home country upon which the tariff is imposed):

$$t_H[E_i\chi_{Fi}M_{Fi}(1+t_H)^{-\sigma}(\frac{\tau p_{Fid}(\overline{\varphi}_{Fix})}{P_i})^{1-\sigma}] \tag{41}$$

To analyse the welfare in terms of productivity under open economy, the magnification effect analyses is used and follows that of Bernard, Redding & Schott (2007). Opening up to trade under the HO setting has the effect of increasing the average productivity in the comparative advantage industry relative to the comparative disadvantage industry and this magnifies the HO-based comparative advantage (Bernard, Redding & Schott, 2007). The magnification effect is measured by the relative productivity in both industries and countries which is defined as the magnification ratio:

$$\frac{\overline{\varphi}_b \diagup \overline{\varphi}_a}{\overline{\varphi}_b^F \diagup \overline{\varphi}_a^F} \tag{42}$$

The variety available to consumers now depends on the mass of firms operating in the domestic industry and the mass of firms operating in the export sector in the foreign country. Therefore, by opening up to trade, the welfare of the country is dependent upon the foreign variables and the level of tar-

iff imposed by the home country. And finally, the real factor reward is the nominal factor reward scaled by the price index.

$$Real\ Wage = \frac{w}{P_a^{\alpha} P_b^{1-\alpha}} \tag{43}$$

and

$$Real\ Rent = \frac{r}{P_a^{\alpha} P_b^{1-\alpha}} \tag{44}$$

4 Open-Economy Equilibrium and Tariff Analyses

Equilibrium under open economy is characterised by endogenous level of productivity cutoff for domestic market and export market and the weighted average productivity in an industry. All the variables are now determined simultaneously in general equilibrium.[6]

4.1 Relative Revenue in Domestic and Export Market

As can be seen from equation (17), the relative revenue in the domestic sector and the export sector depends on the trade costs, relative price indices of the sector between the two countries, and the relative total expenditure in the sector between the two countries. The price indices vary between the countries due to the difference in the mass of firms producing in the sectors and nominal factor rewards. Additionally, if there is a tariff being charged on imported goods at home (abroad), then the price index of home (foreign) will also include the tariff imposed. There is a wedge between the revenue of an exporting firm and a firm operating only in the domestic market which determines the changes in the expected value of entry after trade liberalization and imposition of tariff.

The relative expenditure in the sector between the countries also varies across countries. This could be attributed to the differences in the total factor income (which depends on labor and capital endowment and wage and rent paid per unit labor and capital), tariff revenue and the preferences of the consumers. In the case that home (foreign) imposes tariff on the

[6]The analyses for closed-economy equilibrium is the same as in Melitz (2003) and Bernard, Redding & Schott (2007) and therefore is not re-discussed here

imported goods, this will increase the revenue of home (foreign) and therefore the industry expenditure of home (foreign). This further drives a wedge between the total industry expenditure. Therefore, imposition of tariff drives a wedge between the revenue of an exporting firm and non-exporting firm (both directly and indirectly through changes in the relative price indices and relative industry expenditure).

4.2 Cutoff Productivity and Average Productivity

The productivity cutoff of the domestic market and the export market is determined by the free entry condition (equation 32). The ratio of the domestic market cutoff productivity and export market cutoff productivity (equation 24) can be substituted into the free entry condition to determine the domestic market cutoff productivity. After the domestic productivity level cutoff is determined, the export market productivity level cutoff is determined using equation (24). As can be seen from the equation, the ratio of the cutoff productivities depends on the relative price indices of the industry of the two countries (which includes any tariff imposed by the government), the aggregate industry expenditure of both the countries (which also includes any revenue from tariff imposed by the government) and the tariff level. Therefore, tariff impacts the ratio of the productivity cutoff levels directly and indirectly (through industry expenditure and price index). The ratio of the productivity cutoff also depends on the fixed costs of production in the export sector relative to the domestic sector (which are exogenous).

V_{ei} is monotonically decreasing as the cutoff productivity increases. An increase in the fixed cost of production, for a given level of entry sunk cost (and therefore given level of expected entry value) leads to an increase in the domestic cutoff productivity level. This is because higher revenue will be required to cover the increase in the fixed cost and this will raise the

productivity required to generate the higher level of revenue. The same applies to an increase in the level of fixed cost in the export market.

An increase in the relative foreign industry expenditure, which could be caused by an increase in the foreign industry expenditure or decrease in the domestic industry expenditure, decreases the relative export productivity cutoff level. This is because an increase (decrease) in the foreign (home) total industry expenditure increases (decreases) the revenues of all the firms at every level of productivity. Therefore, the increase (decrease) in revenue at every productivity level allows some firms with relatively lower productivity level to enter (exit) the market since the increased (decreased) revenue makes it profitable (unprofitable) now to operate, decreasing (increasing) the export (domestic) market productivity cutoff level and therefore decreasing the ratio between the export productivity cutoff level and the domestic productivity cutoff level. An increase in the relative fixed cost of production, due to increase in the fixed cost of production in the export market or decrease in the fixed cost of production in the domestic market, increases the ratio of the export productivity cutoff level relative to the domestic productivity cutoff level. This is because increasing (decreasing) productivity is required in order to cover the increasing (decreasing) cost of production.

The overall weighted average productivity depends now on the productivity of the firms operating in the domestic market and productivity of the firms operating in the export sector. The overall weighted average productivity depends on the mass of firms operating and the average weighted productivity in the domestic market of the home country, and the mass of firms operating and the average weighted productivity of the export market (corrected by the iceberg cost). An increase in the weighted average productivity in the domestic market or in the weighted average productivity of the export market increases the overall weighted average productivity.

4.3 Factor Markets

The labor and capital mix used in the industry depends on the production technology and the relative factor reward (rent relative to wage) (equation 37). An increase in the rent relative to wage leads to an increase in the proportion of labor used in the production relative to capital. The effect is more pronounced when the industry is capital intensive. An increase in the wage relative to rent decreases the proportion of labor used in the production relative to capital. The effect is more pronounced when the industry is labor intensive. Based on equations (35) and (36),an increase in the mass of firms operating in an industry or an increase in the mass of firms entering the industry leads to an increase in demand for both the factors of production and the intensity of the change would depend on the level of β_i. For changes in the amount of labor used in the industry, the higher the level of β_i, the more intense will be the effect. The opposite holds true for capital reallocation.

However, opening up to trade increases the demand for both the factors of production in order to serve the foreign market. An increase in the probability of serving the export market conditional upon successful entry increases the number of firms active in the export market, which would increase the demand for both the factors of production. Depending on which industry undergoes a bigger expansion in the markets that they serve by opening up to trade, the relative factor demand and subsequently relative factor rewards will also change.

4.4 Mass of Firms

The mass of firms is combination of firms operating in the domestic market and export market). Mass of firms is equal to the total industry expenditure divided by the average firm revenue. In this case, the total industry expen-

diture depends on the preference of the consumers between the goods of the two industries, the total factor rewards for both labor and capital and the tariff revenue collected by the government which is then redistributed to the consumers.

As can be seen from equation (34), the average revenue of a firm depends on the ex-ante probability of successful entry, the probability of a firm exporting conditional upon the successful entry, the nominal rent and wage, the fixed cost of production in the domestic and export sector and the sunk entry cost. An increase in the level of overall industry expenditure leads to an increase in the mass of firms operating in the industry. This is because an increase in the overall industry expenditure has a positive effect on the average profits of the industry (for a given number of firms actively operating); this increases the expected value of entry and the present value of lifetime profits, making entry more attractive leading to an increase in the number of firms operating.

An increase in the wage or rent paid per unit labor or capital is ambiguous. First, an increase in wage or rent paid per unit labor or capital increases the marginal cost. Some of the firms with relatively lower productivity levels (but above the productivity cut-off level) that previously found it profitable to operate are now unable to generate enough revenue to cover the increased variable and fixed costs of production and therefore they cease production and exit the industry. Second,an increase in the wage and rent paid per unit labor and capital increases the level of sunk cost that the firms have to incur to enter the industry and this will deter some firms from entering the industry. Therefore an increase in the wage or rent paid per until labor and capital decreases the number of firms currently operating in the market and decreases the number of potential entrants. However, an increase in the nominal wage or rent also increases the industry expenditure, which has a positive impact on the number of firms. Therefore, the overall effect of an

increase in the nominal factor reward is ambiguous.

An increase in the cost of production has a negative impact on the number of firms operating in the market. It also holds true for an increase in the level of fixed sunk entry cost. This is intuitive - an increase in the fixed costs of production means that some firms with relatively lower productivity (but still higher than the initial productivity cutoff level) that previously still manage to generate enough revenue to cover the fixed costs of production may no longer be able to do so, and therefore exit the industry. An increase in the level of sunk cost for entry means some firms that previously found it profitable to enter may no longer find it profitable to do so. Therefore, this reduces the mass of entrants into the industry. Both the cases result in lower number of firms operating in the industry. An increase in the probability of a firm exporting conditional upon the successful entry decreases the number of firms operating in the industry. This could be attributed to the fact that an increase in the number of firms exporting (and therefore producing domestically) means that there are now more firms with relatively high productivity levels. There is a shift of profits towards the firms with relatively high productivity levels, causing some firms with relatively low productivity levels to exit. An increase in the level of fixed exporting cost reduces the average profits and therefore reduces the expected value of entry; some firms now find it unprofitable to enter, which reduces the number of firms active in the market.

4.5 Firm Profits

The profit of a firm consist of the profit from serving the domestic market and, if the productivity is high enough, from serving the exports market (equations 15,16,19 and 20).

As the total industry expenditure increases, the profits of a firm with a given level of productivity also increases, ceteris paribus. The total industry expenditure in turn depends on the preference of the consumers between the two industries and the total revenue of the country. The profit of a firm decreases as the number of firms increases, this is intuitive as when the number of firms increase, the profits that a firm can earn decreases (for a fixed level of industry expenditure) due to increased competition. The firm profit also depends on the productivity of the firm relative to the weighted average productivity of the firms in the industry. As the relative productivity of the firm (with respect to weighted average productivity in the industry) increases, the profit of the firm increases.

The profit of a firm is also affected by the level of wage per unit labor or rent per unit capital. However, the impact of an increase in the level of wage per unit labor and rent per unit capital on the firm profit is ambiguous. An increase in the wage per unit labor or rent per unit capital increases the cost incurred by the firm and therefore reducing the firm profit, but it also leads to an increase in the level of industry expenditure which will increase the profit of the firm. The negative effect of an increase in wage is more pronounced when the industry is labor intensive; similar reasoning for an increase in nominal rent.

The profits from serving the export market depends on the total industry expenditure in the foreign country, the number of firms active in the export market at home, the productivity level relative to the average productivity level in the export market, the wage and rent at home and the fixed production cost in the export market. An increase in the level of total industry expenditure in the foreign country increases the profits of a firm in the exporting sector. The total industry expenditure, in turn, depends on the preferences of the consumers in the foreign country and the total revenue of the foreign country. As the number of firms operating in the export market at

home increases, the profits of a firm decreases due to increased competition (ceteris paribus). The firm profits in the export sector also depend on the fixed cost of production in the export sector, with the firm profits decreasing as the fixed cost of production in the export sector increases (same reasoning as increase in the fixed cost of production in the domestic market). An increase in the relative productivity of the firm in the export sector (due to an increase in the firm productivity or decrease in the average productivity level in the export sector) increases the profits of the firm. Finally, an increase in the probability of serving the export market conditional upon successful entry increases the average firm profits in the sector, and therefore the overall average profits in the industry.

4.6 Expected Value of Entrants

As can be seen from equation (28), the expected value of entrants depends upon the ex-ante probability of successful entry, the average profits in the industry and the probability of exogenous death. The partial deriative of the expected value of entrants with respect to the probability of exit (exogenously determined) is negative, which is intuitive because if there is an exogenous increase in the probability of death due to factors beyond the control of the firm, the present value of of the profit flow of all the firms decrease and the value of entering the industry decreases. The expected value of entry also depends on the average profits in the industry; as the average profits in the industry increases, the expected value of entry increases and it becomes more attractive for firms to enter the industry [7]

[7]The factors affecting average industry profits are discussed in previous section

4.7 Price Index

The industry price index can be written in terms of the price index for the domestic variety, the price index for the imports and the tariff imposed by the government. As can be seen from equation (33), the price index of an industry depends price (and therefore nominal wage and rent) at home and foreign, the weighted average productivity of the home domestic sector and the foreign export sector, the number of firms in the domestic market and the foreign export market, the iceberg trade cost and the probability of foreign firms serving the export market conditional upon successful entry.

An increase in the average productivity of the firms decreases the overall price index. Firms with higher productivity is modeled as firms that can produce symmetric variety at lower marginal cost (Melitz, 2003). Therefore, as average productivity increases, the average of the marginal costs faced by the firms decrease and this translates to lower prices charged by the firms (firms charge price equal to the marginal cost) and lower price index.

An increase in the wage per unit labor leads to an increase in the price index. This is because an increase in the wage paid per unit labor leads to an increase in the marginal cost which is translated to higher prices charged by the firms and therefore higher price index. The increase is more pronounced if the industry is more labor intensive. The same also applies to rent per unit capital, the impact of which will be more pronounced in the capital-intensive industry sector.

The price index is also impacted by the number of firms operating in the market. As the number of firms operating in the market increases, the price index decreases. This could be due to the fact that as more firms operate, there is an increase in the variety of goods which decreases the price index.

An increase in the probability of serving the export market conditional

upon successful entry in the foreign country increases the number of firms active in the export market at foreign and decreases the price index. This is because as more firms in the foreign country are active in the export market, there will be more variety of goods imported at home and this decreases the price index of the imports in the home country, therefore reducing the overall price index. This is similar to the reasoning behind the decrease in the price index due to increase in the number of foreign firms active in the market.

An increase in the average productivity of the foreign firms decreases the overall price index; the average marginal cost faced by exporters in the foreign country decreases which lowers the price index (similar to increase in average productivity in the domestic market). The price index increases when the foreign nominal factor rewards increase. This is because the increase in the nominal factor rewards increases the marginal cost and therefore the price charged by the firm, increasing the price index. The increase is more pronounced when the increase in the nominal factor reward happens in the industry in which the factor reward is used intensively.

4.8 Trade Liberalization and Comparative Advantage

4.8.1 Productivity Level Cutoff

Equation (32) shows the free entry condition that determines the domestic and export market productivity cutoff levels. The expected entry value is decreasing monotonically as the productivity level cutoff increases. In autarky, only the first term of the LHS exist in the equation. For open economy condition, there is an additional term regarding the export sector. Therefore, for the same value of entry sunk cost (same value of expected entry), the first term on the LHS must decrease in value, which indicates an increase in the domestic productivity cutoff level. Therefore, opening up to trade in-

creases the domestic productivity cutoff level (and consequently the weighted average productivity level in the domestic industry) compared to autarky.

To analyse the impact of comparative advantage on the increase in the domestic productivity cutoff, it is assumed that the fixed costs of production in both domestic and export sector and the fixed sunk entry cost is equal in both industries (but can vary between countries). In equilibrium, before opening up to trade, since home is labor-abundant, the nominal wage is lower compared to nominal rent and vice versa for foreign country since foreign is capital abundant. Since home is labor abundant, this implies that the relative price index of good A (which is labor intensive) is lower than foreign which is capital abundant. By using equation (33), the relative price indices in a country can be expressed.

The resulting equation shows that as the trade cost and fixed production cost in the export sector increases (goes towards infinity), the relative price index in open economy converges to that in autarky. In the absence of industry-differences in the trade, fixed production and sunk entry costs, the equation shows that the ratio of domestic productivity cutoff and export productivity cutoff is closer in the industry in which the country has comparative advantage. Since the ratio is less in the comparative advantage industry, assuming everything else is the same between the industries, opening up to trade causes a more intense increase in the domestic productivity cutoff in the industry with comparative advantage than comparative disadvantage. Therefore, opening up to trade increases the average productivity level in both industries, but more so in the industry with the comparative advantage; the differences in the average productivity in the domestic and export sector is less in the industry with comparative advantage than comparative disadvantage.

4.8.2 Expected Value of Entry and Industry Profits

Opening up to trade increases the industry profit due to the positive ex-ante probability of exporting to foreign market (overall market that could be served increases). This increases the expected value of entry and this induces entry into the industry (both industries). As more firms enter the market, domestic industry profits are shifted to firms with relatively high productivity level, causing firms with relatively lower productivity levels that are only serving the domestic market to exit the industry since they are no longer able to generate enough revenue to at least break even. This increases the productivity cutoff level and average productivity level even more.

Since the relative export to domestic market profits is larger in the industry with comparative advantage, this causes the increase in the expected value of entry to be larger in the comparative advantage industry than comparative disadvantage industry. This induces more entry into the comparative advantage industry and increases the cutoff productivity and average industry productivity even more. Firms with lower level of productivity are less likely to survive in the industry with comparative advantage than the industry with comparative disadvantage.

4.8.3 Factors of Production

Due to an increase in the market of the industries, there is an increase in the demand for the factors of production. The demand for a factor is more intense in the industry in which the factor intensity is relatively larger than the other factor of production. The increase in the export market is larger in the comparative advantage industry, therefore the factor which is used intensively in that industry experiences a larger increase in demand. This increases the relative factor reward of that factor. Due to this increase, this

impacts the firms operating both domestically and in the export market of the industry which uses the factor intensively, causing an increase in the cutoff productivity level and subsequently average productivity level in that industry (which is the comparative industry). The increase in the relative factor reward is in line with the Stolper-Samuelson Theorem.

4.8.4 Welfare: Mass of Goods

Opening up to trade allows consumers to gain access to a variety of goods from foreign country. The increase in the product variety decreases the price indices and subsequently increases the real income of the consumers. In the sector with comparative advantage, there is an increase in the variety of goods from foreign, but there is also a decrease in the domestic variety due to some firms exiting the domestic industry. In the comparative disadvantage industry, there is a larger increase in the variety of goods from foreign country (since it is a comparative advantage industry in the foreign country) than in the comparative advantage industry. However, since some firms also exit the comparative disadvantage industry, the effect is ambiguous (but more likely to be positive).

4.8.5 Welfare: Real Factor Reward

The factor that is relatively abundant experiences an increase in the real factor reward when the country opens up to trade. First, due to comparative advantage, the factor faces increased demand and therefore the nominal reward rises, increasing the real reward for a given level of price indices. Additionally, the decrease in the price indices of both the industries increases the real reward for a given level of nominal reward. Therefore, the factor that is relatively abundant and is used more intensively in the comparative

advantage industry gains more from opening to trade. The factor that is intensively used in comparative disadvantage industry might experience a decline in the nominal reward due to decreased relative demand (although this is not necessary). This reduces the real wage for given level of price indices. The decrease in the price indices, however, increases the real reward for a given level of nominal reward. Therefore, the effect of opening up to trade on this factor is ambiguous.

4.8.6 Welfare: Magnification Ratio

Opening up to trade causes an increase in the relative average productivity level of the comparative advantage industry relative to comparative disadvantage industry (as discussed previously). By using the magnification ratio (equation 42), it can be seen that due to the change in the relative average productivity level in the industries, the differences in the opportunity cost of production between countries is magnified even more (countries become more specialized) - this provides an indication of the gains in welfare in terms of opportunity cost of production.

4.9 Imposition of Tariff

An imposition of tariff by the government causes differences in the price charged by the firms in the export market and the price faced by the consumers. An imposition of tariff causes an increase in the price faced by consumers in the industry in which the tariff is levied. All analyses are carried out by comparing the situation to free trade condition. As mentioned before, Home is labor abundant and has comparative advantage in industry B which is labor intensive; Foreign is capital abundant and has comparative advantage in industry A which is capital -intensive.

4.9.1 Comparative Advantage Industry

Productivity Level Cutoff The relative productivity cutoff level ratio of domestic firms and foreign firms that are active in the export market can be written as follows:

$$\frac{\varphi^*_{Fix}}{\varphi^*_{id}} = (\frac{f_{ix}}{f_i})^{\frac{1}{\sigma-1}} \tau(1+t_H)^{\frac{\sigma}{\sigma-1}} (\frac{w_F^{\beta_i} r_F^{1-\beta_i}}{w^{\beta_i} r^{1-\beta_i}})^{\frac{\sigma}{\sigma-1}} \tag{45}$$

There are two opposing effects on the productivity level cutoffs due to an imposition of tariff. The first effect of an imposition of tariff by home country is that it causes an increase in the ratio of the foreign export productivity cutoff (the productivity cutoff of the export market in the foreign country) and home domestic market cutoff productivity cutoff (the productivity cutoff of the domestic market in the home country). Ceteris paribus, an increase in the tariff level will cause an increase in this ratio. This can be caused either by a reduction in the domestic cutoff level, therefore there are domestic firms with relatively lower productivity levels that are now able to survive, or an increase in the foreign export cutoff level which means the domestic firms now face less foreign competition, or a combination of both.

An imposition of tariff by home country means that there is an increase in the price index of the industry. This is due to the additional tariff that has to be paid by the domestic customers on the imported variety. The tariff charged by the government is now additional revenue that is spent on the industry. The additional revenue goes to both exporting firms and domestic firms. This decreases the cutoff level for both as there is now additional revenue for every productivity level and some firms with relatively low productivity levels can now find it profitable to survive. Therefore, trade liberalisation leads to an increase in the domestic productivity cutoff. Imposition of tariff could potentially dampen the increase, thereby allowing some

domestic firms with relatively lower productivity levels to still survive. This results in lower level of average productivity compared to free trade situation. However, imposition of tariff leads to an increase in the export productivity cutoff of the foreign firms. Therefore, the positive effect of trade liberalisation for foreign is dampened by the imposition of tariff by the home country. Since trade liberalisation increased the average productivity level in foreign, an imposition of tariff by home leads to an additional increase in the average productivity level in the industry in the foreign country.

Expected Value of Entry Imposition of tariff by home generates tariff revenue which is then spent by the consumers in at home on both domestically produced and imported goods. This combined with the opening up of foreign market increases the expected value of entry at home and this induces more firms to enter the industry at home. The combination of access to bigger export market and larger revenue due to tariff means that the increase in the expected value of entry is larger in industry B than in industry A. This increases entry even more, increasing the productivity cutoff level and average industry productivity. Therefore, imposition of tariff increases entry of firms that will increase the productivity cutoff level and average productivity level in the industry, but it also increases the revenue which may allow some firms with relatively lower productivity levels to survive in the export market, thereby reducing the productivity cutoff level and average productivity. Therefore, opening up to trade with a certain tariff level will still increase the productivity cutoff level and average productivity level in the industry, but not as much as in the free trade.

Factors of Production Imposition of tariff means that some of the firms with lower productivity level are still surviving due to less competition from foreign exporters. But there is an increase in the number of firms operating

and there is also access to foreign market. This increases the relative demand for labor. Imposition of tariff increases the relative wage more than free trade condition, this however, will increase the cutoff productivity level in the industry causing some firms to exit and reducing the relative demand for labor. The increase in wage and tariff revenue would mean that there is now more profits in the industry, which means that some firms with relatively lower levels of productivity would still survive. Therefore, this has the same impact as the changes in the increased expected value of entry: opening up to trade with a certain tariff level will still increase the productivity cutoff level and average productivity level in the industry, but not as much as in the free trade. The impact of tariff on the wage is ambiguous: it could increase if less firms exit the industry under tariff condition, but it could also be the same (it would not decrease).

Welfare: Mass of Goods Imposition of tariff by home country means that there will be some firms in the foreign country (their comparative disadvantage industry) that will not be able to export due to increase in their export productivity cutoff level. This means that the variety of goods from foreign will decrease. However, the increased tariff revenue at home means that ceteris paribus, there is a decrease in the export cutoff productivity level, increasing the mass of firms exporting in the foreign country and increasing the variety of imports at home. The increase will be more pronounced if the tariff revenue at home is large (although this will also have a negative impact due to increase in the price index) and consumers have a high preference for the good of this sector.

Welfare: Real Factor Reward Opening up to trade with tariff can increase the relative wage. There is an increased demand due to more firms being active in the industry, increasing the nominal wage and real wage for

a given level of price indices. However, an imposition of tariff will increase the price index of the industry as well. Therefore, it is ambiguous on what the overall impact of tariff imposition is since it could increase the wage, but not as much as the price index, leading to a reduction in real wage; it could lead to no changes in wage but an increase in price indices leading to a deterioration of real wage; or it could increase the nominal wage by more than the price index, improving the real wage. The increased relative demand for labor and therefore decreased relative demand for capital mean that the relative rent would decrease. It could be the case that rent increases (nominally) due to increased production activities in both sectors due to opening up to trade, leading to a possible improvement in real wage, relative rent would still decrease.

Welfare: Magnification Ratio Imposition of tariff by home in the comparative industry would reduce the magnification ratio due to the fact that the average productivity in this industry at home is now less (relative to free trade) and the average productivity in this industry in foreign is now more (relative to free trade). Therefore, the cross-country differences in relative opportunity costs is not as magnified as it would be under free trade conditions. Therefore, there is loss of welfare. When foreign imposes tariff in this industry, the opposite happens. The average productivity rises in this industry at home and falls in this industry in foreign. This increases the magnification ratio relative to free trade and magnifies the cross-country differences in relative opportunity costs - additional welfare gain is observed under this circumstance.

4.9.2 Comparative Disadvantage Industry

Productivity Level Cutoff In the comparative disadvantage industry, an imposition of tariff would have similar impact on the productivity cutoff levels and average productivity levels as tariff imposed in the comparative advantage industry. It reduces the relative differences in the ratio of the productivity cutoffs between the foreign export productivity cutoff level and the domestic productivity cutoff level. As in the comparative advantage industry, opening up to trade with a certain tariff level will increase the productivity cutoff level and average productivity level in the industry, but not as much as in the free trade. Some firms with relatively lower levels of productivity are still able to survive and operate profitably. When foreign imposes a tariff in this industry, this causes an increase in the cutoff productivity level in the export market of this industry. Considering that this is the comparative disadvantage industry of the home country, the exporting firms are competing with foreign domestic firms that have comparative advantage. Therefore, if foreign imposes a tariff in this industry, less firms at home will be able to export leading to an increase in the productivity level of exporters (and overall) at home due to increased export productivity cutoff level.

Expected Value of Entry and Industry Profits Imposition of tariff by home generates tariff revenue which is then spent by the consumers at home on both domestically produced and imported goods. This combined with the opening up of foreign market increases the expected value of entry at home and this induces more firms to enter the industry at home. However, firms in this industry face severe competition from foreign exporters and this could decrease the number of firms entering and operating in the industry. Imposing a tariff in this industry could dampen number of firms exiting due to trade liberalization. In the case that foreign imposes a tariff in this industry, less firms will enter the industry due to a reduction in the expected

value of entry (due to increased in the export productivity cutoff level) and a reduction in the expected industry profits (tariff would mean that imported goods in foreign are more expensive, which would mean that consumers would prefer domestic goods, reducing the export market from the point of view of home exporters).

Factors of Production Imposition of tariff in this industry means that there will be increased demand for capital due to more firms active in both the domestic market and export market of this industry. The increase in the nominal rent will increase the productivity cutoff level, therefore causing some firms to exit. However, the increase in rent and tariff revenue will increase industry expenditure, allowing some firms with relatively lower productivity levels to survive. Therefore, the effect on the nominal rent is ambiguous. Opening up to trade will increase the demand for labor regardless of whether tariff is imposed in this industry or not. Therefore, the relative demand for labor would still increase compared to capital (even if the tariff is so high that the condition in this industry mimics autarky). Therefore, the relative rent would still decrease. However, imposing tariff would mean that it is possible that the decrease in the relative rent is not as much as under free trade condition.

Welfare: Mass of Goods Due to imposition of tariff, some firms in the foreign country will not be able to export their goods to home country, thereby reducing the variety of goods available to consumers at home. However, since there are now more firms operating in the domestic market, the variety of domestic goods available increases. Additionally, the tariff revenue and increase in nominal reward might increase overall level of industry expenditure and allowing some exporters with relatively lower productivity levels in foreign country to export to home country. Therefore, the overall

effect is ambiguous. The positive effect of tariff on industry expenditure is more pronounced if consumers have a high preference for this sector (increase in tariff revenue is higher). When foreign imposes a tariff in this industry, this reduces the expected value of entry in this industry, leading to less firms entering the industry. This would mean that there will be less domestic firms, but more foreign firms (since there are now more firms that are able to export). This would mean that the domestic firms are facing even more severe competition and the variety of domestic goods available will decrease and the variety of foreign goods available will increase. The overall effect is ambiguous and would depend on the preferences of the consumers between domestic variety and foreign variety.

Welfare: Real Factor Reward Similar to the comparative advantage industry, the imposition of tariff will either have no effect on the nominal rent or a positive impact on the nominal rent. However, the price index of this industry will increase. Therefore, the impact is ambiguous for the rent compared to free trade condition, either the nominal rent will increase more than the price index (increase in real rent), the nominal rent will increase less than the price index (decrease in real rent), or only the price index increases (decrease in real wage).

Welfare: Magnification ratio When home imposes a tariff in this industry, the average productivity level at home in this industry falls, while the opposite happens at home. The magnification ratio increases and there is additional welfare gain. Countries become more specialised. When foreign imposes a tariff in this industry, the opposite happens. The average productivity at home increases and the average productivity in foreign decreases, thereby decreasing the magnification ratio and welfare losses is observed.

5 Discussion

An imposition of tariff will reduce productivity cutoff levels in the industry thereby reducing the average productivity level of the industry the tariff is levied on. It will also allow more firms to find it profitable to enter the export market. In this case, a tariff can be considered as a subsidy given to firms in order to allow them to operate profitably in the industry. However, unlike a subsidy which is given directly to the producers, tariff generates revenue which is redistributed to the consumers, thereby increasing the overall industry expenditure. The results corroborate the notion that tariff is imposed in order to reduce the level of competition from foreign industry in order to allow domestic firms to operate.

When countries impose high levels of tariff in the industries that they have a comparative advantage in, it acts as a subsidy and countries become more and more specialized. In this case, tariff does not necessarily act as protection, but more as an inducement. When countries impose high levels of tariff in the industries that they have comparative disadvantage in, this protects their domestic industry and, with sufficiently high level of tariff, import-export in that industry ceases to exist, mimicking conditions of autarky. In this case, tariff acts as a protection. In both the cases, imposition of tariff reduces the aggregate country productivity relative to free trade condition and therefore is a loss to welfare in absolute terms (in terms of productivity). In terms of the magnification ratio, an imposition of tariff in the comparative disadvantaged industry increases the magnification ratio. When welfare is measured using this measure, tariff should be imposed in the comparative disadvantaged industry - protectionism.

Tariff also protects the relative income of the factor that is not relatively abundant in the country. Opening up to trade allows firms to specialize and the industry with comparative advantage will thrive, however the indus-

try with comparative disadvantage will face severe competition from foreign firms. Therefore, the relative demand for the factor that is relatively less abundant will decrease and they might find their real income decreasing due to decrease in nominal reward. Imposition of tariff prevents severe deterioration of the nominal income (even though there might be an increase in the price index), thereby dampening the negative effect of opening up to trade. However, opening up to trade might lead to a tremendous reduction in the price indices, therefore, even though the nominal income might decrease, the real income might still increase. The real income for the abundant factor will increase regardless. Tariff should be imposed such that the real income increases under tariff (due to more firms operating in the comparative disadvantage sector) compared to under free trade (even though it will still be lower than under autarky).

Imposition of tariff have an ambiguous impact on the variety available to consumers. By imposing tariff on the comparative advantage sector, consumers lose out on the foreign variety (which was already relatively less since it is the sector that foreign has a comparative disadvantage in). However, they might gain some variety in the domestic goods. Similar results on the comparative disadvantaged sector, with the difference being that in the comparative disadvantaged industry, there is a large variety of exports since it is foreign's comparative advantage industry. The overall effect of tariff would depend on the consumer preferences between the two goods and their preferences for domestic or export variety.

Tariff would have maximum positive welfare effect when it is levied on the comparative disadvantaged industry and the industry in which consumers have a higher preference. This is because tariff depends on the imported goods, which will be more in the comparative disadvantaged sector and industry expenditure, which depends on consumer preference. However, placing tariff on the sector that consumers prefer more would have more negative

impact on the real factor rewards.

Therefore, the overall impact of tariff on welfare is ambiguous. Tariff is good for firms since more of them can operate in both the industries, they have ambiguous impact on real factor income (the impact depends on the conditions of the endogenous variables), they have an ambiguous impact on consumer utility (depends on preferences) and they have a negative impact on absolute level of efficiency of the country since they decrease the average productivity level of the industry of the country imposing the tariff but increases the average productivity level of the same industry in the other country. The impact on relative opportunity cost on production is ambiguous (ambiguous effect on magnification ratio).

It is also important to mention that imposition of tariff in one industry has a spillover effect on the other industry through three channels:

1. Nominal factor rewards: imposition of tariff causes changes in the production level in the industry, therefore changing the nominal factor rewards. Since there is factor rewards equalization, a tariff in one industry will have spillover effect on the other industry

2. Tariff revenue generated from tariff in one industry would also increase industry expenditure in the other industry due to consumer preferences. Therefore, imposition of tariff in one industry will also impact variables in the other industry.

3. Changes in the variables mentioned above will have an impact on the cutoff productivity levels in both industries.

There are several ways in which the impact of an increase in the tariff imposed differs from the impact of an increase in the iceberg transportation cost. The first difference is that an increase in the iceberg transportation cost

increases the marginal cost incurred by the firms and therefore the the price charged by the exporting firms. The quantity demanded by the consumers in the foreign country is impacted indirectly by the change in the price due to an increase in the iceberg cost. The revenue of the firm changes through changes in the marginal cost and therefore price charged by the firms and the quantity demanded. On the other hand, firms do not take into account tariff imposed when calculating marginal cost and therefore the price charged, but the tariff is considered by the consumers which will alter their demand. Therefore, the revenue of the firm is altered through the changes in the quantity demanded as opposed to the price charged by the firm, as in the case of the iceberg transport cost.

Second, iceberg transport cost means that more than one unit is required to be produced in order to transport and sell one unit of good in the foreign country. The extra units reflect a variable cost and therefore the unit cannot be consumed. An increase in the iceberg transport cost means that more units need to be produced in order to sell one unit in the foreign market, the impact of which only changes the marginal cost and price of the good. An increase in tariff on the other hand increases the price that the consumers have to pay in order to consume imports and increases the revenue that the government obtains. Therefore, the second main difference is that an increase in tariff means that there is an increase in government revenue while this is not the case with an increase in the iceberg transport cost.

The third difference is that an increase in the iceberg transportation cost means that the foreign consumers pay the price of more than one unit of good when they purchase one unit of good and this payment goes to the producers. The increase in the cost is calculated in terms of units of goods. On the other hand, an increase in tariff means that consumers pay more to the government to consume imported goods and the increase in the cost is calculated in terms of the quantity and the price of the variety consumed.

6 Conclusion

This paper aimed at analysing the impact of tariff on industry productivity and overall welfare under conditions of firm heterogeneity as described in Melitz (2003). It builds on the paper of Bernard, Redding & Schott (2007) which analysed comparative advantage under Melitz - type setting of heterogeneous firms and CES utility preferences. It investigates the impact of tariff on firms, consumers, factor income and production efficiency.

The main findings can be summarised as follows: while tariff can be beneficial in terms of preventing severe deterioration of factor income and it is beneficial in order to allow domestic firms to operate, the impact on absolute domestic efficiency is negative as the average productivity decreases in the country imposing the tariff. Imposition of tariffs in comparative disadvantage industry increase ratio of average industry productivity level in the comparative industry relative to comparative disadvantage industry (which magnifies the cross-country differences). The impact of it on consumers is ambiguous as it depends on the preferences of the consumers. Optimally, tariff should be imposed in the comparative disadvantage country in order to allow specialization to still occur but reducing the negative side effect of trade liberalization in the comparative disadvantage industry.

There are several assumptions made in this paper. First, in analysing comparative advantage, it is assumed that the fixed costs of production in domestic and export sector and sunk entry costs are identical between industries. An extension could be made analysing the impact of differences in these costs on the existing analyses.

This paper did not analyse the impact of asymmetric liberalization. Although it is briefly mentioned that high levels of tariff in the comparative disadvantaged industry would lead to autarkic conditions in that sector and

would mimic asymmetric trade liberalization, the analyses of such asymmetric liberalization is beyond the scope of this paper and could be a potential area of future research. Finally, the analyses of this paper are done based on partial equilibrium and the analyses are done by changing one variable while keeping the other values constant. However, the endogenous factors impact each other and therefore need to be established simultaneously. Therefore, an additional general equilibrium analyses could be an area of future research.

7 Appendix

7.1 Appendix A: Consumers - Autarky

7.1.1 Within Sector Consumer Optimization Problem

The consumers solve the within-sector consumer optimization problem, formulated as follows:

$$max\ C_i = [\int_{\omega \in \Omega_i} q_i(\omega)^\rho d\omega]^{\frac{1}{\rho}} \tag{1}$$

subject to

$$\int_{\omega \in \Omega_i} p_i(\omega) q_i(\omega) d\omega = E_i \tag{2}$$

This can be solved by rewriting the equation as maximization problem formulated as follows:

$$max\ q_i(.) = [\int_{\omega \in \Omega_i} q_i(\omega)^\rho d\omega]\ subject\ to\ \dot{y}_i(\omega) = p_i(\omega) q_i(\omega) \tag{3}$$

The La Grange function for this maximization problem is as follows:

$$L = q_i(\omega)^\rho + \lambda p_i(\omega) q_i(\omega) \tag{4}$$

The first order condition is:

$$\begin{aligned} \frac{\partial L}{\partial q_i} &= \rho q_i(\omega)^{\rho-1} + \lambda p_i(\omega) = 0 \\ q_i &= [\frac{\rho}{-\lambda p_i(\omega)}]^{\frac{1}{1-\rho}} \end{aligned} \tag{5}$$

Substituting equation (5) into equation (2),

$$\frac{E_i}{\int_{\omega\in\Omega_i} p_i(\omega)^{1-\sigma}d\omega} = (\frac{\rho}{-\lambda})^{\frac{1}{1-\rho}} \tag{6}$$

Combining equation (5) and (6),

$$q_i(\omega) = \frac{E_i}{P_i^{1-\sigma}} p_i(\omega)^{-\sigma} where\ P_i^{1-\sigma} = \int_{\omega\in\Omega_i} p_i(\omega)^{1-\sigma}d(\omega) \tag{7}$$

Substituting (7) into (1), and using $\sigma = \frac{1}{1-\rho}$,

$$C_i = \frac{E_i}{P_i^{1-\sigma}}[\int_{\omega\in\Omega_i} p_i(\omega)^{1-\sigma}d(\omega)]^{\frac{1}{\rho}} \tag{8}$$

$$C_i = \frac{E_i}{P_i} \tag{9}$$

7.1.2 Across Sector Consumer Optimization Problem

Next, the consumers solve between-sector consumer optimization problem, formulated as follows:

$$max\ U = C_a^{\alpha_a} C_b^{\alpha_b} \tag{10}$$

subject to

$$E = E_a + E_b \tag{11}$$

The consumer optimization problem is therefore formulated as:

$$L = C_a^{\alpha_a} C_b^{\alpha_b} + \lambda(E - E_a - E_b) \tag{12}$$

The first order conditions are as follows:

$$\begin{aligned} \frac{\partial L}{\partial C_a} &= \alpha_a C_a^{\alpha_a - 1} C_b^{\alpha_b} - \lambda P_a = 0 \\ \frac{\partial L}{\partial C_b} &= \alpha_b C_b^{\alpha_b - 1} C_a^{\alpha_a} - \lambda P_b = 0 \end{aligned} \tag{13}$$

Combining both the first order conditions in equation (13) and equation (9) yielded the following equation:

$$E_i = \alpha_i E \tag{14}$$

7.2 Appendix B : Producers - Autarky

The production cost faced by the producers is formulated as follows:

$$\nu_i = (f_i + \frac{q_i(\varphi_i)}{\varphi_i}) w^{\beta_i} r^{1-\beta_i} \tag{15}$$

By taking the first-order condition with respect to q_i, the marginal cost can be found and is as follows:

$$\frac{\partial \nu_i}{\partial q_i(\varphi_i)} = \frac{w^{\beta_i} r^{1-\beta_1}}{\varphi_i} \tag{16}$$

Revenue of the firm can be found by multiplying the price and the quantity sold by a firm. Using equation (14) and equation (7), the revenue of the firm can be written as:

$$p_i(\varphi_i) q_i(\varphi_i) = \frac{p_i^{1-\sigma}(\varphi_i) \alpha_i E}{P_i^{1-\sigma}} \tag{17}$$

The profit function of the firm is therefore:

$$\pi_i(\varphi_i) = \frac{p_i^{1-\sigma}(\varphi_i) \alpha_i E}{P_i^{1-\sigma}} - (f_i + \frac{q_i(\varphi_i)}{\varphi_i}) w^{\beta_i} r^{1-\beta_i} \tag{18}$$

By taking the first derivative and using the identity $\sigma = \frac{1}{1-\rho}$, the price charged by the firm can be derived and is as follows:

$$p_i(\varphi_i) = \frac{w^{\beta_i} r^{1-\beta_i}}{\varphi_i \rho} \tag{19}$$

By combining equations (7), (14) and (19), the revenue of the firm can be derived and is as follows:

$$r_i(\varphi_i) = \alpha_i E[\frac{\rho \varphi_i P_i}{w^{\beta_i} r^{1-\beta_i}}]^{\sigma-1} \tag{20}$$

By combining equations (7), (14) (18) and (19), the profit function of the firm can be written as:

$$\pi_i(\varphi_i) = \frac{r(\varphi_i)}{\sigma} - f_i w^{\beta_i} r^{1-\beta_i} \tag{21}$$

From this, the zero-profit condition can be derived:

$$r_i(\varphi_i^*) = \sigma f_i w^{\beta_i} r^{1-\beta_i} \tag{22}$$

From equation (22), the revenue of any firm can be derived.

$$r_i(\varphi_i) = \alpha_i E[\frac{\rho P_i \varphi_i}{w^{\beta_i} r^{1-\beta_i}}]^{\sigma-1}$$
$$r_i^*(\varphi_i^*) = \alpha_i E[\frac{\rho P_i \varphi_i^*}{w^{\beta_i} r^{1-\beta_i}}]^{\sigma-1}$$

$$r_i(\varphi_i) = (\frac{\varphi_i}{\varphi_i^*})^{\sigma-1} \sigma f_i w^{\beta_i} r^{1-\beta_i} \tag{23}$$

From equation (23), the profit of any firm can be found:

$$\pi_i(\varphi_i) = \frac{(\frac{\varphi_i}{\varphi_i^*})^{\sigma-1} \sigma f_i w^{\beta_i} r^{1-\beta_i}}{\sigma} - f_i w^{\beta_i} r^{1-\beta_i}$$

$$\pi_i(\varphi_i) = [(\frac{\varphi_i}{\varphi_i^*})^{\sigma-1} - 1] f_i w^{\beta_i} r^{1-\beta_i} \tag{24}$$

The steady-state equilibrium is as follows:

$$[1 - G(\varphi_i^*)] M_{ei} = \delta M_i \tag{25}$$

The value of a firm is equal to:

$$V_i(\varphi_i) = max\ [0, \frac{\pi_i(\varphi_i)}{\delta}] \tag{26}$$

Equilibrium in the industry would mean that the average value of firms conditional on successful entry is equal to 0. The free entry-condition implies that the expected value of entry is equal to the sunk cost entry, i.e:

$$\begin{aligned} prob\ (\varphi_i > \varphi_i^x)\ \overline{V}_i &= f_{ei} w^{\beta_i} r^{1-\beta_i} \\ V_i(\varphi_i) = \frac{1 - G(\varphi_i^*)\overline{\pi}}{\delta} &= f_{ei} w^{\beta_i} r^{1-\beta_i} \end{aligned} \tag{27}$$

The expected profits that is conditional upon successful entry of a firm is equal to:

$$\begin{aligned} \overline{\pi}_i &= \int_{\varphi^*}^{\infty} [\pi_i(\varphi_i) \mu_i(\varphi_i) d\varphi] \\ \overline{\pi}_i &= \int_{\varphi^*}^{\infty} \pi_i(\varphi_i) \frac{g\varphi}{1 - G(\varphi_i^*)} d\varphi \end{aligned} \tag{28}$$

Combining equations (24) and (27), the expected profits of the firm is equal to:

$$\overline{\pi}_i = f_i w^{\beta_i} r^{1-\beta_i} \int_{\varphi^*}^{\infty} [(\frac{\varphi_i}{\varphi_i^*})^{\sigma-1} - 1] \frac{g\varphi}{1 - G(\varphi_i^*)} d\varphi \tag{29}$$

Combining equation (29) with the free-entry condition in equation (27),the free-entry condition can be re-written as:

$$V_i = \frac{f_i}{\delta} \int_{\varphi_i^*}^{\infty} [\frac{\varphi_i}{\varphi_i^*} - 1]^{\sigma-1} g(\varphi) d\varphi = f_{ei} \tag{30}$$

Combining equations (24) and (29), the weighted average productivity of the active firms can be derived:

$$\overline{\varphi}_i(\varphi_i^*) = [\frac{1}{1-G(\varphi_i^*)} \int_{\varphi_i^*}^{\infty} \varphi_i^{\sigma-1} g(\varphi) d(\varphi)]^{\frac{1}{\sigma-1}} \tag{31}$$

7.3 Appendix C: Goods Markets - Autarky

The price index is the weighted average of the prices charged by all the operating firms according to their own productivity level, which means that the price index is equal to the price charged by the firms with weighted average productivity multiplied by the number of firms that are active.

$$P_i = [\int_{\varphi_i^*}^{\infty} [p(\varphi_i)^{1-\sigma} M_i \frac{g(\varphi)}{1-G(\varphi_i^*)}]^{\frac{1}{1-\sigma}}$$

$$P_i = [\int_{\varphi_i^*}^{\infty} [(\frac{w^{\beta_i} r^{1-\beta i}(\varphi_i)}{\rho \varphi_i})^{1-\sigma} M_i \frac{g(\varphi)}{1-G(\varphi_i^*)}]^{\frac{1}{1-\sigma}}$$

$$P_i^{1-\sigma} = M_i p(\overline{\varphi}_i)^{1-\sigma}$$

$$P_i = M_i^{\frac{1}{1-\sigma}} p(\overline{\varphi}_i)$$

$$P_i = M_i^{\frac{1}{1-\sigma}} (\frac{w^{\beta_i} r^{1-\beta_i}}{\rho \overline{\varphi}_i}) \tag{32}$$

7.4 Appendix D: Factor Markets - Autarky

The unit labor requirement involved in the variable part of production can be derived using the wage and marginal cost of production.

$$MC_i = \frac{w^{\beta_i} r^{1-\beta_i}}{\varphi_i} \tag{33}$$

The partial derivative of the marginal cost with respect to the wage (rent) is as follows:

$$\frac{\partial MC_i}{\partial w} = \beta_i (\frac{r}{w})^{1-\beta_i} \frac{1}{\varphi_i} \tag{34}$$

$$\frac{\partial MC_i}{\partial r} = 1 - \beta_i (\frac{w}{r})^{\beta_i} \frac{1}{\varphi_i} \tag{35}$$

The partial derivative of the fixed cost of production with respect to wage (rent) is equal to:

$$\beta_i (\frac{r}{w})^{1-\beta_i} \tag{36}$$

$$(1-\beta_i)(\frac{w}{r})^{\beta_i} \tag{37}$$

The partial derivative of the fixed cost of entry with respect to wage (rent) is equal to:

$$\beta_i (\frac{r}{w})^{1-\beta_i} \tag{38}$$

$$(1-\beta_i)(\frac{w}{r})^{\beta_i} \tag{39}$$

The total labor used in a sector is equal to :

$$[\int_{\varphi_i^*}^{\infty} f_i \beta_i (\frac{r}{w})^{1-\beta_i} M_i \frac{g(\varphi)}{1-G(\varphi_i^*)} + \frac{q_i(\varphi_i)}{\varphi_i} \beta_i (\frac{r}{w})^{1-\beta_i} \frac{1}{\varphi_i} M_i \frac{g(\varphi)}{1-G(\varphi_i^*)}] + f_{ei} \beta_i (\frac{r}{w})^{(1-\beta_i)} M_{ei} \tag{40}$$

The total capital used in a sector is equal to:

$$[\int_{\varphi_i^*}^{\infty} f_i[1-\beta_i](\frac{w}{r})^{\beta_i} M_i \frac{g(\varphi)}{1-G(\varphi_i^*)} + \frac{q_i(\varphi)}{\varphi_i}[1-\beta_i](\frac{w}{r})^{\beta_i}\frac{1}{\varphi_i} M_i \frac{g(\varphi)}{1-G(\varphi_i^*)}]+ f_{ei}[1-\beta_i](\frac{w}{r})^{\beta_i} M_{ei} \tag{41}$$

7.5 Appendix E: Closed Economy Equilibrium

From equation (32), the mass of active firms can be derived:

$$M_i = [\frac{P_i \rho \overline{\varphi}_i}{w^{\beta_i} r^{1-\beta_i}}]^{1-\sigma} \tag{42}$$

The mass of active firms is also equal to:

$$M_i = \frac{R_i}{\overline{r}_i} = \frac{E_i}{\overline{r}_i} \tag{43}$$

Combining equation (20), (30), and (32), in equilibrium, the profit of any firm and the expected value for an entrant can be written as :

$$\pi_i(\varphi_i) = \frac{\alpha_i E}{\sigma M_i}(\frac{\varphi_i}{\overline{\varphi}_i})^{\sigma-1} - f_i w^{\beta_i} r^{1-\beta_i} \tag{44}$$

$$\overline{v}_i = \frac{1-G(\varphi_i^*)\overline{\pi}_i}{\delta} = \frac{1-G(\varphi_i^*)}{\delta}[\frac{\alpha_i E}{\sigma M_i} - f_i w^{\beta_i} r^{1-\beta_i}] \tag{45}$$

Using the equations (23), (29) and (31), the cutoff-productivity can be written as:

$$\frac{\alpha_i E}{M_i}(\frac{\varphi_i^*}{\overline{\varphi}_i})^{\sigma-1} = f_i w^{\beta_i} r^{1-\beta_i} \tag{46}$$

Combining equations (30) and (31), FEC can be written as:

$$(\frac{\varphi_i^*}{\overline{\varphi}_i}) - 1 = \frac{f_{ei}\delta}{f_i[1 - G(\varphi_i^*)]} \tag{47}$$

Combining equations (43), (44) and (45), the number of firms operating can be written as:

$$M_i = \frac{\alpha_i E}{[f_i w^{\beta_i} r^{1-\beta_i} + \frac{\delta f_{ei} w^{\beta_i} r^{1-\beta_i}}{1 - G(\varphi_i^*)}]\sigma} \tag{48}$$

The labor and capital used in a sector is equal to :

$$[\int_{\varphi_i^*}^{\infty} f_i \beta_i (\frac{r}{w})^{1-\beta_i} M_i \frac{g(\varphi)}{1 - G(\varphi_i^*)} + \frac{q_i(\varphi_i)}{\varphi_i} \beta_i (\frac{r}{w})^{1-\beta_i} \frac{1}{\varphi_i} M_i \frac{g(\varphi)}{1 - G(\varphi_i^*)}] + f_{ei} \beta_i (\frac{r}{w})^{(1-\beta_i)} M_{ei} \tag{49}$$

$$[\int_{\varphi_i^*}^{\infty} f_i [1-\beta_i] (\frac{w}{r})^{\beta_i} M_i \frac{g(\varphi)}{1 - G(\varphi_i^*)} + \frac{q_i(\varphi_i)}{\varphi_i} [1-\beta_i] (\frac{w}{r})^{\beta_i} \frac{1}{\varphi_i} M_i \frac{g(\varphi)}{1 - G(\varphi_i^*)}] + f_{ei} [1-\beta_i] (\frac{w}{r})^{\beta_i} M_{ei} \tag{50}$$

As mentioned before, the total labor and capital used is equal to the labor and capital used by firms with weighted average productivity multiplied by the number of active firms:

$$f_i \beta_i (\frac{r}{w})^{1-\beta_i} M_i + \frac{q_i(\overline{\varphi}_i)}{\overline{\varphi}_i} \beta_i (\frac{r}{w})^{1-\beta_i} \frac{1}{\overline{\varphi}_i} M_i + f_{ei} \beta_i (\frac{r}{w})^{(1-\beta_i)} M_{ei} \tag{51}$$

$$f_i [1 - \beta_i] (\frac{w}{r})^{\beta_i} M_i + \frac{q_i(\overline{\varphi}_i)}{\overline{\varphi}_i} [1 - \beta_i] (\frac{w}{r})^{\beta_i} \frac{1}{\overline{\varphi}_i} M_i + f_{ei} [1 - \beta_i] (\frac{w}{r})^{\beta_i} M_{ei} \tag{52}$$

Dividing equation (51) by equation (52),

$$\frac{L_i}{K_i} = \frac{\beta_i}{1 - \beta_i} \frac{r}{w} \tag{53}$$

7.6 Appendix F: Consumers - Open Economy

7.6.1 Within Sector Optimization of Imported Goods

The consumers solve the within-sector consumer optimization problem, formulated as follows:

$$max\ C_i = [\int_{\omega\in\Omega_i} q_i(\omega)^\rho d\omega]^{\frac{1}{\rho}} \tag{54}$$

subject to

$$\int_{\omega\in\Omega_i} (1+t)p_i(\omega)q_i(\omega)d\omega = E_i \tag{55}$$

This can be solved by rewriting the equation as a maximization problem formulated as follows:

$$max\ q_i(.) = [\int_{\omega\in\Omega_i} q_i(\omega)^\rho d\omega]\ subject\ to\ \dot{y}_i(\omega) = (1+t)p_i(\omega)q_i(\omega) \tag{56}$$

The La Grange function for this maximization problem is as follows:

$$L = q_i(\omega)^\rho + \lambda(1+t)p_i(\omega)q_i(\omega) \tag{57}$$

The first order condition is:

$$\begin{gathered}\frac{\partial L}{\partial q_i} = \rho q_i(\omega)^{\rho-1} + \lambda(1+t)p_i(\omega) = 0 \\ q_i = [\frac{\rho}{-\lambda(1+t)p_i(\omega)}]^{\frac{1}{1-\rho}}\end{gathered} \tag{58}$$

Substituting equation (58) into equation (55),

$$\frac{E_i}{\int_{\omega\in\Omega_i}(1+t)p_i(\omega)^{1-\sigma}d\omega} = (\frac{\rho}{-\lambda(1+t)})^{\frac{1}{1-\rho}} \tag{59}$$

Combining equation (58) and (59),

$$q_i(\omega) = \frac{E_i}{P_i^{1-\sigma}} p_i(\omega)^{-\sigma}(1+t)^{-\sigma} where\ P_i^{1-\sigma} = \int_{\omega\in\Omega_i} (1+t)^{1-\sigma} p_i(\omega)^{1-\sigma} d(\omega) \tag{60}$$

The price index of an imported good with tariff is therefore equal to:

$$P_i^{1-\sigma} = (1+t)^{1-\sigma} P_{Fix}^{1-\sigma} \tag{61}$$

7.7 Appendix G: Producers - Open Economy

The production cost faced by the producers in the export market is formulated as follows:

$$\nu_{ix} = (f_{ix} + \frac{\tau q_{ix}(\varphi_i)}{\varphi_i}) w^{\beta_i} r^{1-\beta_i} \tag{62}$$

By taking the first-order condition with respect to q_i, the marginal cost can be found and is as follows:

$$\frac{\partial \nu_{ix}}{\partial q_{ix}(\varphi_i)} = \frac{\tau w^{\beta_i} r^{1-\beta_1}}{\varphi_i} \tag{63}$$

The revenue of the firm can be found by multiplying the price charged abroad and the quantity sold by an exporting firm. The revenue of the firm can be written as:

$$p_{ix}(\varphi_i) q_{ix}(\varphi_i) = \frac{p_i^{1-\sigma}(\varphi_i) \alpha_i E_F (1+t_F)^{-\sigma}}{P_{Fi}^{1-\sigma}} \tag{64}$$

The profit function of an exporting firm is therefore:

$$\pi_{ix}(\varphi_i) = \frac{p_i^{1-\sigma}(\varphi_i) \alpha_i E_F + (1+t_F)^{-\sigma}}{P_{Fi}^{1-\sigma}} - (f_{ix} + \frac{\tau q_{ix}(\varphi_i)}{\varphi_i}) w^{\beta_i} r^{1-\beta_i} \tag{65}$$

By taking the first derivative and using the identity $\sigma = \frac{1}{1-\rho}$, the price charged by an exporting firm can be derived and is as follows:

$$p_{ix}(\varphi_i) = \frac{\tau w^{\beta_i} r^{1-\beta_i}}{\varphi_i \rho} \tag{66}$$

By combining equations (64) and (66), the revenue of an exporting firm can be derived and is as follows:

$$r_{ix}(\varphi_i) = \alpha_i E_F [\frac{\rho \varphi_i P_{Fi}}{\tau w^{\beta_i} r^{1-\beta_i}}]^{\sigma-1} (1+t_F)^{-\sigma} \tag{67}$$

By combining equations (65) and (67), the profit function of an exporting firm can be written as:

$$\pi_{ix}(\varphi_i) = \frac{r_{ix}(\varphi_i)}{\sigma} - f_{ix} w^{\beta_i} r^{1-\beta_i} \tag{68}$$

From this, the zero-profit condition for an exporting firm can be derived:

$$r_{ix}(\varphi_{ix}^*) = \sigma f_{ix} w^{\beta_i} r^{1-\beta_i} \tag{69}$$

From equation (23) and (67), the relative revenue of overseas trade relative to domestic trade of an exporting firm can be derived and is equal to:

$$\frac{r_{ix}(\varphi_i)}{r_{id}(\varphi_i)} = \tau^{1-\sigma} (1+t_F)^{-\sigma} (\frac{P_F}{P})^{\sigma-1} (\frac{E_{Fi}}{E_i}) \tag{70}$$

The total revenue of a firm is as follows:

$$r_i(\varphi_i) = \begin{cases} r_{id}(\varphi) & \textit{firm does not export} \\ r_{ix}(\varphi) + r_{id}(\varphi) & \textit{if the firm exports} \end{cases} \tag{71}$$

The profit for any firm is equal to:

$$\pi_i(\varphi_i) = \pi_{id}(\varphi_i) + max\,[0, \pi_{ix}(\varphi_i)] \tag{72}$$

The free entry-condition implies that the expected value of entry is equal to the sunk cost entry, i.e:

$$V_i(\varphi_i) = \frac{1 - G(\varphi_i^*)}{\delta}[\overline{\pi}_{id} + \chi\overline{\pi}_{ix}] = f_{ei}w^{\beta_i}r^{1-\beta_i} \tag{73}$$

where χ_i is equal to the ex-ante probability of exporting conditional upon successful entry, which is equal to:

$$\chi_i = \frac{1 - G(\varphi_{ix}^*)}{1 - G(\varphi_i^*)} \tag{74}$$

The ratio of the cutoff productivity level of a domestic-only firm and an exporting firm is as follows:

$$\frac{\varphi_{ix}^*}{\varphi_{id}^*} = (\frac{P_i}{P_i^F})(\frac{E_i}{E_i^F}\frac{f_{ix}}{f_i})^{\frac{1}{\sigma-1}}\tau(1+t_F)^{\frac{\sigma}{\sigma-1}} \tag{75}$$

The ratio of the cutoff productivity level of a foreign exporting firm and domestic cutoff level:

$$\frac{\varphi_{Fix}^*}{\varphi_{id}^*} = (\frac{f_{ix}}{f_i})^{\frac{1}{\sigma-1}}\tau(1+t_H)^{\frac{\sigma}{\sigma-1}}(\frac{w_F^{\beta_i}r_F^{1-\beta_i}}{w^{\beta_i}r^{1-\beta_i}})^{\frac{\sigma}{\sigma-1}} \tag{76}$$

Following equation (29), the expected profits that is conditional upon successful entry of a firm is equal to:

$$\begin{aligned}\overline{\pi}_{id} &= f_i w^{\beta_i} r^{1-\beta_i}\int_{\varphi^*}^{\infty}[(\frac{\varphi_i}{\varphi_i^*})^{\sigma-1} - 1]\frac{g\varphi}{1 - G(\varphi_i^*)}d\varphi \\ \overline{\pi}_{ix} &= f_{ix} w^{\beta_i} r^{1-\beta_i}\int_{\varphi_x^*}^{\infty}[(\frac{\varphi_i}{\varphi_x^*})^{\sigma-1} - 1]\frac{g\varphi}{1 - G(\varphi_i^*)}d\varphi\end{aligned} \tag{77}$$

Combining equation (77) with the free-entry condition in equation (73),the free-entry condition can be re-written as:

$$V_{ei} = \frac{f_i}{\delta}\int_{\varphi_i^*}^{\infty}[\frac{\varphi_i}{\varphi_i^*}-1]^{\sigma-1}g(\varphi)d\varphi + \frac{f_{ix}}{\delta}\int_{\varphi_{ix}^*}^{\infty}[\frac{\varphi_i}{\varphi_{ix}^*}-1]^{\sigma-1}g(\varphi)d\varphi = f_{ei} \quad (78)$$

The weighted average productivity of exporting firms is equal to:

$$\overline{\varphi}_{ix}(\varphi_{ix}^*) = [\frac{1}{1-G(\varphi_{ix}^*)}\int_{\varphi_{ix}^*}^{\infty}\varphi^{\sigma-1}g(\varphi_i)d\varphi]^{\frac{1}{\sigma-1}} \quad (79)$$

Average revenue of a firm is equal to:

$$\overline{r}_i = r_{id}(\overline{\varphi}_i) + \chi_i r_{ix}(\overline{\varphi}_{ix}) \quad (80)$$

Average profit of a firm is equal to:

$$\overline{\pi}_i = \pi_{id}(\overline{\varphi}_i) + \chi_i \pi_{ix}(\overline{\varphi}_{ix}) \quad (81)$$

7.8 Appendix H: Goods Markets - Open Economy

The price index is the weighted average of the prices charged by all the operating firms according to their own productivity level.

$$P_i^{1-\sigma} = \int_{\varphi_i^*}^{\infty}[\frac{w^{\beta_i}r^{1-\beta_i}(\varphi_i)}{\rho\varphi_i}]^{1-\sigma}M_i\frac{g(\varphi)}{1-G(\varphi_i^*)} + \int_{\varphi_{Fix}^*}^{\infty}(1+t_H)^{1-\sigma}[\frac{\tau w_F^{\beta_i}r_F^{1-\beta_i}(\varphi_i)}{\rho\varphi_i}]^{1-\sigma}M_{Fi}\frac{g(\varphi)}{1-G(\varphi_{Fi}^*)} \quad (82)$$

This can be simplified to:

$$P_i^{1-\sigma} = [M_i(\frac{w^{\beta_i}r^{1-\beta_i}}{\overline{\varphi}_i\rho})^{1-\sigma} + \chi_{Fi}M_{Fi}(1+t_H)^{1-\sigma}(\frac{\tau w_F^{\beta_i}r_F^{1-\beta_i}}{\overline{\varphi}_{Fix}\rho})^{1-\sigma}] \quad (83)$$

7.9 Appendix I: Factor Markets - Open Economy

The unit labor requirement for an exporting firm can be derived from the marginal cost

$$MC_{ix} = \frac{\tau w^{\beta_i} r^{1-\beta_i}}{\varphi_i} \tag{84}$$

Marginal cost for a firm is therefore:

$$MC_i = \frac{w^{\beta_i} r^{1-\beta_i}}{\varphi_i} + \chi_i \frac{\tau w^{\beta_i} r^{1-\beta_i}}{\varphi_i} \tag{85}$$

The partial derivative of the marginal cost of the variable part of production with respect to the wage (rent) is as follows:

$$\frac{\partial MC_i}{\partial w} = \beta_i (\frac{r}{w})^{1-\beta_i} \frac{1}{\varphi_i} + \beta_i \tau \chi_i (\frac{r}{w})^{1-\beta_i} \frac{1}{\varphi_i} \tag{86}$$

$$\frac{\partial MC_i}{\partial r} = 1 - \beta_i (\frac{w}{r})^{\beta_i} \frac{1}{\varphi_i} + 1 - \beta_i \tau \chi_i (\frac{w}{r})^{\beta_i} \frac{1}{\varphi_i} \tag{87}$$

The total labor used in a sector is equal to :

$$\begin{gathered}
[\int_{\varphi_i^*}^{\infty} f_i \beta_i (\frac{r}{w})^{1-\beta_i} M_i \frac{g(\varphi)}{1 - G(\varphi_i^*)} + \frac{q_i(\varphi_i)}{\varphi_i} \beta_i (\frac{r}{w})^{1-\beta_i} \frac{1}{\varphi_i} M_i \frac{g(\varphi)}{1 - G(\varphi_i^*)}] \\
+ \\
[\int_{\varphi_{ix}^*}^{\infty} f_{ix} \beta_i (\frac{r}{w})^{1-\beta_i} M_{ix} \frac{g(\varphi)}{1 - G(\varphi_i^*)} + \frac{q_{ix}(\varphi_{ix})}{\varphi_{ix}} \beta_i (\frac{r}{w})^{1-\beta_i} \frac{1}{\varphi_{ix}} M_{ix} \frac{g(\varphi)}{1 - G(\varphi_i^*)}] \\
+ \\
f_{ei} \beta_i (\frac{r}{w})^{(1-\beta_i)} M_{ei}
\end{gathered} \tag{88}$$

The total capital used in a sector is equal to:

$$[\int_{\varphi_i^*}^{\infty} f_i[1-\beta_i](\frac{w}{r})^{\beta_i} M_i \frac{g(\varphi)}{1-G(\varphi_i^*)} + \frac{q_i(\varphi_{ix})}{\varphi_i}[1-\beta_i](\frac{w}{r})^{\beta_i}\frac{1}{\varphi_i}M_i\frac{g(\varphi)}{1-G(\varphi_i^*)}]$$
$$+$$
$$[\int_{\varphi_{ix}^*}^{\infty} f_{ix}[1-\beta_i](\frac{w}{r})^{\beta_i} M_{ix} \frac{g(\varphi)}{1-G(\varphi_i^*)} + \frac{q_{ix}(\varphi_{ix})}{\varphi_{ix}}[1-\beta_i](\frac{w}{r})^{\beta_i}\frac{1}{\varphi_{ix}}M_{ix}\frac{g(\varphi)}{1-G(\varphi_i^*)}]$$
$$+$$
$$f_{ei}[1-\beta_i](\frac{w}{r})^{\beta_i} M_{ei} \tag{89}$$

As mentioned before, the total labor and capital used is equal to the labor and capital used by firms with weighted average productivity multiplied by the number of active firms: The total labor used in a sector is equal to :

$$[f_i\beta_i(\frac{r}{w})^{1-\beta_i}M_i + \frac{q_i(\overline{\varphi}_i)}{\overline{\varphi}_i}\beta_i(\frac{r}{w})^{1-\beta_i}\frac{1}{\overline{\varphi}_i}M_i] + [f_{ix}\beta_i(\frac{r}{w})^{1-\beta_i}M_{ix} + \frac{q_{ix}(\overline{\varphi}_{ix})}{\varphi_{ix}}\beta_i(\frac{r}{w})^{1-\beta_i}\frac{1}{\varphi_{ix}}M_{ix}]$$
$$+$$
$$[f_{ei}\beta_i(\frac{r}{w})^{(1-\beta_i)}M_{ei}] \tag{90}$$

The total capital used in a sector is equal to:

$$f_i[1-\beta_i](\frac{w}{r})^{\beta_i}M_i + \frac{q_i(\overline{\varphi}_{ix})}{\overline{\varphi}_i}[1-\beta_i](\frac{w}{r})^{\beta_i}\frac{1}{\overline{\varphi}_i}M_i]$$
$$+$$
$$[f_{ix}[1-\beta_i](\frac{w}{r})^{\beta_i}M_{ix} + \frac{q_{ix}(\overline{\varphi}_{ix})}{\overline{\varphi}_{ix}}[1-\beta_i](\frac{w}{r})^{\beta_i}\frac{1}{\varphi_{ix}}M_{ix}]$$
$$+$$
$$f_{ei}[1-\beta_i](\frac{w}{r})^{\beta_i}M_{ei} \tag{91}$$

Dividing equation (89) by equation (90),

$$\frac{L_i}{K_i} = \frac{\beta_i}{1-\beta_i}\frac{r}{w} \tag{92}$$

7.10 Appendix J: Open Economy Equilibrium

By combining equations (31), (77) and (79), the average profit of a firm serving the domestic market and the average profit of a firm serving the export market can be written as follows: In equilibrium, the zero-profit conditions and average profits are as follows:

$$\overline{\pi}_{id}(\varphi^*) = f_i w^{\beta_i} r^{1-\beta_i}[(\frac{\overline{\varphi}_i(\varphi_i^*)}{\varphi_i^*})^{\sigma-1} - 1] \tag{93}$$

$$\overline{\pi}_{ix}(\varphi^*) = f_{ix} w^{\beta_i} r^{1-\beta_i}[(\frac{\overline{\varphi}_{ix}(\varphi_{ix}^*)}{\varphi_{ix}^*})^{\sigma-1} - 1] \tag{94}$$

By combining equation (81), (93)and (94), the average profit of any firm can be derived as follows:

$$\overline{\pi}_i(\varphi^*) = [(\frac{\overline{\varphi}_i}{\varphi_i^*})^{\sigma-1} - 1] f_i w^{\beta_i} r^{1-\beta_i} + \chi_i [(\frac{\overline{\varphi}_{ix}}{\varphi_{ix}^*})^{\sigma-1} - 1] f_{ix} w^{\beta_i} r^{1-\beta_i} \tag{95}$$

The revenue from the industry consists of the revenue from domestic sales, the revenue from exports and the tariff revenue for the imports in the industry. The industry revenue is derived as follows:

$$\begin{aligned} R_i = E_i M_i [\frac{p_i(\overline{\varphi}_i)}{P_i}]^{1-\sigma} + \chi_i E_{Fi} M_i [\frac{\tau p_{id}(\overline{\varphi}_{ix})}{P_{Fi}}]^{1-\sigma} (1+t_F)^{-\sigma} \\ + t_H [E_i \chi_{Fi} M_{Fi} (1+t_H)^{-\sigma} (\frac{\tau p_{Fid}(\overline{\varphi}_{Fix})}{P_i})^{1-\sigma}] \end{aligned} \tag{96}$$

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